

Adrian, Tobias; Boyarchenko, Nina; Giannone, Domenico

**Working Paper**

## Multimodality in macro-financial dynamics

Staff Report, No. 903

**Provided in Cooperation with:**

Federal Reserve Bank of New York

*Suggested Citation:* Adrian, Tobias; Boyarchenko, Nina; Giannone, Domenico (2019) : Multimodality in macro-financial dynamics, Staff Report, No. 903, Federal Reserve Bank of New York, New York, NY

This Version is available at:

<https://hdl.handle.net/10419/210755>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

Federal Reserve Bank of New York  
Staff Reports

# Multimodality in Macro-Financial Dynamics

Tobias Adrian  
Nina Boyarchenko  
Domenico Giannone

Staff Report No. 903  
November 2019



This paper presents preliminary findings and is being distributed to economists and other interested readers solely to stimulate discussion and elicit comments. The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System. Any errors or omissions are the responsibility of the authors.

## **Multimodality in Macro-Financial Dynamics**

Tobias Adrian, Nina Boyarchenko, and Domenico Giannone

*Federal Reserve Bank of New York Staff Reports*, no. 903

November 2019

JEL classification: C14, E17, E37, G01

### **Abstract**

We estimate the evolution of the conditional joint distribution of economic and financial conditions in the United States, documenting a novel empirical fact: while the joint distribution is approximately Gaussian during normal periods, sharp tightenings of financial conditions lead to the emergence of additional modes—that is, multiple economic equilibria. Although the U.S. economy has historically reverted quickly to a “good” equilibrium after a tightening of financial conditions, we conjecture that poor policy choices under these circumstances could also open a pathway to a “bad” equilibrium for a prolonged period. We argue that such multimodality arises naturally in a macro-financial intermediary model with occasionally binding intermediary constraints.

Key words: density impulse response, multimodality, nonparametric density estimator

---

Boyarchenko: Federal Reserve Bank of New York (email: [nina.boyarchenko@ny.frb.org](mailto:nina.boyarchenko@ny.frb.org)).  
Adrian: International Monetary Fund (email: [tadrian@imf.org](mailto:tadrian@imf.org)). Giannone: Amazon.com, Inc.  
(email: [dgiannon2@gmail.com](mailto:dgiannon2@gmail.com)). The authors thank Patrick Adams and Eric Qiang for providing excellent research assistance. They also thank seminar and conference participants at the Federal Reserve Bank of New York, the Federal Reserve Board, the Bundesbank, the European Central Bank, the International Monetary Fund, Chicago Booth School of Business, Princeton University, Warwick Business School, and London Business School for helpful feedback. This work was completed while Giannone was at the Federal Reserve Bank of New York. The views expressed in this paper are those of the authors and do not necessarily reflect the position of the International Monetary Fund, the Federal Reserve Bank of New York, or the Federal Reserve System.

To view the authors’ disclosure statements, visit  
[https://www.newyorkfed.org/research/staff\\_reports/sr903.html](https://www.newyorkfed.org/research/staff_reports/sr903.html).

# 1 Introduction

“Tout le monde y croit cependant [que les erreurs sont Gaussien], me disait un jour M. Lippmann, car les expérimentateurs s’imaginent que c’est un théorème de mathématiques, et les mathématiciens que c’est un fait expérimental”

