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Tuomas Takalo – Otto Toivanen  
Research Department  
4.3.2003

## Equilibrium in financial markets with adverse selection

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## Equilibrium in financial markets with adverse selection

The views expressed are those of the authors and do not necessarily reflect the views of the Bank of Finland.

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# Equilibrium in financial markets with adverse selection

Bank of Finland Discussion Papers 6/2003

Tuomas Takalo – Otto Toivanen  
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## Abstract

We study a financial market adverse selection model where all agents are endowed with initial wealth and choose to invest as entrepreneurs or financiers, or not to invest. We show that often a lack of outside finance leads to the emergence of financial markets where availability of outside finance leads to autarky. We find that i) there exist Pareto-efficient and inefficient equilibria; ii) adverse selection has more severe consequences for poorer economies; iii) increasing initial wealth may cause a shift from Pareto-efficient to inefficient equilibrium; iv) increasing the proportion of agents with positive NPV projects causes a shift from inefficient to efficient equilibrium; v) equilibrium financial contracts are either equity-like or ‘pure’ debt contracts; vi) agents with negative (positive) NPV projects earn rents only in (non-)wealth-constrained economies; vii) agents earn rents only when employing pure debt contracts; and viii) removing storage technology destroys the only Pareto-efficient equilibrium in non-wealth-constrained economies. Our model enables analysis of various policies concerning financial stability, the need for sophisticated financial institutions, development aid, and the promotion of entrepreneurship.

Key words: financial market efficiency, adverse selection, financial contracts, creation of firms.

JEL classification numbers: D58, G14, G20, G28, G32

# Rahoitusmarkkinoiden tehokkuus ja epäsuotuisa valikoituminen

Suomen Pankin keskustelualoitteita 6/2003

Tuomas Takalo – Otto Toivanen  
Tutkimusosasto

## Tiivistelmä

Tässä tutkimuksessa tarkastellaan rahoitusmarkkinoiden kykyä toimia epäsymmetrisestä informaatiosta huolimatta. Lisäksi työssä tutkitaan, millaisissa olosuhteissa rahoitusmarkkinat suosivat osake- tai velkarahoitusta. Karkeasti ottaen osakerahoitusta käytetään, jos taloudessa on paljon tuottavia projekteja suhteessa alkuvarallisuuteen, ja velkarahoitusta käytetään, jos alkuvarallisuutta on runsaasti suhteessa tuottaviin projekteihin. Mallin avulla voidaan lisäksi tutkia mm. likviditeetin tarjontaa, yrittäjyyspolitiikkaa, rahoitusmarkkinoiden vakautta, ja säännötelyn tarvetta.

Avainsanat: rahoitusmarkkinoiden tehokkuus, epäsuotuisa valikoituminen, rahoitussopimukset, uusien yritysten syntyminen

JEL-luokittelu: D58, G14, G20, G28, G32

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# 1 Introduction

The aim of this paper is to explore the functioning of financial markets with asymmetric information when the roles of agents are determined within the model. In a departure from most of the existing literature, in our model all agents are endowed with some initial wealth and an investment project whose quality is their private information. To initiate a project, a would-be entrepreneur needs outside financing. There is an occupational choice in the sense that agents choose whether to participate and, if they participate, whether to become entrepreneurs or financiers. The set-up creates a natural environment to study whether a market for financial claims emerges in equilibrium, whether the eventual markets are efficient, and what kind of financial contracts are employed. Our model allows us to analyze the effects of different shocks to the economy, the need for more sophisticated financial institutions, and the usefulness of various policies such as development aid and the promotion of entrepreneurship.

We build on a strong foundation: Since Akerlof's (1970) seminal article, a large literature on the effects of asymmetric information on the functioning of credit markets has emerged (for surveys, see, eg, Clemenz and Ritthaler, 1992, and chapter 5 in Freixas and Rochet, 1997). While a substantial part of the literature is in a partial equilibrium setting, the supply of funds is explicitly modeled in a few influential studies such as Bernanke and Gertler (1989, 1990) and Holmström and Tirole (1997, 1998). The literature shares some common elements, and in particular, two key assumptions: First, there are potential borrowers with investment projects and potential lenders with funds, but without projects. Second, potential borrowers have private information about the ex-ante or ex-post value of their projects, or their choice of effort. In such an environment, the well-known problems of adverse selection, moral hazard, credit rationing (Stiglitz and Weiss, 1981, Williamson, 1987) and inefficient investment levels (de Meza and Webb, 1987) may emerge. Financial markets may even collapse. It is the first of the above two assumptions that we relax by allowing the agents to choose between becoming entrepreneurs or financiers, or not participating.<sup>1</sup>

It turns out that the occupational choice of agents mitigates the adverse effects of asymmetric information. As in standard models under adverse selection, the financial market may yield interim or Pareto inefficient outcomes, or the market may collapse to autarky. These observations provide a rationale for more sophisticated financial institutions than the one that we allow for. But, in stark contrast, we show that if the economy is initially sufficiently wealthy or has

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<sup>1</sup> In the recent literature dealing with belief asymmetry, (eg, de Meza and Southey, 1996, Manove and Padilla, 1999, and Hyytinen, 2002) the impact of the second assumption on the performance of credit markets has been scrutinized. Surprisingly little has, however, been said about the first basic assumption, although its importance is evident from the work of Boyd and Prescott (1986).

sufficiently productive projects, a simple form of financial market will emerge endogenously as an equilibrium institution and, for a wide range of parameter values, the financial market yields a Pareto efficient outcome.

Paralleling Holmström and Tirole (1998), we consider both economies where the total initial wealth is sufficient to implement all (positive NPV) projects, and wealth-constrained economies where there is an aggregate shortage of liquidity. We find that Pareto efficient and inefficient equilibria exist both in wealth-constrained and unconstrained economies, as does autarky. Contrary to what one might expect, wealth constraints do not necessarily dilute the performance of the financial market: Relaxing the economy level wealth constraint may induce the agents with low quality projects to seek funding, reducing the efficiency of financial markets (this is reminiscent of de Meza and Webb's (1987) partial equilibrium results). Wealth constraints, however, may affect the distribution of economic rents and forms of financial contracts between agent types: Agents with good projects may earn rents and use debt contracts in non-wealth constrained economies, whereas agents with bad projects may earn rents and use debt contracts in wealth constrained economies.

We also consider how financial market performance depends on the efficiency of the storage technology. It transpires that the efficiency of storage technology has only minor effects, except that the scope of the Pareto-efficient equilibrium in non-wealth constrained economies is increasing in the efficiency of the storage technology. At the limit where there is no storage technology, the Pareto-efficient equilibrium disappears.

In addition to the aforementioned articles, our study is also inspired by Aghion, Bacchetta, and Banerjee (2000) and Caballero and Krishnamurthy (2001). They point out that both the microeconomic and economy-wide financial constraints matter for the performance of financial markets, and that the consequences of traditional financial market frictions – such as moral hazard – are at their worst at an intermediate level of financial market development or initial wealth. Our model has a link also to the literature on financial market imperfections and the occupational choice between becoming an entrepreneur or a worker (eg, Kihlström and Laffont, 1979, Banerjee and Newman, 1993, Ghatak, Morelli, Sjöström, 2001, and Kannianen and Leppämäki, 2002). To the best of our knowledge, however, Boyd and Prescott (1986) is the only study besides ours where there is a choice between investing as an entrepreneur and a financier. In their model agents i) have investment projects whose quality is their private information and ii) can choose whether to invest in their project or evaluate the quality of a project. Our model is simpler than theirs in that we do not allow information acquisition.

We introduce the two departures from the standard partial equilibrium models of adverse selection sequentially. In the next Section we assume that potential entrepreneurs have positive initial wealth but, as in the standard models, that there

are outside investors who do not have a potential project of their own. The model of this Section comes close to the one in de Meza and Webb (1987). In the third Section, we remove outside investors and characterize the equilibria and the necessary conditions for their existence. Most of our discussion rests on a simple graphical representation of the results: the analytical details are deferred into the Appendix. In the fourth Section we discuss the effects of different storage technologies. Policy implications are collected into Section five, and conclusions into Section six.

## 2 The model with outside investors

Except for allowing potential entrepreneurs with positive initial wealth, the model in this Section contains features that are generic in the literature. The economy consists of a unit mass of entrepreneurs who are each endowed with a potential project, and a large number of outside investors without a project of their own. Each projects needs financing of  $I$  to be implemented. All agents are risk-neutral and endowed with assets  $A$ ,  $0 < A < I$ . We assume that the number of outside investors is sufficiently large to satisfy the financing needs of all potential entrepreneurs. This means that the cost of financing is driven to a level where outside investors are indifferent between investing and not investing. For the moment we assume that there is a storage technology that converts the assets to a consumption good at a zero rate of return. In Section 4 we consider an imperfect storage technology under which  $A$  depreciates at rate  $1 - \delta$ ,  $\delta \in [0, 1]$ .

Entrepreneurs' projects have different success probabilities and conditional returns. A project that fails yields zero to all agents. A proportion  $h$  ( $0 < h < 1$ ) of agents are High (H) types who are endowed with a positive NPV project, the rest are Low (L) types with a negative NPV project. We assume that  $p_H R_H > I > p_L R_L$  and  $R_L > R_H$ , where  $p_i$  is the success probability and  $R_i$  the return (conditional on success) of an entrepreneur of type  $i$ ,  $i \in \{H, L\}$ . Project success and wealth ( $A$ ) are verifiable, but project type is private information following, eg, Bolton and Scharfstein (1990). In what follows, we present most of our analysis using a graph in the  $(A, h)$ -space. Natural parameter boundaries are given by  $h \leq 1$ , and  $A < I$ .

The financial market works as follows. In the first stage, potential entrepreneurs decide whether to approach outside investors or to resort to the storage technology.<sup>2</sup> Contract terms stipulate the amount of financing per project ( $I - A$  for all), and the conditional payment from the entrepreneur to outside

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<sup>2</sup> When there are outside investors, agents who do not become entrepreneurs are indifferent between using the storage technology and financing entrepreneurs' projects. Saying that all of them resort to the storage technology is an expositional decision without implications.

investors in case of success. Once financing needs have been settled, entrepreneurs execute their projects in the second stage. Project success is verified, successful entrepreneurs compensate outside investors according to contract terms, and consumption takes place.

