

Haavio, Markus; Kauppi, Heikki

Working Paper

Housing markets, liquidity constraints and labor mobility

Bank of Finland Discussion Papers, No. 8/2000

Provided in Cooperation with:

Bank of Finland, Helsinki

Suggested Citation: Haavio, Markus; Kauppi, Heikki (2000) : Housing markets, liquidity constraints and labor mobility, Bank of Finland Discussion Papers, No. 8/2000, ISBN 951-686-663-8, Bank of Finland, Helsinki,
<https://nbn-resolving.de/urn:nbn:fi:bof-20140807350>

This Version is available at:

<https://hdl.handle.net/10419/211862>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

BANK OF FINLAND DISCUSSION PAPERS

8/2000

Markus Haavio – Heikki Kauppi

Research Department
30.6.2000

Housing Markets, Liquidity Constraints
and Labor Mobility

**Suomen Pankki
Bank of Finland
P.O.Box 160, FIN-00101 HELSINKI, Finland
☎ + 358 9 1831**

Markus Haavio* – Heikki Kauppi**

Research Department
30.6.2000

Housing Markets, Liquidity Constraints and Labor Mobility

The views expressed are those of the authors and do not necessarily correspond to the views of the Bank of Finland

* University of Helsinki, Department of Economics, P.O. Box 54 (Unioninkatu 34), FIN-00014 University of Helsinki. E-mail: markus.haavio@helsinki.fi.

** Bank of Finland, Research Department and University of Helsinki, Department of Economics. E-mail: heikki.kauppi@helsinki.fi.

We would like to thank Juha Tarkka, Jouko Vilmunen and Matti Virén for useful comments and suggestions.

ISBN 951-686-663-8
ISSN 0785-3572
(print)

ISBN 951-686-664-6
ISSN 1456-6184
(online)

Suomen Pankin monistuskeskus
Helsinki 2000

Housing Markets, Liquidity Constraints and Labor Mobility

Bank of Finland Discussion Papers 8/2000

Markus Haavio – Heikki Kauppi
Research Department

Abstract

Recent European data indicate that countries where a large proportion of the population lives in owner-occupied housing are experiencing higher unemployment rates than countries where the majority of people live in private rental housing, which might suggest that rental housing enhances labor mobility. In this paper, we develop a simple intertemporal two-region model that allows us to compare owner-occupied housing markets to rental markets and to analyze how these alternative arrangements allocate people in space and time. Consistent with the empirical observations, we find that the interregional labor market is more fluid under rental housing than under owner-occupation. As a result of greater mobility, the rental arrangement also results in better allocational efficiency than owner-occupation. When dwellings are rented, the decision to move to a booming region is largely based on current productivity, whereas under owner-occupation random wealth effects encourage deviations from this optimal behavior.

Keywords: labor mobility, liquidity constraints, owner-occupation, rental housing

Asuntomarkkinat, likviditeettirajoitteet ja työvoiman liikkuvuus

Suomen Pankin keskustelualoitteita 8/2000

Markus Haavio – Heikki Kauppi
Tutkimusosasto

Tiivistelmä

Euroopan maista koottu aineisto osoittaa, että suuren omistusasumisosuuden maissa on korkeampi työttömyysaste kuin maissa, joissa suurin osa ihmisistä asuu yksityisissä vuokra-asunnoissa. Tämä viittaa siihen, että vuokralla asuminen edistää työvoiman tehokasta liikkuvuutta. Tässä tutkimuksessa kehitetään yksinkertainen intertemporaalinen kahden alueen malli, jonka avulla voidaan teoreettisesti analysoida kuinka hyvin nämä vaihtoehtoiset asumisjärjestelyt allokoivat työvoiman alueellisesti ja ajallisesti. Malli osoittaa, että alueiden väliset työmarkkinat toimivat paremmin vuokra- kuin omistusasumisjärjestelyssä. Työvoiman paremman liikkuvuuden ansiosta vuokra-asuminen tuottaa alueellisesti tehokkaamman työvoiman allokaation kuin omistusasuminen. Vuokra-asunnoissa asuvat muuttavat nopean kasvun alueille pääosin työnsä tuottavuuden perusteella. Omistusasumiseen perustuvassa järjestelmässä poiketaan tästä yhteiskunnan kannalta optimaalisesta päätössäännöstä asuntoihin liittyvien satunnaisten varallisuusvaikutusten vuoksi.

Asiasanat: työvoiman liikkuvuus, likviditeettirajoitteet, omistusasuminen, vuokra-asuminen

Contents

| | |
|--|----|
| Abstract..... | 3 |
| 1 Introduction | 7 |
| 2 The environment..... | 10 |
| 3 Housing arrangement and the evolution of wealth..... | 10 |
| 4 The agent's problem | 13 |
| 5 Stationary wealth distribution..... | 18 |
| 6 Mobility and welfare..... | 20 |
| 7 Concluding remarks..... | 22 |
| Appendix..... | 24 |
| References..... | 25 |

1 Introduction

A well-functioning housing market, providing an adequate turnover of residential mobility, is important for the efficient matching of jobs within the labor market. Immobility caused by frictions in the housing market can seriously inhibit the ability of the labor market to match vacancies with potential employees. Recently, many European researchers have asked whether the relatively high unemployment rates in countries like the UK and Spain can be explained by the fact that the majority of people in these nations owner-occupy their homes, while a small minority live in commercial rental housing (e.g. Oswald (1999)). This question arises naturally, since the lowest rates of unemployment can be found from such contrasting countries like Germany and Switzerland where most people live in privately rented homes. A number of microeconomic studies also indicate that home-owners are more sluggish to move in response to changing labor market conditions than people who rent their homes (see e.g. Hughes and McCormick (1985, 1987), Henley (1998), and Gardner et al. (2000)). Despite the importance of the underlying policy questions, there does not yet exist a satisfactory theoretical analysis about the relative merits of owner occupation and rental housing in enhancing interregional mobility of households and labor.

This paper attempts to fill a part of this gap. We develop a simple intertemporal two-region model which allows us to compare owner-occupied housing to rental markets, and analyze how these alternative arrangements allocate people in space and time. Consistent with the above cited empirical evidence, we find that under the rental arrangement there is more mobility than under owner occupation. Higher mobility also implies that allocational efficiency is better when houses are rented. Roughly speaking, the rental market allocates the most productive people to the growing regions, while under owner-occupation the wealthiest people live there.

There are several reasons why owner-occupation may render the labor market less fluid. Because houses are not accepted as collateral for their whole value, a positive down-payment is required, and wealth affects a household's ability to move. Moreover, a large fraction of a household's wealth is often bound to the previous house. Thus it may be difficult to move from a depressed area, where house prices are low, to a booming area, where prices are high. Obviously when houses are rented, the hurdle to move to the booming area is lower, and the decision to do so can be based on job prospects. Looking at the booming city, a worker whose match with the current job becomes poor faces strong incentives to leave the city, if he has to pay a high rent there. By contrast a person owning his house may not feel an urge to move out even when he loses his job or retires.

Behind these observations, we find a more general pattern. Under the rental arrangement, a house is definitely a consumption good which is bought in every period for the services it offers. At each moment of time there is also an intimate connection between benefits received and costs incurred: people residing in the booming area pay for the privilege in the form of higher rents. The rental market thus offers the agents strong incentives to choose their location based on their productivity, which is also the socially optimal decision rule.

By contrast, when bought, a house not only entitles its owner to a stream of services, but it is also an asset which may either appreciate or depreciate in value. In particular, an agent who buys a house when the region is depressed and sells it when the region is booming, is likely to make a capital gain. A less fortunate agent

who buys in the boom and sells during the recession, is more likely to make a capital loss, however. Thus at least a part of the costs for living in a booming area take the form of random punishments which are incurred by those who happen to own a house there when the boom ends. Likewise the compensation for those settling for a low growth region, partially takes the form of a random gain.

