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Macroeconomic effects of looming policy shifts: Non-falsified expectations and Peso problems

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Research Department
2.7.1998

Macroeconomic Effects of Looming Policy Shifts:
Non-falsified Expectations and Peso Problems

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Macroeconomic Effects of Looming Policy Shifts: Non-falsified Expectations and Peso Problems*

The views expressed are those of the author and do not necessarily correspond to the views of the Bank of Finland.

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Macroeconomic Effects of Looming Policy Shifts: Non-falsified Expectations and Peso Problems

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Abstract

Using a standard model as a basis, we analyse the rational expectations macroeconomic equilibrium for an open economy with flexible exchange rates, in which expectations are affected by a perceived possibility of discrete shifts in monetary policy. These discrete shifts are modelled as possible jumps in the money supply process, which is otherwise a smooth random walk. Two such jump models are analysed. In equilibrium, the distribution of endogenous variables is (apparently systematically) affected by peso problems (premia), which reflect distributional peculiarities associated with expectations of possible future policy shifts. It turns out that the macroeconomic effects of peso premia accord closely with intuition regarding the effects of poor credibility of a policy regime: the output gap widens; the levels of real interest rates and domestic prices rise; and the domestic currency appreciates in real terms due to anticipated expansionary shifts in the money supply. Moreover, the key macro-aggregates become more volatile. The effects of peso premia on the nominal interest rate and the exchange rate turn out to be ambiguous.

Keywords: poisson process, policy shifts, peso problem, rational expectations

Uhkaavien politiikkamuutosten kokonaistaloudelliset vaikutukset: Ei-falsifioidut odotukset ja peso-ongelmat

Suomen Pankin keskustelualoitteita 13/98

Jouko Vilmunen
Tutkimusosasto

Tiivistelmä

Keskustelualoitteessa tarkastellaan avotalouden rationaalisten odotusten tasapainon ominaisuuksia, kun taloudelliset toimijat kokevat suurehkot muutokset tulevassa rahapolitiikassa mahdolliseksi. Suurehkot muutokset tulevassa rahapolitiikassa mallinnetaan mahdollisina hyppäyksiä keskuspankin hallinnoimassa rahan määrässä kahdella vaihtoehtoisella tavalla. Peso-ongelma eli peso-premio, eli odotusten näennäinen harhaisuus tilanteissa, jotka liittyvät rahan määrän mahdolliseksi koettuihin hyppäyksiin tulevaisuudessa, vaikuttaa kokonaistaloudellisten suureiden käyttäytymiseen talouden tasapainossa. Tarkastelut tukevat hyvin niitä ennakkokäsityksiä peso-ongelman mahdollisista kokonaistaloudellisista vaikutuksista, joiden ansiosta sitä käytännössä pidetään talouspoliittisena ongelmana: kun rahapolitiikan pelätään keventyvän tulevaisuudessa, kokonaistuotanto laskee suhteessa potentiaaliinsa, kotimainen hintataso ja reaalikorko nousevat, odotettu inflaatio voimistuu ja talouden hintakilpailukyky heikkenee. Toisaalta peso-premion vaikutusten suunta kotimaiseen nimelliskorkoon ja nimelliseen valuuttakursiin on ilman lisäoletuksia epäselvä. Keskeisten kokonaistaloudellisten suureiden vaihtelevuus voimistuu.

Asiasanat: hyppyprosessit, rahapolitiikan vaihdokset, peso-ongelma, rationaaliset odotukset

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1 Introduction

One of the most established features of modern macroeconomic models is that current macroeconomic variables, including, in particular, asset prices, incorporate market participants' expectations of future economic variables. When agents in the economy (inter)act in a stable and stationary economic environment, it is intuitively plausible that their rational expectations and beliefs are based on the subjective probability distribution for shocks hitting the economy that coincides with the distribution generating the past realizations of the variables. In an unstable environment, on the other hand, there may be differences between the subjective probability distribution underlying expectations formation by the markets and the distribution generating past realizations, if *markets rationally anticipate discrete shifts in the distribution of future shocks*. The literature has come to call this phenomenon or the behaviour of the relevant variables under the influence of these expectational relations, as the 'peso problem'.¹ 'A peso problem' still lacks a uniform interpretation or definition among economists, but the following reasoning hopefully serves to clarify the meaning given to it above in the context of possible future regime shifts.²

Think of the data generating process underlying the observed values of the relevant economic variables as including a low probability, perhaps catastrophic, state or outcome that generates extreme disutility to economic agents. Because this state has low probability, it is unlikely to be observed in a given small sample of data. Because this state is an extreme one, the possibility that this state may occur substantially affects agents' decisions, which in turn determines equilibrium allocations, prices and rates of return. This description has, in the present context, a couple of interesting elements. First of all, the data generating process incorporates an extreme state – a large discrete jump in an asset price, for example – that occurs with low probability, ie it is a *rare* event. Thus, the underlying distribution of states is *skewed* in a highly specific way, while the sample distribution, ie the distribution of the observed values of the variable(s) in a typical (small) sample may not adequately represent this skewness due to the low probability of observing an extreme state. What this implies, in turn, is that the sample moments (mean, variance etc.) calculated from the available data *do not coincide with the population moments* (ie moments of the underlying distribution of the data generating process) *that agents actually use to make their decisions*. In short, then, the way 'a peso problem' is understood here is that it is essentially one

¹Evans (1995 p. 1) argues that the precise origin of the term "peso problem" is unknown, but a number of economists attribute its first use to Milton Friedman in his examination of the Mexican peso market during the early 1970's. During the period, Mexican deposit rates remained substantially above U.S. dollar interest rates even though the exchange rate remained fixed at 0.08 dollars per peso. Friedman argued that this interest rate differential reflected the market's expectation of a devaluation of the peso, which expectation became subsequently, in August 1976, justified when the peso was allowed to float and it fell in value by 46 % to a new rate of 0.05 dollars per peso. The first written discussion of the peso problem, however, appears in Rogoff (1980) in the context of a study of the behaviour of Mexican peso futures prices and spot exchange rates.

²Bekaert *et al.* (1997 p. 2) is followed here. See also Rietz's explanation of the equity premium puzzle (1988).

of small sample inference, and actually results from distributional peculiarities, as Obstfeld (1987 p. 2) aptly puts it.

A highly stylized example of an "announcement" effect that could give rise to a peso type problem (Obstfeld, 1987 p. 3) argues that an announcement from a top decision maker that the country's currency is "too strong" may lead the market to repeatedly underestimate the currency's future strength if monetary policy is not promptly adjusted to push the currency down. A more inspiring example is provided by a simple formalization of the original problem itself, ie anticipated devaluation of the peso in the fixed exchange regime (Evans, 1995 p. 2); let s_{t+1} denote the log of the spot peso price of the dollar in period $t+1$ (pesos per dollar). From April 1954 to August 1976 this spot exchange rate was fixed at $s_t = s^0 = 1/0.08 = 12.5$ pesos per dollar. If $s^1 (> s^0)$ is the (perceived) post-devaluation level of the spot rate, the expected spot rate can be written as

$$E_t[s_{t+1}] = \pi_t s^1 + (1 - \pi_t) s^0,$$

where π_t is the (potentially time-varying) probability that the peso will be devalued between period t and $t+1$, as perceived by the market. While the peso remained fixed at s^0 , the realized difference between the spot rate s^0 and the rate expected by the market, $E_t[s_{t+1}]$, ie the ex post forecast error, was

$$s^0 - E_t[s_{t+1}] = \pi_t (s^0 - s^1) < 0$$

as long as market participants believed that the peso would be devalued with positive probability, $\pi_t > 0$. Hence, ex post forecast errors appear systematically negative! This, in turn, would have a positive contribution to the interest rate differential between Mexico and the U.S., since, via the (uncovered) interest rate parity, this differential equals $i_t^{\text{Mex}} - i_t^{\text{US}} = E_t[s_{t+1} - s_t] = \pi_t (s^1 - s^0) > 0$. Clearly, then, the presence of these persistent devaluation expectations have the potential to solve the original puzzle of a positive (and possibly time-varying) interest rate differential between Mexico and the U.S.

This last example suggests an other important feature of a peso problem, which has been much discussed in the literature on foreign exchange market efficiency and rationality of exchange rate expectations³. Namely, ex post forecast errors appear biased and correlated with the available information at the time forecasts are made, thus militating against the tenet of rational expectations. However, the word 'appears' should be emphasized here, since it is well understood nowadays that peso problems as such are not inconsistent with rational expectations. Actually, it can be readily shown in the above example on the peso's anticipated devaluation that the forecast errors are conditionally, on period t information, unbiased, ie. their conditional date- t expectation is zero. This is an

³See eg Evans (1995), Kaminsky (1993), Lewis (1989a, b). A peso problem in the context of the term structure of interest rates is also relevant, see eg Lewis (1991) and also Balduzzi *et al* (1993, 1997) on the effects of official targeting policy on the term structure of interest rates; insights from the theoretical work on peso problems to account for realistic infrequency of target changes underlie their idea of anticipated policy dynamics. A peso problem is also argued for in and inspiring work by Gourinchas & Tornell (1995) as an underlying factor in the so called *delayed overshooting* of exchange rates to monetary expansion.

important point and we will have many occasions to return to it in this study. That forecast errors are conditionally unbiased, but ex post systematically non-zero may be a source of conceptual confusion and certainly lies at the heart of a peso problem; conditional unbiasedness is evaluated using the underlying population distribution or, more interestingly, the distribution underlying expectations calculation of the market participants, which incorporates an extreme, low probability state. When sampling from this underlying distribution, it is highly unlikely that one obtains a sample where the particular extreme state occurs. Since the mirror image of this is that there is a relatively high probability of obtaining samples excluding the extreme state, it follows, as a matter of inference, that expectations incorporating the potential realization of the extreme state (or the potential future "regime shift" corresponding to the extreme state) have a tendency *not to be falsified by the data!* This observation motivates the title of this study.⁴

Typically, peso type problems have been proposed as an explanation for specific anomalies or puzzles in the observed behaviour of asset prices, like the original peso problem itself analyzed by Friedman. This is quite understandable, given that asset pricing is so forward looking, with the dynamic behaviour of asset prices being strongly affected by self-fulfilling expectations. Furthermore, specific asset pricing models can be usefully combined with a peso problem, particularly in the form of possible future regime switching, to yield a highly tractable analysis of the effects of anticipated policy dynamics on eg the term structure of interest rates and exchange rates and to give suggestions as to how to interpret the evidence of various asset pricing anomalies and puzzles.⁵ Also, learning and a peso problem can be combined fairly straightforwardly using simple asset pricing models.⁶ Certainly research works thus far has enhanced our understanding of the potential effects of peso problem on the dynamic behaviour of asset prices, particularly of exchange rates. At the same time it must be admitted, however, that research on the implications of a peso problem on the rest of the economy, most notably on the real sector, still is surprisingly scarce⁷. A formal account of these

⁴The work of Danthine and Donaldson (1996) on non-falsified expectations and the peso problem in the context of general equilibrium asset pricing has clearly been a source of inspiration here. At various points in his contribution he revitalizes the note that the *possibility* of the occurrence of 'disaster states' may have important consequences to economic behaviour; "Although it did not materialize, the crash of 1987 immediately raised fears of a repeat of 1929. But the fact that these events have not materialized does not prove the *possibility* of their occurrence has not affected economic behavior" (p. 1), "When our attention is restricted to a subset of the series in which z_3 (= disaster state) is not observed [...] A first conclusion is thus as follows; even though our statistical summary of model dynamics may be derived from data for which the disaster state is not present, it is *not* possible to preclude, on the basis of that summary, the *expectation of a disaster state on the part of economic agents*." (p. 10).

⁵See the excellent survey by Evans (1995) of the theoretical and empirical implications of a peso problem. Evans conducts the analysis of a generalized peso problem within a discrete time asset pricing framework. See also Obstfeld (1987) for an extremely clear account of the peso problem in the context of exchange rate determination.

⁶See Evans (1995).

⁷The analysis in Danthine and Donaldson (1996) makes the set of research contributions sought-after here non-empty.

implications would certainly usefully complement the existing analysis, so that the present state of the art need not be entirely satisfactory.⁸

More importantly, peso type problems must be particularly important to policy makers or a policy perspective must enhance the importance of peso type problems. Note that peso type problems can be highly persistent and are more often than not related to perceived future regime shifts. One often hears that expectations are "misaligned" from the policy point of view. Anyway, even learning about the prevailing regime takes time and this may show up adversely in eg asset prices (exchange rates, interest rates etc.), but learning aside, expectations driven by peso type problems are not easily falsified, as explained above. It is the inherent persistence, with fears that it increases the likelihood for real consequences from peso type problems, and the apparent implications that some "structural" changes have to be implemented in the economy or policy making to undo these "misaligned" expectations that seems to underlie the policy concern. Hence, this policy perspective clearly gives the research on potential macroeconomic effects of peso type problems, the focus of this study, additional relevance, on top of the problem's theoretical interest.

