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BANK OF FINLAND DISCUSSION PAPERS

19 • 2002

Kenneth Leong
Research Department
3.9.2002

Reconciling the New Keynesian model with observed persistence

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Reconciling the New Keynesian model with observed persistence

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Kenneth Leong
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Abstract

Despite sound theoretical foundations a drawback of the New Keynesian model is its inability to generate adequate persistence in the variables it seeks to explain. A common solution is to modify the model to include lagged variables. However, this is unsatisfactory as many such modifications depart from the microeconomic underpinnings of the original model. This paper presents results from simulation exercises that support the fully forward-looking New Keynesian model. In particular, we show that the exchange rate channel of monetary policy, which has been largely overlooked in existing studies of persistence, is instrumental in generating inflation persistence. However, the combination of full forward-looking behaviour and an open economy is unable to generate sufficient persistence in the output gap. By including an autocorrelated noise term to the assumption of rational expectations, the model is capable of generating persistence that match those of US inflation, the output gap, the nominal interest rate, as well as the real exchange rate.

Key words: New Keynesian model, rational expectations, persistence, open economy

JEL classification numbers: E31, E52

Makrotalouden persistenssi ja uusi keynesiläinen makromalli

Suomen Pankin keskustelualoitteita 19/2002

Kenneth Leong
Tutkimusosasto

Tiivistelmä

Nykyisin laajasti käytetty uusi keynesiläinen makromalli ei vahvoista teoreettisista perusteistaan huolimatta pysty tyydyttävästi selittämään keskeisissä makrotaloudellisissa muuttujissa, erityisesti inflaatiossa, esiintyvää persistenssiä eli muutosten hitautta. Tavallisesti tämä ongelma ratkaistaan muuttamalla mallia niin, että lisätään muuttujien viipeitä. Tämä ratkaisu on kuitenkin epätydyttävä, koska siinä luovutaan mallin alkuperäisistä teoreettisista perusteista. Tässä työssä esitetään simulointikoetuloksia, jotka osoittavat, että rahapolitiikan valuuttakurssikanava ottaminen mukaan malliin auttaa merkittävästi inflaation persistenssin selittämisessä. Sekään ei kuitenkaan riitä tuottamaan riittävää persistenssiä kokonaistuotannon aikasarjalle. Tutkimuksessa osoitetaan kuitenkin, että malli pystyy tuottamaan Yhdysvaltain tilastoja vastaavan persistenssin sekä kokonaistuotannolle että inflaatiolle, korolle ja reaaliselle valuuttakurssille, kun malliin sisältyvää oletusta odotusten muodostumisesta täsmennetään. Tavallisesti odotukset näissä malleissa oletetaan täysin rationaalisiksi, mutta tässä tutkimuksessa näytetään, että kun odotusten muodostumiseen lisätään autokorreloitunut virhetermi, persistenssi voidaan selittää.

Asiasanat: uusi keynesiläinen makromalli, rationaaliset odotukset, persistenssi, avoin talous

JEL-luokittelu: E31, E52

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1 Introduction

The forward-looking New Keynesian model, with its appealing microeconomic foundations, is often shown to be inconsistent with observed data. Perhaps as noteworthy as the assertion in McCallum (1997) that the New Keynesian Phillips curve (an equation in the more general New Keynesian model) is “the closest thing there is to a standard specification” is the widespread consensus that it is unable to produce sufficient inflation persistence. In the important study by Fuhrer and Moore (1995), which focuses on the Taylor (1980) overlapping contracts model, it is demonstrated that any persistence in inflation is derived from persistence in the excess demand term, typically the output gap. In itself, New Keynesian Phillips curves do not generate any inertia in inflation beyond that of the output gap. Simulations in Fuhrer and Moore support this. There is an important distinction between the notions of “sticky-price” and “sticky-inflation”: the former is a feature of contracting models such as Taylor’s which underlies the New Keynesian Phillips curve¹, while the latter is an empirical regularity which New Keynesian models have, to date, failed to adequately explain.

The ability to generate persistence is linked to the backward- versus forward-looking nature of the models. Under rational expectations, the New Keynesian model is purely forward looking, eg in the New Keynesian Phillips curve, inflation is a function of the mathematical expectation of future inflation and an excess demand term. As a result of its empirical failure, modifications have been made to the model in such a way that backward-looking components appear. In the Fuhrer and Moore case, the relative contracting model that they develop is a hybrid Phillips curve which includes a lagged inflation component. Thus, their model is both a sticky-price and sticky-inflation model. Though these attempts have produced positive results, it is generally regarded that such modifications lack theoretical underpinnings that are as convincing as the original New Keynesian specification.² In effect, many estimated New Keynesian Phillips curves resemble the traditional Friedman-Phelps expectations-augmented Phillips curve more so than the New Keynesian inflation model under rational expectations (see, for example, Fuhrer (1997) and Rudebusch (2002)).

The purpose of this paper is to reconcile the basic New Keynesian model, namely the purely forward-looking specification, with the variability and persistence observed in several macroeconomic variables. This paper is only one in an expanding literature on New Keynesian models and persistence. Two recent

¹ Though there are numerous derivations of the New Keynesian Phillips curve, for example the Taylor model, the Rotemberg (1982) quadratic price adjustment model, and the Calvo (1983) staggered contracts model, Roberts (1995) shows that these models are observationally equivalent.

² For example, Roberts (1998) noted that “Fuhrer and Moore’s interpretation of their results requires abandoning [the microeconomic foundations] of the New Keynesian model”.

studies of inflation persistence are worth mentioning. Rudd and Whelan (2001) present new tests of the New Keynesian Phillips curve and find very little role for forward-looking behaviour. Roberts (2001), like Rudd and Whelan, also carefully conducts econometric tests of the Phillips curves using a unifying framework. He finds some support for forward-looking behaviour. In this paper, we attempt to go all the way to find support for full forward-looking behaviour, as predicted by the basic New Keynesian model. To do this, we conduct simulation experiments of various versions of a small New Keynesian macroeconomic model. Our model departs from that used in Fuhrer and Moore's study by adopting a consistent New Keynesian structure throughout. Hence, instead of combining the New Keynesian Phillips curve with vector autoregressions for the output gap and nominal interest rate as they do, both the output gap equation and the interest rate policy rule are forward looking in our model. Therefore, the modelling methodology in this paper belongs to the New Neoclassical Synthesis paradigm of Goodfriend and King (1997). The modifications we make in the interest of generating persistence retain this feature. The approach we take is to calibrate theoretically plausible parameter values for the model so that the stochastically simulated behaviour of its endogenous variables corresponds to those observed in actual data.