The randomness introduced by the asset aspect of owner-occupied housing then delude the link between services received and payments made, and the incentives to choose one's location in every period based on the goodness of match are weaker than under the rental arrangement. In particular we find that the uncertain punishment embodied in capital losses does not provide a strong enough deterrent for wealthy low productivity agents to move out of the booming city. Consequently a part of the mechanism equating housing supply and demand must assume the form of endogenously arising debt traps or liquidity constraints, which prevent some high productivity agents from working in the growth center. This, of course, is a source of inefficiency.

Let us take a closer look at the model from which we obtain the above findings, and comment some of the assumptions made. We consider an economy where random business cycles fluctuate asynchronously across two regions or cities. This assumption tries to capture the fact that different regions tend to grow at different rates, but the relative growth rate may also vary over time, with the low growth, low employment region becoming the high growth region, and vice versa. Special cases of this modelling strategy include a constant center-periphery structure where the boom never shifts, and deterministic cycles. The stock of houses in each city is fixed and there are no vacant homes. This is a simple way to model the shortage of housing in the booming region. In this economy optimal spatial allocation of workers requires that in every period some workers in one city move crosswise with others in the other city. The need for this arises because the efficiency or quality of each worker changes from time to time in a random manner. Specifically, there are only two possible qualities, high or low, such that a high quality worker compare always better than a low quality worker in the booming city, while both types of workers earn equally well (badly) in the non-booming city. Therefore, all the high quality workers should always live in the booming city, while the low quality workers should populate the non-booming city.

While in the social optimum the place where an agent lives should only depend on his type, in the market outcome the choice is also influenced by wealth. In particular if an agent is liquidity constrained, he cannot move to the booming city where house prices and rents are higher. When modelling liquidity constraints we adopt particularly simple and mild assumptions: we just require that nobody can take credit above a finite limit. Due to the prospect of facing the liquidity constraint, the agent's decision then involves a trade-off between present benefits and future options. Choosing the booming city when productivity is low entails the risk that this option may not be available in the future when productivity is high.

In the environment outlined above an agent's maximization problem usually involves two kinds of decisions: where to live, and how much to consume and save. Then the evolution of wealth depends on a number of factors, including the wage rate, the optimal level of consumption, and costs and gains from housing. In more technical terms, one has to analyze a stochastic optimization problem where the random process evolves in varying step lengths. It is well known that this class of problems is plagued with the 'curse of dimensionality' (see e.g. Ljungqvist and

Sargent (2000)), and finding a solution becomes rather intricate. There are then two main responses to this challenge: numerical methods and simplifying assumptions. In this paper we adopt the latter strategy.

As the paper studies how different housing market arrangements allocate people in space and time, a fairly natural simplification involves focusing on the choice of location, while abstracting from the consumption-savings decision. This can be done by adopting a reduced form model, where living in the booming area simply yields an immediate benefit, which cannot be saved. This payoff may reflect the present value of receiving a higher wage, or alternatively one can think of housing as a source of services, the quality of which varies regionally, depending on public goods, environmental factors and other characteristics of the city. In this reduced form the evolution of wealth then only depends on losses and gains made in the housing sector. The model becomes highly tractable and transparent, clearly highlighting the key differences between renting and owner-occupation. Preliminary results in an on-going research, Haavio and Kauppi (2000), suggest that our main findings survive in a more complex framework where consumption and savings are determined endogenously.

Apart from the empirical studies cited above, this paper does not have any close predecessor at the theoretical front. However, there are some interesting connections to earlier analytical studies on housing markets and labor mobility. For example, our analysis on the role of wealth effects in the decision making problem of an owner-occupier can provide interesting additional insights into earlier studies on the choice of housing tenure (see e.g. Artle and Pravin (1978), Weiss (1978), Henderson and Ioannides (1983), Fu (1995); for a survey see Smith et al. (1988)). It is also worth noting that in the economy of this paper workers and houses can become mismatched due to similar reasons as in the recent searching models for housing markets by Wheaton (1990) and Williams (1995). Yet, the liquidity constraint theme of this paper is closely related to that of Stein (1995) who studied the effects of a down-payment on the formation of house price movements. Most interestingly, it is to be noted that Stein (1995) considered a static economy with an exogenous debt distribution, while in this paper we analyze a dynamic model and derive an endogenously arising wealth distribution across the agents. Finally, from the vast literature on labor mobility the Lucas and Prescott (1974) paper is perhaps the most relevant to our study. While they showed how job searching frictions can result in an important component to the natural rate of unemployment, the present paper reveals another significant component caused by housing market frictions.

The plan of the paper is as follows. Section 2 lays down the model basics and presents the physical environment where the moving of workers or agents takes place. Section 3 shows how an agent's wealth evolves under different housing arrangements. It turns out that the wealth accumulation of an agent in the rental market can be represented as special case of the one of the owner-occupied housing arrangement. In Section 4, we solve the agents' optimal moving policy that depends on their wealth. Section 5 proceeds from the individual to the aggregate level and derives the aggregate wealth distribution of the economy. Section 6 provides labor mobility and welfare comparisons between the two housing market arrangements. Concluding remarks are given in Section 7.

2 The environment

This section describes the physical environment where and the spatial economic dynamics under which the moving of workers takes place.

The economy has two cities. Both cities have an equal, fixed, stock of identical houses. Each house is occupied by a single economic agent and no one agent is ever homeless. For convenience, assume that in both cities the stock of houses is a continuous set of size unity. Then, at each point of these sets there is a house occupied by an agent.

There are infinite discrete time periods indexed by $t = 0, 1, \dots$. In each period, one of the cities is booming and the other is not. When a period changes, the boom removes to the other city through the transition probability $\pi \in [0, 1]$. The duration of the boom has a geometric distribution with the mean $1/\pi$. Note that if $\pi = 1$, the boom shifts deterministically in every period, while if $\pi = 0$, the boom remains in one city.

The agents can and will always work when they live in the booming city. When an agent is working he receives a utility equal to θ , where θ takes one of the two values, θ_L and θ_H such that $0 \leq \theta_L \leq \theta_H$. The higher utility level θ_H is earned when the skills of a working agent matches well with the current production technology, while in the case of mismatching the low value θ_L is earned. At the start of each period, the matching quality of every agent is independently drawn from the symmetric probability distribution, $\Pr(\theta_L) = \Pr(\theta_H) = 1/2$. When an agent lives in the non-booming city, he cannot work and receives zero utility no matter how well his skills match with the current technology. The agents live forever and discount future utilities by a factor $\beta \in (0, 1)$.