The main objective of the present study is to take an initial step toward contributing to the analysis of the implications of a peso type problem on the aggregate economy. It uses a standard open economy IS-LM model under flexible exchange rates and an expectations augmented Phillips curve as the workhorse of the analysis to derive the full macroeconomic effects of a peso problem modelled either as infrequent (perceived) future discrete jumps (generated by a Poisson process) or as a once and for all future shift in the money supply process. The main justification for choosing this particular framework for the analysis is the initial need to gain insights into the nature of the macroeconomic effects of a peso problem. Furthermore, IS-LM-AS models are relatively tractable, their properties as well as limitations are well understood and, once modified to incorporate an "expectations augmented IS-curve" along the lines suggested by McCallum and Nelson (1996), provide a sound basis for macroeconomic analysis broadly consistent with the first principles of microeconomics. The emphasis is here on the first point, ie on deriving a set of initial results where further, and possibly more refined, analysis can improve upon. To this end, interesting results do emerge from the analysis. To anticipate them, it is shown that peso problems (or 'peso premia' as they will be called) tend to lower aggregate output relative to the potential one, increase real interest rates, strengthen the domestic currency in real

⁸Of course, under the classical assumption of costless price adjustment of infinite speed, so that the classical dichotomy prevails, it is doubtful whether existing analyses of peso problems in asset prices can be *usefully* extended to incorporate the potential real effects called for here. But such costless price adjustment cannot be relied upon; in fact promising models with solid microeconomic foundations have begun to appear, that explicitly recognize and build on price stickiness and output effects from (aggregate) demand variations (an excellent account of these new models of imperfect competition can be found in Obstfeld and Rogoff, 1996, ch. 10; on a specific application of their framework to the analysis of exchange rate dynamics, see Obstfeld and Rogoff, 1995). asset prices can be *usefully* extended to incorporate the potential real effects called for here. But such costless price adjustment cannot be relied upon; in fact promising models with solid microeconomic foundations have begun to appear, that explicitly recognize and build on price stickiness and output effects from (aggregate) demand variations (an excellent account of these new models of imperfect competition can be found in Obstfeld and Rogoff, 1996, ch. 10; on a specific application of their framework to the analysis of exchange rate dynamics, see Obstfeld and Rogoff, 1995).

terms and increase the domestic price level. Furthermore, aggregate volatility also increases due to peso premia. These macroeconomic effects are actually what one would expect peso problems to have and must be part of the explanation as to why peso problems are 'problems' from the macroeconomic point of view and why they should be of concern to policy makers.

This paper has the following structure. Chapter 2 presents the macroeconomic model and derives its rational expectations solution for an arbitrary bounded sequence of the money supply process. The solution has the typical structure of a rational expectations model; changes of the variables are driven by the flow of new information that are intimately related to revisions of expectations. Chapter two models the money supply as the sum of a random walk component and a Poisson process with low intensity. The intended interpretation is that discrete money supply jumps occur very infrequently and the cumulative number of discrete money supply jumps from time $t = 0$ to $t = T$ is registered by the value of the Poisson process at time T . The decomposition of the expectations revisions term into what will be called the peso premium and (the source of the) liquidity effect of expansionary monetary policy comes out from the analysis very clearly with the Poisson process. Furthermore, the stationarity of the Poisson process clearly contributes to the simplicity of the analysis; the peso premium is constant as is the contribution of jump uncertainties on aggregate volatility or variability. Chapter 4 modifies the model of the money supply; instead of infrequent discrete jumps the money supply possibly undergoes a once and for all discrete jump in the future corresponding to the notion of a regime shift in the money supply process (or monetary policy, for that matter). Chapter 4 then rederives the rational expectations solution of the model and compares its main features with those of the Poisson process in the previous chapter. The main difference between the two models of the money supply seems to be that the peso premium as well the contribution of jump uncertainties on aggregate volatility is time invariant, whereas they are time varying in the model of chapter 4. Hence, heteroscedastic prediction errors can be given an explanation in terms of the regime switching model of chapter 4. Finally, chapter 5 concludes and discusses further research and extensions of the theoretical framework of the paper, and some of the solution equations are relegated to appendices A and B.

2 The model and its solution

2.1 Description of the model

As argued above one does not need to invoke irrationality of market behaviour to explain the observed, apparently anomalous behaviour of key asset prices like exchange rates, since these prices may suffer from an underlying peso type problem sustained eg by non-trivial beliefs about a future shift in the relevant policy regime. Since this paper is not concerned with asset prices inflicted by an underlying peso problem per se, but rather with the macroeconomic consequences, if any exist, of the underlying peso problem itself, it is useful to adopt a theoretical framework in which nominal variability affects the real equilibrium of the economy. To this end, a model of an open economy under a floating exchange rate regime subject to a peso type problem generated by

perceived discrete future jumps or regime shifts in the process controlled by the monetary authority will be introduced below based on Dornbusch's (1976) model of exchange rate overshooting. This is a generalization of Obstfeld's (1987) analysis of a peso problem in that he adopts the monetary approach to exchange rate determination as exemplified by eg Mussa (1976). The modern monetary approach (under rational expectations) provides an extremely simple, useful and powerful analytical tool to set up the peso problem in formal terms and to display the internal structure of the problem. But it also suffers from a number of limitations, the most important one, from the point of view of the present, being that the peso problem has no real consequences within the monetary model of exchange rate determination. This particular feature is, of course, the consequence of assuming perfect price flexibility so that the real income in the economy is (absent growth) fixed at the full-employment level. One could plausibly argue, however, that it is precisely the potential real costs that give peso problems practical importance, ie. the possibility that there will be output costs due to presence of a specific type of a policy credibility problem not only makes the problem highly interesting but also enhances its relevance from the point of view of economic policy.

The discrete time log-linear version of the Dornbusch model incorporates a real interest rate effect on aggregate demand and an expectations augmented Phillips-curve on the supply side;

$$\begin{aligned}
 y_t - y^n &= \delta(s_t + p_t^* - p_t) - \sigma(i_t - E_t \Delta p_{t+1}) \\
 \Delta p_t &= p_t - p_{t-1} = E_{t-1} \Delta p_t + \theta(y_t - y^n) \\
 m_t - p_t &= y_t - \lambda i_t \\
 i_t - i_t^* &= E_t \Delta s_{t+1}
 \end{aligned} \tag{1}$$

The model as well as the notation adopted here is very standard; y , p , s , i , y^n , p^* and i^* denote, respectively domestic output, price level, nominal exchange rate (domestic price of foreign currency), domestic interest rate, full employment output, foreign price level and foreign interest rate. The expectations augmented Phillips-curve may be given a contract interpretation; the nominal wage rate for the production period t is set at the end of the previous period at a level where labour markets are expected to clear, so that deviations of actual output from the expected (full employment) level occur once the output price differs from the

corresponding equilibrium level.⁹ The structure of the model has been further simplified, essentially without loss of generality, by abstracting away from exogenous disturbances in the equations.

2.2 Solution to rational expectations

We are searching for the rational expectations solution to the exchange rate and for this we need to express the price level p_t in terms of the exchange rate. To start, then, substitute the aggregate demand and interest rate equations into the Phillips-curve

$$\Delta p_t = E_{t-1} \Delta p_t + \theta[\delta(s_t + p_t^* - p_t) - \sigma(i_t^* + E_t \Delta s_{t+1} - E_t \Delta p_{t+1})] \quad (2)$$

which, after some manipulations, results in the following difference equation;

$$\begin{aligned} [1 + \theta(\sigma + \delta)] p_t - E_{t-1} p_t - \sigma \theta E_t p_{t+1} &= \theta[(\sigma + \delta) s_t - \sigma E_t s_{t+1} + \delta p_t^* - \sigma i_t^*] \\ &= \theta(\sigma + \delta) s_t - \theta \sigma E_t s_{t+1} + \theta(\sigma + \delta) p_t^* - \sigma \theta E_t p_{t+1} - \sigma \theta r_t^* \end{aligned} \quad (3)$$

where r_t^* denotes the foreign (ex ante) real interest rate $i_t^* - E_t \Delta p_{t+1}^*$. Taking conditional period $t-1$ expectations, and making use of the law of iterated expectations $E_s[E_t x_{t+k}] = E_s x_{t+k}$, $s < t$, as well as of the rule of forward operation $L^{-k} E_t x_t = E_t x_{t+k}$, (3) can be rewritten as

$$\begin{aligned} [1 - \Psi L^{-1}] E_{t-1} p_t &= [1 - \Psi L^{-1}] E_{t-1} s_t + [1 - \Psi L^{-1}] E_{t-1} p_t^* - \Psi E_{t-1} r_t^* \\ &\text{ie} \\ E_{t-1} p_t &= E_{t-1} s_t + E_{t-1} p_t^* - [1 - \Psi L^{-1}]^{-1} \Psi E_{t-1} r_t^* \end{aligned} \quad (4)$$

⁹This particular model of the supply behaviour, ie one period contract lag along the lines of Gray (1976) and Fischer (1977) plays an important role as far as the form of the rational expectations solution of the model is concerned. Nelson (1997) argues that these Gray-Fischer contracts appear less likely to account for strongly autoregressive price level behaviour often found (see eg Taylor, 1993, and Fuhrer and Moore, (1995) in the US price level data. Instead, Nelson argues in favour of using Taylor's (1980) model of staggered price contracts or, actually its generalization in Fuhrer and Moore. Yet another possible framework for the analysis of supply side behaviour builds on imperfect competition model of price stickiness (Rotemberg 1982 and Calvo 1983; on models of imperfect competition as microfoundations for price stickiness and aggregate demand effects, see Obstfeld and Rogoff 1996, ch. 10). Rotemberg (1987) showed that Calvo-Rotemberg models are observationally equivalent, while Roberts (1995) shows that Taylor and Calvo-Rotemberg price specifications imply similar dynamics. Anyway, these alternative models of supply behaviour, especially that of Fuhrer and Moore (1995), also an essential part of Nelson's (ibid.) analysis, appear to imply that on top of output gap dynamics, the price level or inflation in period t should depend on the conditionally, on date- t information, expected period $t+1$ price level or inflation, ie $p_t = a E_t p_{t+1} + \dots$, where a is positive constant. This specification implies autoregression in the reduced form solution of the model, the degree and form of which depends, among other things, on the exact form of the output dynamics entering the price level (or supply) equation.

where $\Psi = \sigma/(\sigma+\delta)$ is positive and less than one. The operator $(1-\Psi L^{-1})^{-1}$ is defined by the forward sum $(1-\Psi L^{-1})^{-1} = \sum \Psi^k L^{-k}$ (k runs from 0 to infinity). Note Equation (4) thus gives us the solution to the conditional expectation of the period t price level as of time $t-1$. We can use this particular solution to derive an expression for any one step price level forecast $E_{t+k}p_{t+k+1}$ simply by shifting the time index forward $k+1$ periods in (4); in particular, for $k = 1$ we have

$$E_t p_{t+1} = E_t s_{t+1} + E_t p_{t+1}^* - [1 - \Psi L^{-1}]^{-1} \Psi E_t r_{t+1}^* \quad (5)$$

Note that (5) implies, in particular, that the sequence of anticipated future foreign real interest rates, which is exogenously given in the present context, act as forcing variables on the conditionally expected terms of trade, $E_{t-1}q_t = E_{t-1}(p_t - s_t - p_t^*)$, ie anticipated terms of trade are derived from future foreign real interest rates by the appropriate "present value" or "discounting" formula

$$E_{t-1} q_t = E_{t-1} [p_t - s_t - p_t^*] = -[1 - \Psi L^{-1}]^{-1} \Psi E_t r_{t+1}^* = -\Psi \sum_{k=0}^{\infty} \Psi^k E_{t-1} r_{t+k}^* \quad (6)$$

abstracting, as before, from explosive bubble dynamics. As is typical in a rational expectations context, for a unique (fundamental) solution it is not enough to have knowledge of what happens to exogenous real interest rates at any particular point in time; to tie down terms of trade expectations uniquely, the entire sequence of future real interest rates needs to be known.

Using the expression for the conditionally expected price level, eqs (4) and (5), we can consult equation (3) once again to derive an expression for the price level in terms of the (expected) exchange rate or foreign price level in terms of domestic currency;

$$p_t = s_t + p_t^* - \frac{1}{1 + \theta(\sigma + \delta)} [(s_t + p_t^*) - E_{t-1}(s_t + p_t^*)] - \frac{1}{1 + \theta(\sigma + \delta)} [\Psi(1 - \Psi L^{-1})^{-1} (E_{t-1} r_t^* + \sigma \theta E_t r_{t+1}^*) + \sigma \theta r_t^*] \quad (7)$$

where $(1-\Psi L^{-1})^{-1} E_{t-1} r_t^* = \sum \Psi^k E_{t-1} r_{t+k}^*$ (k runs from zero to infinity). Note that according to (7), there are two sources of terms of trade variability; (realized and anticipated future shocks to the) foreign real interest rate and conditional forecast errors from forecasting foreign prices in domestic currency. Most interestingly, it is in the very nature of peso type (policy) problems (Obstfeld 1987, Evans 1995) that ex post (exchange rate) forecast errors appear conditionally biased. Hence, an econometrician, in particular, may find significant and predictable deviations from PPP, even if the conditional mean of the foreign real interest rate appears constant during the time spanned by the sample under consideration.