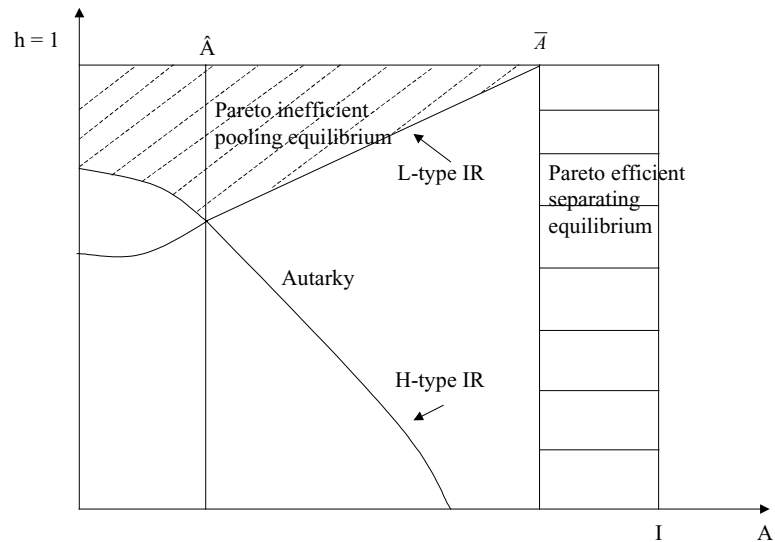
The choice of potential entrepreneurs can be formalized as an individual rationality condition. Denoting expected profits for a type  $i$  entrepreneur by  $\pi_i^e$ , the IR constraint is

$$\pi_i^e \equiv p_i(R_i - R_B) \geq A \quad \forall i, \quad i \in \{H, L\} \quad (2.1)$$

where superscript  $e$  denotes entrepreneurship and  $R_B$  is the (fixed) payment that a successful entrepreneur pays to her investors.

Allowing the entrepreneurs positive initial wealth adds one equilibrium type to the model compared to the standard model. In addition to autarky and a pooling equilibrium where all L- and H-type agents become entrepreneurs, a Pareto-efficient equilibrium exists where only H-type agents become entrepreneurs.<sup>3</sup> In Figure 1 we have depicted the  $(A, h)$ -values for which each of the three different equilibria exist. The standard model where entrepreneurs have no initial wealth lies on the y-axis.

Figure 1.



There are two vertical lines in Figure 1: the  $\hat{A} \equiv p_L p_H (R_L - R_H) / (p_H - p_L)$  -line and the  $\bar{A} \equiv \hat{A} + p_L (p_H R_H - I) / (p_H - p_L) = p_L (p_H R_L - I) / (p_H - p_L)$  -line, and two IR constraints. Using the terminology of Holmström and Tirole (1997, 1998), the

<sup>3</sup> This is unlike de Meza and Webb (1987), because they assume that  $R_H = R_L$ .

pledgeable income of the L-type entrepreneurs exceeds that of H-type agents to the left of  $\hat{A}$ , and vice versa to the right of the line. This means that no separating equilibrium is possible to the left of this line. But once the initial wealth becomes large enough, L-type entrepreneurs' expected profit from entrepreneurship becomes smaller than their initial endowment. This happens trivially when  $A$  exceeds  $p_L R_L$ , but, given the equilibrium  $R_B$ , it turns out that the NPV of L-type projects is less than  $A$  in the whole region to the right of  $\bar{A} < p_L R_L$ . Thus, the L-type IR constraint can be satisfied only to the left of this line. As Figure 1 shows, the L-type IR constraint is an upward sloping curve, whereas the H-type IR constraint slopes downwards.

In sum, the pooling equilibrium exists in the upper left-hand corner of Figure 1, autarky exists below the pooling equilibrium, and the Pareto efficient equilibrium exists to the right of  $\bar{A}$ . Competition between outside investors prevents the emergence of a separating equilibrium to the left of  $\bar{A}$ . This suggests that if finance was provided by a monopoly, the economy would reach Pareto efficiency everywhere to the right of  $\hat{A}$ . Note also that as long as (2.1) is satisfied, both types of agents invest all their funds to their own projects. This rules out using  $A$  as a signaling device to the left of  $\bar{A}$ .

### 3 The model without outside investors

In the absence of outside investors the amount of funds available for investment is limited. A natural consequence is that all agents face a choice between becoming entrepreneurs or financiers. We think of an economy as being wealth constrained if the total wealth of all agents is not sufficient to finance all H-types' projects. Identically, an economy is not wealth constrained if the total initial wealth exceeds the financing needs of all H-type projects. The diagonal naturally divides economies into wealth constrained (above the  $h = A/I$  -line), and the non-wealth constrained ones (below).

The financial market works as before. Since we do not allow for financial institutions that gather and process information, the financial market in our model could be interpreted as a frictionless (stock) market. In other words, after the projects have been implemented, the total payments from all entrepreneurs are divided evenly among all financiers. Thus it is as if a financier buys a stake in the average implemented project, instead of implementing her private project. Loosely speaking, it makes no difference whether one envisions a financial market where some potential financiers come together to finance one or a few projects (to equate demand and supply within the "coalition"), or a market where all financiers buy a similar stake in every implemented project. Both result in the same expected payment to financiers.

An advantage of such a simple financial market is that it is easy to evaluate its performance. The market collapses to autarky when all agents resort to the storage technology and there are neither entrepreneurs nor financiers. In the Pareto-efficient allocation all or as many H-type projects as possible are financed whilst no L-type projects are financed.

We define equilibria through the proportion of type  $i$  agents that become entrepreneurs, denoted by  $\mu_i$ . As  $\mu_i \in [0, 1]$ , we have a  $3 \times 3$  matrix of equilibria as shown in Table 1.<sup>4</sup> It is immediately clear that three out of the nine cannot exist. If no H-type agent becomes an entrepreneur, the potential financiers' individual rationality constraint is violated. Similarly, due to our assumption that  $A < I$ , it is not possible that all agents become entrepreneurs. The remaining six configurations cannot be ruled out a priori. They consist of autarky and five cases where financial markets may emerge as an equilibrium outcome. We have named the five potential equilibria with financial markets according to what occupations (e = entrepreneur, f = financier, s = storage technology) agents of type  $i$  choose. For example,  $H^e L^{fs}$  (column one, row one in Table 1) is the equilibrium where all H-type agents become entrepreneurs, and L-types split between becoming financiers, and using the storage technology.

Table 1. **Types of equilibria**

	$\mu_L = 0$	$0 < \mu_L < 1$	$\mu_L = 1$
$\mu_H = 0$	AUTARKY	Not possible	Not possible
$0 < \mu_H < 1$	$H^{ef} L^f$	$H^{ef} L^{ef}$	$H^{ef} L^e$
$\mu_H = 1$	$H^e L^{fs}$	$H^e L^{ef}, H^e L^{efs}$	Not possible

Notes:  $\mu_i$  = the proportion of  $i$  type agents that become entrepreneurs in equilibrium.

Both Pareto-efficient equilibria are in the first column of Table 1. Of these, the one in the last row is strictly better than the one in the middle row. Similarly, in the middle column, the equilibrium in the last row is more desirable than the one in the middle row. The equilibrium in the last column is the worst of the five equilibria with economic activity.

Removing the outside investors changes the adverse selection problem. An equilibrium is now constrained by four conditions. The first are the individual rationality (IR) constraints we saw before, with a slight modification. Now the agents compare the expected profits from becoming active, either as an entrepreneur or as a financier, to resorting to the storage technology. In the model with outside investors, entrepreneurs effectively only chose between the storage technology and entrepreneurship. Second are the incentive compatibility (IC)

<sup>4</sup> These nine categories can be split further according to whether all type  $i$  agents participate or not.

constraints of both types' of agents. By IC constraints we mean the choice between entrepreneurship and being a financier. The third relationship equalizes the supply of funds from financiers with the demand of funds by entrepreneurs. Finally, the contracts used in the financial market are determined by equating the payments by successful entrepreneurs to the expected compensation for financiers.

Denoting expected profits for a type  $i$  agent from activity  $j$  by  $\pi_i^j$ , the IR constraints are

$$\pi_i^j \geq A \quad \forall i, j, \quad i \in \{H, L\}, \quad j \in \{e, f\} \quad (3.1)$$

where superscript  $e$  denotes entrepreneurship and  $f$  financiership. The IC constraints are written as

$$\pi_i^j \geq \pi_i^k \quad \forall i, j, k, \quad j \neq k, \quad i \in \{H, L\}, \quad j, k \in \{e, f\} \quad (3.2)$$

Depending on the equilibrium (see Table 1) and agent type, the IC or IR constraint or both may bind, and the IC constraint may hold strictly one way (eg, all H-type agents become entrepreneurs) or the other (eg, all L-type agents become financiers).

The equality of supply (left hand side) and demand (right hand side) of funds is given by

$$A[(1 - \mu_H - \chi_H)h + (1 - \mu_L - \chi_L)(1 - h)] = (I - A)[\mu_H h + \mu_L(1 - h)] \quad (3.3)$$

where  $\mu_i$  and  $\chi_i \in [0, 1]$  denote the proportion of type  $i$  agents who become entrepreneurs and who employ the storage technology.

Finally, the expected payment made by entrepreneurs must equal the expected return received by financiers:

$$[\mu_H h p_H + \mu_L(1 - h) p_L] R_B = R_F [(1 - \mu_L - \chi_L)(1 - h) + (1 - \mu_H - \chi_H)h] \quad (3.4)$$

The term in the square brackets on the left hand side is the expected (equilibrium) number of successful entrepreneurs while the term in the square brackets on the right hand side is the (equilibrium) number of financiers. In (3.4)  $R_F$  is the expected payment received by a financier that is independent of the agent type. Substituting  $\pi_H^f = \pi_L^f = R_F$  for the IC constraints (3.2) shows that  $R_F$  also captures the opportunity cost of entrepreneurship.

Since solving the range of parameters where conditions (3.1)–(3.4) hold for all five equilibria with economic activity (see Table 1) is a straightforward but tedious exercise, we relegate the details of the calculations to the Appendix.



Instead, we next consider the equilibrium  $H^e L^{ef}$  as an example and then graphically describe the remaining equilibria.

### 3.1 Example: $H^e L^{ef}$

In  $H^e L^{ef}$ ,  $\mu_H = 1$  and  $0 < \mu_L < 1$ , ie, all H-type agents are entrepreneurs and L-type agents become either entrepreneurs or financiers (ie, nobody chooses the storage technology,  $\chi_i = 0$ ,  $i \in \{L, H\}$ ). Since all agents are active, both types' IR constraints are satisfied in equilibrium. L-type agents' IC constraint holds with equality, whereas H-type agents (weakly) prefer entrepreneurship to being financiers.

To guarantee that all agents participate, we require that

$$R_F \geq A \quad (3.5)$$

The L-type agents' IC constraint is given by

$$p_L(R_L - R_B) = R_F \quad (3.6)$$

The left hand side gives the expected return for an L-type agent from becoming an entrepreneur and the right hand side gives the expected return from becoming a financier. As L-type agents split between the two choices, they must be indifferent between them.

