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## Nonlinear dynamics of interest rate and inflation

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21 • 2002

Markku Lanne  
Research Department  
17.9.2002

## Nonlinear dynamics of interest rate and inflation

Suomen Pankin keskustelualoitteita  
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# Nonlinear dynamics of interest rate and inflation

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## Abstract

According to several empirical studies, US inflation and nominal interest rates, as well as the real interest rate, can be described as unit root processes. These results imply that nominal interest rates and expected inflation do not move one-for-one in the long run, which is not consistent with the theoretical models. In this paper we introduce a nonlinear bivariate mixture autoregressive model that seems to fit quarterly US data (1952 Q1 – 2000 Q2) reasonably well. It is found that the three-month treasury bill rate and inflation share a common nonlinear component that explains a large part of their persistence. The real interest rate is devoid of this component, indicating one-for-one movement of the nominal interest rate and inflation in the long run and thus stationarity of the real interest rate. Comparisons with a linear vector autoregressive model reveal that in policy analysis the consequences of neglecting nonlinearities can be substantial.

Key words: nonlinear models, interest rate, inflation, cointegration analysis

JEL classification numbers: C32, E43

# Koron ja inflaation epälineaarinen dynamiikka

Suomen Pankin keskustelualoitteita 21/2002

Markku Lanne  
Tutkimusosasto

## Tiivistelmä

Useiden empiiristen tutkimusten mukaan Yhdysvaltojen inflaatiovauhti ja sekä nimellis- että reaalikorko ovat kuvattavissa yksikköjuuriprosesseina. Tämä tarkoittaa, että nimelliskorot ja odotettu inflaatio eivät pitkällä aikavälillä vaihtelisi yhden suhteessa yhteen, mikä tulos on kuitenkin ristiriidassa teorian kanssa. Tässä tutkimuksessa otetaan käyttöön epälineaarinen kahden aikasarjan autoregressiivinen malli, joka näyttäisi sopivan varsin hyvin yhdysvaltalaiseen neljännesvuosiaineistoon (1952:I–2000:II). Osoittautuu, että 3 kuukauden valtionvekseleiden korolla ja inflaatiolla on yhteinen, epälineaarinen komponentti, joka selittää suuren osan niiden pitkän aikavälin vaihteluista. Tämä yhteiskomponentti ei vaikuta reaalikorkoon, mikä tarkoittaa, että nimelliskorko ja inflaatio vaihtelevat pitkällä aikavälillä yhden suhteessa yhteen niin, että reaalikorko on stationaarinen. Vertailu lineaarisiin vektoriautoregressiivisiin malleihin osoittaa, että epälineaarisuuk-sien huomiotta jättämisellä voi olla huomattavia vaikutuksia politiikka-analyysiin.

Asiasanat: epälineaariset mallit, korko, inflaatio, kointegraatioanalyysi

JEL-luokittelu: C32, E43

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# 1 Introduction

The ex ante real interest rate is the key variable in determining the dynamics of asset prices over time, which explains the widespread interest in its time series properties. According to several empirical studies (see, eg Rose (1988) and Mishkin (1992)) the real rate has been found to be integrated of order one, and this is puzzling from the point of view of economic theory. The classical framework for studying the behavior the real rate is the Fisher identity which states that the ex ante real rate is the difference between the nominal rate and expected inflation. The unit root hypothesis cannot typically be rejected for nominal interest rates or inflation, and thus, nonstationarity of the real rate would, according to this identity, indicate that these variables are not cointegrated such that they move one-for-one in the long run. In addition to violating the predictions of economic theory, this outcome is also counterintuitive, and therefore, some authors have offered nonlinearities as an explanation.

Garcia and Perron (1996) estimated univariate Markov switching autoregressive models for the U.S. monthly and quarterly real interest rate and inflation from 1961 to 1986, allowing for three different regimes. They found the real rate random with shifts in the mean and variance in 1973 and 1981, and attributed the shifts to the rise in the oil price and the current and expected federal budget deficits, respectively. Hence, according to their results, there were three different consecutive regimes, each occurring once, and the shifts in mean made the real rate series seem nonstationary. Because of this structure and because the regimes are not linked to any observable variables, the model does not necessarily lend itself to further applications such as forecasting or impulse response analysis, although it provides an ex post characterization of the statistical properties of the real rate.

Evans and Lewis (1995) suggested that the finding of less than one-for-one movement of the nominal interest rate and inflation in the previous literature may result from peso effects. They estimated a Markov switching model with two regimes for inflation using U.S. data from 1947 to 1987; potential nonlinearity of the interest rate was not considered. They pointed out that the presence of regime switching may cause problems for inference on the relationship between the nominal interest rate and inflation if the shifts in inflation occur less frequently in the observed sample than implied by the model (that agents use to form expectations). This explanation was supported by the fact that the forecast errors produced by their model were autocorrelated in spite of rational expectations. To take account of this peso effect, Evans and Lewis (1995) used Monte Carlo simulation to obtain appropriate finite-sample distributions of cointegration test statistics, and could not reject the null hypothesis of long-run one-for-one movement of the variables, ie stationarity of the ex ante real rate. The basic assumption, namely nonstationarity of the nominal interest rate and inflation, is however, difficult to square with economic theory. It is more likely that these variables can be characterized as near unit root processes, in which case also simulation methods may not be appropriate for computing finite-sample distributions of test statistics (see, eg Stock (1997)).

Recently Bierens (2000) has studied the comovement of U.S. nominal interest rate and inflation nonparametrically. His methods are designed for modeling highly persistent time series and testing for nonlinear cotrending, and they are motivated by the fact that albeit the dynamics of many economic time series can be approximated by unit root processes and their comovements resemble cointegration relations, the presence of unit roots in their DGPs is not plausible. In the empirical application to U.S. data from 1954 to 1994 the interest rate variable was the Federal Funds rate, and the results lend support to the one-for-one movement of nominal interest rate and inflation in the long run.

In this paper we also analyze the relationship between U.S. nominal interest rate and inflation, employing a nonlinear model, but the approach is different from that of Garcia and Perron (1996), Evans and Lewis (1995) or Bierens (2000). As in Garcia and Perron (1996), the basic idea is to formulate a model in which the apparent nonstationarity is brought about by shifts in the level and conditional variance. However, instead of a univariate model for the ex post real rate, we consider a bivariate model for the nominal interest rate and inflation. If this kind of nonlinearity really lies behind the observed persistence of these time series, there may exist a linear combination of them devoid of the nonlinearities. Should the real rate be such a linear combination, then this can be interpreted in favor of one-for-one movement of the nominal interest rate and expected inflation in the long run. Our model is a bivariate generalization of the mixture autoregressive model of Lanne and Saikkonen (2000), where the regime switches are directly linked to observable variables. This facilitates interpretation of the results in comparison to the Markov switching models discussed above. Moreover, the approach is akin to cointegration analysis, yet avoids having to resort to simulation methods, as in Evans and Lewis (1995), which are potentially invalid in the presence of near unit root variables. As far as comovement is concerned, our conclusions are the same as those of Bierens (2000), but the fact that we specify a fully parametric model, is interpretationally more convenient and facilitates further analysis. If the testing for one-for-one movement of nominal interest rate and inflation would have been our sole target, the tests due to Anderson and Vahid (1998) that do not require the exact form of nonlinearity to be specified, could potentially also have been employed. Their tests are, however, derived for strictly stationary variables and are, hence, not expected to work well for time series exhibiting unit root type behavior.

The plan of the paper is as follows. In Section 2 the econometric model is introduced and parameter estimation and testing, including diagnostic checks, are discussed. In Section 3 the model is fitted to quarterly U.S. data from 1952 to 2000. In addition to the estimates, the results of tests for common nonlinearity as well as several diagnostic checks are presented. In addition, the performance of the model is compared to linear autoregressive models with and without conditional heteroskedasticity. Finally, Section 4 concludes.

