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## Should UI Benefits Really Fall Over Time?

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November 2002

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## **ABSTRACT**

### **Should UI Benefits Really Fall Over Time?**

The issue of whether unemployment benefits should increase or decrease over the unemployment spell is analyzed in a tractable model allowing moral hazard, adverse selection and hidden saving. Analytical results show that when the search productivity of unemployed is constant over the unemployment spell, benefits should typically increase or be constant. The only exception is when there is moral hazard and no hidden saving. In general, adverse selection problems tend to generate increasing benefits, moral hazard problems constant benefits and decreasing search productivity falling benefits.

JEL Classification: J65, J64, E24

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# 1 Introduction

The seminal paper by (Shavell and Weiss 1979) used optimal contract theory to characterize optimal design of unemployment insurance (UI) when search activity is unobservable. Under such a moral hazard problem, they concluded that unemployment insurance should decline as long as the individual remains unemployed. Intuitively, the fear of lower future consumption increases the incentives for engaging in costly search since active search reduces the probability of having to endure this future lower consumption.

Much more recently, the analysis in (Shavell and Weiss 1979) was extended by (Hopenhayn and Nicolini 1997), allowing the insurer to control the consumption profile also for working individuals using a history dependent wage tax. (Hopenhayn and Nicolini 1997) confirm the previous results that optimal unemployment benefits should decline over time. A key assumption in both papers above is that the insurer can fully control the consumption of the individual – an assumption usually interpreted to mean that the individual has no access to markets for saving and borrowing and no alternative sources of income. A large part of the welfare gain from introducing the optimal UI plan comes from the insurer acting as a substitute bank vis-a-vis the individual. More importantly, there are *a-priori* reasons to believe that assumption that consumption can be fully controlled by the insurer is important for the result that UI benefits should decrease over time. First, it is well known that self-insurance through a market for saving and borrowing is a good substitute for insurance against short spells of unemployment (see, e.g., (Hassler and Rodríguez Mora 1999)). Second, when individuals self-insure by building precautionary buffers, consumption follows a profile that is qualitatively similar to the optimal path in (Hopenhayn and Nicolini 1997) – i) falling during unemployment as the buffer gets depleted and ii) lower consumption paths for individuals with a history of many and long unemployment spells.

We argue that the assumption that the insurer can perfectly control individual consumption is neither realistic nor innocuous. Unfortunately, it has proven

difficult to relax this assumption. Only recently, some progress have been made using the optimal contract framework (see (Pavoni 2001) and (Werning 2002)). Instead, numerical analysis has been used and (Abdulkadiroglu, Kuruscu, and Sahin 2002) show that benefits should not necessarily be decreasing while (Heer 2000) come to the opposite conclusion. Although numerical analysis allows more realistic models and quantitative predictions, it is typically difficult to understand the results arise and to make conjectures about their generality.

In this paper, we will therefore follow another route. We will assume that individual preferences are characterized by constant absolute risk aversion and like, e.g., (Fredriksson and Holmlund 2001) focus on two-tier benefit systems that allow different benefits for short-run and long-run unemployed. While these assumptions come at some cost of reduced generality they also provide substantial benefits.<sup>1</sup> We can analytically characterize optimal benefits using standard economic tools under various forms of asymmetric information. We will consider the moral hazard problem arising from costly but unverifiable search activity. To this we will add the case when some, but not all, unemployed can increase the probability of getting hired by undertaking a costly investment, e.g., by retraining or moving to a more favorable location. Under the assumption that the insurer is unable to observe who has this option and who has not, a realistic adverse selection problem that largely has been ignored in UI design arises. In all cases, also when both types of asymmetric information are jointly present, the results are analytical and allow graphical presentation and straightforward interpretation.

The key to analytical tractability under hidden savings is that with constant absolute risk-aversion, search incentives in our model will be independent of asset holdings. However, they will crucially depend on risk-aversion and access to markets for saving and borrowing and can provide qualitative insights that may prove to be valuable also in more general cases. Therefore, we believe that the results under these assumptions can complement numerical analysis under

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<sup>1</sup>The restriction to two-tier systems can, however, be interesting on its own since many real world UI systems have this feature, possibly due to political restrictions on system complexity.

(arguably) more realistic assumptions about preferences.

Our paper is organized as follows; the basic structure of the model is presented in section 2, the cases of moral hazard and adverse selection separately are analyzed in sections 3 and 4 respectively. In section 5 moral hazard and adverse selection are allowed simultaneously and section 6 concludes. Some proofs are provided in the appendix while others are available upon request.

## 2 The model

Consider an economy in continuous time where individuals can be employed or unemployed. Individuals have access to a capital market with an exogenous return  $r$ , equal to the subjective discount rate (including, possibly, a positive probability of dying). An employed individual is said to be in state 1, receiving an exogenous income  $w$ . She loses her job with instantaneous probability  $q$ , and enters into state 2, where she receives benefits denoted  $b_2$ . To analyze the issue of whether unemployment benefits should be increasing or decreasing, we allow two benefit levels  $b_2$  and  $b_3$ .<sup>2</sup> The latter benefit level is given to individuals in state 3, who are denoted long-term unemployed, while those in state 2 are called short-term unemployed. To facilitate simple presentation of the results, we assume that an individual in state 2, enters state 3 with a constant instantaneous probability  $f$ .<sup>3</sup> As a baseline case, we assume that individuals who search, have the same hiring rates in the two unemployment states, denoted by  $h$ . If  $b_2 > (<) b_3$ , we say that benefits are decreasing (increasing) over time.

Unemployed individuals can affect their hiring rate by costly action. This action is, however, unobservable, creating moral hazard problems making full insurance infeasible. Specifically, we will consider two cases. The first case is that search is costly, and unless individuals search, they will remain unemployed. The second case is that unemployed individuals can make costly investment that

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<sup>2</sup>Extending the analysis to any finite number of benefit levels is straightforward.

<sup>3</sup>Assuming, more realistically, that individuals become long-term unemployed after some fixed duration of time complicates the analysis considerably.

increases their chances of becoming employed. However, some individuals have prohibitively high costs for this investment. Therefore, there is in this case an adverse selection, where individuals with high costs needs insurance but the ones with low costs should be induced to search.

Individuals maximize intertemporal utility, given by

$$-E \int_0^{\infty} e^{-rt} U(c_t) dt,$$

where  $c_t$  is consumption at time  $t$  and  $r$  is the subjective discount rate. In order to facilitate analytical solutions when individuals have access to markets for saving and borrowing, we choose the CARA utility function

$$U(c_t) \equiv -e^{-\gamma c_t}$$

where  $\gamma$  is the coefficient of absolute risk aversion.