Since all H-type agents become entrepreneurs, their expected return from that choice must (weakly) exceed that from becoming a financier, ie,

$$p_H(R_H - R_B) \geq R_F \quad (3.7)$$

The aggregate supply and demand for finance is balanced when

$$(1 - \mu_L)(1 - h)A = [h + \mu_L(1 - h)](I - A) \quad (3.8)$$

holds. Finally, our assumptions on financial market transactions yield the following equilibrium relationship between the payment by a successful entrepreneur, and the expected payment received by a financier:

$$[hp_H + \mu_L(1 - h)p_L]R_B = R_F(1 - \mu_L)(1 - h) \quad (3.9)$$

Conditions (3.6), (3.8) and (3.9) determine the endogenous variables  $\mu_L$ ,  $R_B$ , and  $R_F$ . Solving first for  $\mu_L$  from (3.8) gives

$$\mu_L^* = \frac{A - hI}{(1 - h)I} \quad (3.10)$$

The proportion of L-types who become entrepreneurs has to be less than unity. This is guaranteed by our assumption  $I > A$ . As  $\mu_L^*$  also has to be nonnegative in  $H^eL^{ef}$ , (3.10) immediately reveals that  $H^eL^{ef}$  can only exist if  $A/I \in (h, 1)$ . In other words,  $H^eL^{ef}$  cannot exist in a wealth constrained economy (see Figure 2).

Our next step is to use (3.10), (3.9) and (3.6) to solve for the equilibrium payments  $R_B^*$  and  $R_F^*$ . They are given by

$$R_B^* = \frac{(I - A)}{I[hp_H + (1 - h)p_L]} p_L R_L \quad (3.11)$$

and

$$R_F^* = \left[ 1 - \frac{p_L(I - A)}{I[hp_H + (1 - h)p_L]} \right] p_L R_L \quad (3.12)$$

Equations (3.11) and (3.12) suggest for H-types, the payments  $R_B^*$  and  $R_F^*$  are independent of project outcome, whereas for L-types the payments are functions of project outcome. One might be tempted to think that the equilibrium financial contract can be interpreted as a debt contract for H-type agents, but as an equity contract for L-type agents. The problem with this interpretation is twofold: first, although entrepreneurs know the type of the contract they have, financiers do not know it at the level of individual entrepreneurs.<sup>5</sup> Second, the payment from a successful L-type entrepreneur to her financiers is fixed, although it is a function of her project outcome (conditional on success). However, we show in Section 3.1 that there is systematic variation of how each type of agents' payments are or are not a function of project outcome. To clarify discussion, we in the following label without implication (and following de Meza and Webb, 1987) contracts that are a function of project outcome as “equity-like” contracts. The contracts that are not such functions we call “pure debt” contracts.

After solving for the endogenous variables, we still need to find the parameter values satisfying the agents' IR and IC constraints (3.5)–(3.7). The L-type IC constraint (3.6) binds as they split in their occupational choices. As H-types strictly prefer entrepreneurship to becoming financiers, and being a financier is at least as rewarding as resorting to the storage technology, their IR constraint (3.5)

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<sup>5</sup> For the same reason, any argument on financial contracting in a pooling equilibrium becomes moot (see, eg, Stiglitz and Weiss, 1981, and de Meza and Webb, 1987).

does not bind. This means that the relevant constraints are the L-type IR constraint (3.5) and the H-type IC constraint (3.7). Substituting (3.12) into (3.5) shows that the L-type IR constraint is satisfied (guaranteeing that no L-type agent stores her initial wealth) if

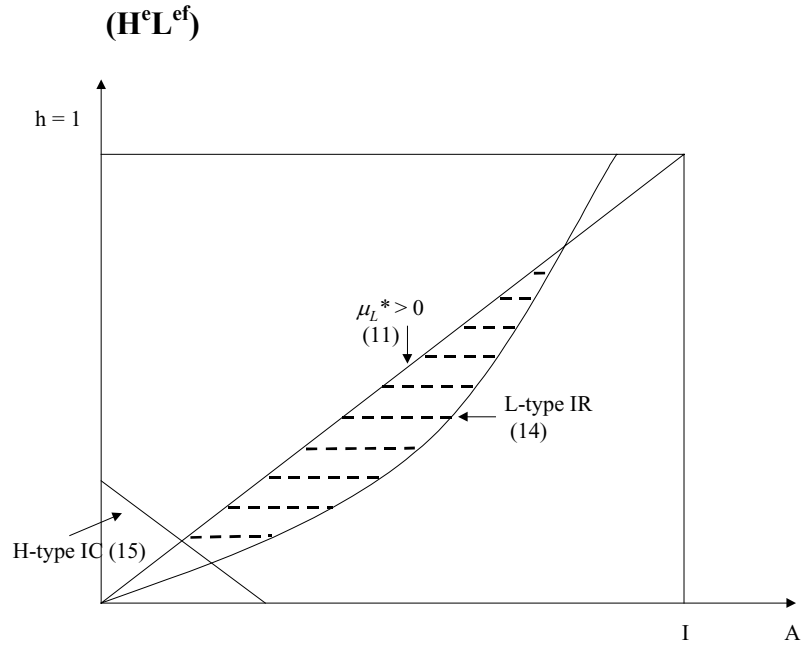
$$A \leq \frac{p_L R_L I h (p_H - p_L)}{I h (p_H - p_L) + p_L (I - p_L R_L)} \quad (3.13)$$

The H-type IC constraint (guaranteeing that no H-type agent prefers becoming a financier over entrepreneurship) is satisfied if

$$A \geq I - \frac{(p_H R_H - p_L R_L) I [h p_H + (1-h) p_L]}{(p_H - p_L) p_L R_L} \quad (3.14)$$

In Figure 2 we use the  $(A, h)$ -space to represent the set of parameter values for which  $H^e L^{ef}$  exists.

Figure 2.



$H^e L^{ef}$  does not exist in wealth constrained economies as suggested by (3.10). The H-type IC constraint (3.14) is a downward sloping line in the  $(A, h)$ -space: it cuts out the vertical and horizontal axes close to the origin, ie, poor economies with a small number of good projects do not satisfy this constraint. L-types' IR constraint (3.13) is a monotonically upward sloping curve that starts at the origin and cuts the  $h = A/I$  diagonal once. Below the curve, some L-types would prefer not to participate. L-types' IC constraint binds in non-wealth constrained

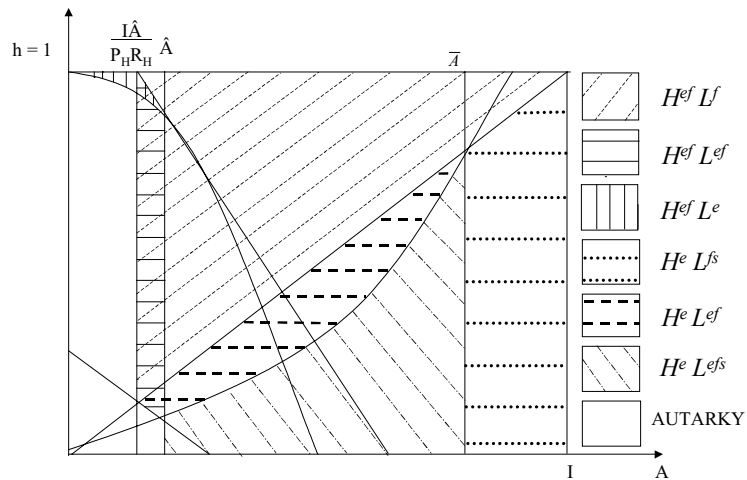
economies (as it should), and H-types' IR constraint does not bind (as it should not).

### 3.2 Existence of equilibria

Following a similar procedure as for the case of  $H^e L^{ef}$  in Section 3.1, we derive the values of the endogenous variables and determine the necessary and sufficient conditions for the existence of the six candidate equilibria (see the Appendix). We first present graphically the equilibria and discuss them. At the end of the Section, we summarize our main results.

In Figure 3, we employ the labeling of Table 1 to indicate the areas in which particular equilibria exist in the  $(A, h)$ -space.<sup>6</sup> There are two key lines: the  $h = A/I$  diagonal and the vertical  $\hat{A} \equiv p_L p_H (R_L - R_H) / (p_H - p_L)$  -line. The diagonal not only divides the economies into wealth and non-wealth constrained ones, but also is a border of various equilibria in many cases. To the right of the vertical  $\hat{A}$ -line the equilibria are unique.

Figure 3.



Let us first examine wealth-constrained economies, ie, the area above the diagonal, starting from the right hand side of the Figure. In wealth constrained economies all available wealth is not enough to finance all available H-type projects. In the upper right hand corner we find economies having a high proportion of H-types, and relatively high initial wealth. There, an equilibrium exists where all L-types become financiers, and H-types mix in their occupations

<sup>6</sup> In the Appendix, we present a separate figure for each equilibrium.

between entrepreneurship and finance ( $H^{ef}L^f$ ). All funds are directed into H-type projects, and therefore the equilibrium is Pareto-efficient.

Moving to the left, entrepreneurship becomes an option to L-types when we reach the vertical  $\hat{A}$ -line. Between it and another vertical line,  $\hat{A}(I/p_H R_H)$ , we have two or three equilibria in the upper part of the Figure. One is the same  $H^{ef}L^f$  as on the right hand side of that line. Another is  $H^{ef}L^{ef}$  where both types mix their occupational choices, ie, both L- and H-types become entrepreneurs and financiers. The third equilibrium is  $H^{ef}L^e$  where all L-types are entrepreneurs and H-types mix their occupations between entrepreneurship and finance. Since there are L-type agents among entrepreneurs in  $H^{ef}L^{ef}$  and  $H^{ef}L^e$ , they are Pareto-inferior to  $H^{ef}L^f$ .

To explain multiple equilibria recall from Section 2 the necessary and sufficient conditions for the existence of a separating equilibrium. The necessary condition is that the opportunity cost of entrepreneurship ( $R_F^*$ ) exceeds  $\hat{A}$ , because then the pledgeable income of H-type agents exceeds that of the L-type agents. The sufficient condition is that the opportunity cost of entrepreneurship exceeds the L-type agents' expected profit from entrepreneurship, ie, that both the equilibrium payments  $R_F^*$  and  $R_B^*$  are sufficiently high. In Section II competition between outside financiers drove down the equilibrium payments and the separating equilibrium emerges only to the right of the vertical  $\bar{A} \equiv p_L(p_H R_L - I)/(p_H - p_L)$ -line. Compared with outside financing, wealth-constraints generate higher equilibrium payments. In particular, in  $H^{ef}L^f$  the equilibrium payments  $R_F^*$  and  $R_B^*$  are so high that the opportunity cost exceeds both  $\hat{A}$  and the L-types' expected profits from entrepreneurship to the right of  $\hat{A}(I/p_H R_H)$ -line and therefore a separating equilibrium also exists between it and  $\hat{A}$ . To the left of  $\hat{A}$  it is also possible to construct the pooling equilibria  $H^{ef}L^{ef}$  and  $H^{ef}L^e$  where the opportunity cost of entrepreneurship remains below  $\hat{A}$  and, accordingly, the H-type agents' pledgeable income below the L-type agents'.