In order to have the reduced form solution to the price level we need to solve for the (actual and expected) exchange rate. To this end, we substitute the semi-reduced price level in (7) into money market equilibrium condition, LM equation; this gives us the following equation for the nominal exchange rate

$$\begin{aligned}
& \left[\lambda + \frac{(1+\theta)(\sigma+\delta)}{1+\theta(\sigma+\delta)} \right] [s_t + p_t^*] + \frac{1-(\sigma+\delta)}{1+\theta(\sigma+\delta)} E_{t-1} [s_t + p_t^*] \\
& - \lambda E_t [s_{t+1} + p_{t+1}^*] = \\
& m_t + \frac{1-(\sigma+\delta)}{1+\theta(\sigma+\delta)} \Psi [1 - \Psi L^{-1}]^{-1} E_{t-1} r_t^* + \left[\lambda + \frac{\sigma(1+\theta)}{1+\theta(\sigma+\delta)} \right] r_t^* \\
& + \frac{(1+\theta)\sigma\Psi}{1+\theta(\sigma+\delta)} [1 - \Psi L^{-1}]^{-1} E_t r_{t+1}^* - y^n
\end{aligned} \tag{8}$$

Following the solution procedure used to derive the semi-reduced price level above, take the conditional expectation as of time $t-1$ on both sides of equation (8); after simplifying, we can express the (bubble free)¹⁰ solution to the conditionally expected exchange rate as

$$\begin{aligned}
E_{t-1} s_t = & -E_{t-1} p_t^* + [1 - \Phi L^{-1}]^{-1} \left[E_{t-1} m_t + \frac{1-(\sigma+\delta)}{1+\theta(\sigma+\delta)} \Psi (1 - \Psi L^{-1})^{-1} E_{t-1} r_t^* \right. \\
& \left. + \left[\lambda + \frac{\sigma(1+\theta)}{1+\theta(\sigma+\delta)} \right] E_{t-1} r_t^* + \frac{(1+\theta)\sigma\Psi}{1+\theta(\sigma+\delta)} (1 - \Psi L^{-1})^{-1} E_{t-1} r_{t+1}^* - y^n \right] / (1+\lambda)
\end{aligned} \tag{9}$$

where $\Phi = \lambda/(1+\lambda)$. As before, the equation (9) gives us the one-step RE forecast function for the nominal exchange rate from any period $t+k$ to period $t+k+1$. Hence, the final step in the solution procedure involves substituting from equation (9) $E_{t-1} s_t$ and $E_t s_{t+1}$ into the exchange rate equation (8); after some manipulation the (bubble-free) solution to the nominal exchange rate is

$$\begin{aligned}
s_t = & -y^n - p_t^* + \frac{1}{1+\lambda} [1 - \Phi L^{-1}]^{-1} E_{t-1} m_t + \chi_s (1 - \Phi L^{-1})^{-1} (E_t - E_{t-1}) m_t \\
& + B_1(L^{-1}) E_{t-1} r_t^* + B_2(L^{-1}) E_{t-1} r_{t+1}^* + B_3(L^{-1}) E_t r_{t+1}^* + B_4(L^{-1}) E_t r_{t+2}^* \\
& + \chi_s \left[\lambda + \frac{\sigma(1+\theta)}{1+\theta(\sigma+\delta)} \right] r_t^*
\end{aligned} \tag{10}$$

where the constant C_s^0 and the lead polynomials on expected future foreign real interest rates, the $B_i(L^{-1})$ s, are given in appendix A. Substantially, they are not too complicated but notationally they are cumbersome and to save space they are relegated to the appendix. We can streamline the presentation of the solution to the nominal exchange rate, (10), drastically, if we make the simplifying assumption that (anticipated) foreign real interest rates as well as foreign price level are constant through time; in this case (10) reduces to

¹⁰A presence of a bubble term of the form $C_0[(1+\lambda)/\lambda]^t$ is ignored in (10); this expectational term satisfies the underlying homogenous expectational difference equation $E_{t-1} s_t - (\lambda/(1+\lambda)) E_{t-1} s_{t+1} = 0$.

$$s_t = -y^n - p^* + (\sigma + \lambda)r^* + \frac{1}{1+\lambda}[1 - \Phi L^{-1}]^{-1} E_{t-1} m_t + \chi_s [1 - \Phi L^{-1}]^{-1} (E_t - E_{t-1}) m_t \quad (11)$$

This simplification of the solution is particularly telling, since it forces one to concentrate on expectations concerning the sequence of future money supplies as the driving force underlying nominal exchange rate dynamics, and, by implications, also other endogenous variables in the model.

(11) signifies important departures from the standard dynamic, bubble-free solution derived from the monetary model of exchange rate determination (see eg Obstfeld 1987). First of all, the standard flexible price monetary approach to exchange rate determination corresponds to (11), where $\sigma = 0$ and $\delta, \theta \rightarrow \infty$. In this case full-employment output y^n is sustained by price flexibility or instantaneous price (level) adjustment; in this case (11) reduces to

$$s_t = C + \frac{1}{1+\lambda} (1 - \Phi L^{-1})^{-1} m_t = C + \frac{1}{1+\lambda} \sum_0^{\infty} \Phi^2 E_t m_{t+k} \quad (12)$$

for some constant C . Interestingly, once we allow for endogenous terms of trade changes and an expectational Phillips-curve (with a one period contract length), the dynamics of the nominal exchange rate does not only reflect optimal use of relevant (forecast) information about future (monetary) policy by financial market participants, who condition their forecast by period t information; rather the dynamic behaviour of the nominal exchange rate now combines the differential use of information in the major markets of the economy. On top of policy anticipations at the time wage contracts are signed, period $t-1$, the nominal exchange rate in period t is affected by the flow of new information incorporated in the changes of expectations, from period $t-1$ to period t , concerning the sequence of future monetary policy. The potential for the financial market to utilize the most recent information prior to taking investment decisions is our most favoured interpretation underlying this latter, expectational revision, effect. Anyway, the structure of the model solution to the nominal exchange rate clearly reflects the strategy that was followed to set up the model, in particular the basic assumptions concerning expectations dating in the model. Also, and more interestingly, possible "credibility premia" affect the (dynamic) performance of the aggregate economy not only through the one period contract length, but also through expectations revisions.

Continuing to assume that the foreign real interest rate is constant, we can also simplify the rest of the solution of the model. From equation (11) we can substitute actual and conditionally (as of time $t-1$) expected nominal exchange rate into the price level equation (7); after simplifying, the (bubble free) solution to the model's price level is

$$\begin{aligned}
p_t &= -y^n + \left[\sigma + \lambda - \frac{\sigma}{\delta} \right] r^* + \frac{1}{1+\lambda} [1 - \Phi L^{-1}]^{-1} E_{t-1} m_t \\
&\quad + \frac{\theta(\sigma + \delta)}{1 + \theta(\sigma + \delta)} \left[\lambda + \frac{(\sigma + \delta)(1 + \theta)}{1 + \theta(\sigma + \delta)} \right]^{-1} [1 - \Phi L^{-1}]^{-1} (E_t - E_{t-1}) m_t \\
&= -y^n + C_p r^* + \frac{1}{1+\lambda} [1 - \Phi L^{-1}]^{-1} E_{t-1} m_t + \chi_p [1 - \Phi L^{-1}]^{-1} (E_t - E_{t-1}) m_t
\end{aligned} \tag{13}$$

with the obvious interpretation of the two constants C_p and χ_p . As it stands, the domestic price level obeys the same underlying logic as the nominal exchange rate, namely that on top of the anticipated future monetary policy at the time wage contracts are signed, the price level reflects revisions in expectations concerning future policy between the time contracts in labour markets are signed and investment decisions in financial markets are taken.

To obtain the solution to the aggregate output, we could work through the Phillips curve; by inverting it, we would obtain an equation for the aggregate output. However, we choose to work through the aggregate demand equation and to this end we still need to solve for the actual terms of trade and ex ante real interest rate. The main reason for the preference over the aggregate demand equation is that by working it out, we obtain valuable information about the real interest rate and terms of trade. Now, equation (7) implies an equation for the terms of trade; substituting in the solution for actual and expected nominal exchange rate, we obtain the following representation of the terms of trade q_t :

$$\begin{aligned}
q_t &= - \frac{\left[\lambda + \frac{(\sigma + \delta)(1 + \theta)}{1 + \theta(\sigma + \delta)} \right]^{-1}}{[1 + \theta(\sigma + \delta)]} [1 - \Phi L^{-1}]^{-1} (E_t - E_{t-1}) m_t - \frac{\sigma}{\delta} r^* \\
&= -C_q r^* - \chi_q [1 - \Phi L^{-1}]^{-1} (E_t - E_{t-1}) m_t
\end{aligned} \tag{14}$$

which essentially says that expansionary revisions, relative to what was previously expected, concerning anticipated future monetary policy will induce a real depreciation of the domestic currency. Using equation (5) and the interest rate parity, the domestic ex ante real interest rate can be expressed as

$$\begin{aligned}
r_t &= i_t - E_t \Delta p_{t+1} = i_t^* + E_t \Delta s_{t+1} - E_t \Delta p_{t+1} \\
&= p_t - s_t - p^* + \Psi^{-1} \frac{\sigma}{\delta} r^* \\
&= r^* - \chi_q [1 - \Phi L^{-1}]^{-1} (E_t - E_{t-1}) m_t
\end{aligned} \tag{15}$$

Finally, using equation (14) for the terms of trade and equation (15) for the domestic real interest rate, we can derive the solution for the domestic output

$$\begin{aligned}
y_t - y^n &= -\delta q_t - \sigma r_t \\
&= \left[\delta \left(\frac{\sigma}{\delta} \right) - \sigma \right] r^* + (\delta + \sigma) \chi_q [1 - \Phi L^{-1}] (E_t - E_{t-1}) m_t \\
&= \chi_y [1 - \Phi L^{-1}] (E_t - E_{t-1}) m_t
\end{aligned} \tag{16}$$

once again with the obvious interpretation of the constant χ_y . Thus, we can conclude from the solution to aggregate output that new information or expectations revisions concerning future monetary policy is the main driving force for fluctuations in output relative to potential output. Of course, unanticipated shocks to the endogenous variables in linear rational expectations models can almost invariably be interpreted as stemming from expectations revisions, and in this sense equation (16) is no exception. However, these expectations revisions in the present context relate to the differential speed of adjustment in the labour and financial markets. Also, eq (16) clearly suggest the form in which credibility problems in the form of potential future policy shifts (including "peso premia") must enter the solution for output (and for other variables as well, of course). This, in turn, suggests in particular that under a credible policy regime, only exogenous factors, like exogenous noise in the data generation process for money supply, giving rise to more or less white noise forecast errors, drive expectations revisions. On average output thus equals potential one under credible policy.

The output solution function (16) clearly suggests that we can make a case for real costs from the pursuit of monetary policy that potentially suffers from a credibility problem. Clearly, moreover, the solutions for the real variables in particular indicate that revisions in the anticipated future policy regime, as embedded in the sequence of conditionally expected money supplies, may be an important mechanism through which (market) perceptions about future policy options, whether well founded in economic fundamentals and government's (announced) policy preferences, impinge on the economy with potential real costs. In short, "anticipate policy dynamics" is what matters.¹¹

Finally, using the uncovered interest rate parity condition together with the solution to the nominal exchange rate, eq (11), we can solve for the domestic nominal interest rate i_t

$$\begin{aligned}
i_t &= i_t^* + E_t [\Delta s_{t+1}] \\
&= i^* + \frac{1}{1+\lambda} [1 - \Phi L^{-1}]^{-1} (E_t m_{t+1} - E_{t-1} m_t) - \chi_s [1 - \Phi L^{-1}]^{-1} (E_t - E_{t-1}) m_t \\
&= i^* + \frac{1}{1+\lambda} [1 - \Phi L^{-1}]^{-1} E_t [\Delta m_{t+1}] + \chi_i [1 - \Phi L^{-1}]^{-1} (E_t - E_{t-1}) m_t \\
\chi_i &= \frac{1}{1+\lambda} - \chi_s
\end{aligned} \tag{17}$$

¹¹This phrase is from Balduzzi *et al* (1997), who use it in a different, but, as they also note, related context. It is similar in spirit to Gourinchas and Tornell (1996) on exchange rate dynamics and learning, where the authors present a model that rationalizes the forward discount puzzle and exhibits delayed overshooting of the nominal exchange rate in response to monetary expansions. More interestingly, they argue that delayed overshooting can result from eg a peso problem.

One of the interesting features of the solution to the domestic nominal interest rate (17) concerns the role of new information in interest rate determination; this is summarized by the last term of equation (17).¹² Not only do these changes in expectations incorporate eg elements of learning and perceptions on possible future switches in policy regimes, as argued above, but they are important also from the perspective of potential liquidity effects of monetary policy, ie how unanticipated changes in (the growth rate of) the money supply impinge on domestic interest rates. Theoretically, it appears to be relatively difficult to rigorously establish a liquidity effect of monetary policy, since more often than not, monetary shocks impinge on interest rates as shocks to expected inflation. Thus, (unexpected) monetary expansion tend to increase nominal interest rate due to higher expected inflation. However, the bulk of empirical research has produced evidence in favour of a robust liquidity effect of monetary policy, at least in the US data.¹³ Above, the coefficient χ_i has an ambiguous sign a priori, but it is easy to show that its sign depends on whether the sum of terms of trade elasticity and real interest rate semi-elasticity of aggregate demand, $\delta+\sigma$, exceeds or falls short of one ($\delta+\sigma \leq \geq 1$). We will return to this matter later, when liquidity effects of monetary policy are taken up in a more specific context.