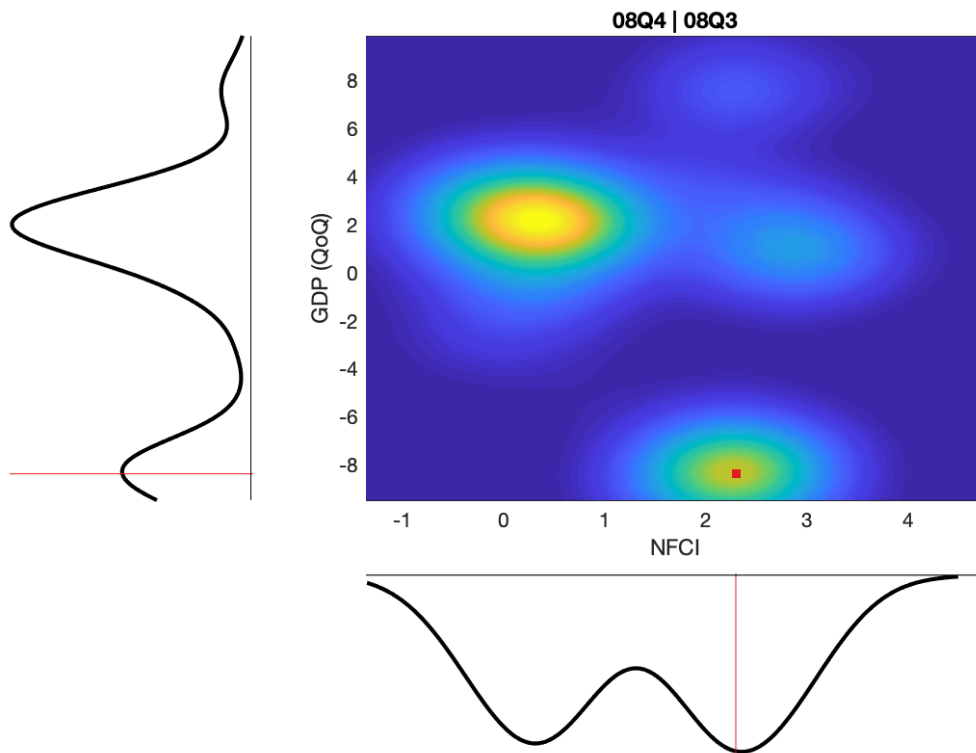
(Henri Poincaré, *Calcul des probabilités*, 2nd ed., 1912, p. 171)

The theoretical literature on macro-financial dynamics has long postulated that the relationship between financial conditions and real activity is nonlinear: while financial conditions are relatively loose, the economy evolves as normal; when financial conditions tighten, the economy can slump into pronounced, lasting, macro-financial feedback loops with (extremely) adverse macroeconomic outcomes. The difficulty in testing and documenting empirically such non-linearities is that the specific nature of the non-linearity is model-specific, with no consensus in the literature on the “right” non-linear specification.

In this paper, we propose a flexible and robust non-parametric approach to characterizing non-linear system dynamics that combines kernel estimation of the one-step-ahead joint distribution (see e.g. Li and Racine, 2007, Chapter 6.2) with an efficient Monte Carlo procedure to generate term structures of joint distributions. Using non-parametric methods allows us to remain agnostic about the nature of dynamic interactions between variables of interest, allowing the data to inform us instead. If we further use independent kernels across variables, the kernel estimation can be easily imbedded into a Gibbs’ sampler, allowing for estimation of joint dynamics of large number of variables of interest, as well as estimation with missing observations of variables of interest.

We apply this approach to estimate the joint dynamics of economic and financial conditions in the United States (U.S.). In our previous work (see Adrian, Boyarchenko, and Giannone, 2019), we documented that the left tail of the conditional distribution of real GDP growth moves together with the tightness of financial conditions. Examining the *joint*

distribution of economic and financial conditions allows us to document a novel empirical fact. While the one-quarter-ahead joint distribution of economic and financial is (roughly) Gaussian and unimodal during “good” times, the shape of the distribution changes dramatically during periods of tight financial conditions, with additional modes emerging.



**Figure 1.** The figure shows the one quarter ahead marginal distributions of economic and financial conditions (real GDP growth and NFCI), together with the contour plot of the joint distribution, for 2008Q4 conditional on data as of 2008Q3. The red square indicates the ex post realization in 2008Q4.

We illustrate this finding in Figure 1, which plots the marginals and the joint one-quarter-ahead distribution of real GDP growth and financial conditions as of Q3 2008. The Figure clearly shows that, in the depth of the financial crisis, both marginal distributions exhibited multimodality: There was a possibility that the economy would resolve within a quarter to a “good” equilibrium with positive economic growth and easier financial conditions, as well as a possibility that the economy would resolve within a quarter to a “bad” equilibrium with

negative economic growth and tight financial conditions.