Between the vertical lines,  $\hat{A}(I/p_H R_H)$  and  $\hat{A}$ , a triangle exists close to the  $h = 1$  border, where only the Pareto-efficient  $H^{ef}L^f$  exists. In this region there are sufficiently many H-types to raise the returns on financing high enough to prevent L-types from becoming entrepreneurs. Proceeding to left, once we cross  $\hat{A}(I/p_H R_H)$  financial markets cease to operate except for a small area close to the  $h = 1$  border. Quite naturally, when  $h$  is sufficiently high, the financial market emerges as an equilibrium institution. The lower is the agents' initial wealth, the higher the needed proportion of H-type agents that prevents the collapse of the financial market.

We then turn to the non-wealth constrained economies – those below the diagonal, and again start from the right. The first equilibrium,  $H^eL^{fs}$ , is Pareto-efficient. All H-type projects are financed, and L-types are indifferent between financing the H-types and using the storage technology. On the right hand side of

the vertical  $p_L R_L$ -line, the storage technology is by assumption more lucrative than entrepreneurship for all L-type agents. Because of costly financing ( $R_B > 0$ ), the storage technology initially remains superior to entrepreneurship for L-type agents on the left of the  $p_L R_L$ -line.

Going further to left, once we hit the vertical  $\bar{A}$ -line, the equilibrium changes. The area to the left of the line supports  $H^e L^{ef}$  which is Pareto-inefficient. Although all H-type agents become entrepreneurs, so do some L-type agents, too. Below the L-type IR-curve (equation (3.13)), the L-type participation constraint is satisfied with equality through some L-types opting for the storage technology. This means that below the curve, the equilibrium is  $H^e L^{efs}$  where all financiers (L-types) earn only  $A$  in expectation, and hence all L-type entrepreneurs also earn only  $A$  in expectation. H-type entrepreneurs' IC (and IR) constraint is given by (3.14).

Above the L-type IR-curve, all L-type agents participate and thus nobody uses the storage technology even though available assets exceed the financing needs of H-type agents. Demand and supply of funds is equated through some L-type agents becoming entrepreneurs. H-types' IC constraint (3.14) and  $\hat{A}(I/p_H R_H)$  cross the diagonal at the same point. To the left these lines, autarky is the only equilibrium in the lower triangle.

A major observation of the above analysis comes from a comparison of the results to the standard model with outside investors (contrast Figure 3 to Figure 1). To the right of  $\bar{A}$  the only difference between outside finance and endogenous finance is in financially constrained economies; there, outside finance allows the execution of all positive NPV projects. To the left of  $\bar{A}$  the difference is stark: apart from the upper left hand corner (close to the y-axis), not having outside investors yields a better outcome than having them. With outside finance, autarky is the equilibrium for a large range of parameter values for which in our model, a financial market emerges in equilibrium. In financially constrained economies, outside finance may lead to autarky where endogenous finance yields Pareto-efficiency.

To conclude the discussion we investigate whether the Pareto-inefficient equilibria are interim efficient in the sense of Holmström and Myerson (1983). In the wealth-constrained economies the pooling equilibria  $H^{ef} L^{ef}$  and  $H^e L^e$  cannot be interim efficient to the right of  $\hat{A}(I/p_H R_H)$  because the Pareto-efficient  $H^{ef} L^f$  exists there. To the left of  $\hat{A}(I/p_H R_H)$ , a social planner cannot induce separation as the pledgeable income of L-type agents exceeds that of H-type agents. Similarly, in the non-wealth constrained economies the pooling equilibria  $H^e L^{ef}$  and  $H^e L^{efs}$  are not interim efficient, because the social planner could achieve Pareto efficiency by imposing a high enough  $R_B (= (p_L R_L - A)/p_L)$ . This would work

because in this region the pledgeable income of H-type agents is higher than that of L-type agents.<sup>7</sup>

We summarize the above discussion in the following proposition:

PROPOSITION 1:

- a. All logically possible six equilibria exist for some set of parameters.
- b. Autarky is the unique equilibrium in both wealth constrained and non-wealth constrained economies
  - 1) To the left of  $\hat{A}(I/p_H R_H)$  and below (A4.18) (see the Appendix for (A4.18)) and
  - 2) Below (3.14) and to the left of  $\hat{A}$ .
- c. Pareto-efficient and inefficient equilibria exist both in wealth constrained and non-constrained economies.
- d. Except for autarky the Pareto-inefficient equilibria to the right of  $\hat{A}(I/p_H R_H)$  are not interim efficient.
- e. Multiple equilibria may only exist between  $\hat{A}(I/p_H R_H)$  and  $\hat{A}$ , and above (3.14).

### 3.3 Rents and contracts

We define economic rents as profits in excess of those that an agent would have earned in her next best occupation; we call informational rents profits to agents of type  $i$  that are in excess of the profits that agents of type  $j$  earn from the same occupation.<sup>8</sup>

A moment of reflection and a look at Figure 3 reveal that H-type agents may only earn economic rents as entrepreneurs in non-wealth constrained economies. There, except in equilibrium  $H^{ef}L^{ef}$ , they strictly prefer entrepreneurship to other occupations, whilst L-type agents are indifferent between at least two occupations. The rents are also informational, as H-type agents earn more from entrepreneurship than L-types.

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<sup>7</sup> If the social planner were allowed to dictate the occupational choices of agents, efficiency could in some cases be further improved. In the region to the left of  $\hat{A}(I/p_H R_H)$ , the planner could raise efficiency by randomly allocating agents into entrepreneurship. This would be feasible when  $h \geq (I - p_L R_L)/(p_H R_H - p_L R_L)$ . Improvements on autarky would also be possible between  $\hat{A}(I/p_H R_H)$  and  $\hat{A}$  and below (3.14), if the social planner were allowed to randomly select agents to use the storage technology. With positive probability, a high-enough proportion of L-type agents would be forced to use the storage technology, pushing the proportion of H-type agents in the remaining population above the threshold  $(A/I)$  needed to obtain economic activity.

<sup>8</sup> These are close, but not necessarily the same as the standard definitions, because in our model, agents can choose their occupation, ie, change from the supply side of a resource to the demand side.

Analogously, it turns out that only L-types may earn rents in wealth-constrained economies. In  $H^{ef}L^f$ , L-type agents earn economic rents as their profits from becoming financiers strictly exceed profits from becoming entrepreneurs, or of not participating. In  $H^{ef}L^e$ , L-types earn both economic and informational rents since they strictly prefer entrepreneurship to financier-ship whereas H-types are indifferent between them.

Concerning contracts forms, different predictions have been obtained in environments reminiscent of ours: in de Meza and Webb (1987), only (pure) debt is an equilibrium contract form, while in Stiglitz and Weiss (1981), equilibrium contracts are equity(-like) (see de Meza and Webb (1987), pp. 291). As already pointed out, our model generates both pure debt and equity-like contracts. As Table 2 shows, there is a clear pattern in how contract form is determined. If neither ICs nor IRs for agents of type  $i$  bind, ie, the agents are on a single occupation, they have pure debt contracts. If agents' IC constraints are binding, their outside option depends on the project return and, consequently, they must have equity-like contracts. If agents' IR constraints are binding, their outside option depends on their initial wealth and, consequently, they must have pure debt contracts. In equilibrium  $H^eL^{efs}$ , where L-types' IC and IR constraints bind, they use equity-like contracts as entrepreneurs and pure debt contracts as financiers. Entrepreneurs and financiers of the same agent type also have different contract forms in equilibrium  $H^{ef}L^{ef}$  where both types' ICs are binding. H-type entrepreneurs and L-type financiers have equity-like contracts whereas H-type financiers and L-type entrepreneurs use "inverse" equity-like contracts.

Table 2. **Expected equilibrium payments**

Equilibrium	$R_B^*$	$R_F^*$
$H^eL^{fs}$	$\frac{I - A}{p_H}$	A
$H^{ef}L^f$	$\frac{R_H(I - A)}{I}$	$p_H R_H \frac{A}{I}$
$H^eL^{ef}$	$\frac{(I - A)}{I[h p_H + (1 - h)p_L]} p_L R_L$	$\left[ 1 - \frac{p_L(I - A)}{I[h p_H + (1 - h)p_L]} \right] p_L R_L$
$H^eL^{efs}$	$\frac{p_L R_L - A}{p_L}$	A
$H^{ef}L^{ef}$	$\frac{p_H R_H - p_L R_L}{p_H - p_L}$	$p_H \left( R_H - \frac{p_H R_H - p_L R_L}{p_H - p_L} \right)$
$H^{ef}L^e$	$\frac{p_H R_H(I - A)}{I(p_L + h(p_H - p_L))}$	$\frac{p_H R_H [p_H A - (p_H - p_L)I(1 - h)]}{I(p_L + h(p_H - p_L))}$



There is also a link between contractual form and rents: agents only earn rents when they use pure debt contracts. H-type entrepreneurs are financed by equity-like contracts in wealth-constrained economies where they do not earn rents but with pure debt contracts in non-wealth constrained economies when they do earn rents. L-types earn rents as financiers in  $H^{ef}L^f$  and as entrepreneurs in  $H^{ef}L^e$  where they employ pure debt contracts, but H-types have equity-like contracts. We can, however, draw no inference about causality relationship between rents and contract form, since they are simultaneously determined as a part of equilibrium.

Since wealth constraints are relative to the amount of positive NPV projects, one can think that in non-wealth constrained economies the high productivity projects are scarce in comparison to initial wealth. Our model predicts that in such economies, financial contracts are pure debt contracts. In contrast, even if absolute initial wealth is high, economies resort to equity-like contracts when the proportion of productive projects becomes “large enough”. If the proportion of high NPV projects is exogenous – as assumed in our model – our model would predict that both over time and over a cross section of economies, those periods or countries that can be characterized by a relatively high proportion of high NPV projects should also be characterized by wide-spread use of equity-like contracts.

We summarize our findings regarding rents and contracts in the following proposition:

**PROPOSITION 2:**

- a. H-type agents may earn rents only in non-wealth constrained economies; L-type agents may earn rents only in wealth-constrained economies.
- b. H-type agents earn rents only as entrepreneurs; L-type agents may earn rents as entrepreneurs and as financiers.
- c. When agents earn rents as entrepreneurs, economic and informational rents coincide.
- d. Agents only earn rents when they use pure debt contracts.
- e. When agents of type  $i$  split in their occupational choices in equilibrium, they use equity-like contracts.
- f. A contract can be an equity-like contract for type  $i$  entrepreneurs while simultaneously being a pure debt contract for type  $i$  financiers.

## 4 Imperfect storage technology

So far we have maintained the standard assumption of an exogenous storage technology that fully converts the initial assets to consumption goods. The existence of such a storage technology is vital in the literature that studies the stability of the banking system. For instance, it is known that removing the storage technology eliminates bank runs in Diamond and Dybvig's (1983) model and its variations. To verify whether our findings are sensitive to the efficiency of the storage technology, we now assume that there is an imperfect storage technology under which  $A$  depreciates at rate  $1 - \delta$ ,  $\delta \in [0, 1]$ .