All individuals are born (enter the labor market) as employed without assets and are at that point identical. The purpose of this paper is to characterize optimal unemployment insurance under moral hazard. To do this, we want to remove other motives for unemployment benefits, in particular transfer motives. We therefore assume that individuals face an actuarially fair insurance. This means that when an individual enter the labor force, the expected present discounted value of the benefits she will receive during her life-time exactly balances the taxes expected present discounted value of her contributions. An alternative interpretation of actuarial fairness is that in a decentralized equilibrium, actuarial fairness is identical to a break-even condition for insurance companies, which would be satisfied under perfect competition.

Without loss of generality, we let individuals pay lump sum taxes, denoted  $\tau$ . We denote the average discounted probabilities (ADP's) of being in state 2 and 3 respectively by

$$\begin{aligned} \Pi_2 &\equiv r \int_0^{\infty} e^{-rt} \mu_{2,t} dt, \\ \Pi_3 &\equiv r \int_0^{\infty} e^{-rt} \mu_{3,t} dt. \end{aligned}$$



where  $\mu_{2,t}$  and  $\mu_{3,t}$  are the probabilities of being short term and long term unemployed at time  $t$ , respectively, conditioned on being employed at time zero. Solving for the ADP's in the base line case when hiring rates are the same in both states yields<sup>4</sup>

$$\begin{aligned}\Pi_2 &= q \frac{h+r}{(r+h+q)(r+h+f)}, \\ \Pi_3 &= \Pi_2 \frac{f}{h+r}.\end{aligned}$$

The actuarial fairness requirement the UI system can then be written

$$\tau = \Pi_2 b_2 + \Pi_3 b_3. \quad (1)$$

## 2.1 Search costs

The insurer's ability to provide insurance is hampered by asymmetric information. First, we assume that search activity is costly – a cost of  $m$  per unit of time has to be paid, otherwise the hiring probability is zero. We may think of this cost as representing the opportunity cost of searching arising from some alternative economic activity. Whether the agent actually search or not is assumed to be the agents private information. Second, we assume that an unemployed individual can undertake a costly investment, (re-training or moving). The cost is either low,  $\tilde{m}$  (with probability  $p$ ) or prohibitively high. For simplicity, we assume that if the unemployed pays the cost, she immediately gets rehired. Otherwise, she remains unemployed and decides whether or not to search for a new job. To make the problem interesting, we assume that parameters are

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<sup>4</sup>In the more general case, when hiring rates are  $h_2$  and  $h_3$ , we have

$$\begin{aligned}\Pi_2 &= \frac{(h_3+r)q}{(\rho_2-r)(\rho_1-r)}, \\ \Pi_3 &= \frac{fq}{(\rho_2-r)(\rho_1-r)},\end{aligned}$$

where  $\rho_1$  and  $\rho_2$  are the roots of the system and given by

$$\rho_{1,2} = -\frac{F \pm \sqrt{F^2 - 4(qf + h_3(f + h_2 + q))}}{2} < 0,$$

where  $F \equiv f + q + h_3 + h_2$

such that it is optimal to induce search for all unemployed and investment for individuals with low costs.

### 3 Moral hazard

We start the analysis by assuming that individuals cannot save or borrow. The value function of an employed individual, conditional on her searching when unemployed, is then given by

$$V_1 = -e^{\gamma\tau} e^{-\gamma w} \frac{1 - \Pi_2 - \Pi_3 + \Pi_2 e^{\gamma\Delta_2} + \Pi_3 e^{\gamma\Delta_3}}{r} \quad (2)$$

where  $\Delta_2 \equiv c_1 - c_2, \Delta_3 \equiv c_1 - c_3$  denotes the reduction in consumption for short- and long-run unemployed relative to employed. It is straightforward to verify that that individuals prefer flat benefit schedules, whatever the tax level. To see this, note that the necessary first-order conditions for choosing  $\Delta_2$  and  $\Delta_3$  optimally subject to (1) and letting  $\lambda$  denote the shadow value on (1) can be written

$$\frac{\partial V_1}{\partial \Delta_2} = \lambda \Pi_2, \quad \frac{\partial V_1}{\partial \Delta_3} = \lambda \Pi_3,$$

Dividing the two first-order conditions yields

$$\frac{-e^{\gamma\tau} e^{-\gamma c_1} \gamma \Pi_2 e^{\gamma\Delta_2} / r}{-e^{\gamma\tau} e^{-\gamma c_1} \gamma \Pi_3 e^{\gamma\Delta_3} / r} = \frac{\Pi_2}{\Pi_3},$$

implying  $\Delta_2 = \Delta_3$  in the optimum. Thus;

**Proposition 1** *When no moral hazard problem exists and there is no market for saving and borrowing, UI benefits should be constant over time.*

Let us now introduce moral hazard by allowing individuals to abstain from searching if the search incentive is too weak. In order to derive the incentive compatibility constraints, we first note the value functions for the two states,  $V_2$  and  $V_3$  conditioning on search, are given by

$$V_2 = -e^{\gamma\tau} e^{-\gamma w} \frac{\frac{h}{h+r} (1 - \Pi_2 - \Pi_3) + \frac{r+q}{q} \Pi_2 e^{\gamma\Delta_2} + \frac{r+q}{q} \Pi_3 e^{\gamma\Delta_3}}{r} \quad (3)$$

$$V_3 = -e^{\gamma\tau} e^{-\gamma w} \frac{\frac{h}{h+r} (1 - \Pi_2 - \Pi_3) + \frac{h}{h+r} \Pi_2 e^{\gamma\Delta_2} + \left(1 + \frac{r(r+h+f+q)}{qf}\right) \Pi_3 e^{\gamma\Delta_3}}{r},$$

where  $\Delta_2 = w - b_2 + m$  and  $\Delta_3 = w - b_3 + m$  since we continue to assume no saving or borrowing.

If the individual is not searching in the current state, the corresponding value functions, conditioned on searching in the future states is

$$V_{3,n} = \frac{-e^{-\gamma(b_3-\tau)}}{r}, V_{2,n} = \frac{-e^{-\gamma(b_2-\tau)} + fV_3}{r+f}.$$

The incentive compatibility constraint for the long-term unemployed (IC3) is then

$$V_3 \geq V_{3,n}.$$

Using the definitions above, this can be rewritten

$$(1 - \Pi_2 - \Pi_3)(1 - e^{-\gamma\Delta_3}) + \Pi_2(1 - e^{\gamma(\Delta_2 - \Delta_3)}) \geq \left(1 + \frac{r}{h}\right)(1 - e^{-\gamma m}). \quad (4)$$

As we see, this implies that higher search costs requires higher  $\Delta_3$  and/or lower  $\Delta_2$  to induce search. The positive effect on search incentives of lower higher slope is the "entitlement" effect (see (Mortensen 1977)) – higher benefits for short-term unemployed *increase* the search incentives for long-term unemployed since the latter need to first become employed to be entitled to these higher benefits.