To anticipate the results, we find that the basic New Keynesian model can indeed generate substantial inflation persistence. The important factor here is the exchange rate channel of monetary policy, which is introduced by an open economy extension to the baseline model. A small degree of openness is all that is required to impart the inertia to inflation that is missing in closed-economy models. However, the combination of an open-economy model and full forward-looking behaviour is unable to produce sufficient autocorrelation in the output gap. To overcome this deficit we turn to the formation of inflation expectations. We introduce an expected inflation formulation that has two components: the rational expectation of inflation and a possibly autoregressive noise term. The latter is motivated in terms of deviation-from-rationality potentially being persistent, thus indicating elements of myopic behaviour. By doing so, we arrive at the following result. The basic New Keynesian model, incorporating an exchange rate channel, and allowing for rationality with autocorrelated noise, is able to mimic the time series behaviour of United States inflation, the output gap, the nominal interest rate, and the real exchange rate.

The remainder of this paper is organised as follows. We first present some stylised facts of the US economy that modern macroeconomic models aim to explain. In Section 3, we provide an overview of the New Keynesian model and briefly discuss reasons for its popularity. A new recursive simulation procedure for rational expectations models is outlined in Section 4. The results of simulating New Keynesian models ranging from the closed-economy version to an open-economy version augmented with a formulation of rationality with autocorrelated

noise are presented in Section 5. Section 6 summarises the findings and concludes with a suggestion for further research.

2 Stylised facts of the US macroeconomy

We begin by noting the behaviour of several macroeconomic variables in the US economy. This serves to provide a benchmark set of values for comparison with the simulations conducted later in this paper.

The focus is on the standard variables used in small macroeconomic models, ie the output gap, inflation rate, nominal interest rate and the real exchange rate. As a first indication of variability and persistence consider the standard deviations and first-order autocorrelation coefficients in Table 1. The time period begins around two years prior to the onset of Federal Reserve Chairman Alan Greenspan's tenure.

Table 1. **Stylised facts of the US macroeconomy – 1985q1 to 2002q1**

Variable	Standard Deviation	Autocorrelation
Output gap, y_t	0.935	0.862
Inflation, π_t	1.011	0.600
Interest rate, i_t	1.553	0.956
Real exchange rate, $100 \ln Q_t$	19.076	0.966

Note: "Autocorrelation" refers to the first-order autocorrelation coefficient. The real exchange rate data end on 2001q4.

Here, the output gap is computed as $y_t = 100[\ln Y_t - (\ln Y_t)^{pot}]$ where Y_t is real GDP and potential output is obtained by applying the Hodrick-Prescott filter to the natural logarithm of output. The inflation rate $\pi_t = 400(\ln P_t - \ln P_{t-1})$ is the annualised quarterly log-difference in the GDP chain price index P_t . The nominal interest rate i_t is the 3-month Treasury bill rate. Finally, the real exchange rate is computed as $Q_t = \frac{S_t P_t^f}{P_t}$ where S_t is the nominal exchange rate and P_t^f is the

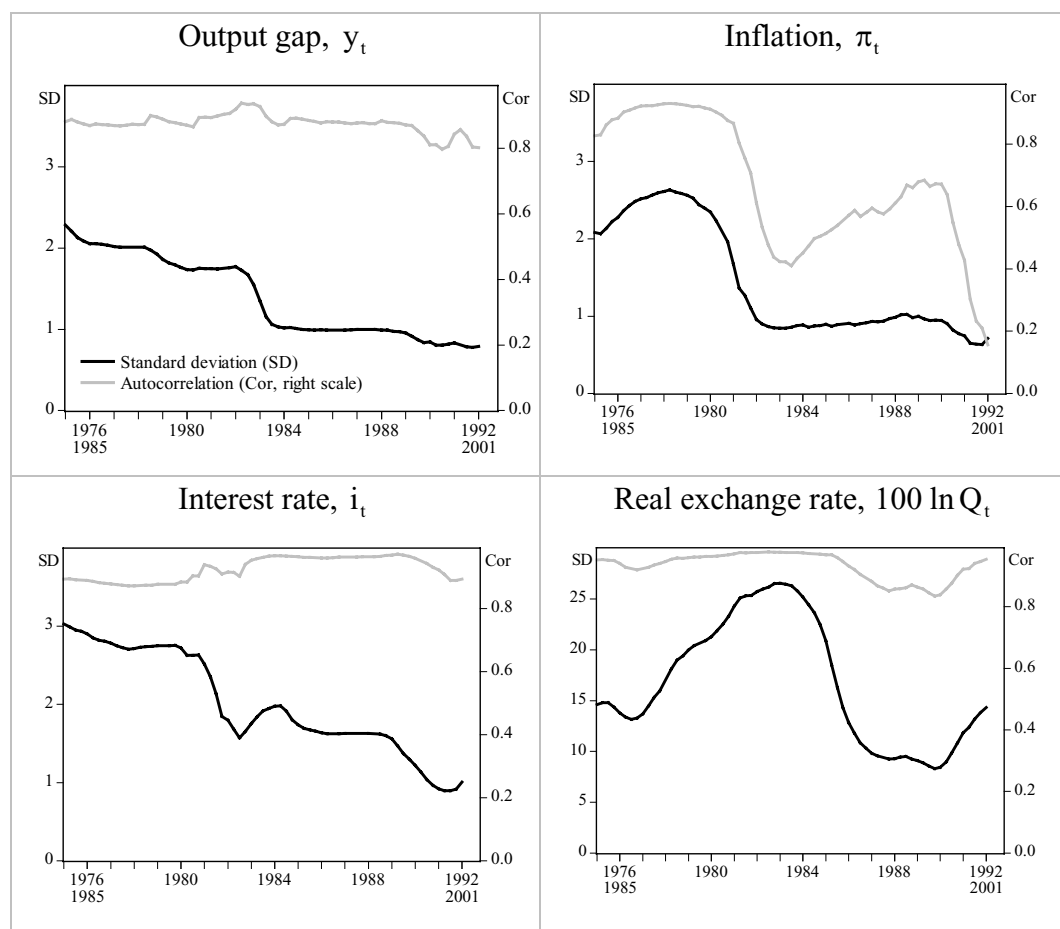
foreign price level. For the nominal exchange rate, where an increase indicates depreciation, we use the USD/Euro spot rate, with the USD/DM rate, converted to Euros using the Euro/DM irrevocable rate, applied prior to 1999q1. As for the foreign price level, the Euro area GDP deflator is used.³

³ The US data are obtained from the Federal Reserve Bank of St. Louis FRED[®] database, and the Euro area data are obtained from European Central Bank publications.