## 2 Econometric methodology

### 2.1 Bivariate mixture autoregressive model

In this section we introduce the multivariate mixture autoregressive model. As the main emphasis of this paper is to model U.S. inflation and short-term interest rate, we concentrate throughout on the bivariate case, which also simplifies notation. The generalization to higher-dimensional models is, however, straightforward. The model is a direct extension of the mixture autoregressive (MAR) model of Lanne and Saikkonen (2000), and in the same way as their model can be interpreted as a threshold autoregressive (TAR) model with smooth switching between the regimes, our model can be seen as a corresponding generalization of the multivariate TAR model (see Tong (1990) and the references therein).

For interpretation, it is convenient to present the model in its error-correction form (for simplicity, we assume throughout that there are only two regimes),

$$\begin{aligned} \Delta \mathbf{y}_t = & \mathbf{\Pi} [\mathbf{y}_{t-1} - \boldsymbol{\mu} - \boldsymbol{\delta} I(\boldsymbol{\gamma}' \mathbf{y}_{t-d} \geq c + \eta_t)] + \mathbf{\Gamma}_1 \Delta \mathbf{y}_{t-1} \\ & + \cdots + \mathbf{\Gamma}_{p-1} \Delta \mathbf{y}_{t-p+1} + \boldsymbol{\varepsilon}_t, \end{aligned} \quad (1)$$

where  $\mathbf{y}_t = (y_{1t}, y_{2t})'$ ,  $\mathbf{\Pi}$  and  $\mathbf{\Gamma}_1$  through  $\mathbf{\Gamma}_{p-1}$  are  $4 \times 4$  coefficient matrices,  $\boldsymbol{\mu}$ ,  $\boldsymbol{\delta}$  and  $\boldsymbol{\gamma}$  are  $2 \times 1$  vectors, and  $I(\cdot)$  is the indicator function. The error term  $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t})'$  is assumed to follow the bivariate normal distribution with mean  $\mathbf{0}$  and covariance matrix  $\mathbf{H}$ ,  $\eta_t \sim N(0, \sigma_\eta^2)$ , and  $\boldsymbol{\varepsilon}_t$  and  $\eta_s$  are independent for all  $t$  and  $s$ . If  $\sigma_\eta^2$  equals zero, the model reduces to a multivariate TAR model where the linear combination  $\boldsymbol{\gamma}' \mathbf{y}_{t-d}$  serves as the threshold variable determining the regime to which observation  $t$  belongs, and  $c$  is the threshold value. Hence, in this special case, observations for which the value of  $\boldsymbol{\gamma}' \mathbf{y}_{t-d}$  is less than  $c$ , belong to the lower regime where the equilibrium value in the square brackets is  $\boldsymbol{\mu}$ , and the rest to the upper regime with equilibrium value  $\boldsymbol{\mu} + \boldsymbol{\delta}$ . The switching between these regimes is abrupt, occurring whenever  $\boldsymbol{\gamma}' \mathbf{y}_{t-d}$  crosses  $c$ . If, on the other hand,  $\sigma_\eta^2 > 0$ , the switching between the regimes becomes smooth in that even with known threshold value  $c$ , regime classification is unknown. For instance, observation  $t$  may belong to the upper regime even if  $\boldsymbol{\gamma}' \mathbf{y}_{t-d} < c$ , provided  $\eta_t$  takes a sufficiently large positive value. In economic applications this kind of additional flexibility is likely to be relevant, as it allows the current regime to depend on (unmodeled) random external factors as well. The presence of extra noise might also be interpreted as representing the heterogeneity of the views held by economic agents.

Model (1) is designed specifically for near unit root processes that cannot really be  $I(1)$  processes on theoretical grounds. The idea is that the regime shifts make a stationary vector process resemble an  $I(1)$  process; it is well known that in the univariate case regime shifts can have this effect (see, eg Perron (1989))<sup>1</sup>. Then, supposing that the differences of the variables are

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<sup>1</sup>Model (1) could, of course, be extended to allow for different autoregressive coefficients across the regimes, but such an extension would not have the simple interpretation entertained in this paper.

not characterized by such persistence, so neither is the term in the square brackets in equation (1), and the term causing the persistence in the series,  $I(\gamma' \mathbf{y}_{t-d} \geq c + \eta_t)$ , can be considered their common nonlinear ("trend") component provided neither  $\delta_1$  nor  $\delta_2$  equals zero. Furthermore, the linear combination  $\delta'_\perp \mathbf{y}_{t-1}$ , where  $\delta'_\perp \delta = 0$ , is devoid of this common component, and this linear combination of the variables parallels the notion of a cointegrating relation for strictly I(1) processes. For instance, if  $\delta_1 = \delta_2$ , the linear combination that is free of I(1) type variation brought about by the common nonlinear component, is the difference  $y_{1,t-1} - y_{2,t-1}$ .

Although  $\mathbf{y}_t$  can conveniently be interpreted as following a generalized multivariate TAR model, it is, in fact, a mixture of two conditional bivariate normal distributions, which can be seen by writing its conditional density function conditional on information at time  $t - 1$ :

$$f_{t-1}(\mathbf{y}_t) = (2\pi)^{-n/2} |\mathbf{H}|^{-1/2} \left\{ \exp \left[ -\frac{1}{2} \boldsymbol{\varepsilon}'_{1t} \mathbf{H}^{-1} \boldsymbol{\varepsilon}_{1t} \right] (1 - \pi_{t-d}) + \exp \left[ -\frac{1}{2} \boldsymbol{\varepsilon}'_{2t} \mathbf{H}^{-1} \boldsymbol{\varepsilon}_{2t} \right] \pi_{t-d} \right\}, \quad (2)$$

where  $\boldsymbol{\varepsilon}_{1t} = \Delta \mathbf{y}_t - \boldsymbol{\Pi}(\mathbf{y}_{t-1} - \boldsymbol{\mu}) - \sum_{j=1}^{p-1} \boldsymbol{\Gamma}_j \Delta \mathbf{y}_{t-j}$  and  $\boldsymbol{\varepsilon}_{2t} = \Delta \mathbf{y}_t - \boldsymbol{\Pi}(\mathbf{y}_{t-1} - \boldsymbol{\mu} - \boldsymbol{\delta}) - \sum_{j=1}^{p-1} \boldsymbol{\Gamma}_j \Delta \mathbf{y}_{t-j}$ . The mixing proportion  $\pi_{t-d} = \Phi(\gamma' \mathbf{y}_{t-d} - c)$ , with  $\Phi(\cdot)$  the cumulative distribution function of the standard normal distribution, gives the conditional probability of the process being in the upper regime in period  $t$ . Hence, equation (1) defines a mixture of two bivariate VAR( $p$ ) models in error-correction form with mixing proportions  $1 - \pi_{t-d}$  and  $\pi_{t-d}$  corresponding to the lower and upper regime, respectively. The conditional mean and covariance are defined analogously:

$$E_{t-1}(\mathbf{y}_t) = (\boldsymbol{\Pi} + \mathbf{I}_2) \mathbf{y}_{t-1} - \boldsymbol{\Pi}[(1 - \pi_{t-d})\boldsymbol{\mu} + \pi_{t-d}(\boldsymbol{\mu} + \boldsymbol{\delta})] + \sum_{j=1}^{p-1} \boldsymbol{\Gamma}_j \Delta \mathbf{y}_{t-j} \quad (3)$$

