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## Welfare Analysis in a Schumpeterian Growth Model with Capital

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## **ABSTRACT**

### **Welfare Analysis in a Schumpeterian Growth Model with Capital**

In this note we compare the laissez-faire steady-state solution in the Howitt and Aghion (1998) model to the social optimum. The analysis offers several new insights in comparison to the welfare analysis in Aghion and Howitt (1992). We find various new distortions between private and optimal solution. First, a monopoly distortion effect generates too little capital accumulation in the private solution because households' gross return per unit of capital will be lower than in the social optimum due to monopoly power. Second, a cost-benefit gap effect leads to excessive research in the private solution because the planner is interested in the average technology whereas the private researcher is interested in the leading edge technology. Third, we decompose the well-known intertemporal spillover effect into three subeffects and clarify why the planner uses the interest rate as discount rate.

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# 1 Introduction

In this note we do the welfare analysis in the Howitt and Aghion (1998) model. We find various distortions in the laissez-faire steady state solution compared to the social optimum. The analysis offers several new insights in comparison to the welfare analysis in Aghion and Howitt (1992).

## 2 The Howitt-Aghion (1998) Model

Recall the basic structure of the Howitt and Aghion (1998) model. There is a final good sector, a continuum of intermediate good sectors, a research sector for each intermediate good and a representative household. We briefly describe these building blocks in turn.

### 2.1 Final Good Sector

Firms in the final good sector produce their good using the fixed labor supply  $L$  of the economy and the intermediate goods  $x_{it}$ ,  $i \in [0, 1]$  according to the production function

$$Y_t = \int_0^1 A_{it} F(x_{it}, L) di, \quad (1)$$

where  $x_{it}$  is the factor input of intermediate input  $i$  in period  $t$  and  $A_{it}$  is the corresponding productivity parameter in sector  $i$  in period  $t$ .  $F$  is a smooth CRS production function. The final good is used for consumption  $C$  and savings  $\dot{B}$ , where  $B$  denotes household wealth and can be interpreted as bond holdings. The savings are allocated to capital accumulation or used as research input,  $\dot{B} = I + N$ . It is assumed that the final good sector is competitive.

### 2.2 Intermediate Good Sectors

Intermediate goods are produced using capital only. The following simple production function reflects the fact that the production of more advanced goods is more capital intensive:

$$x_{it} = \frac{K_{it}}{A_{it}} \quad \forall i. \quad (2)$$

This implies that the production of  $x_{it}$  units of the intermediate good requires  $A_{it}x_{it}$  units of capital. It is assumed that each intermediate good sector is monopolized. This assumption is related to the description of the research sectors.

### 2.3 Research Sectors

There is a different research sector for each intermediate good. Firms in the research sector attempt to innovate, i.e. discover the next generation of the intermediate good. Innovation is stochastic. If in sector  $i$   $N_{it}$  is spent as research inputs, innovation occurs with Poisson probability.

$$\phi_{it} = \lambda n_{it} \quad \text{where} \quad n_{it} = \frac{N_{it}}{A_t^{max}} \quad \forall i, \quad (3)$$

where  $\lambda$  is the productivity of research,  $A_t^{max} \equiv \max \{ A_{it} \mid i \in [0, 1] \}$  is the “leading-edge” productivity parameter and  $n_{it}$  is the productivity adjusted research intensity.

Innovation has two effects, one is sector specific, the other is aggregate. The sector specific effect improves the existing intermediate good  $i$  (vertical innovation).

The improved intermediate good enters the production of the final good with higher productivity:  $A_{it}$  jumps to  $A_t^{max}$ . The succesful innovator is granted a patent which gives him the exclusive right to produce the intermediate good. This monopoly position is held until a new innovation in sector  $i$  occurs and the incumbent is replaced. This effect of innovation describes innovation as a private good. The patent system ensures the exclusive use of the innovation on the intermediate good market.