Now, imagine the basic setup of the economy in the beginning of any time period. First, a random process determines which city is booming and which is not. Next, in both cities, all agents are randomly divided into two groups of the same size, a half. One of the groups consists of low matching agents, and the other of well matching agents. When the agents have observed their type, they choose the city in which they want to live. In every period, the aggregate welfare would be maximized, if all high quality agents were allocated to the booming city, while the low quality workers would live in the non-booming city. Should this settlement take place in every period, the expected utility of a representative agent would be

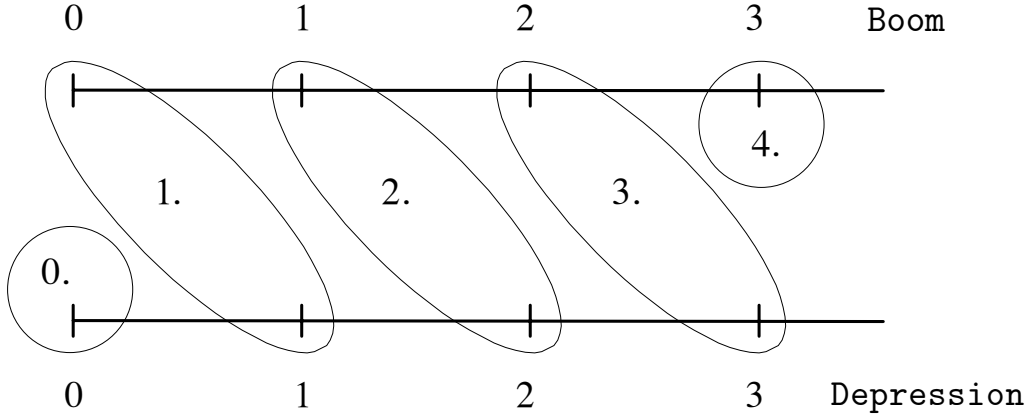
$$\sum_{t=0}^{\infty} \beta^t \Pr(\theta_H) \theta_H = \frac{1}{2} \frac{\theta_H}{1 - \beta}.$$

Note that in this arrangement the expected utility does not depend on the probability at which the boom shifts. In what follows, we incorporate two different housing market arrangements to the model through which the agents can move from one city to the other.

3 Housing arrangement and the evolution of wealth

While in the social optimum the place where an agent lives should only depend on his type, in the market outcome the choice is also influenced by wealth. In particular an agent's moving decisions may be radically restricted by liquidity constraints. This section studies how an agent's wealth evolves under different housing arrangements.

Figure 1. **Wealth class, bank savings, and house location**



We begin by owner-occupied housing. Let p_b and p_d , respectively, denote the market prices of the houses in the booming and non-booming cities. These steady state prices satisfy the inequality $p_b > p_d$. A move from the booming city to the non-booming city by an agent entails that he sells his current house in the booming city and buys another home in the non-booming city. The difference in house prices $p = p_b - p_d$ is then deposited to the agent's bank account. In the opposite case, an agent moving from the depressed city to the booming city withdraws p units from his account to make up for the difference between house prices. For a move from the depressed to the booming city to be possible, the agent's bank balance has to be large enough to cover the costs. To be more specific we assume that there is a maximum level of debt which cannot be exceeded. If this requirement is not met, the agent is liquidity constrained and cannot move. To keep the model as simple as possible we assume that the bank is just a clearinghouse which keeps track of agents' balances. In particular, the bank does not pay any interest to the deposits. Note that the savings of an agent then always comprises a multiple of p .

Take two agents i and j . At the beginning of a period agent i has $n \times p$ units of wealth in his bank account and a house in the depressed city, while agent j has $(n - 1) \times p$ units in the bank and a house in the booming city. Now it should be fairly easy to see that there is no real difference between the two agents' financial position. As housing markets are assumed to be frictionless i can sell his house in the depressed city and buy another in the booming city. After this transaction he has exactly the same assets as j . The same argument also applies the other way round. We can then define a wealth class consisting of two financial states:

$$\text{Wealth class } n : \begin{cases} (n - 1) \times p \text{ units in bank and a house in the booming city} \\ n \times p \text{ units in bank and a house in the non-booming city.} \end{cases} \quad (1)$$

In the above example agents i and j belong to the same wealth class n .

A case with five wealth classes, ranging from 0 to 4, is illustrated in Figure 1. Bank savings of people living in the booming and the depressed city are measured on the two horizontal axes. For simplicity, we denote the lowest permitted bank balance by 0. This is, however, only a convenient normalization, and agents with this lowest possible balance may be deeply in debt. Wealth classes 1, 2, and 3, delimited by ellipsoids, each consist of two financial states, in accordance with the

Table 1. Wealth class in period $t + 1$ depending on house location and housing arrangement, when the initial wealth class is n

| | Booming city | Depressed city |
|------------------------|--|--|
| Owner-occupied housing | n with probability $1 - \pi$ $n - 1$ with probability π | n with probability $1 - \pi$ $n + 1$ with probability π |
| Rental housing | $n - 1$ with probability 1 | $n + 1$ with probability 1 |

definition (1) given above. In the lowest wealth class 0, the agents are liquidity constrained and cannot move to the booming region even if they wanted. Thus this wealth class is a singleton. Also the highest wealth class, in our example class 4, contains only one financial state. Agents belonging to this class are so wealthy that they never want to move to the depressed region.

If an agent is in wealth class n in period t , what is his class in the next period? Suppose agent i decides to live in the booming city. Then with probability $1 - \pi$ the boom remains and the agent begins the next period with the same possessions. With probability π the boom shifts, and the agent suffers a capital loss. In the next period the agent has $(n - 1) \times p$ units of wealth in the bank and a house in a depressed city. Thus he has fallen one step down to wealth class $n - 1$.

Next suppose the agent chooses the depressed city, instead. With probability $1 - \pi$ the city is depressed also in the subsequent period, and the agent stays in wealth class n . With probability π there is a change in regional fortunes, and the agent makes a capital gain. With $n \times p$ units of wealth in the bank and a house in a booming city, the agent has succeeded to climb to wealth class $n + 1$.

Next we turn to rental housing. Assume the housing stock in both cities is managed by a real estate company, owned by the agents. Let r_b and r_d denote per period level of rents in the booming and in the depressed city, respectively. These steady state rents satisfy the inequality $r_b > r_d$. In every period the total revenue collected by the real estate company is $r_b + r_d$. This revenue is then distributed to the agents, with each of them receiving $(r_b + r_d)/2$ units. Now the net change in an agent's balances depends on where he lives. Those residing in the booming city pay $r = (r_b - r_d)/2$ units more than they earn, while those choosing the depressed city gain r units. The savings of an agent then always constitute a multiple of r . Since the housing stock is in common ownership, knowing an agent's bank balance also tells his level of wealth. The transition from one wealth class to another is simple. Those agents who choose the booming area always pay a net price r and fall one class down, while others who settle for the depressed city climb one ladder. In accordance to owner-occupation, we also assume that there is a maximum amount of debt which cannot be exceeded. An agent reaching the lowest allowed bank balance can only live in the depressed city, no matter what his type is.

The differences between owner-occupied and rental housing are summarized in Table 1. Three points are worth emphasizing. First, in the rental arrangement, those who want to live in the booming city always pay for the privilege, and likewise those who stay in the depressed city are duly compensated. In contrast, when houses are owner-occupied, both payment and compensation take place only with probability π . In what follows we demonstrate that the closer link between costs and benefits implies that rental housing does a better job in allocating agents in space and time. Second, while the workings of the rental market are independent

of the transition probability π , the market with owner-occupation is affected by the length of the cycle. Finally, rental markets are equivalent to owner-occupation with deterministic cycles. This fact considerably simplifies the subsequent analysis: instead of analyzing two separate model variants, each corresponding to one housing arrangement, we can simply construct a model of owner-occupation, and then study renting by setting π equal to one.

4 The agent's problem

Consider the optimization problem of any one agent. In every period he chooses his location so as to maximize the expected discounted utility stream

$$E_\theta \sum_{t=0}^{\infty} \beta^t \theta I_t,$$

where I_t denotes an indicator function which is equal to one, if the agent lives in the booming city in period t , and zero otherwise. This is a stochastic dynamic control problem in which the state variable is the level of wealth. The problem can be conveniently presented in a recursive form.

We find it instructive to first define separate value functions for agents owning a house in the booming city and in the depressed city. Then we demonstrate that both value functions can be reduced into a single function, having the wealth class as its argument. This unified value function has the additional advantage that it can be also used in the analysis of rental markets.

