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Population ageing in a small open economy: some policy experiments with a tractable general equilibrium model

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Juha Kilponen – Helvi Kinnunen – Antti Ripatti

**Population ageing in a small open
economy – some policy
experiments with a tractable
general equilibrium model**




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Population ageing in a small open economy – some policy experiments with a tractable general equilibrium model

The views expressed are those of the authors and do not necessarily reflect the views of the Bank of Finland.

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Population ageing in a small open economy – some policy experiments with a tractable general equilibrium model

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Abstract

This paper extends Gertler's (1999) tractable overlapping generations model with life-cycle features by allowing for distortionary taxation, demographic transition and stochastic variation in demographic structure. The model is then used to study demographic change in the small open economy of Finland. Simulations highlight the key role played by labour market responses to ageing. When the responses of labour supply, wages, and hence private consumption, to higher taxation are consistently accounted for, population ageing has clearly much larger effects on public finance, when compared to mechanical sustainability calculations. Stochastic simulations suggest that lengthening of working time has only a modest alleviating effect on the fiscal burden of ageing. This is due to the fact that stochastic variation in the length of working time has only a relatively small effect on the model's dependency ratio. Variation in life expectancy is clearly much more important.

Key words: ageing, general equilibrium, public finance, demographic uncertainty

JEL classification numbers: E13, H55, J11, J26

Väestön ikääntyminen pienessä avotaloudessa – politiikkavaihtoehtojen tarkastelua yleisen tasapainon mallilla

Suomen Pankin tutkimus
Keskustelualoitteita 28/2006

Juha Kilponen – Helvi Kinnunen – Antti Ripatti
Rahapolitiikka- ja tutkimusosasto

Tiivistelmä

Tässä tutkimuksessa kehitetään edelleen Gertlerin (1999) sukupolvimallia siten, että verojen haittavaikutukset, ikärakenteen muutos sekä stokastinen vaihtelu ikärakenteessa voidaan ottaa huomioon mallilaskelmissa. Kehitettyä mallia käytetään hyväksi arvioitaessa ikääntymisen kokonaistaloudellisia vaikutuksia Suomen kaltaisessa pienessä avotaloudessa. Laskelmat osoittavat, että kun ikääntymisen vaikutukset työn tarjontaan, palkkoihin ja yksityiseen kulutukseen otetaan huomioon, nämä vaikutukset julkisen talouden tasapainoon ovat selkeästi suuremmat kuin mekanistisissa kestävyysarvioinneissa yleensä. Stokastisissa simuloinneissa osoitetaan, että työuran pidentyminen lieventää vain hieman ikääntymisen luomaa painetta nostaa veroja. Tämä johtuu siitä, että vain suhteellisen pieni osa stokastisesta vaihtelusta mallin ikärakennemuuttujassa aiheutuu vaihtelusta työuran pituudessa. Vaihtelu eläkkeelläoloajassa on sen sijaan huomattavasti merkittävämpi tekijä.

Avainsanat: ikääntyminen, yleinen tasapaino, julkinen talous, demografinen epävarmuus

JEL-luokittelu: E13, H55, J11, J26

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1 Introduction

Populations in most of the industrialised countries are ageing. Demographic patterns are especially pronounced in some European economies where old-age dependency ratio are set to rise considerably. The key factors behind this trend are a slowdown of population growth and a substantial increase in longevity. Many European countries have begun to prepare for demographic change. In many European countries where tax rates are already at very high levels, an additional fiscal burden on labour could have detrimental effects on future economic performance.

One of the main ideas for resolving the issue has been to design pension reforms that increase incentives to work longer and/or to cut pension benefits. One prominent example from Europe is Finland, where the pension reform effective from the start of 2005 introduced a flexible retirement age of from 63 to 68. Under the latest pension reform, the pension level is now linked more explicitly than before to exit age and thus with to contributions made by the insured. This is expected to increase incentives to work longer. Furthermore, starting from 2009, changes in life expectancy will effect pension levels. In addition, pension indexation has been changed so that all pensions will be indexed using a weight of 0.8 for living costs and 0.2 for wages, in contrast to the earlier ‘midway index’ applied up to age 65. This will contribute to some erosion of the value of pensions compared to wage level in the future. The latter reforms are clearly attempts to moderate the fiscal impact of growing pension expenditures.

In this paper, we develop a tractable dynamic general equilibrium macroeconomic model that allows us to account for demographic transition. The model features dynamic optimization of a small open economy with an internationally given real interest rate and a non-stochastic balanced growth path determined by labour saving technological development and population growth. Optimal consumption and labour supply decisions are based on Gertler’s (1999) tractable overlapping generation model, extended for distortionary taxes and time-varying retirement and death probabilities¹. Pensions are conditioned on aggregate wage level and on demographic trend, and the model features a partially funded pension system. The funded part of the pension system is considered contractual saving (assets accumulated by the pension fund) and the PAYG part as a transfer from workers to pensioners. These transfers are financed by collecting pension contributions from firms and workers. Finally, we quantify the uncertainty associated with demographic projections, by allowing for stochastic variation in the key variables driving demographic trends.

The results of simulations highlight the key role played by feedback effects from taxation in the assessment of economic costs of ageing. When the responses of labour supply, real wages, and hence private consumption, to higher taxation are consistently accounted for, the economy settles at a level

¹Keuschnigg and Keuschnigg (2004), Ferrero (2005), Roeger (2005), Kara and Thadden (2006), Fujivara and Teranish (2006), and Grafenhofer, Jaag, Keuschnigg and Keuschnigg (2006) develop the Gertler (1999) model further and study different aspects of population ageing in their models.

of taxation clearly above that generally estimated in mechanical sustainability calculations. Even if the effective retirement age were to increase as expected, the burden from pension payments alone would cause the tax rate to rise to a level above that witnessed in the worst years of Finland's recession of the mid-1990s. Results from stochastic simulations support this view, by showing that a lengthening of working time has only a minor alleviating impact on the fiscal burden of ageing: Only a small fraction of the stochastic variation in an endogenously determined income tax rate is explained by the stochastic variation in the length of working time. Finally, if the pension replacement rate falls, as seems likely under the current pension regime, there would be a much smaller increase in taxation in response to rising dependency ratio. The average decline in the level of pensions relative to wage developments, however, raises a concern that the falling purchasing power of pensions relative to wage developments will exert pressure on other social security schemes. The current pension scheme, which appears to be financially sustainable, may thus in practice generate costs to local and central government in the form of higher expenditures on income support and other benefits.

The rest of the paper is organized as follows. Section 2 introduces the model, including a description of the pension system. Sections 4–6 discuss the results from various policy experiments including a sensitivity analysis and stochastic simulations. Section 7 concludes.

2 The model

2.1 General features

The model developed in this paper features dynamic optimization of a small open economy with an internationally given real interest rate and non-stochastic balanced growth path. On the balanced growth path, economic growth is determined by exogenously given growth of labour-saving technology and population.

Households' saving decisions, and hence accumulation of financial assets, are influenced by households' desire to smooth consumption over time. Individuals are expected to have finite lives consisting of two distinct periods. Following Gertler (1999), we label the households living in these two different periods 'workers' and 'retirees'.

The likelihood that a worker will lose part of his labour income at retirement, induces her to discount the future income stream at a higher rate than otherwise. This reduces consumption and increases saving. In this sense, workers save for a rainy day and for retirement.² The planning horizon of pensioners is shorter than that of workers due to the periodic probability of

²This view is generally consistent with the study of Gourinchas and Parker (2002), who find that empirically observed saving patterns accord with forward-looking optimizing behaviour in a life-cycle setup with income uncertainty. Their study suggests that the precautionary saving motive in early life implies that between 60 and 70 per cent of non-pension wealth is due to precautionary saving

death. Therefore pensioners' propensity to consume out of wealth is greater than that of the working-age population.

In order to capture changing labour supply incentives of the elderly, we assume that 'retirees' participate in the labour markets. However, compared to workers, their labour efficiency is lower. Lower labour efficiency can capture issues such as part-time work and possibly lower productivity. A more general, elastic labour supply allows demographic change to feed into adjustment of capital and investment via the capital-labour substitution effect.

Individuals receive transfers from both the central government and pension funds. However, in order to maintain analytical tractability, pensions are related to the prevailing aggregate wage level, and not to individual characteristics³. The pension system is characterised as partially funded. We consider the funded part of the pension system contractual saving (assets accumulated by the pension fund) and the PAYG part as a transfer from workers to pensioners. These transfers are financed by collecting pension contributions from firms and workers as noted above.

The model is closed by fiscal rules. Given that we distinguish between central government and the pension fund, one fiscal rule determines the pension fund's long-term net lending rate, while the other determines the central government 'debt ratio'. Supply side (production structure) is based on CES production technology with factor augmentation in the underlying technological progresses and nominal and real rigidities.

2.2 Demographics

Consumers are assumed to be born as workers. Conditional on being a worker in the current period, the probability of remaining one in the next period is ω_t , while the probability of retiring is $1 - \omega_t$. These transition probabilities are independent of individuals' employment tenure, so that the average tenure of working is $\frac{1}{1 - \omega_t}$. In order to allow for non-stationary demographic structure, we subindex the probabilities with t . Once an individual has retired she faces a periodic probability of death of $(1 - \gamma_t)$. Given that the survival probability γ_t is assumed to be independent of retirement tenure, but that it may depend on calendar time, the average retirement period at each point of time is $\frac{1}{1 - \gamma_t}$. Allowing for time variation in the retirement and death probabilities enables us to generate demographic 'shocks' that feed into the dependency ratio gradually rather than instantly.

³In contrast to large scale overlapping generations models, such as Auerbach-Kottlikoff (1987), we do not follow individual cohorts within the two age groups. This limits our ability to model the demographic change and pension system in a very detailed manner. We also need to abstract from many other potential sources of heterogeneity in consumption and labour supply behaviour, such as differing educational levels. However, we can still specify the retirement and death probabilities, as well as the growth rate of working-age population, in such a way that demographic transition can be captured with reasonable accuracy at the aggregate level. Similarly, linking pensions to demographics, we can roughly mimic the associated trends in pensions and public expenditures.