Over this period, average inflation is 2.4 per cent and the average value of an ex post measure of the real interest rate, $i_t - \pi_t$, is 2.8 per cent. Accordingly, we will use the values 2.5 and 2.8 for the inflation target and neutral real interest rate, respectively, in our model specification.⁴

It may not be sensible to take the standard deviation and autocorrelation coefficient of the variables at face value, especially since they are based on a particular sample period. Indeed, in Figure 1, where the statistics are calculated over a 10-year rolling window starting from the mid-1970s, we observe a striking change.

Figure 1. **10-year rolling descriptive statistics – United States**



Compared with the values in Table 1, in general, not only has the variability decreased over time, but so too has the measure of persistence. This is most evident with the autocorrelation coefficient of inflation currently being as low as

⁴ Since the focus is on measures of variability and persistence the precise values chosen for the inflation target and neutral real rate do not matter. They simply affect the mean of the simulated variables.

0.2, as opposed to around 0.7 in the decade beginning 1990. But in line with other studies, some of which use the CPI (instead of a deflator) as a measure of inflation, and the common finding that inflation is an I(1) process, we concur with inflation being a highly persistent process. The standard deviation of the real exchange rate has been both high and variable over time. Given the variability and persistence, the task facing macroeconomists is to build models in attempts to explain these facts.⁵

3 New Keynesian models

New Keynesian models are increasingly used in the literature. These models belong to the class of dynamic general equilibrium models that are derived from individual optimising behaviour, where households maximise utility and firms maximise profits under monopolistic competition.⁶ Features of these models include temporary rigidities in price setting so that policy is non-neutral in the short run, forward-looking behaviour arising from optimising agents with rational expectations, and parameters which have structural interpretations, which make the models less vulnerable to the Lucas Critique compared to backward-looking models.

For the purpose of this paper, a general New Keynesian model is given by

$$y_t = \alpha_1 E_t y_{t+1} - \alpha_2 [(i_t - E_t \pi_{t+1}) - r^*] + \alpha_3 (100 \ln Q_t) + u_t \quad (3.1)$$

$$\pi_t = \beta_1 E_t \pi_{t+1} + \beta_2 y_t + \beta_3 (100 \Delta \ln Q_t) + v_t \quad (3.2)$$

$$i_t = E_t \pi_{t+1} + r^* + \lambda_1 (E_t \pi_{t+1} - \pi^*) + \lambda_2 y_t \quad (3.3)$$

$$400 E_t \Delta \ln Q_{t+1} = i_t - E_t \pi_{t+1} - (i_t^f - E_t \pi_{t+1}^f). \quad (3.4)$$

The endogenous variables, ie the output gap y_t , inflation rate π_t , nominal interest rate i_t and the real exchange rate Q_t , as well as the neutral real interest rate r^* and inflation target π^* , were described in the previous section.⁷ All parameters are expected to be positive. The notation $E_t x_{t+1}$ represents the rational expectation of x_{t+1} conditional on the information set at time t .

⁵ Some studies also attempt to explain the variability in the standard deviation of the variables, ie heteroscedasticity.

⁶ See, for example, Goodfriend and King (1997) and Clarida, Gali and Gertler (1999).

⁷ As indicated, parts of the model are multiplied by 100 or 400 to be consistent with the “percentage” definition of the variables as provided in Section 2.

Equation (3.1) is the optimising IS equation in which the output gap is a function of the expected future output gap, the deviation of the ex ante real interest rate from its neutral level, the level of the real exchange rate, and a white noise demand shock u_t .

Equation (3.2) is a stylised representation of the New Keynesian Phillips curve, where inflation depends on expected future inflation, the output gap, the growth rate of the real exchange rate, and a white noise supply shock v_t . We have, for completeness, included the term $\beta_3(100\Delta\ln Q_t)$ in the equation. There is currently no consensus as to whether it should be the growth rate or the level of the exchange rate, if either, that should enter the New Keynesian Phillips curve.⁸ Should it be the growth rate of the exchange rate that is included in the model, a lagged endogenous variable would be present, rendering the system (partially) backward looking. It is also worthwhile to note that, since the GDP deflator is used instead of the CPI to construct the inflation rate, it is not clear what the magnitude of the coefficient β_3 should be. For these reasons we will attempt to abstract from the exchange rate term in the inflation specification where possible.

Equation (3.3) is a forward-looking policy/Taylor rule in the spirit of Clarida, Gali and Gertler (2000) and Batini and Haldane (1999). Though it is expressed in terms of the nominal interest rate, it specifies that the ex ante real interest rate will deviate from its neutral level when expected inflation differs from the central bank's inflation target and/or real output deviates from its potential. Furthermore, this interest rate rule is Wicksellian as it can be expressed as $i_t - i^* = (1 + \lambda_1)(E_t \pi_{t+1} - \pi^*) + \lambda_2 y_t$, where the central bank responds aggressively, ie with a coefficient greater than unity, to shocks that drive expected inflation above target (i^* is the neutral nominal interest rate; see footnote 9). Following Clarida et al (2000) we make the assumption that the Federal Reserve's target horizon for its implicit inflation target is greater (one-period ahead) than that for the output gap (contemporaneous).

Finally, the real exchange rate evolves according to an uncovered interest rate parity condition, in real terms, given by equation (3.4). Thus, each of the four equations in the model contains an expectational term. Apart from the two error terms u_t and v_t the model also contains two exogenous variables, namely the foreign interest rate i_t^f and the foreign inflation rate π_t^f . A closed-economy version of the model is obtained by setting $\alpha_3 = \beta_3 = 0$ and removing the interest parity condition from the model.

⁸ For example, in the Batini and Haldane (1999) (hybrid) New Keynesian Phillips curve, the growth rate of the real exchange rate enters, while the model in Svensson (2000) contains the level of the real exchange rate. Another widely cited simple open-economy model is that of Ball (1999), in which the Phillips curve (though backward-looking) contains the growth rate of the exchange rate.

