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The effects of transmission uncertainty on the flexibility-credibility tradeoff in monetary policy

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14/99

Marc-Alexandre S  n  gas – Jouko Vilmunen

Research Department
8.3.2000

The Effects of Transmission Uncertainty on the Flexibility-Credibility Tradeoff in Monetary Policy

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The views expressed are those of the authors and do not necessarily correspond to the views of the Bank of Finland

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Abstract

In this paper we address the issue of how parameter uncertainty affects the optimal degree of central bank conservatism. The analysis is conducted in the standard macroeconomic model of a monetary policy game embedding an expectational Phillips-curve. Multiplicative “Brainard” uncertainty is added to the model. This means that the central bank’s policy instrument has a stochastic impact on inflation. This type of uncertainty is particularly interesting, since it affects the credibility–flexibility tradeoff in monetary policymaking.

We show that if the flexibility problem dominates, an increase in uncertainty reduces optimal conservatism. However, increases in uncertainty can also require increases in the optimal degree of conservatism. This happens when the central bank has a sufficiently large credibility problem. This is particularly clear in the case of the introduction of uncertainty at the margin. Furthermore, the coefficient of variation of inflation appears to contain useful information about the relative size of the credibility problem and, hence, about how incipient uncertainty can affect optimal conservatism in actual economies.

Keywords: credibility, flexibility, monetary policy, conservatism, uncertainty

Transmissioepävarmuuden vaikutus rahapolitiikan uskottavuuden ja joustavuuden väliseen ristiriitaan

Suomen Pankin keskustelualoitteita 14/99

Marc-Alexandre Sénégas – Jouko Vilmunen
Tutkimusosasto

Tiivistelmä

Keskustelualoitteessa tarkastellaan, miten rahapolitiikan vaikutuksiin liittyvä epävarmuus vaikuttaa siihen, kuinka inflaatiota kaihtava keskuspankin tulisi yhteiskunnan kannalta olla. Analyysi perustuu rahapolitiikkaa strategisena pelinä tarkastelevassa kirjallisuudessa vakiintuneeseen kokonaistaloudelliseen malliin, jonka keskeinen osa on odotuksilla täydennetty Phillips-käyrä. Brainardin klassisia, parametripävarmuuksiin liittyviä tarkasteluja mukaillen mallia täydennetään transmissioepävarmuudella. Tällä tarkoitetaan rahapolitiikan vaikeasti ennakoitavia, satunnaisia inflaatiovaikutuksia. Tämä rahapolitiikan välittymis- eli transmissioepävarmuus on tärkeä, koska se vaikuttaa uskottavuuden ja joustavuuden väliseen ristiriitaan rahapolitiikassa.

Tulokset riippuvat siitä kuinka vakava rahapolitiikan uskottavuusongelma on suhteessa joustavuusongelmaan eli siihen, miten suuria kokonaistaloudelliset tarjontahäiriöt ovat. Jos kokonaistaloudelliset tarjontahäiriöt ovat suuria, mikä korostaa joustavuuden tarvetta rahapolitiikassa, transmissioepävarmuuden kasvu vähentää keskuspankin optimaalista inflaation vastaisuutta. Jos taas rahapolitiikan uskottavuusongelma on hallitseva, transmissioepävarmuuden osoitetaan kasvattavan keskuspankin optimaalista inflaation vastaisuutta. Nämä tulokset ovat selkeimpiä tilanteessa, jossa transmissioepävarmuus on pieni. Inflaatiovauhdin variaatiokertoimen eli keskihajonnan ja keskiarvon suhteen avulla voidaan karkeasti arvioida, miten välittymiseen liittyvän epävarmuuden kasvu voisi vaikuttaa keskuspankin optimaaliseen inflaation vastaisuuteen.

Asiasanat: uskottavuus, joustavuus, rahapolitiikka, konservatiivisuus, transmissioepävarmuus

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1 Introduction

Since the start of the European monetary union on January 1, 1999, the European System of Central Banks (ESCB) has been responsible for formulating and implementing the common monetary policy of the Euro area. These new monetary (policy) arrangements in Europe give rise to a number of interesting questions and problems that the European monetary authorities will be faced with or have already been faced with and which, in light of the recent literature on (perceived tradeoffs in) monetary policymaking, will bear on how to formulate (and implement) optimal monetary policy for the Euroarea and on the likely outcomes of policies adopted by the European Central Bank (ECB). Two of these problems, originating from the seminal contributions of Rogoff (1985) and Brainard (1967), have already been discussed in the recent literature.¹

First of all, there is the question of delegation of monetary policy decisions to an independent central bank as a means of sustaining and enhancing credibility of monetary policy. In the European context this issue will probably stay topical for an extended period of time in so far as actions and decisions taken by the ECB will certainly be scrutinized by different quarters to see whether they correspond to those of a truly independent² institution and decision maker. Thus the issue of independence will probably be of importance when markets take a perspective on ECB's monetary policymaking and will probably affect the economic outcome of the policy actions taken by the ECB. The literature has identified two aspects of monetary policy which are likely to be affected by the decision to delegate monetary policy to an independent central bank. The first one is the inflationary bias associated with discretionary monetary policy. Rogoff (1985) demonstrated that the bias can be lowered if monetary policy is delegated to an independent central bank which is more inflation averse than the society, ie. which is more conservative in preferences than the society. Hence, independence *cum* conservatism is an institutional mechanism that mitigates the inflationary consequences of discretionary behaviour of the monetary authority. However, delegation of monetary policy to an independent, more conservative central bank does not offer the society a 'free lunch' because it entails reduced flexibility, ie. less room for manoeuvre in the stabilization domain. A more conservative central bank will accommodate output supply shocks to a lesser extent than would be implied by the preferences of the society. This implies, in turn, that the variability of aggregate output will be higher under the more conservative monetary policy regime, ie. the cost of a reduced inflation bias is increased output variance. As a consequence, there is a tradeoff between flexibility and credibility and when choosing an optimal (independent) central banker or when choosing the optimal degree of central bank (weight-) conservatism the society balances the marginal costs from reduced flexibility and marginal benefits from increased credibility. The exact nature of this tradeoff in the context of the ECB or ESCB is, of course, an open question, but since the newly created institution appears to have a reputation to build, it is not at all unlikely that it is willing to take unnecessarily strong

¹ See Bordes (1998) for a concise overview of these issues.

² Independence should more often than not be interpreted as a relative concept. Society can change the ultimate goal of monetary policy, so it would be better to talk about 'independence in a given (monetary) policy regime'. Also, there is this distinction between 'instrument' and 'goal' independence drawn e.g. by Svensson (1996).

decisions in its early days, so as to signal its strong dislike for inflation. Monetary policy could, therefore, during these early days be less flexible than in the case of a monetary authority with a well-established reputation for being able to pursue policies of the required type.

In addition to the issue of credibility, or reputation building, another element that has to be taken into account in the formulation and implementation of monetary policy by the ECB as well as in the assessment of the policy is the (perceived) uncertainty associated with effects of monetary policy effects on the economies of the Euroland and, hence, on the aggregate Euroeconomy itself. In particular, several studies have emphasised the likely differences in the transmission channels of monetary policy in the member countries (e.g. Dornbusch et al (1998)). Plausible theoretical arguments, perhaps also practical experience in central banking, seem to suggest that in such a context, the ECB should act with caution, since due to the uncertainty of the (short-run) effects of its policy decisions and actions, activism could interact with the structural features of the economy as well as of the policy problem so that the outcome is poorer performance of the aggregate Euroeconomy as well as of the individual economies of the member countries. The underlying reasoning here builds, of course, on the pioneering results of Brainard (1967), according to which monetary policy should, in the relevant case of *multiplicative parameter uncertainty*, also called *transmission uncertainty* in the sequel, react less to shocks than in the reference case of full knowledge of the economic structure, ie. in the case where there is no transmission uncertainty. As a consequence, the stabilisation potential of (optimal) monetary policy is reduced in this case of transmission uncertainty.

Transmission uncertainty will thus fundamentally affect the relationship between credibility and flexibility of monetary policymaking. Or, more interestingly, it interacts with the credibility - flexibility tradeoff in a way that has important implications for e.g. the optimal degree of central bank (weight-) conservatism.³ In this paper we are interested in the precise nature of this interaction and in its implications for optimal conservatism. We try to find (hopefully simple) conditions under which the introduction of or increases in transmission uncertainty will increase (decrease) optimal conservatism. We address this question in the framework of a Barro-Gordon model⁴ in which transmission uncertainty (multiplicative parameter uncertainty à la Brainard) is introduced. Because of its simplicity, the Barro-Gordon set up is particularly attractive in the present context: closed form solutions for optimal policies as well as for the equilibria of the economy can be easily derived and have straightforward interpretations. The ensuing analysis will put much emphasis on two fundamental features of the model that will bear, most notably, on the economic interpretation of optimal conservatism and its comparative statics behaviour. On the one hand, there is, from Brainard, what we call *instrument*

³ We shall henceforth simply talk about 'conservatism' without the reference to weight-conservatism. This should cause no confusion, since the formal analysis of the paper does not recognise any other forms of conservatism.

⁴ See Barro and Gordon (1983a, b). To be a little more precise, the linear-quadratic framework applied in the present paper was analysed by Kydland-Prescott (1977) in an example in their seminal contribution on time inconsistency of optimal policies. In their model, social loss is quadratic in the deviations of output from the target. In the Barro-Gordon models, on the other hand, the social loss function is actually linear in these deviations.

variability that will affect the extent to which sources of aggregate variability will affect the social loss from different policies. On the other hand, there is, from Rogoff, the size of the credibility problem relative to the flexibility problem of monetary policy making, or the size of the *relative bias* as we will call it, that will fundamentally bear on the question of how favourable the conditions are for the delegation of monetary policy to an independent, (more) conservative central bank. It turns out that we can give a systematic account of the effects of transmission uncertainty of the optimal degree of central bank conservatism in terms of these two features of the model.

