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## The rigidity bias

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31 • 2003

Risto Herrala  
Economics Department  
12.11.2003

## The rigidity bias

Suomen Pankin keskustelualoitteita  
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The views expressed are those of the author and do not necessarily reflect the views of the Bank of Finland.

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# The rigidity bias

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## Abstract

We study the basic economic problem of choice between long-term and short-term commitments under a general characterization of uncertainty (aggregate uncertainty). When contingencies are contractible, a perfect market of Arrow-Debreu contingent claims implements the social optimum. When contingencies are not contractible, long-term commitments receive too much weight in individual portfolios. The economy as a whole is too rigid during periods of high aggregate shocks. The model links a rigidity bias with the operation of the price mechanism and the monetary system.

Key words: liquidity, central banking, monetary system

JEL classification numbers: G0, E0

# Talouden liiallinen jäykkyys

Suomen Pankin keskustelualoitteita 31/2003

Risto Herrala  
Kansantalousosasto

## Tiivistelmä

Tutkimuksessa tarkastellaan pitkien ja lyhyiden sijoitusten välistä valintatilannetta yleisen epävarmuuden vallitessa. Talous saavuttaa sosiaalisen hyvinvoinnin kannalta parhaan mahdollisen tasapainon, mikäli Arrow'n-Debreun futuurimarkkinat toimivat täydellisesti. Mikäli futuurimarkkinat toimivat epätäydellisesti, talouden toimijat suosivat sosiaalisen hyvinvoinnin kannalta liian suuressa määrin maturiteetiltaan pitkiä sijoituksia. Tästä seuraa, että kansantalous on liian jäykkä kohdatessaan voimakkaita sokkeja. Talouden liika jäykkyys liittyy hintamekanismin sekä rahoitusjärjestelmän toimintaan.

Avainsanat: likviditeetti, keskuspankki, rahoitusjärjestelmä

JEL-luokittelu: G0, E0

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# 1 Introduction

Reconsider the basic economic problem of choice in between long-term and short-term commitments. An entrepreneur faces this problem when planning investment projects, a worker when considering terms of employment, and a banker/financier when planning the maturity composition of a financial portfolio. A defining characteristic of a long-term commitment is that it cannot be costlessly broken on short notice. It may be passed on to others, but someone needs to bear it.

Is it feasible to think that, in market economies, these choices are governed by the ‘invisible hand’ of the market to the best interests of the average man? In accordance with classical welfare economics, the optimal outcome can, indeed, be reached with a perfect Arrow-Debreu contingent claims market. We argue, however, that in the realistic scenario where a contingent claims market is not operational, people tend to place too much weight on long-term commitments at a cost to maneuverability of short-term contingencies. Too much weight on long-term commitments leads to excess ‘rigidity’ on aggregate, and too little ‘liquidity’, contemporaneously reallocable resources, when unforeseen contingencies arise.

The proof of this rigidity bias is that, for individuals, the desirability of any short-term asset depends on the expected price of liquid wealth at its maturity. Even if the spot market for liquid wealth is perfect, still the market clearing price of liquidity must at times fall short of the marginal benefit gained from its use. The reason is that aggregate demand is sometimes dampened by individual budget constraints. The expected market price of liquid wealth must, then, fall short of the expected marginal benefit of its use. The bias towards rigidity is an optimal response of individuals to this rational expectation.

The idea that the liquidity creation mechanism is biased from the social optimum, is not novel. Among the pioneering studies of liquidity creation are Bryant’s (1980) and Diamond and Dybvig’s (1983) studies of banking, Bhattacharya and Gale’s (1989) study on the operation of inter bank markets, and Holmström and Tirole’s (1998) study of the role of the public sector.<sup>1</sup> These and other contributions in the genre show how an imperfect contracting environment, brought about by non observable states and/or actions, leads to deviations from the ideal Arrow-Debreu contingent claims equilibrium of classical welfare economics. The resulting ‘second best’ equilibria correspond with observed phenomena in financial systems.

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<sup>1</sup> Other examples of this are Freixas and Parigi (1997) on payment systems, Allen and Gale (2001) on systemic risk, Diamond and Rajan (2001) on banking, Rochet and Vives (2002) on LOLR, and Caballero and Krishnamurthy (2001) on the role of sterilization in monetary policy. Kiyotaki and Moore (eg 1997 and 2001) utilise an alternative approach based on the assumption of constrained actions.

The intuition of why contracting issues are important for policy is that, in the absence of a perfect contracting environment, it matters who controls wealth. From this premise, it is possible to explain why depositors have powers of immediate withdrawal, even though these powers make financial systems prone to bank runs (See eg Diamond and Dybvig 1983 and Diamond and Rajan 2001). Also, it is possible to explain why a central banking arrangement may lead to a superior allocation of liquidity compared with an inter bank market (Bhattacharya and Gale 1987). Caballero and Krishnamurthy (2001) argue that the special ability of central banks to promote wealth transfers across borders is key to understanding the transmission mechanism of central bank sterilization operations. The approach holds promise of bridging the gap in between monetary policy theory, which traditionally pays no attention to financial systems issues, and central banking practise.

Deviations from the classical assumptions of perfect contractibility come at the cost of analytical complexity. For tractability, models in this genre are typically staged in a single good economy with three periods, the minimum to enable differentiation of short and long-term assets. Also the state space is truncated to reduce the number of possible contingencies. Much of the literature on liquidity creation pertains to the special case of aggregate certainty, or aggregate uncertainty in the very restricted sense that all agents experience the same contingency. While the assumptions about a three stage economy and a truncated state space may at first sight appear purely technical matters, they in effect reduce the challenges related to contracting in the models and thus, potentially, hide important policy issues.