We show that, more generally, deviations from Gaussianity only emerge during times of unusually tight financial conditions. When the tightness in financial conditions is relatively mild, such as during the 1987 stock market crash, multimodality only appears in short horizon forecasts and resolves within a couple of quarters. When the tightness in financial conditions is more widespread, such as during the 2007-2009 financial crisis, multimodality is evident in longer horizon forecasts and can take up to a year to resolve. Although in the long-run the economy always reverts back to a “good” equilibrium, with positive GDP growth and looser financial conditions, prolonged periods of multimodality or even selection of the “bad” equilibrium can lead to large losses in the level of GDP. We speculate that appropriate policy accommodation is crucial in reverting quicker to the good equilibrium but leave a formal exploration of this hypothesis for future work.

Our methodology allows us to estimate the term structure of joint distributions which, in turn, allow us to estimate density impulse response functions (DIRs) in the spirit of Gallant, Rossi, and Tauchen (1993). DIRs compute the change in the full distribution over horizons of interest, conditional on a perturbation in the one-step-ahead distribution. Analogously to impulse response functions in the linear vector autoregression setting, DIRs provide (time-varying) target moments against which models should be evaluated.

We show that our estimator performs well out-of-sample. In particular, the estimated in-sample distributions for real GDP growth and financial conditions coincide with those estimated out-of-sample. Comparing the overall out-of-sample performance of our non-parametric estimator to two linear vector auto-regressive (VARs) models – one with Gaussian standard errors and one with non-parametric standard errors – our estimator out-performs both alternatives in terms of standard performance metrics, such as probability integral transforms (PITs) and log-scores. Focusing on the forecasts during the 2007–2009 financial crisis, our out-of-sample forecast also exhibits multimodality, though the “bad” equilibrium is not as severe as that predicted in-sample. In contrast, even the VAR with non-parametric

standard errors predicts a unimodal distribution, suggesting the multimodality arises from non-linear propagation of shocks, rather than non-Gaussian innovations.

The remainder of the paper is organized as follows. Section 2 discusses the theoretical foundations of multimodality. Section 3 presents our algorithm to compute the term structures of joint distributions, and discusses our contribution from an econometric perspective. Section 4 shows the empirical results for the joint distribution of economic and financial conditions in the U.S. Section 5 presents the density impulse responses. Section 6 undertakes out-of-sample forecast evaluation. Section 7 concludes.

## 2 The theoretical case for multimodality

Theoretical literature on economic dynamics has long postulated the possibility of multiple equilibria. In a seminal contribution, Diamond (1982) points out that thick market externalities in the labor market can generate a market failure (the so-called “coconut model”). Bryant (1983) develops a model where technological complementarities can similarly give rise to a persistent market failure via bad equilibrium selection. In the fundamental contribution of Diamond and Dybvig (1983), the possibility of bank runs give rise to multiple equilibria. And in Murphy, Shleifer, and Vishny (1989), demand spillovers generate multiple equilibria.

More recently, the macro-finance literature has focused on the wholesale banking sector as a source of equilibrium multiplicity. Gertler and Kiyotaki (2015) and Gertler, Kiyotaki, and Prestipino (2016) study models that account for the buildup and collapse of wholesale banking, and sketch out the transmission of the crises to the real sector by characterizing the sudden and discrete nature of the banking panics as well as the circumstances that makes an economy vulnerable to such panics. Sunspot runs can arise that are harmful to the economy. Whether a run equilibrium exists depends on fundamentals. The probability of a sunspot run is the outcome of a rational forecast based on fundamentals. The Gertler et al. (2016) model thus captures the movement from slow to fast runs that was a feature of the Great

Recession: A weakening of banks' balance sheets increases the probability of a run, leading depositors to withdraw funds from banks.

Models with multiple equilibria naturally translate to multimodal predictive densities, with the “good” equilibrium assigned most of the probability distribution most of the time, but the “bad” equilibrium also receiving positive probability mass during times of crisis. Multimodality on its own, however, is not evidence of multiple equilibria. Instead, multimodality in the conditional distribution can arise due to sharp amplification mechanisms. Indeed, in two fundamental contributions, Morris and Shin (2000, 2002) propose unique equilibrium via imperfect knowledge while preserving amplification in models of strategic complementarity, thereby allowing for amplification mechanisms without multiplicity. In the aftermath of the financial crisis, the macro-finance literature has used this insight to model nonlinear amplification mechanisms through financial constraints, generating strong macro-financial linkages and extreme negative skewness in real outcomes in “bad” times but linear Gaussian dynamics during normal periods.

