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IFN Working Paper No. 1309, 2019

# **Power Against Random Expenditure Allocation for Revealed Preference Tests**

Per Hjertstrand

# POWER AGAINST RANDOM EXPENDITURE ALLOCATION FOR REVEALED PREFERENCE TESTS

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## Abstract

This paper proposes new power indices for revealed preference tests. The indices are based on a model of irrational consumption behavior where the consumer randomly allocates a certain fraction of expenditure. The methods allow a researcher to trace out the entire power curve against random expenditure allocation. The power indices are illustrated on altruistic choices in experimental data.

**JEL Classification:** C14; D12

**Keywords:** GARP; Power; Revealed preference

## 1 Introduction

The standard way of testing whether consumer choice data is consistent with rationality, i.e., the hypothesis of utility maximization, is to apply revealed preference tests. These procedures test whether the data satisfies certain axioms, such as the generalized axiom of revealed preference (GARP). If the data satisfies GARP, then there exists a continuous, strictly increasing and concave utility function rationalizing the data (Varian, 1982).

However, when GARP is violated it does not tell us how well the utility maximization model fits the data, which is usually referred to as goodness-of-fit.<sup>1</sup> Conversely, when GARP is satisfied it does not tell us how stringent the test is, that is, if GARP has enough bite to reject rationality if the data was generated by some type of irrational behavior. This is commonly referred to as the *power of the test*.

For this reason, empirical studies most often accompany the GARP test with diagnostic measures, such as goodness-of-fit and power, to assess how well the test performs. This paper proposes simple procedures to calculate the power against random expenditure allocation. A novel feature of these procedures is that it allows a researcher to trace out the entire power curve against random expenditure allocation.

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<sup>1</sup>Several goodness-of-fit measures has been proposed in the literature. The most prominent is the Afriat efficiency index proposed by Afriat (1972) and Varian (1990). For other measures, see Houtman and Maks (1985) and more recently, Echenique et al. (2012) and Dean and Martin (2016). Varian (1985) proposed to deal with violations of GARP by testing whether these can be attributed to measurement errors in the data (Jones and Edgerton, 2009, provide a recent survey of the literature on measurement errors in revealed preference testing; See also Demuyneck and Hjertstrand, 2019).

Our procedures are generalizations of the uniformly random power index proposed by Bronars (1987). This index is based on Becker’s (1962) model of irrationality, where consumption choices are uniformly distributed on the frontier of the budget set.<sup>2</sup> Bronars’ uniformly random power index is based on the assumption that the consumer randomly allocates the entire expenditure among the goods. However, this type of irrational behavior has been criticized as being too naive, and consequently, not a very appealing alternative hypothesis to rationality (Breunig et al., 1998; Varian, 2006; Andreoni et al., 2013). Nevertheless, as Varian (2006) notes, there seem to be few alternative hypotheses other than uniformly random behavior that can be applied using the same sorts of data used for revealed preference analysis. For that reason, Bronars’ index has been widely used in empirical applications and is unequivocally the most popular power index in the literature.<sup>3</sup>

Instead, we propose a notion of irrational behavior where the consumer only randomly allocates a certain *fraction* of the expenditure. This corresponds to a model of irrationality given by the weighted average of the observed data and Bronars’ purely uniformly random data. The weight can be interpreted as the fraction of expenditure that is being randomly allocated. We propose a new power index, called the partial uniform random power (PURP) index, based on this model of irrational behavior. By varying the weight and, consequently, the fraction of expenditure that is randomly allocated, our model can be used to trace out the entire power curve against uniformly random expenditure allocation.

A potentially unappealing property of the PURP index is that it generates budget shares that are not centered on the actual (observed) budget shares. We therefore introduce a generalization of the PURP index, called the centered partial uniform random power (cPURP) index, where budget shares are centered on the observed shares. Thus, while the cPURP index still is based on the notion that the consumer only randomly allocates a certain fraction of the expenditure, it differs from the PURP index in the sense that the expected value of the budget shares are equal to the observed shares. As such, the cPURP index relates to another power index proposed by Bronars (1987), which also generates shares that are centered on the observed shares.<sup>4</sup>

Revealed preference procedures are often used in experimental economics to test whether the choices of subjects are rational. Andreoni and Miller (2002) analyzed if altruistic choices of subjects are consistent with revealed preference in an experimental framework. We illustrate the PURP and cPURP indices to their data, and find that although GARP have good power against purely uniform random behavior for many subjects, the power is sensitive to the amount of random expenditure allocation. Hence, we find that the power can be rather low for more realistic random expenditure allocations.

This paper is organized as follows. Section 2 briefly recalls the generalized axiom of revealed preference (GARP) from Varian (1982). Sections 3 and 4 introduces the PURP and cPURP indices. Section 5 contains the empirical application and Section 6 concludes.