Our contribution to the literature is to extend the state space to normal dimensions to allow for real aggregate uncertainty under non-contractibility of states. A problem with an extended state space comparable to the one used in this paper has been studied previously by Holmström and Tirole in their theory on asset prices (2001). In their model, states are contractible but asset returns are not fully pledgeable: the challenge of liquidity creation is less than full pledgeability of long-term investment returns. In contrast to Holmström and Tirole, we study the case of non contractible states. Surprisingly (at least to us), a study of this case suggests that contractibility problems lead to a rigidity bias in an economy, even if returns of the long-term technology are fully pledgeable.

In the following chapter, we at first study the allocation of wealth by a single agent in between long and short-term assets. The single agent is then replaced by a continuum of agents, whose situation is studied from the point of view of a social planner. The benevolent social planner's solution gives a benchmark for an analysis of the situation, where the actions of agents are governed by the invisible hand of the price mechanism. In the final chapters, we discuss the issues of forwards markets, a different class of target functions, and aggregate certainty.

We also present our views on the links of the model with the development of monetary systems, and the agenda for further research.

## 2 The liquidity problem

We proceed to solve a problem by a single agent concerning allocation of wealth in between short-term storage and long-term commitments. The relationship of our formulation with earlier work is discussed at the end of this chapter, once the basic concepts have been introduced.

There are three planning periods,  $t \in \{0,1,2\}$ , and the following order of events:

- At  $t=0$  one unit of perfectly divisible good is allocated in between long-term investment  $L$  and short-term storage  $Z$ .
- In between  $t=0$  and  $t=1$ , nature chooses a shock  $S$  from domain  $I = [0,1]$  with cumulative distribution  $F$  and p.d.  $f$ .
- At  $t=1$ , storage is allocated to intermediate consumption  $C_1$ .
- At  $t=2$ , return of long-term investment  $RL$  is allocated to final consumption  $C_2$ .

The target, ‘expected utility at  $t=0$ ’ is:

$$\int_I (C_1 + C_2 - B \max\{S - C_1, 0\}) f[S] dS, \quad (2.1)$$

where  $B$  is a parameter ‘penalty’. The two parameters of relevance, return from long-term investment and penalty, satisfy:

$$0 < R - 1 < B < 2R - 1 \quad (2.2)$$

The p.d. is differentiable and positive throughout its domain:

$$f[S] > 0 \quad \forall S \in I \quad (2.3)$$

The ‘linear kinked’ target implies that, while agents are risk neutral locally everywhere except in the vicinity of the kink, they are risk averse globally. The

shock imposes a jump in agents' marginal utility in consumption at the level of the shock.<sup>2</sup>

Define 'liquidity' as contemporaneously reallocable amount of the good. The periodic allocation decisions of the planner are constrained by liquidity (right side of the following equations) as follows:

$$\begin{aligned}
 (t = 0) \quad & L + Z = 1 \\
 (t = 1) \quad & C_1 = Z \quad \forall S \in I \\
 (t = 2) \quad & C_2 = RL \quad \forall S \in I
 \end{aligned} \tag{2.4}$$

All decision variables  $\{L, Z, C_1, C_2\}$  are defined on the nonnegative real axis. Solving  $L$  and  $C_2$  from the liquidity constraints, the problem becomes:

$$\begin{aligned}
 & \max_{Z, C_1} R - (R - 1)Z - B \int_I \max\{S - C_1, 0\} f[S] dS \\
 (i) \quad & 1 - Z \geq 0 \\
 (ii) \quad & Z - C_1 \geq 0 \quad \forall S \in I \\
 (iii) \quad & R - (R - 1)Z - C_1 \geq 0 \quad \forall S \in I \\
 (iv) \quad & Z, C_1 \geq 0
 \end{aligned} \tag{2.5}$$

Consider the solution to intermediate consumption first, given any shock realisation. Ignore constraints (i), (iii) and (iv) for now, it is verified later that the proposed solution does not violate them. Under parametrisation (2.2), the target is increasing in intermediate consumption below the shock, and horizontal above it.<sup>3</sup> So, the proposed solution (denoted by asterisks) for intermediate consumption is:

$$\forall S \in I: \begin{cases} \text{If } S < Z \text{ then } S \leq C_1^* \leq Z \\ \text{Else} & C_1^* = Z \end{cases} \tag{2.6}$$

Inserting these equilibrium values into (2.5), the planner's problem simplifies to:

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<sup>2</sup> The shock could be interpreted as the price of the cheapest consumption bundle that feeds a family, the price of the cheapest house, or the minimum outlay needed to finish a project etc.

<sup>3</sup> The target is not differentiable in  $C_1$  at the 'kink' ( $C_1 = S$ ) but one can formally derive the equilibrium conditions for intermediate consumption by utilising the Kuhn-Tucker method locally for the two areas separated by the kink.

$$\begin{aligned} & \text{Max}_Z R - (R - 1)Z - B \int_Z^1 (S - Z) f[S] dS \\ & \text{st} \\ & \text{(i) } 1 - Z \geq 0 \\ & \text{(ii) } Z \geq 0 \end{aligned} \tag{2.7}$$

This is a quasi concave program. The Kuhn-Tucker method gives:

$$F[Z^*] = \frac{1 + B - R}{B} \tag{2.8}$$

To complete the analysis, it is verified that expected utility is positive in equilibrium, and no constraints are violated by the proposed solution.<sup>4</sup>

This problem is a variant of the type of problem studied in the literature on liquidity, references of which are given in the introductory section. Among the common factors are the three periods, the single good, the two technologies, and that the planner receives relevant information after the initial allocation in between the alternative technologies has been fixed. The long-term commitment may be interpreted as a physical or a financial investment. The contingency (shock) could be an intermediate consumption or investment need, or a refinancing need of the long-term project (see also footnote 2).