The interaction of the time inconsistency problem with transmission uncertainty has, of course, already been emphasised in the literature on strategic monetary policymaking. In the context of the present paper, however, the analysis conducted by Schellekens (1998) on caution and conservatism in monetary policymaking is perhaps the most important one.⁵ First of all, his model is slightly more general than ours in that it allows for additive control errors in the inflation equation. Furthermore, the multiplicative and additive control errors need not be independent of each other, although the control errors and other, economic errors or shocks are. Since we are not explicitly concerned about the hedging possibilities or signalling problems in monetary policymaking, we abstract from the additive source of control errors and, hence, from the correlated control errors. We do think, however, that the issue of the interaction of uncertainty generated by policymaking and transmission uncertainty is of importance and would need further analysis. However, we do think that, in relation to Schellekens, we provide a different perspective to the question of how Brainard affects Rogoff's delegation problem by systematically emphasising the interaction between the Rogoffian relative bias and Brainardian transmission uncertainty. Furthermore the (3D) graphical technique, which we use extensively in the core parts of our analysis, also clearly suggests that as far as the effects of transmission uncertainty on optimal central bank conservatism are concerned, the size of the relative bias is the critical factor to control for.

The structure of the paper is as follows. In the next section, we present the model and discuss the two separate cases usually considered in the literature; delegation in the absence of transmission uncertainty and the effects of transmission uncertainty on optimal monetary policy without delegation. Section three and four then combine the two cases in an analysis of the determination of optimal degree of central bank conservatism under transmission uncertainty. The analysis in these sections strongly emphasises the size of the relative bias in determining the comparative statics effects of increases in transmission uncertainty on optimal central bank conservatism. Section five concludes and discusses possibilities of using data in an attempt to pin down the (sign of the) likely effect of introducing or increasing transmission uncertainty on (optimal) central bank conservatism in

⁵ The literature on (public's) uncertainty about the preferences of the policy maker provides another perspective on the determination of optimal monetary policy and inflation in a strategic context (See e.g. Cukierman and Meltzer 1986, Cukierman 1992 and Söderström 1999). The focus of this strand of literature is more on the effects of the interaction of asymmetric information (about the policy maker's preferences) with the underlying sources of random shocks and policy errors. Models in this appear to be particularly useful e.g. on issues related to the optimal degree of monetary policy ambiguity, or of central bank secrecy.

practice. Finally, an appendix contains parts of the formal analysis referred to in the main text.

2 Monetary policy, conservatism and uncertainty

The model to be presented is a standard one in the literature on monetary policymaking. The supply side of the economy is described by an expectations augmented Phillips-curve: output deviations from the natural level are driven by expectations errors, which, in turn, arise from rigidities due to nominal contracting in the labour market.⁶ We have then:

$$y = \bar{y} + \alpha (\pi - \pi^e) + u, \alpha \geq 0 \quad (1)$$

where y denotes the log of actual level and \bar{y} the log of the natural level of output,⁷ π and π^e are the current and rationally expected inflation rates and u is a supply shock whose distribution is common knowledge in the economy: in particular, $E[u] = 0$ and $var[u] = \sigma_u^2$

In order to introduce transmission uncertainty, we assume that the monetary authority - central bank - has only an imperfect control over the inflation rate. More specifically, we suppose that the money supply is its monetary policy instrument and that the inflation effects of operating it are stochastic, ie. we assume the existence of control errors. The latter may be of two types (Brainard 1967). On the one hand, control errors may be additive representing stochastic intercept shifts in the linear money supply control rule. These shifts may originate from e.g. exogenous money demand shocks impinging additively on the quantity theoretic relationship between inflation and monetary growth. The effect on inflation and output volatility depends then on whether the policy-maker can observe this disturbance before implementing monetary policy. On the other hand, the impact effects of changes in the money supply (growth rate) on inflation may be stochastic. In a log-linear context, this means that the elasticity of the inflation rate w.r.t. the changes in the (growth rate of) money supply is a random variable. Inflation may be a purely monetary phenomenon in the long-run, but in the short-run there may be deviations from the unit elasticity (of inflation w.r.t. changes in money growth) due e.g. stochastically changing lags in the transmission of monetary impulses to the inflation rate. As a consequence, the multiplicative effects of monetary impulses⁸ on inflation become stochastic. In summary, then, the introduction of additive and multiplicative transmission uncertainty into the model amounts to assuming:

$$\pi = \mu \cdot \Delta m - v \quad (2)$$

⁶This is the contracting interpretation of the Phillips-curve, initially formalised by Gray (1976) and Fischer (1977) in the context of macroeconomic models. Lucas's (1972) interpretation, on the other hand, of the Phillips-curve relies on agents' misperception between relative price and general price level movements in a rational expectations (, flexible price) equilibrium of the economy.

⁷Time subscripts are dropped for convenience.

⁸In the framework of the model, these impulses amount to changes in the money supply growth rate.

where Δm denotes the growth rate in the quantity of money (m is the log of nominal money balances) and v is the zero mean additive control error - velocity shock, for short - with a constant variance: $E[v] = 0$ and $var[v] = \sigma_v^2$. Multiplicative transmission uncertainty is represented by the random variable μ , whose two first moments are given by $E[\mu] = 1$ and $var[\mu] = \sigma_\mu^2$. As such equation (2) may be derived from a (linear) money market equilibrium with additive money demand shocks and stochastic elasticity of inflation w.r.t. the changes in the money growth rate.

In what follows, we focus, for simplicity, on multiplicative transmission uncertainty. This assumption simplifies the notation nicely and makes the calculation more straightforward. We can make our case in this simpler set up. Furthermore, if μ and v are assumed to be independent random variable, our results do not change qualitatively, because additive transmission uncertainty affects the stabilization dimension of monetary policy, which is still present in the model through the occurrence of supply shocks.⁹ Finally, we suppose that the random variables u and μ are independent with each other.

As for the monetary policy game, we assume that in each period, the private sector minimises the mean squared error of its inflation forecasts. Absent delegation, monetary policy choices are ranked according to the society's preferences; using the standard macroeconomic formulation, optimal monetary policy is found by minimising a quadratic social loss function, according to which social loss (L) depends on squared deviations of current inflation and output from some specified targets π^* and y^* :

$$L = (y - y^*)^2 + \gamma(\pi - \pi^*)^2 \quad (3)$$

We assume that π^* is zero and that the output target is higher than the natural level: $y^* = k \cdot \bar{y}$ with $k > 1$. The latter assumption reflects the view that there are distortions that prevent the labor market to attain full employment; the existence of distorting wage taxes or monopoly power in labour pricing are mostly quoted to sustain this assumption. These externalities are taken into account at the aggregate level in the policy process. However, if monetary authorities have an incentive to expand output beyond its natural level, they can only do so by fooling the private sector inflationary expectations (see equation (1)). γ reflects the relative aversion of the monetary authorities towards inflation. The more the policy-maker dislikes inflation, the higher is the value of γ .

Throughout the paper, we assume that monetary authority cannot precommit in monetary policy. Thus, in the discretionary policy game, the policy maker moves after the private sector. This is a four stages sequential game. At the beginning

⁹For an explicit account of monetary policy additive uncertainty, see Swank (1994), Lettierie (1997) and Lettierie and Lippi (1997). In these studies, however, additive uncertainty implicitly pertains to a demand shock, the reaction to which does not entail a trade-off between output and price fluctuations. In this respect, the authors do not fully examine the stabilisation properties of monetary policy in such a context. For the general case of multiplicative and additive transmission uncertainty, see Schellekens (1998).

of the period, private agents form their expectations about the inflation rate (stage 1). Supply shocks are then realised and determine the need for stabilization or accommodation through monetary policy (stage 2). Because the private sector is “locked” in expectations, it cannot react to those disturbances, but it knows that the monetary authorities will do it. Furthermore, we suppose that information is symmetric so that the distribution of the shocks is common knowledge.¹⁰ Next, the policy maker chooses the growth rate of money, given agents’ inflation expectation and the supply shock.(stage 3). Under transmission uncertainty, however, the policy-maker does not at this stage know what the precise impact of her choice of the growth rate of money (Δm) on the inflation rate will ultimately be; that is, at the time of the decision, μ is a random variable. At the last stage of the game, transmission uncertainty is resolved and the realised value of the transmission shock μ is known by all the players.

The unconditional expectation of the social loss serves as the benchmark whereupon the comparison of different policy outcomes is based in what follows. The expected social loss can be decomposed into three components. The first one is related to size of the distortion b which cannot be eliminated by monetary policy. The second component reflects the lack of inflation discipline in discretionary policymaking. The last one, which is associated with the variances of the endogenous variables, measures costs due to lack of sufficient stabilisation in the presence of stochastic shifts in the output supply. The introduction of transmission uncertainty will affect the decomposition of the loss, since we then have to take instrument variability into account. If we substitute equations (1)-(2) into (3) and assume independence of μ and u , we may write:

$$E[L] = b^2 + \gamma \cdot (E[\pi])^2 + \gamma \cdot var[\pi] + var[y]$$

where $b \equiv (k - 1) \bar{y}$.

In the following, we first solve for the basic non-cooperative Nash equilibrium of the monetary policy game assuming no delegation or transmission uncertainty. We then consider delegation of monetary policy to an independent and conservative central banker (as a means to contain the discretionary inflation bias). Finally, we introduce transmission uncertainty and rederive the optimal degree of central bank conservatism. The main question we want to focus on concerns the effects of increased transmission uncertainty on the optimal degree of central bank conservatism. To interpret those effects, we strongly emphasise the size of the credibility problem relative to the flexibility problem.

2.1 The basic case of pure discretion

In this case, there is no transmission uncertainty: $\mu = 1$ and $\sigma_\mu^2 = 0$. Hence in the last stage of the game, the policy-maker chooses (Δm) to minimize the social loss L , taking agents’ inflation expectations and the value of the supply shock as given. The policy maker consequently solves:

$$\min_{\Delta m} L$$

¹⁰See e.g. Canzoneri (1985) for an analysis of the role of private information in monetary policy games.

$$s.t. \begin{cases} (1) - (2) \\ \pi^e = E[\pi | I_{PS}] \\ u \end{cases}$$

From the first order conditions for this problem, we can solve for the reaction function of the policy maker, $\Delta m_D = \Delta m_D(\pi^e)$ (D for discretion):

$$(\Delta m)_D = \frac{\alpha}{\alpha^2 + \gamma} b + \frac{\alpha^2}{\alpha^2 + \gamma} \pi^e - \frac{\alpha}{\alpha^2 + \gamma} u \quad (4)$$

In the first stage, the private sector forms rational expectations about the inflation rate. Given equation (2), this takes the form of the best forecast of the optimal value Δm_D , that is:

$$\pi^e = E[\Delta m_D | I_{PS}] \quad (5)$$

where $E[. | I_{PS}]$ denotes the conditional expectation operator with respect to the information set of the private sector, I_{PS} .