Now, consider the short-term unemployed. The incentive compatibility constraint for them (IC2) is

$$V_2 \geq V_{2,n}$$

which we can write

$$(1 - \Pi_2 - \Pi_3)(1 - e^{-\gamma\Delta_2}) + \frac{f}{q}\Pi_2(e^{\gamma(\Delta_3 - \Delta_2)} - 1) \geq \left(1 + \frac{r}{h}\right)(1 - e^{-\gamma m}). \quad (5)$$

Note that when  $\Delta_3 = \Delta_2$ , the two constraints coincide at  $(1 - \Pi_2 - \Pi_3)(1 - e^{-\gamma\Delta_2}) \geq (1 - e^{-\gamma m})\frac{r+h}{h}$ , implying

$$\Delta_2 \geq \frac{-\ln(1 - (1 - e^{-\gamma m})(1 + \frac{q+r}{h}))}{\gamma} \equiv \bar{\Delta}.$$

Along the IC3 constraint, we have

$$\frac{d\Delta_2}{d\Delta_3}|_{IC3} = \frac{1 - \Pi_3}{\Pi_2} - \left(\frac{1 - \Pi_2 - \Pi_3}{\Pi_2}\right)(1 - e^{-\gamma\Delta_2}) > 1 \forall \Delta_2 > 0,$$

where we should note that  $\frac{1-\Pi_3}{\Pi_2}$  is the slope of the IC3 constraint under risk neutrality and/or under perfect insurance.

Along the IC2 constraint, we have

$$\begin{aligned} \frac{d\Delta_2}{d\Delta_3}|_{IC2} &= -\frac{\Pi_3}{\Pi_2} \frac{1}{1 - \left(1 + \frac{\Pi_3}{\Pi_2}\right) (1 - e^{-\gamma\Delta_3})} \\ &< -\frac{\Pi_3}{\Pi_2} \forall \Delta_3 \in \left(0, \frac{1}{\gamma} \ln \left(1 + \frac{\Pi_2}{\Pi_3}\right)\right), \end{aligned} \quad (6)$$

where  $-\frac{\Pi_3}{\Pi_2} = -\frac{f}{h+r}$  is the slope of the IC3 constraint under risk-neutrality and/or under perfect insurance.<sup>5</sup>

The optimal contract maximizes  $V_1$  as given by (2) over  $\Delta_2$  and  $\Delta_3$ , subject to the incentive constraints (4), (5) and the fairness constraint (1). Clearly, indifference curves in the  $\{\Delta_2, \Delta_3\}$  space are ovals around origo. The slope of an indifference curve is given by

$$\frac{d\Delta_2}{d\Delta_3}|_{V_1=\bar{V}} = -\frac{\Pi_3(1-\Pi_3)e^{\gamma\Delta_3} - (1-\Pi_2-\Pi_3+\Pi_2e^{\gamma\Delta_2})\Pi_3}{\Pi_2(1-\Pi_2)e^{\gamma\Delta_2} - (1-\Pi_2-\Pi_3+\Pi_3e^{\gamma\Delta_3})\Pi_2}.$$

When  $\Delta_3 = \Delta_2$ , this simplifies to

$$-\frac{\Pi_3}{\Pi_2},$$

which is the slope of an actuarially fair increase in  $b_2$  financed with a decrease in  $b_3$ .

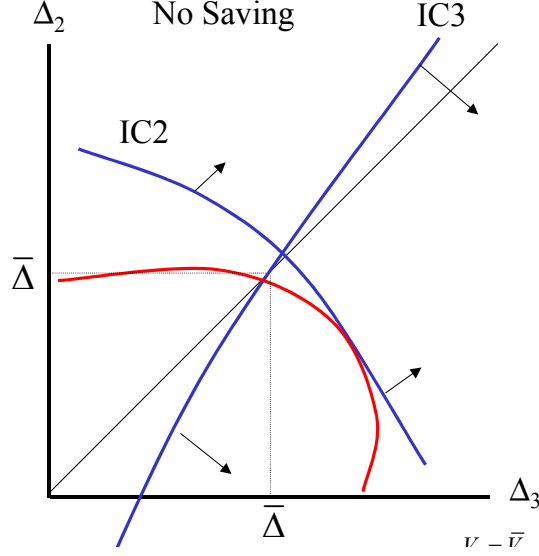
As we see from (6), the slope of IC2 is necessarily steeper than the indifference curve at  $\Delta_2 = \Delta_3 = \bar{\Delta}$ , whenever risk-aversion is strictly positive and a strictly positive  $m$  prevents perfect insurance. Therefore,  $\Delta_3$  should optimally be larger than  $\Delta_2$  requiring a downward-sloping benefit schedule.

We depict our results in figure 3 and summarize in the following proposition;

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<sup>5</sup>As  $\Delta_3$  approach  $\frac{1}{\gamma} \ln \left(1 + \frac{\Pi_2}{\Pi_3}\right)$ , the IC2 curve becomes vertical. However, if  $m < -\frac{1}{\gamma} \ln \left(1 - \frac{h}{q}\Pi_2\right)$ , the IC2 curve has a finite slope in the positive quadrant of the space  $\Delta_2, \Delta_3$ . To see this, set  $\Delta_2 = 0$  in the IC2 constraint, solve for  $\Delta_3$  as a function of  $m$  and set this expression equal to  $\frac{1}{\gamma} \ln \left(1 + \frac{\Pi_2}{\Pi_3}\right)$ .

**Proposition 2** *Under moral hazard and without markets for saving and borrowing, UI benefits should be decreasing over time.*



This result is qualitatively similar to the results in (Shavell and Weiss 1979) and (Hopenhayn and Nicolini 1997) but the intuition is a bit different. Consider an actuarially fair reduction in long-term benefits, financed with increases in short-term benefits, i.e.,  $d\Delta_2 = -\frac{\Pi_3}{\Pi_2} d\Delta_3$ . Whenever,  $\Delta_2 = \Delta_3$  such a change only has second order effects on  $V_1$ . Furthermore, it does neither have any first order effects on  $V_2$  since it is actuarially fair also from the point of view of short-term unemployed. However, the temptation not to search is not invariant to an actuarially fair increase in  $b_2$ . To see this, note first that the IC2 constraint is calculated conditional on search in state 3. When the individual deviates in state 2 but not in state 3, her consumptions levels are  $b_2 - \tau$  versus  $b_3 - \tau - m$ . An actuarially fair transfer from state 3 to state 2 therefore reduces the value  $V_{2,n}$ .<sup>6</sup> We conclude that if the IC2 constraint was satisfied with equality at

<sup>6</sup>Formally, we have  $V_{2,n} = \frac{-e^{-\gamma(w_2-\tau)} + fV_3}{r+f} = -\frac{1}{r+f} \left( e^{-\gamma(w_2-\tau)} + \frac{\Pi_3}{\Pi_2} e^{-\gamma(w_3-\tau-m)} \right) + \frac{f}{r+f} \frac{h}{r+h} V_1$ .

This is invariant to an actuarial change in  $w_3$  and  $w_2$  at  $w_3 = w_2$  only if  $m = 0$ .