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<sup>2</sup>Bronars (1987) calls his uniformly random power index Algorithm 1.

<sup>3</sup>Andreoni et al. (2013) propose three alternative power indices. Their Afriat power index is a measure of how much budget sets would need to be shifted outward in order to generate a violation of GARP. The optimal placement index is a measure of how well an experimental design (i.e., the exogeneous choice of prices and expenditure in the data) performed relative to the best possible design that could have been generated ex post. The jittering index is a measure of how noisy the data must be in order for an experimental design to have any power. Heufer (2014) propose a conditional index, which calculates the power against uniformly random behavior, where GARP is a necessary but not sufficient condition (e.g., homotheticity and separability).

<sup>4</sup>Bronars calls this power index Algorithm 3.

## 2 The generalized axiom of revealed preference

Suppose a consumer chooses from  $K \geq 2$  goods observed in  $T \geq 2$  time periods. The goods and time periods are indexed by  $\mathbb{K} = \{1, \dots, K\}$  and  $\mathbb{T} = \{1, \dots, T\}$ , respectively. Let  $\mathbf{x}_t = (x_{1t}, \dots, x_{Kt}) \in \mathbb{R}_+^K$  denote the observed quantity-vector at time  $t \in \mathbb{T}$  with corresponding price-vector  $\mathbf{p}_t = (p_{1t}, \dots, p_{Kt}) \in \mathbb{R}_{++}^K$ . We assume that budgets are exhaustive, so that budget shares can be expressed as (for all  $k \in \mathbb{K}$  and  $t \in \mathbb{T}$ ):

$$w_{kt} = \frac{p_{kt}x_{kt}}{\mathbf{p}_t\mathbf{x}_t}. \quad (1)$$

Consider the concept of revealed preferences: for two observations  $s, t \in \mathbb{T}$ ,  $\mathbf{x}_t$  is directly revealed preferred to  $\mathbf{x}_s$  written  $\mathbf{x}_t R^D \mathbf{x}_s$  if  $\mathbf{p}_t(\mathbf{x}_t - \mathbf{x}_s) \geq 0$ , which means that  $\mathbf{x}_t$  was chosen when in fact  $\mathbf{x}_s$  also was affordable. We say that  $\mathbf{x}_t$  is revealed preferred to  $\mathbf{x}_s$ , written  $\mathbf{x}_t R \mathbf{x}_s$ , if there exists a sequence of observations  $(t, u, v, \dots, w, s) \in \mathbb{T}$  such that  $\mathbf{x}_t R^D \mathbf{x}_u, \mathbf{x}_u R^D \mathbf{x}_v, \dots, \mathbf{x}_w R^D \mathbf{x}_s$ . The data  $\mathbb{D} = \{\mathbf{p}_t, \mathbf{x}_t\}_{t \in \mathbb{T}}$  satisfies the Generalized Axiom of Revealed Preference (GARP) if  $\mathbf{x}_t R \mathbf{x}_s$  implies  $\mathbf{p}_s(\mathbf{x}_s - \mathbf{x}_t) \leq 0$  (Varian, 1982).

The importance of GARP for empirical and theoretical work can be summarized in what has become known as Afriat's theorem (Afriat, 1967). We say that a utility function  $U(\mathbf{x})$  rationalizes a data set  $\mathbb{D} = \{\mathbf{p}_t, \mathbf{x}_t\}_{t \in \mathbb{T}}$  if  $\mathbf{p}_t\mathbf{x}_t \geq \mathbf{p}_t\mathbf{x}$  implies  $U(\mathbf{x}_t) \geq U(\mathbf{x})$  for any  $\mathbf{x} \in \mathbb{R}_+^K$  and all  $t \in \mathbb{T}$ . Varian's (1982) formulation of Afriat's theorem states that there exist a continuous, strictly increasing and concave utility function rationalizing the data  $\mathbb{D}$  if and only if  $\mathbb{D}$  satisfies GARP. In practice, GARP can be efficiently implemented in standard statistical and mathematical software, and has become the most popular test-method in empirical applications of consumer rationality.<sup>5</sup>

## 3 The partial uniform random power (PURP) index

The power of a revealed preference test, such as GARP, is defined as the probability of rejecting GARP, given that the consumption choices were generated from some type of irrational behavior. Bronars (1987) proposes a power index, where the irrational behavior is based on Becker's (1962) uniformly random model. In his model, budget shares are uniformly distributed on the unit simplex, which implies that the consumption choices are uniformly distributed on the frontier of the budget set.