The full equilibrium of the game can now be derived by solving equations (4)-(5) and using (1)-(2). We find:

$$\begin{aligned} (\pi^e)_D &= \frac{\alpha}{\gamma} b \\ \pi_D &= \frac{\alpha}{\gamma} b - \frac{\alpha}{\alpha^2 + \gamma} u \\ y_D &= \bar{y} + \frac{\gamma}{\alpha^2 + \gamma} u \end{aligned} \quad (6)$$

As usual, discretionary monetary policymaking entails an inflation bias which reduces social welfare relative to the case of full pre-commitment. Optimal stabilisation of supply shocks depend inversely on the degree of policy maker's inflation aversion. This implies, in particular, that delegation to a more conservative central bank entails a cost in terms of reduced stabilisation of the supply shocks. Expected social loss can now be written as:

$$\Lambda_D \equiv E(L_D) = b^2 + b^2 \frac{\alpha^2}{\gamma} + \frac{\gamma}{\alpha^2 + \gamma} \sigma_u^2 \quad (7)$$

2.2 Conservatism and delegation of monetary policy

According to the solution proposed by Rogoff (1985), the society can alleviate the discretionary inflation bias by delegating monetary policy to a (weight-) conservative central banker which has complete instrument independence. The Rogoff solution corresponds to the discretionary solution derived by minimising the expected value of the loss:

$$L^C = (y - y^*)^2 + \chi \cdot \pi^2 \quad (8)$$

(C for conservative) where $\chi > \gamma$ denotes the degree of inflation aversion of the central banker; by assumption, then, the central banker is more inflation averse than the society. We denote by ε the degree of conservatism of the central banker: $\varepsilon \equiv \chi - \gamma$.

The implied reaction function, $[\Delta m]_D^C = [\Delta m]_D^C(\pi^e)$ of the independent central banker is given by:

$$[\Delta m]_D^C = \frac{\alpha}{\alpha^2 + \chi} b + \frac{\alpha^2}{\alpha^2 + \chi} \pi^e - \frac{\alpha}{\alpha^2 + \chi} u$$

Equilibrium output and inflation as well as inflation expectations are consequently given by:

$$\begin{aligned} (\pi^e)_D^C &= \frac{\alpha}{\chi} b \\ \pi_D^C &= \frac{\alpha}{\chi} b - \frac{\alpha}{\alpha^2 + \chi} u \\ y_D^C &= \bar{y} + \frac{\chi}{\alpha^2 + \chi} u \end{aligned} \tag{9}$$

If we compare the discretionary solutions under the two cases of delegation and no-delegation, it is obvious that average inflation and the variability of inflation will be lower under delegation. Thus credibility of monetary policy is enhanced by delegation to a more conservative central banker. However, the cost of reducing the inflation bias is more variable output. This is the reason why the optimal degree of inflation aversion of the central banker cannot be infinite and that delegation does not offer the society a free lunch. The choice of an optimal central banker therefore involves a trade-off between credibility and flexibility. This can be seen particularly clearly by looking at the value of the social loss function:

$$\Lambda_D^C \equiv E(L_D^C) = b^2 + b^2 \frac{\alpha^2 \gamma}{\chi^2} + \frac{\chi^2 + \gamma \alpha^2}{(\alpha^2 + \chi)^2} \sigma_u^2 \tag{10}$$

Therefore, by defining $A(\varepsilon) \equiv b^2 \frac{\alpha^2 \gamma}{\chi^2}$ and $B(\varepsilon) \equiv \frac{\chi^2 + \gamma \alpha^2}{(\alpha^2 + \chi)^2} \sigma_u^2$, we have $\frac{\partial A(\varepsilon)}{\partial \varepsilon} < 0$ and $\frac{\partial B(\varepsilon)}{\partial \varepsilon} > 0$.

These two conditions reflect the trade-off that the society faces when it ponders on the prospect of delegating monetary policy to a conservative central banker. The optimal degree of conservatism is such that the associated benefits and costs of increased credibility are equalised at the margin. In other terms, ε^* solves:

$$\varepsilon^* = \arg \min_{\varepsilon} \Lambda_D^C$$

or, equivalently,

$$\left. \frac{\partial A(\varepsilon)}{\partial \varepsilon} \right|_{\varepsilon=\varepsilon^*} = - \left. \frac{\partial B(\varepsilon)}{\partial \varepsilon} \right|_{\varepsilon=\varepsilon^*} \tag{11}$$

As Eijffinger *et al* (1995) show, the last equality implies that ε^* solves the fixed point problem:

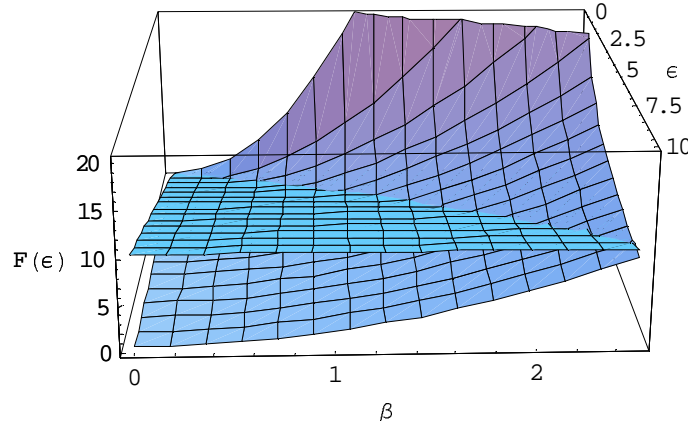


Figure 1. **The dependence of F on the degree of conservatism ε and relative bias b/σ_u ($\sigma = \sigma_u$), and the graphical solution to the fixed point problem $F(\varepsilon^*) = \varepsilon^*$.**

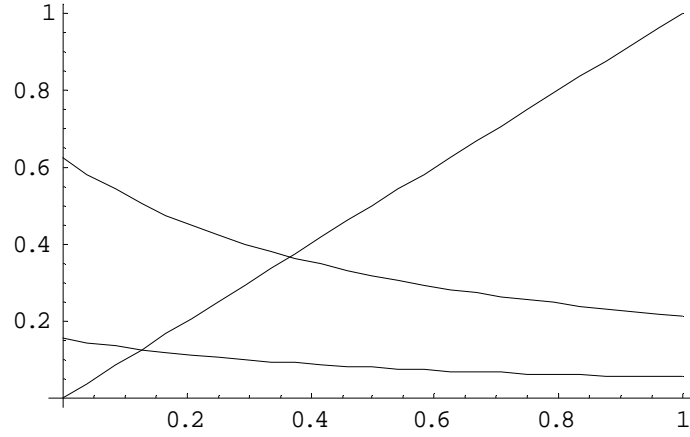


Figure 2. **Higher relative bias b/σ_u implies higher optimal degree of conservatism ε^* .**

$$F(\varepsilon^*) = \varepsilon^* \quad (12)$$

with

$$F(\varepsilon) = \frac{b^2\gamma}{\sigma_u^2} \cdot \left(1 + \frac{\alpha^2}{\gamma + \varepsilon}\right)^3 \quad (13)$$

There is a solution,¹¹ since $\frac{\partial F(\varepsilon)}{\partial \varepsilon} < 0$ and $F(0) > F(\infty) > 0$ where $F(\infty) \equiv \lim_{\varepsilon \rightarrow +\infty} F(\varepsilon) = \frac{b^2\gamma}{\sigma_u^2}$. Figure 1 shows, in three dimensions, the dependence of the l.h.s. of equation (10) the degree of conservatism, ε , and on the relative bias b/σ_u , as well as the solution to the fixed point problem. Graphically, the latter can be read off from the intersection of the graph of the 45° plane and of the function F .

¹¹ We restrict the solution set to positive values.

Figure 2, on the other hand, is obtained from figure 1 by projecting the intersection of the surface in figure 1 with two planes, $b/\sigma_u = 0.05$ and $b/\sigma_u = 0.1$, corresponding to two values of the relative bias onto the $(\varepsilon, F(\varepsilon))$ - plane.¹² Together with the 45° line figure 2 vividly illustrates the solution to the fixed point problem in equation (12). The upper F-curve in figure 2 corresponds to the higher level of the relative bias b/σ_u ; hence an increase in the relative bias increases the optimal degree of conservatism, ε^* .

To sum up, designing the optimal central banker will provide the economy with a lower inflation rate but at the cost of higher output variability. Such an outcome, however, can also arise in a context where uncertainty surrounds the implementation of monetary policy. To this issue, we now turn.

2.3 Introducing transmission uncertainty: no delegation

In the case where monetary policy has uncertain effects on the economy, the policy-maker does not know the very value of μ when she chooses the optimal level of the monetary policy instrument. As a consequence, the choice is made by minimising the expected value of the social loss, where the expectation is taken w.r.t. to the distribution of the transmission shock μ . In other terms, $[\Delta m]_D^U = [\Delta m]_D^U(\pi^e)$ (U for uncertainty) solves:

$$\begin{aligned} & \min_{\Delta m} E_{\mu} [L] \\ & s.t. \begin{cases} (1) - (2) \\ \pi^e = E[\pi | I_{PS}] \\ u \end{cases} \end{aligned}$$

The expectations operator $E_{\mu}[\cdot]$ signifies that the expectation is taken w.r.t. the distribution of the transmission shock μ . The reaction function of monetary authorities under uncertainty is thus given by:

$$\begin{aligned} [\Delta m]_D^U &= \frac{\alpha}{(\alpha^2 + \gamma)(\sigma_{\mu}^2 + 1)} \cdot b + \frac{\alpha^2}{(\alpha^2 + \gamma)(\sigma_{\mu}^2 + 1)} \cdot \pi^e \\ &\quad - \frac{\alpha}{(\alpha^2 + \gamma)(\sigma_{\mu}^2 + 1)} \cdot u \end{aligned} \quad (14)$$

If we compare (14) and (4), we conclude that uncertainty implies a more cautious use of the monetary instrument. The resulting caution is not due to a credibility problem, but to a partial knowledge of the effects of monetary policy decisions on the economy.¹ Transmission uncertainty induces the policy maker to act more cautiously simply because more aggressive use of the instrument can contribute to an increase in aggregate variability and, hence, to a loss of welfare.