$\Delta_2 = \Delta_3$ , a positive change in  $\Delta_3$  and a negative  $d\Delta_2 = -\frac{\Pi_3}{\Pi_2}d\Delta_3$  implies that the constraint is satisfied with strict inequality, in other words, IC2 is steeper than  $-\frac{\Pi_3}{\Pi_2}$ .

### 3.1 Saving

Consider now the case when individuals can self-insure via precautionary savings. As above, we assume that there is a cost of searching and that the search only takes place if there are incentive for search. The value function for the three types, conditional on searching is

$$V_j(A_t) = -\frac{1}{r}e^{-\gamma r A_t}e^{-\gamma c_j}, j \in \{1, 2, 3\},$$

and consumption is

$$c_{t,j} = rA_t + c_j, j \in \{1, 2, 3\}.$$

Note our abuse of notation; from now on we let  $c_j$  denote consumption net of permanent income from current asset holding. The Bellman equation for the employed is satisfied if the constants  $c_j$ , satisfy

$$\begin{aligned} c_1 &= w - \tau - \frac{q(e^{\gamma\Delta_2} - 1)}{\gamma r}, & (7) \\ c_2 &= b_2 - m - \tau + \frac{h(1 - e^{-\gamma\Delta_2})}{\gamma r} - \frac{f(e^{\gamma(\Delta_3 - \Delta_2)} - 1)}{\gamma r}, \\ c_3 &= b_3 - m - \tau + \frac{h(1 - e^{-\gamma\Delta_3})}{\gamma r}. \end{aligned}$$

It is convenient to rewrite the second two equations of (7) as

$$\begin{aligned} b_2 &= w - \Delta_2 + m - \frac{q(e^{\gamma\Delta_2} - 1)}{\gamma r} - \frac{h(1 - e^{-\gamma\Delta_2})}{\gamma r} + \frac{f(e^{\gamma(\Delta_3 - \Delta_2)} - 1)}{\gamma r}, & (8) \\ b_3 &= w - \Delta_3 + m - \frac{q(e^{\gamma\Delta_2} - 1)}{\gamma r} - \frac{h(1 - e^{-\gamma\Delta_3})}{\gamma r}. \end{aligned}$$

Consider first the problem of choosing benefits, disregarding the incentive compatibility constraints. The optimal contract maximizes  $c_1$  (and therefore  $V_1(A)$  for any  $A$ ) as given by (7) over  $\Delta_2$  and  $\Delta_3$  subject to individual consumption choices as given by (8) and the fairness constraint (1). It follows

immediately that whenever  $\Delta_2$  is positive in optimum (less than full insurance) individuals prefer upward sloping benefits. To see this, note that the first-order condition for  $\Delta_3$  simply minimizes taxes over  $\Delta_3$ , i.e., sets

$$\frac{\partial \tau}{\partial \Delta_3} = \Pi_2 \left( \frac{f}{r} e^{\gamma(\Delta_3 - \Delta_2)} \right) - \Pi_3 \left( 1 + \frac{h}{r} e^{-\gamma \Delta_3} \right) = 0.$$

Clearly, this is not satisfied at  $\Delta_3 = \Delta_2$ , unless there is full insurance ( $\Delta_3 = \Delta_2 = 0$ ). Instead  $b_3$  should be raised until

$$\Delta_3 = \Delta_2 + \frac{1}{\gamma} \ln \left( 1 - \frac{h}{h+r} (1 - e^{-\gamma \Delta_3}) \right) < \Delta_2.$$

Now, let us consider the incentive constraints. A long-run unemployed who does not search, consumes  $b_3 - \tau + rA_t$  for ever, yielding a value of  $-\frac{1}{r} e^{-\gamma r A_t} e^{-\gamma(b_3 - \tau)}$ . The IC3 constraint is therefore

$$-\frac{1}{r} e^{-\gamma r A_t} e^{-\gamma c_3} \geq -\frac{1}{r} e^{-\gamma r A_t} e^{-\gamma(b_3 - \tau)},$$

$$c_3 \geq b_3 - \tau.$$

As we see, total consumption ( $c_3 + rA_t$ ), has to be at least as large as net income ( $b_3 - \tau + rA_t$ ). This means that incentives have to be at least large enough to make the individual willing to borrow to finance the search cost. This in turn, means that consumption necessarily falls as long as the individual remains unemployed.

Using (7), IC3 can be written

$$\Delta_3 \geq \frac{\ln \left( \frac{h}{h - \gamma r m} \right)}{\gamma} \equiv \hat{\Delta}(h). \quad (9)$$

For the short term unemployed, we compute the value associated with no search in state 2, conditioned on searching in state 3. This is  $-\frac{e^{-\gamma r A_t} e^{-\gamma c_{2,n}}}{r}$  where  $c_{2,n}$  satisfies,

$$c_{2,n} = b_2 - \tau + \frac{f(1 - e^{-\gamma(c_3 - c_{2,n})})}{\gamma r}$$

Thus, the IC2 constraint is  $c_2 \geq c_{2,n}$ , which can be written

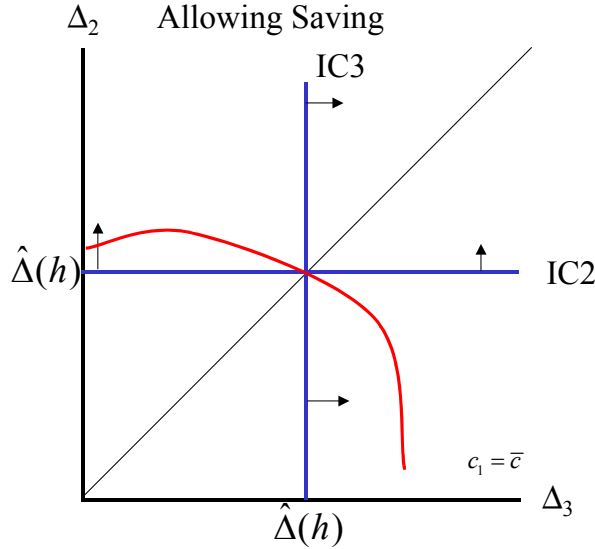
$$\Delta_2 \geq \hat{\Delta}(h). \quad (10)$$

The optimal insurance contract should then be chosen to maximize  $c_1$  as given by (7) over  $\Delta_2$  and  $\Delta_3$  subject to the incentive constraint (9) and (10), individual consumption choice (8) and the fairness constraint (1). The slope of an indifference curve, evaluated at  $\Delta_3 = \Delta_2$  is given by

$$\frac{d\Delta_2}{d\Delta_3}\Big|_{c_1=\bar{c}} = -\frac{\Pi_3 h (1 - e^{-\gamma\Delta_2})}{\Pi_2 f + (1 - \Pi_2 - \Pi_3) q (e^{\gamma\Delta_2} - 1) + \Pi_2 h (1 - e^{-\gamma\Delta_2})} \leq 0.$$