Bronars' uniformly random power index imposes the restriction that the *entire* expenditure is randomly allocated among the goods. A more realistic scenario is that a consumer only randomly allocates a *fraction* of the expenditure. In this model, the budget share  $w_k^q$  for any good  $k \in \mathbb{K}$  corresponds to the weighted average between the observed budget share  $w_k$  and the uniformly random budget share  $w_k^U$ , i.e.,

$$w_k^q = (1 - \lambda)w_k + \lambda w_k^U. \quad (2)$$

The parameter  $\lambda \in [0, 1]$  can be interpreted as the fraction of the expenditure that is randomly allocated.<sup>6</sup> This model encompasses a continuum of sub-models. At the one extreme  $\lambda = 0$ , it conform to when no part of the expenditure is being randomly allocated, in which case  $w_k^q$  is equal to the observed budget

<sup>5</sup>See Demuyne and Hjertstrand (2019) for a recent survey of Afriat's theorem.

<sup>6</sup>Assuming that  $w_k^U$  are Dirichlet distributed random variables with parameters  $(1, 1, \dots, 1)$ , the expected value, variance, covariance and correlation of the budget shares in the model (2) are:

$$\begin{aligned} E[w_k^q] &= w_k + \lambda \left( \frac{1}{K} - w_k \right), & \text{and} & & Var[w_k^q] &= \lambda^2 \frac{(K-1)}{K^2(K+1)}; \\ Cov[w_k^q, w_j^q] &= -\lambda^2 \frac{1}{K^2(K+1)}, & \text{and} & & Corr[w_k^q, w_j^q] &= -\frac{1}{(K-1)} \quad \text{for } k \neq j. \end{aligned}$$

share, i.e.,  $w_k^q = w_k$ . At the other extreme  $\lambda = 1$ , the model reduces to Bronars' pure model of uniformly random behavior, implying that  $w_k^q$  is equal to the uniformly random budget share, i.e.,  $w_k^q = w_k^U$ . Thus, in Bronars' power index equal weight is put on all budget shares, i.e.,  $E[w_k^q] = E[w_k^U] = \frac{1}{K}$ .

The model (2) is illustrated in the four graphs on the left-hand side of Figure 1. We assume  $K = 3$  and that the observed budget shares are given by  $(w_1, w_2, w_3) = (0.6, 0.3, 0.1)$  (marked with a red point). Each graph contain 5000 simulated points of  $(w_1^q, w_2^q, w_3^q)$  calculated from (2) for  $\lambda = (0.25, 0.5, 0.75, 1.0)$ .<sup>7</sup> We see that as  $\lambda$  increases, the area containing the observed shares grows and for  $\lambda = 1$  (bottom left graph) it covers the entire unit simplex.

We propose to use (2) as the model of irrational behavior when calculating the power of GARP. The implementation of this power index, which we call the partial uniform random power (PURP) index, takes the following 8 steps.

