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## Extracting growth and inflation expectations from financial market data

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# BANK OF FINLAND DISCUSSION PAPERS

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2 • 2004

Lauri Kajanoja  
Research Department  
20.1.2004

Extracting growth and  
inflation expectations from  
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Suomen Pankin keskustelualoitteita  
Finlands Banks diskussionsunderlag

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# Extracting growth and inflation expectations from financial market data

Bank of Finland Discussion Papers 2/2004

Lauri Kajanoja  
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## Abstract

This study presents a framework for extracting long-run GDP growth and inflation expectations from financial market data on a real-time basis. The framework uses information from both stock and bond markets. It builds on a dividend discount model of stock valuation and on a linearized consumption Euler equation. Furthermore, expected long-run dividend growth for a broad equity index is assumed to be related to expected long-run GDP growth. Short-run and long-run dividend growth expectations are allowed to differ. The former are measured using equity index futures. We extract growth and inflation expectations for the euro area and for the United States.

Key words: inflation expectations, growth expectations, equity index futures

JEL classification numbers: E31, E44, E66

# Kasvu- ja inflaatio-odotusten johtaminen rahoitusmarkkinahinnoista

Suomen Pankin keskustelualoitteita 2/2004

Lauri Kajanoja  
Tutkimusosasto

## Tiivistelmä

Tämä tutkimus esittelee tavan, jolla pitkän aikavälin kasvu- ja inflaatio-odotukset voidaan johtaa rahoitusmarkkinahinnoista. Odotukset voidaan mitata reaaliaikaisesti käyttäen tietoja koroista ja osakkeiden hinnoista. Menetelmä lähtee liikkeelle mallista, jonka mukaan osakkeen hinta on sen odotetun osinkovirran diskontattu arvo. Toinen lähtökohta on kulutuksen linearisoitu Eulerin yhtälö. Lisäksi menetelmässä oletetaan, että laajan osakeindeksin odotettu osinkojen pitkän aikavälin kasvuvauhti on yhteydessä odotettuun pitkän aikavälin BKT-kasvuvauhtiin. Osinkojen lyhyen aikavälin kasvuodotuksia mitataan osakeindeksifutuuriin avulla. Tutkimuksessa mitataan euroalueen ja Yhdysvaltain kasvu- ja inflaatio-odotuksia.

Avainsanat: inflaatio-odotukset, kasvuodotukset, osakeindeksifutuurit

JEL-luokittelu: E31, E44, E66

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# 1 Introduction

Financial market prices are affected by the market participants' expectations concerning future macroeconomic developments. However, expectations on real GDP growth, for example, cannot be directly observed in the price quotations for any financial market instrument. In order to gain information on such expectations, one needs to employ economic models, in addition to financial market data.

Financial market participants' expectations concerning macroeconomic developments are obviously of great interest, not least to economic policy makers. The expectations, as they are reflected in financial market prices, are based on a huge amount of information. Naturally, the expectations can be wrong, and each individual may disagree with them. In any case, the knowledge on market expectations makes it easier to understand current economic developments and to form one's own expectations concerning the future.

Various measures of market expectations concerning macroeconomic developments have been put forward. A widely used measure of inflation expectations is the so called break-even inflation rate derived from the yield of an inflation-indexed bond. Break-even inflation rates are discussed by eg Sack (2000) and Scholtes (2002). Another market-based measure of inflation expectations can be derived from inflation-linked swaps, as reported by the European Central Bank (2003). Measuring market expectations of real output growth seems to be a more formidable task. For measuring the market's perception of the output gap, Martin & Sawicki (2003) propose a method based on an inverted Taylor rule. Taking a broader view on measuring market expectations, one can also consider indicator models of growth and inflation that use financial market variables as inputs. Such models are widely used in short-term macroeconomic forecasting.

Stock prices and interest rates can be interpreted to yield information concerning market expectations of future output growth and inflation. High stock prices indicate fast expected growth of companies' earnings and dividends in the future. Long term interest rates reflect expectations concerning inflation as well as output in the long run. The long term real interest rate is, according to standard macroeconomic theory, related to the expected long run output growth. However, stock prices and interest rate do not as such provide measures of growth or inflation expectations.

This paper presents a new framework for measuring market expectations concerning long run inflation and real output growth. The method combines pieces of information contained in stock prices and interest rates. The framework presented here can directly be applied to measuring expectations in real time. As inputs, the framework uses interest rates and dividend-price ratios for equity indices. In addition, equity index futures are utilized in gauging short run expectations. The framework is based on economic theory. It builds on three elements. First, a dividend discount model of stock prices is used. Second, it is assumed that expected long run dividend growth is proportional to expected long run GDP growth. Third, it is assumed that

there is a stable linear relationship between the long term real interest rate and the expected long run real GDP growth.

The paper is organized as follows. Section 2 describes the method. Section 3 presents the results, that is, the series of extracted growth and inflation expectations for the euro area and the United States. Section 4 concludes.

## 2 Framework

In this study, market expectations for the long run GDP growth and inflation are measured using the following data as inputs: interest rates, dividend-price ratios of equity indices, and equity index futures. This section presents the framework used in the measurement. Section 2.1 describes how the dividend discount model is used in the framework. The method for deriving long run expectations is further developed in Section 2.2. Section 2.3 describes how the near term dividend growth expectations are measured using data on equity index futures. The near term expectations are measured in order to improve the measurement of long run expectations.

### 2.1 Expected dividend growth

Following the dividend discount model, we start from the assumption that stock prices equal expected discounted future dividends. The discount rate is the expected return on equity capital, which can be approximated by the risk free interest rate plus an equity premium. Here, the latter is assumed to be constant. Therefore, the end-of-period- $t$  price of a stock  $P_t$  can be expressed as

$$P_t = \sum_{j=1}^{\infty} \frac{D_{t+j|t}}{(1 + i_{j,t} + \omega)^j} = D_t \sum_{j=1}^{\infty} \frac{(1 + n_{j,t+j|t})^j}{(1 + i_{j,t} + \omega)^j}, \quad (2.1)$$

where  $D_{t+j|t}$  denotes dividends paid during period  $t + j$  as expected at the end of period  $t$ . Symbol  $i_{j,t}$  denotes the risk free interest rate in maturity  $j$  at the end of period  $t$ , and  $\omega$  denotes the equity premium. Symbol  $n_{j,t+j|t}$  denotes the end-of-period- $t$  expectation of the growth rate of nominal dividends from period  $t$  till period  $t+j$ , in annual terms. That is, the first subscript denotes the length of the time horizon for the variable, and the second subscript indicates when the value of the variable is realized. We take the length of a time period to be one year.