The solution to private sector expectations involves optimal forecasting of monetary growth under transmission uncertainty:

¹² See also Eijffinger et al (1995, p. 10) for a similar graphical exposition.

¹ For a more explicit distinction between these two notions of activism, see Letterie and Lippi (1997).

$$\pi^e = E \left[[\Delta m]_D^U | I_{PS} \right]$$

which gives in equilibrium:

$$[\pi^e]_D^U = \frac{\alpha}{\sigma_\mu^2 \cdot \alpha^2 + \gamma (\sigma_\mu^2 + 1)} b$$

Since transmission uncertainty induces more cautious policy behaviour, the discretionary inflation bias is reduced. This is the sense in which transmission uncertainty enhances credibility. Equilibrium inflation and output are now given:

$$\begin{aligned} \pi_D^U &= \mu \cdot \left[\frac{\alpha \cdot b}{\sigma_\mu^2 \cdot \alpha^2 + \gamma (\sigma_\mu^2 + 1)} - \frac{\alpha}{(\alpha^2 + \gamma) (\sigma_\mu^2 + 1)} u \right] \\ y_D^U &= \bar{y} + (\mu - 1) \cdot \frac{\alpha^2}{\sigma_\mu^2 \cdot \alpha^2 + \gamma (\sigma_\mu^2 + 1)} b \\ &\quad + \frac{\alpha^2 \sigma_\mu^2 + \gamma (\sigma_\mu^2 + 1) + \alpha^2 (1 - \mu)}{(\alpha^2 + \gamma) (\sigma_\mu^2 + 1)} u \end{aligned} \quad (15)$$

As can be immediately seen, transmission uncertainty impinges on the equilibrium values of output and inflation both through the inflation bias and the impact effects of the supply shock on output and inflation..

As for the former channel, increased transmission uncertainty will reduce average inflation. The underlying reason is the induced caution, or reduced activism, in the use of the monetary instrument. Indeed, as the private sector rationally expects that the growth rate of money is reduced due to transmission uncertainty, inflation expectations are revised downwards. Average output, on the other hand, is not affected by transmission uncertainty (surprise inflation still averages out to zero):

$$\begin{aligned} E [\pi_D^U] &= \frac{\alpha \cdot b}{\sigma_\mu^2 \cdot \alpha^2 + \gamma (\sigma_\mu^2 + 1)} \\ E [y_D^U] &= \bar{y} \end{aligned}$$

As for aggregate variability, note that under transmission uncertainty case there are now two sources of variability that need to be accounted for. The first stems from the presence of stochastic (supply) shocks; we call this source of variability *economic or structural variability*. The other source of aggregate variability relates to the stochastic effects of monetary policy and it will be called *instrument(al) variability*. The two sources interact or are complementary.

$$\begin{aligned} var [\pi_D^U] &= \sigma_\mu^2 \cdot B^2 + \frac{\alpha^2}{(\alpha^2 + \gamma)^2 \cdot (\sigma_\mu^2 + 1)} \cdot \sigma_u^2 \\ var [y_D^U] &= \alpha^2 \cdot \sigma_\mu^2 \cdot B^2 + \frac{\gamma^2}{(\alpha^2 + \gamma)^2 \cdot (\sigma_\mu^2 + 1)} \cdot \sigma_u^2 \end{aligned}$$

$$+\frac{\sigma_{\mu}^2}{\sigma_{\mu}^2+1} \cdot \sigma_u^2$$

$$\text{with: } B \equiv \frac{\alpha b}{\sigma_{\mu}^2 \cdot \alpha^2 + \gamma (\sigma_{\mu}^2 + 1)}$$

We furthermore define:

$$\begin{aligned} \text{var}[y|_{EV}] &\equiv \left[\frac{\sigma_{\mu}^2}{\sigma_{\mu}^2+1} + \frac{\gamma^2}{(\alpha^2 + \gamma)^2 \cdot (\sigma_{\mu}^2 + 1)} \right] \cdot \sigma_u^2 \\ \text{var}[\pi|_{EV}] &\equiv \frac{\alpha^2}{(\alpha^2 + \gamma)^2 \cdot (\sigma_{\mu}^2 + 1)} \cdot \sigma_u^2 \\ \text{var}[y|_{IV}] &\equiv \alpha^2 \sigma_{\mu}^2 \cdot B^2 \\ \text{var}[\pi|_{IV}] &\equiv \sigma_{\mu}^2 \cdot B^2 \end{aligned}$$

$\text{var}[\cdot]|_{IV}$ represents contribution to aggregate variability due to instrument variability whereas $\text{var}[\cdot]|_{EV}$ refers to the contribution from economic shocks.

Note that the presence of transmission uncertainty has an ambiguous effect on the variance of inflation. On the one hand, instrument variability reduces the effect of economic variability on the variance of inflation (relative to the benchmark case). The underlying reason has already been given a number of times: transmission uncertainty induces caution in the use of the policy instrument.

On the other hand, transmission uncertainty interacts with the discretionary inflation bias ($\sigma_{\mu}^2 \cdot B^2$); a bias of a given size will contribute positively to inflation variability due to the randomness in the transmission of policy impulses to the inflation rate. That is, at a given mean, transmission uncertainty increases the probability of observing (larger) inflation deviations from the mean. However, the size of the inflation bias, ie. mean inflation, is reduced due to transmission uncertainty. As a consequence, the effects of the discretionary inflation bias on the variance of inflation are ambiguous. Which effect dominates?

$$\begin{aligned} \text{The sign of the derivative } &\frac{\partial (\sigma_{\mu}^2 \cdot B^2)}{\partial \sigma_{\mu}^2} \\ &\frac{\partial (\sigma_{\mu}^2 \cdot B^2)}{\partial \sigma_{\mu}^2} = \frac{\gamma - \sigma_{\mu}^2 (\alpha^2 + \gamma)}{[\sigma_{\mu}^2 \cdot \alpha^2 + \gamma (\sigma_{\mu}^2 + 1)]^3} \end{aligned}$$

is positive when the degree of transmission (σ_{μ}^2) uncertainty is low. This comes out most clearly in the case of the introduction of transmission uncertainty: $\frac{\partial (\sigma_{\mu}^2 \cdot B^2)}{\partial \sigma_{\mu}^2} \Big|_{\sigma_{\mu}^2=0} = \frac{1}{\gamma^2} > 0$. When uncertainty increases, on the other hand, the lower bias starts to dominate.

Second, as far as output variability is concerned, the two effects operate again.² Note, however, that in addition to its effects on inflation variability the supply shock

²Note that $\text{var}[y]|_{IV} = \alpha^2 \cdot \text{var}[\pi]|_{IV}$.

will also affect aggregate output variability directly and through the correlation of inflation surprises and supply shocks:

$$\text{var} (y_D^U) = \alpha^2 \cdot \text{var} [\pi |_{IV}] + \alpha^2 \cdot \text{var} [\pi |_{EV}] + \sigma_u^2 - \frac{2a^2}{(1 + \sigma_\mu^2) \cdot (\gamma + \alpha^2)} \cdot \sigma_u^2 \quad (16)$$

Increased transmission uncertainty means that actual inflation carries less information about the supply shock; hence, as σ_μ^2 increases without bound, the last term in (16) vanishes. Quite intuitively, it also gets smaller, in absolute value, as the degree of inflation aversion of the policy maker increases. But the direct effect of the variance of the supply shock remains. This is the underlying reason why output variance ultimately increases if transmission uncertainty or inflation aversion of the policy maker increases.

Therefore, the two well-known effects of the presence of transmission uncertainty are that, first, it induces caution to the policy maker's behaviour and, second, that it reduces the degree of accommodation of the supply shocks. Increased caution involves less activism which reduces the discretionary inflation bias and, hence, enhances credibility of monetary policy. In the presence of multiplicative transmission uncertainty, a fall in the inflation bias contributes to a more stable aggregate output (and inflation). Reduced accommodation, on the other hand, means less stabilisation of the supply shocks and, consequently, more variable aggregate output.

All in all, these different elements interfere with each other and the expression of the expected social loss reflects the result of these crossed effects:

$$\Lambda_U \equiv E(L_U) = b^2 + \frac{\alpha^2 b^2}{[\alpha^2 \sigma_\mu^2 + \gamma (\sigma_\mu^2 + 1)]} + \frac{\alpha^2 \sigma_\mu^2 + \gamma (\sigma_\mu^2 + 1)}{(\sigma_\mu^2 + 1) (\alpha^2 + \gamma)} \sigma_u^2 \quad (17)$$

3 Transmission uncertainty and delegation: a tradeoff? A first comparison

In terms of the stabilisation of the supply shock, the introduction of uncertainty has, on a qualitative basis, the same effect on the distribution of the equilibrium quantities as the appointment of a more conservative central banker. An higher degree of transmission uncertainty - a more conservative central banker - implies a lower variance of inflation and a higher variance of aggregate output. In terms of the expected social loss, this implies an increase in the component associated with the variance of the shocks.

As for the credibility issue, matters are somewhat more complicated because of the presence of transmission uncertainty. In the model with delegation, a more conservative central banker lowers the inflationary bias and thus *directly* contributes to the solution of the credibility problem that poolcy-makers acting on discretion face.

This disciplinary mechanism is also present when transmission uncertainty is introduced into the model. However, although transmission uncertainty

reduces, through caution, the inflation bias, it contributes, through instrument variability, to increasing aggregate variability by increasing the effects of a given bias on aggregate variability. As a consequence, generally an increase in transmission uncertainty may or may not reduce social loss due to the credibility problem of monetary policy, although the *introduction* of transmission uncertainty unambiguously reduces social welfare due to the credibility problem.