The second, aggregate effect of innovation is a technology spillover effect. Each innovator can build on the stock of technology as embodied in the leading-edge technology  $A_t^{max}$ , independent of the technology in its corresponding intermediate good sector,  $A_{it}$ . Innovation is thus described as a public good - it is nonrival and not exclusive for the use of research. Since there is a continuum of intermediate goods,  $A_t^{max}$  will grow continuously. Its growth rate is assumed to be proportional to the aggregate rate of innovation  $\lambda n_t$ , where  $n_t$  is the aggregate research input and  $\sigma$  is the factor of proportionality. Thus, the growth rate of  $A_t^{max}$  will be

$$g_t \equiv \frac{\dot{A}_t^{max}}{A_t^{max}} = \sigma \lambda n_t. \quad (4)$$

### 2.4 Households

There is a representative infinitely lived Ramsey household who has a utility function with constant elasticity of intertemporal substitution and faces an intertemporal consumption-savings decision subject to a standard budget constraint,  $\dot{B}_t = rB + w - C$  given initial wealth  $B_0$ , where  $w$  denotes the competitive wage rate determined

in the final good sector and labor supply has been normalized to one. The result is the familiar Euler equation:

$$\frac{\dot{C}}{C} = \frac{r(.) - \rho}{\varepsilon}. \quad (5)$$

where  $C$  is consumption,  $\rho$  is the rate of time preference,  $\varepsilon$  is the elasticity of intertemporal substitution and  $r$  is the interest rate.

### 3 The Decentralized Problem

#### 3.1 Decisions in the Three Sectors

The decentralized problem is to find the market allocation of output in each period between consumption, investment and research given the consumption-saving pattern of the representative household. The decision by the research sector of how much to invest into research depends on the profits in the intermediate good sector, which in turn depend on the demand in the final good sector. We briefly describe the decision problems of the three sectors. A decentralized equilibrium in the model is an allocation satisfying the following conditions which will be explained below.

1. Profit maximization

$$\begin{aligned} \text{Final good sector:} \quad x_{it}^D &= \arg \max_{x_{it}} \left[ \int_0^1 (A_{it}F(x_{it}, L) - p_{it}x_{it}) di \right] && \forall t \\ \text{Intermediate good sectors:} \quad x_{it}^S &= \arg \max_{x_{it}} [p_{it}(x_{it})x_{it} - \zeta_t A_{it}x_{it}] && \forall i \quad \forall t \\ \text{Research sectors:} \quad n_{it} &= \arg \max_{n_{it}} [\lambda n_{it}V_{it}(\pi_t) - n_{it}A_t^{max}] && \forall i \quad \forall t \end{aligned}$$

2. Utility maximization

$$\text{Household:} \quad C_t = \arg \max_{\tilde{C}_t} \left[ \int_0^\infty e^{-\rho t} \frac{\tilde{C}_t^{1-\varepsilon} - 1}{1-\varepsilon} dt \right] \quad \text{s.t.} \quad \dot{B}_t = r_t B_t + w_t - \tilde{C}_t$$

3. Market clearing

$$\text{Final good:} \quad Y_t = C_t + \dot{B}_t \quad \forall t$$

$$\text{where} \quad \dot{B}_t = I_t + N_t \quad \text{and} \quad N_t = A_t^{max} \int_0^1 n_{it} di$$

$$\text{Intermediate good:} \quad x_{it}^S = x_{it}^D \quad \forall i \quad \forall t$$

$$\text{Capital market:} \quad K_t = \int_0^1 A_{it}x_{it} di \quad \forall t$$

4. Equations of motion

$$\text{Capital:} \quad \dot{K}_t = I_t - \delta K_t \quad \forall t$$

$$\text{Technology:} \quad \dot{A}_t^{max} = \lambda n_t \sigma A_t^{max} \quad \forall t$$

### 3.2 Decisions in the Final-Good Sector

Firms in the competitive final good sector take intermediate good prices as given and demand quantities of the intermediate goods such that price equals marginal product:

$$p_{it} = A_{it}F_1(x_{it}, L) \quad \forall i \quad \forall t. \quad (6)$$

### 3.3 Decisions in the Intermediate-Good Sectors

Intermediate goods are produced by a monopolist who maximizes profits

$$\pi_{it} = A_{it}F_1(x_{it}, L)x_{it} - \zeta_t K_{it},$$

where  $K_{it} = A_{it}x_{it}$ . He takes as given the user costs of capital  $\zeta_t = r_t + \delta$  which consist of the interest rate  $r$  and the rate of depreciation  $\delta$ .