We do not assume that dividend growth is expected to be constant in the future. Instead, we decompose the expected dividend growth into short run and long run expectations. We use the following ‘term structure’ assumption for the expected nominal dividend growth.

$$1 + n_{j,t+j|t} = (1 + n_{1,t+1|t})^{\frac{1}{j}} (1 + n_{LR|t})^{\frac{j-1}{j}}, \quad (2.2)$$

where  $n_{LR|t}$  denotes expected long run nominal dividend growth. In addition, let us use a similar approximation for the term structure of the discount rate, that is

$$1 + i_{j,t} + \omega = (1 + i_{1,t} + \omega)^{\frac{1}{j}} (1 + i_{LR,t} + \omega)^{\frac{j-1}{j}}, \quad (2.3)$$

where  $i_{LR,t}$  denotes the long term risk free interest rate at the end of period  $t$ . The empirical definition of this variable will be given in Section 2.4 below.

Equations (2.1) through (2.2) imply, as an approximation, that

$$n_{LR|t} = i_{LR,t} + \omega - \frac{D_t}{P_t} \frac{1 + i_{LR,t}}{1 + i_{1,t}} (1 + n_{1,t+1|t}). \quad (2.4)$$

According to equation (2.4), the market expectations concerning long run nominal dividend growth can be inferred from current financial market prices, past dividends, and estimates of the near term dividend growth expectations and the equity premium.

## 2.2 Measuring GDP growth and inflation expectations

We assume that the expected long run dividend growth varies in proportion to the expected long run GDP growth. For an imaginary stock price index covering all firms in an economy, one could argue that these two should move one-to-one. For the stock price indices used here it is natural to assume that the expected long run dividend growth rate varies more than the expected long run GDP growth rate for the whole economy. Therefore, we assume that

$$n_{LR|t} - \pi_{LR|t} = \alpha + \beta g_{LR|t}, \quad (2.5)$$

where  $\pi_{LR|t}$  denotes expected long run inflation, and  $g_{LR|t}$  denotes expected long run real GDP growth. Symbols  $\alpha$  and  $\beta$  denote positive constants. Equation (2.5) states the relationship between the expected real long run dividend growth and the expected real long run GDP growth. As discussed in Section 2.4 below, we will set  $\beta$  close to 2, when the S&P 500 index for the United States is considered.

Let us next introduce an assumption concerning the relationship between expected long run real GDP growth and the long term real rate of interest. A standard consumption Euler equation from a representative consumer model combined with a market clearing condition, saying that consumption equals output, yields

$$r_{LR,t} + \omega_Y \approx \frac{1}{\delta} \frac{u'[Y_t]}{u'[(1 + g_{LR|t}) Y_t]} - 1, \quad (2.6)$$

where  $\delta$  denotes the discount factor,  $u'$  is the first derivative of the period utility function, and  $Y_t$  denotes period  $t$  real consumption, which equals real output. Symbol  $\omega_Y$  denotes a risk premium. It is not assumed to be equal to  $\omega$  of equation (2.1), since  $\beta$  is allowed to differ from one in equation (2.5). The long term real interest rate is denoted by  $r_{LR|t}$ , and it is defined as

$$r_{LR,t} = i_{LR,t} - \pi_{LR|t}. \quad (2.7)$$

Equation (2.6) can be linearized to yield, as an approximation,

$$r_{LR,t} = \rho + \lambda g_{LR|t}. \quad (2.8)$$

This linearization holds for some positive constants  $\rho$  and  $\lambda$ , the latter of which represents the inverse of the elasticity of intertemporal substitution.

Equations (2.4), (2.5), (2.7) and (2.8) can be combined to yield the following system of equations.

$$g_{LR|t} = \frac{\rho + \gamma}{\beta - \lambda} - \frac{1}{\beta - \lambda} \frac{D_t}{P_t} \frac{1 + i_{LR,t}}{1 + i_{1,t}} (1 + n_{1,t+1|t}), \quad (2.9)$$

$$\pi_{LR|t} = i_{LR,t} - \rho - \frac{\lambda(\rho + \gamma)}{\beta - \lambda} + \frac{\lambda}{\beta - \lambda} \frac{D_t}{P_t} \frac{1 + i_{LR,t}}{1 + i_{1,t}} (1 + n_{1,t+1|t}), \quad (2.10)$$

where symbol  $\gamma$  denotes a constant defined as  $\gamma = \omega - \alpha$ . Equations (2.9) and (2.10) express the expected long run real GDP growth and inflation in terms of 1) current observable variables:  $D_t$ ,  $P_t$ ,  $i_{LR,t}$ ,  $i_{1,t}$ , 2) parameters:  $\lambda$ ,  $\rho$ ,  $\gamma$ , and 3) expected one-year-ahead growth in nominal dividends  $n_{1,t+1|t}$ .

In the next section we deal with the near term expectations  $n_{1,t+1|t}$ . After that, we set values for the parameters  $\lambda$ ,  $\rho$ ,  $\gamma$  and  $\beta$ . Then we are ready to use equations (2.9) and (2.10) empirically to extract market expectations for the euro area and for the United States.

## 2.3 Measuring near future dividend growth expectations

The framework presented in this paper is constructed in order to extract market expectations concerning future long run developments. Sometimes when the dividend discount model is utilized in extracting market expectations, the expected dividend growth rate is assumed to be constant in the future. Expectations derived in such a way reflect, to a large extent, short run expectations. This is because short run expectations seem to vary more than long run expectations, and because they have a larger weight in the dividend discount model, because of the discounting. Therefore, we deal with long and short run expectations separately, as shown in Section 2.1.

Regarding expectations concerning near term stock returns, one way to proceed would be to use stock analysts' bottom-up predictions. However, in the current context this approach would have an obvious drawback: the predictions are not available on a real time basis. In addition, such predictions are known to have a significant upward bias. The approach chosen here is to use the information contained in equity index futures quotations.

### 2.3.1 Expectations and equity index futures: the idea

Near future dividend growth expectations can be extracted from financial market data using the principle of equation (2.1) and the prices of equity index futures. They are financial derivatives, whose underlying assets are equity indices. The value of an equity index future reflects the market expectations

concerning the value of the index in the future as well as expectations concerning dividends paid out before the future matures.