Define

$$\begin{aligned} A(\sigma_\mu^2) &\equiv \frac{\alpha^2 b^2}{[\alpha^2 \sigma_\mu^2 + \gamma(\sigma_\mu^2 + 1)]} \\ B(\sigma_\mu^2) &\equiv \frac{[\alpha^2 \sigma_\mu^2 + \gamma(\sigma_\mu^2 + 1)] \cdot \sigma_u^2}{(\sigma_\mu^2 + 1)(\alpha^2 + \gamma)} \end{aligned} \quad (18)$$

We then have $\frac{\partial A(\sigma_\mu^2)}{\partial \sigma_\mu^2} < 0$ and $\frac{\partial B(\sigma_\mu^2)}{\partial \sigma_\mu^2} > 0$. This greatly resembles the trade-off observed previously in the case of delegation of monetary policy to a more conservative central banker.

It is important to note, however, that in the two cases considered, the trade-off between credibility and flexibility has quite different characteristics.

- Under delegation, the conservativeness of the central banker directly disciplines monetary policy and directly impinges on the stabilisation potential of the optimal monetary policy.
- Under transmission uncertainty there are both direct and indirect effects. The former are similar to those under delegation. The indirect effects are, however, specific to the presence of transmission uncertainty, since they are tightly related to instrument variability. The latter, nevertheless, has an ambiguous effect on aggregate variability.

Table 1 summarises the different channels through which the relationship between credibility and flexibility is effected by the degree of conservatism and transmission uncertainty.¹⁵ As can be noted, the introduction or increase of uncertainty implies the same effects as the appointment of a conservative central banker, but gives rise to new sources of output and inflation variability.

Table 1		
Expected loss	Conservatism (\uparrow)	Uncertainty (\uparrow)
$(E[\pi])^2$	(−)	(−)
$var[\pi]_{IV}$		(?)
$var[y]_{IV}$		(?)
$var[y]_{EV}$	(+)	(+)
$var[\pi]_{EV}$	(−)	(−)

¹⁵ We look at the different components of the unconditional expected social loss. Inflation and output variances can be decomposed into ‘instrumental’ and ‘economic’ components in a straightforward manner, because we assume that the supply shock and the transmission shock are independent random variables. $var[\cdot]_{IV}$ represents the variance attributable to instrument variability whereas $var[\cdot]_{EV}$ is the one which directly stems from the presence of economic disturbances (supply shocks).

The introduction of new sources of aggregate variability implies, in turn, that the nature of the credibility-flexibility trade-off changes. The credibility issue indeed involves the compound effect of the level of the bias and instrument variability (in the variance of inflation and output)¹⁶ whereas the stabilization issue depends only on the presence of economic shocks as in the benchmark case. Table 2 summarises the effects of uncertainty and delegation on credibility and flexibility components, using the notions of economic and instrument variability.

Table 2		
Model	Loss components	
	Credibility component	Flexibility component
Conservatism (\uparrow)	$(E[\pi])^2 \} (-)$	$\left. \begin{array}{l} var[\pi]_{EV} \quad (-) \\ var[y]_{EV} \quad (+) \end{array} \right\} (+)$
Uncertainty (\uparrow)	$\left. \begin{array}{l} (E[\pi])^2 \quad (-) \\ var[\pi]_{IV} \quad (?) \\ var[y]_{IV} \quad (?) \end{array} \right\} (-)$	$\left. \begin{array}{l} var[\pi]_{EV} \quad (-) \\ var[y]_{EV} \quad (+) \end{array} \right\} (+)$

This classification helps us to assess how, from the point of view of social welfare, conservatism combines with transmission uncertainty. To this end, suppose, for example, that a more inflation averse (ie. more conservative) central banker is suddenly faced with transmission uncertainty. From table 2, we see that in this case, transmission uncertainty reinforces the increase in aggregate variability generated by less stabilisation oriented, more "hawkish" monetary policy. At the same time, however, benefits from the reduced credibility problem accrue to the society. The net effect on the social loss depends, then, on the balance between credibility and flexibility, ie. how favourable the tradeoff between credibility and flexibility is. in the end, to the delegation of monetary policy to a more conservative central banker.

4 Delegation and transmission uncertainty: combining the two in a more formal analysis

We now combine the two previous cases and look at the outcome of the game where monetary policy is delegated to a conservative central banker in the presence of transmission uncertainty. The model is given by the following equations:

$$y = \bar{y} + \alpha(\pi - \pi^e) + u \quad (19)$$

$$\pi = \mu \cdot \Delta m \quad (20)$$

$$L^C = (y - y^*)^2 + \chi \cdot \pi^2 \quad (21)$$

$$L = (y - y^*)^2 + \gamma \cdot \pi^2 \quad (22)$$

¹⁶We chose to associate output variance component associated with instrument variability with the credibility loss of the central bank. Indeed, this element directly stems from the fact that the use of a noisy instrument affects unexpected inflation and thereby, through the Phillips curve, the current level of output.

In the implied reaction function of the central banker both the level of uncertainty and the degree of conservatism now play a role in determining the nature of optimal monetary policy (C, U for conservatism and transmission uncertainty):

$$[\Delta m]_D^{C,U} = \frac{\alpha}{(\alpha^2 + \chi)(\sigma_\mu^2 + 1)} \cdot b + \frac{\alpha^2}{(\alpha^2 + \chi)(\sigma_\mu^2 + 1)} \cdot \pi^e - \frac{\alpha}{(\alpha^2 + \chi)(\sigma_\mu^2 + 1)} \cdot u \quad (23)$$

The expression of the unconditional expected social loss function obviously depends on ε and σ_μ^2 . We denote it by $\Lambda_D^{C,U}(\varepsilon, \sigma_\mu^2)$ and we have:

$$\begin{aligned} \Lambda_D^{C,U} \equiv E[L_D^{C,U}] &= b^2 + \alpha^2 b^2 \cdot \frac{\alpha^2 \sigma_\mu^2 + \gamma(\sigma_\mu^2 + 1)}{[\alpha^2 \sigma_\mu^2 + \chi(\sigma_\mu^2 + 1)]^2} \\ &+ \frac{(\chi^2 + \alpha^2 \gamma) + \sigma_\mu^2(\alpha^2 + \chi)^2}{(\sigma_\mu^2 + 1)(\alpha^2 + \chi)^2} \sigma_u^2 \end{aligned} \quad (24)$$

The relationship between social loss and the various parameters of interest is, as can be readily seen from equation (24), nonlinear and quite involved. Consequently, an increase in transmission uncertainty and/or degree of central bank conservatism has, in general, an ambiguous effect on social welfare.¹⁷

Our objective is in the following two-fold. First, we solve for the optimal level of central bank conservatism, where optimality means minimum loss to the society (which delegates monetary policy) from delegation under transmission uncertainty. Secondly, we try to find out the comparative statics effect of an increase in transmission uncertainty on the optimal degree of central bank conservatism; for future reference we denote the optimal degree of central bank conservatism by $\tilde{\varepsilon} = \tilde{\varepsilon}(\sigma_\mu^2)$, to emphasise the dependence on the degree of transmission uncertainty. Finally, an interpretation of the results is given in terms of the relative size of the credibility problem of monetary policymaking.

4.1 Optimal degree of conservatism under transmission uncertainty

Minimising the loss function $\Lambda_D^{C,U}$ w.r.t. χ and rearranging terms, we find that $\tilde{\varepsilon}$ solves the following fixed point problem:

$$G(\tilde{\varepsilon}) = \tilde{\varepsilon} \quad (25)$$

with

¹⁷ The relationships we derived in the previous sections are, of course, special cases of the present one; either $\mu = 1$ and $\sigma_\mu^2 = 0$ (no transmission uncertainty) or for $\varepsilon = 0$ (no delegation).

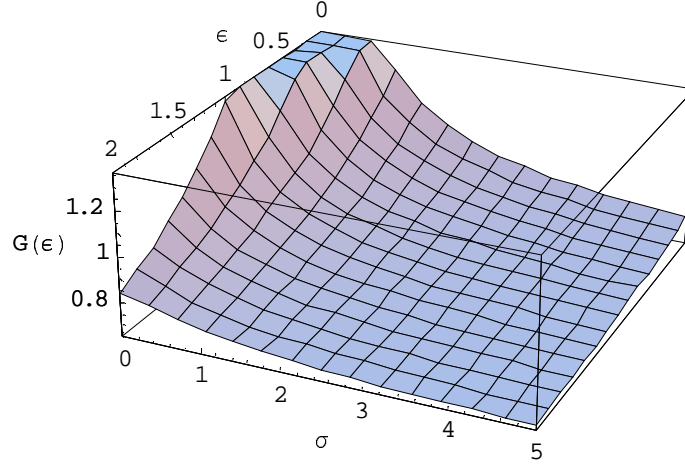


Figure 3. The dependence of G on the degree of conservatism ε and transmission uncertainty σ_μ ($\sigma(\mu) \equiv \sigma_\mu$) ($b/\sigma_\mu = 0.5$).

$$G(\varepsilon) \equiv \frac{b^2}{\sigma_u^2} \cdot (\alpha^2 + \chi)^3 \cdot \frac{[\alpha^2 \sigma_\mu^2 + \gamma (\sigma_\mu^2 + 1)] (1 + \sigma_\mu^2)^2}{[\alpha^2 \sigma_\mu^2 + \chi (\sigma_\mu^2 + 1)]^3} \quad (26)$$

or

$$G(\varepsilon) = F(\varepsilon) \cdot \left(\frac{1}{1 + (\frac{\alpha^2}{\gamma + \varepsilon})(\frac{\sigma_\mu^2}{1 + \sigma_\mu^2})} \right)^2 \cdot \left(\frac{1 + \frac{\alpha^2}{\gamma}(\frac{\sigma_\mu^2}{1 + \sigma_\mu^2})}{1 + (\frac{\alpha^2}{\gamma + \varepsilon})(\frac{\sigma_\mu^2}{1 + \sigma_\mu^2})} \right) = F(\varepsilon) \cdot H_1 \cdot H_2 \quad (27)$$

The $G(\cdot)$ function depends, most importantly, on ε , b/σ_u and σ_μ^2 ; this dependence is simply written as $G(\varepsilon; \sigma_\mu^2)$, thus ignoring the relative bias b/σ_u in the notation. Figure 3 plots the dependence of G on ε and σ_μ for a given value of the relative bias b/σ_u .