The parameter space (2.2) is restricted to guarantee an intermediate solution for the initial choice of technology. If the penalty were below the long-term net return, then storage would be useless. The upper bound of penalty guarantees that there is an incentive for some long-term investment. The rate of return of long-term investment may be considered either stochastic or deterministic. Assumption (2.3) about the continuity of the distribution of contingencies allows the use of standard analytical methods to infer the solution to the choice of technology. However, in the analysis that follows, the smoothness of the shock distribution appears to play a deeper role than just analytical tractability. As regards the liquidity constraints (2.4), we abstract from the possibility of consumption at  $t=0$ , and intermediate storage at  $t=1$ , but these omissions merely economise on notation. Also, we could allow for partial (but not full) recovery of the long-term

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<sup>4</sup> In parameter space (2.2), the right hand side of (2.8) is strictly in between zero and one. By continuity of the shock distribution, the equilibrium value of initial reserves must also be in between zero and one. It is then verified, that the equilibrium value of initial reserves does not violate constraint (i) or (iv). By (2.6) it may then be verified that the equilibrium values of intermediate consumption do not either violate (i) or (iv). In parameter space (2.2), constraint (iii) cannot bind in equilibrium. In equilibrium, expected utility at  $t=0$  is  $R - B \int_{Z^*}^1 S f[S] dS$ . By domain of shocks, this is positive when  $2R - 1 - B > 0$ .

investment project at  $t=1$ . This would complicate some of the liquidity constraints, but it would not remove the rigidity bias which is given in result 3 below.

A variety of targets have been utilised in the literature. To our knowledge, (2.1) is a novelty. The implications of using a different target is discussed in chapter 8.

### 3 The macroeconomic planning problem

In this chapter, the single agent context is abandoned in favour of a continuum of individuals  $i$ ,  $i \in I$ , each with unit mass. Individual investment, reserves and consumption are denoted by  $l[i]$ ,  $z[i]$ ,  $c_t[i]$  respectively, and the shock of any individual is  $s[i]$ . Denote by  $s$  a vector of individual shocks (one shock for each agent), randomly chosen by nature from  $I \times I$ , with cumulative distribution  $F_s$  and p.d.  $f_s$ .<sup>5</sup>

This chapter considers the p.o.w. of a benevolent social planner: the analysis gives a benchmark for assessment of how the economy behaves in the absence of one. The benevolent social planner's objective is the expected average utility of agents:

$$\int_{I \times I} (c_1[i] + c_2[i] - B \max\{s[i] - c_1[i], 0\}) f_s[s] ds \quad (3.1)$$

For the benevolent social planner, liquidity constraints and non negativity conditions hold only at the aggregate level.

To facilitate comparison of this 'macroeconomic planning problem' with the liquidity problem, redefine  $L$ ,  $Z$  and  $C$  as average investment, reserves, and consumption, and  $S$  as the average shock:

$$\begin{aligned} L &\equiv \int_I l[i] di \\ Z &\equiv \int_I z[i] di \\ C_t &\equiv \int_I c_t[i] di \quad \forall t \in \{1, 2\} \\ S &\equiv \int_I s[i] di \quad \forall s \in I \times I \end{aligned} \quad (3.2)$$

Finally, redefine  $F$  as the cumulative probability distribution of the average shock. Denote by  $V$  a variable from the unit line, and by  $v$  an (infinite) vector of

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<sup>5</sup> The sample space has one dimension for agents and one for shocks.

variables, with  $v[i] \in I$  as the variable in position  $i$ . Assumption (2.3) about continuity of the probability density of the average shock may, then, be restated as follows:

$$f[S] \equiv \frac{\partial F[S]}{\partial S} \equiv \frac{\partial \left( \int_{v \leq S} f[v] dv \right)}{\partial S} = \frac{\partial \left( \int_{\int_i v[i] di \leq S} f_s[v] dv \right)}{\partial S} > 0 \quad \forall S \in I \quad (3.3)$$

The domain of the rightmost integral is shorthand for the set  $\left\{ v \in I \times I : \int_i v[i] di \leq S \right\}$ . This notation about sets is used also elsewhere in the paper.

Certain other smoothness assumptions about the joint probability distribution are also utilised for derivation of the results of this paper (These are needed in connection with the proofs only). Denote by  $i$  some given agent in the sample (to be considered a constant and not a variable in the calculations). The shock generating mechanism implies that the cumulative probability of idiosyncratic shocks of some agent  $i$  at some shock value  $x$  is  $\int_{v[i] \leq x} f_s[v] dv$ . It is interesting, that

the following assumption about continuity of conditional cumulative probabilities appears necessary for deriving the rigidity bias.

$$\frac{\partial \left( \int_{\int_i v[i] di = S} f_s[v] dv \right)}{\partial x} > 0 \quad \forall i, S \in I, x \in [0,1) \quad (3.4)$$

The domain of the integral is the intersection of the two sets. The shock generating mechanism also implies that the cumulative frequency of realised shocks at some  $x$  is  $\int_{s[i] \leq x} di$ . It is assumed that the cumulative shock frequency is

differentiable and increasing throughout its domain in all states:

$$\frac{\partial \left( \int_{s[i] \leq x} di \right)}{\partial x} > 0 \quad \forall 0 \leq x < 1, s \in I \times I \quad (3.5)$$



After these assumptions and definitions, we will now proceed to solve the benevolent social planner's problem. By utilising the liquidity constraints (2.4), and the definition of averages (3.2), the macroeconomic planning problem may be formulated as follows:

$$\begin{aligned} & \text{Max}_{z[i], c_1[i]} R - (R - 1)Z - B \int_{I \times I} \max\{s[i] - c_1[i], 0\} f_s[s] ds \\ & \text{st} \\ & \text{(i)} \quad 1 - Z \geq 0 \\ & \text{(ii)} \quad Z - C_1 \geq 0 \quad \forall s \in I \times I \\ & \text{(iii)} \quad R - (R - 1)Z - C_1 \geq 0 \quad \forall s \in I \times I \\ & \text{(iv)} \quad Z, C_1 \geq 0 \end{aligned} \tag{3.6}$$

The following observation may be used to infer the solution. In equilibrium, given any realisation of the shock vector, all agents need to consume on the same side of their shocks at  $t=1$ . Else, by marginal utilities, welfare can be increased by reallocating intermediate consumption from agents that consume more than their shock to agents that consume less than their shock. Formally:

$$\forall s \in I \times I: \begin{cases} \text{either} & c_1[i]^* \leq s[i] \quad \forall i \in I \\ \text{or} & c_1[i]^* \geq s[i] \quad \forall i \in I \end{cases} \tag{3.7}$$

Under (3.7) the following holds in all states:  $\int_I \max\{s[i] - c_1[i]^*, 0\} di = \max\{S - C_1^*, 0\}$ . But then the macroeconomic planning problem (3.6) is transformed into the liquidity problem (2.5), for which the solution is already known.