He does not take prices as given but faces the marginal price schedule from the final good sector. Thus he will supply a quantity such that marginal revenue (scaled by the productivity level)  $F_1(x_{it}, L) + x_{it}F_{11}(x_{it}, L)$  equals the (scaled) marginal cost  $\zeta_t$ . Note that because  $K_{it} = A_{it}x_{it}$ , the productivity parameter  $A_{it}$  enters both revenue and costs. Hence all monopolists in the different sectors will produce the same quantity  $x_{it} = x_t$ . This simplifies the equilibrium condition on the capital market:  $x_t = k_t L$ , where  $k_t = \frac{K_t}{A_t L}$  is the capital stock per effective unit of labor and  $A_t = \int_0^1 A_{it} di$  is the average productivity parameter. Furthermore, the aggregate production function simplifies:  $Y_t = F(K_t, A_t L)$ , or in intensity notation  $y_t = f(k_t)$ . The "marginal revenue equals marginal cost" condition for the monopolist can then be rewritten:

$$R(k_t) = r_t + \delta = \zeta_t, \quad (7)$$

where  $R(k_t) = F_1(k_t, 1) + k_t F_{11}(k_t, 1)$  is the productivity adjusted marginal revenue of the monopolist. It is assumed that this marginal revenue decreases in the capital stock. Thus profits in the intermediate goods sector can be expressed as a function of the capital stock:

$$\pi_{it} = A_{it}\pi(k_t)L, \quad (8)$$

where  $\pi(k_t) = F_1(k_t, 1)k - [F_1(k_t, 1) + k_t F_{11}(k_t, 1)]k_t = -k_t^2 F_{11}(k_t, 1)$ . Because of the assumption that marginal revenue  $R(k)$  (and thus in equilibrium: user cost of capital) decreases in  $k$ , equilibrium profits increase in the equilibrium capital stock  $k$ .



### 3.4 Decisions in the Research Sector

Productivity-adjusted research input in each sector  $n_{it} = \frac{N_{it}}{A_t^{max}}$  is chosen in order to maximize expected profits. Profits in case of a successful innovation are  $\pi_{it} = \pi_t = A_t^{max} \pi(k_t)L$ . Thus the sectors will choose  $n_{it} = n_t$  such that marginal costs equal expected marginal revenue:

$$A_t^{max} = \lambda V_t \quad \text{with} \quad V_t = \int_t^\infty e^{-\int_t^\tau (r_s + \lambda n_s) ds} A_t^{max} \pi(k_t)L \quad d\tau. \quad (9)$$

$V_t$  is the expected revenue. This condition can be simplified to yield the familiar research arbitrage equation:

$$1 = \lambda \frac{\pi(k_t)L}{r_t + \lambda n_t}. \quad (10)$$

Using the equilibrium interest rate as a function of  $k$ , this expression defines a monotone increasing relationship:

$$n_t = \hat{n}(k_t) \quad \text{with} \quad \hat{n}'(k_t) \geq 0. \quad (11)$$

Using the solution for  $n_t$  and the relation<sup>1</sup>  $A_t/A_t^{max} = 1/(1 + \sigma)$  as well as the equilibrium growth rate and the capital market equilibrium, the law of motion for the capital stock  $\dot{K}_t = F(K_t, A_t L) - C_t - n_t A_t^{max} - \delta K$  and the consumption Euler equation can be expressed in intensity notation:

$$\dot{k}_t = f(k_t) - c_t - \frac{\hat{n}(k_t)(1 + \sigma)}{L} - k_t(\delta + \sigma \lambda \hat{n}(k_t)), \quad (12)$$

$$\dot{c}_t = c_t \left[ \frac{R(k_t) - \delta - \rho}{\varepsilon} - \sigma \lambda \hat{n}(k_t) \right]. \quad (13)$$