$$\text{Cov}_{t-1}(\mathbf{y}_t) = \mathbf{H} + (1 - \pi_{t-d}) [\boldsymbol{\Pi}(\boldsymbol{\mu} - \bar{\boldsymbol{\mu}})(\boldsymbol{\mu} - \bar{\boldsymbol{\mu}})' \boldsymbol{\Pi}'] + \pi_{t-d} [\boldsymbol{\Pi}(\boldsymbol{\mu} + \boldsymbol{\delta} - \bar{\boldsymbol{\mu}})(\boldsymbol{\mu} + \boldsymbol{\delta} - \bar{\boldsymbol{\mu}})' \boldsymbol{\Pi}'], \quad (4)$$

where  $\mathbf{I}_2$  is the identity matrix and  $\bar{\boldsymbol{\mu}} = (1 - \pi_{t-d})\boldsymbol{\mu} + \pi_{t-d}(\boldsymbol{\mu} + \boldsymbol{\delta})$ . It is noteworthy that in addition to the covariance matrix  $\mathbf{H}$  of the innovation  $\boldsymbol{\varepsilon}_t$ , the conditional covariance includes two terms dependent on the mixing proportions. Thus, although  $\boldsymbol{\varepsilon}_t$  in model (1) is assumed to be homoskedastic, the model implies conditional heteroskedasticity (and correlation), which is prominent when  $\pi_{t-d}$  is not close to 0 or 1, ie when  $\gamma' \mathbf{y}_{t-d}$  lies close to the value of the threshold parameter  $c$ . Larger conditional variance near the threshold reflects the uncertainty in the regime switches modeled by the random factor  $\eta_t$ .

In most economic applications, including interest rates and inflation, the heteroskedasticity inherent in specification (1) is likely to be inadequate. Fortunately, the model can relatively easily be extended to allow for various forms of conditional heteroskedasticity. The simplest kind of extension consists only

of letting the covariance matrix of the innovations,  $\boldsymbol{\varepsilon}_t$ , to depend on the regime in the same way as the equilibrium value of the error-correction term. In this case the covariance matrix  $\mathbf{H}$  would be replaced by  $\mathbf{H}_i$ ,  $i = 1, 2$  in the conditional density function (2), giving

$$\begin{aligned} f_{t-1}(\mathbf{y}_t) = & (2\pi)^{-n/2} |\mathbf{H}_1|^{-1/2} \left\{ \exp \left[ -\frac{1}{2} \boldsymbol{\varepsilon}'_{1t} \mathbf{H}_1^{-1} \boldsymbol{\varepsilon}_{1t} \right] (1 - \pi_{t-d}) \right\} \\ & + (2\pi)^{-n/2} |\mathbf{H}_2|^{-1/2} \left\{ \exp \left[ -\frac{1}{2} \boldsymbol{\varepsilon}'_{2t} \mathbf{H}_2^{-1} \boldsymbol{\varepsilon}_{2t} \right] \pi_{t-d} \right\}, \end{aligned} \quad (5)$$

and in the conditional covariance matrix (4)  $(1 - \pi_{t-d})\mathbf{H}_1 + \pi_{t-d}\mathbf{H}_2$  would replace  $\mathbf{H}$ . Both  $\mathbf{H}_1$  and  $\mathbf{H}_2$  would still consist of constant parameters, but the model also lends itself to extensions with autoregressive conditional heteroskedasticity. The simplest possible bivariate GARCH specification, considered also in the empirical application of this paper, is Bollerslev's (1990) constant conditional correlation model in which each component of  $\mathbf{y}_t$  follows its own GARCH( $r, q$ ) specification. Coupled with model (1) this implies a conditional density function of the form (5) but with  $\mathbf{H}_1$  and  $\mathbf{H}_2$  replaced by  $\mathbf{H}_{1t}$  and  $\mathbf{H}_{2t}$  defined as

$$\mathbf{H}_{it} = \begin{pmatrix} \frac{h_{i11,t}}{\rho_i \sqrt{h_{i11,t} h_{i22,t}}} & \rho_i \sqrt{h_{i11,t} h_{i22,t}} \\ \rho_i \sqrt{h_{i11,t} h_{i22,t}} & h_{i22,t} \end{pmatrix} \quad i = 1, 2 \quad (6)$$

where

$$\begin{aligned} h_{ijj,t} &= \sigma_{ij} + \beta_{1j} h_{ijj,t-1} + \cdots + \beta_{rj} h_{ijj,t-r} + \alpha_{1j} \varepsilon_{j,t-1}^2 + \cdots + \alpha_{qj} \varepsilon_{j,t-q}^2 \\ i, j &= 1, 2 \end{aligned} \quad (7)$$

Thus the conditional correlation in regime  $i$  is indeed constant,  $\rho_i$ . Note that we have here assumed that the GARCH dynamics are the same across the regimes but the extension to regime-dependent alphas and betas is, of course, straightforward. Likewise, even more general GARCH formulations instead of the constant conditional correlation specification discussed here, are possible. Henceforth the bivariate model defined by equations (1), (6) and (7) will be called the BVMAR( $m, p, d$ )-GARCH( $r, q$ ) model, where  $m$  is the number of regimes.

Above we have assumed that there are only two regimes, but it is straightforward to extend the model by augmenting the expression in the square brackets in equation (1) with additional indicator functions, each with a different threshold parameter and coefficient vector. It is also assumed that equation (1) defines a stationary vector process. However, even for vector TAR models the conditions ensuring stationarity are only known in some special cases (see, eg Chan et al (1985)), and for our multivariate MAR model such conditions are not available. Therefore, in any empirical application the practical approach of examining stability by simulation methods should be taken. In Section 3 we employ the methods suggested by Gallant et al (1993).

## 2.2 Parameter estimation and inference

Having fixed the number of regimes,  $m$ , the orders of the VAR and GARCH parts,  $p$ ,  $r$  and  $q$ , respectively, and the delay parameter  $d$ , the BVMAR( $m, p, d$ )-GARCH( $r, q$ ) model can be estimated by the method of maximum likelihood (ML) assuming normality. It is, however, easy to see that without additional parameter restrictions the model is not identified because infinitely many combinations of  $\gamma$  and  $\sigma_\eta^2$  yield the same value of the likelihood function. Restricting  $\sigma_\eta^2$  to unity would guarantee identification, but, alternatively, restrictions could be imposed on  $\gamma$ . Suitable identification restrictions are often suggested by the economic theory under study; for instance, in Section 3 we set  $\gamma = (0, 1)'$ , meaning that  $y_{2,t-d}$  (the lagged inflation rate) acts as the threshold variable. Letting  $\theta$  denote the vector of the parameters of the model, the conditional log-likelihood function can be written as

$$l_T(\theta) = \sum_{t=1}^T \ln f_{t-1}(\mathbf{y}_t; \theta)$$

and maximized numerically to obtain the ML estimates. It is clear that the likelihood function is twice continuously differentiable so that, assuming stationarity and ergodicity, it seems a reasonable approach to apply standard asymptotic theory in statistical inference on the parameters. In particular, asymptotic standard errors are given by the diagonal elements of the matrix  $-\left(\partial^2 l_T(\hat{\theta}) / \partial \theta \partial \theta'\right)^{-1}$  where  $\hat{\theta}$  denotes the ML estimate of  $\theta$ . Also, the hypothesis of  $y_{1,t-d} - y_{2,t-d}$  being devoid of the nonlinear component, ie  $\delta_1 = \delta_2$  in the two-regime model, that is of great interest in the current application to the real rate, can be tested using the standard likelihood ratio test.