*Procedure to implement the PURP index*

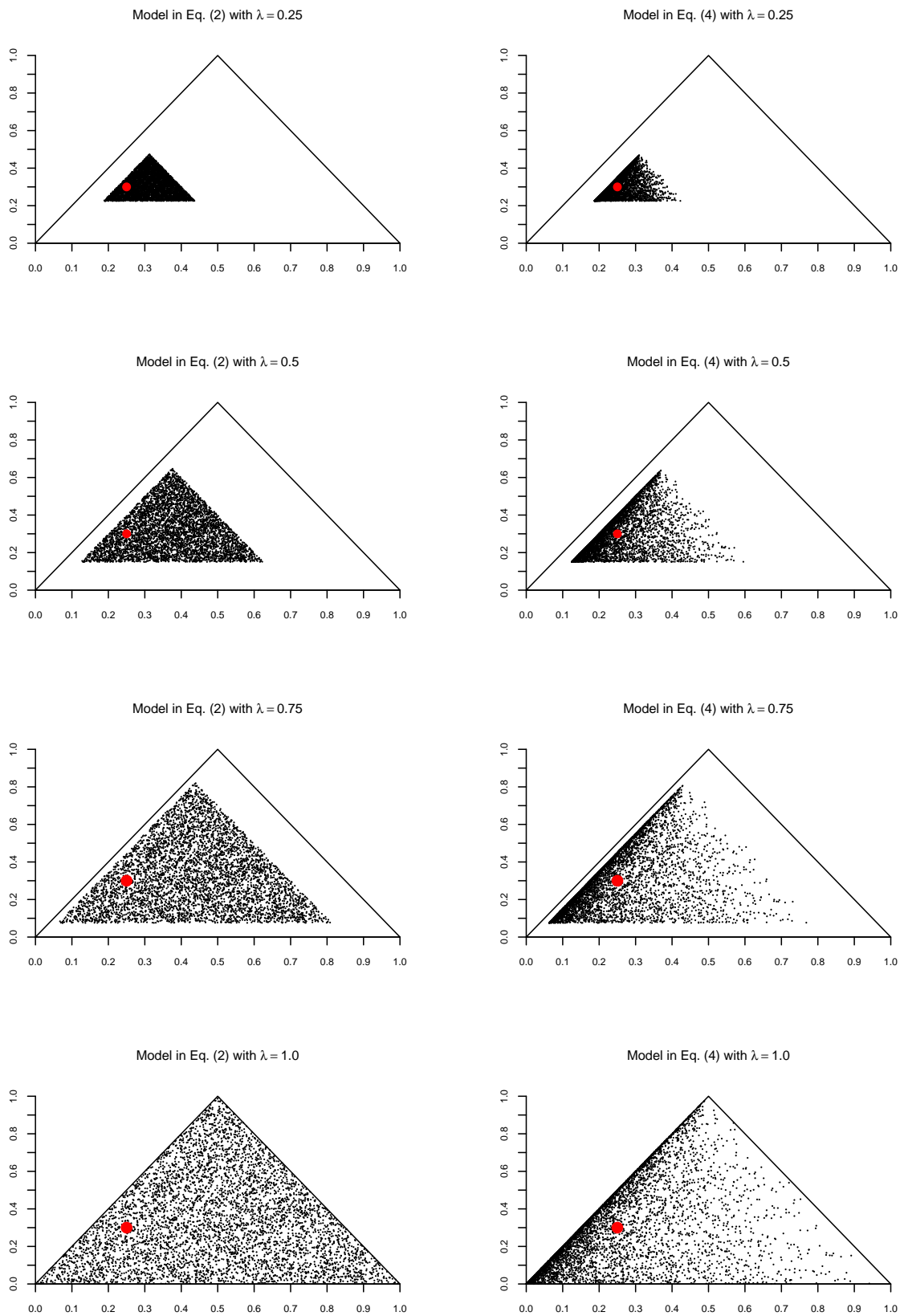
1. Set  $\lambda \in [0, 1]$ .
2. Choose the number of simulations  $S$  and set  $m = 0$ .
3. For each  $t \in \mathbb{T}$ , generate the uniformly random budget shares  $\{w_{kt}^U\}_{k \in \mathbb{K}}$  on the unit simplex from the Dirichlet distribution with all parameters set to one, i.e.,  $(w_{1t}^U, \dots, w_{Kt}^U) \sim \text{Dir}(1, \dots, 1)$ . This can easily be done by first generating Gamma(1,1) random variables, denoted as  $\{G_{kt}\}_{k \in \mathbb{K}}$ , by drawing  $K$  independent uniformly random numbers  $u_{kt} \sim U_{(0,1)}$ , and calculate  $G_{kt} = -\ln u_{kt}$ . The Dirichlet random numbers are then given by  $w_{kt}^U = G_{kt} / \sum_{j=1}^K G_{jt}$  for all  $k \in \mathbb{K}$ .
4. For each  $t \in \mathbb{T}$ , calculate  $\{w_{kt}^q\}_{k \in \mathbb{K}}$  from (2), where the observed budget shares  $\{w_{kt}\}_{k \in \mathbb{K}}$  are calculated from (1).
5. Given prices,  $p_{kt}$ , and expenditure,  $\mathbf{p}_t \mathbf{x}_t$ , solve for the random quantities,  $q_{kt}$ , corresponding to  $w_{kt}^q$  as:
$$q_{kt} = w_{kt}^q \frac{\mathbf{p}_t \mathbf{x}_t}{p_{kt}}, \quad (3)$$
and define  $\mathbf{q}_t = (q_{1t}, \dots, q_{Kt})$  for all  $t \in \mathbb{T}$ .<sup>8</sup>
6. If  $\{\mathbf{p}_t, \mathbf{q}_t\}_{t \in \mathbb{T}}$  violates GARP then set  $m = m + 1$ .
7. Repeat steps 3-6  $S$  times.
8. The power is equal to  $m/S$ .

The PURP index has two important properties. First, it is just as easy to implement as Bronars' uniformly random power index, and can be applied using the same sorts of data used for revealed preference analysis. Second, it can trace out the entire power curve against uniformly random expenditure allocation by implementing the model at each node in an equally-spaced grid for  $\lambda \in [0, 1]$ .

<sup>7</sup>Each simulated three-dimensional point represents a vector that is projected on the two-dimensional simplex in every graph. The vertices in the simplex are given by:  $(1, 0, 0) \rightarrow (0, 0)$ ,  $(0, 1, 0) \rightarrow (1, 0)$  and  $(0, 0, 1) \rightarrow (0.5, 1)$ .

<sup>8</sup>This step transforms the random budget shares  $w_{kt}^q$  to random quantities which are located at the frontier of the budget set.