3 Modelling the money process; perceived policy future shifts as sources for potential peso premia

3.1 Macroeconomic effects of jump uncertainties in the money supply process; introducing the Poisson process

Obstfeld's (1987) analysis, reintroduced into the present macroeconomic context later in section 4, builds on the intuition originally underlying the peso problem, namely that markets anticipate a once and for all permanent regime shift which, given the short time span covered by the available data, has not hitherto realized. This analysis has the potential to explain certain features or anomalies encountered in exchange rate data, including apparently systematic bias in forecast errors or the ex post correlation of exchange rate forecast errors with lagged information, exchange rate misprediction and conditional heteroscedasticity of forecast errors (see Obstfeld 1987, p. 7–9). Its theoretical appeal comes from its simple and manageable structure, sustaining a closed form analysis of exchange rate determination within the monetary model used by Obstfeld. Also the specific structure of the money supply jump process has, for many at least, the intuitively appealing feature that market expects to put progressively less weight on the significance of a possible policy change the longer the interval over which the is forecasted not to occur.

¹²Note that $(E_t - E_{t-1})m_t$ represents, *on date-(t-1) information*, unexpected changes in the money supply.

¹³See eg Nelson (1997) and the references therein, Stockman and Ohanian (1995) and Gourinchas and Tornell (1996).

We shall, however, begin the analysis in this section by taking a different approach to modelling possible jumps in the money supply process (or, more generally, in the policy process). Instead of focusing on a once and for all possible future jump in the money supply process, which is interpreted as a future regime or policy shift, we shall analyze the implications of uncertainties resulting from (relatively infrequent) discrete future jumps in the money supply process; specifically, we assume that the observed money supply is generated by the sum of a random walk and a Poisson process with jump intensity (per unit time) α ;

$$m_t = m_t^B + N_t \quad (18)$$

$m_{Bt} = m_{t-1}^B + \omega_t$ $\omega_t \sim WS(0, \sigma_\omega^2)$ and where where the Poisson process N_t registers the cumulative number of jumps up to and including time t , ie it gives us the cumulative number of jumps in the time interval $[0, t]$ with the probability of observing k jumps given by the Poisson density

$$\Pr[N_t = k] = e^{-\alpha t} \frac{(\alpha t)^k}{k!} \quad (19)$$

for $k = 0, 1, 2, \dots$. Thus, the expected number of jumps in $[0, t]$, $E(N_t)$, is αt . Furthermore, we know that the Poisson process has independent increments (see eg Karlin and Taylor 1975, p. xx), ie for two non-overlapping time intervals $[t_1, t_2]$ and $[t_3, t_4]$, $t_1 < t_2 < t_3 < t_4$, say, the corresponding increments of the Poisson random process $N_{t_2} - N_{t_1}$ and $N_{t_4} - N_{t_3}$ are independent. This particular feature of the Poisson process will be of help in the ensuing calculations, since it implies that

$$E_t[N_{t+k}] = E_t[N_{t+k} - N_t + N_t] = N_t + E[N_k] = N_t + \alpha k \quad (20)$$

where the (realized) number of jumps up to and including time t , N_t , belongs to agents' conditioning information set underlying the conditional expectations operator E_t .¹⁴ The next fact that we need for our calculations is the expected money supply in period $t+k$, $k = 1, 2, 3, \dots$ conditional on information in period t (or $t-1$); using (20) we have¹⁵

$$E_t[m_{t+k}] = m_t^B + N_t + \alpha k = m_t + \alpha k$$

$$[1 - \Phi L^{-1}]^{-1} E_t[m_t] = \frac{1}{1 - \Phi} m_t + \frac{\alpha \Phi}{(1 - \Phi)^2} \quad (21)$$

Furthermore, the term reflecting the present value of the flow of new information between period $t-1$ and t concerning the anticipated sequence of future money

¹⁴Ie N_t is measurable w.r.t. time- t information. Note that the notation does not make difference between the random variable and its values. Notation that keeps these two separate would perhaps be preferable, but context should make the meaning of the notation clear.

¹⁵By differentiating the sum of a geometric series, $\sum x^k = 1/(1-x)$, we obtain $\sum kx^{k-1} = 1/(1-x)^2$. This fact is used in the calculations.

supplies, earlier called expectational revisions, can, after some manipulations, be written in the form

$$[1 - \Phi L^{-1}]^{-1} (E_t - E_{t-1}) m_t = \frac{1}{1 - \Phi} [m_t - m_{t-1}] - \frac{\alpha}{1 - \Phi} \quad (22)$$

Note that the first term¹⁶ in (22) is related to for what could be called *liquidity effects of changes in money supply*, while the second term summarizes the effects from changes in expectations concerning the number of future jumps in the money supply process, or effects from *jump uncertainties* for short; it acts negatively on revisions in expectation about the sequence of future money supplies, since by time $t+h$ more jumps in money supply is forecasted on date- $(t-1)$ relative to date- t information. Using (22) in equations (14)–(16) we can explicitly solve for the underlying determinants of the terms of trade, ex ante real interest rate and aggregate output movements;

$$\begin{aligned} q_t &= -\frac{\sigma}{\delta} r^* - \frac{\chi_q}{1 - \Phi} [m_t - m_{t-1}] + \frac{\chi_q}{1 - \Phi} \alpha \\ r_t &= r^* - \frac{\chi_q}{1 - \Phi} [m_t - m_{t-1}] + \frac{\chi_q}{1 - \Phi} \alpha \\ y_t - y^n &= \frac{\chi_q (\sigma + \delta)}{1 - \Phi} [m_t - m_{t-1}] - \frac{\chi_q (\sigma + \delta)}{1 - \Phi} \alpha \end{aligned} \quad (23)$$

Clearly, then, (23) suggests that on top of the liquidity (related) effect,¹⁷ $m_t - m_{t-1}$, of monetary policy, negative effects from inherent jump uncertainties in the (future) money supply process will affect the real equilibrium of the economy. Of course, under the "classical assumptions" of instantaneous price adjustment and continuous PPP ($\theta, \delta \rightarrow \infty$ and $\sigma = 0$) both of these effects on the real economy will vanish (since $\chi_q \rightarrow 0$ in this case). Furthermore, as the intensity per unit time or expected number of discrete money supply jumps within a unit time interval, α , increases, *cet. par.*, real costs, in terms of larger real appreciation of the domestic currency, higher real interest rates and lower output relative to potential, of jump uncertainties will also go up. Note further, that the existence of "jump premia" in the real variables does not mean that expectations are biased or irrational; in fact by taking expected values of (23), we have

¹⁶ That changes in money supply have real effects in an open economy IS-LM model under flexible exchange rates should be no surprise.

¹⁷ We use the notion of a 'liquidity (related) effect' rather generously here to cover the whole range of effects; a reduction in real interest rate r , real depreciation of the domestic currency and expansion in output. On a more narrower interpretation, 'liquidity effect' means a reduction in the interest rate following an (unanticipated) increase, *cet. par.*, in the growth rate of money supply (see eg Christiano 1995; Nelson 1997, provides a fairly balanced discussion of the theoretical ideas underlying the liquidity effect as well as empirical evidence pertaining to it).

$$\begin{aligned}
E_{t-1}[q_t] &= -\frac{\sigma}{\delta} r^* - \frac{\chi_q}{1-\Phi} \alpha + \frac{\chi_q}{1-\Phi} \alpha = -\frac{\sigma}{\delta} r^* \\
E_{t-1}[r_t] &= r^* - \frac{\chi_q}{1-\Phi} \alpha + \frac{\chi_q}{1-\Phi} \alpha = r^* \\
E_{t-1}[y_t - y^n] &= \frac{\chi_q(\sigma + \delta)}{1-\Phi} \alpha - \frac{\chi_q(\sigma + \delta)}{1-\Phi} \alpha = 0
\end{aligned} \tag{24}$$

which implies that forecast errors $x_t - E_{t-1}[x_t]$ have zero conditional date-(t-1) expectations. Thus there is no conditional bias in expectations.

Now, to solve for the nominal exchange rate, price level and nominal interest rate variables, we use equations (13), (15), (19), (23) and (24); this gives us

$$\begin{aligned}
s_t &= -y^n + (\sigma + \lambda) r^* + m_{t-1} + \frac{\chi_s}{1-\Phi} [m_t - m_{t-1}] + \frac{1-\chi_s}{1-\Phi} \alpha \\
p_t &= -y^n + C_p r^* + m_{t-1} + \frac{\chi_p}{1-\Phi} [m_t - m_{t-1}] + \frac{1-\chi_p}{1-\Phi} \alpha \\
i_t &= i^* + [1 - \frac{\chi_s}{1-\Phi}] [m_t - m_{t-1}] + \frac{\chi_s}{1-\Phi} \alpha
\end{aligned} \tag{25}$$

There are several interesting features in the set of solutions to the nominal variables in (25). First of all, while jump uncertainties tend to impose real costs on the economy in terms of higher ex ante real interest rates, real appreciation of the domestic currency and, hence, lower aggregate output relative to potential one, their effects on the price level and nominal exchange rate appear to be ambiguous, but it turns out that only the effects on the nominal exchange rate remains ambiguous a priori. As far as the price level effects are concerned, the reasoning here is that although jump uncertainties shift both aggregate supply and demand,¹⁸ the supply shift, working mainly through the higher expected price level that is sustained by the perceived number of policy jumps¹⁹ in the future, will dominate in the present model.²⁰ Output will fall relative to the potential one, as already noted above, see eqs. 25, due to perceived future policy jumps, but not to the extent to (over)compensate for the higher expected price level.

Secondly, it can be shown that under finite speed of price level adjustment (ie $\theta < \infty$), the nominal exchange rate will depreciate due to perceived future policy jumps if and only if $\lambda > 1 - \eta$, where $\eta = (1 + \theta)(\sigma + \delta) / [1 + \theta(\sigma + \delta)]$. Hence, a sufficient condition for a spot nominal depreciation due to jump uncertainties is that that aggregate demand be sufficiently responsive to terms of trade and real interest rate movements, ie $\sigma + \delta > 1$, so that $\eta > 1$. Actually the condition $\lambda > 1 - \eta$ guarantees that expected nominal depreciation will weaken the current nominal

¹⁸Here we are using 'demand and supply schedules' in reference to the (y, p) – plane.

¹⁹"Policy jumps" is used here and elsewhere interchangeably with "money supply jumps".

²⁰Direct calculation reveals that the coefficient χ_p is less than one if and only if $\lambda + (\sigma + \delta)(1 + \lambda\theta) > 0$, which, of course, we assume holds. On the other hand, the exchange rate coefficient χ_s is less than one if and only if $[1 - (\sigma + \delta)] / [1 + \theta(\sigma + \delta)] < \lambda$, so that sufficiently high aggregate demand elasticities will sustain exchange rate depreciation due to anticipated future money supply jumps.

spot exchange rate, see eq (10) p. 10. Thus, the spot rate will weaken in response to factors that generate expectations of a weaker currency, like expected policy jumps (eq (25), wherefrom it follows immediately that the currency is expected to weaken as the expected number of jumps per unit time, α , increases). An alternative way to see that the dynamic response of the nominal exchange rate to jump uncertainties is a priori ambiguous is to utilize the relationship between the spot exchange rate, expected price level and surprises in the price level (ie forecast errors) implied by the model:

$$s_t = E_{t-1}[p_t] + \kappa (p_t - E_{t-1}[p_t])$$

$$\kappa = \frac{1 + \theta(\sigma + \delta)}{\theta(\sigma + \delta)} \quad (26)$$

Now, jump uncertainties will raise the expected price level, as argued above. However, they affect price level surprises or forecast errors negatively (a feature that imparts the impression that forecast errors are systematically biased, which, of course they are not), as can be readily verified from the solution equation for the price level in eqs (26).

The second equation in (25) gives us the reduced form solution to the domestic price level, which, of course, can be used to derive the solution to expected and actual inflation. Inflation, most notably expected inflation, appears to be the relevant variable from the point of view of the (practical) conduct of monetary policy, so it is worthwhile to derive the model's solution to it explicitly. To this end, the second equation in (25) implies that the ex post inflation from period t to period $t+1$ is given by

$$\Delta p_{t+1} = \Delta m_t + \beta[\Delta m_{t+1} - \Delta m_t] = (1 - \beta)\Delta m_t + \beta\Delta m_{t+1}, \quad \beta = \chi_p / (1 - \Phi) \quad (27)$$

ie weighted average of the growth rate of the money supply in period t and $t+1$. Expected inflation, conditional on period t information, on the other hand, satisfies

$$E_t[\Delta p_{t+1}] = \Delta m_t + \beta[\alpha - \Delta m_t] = (1 - \beta)\Delta m_t + \beta\alpha \quad (28)$$

given that forecasts of the changes in the smooth component of the money supply in period $t+1$, ω_{t+1} , cannot be, on period t information, improved upon from their unconditional mean of zero. Note that the expected number of jump increases from period t to $t+1$ enters in the forecast for period $t+1$ inflation. The difference between the realized and expected inflation, the prediction error, is therefore

$$\Delta p_{t+1} - E_t[\Delta p_{t+1}] = \beta\Delta m_{t+1} - \beta\alpha = \beta[\omega_{t+1} + (\Delta N_{t+1} - \alpha)] \quad (29)$$

The interesting possibility in (29) comes from interpreting the poisson process as a formal model for an infrequently occurring jump in the money supply; the increment of the poisson process in period $t+1$, ΔN_{t+1} , is, with high probability, zero, so that the last term in equation (29), reflecting "surprise jumps" in money

supply, tends to be negative.²¹ Hence, in typical samples encountered eg in economics, ex post prediction errors tend to have a negative bias, reflecting the fact that markets view future discrete jumps in the money supply as a distinct possibility and incorporate this eventuality in their expectations. Of course, the possibility for persistent differences realized inflation and expected one is a source of a credibility problem, and should be so interpreted, and shows up in eg persistent differences in ex post and ex ante real interest rates. We will return to this point more systematically later in section 4.2, which goes deeper into the apparent conditional bias in ex post forecast errors.