A problem common to all regime-switching models like our BVMAR-GARCH model, is the inapplicability of standard asymptotic inference in testing hypotheses that imply a reduced number of component distributions. This is due to the fact that under such null hypotheses there are unidentified nuisance parameters (see Davies (1977)). For instance, assuming homoskedasticity, parameters  $c$  and  $\sigma_\eta^2$  do not appear in model (1) under the null hypothesis  $\delta = \mathbf{0}$  and, therefore, are not identified.<sup>2</sup> However, assuming conditional heteroskedasticity dependent on the regime, there is no reason to expect standard inference not to work satisfactorily because the parameters included in the conditional covariance matrices are sufficient to identify  $c$  and  $\sigma_\eta^2$ . Conversely, it is not possible to formulate a hypothesis that would imply only a reduced number of regimes in the conditional covariance but no corresponding reduction in the conditional mean because the conditional covariance depends on  $\delta$ . Hence, there is nonlinearity in the conditional covariance matrix whenever  $\delta$  differs from zero. In addition to a formal test, informal checks discussed below as well as information criteria can be used to select the number of component models.

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<sup>2</sup>Recently computer-intensive methods to attack this problem have been suggested for certain nonlinear models (see, eg Hansen (1996)), but their implementation to higher dimensional models in the present context does not seem practically feasible.

### 2.3 Diagnostic checks

After estimation, diagnostic checks are required to assess the adequacy of the model and reveal potential misspecification. Typically, such checks are based on visual inspection of and formal diagnostic tests based on the residuals of the fitted model. In the case of the BVMAR-GARCH model (and its special cases), however, the computation of the residuals is not straightforward. Intuitively, an empirical counterpart of  $\varepsilon_t$  would be needed, but it is clear that it cannot be obtained by plugging in the estimates into equation (1) because  $\varepsilon_t$  depends on the unobservable error term  $\eta_t$ . Therefore, we must be content with the following residuals that are only useful for checking the capability of the model to describe the first two conditional moments of the observed process  $\mathbf{y}_t$ . These residuals are obtained as

$$\hat{e}_{it} = \frac{y_{it} - \hat{E}_{t-1}(y_{it})}{\sqrt{\widehat{Var}_{t-1}(y_{it})}}, \quad i = 1, 2$$

where  $\hat{E}_{t-1}(y_{it})$  and  $\widehat{Var}_{t-1}(y_{it})$  denote the empirical counterparts of the conditional expectation and variance of the  $i$ th variable computed at the ML estimates, respectively (cf. Wong and Li (2000)). If the model is correctly specified,  $e_{it}$  is a martingale difference sequence with unit variance so that pronounced autocorrelation or heteroskedasticity in  $\hat{e}_{it}$  indicate misspecification of the first two conditional moments. Note, however, that even if the model is correctly specified,  $\mathbf{e}_t = (e_{1t}, e_{2t})'$  is not normally distributed.

Apart from visual inspection of the series  $\hat{e}_{it}$ , diagnostic tests can be used. LM type misspecification tests robust to deviations from normality can be derived following the approach of Wooldridge (1990), and in this paper we present results for three types of hypotheses. First, to examine the adequacy of conditional mean, we consider, for each equation in the model, an alternative that has  $p - 1 + s$  lags of  $\Delta \mathbf{y}_t$  in equation (1). In the BVMAR-GARCH model the lag length also affects the conditional variance, but otherwise it is assumed to be correctly specified. Under the null hypothesis that the lag length is  $p - 1$  the LM test statistics asymptotically follows the  $\chi^2_{2s}$  distribution. Second, a similar test for the adequacy of the conditional variance in the BVMAR-GARCH model is devised by assuming that the conditional mean is correctly specified, and testing for an augmented GARCH specification with  $q + s$  lags of  $\varepsilon_{jt}^2$  in equation (7) for each  $j$  against the specification with  $q$  lags. The asymptotic null distribution of the LM test statistic is the  $\chi^2_s$  distribution. Finally, in the same way as the GARCH equations can be augmented with lags of  $\varepsilon_{jt}^2$ , they can also be augmented with dummy variables to test whether the conditional variance is modeled adequately to take into account known exceptional periods. Each of the test statistics can easily be computed from a sequence of least squares regressions (for details, see Wooldridge (1990, Example 3.3) and Lanne and Saikkonen (2000)).

In addition to the diagnostic tests and visual inspection of the residuals, there are a number of informal procedures that can be employed to evaluate the performance of the estimated BVMAR-GARCH model. Probably the most important of these involve checking for stability, because no general conditions



are known. First, conditional moment profiles introduced by Gallant et al (1993) give information concerning the dependence of the first two conditional moments on initial values, and such strong dependences would, of course, imply nonstationarity. Second, simulating long realizations gives an idea of whether the model is capable of generating series resembling the observed data. Such a capability is not important only as a descriptive measure of the fit of the model but also because simulation methods may subsequently be employed in computing measures such as impulse response functions and unconditional moments implied by the model.

### 3 Empirical results

In this section we study the time series properties of the U.S. nominal interest rate and inflation, and provide evidence for the stationarity of the real rate. A BVMAR-GARCH model is estimated and compared to linear specifications. The data are quarterly observed and span the period from 1952:I until 2000:III (195 observations). The interest rate variable is the U.S. Treasury bill rate extracted from the H.15 release of the Federal Reserve System, and the annualized inflation rate is computed from the seasonally adjusted<sup>3</sup> consumer price index (for all urban consumers) provided by the Bureau of Labor Statistics. Subsequently the inflation and interest rate will be denoted by  $i_t$  and  $r_t$ , respectively.

To describe the time series properties of the variables, unit root and cointegration test results are presented in Table 1. In accordance with the previous literature, the unit root null cannot be rejected for  $i_t$  or  $r_t$  and is only barely rejected for the ex post real rate  $r_t - i_t$  at the 5% level by the ADF test. The results for the cointegration tests are less clear-cut. Johansen's trace test indicates the presence of one cointegrating relation, and the hypothesis of this relation being  $r_t - i_t$  (allowing for a nonzero constant) cannot be rejected at any reasonable significance level. This kind of two-stage procedure may, however, lack power when it is not known that the variables are I(1) processes, as pointed out by Elliott (1995). Theoretically, the nominal interest rate and inflation are expected to be stationary, and therefore, we also consider a joint test of the two hypotheses derived by Horvath and Watson (1995). In this case the joint test is likely to be more powerful than the two-stage test or the unit root test for  $r_t - i_t$  (see, Horvath and Watson (1995, 1004)). According to this test, stationarity of the ex post real rate is rejected at the 10% level. In general, this evidence motivates the application of the BVMVAR-GARCH model to these data.

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<sup>3</sup>Results obtained using data without seasonal adjustment turned out to be virtually identical.

### 3.1 BVMAR-GARCH model

Because it is difficult to devise an algorithmic model selection strategy for the mixture autoregressive models, we experimented with different specifications and, using a combination of diagnostic tests and informal checks, finally selected a BVMAR(2,4,1)-GARCH(1,1) model that adequately seems to describe the dynamics in the data. The four lags in the conditional mean specification seem to adequately capture all seasonalities in the data; specifications with seasonal dummies were also considered but the dummies turned out to be insignificant. To keep the model practically manageable, two assumptions were made at the outset. First, to identify the model, it was assumed that the inflation rate is the threshold variable, ie  $\gamma = (0, 1)'$ . Although other identifying restrictions for  $\gamma$  and  $\sigma_\eta^2$  were also entertained, this seemed to produce the most easily interpretable results, classifying the data into low and high inflation regimes. Second, it was assumed that the  $\alpha$  and  $\beta$  coefficients of the GARCH specifications are independent of the regime, ie only level shifts in the conditional variance were allowed for. In addition, the value of the delay parameter  $d$  was set equal to 1 which seemed the most natural choice for quarterly data.