He and Krishnamurthy (2013) model the dynamics of risk premia during crises in asset markets where the marginal investor is a financial intermediary that faces an equity capital constraint. Risk premia rise when the constraint binds, reflecting capital scarcity. The calibrated model matches the nonlinearity of risk premia during crises and the speed of reversion in risk premia from a crisis back to precrisis levels. Brunnermeier and Sannikov (2014) also study an economy with financial frictions in the intermediation sector. Due to highly nonlinear amplification effects, the economy is prone to instability and occasionally enters volatile crisis episodes. Endogenous risk, driven by asset illiquidity, persists in crisis even for very low levels of exogenous risk. In both He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014), all shocks are conditionally Gaussian, but volatility and drift depend on state variables in highly nonlinear ways due to occasionally binding constraints.

Adrian and Boyarchenko (2012) also develop a theory of financial intermediary leverage



cycles in the context of a dynamic model of the macroeconomy, but in their approach, intermediaries are subject to Value-at-Risk constraints. The interaction between a production sector, a financial intermediation sector, and a household sector gives rise to amplification of fundamental shocks that affect real economic activity. The leverage of the intermediaries is procyclical. Tightening intermediaries' risk constraints affects the systemic risk-return trade-off, by lowering the likelihood of systemic crises at the cost of higher pricing of risk. Adrian and Duarte (2016) also feature an intermediary sector with a Value-at-Risk constraint, and additionally assume sticky prices of goods, as in standard New Keynesian models. They derive a reduced form solution in which the volatility of consumption growth is endogenous and proportional to the price of risk in the economy. Hence the Value-at-Risk constraints ties the conditional mean and the conditional volatility of the economy together.

Finally, most closely related to our results, Fernández-Villaverde, Hurtado, and Nuño (2019) study a heterogeneous households model with a representative intermediary where idiosyncratic risks to household income lead to a multiplicity of stochastic steady states. In the high intermediary leverage stochastic steady state, erosion of intermediary wealth leads to prolonged periods of low wages and low capital, while shocks to intermediary wealth in the low leverage steady state resolve quickly.

### 3 Methodology

We are interested in constructing multi-period-ahead conditional joint distributions of economic and financial conditions. In this section, we describe how to construct the one-period-ahead conditional joint distribution. We then use efficient Markov Chain Monte Carlo (MCMC) to construct multi-period-ahead distributions from the one-period-ahead distribution.

### 3.1 One-period-ahead distribution

Consider a time series dataset of  $n_y$  endogenous variables  $y_{i,t}$ ,  $i = 1, \dots, n_y$  and denote by  $y_t = (y_{1,t}, \dots, y_{n_y,t})'$  the vector of date  $t$  realizations of the  $n$  variables. In addition to the endogenous variables  $y_t$ , suppose that we have  $n_x$  exogenous predictors  $x_t$ . In this paper, we are interested in the case when the exogenous predictors are  $p$  lags of  $y$ , so that  $n_x = p \times n_y$  and

$$x_t = (y'_{t-1}, \dots, y'_{t-p})'.$$

That is, we are interested in estimating a distributional equivalent to vector autoregressions.

We are now ready to write down our kernel estimator (see Li and Racine, 2007, chapter 6.2). Let  $T$  be the number of observations of  $y_t$  that we have available. Then the joint distribution of  $y$  conditional on  $x$  can be estimated as

$$\hat{p}(y|x) = \frac{\frac{1}{T-p} \sum_{t=p+1}^T \mathcal{K}_{\omega_y}^y(y - y_t) \mathcal{K}_{\omega_x}^x(x - x_t)}{\frac{1}{T-p} \sum_{t=p+1}^T \mathcal{K}_{\omega_x}^x(x - x_t)}, \quad (1)$$