Let us start by stating that

$$\frac{D_{t+1|t} + F_{t+1,t}}{P_t} = 1 + i_{1,t} + \omega_{D1}, \quad (2.11)$$

where  $F_{t+1,t}$  denotes the end-of-period- $t$  market delivery price for an equity index future concerning a contract maturing at the end of period  $t+1$ .<sup>1</sup> Symbol  $\omega_{D1}$  denotes a risk premium.

The left hand side of equation (2.11) is the expected gross return from an investment strategy where one buys equities underlying the index in period  $t$  and sells them in period  $t + 1$  for a price set in a futures contract made in period  $t$ . In practice this means holding the stocks for one period and hedging against stock price movements by selling short equity index futures in period  $t$ . The expected return from this strategy must equal the right hand side of the equation, that is, one plus the risk free interest rate plus the risk premium  $\omega_{D1}$  related to the uncertainty concerning  $D_{t+1}$  as of time  $t$ . This risk premium is related to but not equal to  $\omega$  of equation (2.1). It is likely to be very small and can be safely ignored in the following calculations.

Equation (2.11) shows that the difference between the current equity index value  $P_t$  and the futures contract price  $F_{t+1,t}$  reflects two things: expected next period dividends and the discount rate. The larger are the expected next period dividends, the smaller is the futures contract price, other things being equal. This reflects the fact that next period dividends will be paid out before the futures contract is settled, and paying out dividends decreases the value of a firm, *ceteris paribus*. Equation (2.11) naturally holds only for equity indices which are not adjusted for cash dividends, that is, those that are not so called total return indices. Most widely-used equity indices are not total return indices, including the ones used in this study.

Using the notation  $1 + n_{1,t+1|t} = D_{t+1|t}/D_t$  and ignoring the risk premium, equation (2.11) can be rewritten as

$$n_{1,t+1|t} = \frac{P_t}{D_t} \left( 1 + i_{1,t} - \frac{F_{t+1,t}}{P_t} \right) - 1. \quad (2.12)$$

Equation (2.12) shows that one can infer expected one period nominal dividend growth  $n_{1,t+1|t}$  from the values of  $F_{t+1,t}$ ,  $P_t$ ,  $D_t$  and  $i_{1,t}$ , all of which are observable at the end of period  $t$ .

### 2.3.2 Expectations and equity index futures: a detailed account

In practice, equity index futures exist only for certain maturity dates. Rather than using equation (2.12) empirically as such, it is simpler to use data on equity index futures with different maturities. In this section, we use daily frequency in time notations. In the empirical analysis we will use data

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<sup>1</sup>Here, we use an index future for which the delivery price is quoted in terms of the value of the index itself.

from money markets which have adopted the so called actual/360 method for interest rate calculations, also known as the 365/360 method. Therefore, using daily frequency and annualized interest rates, equation (2.11) must be written as

$$P_t = \frac{F_{T,t} + D_{(t,T)|t}}{1 + \frac{T-t}{360} \cdot i_{T-t,t}}, \quad (2.13)$$

where  $T$  denotes the maturity date of the future, and  $D_{(t,T)|t}$  denotes the day  $t$  expectation of the day  $T$  value of dividends that will be paid during the days  $t + 1, t + 2, \dots, T$ . The maturity of the relevant interest rate is now  $T - t$  days. When equation (2.13) is written for two different future dates  $T = T_1$  and  $T = T_2$ , combining these two equations by eliminating  $P_t$  yields

$$\frac{D_{(t,T_2)|t}}{1 + \frac{T_2-t}{360} \cdot i_{T_2-t,t}} - \frac{D_{(t,T_1)|t}}{1 + \frac{T_1-t}{360} \cdot i_{T_1-t,t}} = \frac{F_{T_1,t}}{1 + \frac{T_1-t}{360} \cdot i_{T_1-t,t}} - \frac{F_{T_2,t}}{1 + \frac{T_2-t}{360} \cdot i_{T_2-t,t}}. \quad (2.14)$$

Equation (2.14) can be rewritten to express the expected dividend stream as

$$D_{(T_1,T_2)|t} = \frac{1 + \frac{T_2-t}{360} \cdot i_{T_2-t,t}}{1 + \frac{T_1-t}{360} \cdot i_{T_1-t,t}} F_{T_1,t} - F_{T_2,t}, \quad (2.15)$$

where  $D_{(T_1,T_2)|t}$  denotes the day  $t$  expectation of the day  $T_2$  value of the dividends that will be paid out during the days  $T_1 + 1, T_1 + 2, \dots, T_2$ .

We approximate the near term annual expected dividend growth rate  $n_{1,t+1|t}$  by

$$n_{1,t+1|t} = \frac{D_{(T_1,T_2)|t}}{D_{(T_1-365,T_2-365)}} - 1, \quad (2.16)$$

where the dividends in the denominator are already observed on day  $t$ . Equation (2.16) shows how the near future dividend growth expectations can be extracted from the prices of equity index futures and money market interest rates.<sup>2</sup> In practice, the equity index futures that are used mature close to the end of each quarter.<sup>3</sup>

Based on equation (2.16), Figures 1 and 2 show the expected near term nominal dividend growth rates for two equity indices. The Standard & Poor's 500 index represents US stocks, and the Dow Jones Euro Stoxx 50 respectively represents euro area stocks. The series depicted in Figures 1 and 2 result from using equity index futures such that the growth rate given by the right hand side of equation (2.16) refers to the one that is expected to take place about half a year into the future.

To be exact, for the S&P 500 index, we use prices for the futures contract that is the next one to mature and for the contract that is the fourth one to mature. This means that we are measuring market expectations concerning

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<sup>2</sup>In the case of a leap year the figure 365 is replaced by 366 in the subscript of the denominator.

<sup>3</sup>They mature on the third Friday of the last month of each quarter.

the dividends that will be paid out during the next three full calendar quarters, approximately. Respectively, for the DJ Euro Stoxx 50 index, we deal with dividends during the next two full calendar quarters, approximately. In the case of DJ Euro Stoxx 50 the futures that mature further in the future either do not exist, and in the case of S&P 500 they have a shorter history. As the empirical counterparts for the interest rates that appear in equation (2.15) we use money market interest rates, linearly interpolated for different maturities when necessary.