Let N_t^w denote the stock of workers alive at time t . We then assume that $(1 - \omega_{t+1} + n_{t+1}^w)$ new workers are born at $t + 1$. This implies that we can set worker population to grow at some exogenous gross growth rate of \hat{N}_{t+1}^w

$$N_{t+1}^w = (1 - \omega_{t+1} + n_{t+1}^w)N_t^w + \omega_{t+1}N_t^w = \hat{N}_{t+1}^w N_t^w \quad (2.1)$$

where $\hat{N}_{t+1}^w \equiv 1 + n_{t+1}^w$. Given age-independent probabilities of retirement and death and that cohorts are large, the retiree population (N_t^r) evolves according to

$$N_{t+1}^r = (1 - \omega_{t+1})N_t^w + \gamma_{t+1}N_t^r \quad (2.2)$$

where N_t^r refers to the stock of retirees population at time t . With some manipulation, it is shown that the retiree-to-worker ratio, $\varphi_t = \frac{N_t^r}{N_t^w}$, evolves according to

$$\varphi_t \equiv \frac{N_t^r}{N_t^w} = \frac{1 - \omega_t}{\hat{N}_t^w} + \gamma_t \frac{\varphi_{t-1}}{\hat{N}_t^w} \quad (2.3)$$

Defining the stock of whole population as $N_t = N_t^w + N_t^r$, we can express the growth rate for the whole population as a function of retiree to worker ratio and growth rate of working age population as follows

$$\hat{N}_t = \frac{(1 + \varphi_t) N_t^w}{(1 + \varphi_{t-1}) N_{t-1}^w} = \frac{(1 + \varphi_t)}{(1 + \varphi_{t-1})} \hat{N}_t^w \quad (2.4)$$

In the steady state the demographic change has ended, so that

$$\varphi = \frac{1 - \omega}{\hat{N} - \gamma} \quad (2.5)$$

$$\hat{N} = \hat{N}^w = \hat{N}^r \quad (2.6)$$

2.3 Households

2.3.1 Preferences

A household's preferences are expressed recursively using the constant elasticity aggregator

$$V_t = [\{u(C_t, l_t)\}^{\rho_c} + \beta \{E_t(V_{t+1})^\mu\}^{\frac{\rho_c}{\mu}}]^{\frac{1}{\rho_c}}. \quad (2.7)$$

V_t is the value function, and β gives the subjective time preference. The parameter $\rho_c < 1$ captures intertemporal substitution. A special case of $\mu = 1$, applied here, corresponds to a type of risk neutrality where agents are indifferent regarding risk but retain a non-trivial preference for the time at which consumption occurs (cf. Farmer, 1990)⁴. This special case is analytically

⁴Assumption of risk-neutrality is important. For instance, analysis of welfare effects of social security reforms are importantly affected by the treatment of risk. A mandatory social security system imposes an implicit tax on households, so that there is a reduction in expected life-cycle income (due to social security contributions). However, if the social security system reduces the variance of life-cycle income by pooling the income risk between young and old generation, there is potentially a tradeoff between a reduction in expected life-cycle income and the variability: The reduction of welfare due to mandatory social security would then be lower for risk-averse households than for risk-neutral ones.

tractable, since it generates linear decision rules even with (idiosyncratic) risk to income, asset return and length of life. In addition to risk neutrality and a recursive structure of preferences, we assume that individuals enjoy utility from consumption, C_t , as well as leisure, $(1 - l_t)$, according to the utility functional

$$u(C_t, l_t) = C_t^v (1 - l_t)^{1-v}. \quad (2.8)$$

where v is the elasticity of period utility with respect to consumption.

Taking into account the two distinct periods of life, as well as retiring and death probabilities, the preferences of a household can be summarized as

$$V_t^z = \left\{ [(C_t^z)^v (1 - l_t^z)^{1-v}]^{\rho_c} + \beta^z [E_t(V_{t+1}|z)]^{\rho_c} \right\}^{\frac{1}{\rho_c}}$$

where

$$E_t(V_{t+1}|w) = \omega_t V_{t+1}^w + (1 - \omega_t) V_{t+1}^r, \quad \beta^w = \beta \quad (2.9)$$

$$E_t(V_{t+1}|r) = V_{t+1}^r, \quad \beta_t^r = \beta \gamma_t. \quad (2.10)$$

$z = w, r$ indicates whether the individual is a worker or retiree respectively. The willingness to smooth consumption over time implies a finite (constant) intertemporal elasticity of substitution $\sigma = 1/(1 - \rho_c)$. The retirees effective discount factor β^r is adjusted to take into account the periodic probability of death. We assume a perfect annuities market in order to eliminate the impact of uncertainty about time of death: A retiree's remaining wealth at death is invested in a mutual fund, which invests it in the available financial assets in each period of time. Those surviving to the following period each receive a return that is proportional to his contribution to the fund. For instance, if R_t is the gross return per unit invested by the fund, the gross return for a surviving retirees is at time t R_t/γ_t .

Workers, in turn, face the risk of decline in wage income at retirement. However, since an individual's preferences are over the mean of the next period's value function, only a desire the smooth consumption over time will affect the consumption pattern in the face of idiosyncratic income risk. Thus a worker simply forms a certainty equivalent of his random utility, as shown in equation (2.9).

2.3.2 Retirees

A retiree born at time j and retiring at time k , and who survives at least until $t + 1$, solves the maximization problem

$$\max_{C_t^{rjk}, l_t^{rjk}} V_t^{rjk} = \left\{ [(C_t^{rjk})^v (1 - l_t^{rjk})^{1-v}]^{\rho_c} + \beta \gamma_t [E_t(V_{t+1}^{rjk})]^{\rho_c} \right\}^{\frac{1}{\rho_c}} \quad (2.11)$$

s.t.

$$A_{t+1}^{rjk} = \frac{1}{\gamma_t} R_t A_t^{rjk} + W_t (1 - \mathbf{t}_t) \xi l_t^{rjk} + \mathcal{T}_t^{rjk} - P_t^c C_t^{rjk} \quad (2.12)$$

where R_t denotes after-tax gross rate of return on financial assets A_t^{rjk} , \mathcal{T}_t^{rjk} denotes pensions, and $\xi < 1$ is the labour efficiency of retirees with respect to workers. P_t^c is a price index of consumption, to be determined later. \mathbf{t}_t is the total labour income tax rate including pension contribution rate, \mathbf{t}_t^{WP} , to be discussed later. From the first order condition for labour, we first derive a standard labour supply condition

$$1 - l_t^{rjk} = \frac{1 - v}{v} \frac{P_t^c C_t^{rjk}}{(1 - \mathbf{t}_t) W_t \xi} \quad (2.13)$$

Solving the retiree's maximization problem with respect to consumption, using (2.13) and then aggregating over retirees results into following aggregate consumption equation

$$P_t^c C_t^r = \epsilon_t \pi_t [R_t A_t^r + \mathcal{H}_t^r + \mathcal{S}_t^r]. \quad (2.14)$$

\mathcal{H}_t^r and \mathcal{S}_t^r denote discounted after-tax values of labour income and pensions, and $\epsilon_t \pi_t$ is retirees marginal propensity to consume out of wealth. More specifically

$$\mathcal{H}_t^r = (1 - \mathbf{t}_t) W_t \xi L_t^r + \frac{\mathcal{H}_{t+1}^r}{\hat{N}_{t+1}^r R_{t+1} / \gamma_{t+1}} \quad (2.15)$$

$$\mathcal{S}_t^r = \mathcal{T}_t^r + \frac{\mathcal{S}_{t+1}^r}{\hat{N}_{t+1}^r R_{t+1} / \gamma_{t+1}}. \quad (2.16)$$

Since total social security payments (pensions) are distributed equally among retirees, the gross growth rate of retirees \hat{N}_{t+1}^r enters into the discount factor. The discount factor for human wealth is similarly augmented by \hat{N}_{t+1}^r . A retiree's marginal propensity to consume out of wealth $\epsilon_t \pi_t$ evolves according to the following non-linear difference equation

$$\epsilon_t \pi_t = 1 - \left(\frac{W_t / P_t^c}{W_{t+1} / P_{t+1}^c} \frac{(1 - \mathbf{t}_t)}{(1 - \mathbf{t}_{t+1})} \right)^{\frac{(1-v)\rho_c}{1-\rho_c}} \beta^{\frac{1}{1-\rho_c}} \left(\frac{R_{t+1}}{\hat{P}_{t+1}^c} \frac{\gamma_t}{\gamma_{t+1}} \right)^{\frac{\rho_c}{1-\rho_c}} \frac{\epsilon_t \pi_t \gamma_{t+1}}{\epsilon_{t+1} \pi_{t+1}} \quad (2.17)$$

where $\hat{P}_{t+1}^c \equiv P_{t+1}^c / P_t^c$. A retiree's marginal propensity to consume varies with the real interest rate $R_{t+1} / \hat{P}_{t+1}^c$ as well as with expected changes in real net wage income. Due to the fact that survival probability can vary over calendar time, it influences on retiree's effective discount rate and introduces additional dynamics into the marginal propensity to consume equation.