### 3.5 Steady State Analysis

In a steady state, the capital market equilibrium and the research arbitrage equation can be simplified (using the household Euler equation and the equilibrium growth rate):

$$R(k) = \rho + \varepsilon \sigma \lambda n + \delta, \quad (14)$$

$$1 = \lambda \frac{\pi(k)L}{\rho + (\varepsilon \sigma + 1) \lambda n}. \quad (15)$$

The capital market equilibrium defines a decreasing function in the  $k - n$  space, whereas the research arbitrage equation defines an increasing function.

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<sup>1</sup>Howitt and Aghion (1998), page 117 and footnote 17 on page 129.

## 4 The Social Optimum

In this section, we will derive the social optimum in the Howitt and Aghion (1998) model and compare it to the laissez-faire steady state solution.

### 4.1 Derivation of the Social Optimum

A social planner maximizes the utility of a representative household. The planner's problem is to maximize utility by choice of consumption  $C_t$  and resources devoted to research  $N_t$ .

$$U = \int_0^{\infty} e^{-\rho t} u(C_t) dt \quad \text{where} \quad u(C_t) = \frac{C_t^{1-\varepsilon} - 1}{1-\varepsilon} \quad (16)$$

$$\begin{aligned} \text{subject to: } Y_t &= \int_0^1 A_{it} F(x_{it}, L) di = C_t + I_t + N_t && \text{final good market clearing,} \\ K_t &= \int_0^1 A_{it} x_{it} di && \text{capital market clearing,} \\ \dot{K}_t &= I_t - \delta K_t && \text{capital accumulation,} \\ \dot{A}_t &= \lambda \frac{N_t}{A_t^{\max}} \sigma A_t && \text{technology accumulation.} \end{aligned}$$

The problem can be simplified by first noting that the planner will ensure an efficient production at any point in time. It can easily be seen that, as in the private solution, this requires that all sectors produce the same amount of the intermediate goods. This again implies that the condition for an equilibrium on the capital market requires  $x_t = k_t L = \frac{K_t}{A_t L} L$  and the production function simplifies to  $Y_t = F(K_t, A_t L)$ .

Furthermore, as in the laissez-faire solution, it can be shown that in the social optimum the following relation holds:  $\frac{A_t}{A_t^{\max}} = \frac{1}{1+\sigma}$ .<sup>2</sup> Therefore, the law of motion for technology accumulation can be written as  $\dot{A}_t = \lambda N_t \frac{\sigma}{1+\sigma}$ . Inserting the condition for an equilibrium on the final goods market into the law of motion for capital accumulation, we face a standard two-dimensional optimal control problem with state variables  $K_t$  and  $A_t$  and control variables  $C_t$  and  $N_t$ . The Hamiltonian is given by:

$$H = \frac{C_t^{1-\varepsilon} - 1}{1-\varepsilon} + \eta_t [F(K_t, A_t L) - C_t - N_t - \delta K_t] + \mu_t \left[ \lambda N_t \frac{\sigma}{1+\sigma} \right].$$

The first order conditions are:

- $\frac{\partial H}{\partial C_t} = C_t^{-\varepsilon} - \eta_t = 0$ . This leads to:  $\eta_t = C_t^{-\varepsilon}$  and  $\dot{\eta}_t = -\varepsilon \frac{\dot{C}_t}{C_t} \eta_t$ .

This is the familiar condition that the marginal gain of an investment in  $K$

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<sup>2</sup>See Howitt and Aghion (1998), page 117 and footnote 17 on page 129 for the private solution. From the proof it can be seen that the relation also holds in the social optimum.

weighted with the shadow price  $\eta_t$  equals the marginal utility of consumption today.