The  $n_{1,t+1|t}$  series shown in Figures 1 and 2 are not given directly by equation (2.16). Two modifications are made to the series. First, to smooth out what seems to be noise, we use moving averages: past 60 days moving average for the S&P 500 and past 30 days for the DJ Euro Stoxx 50. In addition, there seem to be some premia affecting the futures prices or some other institutional factors unaccounted for, so that the variances of the series given by equation (2.16) are implausibly large. Therefore, we regress realized ex-post dividend growth series on the series given by equation (2.16) and use the fitted values from those linear regressions as the  $n_{1,t+1|t}$  series shown in Figures 1 and 2.<sup>4</sup>

## 2.4 Parameter values

Now we are almost ready to use equations (2.9) and (2.10) in extracting expectations. What remains to be done is to set the values for the parameters  $\lambda$ ,  $\rho$ ,  $\gamma$  and  $\beta$ . In doing that, we make use of published macroeconomic forecasts. In the case of the parameter  $\lambda$ , we also make use of restrictions stemming from economic theory. The financial market data used here are daily series provided by Bloomberg.

For the US, we start by setting  $\beta$  so that it makes equal the standard deviations of the two sides of equation (2.5). We use the right hand side of equation (2.4) as the empirical counterpart of  $n_{LR|t}$ . The series for the long term interest rate  $i_{LR,t}$  is calculated solving equation (2.3) for it with  $j = 10$ , with  $i_{10,t}$  being the 10 year government bond yield and  $i_{1,t}$  the 12 month money market interest rate.<sup>5</sup> Here, for  $\pi_{LR|t}$  and  $g_{LR|t}$  we use the inflation and real GDP growth forecasts by the Congressional Budget Office (CBO). We use the forecasts for 3–5 years ahead, so that they do not include the forecasts for the first two years. The standard deviations of the two sides of equation (2.5) are then calculated for the period 1991 through 2002, as the data on equity index futures starts in 1991. For  $n_{LR|t}$  we use end-of-the-year values, since CBO publishes its forecasts close to the end of the year. The resulting  $\beta$  equals 1.98.

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<sup>4</sup>All available data is used in the estimations. For the SP 500 the estimation period extends from 1991Q3 through 2002Q3, and for the DJ Euro Stoxx 50 from 1999Q1 through 2002Q3. The data for the explanatory variable are mid-quarter values. We use ordinary least squares. The slope estimates are greater than zero, as expected, and in the case of S&P 500 the estimate is statistically significant.

<sup>5</sup>Here, we make the approximation that  $\omega = 0$ .



Once the value of  $\beta$  is set, we obtain the rest of the parameter values for the US by estimating equations (2.9) and (2.10) as a system with parameter restrictions. As left hand side variables, we use the CBO long run forecasts described above. As such, the statistical model of the forecasts is of no interest to us. The purpose of the estimation exercise, loosely speaking, is to set the values for the parameters so that equations (2.9) and (2.10) produce expectation series with averages and variances in line with the published forecasts that are used as benchmarks here. The data set used in the estimation extends from 1991 through 2002 in annual frequency, and again we use end-of-the-year values for the financial market variables.

We estimate the system using the method of maximum likelihood, with the assumption that the error terms are normally distributed, assuming the parameter restrictions given below. The estimation is performed numerically. The parameter restrictions include a restriction of having the same values for  $\lambda$ ,  $\rho$ , and  $\gamma$  in both equations. In addition, we restrict the value of  $\lambda$  to be greater than or equal to 0.5. This restriction for the elasticity of intertemporal substitution is based on macroeconomic literature. Unrestricted, the estimate of  $\lambda$  would be lower and thus inconsistent with the theoretical starting point of the framework. The restriction for  $\lambda$  turns out to be binding, and the estimate of  $\lambda$  is thus 0.5. The estimate of  $\rho$  equals 0.022, and the estimate of  $\gamma$  is 0.0375. The fitted values of the equations are depicted in Figures 3 and 4.

For the euro area, data on futures prices are available only since the beginning of 1999. The value of  $\beta$  is set as for the US, now based on the forecasts reported by Consensus Economics Inc. We use the forecasts 3–7 years ahead, so that they do not include the forecasts for the first two years. The forecasts for euro area averages are approximated by weighted averages of the five largest euro area countries. The inflation forecasts are calculated from the real and nominal GDP growth forecasts. The long run Consensus Forecasts are published August each year. Therefore, we use end-of-July values for  $n_{LR|t}$ . The resulting  $\beta$  parameter for the euro area is 4.98.

With the data series for the euro area being very short, we do not estimate equations (2.9) and (2.10) for the euro area. Rather, we set  $\lambda$  at 0.5, following the US value. The values for  $\rho$  and  $\gamma$  are then calculated using equations (2.9) and (2.10) with  $g_{LR}$ ,  $\pi_{LR|t}$ ,  $i_{LR,t}$ , and  $\frac{D_t}{P_t} \frac{1+i_{LR,t}}{1+i_{1,t}} (1 + n_{1,t+1|t})$  set at their average values during 1999–2002. For  $g_{LR|t}$  and  $\pi_{LR|t}$  we use the Consensus Forecasts described above. The resulting values for  $\rho$  and  $\gamma$  are, respectively, 0.0214 and  $\gamma = 0.103$ .

### 3 Results: long run expectations

The long run expectations given by the framework are presented in Figures 5 through 12. That is, these figures show the  $g_{LR|t}$  and  $\pi_{LR|t}$  series given by equations (2.9) and (2.10), using the parameter values described in the previous section. The series are derived from daily observations of equity indices, dividends, interest rate and equity index futures. The data are provided by

Bloomberg. The last data points in the figures refer to 30 December 2003 for the US, and to 20 December 2003 for the euro area.

Figures 5 and 7 present the expected long run inflation and GDP growth rates for the United States. According to the results, the markets' long run inflation expectations have been on a declining trend since 1991. This is not surprising, since in the 1990s the inflation rate slowed considerably in the United States, as seen in Figure 7. The credibility of US monetary policy regarding price stability seems to have increased since the early 1990s. Figure 8 shows that inflation expectations evolve to some extent similarly to long term inflation forecasts and to a break-even inflation rate derived from an inflation-indexed bond. Recently, however, the level of inflation expectation has been very low. In terms of the framework, this mainly reflects the current low level of long term interest rates.

While inflation expectations were lowered in the 1990s, the expected long run GDP growth rate increased during the same time, according to the results. This is in line with the accelerated productivity growth in the US, and with the upward revisions in growth forecasts, such as those presented in Figure 6. The strong upward movement in the expected real GDP growth implied by the framework largely reflects the increases in stock prices seen in the late 1990s. Similarly, the recent fall in stock prices implies a fall in the growth expectation series. The turning point toward lower growth expectations is earlier than the corresponding turning point in the forecasts shown in Figure 6.

The results for the euro area are presented in Figures 9 through 12. The results cover only the period since 1999, because the data for the equity index futures is unavailable before that. Similarly to the US, euro area growth and inflation expectations have diminished somewhat since the year 2000. Again, Figure 10 shows that the turn for the worse in the growth expectation series takes place earlier than in the published growth forecasts.