As in the standard Yaari (1965) and Blanchard (1985) models, likelihood of death $(1 - \gamma_t)$ in (2.17) raises the retirees' marginal propensity to consume. This can be seen easily by considering a limiting case of logarithmic preferences ($\sigma \rightarrow 1$) and when survival probability is constant. In this case

$$\epsilon \pi = 1 - \beta \gamma \quad (2.18)$$

2.3.3 Workers

As regards workers, the decision problem for the worker born at time s , is

$$\max_{C_t^{ws}, l_t^{ws}} V_t^{ws} = \left\{ [(C_t^{ws})^v (1 - l_t^{ws})^{1-v}]^{\rho_c} + \beta [E_t(V_{t+1}^{ws})]^{\rho_c} \right\}^{\frac{1}{\rho_c}} \quad (2.19)$$

s.t.

$$A_{t+1}^{ws} = R_t A_t^{ws} + (1 - \mathbf{t}_t) W_t l_t^{ws} + \mathcal{T}_t^{ws} - P_t^c C_t^{ws} \quad (2.20)$$

\mathcal{T}_t^{ws} denotes financial transfers to working age and \mathbf{t}_t is the total labour income tax rate. The first order condition for labour yields standard labour supply condition

$$1 - l_t^{ws} = \frac{\frac{1-v}{v} P_t^c C_t^{ws}}{(1 - \mathbf{t}_t) W_t} \quad (2.21)$$

Intertemporal maximization in turn gives rise to a fairly complicated Euler equation, but again the workers' consumption plan aggregates to

$$P_t^c C_t^w = \pi_t [R_t A_t^w + \mathcal{H}_t^w + \mathcal{S}_t^w]. \quad (2.22)$$

π_t is a worker's marginal propensity to consume and \mathcal{H}_t^w and \mathcal{S}_t^w denote her human and social security wealth. Marginal propensity to consume out of wealth is a non-linear first order difference equation

$$\pi_t = 1 - \left(\frac{(1 - \mathbf{t}_t) W_t / P_t^c}{W_{t+1} / P_{t+1}^c} \right)^{\frac{(1-\nu)\rho_c}{1-\rho_c}} \beta^{\frac{1}{1-\rho_c}} \left(\frac{\Omega_{t+1} R_{t+1}}{\hat{P}_{t+1}^c} \right)^{\frac{\rho_c}{1-\rho_c}} \frac{\pi_t}{\pi_{t+1}} \quad (2.23)$$

where Ω_{t+1} is the factor that weights the gross real return $R_{t+1} / \hat{P}_{t+1}^c$. This factor evolves according to

$$\Omega_{t+1} = \left(\frac{1}{1 - \mathbf{t}_t} \right)^{1-\nu} \left[\omega_t + (1 - \omega_t) \epsilon_{t+1}^{\frac{-1-\rho_c}{\rho_c}} \left(\frac{1}{\xi} \right)^{1-\nu} \right] \quad (2.24)$$

$\epsilon_{t+1} > 1$ is the ratio of a retiree's marginal propensity to consume to that of a worker. The enlarged discount rate due to the presence of $\Omega_{t+1} > 1$ in the denominator of (2.25)–(2.26) means that workers value human wealth and social security *less* than in the infinite horizon case. This in turn tends reduce worker's consumption and increase saving. Importantly, notice also that distortionary taxes increase the workers' discount factor.

\mathcal{H}_t^w in (2.22) is a discounted sum of worker's wage bill (in net terms) and \mathcal{S}_t^w is the sum across workers alive at t of the capitalized value of social security (in net terms). Both of these measures take into account of corresponding

discounted values at the time of retirement. Formally,

$$\mathcal{H}_t^w = \frac{\omega_t \left(\frac{1}{1-t_{t+1}} \right)^{1-v} \mathcal{H}_{t+1}^w}{R_{t+1} \Omega_{t+1} \hat{N}_{t+1}^w} + (1 - t_t) W_t L_t^w \quad (2.25)$$

$$\begin{aligned} & + \frac{(1 - \omega_t) (\epsilon_{t+1})^{-\frac{1-\rho_c}{\rho_c}} \left(\frac{1}{\xi(1-t_{t+1})} \right)^{1-v} \varphi_{t+1}^{-1} \mathcal{H}_{t+1}^{r(t+1)}}{R_{t+1} \Omega_{t+1} \hat{N}_{t+1}^r} \\ \mathcal{S}_t^w &= \mathcal{T}_t^w + \frac{\omega_t \left(\frac{1}{1-t_{t+1}} \right)^{1-v} \mathcal{S}_{t+1}^w}{R_{t+1} \Omega_{t+1} \hat{N}_{t+1}^w} \\ & + \frac{(1 - \omega_t) (\epsilon_{t+1})^{-\frac{1-\rho}{\rho}} \left(\frac{1}{\xi(1-t_{t+1})} \right)^{1-v} \varphi_{t+1}^{-1} \mathcal{S}_{t+1}^{r(t+1)}}{R_{t+1} \Omega_{t+1} \hat{N}_{t+1}^r} \end{aligned} \quad (2.26)$$

$\mathcal{H}_{t+1}^{r(t+1)}$ and $\mathcal{S}_{t+1}^{rj(t+1)}$ are the values of human wealth and social security for a working retiree who retired at time $t + 1$ but was still working at time t . The factor \hat{N}_{t+1}^w augments the discount rate of the capitalized value of a worker's social security because with finite lives, the share of total social security entitlements going to those currently alive declines over time as the working-age population grows. By similar argument, \hat{N}_{t+1}^w enters into the discount factor of human wealth.⁵

2.4 Distribution of wealth and aggregate consumption

Given our assumptions on preferences and population dynamics, there is no need to keep track of how assets and consumption are distributed within the groups of retirees and workers. However, since marginal propensities to consume differ as between the two groups, we must keep track of the distribution of assets *between* the two groups. Consequently, we need a state equation for distribution of wealth.

Let $\lambda_{t+1}^r \equiv \frac{A_{t+1}^r}{A_{t+1}}$ be the share of financial assets held by retirees and let $1 - \lambda_{t+1}^r \equiv \frac{A_{t+1}^w}{A_{t+1}}$ be the share of financial assets held by workers. It can be shown that retirees' share of financial wealth evolves according to

$$\begin{aligned} \lambda_{t+1}^r &= \left(1 - \frac{\epsilon_t \pi_t}{\nu}\right) \frac{R_t \lambda_t^r A_t}{A_{t+1}} \\ &+ \frac{(1 - \tau_t) \xi W_t N_t^r + \mathcal{T}_t^r - \frac{\epsilon_t \pi_t}{\nu} (\mathcal{S}_t^r + \mathcal{H}_t^r)}{A_{t+1} / \omega_t} + \frac{(1 - \omega_t)}{\omega_t} \end{aligned} \quad (2.27)$$

Aggregate private consumption can then be obtained simply by summing up (2.14) and (2.22), using $\lambda_{t+1}^f \equiv \frac{A_{t+1}^r}{A_{t+1}}$ and that all assets are eventually held by domestic consumers

$$P_t^c C_t^H = \pi_t \left([(1 - \lambda_t^r) R_t A_t + \mathcal{H}_t^w + \mathcal{S}_t^w] + \epsilon_t [\lambda_t^r R_t A_t + \mathcal{H}_t^r + \mathcal{S}_t^r] \right) \quad (2.28)$$

⁵Note that in the limiting case of logarithmic preferences ($\sigma \rightarrow 1$) marginal propensity to consume is constant and equal to $1 - \beta$.

2.5 Assets

There are several types of financial assets available for consumers: domestic government bonds A_t^S , foreign bonds A_t^W , and stocks issued by domestic firms A_t^F . In addition, it is assumed that all the assets accumulated by the pension fund A_t^p are hold by domestic consumers.

Domestic one-period bonds pay a nominal return r_t , while the gross return on stocks is determined according to the profits of the firms in the model. Foreign bonds pay a return r_t^F , which is exogenously given. The arbitrage condition equates ex ante returns of domestic and foreign bonds, to give rise to uncovered interest rate parity (UIP) condition. The share price is the nominal price (ex-dividend) of a unit of equity in period t . The factor defining the gross return on stocks is the firms' Π_t^D . This gross return is defined as

$$1 + r_t^D = [A_{t+1}^F + (1 - \mathfrak{t}_t^K)\Pi_t^D]/A_t^F \quad (2.29)$$

where \mathfrak{t}_t^K denotes the corporate tax rate. Optimal consumption plans can be combined with the arbitrage equation for holding different assets. This yields two equations that relate the after-tax interest rates to each other

$$r_t^D = r_t^S(1 - \tau_t^S) + \mathfrak{T}_t \quad (2.30)$$

$$1 + r_t^S = (1 + r_t^F) \frac{S_{t+1}}{S_t} \quad (2.31)$$

S_t denotes the nominal exchange rate, r_t^S denotes domestic short-term nominal interest rate and r_t^F denotes corresponding foreign short term interest rate. \mathfrak{t}_t^S is tax rate at source. The latter is a standard UIP condition. In addition to this, we assume an exogenously determined risk-premium \mathfrak{T}_t vs. domestic bonds for stocks issued by the domestic firms.

2.6 Labour markets

The model features nominal wage rigidity in the form of quadratic adjustment costs. A worker faces quadratic adjustment if when it has been allowed to re-set her wage. For those not able to optimize in period t , the wage is adjusted using the steady state growth rate of wages. This steady state growth rate of wages, denoted $d\bar{w}$, equals steady state productivity growth plus inflation. The behaviour of aggregate nominal wages is then characterized by the following two wage rate equations

$$W_t^* = \frac{\frac{(1-v)}{v} P_t^c C_t^w / (1 - \mathfrak{t}_t)}{[N_t^w - L_t^w]} \quad (2.32)$$

$$W_t = \frac{(1-q)\beta d\bar{w}}{(1+\beta(1-q)^2 d\bar{w}^2)} E_t W_{t+1} + \frac{(1-q)d\bar{w}}{(1+\beta(1-q)^2 d\bar{w}^2)} W_{t-1} + \frac{q(1-(1-q)\beta d\bar{w}^2)}{(1+\beta(1-q)^2 d\bar{w}^2)} W_t^* \quad (2.33)$$

where $P_t^c C_t^w$ is consumption of workers, N_t^w is worker population, L_t^w denotes the number of workers demanded, $q \in (0, 1)$ is the exogenous probability that determines how often a randomly chosen worker is allowed to re-set her wage. The equation for optimal wage W_t^* is derived directly from the aggregate version of worker's labour supply decision.

Given that workers' and retirees' labour efficiency differ, we define aggregate effective labour supply index L_t as

$$L_t = L_t^w + \xi L_t^r \quad (2.34)$$

Here $\xi \in (0, 1)$ denotes the relative efficiency of a unit of retirees' labour. Labour demand for workers L_t^w is derived from (2.34) by assuming that retirees are always on their labour supply curve at prevailing wage (W), and that the domestic intermediate goods producer is always on its labour demand curve.