Figure 1: 5000 simulated points on the unit simplex from the model in Eq. (2) and the model in Eq. (4) for different values of  $\lambda$ . The red points are the observed budget shares given by  $(w_1, w_2, w_3) = (0.6, 0.3, 0.1)$ .



## 4 The centered partial uniform random power (cPURP) index

A possibly unappealing feature of the PURP index is that it does not generate budget shares that are centered on the observed budget shares.<sup>9</sup> In this section, we propose a generalization of the PURP index that satisfies this property. As above, we consider a model where the budget shares, in this case denoted by  $w_k^z$ , are a linear combination of the observed budget share  $w_k$  and a random budget share denoted by  $w_k^C$ , i.e.,

$$w_k^z = (1 - \lambda) w_k + \lambda w_k^C. \quad (4)$$

However, in contrast to the PURP index where the random budget shares follow a Dirichlet distribution with all parameters set to one, in this model, the random budget shares  $(w_1^C, \dots, w_K^C)$  follow a Dirichlet distribution with parameters  $(K \times w_1, \dots, K \times w_K)$ . This ensures that the expected value of the random budget shares is equal to the observed shares, i.e.,  $E[w_k^C] = w_k$  for all  $k \in \mathbb{K}$ . Hence, the budget shares  $w_k^z$  in the model (4) are *always* centered on the observed shares since,

$$\begin{aligned} E[w_k^z] &= E[(1 - \lambda) w_k + \lambda w_k^C] \\ &= (1 - \lambda) w_k + \lambda E[w_k^C] \\ &= (1 - \lambda) w_k + \lambda w_k \\ &= w_k. \end{aligned}$$

As before, the parameter  $\lambda \in [0, 1]$  is the fraction of expenditure that is randomly allocated.<sup>10</sup> The graphs on the right-hand side of Figure 1 plots 5000 simulated shares of  $w_k^z$  from the model (4) for different values of  $\lambda$ . We see that the simulated shares for the model (4) are more scattered around the observed shares  $(w_1, w_2, w_3) = (0.6, 0.3, 0.1)$  than in the model (2).

We propose the centered partial uniform random power (cPURP) index based on the model of irrationality in (4). The cPURP index is implemented as follows.

*Procedure to implement the cPURP index*

1-2. *Same as above.*

3. *For each  $t \in \mathbb{T}$ , generate the random budget shares  $\{w_{kt}^C\}_{k \in \mathbb{K}}$  on the unit simplex from the Dirichlet distribution  $(w_{1t}^C, \dots, w_{Kt}^C) \sim \text{Dir}(K \times w_{1t}, \dots, K \times w_{Kt})$ . These variables can be simulated by first drawing  $G_{kt} \sim \text{Gamma}(K \times w_{kt}, 1)$  and then setting  $w_{kt}^C = G_{kt} / \sum_{j=1}^K G_{jt}$  for all  $k \in \mathbb{K}$ .*

4. *For each  $t \in \mathbb{T}$ , calculate  $\{w_{kt}^z\}_{k \in \mathbb{K}}$  from (4), where the observed budget shares  $\{w_{kt}\}_{k \in \mathbb{K}}$  are calculated from (1).*

5. *Given prices,  $p_{kt}$ , and expenditure,  $\mathbf{p}_t \mathbf{x}_t$ , solve for the random quantities,  $z_{kt}$ , corresponding to  $w_{kt}^z$  as:*

$$z_{kt} = w_{kt}^z \frac{\mathbf{p}_t \mathbf{x}_t}{p_{kt}}, \quad (5)$$

*and define  $\mathbf{z}_t = (z_{1t}, \dots, z_{Kt})$  for all  $t \in \mathbb{T}$ .*

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<sup>9</sup>The budget shares in the PURP index are centered on the observed budget shares only if  $w_k = \frac{1}{K}$  for all  $k \in \mathbb{K}$ , which implies  $E[w_k^z] = w_k = \frac{1}{K}$ .

<sup>10</sup>The variance, covariance and correlation of the budget shares in the model (4) are:

$$\text{Var}[w_k^z] = \lambda^2 \frac{w_k(1 - w_k)}{(K + 1)}, \quad \text{Cov}[w_k^z, w_j^z] = -\lambda^2 \frac{w_k w_j}{(K + 1)} \quad \text{and} \quad \text{Corr}[w_k^z, w_j^z] = -\sqrt{\frac{w_k w_j}{(1 - w_k)(1 - w_j)}} \quad \text{for } k \neq j.$$

6. If  $\{\mathbf{p}_t, \mathbf{z}_t\}_{t \in \mathbb{T}}$  violates GARP then set  $m = m + 1$ .

7-8. Same as above.