In order to consider longer time series for the euro area, we consider a version of the framework that differs from the one presented in the preceding sections. Figure 13 depicts growth and inflation expectations derived under the assumption that growth and inflation rates are expected to be constant in the future. That is, the series shown in Figure 13 are derived making the assumptions that  $n_{1,t+1|t} = n_{LR|t}$  in equation (2.2), and  $i_{1,t} = i_{LR,t}$  in equation (2.3). Since near term expectations are not treated explicitly in this version of the framework, data on equity index futures are not required. Therefore, the results can be derived for a longer time span than in the case presented in Figures 9 through 12. According to Figure 13, changes in growth and inflation expectations in the euro area since the early 1990s have been to some extent similar to those in the US. However, changes in growth expectations have been somewhat smaller in the euro area, and the drop in inflation expectations has been greater.

## 4 Conclusions

This study presents a framework for measuring financial markets' expectations concerning long run real GDP growth and inflation. The framework is based on economic theory, and it uses as inputs data on equity indices, dividends, interest rates, and equity index futures prices. Using the framework, market expectations can be measured in real time.

Obviously, it is impossible to determine what are 'the true market expectations'. First, there is no unique set of market expectations, in the sense that the expectations of individual market participants differ from each other. What is being measured in all attempts to gauge market expectations is some sort of a noisy weighted average of the individual market participants' expectations. Second, that weighted average can only be observed with limited accuracy: We do not know which of the different measures presented in Figures 6, 8, 10 and 12 are closest to the 'truth'.

To some extent, the measures of market expectations produced by the framework presented here are similar to some other measures and published forecasts. However, in some instances this is not the case. For example, our measures of US growth expectations differ from other measures in the early 1990s and again during the years 2002 and 2003, as shown in Figure 6. During the latter period the same is true for US inflation expectations, as shown in Figure 8.

Regarding recent developments, one can speculate whether the results of this framework imply that the growth forecasts of Figure 6 will be revised further down in the near future, following the downturn in the expectation series. After all, the level of stock prices compared with past dividends has settled at a level considerably lower than that which prevailed in the late 1990s. Generally, the measures of growth expectations presented here have been quite strongly influenced by changes in stock prices.

The recent low inflation expectations shown in Figure 8, in turn, reflect low long term nominal interest rates. This suggests that the fall in interest rates during the recent years has been large even relative to the fall in stock prices. This interpretation is based on the fact that our measure of inflation expectations is affected by both nominal interest rates and stock prices as shown in equation (2.10): stock prices have not fallen enough to counteract the effect of the fall of interest rates. If this result is taken seriously, then one of the following must be true: 1) the market currently expects long term inflation to be lower than the published forecasts indicate, or 2) stock prices are still too high compared with expectations concerning the macroeconomy, or 3) inflation expectations are not, for some reason, fully priced in long term interest rates.

The framework presented here is new and experimental. In addition to the discussion above, one way to interpret the differences between the measures of expectations is that the framework is flawed in one way or another. Naturally, one can identify some potential problems with the approach. One is that equation (2.5), presenting the relationship between the expected GDP and dividend growth rates, may not hold empirically. It is difficult to assess how severely this equation may be misspecified.

In addition, it is possible that some other parameters of the model framework are not stable. For example, one might think that the assumption of a constant equity premium does not hold, even though this assumption is often made in applied work. In this respect, one way to try improve the framework presented here would be to consider modelling the variation in the equity premium. Further, it is possible that the relationship between the long term real interest rate and the expected long run real GDP growth is not stable. Finally, international linkages in the bond and equity markets have not been taken into account in the framework. For example, the real interest rates in the US and in the euro area surely reflect developments also in other parts of the world. Dealing explicitly with such international linkages would be another way to possibly improve this framework in future work.

While the framework presented here provides measures of market expectations, it does not attempt to determine whether the expectations later turn out to be correct or not. Therefore, we not need to take a stand when it comes to the question whether there are bubbles in financial markets. We simply interpret market prices to reflect market expectations. However, if bubbles exist, they can be problematic for the method we use. This is because we assume that similar growth and inflation expectations are reflected in both stock prices and bond prices. When talking about bubbles, macroeconomists often have stock prices in mind more than interest rates. After all, there is strong evidence that the value of the S&P 500 index, for example, has tended to vary too much with respect to the subsequent changes in dividends. Large swings in the dividend-price ratio have been followed by large movements in stock prices and not in dividends, as documented by Campbell and Shiller (2001), among others. In addition to stock prices, however, it is naturally possible to argue that there are bubbles in bond prices as well. For example, some economists explained the very low level of long term interest rates in mid-2003 in terms of a bond market bubble.

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### S&P 500: Short run growth in nominal dividends

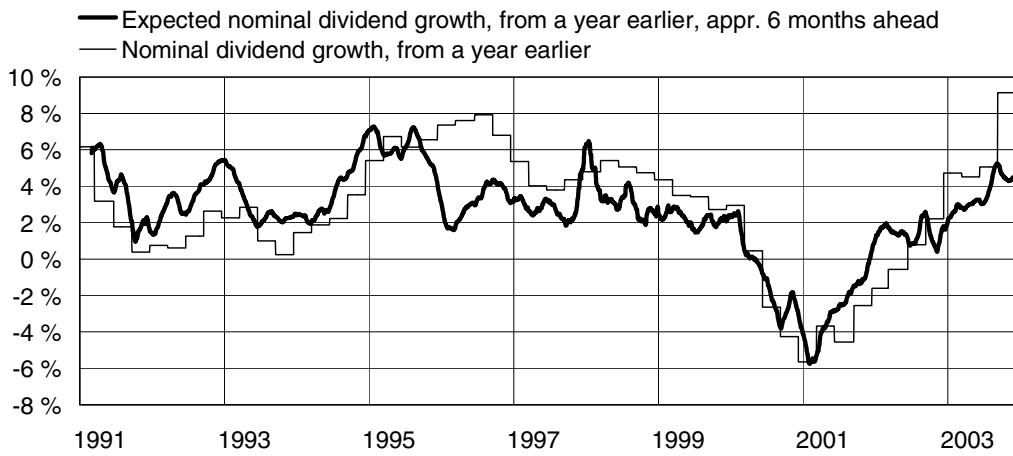


Figure 1:

### DJ Euro Stoxx 50: Short run growth in nominal dividends

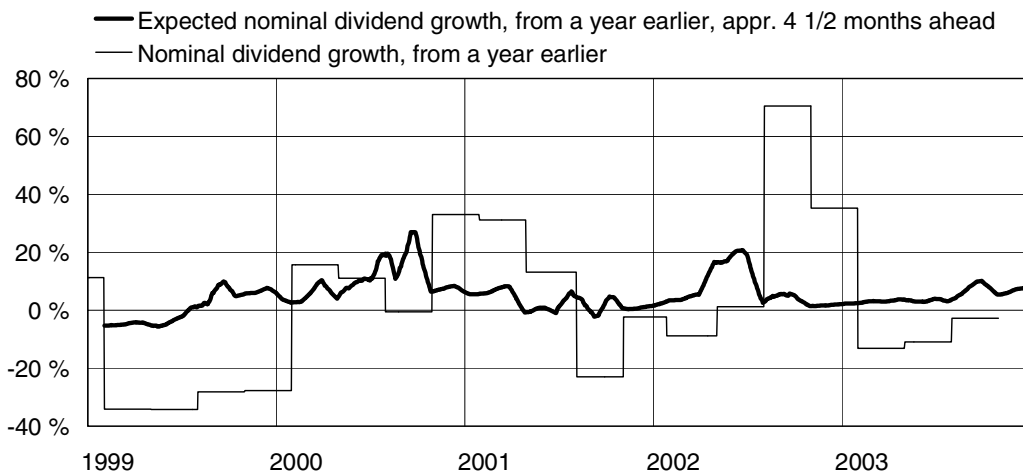


Figure 2:

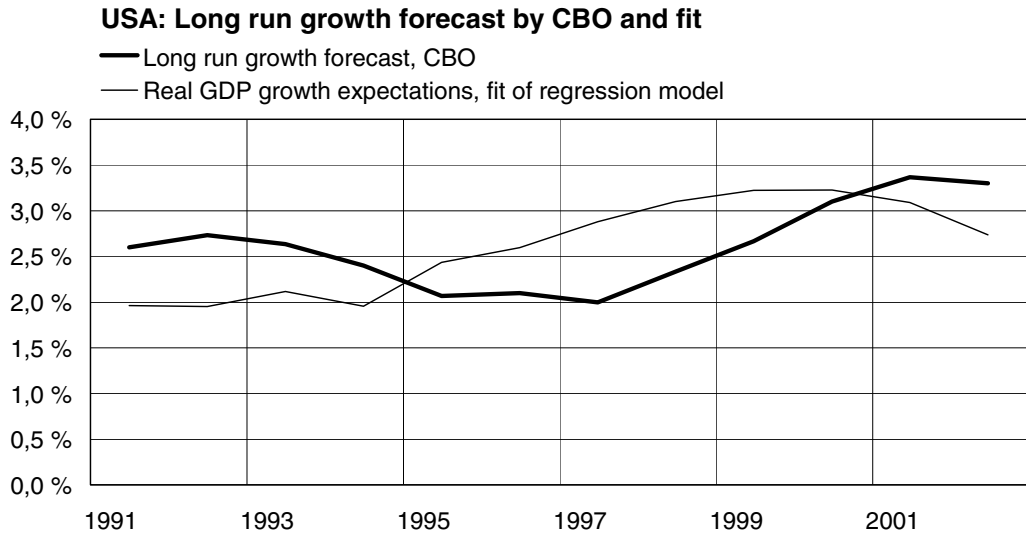


Figure 3:

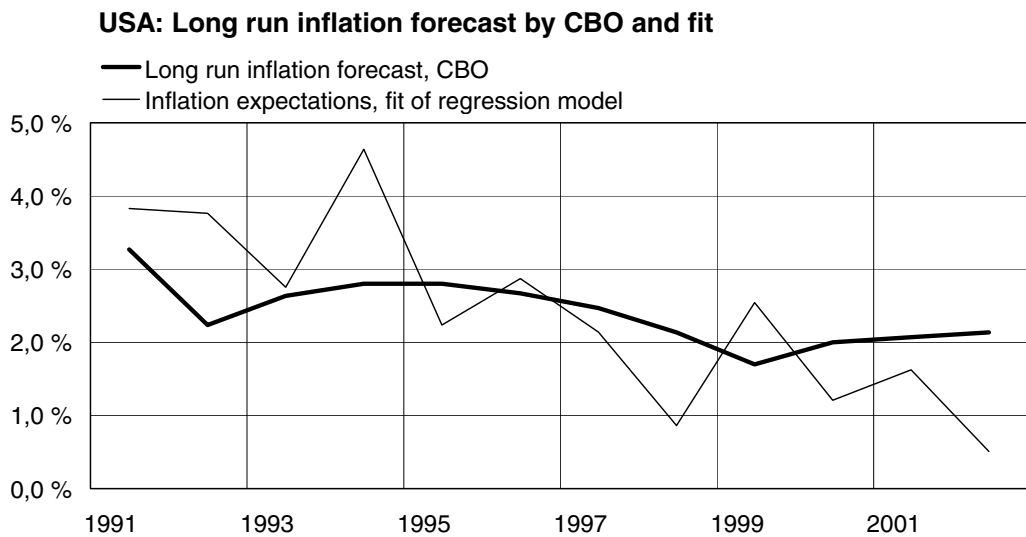


Figure 4:

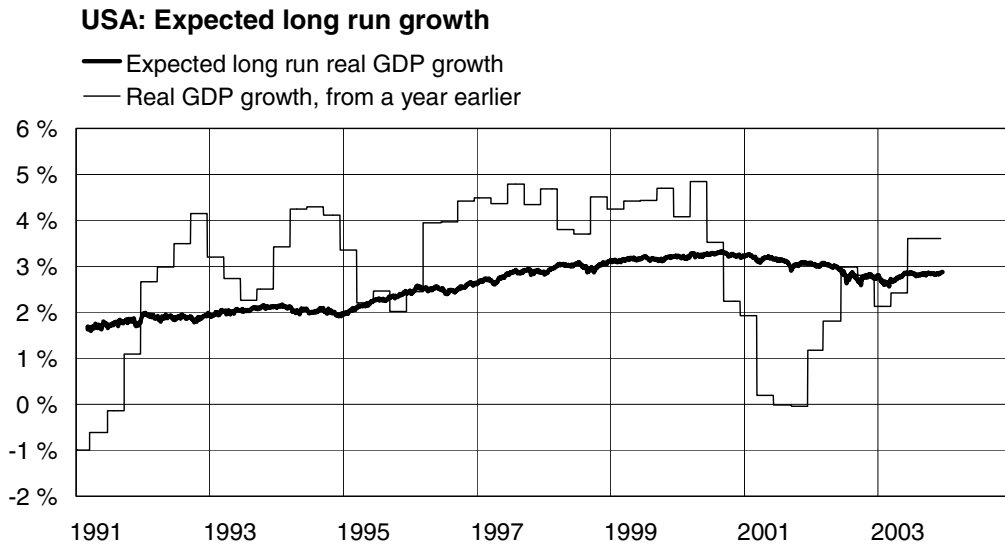


Figure 5:

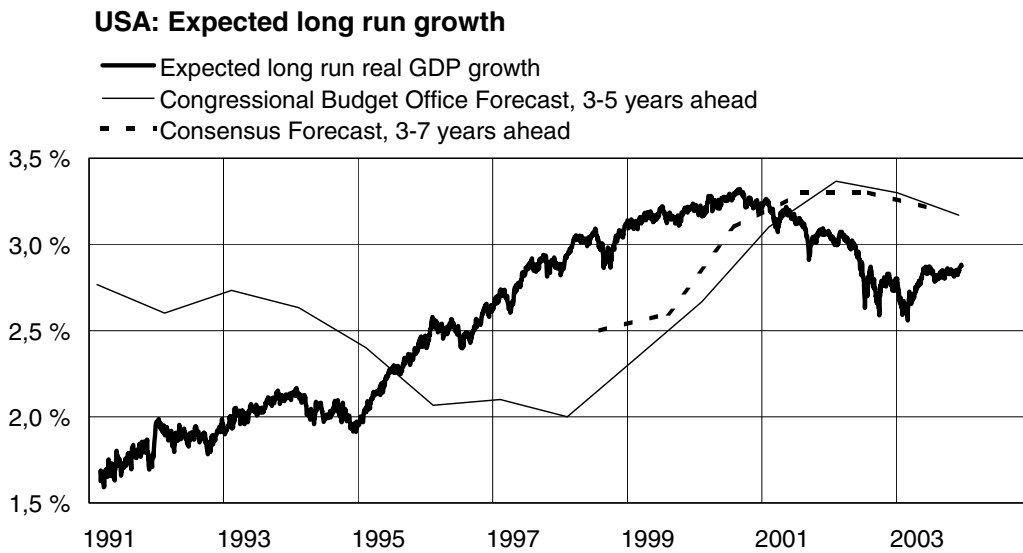


Figure 6:



**USA: Expected long run inflation**

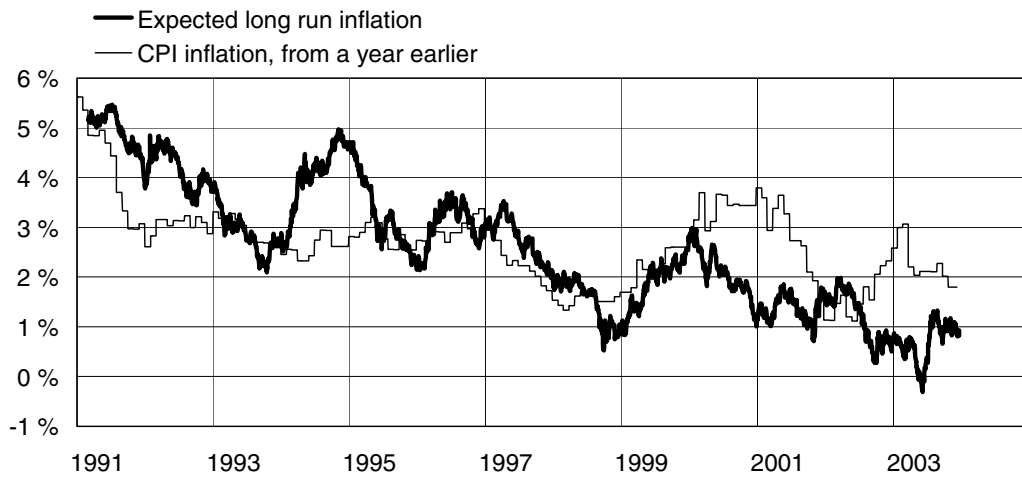


Figure 7:

**USA: Expected long run inflation**

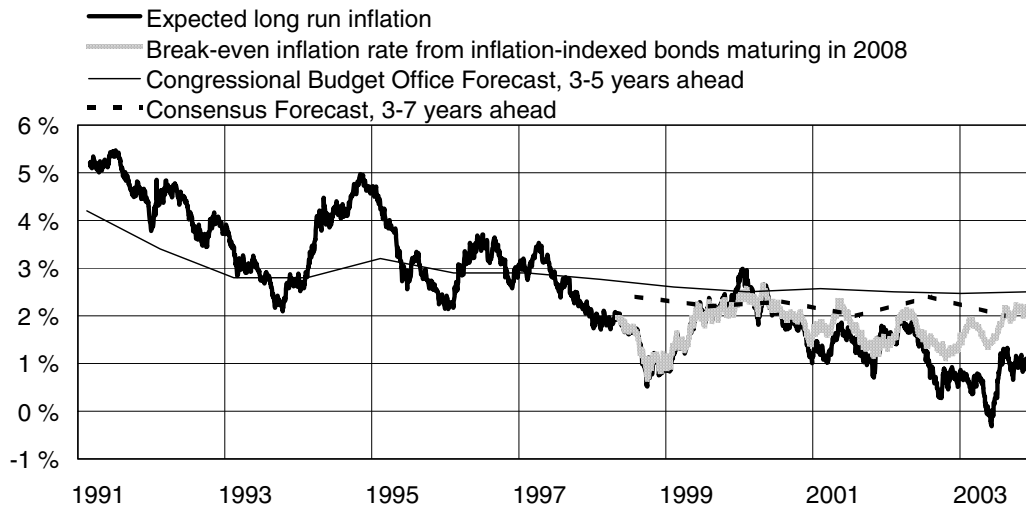


Figure 8:

### Euro area: Expected long run growth

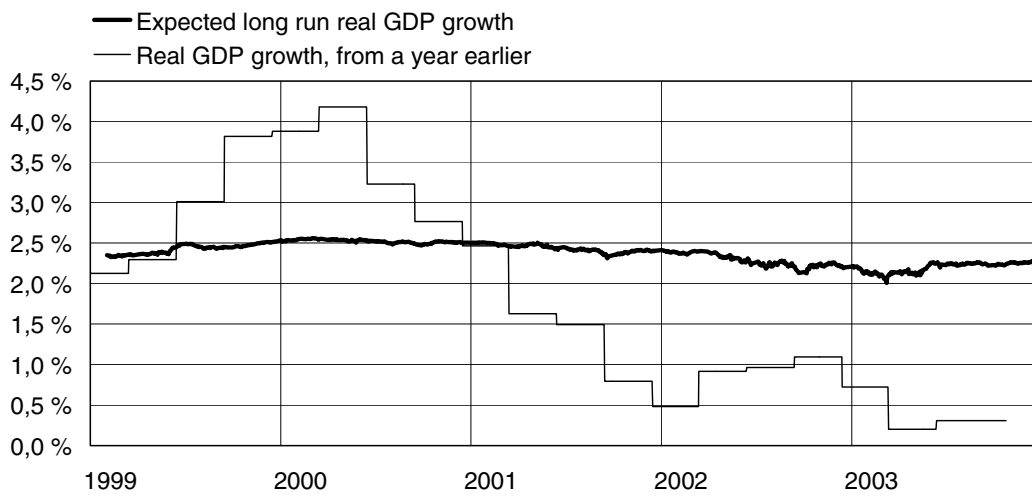


Figure 9:

### Euro area: Expected long run growth



Figure 10:

### Euro area: Expected long run inflation

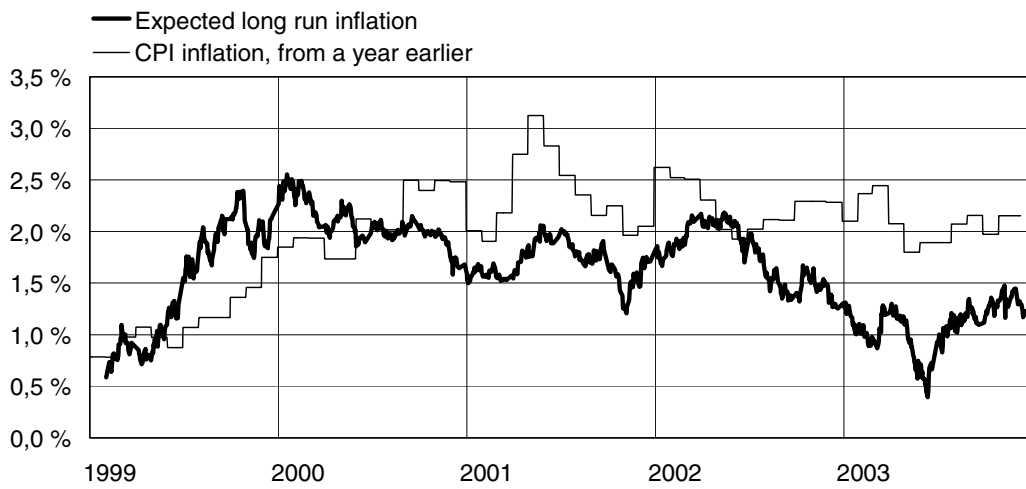


Figure 11:

### Euro area: Expected long run inflation

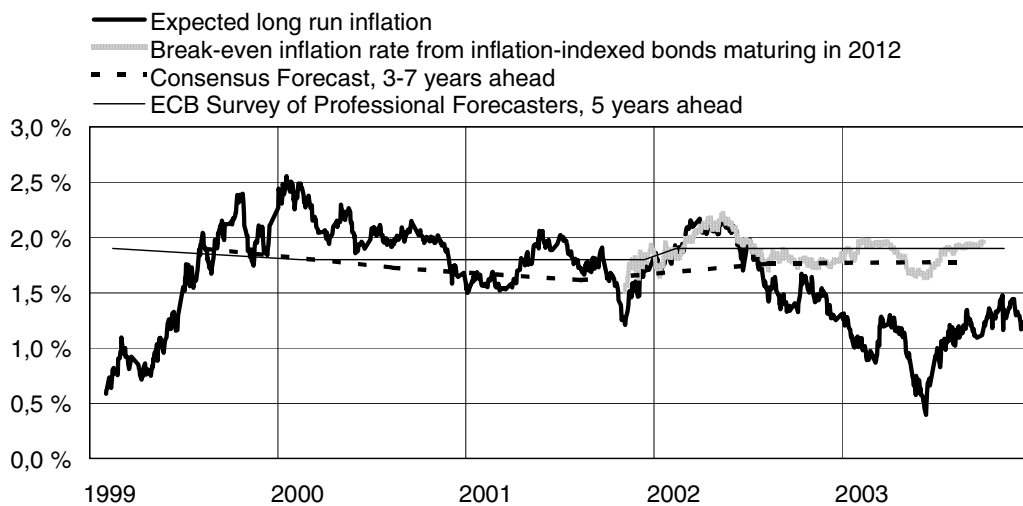


Figure 12:

**Euro area: Long run expectations, assuming constant expected rates of growth and inflation**

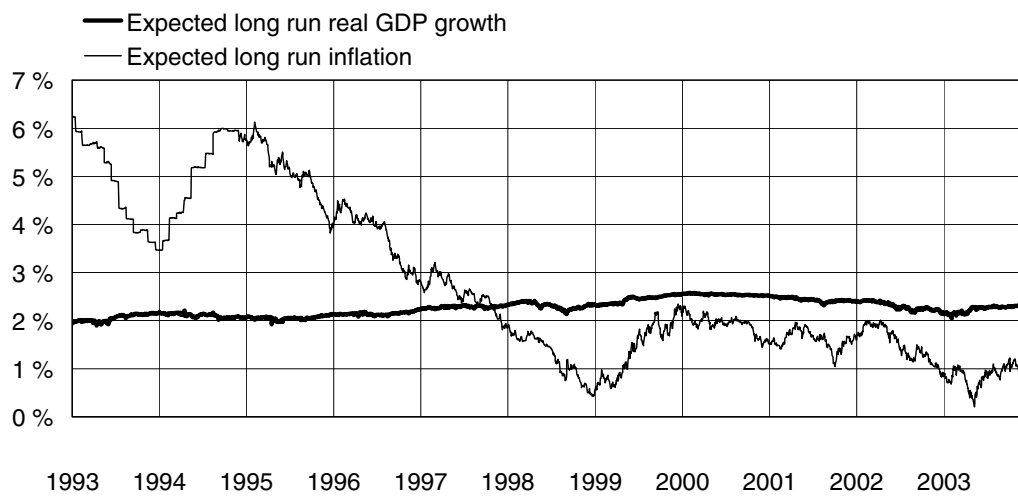


Figure 13:

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