Let $V^i(n)$, ($i = b, d$) denote the optimal value of the problem for an agent who owns a house in the booming city ($i = b$) or the non-booming city ($i = d$), respectively, and has a balance of $n \times p$ units in his bank account. Then, $V^b(n)$ satisfies the Bellman equation

$$V^b(n) = E_\theta \left[\max \left\{ \theta + \beta \left[(1 - \pi)V^b(n) + \pi V^d(n) \right], \right. \right. \\ \left. \left. \beta \left[(1 - \pi)V^d(n + 1) + \pi V^b(n + 1) \right] \right\} \right].$$

Note that the value function $V^b(n)$ is evaluated after the location of the boom is known but before the type θ has been revealed to the agent. The maximization problem then defines the optimal moving decision that takes place when the value of θ becomes known. Inside the maximum operator, the first expression is the value of staying in the booming city, while the second expression is the value of moving to the depressed city. If the agent's optimal decision is to stay in the booming city, he can immediately 'eat' whatever value θ realized. His prospects for the next period are discounted by β and are given in the square brackets in the first argument of the maximization problem. There is a probability $1 - \pi$ that the boom will stay in his city tomorrow so that he will be facing the same value function as today, while with probability π the boom shifts to the other city giving him the prospects of an agent in the non-booming city. If the agent moves to the depressed city, his utility in the current period is zero. With probability $(1 - \pi)$ his city will be depressed also in the next period, and with probability π the boom shifts. In the same way, it is easy to see that the value function for an agent who owns a house in the non-booming city satisfies the recursion

$$V^d(n) = E_\theta \left[\max \left\{ \theta + \beta \left[(1 - \pi)V^b(n - 1) + \pi V^d(n - 1) \right], \right. \right. \\ \left. \left. \beta \left[(1 - \pi)V^d(n) + \pi V^b(n) \right] \right\} \right].$$

Comparing $V^b(n)$ with $V^d(n)$ immediately reveals that $V^d(n) = V^b(n - 1)$. That is, at the start of a period, an agent who has his house in the non-booming city and whose bank savings equal n units has equal future prospects with an agent in the booming city with $n - 1$ units of bank savings. Thus we can move to a common value function having the wealth class, defined in Section 3, as its argument. Let $V^b(n) = V(n)$ so that $V^d(n) = V(n + 1)$. The recursion in terms of the common value function $V(n)$ is then

$$V(n) = E_\theta [\max \{ \theta + \beta [(1 - \pi)V(n) + \pi V(n - 1)], \beta [(1 - \pi)V(n) + \pi V(n + 1)] \}]. \quad (2)$$

It is worth noting that in the special case with $\pi = 1$ the recursion (2) captures the agent's maximization problem under rental markets. That is

$$V(n) = E_\theta [\max \{ \theta + \beta V(n - 1), \beta V(n + 1) \}].$$

The maximization problem in (2) essentially boils down to the following choice: Living in the booming area involves an immediate benefit in the form of a job which pays θ . On the cost side there is an uncertain capital loss which materializes with probability π . Choosing the depressed area entails no immediate benefits, but there is a possible capital gain, again with probability π . If the housing arrangement is that of renting, costs and gains occur with certainty. On a more fundamental level, there is a trade-off between present benefits and future options. The agent wants to avoid the situation where his choices are limited by the liquidity constraint. By choosing the booming city when productivity is low entails the risk that this option may not be available in the future when productivity is high.

There is then a cut-off level

$$\theta^* = \pi\beta [V(n + 1) - V(n - 1)] \quad (3)$$

that equates the two arguments in the maximization operator in (2). For an agent in wealth class n to be willing to live in the booming city, his productivity must be at least θ^* . We can distinguish three regimes for the values of θ^* given by: (i) $\theta^* \leq \theta_L$, (ii) $\theta_L < \theta^* < \theta_H$, and (iii) $\theta^* \geq \theta_H$.

In the subsequent analysis, we mainly focus on regime (ii). Regime (iii), where an agent always chooses the depressed city, cannot be part of an optimal strategy. The only reason not to live in the booming city is to keep the option to do so in a later period when the match is better. Obviously this motivation has no bite, if the current match is the best possible (θ_H). In equilibrium, an agent with high productivity fails to move to the booming city only if he is liquidity constrained. Regime (i), which applies for high wealth levels, only appears as a boundary condition in the subsequent analysis. In this regime the agent wants to be in the booming city, no matter what his type is. Given this policy, only capital losses are possible, and the agent's wealth eventually erodes. In equilibrium, only the lowest wealth class in the regime is ever reached. We denote this class by \bar{n} ; it constitutes the ceiling for the equilibrium wealth distribution.

Consider regime (ii). In this case, every agent whose quality realization is high will want to choose to live in the booming city irrespective of whether that requires moving or not. On the other hand, everybody with low quality match will want to live in the non-booming city and will move there if currently in the booming city. It is worth noting that this strategy is identical to the socially optimal decision rule,

described in Section 2. Given that the low and high qualities, θ_L and θ_H , are equally probable we see from (2) that the value function satisfying these policies is

$$V(n) = \frac{1}{2} \{ \theta_H + \beta [(1 - \pi)V(n) + \pi V(n - 1)] \} \\ + \frac{1}{2} \beta [(1 - \pi)V(n) + \pi V(n + 1)],$$

which can be rewritten as

$$V(n) = \frac{1}{2}(1 - \delta) \frac{\theta_H}{1 - \beta} + \frac{1}{2} \delta [V(n - 1) + V(n + 1)], \quad (4)$$

where

$$\delta = \frac{\beta\pi}{1 - \beta(1 - \pi)} \quad (5)$$

is the uncertainty adjusted discount factor with the properties

$$\frac{\partial \delta}{\partial \beta} = \frac{\pi}{[1 - (1 - \pi)\beta]^2} > 0, \quad \frac{\partial \delta}{\partial \pi} = \frac{\beta(1 - \beta)}{[1 - (1 - \pi)\beta]^2} > 0.$$

The relation between β and δ is obvious; the connection between π and δ then essentially tells that the weight given to future changes in wealth depends on their probability. Also, note that when $\pi = 1$, i.e. when the boom switches deterministically in every period, then $\delta = \beta$. This is also the applicable discount factor under rental housing.

We turn to solving the second order difference equation in (4). The general form of the solution is given by

$$V(n) = A_1 q_1^n + A_2 q_2^n + \frac{1}{2} \frac{\theta_H}{1 - \beta}, \quad (6)$$

where

$$q_1 = \frac{1 + \sqrt{1 - \delta^2}}{\delta}, \quad q_2 = \frac{1 - \sqrt{1 - \delta^2}}{\delta}$$

are the roots of the fundamental quadratic

$$Q(q) \equiv \frac{1}{2} \delta q^2 - q + \frac{1}{2} \delta.$$

Both roots are positive, with $q_1 > 1$, $q_2 < 1$, and $q_1 q_2 = 1$. Notice that the special solution in (6) coincides with the expected utility of a representative agent in the optimal settlement discussed in Section 2 where all high quality agents always live in the booming city. The homogenous part of the solution comprises any deviations from this optimal allocation. Recall that these deviations derive from two sources: First, when the agent is liquidity constrained he cannot move to the booming city. Second, when the agent has accumulated enough wealth he wants to live in the booming city even when his productivity is low.