To expand on the issue raised in the introduction we now discuss one of the more common modifications to the New Keynesian model to generate persistence in the endogenous variables. A strategy has been to assume that the demand and supply shocks, u_t and v_t in our notation, are autocorrelated processes (see, for example, Clarida et al (1999)). This would be expected to generate persistence since backward substitution performed on the equations would yield successive lags of the dependent variable. Roberts (2001) shows that the assumption of autocorrelated shocks is tantamount to an inflation model that bears a strong resemblance to the hybrid model, ie a model with both backward- and forward-looking behaviour. Since the relationship to the hybrid model is not perfect, and given that the presence of autocorrelated shocks is frequently not justified, we prefer to exercise caution in interpreting results from such models.

4 A recursive simulation procedure

Before simulating the model, the treatment of expectations needs to be resolved. In this section, we describe a recursive simulation procedure to stochastically solve the rational expectations model. The procedure utilises the certainty equivalence principle to solve for expectations, which is possible since the model is linear in its variables and stochastic terms.

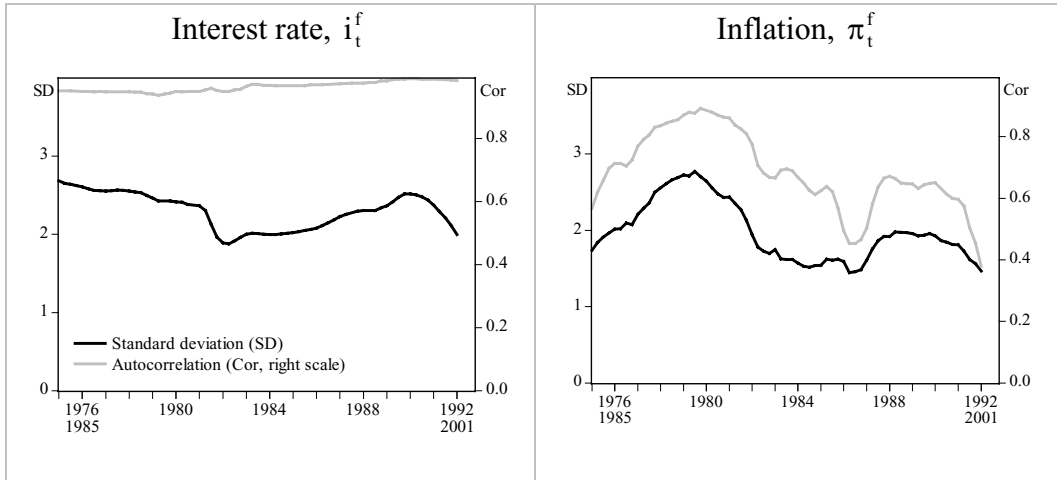
Our procedure involves simulating the model given by equations (3.1) to (3.4) for 1000 periods, which, for sake of exposition, span 2000q1 to 2249q4. The initial values (for the lags) and terminal values (for the expected leads) are set equal to the steady state values of the model. In the steady state, we let the endogenous variables take the values $y = 0$, $\pi = \pi^* = 2.5$ and $Q = 1$ ($\ln Q = 0$).⁹ The steady state value of the exogenous foreign inflation rate is $\pi^f = \pi$.

We now discuss the way the exogenous variables are treated over the simulation horizon. Values for the two exogenous foreign variables i_t^f and π_t^f should not be constant, for example set equal to the steady state values of their domestic counterpart or normalised to zero, as this would remove an important source of variation for the model's endogenous variables. Therefore, the stochastic nature of these exogenous variables is explicitly modelled.

Figure 2 plots the time series properties of the two foreign variables, represented by the 3-month EURIBOR rate (with the German rate used prior to 1999q1) and the Euro area GDP-deflator inflation rate, over time again using a 10-year rolling window.

⁹ Given that the neutral real interest rate is assumed to be 2.8 per cent, and the inflation target is 2.5 per cent, the “steady state” value of the nominal interest rate is $i = r^* + \pi^*$ which equals 5.3 per cent.

Figure 2.

10-year rolling descriptive statistics – Euro area

As would be expected, the variability in the standard deviation and autocorrelation coefficient of the inflation rate is greater than those of the interest rate. Furthermore, interest rates are highly persistent. The standard deviations and autocorrelation coefficients have not always been constant. However, from around the mid-1980s the statistics have been relatively stable and have magnitudes similar to the US data.

We use first-order autoregressive processes to calibrate the standard deviations of i_t^f and π_t^f to match those of the EURIBOR rate and Euro area inflation for the period 1985q1 to 2001q2. The mean values are set equal to the steady state values of the corresponding domestic variables. Our preferred specifications are:

$$i_t^f = 5.3 + \psi_t, \quad \psi_t = 0.95\psi_{t-1} + v_t, \quad v_t \sim N(0, 0.80) \quad (4.1)$$

$$\pi_t^f = 2.5 + \zeta_t, \quad \zeta_t = 0.65\zeta_{t-1} + u_t, \quad u_t \sim N(0, 1.50) \quad (4.2)$$

where it is the standard deviation (rather than the variance) that is used in the distribution notation.¹⁰

In Table 2, we see that over the simulation horizon, ie 2000q1 to 2249q4, the exogenous foreign variables that have been generated using equations (4.1) and (4.2) behave in a similar manner to observed behaviour in actual Euro area data, which constitute the foreign sector.

Table 2. **Properties of the exogenous foreign variables – actual and generated**

Variable	SD(actual)	SD(generated)	Cor(actual)	Cor(generated)
Interest rate, i_t^f	2.106	2.230	0.977	0.931
Inflation, π_t^f	1.894	1.824	0.679	0.609

Note: “SD” refers to standard deviation and “Cor” refers to the first-order autocorrelation coefficient. The generated data are calibrated to match actual interest rate data from 1985q1 to 2002q1, and inflation data from 1985q1 to 2001q4.