- $\frac{\partial H}{\partial N_t} = 0 - \eta_t + \mu_t \lambda \frac{\sigma}{1+\sigma} = 0$ . This leads to:  $\mu_t = \frac{\eta_t}{\lambda \frac{\sigma}{(1+\sigma)}}$  und  $\dot{\mu}_t = \frac{\dot{\eta}_t}{\lambda \frac{\sigma}{(1+\sigma)}}$ . This condition says that the gains from an investment in  $K$  equal the gains of an investment in  $A$ . It is interesting to note that in contrast to Aghion and Howitt (1992), the marginal loss of an investment in  $N_t$  (which is  $-\mu_t \lambda \frac{\sigma}{1+\sigma}$ ) in terms of the marginal loss of consumption (which is  $\eta_t$ ) is constant and equal to  $\frac{-\mu_t \lambda \frac{\sigma}{1+\sigma}}{\eta_t} = -1$ , independent of the spending on research. The reason is that in Aghion and Howitt (1992) only labor serves as an input for research, but labor is also used in the intermediate good sectors. If the level of research is high there, employment in the intermediate good sectors is low, hence marginal productivity in the intermediate good sectors is high. Higher research is then particularly costly in terms of consumption.

Taken together the conditions say that the marginal gain from consuming / saving / research must be equal in an optimal solution.

Next we calculate the multipliers  $\mu_t$  and  $\eta_t$  using the Euler equations:

- $\eta_t = \rho \eta_t - \eta_t F_1(K_t, A_t L) + \delta \eta_t$ .

This leads to

$$\frac{\dot{C}_t}{C_t} = \frac{F_1(k_t, 1) - (\rho + \delta)}{\varepsilon}. \quad (17)$$

This is basically a standard consumption Euler equation. The only difference to the Euler equation in the private problem (5) is that the interest rate is replaced by the marginal product of capital.

- $\dot{\mu} = \rho \mu - \eta F_2(K_t, A_t L) L$ .

Inserting the expressions for  $\mu$  and  $\dot{\mu}$  and dividing by  $\eta_t$  gives us

$$\frac{\dot{C}_t}{C_t} = \frac{\lambda \frac{\sigma}{1+\sigma} F_2(K_t, A_t L) L - \rho}{\varepsilon}. \quad (18)$$

This equation differs from (17) by replacing the (social) marginal product of capital with that of the factor technology. Instead of maximizing over the research expenditure  $N$ , the planner could alternatively maximize over the research intensity  $n$ . The Euler equation would then become:  $\dot{\mu} = \rho \mu - \eta F_2(K_t, A_t L) L + \eta n_t (1 + \sigma) - \mu \lambda n_t \sigma$ . This highlights an important point: Inserting the shadow costs  $\mu$  and  $\eta$ , the last two terms cancel out. The last term says that the planner appreciates a higher research intensity, because

technology is accumulated. The penultimate term, however, takes into account that this is at the cost of consumption / saving. In the optimal solution both effects cancel out.

The system of equations (17, 18) describes the social optimum. It says that the growth rate of consumption must be proportional to the social marginal product of the two accumulable factors (adjusted by the discount rate), where the factor of proportionality is the inverse of the elasticity of intertemporal substitution.

## 5 Welfare Properties of the Steady State

In this section we compare the steady state laissez-faire solution to one that would be chosen by a social planner. Note that in the planner's problem, as in the private steady state, the growth rate of consumption will be  $g = \lambda n \sigma$ . Furthermore, we define the productivity adjusted surplus which can be used for the remuneration of the factor technology  $\pi^*(k_t) = F(k_t, 1) - kF_1(k_t, 1)$ . Dropping time indices, the system of equations (17, 18) then simplifies.<sup>3</sup> Table 1 summarizes the steady state laissez faire solution and the social optimum.