Clearly, the optimal contract is the point where IC3 crosses IC2. There we have  $\Delta_2 = \Delta_3 = \bar{\Delta}$  and  $b_2 = b_3 = w - \bar{\Delta} - \frac{mq}{h-\gamma rm}$ . We depict our results in Figure 3.1 and summarize as follows;

**Proposition 3** *Under moral hazard and with markets for saving and borrowing, UI benefits should be constant over time at  $w - \bar{\Delta} - \frac{mq}{h-\gamma rm}$ . Consumption of unemployed is equal to income. The search cost is financed with negative savings, implying falling consumption over the unemployment spell.*



To get some intuition for the results we first note that in general, search incentives arise from a comparison of expected lifetime utility (the value function) under different search strategies. When individuals have access to a capital market for saving and borrowing, however, there is a one-to-one mapping between



consumption and the value function. In contrast to the no savings case, we cannot increase search incentives in state 3 for a given level of  $\Delta_3$  by reducing  $\Delta_2$ . Similarly, in state 2, and given  $\Delta_2$  search incentives cannot be strengthened by increasing  $\Delta_3$ . In other words, the two constraints are independent of each other and both should be satisfied with equality. In this case, here it is assumed that hiring rates are the same in both states, which calls for constant benefits.<sup>7</sup>

## 4 Adverse selection

As above, we the value functions are of the form

$$-\frac{1}{r}e^{-\gamma(rA_t+c_s)}$$

for the three states and consumption given by

$$rA_t + c_s,$$

unless the individual invests, in which case assets fall discontinuously by  $\tilde{m}$ .

The consumption constants satisfy

$$\begin{aligned} c_1 &= w - \tau - q \frac{pe^{\gamma r \tilde{m}} + (1-p)e^{\gamma \Delta_2} - 1}{\gamma r} \\ c_2 &= b_2 - \tau + h \frac{1 - e^{-\gamma \Delta_2}}{\gamma r} - f \frac{e^{\gamma(\Delta_3 - \Delta_2)} - 1}{\gamma r} \\ c_3 &= b_3 - \tau + h \frac{1 - e^{-\gamma \Delta_3}}{\gamma r}. \end{aligned}$$

Noting that , when individuals who get the low mobility cost move, the flow into unemployment is  $(1-p)q$ , we find that the ADP of being short term and long term unemployed, are respectively given by

$$\begin{aligned} \bar{\Pi}_2 &\equiv q(1-p) \frac{h+r}{(r+h+q(1-p))(r+h+f)}, \\ \bar{\Pi}_3 &\equiv \bar{\Pi}_2 \frac{f}{h+r}. \end{aligned} \tag{11}$$

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<sup>7</sup>In fact, in an unpublished paper, Werning (2002) shows in a smiliar setting that constant benefits are optimal under CARA utility in a general class of UI-schemes.

Now, the incentive compatibility constraint under adverse selection (ICA) is that individuals with a low cost should pay the investment cost. This can be written

$$-\frac{1}{r}e^{-\gamma(r(A_t-\tilde{m})+c_1)} \geq -\frac{1}{r}e^{-\gamma(rA_t+c_2)}, \quad (12)$$

$$\Delta_2 \geq r\tilde{m},$$

which is independent of assets. Now, since the insurance is actuarially fair and individuals are risk averse, the ICA condition will surely to bind at the optimal tax rate, in which case  $r\tilde{m} = \Delta_2$ , giving  $c_1 = w - \tau - q\frac{e^{\gamma r\tilde{m}}-1}{\gamma r}$ , and

$$b_2 = w - r\tilde{m} - q\frac{e^{\gamma r\tilde{m}} - 1}{\gamma r} - \left( h\frac{1 - e^{-\gamma r\tilde{m}}}{\gamma r} - f\frac{e^{\gamma(\Delta_3 - r\tilde{m})} - 1}{\gamma r} \right), \quad (13)$$

$$b_3 = w - \Delta_3 - q\frac{e^{\gamma r\tilde{m}} - 1}{\gamma r} - h\frac{1 - e^{-\gamma\Delta_3}}{\gamma r}.$$

The problem is then to solve  $\max_{\Delta_3} w - \tau - q\frac{e^{\gamma r\tilde{m}}-1}{\gamma r}$ , subject to the incentive constraint (12), individual consumption choice (13) and the fairness  $\tau = \bar{\Pi}_2 b_2 + \bar{\Pi}_3 b_3$ . As we see, the problem reduces to minimize taxes, given the constraints. The first-order condition for this problem is

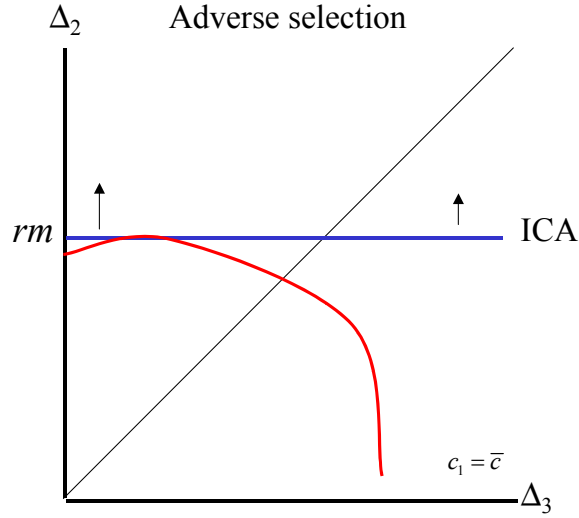
$$\bar{\Pi}_2 \left( \frac{f}{r} e^{\gamma(\Delta_3 - r\tilde{m})} \right) = \bar{\Pi}_3 \left( 1 + \frac{h}{r} e^{-\gamma\Delta_3} \right).$$

Evaluating this at  $\Delta_3 = r\tilde{m}$  we get

$$\bar{\Pi}_2 \frac{f}{r} - \bar{\Pi}_3 \left( 1 + \frac{h}{r} e^{-\gamma r\tilde{m}} \right) = \bar{\Pi}_3 \frac{h}{r} (1 - e^{-\gamma r\tilde{m}}) \geq 0,$$

implying that taxes could be reduced by lowering  $\Delta_3$  below  $\Delta_2$ . However, it is immediate to show that the slope of the indifference curve at  $\Delta_3 = 0$  is strictly positive. Therefore, full insurance of the long term unemployed is not optimal. Our results are depicted in Figure 3, and summarized in the following proposition;

**Proposition 4** *Under adverse selection and access to markets for saving and borrowing, benefits should increase over time but not to full insurance.*



The intuition here is that the IC constraint associated with the adverse selection problem puts a wedge between the value of being employed and short-run unemployed and therefore between consumption in these states. However, this wedge provides no argument for not satisfying the preference for upward sloping benefits in the case when investment costs are high. In a sense, insurance should be (constrained) efficient in the choice of relative insurance for long-term and short-term insurance. However, full insurance for long-term unemployed cannot be optimal since a marginal reallocation from long-term to short-term unemployed, when the former have full insurance, but not the latter, must improve the constrained efficiency of the insurance.