Note finally that jump uncertainties tend to generate an upward pressure on the domestic nominal interest rate i_t . The underlying intuition here comes from the fact that the perceived future policy jumps contribute to the expected rate of depreciation of the domestic currency by an amount of $\chi_s \alpha / (1 - \phi)$. Finally, note that the liquidity effect, ie unexpected²² changes in the growth rate of money, $m_t E_{t-1} m_t$, is equal to $m_t - m_{t-1} - \alpha$;²³ hence, a positive liquidity effect will drive the nominal interest rate down,²⁴ if $\sigma + \delta < 1$;²⁵ the reason is that, although money demand also increases, it does so insufficiently so that the domestic interest rate has to come down to eliminate the excess supply in the money market induced by money supply expansions.²⁶ Note also that a corresponding liquidity effect will weaken the domestic currency as well as increase the domestic price level.

3.2 Jump uncertainties and aggregate volatility

In the preceeding section we saw that (stationary Poisson) jump uncertainties give rise to what could be characterized as (constant) premia that will affect the behaviour of the aggregate economy. Most notably, the possibility of jump uncertainties in the money supply process will impose real costs on the economy in terms of higher ex ante real interest rates, real appreciation and, hence, lower output relative to the potential one. Furthermore, if an econometrician works with an unrepresentative sample – no discrete jumps in the money supply occurred

²¹ Actually, with the intensity parameter α as high as 0.5 (one would expect one jump in two periods), $P\{\Delta N = 0\} = 0.607$, while for α as low as 0.001 (one would expect one jump in 1000 periods), $P\{\Delta N = 0\} = 0.999$. These calculations follow from the fact that the increments ΔN_t are distributed as Poisson (α), if N_t is Poisson (αt). This, in turn, can be proved by noting that the moment generating function, mgf, of the distribution of the increments ΔN_t equals the ratio of the mgf of the distribution of N_t and N_{t-1} . The mgf of the distribution of N_t , $M_N(\theta)$ say, for real θ , on the other hand, is $\exp[\alpha t(v-1)]$, where $v = \exp[\theta]$.

²² On date $t - (t-1)$ information, that is.

²³ Note that $E_{t-1}[m_t - m_{t-1} - \alpha] = E_{t-1}[\omega_t] + E_{t-1}[N_t - N_{t-1}] - \alpha = 0 + \alpha - \alpha = 0$.

²⁴ Since $1 < \chi_s / (1 - \phi)$ if and only if $\sigma + \delta < 1$.

²⁵ The condition $\sigma + \delta > 1$ is equivalent to an equilibrium fall in nominal income $p + y$.

²⁶ Note that this particular condition for the sum of terms of trade and real interest rate elasticity of aggregate demand ultimately derives, via the interest rate parity, from the equivalent condition for an expected appreciation of the domestic currency.

within the time interval spanned by the sample – he or she may erroneously conclude in favour of irrationality of expectations, since forecast errors appear to have a non-zero mean in the sample. But, as already noted above, deviations from rational expectation need not occur; in particular, expectations in the above model are conditionally unbiased.

The possibility of future policy switches or jumps does not only affect the conditional mean of the multivariate distribution of the model variables, but also the variance-covariance structure of that distribution. This is the subject we will focus on this section.²⁷ From the system of solutions to the endogenous real variables, (23), and nominal variables, (25), we can see that the kernel of the conditional, date-(t-1) (co)variance structure of system of endogenous variable is

$$[\sigma_{\omega}^2 + \alpha] \quad (31)$$

Hence, we can immediately see that jump uncertainties of the Poisson type will contribute positively to overall aggregate volatility.²⁸ Moreover, (27) highlights the general structure of the (conditional) variance of the endogenous variables in the model, both real and nominal; that the kernel of the variance is simply $\sigma_{\omega}^2 + \alpha$, which principally comes from the independence of the innovations to the random walk part of the money supply process and increments of the Poisson part. Also, aggregate volatility tends to increase as the perceived number of discrete future money supply jumps or policy shifts, α , increases. The underlying intuition can perhaps best be explained by noting that the variance of a distribution of a random variable \underline{x} , $D^2 \underline{x}$, say, can be decomposed, by conditioning on another random variable \underline{y} , as follows

$$D^2 \underline{x} = E_y[D^2(\underline{x} | \underline{y})] + D_y^2[E(\underline{x} | \underline{y})] \quad (32)$$

Think of the variance on the l.h.s. of equation (28) as the conditional date-(t-1) variance of an endogenous variable. Now, condition the distributional variability of an endogenous variable, say of aggregate output, on the number of discrete money supply jumps between time t-1 and t, $N_1 = N_t - N_{t-1}$. The conditional variance of aggregate output, conditional on observing N_1 is proportional to the variance of the innovation to the random walk part of the money supply, σ_{ω}^2 , and is thus independent of N_1 . Hence, the first term on the r.h.s. of (28) is proportional to σ_{ω}^2 . The conditional expectation of aggregate output, conditional on observing N_1 , is proportional to the "surprise jumps" generated by the Poisson process,

²⁷Note that in ensuing calculations we use the facts that the Poisson process has independent increments and that the variance of the Poisson distribution is α .

²⁸Simulations by Danthine and Donaldson (1996) are also relevant here; in the context of a real business cycle model, he formalizes the peso problem by introducing a 'depression or disaster state' as one of the states the underlying productivity process can go into or come out of. Quite plausibly, the occurrence of this state has low probability, but can be given a high degree of persistence. Table 1 in Danthine and Donaldson reveals that the introduction of a 'depression state' tends to increase overall aggregate volatility, at best even double it. We will return to this point later when discussing peso problem in the context of our model. Their results also suggest (1996, Table 1, last 2 columns) that conditional on the non-occurrence of the 'depression state', overall aggregate volatility need not increase!

$N_1 - \alpha$, the variance of the distribution of which (the second term on the r.h.s. of (28)) is simply α , ie the variance of the Poisson distribution. We can interpret the contribution of the Poisson process to the variance of aggregate output or, more generally, of the aggregate economy *as if* resulting from the (additional) variability of the conditional mean, where the conditioning event takes the form of observing the increase in discrete money supply jumps over a unit interval.

4 Macroeconomic effects of regime switches; perceived future one time shift in the money supply process

As we saw in the last sections, the Poisson process provides a stationary, relatively simple and tractable model to analyze the macroeconomic effects of infrequent discrete jumps in the underlying money supply process. Clearly, this simplicity and tractability must be appreciated, but at the same time it must be admitted that time-invariant effects of perceived future jumps in the policy process maybe a (major) theoretical limitation of the Poisson model and it may not be able to explain certain robust observations from eg foreign exchange markets, like the forward discount puzzle (ie partially predictable, time varying excess returns) and conditional heteroscedasticity in forecast errors.²⁹ Furthermore, the Poisson model does not necessarily conform well with the idea underlying peso type problems; that of a possible regime switch which has not hitherto realized. So, in the present section we shall a different approach to modelling the money process; to be more specific, we shall in the present section follow Obstfeld (ibid.) and postulate that the money supply process has the following representation

²⁹The reference literature here is very large, but see eg Obstfeld (1987), Lewis (1994), Evans (1995) and Gourinchas and Tornell (1996).

$$\begin{aligned}
m_t &= m_t^B + d_t \mu_t \\
m_t^B &= m_{t-1}^B + \omega_t, \quad \omega_t \sim \text{WS}(0, \sigma^2)
\end{aligned}
\tag{33}$$

Hence, the money supply process is the sum of two components; the first is a random walk process, m^B , with a wide sense stationary (ie zero mean, constant variance) prediction error ω . The second component contributing to the money supply process is itself the product of two random variables, denoted by d_t (dummy for the occurrence of the switch) and μ_t (the size of the jump) respectively. d_t has the distribution:

$$\begin{aligned}
d_t &= 1 \text{ w. pr. } \pi \text{ and} \\
d_t &= 0 \text{ w. pr. } 1 - \pi, \quad \text{conditional on } d_{t-1} = 0 \\
d_t &= 1, \text{ w. pr. } 1, \text{ conditional on } d_{t-1} = 1
\end{aligned}
\tag{34}$$

Furthermore, conditional on $d_{t-1} = 0$, μ_t is generated by the first-order autoregressive process $\mu_t = \rho\mu_{t-1} + \varepsilon_t$, where $0 \leq \rho \leq 1$ and $\varepsilon_t \sim \text{WS}(0, v^2)$, whereas conditional on $d_t = 1$, μ_{t+j} ($j > 0$) is constant at μ_t .

The interpretation of (18) of the money supply process is that one component of it is a random walk, which in itself would lead to no expected future changes in money. It is the second component, the product $d_t\mu_t$, which causes a peso problem. Each period, there is a probability π that the money supply will jump permanently by the amount μ_t , given that no such jump has yet taken place. The expected value, in period $t-1$, of this possible jump is $\rho\mu_{t-1}$. The μ_t process is assumed to damp out monotonically in mean to capture the intuitive notion that the market expects to put progressively less weight on the significance of a possible policy change the longer the interval over which the change has not occurred.³⁰

When analysing the effects of a peso problem, we need to make further assumptions concerning agents' knowledge of the prevailing policy regime. In what follows we shall assume that agents know the currently prevailing regime so that they do not have to enter into a learning process, whereby they use observable information to make inferences about the prevailing regime and to update, via

³⁰As also noted by Obstfeld (1987 fn. 2, p. 6), one could alternatively assume a time-varying switching probability π to capture this type of effect. One could, on the other hand, argue that, due to reasons elaborated by eg Drazen and Masson (1993), a longer interval of no policy change would actually increase the size of the jump, the argument being that larger disequilibria would have to be then adjusted. In this light, then, the present formalism should perhaps be interpreted as a short-cut to modelling complex structural changes in the economy that would reduce the need for future policy adjustment via regime changes.

Bayes's rule, previous estimates of the regime probabilities.³¹ In particular we shall assume that $d_t = 0$ for all t , so that eg the available data from this model economy are conditioned on the non-occurrence of a regime shift, even though the market viewed a major policy switch as a possibility.

4.1 Solving for the money supply process

As explained above, markets took into account, in forecasting future values, that d might equal 1 in some future date, even though the event $d = 1$ turned not to occur ex post. The possibility that d might equal 1 is reflected in the sequence of expected future money supplies, $E_t[m_{t+k}]$, $k > 0$. This sequence is computed as follows. If $d_t = 0$, the event $d = 1$ will have occurred by date $t+i$ with probability $[1-(1-\pi)^k] = \pi + (1-\pi)\pi + (1-\pi)^2\pi + \dots + (1-\pi)^{k-1}\pi$.³² Hence, on date t , the expected value of m_t is the sum of m_t^B and

$$\{\pi + \pi(1-\pi)\rho + \pi[(1-\pi)\rho]^2 + \dots + \pi[(1-\pi)\rho]^{k-1}\} \rho \mu_t \quad (35)$$

(cf. the autoregression for the jump size process μ), where the latter is just the expected date $(t+k)$ value of the second component of the money supply process. Hence, conditional on $d_t = 0$ ³³

$$E_t m_{t+k} = m_t^B + \frac{1 - [(1-\pi)\rho]^k}{1 - (1-\pi)\rho} \pi \rho \mu_t \quad (36)$$

In particular, on time $t-1$, when wage contracts are signed, conditional on $d_{t-1} = 0$

$$E_{t-1} m_{t+k} = m_{t-1}^B + \frac{1 - [(1-\pi)\rho]^{k+1}}{1 - (1-\pi)\rho} \pi \rho \mu_{t-1} \quad (37)$$

Now we have the necessary material to calculate the expectational revision term that affects the aggregate real economy, $(E_t - E_{t-1})m_{t+i}$. But first, let us remind that

³¹Evans (1995) combines learning about the current regime and peso problems in an asset pricing context. Using the expectations theory of the term structure of interest rates as the workhorse of the analysis he shows, interestingly, how the perceived persistence of policy regimes, in the sense of a probability of remaining in a particular regime, is critical for future expected changes in short term interest rates to have an effect on the current yield spread. Specifically, these probabilities affect the discount factor, which can also be negative, used to discount future expected short rates changes and if the probability of remaining in a regime is as large as leaving it, ie entropy in regime switching distribution is maximized, the discount factor goes to zero.

³² Let A_t denote the event that the jump occurred at time t , and A_t^c its complement; then the above probability comes from the geometric distribution; $P\{A_{t+1}\} + P\{A_{t+1}^c\}P\{A_{t+2}\} + \dots + P\{A_{t+1}^c, A_{t+2}^c, \dots, A_{t+i-1}^c\}P\{A_{t+i}\}$.