The solution to the macroeconomic planning problem is summarised in the following result 1. We use the support MP to indicate that the equilibrium values refer specifically to this problem.

**Result 1.** *The solution to the macroeconomic planning problem*

$$\text{a) } \forall s \in I \times I \begin{cases} \text{If } S < Z_0^{\text{MP}*} \text{ then } S \leq C_1^{\text{MP}*} \leq Z^{\text{MP}*} \text{ and } c_1^{\text{MP}*}[i]^* \geq s[i] \quad \forall i \in I \\ \text{Else} & C_1^{\text{MP}*} = Z^{\text{MP}*} \text{ and } c_1^{\text{MP}*}[i]^* \leq s[i] \quad \forall i \in I \end{cases}$$

$$\text{b) } F[Z^{\text{MP}*}] = \frac{1 + B - R}{B}$$

To comment, the move from a single agent setting into the macroeconomic context, is straightforward under the chosen target. The only additional consideration that arises in the macroeconomic context is that the allocation of liquidity at  $t=1$  across agents must accord with (3.7). The social planner abstracts from all other issues related to allocation of consumption across agents.

In a sense, the target is in this context consistent with the ‘equality of opportunity’ view of the ideal society. In the model, all agents start with the same endowment, but their proneness to shocks varies. The social planner neutralises the consequences of the effects of these ‘born with’ differences in idiosyncratic shock probabilities, by equalising intermediate marginal utilities in accordance with (3.7). However, the social planner is not interested the differences in the realised utility levels across agents.

This neglect of equality of outcomes makes the policy induced by the linear kinked target undemanding as regards implementation. We now proceed to study, whether this optimal policy may be implemented by a price mechanism.

## 4 Individual choices

In this chapter we study the situation of some given individual  $i$ . The boldface, again, indicates that individual is not variable, but some given individual in the sample. The following assumptions are made, part of which will be relaxed later:

Assumption 4.1 The agent does not at  $t=0$  make any commitments as regards liquidity allocation in  $t=1$ . This assumption is relaxed in chapter 7.

Assumption 4.2 There exists at  $t=1$  in (almost) all states a unit price of liquidity  $r \in [1, 1+B]$ . It ( $r$ ) is the amount the agent needs to pledge at  $t=1$  in period  $t=2$  liquidity to purchase one unit of period  $t=1$  liquidity. This assumption is replaced by an assumption about the price mechanism in chapter 5.

Assumption 4.3 The agent has rational expectations about idiosyncratic shocks and prices of liquidity. Accordingly, the agent’s target at  $t=0$  is expected utility across states:

$$\int_{s \in I} (c_1[i] + c_2[i] - B \max\{s[i] - c_1[i], 0\}) f_s[s] ds, \quad (4.1)$$

where  $i$  is considered fixed and not variable (this contrasts 4.1 from the aggregate target 3.1).

Assumption 4.4 Agents can fully commit at  $t=1$  to future delivery of liquidity. Accordingly, the full set of individual liquidity constraints is:

$$\begin{aligned}
\text{(i)} \quad & l[\mathbf{i}] + z[\mathbf{i}] = 1 \\
\text{(ii)} \quad & c_1[\mathbf{i}] \leq z[\mathbf{i}] + \frac{R}{r} l[\mathbf{i}] \quad \forall s \in I \times I \\
\text{(iii)} \quad & c_2[\mathbf{i}] = r(z[\mathbf{i}] - c_1[\mathbf{i}]) + Rl[\mathbf{i}] \quad \forall s \in I \times I
\end{aligned} \tag{4.2}$$

Non negativity of variables holds at individual level. With some manipulation, the agent's problem may be expressed as follows:

$$\begin{aligned}
& \text{Max}_{z[\mathbf{i}], c_1[\mathbf{i}]} \int_{I \times I} \{R - (R - r)z[\mathbf{i}] - (r - 1)c_1[\mathbf{i}] - B \max\{s[\mathbf{i}] - c_1[\mathbf{i}], 0\}\} f_s[s] ds \\
\text{(i)} \quad & 1 - z[\mathbf{i}] \geq 0 \\
\text{(ii)} \quad & c_1[\mathbf{i}] \leq z[\mathbf{i}] + \frac{R}{r}(1 - z[\mathbf{i}]) \quad \forall s \in I \times I \\
\text{(iii)} \quad & R - (R - r)z[\mathbf{i}] - c_1[\mathbf{i}] \geq 0 \quad \forall s \in I \times I \\
\text{(iv)} \quad & z[\mathbf{i}], c_1[\mathbf{i}] \geq 0
\end{aligned} \tag{4.3}$$

The solution proceeds like before, so that we omit the intermediate steps. The solution to intermediate consumption is:

$$\forall s \in I \times I: \begin{cases} r = 1 & \Rightarrow s[\mathbf{i}] \leq c_1[\mathbf{i}]^* \leq z[\mathbf{i}] + R(1 - z[\mathbf{i}]) \\ 1 < r < 1 + B & \Rightarrow c_1[\mathbf{i}]^* = \min\left\{s[\mathbf{i}], z[\mathbf{i}] + \frac{R}{r}(1 - z[\mathbf{i}])\right\} \\ r = 1 + B & \Rightarrow 0 \leq c_1[\mathbf{i}]^* \leq \min\left\{s[\mathbf{i}], z[\mathbf{i}] + \frac{R}{1 + B}(1 - z[\mathbf{i}])\right\} \end{cases} \tag{4.4}$$

Individual demand schedules for consumption (4.4) are non increasing, and each has a horizontal segment. Indeed, the schedule is horizontal throughout the domain of  $r$  in some states. This horizontal segment reflects the assumption that there is a threshold in between the marginal utilities related to the two uses of the good: The marginal benefit of consumption is unity above the shock, and penalty below it. Thus, whenever the price of liquidity is above unity and below the penalty, agents want to consume up to the shock if they can. The only limiting consideration is the liquidity constraint. Once the period  $t=1$  liquidity constraint binds, individual consumption becomes responsive to the price of liquidity, because an increase in the price of liquidity lowers the liquidity constraint.