#### 4.1 Adverse selection and no saving

To understand the results on adverse selection, we want to analyze the case of no savings in a setting as close as possible to the case of savings. This poses a technical problem, since investments is hard to model when there is no savings. To keep as close as possible to the savings, in particular that the investment costs is monetary and that there are three employment states only, we make the

following assumptions; the investment cost is a loss of income  $\tilde{m}$  during a unitary masspoint of time. In other words, we assume that the value function falls by an amount  $(e^{-\gamma(w-\tau-\tilde{m})} - e^{-\gamma(w-\tau)})$  if the individual decides to undertake he investment and has the low cost of doing so. This assumption is isomorphic to a discrete time case when consumption falls by an amount  $\tilde{m}$  during one period if the investment is undertaken.

Now, the incentive compatibility constraint  $V_1 - (e^{-\gamma(w-\tau-\tilde{m})} - e^{-\gamma(w-\tau)}) \geq V_2$  can be written<sup>8</sup>

$$\bar{\Pi}_2 (e^{\gamma\Delta_2} - 1) + \bar{\Pi}_3 (e^{\gamma\Delta_3} - 1) \geq (e^{\gamma\tilde{m}} - 1) q(1-p) \frac{h+r+q}{h+r+q(1-p)}$$

Clearly, the slope of this constraint is the same as the indifference curve if and only if  $\Delta_3 = \Delta_2$ . Thus, benefits should be flat under adverse selection and no savings.

**Proposition 5** *Under adverse selection and no access to markets for saving and borrowing, benefits should be constant over time.*

The intuition for our results is the same as under savings; the IC constraint puts a wedge between the value of employed and short-term unemployed. However, this does not call for not satisfying the preferences of unemployed with high investment costs, which, in the case of no savings, is to have constant benefits. The insurance should be efficient in the relative insurance of long- and short-term unemployed.

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<sup>8</sup>The expressions for the value functions are given in appendix.

## 5 Moral hazard, adverse selection and different hiring rates

Let us now analyze the more general case when hiring rates are allowed to be different in the two unemployment states (denoted  $h_2$  and  $h_3$ ) and when we have both moral hazard and adverse selection. The consumption constants satisfy

$$\begin{aligned} c_1 &= w - \tau - q \frac{pe^{\gamma r \tilde{m}} + (1-p)e^{\gamma \Delta_2} - 1}{\gamma r}, \\ c_2 &= b_2 - m - \tau + h_2 \frac{1 - e^{-\gamma \Delta_2}}{\gamma r} - f \frac{e^{\gamma(\Delta_3 - \Delta_2)} - 1}{\gamma r}, \\ c_3 &= b_3 - m - \tau + h_3 \frac{1 - e^{-\gamma \Delta_3}}{\gamma r}, \end{aligned} \quad (14)$$

the ICA constraint remains  $\Delta_2 \geq r\tilde{m}$  and the IC2 and IC3 constraint are  $\Delta_2 \geq \hat{\Delta}(h_2)$  and  $\Delta_3 \geq \hat{\Delta}(h_3)$ , respectively.

We now have two cases. First, when the adverse selection problem is small, specifically, if  $\hat{\Delta}(h_2) \geq r\tilde{m}$ , the ICA constraint is satisfied whenever, IC2 is satisfied. Then, the optimal contract sets  $\Delta_2 = \hat{\Delta}(h_2)$  and  $\Delta_3 = \hat{\Delta}(h_3)$ , implying from (14) that

$$\begin{aligned} b_3 &= w - \hat{\Delta}(h_3) - q \frac{pe^{\gamma r \tilde{m}} + (1-p) \left( \frac{h_2}{h_2 - \gamma r m} \right) - 1}{\gamma r} \\ b_2 &= w - \hat{\Delta}(h_2) - q \frac{pe^{\gamma r \tilde{m}} + (1-p) \left( \frac{h_2}{h_2 - \gamma r m} \right) - 1}{\gamma r} + fm \frac{h_2 - h_3}{h_2 h_3 - mr \gamma h_2}. \end{aligned}$$

Since  $\hat{\Delta}'(h) < 0$ , we have that  $h_2 > (\leq) h_3 \implies b_2 - b_3 = \hat{\Delta}(h_3) - \hat{\Delta}(h_2) + fm \frac{h_2 - h_3}{h_2 h_3 - mr \gamma h_2} > (\leq) 0$ .

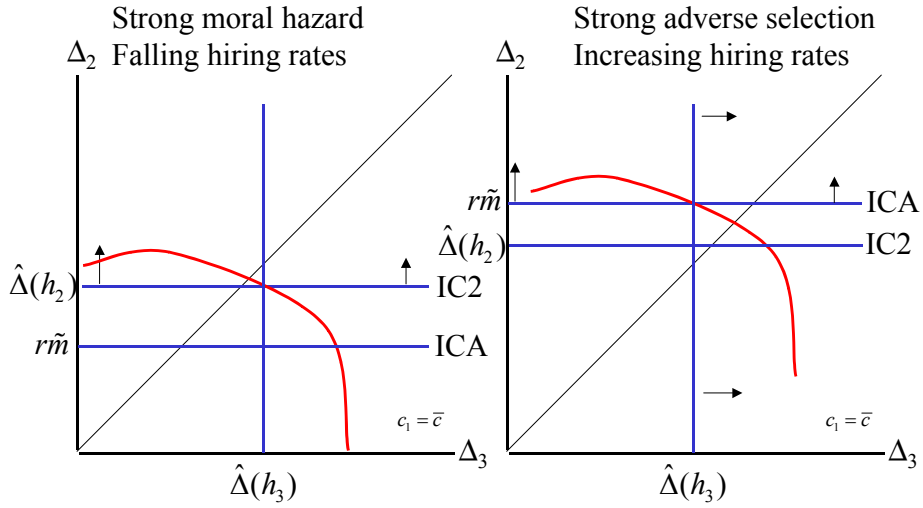
Second, if the adverse selection problem is relatively strong, i.e.,  $\hat{\Delta}(h_2) < r\tilde{m}$ , IC2 is satisfied when ICA is satisfied. Then, the optimal contract is  $\Delta_2 = r\tilde{m}$  and  $\Delta_3 = \hat{\Delta}(h_3)$ . Using this in (14) implies,

$$\begin{aligned} b_3 &= w - q \frac{e^{\gamma r \tilde{m}} - 1}{\gamma r} - \hat{\Delta}(h_3) \\ b_2 &= w - r\tilde{m} + m - h_2 \frac{1 - e^{-\gamma r \tilde{m}}}{\gamma r} + f \frac{\frac{h_3}{h_3 - \gamma r m} e^{-\gamma r \tilde{m}} - 1}{\gamma r} - q \frac{e^{\gamma r \tilde{m}} - 1}{\gamma r} \end{aligned}$$