In the model, it is not the level of the foreign inflation rate π_t^f but rather the rational expectation $E_t \pi_{t+1}^f$ that enters the interest parity condition. It would be a mistake to substitute the actual lead of foreign inflation in place of the expectation term in the model. This is because the evolution of π_t^f is explicitly given by equation (4.2), where $E_t \pi_{t+1}^f \neq \pi_{t+1}^f$. Therefore, we analytically solve the model-consistent expectation of foreign inflation via the following steps, noting that π_t^f is part of the information set at time t and v_t is a zero-mean process:

¹⁰ It may be preferable to build a model of the foreign economy, one that captures the linkages between output, inflation and the interest rate, to generate the stochastic series for the two foreign variables. Another alternative may be to use a vector autoregression of the Euro area in place of equations (4.1) and (4.2). These alternatives are, however, reasonably involved with respect to specification and would involve an assumption regarding the conduct of the central bank in response to disequilibrium in the output gap and inflation. As it is the variability of the foreign interest and inflation rates that is of importance in the simulations conducted in this paper these avenues are not pursued. For the simulations in Sections 5.2 and 5.3, where these variables are used, the same stochastic series are used for the numerous simulations, ie the variables are not re-generated each time a simulation is performed.

$$\begin{aligned}
E_t \pi_{t+1}^f &= E_t(2.5 + \zeta_{t+1}) \\
&= E_t[2.5 + (0.65\zeta_t + \upsilon_{t+1})] \\
&= E_t[2.5 + 0.65(\pi_t^f - 2.5) + \upsilon_{t+1}] \\
&= 2.5 + 0.65(\pi_t^f - 2.5) \\
&= 0.875 + 0.65\pi_t^f.
\end{aligned}$$

For the other exogenous variables u_t and v_t , as well as the appropriate treatment of expectations we propose a recursive simulation procedure. This procedure also allows us to include a more general expectational equation to the model (as we do in Section 5.3). Under certainty equivalence the expected value of the stochastic terms of the model is zero. This is achieved when the system is treated as deterministic beyond the current time period so that the stochastic terms are zero except for the current period.

We solve the model 1000 times to obtain one set of time series with which to compute the variability and persistence statistics. In our procedure, the simulations are done in a recursive manner: in the first round, the model is simulated for 1000 periods, in the second round, the model is simulated for 999 periods, etc. The simulation end date is fixed at 2249q4 whilst the simulation start date moves forward by one quarter in each round, starting in 2000q1, and ending at 2249q4. In each simulation round, the current-period shocks u and v are drawn from $N(0, \sigma_u)$ and $N(0, \sigma_v)$ distributions, but for subsequent periods their values are set to zero. Only the current period values of the simulated endogenous variables are of interest in each round. As mentioned above, the setting of period $t+1$ shocks equal to zero in each period allows us to treat the expected values equal to the realised values, for example $E_t y_{t+1} \equiv y_{t+1}$. This procedure is then successively repeated in order to obtain the values of the endogenous variables in the following quarters. Where lagged values are required, for example the $\beta_3 [100(\ln Q_t - \ln Q_{t-1})]$ term in equation (3.2), the previously simulated values are used.

5 Simulation results

We present the simulation results in a series of steps, progressing from the basic New Keynesian model to modifications that include an open-economy variant with a new expected inflation formulation. The purpose of presenting the results in this sequence is to highlight the modifications that are required to replicate the observed persistence in inflation and other variables.

The approach taken is to calibrate plausible parameter values for each model so that the simulated endogenous variables possess similar time series properties

to observed US data. Since the purpose of this paper is to reconcile the basic New Keynesian model with the data the coefficients of the expected lead terms (ie α_1 and β_1) are set equal to one. Wherever possible, similar parameters are used for successive simulations to facilitate comparison between alternative models. The standard deviations of the demand and supply shocks are also calibrated in the same manner. Following the discussion in Section 3 we try to abstract from the exchange rate term in the New Keynesian Phillips curve. As for the policy rule, equal weights are placed on the deviation of expected inflation from target and the output gap. Following Taylor (1993), we use weights of 0.5 and 0.5 in the rule. All simulations are solved over 1000 periods from the steady state using the recursive procedure described in the previous section.

5.1 The closed-economy model

The first simulation shows the inability of the pure forward-looking model to explain macroeconomic persistence. The calibrated model is given by:

$$y_t = E_t y_{t+1} - 0.15[(i_t - E_t \pi_{t+1}) - 2.8] + u_t \quad (5.1)$$

$$\pi_t = E_t \pi_{t+1} + 0.15y_t + v_t \quad (5.2)$$

$$i_t = E_t \pi_{t+1} + 2.8 + 0.5(E_t \pi_{t+1} - 2.5) + 0.5y_t \quad (5.3)$$

where $u_t \sim N(0, 1.80)$ and $v_t \sim N(0, 1.50)$. A summary of the simulation results is contained in Table 3.

Table 3.

Summary of simulation results – closed-economy model

Variable	Standard deviation	Autocorrelation
Output gap, y_t	1.662	0.073
Inflation, π_t	1.509	0.021
Interest rate, i_t	0.831	0.073

Note: “Autocorrelation” refers to the first-order autocorrelation coefficient.

By comparing these values to the range of values in Figure 1 we note that the only aspects of the stylised facts that this model can generate is the variability in the output gap and inflation. The statistics in this simulation can be explained intuitively. Observe that the autocorrelation coefficient of the output gap and the

interest rate are identical. This can be rationalised as follows. First, the equilibrium IS equation is purely forward looking, ie its roots are greater than unity, implying that the output gap is solely a function of the demand shock u_t . Second, the only expectations term that enters the policy rule is expected inflation. By leading the Phillips curve by one period and taking expectations conditional on information at time t , the supply shock v_t vanishes. Hence, the equilibrium policy rule is also a function of the demand shock. This is the reason that both the output gap and the interest rate have the same autocorrelation coefficient. The magnitude of the coefficient is small since the demand shock is an independently and identically distributed error term. Furthermore, note that the standard deviation of the interest rate is half that of the output gap. This can be attributed to the weight of 0.5 on the output gap in the policy rule.

These results are consistent with the current consensus, ie the basic New Keynesian model is not capable of fitting observed data, especially with regards to inflation persistence. As Fuhrer and Moore (1995) point out, the Taylor-style contracting specification upon which the forward-looking New Keynesian Phillips curve is based implies persistence in the price (or wage, given a mark-up relationship) level but not the rate of inflation. Under the Taylor equation, any persistence to inflation derives from the persistence in the excess demand term. Since the output gap is almost random in our simulated model, there is consequently little persistence in inflation.

In Fuhrer and Moore's study, the output gap and the nominal interest rate evolve according to vector autoregressive processes, which in part ensures that the output gap would be a reasonably persistent process. In the following experiment, we use the two-period version of the Fuhrer and Moore inflation equation together with New Keynesian equations for the output gap and the interest rate. The calibrated model becomes:

$$y_t = 1E_t y_{t+1} - 0.15[(i_t - E_t \pi_{t+1}) - 2.8] + u_t \quad (5.4)$$

$$\pi_t = 0.5(\pi_{t-1} + E_t \pi_{t+1}) + 0.15y_t + v_t \quad (5.5)$$

$$i_t = E_t \pi_{t+1} + 2.8 + 0.5(E_t \pi_{t+1} - 2.5) + 0.5y_t \quad (5.6)$$

where $u_t \sim N(0, 0.80)$ and $v_t \sim N(0, 0.60)$. The simulation results are presented in Table 4.