Thus, the cPURP index differs from the PURP index only by step 3. Specifically, it is easy to see that the cPURP index reduces to the PURP index when the observed budget shares  $(w_{1t}, \dots, w_{Kt})$  are equal across goods, i.e., if  $w_{kt} = \frac{1}{K}$  for all  $k \in \mathbb{K}$  and  $t \in \mathbb{T}$ , in which case, we have  $\text{Dir}(K \times w_{1t}, \dots, K \times w_{Kt}) \stackrel{d}{=} \text{Dir}(1, \dots, 1)$ . In the next section, we apply the PURP and cPURP indices to trace out the power curves for consumer choice data from an experiment of altruistic preferences.

## 5 Application

Andreoni and Miller (2002) tested whether the altruistic choices of experimental subjects are rational. They employed a generalized dictator game in which one subject (the dictator) allocates token endowments between himself and an anonymous other subject with different transfer rates. The payoffs of the dictator and the beneficiary are interpreted as two distinct goods and the transfer rates as the price ratio. The experiment was split in two parts, where 142 subjects (Group 1) faced  $T = 8$  decision rounds while 34 subjects (Group 2) faced  $T = 11$  decision rounds. They used GARP to test for rationality and found that 13 subjects (9%) in Group 1 and 5 subjects (15%) in Group 2 violated GARP. They also reported a Bronars' uniformly random power index of 78.1% for Group 1 and 94.7% for Group 2.<sup>11</sup> Andreoni et al. (2013) provided a more detailed power analysis by applying their own three indices and all three of Bronars' power indices.

We complement the analyses in Andreoni and Miller (2002) and Andreoni et al. (2013) by calculating the PURP and cPURP indices for those subjects in Groups 1 and 2 that satisfy GARP. By excluding subjects that violate GARP, our indices are based on irrational models where data consistent with GARP is mixed with random data. We implement the indices at each node using a grid of 0.02 for  $\lambda$  (starting at 0 and ending at 1) and the number of simulations in step 2 is set to  $S = 10,000$ .

**Results from the PURP index.** Figure 2(a) traces out the power curves for the 129 subjects in Group 1 that satisfy GARP using the PURP index (i.e., with  $E[w_{kt}^U] = 1/K$ ).

Table 1 gives summary statistics over these 129 subjects for different values of  $\lambda$ . The results are mixed. The power curves for most subjects are essentially zero up to a random expenditure allocation of 20% ( $\lambda = 0.2$ ). For a random expenditure allocation of 50% ( $\lambda = 0.5$ ), we see that 75% (3rd quartile) of all subjects have a power of 34.6% or less. The average power over all subjects is 46.7% for a random expenditure allocation of 70%.

Figure 2(c) traces out the power curves for the 29 subjects in Group 2 that satisfy GARP using the PURP index. Table 2 gives summary statistics over the 29 subjects for different values of  $\lambda$ . The power is generally higher for subjects in this group. However, notice that it is still equal or less than 38% for 75% (3rd quartile) of all subjects at a random expenditure allocation of 50%.

**Results from the cPURP index.** Figure 2(b) traces out the power curves for the 129 subjects in Group 1 that satisfy GARP using the cPURP index (i.e., with  $E[w_{kt}^z] = w_{kt}$ ). Table 3 gives summary statistics over the 129 subjects for different values of  $\lambda$ . These results show that the power decreases

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<sup>11</sup>Because the subjects in each group faced the same prices and budgets, Bronars' uniformly random power index is the same for every subject in each group.

Figure 2: Power plots for the PURP and cPURP indices over all subjects that satisfy GARP in Groups 1 and 2.