In this case,

$$b_2 - b_3 = m - r\tilde{m} + h_2 \frac{e^{-\gamma r \tilde{m}} - 1}{\gamma r} + f \frac{\frac{h_3}{h_3 - \gamma r m} e^{-\gamma r \tilde{m}} - 1}{\gamma r} + \hat{\Delta}(h_3).$$

As we see, the benefit profile becomes more increasing (more negative  $b_2 - b_3$ ) the smaller is  $m$  and  $h_2$  and the larger is  $\tilde{m}$  and  $h_3$ . In other words, an adverse selection problem that is strong relative to the moral hazard problem and hiring rates that are increasing (decreasing), calls for increasing benefits. When,  $\hat{\Delta}(h_2) = r\tilde{m}$  and  $h_2 = h_3$ , we already know that optimally  $b_2 = b_3$ . Consequently, if  $\tilde{m}$  is increased from this point, the benefit profile becomes upward-sloping. Furthermore, for sufficiently low  $h_3$ , benefits should be downward sloping. The results are depicted in Figure 4. An increase in  $\tilde{m}$  shifts the ICA constraint upwards, similarly, a decrease in  $h_2$  shifts IC2 upwards.



Summarizing;

**Proposition 6** *When the adverse selection problem is relatively small ( $\hat{\Delta}(h_2) \geq r\tilde{m}$ ), benefits should be decreasing iff hiring rates are decreasing ( $h_2 > h_3$ ).*

*When the adverse selection problem is relatively high,  $\hat{\Delta}(h_2) < r\tilde{m}$ , benefits should be increasing for non-decreasing hiring rates. For sufficiently decreasing*

*hiring rates, benefits should be decreasing.*

## 6 Conclusion

We have in this paper provided a tractable model where risk averse individuals face unemployment risk that cannot be completely insured due to various forms of asymmetric information. It has been shown that access to savings has important qualitative effects on the time profile of optimal unemployment benefits. The model provided a number of analytical results.

First, access to savings imply that individuals tend to prefer increasing benefits since precautionary savings is a good (bad) substitute for short (long) spells of unemployment.

Second, moral hazard problems arising from unobservable search effort may call for decreasing benefits if the insurer can control individual consumption, i.e., when there is no (hidden) savings. However, if, realistically, the insurer cannot control consumption, this is no longer necessarily the case. The reason is that individual consumption choices imply that search incentives have a one-to-one relation to the expected consumption increase associated with finding a job. If search productivity and the cost of search are constant over time, the incentive to search should also be constant. This calls for a consumption increase at employment that is independent of the duration of the unemployment period and this is implemented with constant benefits. It seems likely that this result would remain if search intensity was a continuous rather than dichotomous variable.

Third, we have analyzed the case when individuals can effect the hiring probability by an up-front investment, e.g., retraining or moving. When the adverse selection problem arising from this is strong, the benefit profile is optimally increasing. The intuition here is straightforward – the adverse selection problem calls for a separation of individuals with low and high investment costs. However, this separation should not be done by using an insurance profile that

provides an inefficient insurance. In other words, unemployed individuals' preference for an increasing benefits should be satisfied.

Finally, the benefit profile is sensitive to how search productivity evolves over the unemployment spell. If search productivity is falling, benefits should also be falling. This result may change if search intensity is modelled as a continuous variable and it is optimal for search intensity to fall as search productivity falls.

Let us conclude by some speculations on the consequences of allowing constant relative risk-aversion. In such a case, the analysis is greatly complicated by the fact that, in general, search incentives would depend on asset holdings. The intuition for the results in this paper appear not to be related to such effects, however. Therefore, the mechanisms we have analyzed could likely be present also under constant absolute risk aversion. However, since search incentives depend on asset holdings and the duration of the unemployment is likely to be correlated with the individual's asset holdings, unobservability of the latter may have consequences for optimal benefit time profiles. For example, if search incentives are strengthened as wealth decumulates and individuals with long unemployment spells are likely to have less wealth, this effect could call for increasing benefits.

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## 7 Appendix

Recursive proof of (2) (3). The value functions must satisfy

$$\begin{aligned} V_1 &= \frac{-e^{\gamma\tau}e^{-\gamma w} + qV_2}{r + q}, \\ V_2 &= \frac{-e^{\gamma\tau}e^{-\gamma b_2} + hV_1 + fV_3}{r + h + f}, \\ V_3 &= \frac{-e^{\gamma\tau}e^{-\gamma b_3} + hV_1}{r + h}. \end{aligned}$$

Solving these equations yields, (2) and (3).

### 7.1 Savings

Guessing that the value function is  $-e^{-\gamma(rA_t+c_j)}$  for  $j \in \{1, 2, 3\}$ , the Bellman equation for the employed is

$$\begin{aligned} -\frac{1}{r}e^{-\gamma(rA_t+c_1)} &= \max_c -e^{-\gamma(rA_t+c)}dt \\ &\quad - (1 - rdt) \left[ (1 - qdt) \frac{1}{r}e^{-\gamma(rA_{t+dt}+c_1)} + qdt \frac{1}{r}e^{-\gamma(rA_{t+dt}+c_2)} \right]. \end{aligned}$$

Using, first order linear approximations and dividing by  $e^{-\gamma rA_t}$ , this becomes

$$\begin{aligned} -\frac{1}{r}e^{-\gamma c_1} &= \max_c -e^{-\gamma c}dt \\ - (1 - rdt) &\left[ (1 - qdt) \frac{1}{r}e^{-\gamma c_1} (1 - \gamma r (w - c - \tau) dt) + qdt \frac{1}{r}e^{-\gamma c_2} (1 - \gamma r (w - \tau - c) dt) \right] \end{aligned}$$

Adding  $\frac{1}{r}e^{-\gamma c_1}$  to both sides, dividing by  $dt$  and letting  $dt$  go to zero, yields

$$0 = \max_c -re^{-\gamma(c-c_1)} + r + \gamma r (w - c - \tau) + q \left( 1 - e^{-\gamma(c_2-c_1)} \right), \quad (15)$$

Similarly, for the short-term and long-run unemployed, we get

$$0 = \max_c -re^{-\gamma(c-c_2)} + r + \gamma r (b_2 - c - m - \tau) + h + f - he^{-\gamma(c_1-c_2)} - fe^{-\gamma(c_3-c_2)}, \quad (16)$$

$$0 = \max_c -re^{-\gamma(c-c_3)} + r + \gamma r (b_3 - c - m - \tau) + h + he^{-\gamma(c_1-c_3)}.$$