³³Use has been made of the fact that for a geometric series, the sum of the first n terms can be written as $(1-x^{n+1})/(1-x)$, where x is less than one in absolute terms.

we shall abstract from the case, where a regime switch occurred at time t , ie the case where labour contracts were signed knowing that $d_{t-1} = 0$, after which the regime switch occurred at time t , when financial market expectations were formed. Given this assumption, we have from equations (20) and (21)

$$\begin{aligned}
[E_t - E_{t-1}] m_{t+k} &= m_t^B + \frac{1 - [(1-\pi)\rho]^k}{1 - (1-\pi)\rho} \pi \rho \mu_t - m_{t-1}^B - \frac{1 - [(1-\pi)\rho]^{k+1}}{1 - (1-\pi)\rho} \pi \rho \mu_{t-1} \\
&= \omega_t + \frac{\rho[1 - [(1-\pi)\rho]^k] - [1 - [(1-\pi)\rho]^{k+1}]}{1 - (1-\pi)\rho} \pi \rho \mu_{t-1} + \frac{1 - [(1-\pi)\rho]^k}{1 - (1-\pi)\rho} \pi \rho \varepsilon_t \\
&= \omega_t - \frac{[(1-\rho) + \pi(1-\pi)^k \rho^{k+1}]}{1 - (1-\pi)\rho} \pi \rho \mu_{t-1} + \frac{1 - [(1-\pi)\rho]^k}{1 - (1-\pi)\rho} \pi \rho \varepsilon_t
\end{aligned} \tag{38}$$

Note that on top of the error in forecasting the jump size and "smooth" money supply growth in money supply, ε and ω respectively, "revisions in expectations", from time $t-1$ to t , are (systematically) affected by lagged values of the jump size, μ_{t-1} . If $\mu_{t-1} > 0$ over some interval, for example, agents in the economy appear systematically to revise expectations about future money supply downwards. (36) is very revealing in the sense that what makes the present model different from the Poisson model in an important way is that in the rational expectations equilibrium the peso premium, ie. the second term on the r.h.s. of (36), is *time-varying*, in contrast to the Poisson model of the previous chapter. This time variability of the peso premium is the source of heteroscedasticity of the model's forecast errors and of time-varying aggregate volatility, to be considered later in this chapter.

Note also that the ex post pattern of expectational changes alluded to above does not imply conditionally biased revisions, since, on time $t-1$ information, agents took into account the likelihood of the event $d_t = 1$, which would have occasioned a sharp increase in money supply. This can be more formally seen by calculating $E_{t-1}[E_t m_{t+1}]$ and comparing it to $E_{t-1}[m_{t+1}]$ from the date $t-1$ forecasting equation (21); conditional on $d_{t-1} = 0$;

$$\begin{aligned}
E_{t-1}[E_t m_{t+1}] &= \pi E_{t-1}[m_t^B + \mu_t] + (1-\pi) E_{t-1}[m_t^B + \pi \rho \mu_t] \\
&= m_{t-1}^B + \pi \rho \mu_{t-1} + (1-\pi) \pi \rho^2 \mu_{t-1} + E_{t-1}[\omega_t + (\pi + (1-\pi)\pi \rho) \varepsilon_t] \\
&= m_{t-1}^B + [1 + (1-\pi)\rho] \pi \rho \mu_{t-1}
\end{aligned} \tag{39}$$

since ε_t and ω_t are zero mean stationary processes and conditional on $d_t = 1$, $\mu_{t+k} = \mu_t$ for all $k > 0$. On the other hand, equation (21) gives, for $k=1$

$$\begin{aligned}
E_{t-1} m_{t+1} &= m_{t-1}^B + \frac{1 - [(1-\pi)\rho]^2}{1 - (1-\pi)\rho} \pi \rho \mu_{t-1} \\
&= m_{t-1}^B + [1 + (1-\pi)\rho] \pi \rho \mu_{t-1}
\end{aligned} \tag{40}$$

Hence, $E_{t-1}\{E_t[m_{t+1}] - E_{t-1}[m_{t+1}]\} = 0$ and the expectational revisions are conditionally unbiased. What this analysis suggests, then, is that ex post these expectational revisions appear to be correlated with period $t-1$ information. This in turn seems to imply data from our model economy subject to a peso problem provides proof in favour of deviations from rational expectations, but in fact, as

can be seen from equations (23) and (24), there are none, since changes in expectations are conditionally unbiased and can thus be treated as (part of the structure of) true rational expectations errors.

Finally, note that in equation (36) the innovation term ω_t is related to, but not equal to what was earlier called the (source of the) liquidity effect of monetary policy. This is so, since we have assumed that $d_t = 0$, so that the unexpected change in the money supply in period t is $\omega_t - \pi\rho\mu_{t-1}$. On the other hand, if $d_t = 1$, the surprise increase in period t money supply equals $\omega_t + \varepsilon_t + (1-\pi)\rho\mu_{t-1}$. As was also the case with the Poisson model, these surprise money injections appear to be correlated with period $t-1$ information, and militate against the notion of a surprise increase in money supply. To reiterate, however, these money surprises average out, across regimes, to zero;

$$\begin{aligned} E_{t-1}[m_t - E_{t-1}(m_t)] &= \pi E_{t-1}[(m_t - E_{t-1}(m_t))|d_t = 1] \\ &\quad + (1-\pi)E_{t-1}[(m_t - E_{t-1}(m_t))|d_t = 0] \\ &= \pi E_{t-1}[\omega_t + \varepsilon_t] + \pi(1-\pi)\rho\mu_{t-1} \\ &\quad + (1-\pi)E_{t-1}[\omega_t] - (1-\pi)\pi\rho\mu_{t-1} = 0, \end{aligned}$$

because ω_t and ε_t are innovations relative to the information available at time $t-1$. This clearly demonstrates that any aggregate effect there is from the peso premia is not the result of irrationality of expectations of the agents in the economy.

4.1.1 Full solution of the model

As can readily be seen from the solution of the various endogenous variables, the full solution to the model requires knowledge of the present discounted value of the "news factor" or revisions in expectations $(E_t - E_{t-1})m_{t+k}$ concerning the sequence of future money supplies. The discount factor is Φ , which equals $\lambda/(1+\lambda)$, where λ is the interest semi-elasticity of the money demand. After simplifying, we have

$$\begin{aligned} &[1 - \Phi L^{-1}]^{-1} [E_t - E_{t-1}] m_t \\ &= - \left[\frac{(1 - \rho\Phi)}{(1 - \Phi)[1 - (1 - \pi)\rho\Phi]} \right] \pi\rho\mu_{t-1} + \frac{\Phi\pi\rho}{(1 - \Phi)[1 - (1 - \pi)\rho\Phi]} \varepsilon_t + (1 + \lambda) \omega_t \end{aligned} \quad (41)$$

since, under $d_t = 0$, $m_t = m_t^B$; also $E_{t-1}m_t = m_{t-1}^B + \pi\rho\mu_{t-1}$, so that $m_t - E_{t-1}m_t = \omega_t - \pi\rho\mu_{t-1}$.

Substituting equation (25) into the aggregate output equation (16), we can see that fluctuations in aggregate output relative to the potential output are given by

$$\begin{aligned} y_t - y^n &= \chi_y [1 - \Phi L^{-1}]^{-1} (E_t - E_{t-1}) m_t \\ &= -\chi_y^\mu \pi\rho\mu_{t-1} + \chi_y^\varepsilon \varepsilon_t \end{aligned} \quad (42)$$

where

$$\begin{aligned}\chi_y^\mu &= \chi_y \left[\frac{(1 - \rho\Phi)}{(1 - \Phi)[1 - (1 - \pi)\rho\Phi]} \right] \\ \varepsilon_t^y &= \chi_y \left[(1 + \lambda) \omega_t + \frac{\lambda\pi\rho}{[1 - (1 - \pi)\rho\Phi]} \varepsilon_t \right]\end{aligned}\quad (43)$$

The rational expectations prediction error, ε^y , has a very familiar, in the context of linear RE models, representations, being a linear combination of the underlying structural errors. As can be seen, it's distribution is heavily influenced by the parameter structure of the model, especially the parameters related to the peso problem. The really interesting, and novel, feature of the output solution is the effect of the "peso premium" on the behaviour of aggregate output; a peso problem, formally working through the term $\pi\rho\mu_{t-1}$ with a positive expected jump size, $\rho\mu_{t-1} > 0$, imposes a real cost on the economy in terms of exercising a contractionary effect on aggregate output relative to the potential one. This is intuitive enough and something that one really expects, once (short-run) real effects from a peso problem are allowed for. Moreover, an increase in the probability of a regime switch, π , or expected value of the jump, $\rho\mu_{t-1}$, *cet. par.*, tend to increase period t output losses due to a peso problem. Once again, this seems plausible enough and something one would expect. Finally, it should be noted the the neutrality of a peso problem vis-a-vis the real economy, as exemplified by eg Obstfeld's analysis (1987), which builds on the monetary approach to exchange rate determination, comes out nicely from (16') by letting θ , $\delta \rightarrow \infty$ and $\sigma = 0$; in this case $\chi_y^\mu = 0$.

From equations (14) and (15) together with equation (25), we obtain the solutions to the terms of trade, q , and domestic ex ante real interest rate, r ;

$$\begin{aligned}q_t &= -\left(\frac{\sigma}{\delta}\right) r^* - \chi_q [1 - \Phi L^{-1}]^{-1} (E_t - E_{t-1}) m_t \\ &= \left(\frac{\sigma}{\delta}\right) r^* + \chi_q^\mu \pi \rho \mu_{t-1} - \varepsilon_t^q \\ r_t &= r^* + \chi_q^\mu \pi \rho \mu_{t-1} - \varepsilon_t^q\end{aligned}\quad (44)$$

where

$$\begin{aligned}\chi_q^\mu &= \chi_q \left[\frac{(1 - \rho\Phi)}{(1 - \Phi)[1 - (1 - \pi)\rho\Phi]} \right] \\ \varepsilon_t^q &= \chi_q \left[(1 + \lambda) \omega_t + \frac{\lambda\pi\rho}{[1 - (1 - \pi)\rho\Phi]} \varepsilon_t \right]\end{aligned}\quad (45)$$

Once again, these results are what one would expect them to be; a peso problem causes a real appreciation of the domestic currency, leading to competitiveness problems, and an increase in ex ante real interest rates. The loss in competitiveness and increase in the real interest rate underly the output losses alluded to above.

As for the nominal variables, we use equations (11) and (13) for the nominal exchange rate and price level, respectively. After simplifications, we obtain

$$\begin{aligned} s_t &= C_s + m_{t-1}^B + [\phi - \chi_s^\mu] \pi \rho \mu_{t-1} + \varepsilon_t^s \\ p_t &= C_p + m_{t-1}^B + [\phi - \chi_p^\mu] \pi \rho \mu_{t-1} + \varepsilon_t^p \end{aligned} \quad (46)$$

where

$$\begin{aligned} \chi_s^\mu &= \chi_s \frac{(1 - \rho \Phi)}{(1 - \Phi)[1 - (1 - \pi)\rho \Phi]} \\ \chi_p^\mu &= \chi_p \frac{(1 - \rho \Phi)}{(1 - \Phi)[1 - (1 - \pi)\rho \Phi]} \\ \varepsilon_t^s &= \chi_s \left[(1 + \lambda) \omega_t + \frac{\lambda \pi \rho}{[1 - (1 - \pi)\rho \Phi]} \varepsilon_t \right], \quad \varepsilon_t^p = \left(\frac{\chi_p}{\chi_s} \right) \varepsilon_t^s \\ C_s &= -y^n - p^* + C_s r^* \\ C_p &= -y^n + C_p r^* \\ \phi &= \frac{1}{1 - (1 - \pi)\rho \Phi} \end{aligned} \quad (47)$$

The standard result from the literature (see eg Obstfeld 1987, p. 7, eq (5)) appears to be that a peso problem ($\mu_{t-1} > 0$) tends to make the domestic currency weaker relative to the situation, where no peso premia exist. Furthermore, by the PPP peso premia also tend to increase domestic price level. From the two equations in (30), we can immediately see the full force of the assumption of instantaneous price level adjustment and, hence, of exogenous aggregate output in these standard models; the presence of a peso problem no longer necessarily implies that the domestic currency depreciates in response to an anticipated future jump in the money supply process. The intuition here for this result was already given in the context of the Poisson model; the relevant parameter constraint here for the peso premium to weaken the currency is $\lambda \rho > 1 - \eta$, which slightly more demanding, for positive first order correlation in the in the jump size ($\rho > 0$), than in the Poisson model. Also, the peso premium will increase the price level, for reasons given for the Poisson model.

Previously, in the context of the Poisson model (p. 22) we demonstrated that the presence of jump uncertainties (of constant intensity) drives a persistent wedge between expected and realized inflation, which, in turn, underlies the differences between ex ante and ex post real interest rates. Similarly, we can show in the present context that these persistent differences between expected and realized inflation do occur, but this time time-varying effects are present in the relevant inflation prediction errors: conditional on no jumps in the money supply at the time expectations are formed, we have

$$\begin{aligned} \Delta p_{t+1} - E_t[\Delta p_{t+1}] &= \chi_p [1 - \Phi L^{-1}]^{-1} (E_{t+1} - E_t) m_{t+1} \\ &= -\chi_p \Theta \pi \rho \mu_t + \xi_{t+1} \end{aligned} \quad (48)$$

where $\Theta = \frac{1 - \rho\Phi}{(1 - \Phi)[1 - (1 - \pi)\rho\Phi]}$ and ξ is a linear combination of the underlying errors in the money supply process. Hence, there is a persistent (time-varying) tendency for expected inflation to exceed actual one. This, in turn, implies among other things that there is persistent tendency for the ex post real interest rate to exceed the ex ante real interest rate; higher ex post real interest rates are a symptom of the presence of an underlying peso problem. This appears to suggest that caution should be exercised when interpreting inflation forecasts from models that do not explicitly account for the presence of an underlying peso problem.