The only difference to the general model is that the agents' IR constraints (3.1) should be rewritten as

$$\pi_i^j \geq \delta A \quad \forall i, j, \quad i \in \{H, L\}, \quad j \in \{e, f\} \quad (4.1)$$

When  $\delta$  is close to unity, our previous analysis is robust to the introduction of imperfect storage technology by continuity. As one might expect, however, the equilibria will change if  $\delta$  becomes small, because all agents are willing to invest either as financiers or as entrepreneurs even if their returns are small. To get an idea of the changes, let us reconsider the example of Section 3.1 ( $H^e L^{ef}$ ) under an imperfect storage technology. To guarantee that all agents participate, we require that

$$R_F \geq \delta A \quad (4.2)$$

All other equations remain unchanged except that the L-type IR constraint (3.13) now takes the form

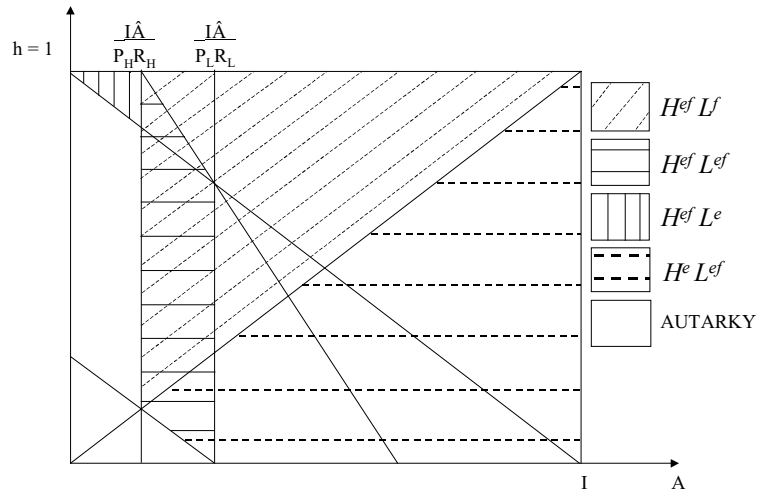
$$A \leq \frac{p_L R_L \text{Ih}(p_H - p_L)}{\text{Ih}(p_H - p_L) + p_L (\delta I - p_L R_L)} \quad (4.3)$$

When  $\delta$  is close to unity, (4.3) remains a monotonically upward sloping curve in the  $(A, h)$ -space. Decreasing  $\delta$  shifts the curve to the right, increasing the range of parameters where  $H^e L^{ef}$  exists. It can be shown that when  $\delta$  approaches zero,  $H^e L^{ef}$  exists for all parameter values in the non-wealth constrained region in so far as H-types' IC constraint (3.14) holds. This is quite natural, since without a storage technology, the L-type IR constraint is trivially satisfied.

We now report the changes for the other equilibria when  $\delta = 0$ . Besides shortening the discussion, letting  $\delta = 0$  further generalizes our model. There is no longer an exogenous storage technology, but the only way to transfer wealth over time, and to transform it from initial wealth to a consumption good, is to invest

either as an entrepreneur or as a financier. Although our model lacks the second investment period, this is similar in spirit to Holmström and Tirole (1998) who evaluate whether financial markets alone are able to supply enough liquidity.

Figure 4.



The results of this exercise are summarized in Figure 4 (the calculations are available from the authors upon request). Obviously,  $H^e L^{fs}$  and  $H^e L^{efs}$ , the equilibria where the storage technology was a viable option, cease to exist. Instead,  $H^{ef} L^f$ ,  $H^{ef} L^{ef}$  and  $H^{ef} L^e$  exist for larger parameter value ranges.  $H^{ef} L^f$  remains unchanged. Thus, removing the storage technology causes only modest changes to financial market performance, suggesting that financial markets alone can take care of transformation of wealth over time. The largest effect is an efficiency effect: the inefficient  $H^e L^{ef}$  exists where the efficient  $H^e L^{fs}$  used to exist. As a result, only Pareto-inefficient equilibria exist in non-wealth constrained economies.

## 5 Policy implications

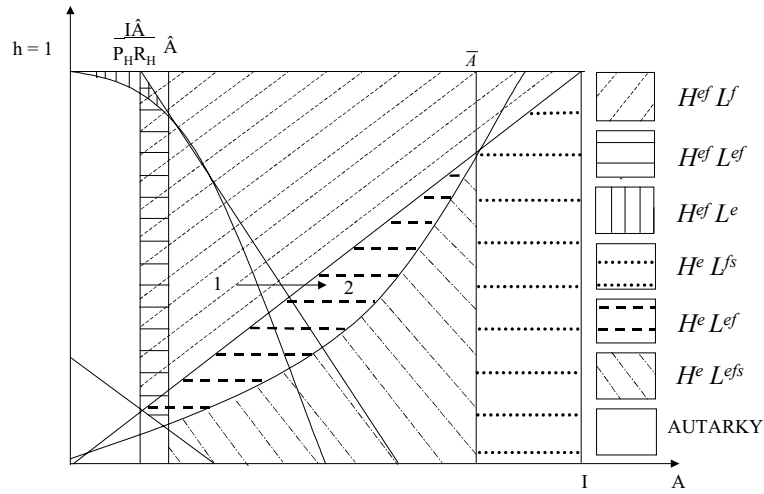
Though there are several limitations<sup>9</sup> to our simple model, we boldly offer some policy recommendations. The first deals with widely adopted policies that seek to promote entrepreneurship (see, eg, European Commission, 2001). A straightforward “policy experiment” is to move the vertical  $\bar{A} \equiv p_L(p_H R_L - I)/(p_H - p_L)$  -line to left. This increases the set of parameter values for which a Pareto-efficient equilibrium exists in non-wealth constrained economies. The worse is either the return on the successful projects of L-type agents or the smaller their probability of success, or the higher is the success probability of H-type agents’ projects, the smaller are the problems created by adverse selection. In the limit, the expected value of the L-type agents’ projects is zero; this is practically equivalent to assuming that agent type is observable ex ante. Our experiment thus suggest that if the policies promoting entrepreneurship raise the returns on the successful L-type projects without succeeding in making them positive NPV projects, they may be misguided. This also means that a progressive tax on firm profits above  $R_H - R_B^*$  may help restore efficiency (for a similar finding, see de Meza and Webb, 1987).

Shocks to model parameters change the values of the endogenous variables even if the equilibrium type remains the same as before the shock. When the parameters initially are close to a border, even small shocks may change the type of equilibrium. It is clear that in our model, decreases in initial wealth may shift the economy from a Pareto-efficient equilibrium to an inefficient one (eg, from  $H^e L^{fs}$  to  $H^e L^{efs}$ ), or even to autarky (eg, from  $H^{ef} L^f$  to autarky). But it is also possible that an increase in initial wealth may have the same effect. Think of an economy that is in a Pareto-efficient  $H^{ef} L^f$  equilibrium (point 1 in Figure 5). Increasing initial wealth may move such an economy to the Pareto-inefficient  $H^e L^{ef}$  (point 2), if the increase is sufficient to turn a wealth constrained economy into an unconstrained one. Even substantial increases in initial wealth may result in such an adverse shift. One could interpret such policies as development aid, as the increase in initial wealth comes from outside the economy in question. Such policies will result in an increase of aggregate production, but possibly at the cost of inefficiency. However, increasing initial wealth is an effective tool in raising an economy out of autarky.

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<sup>9</sup> For instance, future work should consider more than two types of agents, allow variation in agents’ initial wealth, and render the model more dynamic.

Figure 5.



In contrast, increases in the proportion of good entrepreneurs always have a beneficial effect on the economy. Keeping initial wealth constant, an increase in  $h$  either keeps the economy in the initial equilibrium category, or moves it from the existing equilibrium into a better one. In other words, it is not necessarily disadvantageous for an economy to be wealth constrained. In our model it is a relative measure, indicating that the economy has a large number of positive NPV projects in relation to its initial wealth. Policies that help raising the proportion of positive NPV projects in the economy are always beneficial, even if they lead to aggregate wealth constraints.

Finally, let us consider the role for financial intermediaries that collect and analyze information. In Bernanke and Gertler (1989, 1990) and Holmström and Tirole (1997, 1998) the financial intermediaries mitigate the moral hazard problem. In our model where the source of asymmetry is adverse selection instead of moral hazard, there is no need for complicated financial institutions in the Pareto efficient equilibria. In this respect our findings contrast with the common view that moral hazard and adverse selection are qualitatively similar problems. However, financial intermediaries could improve the functioning of the economy when we have low initial wealth (autarky), moderate initial wealth ( $H^{ef}L^{ef}$ ), or moderate to high initial wealth and a non-wealth constrained economy ( $H^eL^{ef}$ ). This is in line with the work in financial development suggesting that banks are important in less developed economies, while financial markets become important in developed economics (see Levine, 1997, and Allen and Gale, 2001, for surveys). The welfare-improving prospects of financial intermediaries also increase when financial markets alone have to perform transformation of wealth over time (ie, when  $\delta = 0$ ).

Perhaps the most surprising rationale for financial intermediaries comes from the observation that competitive financial markets can drive interest rates too low from an efficiency point of view. As in de Meza and Webb (1987), we show in Section 2 how competition between outside financiers results in the oversupply of funds for a wide range of parameter values. Such oversupply of funds also occurs with endogenous finance in non-wealth constrained economies where competition between agents who become financiers generates the pooling equilibria  $H^eL^{ef}$  and  $H^eL^{efs}$ .

## 6 Conclusions

In this paper we study whether financial markets can endogenously emerge in equilibrium, whether the eventual markets are efficient, and what the equilibrium forms of financial contracts are. We take the basic building blocks of our model from the literature and supplement them by endowing all agents with initial wealth and allowing them to choose their occupations. In the usual partial equilibrium setting, the only equilibria are autarky and a Pareto-inefficient equilibrium where all agents with a project become entrepreneurs. We first show that merely allowing potential entrepreneurs to have initial wealth may lead to a Pareto-efficient equilibrium if the initial wealth is high enough. We then exclude outside investors and find that Pareto-efficient equilibria exist both in wealth constrained and non-wealth constrained economies, as do Pareto-inefficient equilibria. Autarky is also an equilibrium, but only for the poorest economies where agents' initial wealth is low. In this respect, it makes only a modest difference whether there are "many" or "few" good projects in the population. We show that excluding outside investors (mostly) improves the performance of financial markets, sometimes to the extent that where existence of outside finance leads to autarky, excluding it yields Pareto-efficiency.

There also turns out to be a link between rents and contracts, and contract type and the proportion of high-type agents in the economy. Agents only earn rents when they use pure debt contracts (ie, contracts where the payment is not a function of the project outcome, conditional on success). When agents split in their occupational choices in equilibrium, they use equity-like contracts where the sum that a successful entrepreneur pays to her financiers is a function of her project's project outcome. High-type agents may earn economic and informational rents and use pure debt contracts only in non-wealth constrained economies. In wealth-constrained economies that have a high proportion of high-type agents, they do not earn rents and are financed by equity-like contracts, whereas low-type agents may earn rents and have pure debt contracts only there.