Inserting this solution into the original problem, and solving for initial reserves, we get:

$$\left\{ \begin{array}{l} \text{if} \quad \int_{s[i] > \frac{R}{r}} \frac{(1+B-r)}{r} (r-R)f_s[s]ds + \int_{I \times I} (r-R)f_s[s]ds < 0 \text{ then } z[i]^* = 0 \\ \text{if} \quad \int_{I \times I} (r-R)f_s[s]ds > 0 \text{ then } z[i]^* = 1 \\ \text{else} \quad \int_{s[i] > z[i]^* + \frac{R}{r}(1-z[i]^*)} \frac{(1+B-r)}{r} (r-R)f_s[s]ds + \int_{I \times I} (r-R)f_s[s]ds = 0 \end{array} \right. \quad (4.5)$$

The integral is the expected net return of reserves (the expected penalty of uncovered shocks, and the opportunity cost of reserves). Individual behaviour may now be summarised:

**Result 2.** *Under assumptions 4.1–4.4:*

- a) Individual agents hoard reserves at  $t=0$  in accordance with (4.5).
- b) Individual agents consume at  $t=1$  in accordance with (4.4).  
(equations not repeated to save space).

## 5 The price mechanism

At  $t=1$ , all liquidity is allocated to intermediate consumption. The aggregate supply of liquidity for consumption at  $t=1$  is, then,  $Z$ . The aggregate demand of liquidity for consumption at  $t=1$  (AD) is an integral of individual consumption demands (4.4) across agents.

$$AD[r,s] \equiv \int_I c_1[i]^* di \quad (5.1)$$

Some properties of AD are stated in the following lemma for further reference.

**Lemma L1** about AD (L1.3 derived in appendix 1)

$$\mathbf{L1.1} \quad AD[r_0, s] \geq AD[r_1, s] \quad \forall 1 \leq r_0 < r_1 \leq 1 + B, s \in I \times I$$

$$\mathbf{L1.2} \quad AD[r, s] = S \quad \forall 1 < r \leq R, s \in I \times I$$

$$\mathbf{L1.3} \quad Z < 1 \Rightarrow \int_1^{\int_1^{v[i]di=S}} AD[r_1, v] f_s[v] dv < \int_1^{\int_1^{v[i]di=S}} AD[r_0, v] f_s[v] dv < S \\ \forall R < r_0 < r_1 < 1 + B, S \in (0, 1]$$

To understand L1, recall that the price of liquidity effects demand via the period 1 liquidity constraint. L1.1 and L1.2 follow directly from the definition of the constraint, and the domain of shocks, and their proofs are only given verbally. AD is non increasing in  $r$  throughout its domain (L1.1), because an increase in the price of liquidity either reduces individual liquidity constraints at  $t=1$  (wherever  $z[i] < 1$ ) or leaves them unaffected (wherever  $z[i] = 1$ ). In the interval  $1 \leq r \leq R$ , aggregate demand is horizontal (L1.2), because then individual liquidity constraints are at or above unity for all agents at  $t=1$ . By domain of shocks, individual liquidity constraints cannot, then, bind anyone. It is proven in appendix 1 that when average liquidity is below unity (L1.3), then by domains of shocks liquidity constraints must bind some agents in some states in the interval  $R < r \leq 1 + B$ .

Assumption (4.2) about the existence of  $r$  may now be replaced by the assumption that, in equilibrium, aggregate demand and supply of liquidity are equalised at the prevailing price of liquidity:

$$r^*: AD[r^*, s] = Z \quad \forall s \in I \times I \quad (5.2)$$

For illustration of the price mechanism (5.2), divide states into two groups as follows

$$\{s\}_{NCONS} = \left\{ s \in I \times I : \int_0^1 \min \left\{ s[i], z[i] + \frac{R}{r} (1 - z[i]) \right\} di = S \quad \forall 1 \leq r \leq 1 + B \right\} \quad (5.3)$$

$$\{s\}_{CONS} = \{s \in I \times I : s \notin \{s\}_{NCONS}\}$$

The support CONS is shorthand for ‘some agents are liquidity constrained’, and NCONS is ‘no one is liquidity constrained’. The following characterisation of prices may, then, be given:

$$\begin{aligned}
\forall s \in \{s\}_{\text{NCONS}} : & \begin{cases} S < Z \Rightarrow r^* = 1 \\ S = Z \Rightarrow r^* \in [1, 1+B] \\ S > Z \Rightarrow r^* = 1+B \end{cases} \\
\forall s \in \{s\}_{\text{CONS}} : & \begin{cases} S < Z \Rightarrow r^* = 1 \\ S = Z \Rightarrow r^* \in [1, 1+B) : \int_0^1 \min \left\{ s[i], z[i] + \frac{R}{r^*} (1-z[i]) \right\} di = S \\ S > Z > \int_0^1 \min \left\{ s[i], z[i] + \frac{R}{1+B} (1-z[i]) \right\} di \\ \Rightarrow r^* \in (R, 1+B) : \int_0^1 \min \left\{ s[i], z[i] + \frac{R}{r^*} (1-z[i]) \right\} di = Z \\ \int_0^1 \min \left\{ s[i], z[i] + \frac{R}{1+B} (1-z[i]) \right\} di \geq Z \geq 0 \\ \Rightarrow r^* = 1+B \end{cases} \tag{5.4}
\end{aligned}$$

It is observed from (5.4) that the price of liquidity is not uniquely determined in states characterised by  $S = Z$ . This problem may be considered insignificant as, by continuity of  $F$ , the probability that the average shock perfectly matches the level of aggregate reserves is nil.