In order to find the unknown coefficients A_1 and A_2 in (6) we need boundary conditions. Let $n = 0$ denote the lowest possible wealth class. An agent belonging to wealth class 0 has a house in the depressed city and is unable to move because of the liquidity constraint. He can only wait until the boom comes to his city. In this case, the value function is given by

$$V(0) = \beta [(1 - \pi)V(0) + \pi V(1)]$$

or

$$V(0) = \delta V(1),$$

where δ is given in (5). From (4), we get a representation for the lowest unconstrained wealth class $n = 1$:

$$V(1) = \frac{1}{2}(1 - \delta) \frac{\theta_H}{1 - \beta} + \frac{1}{2}\delta[V(0) + V(2)].$$

The latter two equations together yield

$$(1 - \frac{1}{2}\delta^2)V(1) = \frac{1}{2}(1 - \delta) \frac{\theta_H}{1 - \beta} + \frac{1}{2}\delta V(2). \quad (7)$$

This is the first boundary condition we need.

Next, let \bar{n} be the highest wealth class ever reached by the agent. In a moment we are going to study how \bar{n} is determined endogenously by optimization. For the time being, however, we take \bar{n} as given. An agent belonging to wealth class \bar{n} has a house in the booming city and wants to live there even when his productivity is low. Thus the agent stays in the booming city until the state of the economy changes. Given his strategy, the agent cannot climb up to the next wealth class; with probability π he falls down into wealth class $\bar{n} - 1$. The rationale for this decision rule is the following: agents accumulate housing wealth just in order to guarantee future options to move to the booming city whenever the high value θ_H realizes while they happen to live in the non-booming city. However, at a certain wealth level, liquidity constraint becomes such a distant and improbable prospect, that it is better to stay and 'eat' the low realization θ_L immediately rather than earn an additional piece of insurance for receiving the high outcome θ_H at some remote future point of time.

The value function for wealth class \bar{n} satisfies the recursion:

$$V(\bar{n}) = \frac{1}{2}(\theta_H + \theta_L) + \beta[(1 - \pi)V(\bar{n}) + \pi V(\bar{n} - 1)]$$

or

$$V(\bar{n}) = \frac{1}{2}(1 - \delta) \frac{\theta_H + \theta_L}{1 - \beta} + \delta V(\bar{n} - 1). \quad (8)$$

Using (4) we get the representation for the second highest wealth class $\bar{n} - 1$, where agents still follow the socially optimal decision rule:

$$V(\bar{n} - 1) = \frac{1}{2}(1 - \delta) \frac{\theta_H}{1 - \beta} + \frac{1}{2}\delta[V(\bar{n}) + V(\bar{n} - 2)]$$

and combining this with (8) yields

$$(1 - \frac{1}{2}\delta^2)V(\bar{n} - 1) = \frac{1}{2}(1 - \delta) \frac{\theta_H}{1 - \beta} + \frac{1}{4}\delta(1 - \delta) \frac{\theta_H + \theta_L}{1 - \beta} + \frac{1}{2}\delta V(\bar{n} - 2). \quad (9)$$

This is the second boundary condition we need.

Now, using the general solution (6) together with the boundary conditions (7) and (9) we can solve for the unknown coefficients A_1 and A_2 . Plugging these coefficients back into the general solution gives the value function in terms of parameters, the wealth class n and the ceiling of wealth \bar{n}

$$V(n; \bar{n}) = \frac{1}{2} \frac{1 - \delta}{(1 - \beta)\sqrt{1 - \delta^2}} \frac{\theta_L (q_1^n + q_2^n) - \theta_H (q_1^{\bar{n}-n} + q_2^{\bar{n}-n})}{q_1^{\bar{n}} - q_2^{\bar{n}}} + \frac{1}{2} \frac{\theta_H}{1 - \beta}. \quad (10)$$

For the sake of clarity we have explicitly expressed the dependence on \bar{n} . As already noted above, the specific solution is the expected present value of well-being in the social optimum, while the homogenous part reflects the deviations from the optimum. It is easy to check that the value function increases with the wealth class n . Also notice that $V(0) < \theta_H/[2(1 - \beta)]$: the liquidity constrained are worse off than in the social optimum, as they evidently have to be.

Next we turn to studying how the upper boundary of wealth \bar{n} is chosen by the optimizing agent. To do so we go back to equation (3), defining the cut-off productivity θ^* . Evaluated at $n = \bar{n}$ this equation reads

$$\theta^* = \pi\beta[V(\bar{n} + 1) - V(\bar{n} - 1)]. \quad (11)$$

Now the optimally chosen upper boundary is the lowest integer \bar{n} such that

$$\theta^* \leq \theta_L.$$

In order to use equation (11) we still have to evaluate $V(\bar{n} + 1)$. As wealth class $\bar{n} + 1$ belongs to regime (i), the agent would follow the same policy as in wealth class \bar{n} , were this state ever to be reached. (Of course, this never happens along the equilibrium path.) Thus, looking back at equation (8), the value function satisfies the following recursion

$$\begin{aligned} V(\bar{n} + 1) &= \frac{1}{2}(1 - \delta)\frac{\theta_L + \theta_H}{1 - \beta} + \delta V(\bar{n}) \\ &= (1 - \delta^2)\frac{1}{2}\frac{\theta_L + \theta_H}{1 - \beta} + \delta^2 V(\bar{n} - 1), \end{aligned} \quad (12)$$

where the latter equality follows from (8). Next set $\theta^* = \theta_L$ and let \bar{n}^* be the solution to

$$\theta_L = \pi\beta[V(\bar{n}^* + 1; \bar{n}^*) - V(\bar{n}^* - 1; \bar{n}^*)].$$

Then the upper boundary of wealth \bar{n} is the smallest integer greater than \bar{n}^* . Finally, the recursion (12) and the solution of the value function (10) allow us to derive an equation for \bar{n}^* in terms of the parameters of the model

$$\frac{\theta_L}{\theta_H} = \frac{1}{2} \frac{q_1 - q_2}{(1 - q_2)q_2^{\bar{n}^*} - (1 - q_1)q_1^{\bar{n}^*}}. \quad (13)$$

There are three factors determining the wealth ceiling \bar{n} : temptation, punishment, and the probability of the punishment. The temptation is to stay in the booming city and 'eat' the lower realization θ_L now rather than to go to the depressed city in the hope of capital gains. The larger the cake θ_L on offer, the stronger the incentives to take it even at a lower level of wealth:

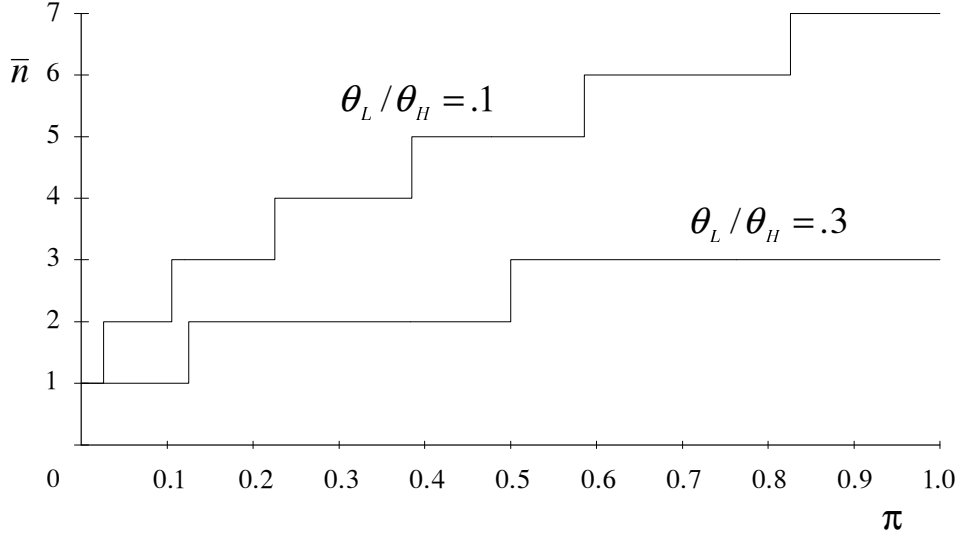
$$\frac{\partial \bar{n}^*}{\partial \theta_L} < 0.$$

The punishment is the possibility that in a future period the liquidity constraint will hamper the move to the booming city, when the match is good. The larger the punishment, the more wealth people are willing to accumulate in order to avoid it:

$$\frac{\partial \bar{n}^*}{\partial \theta_H} > 0.$$

The probability of the punishment depends on the length of the cycle, captured by π : recall that capital losses only materialize when the boom shifts. When transitions

Figure 2. The dependency of \bar{n} on π for two values of θ_L/θ_H when $\beta = .95$



from one wealth class to another become more frequent, people find it optimal to build a larger wealth buffer:

$$\frac{\partial \bar{n}^*}{\partial \pi} > 0.$$

Remember that under the rental arrangement a transition up or down the wealth ladder occurs with full certainty, $\pi = 1$. Thus with rental markets low quality agents are less inclined to go to the booming city than under owner-occupation.