Our model shows that, in the face of asymmetric information, the simplest type of financial markets may perform their role in resource allocation and asset transformation surprisingly well, and that while increasing the proportion of high-quality entrepreneurs is a remedy for removing inefficiency, increasing the initial wealth of an economy may not be.

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# Appendix

In this appendix, we go through all possible equilibria besides autarky. For each equilibrium, we present

- the constraints,
- the equilibrium values of endogenous variables, and
- the equilibrium existence conditions.

We shorten the exposition by using the following notation:  $\Delta p = p_H - p_L$ ,  $\Delta R = R_L - R_H$ ,  $\gamma = p_H R_H - I$ ,  $\lambda = I - p_L R_L$ , and  $\Delta W = \gamma + \lambda = p_H R_H - p_L R_L$ . The definitions have obvious interpretations. Since our approach to solve the model is rather mechanical, we explain the solution for the first equilibrium in more detail than for the subsequent equilibria. We also omit intermediate steps as these are straightforward (albeit sometimes tedious).

## Appendix 1

### $H^e L^{ef}$ and $H^e L^{efs}$

$H^e L^{ef}$  is described in the example in Section 3.1, so we first define  $H^e L^{efs}$  and then merely characterize its relation to  $H^e L^{ef}$ . In  $H^e L^{efs}$ , all H-types are entrepreneurs and L-types are indifferent among entrepreneurship, financing, and using the storage technology, ie,  $\mu_H = 1$ ,  $\mu_L \in (0, 1)$ ,  $\chi_H = 0$  and  $\chi_L \in (0, 1)$ . The situation here is otherwise similar to  $H^e L^{ef}$  described in Section 3.1 except that  $\chi_L$  is strictly positive. This means that (3.5) must hold as an equality, ie,

$$R_F = A. \quad \text{”L- and H-type IR”} \quad (A1.1)$$

The agents’ IC constraints are as before in (3.5) and (3.7), ie,

$$p_L(R_L - R_B) = R_F. \quad \text{”L-type IC”} \quad (A1.2)$$

$$p_H(R_H - R_B) \geq R_F. \quad \text{”H-type IC”} \quad (A1.3)$$

The economy level “budget constraint” (3.8) becomes

$$(1 - \mu_L - \chi_L)(1 - h)A = [h + \mu_L(1 - h)](I - A) \quad (A1.4)$$

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and, similarly, the financial market equilibrium condition (3.9) is

$$[hp_H + \mu_L(1-h)p_L]R_B = R_F(1-\mu_L-\chi_L)(1-h) \quad (A1.5)$$

“Financial market transactions”

Conditions (A1.1)–(A1.5) constrains  $H^eL^{efs}$ . Equation (A1.3) restricts the range of parameters and an equation system consisting of (A1.1), (A1.2), (A1.4) and (A1.5) determines the values of the endogenous variables  $R_B$ ,  $R_F$ ,  $\chi_L$ , and  $\mu_L$ . The equilibrium value of the expected payment received by financier,  $R_G^*$ , equals  $A$  by (A1.1). Then, solving (A1.1) and (A1.2) for  $R_B$  gives.

$$R_B^* = \frac{p_L R_L - A}{p_L} \quad (A1.6)$$

Upon substituting (A1.1) and (A1.6) into (A1.5) we have two equations, (A1.4) and (A1.5), that determine the remaining two endogenous variables,  $\chi_L$ , and  $\mu_L$ . After somewhat involved algebra they can be written as

$$\mu_L^* = \frac{h}{1-h} \left[ \frac{(p_L R_L - A)\Delta p}{p_L \lambda} - 1 \right] = \frac{h}{1-h} \left( \frac{R_B^* \Delta p}{\lambda} - 1 \right) \quad (A1.7)$$

and

$$\chi_L^* = \frac{1}{1-h} \left[ 1 - \frac{(p_L R_L - A)\Delta p h}{p_L \lambda A} \right] = \frac{1}{1-h} \left( 1 - \frac{R_B^* \Delta p h}{\lambda A} \right) \quad (A1.8)$$

where the last equalities come from (A1.6).

The equilibrium exists if  $\chi_L^*$  and  $\mu_L^*$  given by (A1.7) and (A1.8) satisfy our initial assumptions that  $\mu_L \in (0, 1)$  and  $\chi_L \in (0, 1)$ , and if the agents' IC and IR constraints are satisfied with  $R_B^*$  given by (A1.6). The first four existence conditions are

$$\mu_L^* < 1 \Leftrightarrow A > p_L \left( R_L - \frac{\lambda}{h\Delta p} \right) \quad (A1.9)$$

$$\mu_L^* > 0 \Leftrightarrow A < p_L \left( R_L - \frac{\lambda}{\Delta p} \right) = \hat{A} + \frac{p_L \gamma}{\Delta p} = \bar{A} \quad (A1.10)$$

$$\chi_L^* < 1 \Leftrightarrow A < \frac{p_L R_L I \Delta p}{I \Delta p + p_L \lambda} = \frac{p_L R_L I \Delta p}{p_H I - p_L^2 R_L} \quad (\text{A1.11})$$

and

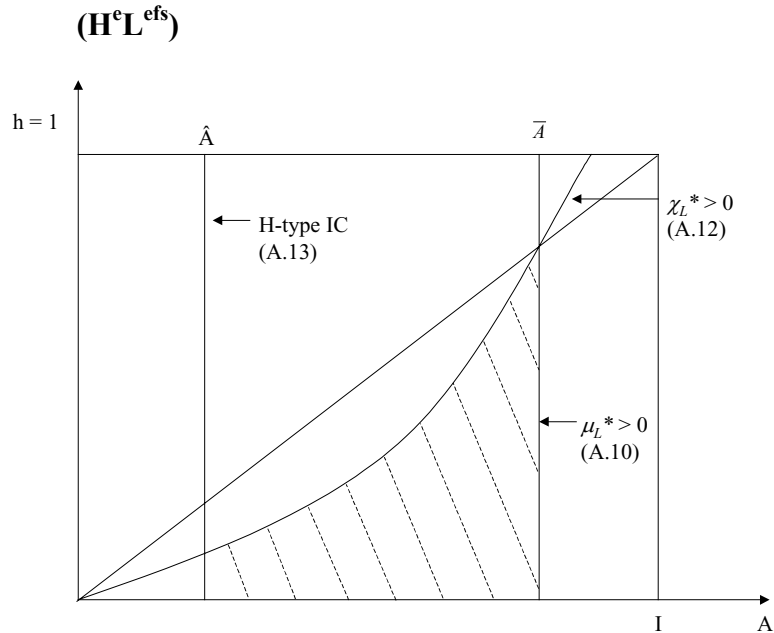
$$\chi_L^* > 0 \Leftrightarrow A > \frac{p_L R_L I h \Delta p}{I h \Delta p + p_L \lambda} = \frac{p_L R_L I h \Delta p}{I(p_L + h \Delta p) - p_L^2 R_L} \quad (\text{A1.12})$$

Since L-type IC and IR bind by (A1.1) and (A1.2), the fifth existence condition comes from H-type IC (A1.3). If it is satisfied, H-type IR (A1.1) also trivially holds. Inserting (A1.1) and (A1.6) into (A1.3) shows that H-type IC holds if

$$A \geq \hat{A} \quad (\text{A1.13})$$

Equations (A1.9)–(A1.13) define the range of parameters for which  $H^e L^{efs}$  exists. Since the critical values of  $A$  in (A1.11) and (A1.12) are strictly larger than the respective critical values in (A1.10) and (A1.9), the binding critical values are given by (A1.10) and (A1.12). They in turn cross each other at the diagonal  $h = A/I$ . This means that  $H^e L^{efs}$  only exists in non-wealth constrained economies. In terms of the  $(A, h)$ -space,  $H^e L^{efs}$  exists in the area between the vertical lines (A1.13) and (A1.10), and below the curve (A1.12), as depicted in Figure A1.1.

Figure A1.1.



When (A1.12) (which is identical to equation (3.13)) is violated, the H-type IC changes from (A1.13) to (3.14), ie, to  $A \geq I - I \Delta W(p_L + h \Delta p) / \Delta p p_L R_L$ . Thus,  $H^e L^{ef}$  exists in the range of parameters described in Section 3.1, ie, in the area

shaped by curve (A1.12), the downward sloping line (3.14) and  $h = A/I$  diagonal. Note also that curve (A1.12), the vertical  $\hat{A}$  line and the downward sloping line cross at the same point where  $h = h_1 \equiv \frac{\hat{A}\lambda}{I\Delta W}$ .

## Appendix 2

$H^{ef}L^f$  and  $H^{efs}L^{fs}$

We first prove that  $H^{efs}L^{fs}$  cannot exist. In this equilibrium  $\mu_H \in (0,1)$ ,  $\mu_L = 0$  and both  $\chi_H$  and  $\chi_L \in (0,1)$ . The equilibrium is constrained by the following five conditions:

$$R_F \geq A \quad \text{”L- and H-type IR”} \quad (A2.1)$$

$$p_L(R_L - R_B) \leq R_F \quad \text{”L-type IC”} \quad (A2.2)$$

$$p_H(R_H - R_B) = R_F \quad \text{”H-type IC”} \quad (A2.3)$$

$$[(1 - \mu_H - \chi_H)h + (1 - \chi_L)(1 - h)]A = \mu_H h(I - A) \quad (A2.4)$$

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and

$$h\mu_H p_H R_B = R_F [(1 - \mu_H - \chi_H)h + (1 - \chi_L)(1 - h)] \quad (A2.5)$$

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In  $H^{efs}L^{fs}$  (A2.1) holds with equality. Solving (A2.4) for  $\mu_H$  yields

$$\mu_H = \frac{A}{hI} [1 - \chi_H h - \chi_L (1 - h)] \quad (A2.6)$$

Using (A2.3) and (A2.6) in (A2.5) yields  $R_B$  as

$$R_B^* = \frac{R_H(I - A)}{I} \quad (A2.7)$$

Inserting (A2.7) back into (A2.3) gives

$$R_F^* = p_H R_H \frac{A}{I} \quad (\text{A2.8})$$

Since  $R_F^*$  in (A2.8) is strictly larger than  $A$ , the initial assumption (A2.1) that the agents' IR constraints binds is invalid. This means that the equilibrium cannot exist for positive  $\chi_H$  and  $\chi_L$ .