The following Lemma concerns the average price of liquidity when  $Z < 1$ .

**Lemma L2** about expected price of liquidity in states where there is a shortage of it (proven in appendix 2)

$$Z < 1 \Rightarrow \int_{s>Z} r f_s[s] ds < (1+B)(1-F[Z])$$

Lemma L2 is important. It implies that the expected price of liquidity falls below the expected marginal benefit of liquidity in the economy. The proof is that when  $Z < 1$ , AD schedules are downwards sloping on average in the interval  $R < r < 1+B$ . In a continuum of states where the average shock is ‘not much above’ the average level of liquidity, the aggregate demand and supply schedules must meet at the downwards sloping part of the AD schedule. By the price mechanism, in these states,  $R < r^* < 1+B$ . From this result, it is a relatively short step to show that, from the social point of view, not enough period  $t=1$  liquidity is created in this economy.

## 6 Liquidity creation in equilibrium

An equilibrium is characterised by individual intermediate consumption equations (4.4), individual initial reserve equations (4.5), and prices (5.4). It is now shown that the equilibrium amount of initial reserves must be below the level preferred by the benevolent social planner given in Result 1. Below, this property of the equilibrium is first derived, and then stated formally.

As a first step of the derivation, the following lemma gives an upper bound for the expected net return of reserves in equilibrium.

**Lemma L3** an upper bound in the expected return of reserves (derived in appendix 3)

$Z < 1 \Rightarrow \forall i \in I$ :

$$\begin{aligned} & \int_{I \times I} (r^* - R) f_s[s] ds + \int_{s[i] > z[i]^* + \frac{R}{r^*}(1-z^*[i])} \frac{(1+B-r^*)}{r^*} (r^* - R) f_s[s] ds \\ & < \int_{S < Z} (1-R) f_s[s] ds + \int_{S > Z} (1+B-R) f_s[s] ds \end{aligned}$$

The following lemma states that, in a situation, where the average level of reserves hoarded at  $t=0$  is at or above the level preferred by the benevolent social planner, the expected return of initial reserves is negative for all agents.

**Lemma L4** about the expected return of reserves

$$Z^* \geq Z^{MP^*} \Rightarrow \forall i \in I: \int_{I \times I} (r^* - R) f_s[s] ds + \int_{s[i] > z[i]^* + \frac{R}{r^*}(1-z^*[i])} \frac{(1+B-r^*)}{r^*} (r^* - R) f_s[s] ds < 0$$

**Proof**

$$Z^* \geq Z^{MP^*}$$

$$\stackrel{R1b}{\Leftrightarrow} F[Z^*] \geq \frac{1+B-R}{B}$$

$$\stackrel{L3}{\Leftrightarrow} \forall i \in I:$$

$$\begin{aligned} & \int_{I \times I} (r^* - R) f_s[s] ds + \int_{s[i] > z[i]^* + \frac{R}{r^*}(1-z^*[i])} \frac{(1+B-r^*)}{r^*} (r^* - R) f_s[s] ds \\ & < \frac{(1+B-R)}{B} (1-R) + \frac{(R-1)}{B} (1+B-R) = 0 \end{aligned}$$

By definition of equilibrium (see 4.5), the net return of reserves cannot be negative for agents if they hoard a nonzero level of them. In conclusion, the average level of reserves cannot be at or above the solution to the macroeconomic planning problem in equilibrium. By an analogous argument, one can rule out zero reserves (then the expected return of liquidity would be  $1 + B$ ). It is, then, established that the range of feasible equilibrium values is at most  $0 < Z^* < Z^{MP*}$ .

**Result 3** *Under assumptions A4.1, A4.3 and A4.4, and the price mechanism (5.3), the range of feasible equilibrium values for reserves is at most:*

a)  $0 < Z^* < Z^{MP*}$ .

Result 3a) is the rigidity bias. To summarize, in this economy, agents will not hoard ‘enough’ reserves in equilibrium from the point of view of the social planner. Socially optimal behavior can be ruled out by the following argument. In some states, the price of consumable goods falls below the marginal utility of consumption for some individuals, because consumption demand is constrained by individual liquidity constraints. This makes the expected returns of reserves, if hoarded at the socially optimal level, negative.

Even if the socially optimal level of reserves were hoarded at  $t=0$ , equilibrium intermediate consumption would not necessarily be identical with the solution to the macroeconomic planning problem, because equilibrium consumption is not always uniquely determined by the price mechanism. The two are equalised on the following assumption:

$$S \leq \int_I \min \left\{ s[i], z[i] + \frac{R}{1+B} (1 - z[i]) \right\} di \Rightarrow C_1^* = Z_0; c_1[i]^* = \min \left\{ s[i], z[i] + \frac{R}{1+B} \right\} \quad (6.1)$$

## 7 Futures markets at $t=0$

Results 2 and 3 utilise assumption 4.1, that agents do not at  $t=0$  make commitments as regards future liquidity allocation, ie that futures markets are not active (possibly do not exist) at  $t=0$ . A market for ‘non state contingent’ claims at  $t=0$  cannot implement the socially optimal allocation, because implementation of 1a) requires information about shock realisations at the level of agents.

However, consider a situation where the realised vector of shocks  $s$  (the state) is freely observable at  $t=1$ , and agents can commit at  $t=0$  to state contingent date  $t=1$  liquidity transfers. Then, agents can set up a contingent claims market at  $t=0$  for period  $t=1$  liquidity. A contingent claim guarantees the claimant liquidity at  $t=1$  in state  $s \in I \times I$ , but not in other states.