Table 4.

**Summary of simulation results –
Fuhrer-Moore inflation specification**

Variable	Standard deviation	Autocorrelation
Output gap, y_t	0.693	0.069
Inflation, π_t	1.571	0.779
Interest rate, i_t	1.844	0.698

Note: “Autocorrelation” refers to the first-order autocorrelation coefficient.

Using a consistent New Keynesian structure throughout, we find that the Fuhrer and Moore relative contracting model generates sensible inflation (and interest rate) dynamics, even in an otherwise forward-looking model. However, persistence in the output gap is still lacking. In Fuhrer and Moore’s study, the IS equation contains lags as it is an equation in a two variable vector autoregression, and it is shown that the model could generate adequate persistence in the output gap.

5.2 Introducing the open-economy

The current consensus seems to be that the pure forward-looking New Keynesian model is unable to replicate the persistence that is observed in the data. The evidence presented in Table 3 supports this. In this section, we show that this result can be overturned by considering an open economy model. We simulate the following open-economy model

$$y_t = E_t y_{t+1} - 0.15[(i_t - E_t \pi_{t+1}) - 2.8] + 0.006(100 \ln Q_t) + u_t \quad (5.7)$$

$$\pi_t = E_t \pi_{t+1} + 0.15y_t + v_t \quad (5.8)$$

$$i_t = E_t \pi_{t+1} + 2.8 + 0.5(E_t \pi_{t+1} - 2.5) + 0.5y_t \quad (5.9)$$

$$400 E_t \Delta \ln Q_{t+1} = i_t - E_t \pi_{t+1} - (i_t^f - E_t \pi_{t+1}^f) \quad (5.10)$$

where $u_t \sim N(0, 1.40)$ and $v_t \sim N(0, 0.60)$. Recall that since the rational expectation of foreign inflation is exogenously determined it is analytically equal to $0.875 + 0.65\pi_t^f$. The implication of using stochastic non-degenerate series for i_t^f and π_t^f , as opposed to normalising them to zero, is that even in the absence of any domestic shocks, ie $u_t = v_t = 0$, the endogenous variables would still exhibit

some variability. In this model, the open economy elements are the real exchange rate term in the IS equation and the uncovered interest rate parity condition. The model is simulated using the procedure described in Section 4 and the results are contained in Table 5.

Table 5. **Summary of simulation results – open-economy model**

Variable	Standard deviation	Autocorrelation
Output gap, y_t	1.316	0.043
Inflation, π_t	1.518	0.840
Interest rate, i_t	2.163	0.911
Real exchange rate, $100 \ln Q_t$	16.326	0.999

Note: “Autocorrelation” refers to the first-order autocorrelation coefficient.

There are five interesting results from this simulation. First, the open economy nature of the model yields a standard deviation of inflation of 1.518 and a persistence coefficient of 0.840, both of which are well within the range of actual behaviour of US inflation. No lags in the Phillips curve are required. Second, only a small degree of openness is required to achieve this. The coefficient on the real exchange rate in the IS equation is only 0.006. Furthermore, an exchange rate term in the Phillips curve is not necessary to obtain these results. Third, the smooth movements in actual interest rates can be replicated using this model without the aid of a lagged interest rate term in the policy rule (see, for example, Judd and Rudebusch (1998)). Fourth, the observed variability and persistence of the real exchange rate is reproduced without a stochastic risk premium term (or the like) in the interest parity condition. Fifth, the combination of an open-economy model and pure forward-looking behaviour, however, is not sufficient to impart adequate autocorrelation in the output gap.¹¹

5.3 Incorporating noise to the rational expectations assumption

The findings from the previous section suggest that a purely forward-looking open-economy model can match the stylised time series properties of three of the four endogenous variables in the model, the exception being the persistence in the

¹¹ A remedy for this problem would be to use a hybrid-type IS equation with a weight of $(1-\alpha_1)$ on the one-period lag of the output gap. This simulation is done and the results are shown in Appendix 1. We find that a very large backward-looking component is required to generate persistence in the output gap.

output gap. Whilst a large backward-looking component in the IS equation can remedy this problem it would be of interest to see whether it is possible to generate persistence in the output gap in a fully forward-looking model.

One interesting avenue that has not been explored in the literature in much depth is the formation of inflation expectations. In this section we provide a new model of expectations based on the existing rational expectations assumption but augmented with an autocorrelated noise term. To begin our discussion note that it is reasonable to expect that the information sets used to condition expected inflation may differ between the private sector and the central bank. Equations (3.1) to (3.4) can hence be modified as:

$$y_t = \alpha_1 E_t y_{t+1} - \alpha_2 [(i_t - E_t^{\text{PS}} \pi_{t+1}) - r^*] + \alpha_3 (100 \ln Q_t) + u_t \quad (5.11)$$

$$\pi_t = \beta_1 E_t^{\text{PS}} \pi_{t+1} + \beta_2 y_t + \beta_3 (100 \Delta \ln Q_t) + v_t \quad (5.12)$$

$$i_t = E_t^{\text{CB}} \pi_{t+1} + r^* + \lambda_1 (E_t^{\text{CB}} \pi_{t+1} - \pi^*) + \lambda_2 y_t \quad (5.13)$$

$$400 E_t \Delta \ln Q_{t+1} = i_t - E_t^{\text{PS}} \pi_{t+1} - (i_t^f - E_t^{\text{PS}} \pi_{t+1}^f) \quad (5.14)$$

where the superscripts PS and CB denote private sector and the central bank, respectively.