In order to compare the laissez-faire solution (equations  $(\hat{K}, \hat{N})$  in table (1) with the social optimum (equations  $(K^*, N^*)$ ), we will discuss the differences in detail. The two solutions differ with respect to five expressions.

	laissez faire solution	social optimum	
$\hat{K}$	$F_1(k, 1) + kF_{11}(k, 1) = \rho + \varepsilon\sigma\lambda n + \delta$	$F_1(k, 1) = \rho + \varepsilon\sigma\lambda n + \delta$	$K^*$
$\hat{N}$	$1 = \lambda \frac{\pi(k)L}{\rho + (\varepsilon\sigma + 1)\lambda n}$	$1 = \lambda \frac{\pi^*(k)L\sigma \frac{1}{1+\sigma}}{\rho + \varepsilon\sigma\lambda n}$	$N^*$

Table 1: Laissez-faire steady state solution and social optimum

1. In the numerator of the N-equations we find the productivity adjusted flow of gains from a successful innovation. These are of course profits in the private case. In the planner's problem the gains constitute the surplus that can be used for the remuneration of research. These gains can be written in the form of revenue minus costs:

$$\begin{array}{rcl}
 \pi(k) & = F_1(k, 1)k & -(F_1(k, 1) + kF_{11}(k, 1))k = \frac{1}{L}[F_1(x, L)x - (F_1(x, L) + xF_{11}(x, L))x] \\
 \pi^*(k) & = F(k, 1) & -F_1(k, 1)k = \frac{1}{L}[F(x, L) - F_1(x, L)x]
 \end{array}$$

<sup>3</sup>This follows from equation (18) using  $F_2(K, AL) = F(k, 1) - kF_1(k, 1)$ .

This decomposition highlights two effects. First, private and social revenue differ. The planner takes into account total output  $F(k)$ , whereas the monopolist can only appropriate total output minus consumer surplus  $F_1(k)k$ . This **appropriability effect** tends to generate too little research under laissez-faire. Second, private and social costs differ. The reason is the possibility of the monopolist to set prices. He will set prices such that his marginal revenue is lower than the marginal revenue in the planner's solution. In equilibrium, however, marginal revenue equals marginal cost. Thus private marginal cost will be lower than social marginal cost in equilibrium. The **monopoly distortion effect** tends to generate too much research under laissez-faire. Note that this effect increases with increasing monopoly power. The more the profit maximizing price deviates from marginal costs, the larger the difference between social and private marginal revenue and thus between social and private marginal cost.

2. The **monopoly distortion effect** can also be found in the K-equations. The monopolist's gain from the lower capital cost is of course borne by the suppliers of capital, the representative household, whose gross return per unit of capital will be lower than in the optimal solution. Accordingly, the effect will induce too little capital accumulation. Obviously, this effect also increases with monopoly power. Note that this effect is absent in Aghion and Howitt (1992).
3. The  $\sigma$  in the numerator of equation  $N^*$  describes the well known business stealing effect also present in Aghion and Howitt (1992). The social planner considers the incremental surplus of an innovation relative to the old technology (and not the size of the innovation per se). This is reflected by the factor  $\sigma$ , which can be interpreted as a measure of the impact of each innovation on the stock of public knowledge. The monopolist, on the other hand, does not take into consideration the loss to the previous monopolist caused by a new innovation. From the point of view of the innovator, this effect corresponds to the creative part of creative destruction. He creates an innovation, becomes a monopolist and gains at the cost of his predecessor without compensating him for the basis of knowledge on which the new innovator builds. He thus earns profits also for the stock of knowledge which was not created by himself (embodied in  $A_t^{max}$ ). Consider the extreme case of  $\sigma = 0$ . Here an innovator drives out his predecessor although he does not produce with a better technology. We will call this the **active business stealing effect**, because the new monopolist actively steals (by innovating) the previous monopolist's