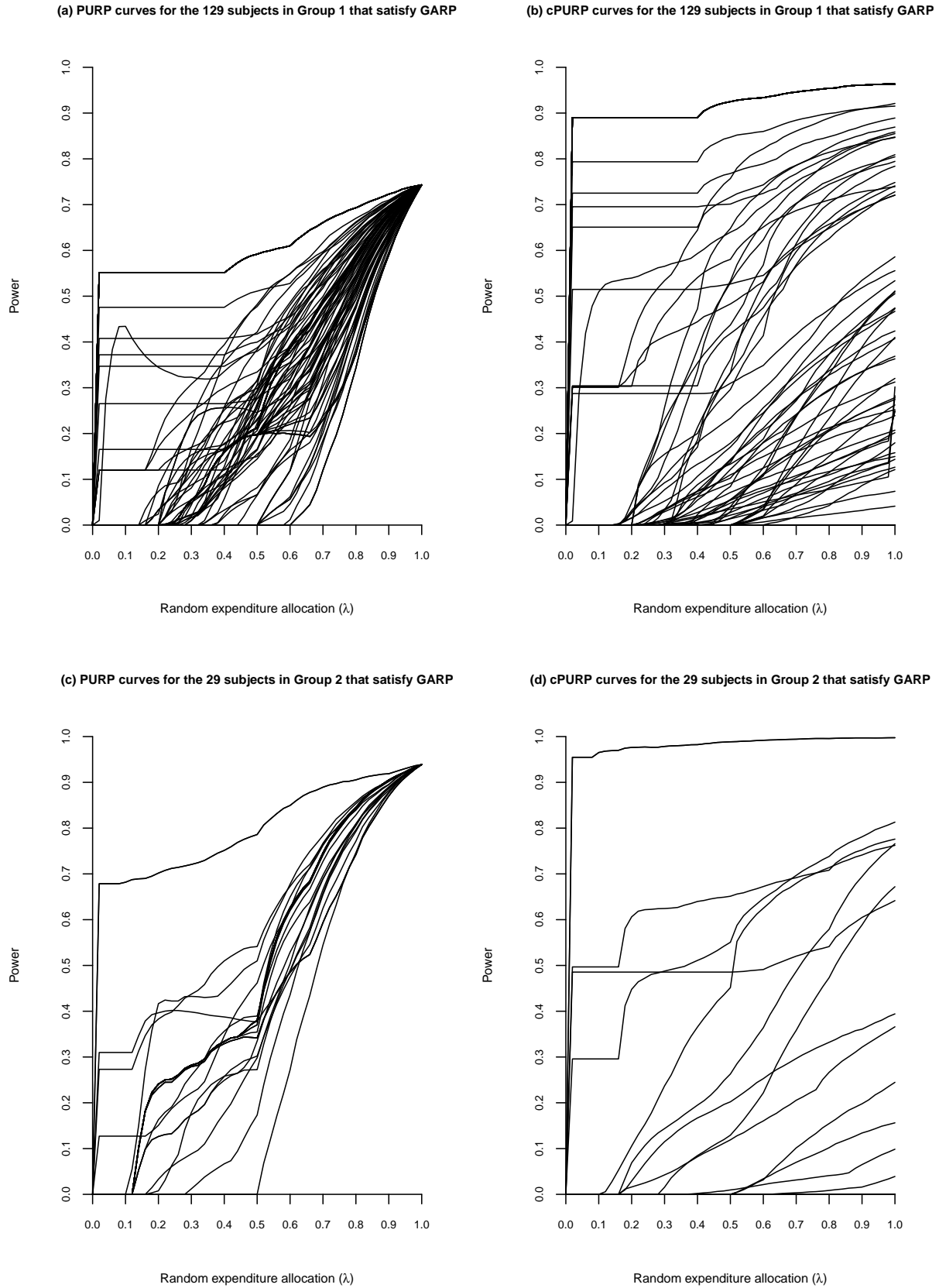


Table 1: Summary statistics for the PURP index over the 129 subjects in Group 1 that satisfy GARP for different values of  $\lambda$ .

| $\lambda$ | Mean  | Min   | 1st quartile | Median | 3rd quartile | Max   |
|-----------|-------|-------|--------------|--------|--------------|-------|
| 0.1       | 0.121 | 0     | 0            | 0      | 0.120        | 0.551 |
| 0.2       | 0.124 | 0     | 0            | 0      | 0.132        | 0.551 |
| 0.3       | 0.163 | 0     | 0.055        | 0.055  | 0.226        | 0.551 |
| 0.4       | 0.225 | 0     | 0.159        | 0.159  | 0.274        | 0.551 |
| 0.5       | 0.279 | 0     | 0.197        | 0.212  | 0.346        | 0.592 |
| 0.6       | 0.371 | 0     | 0.272        | 0.386  | 0.454        | 0.610 |
| 0.7       | 0.467 | 0.120 | 0.393        | 0.489  | 0.547        | 0.661 |
| 0.8       | 0.572 | 0.343 | 0.528        | 0.583  | 0.631        | 0.693 |
| 0.9       | 0.675 | 0.589 | 0.658        | 0.681  | 0.696        | 0.723 |
| 1         | 0.743 | 0.743 | 0.743        | 0.743  | 0.743        | 0.743 |

Table 2: Summary statistics for the PURP index over the 29 subjects in Group 2 that satisfy GARP for different values of  $\lambda$ .