Denote by  $p_{c_1}[s]$  the unit price of a state contingent claim that returns liquidity at  $t=1$  to the claimant if state  $s$  is realised, and zero in other states. By technology, the supply of liquidity at  $t=1$  is  $Z$  in all states, so that  $p_{c_1}[s]$  is determined by the following market clearing condition:

$$p_{c_1}[s]^* : \int_I c_1[i, s]^* di = Z^* \quad \forall s \in I \times I \quad (7.1)$$

At  $t=0$ , agents choose reserves and state contingent consumption by maximising expected utility (4.1), subject to the liquidity constraint:

$$\int_{I \times I} c_1[i, s] p_{c_1}[s] f_s[s] ds + c_2[i] = z[i] \int_{I \times I} p_{c_1}[s] f_s[s] ds + R(1 - z[i]) \quad (7.2)$$

Solving the agents' program, and inserting the solution to the market clearing condition (7.1) reveals that the equilibrium in this economy accords with the solution to the macroeconomic planning problem given in Result 1. The equilibrium price of state contingent bonds satisfies:

$$p_{c_1}[s]^* : \begin{cases} 1 & \text{if } S < Z \\ 1 + B & \text{if } S > Z \end{cases} \quad (7.3)$$

This analysis establishes a link in between the rigidity bias and standard welfare economics. While the first theorem in welfare economics is applicable to the case of uncertainty when states are observable (see Arrow and Hahn 1971), the rigidity bias arises under the key assumption of non-observability of states.

Why does a classical Arrow-Debreau futures market succeed where even a perfect spot market for liquidity cannot? The initial endowment, and the target of agents are naturally the same in both cases. Also the market clearing condition, which determines the state contingent return of liquidity, is essentially the same in both cases (just replace  $p_{c_1}[s]$  with  $r$ ). The difference is how the liquidity constraints are written. In this classical case, liquidity demand in any state is effected only by the average price of consumable goods across states in accordance with (7.2). In the absence of the futures market, there is a separate constraint for each state.

## 8 Extensions and concluding remarks

The key result in this paper is that a rigidity bias arises under aggregate uncertainty, given non observability of shocks, and certain continuity assumptions related to the joint distribution of shocks. The prediction of the model is that people tend to commit to long-term projects and invest in long-term assets too heavily, so that the economy is ‘too rigid’. To bring out the empirical implications of the hypothesis, it is useful to restate the proof as follows.

The explanation for the rigidity bias is that, even under ideal conditions, not enough money can be created by the private sector to equalise the price of consumable goods with the marginal benefit of consumption under all contingencies. During periods when the economy is hit by relatively high shocks, some people are budget constrained while some are not. In these circumstances, the market price of consumable goods falls below marginal benefit of consumption of the credit constrained agents. The real insight of the model is that this property of the price mechanism causes the rigidity bias. In anticipation of the dampened effect of individual budget constraints on prices, people prefer to invest long-term, at the cost that the reserve of the economy is insufficient. Special government powers, such as the power to impose reserve requirements on agents, are welfare improving in this economy.

The rigidity bias is derived under a linear kinked target. In that case the social planner abstracts from issues related to allocation of consumption across agents to a high degree. We have made some preliminary inquiries into the case of a strictly concave utility function. In this case, the social planner is interested in ‘equality of outcomes’ (ie equality of individual levels of utility within periods). The first best allocation is then, not implementable by a price mechanism, because the equality of outcomes requirement imposes an implicit price on liquidity. It appears that in this case there are forces that bias liquidity creation in opposite directions. One is the rigidity bias caused by the dampening effect of individual budget constraints on prices of liquidity. The other is a ‘reserve bias’, which arises from the fact that agents are in this case risk averse wrt period  $t=2$  consumption. Under risk aversion, uncertainty about the cost of covering shocks in terms of period  $t=2$  consumption makes individuals biased towards holding large reserves.

Another interesting extension is to the case of aggregate certainty, which links this model with mainstream liquidity models. Under aggregate certainty, and the linear kinked target, the first best allocation is implemented by the price mechanism if and only if agents can pledge at  $t=1$  up to  $R$  in period  $t=2$  returns. If investment returns are not fully pledgeable, then the rigidity bias emerges.

In principle at least, the rigidity bias is testable by experiment in a finite population of agents. Empirical applications of the theory to forecasting and analysis of real economies requires a more detailed analysis about the outcome

that will, indeed, be reached by agents. The reason that we focus on ruling out certain outcomes, in stead of describing the equilibrium, is that the equilibrium seems to in this economy be very dependent on circumstance (specifics of the shock distribution etc). To develop a forecasting model one needs to choose some approximation about the equilibrium, and enrich the dynamics of the model. Both of these challenges we leave for future research.

In the absence of an empirical model, it is risky to make strong inferences about the relationship in between the rigidity bias and specific historical phenomena. However, to motivate future research, it may be useful to make certain suggesting about the empirical relevance of the theory, based on intuition.

Firstly, we suggest that the rigidity bias could explain the classical boom-bust cycle (Kindleberger 1989). The boom periods could be interpreted as periods of stark 'long-term visions' and arrangements that permit commitment to them, with insufficient regard for near term challenges related to their implementation. A crisis occurs once aggregate risk is realised in the economy.

In a related vein we suggest that the rigidity bias might in part explain the evolution of financial systems in the post-industrialisation period. Industrial projects are typically long-term enterprises that require commitment by both the entrepreneurs, and the financiers in the projects. Contingencies are varied and the contracting environment may be very limited. The tentative conclusion from the model is, then, that once such technology becomes available, people tend to invest too much on it. Economies become too rigid, and macroeconomic shocks are too costly from the social point of view. The frequency of financial crises in the 1800's (Bordo 1990) could be symptomatic of such excess rigidity. The response of economies to excess rigidity was the evolution of institutions of central banking and financial regulation. In this way, the institutional and regulatory development in the late 1800's and 1900's could perhaps be interpreted as a response to the rigidity bias, a failure by the market mechanism to govern liquidity creation.