profits. Jones and Williams (2000) call this part of creative destruction the “carrot”, because it constitutes an incentive for new innovators. Note that there is a difference between this model and Aghion and Howitt (1992) concerning the active business stealing effect. In Aghion and Howitt (1992)  $\sigma$  could be interpreted as the size of an innovation. The size matters for the private monopolist because he appropriates not only the old monopolist’s profits but also the gains from a discontinuous jump (from  $A_t$  to  $A_{t+1}$ ) in the stock of knowledge due to an innovation. The reason is that he incurs research costs based on the old technology (which is the stock of knowledge at the time he carries out research) but earns benefits (conditional on innovating) based on the new technology (invented by himself). In Howitt and Aghion (1998) no such discontinuous jump occurs. A researcher calculates with the same  $A_t^{max}$  both on the cost and benefit side. The reason is the assumption of a continuously growing leading-edge technology which is used as research input. This difference has the consequence that the active business stealing effect is ambiguous in this model whereas it generates too much research in Aghion and Howitt (1992). The size and direction of this effect is determined by the growth rate of the technology parameter per innovation. For  $\sigma < 1$  there is too much research under laissez-faire, for  $\sigma > 1$  the opposite is true, and for  $\sigma = 1$  the laissez-faire solution is socially optimal. Intuitively, from a social point of view, a high research productivity  $\sigma$  makes high a research intensity desirable and vice versa. But the monopolist is only concerned with stealing the incumbent’s profits, thus ignoring the issue of technological progress of the society. Note, however, that for the realistic case of a small growth rate of technological progress ( $\sigma < 1$ ) the effect induces too much research under laissez-faire.

4. The term  $\frac{1}{1+\sigma}$  in the numerator of the  $N^*$ -equation corresponds to  $\frac{A}{A^{max}}$  and constitutes a **cost-benefit gap effect**. This effect reflects the fact that the researcher calculates the benefits of an innovation based on  $A^{max}$  whereas the planner is only interested in the average technology  $A$ . But both incur research costs of  $A^{max}$ . The magnitude of this effect is small if the average technology does not differ much from the leading-edge technology. This is true if  $\sigma$  is small. The cost-benefit gap effect leads to excessive private research. Note that this effect is absent in Aghion and Howitt (1992).
5. As in Aghion and Howitt (1992), the social and private discount rates differ. There, this effect was called **intertemporal spillover effect**. We will decompose the effect into 3 subeffects, two of which are present in both Aghion

and Howitt (1992) and Howitt and Aghion (1998), and one is new in Howitt and Aghion (1998). First, the research firm discounts profits at a rate higher than the interest rate  $\rho + \varepsilon\sigma\lambda n$ . The reason is that it takes into account the Poisson probability  $\lambda n$  of losing its monopoly. This effect is the destructive part of creative destruction. Jones and Williams (2000) call it the “stick” because it constitutes a disincentive for innovators. We will call this effect **passive business stealing effect** because the monopolist fears that its profits are stolen by a successive innovator. Note that this effect is the backside of the active business stealing effect discussed above. The planner considers this destructive effect  $+\lambda n$ , but also two other effects. He considers that the destroyed old profits are overcompensated by the larger profits of the new innovator  $-(1 + \sigma)\lambda n$ . This fact leads to a social discount rate that is lower than the interest rate in the Aghion and Howitt (1992) model. We will call this second effect the **standing on giant’s shoulders effect** because it reflects the fact that an innovator builds upon the stock of knowledge in the economy generated by past innovations. In the Howitt and Aghion (1998) paper, however, the additional profit of a new innovation has to be used to enlarge the research input in order to support the growth rate of the larger  $A^{max}$ . This is due to the assumption that the growth rate  $g$  is proportional to the research intensity  $n = \frac{N}{A^{max}}$  so that a growing  $A^{max}$  implies a growing  $N$  in steady state, which in turn lowers resources available for consumption. Therefore, we label this new effect **consumption dilution effect**. Of course the planner takes this effect into account. It turns out that in the planner solution the consumption dilution effect exactly offsets the net effect  $\sigma\lambda n$  of the standing on giant’s shoulders effect and the passive business stealing effect so that all three cancel out. Concerning the discount rate, the planner thus uses the interest rate and is consequently indifferent towards growth. Recall that the monopolist considers only the passive business stealing effect. Therefore, the discount rate in the private solution differs from the interest rate by  $\lambda n$ . The size of the passive business stealing effect is small if the rate of creative destruction  $\lambda n$  is low. In this case, the social and private discount rates are similar.