2.7 Public sector

The general government (public sector) is divided into two sectors, labelled state (central government) and pension funds. The state collects taxes on labour income at the rate \mathfrak{t}_t^{WS} from capital gains at \mathfrak{t}_t^K and from consumption at \mathfrak{t}_t^C . The state consumes C_t^S and pays transfers to workers and to retirees. In the budget constraint these total transfers are denoted \mathcal{T}_t^S . In addition, the state issues one-period government bonds amounting to A_t^S and paying a nominal return of r_t . In each period, the following budget constraint holds

$$\begin{aligned} & - (A_t^S - A_{t-1}^S) \text{ (net lending)} \\ & = \mathfrak{t}_t^{WS} (W_t L_t^w + \xi W_t L_t^r) \text{ (income tax revenues)} \\ & + \mathfrak{t}_t^K \Pi_t \text{ (corporate income tax revenues)} \\ & + \mathfrak{t}_t^C P_t^C C_t^F \text{ (indirect taxes)} \\ & + \mathfrak{t}_t^{FS} W_t L_t \text{ (firms' social security contributions)} \\ & - P_t^C C_t^S \text{ (government consumption)} \\ & - \mathcal{T}_t^S \text{ (total net transfers)} \\ & - r_t A_{t-1}^S \text{ (interest payments)} \end{aligned} \quad (2.35)$$

2.8 Fiscal policy rule

Typically, models like this are closed by either a tax rule or by a lump sum transfer rule. We use a rather general form of tax rule that stabilizes the evolution of government debt via labour income tax. Formally, the tax rule takes a partial adjustment form⁶

$$\tau_t^{ws} = \tau_{t-1}^{ws} - \theta_1 (\tau_{t-1}^{ws} - \bar{\tau}^{ws}) + \theta_2 (A_t^S - \bar{A}^S) / Y_t \quad (2.36)$$

⁶See eg Railavo (2004) for discussion of alternative fiscal policy rules and their stability properties.

The tax rule has two attractors $\bar{\tau}$ and \bar{A}^S/Y_t towards which the tax, and consequently the debt to output ratio are stabilized. Benchmark values for the parameters θ_1 and θ_2 have been calibrated to 0.3 and 0.1 respectively. $\bar{\tau}$ and \bar{A}^S/Y can be set such that the public debt to output ratio reach wanted steady state values.

2.9 Statutory pension fund

There are several motivations for considering the pension fund(s) separately from the central government. First, when the pension scheme is defined benefit and partly funded, we should consider the funded part of the pension system as contractual saving, as opposed to discretionary saving, while the PAYG part should be considered a direct transfer from young generation (workers) to old generation (pensioners).⁷ In Finland, where approximately 25% of the pensions are funded both features are quantitatively important.

Second, pension contributions are considered at least partly as taxes. Analogously with the previous section, this means that the way in which the increasing fiscal burden of aging is financed along the demographic transition path is of crucial importance for the economy's labour market responses to ageing. The tradeoff is clearly between proportions of demographic transition financed by changing the pension fund asset position and by changing the contribution rates.⁸

Accordingly, we assume that the economy's pension fund are administrated separately from the central government. The fund collects pension contributions from the private sector and distributes pensions to retirees totalling \mathcal{T}_t^{PR} . Pension funds accumulate financial assets A_t^P . In each period, therefore, the following flow budget constraint holds for the pension fund

$$\begin{aligned}
& - (A_t^P - A_{t-1}^P) \text{ (net lending)} \\
& = \mathfrak{t}_t^P W_t L_t \text{ (social security contributions of employer and employee)} \\
& - \mathcal{T}_t^{PR} \text{ (total transfers paid to retirees)} \\
& - r_t A_{t-1}^P \text{ (interest payments)}
\end{aligned} \tag{2.37}$$

where $\mathfrak{t}_t^P = \mathfrak{t}_t^{FP} + \mathfrak{t}_t^{WP}$ is overall pension contribution rate, consisting of employer and employee contributions. Finally \mathcal{T}_t^{PR} denotes pensions and other transfers from pension funds to retirees.

⁷In Finland, approximately 25% of the pensions are funded.

⁸A third important consideration is that pension funds hold the savings for the households that are completely illiquid. Households are thus not able to borrow, or are limited to a small amount, against their savings accumulated in the pension funds. This means that households do not see pensions accumulated in pension funds as perfect substitutes for more liquid forms of saving, such as bonds and equity. This is supported by empirical findings according to which growth in partially funded pension schemes does boost personal saving, but not one-to-one. The current version of the model, however, does not address this issue.

2.10 Contribution rule

Inter-generational distribution of the fiscal burden of ageing is not actively managed by institutional control built into current pension schemes in general. According to Fenge and Werding (2003) ‘it merely just happens’. Consequently, inter-generational distribution is hardly an issue in devising the contribution rule for a model like ours. In a defined benefit pension system, such as in Finland, the contribution rate is adjusted to maintain the pension fund’s fiscal balance. One way to write the pension fund’s contribution rule, is to assume that the contribution rule stabilizes the net lending-to-total wage ratio at some predetermined long-run level.⁹ Consequently, we write a simple ‘net lending’ rule that stabilizes the net lending-to-total-wages ratio at a pre-specified target level. Formally

$$\mathfrak{t}_t^P = \mathfrak{t}_{t-1}^P + \theta_3 \left(\frac{(A_t^P - A_{t-1}^P)}{W_t L_t} + \bar{A}_{WL}^P \right) \quad (2.38)$$

where \bar{A}_{WL}^P is the target level for net lending-to-total-wages and θ_3 is an adjustment parameter. This form is flexible enough for policy option experiments. For instance setting \bar{A}^P at zero and θ_3 high represents a pure PAYG system. In our benchmark simulations, we set $\theta = 0.15$ and $\bar{A}^P = 0.07$.

2.11 Pension expenditures

Allowing idiosyncratic historical dependence in social security and pension payments would make the model more realistic, but it would sacrifice analytical tractability. We thus link the model’s pension expenditures/transfers to the demographic structure and aggregate wages, by writing

$$\mathcal{T}_t^R = \mu_t N_t^r W_t \quad (2.39)$$

where $\mu_t = \bar{e}_t / \bar{W}$ is the average pension rate evaluated at the initial steady state level of aggregate wages \bar{W} . Since the wage rate W_t is endogenous, we obtain projections for pension expenditures once we set a deterministic path to average pension rate μ_t . Total pension expenditures \mathcal{T}_t^R are thus linked to average wages and number of pensioners in the model. Furthermore, making use of our demographic assumptions, we can express pension expenditures per capita in terms of the dependency ratio, wages and pension rate:

$$\frac{\mathcal{T}_t^R}{N_t} = \mu_t \frac{N_t^r}{N_t} W_t = \mu_t \frac{\varphi_t}{1 + \varphi_t} W_t \quad (2.40)$$

2.12 Production sector

The supply side is based on a single intermediate good that can be used in the production of final goods. Producers of this intermediate goods

⁹Indeed, in Finland, pension funds’ total net lending relative to aggregate wages has been rather stable during the last 25 years.

combine capital and labour using a constant-elasticity-of- substitution (CES) production function and operate in monopolistic product markets. Prices of final goods are sticky in the form of Calvo-pricing. Domestic producers of intermediate products purchase their capital inputs (capital services) in a competitive capital market (from companies providing capital services) in which capital is freely sold and transferable for use by other companies. Building up new capital generates cost – adjustment costs – in the form of lost capital stock.

Domestic intermediate goods are combined with imported intermediate goods to produce final goods of three kinds: consumption goods, capital goods and exported goods. The production function – or, rather, aggregator – is CES. The production differs across final goods in terms of elasticity of substitution. All three types of final producers operate in competitive product markets, in which they take the market price for their products as given in their own decision-making.

Finally, nominal import prices are assumed to be sticky in a manner corresponding to domestic intermediate goods prices. It is also assumed that, in the short run exchange rate pass-through to import prices is incomplete. This is due to the fact that a fixed portion of importing companies price their products in the local currency.

2.12.1 Domestic intermediate goods producer

The domestic composite intermediate good, Y_t , is produced according to a constant elasticity of substitution (CES) production function, which combines a continuum of individual goods $Y_t(j)$ ($j \in [0, 1]$) (Dixit and Stiglitz, 1977)

$$Y_t = \left[\int_0^1 Y_t(j)^{-\rho^z} dj \right]^{\frac{1}{-\rho^z}}.$$

The parameter $\rho^z \in [-1, \infty)$ determines the elasticity of substitution $1/(1+\rho^z)$. Cost minimization implies the following conditional demand function for the individual good j

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\frac{1}{1+\rho^z}} Y_t$$

The price index for the composite domestic intermediate good is

$$P_t = \left[\int_0^1 P_t(j)^{\frac{\rho^z}{1+\rho^z}} dj \right]^{\frac{1+\rho^z}{\rho^z}}$$

Domestic intermediate goods, $Y_t(j)$, are produced by producers operating in monopolistic market. The production technology and the factor augmenting technical trends are exogenously given. The production function is of the CES type and takes a specific constant-returns-to-scale form¹⁰

$$Y_t(j) = \left[\delta (\Lambda_t^K K_t)^{-\rho} + (1 - \delta) (\Lambda_t^L L_t^F)^{-\rho} \right]^{-1/\rho}$$

¹⁰Jalava, Pohjola, Ripatti and Vilminen (2006) provide evidence that this may be a reasonable approximation for the Finnish post-WWII data.

The factors of production include homogeneous capital services, K_t , and labour input L_t^F . Λ_t^K and Λ_t^L denote time-varying¹¹ capital and labour-augmenting technical progress respectively. The elasticity of technical substitution is given by $1/(1+\rho)$, where ρ is the substitution parameter and δ the share parameter in the production function. The technical change is labour-augmenting, Λ_t^L , in the balanced growth path.

Cost minimization implies the following real marginal costs

$$\frac{MC_t(j)}{P_t(j)} = \left[\delta^{\frac{1}{1+\rho}} \left(\frac{R_t}{\Lambda_t^K P_t(j)} \right)^{\frac{\rho}{1+\rho}} + (1-\delta)^{\frac{1}{1+\rho}} \left(\frac{W_t^F}{\Lambda_t^L P_t(j)} \right)^{\frac{\rho}{1+\rho}} \right]^{\frac{1+\rho}{\rho}}$$

where R_t denotes the nominal rental price of capital services and $W_t^F = (1 + \tau_t^F)W_t$ represents nominal labour costs including employers' pension and social security contributions. In the steady-state prices $P(j)$ are determined by markup, $\Upsilon (= -\frac{1}{\rho^z})$ over marginal costs

$$P(j) = \Upsilon MC(j) \quad (2.41)$$

The first order conditions (in logs) with respect to capital services and labour are given by

$$r_t - p_t = \log \delta - \log(\Upsilon) - \rho \log \Lambda_t^K + (1+\rho)(y_t - k_t) \quad (2.42)$$

$$w_t^F - p_t = \log(1-\delta) - \log(\Upsilon) - \rho \log \Lambda_t^L + (1+\rho)(y_t - l_t). \quad (2.43)$$

Due to the monopolistic competition in the market for output, the slope of the demand curve, $v \equiv \log(\Upsilon)$, enters into both first order conditions.