Equations (15) and (16) are maximized at  $c = c_j$ , implying that for the Bellman equation to be satisfied the constants  $c_j$ , has to satisfy

$$\begin{aligned} c_1 &= w - \tau - \frac{q(e^{\gamma\Delta_2} - 1)}{\gamma r} \\ c_2 &= b_2 - m - \tau + \frac{h(1 - e^{-\gamma\Delta_2})}{\gamma r} - \frac{f(e^{\gamma(\Delta_3 - \Delta_2)} - 1)}{\gamma r} \\ c_3 &= b_3 - m - \tau + \frac{h(1 - e^{-\gamma\Delta_3})}{\gamma r}. \end{aligned}$$

The IC2 constraint is given by

$$c_2 \geq c_{2,n}$$

$$\begin{aligned} b_2 - m - \tau + \frac{h(1 - e^{-\gamma(c_1 - c_2)}) - f(e^{\gamma(c_2 - c_3)} - 1)}{\gamma r} &\geq b_2 - \tau - \frac{f(e^{\gamma(c_{2,n} - c_3)} - 1)}{\gamma r} \\ h(1 - e^{-\gamma(c_1 - c_2)}) - f(e^{\gamma(c_2 - c_3)} - 1) - \gamma r m &\geq -f(e^{-\gamma(c_3 - c_{2,n})} - 1) \\ h(1 - e^{-\gamma\Delta_2}) &\geq \gamma r m + f(e^{\gamma(c_2 - c_3)} - e^{-\gamma(c_3 - c_{2,n})}) \\ &= \gamma r m + f e^{-\gamma c_3} (e^{\gamma c_2} - e^{\gamma c_{2,n}}) \\ &\geq \gamma r m + f e^{-\gamma c_3} (e^{\gamma c_{2,n}} - e^{\gamma c_{2,n}}) \\ &= \gamma r m \end{aligned}$$

Which can be written,

$$\Delta_2 \geq -\frac{\ln(1 - \frac{\gamma r m}{h})}{\gamma}$$

To find the indifference curves we look at the problem

$$\begin{aligned} \max_{\Delta_2, \Delta_3} \quad & w - \tau + \frac{q(1 - e^{\gamma\Delta_2})}{\gamma r} \\ \text{s.t.} \quad & \text{IC2, IC3} \\ & \tau = \Pi_2 b_2 + \Pi_3 b_3 \\ b_2 = w - \Delta_2 + m - \frac{q(e^{\gamma\Delta_2} - 1)}{\gamma r} - \frac{h(1 - e^{-\gamma\Delta_2})}{\gamma r} + \frac{f(e^{\gamma(\Delta_3 - \Delta_2)} - 1)}{\gamma r} \\ b_3 = w - \Delta_3 + m - \frac{q(e^{\gamma\Delta_2} - 1)}{\gamma r} - \frac{h(1 - e^{-\gamma\Delta_3})}{\gamma r} \end{aligned}$$

Implying

$$\begin{aligned}\frac{\partial \tau}{\partial \Delta_2} &= -\Pi_2 \left( 1 + \frac{q}{r} e^{\gamma \Delta_2} + \frac{h}{r} e^{-\gamma \Delta_2} + \frac{f}{r} e^{\gamma(\Delta_3 - \Delta_2)} \right) - \Pi_3 \frac{q}{r} e^{\gamma \Delta_2} \\ \frac{\partial \tau}{\partial \Delta_3} &= \Pi_2 \left( \frac{f}{r} e^{\gamma(\Delta_3 - \Delta_2)} \right) - \Pi_3 \left( 1 + \frac{h}{r} e^{-\gamma \Delta_3} \right)\end{aligned}$$

and

$$\begin{aligned}\frac{d\Delta_2}{d\Delta_3} \Big|_{\bar{v}} &= \frac{-\frac{\partial \tau}{\partial \Delta_3}}{\left( \frac{\partial \tau}{\partial \Delta_2} + q \frac{e^{\gamma \Delta_2}}{r} \right)} \\ &= -\frac{\Pi_2 \left( \frac{f}{r} e^{\gamma(\Delta_3 - \Delta_2)} \right) - \Pi_3 \left( 1 + \frac{h}{r} e^{-\gamma \Delta_3} \right)}{-\Pi_2 \left( 1 + \frac{q}{r} e^{\gamma \Delta_2} + \frac{h}{r} e^{-\gamma \Delta_2} + \frac{f}{r} e^{\gamma(\Delta_3 - \Delta_2)} \right) - \Pi_3 \frac{q}{r} e^{\gamma \Delta_2} + q \frac{e^{\gamma \Delta_2}}{r}}\end{aligned}$$

When  $\Delta_3 = 0$ , this is  $> 0$ , given that  $\Delta_2 > 0$ .

## 7.2 Adverse selection and no saving

We can write the value functions when the individuals are undertaking the investments as

$$\begin{aligned}V_1 &= -e^{\gamma \tau} e^{-\gamma w} \frac{1 - qp(1 - e^{\gamma m})}{r + q} + q \frac{(1 - p)V_2}{r + q} \\ V_2 &= -\frac{e^{\gamma \tau} e^{-\gamma b_2}}{r + h + f} + \frac{hV_1 + fV_3}{r + h + f} \\ V_3 &= -\frac{e^{\gamma \tau} e^{-\gamma b_2}}{r + h} + \frac{hV_1}{r + h}.\end{aligned}$$

Solving this yields

$$\begin{aligned}V_1 &= -e^{\gamma \tau} e^{-\gamma w} \frac{(1 - \bar{\Pi}_2 - \bar{\Pi}_3) (1 - qp(1 - e^{\gamma \bar{m}})) + \bar{\Pi}_2 e^{\gamma \Delta_2} + \bar{\Pi}_3 e^{\gamma \Delta_3}}{r} \\ V_2 &= -e^{\gamma \tau} e^{-\gamma w} \left( \frac{\frac{h}{h+r} (1 - qp(1 - e^{\gamma \bar{m}})) (1 - \bar{\Pi}_2 - \bar{\Pi}_3)}{r} \right) \\ &\quad - e^{\gamma \tau} e^{-\gamma w} \left( \frac{\frac{r+q(1-p)}{q(1-p)} \bar{\Pi}_2 e^{\gamma \Delta_2} + \frac{r+q(1-p)}{q(1-p)} \bar{\Pi}_3 e^{\gamma \Delta_3}}{r} \right), \\ V_3 &= -e^{\gamma \tau} e^{-\gamma w} \left( \frac{\frac{h}{h+r} (1 - qp(1 - e^{\gamma \bar{m}})) (1 - \bar{\Pi}_2 - \bar{\Pi}_3)}{r} \right) \\ &\quad - e^{\gamma \tau} e^{-\gamma w} \left( \frac{\frac{h}{h+r} \bar{\Pi}_2 e^{\gamma \Delta_2} + \left( 1 + \frac{r(h+f+q(1-p))}{(1-p)qf} \right) \bar{\Pi}_3 e^{\gamma \Delta_3}}{r} \right),\end{aligned}$$

where  $\bar{\Pi}_2$  and  $\bar{\Pi}_3$  are defined in (11).

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