After having described the distortions in the private solution, we discuss how they affect the graphs of the research arbitrage equation and the capital market equilibrium. The monopoly distortion effect tends to generate too little capital accumulation and thus implies that the  $K^*$  graph is always above the  $\hat{K}$  graph. The shift of the  $N^*$  graph is not uniquely determined. The appropriability and passive business stealing effect generate too little research ( $\hat{N}$  curve below  $N^*$  curve),

whereas the monopoly distortion effect and the technology spillover effect lead to excessive research under laissez-faire ( $\hat{N}$  curve above  $N^*$  curve). Finally, the active business stealing effect is a priori ambiguous, depending on the parameter  $\sigma$ . For the realistic case of a small growth rate of technological progress the effect induces too much research under laissez-faire.

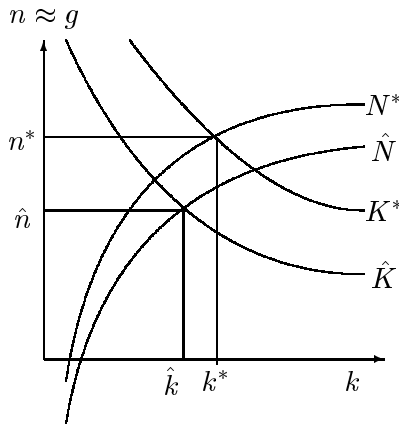


Figure 1: Laissez-faire versus social optimum (example)

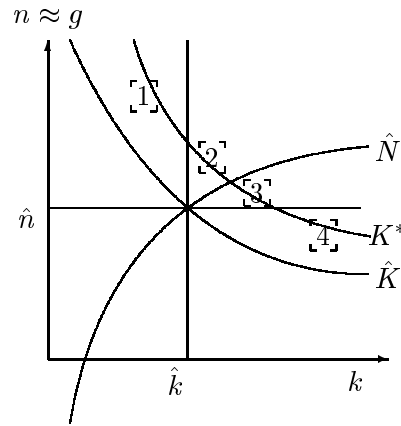


Figure 2: Possible cases for social optimum

For small distortions of the  $N^*$  curve relative to the  $\hat{N}$  curve, the private solution will have too little capital intensity ( $\hat{k} \leq k^*$ ) and too little research intensity ( $\hat{n} \leq n^*$ ). This case is depicted in figure 1. But for larger distortions, other cases are also possible. Figure 2 characterizes the four possible combinations of too much (too little) capital (research). Sectors 1 to 4 in the figure indicate where the social optimum can lie.

An important policy conclusion was already derived in Howitt and Aghion (1998). Policy measures which lower capital cost like a capital subsidy accelerate technological progress, because a lower interest rate increases the value of an innovation via higher profits and a lower discount rate. Technically, such policy measures shift the  $\hat{K}$  curve to the right. This constitutes an indirect way of subsidizing research and avoids agency problems of a direct research subsidy. Our analysis answers the questions whether such a policy measure is desirable. The welfare effects of such a subsidy are unambiguously positive if the optimum is in sectors 2 or 3 in figure 2. Then private capital and research intensity are moved towards the social optimum. However, if the economy is in sector 1, i.e if the economy oversaves in physical capital (dynamic inefficiency) the welfare effects of a subsidy are not clear a priori because the capital stock is further enlarged (which is undesirable) but the research intensity increases (which is desirable). An analogous argument holds for an optimum in



sector 4. There, a capital subsidy leads to a larger capital stock (which is positive in that case). But on the other hand, research intensity increases although in this sector research is already excessive. Neither case can be ruled out on theoretical grounds in this model.

## 6 Conclusion

We have analyzed the welfare properties of the laissez-faire steady state solution in the Howitt and Aghion (1998) model. We isolated various distortions compared to a social planner solution. The private and social solution differ because of an appropriability effect, a monopoly distortion effect, an (active) business stealing effect, a cost-benefit gap effect, and an intertemporal spillover effect. The latter can be decomposed into a passive business stealing effect, a standing on giant's shoulders effect and a consumption dilution effect.

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