The dynamics of the price level $P_t(j)$ of producer j arise from the assumption that a firm changes its price level when it receives a random 'price-change signal'. A constant probability of receiving a price change signal is given by $1 - \zeta$ ($\zeta \in [0, 1]$). Since there is a continuum of intermediate producers, $1 - \zeta$ also represents the share of producers that have received such a signal and thus are able to change their prices. The average time between price changes is given by $1/(1 - \zeta)$. Solving the first order condition and linearizing in a standard way yields the following aggregate pricing equation for the intermediate goods producer

$$\Delta p_t = \beta E_t \Delta p_{t+1} + \frac{(1-\zeta)(1-\zeta\beta)}{\zeta} [v + mc_t - p_t]. \quad (2.44)$$

Inflation is determined by expected inflation and log markup v over the real marginal costs $mc_t - p_t$.

2.12.2 Capital rental firms

Capital is a homogeneous factor of production that is owned by a firm that rents capital to producers of domestic intermediate goods. It operates under

¹¹See Ripatti and Vilminen (2001) for further discussion of their properties and estimates using aggregate Finnish data.

perfect competition. Physical capital accumulation generates real adjustment costs in the form of lost capital stock. Capital accumulation is given by

$$K_t^p = I_t - \mathcal{S}(K_t^p, K_{t-1}^p, K_{t-2}^p) + K_{t-1}^p (1 - \delta_K) \quad (2.45)$$

where $\mathcal{S}(\cdot)$ denotes the adjustment costs of physical capital stock and δ_K is the capital depreciation factor. The capital rental firm maximizes its expected discounted profits

$$\max_{\{I_t\}} E_t \sum_{s=0}^{\infty} M_{t,t+s} \Pi_{t+s}^K \quad (2.46)$$

subject to the capital accumulation equation (2.45) and the definition of capital services,¹² $K_t = K_{t-1}^p$. Its momentary profits are given by

$$\begin{aligned} \Pi_t^K &= R_t K_t - P_t^I I_t \\ &= R_t K_{t-1}^p - P_t^I (K_t^p + \mathcal{S}_t(K_t^p, K_{t-1}^p, K_{t-2}^p) - K_{t-1}^p (1 - \delta_K)) \end{aligned} \quad (2.47)$$

The price index of investment goods, P_t^I , is the price index of the domestic investment good retailer and R_t denotes rental rate for capital. Future profits are discounted using the nominal stochastic discount factor (pricing kernel) $M_{t,t+s} = \beta^s U'(C_{t+s}) P_t^C / [U'(C_t) P_{t+s}^C]$. The first order condition with respect to capital stock K_t^p is given by

$$\begin{aligned} & - P_t^I E_t [1 + \mathcal{S}'_t(K_t^p, K_{t-1}^p, K_{t-2}^p)] \\ & + E_t M_{t,t+1} \{ R_{t+1} - P_{t+1}^I [\mathcal{S}'_{t+1}(K_{t+1}^p, K_t^p, K_{t-1}^p) - (1 - \delta^K)] \} \\ & - E_t M_{t,t+2} [P_{t+2}^I \mathcal{S}'_{t+2}(K_{t+2}^p, K_{t+1}^p, K_t^p)] = 0. \end{aligned} \quad (2.48)$$

Due to the end-of-period timing of physical capital stock, the accumulated physical capital is in use in the following period. Hence, the expected following period's rental rate R_{t+1} governs the current period investment decision. The adjustment cost function is quadratic in changes of the physical capital stock

$$\mathcal{S}_t(\cdot) = \frac{\gamma_1}{2} \frac{(\Delta K_t^p - \gamma_2 \Delta K_{t-1}^p)^2}{K_{t-1}^p}. \quad (2.49)$$

The usual ‘investment equation’ can be obtained by substituting the parametric version of adjustment costs into the first order condition.

2.12.3 Production of final goods

The economy is inhabited by two retailers that produce final goods, one producing consumer goods and the other capital goods. They combine domestic intermediate inputs, produced by intermediate goods producers, and

¹²For simplicity we assume that capital services obtains a lagged value of physical capital stock. In a more general case by Ripatti and Vilmunen (2004) capital services depend also on the endogenous utilization rate. This extension alters the results only in business cycle frequencies and is thus beyond the interest of this study.

imported goods and services, and operate under perfect competition. The demand for retailers' output is given by consumption and investment of the private and general government sectors. The output of the consumption-goods retailer divides into the private consumption and public purchases of market goods, $C_t^T \equiv C_t^H + C_t^{SF}$. The capital-goods retailer faces similar demand comprising private sector and public sector investment, $I_t^T \equiv I_t + I_t^S$. The production technology is CES for both retailers¹³

$$Q_t^j = \left[\delta^j (Y_t^j)^{-\rho^j} + (1 - \delta^j) (M_t^j)^{-\rho^j} \right]^{-1/\rho^j}, \quad j = I, C^T$$

δ^j is the respective share parameter and ρ^j the respective substitution parameter ($\sigma^j = 1/(1 + \rho^j)$). M^j denotes imports and Y^j the domestic intermediate good. Cost minimization generates following price indices,

$$P_t^j = (1 - \tau_t^c)^{-1} \left[(\delta^j)^{\frac{1}{1+\rho^j}} (P_t)^{\frac{\rho^j}{1+\rho^j}} + (1 - \delta^j)^{\frac{1}{1+\rho^j}} \left(P_t^{Mj} \right)^{\frac{\rho^j}{1+\rho^j}} \right]^{\frac{\rho^j+1}{\rho^j}}$$

and the conditional factor demands,

$$Y_t^j = (\delta^j)^{1+\rho^j} \left(\frac{P_t}{(1 - \tau_t^c) P_t^j} \right)^{\frac{-1}{1+\rho^j}} Q_t^j$$

$$M_t^j = (1 - \delta^j)^{1+\rho^j} \left(\frac{P_t^{Mj}}{(1 - \tau_t^c) P_t^C} \right)^{\frac{-1}{1+\rho^j}} Q_t^j.$$

The consumption-goods retailer pays the indirect taxes, τ_t^C . Hence the tax base for indirect taxes consists of private consumption and government purchases. No indirect taxes are levied on investment goods.

The elasticity of substitution between imported consumption goods and domestic intermediate products has been estimated by Ripatti and Vilmunen (2004). Using cointegration techniques they find that $\hat{\rho}^C = -0.7731$ (standard error 0.049) implying elasticity of substitution of 4.4. δ^C is calibrated at 0.87. The estimation of elasticity of substitution between imported investment goods and domestic intermediate goods suggests a value of 2.2, which is given by the estimate of $\hat{\rho}^I = -0.538$ (standard error 0.183). This means that the factors are gross-substitutes. The calibrated value of the share parameter δ^I is 0.67.

The exporter is a firm that combines domestic intermediate inputs Y_t^X and imported raw materials M_t^R to produce export good X_t in competitive markets. Technology and preferences are identical to those of the retailers. Ripatti and Vilmunen (2004) assume that the imported-raw-materials-augmenting technical change may contain a deterministic linear time trend. This trend captures the structural change in input usage of exports. The estimate of the elasticity of substitution is 0.45, implying that $\rho^X = 1.217$ (standard error 0.378). Not surprisingly, the point estimate suggests that the imported raw materials and the domestic intermediate input are gross-complements in the production of exported goods and services. The calibrated value of the share parameter δ^X is 0.51.

¹³We abstract from the time-varying factor augmenting technical progress, which reflects changes in preferences for domestic and imported intermediate goods.

2.12.4 Importing firms

Imported goods and services are used by the retailers and the exporter. They combine imported and domestically produced intermediate goods to produce final consumption, capital and exported goods. The consumer goods and services (including 5 per cent of imported energy) are used by the consumption-goods retailer, capital goods and services are used by the capital-goods retailer, and the exporter uses energy and intermediate goods in producing exported goods. Both retailers operate under perfect competition in the output markets. A model for import prices of imports by main use, ie for the retailer sector, is derived by via the approach of Betts and Devereux (1996) and (2000) applied to the Finnish aggregate import data by Freystätter (2003). A portion of importers price their products in local (Finnish) currency and the other importers in producers' own currencies. Their pricing contains identical frictions in the form of Calvo (1983b), ie they can change their price only if a random price-change-signal is received. Their marginal costs are identical. The aggregation of pricing behaviour over these two types of importers yields an import-price Euler equation where import prices depend on expected future import price inflation, current and expected future changes in foreign exchange rates, and on importers' real marginal costs. This brings incomplete exchange rate pass-through into the model.¹⁴

2.13 Market equilibrium

All markets are in equilibrium at all point of times. The capital goods market is in equilibrium when the supply of capital services by the capital-rental firm equals the demand for capital services by intermediate goods producers. Similarly the labour markets are in equilibrium when the demand for labour equals its supply, $L_t^s = L_t^D$. In the intermediate goods sector, the demand for intermediate goods by retailers and exporters equals total supply

$$Y_t^C + Y_t^I + Y_t^X = Y_t. \quad (2.50)$$

Markets for final goods clear when

$$\begin{aligned} C_t^S + C_t^H &= C_t^T \\ I_t^G + I_t &= I_t^T \\ \left(\frac{P_t^X}{S_t P_t^W} \right)^{-\rho^W} M_t^W &= X_t, \end{aligned} \quad (2.51)$$

where P_t^W is the aggregated export price of competing economies and M_t^W is aggregate imports of export markets. When market clearing conditions hold, the workers' and pensioners' budget constraints (2.12) and (2.20), the general

¹⁴As incomplete exchange rate pass-through is not essential for the ageing simulations provided in this paper, we abstract from explicit derivation. Derivation is provided in Ripatti and Viertola (2004).

government budget constraint (2.35) and the pension fund's budget constraint (2.37) imply the following equation for the accumulation of foreign assets

$$S_t A_t^W = (1 + r_t^F) S_t A_{t-1}^W + \underbrace{P_t^X X_t - P_t^{MR} M_t^R - P_t^{MC} M_t^C - P_t^{MI} M_t^I}_{\text{trade balance}} \quad (2.52)$$

The current account balance is given by $S_t(A_t^W - A_{t-1}^W)$ and the factor income account by $r_t^F S_t A_t^W$.