The above comparative static effects are illustrated in Figure 2 where the two stepping lines show how the ceiling of wealth \bar{n} depends upon the cycle parameter π and the ratio of θ_L and θ_H . Note that the optimal wealth buffer under the rental arrangement is given at the points of the lines where $\pi = 1$ irrespective of the actual value of π . Also, note that owner-occupied and rental housing arrangements can lead to the same value of \bar{n} . For example, in the case where $\theta_L/\theta_H = .3$ and $\beta = .95$, the same value $\bar{n} = 3$ follows under both housing arrangements as long as there is at least a fifty-fifty chance for the shift of the boom.

Finally, it is worth pointing out that as the punishment takes place in the future, the deterrent is more efficient when people are patient. Evidently we have

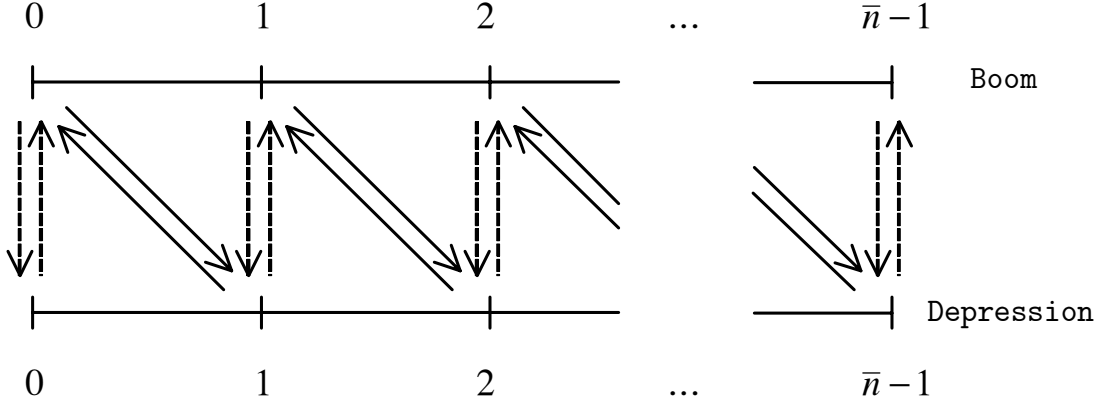
$$\frac{\partial \bar{n}^*}{\partial \beta} > 0.$$

The derivations underlying the comparative statics are in the Appendix.

5 Stationary wealth distribution

The agents' optimal strategies depend on their wealth. Thus to proceed from the individual level to the aggregate level we need to know the wealth distribution in the economy. In particular, we are interested in the size of the liquidity constrained group and the group of agents who want to be in the booming city even when their productivity is low.

Figure 3. **Wealth distribution**



It turns out that the prescribed optimal moving policy of a representative agent implies a stationary housing wealth distribution across all agents in the economy. When subdividing agents into different groups, wealth classes do not provide the most practical classification system. Instead we revert to the alternative characterization based on bank savings and location. There are \bar{n} different levels (i) of bank savings ranging from $i = 0$ to $i = \bar{n} - 1$. Notice that agents belonging wealth class \bar{n} have $\bar{n} - 1$ units of savings. As an agent resides either in the booming or the non-booming city, we can classify all agents into $2 \times \bar{n}$ groups depending on the levels of their bank savings and the city they live in. This classification is illustrated in Figure 3.

Let us denote the frequency of agents having i units of savings in the bank account and living in the booming city by f_i^b and let f_i^d be the corresponding notation for the depressed city. What are the stationary frequencies that do not change over time as people migrate and the boom shifts from one city to the other? Recall that in every period a random process determines the matches of the agents such that both cities have a continuum of agents of each quality all equal in size. It also follows that in each of the savings classes one half consists of low quality and the other half of high quality agents. Then recall the basic moving policy of the agents in a given period. First, all low quality agents in the non-booming city will stay there, while the high quality workers want to move to the booming city and also will do it as long as they are not liquidity constrained. Second, all high quality workers in the booming city will stay there, while the low quality workers want to move to the non-booming city unless their bank savings are large enough. To sum up, for each level of bank savings $i = 0, \dots, \bar{n} - 2$, there are $\frac{1}{2}f_i^b$ agents who begin the period with i units of savings and a house in the booming city, and end the period having $i + 1$ units of savings and a house in the non-booming city. The frequency of those making the opposite move is $\frac{1}{2}f_{i+1}^d$. In Figure 3, this two-way migration is depicted by diagonal solid arrows. In the steady state, the labor flows between the two cities have to cancel out each other at every level of bank savings:

$$\frac{1}{2}f_i^b = \frac{1}{2}f_{i+1}^d, \quad i = 0, \dots, \bar{n} - 2. \quad (14)$$

The condition (14) also guarantees that the demand for housing equals the supply of housing in both cities and in every period.

Next, suppose the period changes and the boom removes so that all agents who previously owned a house in the booming city now find that their possession is in the depressed area. Thus the frequency f_i^d at the beginning of period $t + 1$ is equal to the frequency f_i^b at the end of period t . This transition is captured by the vertical dashed arrows in Figure 3. Now, for the stationary distribution to be unaltered by changes in regional fortunes we must have

$$f_i^b = f_i^d, \quad i = 0, \dots, \bar{n} - 1. \quad (15)$$

Finally, combining (14) and (15) yields

$$f_i^k = f_j^m \text{ for } \forall i, j \in \{0, \dots, \bar{n} - 1\} \text{ and } \forall k, m \in \{b, d\}.$$

In words, all groups must be equal in size. As the total population of the economy is 2 and there are $2 \times \bar{n}$ groups, the size of each group is $1/\bar{n}$. In particular, there are $1/\bar{n}$ liquidity constrained agents, and $1/\bar{n}$ agents who choose the booming city no matter what their types are.

6 Mobility and welfare

Now we can characterize the workings of the economy and compare the market outcome to the first best. In particular, we want to study how different housing arrangements create hurdles for labor mobility and thereby generate welfare losses.

If the socially optimal decision rule were followed, half of the labor force would move in every period: a measure of $1/2$ of high quality workers would migrate from the depressed city to the booming city, and a measure of $1/2$ of low quality workers would make the trip the other way round. Thus in social optimum the measure of per period labor mobility is

$$M^* = 1.$$

In the market outcome, however, the liquidity constrained agents cannot move to the booming city, although half of them, those with the high realization θ_H , optimally should. On the other hand no one of those belonging to the highest wealth class \bar{n} wants to leave the booming city, although half of them should. As there are $1/(2\bar{n})$ agents in each of the non-optimally behaving groups, and rest of the agents follow the socially optimal rule, the per period measure of mobility is

$$M^e = 1 - \frac{1}{\bar{n}}.$$

The difference between these two migration streams gauges the rigidities caused by the housing market. We define the measure of stiffness S :

$$S = M^* - M^e = \frac{1}{\bar{n}}.$$

Note that the 'stiffness' of the market simply reflects the size of the groups which are unable or unwilling to migrate optimally.