$H^{ef}L^f$  can be characterized by setting  $\chi_H = \chi_L = 0$  in (A2.6). This means that

$$\mu_H^* = \frac{A}{hI} \quad (\text{A2.9})$$

Equation (A2.9) gives two equilibrium existence conditions:

$$\mu_H^* < 1 \Leftrightarrow A < hI \quad (\text{A2.10})$$

and

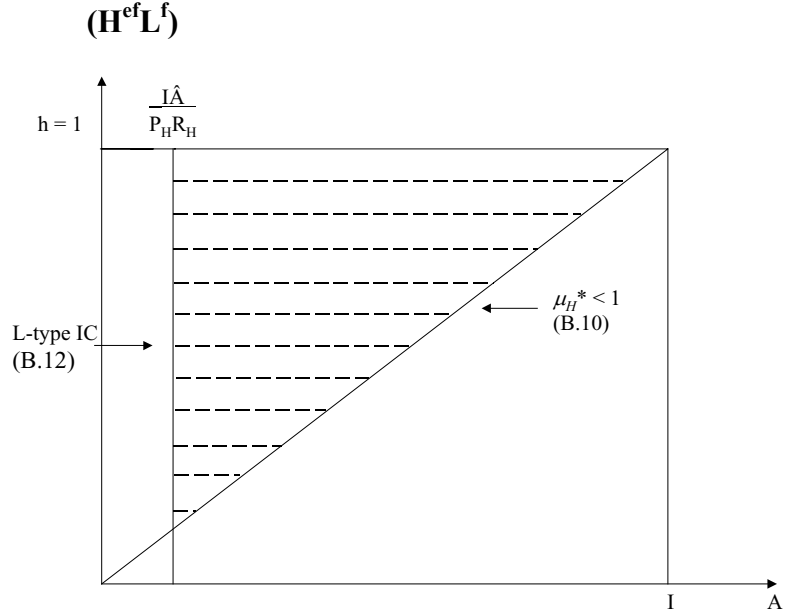
$$\mu_H^* > 0 \Leftrightarrow A > 0 \quad (\text{A2.11})$$

By means of (A2.7) and (A2.8) the third existence condition, the L-type IC constraint (A2.2), can be written as

$$A \geq \frac{I}{p_H R_H} \hat{A} \quad (\text{A2.12})$$

Equations (A2.10)–(A2.12) define the range of parameters for which  $H^{ef}L^f$  exists. As shown in Figure A2.1, the equilibrium exists in wealth constrained economies for  $A \in [\hat{A}/p_H R_H, I)$ .

Figure A2.1



### Appendix 3

$H^e L^{fs}$

In this equilibrium,  $\mu_H = 1$ ,  $\mu_L = 0$ ,  $\chi_H = 0$  and  $\chi_L \in (0, 1)$ . In words, all H-types are entrepreneurs and L-types are either financiers or use the storage technology. The five basic conditions constraining the equilibrium are

$$R_F = A \quad \text{“L-type IR”} \quad (\text{A3.1})$$

$$p_L(R_L - R_B) \leq R_F \quad \text{“L-type IC”} \quad (\text{A3.2})$$

$$p_H(R_H - R_B) \geq R_F \quad \text{“H-type IC and IR”} \quad (\text{A3.3})$$

$$(1 - \chi_L)(1 - h)A = h(I - A) \quad (\text{A3.4})$$

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and

$$hp_H R_B = R_F(1 - \chi_L)(1 - h) \quad (\text{A3.5})$$

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The equilibrium value of  $R_F$  trivially equals  $A$  by (A3.1). By substituting (A3.1) into (A3.5), the other endogenous variables,  $\chi_L$ , and  $R_B$ , can be solved from (A3.4) and (A3.5). They are given by



$$\chi_L^* = \frac{A - hI}{A(1 - h)} \quad (\text{A3.6})$$

and

$$R_B^* = \frac{I - A}{P_H} \quad (\text{A3.7})$$

From (A3.6) we see that  $\chi_L^* < 1$  by assumption  $A < I$ . Similarly, inserting (A3.1) and (A3.7) into (A3.3) shows that H-types' IC and IR constraints are equivalent to  $p_H R_H > I$  which holds by assumption. Thus,  $H^e L^{fs}$  is defined by two existence conditions. First,

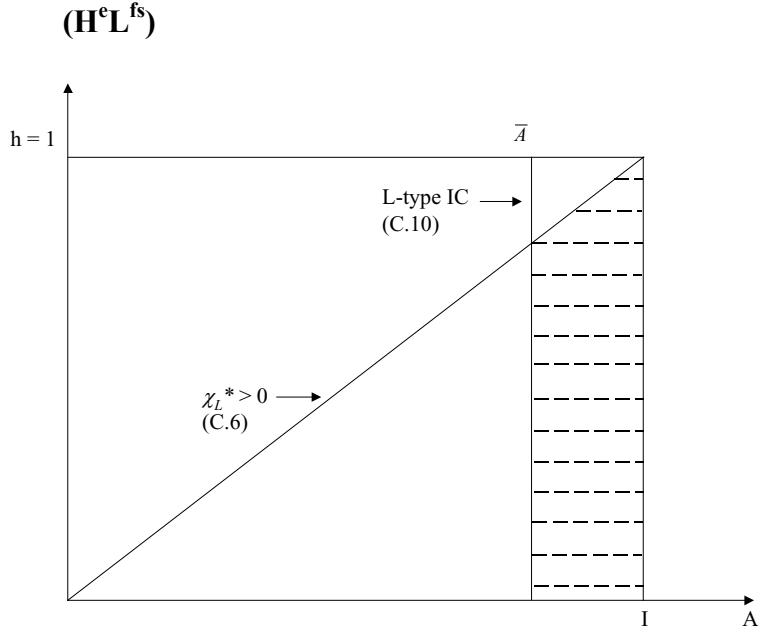
$$\chi_L^* \geq 0 \Leftrightarrow A \geq hI \quad (\text{A3.8})$$

must hold. Second, the L-type IC constraint (A3.2) must hold. Employing (A3.1) and (A3.7), it can be rewritten as

$$A \geq \hat{A} + \frac{p_L \gamma}{\Delta p} = \bar{A} \quad (\text{A3.9})$$

where the right hand side equals (A1.10). Equations (A3.8) and (A3.9) show that  $H^e L^{fs}$  only exists in non-wealth constrained economies for  $A \in [\hat{A} + p_L \gamma / \Delta p, I)$  (see Figure A3.1).

Figure A3.1



## Appendix 4

$H^{ef}L^e$  and  $H^{efs}L^e$

We first prove that  $H^{efs}L^e$  cannot exist. The set-up of  $H^{efs}L^e$  practically mirrors  $H^eL^{efs}$  of Appendix 1, because here  $\mu_H \in (0,1)$ ,  $\mu_L = 1$ ,  $\chi_H \in (0,1)$  and  $\chi_L = 0$ . In words, all L-types are entrepreneurs, and H-types are indifferent between entrepreneurship, financing, and using the storage technology. The five basic constraints in  $H^{efs}L^e$  are

$$R_F = A \quad \text{“L- and H-type IR”} \quad (A4.1)$$

$$p_L(R_L - R_B) \geq R_F, \quad \text{“L-type IC”} \quad (A4.2)$$

$$p_H(R_H - R_B) = R_F, \quad \text{“H-type IC”} \quad (A4.3)$$

$$(1 - \mu_H - \chi_H)hA = [1 - h + \mu_H h](I - A) \quad (A4.4)$$

“Equality of supply and demand for funds”,

and

$$[h\mu_H p_H + (1 - h)p_L]R_B = R_F(1 - \mu_H - \chi_H)h \quad (A4.5)$$

“Financial market transactions”

An equation system consisting of (A4.1) and (A4.3)–(A4.5) determines the values of the endogenous variables,  $R_B$ ,  $\chi_H$ , and  $\mu_H$ . Solving (A4.1) and (A4.3) for  $R_B$  gives

$$R_B^* = \frac{p_H R_H - A}{p_H} \quad (\text{A4.6})$$

Upon substituting (A4.1) and (A4.6) into (A4.5) we have two equations (A4.4) and (A4.5) that determine the remaining two endogenous variables,  $\chi_H$ , and  $\mu_H$ . After somewhat involved algebra they can be written as

$$\mu_H^* = \frac{1-h}{h} \left[ 1 - \frac{\Delta p (p_H R_H - A)}{p_H \gamma} \right] = \frac{1-h}{h} \left( 1 - \frac{R_B^* \Delta p}{\gamma} \right) \quad (\text{A4.7})$$

and

$$\chi_H^* = \frac{1}{h} \left[ 1 - \frac{\Delta p (p_H R_H - A) I (1-h)}{p_H \gamma A} \right] = \frac{1}{h} \left[ 1 - \frac{R_B^* \Delta p I (1-h)}{\gamma A} \right] \quad (\text{A4.8})$$

Equations (A4.7) and (A4.8) provide four equilibrium existence conditions:

$$\mu_H^* \leq 1 \Leftrightarrow A \leq \frac{p_H [h\gamma + (1-h)\lambda]}{\Delta p (1-h)} \quad (\text{A4.9})$$

$$\mu_H^* \geq 0 \Leftrightarrow A \geq \frac{p_H (I - p_L R_H)}{\Delta p} \quad (\text{A4.10})$$

$$\chi_H^* \leq 1 \Leftrightarrow A \leq \frac{p_H R_H I \Delta p}{I \Delta p + p_H \gamma} = \frac{p_H R_H I \Delta p}{p_H^2 R_H - p_L I} \quad (\text{A4.11})$$

and

$$\chi_H^* \geq 0 \Leftrightarrow A \geq \frac{p_H R_H I (1-h) \Delta p}{I (1-h) \Delta p + p_H \gamma} = \frac{p_H R_H I (1-h) \Delta p}{p_H^2 R_H - I (p_L + h \Delta p)} \quad (\text{A4.12})$$

Since H-types' IC and IR bind, and L-types' IR is satisfied through their IC, the L-type IC is the fifth equilibrium existence condition. It is satisfied if

$$A \leq \hat{A} \quad (\text{A4.13})$$

The equilibrium may exist between the vertical lines (A4.10) and (A4.11), which is a nonempty set. However, the vertical line (A4.13) is smaller in value than the vertical line (A4.10). This means that the equilibrium cannot exist for positive  $\chi_H$ .