According to the uncovered interest rate parity, the domestic interest rate is equal to the sum of the foreign interest rate and expected depreciation of the domestic currency; hence

$$\begin{aligned}
i_t &= i_t^* + E_t[\Delta s_{t+1}] \\
&= i^* + \frac{1}{1 + \lambda} [1 - \Phi L^{-1}]^{-1} (E_t m_{t+1} - E_{t-1} m_t) - \chi_s [1 - \Phi L^{-1}]^{-1} (E_t - E_{t-1}) m_t \\
&= i^* - [(1 - \rho)\phi - \chi_s^\mu] \pi \rho \mu_{t-1} + \left[\omega_t + \frac{\pi \rho}{1 - (1 - \pi)\rho\Phi} \varepsilon_t - \varepsilon_t^s \right] \\
&= i^* - \psi_i \pi \rho \mu_{t-1} + \varepsilon_t^i
\end{aligned} \tag{49}$$

where, it can be shown that, $\psi_i < 0$ for $\rho > 0$. Hence, as before in the Poisson model, the peso premium puts an upward pressure on domestic nominal interest rate. This result implies, given the UIP, that the expected rate of depreciation of the domestic currency, $E_t[\Delta s_{t+1}]$, is positive due to the presence of a peso problem. Note also, in particular, that the interest rate differential $i_t - i^*$ has a time-varying factor as a result of the peso premium.

4.2 Prediction errors; apparent conditional bias in forecasts

Having derived the full solution of the model, we shall now take a closer look at some of the implications of a peso problem. The first one, apparent conditional bias in forecasts, stems from the observation that one of the drivers for real variables in the system is determined by expectational revisions, which depend on the expected jump size, $\rho \mu_{t-1}$, whereas for nominal variables these revisions come on top of a conditional expectations term, which also depend on the jump size. Hence, date- t forecast errors for the model's variables depend, via expectational revisions, on the forecasted jump size $\rho \mu_{t-1}$. Furthermore, the way the model's solution is parameterized above suggests that there is an apparent symmetry in the forecast errors in that for $d_t = 0$ the forecast error of an endogenous variable x_t in the system is either

$$x_t - E_{t-1}[x_t] = -\chi_x^\mu \pi \rho \mu_{t-1} + \varepsilon_t^x \tag{50}$$

or the sign of the coefficient of the term $\pi\rho\mu_{t-1}$ is χ_x^μ , while for $d_t = 1$ the forecast error is either

$$x_t - E_{t-1}[x_t] = -\chi_x^\mu (1-\pi)\rho\mu_{t-1} + \chi_x (1+\lambda)[\omega_t + \varepsilon_t] \quad (51)$$

or the coefficient of the term $(1-\pi)\rho\mu_{t-1}$ is χ_x^μ . See appendix B for details. Hence, the unbiasedness of the forecast errors follows immediately from this apparent symmetry, since the conditional expectation of the forecast error, conditional on date- $(t-1)$ information is obtained by weighting (43) and (44) with $(1-\pi)$ and π respectively. To take a couple of concrete examples, the forecast errors for the domestic output is

$$\begin{aligned} y_t - E_{t-1}[y_t] &= -\chi_y^\mu \pi \rho \mu_{t-1} + \varepsilon_t^y, \text{ for } d_t = 0 \\ y_t - E_{t-1}[y_t] &= \chi_y^\mu (1-\pi)\rho\mu_{t-1} + \chi_y (1+\lambda)[\omega_t + \varepsilon_t], \text{ for } d_t = 1 \end{aligned} \quad (52)$$

In particular, this pattern of *ex post* forecast errors indeed appears to suggest that agents (in the labour market) systematically overstate the future development of the aggregate output, given when $d_t = 0$. But, we can immediately verify from (45) that the conditional expectation of the forecast error of domestic output is

$$\begin{aligned} E_{t-1}[y_t - E_{t-1}[y_t]] &= -(1-\pi)\chi_y^\mu \pi \rho \mu_{t-1} + \pi \chi_y^\mu (1-\pi)\rho\mu_{t-1} \\ &\quad + E_{t-1}[(1-\pi)\varepsilon_t^y + \pi \chi_y (1+\lambda)(\omega_t + \varepsilon_t)] \\ &= E_{t-1}[(1-\pi)\varepsilon_t^y + \pi \chi_y (1+\lambda)(\omega_t + \varepsilon_t)] = 0 \end{aligned} \quad (53)$$

since $E_{t-1}[\omega_t] = 0 = E_{t-1}[\varepsilon_t]$. This is well in line what was explained earlier about the conditional unbiasedness of expectational revisions. So that it is wrong to infer from the pattern of *ex post* forecast errors in (45) that conditional forecasts are biased generating deviations from rationality. The point is that markets take into account the likelihood of the event $d_t = 1$, when forming expectations at time $t-1$, which, of course, would have occasioned a sharp increase in the money supply.

As a second example, we can see that similar reasoning applies to the conditional forecast error of the nominal exchange rate;

$$\begin{aligned} s_t - E_{t-1}[s_t] &= -\chi_s^\mu \pi \rho \mu_{t-1} + \varepsilon_t^s, \text{ for } d_t = 0 \\ s_t - E_{t-1}[s_t] &= \chi_s^\mu (1-\pi)\rho\mu_{t-1} + \chi_s (1+\lambda)[\omega_t + \varepsilon_t], \text{ for } d_t = 1 \end{aligned} \quad (54)$$

wherefrom conditional unbiasedness immediately follows. Once again this pattern of *ex post* forecast errors appears to suggest, in particular, that for $d_t = 0$ (forex) markets systematically overestimate (given that $\mu_{t-1} > 0$) the domestic currency's future depreciation. That this conclusion is incorrect follows directly from conditional unbiasedness and is also emphasized by Obstfeld (1987, p. 7–8).

4.3 Macroeconomic volatility reconsidered

Earlier, in the case of Poisson jumps in the money supply process, we saw that the distributional volatility of an endogenous variable in the model, a measure of volatility, was proportional to the sum of the innovation variance of the random walk component and the variance of the Poisson component of the money supply. Hence, what the Poisson model implies is that, while macroeconomic volatility increases due to jump uncertainties in the policy process, the increase in volatility is not time-varying. This is, of course, well rooted in the structure of the Poisson process³⁴ in that it has independent increments, which, in turn, induces (second order) stationarity into the changes in the money supply process. From the modelling point of view, the resulting simplicity of the model structure awarded by the Poisson process is of course a real virtue; we have seen that the resulting framework for analyzing the macroeconomic effects of jump uncertainties is surprisingly tractable. However, peso type problems can result in time-dependence in the conditional variance of a macroeconomic variable, particularly in the model of the previous section,³⁵ the dependence of the conditional variance on lagged information reflects the dependence of conditional forecast errors on lagged information.

So, if the discrete component of the money supply is of the regime switching type introduced in the previous section, the kernel of the covariance structure of the system of endogenous variables in our model can, by combining the previous results with those of the case $d_t = 1$, be shown to be

$$\chi_x^\mu \chi_z^\mu (1-\pi) \pi \rho^2 \mu_{t-1}^2 + \chi_x \chi_z (1+\lambda)^2 \pi \sigma_{\omega+\varepsilon}^2 + (1-\pi) \sigma_{\varepsilon^x \varepsilon^z} \quad (55)$$

for the pair (q, r) or a pair of the variables $x, z = y, s, p$ and i , where $\sigma_{\omega+\varepsilon}^2$ and $\sigma_{\varepsilon^x \varepsilon^z}$ denote, respectively, the variance of the distribution of the sum of ω and ε and the covariance between ε^x and ε^z . The coefficients χ_x^μ are reported in equations (37), (39), (41) and (B4) (appendix B). For a pair of variables $x = q$ and r and any of $z = y, s, p$ or i , the kernel of the covariance structure is

$$-[\chi_x^\mu \chi_z^\mu (1-\pi) \pi \rho^2 \mu_{t-1}^2 + \chi_x \chi_z (1+\lambda)^2 \pi \sigma_{\omega+\varepsilon}^2 + (1-\pi) \sigma_{\varepsilon^x \varepsilon^z}] \quad (56)$$

It is evident from (48) and (49) that the stochastic properties of the switching process $d_t \mu_t$ greatly affect the macroeconomic volatility implied by our model; in particular, in contrast to the previous case of Poisson jumps, there is now a potential for time-varying aggregate volatility, the importance of which depends,

³⁴Coupled with the innovation process in the random walk component as well as the independence of the components of the money supply.

³⁵Obstfeld (1987, p. 9–10) notes that peso problems can result in time-varying conditional variance of exchange rates' forecast errors. He also quotes evidence of such conditional heteroscedasticity in forward-rate forecast errors (Cumby and Obstfeld 1984, Hodrick and Srivastava 1984, Giovannini and Jorion 1987). Note that not only does conditional heteroscedasticity pose problems for statistical inference, but, in a world of risk averse investors, it has a crucial implication about the nature of foreign exchange risk premia, as also noted and further detailed by Obstfeld (1987, p. 9 and p. 15–).

on top of the stochastic structure of the money supply process, on the whole macroeconomic structure under consideration, as embedded in the coefficients χ_x^μ . Note that not only does the stochastic structure of the regime switching process affect the time varying part of the covariance structure, but it also affects the transmission of the shocks, ω_t and ε_t .

Danthine (1997) demonstrates that among the general equilibrium effects of the introduction of a 'depression state' is an increase in aggregate volatility, but that conditional on the non-occurrence of the 'disaster state' (the presence of a peso problem) aggregate volatility need not increase.³⁶ His simulations³⁷ also suggest that the increase in the probability of the productivity process entering the 'disaster state' results in a reduction in aggregate volatility.

According to our results aggregate variability, ie the variance of an endogenous variable, obtained by letting $x = z$ in (48), need not decrease as the probability of a regime switch, π , increases. It may, of course, decrease, but this depends critically on the level of the transition probability π . First of all, it can be shown by straightforward differentiation³⁸ that the time-varying part of aggregate volatility increases as long the transition probability π remains in the range $[0, 0.5)$. For transition probabilities higher than 0.5, the time-varying component of aggregate volatility decreases as (perceived) future regime switch becomes more likely. Finally, at $\pi=0.5$, marginal increases in the likelihood of a future regime switch has no effect on the time-variability of aggregate volatility. These results are not counter-intuitive, once it is fully taken into account that it is the inherent degree of randomness, or entropy, if one likes, in the distribution of the regime switching process, as implied by the transition probabilities, that really underlies these results;³⁹ the degree of randomness in the conditional distribution of d_t (conditional on $d_{t-1} = 0$), or the variance of that distribution, reaches its maximum at $\pi = 0.5$.

To be more concrete about the effects of varying the (conditional) probability of a regime switch on the *transmission* of the money supply shocks, assume, for simplicity, that ε_t and ω_t are uncorrelated at all lags and leads. Then it follows from (48) that the probability of a (future) regime switch, π , does not affect the contribution to the aggregate volatility of the innovation variance of the random walk component of the money supply, σ_ω^2 . Hence, the contribution of the variability of ω_t to aggregate volatility factors out completely from the contribution generated by the innovation to the jump size, ε_t . Furthermore, an increase in the probability of a regime switch will generally have ambiguous effects on the contribution of the innovation of the jump size to aggregate

³⁶See also fn. 16.

³⁷Danthine and Donaldson (1996, Table 2).

³⁸The coefficients χ_x^μ also depend on π through the term $1-(1-\pi)\rho\Phi$.

³⁹Entropy, used as a measure of information value of an event by information theorists, can be regarded as a second moment of a distribution of categorical data (ie non-numerical data). Literally, it measures the degree of randomness in the distribution, and reaches its maximum value in a (discrete) rectangular distribution, ie in a distribution where all the categories used to classify the outcomes of a random experiment have equal probabilities.

volatility. But for a sufficiently small probability of a regime switch to start with, a marginal increase in it will increase aggregate volatility due to the innovations in the jump size.

This last sentence points to an important possibility; a peso problem typically corresponds to the case of a small, but positive, probability, π , of a regime switch. The idea is that a discrete regime switch is regarded as a "rare" event and, although the discrete future jump in the money supply could be large, if it materializes, it occurs, in the eyes of the markets, with (possibly very) low probability. What this line reasoning suggests, then, is that in the context of a peso problem, marginal increase in the (perceived) probability of a regime switch will contribute to increasing aggregate volatility; this increase will be the result of an increasing contribution from the time-varying component of aggregate volatility, which comes from the time-varying estimate of the jump size of the money supply, as well as from an increased contribution from the innovations to the two components of the money supply process. Finally, from (48) and (49) we can deduce that under these "peso conditions", the covariation between the macroeconomic variables will, in absolute terms, also strengthen.

5 Concluding remarks

The starting point of the analysis in this study is what is taken as the central feature of peso type problems as documented and formalized in the existing literature on the subject matter, namely that peso problems arise mainly because expectations become anchored in perceived future regime shifts and that once the economy suffers from a peso type problem, the effects can be persistent and felt over an extended period of time.⁴⁰ Part of the underlying motivation of the present study is that this persistence most likely contributes to the real consequences of peso type problems, or that these persistent effects must increasingly become a burden also from the point of view of the real sector of the economy, otherwise it is somewhat difficult to understand why policy makers should be so concerned with peso type problems. Being able to derive a full set of macroeconomic effects of a peso type problem has, of course, also considerable theoretical interest, which together with the policy concern, drives the motivation of this study. The analysis of this paper tries to contribute to the existing literature by utilizing a standard discrete time open economy macroeconomic model of the Dornbusch type to derive an initial set of results concerning the potential macroeconomic effects of peso type problems.