Remembering the result $\partial \bar{n}^* / \partial \pi > 0$ derived above in Section 4, we observe that there is less mobility, or more stiffness, under owner-occupation than when houses are rented. Under the rental arrangement those residing in the booming city always pay for the privilege. This prospect is not particularly tempting for low quality agents. By contrast, under owner-occupation, the payment, taking the

form of a capital loss, is incurred only with probability π . Thus there are more agents who are willing to stay in the booming city no matter what their type. The downside of owner-occupiers' reluctance to leave the booming city is then that for the housing market to clear there must be a sizable group of liquidity constrained agents who are unable to move in even when they have a good match. It is also worth noting that the stiffness of the owner-occupied housing market is accentuated when business cycles become longer, and the economy assumes a relatively constant center-periphery pattern.

Before proceeding to welfare analysis, we briefly pause to analyzing how productivity differences influence labor mobility. Using the results presented in Section 5, it is easy to conclude that better the poor match θ_L fares against the good match θ_H , the more agents there are, who do not want to quit the booming city when they should. Through the equilibrium of the housing market, this is also mirrored in the increasing size of the liquidity constrained group. In other words, the smaller the difference between the two productivity levels, the less there is labor mobility.

What is the welfare of the economy in the social optimum and under different housing arrangements? As we have repeated ad nauseam, it would be socially optimal in all periods to allocate all well matching agents to the booming city. In this case, in a given period, all high quality agents live in the booming city and receive θ_H , while the rest of the agents live in the non-booming city and receive nothing. Consequently, the aggregate welfare in a period is given by

$$W^* = \theta_H.$$

In the market equilibrium there are $1/(2\pi)$ liquidity constrained high productivity agents and $1/(2\pi)$ wealthy low productivity agents who live in the wrong city. The aggregate per period welfare is

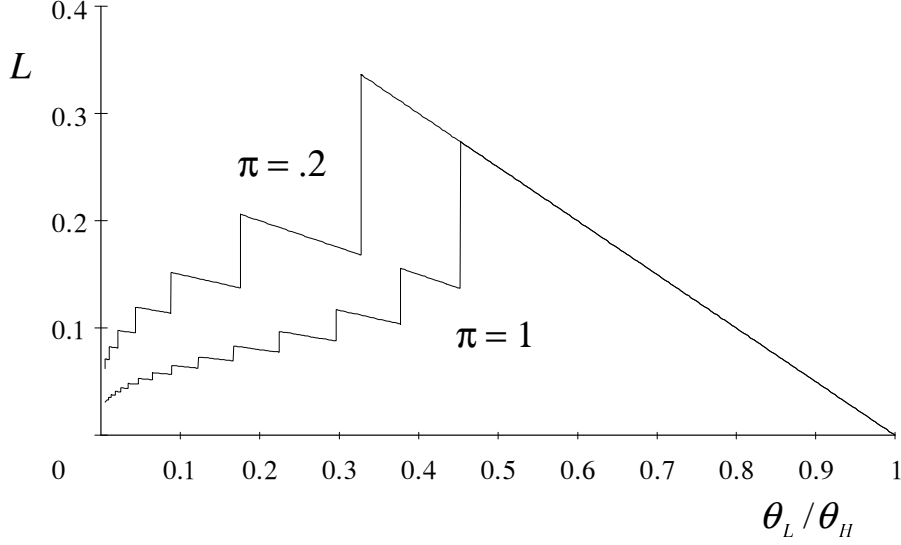
$$W^e = \theta_H - \frac{1}{2\pi}(\theta_H - \theta_L).$$

The difference between the social optimum and the market outcome is the welfare loss generated by the housing arrangement

$$L = \frac{W^* - W^e}{W^*} = \frac{1}{2}S(1 - \frac{\theta_L}{\theta_H}).$$

The welfare loss is a product of two factors: the productivity difference between high and low types $\theta_H - \theta_L$, and the stiffness of the market S , impeding mobility and causing mismatch. We already noted above that there is more stiffness under owner-occupied housing than in the rental arrangement. Thus also the welfare loss is higher when people own their houses. The intuition behind this finding should be clear by now. Under rental housing, there is an intimate connection between costs and benefits. Those who reside in the booming city always pay for the services they receive. By contrast, when houses are own, the payment is uncertain. The agent can then reap the benefits and, with some chance, avoid the costs. Thus the rental arrangement provides the agents with stronger incentives to act as they should. As the weaker incentive system embodied in owner-occupation cannot keep enough agents out of the booming city, a larger part of this task is left to liquidity constraints, preventing some high productivity agents from working in the booming city. This mismatch is then a source of inefficiency. Also notice that owner-occupied housing fares the worse, the longer the business cycle. Thus renting should be the preferred housing arrangement especially if there is a relatively stable

Figure 4. **The dependency of the welfare loss on the ratio θ_L/θ_H under owner-occupied housing ($\pi = 1$) and rental market ($\pi = .2$) arrangement when $\beta = .95$**



center-periphery pattern in the economy, but variations in match require that the identity of people working in the center changes over time.

Figure 4 draws the welfare loss, L , as a function of the ratio of θ_L and θ_H when the discount factor $\beta = .95$. The line denoted by $\pi = 1$ shows the welfare loss under the rental market arrangement. Notice the same line applies irrespective of the actual value of π . In contrast, under owner-occupation the welfare loss depends on the length of the cycle. In Figure 4, the line denoted by $\pi = .2$ shows the welfare loss under owner-occupied housing when the expected length of the cycle is five periods. The gap between the two lines comprises the welfare loss that results in if houses are owned rather than rented.

7 Concluding remarks

Recent European data indicate that countries with high shares of owner-occupation have larger unemployment rates on average than countries where most people live in private rental housing, suggesting that owner-occupation is inferior to rental housing in enhancing efficient spatial matching of labor and jobs. In this paper, we developed a simple intertemporal two-region model, which allowed us to compare owner-occupied housing markets to rental markets, and analyze how these alternative arrangements allocate people in space and time. Consistent with the empirical observations, we found that the interregional labor market is more fluid under rental housing than under owner-occupation. As a result of greater mobility, the rental arrangement also results in better allocational efficiency than owner-occupation.

The first step towards understanding the results of the paper is to recognize that the two housing markets have different implications for the optimal moving policy of an individual agent. Under the rental arrangement, people residing in the booming area always pay for the privilege in the form of higher rents, offering them strong incentives to choose their location based on their productivity. In contrast,

under owner occupation the payment occurs only with uncertainty; those living in the booming region are partly insured against high prices in housing. They are only worried about capital losses they could make, if the boom removed to somewhere else. The smaller is the likelihood of the shifting of the boom the less they care about this possibility, and as we showed in this paper, the weaker are their incentives to choose their location based on the goodness of the match.

The completing step is to see how the optimal moving policy of an individual agent affects the steady state wealth distribution across all agents. It is useful to recall that the wealth distribution in our model consisted of a simple debt trap, where anybody currently at the lowest ladder had the highest risk to run into a liquidity constraint where it is no longer possible to move to the booming city although the match would be the best possible. On the other hand, anyone at the highest ladder of the debt trap had the best insurance against such an undesirable outcome. Given the prescribed incentive schemes owner-occupiers accumulate less wealth than those who rent their homes, and thus, there are fewer ladders in the debt trap under owner-occupation than under rental markets. Therefore, the equilibrium share of well-matching people who are liquidity constrained and cannot work in the booming area is larger under owner-occupation than under rental markets. This is the basic source of inefficiency and results in needless spatial mismatching between skills and jobs.