The estimation results are presented in Table 2. Where applicable, the estimates and standard errors of the parameters of the equation for  $r_t$  appear on the left and those of the equation for  $i_t$  on the right side of the table. The specification assumes that  $\rho_1 = 0$  and  $\delta_1 = \delta_2$ , and these restrictions cannot be rejected (in likelihood ratio tests the p-value for the first restriction equals 0.11 and that of the second 0.21). The support for the latter restriction is important because it implies that  $r_t - i_t$  is devoid of the nonlinear component such that the real rate is the long-run equilibrium relation between  $r_t$  and  $i_t$ . Although not all the remaining parameters seem significant, judged by standard significance tests, further restrictions turned out to lead to inferior performance in diagnostic checks. The 'nonlinearity' parameters  $\delta_1$ ,  $\delta_2$ ,  $c$  and  $\sigma_\eta$  are accurately estimated. The estimated value of the threshold between the low and high inflation regimes is 4.01 and the estimate of the standard deviation of the extra error term  $\eta_t$ , 0.82, is small relative to the variation of the inflation rate series. Hence, the asymptotic 95% confidence interval for  $c$  is [2.40, 5.62], indicating rather clear regime classification. In the high inflation regime also the conditional variance of both variables as well as the conditional correlation are much higher. The long-run equilibrium levels of  $r_t$  and  $i_t$  are 3.87 and 2.74 in the low inflation regime and 7.44 and 6.31 in the high inflation regime, respectively. The diagonal elements of the  $\Pi$  matrix are highly significant and negative as expected, whereas the nondiagonal elements do not significantly differ from zero. Thus, within the regimes, when either  $r_t$  or  $i_t$  lies below its equilibrium value, it tends to increase, and conversely, when it lies above the equilibrium value, it tends to decrease. Moreover, the probability of a switch to the other regime is the higher the closer  $i_{t-1}$  is to the threshold value 4.01.

The nonlinear component in the model can be computed as the conditional expectation of the estimated indicator function  $I(i_{t-1} \geq \hat{c} + \eta_t)$ , and it can also be interpreted as the smoothed conditional probability of the upper

regime. This time series is graphed in Figure 1. The advantage of our model is that the regimes are linked to observable inflation unlike in the Markov switching models of Evans and Lewis (1995) and Garcia and Perron (1996), which facilitates interpretation. In general, the probability of high inflation was particularly low before the year 1966 and again after the year 1991. In the period from 1972–1981 this probability was very close to one with the exception of the first quarter of 1976. In other periods the probabilities more or less tend to switch between very low and very high values quite frequently. The same pattern is discernible in Figure 2 which depicts the conditional variance of  $r_t$ ,  $i_t$  and  $r_t - i_t$ . For both variables the conditional variance is highest during the Fed experiment period 1979–1982, but for the interest rate the difference between this period and the rest of the sample is conspicuous. The conditional variance of inflation has been steadily declining since 1983 until the very late 1990's where a slight increase is to be seen. The pattern of conditional variance of the real rate resembles that of the nominal interest rate, but the peak supposedly caused by the Fed experiment is less distinguishable due to high conditional correlation during this period.

We also computed two further specification tests. First, the null hypothesis of linearity in the conditional mean,  $\delta_1 = \delta_2 = 0$ , was clearly rejected (p-value equals 6.00e-6). Second, a MVMAR(3,4,1)-GARCH(1,1) model was estimated to find out whether a third regime would be significant. The null hypothesis that reduces this model to the simpler MVMAR(2,4,1)-GARCH(1,1) model involves four restrictions ( $\delta_3 = 0$ ,  $\sigma_{12} = \sigma_{13}$ ,  $\sigma_{22} = \sigma_{23}$  and  $\rho_2 = \rho_3$ ), and the obtained value of the likelihood ratio test statistic, 1.53, clearly falls short of the 95th percentile of the  $\chi^2_4$  distribution, 9.49. As discussed in Section 2.2, using critical values from the  $\chi^2$  distribution is not, in fact, valid because under the null hypothesis the second threshold parameter is not identified. However, the appropriate critical value necessarily exceeds that from the  $\chi^2_4$  distribution, and, therefore, nonrejection using the  $\chi^2_4$  distribution can definitely be interpreted in favor of the null hypothesis. Hence, a two-regime BVMAR-GARCH model seems to be sufficient.

Some results of robust diagnostic LM tests are presented in Table 3. In the first two tests, labeled AR(5) and AR(6), the sufficiency of the conditional mean specification is considered by testing the significance of one and two additional lags, respectively. In neither equation of the model can the null hypothesis be rejected at reasonable significance levels which lends support to the adequacy of the specification. The remaining tests consider the conditional covariance specification. The GARCH(1,2) and GARCH(1,3) tests test for the significance of one and two additional lags of the squared error term in each equation, respectively, and the results favor the BVMAR(2,4,1)-GARCH(1,1) specification. Finally, the purpose of the structural break test is to test whether the model manages to capture the effect of the "new operating procedures" of the Federal Reserve (1979:IV–1982:III), especially the increased conditional variance of both variables. According to the test results there is no remaining unmodeled shift in the conditional variance due to this period.

The model can be used to compute forecasts of inflation and interest rate, but only forecasts one quarter ahead can be obtained in closed form. Of particular interest is the difference between the ex ante real rate implied by the

model, ie  $r_t - \hat{E}_{t-1}i_t$  where  $\hat{E}_{t-1}$  denotes conditional expectation evaluated at the estimated parameter values, and the ex post real rate  $r_t - i_t$ . These series are graphed in Figure 3. Contrary to the findings of Evans and Lewis (1995), the differences are not conspicuously large, except, maybe in the late 70's. In particular, Evans and Lewis (1995) pointed out that for their Markov switching model the discrepancy became very wide during the period beginning in 1980 when their model kept giving high inflation forecasts although inflation had turned down. Consequently, the forecast error turned out to be strongly serially correlated, lending support to their peso effect explanation. For our model, however, no such discrepancy is visible, and according to Ljung-Box tests (up to 10th order) the forecast residual is not significantly autocorrelated at the 5% level. Hence, rationality of inflation expectations is supported even without having to resort to peso effect explanations once nonlinearities are taken into account.

### 3.2 Dynamic properties

In addition to formal tests, informal checks are required to assess the adequacy of the selected BVMAR-GARCH specification. First, simulation methods must be employed to examine the stability of the model because no theoretical results are available. Second, by comparing simulated realizations and implied unconditional moments from the BVMAR-GARCH and linear alternative models, the potential advantages of introducing nonlinear dynamics can be judged. Finally, differences in measures such as impulse responses across different models should reveal how tenable the implied dynamics really are.

In order to check the stability, long realizations of the estimated models were visually inspected, and no obvious deviations from stationarity were detected. For a more systematic stability check Gallant et al (1993) have suggested computing the conditional mean and volatility profiles with a large range of initial values, each profile obtained by computing the mean at every point  $t$  ( $t = 1, \dots, T$ ) of the respective moment from a large number of realizations. By over-plotting all these profiles we obtain a profile bundle whose shape should reveal potential excessive dependence on initial conditions. If the bundle retains its thickness, instability of the model is suspected, while if it gets narrower over the sequence, this lends support to stability. The conditional profile bundles for  $r_t$  and  $i_t$  implied by the BVMAR(2,4,1)-GARCH(1,1) model are graphed in Figure 4. Each bundle consists of 100 profiles with  $T = 100$  corresponding to different initial values randomly drawn from the observed inflation and interest rate series. For the conditional variance bundles, the initial values were drawn from the conditional variances given by the estimated model. Regardless of the initial value, all the conditional moment profiles relatively quickly converge, which implies stability of the model.