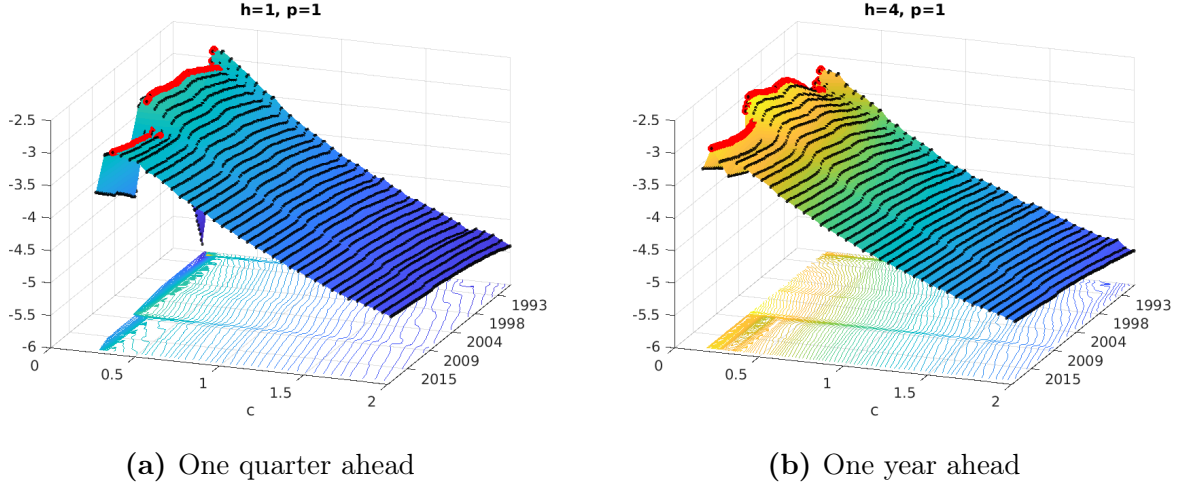
where  $\mathcal{K}_{\omega_y}^y$  and  $\mathcal{K}_{\omega_x}^x$  are independent kernels for  $y$  and  $x$ , given by

$$\mathcal{K}_{\omega_y}^y(y - y_t) = \prod_{i=1}^{n_y} \frac{1}{\omega_{y_i}} \varphi\left(\frac{y_i - y_{i,t}}{\omega_{y_i}}\right) \equiv \prod_{i=1}^{n_y} \mathcal{K}_{\omega_{y_i}}^{y_i}(y_i - y_{i,t}) \quad (2)$$

$$\mathcal{K}_{\omega_x}^x(x - x_t) = \prod_{i=1}^{n_x} \frac{1}{\omega_{x_i}} \varphi\left(\frac{x_i - x_{i,t}}{\omega_{x_i}}\right) \equiv \prod_{i=1}^{n_x} \mathcal{K}_{\omega_{x_i}}^{x_i}(x_i - x_{i,t}). \quad (3)$$

For our baseline results, we use multivariate normal kernels, so that  $\varphi(\cdot)$  is the normal probability distribution function, but the kernel estimation can easily be used with alternative kernels (such as multivariate Student kernels) or be modified to accommodate dependent kernels for the endogenous and exogenous variables.

We parameterize the bandwidths as being proportional to the in-sample unconditional standard deviation of the corresponding variable:  $\omega_{y_i} = c_{y_i} \sigma_{y_i}$ ,  $\omega_{x_i} = c_{x_i} \sigma_{x_i}$ , and choose



**Figure 2. Optimal bandwidth selection.** This figure plots the out-of-sample log predictive scores for the one-period-ahead conditional joint distribution of real activity and financial conditions, as a function of the bandwidth proportionality constant  $c$ . Predictor variables: one lag of economic and financial conditions; time period: Q1 1973 – Q1 2019.

a single proportionally constant  $c = c_{y_i} = c_{x_j}$  for all  $i = 1, \dots, n_y$ ,  $j = 1, \dots, n_x$  to maximize the predictive accuracy of the resultant one-period-ahead conditional joint distribution. We plot the relationship between the bandwidth proportionality constant  $c$  and the out-of-sample (log) predictive score for the one-quarter-ahead and one-year-ahead distribution in Figure 2. The figure shows that the bandwidth selection is in-line with values that would be predicted by asymptotic criteria,<sup>1</sup> that there is an interior solution for the optimal constant of proportionality, and that the performance is maximized for  $c \approx 0.5$ .