The model represented by equations (3.1) to (3.4) frequently assumes that expectations of inflation are formed rationally, within the confines of the model at hand. It is entirely possible that expectations can deviate from full rationality by a zero-mean noise term. Furthermore, this noise term may be autocorrelated. We therefore propose that the inflation expectations formation of the private sector and the central bank follow:

$$\begin{aligned} E_t^{\text{PS}} \pi_{t+1} &= E_t \pi_{t+1} + \eta_t, \\ \eta_t &= \theta \eta_{t-1} + \varepsilon_t, \\ \varepsilon_t &\sim N(0, \sigma_\varepsilon), 0 \leq \theta < 1. \end{aligned} \quad (5.15)$$

$$\begin{aligned} E_t^{\text{CB}} \pi_{t+1} &= E_t \pi_{t+1} + \zeta_t, \\ \zeta_t &= \phi \zeta_{t-1} + \omega_t, \\ \omega_t &\sim N(0, \sigma_\omega), 0 \leq \phi < 1. \end{aligned} \quad (5.16)$$

In equations (5.15) and (5.16), expected inflation has two components. The first component $E_t \pi_{t+1}$, which is common to both sectors, is the full rational expectation of inflation. If this is the only component in the expectations formulation, the model is identical to the model with rational expectations. Since

we have not been able to impart persistence to the output gap by assuming strict rationality, we introduce a second component to the expectations formation mechanism. The second component, η_t in the case of the private sector and ζ_t in the case of the central bank, is a general stationary first-order autoregressive process where ε_t and ω_t are white noise processes. Note that we are incorporating dynamics in the expectational formation (information set) side of the model rather than in the demand and supply shocks. This distinguishes the specification in equations (5.15) and (5.16) from other studies in the literature, such as Clarida et al (2000).

The interpretation of these expectations processes is as follows. Consider the formation of private sector inflation expectations as given by equation (5.15). Here, the deviation of private sector inflation expectations from full rationality is $E_t^{\text{PS}}\pi_{t+1} - E_t\pi_{t+1}$, where this quantity η_t may be an autocorrelated process (in the case of $0 < \theta < 1$) or purely random (if $\theta = 0$). Note that the η -process can be decomposed into two forecast errors: $\eta_t = E_t^{\text{PS}}\pi_{t+1} - E_t\pi_{t+1} = (E_t^{\text{PS}}\pi_{t+1} - \pi_{t+1}) - (E_t\pi_{t+1} - \pi_{t+1})$, where $E_t^{\text{PS}}\pi_{t+1} - \pi_{t+1}$ is the private sector inflation forecast error and $E_t\pi_{t+1} - \pi_{t+1}$ is the rational expectation forecast error. Since the rational expectation forecast error is uncorrelated with elements of the information set at time t , autocorrelation in the η -process derives solely from the private sector forecast error.

Hence, if the autocorrelation coefficient θ is zero, private sector inflation expectations are a random deviation from full-rationality; otherwise, there are elements of persistence in the noise term. If $\theta = 0$, $E_t^{\text{PS}}\pi_{t+1} = E_t\pi_{t+1} + \eta_t$, $\eta_t \sim N(0, \sigma_\eta)$, which differs from $E_t^{\text{PS}}\pi_{t+1} = E_t\pi_{t+1}$, ie the rational expectation of inflation that is commonly utilised. Though the former specification may appear unusual it is consistent with the definition of rational expectation since η_t is a zero mean disturbance term and is uncorrelated with elements in the information set at time t . A similar interpretation also applies to equation (5.16) in the case of the central bank.

Substituting equations (5.15) and (5.16) into (5.11) to (5.14) gives

$$y_t = \alpha_1 E_t y_{t+1} - \alpha_2 \{ [i_t - (E_t \pi_{t+1} + \eta_t)] - r^* \} + \alpha_3 (100 \ln Q_t) + u_t \quad (5.17)$$

$$\pi_t = \beta_1 (E_t \pi_{t+1} + \eta_t) + \beta_2 y_t + \beta_3 (100 \Delta \ln Q_t) + v_t \quad (5.18)$$

$$i_t = (E_t \pi_{t+1} + \zeta_t) + r^* + \lambda_1 [(E_t \pi_{t+1} + \zeta_t) - \pi^*] + \lambda_2 y_t \quad (5.19)$$

$$400 E_t \Delta \ln Q_{t+1} = [i_t - (E_t \pi_{t+1} + \eta_t)] - (i_t^f - E_t \pi_{t+1}^f) \quad (5.20)$$

$$\eta_t = \theta \eta_{t-1} + \varepsilon_t \quad (5.21)$$

$$\zeta_t = \phi\zeta_{t-1} + \omega_t. \quad (5.22)$$

In this system, the number of error terms is equal to the number of endogenous variables. Regarding expected foreign inflation, we assume that $E_t^{\text{PS}}\pi_{t+1}^f = E_t\pi_{t+1}^f$, where the expression for the rational expectation of foreign inflation is given in Section 4.

The advantage of considering the more general expected inflation formulation introduced in this section can be discussed by focussing on the IS equation, ie equation (5.17). Where previously we could not generate sufficient persistence in the output gap we now have an additional equation, ie equation (5.21), that is able to impart inertia in the IS equation when the value of θ is high.

With this new specification of inflation expectations we simulate the following calibrated system:

$$y_t = 1E_t y_{t+1} - 0.2\{[i_t - (E_t\pi_{t+1} + \eta_t)] - 2.8\} + 0.005(100 \ln Q_t) + u_t \quad (5.23)$$

$$\pi_t = 1(E_t\pi_{t+1} + \eta_t) + 0.05y_t + v_t \quad (5.24)$$

$$i_t = (E_t\pi_{t+1} + \zeta_t) + 2.8 + 0.5[(E_t\pi_{t+1} + \zeta_t) - 2.5] + 0.5y_t \quad (5.25)$$

$$400 E_t \Delta \ln Q_{t+1} = [i_t - (E_t\pi_{t+1} + \eta_t)] - (i_t^f - E_t\pi_{t+1}^f) \quad (5.26)$$

$$\eta_t = 0.925\eta_{t-1} + \varepsilon_t \quad (5.27)$$

$$\zeta_t = \eta_t \quad (5.28)$$

where $u_t \sim N(0, 0.55)$, $v_t \sim N(0, 0.06)$ and $\varepsilon_t \sim N(0, 0.08)$. The results are presented in Table 6. The simulations are based on setting $\zeta_t = \eta_t$. This suggests that the deviation-from-rationality of the private sector is identical to that of the central bank, suggesting a situation of perfect transparency in the formulation of monetary policy in such a way that both parties form their expectations of inflation in the same manner.

Table 6.

**Summary of simulation results –
rationality with autocorrelated noise**

Variable	Standard deviation	Autocorrelation
Output gap, y_t	1.459	0.812
Inflation, π_t	2.110	0.934
Interest rate, i_t	2.683	0.938
Real exchange rate, $100 \ln Q_t$	18.606	0.999

Note: “Autocorrelation” refers to the first-order autocorrelation coefficient.

These results are very similar to those in Table 5, with one notable exception: the observed persistence in the US output gap is fully replicated in the open-economy model that incorporates the more general expected inflation formulation. The model used here retains the hallmarks of the basic New Keynesian model with rational expectations. No lags resulting from the manipulation of the basic structure (or deep parameters) of the model is required to replicate the time series properties of the output gap, inflation, the nominal interest rate, and the real exchange rate.