It is reasonable to ask how robust are the findings of this paper. In our economy, people consumed all their labor incomes immediately. The only way to accumulate saving was through moving from the depressed to the booming city, while these housing market gains could not be directly consumed. Of course, in reality people save their labor incomes in order to be able to move, and may consume whatever capital gains they earn in the housing market. Taking account of these additional features in the modelling brings in complications to the analysis. First, when labor incomes can be saved, the wealth of an individual agent may evolve in various step lengths, making the underlying discrete time optimization problem insuperable. Second, when labor incomes can be saved and housing capital gains be consumed, the solution of the model requires deriving explicitly the equilibrium house price difference between the two cities and how this relates to the labor incomes. In a forthcoming paper, Haavio and Kauppi (2000), we attempt to resolve these complications in a continuous time model and show that similar results follow even when consumption and savings are determined endogenously.

Appendix

We adopt the notation

$$v(q_1, \bar{n}^*) \equiv \frac{1}{2} \frac{q_1 - q_2}{(1 - q_2) q_2^{\bar{n}^*} - (1 - q_1) q_1^{\bar{n}^*}}.$$

Notice that $q_2 = q_1^{-1}$. Now the equation (13) determining the upper boundary of wealth as a function of the parameters can be expressed in the form

$$\frac{\theta_L}{\theta_H} = v(q_1, \bar{n}^*). \quad (16)$$

Next, taking the derivatives of $v(q_1, \bar{n}^*)$ with respect to its arguments yields

$$v_{\bar{n}^*} \equiv \frac{v(q_1, \bar{n}^*)}{\partial \bar{n}^*} = v(q_1, \bar{n}^*) \frac{\ln(q_1) [(1 - q_2) q_2^{\bar{n}^*} + (1 - q_1) q_1^{\bar{n}^*}]}{(1 - q_2) q_2^{\bar{n}^*} - (1 - q_1) q_1^{\bar{n}^*}} < 0$$

and

$$v_{q_1} \equiv \frac{\partial v(q_1, \bar{n}^*)}{\partial q_1} = \frac{1}{2} \frac{1 + q_2^2}{(1 - q_2) q_2^{\bar{n}^*} - (1 - q_1) q_1^{\bar{n}^*}} - v(q_1, \bar{n}^*) \frac{q_1^{\bar{n}^*} + q_2^{\bar{n}^*+2} - \bar{n}^* [(1 - q_2) q_2^{\bar{n}^*+1} + (1 - q_1) q_1^{\bar{n}^*-1}]}{(1 - q_2) q_2^{\bar{n}^*} - (1 - q_1) q_1^{\bar{n}^*}} < 0.$$

Then totally differentiating (16) gives the comparative statics

$$\frac{d\bar{n}^*}{d\theta_H} = -\frac{\theta_L}{\theta_H^2 v_{\bar{n}^*}} > 0; \quad \frac{d\bar{n}^*}{d\theta_L} = \frac{1}{\theta_H v_{\bar{n}^*}} < 0; \quad \frac{d\bar{n}^*}{d\delta} = \frac{v_{q_1}}{v_{\bar{n}^*}} \frac{q_1}{\delta \sqrt{1 - \delta^2}} > 0.$$

As δ grows monotonically with both π and β , the relation between \bar{n}^* , π and β follows in the obvious way.

References

- Artle, R. — Pravin, V. (1987) **Life cycle consumption and ownership**. Journal of Economic Theory, 18, 35.
- Fu, Y. (1995) **Uncertainty, liquidity, and housing choices**. Regional Science and Urban Economics, 25(2), 223.
- Gardner, J. — Pierre, G. A. — Oswald, J. (2000) **Moving for job reasons**. Manuscript, University of Warwick.
- Haavio, M. — Kauppi, H. (2000) **Housing markets and labor mobility in a continuous time model**. Preliminary manuscript, University of Helsinki.
- Henderson, P. H. — Ioannides, Y. M. (1983) **A model of housing tenure choice**. American Economic Review, 73, 98.
- Henley, A. (1998) **Residential mobility, housing equity and the labour market**. The Economic Journal, 108, 414.
- Hughes, G. — McCormick, B. (1985) **An empirical analysis of on-the-job search and job mobility**. Manchester School, 53, 76.
- Hughes, G. — McCormick, B. (1987) **Housing markets, unemployment and labour market flexibility in the UK**. European Economic Review, 31, 615.
- Ljungqvist, L. — Sargent, T. J. (2000) **Recursive macroeconomic theory**. Manuscript, Stanford University.
- Lucas, R. E. J. (1988) **On the mechanics of economic development**. Journal of Monetary Economics, 22, 43.
- Lucas, R. E. J. — Prescott, E. C. (1974) **Equilibrium Search and Unemployment**. Journal of Economic Theory, 7(2), 188.
- Oswald, A. J. (1999) **The housing market and Europe's unemployment: A non-technical paper**. Manuscript, University of Warwick.
- Smith, L. B. — Rosen, K. T. (1988) **Recent developments in economic models of housing markets**. Journal of Economic Literature, 26, 29.
- Stein, J. (1995) **Prices and trading volume in the housing market: a model with down-payment effects**. Quarterly Journal of Economics, 110, 379.
- Weiss, Y. (1978) **Capital gains, discriminatory taxes, and the choice between renting and owning a house**. Journal of Public Economics, 10, 45.
- Wheaton, W. C. (1990) **Vacancy, search, and prices in a housing market matching model**. Journal of Political Economy, 98(6), 1270.

Williams, J. T. (1995) **Pricing real assets with costly search.** The Review of Financial Studies, 8(1), 55.

BANK OF FINLAND DISCUSSION PAPERS

ISSN 0785-3572, print; ISSN 1456-6184, online

- 1/2000 Jussi Snellman – Jukka Vesala – David Humphrey **Substitution of Noncash Payment Instruments for Cash in Europe**. 2000. 39 p. ISBN 951-686-647-6, print; ISBN 951-686-648-4, online. (TU)
- 2/2000 Esa Jokivuolle – Samu Peura **A Model for Estimating Recovery Rates and Collateral Haircuts for Bank Loans**. 2000. 22 p. ISBN 951-686-649-2, print; ISBN 951-686-650-6, online. (RM)
- 3/2000 Risto Herrala **Markets, Reserves and Lenders of Last Resort as Sources of Bank Liquidity**. 2000. 24 p. ISBN 951-686-653-0, print; ISBN 951-686-654-9, online. (TU)
- 4/2000 Pekka Hietala – Esa Jokivuolle – Yrjö Koskinen **Informed Trading, Short Sales Constraints and Futures' Pricing**. 2000. 29 p. ISBN 951-686-655-7, print; ISBN 951-686-656-5, online. (RM)
- 5/2000 Mika Kuismanen **Labour Supply and Income Tax Changes: A Simulation Study for Finland**. 2000. 36 p. ISBN 951-686-657-3, print; ISBN 951-686-658-1, online. (TU)
- 6/2000 Ralf Pauli **Payments Remain Fundamental for Banks and Central Banks**. 2000. 40 p. ISBN 951-686-659-X, print; ISBN 951-686-660-3, online. (RM)
- 7/2000 Yuksel Gormez – Forrest Capie **Surveys on Electronic Money**. 2000. 46 p. ISBN 951-686-661-1, print; ISBN 951-686-662-X, online. (TU)
- 8/2000 Markus Haavio – Heikki Kauppi **Housing Markets, Liquidity Constraints and Labor Mobility**. 2000. 26 p. ISBN 951-686-663-8, print; ISBN 951-686-664-6, online. (TU)