| $\lambda$ | Mean  | Min   | 1st quartile | Median | 3rd quartile | Max   |
|-----------|-------|-------|--------------|--------|--------------|-------|
| 0.1       | 0.071 | 0     | 0            | 0      | 0            | 0.682 |
| 0.2       | 0.243 | 0     | 0.160        | 0.241  | 0.241        | 0.702 |
| 0.3       | 0.287 | 0     | 0.250        | 0.282  | 0.282        | 0.721 |
| 0.4       | 0.341 | 0     | 0.329        | 0.334  | 0.337        | 0.751 |
| 0.5       | 0.385 | 0     | 0.342        | 0.380  | 0.380        | 0.786 |
| 0.6       | 0.587 | 0.272 | 0.488        | 0.623  | 0.626        | 0.850 |
| 0.7       | 0.717 | 0.545 | 0.664        | 0.746  | 0.746        | 0.889 |
| 0.8       | 0.824 | 0.742 | 0.803        | 0.841  | 0.845        | 0.905 |
| 0.9       | 0.896 | 0.870 | 0.887        | 0.903  | 0.905        | 0.919 |
| 1         | 0.939 | 0.939 | 0.939        | 0.939  | 0.939        | 0.939 |

Table 3: Summary statistics for the cPURP index over the 129 subjects in Group 1 that satisfy GARP for different values of  $\lambda$ .

| $\lambda$ | Mean  | Min | 1st quartile | Median | 3rd quartile | Max   |
|-----------|-------|-----|--------------|--------|--------------|-------|
| 0.1       | 0.201 | 0   | 0            | 0      | 0.301        | 0.890 |
| 0.2       | 0.203 | 0   | 0            | 0      | 0.303        | 0.890 |
| 0.3       | 0.217 | 0   | 0            | 0      | 0.331        | 0.890 |
| 0.4       | 0.234 | 0   | 0            | 0      | 0.451        | 0.890 |
| 0.5       | 0.261 | 0   | 0            | 0.022  | 0.530        | 0.925 |
| 0.6       | 0.285 | 0   | 0            | 0.044  | 0.630        | 0.934 |
| 0.7       | 0.313 | 0   | 0            | 0.098  | 0.702        | 0.946 |
| 0.8       | 0.334 | 0   | 0            | 0.136  | 0.746        | 0.954 |
| 0.9       | 0.353 | 0   | 0            | 0.163  | 0.777        | 0.961 |
| 1         | 0.370 | 0   | 0            | 0.239  | 0.806        | 0.963 |

substantially when the random budget shares are centered on the observed shares. For example, half of all subjects only have a power of at most 2.2% for a random expenditure allocation of 50%. Moreover, the average power is 37% for a random expenditure allocation of 100%. However, notice from the last column in Table 3 that GARP for a few subjects has a very high power for low random expenditure allocation.

Finally, we consider the cPURP results for the 29 subjects who pass GARP in Group 2. Figure 2(d) traces out the power curves and Table 4 gives summary statistics. The power usually increases with the number of observations. However, in this case, the power is generally lower for the subjects in Group 2 than in Group 1. For example, 75% of all subjects (3rd quartile) has a power lower than 21.7% at a random expenditure allocation of 50%, and the power only increases to 64.9% for an allocation of 100%. As in Group 1, there are a few subjects with very high power at low random expenditure allocations.

## 6 Conclusions

This paper has proposed new power indices for revealed preference tests. The indices are based on a model of irrationality where the consumer only randomly allocates a certain fraction of expenditure. This is distinct from the common power indices, where the consumer is assumed to randomly allocate the entire expenditure. A novel feature of our power indices is that they can be used to trace out the entire power curve against random expenditure allocation. We apply the indices to experimental data and find that the power for subjects may be low for more realistic amounts of random expenditure allocation. The indices are easy to implement and can be readily combined with other power indices such as Heufer’s (2014) conditional power index.

Table 4: Summary statistics for the cPURP index over the 29 subjects in Group 2 that satisfy GARP for different values of  $\lambda$ .

| $\lambda$ | Mean  | Min | 1st quartile | Median | 3rd quartile | Max   |
|-----------|-------|-----|--------------|--------|--------------|-------|
| 0.1       | 0.110 | 0   | 0            | 0      | 0            | 0.965 |
| 0.2       | 0.128 | 0   | 0            | 0      | 0.039        | 0.976 |
| 0.3       | 0.142 | 0   | 0            | 0      | 0.122        | 0.979 |
| 0.4       | 0.155 | 0   | 0            | 0      | 0.174        | 0.982 |
| 0.5       | 0.167 | 0   | 0            | 0      | 0.217        | 0.989 |
| 0.6       | 0.190 | 0   | 0            | 0      | 0.281        | 0.992 |
| 0.7       | 0.211 | 0   | 0            | 0      | 0.390        | 0.994 |
| 0.8       | 0.229 | 0   | 0            | 0      | 0.490        | 0.996 |
| 0.9       | 0.250 | 0   | 0            | 0      | 0.593        | 0.997 |
| 1         | 0.266 | 0   | 0            | 0      | 0.649        | 0.998 |

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