### 3.2 Efficient Monte Carlo

Given an estimated one-period-ahead distribution  $\hat{p}(y|x)$ , we can use Monte Carlo simulations to estimate  $h$ -period-ahead distributions by sequentially drawing paths of  $y$ . In principle, these draws can be directly from the inverse CDF implied by  $\hat{p}(y|x)$  by drawing  $u$  from a (multinomial) uniform distribution and finding  $y$  that solves  $y = \hat{P}^{-1}(u|x)$ . We

<sup>1</sup>Li and Racine (2007) apply a version of the Silverman (1986) “rule-of-thumb” for joint unconditional density estimation where  $w_j = 1.06\sigma_j T^{-1/(4+M+N)}$  for variable  $j$ , bandwidth  $w_j$ , standard deviation  $\sigma_j$ , sample size  $T$ , number of independent variables  $M$ , and number of dependent variables  $N$ . The rule is derived from minimizing asymptotic mean integrated square error for a Gaussian reference distribution.

increase the efficiency of this procedure by discretizing the state space as follows.

**Algorithm 1. *Simulating paths of  $y$ .***

*To estimate the  $H$ -period-ahead distribution of  $y$ , generate  $n_{sim}$  paths of  $y$  as follows.*

1. Discretize the state-space. Set  $\kappa = 0.1$ . For each variable  $j$ , loop through:
  - (a) Initialize grid with bound  $[\min(y_j) - \kappa\sigma_{y,j}, \max(y_j) + \kappa\sigma_{y,j}]$  and grid point increments of  $\sigma_{y,j}/20$ .
  - (b) For each grid point  $y_{j,i}$ , compute the kernel CDF  $\Phi\left(\frac{y_j - y_{j,i}}{\omega_{y_j}}\right)$ .
  - (c) Verify that the kernel PDF integrates to one. Verify that the kernel CDF has a maximum of 1 within a tolerance of  $10^{-20}$ . If not, set  $\kappa = 1.05 \times \kappa$  and repeat.
2. For each simulated path  $k = 1, \dots, n_{sim}$ , loop through each horizon  $h = 1, \dots, H$  by drawing  $y_{t+h}^k | y_{t+h-1}^k$  from the grid established in Step 1 and verify the normalization condition.

### 3.3 Alternative estimators and other considerations

Multi-period conditional distributional forecasts provide a natural non-parametric counterpart to traditional VARs. Though “standard” VARs are linear, they have proven extraordinarily useful in forecasting and scenario design, helping establish many stylized facts to guide and validate economic modeling. More recent literature has extended this traditional VAR literature to nonlinear but parametric settings, as reviewed extensively in Kilian and Lutkepohl (2018). Terasvirta and Anderson (1992); van Dijk, Terasvirta, and Franses (2002); Kilian and Taylor (2003) propose smooth transition models. Cogley and Sargent (2005); Primiceri (2005); Cogley and Sbordone (2008); D’Agostino, Gambetti, and Giannone (2013); Del Negro and Primiceri (2015); Carriero, Clark, and Marcellino (2018b) develop time varying parameters VARs with stochastic volatility. Altissimo and Violante (2001) propose thresholded VARs where recent applications include Ascari and Haber (2019); Auerbach and

Gorodnichenko (2013); Aikman, Lehnert, Liang, and Modugno (2016); Caggiano, Castelnovo, and Figueres (2019). Hamilton (1989); Sims and Zha (2006); Chang, Choi, and Park (2017); Hubrich and Tetlow (2015) offer Markov switching VARs. Aruoba, Bocola, and Schorfheide (2017) use an approach of quadratic autoregressions with pruning. Relative to the approach proposed in our paper, this literature has the potential draw back of relying on particular parametrizations of the non-linearity, which may not correspond to any particular theoretical model of non-linearity. We examine the statistical performance of one particular linear VAR that relaxes the Gaussianity assumption in Section 6.

The first step in our methodology – the estimation of the one-period-ahead conditional joint distribution – can, of course, be implemented using alternative estimation methods. For example, Gallant et al. (1993) propose using splines. Norets and Pati (2017) and Sims (2000) deploy Bayesian non-parametric methods. Koenker, Leorato, and Peracchi (2013), and Adrian et al. (2019) use quantile or distributional regressions. We focus on kernel density estimation as it provides a relatively straightforward and easily implementable method for characterizing the distribution. An additional advantage is that the kernel density estimator can be easily modified to accommodate a Gibbs’ sampling approach; that is, we can easily use a combination of the kernel density estimator and Gibbs’ sampling to estimate joint distributions of large numbers of variables.