Seen from the origin in figure 3, the effect of an increase in the relative bias b/σ_u is to shift the function surface up- and outward. In particular, if we project the curve of intersection of the 45° plane and the function surface onto the $(\sigma_\mu, G(\tilde{\varepsilon}; \sigma_\mu))$ plane, we can see how the increase of the relative bias affects the slope of this curve in the $(\sigma_\mu, G(\tilde{\varepsilon}; \sigma_\mu))$ plane; if the relative bias is small, the curve has a negative slope, where for a large relative bias, the slope is positive. Thus the effects of increased transmission uncertainty on the optimal degree of central bank conservatism depends critically on the relative importance of the inflation bias - relative to the stabilisation needs - in the economy. This is intuitive enough, and reflects the underlying feature of the model that the optimal degree of central bank conservatism is ultimately the outcome of the interaction of the transmission uncertainty with the relative bias or credibility - flexibility trade off.¹⁸

¹⁸ Schellekens (1998) also emphasises the fact multiplicative Brainard uncertainty affects the credibility-flexibility tradeoff.

It is easy to check that this formula is consistent with the one obtained in the model without uncertainty, *i.e.* that $G(\varepsilon; 0) = F(\varepsilon)$. Moreover, as the graphical argument given above in the context of figure 3 suggests, we can prove that equation (25) admits, for all nonnegative σ_μ^2 , a unique solution, which can be illustrated in $(\varepsilon; G(\cdot))$ plane (see Appendix 1). That is using the graphical method in the context of figure 3, the optimal degree of conservatism is found by projecting the intersection of the function surface with the plane $\sigma_\mu = \text{constant}$ onto to the $(\varepsilon, G(\cdot))$ plane to see where it crosses the 45° line. The relevant graphical solutions will be presented in the context of the following analysis of how the introduction or presence of uncertainty affects the optimal degree of conservatism. The analysis makes use of equation (25) and the ensuing implicit function between $\tilde{\varepsilon}$ and σ_μ^2 , *i.e.* $\tilde{\varepsilon}(\sigma_\mu^2)$.

According to the implicit function theorem and the equation (26), the following result applies:

$$\frac{\partial \tilde{\varepsilon}(\sigma_\mu^2)}{\partial \sigma_\mu^2} \propto \frac{\partial G(\tilde{\varepsilon}; \sigma_\mu^2)}{\partial \sigma_\mu^2}$$

- Proof: see Appendix 2.

Looking at the decomposition of $G(\cdot)$, we can deduce, relative to the case of no transmission uncertainty, the qualitative behaviour of the optimal degree of conservatism w.r.t. increasing transmission uncertainty. To this end, suppose that $\tilde{\varepsilon}$ is initially small; then, from (27), we can see that H_2 is approximately equal to 1 and, hence, the local behaviour of $G(\cdot)$ w.r.t. to σ_μ^2 is dominated by H_1 , which is decreasing w.r.t. σ_μ^2 . As a consequence, increased transmission uncertainty will reduce the optimal degree of central bank conservatism. If, on the other hand, $\tilde{\varepsilon}$ is large to start with, the local behaviour of $G(\cdot)$ is dominated by $H_2 - H_1$ is close to 1 - which is increasing in σ_μ^2 . In this case, then, increases in transmission uncertainty will increase the optimal degree of central bank conservatism.

From (27) we can also immediately see that $\tilde{\varepsilon}$ increases as the relative bias b/σ_u increases; thus $\tilde{\varepsilon}$ will, most interestingly, be high when the relative bias is high. This, in turn, implies that when the credibility - flexibility tradeoff in monetary policy favours a very conservative central banker relative to the society (high b/σ_u), increases in transmission uncertainty will increase the optimal degree of central bank conservatism further. If, on the other hand, the stabilisation problem of monetary policymaking is relatively more important (low b/σ_u and, hence, low $\tilde{\varepsilon}$), then increases in transmission uncertainty would sustain delegating monetary policy to a less conservative central banker.

Graphically these results mean that when the relative bias is small, then in the $(\varepsilon, \sigma_\mu, G(\varepsilon; \sigma_\mu))$ space the set of points in the intersection of the 45° degree plane and the relevant portion of the function surface determined by the G function, which represents the solutions to the fixed point problem in (25) involves a negative relationship between the optimal degree of central bank conservatism $\tilde{\varepsilon}$ and the degree of transmission uncertainty σ_μ , *i.e.* $\tilde{\varepsilon}'(\sigma_\mu^2) < 0$. This is illustrated in figure 4. Figure 5, on the other hand, displays projections from the space in figure 4 onto the $(\varepsilon, G(\varepsilon; \sigma_\mu^2))$ plane at prespecified levels of the degree of transmission uncertainty σ_μ ; the curves representing the $G(\cdot)$ -functions for strictly positive σ_μ^2

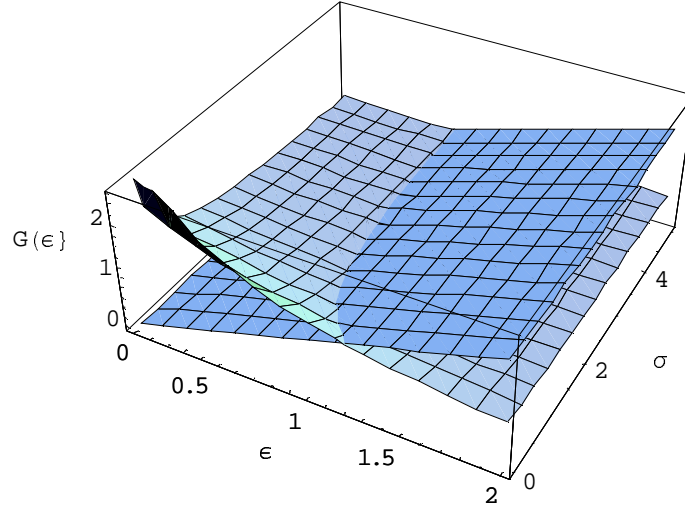


Figure 4. **Graphical solution to the fixed point problem $G(\tilde{\varepsilon}; \sigma_\mu^2) = \tilde{\varepsilon}$, when the relative bias b/σ_u is small; $\tilde{\varepsilon}$ falls when the degree of transmission uncertainty σ_μ^2 increases ($\sigma(\mu) \equiv \sigma_\mu$).**

values lie below the graph of the $F(\cdot)$ -function, which corresponds to the case of no transmission uncertainty, in the $(\varepsilon; G(\cdot))$ plane and would therefore cross the 45° line at a point where $\tilde{\varepsilon}$ would be lower.

Analogously, the effects of the alternative case of a high relative bias are illustrated in figures 6 and 7. In figure 6 the set of points in the intersection of the 45° plane and the relevant portion of the function surface involves a negative relationship between optimal central bank conservatism and transmission uncertainty. The projections in figure 7 illustrate this negative relationship between the two in the $(\varepsilon, G(\varepsilon, \sigma_\mu^2))$ plane; the lowest point on the 45° line corresponds to the $F(\tilde{\varepsilon})$ curve and the middle and the upmost point to the $G(\tilde{\varepsilon}; \sigma_{\mu,l}^2)$ and $G(\tilde{\varepsilon}; \sigma_{\mu,h}^2)$ curves respectively, where $\sigma_{\mu,l}^2 < \sigma_{\mu,h}^2$.³

A more formal analysis of the impact of an increase in uncertainty on $G(\varepsilon; \sigma_\mu^2)$ also demonstrates that in general transmission uncertainty impinges ambiguously on the optimal level of conservatism:

$$\frac{\partial G(\tilde{\varepsilon}; \sigma_\mu^2)}{\partial \sigma_\mu^2} = H(\tilde{\varepsilon}; \sigma_\mu^2) \cdot \Psi(\tilde{\varepsilon}; \sigma_\mu^2) \quad (28)$$

$$\text{with } H(\tilde{\varepsilon}; \sigma_\mu^2) \equiv \frac{F(\tilde{\varepsilon}) \cdot H_1 \cdot (H_2 - 1)}{\left[1 + \frac{\alpha^2 \sigma_\mu^2}{\tilde{\chi}(1 + \sigma_\mu^2)}\right] \cdot \tilde{\chi} \cdot (1 + \sigma_\mu^2)^3}$$

³Note that $F(0) > G(0; \sigma_{\mu,l}^2) > G(0; \sigma_{\mu,h}^2)$. This implies, in particular, that the graphs of the F and G functions can cross each other before the 45° line, as figure 5 also suggests. Thus, figure 6.2 in Schellekens (1998, appendix 6.2) is not entirely correct, since the shifts in his F curves are not as uniform as figure 6.2 suggests.