3 Policy designs

3.1 An overview of the pension system and pension reform in Finland

The model developed above will be used to study population ageing and policy experiments in the small open economy of Finland. First, we set out the Finnish pension scheme and pension reform effective since 2005.

The pension scheme in Finland is defined benefit in the sense that pension benefits are not directly dependent on contributions of workers to employment pension schemes or/and the yields of pension funds. Instead, of total benefits being changed, it has been contribution rates that have been altered in response to possible shortfalls in the balance. When there has been a danger of a shortfall in the agreed funding rate, the level of contributions has been raised. The second general feature of the Finnish system is the negligible role of private and occupational pension schemes of private firms. Nearly all old age pensions are provided by employment pension institutions or national pension institutions closely controlled by the state.

The latest reform, effective from the start of 2005, changed the pension system in line with more actuarial principles. With the introduction of flexible retirement age, 63 to 68, the level of pensions becomes linked more explicitly than before with exit age and thus with contributions made by the insured. Furthermore, starting from 2009, changes in life expectancy will have an effect on pension levels. In addition, pension indexation has been changed so that all pensions will be indexed using weights of 0.8 for living costs and 0.2 for wages, whereas before a 'midway index' was applied up to age 65. This will contribute to a comparative erosion in pension values, and thus to a decline in the pension replacement ratio μ , compared to wage level. Contributions are collected from employers and employees. Currently, 16.8% is collected on average from employers and 4.6 % from employees. It has been agreed that further changes in contribution rates will be shared equally (50-50) between employers and employees. Due to the benefit-based nature of the Finnish employment pension system, long-term aggregate pension expenditure can be approximated simply from demographic factors and parameters determining average pension benefits. The most important factors are naturally demographic trends and the average age at which people start to draw pensions, which together determine the total number of pensioners. The average level of pensions is affected by wage levels and the pension index, and thus by consumption prices and

productivity developments. In addition, changes in life expectancy will have an effect on level of benefits so that the replacement rate became dependent on exit age.

3.2 Model's calibration – initial steady state

For the purpose of simulating the model, the quantity variables are de-trended by the labour-augmenting technical factor Λ_L , population N_t and the numeraire price level P_t . As for labour market variables, the nominal wage is scaled by the labour augmenting technical change Λ_L and consumption deflator P_t^C . Labour supply indices are scaled by population N_t . Finally, all the other price variables are de-trended by the numeraire price level P_t .

In order to set up the model for demographic simulations and policy experiments, we assume that the economy is initially in the steady state. The economy is growing on the balanced growth path and the demographic structure is stable. The model's key parameters are then calibrated to reflect the main features of the Finnish economy. The key parameters are calibrated so that the key 'great ratios', factor shares, participation rates, demographic structure as well as fiscal variables reflect the situation in the Finnish economy on average during the last 10 years. The parameters of the production functions were estimated using cointegration methods (Johansen, 1995), and the parameters related to capital stock adjustment costs, depreciation function and import prices are estimated using GMM.¹⁵ Tables (1)–(4) characterize the calibration of some of the key parameters and resulting values of the key endogenous variables. We next comment on the calibration of key parameters.

In order to fit the participation rates observed we set the relative efficiency of retirees at roughly 30%. Elasticity of periodic utility with respect to consumption was at 0.8 and inter-temporal elasticity of substitution at 0.4. The public-finance literature tends to use values well below unity for the inter-temporal elasticity of substitution, while the RBC literature prefers large value of σ . Typically, smaller value of σ makes the economy react more strongly to fiscal stimulus (table 1).

Reflecting a calibrated value of periodic utility of consumption, the value for the Frisch elasticity of labour supply for workers is in line with international microeconomic studies, which report the values of 0.15–0.32.¹⁶ Kuusmanen (2005a, 2005b) has also estimated compensated labour supply elasticities using Finnish Labor Force survey data. Depending on the data sample and methods used, his estimates range from 0.08 to 0.30. The distribution of wealth between working-age and elderly persons seems reasonable in light of the demographic structure: 17 per cent of the model economy's total financial wealth is owned by retirees. Ω is an additional factor that multiplies the worker's discount factor. This additional discount factor is clearly higher than those obtained in Gertler (1999). The difference is largely explained by the distorting nature of

¹⁵For detailed description of estimation strategies, see Ripatti and Vilminen (2004).

¹⁶Bayomi, Laxton and Pesenti (2004) use the value 0.33 in the standard calibration of GEM for the euro area.

taxes. Similarly, retirees marginal propensity to consume is ‘only’ 60% higher than that of the workers.¹⁷

Variable	Explanation	Value
β	subjective discount factor, parameter	0.99
v	elasticity of periodic utility with respect to consumption	0.80
σ	Inter-temporal elasticity of substitution, parameter	0.45
ξ	Labour efficiency of retirees, parameter	0.30
ϵ	Relative marginal propensity to consume, variable	1.59
π	Worker’s marginal propensity to consume, variable	0.019
λ^F	Distribution of financial asset wealth, variable	0.17
Ω	Additional discounting factor	1.10
$\bar{\nu}^w$	Frisch elasticity of labour supply (workers), variable	0.26
$\bar{\nu}^r$	Frisch elasticity of labour supply (pensioners), variable	0.23

Table 1: Calibration of key demand side parameters

Regarding parameters affecting most strongly to pension system and demographic structure, we assume that individuals work on average 43 years and stay in retirement for about 15 years. Annual growth rate of working age population is set at 0.18% per annum (table 2). The steady state retiree worker ratio then amounts to 33 per cent, in accord with recent data. The pension rate is calibrated at 0.55, which, given endogenous average wage and demographic structure means, that pension expenditures amount to 20% per cent of total wages, also roughly in accord with recent data. Finally, the pension contribution rate to employees is 3.9 per cent, while the employees’ contribution rate amounts to 15.4 per cent, so that overall pension contribution rate is 19.3 per cent.

Variable	Explanation	Value
$\frac{1}{1-\gamma}$	Expected time of retirement, in years, parameter	14.8
$\frac{1}{1-\omega}$	Expected time of working, in years, parameter	43.1
\bar{N}^w	Annual growth rate of working age %, parameter	0.18
φ	Old age dependency ratio, variable	0.33
τ^{WP}	Pension contribution rate of the workers %, implicit	3.9
τ^{FP}	Pension contribution rate of the firms %, parameter	15.4
\mathcal{T}^R	Pensions, % of wage sum, variable	20.5
μ	Pension rate	0.55

Table 2: Calibration of key parameters of the pension system

Regarding public sector (excluding pension fund) it is assumed that the steady state debt to output ratio is 60 percent, while total public consumption

¹⁷We assume that average expected time between price changes is 5 quarters, while wage changes occur on average only every 8 quarters. In comparison to the literature, wage changes occur seldom, but the values reflect the Finnish centralized wage setting system and the fact that centralized wage contracts are usually set for 1–2 years time in Finland. Similarly, the Calvo parameter for prices is on the high side.

amounts to 20.8 percent of output (table 3). Total social security transfers to workers are 6.4 percent of output. Finally labour income tax rate is calculated endogenously so as to satisfy the state's budget constraint, while the rest of the tax parameters have been set exogenously reflecting the current tax system in Finland.

Variable	Explanation	Value
κ	Fiscal rule adjustment, parameter	0.1
a^S	Debt (% of output), variable	60.0
τ^{WS}	Income tax rate % of workers, implicit	30.2
τ^K	Corporate tax rate %, parameter	19.2
τ^C	Indirect tax rate %, parameter	21.0
τ^{FS}	Firm's social security contribution rate, parameter	4.0
c^S	Public consumption (% of output), variable	20.8
\mathcal{T}^{SW}	Total transfers to workers (% of output), variable	6.4

Table 3: Calibration of key parameters of the state

Regarding the supply side of the economy, we assume that the real interest rate is about 2.4 per cent per annum (table 4). The economy grows on the balanced growth path at the rate of 2.3 per cent per annum, reflecting the labour-saving technical change and steady state growth rate of the population. Given the small size of Finland in the euro area, the feedback from Finnish economy to euro area level is very modest. Thus a reasonable approximation is that the euro area policy rate is exogenous and that the nominal exchange rate is fixed. The foreign nominal interest rate determines the domestic nominal interest rate up to the UIP condition. On the balanced growth path, inflation is set at 2.0% in accord with the inflation target of the ECB.

Price markup and the share price-to-equity ratio are roughly in accord with empirical evidence,¹⁸ while capital share parameter and elasticity of substitution between capital and labour have been estimated from historical data by Ripatti and Vilmunen (2001). Capital depreciation rate is set to 8 % per annum.

Finally, table (5) summarizes key 'great ratios', factor shares, net foreign asset position and employment rates at initial equilibrium. Comparison with recent data shows that the model fits the data reasonable well. The model's current calibration, however, involves some difficulties in capturing the relatively low investment and private consumption shares observed in the data.

3.3 Policy experiments

In order to facilitate analysis of fairly complex general equilibrium effects associated with ageing and different assumptions that can be made as to

¹⁸Kilponen and Santavirta (2005) estimate from microdata that the percentage share of operating profits in the value of gross output in Finnish manufacturing firms was roughly 8% during the last decade.

Variable	Explanation	Value
\bar{R}^F/\hat{P}^c	Real interest rate, p.a., variable	2.4
\hat{Y}	Output growth rate, p.a., variable	2.3
$\hat{\Lambda}_L$	Labour saving technical change p.a., parameter	2.08
δ^K	Capital depreciation rate, parameter	0.08
Υ	Price markup, parameter	1.08
$\bar{A}^F/\bar{\Pi}$	Price to equity ratio, variable	15.0
δ	capital share parameter	0.1
ρ	El. of substitution between capital and labour	0.72

Table 4: Calibration of key parameters for the supply side

Variable	Explanation	The data 1995-2005	Steady State
c^H	Private consumption share,%	51.3	54.0
i	Private investment rate,%	15.8	18.6
x	Export share,%	39.8	34.0
m	Import share,%	31.1	34.0
ls	Labour share,%	49.5	51.5
k/l	Capital intensity	4.8	4.3
a^w	Net foreign assets, %	-42.5	-23.3
l	Employment rate, %	58.6	55.4
l^w	Workers	56.6	53.8
l^r	Retirees	2.1	5.2

Shares calculated in nominal terms except for capital intensity

Table 5: Great ratios, factor shares and participation rates

pension parameters, we designed some alternative demographic scenarios and policy experiments. The long-run implications of ageing population are treated as shocks to the initial state of unchanged demographic age structure and steady pace of population growth. We performed five different simulations altogether, as reported in table 6.