In addition to checking stability, simulated long realizations can be used to compute unconditional moments implied by models. This serves as a complementary check of the reasonableness of specification and gives information on the existence of unconditional moments that cannot, in general, be established analytically. Series with at least one million observations were required to

nail down the first two digits of the unconditional expectation and variance of the estimated BVMAR-GARCH model; for skewness and kurtosis no convergence was reached at even much longer realizations which suggests that higher moments do not exist for these models. The unconditional mean and variance of  $r_t$ ,  $i_t$  and  $r_t - i_t$  implied by the estimated BVMAR-GARCH, VAR(4), VAR(4)-GARCH(1,1)<sup>4</sup> models as well as estimated from the observed sample are presented in Table 4. Although the unconditional variance of  $r_t$  or  $i_t$  is not most accurately matched by the BVMAR-GARCH model, its implied moments are still relatively close to the sample moments, and for the real rate it provides by far the best match among the models considered. This suggests that due to the common nonlinear component, the nonlinear model is superior in modeling the comovements.

The differences between the BVMAR-GARCH and linear VAR model can be highlighted by computing the impulse response functions implied by them. Simulation methods are required to compute the impulse response functions for the nonlinear model, and they can depend on both the sign and size of the shock as well as the initial regime. Furthermore, it is important to ascertain that the shocks to be considered are consistent with the historical range of the data. In a bivariate model this means that, when tracing out the effects of a shock in one variable, the shock to the other variable must be adjusted to take account of their contemporaneous covariance. Gallant et al (1993) suggested visually examining a scatter plot of the data to determine the shocks that appear typical relative to the historical dispersion. For simplicity, and to be able to compare the impulse response functions to those of the linear VAR, we only consider one standard deviation shocks (positive and negative) to both  $r_t$  and  $i_t$ , and the scatter plot suggests that it is reasonable to accompany such a shock by a zero shock to the other variable. For the linear VAR model orthogonalized shocks obtained by setting  $r_t$  the latter variable are computed, but the ordering turned out to be irrelevant for the conclusions.

The impulse response functions of the BVMAR-GARCH model graphed in Figure 5 are obtained as follows. First, 1,000 realizations of the estimated model are simulated, using all the observed values in the sample as initial values. Second, another 1,000 realizations are simulated, adding a one standard deviation shock to the value of either of the variables at time 0. Finally, the functions are obtained by computing the mean of the differences between these two realizations at each lead 1,2,...,24. In fact, the graphs in Figure 5 correspond to the low inflation regime (i.e. in their computation the initial value is selected among those observations for which  $i_0$  is less than the estimated threshold value 4.01), but with one exception, the differences between the regimes are negligible. In general, the impulse responses converge to zero relatively fast, and the effects of positive and negative shocks only differ in sign. However, the effect of a negative shock to the interest rate on inflation is initially smaller than that of a positive shock. In the high inflation regime this outcome is reversed, which is also the only noticeable difference in the impulse response functions across the regimes. The corresponding orthogonalized im-

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<sup>4</sup>The GARCH part of the VAR-GARCH model is Bollerslev's (1990) constant conditional correlation specification.

pulse response functions for a positive one standard deviation shock for the linear VAR model are graphed in Figure 6. The functions for corresponding negative shocks, of course, only differ in sign. The most striking feature in comparison to Figure 5 is the slow convergence of the effects of shocks to zero, i.e. according to the VAR model the effects seem to persist for a very long time. As a matter of fact, over 20 years are required for all the functions to attain zero which seems very unrealistic, and suggests the inapplicability of the VAR model for policy analysis. Thus, although there are only minor difference between the regimes, ignoring regime switching can strongly affect the conclusions drawn from further analyses.

## 4 Conclusion

In this paper we have introduced a bivariate mixture autoregressive model and applied it to model the relationship between the nominal interest rate and inflation in quarterly U.S. data from 1952 to 2000. Two regimes, interpreted as low and high inflation regimes, seemed to be adequate to capture the nonlinearities in the system. The model seems to fit the data relatively well which lends support to the conjecture that the time series are stationary and the apparent nonstationarity, implied by unit root tests, is brought about by shifts in the level and conditional variance. Furthermore, it could not be rejected that the two variables share a common nonlinear component such that the real rate is devoid of nonlinearities in the level. This, in turn, suggests stationarity of the real rate contrary to some findings in the previous literature, but in accordance with economic theory. The finding of a common nonlinear component in the nominal interest rate and inflation is a long-run phenomenon, ie the variables move one-for-one only in the long run. However, there is no such implication for the short-run movements, which accords with much of recent literature; see eg Lanne (2001) where support is found for the short-run comovement only for the period prior to the 'new operating procedures' of the Federal reserve that started in November 1979.

The new BVMAR-GARCH model turned out to perform relatively well compared to linear alternatives. In particular, simulated realizations closely resembled the observed time series and the unconditional moments closely matched those actually observed. Furthermore, the impulse response functions implied by the nonlinear model display much quicker decay towards zero than the ones implied by the corresponding linear VAR model which is an intuitively appealing feature. Although the model is very reduced, these differences suggest that neglecting nonlinearities can give misleading conclusion even in larger models.

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Figure 1: Smoothed conditional probability of the high inflation regime of the BVMAR(2,4,1)-GARCH(1,1) model.

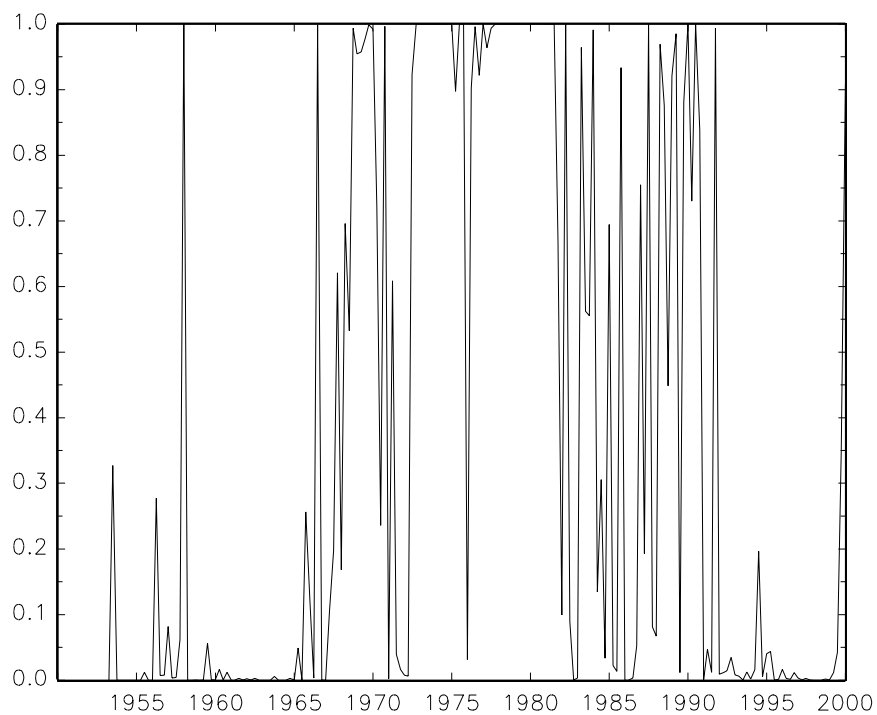


Figure 2: The conditional variance of  $r_t$  (top panel) and  $i_t$  (middle) and  $r_t - i_t$  (bottom).