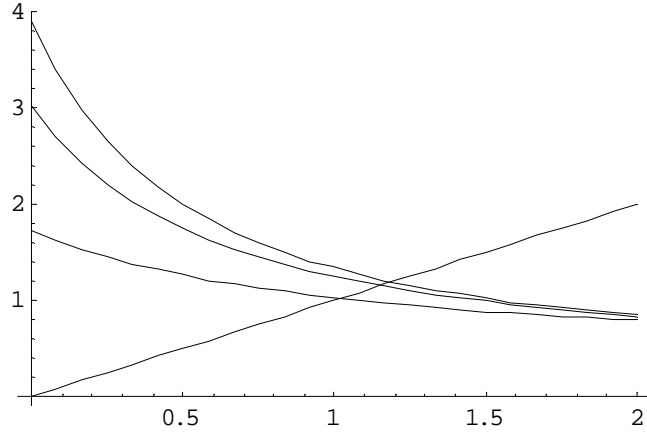


Figure 5. **The effect of an increase in transmission uncertainty σ_μ : ε decreases when the relative bias b/σ_u is small.**

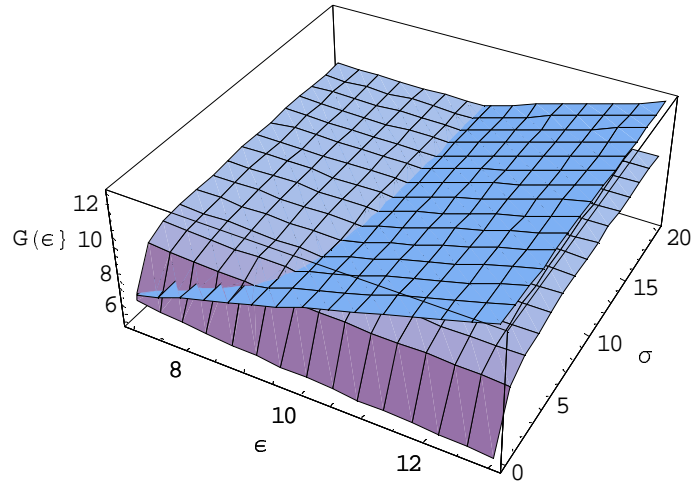


Figure 6. **Graphical solution to the fixed point problem $G(\tilde{\varepsilon}; \sigma_\mu^2) = \tilde{\varepsilon}$, when the relative bias b/σ_u is large; $\tilde{\varepsilon}$ increases when the degree of transmission uncertainty σ_μ^2 increases ($\sigma(\mu) \equiv \sigma_\mu$).**

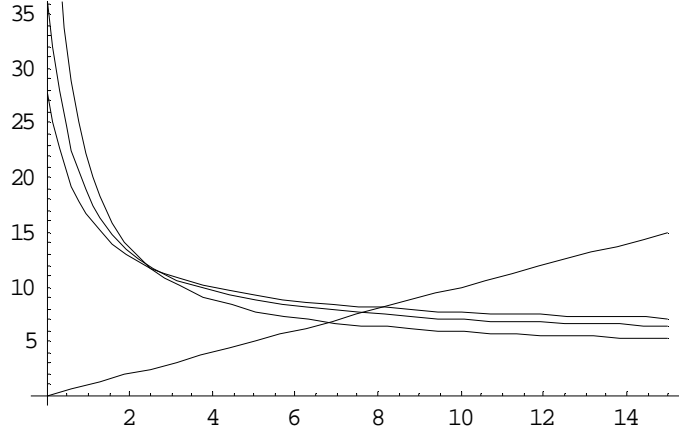


Figure 7. **Graphical solution to the fixed point problem $G(\tilde{\varepsilon}; \sigma_\mu^2) = \tilde{\varepsilon}$, when the relative bias b/σ_u is large; $\tilde{\varepsilon}$ increases when the degree of transmission uncertainty σ_μ^2 increases ($\sigma(\mu) \equiv \sigma_\mu$).**

$$\text{and } \Psi(\tilde{\varepsilon}; \sigma_\mu^2) \equiv (1 + \sigma_\mu^2) \tilde{\varepsilon} - [2\sigma_\mu^2(\alpha^2 + \gamma) + 2\gamma]$$

$$\text{or } \Psi(\tilde{\varepsilon}; \sigma_\mu^2) \equiv (1 + \sigma_\mu^2)(\tilde{\varepsilon} - 2\gamma) - 2\alpha^2\sigma_\mu^2$$

From equation (28), it is easy to see that:

$$\frac{\partial G(\tilde{\varepsilon}; \sigma_\mu^2)}{\partial \sigma_\mu^2} \propto \Psi(\tilde{\varepsilon}; \sigma_\mu^2)$$

the latter being alternatively positive or negative. However, we can immediately see that $\text{sgn}[\Psi(\tilde{\varepsilon}; \sigma_\mu^2)] = \text{sgn}[(1 + \sigma_\mu^2)(\tilde{\varepsilon} - 2\gamma) - 2\alpha^2\sigma_\mu^2]$, ie. $\Psi(\tilde{\varepsilon}; \sigma_\mu^2) \geq 0 \Leftrightarrow \tilde{\varepsilon} \geq 2(\gamma + \alpha^2) - \frac{2\alpha^2}{1+\sigma_\mu^2}$.²⁰ The r.h.s. of this comparison falls as the degree of transmission uncertainty, σ_μ^2 , increases; so for *moderately conservative* central bankers, increases in transmission uncertainty will act to lower further the optimal degree of central bank conservatism. In this case, there is essentially a tradeoff between conservatism and *caution*, as also emphasised by Schellekens (1998, prop. 3.2, p. 27), who also provides the intuition underlying this result. Essentially, the result comes out because an increase in Brainard uncertainty decreases the marginal effect of weight-conservatism on output variability, more so in economies with more weight-conservative central bankers. On the other hand, in economies where optimality sustains very conservative central bankers, increases in Brainard uncertainty will act to increase optimal central bank conservatism further. Hence, increased transmission uncertainty will *reinforce* central bank conservatism.

In interpreting these results concerning the effects of Brainard uncertainty on the optimal degree of central bank conservatism, we prefer to focus on the extent of the credibility problem of monetary policy making, ie. on size of the relative bias b/σ_u . Since optimal central bank conservatism is increasing in the relative bias, increases in transmission uncertainty tends to increase the optimal degree of

²⁰ See also Schellekens (1998, p. 26).

central bank conservatism in economies where the credibility problem of monetary policymaking is large. If, on the other hand, the flexibility problem dominates monetary policymaking (small b/σ_u), increased transmission uncertainty tends to lower optimal central bank conservatism. In this case, there is a tradeoff between weight-conservatism and caution; increased transmission uncertainty will induce a central bank to use its instrument more cautiously, creating an opportunity (for the society) to lower optimal central bank conservatism to sustain a given level of caution. **In any case, the general flavour is that Brainard uncertainty will reinforce the effects of the balance of the credibility-flexibility problem on central bank conservatism.**

4.2 A interesting special case: the effects of introducing transmission uncertainty

The general case analysed above can be specialised to a situation, where we *introduce* transmission uncertainty into our model economy and try to see its effects on optimal central bank conservatism. It turns out that the role of the relative bias in determining these effect is particularly simple and clear. This special case is, to our minds, important, particularly in the European context, since we could argue that transmission uncertainty is currently present in the monetary union, even if no such uncertainty existed in the individual member countries prior to the start of the monetary union in the beginning of 1999. There may have been country specific differences in e.g. the structure of money markets that generate uncertainties about the areawide effects of the common monetary policy pursued by the ECB. This would be one possible interpretation of the present model in the context of the monetary union.

Let suppose that the current central banker was appointed in times when there was no uncertainty about the effects of monetary policy on inflation. Then, in the model, $\tilde{\varepsilon} = \tilde{\varepsilon}(0) = \varepsilon^*$. Now, suppose further that due to some structural change in the economy, uncertainties in the effects of monetary policy on inflation start to emerge. To evaluate the effects of the introduction of transmission uncertainty on the optimal degree of central bank conservatism, we use the optimality condition corresponding to the preferences of the still-in-place and previously optimal central banker and, in particular his/her degree of inflation aversion (relative to output). Thus, from (28) we deduce:

$$\frac{\partial G(\tilde{\varepsilon}; \sigma_\mu^2)}{\partial \sigma_\mu^2} \Big|_{\tilde{\varepsilon}=\varepsilon^*; \sigma_\mu^2=0} = H(\tilde{\varepsilon}; 0) \cdot (\tilde{\varepsilon}(0) - 2\gamma)$$

Recall that, by convention, $\varepsilon^* = \tilde{\varepsilon}(0) = \gamma \cdot \left(\frac{b}{\sigma_u}\right)^2 \cdot \left(1 + \frac{\alpha^2}{\gamma + \varepsilon^*}\right)^3$. In other terms, we obtain:

$$\frac{\partial G(\tilde{\varepsilon}; \sigma_\mu^2)}{\partial \sigma_\mu^2} \Big|_{\tilde{\varepsilon}=\varepsilon^*; \sigma_\mu^2=0} = H(\varepsilon^*; 0) \cdot \gamma \cdot \left[\left(\frac{b}{\sigma_u}\right)^2 \cdot \left(1 + \frac{\alpha^2}{\gamma + \varepsilon^*}\right)^3 - 2 \right]$$

where we make use of equation (13).

We can immediately see that the comparative statics effect of introducing transmission uncertainty on optimal central bank conservatism depends, above all,

on the size of the relative bias $\left(\frac{b}{\sigma_u}\right)$ or the size of the credibility problem of monetary policy making relative to the flexibility problem.

To take a concrete example, assume two parameter vectors that differ only in the size of the relative bias: $(\alpha, \gamma, b/\sigma_u) = (0.5, 1, 0.5)$ and $(\alpha, \gamma, b/\sigma_u) = (0.5, 1, 1.5)$. Then $\varepsilon^* = 0.408$ and $\varepsilon^* = 2.733$ respectively. Hence, in the former (latter) case the introduction of Brainard uncertainty reduces (increases) the optimal degree of central bank conservatism. These parameter values do not to us seem be extreme ones, so neither of the case seem (too) unpalatable a priori.

This last result suggest a way we could try to capture the effect of introducing transmission uncertainty on the optimal degree of central bank conservatism using actual data, ie. in an empirical analysis. From equation (9) we can see that the coefficient of variation of inflation, $V(\pi) = \frac{\sqrt{Var(\pi)}}{E\pi} = \frac{\sigma_u}{b} \cdot \left(1 + \frac{\alpha^2}{\chi}\right)^{-1}$. Consequently, the introduction of transmission uncertainty will increase optimal central bank conservatism, iff

$$\left(\frac{1}{V(\pi)}\right)^2 \cdot \left(1 + \frac{\alpha^2}{\chi}\right) > 2 \quad (29)$$

A sufficient condition - given the assumption of a nonnegative $\tilde{\varepsilon}$ - is that $\left(\frac{1}{V(\pi)}\right) > \sqrt{2}$, ie. the inverse of the coefficient of variation of inflation exceeds the square root of 2.²¹ Although perhaps not precise enough, this condition certainly gives a starting condition in any empirical search trying to estimate the effects of transmission uncertainty on optimal central bank conservatism.