The shock simulating population ageing incorporated both the higher life expectancy projected in the population forecast and the slowdown in the growth of working-age population. Higher life expectancy was simulated by increasing the expected period of retirement by 5 years (table 6, columns I–II) and then supplementing the shock with a longer expected working time of 2.5 years, while taking into account the slow down of the growth of working age population (table 6, column III).

Experiment II is a pure demographic shock that captures the main demographic trends. According to the latest population prognoses, the fastest growth in the old age dependency ratio will take place in the 2020s and 2030s, when the baby boom generation, born after the second world war, retires. After that, the ratio will grow at a slower pace. The overall dependency ratio is projected to level off by the 2030s. Immigration is forecast to remain

Change	Experiment				
	I	II	III	IV	V
Expected period of retirement, years	5.0	5.0	5.0	5.0	5.0
Growth rate of workers, % annually		-0.5	-0.5	-0.5	-0.5
Expected working time, years			2.5	2.5	2.5
Pension replacement rate, % -points				-10.0	μ^*

*endogenous

Table 6: Demographics shocks relative to initial steady state

insignificant, at only 5000 persons a year net. Finally, according to Statistics Finland estimates, the life expectancy for men will rise from the current 75.1 years to 82.1 years by 2040. For women the corresponding figures are 81.8 and 86.8 years.

Features inherent in the pension scheme, ie deferred retirement in response to accelerated accrual and a lower replacement rate, were built into the demographic effects by lowering the pension replacement rate by 10 percentage points (column IV). The analysis was completed with an equilibrium calculation where the replacement rate was endogenously defined while the rate of employee contribution was kept unchanged, at its initial steady state level (column V).

4 Long-run effects of ageing

4.1 Increasing life expectancy

Increase in life expectancy projected in the demographic forecast for Finland alone would have a pronounced effect on the economy's long-run equilibrium. Extension of the period of retirement by five years, as assumed in the calculation, would imply an increase in the old-age dependency ratio of 11 percentage points. This would lead to 6.3 percentage point increase in total pension expenditures relative to total wages. Tax burden on labour would raise by more than 3.7 percentage points and would be reflected in falling employment rates. The total contribution rate would rise by 6.7 percentage points. (table 7, column I). Given age cohorts of the same size and a stable fertility rate, higher life expectancy alone would impose a considerable burden on the economy. The falling employment rate of workers is attributable to a higher tax burden on labour, so that the after-tax wage rate declines slightly. These factors are reflected in rising real wages in the long-run. The consumption response, measured as a share of output, reflects households' different marginal propensities to consume. Retirees own a larger share of the economy's financial wealth and are more willing than workers to consume it. Mirror effect of this is that capital share is higher in the aged economy.

Variable	Explanation	Final steady-state, change				
		I	II	III	IV	V
φ	Old-age dependency ratio ^{**}	10.9	15.5	13.0	13.0	13.9
t^P	Contribution rate ^{**}	6.7	8.8	7.0	1.3	a)0.0
	employee	1.4	1.8	1.4	.3	a)0.0
	employer	5.4	7.0	5.6	1.0	a)0.0
t^{WS}	Labour income tax rate ^{**}	2.3	5.0	3.4	0	-0.7
$\frac{T^R}{WL}$	Pension expenditure/wage sum ^{**}	5.3	8.0	6.7	2.0	1.0
w	c)Real wage [*]	5.4	5.3	6.0	9.5	10.3
$(1 - t)w$	c)After-tax real wage [*]	-0.38	-5.6	-1.6	9.5	11.5
c^H	Private consumption share [*]	5.1	6.2	6.4	6.8	13.6
λ	Wealth distribution ^{**}	3.4	6.1	5.2	5.5	5.6
l	Employment rate ^{**}	-3.7	-5.7	-4.7	-3.3	-3.0
l^w	Workers	-4.3	-6.2	-5.2	-4.3	-4.2
l^r	Retirees	2.1	1.8	1.6	3.5	3.7
k	Capital share [*]	21.0	23.5	22.4	21.1	21.0
a^w	Net foreign assets ^{**}	4.8	21.1	27.1	51.2	56.7
μ	Pension replacement rate ^{**}	0	0	0	-10.0	b)-12.0
y	Output, efficiency units	-1.7	-5.5	-3.4	-0.5	0.12

* Percentage change relative to initial steady state
** Change, percentage points relative to initial steady state
a) Contribution rate kept fixed at initial steady state, b) endogenous, c) in efficiency units

Table 7: Long-run effects of ageing

4.2 Declining fertility rate

Considering that age cohorts entering the labour market are smaller than those withdrawing, the old-age dependency ratio would increase by almost 16 percentage points (table 7, column II). Tax rate and employment responses would also be pronounced, in that the pension contribution rate would increase by 8.8 percentage points and the income tax rate by 3.4 percentage points. The employment rate would fall by almost 6 percentage points reflecting a marked increase in real wages. The private consumption share would increase by 6.2 percentage points relative to the initial state of constant population growth. The major increase in tax rates is related not only to pension expenditures but also to the strong response of workers' employment rates. This reduces tax bases, even if retirees supply more labour. Retirees positive labour supply response is attributable to a longer retirement period.

4.3 Pension reform and a lower replacement rate

It has been estimated that the pension reform will extend labour force participation by an average of 2–3 years. This alone would clearly ease the burden of an ageing population (table 7, column III). The need to increase the contribution would be reduced by 1.8 percentage points. Similarly, the need

to raise the labour income tax rate would decrease almost as much, compared to the pure demographic shock. This would in turn be reflected in smaller employment losses in the long-run.

The decline in average replacement induced by the rules of pension indexation also has a considerable effect on sustainability of the pension scheme. This effect was measured by the assumption that the ratio of average pension to average wage would decline by 10 percentage points, ie. clearly less than that suggested by the calculation discussed above. This would considerably reduce the need to raise the contribution rate in response to population ageing in the long-run. The labour income tax rate could be kept at the current level, and contribution rates would need to be raised only by 1.3 percentage points. (7, column IV). Pension expenditure would raise 4.7 percentage points less than the figure returned by the previous simulation (column III).

According to the results of the equilibrium calculation based on given rate of contribution, we find that pensions would, in the long-run, have to decline from their current levels by 12 percentage points relative to average wages (table 7, column V). This would ensure that the ratio of pension expenditure would increase only by 1% percentage point relative to total wages in response to population ageing. However, real wages would still increase by 10%, while the employment rate would fall by 3 percentage points. Once more, retirees incentives to work would be improved, primarily because of the wealth effect compared to the previous experiment. Finally output would remain virtually intact in this case.

5 Dynamic effects

Dynamic optimization and perfect foresight assumption implies that the transition paths of the macroeconomic variables reflect the way in which optimizing agents prepare for a new demographic situation. These adjustment paths are further complicated by our assumption that demographic structure is changing over time. The adjustment paths can typically involve over- or undershooting of forward looking variables, even though the transitional path of a demographic shock is a smooth slowly evolving process. One such variable is consumption, which typically initially jumps and then begins gradually to adjust to the new long-run steady state. This is reflected in over- or undershooting of wages and employment, given that leisure and consumption are non-separable in the periodic utility.

Figure 1 shows how the model economy responds to a pure anticipated shock to retirement age (I) and to the shock where the fertility rate drops as described in experiment II.¹⁹

After a temporary drop in distribution of wealth (in favour of workers), retirees eventually hold a larger share of the economy's financial wealth. The worker's tax burden on labour follows a similar pattern. Aggregate consumption reacts in advance of anticipated demographic change and starts

¹⁹ All the simulations were done using *DYNARE*, version 3.051.

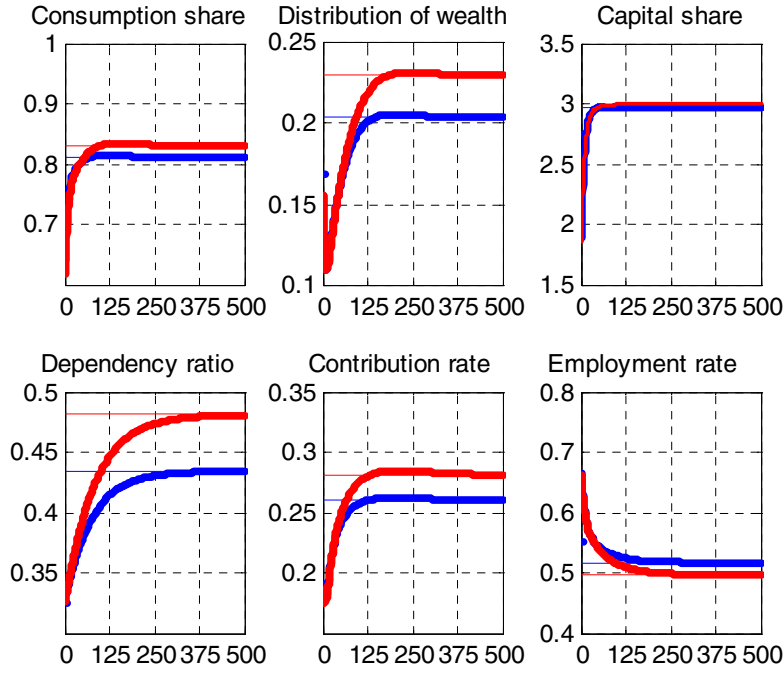


Figure 1: Pure demographic shocks and transition.

Blue line corresponds to experiment I and red line (lighter grey) to experiment II. Periods are quarters.

stabilizing well before the demographic change is over. Beyond the response of aggregate consumption, there are opposite responses of retirees and workers. While the workers increase saving for retirement by reducing their consumption, and also because of a higher tax burden, retirees respond positively. Retirees are also now more numerous as shown by the gradual rise in the dependency ratio. A lower fertility rate, besides increasing life expectancy, gradually increases the dependency ratio and leads to a further increase in the pension contribution rate. This effect is also associated with a slightly higher consumption share and lower employment rate.²⁰

Figure 2 illustrates how an extension of working life and a lower replacement rate alleviate the effects of a pure demographic shock. Consumption share is practically intact (red (lighter gray) line), while the need to increase contribution rates gradually ease along with a less pronounced increase in the dependency ratio. Capital share increases initially with the extension of working life but eventually returns to a very close to the same level as with a pure demographic shock. Longer working time means that there is a roughly one percentage point improvement in employment rate compared to a pure demographic shock. Extending the simulation by taking into account also a lower pension replacement rate shows that the consumption share may

²⁰Higher capital and consumption shares in the long-run are compatible with the Blanchard-Yaari asset market equilibrium condition. In essence, the condition produces a positive relationship between consumption and capital stock, as required by the current account balance in an open economy.