In contrast,  $H^{ef}L^e$  does exist.<sup>10</sup> To see this, let  $\chi_H = 0$  in (A4.4) to get

$$\mu_H^* = \frac{A - (1-h)I}{hI} \quad (A4.14)$$

Substituting (A4.14) and (A4.3) for (A4.5) and letting  $\chi_H = 0$  yields

$$R_B^* = \frac{p_H R_H (I - A)}{I(p_L + h\Delta p)} \quad (A4.15)$$

Inserting (A4.15) back into (A4.3) gives

$$R_F^* = \frac{p_H R_H [p_H A - \Delta p I (1-h)]}{I(p_L + h\Delta p)} \quad (A4.16)$$

From (A4.14) we see that  $\mu_H^* < 1$  holds by our assumption that  $A < I$ . An equilibrium existence condition is thus

$$\mu_H^* \geq 0 \Leftrightarrow A \geq (1-h)I \quad (A4.17)$$

The H-type IR is now  $R_F \geq A$ , which – using (A4.16) – can be expressed as

$$A \geq \frac{p_H R_H I \Delta p (1-h)}{I \Delta p (1-h) + p_H \gamma} = \frac{p_H R_H I \Delta p (1-h)}{p_H^2 R_H - I[p_L + h\Delta p]} \quad (A4.18)$$

Similarly, by means of (A4.15) and (A4.16) the L-type IC (A4.2) is given by

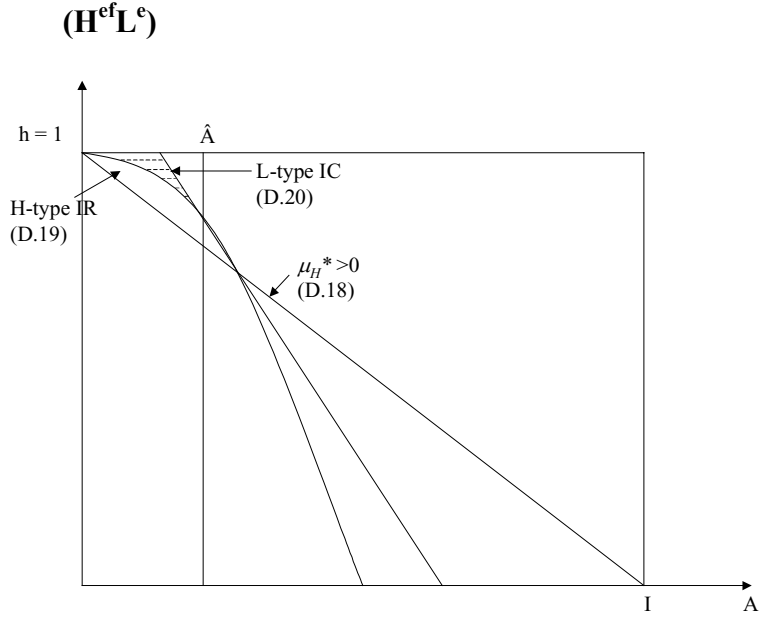
$$A \leq I - \frac{I \Delta W (p_L + h\Delta p)}{\Delta p p_H R_H} = \frac{I}{p_H R_H} [\hat{A} + \Delta W (1-h)] \quad (A4.19)$$

Conditions (A4.17)–(A4.19) define the range of parameters for which  $H^{ef}L^e$  exists. This is shown in the  $(A, h)$ -space in Figure A4.1.

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<sup>10</sup> Note that in  $H^{ef}L^e$ , (A4.1) holds with a weak inequality.

Figure A4.1



Conditions (A4.17)–(A4.19) practically mirror those of  $H^e L^e$  described in Section 3.1. Equation (A4.17) defines the downward sloping  $h = 1 - A/I$  diagonal that starts from the  $(A = 0, h = 1)$  corner and ends in the  $(A = I, h = 0)$  corner. The L-type IC constraint (A4.19) is a downward sloping line that cuts the  $h = 1 - A/I$  diagonal at the same point as the vertical  $\hat{A}/p_L R_L$  line. H-types’ IR constraint (A4.18) is a monotonically downward sloping curve that starts from the  $(A = 0, h = 1)$  corner and cuts the  $h = 1 - A/I$  diagonal once. H-types’ IR and L-types’ IC constraints and the vertical  $\hat{A}$  line cross at the same point at  $h = h_2 \equiv 1 - \hat{A} \frac{\gamma}{\Delta W I}$ . In sum,  $H^e L^e$  exists above the H-type IR curve (A4.18) and below the L-type IC line (A4.19). This area exists in the upper-left corner of the  $(A, h)$ -space where  $A \in [0, \hat{A}]$  and  $h \in [h_2, 1]$ .

## Appendix 5

### $H^e L^e$ and $H^{efs} L^{efs}$

We first prove that  $H^{efs} L^{efs}$  cannot exist for a non-trivial set of parameters. In this equilibrium  $\mu_H \in (0, 1)$ ,  $\mu_L \in (0, 1)$ ,  $\chi_H \in (0, 1)$ ,  $\chi_L \in (0, 1)$ . In words, all agents are indifferent between entrepreneurship, financing, and using the storage technology. The agents’ IR and IC constraints bind, ie, it must hold that

$$R_F = A, \quad \text{“L- and H-type IR”} \quad (\text{A5.1})$$

$$p_L(R_L - R_B) = R_F, \quad \text{“L-type IC”} \quad (\text{A5.2})$$

and

$$p_H(R_H - R_B) = R_F. \quad \text{“H-type IC”} \quad (\text{A5.3})$$

Solving (A5.2)–(A5.3) for  $R_B$  gives

$$R_B^* = \frac{\Delta W}{\Delta p} \quad (\text{A5.4})$$

Thus, there is a unique value of

$$A = p_H \left( R_H - \frac{\Delta W}{\Delta p} \right) = p_L \left( R_L - \frac{\Delta W}{\Delta p} \right) = \hat{A} \quad (\text{A5.5})$$

for which this equilibrium can exist. This means that only  $H^{ef}L^{ef}$  (where  $\mu_H \in (0,1)$ ,  $\mu_L \in (0,1)$ , and  $\chi_H = \chi_L = 0$ ) may exist for a non-trivial range of parameters.

$H^{ef}L^{ef}$  is constrained by the following five basic conditions:

$$R_F \geq A, \quad \text{“L- and H-type IR”} \quad (\text{A5.6})$$

$$p_L(R_L - R_B) = R_F, \quad \text{“L-type IC”} \quad (\text{A5.7})$$

$$p_H(R_H - R_B) = R_F, \quad \text{“H-type IC”} \quad (\text{A5.8})$$

$$[(1 - \mu_H)h + (1 - \mu_L)(1 - h)]A = [\mu_L(1 - h) + \mu_H h](I - A) \quad (\text{A5.9})$$

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and

$$[(1 - \mu_H)h + (1 - \mu_L)(1 - h)]R_F = [p_L \mu_L(1 - h) + p_H \mu_H h]R_B \quad (\text{A5.10})$$

“Financial market transactions”

Equation system (A5.7)–(A5.10) determines the values of the endogenous variables,  $R_F$ ,  $R_B$ ,  $\mu_L$ , and  $\mu_H$ . Solving (A5.7) and (A5.8) for  $R_B$  and  $R_F$  gives

$$R_B^* = \frac{\Delta W}{\Delta p} \quad (\text{A5.11})$$

and

$$R_F^* = p_H \left( R_H - \frac{\Delta W}{\Delta p} \right) = p_L \left( R_L - \frac{\Delta W}{\Delta p} \right) = \hat{A} \quad (\text{A5.12})$$

Substituting (A5.11) and (A5.12) into (A5.10) and solving (A5.9) and (A5.10) for  $\mu_L$  and  $\mu_H$  yields

$$\mu_H^* = \frac{1}{h\Delta W} \left( \hat{A} - \frac{Ap_L R_L}{I} \right) \quad (\text{A5.13})$$

and

$$\mu_L^* = \frac{1}{(1-h)\Delta W} \left( \frac{Ap_H R_H}{I} - \hat{A} \right) \quad (\text{A5.14})$$

Equations (A5.13) and (A5.14) yield four equilibrium existence conditions:

$$\mu_H^* < 1 \Leftrightarrow A > \frac{I}{p_L R_L} (\hat{A} - h\Delta W) \quad (\text{A5.15})$$

$$\mu_H^* > 0 \Leftrightarrow A < \frac{I}{p_L R_L} \hat{A} \quad (\text{A5.16})$$

$$\mu_L^* < 1 \Leftrightarrow A < \frac{I}{p_H R_H} [\hat{A} + (1-h)\Delta W] \quad (\text{A5.17})$$

and

$$\mu_L^* > 0 \Leftrightarrow A > \frac{I}{p_H R_H} \hat{A} \quad (\text{A5.18})$$

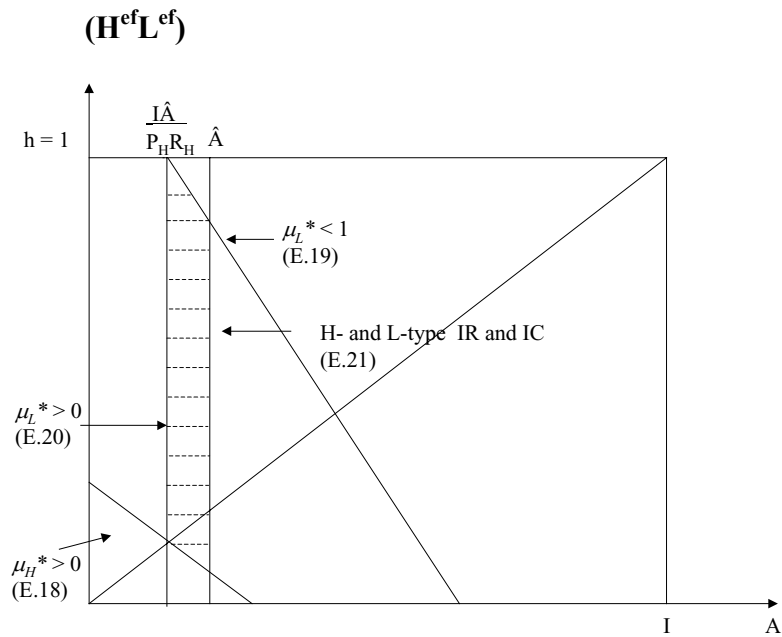
From equations (A5.6)–(A5.8) we see that agents' IC constraints bind and IR constraints are satisfied if

$$A \leq \hat{A} \quad (\text{A5.19})$$

This is the fifth equilibrium existence condition. However, we see that if condition (A5.19) holds, (A5.16) also holds. The equilibrium is thus defined by equations (A5.15), and (A5.17)–(A5.19). Since (A5.17) is identical to (A4.19) we know that

it cuts the vertical  $\hat{A}$ -line at  $h = h_2$  where  $h_2 \equiv 1 - \frac{\hat{A}\gamma}{\Delta W I}$  as defined in Section 4 of the appendix. This means that when  $h$  is large, ie,  $h \in [h_2, 1]$ , the downward sloping line (A5.17) and the vertical line (A5.18) are the binding constraints. For  $h \in [h_2, h_3]$  where  $h_3 \equiv \frac{\hat{A}}{p_H R_H}$ , the binding constraints are the vertical lines (A5.18) and (A5.19). For  $h \in [h_1, h_3]$ , where  $h_1 \equiv \frac{\hat{A}\lambda}{I\Delta W}$  as defined in Appendix 1, the binding constraints are (A5.15) (which is identical to (3.13)) and (A5.19). For  $h < h_1$ , the equilibrium does not exist, since (A5.15) is violated.

Figure A5.1



In Figure A5.1 we illustrate how in terms of the  $(A, h)$ -space,  $H^{ef}L^{ef}$  exists in a parallelogram between the vertical lines (A5.18) and (A5.19) and the downward sloping lines (A5.15) and (A5.17). This parallelogram exists for  $A \in (\hat{A}/p_H R_H, \hat{A})$  and  $h \in [h_1, 1]$ .



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