5 Conclusion

Currently it is very well understood that (multiplicative) transmission uncertainty induces caution to monetary policymaking. The underlying reason is that the policy maker knows that transmission uncertainty can contribute to increasing aggregate variability. Hence, aggressive use of the policy instrument in such a context can generate welfare losses so that policy activism is not sustained by optimality. It is also well understood that transmission uncertainty enhances policy credibility; due to the induced caution - well understood by private agents - the effect of the distortion that ultimately gives rise to the inflation bias on inflation expectations and, hence, on actual inflation and output is reduced. But, also due to the induced caution, the effectiveness of monetary policy to stabilise the effects of aggregate output supply shocks is also reduced in the presence of transmission uncertainty.

Delegation of monetary policy to an independent and more conservative central bank(er) also has the well known effects on optimal (discretionary) monetary policy; credibility is enhanced, but only at the cost of a reduction in the stabilisation potential of monetary policy. Hence, there can exist a tradeoff between delegation

²¹ To invite further speculation in this context annual data on consumer price inflation in Austria, Belgium, Finland, France, Germany, Italy, the Netherlands, Spain and Portugal from 1971 to 1998 and calculated $V(\pi)$ for each of these countries. The Netherlands was the only country where the condition $\frac{1}{V(\pi)} > \sqrt{2}$ did not hold.

and transmission uncertainty. From the point of view of the present paper, the emphasis here should be strongly placed on the word 'can', since we have not gone deep into the question of the extent to which transmission uncertainty - ambiguity or secrecy in a sense - is also subject to the choice by the society, along the lines of the degree of central bank conservatism.²²

The focus of the present paper is on the effects of transmission uncertainty on the optimal degree of central bank conservatism or, 'how Brainard (1967) affects Rogoff (1985)'. This is exactly the question that Schellekens (1998) also analyses. He, to our minds, emphasises the tradeoff between conservatism and caution (transmission uncertainty), whereas we want to emphasise the balance between the credibility problem and flexibility problem of monetary policy as the key determinant of the effect of an increase in transmission uncertainty on optimal central bank conservatism. More specifically, we argue that if the relative bias - credibility problem relative to the flexibility problem - dominates monetary policymaking, then an increase in transmission uncertainty leads to an increase in optimal central bank conservatism. This is just another way of saying that the compound effects of increased transmission uncertainty and large relative bias dominate aggregate variability, so that optimality sustains an increase in the conservativeness of the central banker. If, on the other hand, the stabilisation problem dominates monetary policymaking, so that the relative bias is small, optimality sustains a reduction in the optimal central bank conservatism as transmission uncertainty increases. So, we think that Rogoff's original idea of checking how favourable the credibility-flexibility tradeoff is for delegation is the unifying principle underlying the effects of an increase in transmission uncertainty on optimal central bank conservatism.

To some extent, our results should be compared to the ones obtained in the literature on uncertainty about the preferences of a (conservative) central banker. In this case, Nohan and Schaling (1996)²³ show that there exists a trade-off between the degree of conservatism and the level of accountability (which is inversely related to the degree of preference uncertainty). This tradeoff arises, however, because of a positive effect of uncertainty on the inflation bias and on inflation variability as well as of an ambiguous effect on the output variance (see Eijffinger *et al* (1997) on this point). This is in sharp contrast with the results obtained under transmission uncertainty. Two reasons account for the contrast in results. First, whereas transmission uncertainty is common knowledge in the previous model, the preferences of the conservative central banker is private knowledge to the central banker: neither the private sector, at the time when expectations are formed, nor the policy-maker, when she decides on delegation, knows these preferences. As a consequence, the impact of uncertainty depends critically on the assumed informational structure of the monetary policy game. Second, the presence of preference uncertainty implies, most importantly, nonlinear (as well as asymmetric) effects on inflation expectations whereas for a given growth rate of money, transmission uncertainty impinges linearly on inflation.

All in all, these different results might deliver some interesting insights into the robustness of optimal monetary policy rules to different types of uncertainty. A

²² On credibility and ambiguity, see e.g. Cukierman (1986). His results do suggest that ambiguity or secrecy can be sustained by optimality.

²³ See also Briault *et al* (1996).

unified framework of analysis would be desirable in an attempt to tackle the issue of robustness, however. To this end, it would also be interesting to see the potential effects of transmission uncertainty on optimal targeting in monetary policymaking as well as on optimal central bank incentive contracts. As it has been emphasised recently,²⁴ under the preference uncertainty, the equivalence between the linear incentive contract (à la Walsh) and the optimal inflation target (à la Svensson) breaks down whereas the case for a conservative central banker with a specific inflation target or an incentive contract can be restored. In our model, the optimal incentive contract would not depend on the level of transmission uncertainty, which contrast sharply with the case of preference uncertainty. Again, as far as the welfare ranking of different delegation schemes goes, the type of uncertainty seems to matter. Further research on this issue is clearly needed.

What are then the main implications of our analysis for the behavior of the ECB? From the foregoing, one conclusion can be readily drawn. The creation of the monetary union may have, plausibly, introduced transmission uncertainty, even if none existed in the individual member countries prior to creation of the monetary union; the foregoing analysis then suggests that the effect of the introduction of transmission uncertainty on the optimal degree of the ECB's conservatism depends on the balance of the credibility-flexibility tradeoff in the Euroland. Optimality sustains caution in the presence of transmission uncertainty, and this will, as we have seen, on its part enhance the credibility of the ECB's monetary policy. But this alone does not solve the problem of the optimal degree of inflation aversion to be embedded in the ECB: we need to check the balance à la Rogoff to contribute to the solution of the problem. Hence, there is plenty of room for empirical research on this matter.

²⁴ See on this point Eijffinger *et al* (1998) and Muscatelli (1999).

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6 Appendix 1: Uniqueness of the solution of $G(\tilde{\varepsilon}) = \tilde{\varepsilon}$ and shape of the $G(\cdot)$ curves.

We restrict the solution set to positive values of ε . We have the following results:

- $\frac{\partial G(\varepsilon; \sigma_\mu^2)}{\partial \varepsilon} = -3 \cdot J(\sigma_\mu^2) \frac{(\alpha^2 + \chi)^2}{[\alpha^2 \sigma_\mu^2 + \chi(\sigma_\mu^2 + 1)]^4} < 0$.
with $\chi \equiv \gamma + \varepsilon$ and $J(\sigma_\mu^2) \equiv \frac{\alpha^2 b^2 [\alpha^2 \sigma_\mu^2 + \gamma(\sigma_\mu^2 + 1)] \cdot (\sigma_\mu^2 + 1)^2}{\sigma_u^2}$.
- $G(0; \sigma_\mu^2) = \frac{b^2}{\sigma_u^2} \cdot \frac{(\alpha^2 + \gamma)^3}{[\alpha^2 \sigma_\mu^2 + \gamma(\sigma_\mu^2 + 1)]^2} \cdot (\sigma_\mu^2 + 1)^2$
- $\lim_{\varepsilon \rightarrow +\infty} G(\cdot; \sigma_\mu^2) = \frac{b^2}{\sigma_u^2} \cdot \frac{[\alpha^2 \sigma_\mu^2 + \gamma(\sigma_\mu^2 + 1)]}{(\sigma_\mu^2 + 1)} \equiv G(\infty; \sigma_\mu^2) < \infty$.

It ensues that $G(0; \sigma_\mu^2) > G(\infty; \sigma_\mu^2) > 0$ (for $\sigma_\mu^2 > 0$) and this implies that we can find a unique positive $\tilde{\varepsilon}$ such that $G(\tilde{\varepsilon}) = \tilde{\varepsilon}$.

Other properties of the $G(\cdot)$ may be worth mentioning too. Indeed, we are able to show that⁴:

- $G(0; \sigma_\mu^2) < F(0)$ whatever the value of σ_μ^2 . (Recall that $F(0) = \frac{b^2(\alpha^2 + \gamma)^3}{\sigma_u^2 \cdot \gamma^2}$).
- Moreover $\frac{\partial G(0; \sigma_\mu^2)}{\partial \sigma_\mu^2} < 0$.
- $\left| \frac{\partial F(\cdot)}{\partial \varepsilon} \right|_{\varepsilon=0} > \left| \frac{\partial G(\cdot; \sigma_\mu^2)}{\partial \varepsilon} \right|_{\varepsilon=0}$. In graphical terms, this means that, in the $(\varepsilon; G(\cdot))$ plane, the slope of the $F(\cdot)$ - curve at the point $(0; F(0))$ is steeper than the one corresponding to the $G(\cdot)$ - curve at $(0; G(0))$.
- Finally, $\frac{\partial^2 G(\cdot; \sigma_\mu^2)}{\partial^2 \varepsilon} > 0$. The $G(\cdot)$ - curve is convex for every (nonnegative) value of σ_μ^2 .

7 Appendix 2: Sign of $\frac{\partial \tilde{\varepsilon}(\sigma_\mu^2)}{\partial \sigma_\mu^2}$

Substitute $\tilde{\varepsilon}(\sigma_\mu^2)$ into equation (25) and differentiate with respect to σ_μ^2 . By rearranging terms, we obtain:

$$\frac{\partial \tilde{\varepsilon}(\sigma_\mu^2)}{\partial \sigma_\mu^2} = -\frac{\partial G(\tilde{\varepsilon}; \sigma_\mu^2)}{\partial \sigma_\mu^2} \cdot \frac{1}{\left[\frac{\partial G(\tilde{\varepsilon}; \sigma_\mu^2)}{\partial \tilde{\varepsilon}} - 1 \right]}$$

⁴More precise calculations are available from the authors upon request.

Now by the second order sufficient conditions for the loss minimisation in (24) $\left[1 - \frac{\partial G(\tilde{\varepsilon})}{\partial \varepsilon}\right]$ is positive; hence, the denominator in the above expression is negative. This, in turn, implies that the sign of $\frac{\partial \tilde{\varepsilon}(\sigma_\mu^2)}{\partial \sigma_\mu^2}$ is the same as the one of $\frac{\partial G(\tilde{\varepsilon}; \sigma_\mu^2)}{\partial \sigma_\mu^2}$.