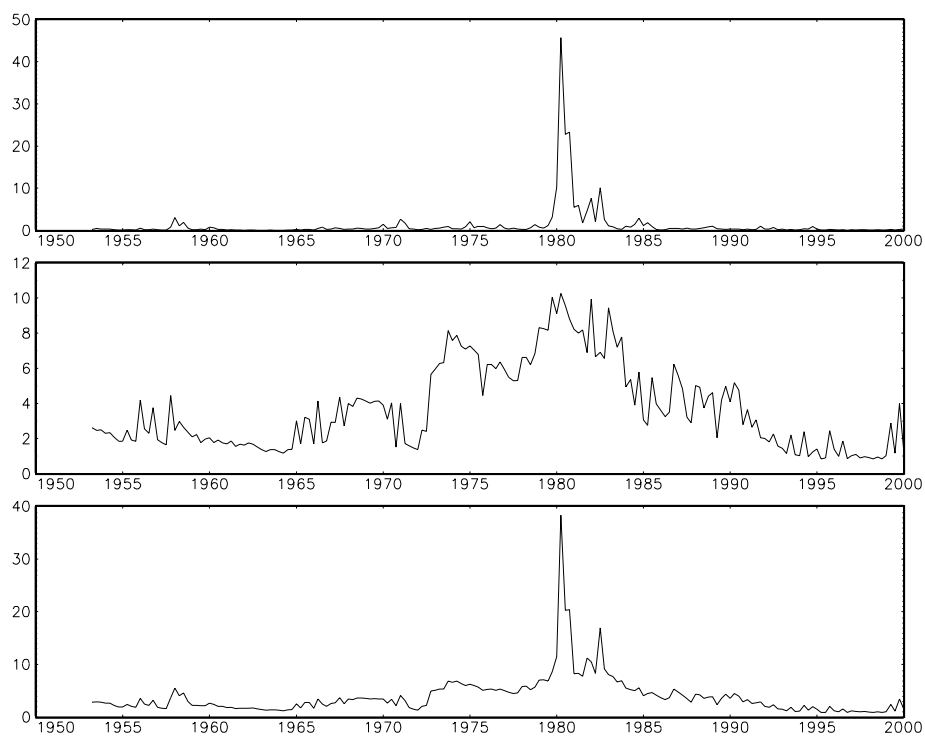


Figure 3: Ex ante (solid line) and ex post (dashes) real rates.

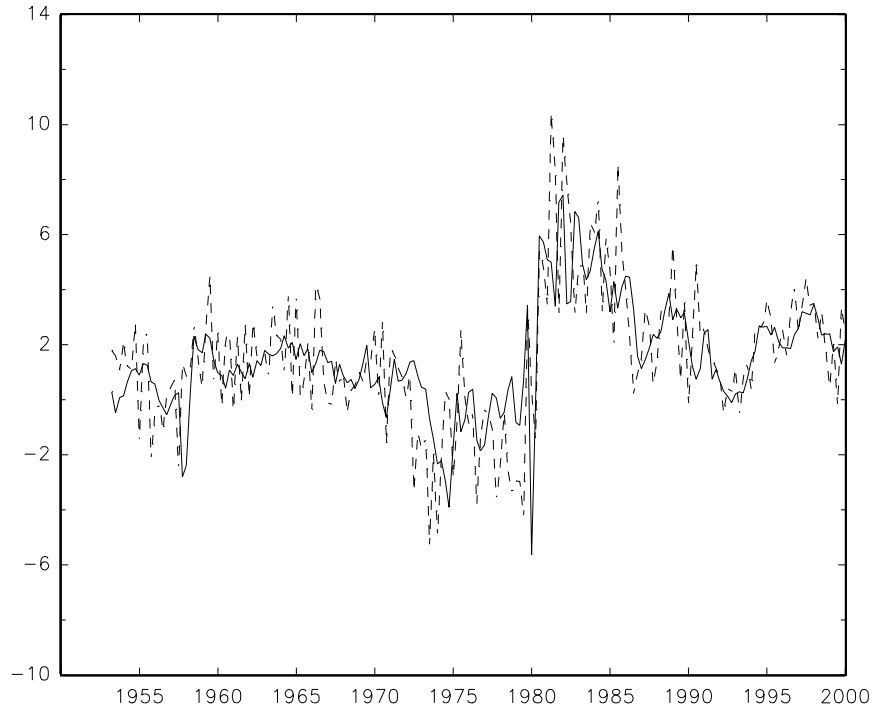


Figure 4: Profile bundles of conditional mean and variance of the BVMAR(2,4,1)-GARCH(1,1) model for  $r_t$  (upper panel) and  $i_t$  (lower panel).

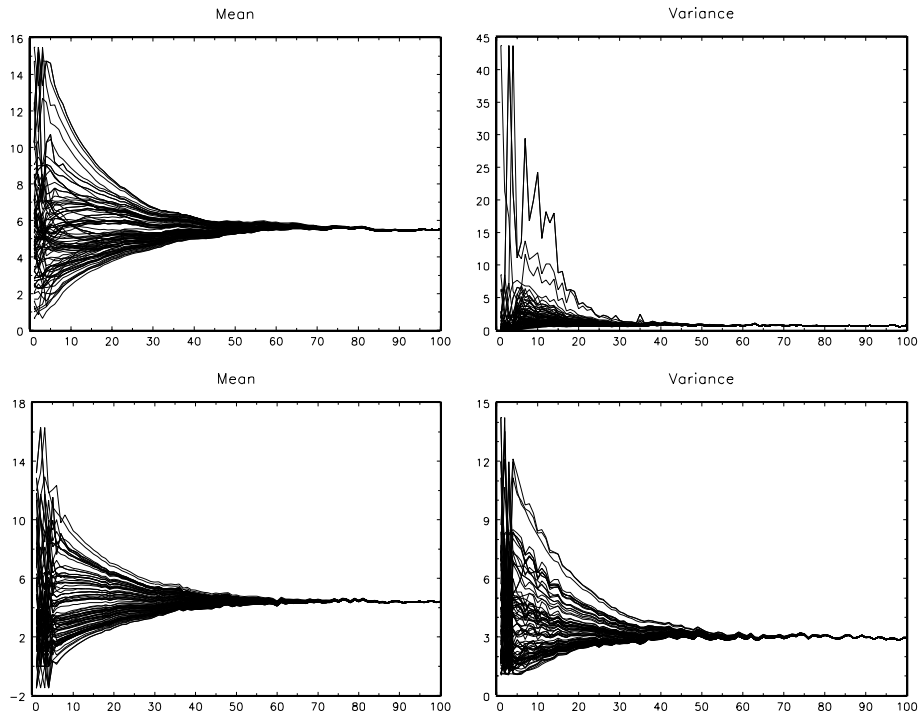


Figure 5: Impulse response functions of one standard deviation shocks to  $r_t$  (upper panel) and  $i_t$  (lower panel) implied by the BVMAR(2,4,1)-GARCH(1,1) model. The solid and dashed lines correspond to positive and negative shocks, respectively.

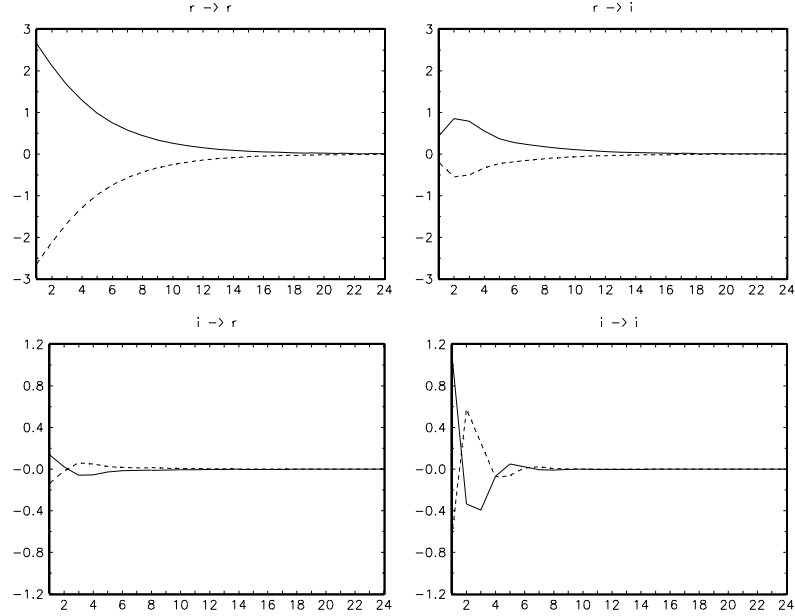


Figure 6: Impulse response functions of one standard deviation shocks to  $r_t$  (upper panel) and  $i_t$  (lower panel) implied by the VAR(4) model.