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# Appendix 1

## Proof of Lemma L1.3

Restate part of L1.3 as follows:

$$\begin{aligned}
 Z < 1 &\Rightarrow \int_1^{v[i]_{di=S}} AD[r, v, z] f_s[v] dv < S \quad \forall R < r < 1+B, S \in (0,1] \\
 &\Leftrightarrow \int_1^{v[i]_{di=S}} \int_{v[i]_{>z[i] + \frac{R}{r}(1-z[i])}}^{\frac{R}{r}(1-z[i])} \left( z[i] + \frac{R}{r}(1-z[i]) - v[i] \right) di f_s[v] dv < 0 \quad \forall R < r < 1+B, S \in (0,1]
 \end{aligned}$$

The following shows that the counterhypothesis of this lemma leads to contradiction with the premise that  $Z < 1$ :

$$\begin{aligned}
 \exists R < r < 1+B, S \in (0,1]: \int_1^{v[i]_{di=S}} \int_{v[i]_{>z[i] + \frac{R}{r}(1-z[i])}}^{\frac{R}{r}(1-z[i])} \left( z[i] + \frac{R}{r}(1-z[i]) - v[i] \right) di f_s[v] dv = 0 \\
 \Leftrightarrow \exists R < r < 1+B, S \in (0,1]: \frac{\partial \left( \int_1^{v[i]_{di=S}} \int_{v[i]_{\leq z[i] + \frac{R}{r}(1-z[i])}}^{\frac{R}{r}(1-z[i])} f_s[v] dv \right)}{\partial \left( z[i] + \frac{R}{r}(1-z[i]) \right)} = 0 \quad \text{a.e. } i \in I \\
 \stackrel{(3.4)}{\Leftrightarrow} \exists R < r < 1+B, S \in (0,1]: z[i] + \frac{R}{r}(1-z[i]) = 1 \quad \text{a.e. } i \in I \\
 \stackrel{z[i] \leq 1}{\Leftrightarrow} z[i] = 1 \quad \text{a.e. } i \in I \\
 \Rightarrow \text{contradiction with premise}
 \end{aligned}$$

A similar proof can be utilised for the remaining part of L1.3.

## Appendix 2

### Proof of Lemma L2

Consider average aggregate demand  $\int_{\int_1^I v[i]di \in (Z, \bar{S})} AD[r]f_s[v]dv$  for some  $Z < \bar{S} \leq 1$  and

$R < r < 1 + B$ . Average aggregate demand is continuous in  $\bar{S}$  (see proof of continuity below) and, by L1.3, it falls strictly below  $Z$  in the limit  $\bar{S} \rightarrow Z$ . From continuity, and this limit, it follows that there exists some  $\bar{S}[r] > Z$  such that average aggregate demand is at or below  $Z$  in the set  $S \in (Z, \bar{S}[r])$ . From L1.3 it then follows, that average aggregate demand at  $r = 1 + B$ , must be strictly below  $Z$  in that set. By continuity of the aggregate shock distribution, this set is not empty. By the price mechanism, the average price of liquidity falls below  $1 + B$  in the set  $S \in (Z, \bar{S}[r])$ . That the price of liquidity is at most  $1 + B$  in all states, and below  $1 + B$  in some states characterised by  $S > Z$ , establishes L2.

### Proof of continuity:

Denote by  $\int_{\int_1^I v[i]di \in (Z, \bar{S})} AD[r]f_s[v]dv$  average aggregate demand in the set

$\left\{ v \in I \times I : Z < \int_1^I v[i]di \leq \bar{S} \right\}$ , when  $0 < Z < \bar{S} \leq 1$ . Average aggregate demand in the two intervals  $(Z, \bar{S} \pm e]$ ,  $e > 0$ , are:

$$\begin{cases} \int_{\int_1^I v[i]di \in (Z, \bar{S})} AD[r]f_s[v]dv + \int_{\int_1^I v[i]di \in (\bar{S}, \bar{S}+e]} AD[r]f_s[v]dv \\ \int_{\int_1^I v[i]di \in (Z, \bar{S})} AD[r]f_s[v]dv - \int_{\int_1^I v[i]di \in (\bar{S}-e, \bar{S})} AD[r]f_s[v]dv \end{cases} \quad (A2.1)$$

By continuity of the distribution  $F$  of the average shock, the second terms on both rows in (A2.1) vanish when  $e \rightarrow 0$ . This establishes continuity of average aggregate demand  $\int_{\int_1^I v[i]di \in (Z, \bar{S})} AD[r]f_s[v]dv$  wrt.  $\bar{S}$  when  $0 < Z < \bar{S} \leq 1$  and  $R < r < 1 + B$ .

## Appendix 3

### Proof of Lemma L3

$$\begin{aligned}
\forall i \in I: & \int_{\mathbb{R}^1} (r^* - R) f_s[s] ds + \int_{s[i] > z[i]^* + \frac{R}{r^*}(1-z^*[i])} (1+B-r^*) \frac{(r^* - R)}{r^*} f_s[s] ds \\
& \stackrel{(5.4)}{=} \int_{S < Z} (1-R) f_s[s] ds + \int_{S > Z} (r^* - R) f_s[s] ds + \int_{\substack{S > Z \\ s[i] > z[i]^* + \frac{R}{r^*}(1-z^*[i])}} (1+B-r^*) \frac{(r^* - R)}{r^*} f_s[s] ds \\
& \stackrel{\text{domains}}{\leq} \int_{S < Z} (1-R) f_s[s] ds + \int_{S > Z} (r^* - R) f_s[s] ds + \int_{S > Z} (1+B-r^*) \frac{(r^* - R)}{r^*} f_s[s] ds \\
& \stackrel{\text{manipulate}}{=} \int_{S < Z} (1-R) f_s[s] ds + \int_{S > Z} \left\{ (1+B) - \frac{(1+B)}{r^*} R \right\} f_s[s] ds \\
& \stackrel{(L2.1)}{<} \int_{S < Z} (1-R) f_s[s] ds + \int_{S > Z} (1+B-R) f_s[s] ds
\end{aligned}$$



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