The peso problem is formalized in two alternative ways; the money supply process, controlled by the monetary authority, but exogenously given under the assumed flexible exchange rate regime (, since the optimum problem underlying the monetary authority's money supply policy is not explicitly modelled), is

⁴⁰Note that since our analysis assumes the current regime is assumed to be known to the agents in the economy, the notion of 'persistence' does not correspond to Evans's (1995, p. 27-) 'serial time dependence of regimes', which results from agents' probability beliefs concerning the probability of remaining in a particular regime, ie Evans's analysis covers the case where the current regime is not observed by the agents. Here 'persistence' is used to refer to the difficulty of falsifying expectations driven by a peso problem using standard statistical inference procedures.

decomposed into two components, a conventional random walk without a drift component and a jump component, whose dynamic behaviour is determined either by the Poisson process (or counter) or by the particular regime switching process of the type analyzed by Obstfeld (1987). The latter builds on the idea that the money supply process potentially undergoes a discrete jump, a regime switch, in the future with a zero probability for the process to revert back to the pure random walk process. The major difference between the two money supply specifications is, apart from the fact that the Poisson process is intended to be a formal counterpart of *infrequent jumps* in the money supply process, not once and for all shift in the money supply process, that the contribution of a peso problem, the peso premium as it is called, on the distribution of a set of macroeconomic variables is time invariant in the Poisson case, in contrast to the Obstfeld's jump specification. Otherwise, both specifications of the money supply have qualitatively similar macroeconomic effects, particularly on the *level* of macroeconomic variables.

The rational expectations solution of the model under flexible exchange rates can be computed in a fairly straightforward manner and certainly has the familiar structure in the sense that the dynamic behaviour of the set of macroeconomic variables is determined by the flow of new information which is intimately related to revisions in expectations. In the particular context of this study these revisions in expectations decompose, under both money supply specifications, into what is called (the source of) the liquidity effect and peso premium, ie. the effect of a perceived future money supply jump. The liquidity effect, ie unexpected change in (the growth rate of) the money supply, works in the usual way; real interest rate falls, domestic currency weakens in real and nominal terms, output expands and domestic price level increases. Domestic nominal interest rate, on the other hand, fall due to the liquidity effect, if the increase in real money demand falls short of the increase in real money stock. This happens, when the sum of terms of trade and real interest rate elasticity of aggregate demand fall short of one.

On the other hand, it is argued that the macroeconomic effects of a peso premium (a perceived positive jump in the future money supply) are actually what one expects adversely affected or "misaligned" expectations would have; in both specifications output falls relative to the potential one, the exchange rate appreciates in real terms, ex ante real interest rate increases as does the domestic price level. Obstfeld shows in his analysis that the nominal exchange rate tends to weaken due to the presence of a peso premium, but the present analysis suggest that the effect on the nominal exchange rate and domestic interest rate depends on which of two effects, the fall in the money supply due to the price level increase or the fall in output, dominates the money market effects, ie whether nominal income fall or increases due to peso premia. If the output effect dominates, the demand for (real) money falls more than the supply, so that the domestic interest rate has to come down to equilibrate the money market. This fall in domestic nominal interest rate is, under uncovered interest parity, sustained by expected appreciation of the domestic currency, ie depreciation of the nominal exchange rate relative to the expected one next period. On the other hand, if the money demand falls less than supply (which happens when the sum of the elasticity of aggregate demand w.r.t. the terms of trade and real interest rate is less than one), the domestic nominal interest rate has to increase to equilibrate the domestic money market, ie the nominal exchange rate has to appreciate relative to the level expected for the next period.

The difference between the two alternative specifications of the money supply process also comes out clearly once we consider the effects of the peso premia on macroeconomic or aggregate volatility, ie (mainly) on the variance of the distribution of the endogenous variables. The Poisson jump process increases aggregate volatility relative to the case where no jumps in the money supply process exist, and the contribution is time invariant. The intuition here comes from the stationary structure of the Poisson process itself, and the assumed independence of the two components of the money supply process. Furthermore, as the expected number of jumps per unit time increases, aggregate volatility also increases. In the alternative specification of a perceived future regime shift, the situation is more complex. First of all, the probabilistic structure of regime switching process affects aggregate volatility directly and indirectly, by affecting the transmission of exogenous shocks into the economy. The direct effect is time varying, since the conditionally expected or forecasted jump size is time varying, being generated by an autoregressive process. Hence, the model has the potentially useful property of time varying volatility due to presence of a peso problem. Secondly, the interaction of the peso problem with exogenous sources of uncertainty may result in the former magnifying the effects of the latter on aggregate volatility, ie given a sufficiently small probability of a regime switch to start with, a marginal increase in this probability will tend to increase aggregate volatility due to exogenous shocks (to the random walk component of money supply and to the jump size). Similar qualification applies to the comparative statics effect of the time varying part of aggregate volatility. Whether this reflects underlying generalities of peso type problems or regime switching, remains to be seen. In any case, however, it is argued that the probabilistic structure of the regime switching process in the present study at least partly explains these comparative statics results.

These results are certainly very interesting and go a long way in explaining the policy concern outlined earlier. On balance, then, a peso type problem or "misaligned" and non-falsified expectations appear to impose a real cost on the economy.⁴¹ However, certain features of the analysis, most notably of the framework of analysis, needs to be reconsidered and developed to provide firmer theoretical foundations for the aggregate effects of peso type problems. The standard open economy IS-LM-AS model has well known limitations, the most important of which appears to be its weak microeconomic foundations, particularly those of aggregate demand and/or supply functions. To this end we could follow McCallum and Nelson (1996) and respecify aggregate demand to include a term reflecting expectations concerning future aggregate output to arrive at what could be called an 'expectations augmented IS-curve'. To develop an approach to the effects of peso type problems with more explicit and, hopefully, workable general equilibrium features, this could be combined with aggregate supply behaviour derived from Calvo (1983) contracts, or from Rotemberg's (1982) model, which is observationally equivalent to Calvo's model.⁴² These models of supply behaviour appear to display different (expectations augmented)

⁴¹Of course, the analysis in the main text is symmetric in the sense that the results are exactly reversed if the money supply is expected to contract in a jump like fashion in the future.

⁴²Nelson (1997) argues in favour of Fuhrer-Moore (1995) price specification, which generalizes Taylor's (1980) staggered wage setting model.

"Phillips curve trade-offs" between inflation and output compared to our model of an open economy, possibly also a richer dynamic structure.⁴³ Alternatively, we could try to formalize the supply behaviour starting from the increasingly popular model of imperfect competition presented in eg Obstfeld and Rogoff (1996, ch. 10). Since modern macroeconomic analysis increasingly relies on models along these lines, and certainly these models have proven very useful in a wide range of macroeconomic issues, further research on the macroeconomic effects of peso type problems using these newly developed general equilibrium models is certainly worth taking.

A second comment on the analysis of the paper concerns the specification of monetary policy. To reiterate, monetary policy is introduced into the analysis via the *exogenous money supply process*. Neither decisions over monetary policy actions nor those over instruments are thus modelled as part of the determination of the macroeconomic equilibrium. The first thing to comment about is that central banks less frequently nowadays use monetary aggregates as their key control instrument. Rather the interest rate, or a set of central bank interest rates, appears the key monetary policy instrument favoured by many central banks of today. Furthermore, it may not, in the end of the day, be the best of all approaches to leave decisions over monetary policy actions exogenous or unmodelled. Monetary policy does respond to the state of the economy and in many cases the level of the (central bank) interest rate(s) or an interest rate rule is best regarded as having been dictated by a well defined constrained optimum problem facing the central bank. Hence, the model analyzed in this study could possibly usefully be modified to reflect these changes in the operating procedures of central banks. Incorporation of an explicit optimum problem facing a central bank could provide valuable information about the possible sources of the peso problem, ie information on those features of monetary policy and the institutional environment that potentially sustain private sector's beliefs about a possible future regime shift. All these extensions and modifications of the theoretical framework entertained in the present study seem well founded and worth pursuing in the future research work.

⁴³See eg Nelson (1997, p. 15-) and also the linearization in Woodford (1996, p. 14).

Appendix A

Solution to the constant and the lead polynomials on expected future foreign real interest rates in equation (10);

$$\begin{aligned}
 B_1(L^{-1}) &= \left(\frac{1}{1+\lambda} - \chi_s\right) \left[\frac{1-(\sigma+\delta)}{1+\theta(\sigma+\delta)} \Psi(1-\Psi L^{-1})^{-1} (1-\Phi L^{-1})^{-1} \right. \\
 &\quad \left. + \left(\lambda + \frac{\sigma(1+\theta)}{1+\theta(\sigma+\delta)}\right) (1-\Phi L^{-1})^{-1} \right] + \chi_s \left[\frac{1-(\sigma+\delta)}{1+\theta(\sigma+\delta)} \Psi(1-\Psi L^{-1})^{-1} \right] \\
 B_2(L^{-1}) &= \left(\frac{1}{1+\lambda} - \chi_s\right) \left[\frac{(1+\theta)\sigma\Psi}{1+\theta(\sigma+\delta)} (1-\Phi L^{-1})^{-1} (1-\Psi L^{-1})^{-1} \right] \\
 B_3(L^{-1}) &= \left(\frac{\lambda}{1+\lambda}\right) \chi_s \left[\frac{1-(\sigma+\delta)}{1+\theta(\sigma+\delta)} \Psi(1-\Psi L^{-1})^{-1} (1-\Phi L^{-1})^{-1} \right. \\
 &\quad \left. + \left(\lambda + \frac{\sigma(1+\theta)}{1+\theta(\sigma+\delta)}\right) (1-\Phi L^{-1})^{-1} \right] + \chi_s \left[\frac{(1+\theta)\sigma\Psi}{1+\theta(\sigma+\delta)} (1-\Psi L^{-1})^{-1} \right] \\
 B_4(L^{-1}) &= \left(\frac{\lambda}{1+\lambda}\right) \frac{(1+\theta)\sigma\Psi}{1+\theta(\sigma+\delta)} \chi_s (1-\Phi L^{-1})^{-1} (1-\Psi L^{-1})^{-1}
 \end{aligned} \tag{A1}$$

$$\chi_s = \left[\lambda + \frac{(1+\theta)(\sigma+\delta)}{1+\theta(\sigma+\delta)} \right]^{-1} \tag{A2}$$

Appendix B

Conditional date-(t-1) forecast errors for the domestic output and nominal exchange rate were given in the main text. This appendix records the corresponding forecast errors for the terms of trade, real interest rate, price level and nominal interest rate. From the equations for the domestic price level, terms of trade and domestic real interest rate, (13), (14) and (15) and, we can immediately see that the relevant forecast errors are

$$\begin{aligned}
 q_t - E_{t-1}[q_t] &= \chi_q^\mu \pi \rho \mu_{t-1} - \varepsilon_t^q, \text{ for } d_t = 0 \\
 q_t - E_{t-1}[q_t] &= -\chi_q^\mu (1 - \pi) \rho \mu_{t-1} - \chi_q (1 + \lambda) [\omega_t + \varepsilon_t], \text{ for } d_t = 1 \\
 r_t - E_{t-1}[r_t] &= \chi_q^\mu \pi \rho \mu_{t-1} - \varepsilon_t^q, \text{ for } d_t = 0 \\
 r_t - E_{t-1}[r_t] &= -\chi_q^\mu (1 - \pi) \rho \mu_{t-1} - \chi_q (1 + \lambda) [\omega_t + \varepsilon_t], \text{ for } d_t = 1 \\
 p_t - E_{t-1}[p_t] &= -\chi_p^\mu \pi \rho \mu_{t-1} + \varepsilon_t^p, \text{ for } d_t = 0 \\
 p_t - E_{t-1}[p_t] &= \chi_p^\mu (1 - \pi) \rho \mu_{t-1} + \chi_p (1 + \lambda) [\omega_t + \varepsilon_t], \text{ for } d_t = 1
 \end{aligned} \tag{B1}$$

Note that, conditional on $d_t = 0$, ie conditional on the non-occurrence of the regime switch, there is a tendency for the agents to underestimate the level of the domestic currency in real terms, underestimate the level of the domestic real interest rate and overestimate the domestic price level in period t.

From the second equality of equation (42), we can see that the conditional date-(t-1) expectation of the period t nominal interest rate is (given that $d_{t-1} = 0$)

$$\begin{aligned}
 i^* + \frac{1}{1 + \lambda} [1 - \Phi L^{-1}]^{-1} (E_{t-1} m_{t+1} - E_{t-1} m_t) \\
 = i^* + \left(\frac{1}{1 + \lambda} \right) \frac{\rho \pi (1 - \pi)}{1 - (1 - \pi) \rho \Phi} \rho \mu_{t-1}
 \end{aligned} \tag{B2}$$

Hence, the conditional forecast error, $i_t - E_{t-1}[i_t]$, is given by

$$\begin{aligned}
 -\chi_i^\mu \pi \rho \mu_{t-1} + \varepsilon_t^i \\
 \chi_i^\mu = \left[\frac{(1 - \pi) \rho \phi}{1 + \lambda} + \psi_i \right]
 \end{aligned} \tag{B3}$$

when $d_t = 0$, and by

$$\begin{aligned}
 \chi_i^\mu (1 - \pi) \rho \mu_{t-1} + [1 - \chi_s (1 + \lambda)] (\omega_t + \varepsilon_t) \\
 = \chi_i^\mu (1 - \pi) \rho \mu_{t-1} + \chi_i (\omega_t + \varepsilon_t)
 \end{aligned} \tag{B4}$$

when $d_t = 1$. Conditional unbiasedness once again follows immediately from the system of forecast errors (B1)–(B4).

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