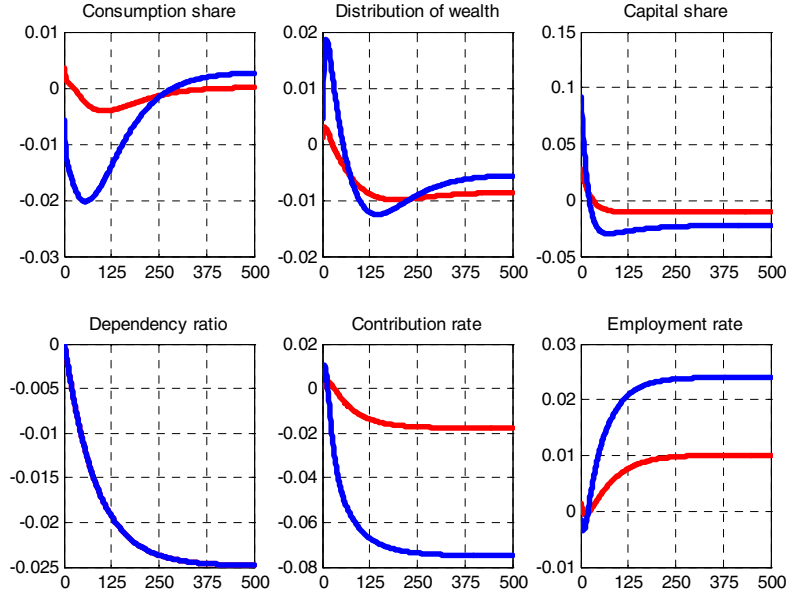


Figure 2: Extensions of working life and lower replacement rate ease fiscal burden of ageing.

Red line (lighter grey) corresponds to experiment III, blue line to experiment IV; baseline is a pure demographic shock; periods are quarters.

now fall in the short and medium term. A drop in the replacement rate reduces pensioners' permanent income, with a subsequent temporary fall in consumption levels. As the need to raise the contribution rate gradually eases, employment improves rather quickly, while consumption bounces back only after a several decades. Mirroring this, there is also a temporary fall in capital share and a permanent fall in distribution of wealth held by the retirees.²¹

5.1 Demographic uncertainty

Population scenarios and ageing reflect projected changes in fertility rates and longevity. These population scenarios are however subject to much uncertainty. OECD (2005) has reported that in the case of Finland, overall

²¹These results are naturally subject to uncertainty regarding calibrated and estimated parameter values of the model, as well as assumptions on the external environment of the economy and demographic projections. In particular intertemporal substitution (σ) and elasticity of periodic utility with respect to consumption (ν) guide the dynamic and long run response of consumption, real wages and employment, which in turn matter how the tax burden on labour evolves in response to population ageing. Subsequent sensitivity analysis shows that a higher value of intertemporal substitution mitigates the effects of pure demographic shock on taxes, while the opposite is true for lower values of σ . Parameter ν affects primarily on the long-run response of the economy to population ageing as well as the initial steady state. Sensitivity analysis suggests that higher values of ν lead to somewhat smaller response of the contribution rate which in turn is associated with a slightly smaller negative response of employment.

uncertainty surrounding longevity and fertility rate projections suggests that the dependency ratio will fluctuate ± 10 percentage points around the official scenario.²² Rather similar results were obtained in the project ‘Uncertain Population of Europe’ (UPE) funded by the EU Commission. In this project, stochastic population forecasts were generated by simulating 3,000 alternative paths of future development for age-specific mortality, fertility and net migration for different European countries. These different scenarios were then combined into population scenarios using the classical cohort-component book-keeping equations. Based on the resulting age-specific cohort tables from different simulations, table 8 summarizes the statics related to the old-age dependency ratio relevant to the model. This helps us anchor the degree of uncertainty related to the old-age dependency ratio in Finland and thus quantify the degree of uncertainty related to demographic trends. The table suggests that risks of making large errors in population forecasts increase after 2020, making the confidence limits expand quite fast (see also Alho (1998)).

Year	Min	Mean	Max	S.E.
2010	0.40	0.42	0.43	0.005
2020	0.48	0.54	0.61	0.019
2030	0.49	0.64	0.74	0.043
2040	0.41	0.65	0.94	0.071
2050	0.36	0.67	1.08	0.099

Table 8: Summary statistics from stochastic population simulations (based on own calculations from <http://www.stat.fi/tup/euupe/>)

In order to quantify how the demographic uncertainty feeds into the model’s outcomes, we take experiment III as a baseline (see table 6), but allow for stochastic variation in retirement probability ($1 - \omega_t$), survival probability (γ_t) and worker’s growth rate \hat{N}_t^w . Allowing for stochastic variation in these variables generates stochastic variation for the model’s endogenous variables along the transition path.²³ We calibrate the uncertainty in retirement probability and survival probability such that the 90% confidence intervals for working time and retirement are respectively ± 2 and ± 4 years at the final steady state of the model. Furthermore, we assume that the population growth rate has a 90% confidence interval of 0.5% in annual terms. This generates empirically plausible variation in dependency ratio, amounting to a 4% standard error. Figure (3) shows the 90% confidence intervals generated by these assumptions on the relevant demographic variables, while table 9 summarizes the results for selected endogenous variables at the final steady state.

²²Assessing the robustness of demographic projections in OECD countries, Frederic Gonand, Economics Department Working Papers, No. 464, OECD)

²³DYNARE’s functions *forecast* and *stoch_simul* allow mixing of deterministic and stochastic shocks. We are indebted to Michel Juillard for developing these functions.

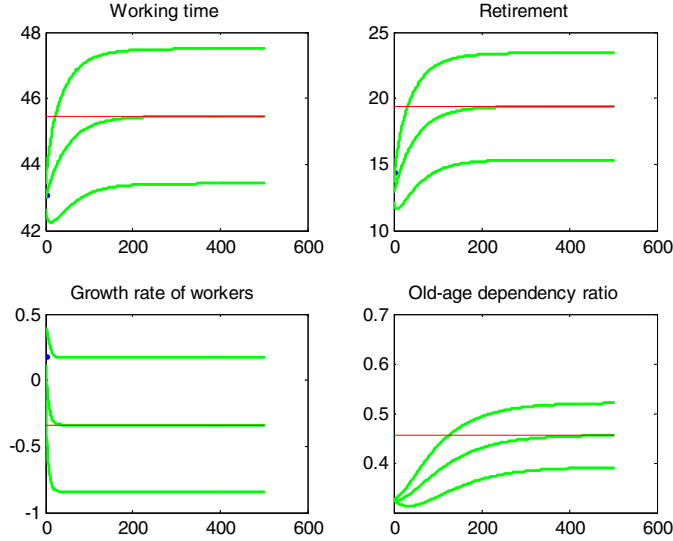


Figure 3: Mean outcome and 90% confidence intervals generated by the stochastic variation in retirement probability, survival probability and population growth rate.

It is evident that there much uncertainty about economic outcomes of the model, once uncertainty as to demographic structure is accounted for. A standard error of 4% for the dependency ratio translates to roughly a 3 percentage point standard error in the labour income tax and contribution rates. Variation in the employment rate is roughly 2 percentage points.

Stochastic simulations allows us also to quantify the relative importance of different demographic shocks to economic outcomes. This can be done by looking at (asymptotic) variance decomposition as generated by the stochastic simulations of the model. Although the results of variance decomposition depend on the assumed relative standard errors of the shocks, columns 5–7 in table 9 clearly suggest that lengthening the working time has rather minor effect on alleviating the fiscal burden of ageing: Only about 3% of the stochastic variation in the contribution rate is explained by the variation in working period. Stochastic variation in the time spent in retirement is clearly more important.

6 Conclusions

As industrialized countries prepare for population ageing, pension reforms have been designed with the aim on increasing incentives to work longer and/or cutting pension benefits. One prominent example from Europe is Finland, where the latest pension reform introduced a flexible retirement age of between 63 and 68.

Using a tractable dynamic general equilibrium model calibrated to Finland, we show that even if the effective retirement age were to increase as expected,

Variable	Explanation	Mean	Standard error	Variance decomposition		
				ω	\hat{N}_t^W	γ
φ	Dependency ratio	0.46	0.04	3.7	3.8	92.5
t^P	Contribution rate	0.28	0.03	3.7	1.6	94.7
t^{WS}	Labour income tax rate	0.42	0.03	3.7	2.1	94.2
$\frac{\mathcal{T}^R}{WL}$	Pension exp/wage sum	0.28	0.02	3.6	3.9	92.4
y	Output, efficiency units	0.18	0.01	3.6	1.9	92.4
L/N	Employment rate	0.49	0.02	3.5	2.8	93.7
$4(1-\omega)^{-1}$	Working time, years	45.5	1.2			
$4(1-\gamma)^{-1}$	Retirement, years	19.4	2.5			
\hat{N}_t^W	Growth rate of workers	-.33	0.3			
a) All the moments are evaluated at final steady state						

Table 9: Asymptotic moments from stochastic model simulations for selected model's variables

the burden from pension payments alone would cause the income tax rate to rise to a level above that witnessed in the worst years of recession in the mid-1990s. A stochastic simulation exercise suggests also that lengthening the working time has a rather minor impact on alleviating the fiscal burden of ageing: only a minor part of the stochastic variation in the contribution rate is explained by the variation in working period.

On the contrary, if the replacement rate falls, as seems likely under the current pension regime, there would be a much smaller increase in taxation in response to growing expenditure on pensions. The average decline in the level of pensions relative to wage developments in turn raises several concerns. In Finland, where supplementary pension funds are of minor importance by international standards there is a risk that the falling purchasing power of pensions relative to wage developments will exert pressure on other social security schemes. The employee pension scheme, which appears to be financially sustainable, may thus in practice generate costs to local and central government in the form of higher expenditure on income support and other benefits.

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