The results in Table 6 are generated under the assumption that $\zeta_t = \eta_t$, ie the inflation expectations formation of the private sector and the central bank are identical. In Appendix 2, we relax this assumption by allowing the shocks ε_t and ω_t to be independent processes. As indicated in Table A2.1, the simulated variability and persistence are very similar to those in Table 6. In particular, persistence in the output gap is still capable of being generated.

6 Conclusion

“Although the New Keynesian Phillips curve has many virtues, it also has one striking vice: It is completely at odds with the facts.”

Mankiw (2000, p 13).

Mankiw outlined three shortcomings with the New Keynesian Phillips curve, which also apply to the New Keynesian model in general: a credible disinflation can cause economic expansions, the model fails to generate inflation persistence, and the impulse response of monetary policy shocks to inflation is implausible. Perhaps the issue that has received the most attention in recent times in the New Keynesian literature is that of persistence, not only in inflation but also in other macroeconomic variables that these models seek to explain. Directions that have been taken to generate persistence from New Keynesian models have generally led to models with additional lags that are not predicted by the original model

under rational expectations. In this paper we sought to reconcile the purely forward-looking New Keynesian model with the persistence in macroeconomic variables that are observed in the data. We appeal to the exchange rate channel and rational expectations with autocorrelated noise to achieve our aim.

The main findings of the paper are summarised as follows. The exchange rate channel is important for generating inflation persistence in a fully forward-looking New Keynesian model. However, a fully forward-looking model is unable to generate sufficient persistence in the output gap. As an alternative to the arguably ad hoc use of lags in the IS equation, the assumption of rational expectations with an autocorrelated noise term is able to generate adequate persistence in the output gap. Once the exchange rate channel has been incorporated gradual interest rate adjustment in the form of lags in the policy rule is not required.

The use of an open economy model and the consideration of rationality with autocorrelated noise led us to identify one avenue for further research – the so-called forward premium puzzle. This refers to the inability of existing models to explain the persistence and variability of the ex post excess return in the foreign exchange market, measured by the growth rate of the exchange rate less international interest rate differentials. In our notation the ex post excess return is

$$100 \left\{ \Delta \ln Q_{t+1} - \left[\frac{i_t - \pi_{t+1}}{400} - \frac{i_t^f - \pi_{t+1}^f}{400} \right] \right\}.$$

Computing 10-year rolling statistics of the US (with the Euro area treated as the foreign sector) ex post excess return from the mid-1970s, we find that the mean of the standard deviation is around 5.2 per cent and the autocorrelation coefficient is around 0.3. Though our model is able to reproduce the time series properties of the level of the real exchange rate, it is unable to generate sufficient variability in the ex post excess return.

To conclude, the stochastic simulations presented in this paper show that the basic New Keynesian model, after incorporating an exchange rate channel and a more general expected inflation formulation to that used in the literature, ie an assumption of rationality with autocorrelated noise, is consistent with the time series properties of US inflation, the output gap, the nominal interest rate, and the real exchange rate.

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Appendix 1

A hybrid open-economy model

The results in Section 5.2 indicate that a fully forward-looking open economy model fails to generate sufficient persistence in the output gap. In the following model, the IS equation contains a weighted average of the expected lead and one-period lag of the output gap.

$$y_t = 0.15E_t y_{t+1} + 0.85y_{t-1} - 0.15[(i_t - E_t \pi_{t+1}) - 2.8] + 0.003(100 \ln Q_t) + u_t$$

$$\pi_t = E_t \pi_{t+1} + 0.15y_t + v_t$$

$$i_t = E_t \pi_{t+1} + 2.8 + 0.5(E_t \pi_{t+1} - 2.5) + 0.5y_t$$

$$400 E_t \Delta \ln Q_{t+1} = i_t - E_t \pi_{t+1} - (i_t^f - E_t \pi_{t+1}^f)$$

where $u_t \sim N(0, 0.70)$, $v_t \sim N(0, 0.30)$, and the model-consistent expectation of foreign inflation is equal to $0.875 + 0.65\pi_t^f$. This model is simulated using the procedure outlined in Section 4, and the results are contained in Table A1.1.

Table A1.1 **Summary of simulation results – open-economy hybrid model**

Variable	Standard deviation	Autocorrelation
Output gap, y_t	1.409	0.867
Inflation, π_t	1.567	0.839
Interest rate, i_t	2.570	0.888
Real exchange rate, $100 \ln Q_t$	18.974	0.998

Note: “Autocorrelation” refers to the first-order autocorrelation coefficient.

Appendix 2

Rational expectations with autocorrelated noise: Independent shocks

The simulation exercise in Section 5.3 incorporates the assumption of rational expectations with autocorrelated noise. There, the inflation expectations formation of the private sector and the central bank are assumed to be the same. The following model treats the noise terms in the inflation expectations formulation as independent processes.

$$y_t = 1E_t y_{t+1} - 0.2\{[i_t - (E_t \pi_{t+1} + \eta_t)] - 2.8\} + 0.005(100 \ln Q_t) + u_t$$

$$\pi_t = 1(E_t \pi_{t+1} + \eta_t) + 0.05y_t + v_t$$

$$i_t = (E_t \pi_{t+1} + \zeta_t) + 2.8 + 0.5[(E_t \pi_{t+1} + \zeta_t) - 2.5] + 0.5y_t$$

$$400 E_t \Delta \ln Q_{t+1} = [i_t - (E_t \pi_{t+1} + \eta_t)] - (i_t^f - E_t \pi_{t+1}^f)$$

$$\eta_t = 0.925\eta_{t-1} + \varepsilon_t$$

$$\zeta_t = 0.925\zeta_{t-1} + \omega_t$$

where $u_t \sim N(0, 0.55)$, $v_t \sim N(0, 0.06)$, $\varepsilon_t \sim N(0, 0.08)$, and $\omega_t \sim N(0, 0.08)$. The results are presented in Table A2.1.

Table A2.1 **Summary of simulation results – rationality with autocorrelated noise**

Variable	Standard deviation	Autocorrelation
Output gap, y_t	1.244	0.753
Inflation, π_t	2.141	0.927
Interest rate, i_t	2.644	0.933
Real exchange rate, $100 \ln Q_t$	18.550	0.999

Note: “Autocorrelation” refers to the first-order autocorrelation coefficient.

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