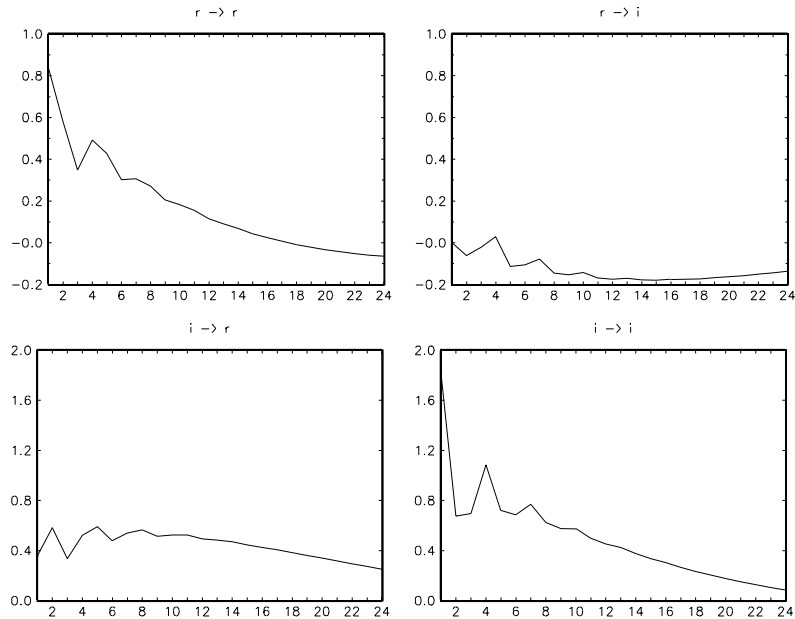


Table 1: Unit root and cointegration tests

| Unit Root Tests                    |                  |                   |       |
|------------------------------------|------------------|-------------------|-------|
| Variable                           | ADF <sup>a</sup> | 5% Critical Value |       |
| $r_t$                              | -2.34            | -2.88             |       |
| $i_t$                              | -2.33            | -2.88             |       |
| $r_t - i_t$                        | -2.90            | -2.88             |       |
| Cointegration Tests <sup>b</sup>   |                  |                   |       |
| Hypothesis                         | Test Statistic   | Critical Values   |       |
|                                    |                  | 5%                | 10%   |
| $rank = 0^c$                       | 23.80            | 20.04             | 17.81 |
| $rank \leq 1^c$                    | 9.07             | 9.11              | 7.51  |
| Real Rate Restriction <sup>d</sup> | 0.23             | 3.84              | 2.71  |
| Joint Restriction <sup>e</sup>     | 9.32             | 10.18             | 8.30  |

<sup>a</sup>The lag length is determined by step-down testing (Ng and Perron (1995)) using a 5% level for each lag.

<sup>b</sup>The results are based on a VAR(4) model.

<sup>c</sup>Johansen's trace test.

<sup>d</sup>Likelihood ratio test of the ex post real rate being the cointegration relation.

<sup>e</sup>Wald test for the cointegration rank being one and the ex post real rate being the cointegration relation.

Table 2: Estimation results of the BVMAR(2,4,1)-GARCH(1,1) model.

| Parameter       | Estimate | Std.err. | Parameter       | Estimate | Std.err. |
|-----------------|----------|----------|-----------------|----------|----------|
| $\Pi_{11}$      | -0.109   | 0.022    | $\Pi_{21}$      | 0.002    | 0.080    |
| $\Pi_{12}$      | 0.007    | 0.023    | $\Pi_{22}$      | -0.237   | 0.089    |
| $\Gamma_{1,11}$ | 0.030    | 0.078    | $\Gamma_{1,21}$ | 0.047    | 0.169    |
| $\Gamma_{1,12}$ | 0.045    | 0.024    | $\Gamma_{1,22}$ | -0.586   | 0.099    |
| $\Gamma_{2,11}$ | -0.116   | 0.062    | $\Gamma_{2,21}$ | -0.052   | 0.163    |
| $\Gamma_{2,12}$ | 0.094    | 0.024    | $\Gamma_{2,22}$ | -0.239   | 0.103    |
| $\Gamma_{3,11}$ | 0.035    | 0.056    | $\Gamma_{3,21}$ | 0.142    | 0.152    |
| $\Gamma_{3,12}$ | 0.067    | 0.018    | $\Gamma_{3,22}$ | 0.050    | 0.079    |
| $\mu_1$         | 3.873    | 0.319    | $\mu_2$         | 2.741    | 0.517    |
| $\delta_1$      | 3.571    | 0.867    | $\delta_2$      | 3.571    | 0.867    |
| $c$             | 4.013    | 0.353    |                 |          |          |
| $\sigma_\eta$   | 0.824    | 0.349    |                 |          |          |
| $\sigma_{11}$   | 0.015    | 0.009    | $\sigma_{21}$   | 0.072    | 0.070    |
| $\sigma_{12}$   | 0.172    | 0.069    | $\sigma_{22}$   | 0.476    | 0.306    |
| $\alpha_1$      | 0.783    | 0.208    | $\alpha_2$      | 0.076    | 0.050    |
| $\beta_1$       | 0.204    | 0.098    | $\beta_2$       | 0.855    | 0.079    |
| $\rho_2$        | 0.409    | 0.104    |                 |          |          |

The standard errors are computed from the inverse of the final Hessian matrix.

Table 3: Diagnostic tests for the BVMAR(2,4,1)-GARCH(1,1) model.

| Hypothesis                    | Equation |       |
|-------------------------------|----------|-------|
|                               | $r_t$    | $i_t$ |
| AR(5) <sup>a</sup>            | 0.546    | 0.331 |
| AR(6)                         | 0.555    | 0.584 |
| GARCH(1,2) <sup>b</sup>       | 0.660    | 0.503 |
| GARCH(1,3)                    | 0.291    | 0.356 |
| Structural Break <sup>c</sup> | 0.606    | 0.885 |

All figures are marginal significance levels. The numerical derivatives required in the calculation of the test statistics are computed by the gradp routine in GAUSS.

<sup>a</sup>In the tests labeled AR( $l$ ) the alternative model is the corresponding BVMAR-GARCH(1,1) model with  $l$  lags in the conditional mean equation in question.

<sup>b</sup>In the tests labeled GARCH(1, $l$ ) the alternative model is the corresponding BVMAR-GARCH(1,1) model with  $l$  lags of the squared error term in the conditional variance equation in question.

<sup>c</sup>The alternative model is the BVMAR-GARCH model with a dummy variable for the period of the "new operating procedures" of the Fed (1979:IV–1982:III) entering the conditional variance equation in question.

Table 4: Comparison of the unconditional moments estimated from the sample and implied by the different models.

|                   | $r_t$ |          | $i_t$ |          | $r_t - i_t$ |          |
|-------------------|-------|----------|-------|----------|-------------|----------|
|                   | Mean  | Variance | Mean  | Variance | Mean        | Variance |
| Sample            | 5.38  | 7.87     | 3.88  | 10.26    | 1.50        | 6.36     |
| VAR(4)            | 1.57  | 5.74     | 4.69  | 10.80    | -3.12       | 10.09    |
| VAR(4)-GARCH(1,1) | 5.32  | 7.64     | 3.40  | 7.90     | 1.92        | 8.79     |
| BVMAR(2,4,1)      |       |          |       |          |             |          |
| -GARCH(1,1)       | 5.61  | 5.07     | 4.06  | 8.27     | 1.54        | 5.84     |

The unconditional moments implied by the models are computed from simulated realizations of 1,000,000 observations.

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