More specifically, construct the distribution of  $y_i$  conditional on  $x$  and all the other endogenous variables  $y_{-i}$  as

$$\hat{p}_i(y_i|x, y_{-i}) = \frac{\frac{1}{T-p} \sum_{t=p+1}^T \mathcal{K}_{\omega_y}^y(y - y_t) \mathcal{K}_{\omega_x}^x(x - x_t)}{\frac{1}{T-p} \sum_{t=p+1}^T \mathcal{K}_{\omega_x}^x(x - x_t) \prod_{k \neq i} \mathcal{K}_{\omega_{y_k}}^{y_k}(y_k - y_{k,t})}. \quad (4)$$

By construction, each  $\hat{p}_i(y_i|x, y_{-i})$  corresponds to the same joint distribution of  $y|x$ : the numerator in (4) is the same for all  $i = 1, \dots, n_y$ , and is identical to the numerator in (1). Thus, a Gibbs’ sampler that draws realizations of  $y_t$  by cycling through the conditional draws

from (4) converges.<sup>2</sup>

To make multiperiod draws with the Gibbs sampler, we modify the procedure in Algorithm 1 as follows.

**Algorithm 2.** *Simulating paths of  $y$  using the Gibbs sampler.* To estimate the  $H$ -period-ahead distribution of  $y$ , generate  $n_{sim}$  paths of  $y$  as follows.

1. Discretize the state-space. Set  $\kappa = 0.1$ . For each variable  $j$ , loop through:
  - (a) Initialize grid with bound  $[\min(y_j) - \kappa\sigma_{y,j}, \max(y_j) + \kappa\sigma_{y,j}]$  and grid point increments of  $\sigma_{y,j}/20$ .
  - (b) For each grid point  $y_{j,i}$ , compute the kernel CDF  $\Phi\left(\frac{y_j - y_{j,i}}{\omega_{y_j}}\right)$ .
  - (c) Verify that the kernel PDF integrates to one. Verify that the kernel CDF has a maximum of 1 within a tolerance of  $10^{-20}$ . If not, set  $\kappa = 1.05 \times \kappa$  and repeat.
2. Initialize  $y_{j,t+1}^1$  at the conditional mean of variable  $j$ .
3. For each simulated path  $k = 1, \dots, n_{sim}$ , loop through each horizon  $h = 1, \dots, H$ :
  - (a) Draw  $y_{1,t+h}^k | y_{2,t+h}^{k-1}, \dots, y_{N,t+h}^{k-1}, y_{t+h-1}^k$  from the grid established in Step 1 and verify the normalization condition.
  - (b) Draw  $y_{2,t+h}^k | y_{1,t+h}^k, y_{3,t+h}^{k-1}, \dots, y_{N,t+h}^{k-1}, y_{t+h-1}^k$  from the grid established in Step 1 and verify the normalization condition.
  - $\vdots$
  - (c) Draw  $y_{N,t+h}^k | y_{1,t+h}^k, \dots, y_{N-1,t+h}^k, y_{t+h-1}^k$  from the grid established in Step 1 and verify the normalization condition.

4. Discard first DISCARD draws.

---

<sup>2</sup>See e.g. Arnold, Castillo, and Sarabia (1999), Ch. 1.

Finally, the kernel density approach combined with a Gibbs sampler can easily accommodate missing observations of the endogenous variables: given a candidate distribution of  $y_i|y_{-i}, x$ , we can draw the missing observations of  $y_i$ , and then re-estimate the conditional distribution (4).

### 3.4 Data

To gauge economic and financial conditions, we use the Chicago Fed National Activity Index (CFNAI) and the National Financial Conditions Index (NFCI).<sup>3</sup> The NFCI provides a weekly estimate of U.S. financial conditions in money markets, debt and equity markets, and the traditional and shadow banking systems. The index is a weighted average of 105 measures of financial activity, each expressed relative to their sample averages and scaled by their sample standard deviations.<sup>4</sup> The methodology for the NFCI is described in Brave and Butters (2012) and is based on the quasi maximum likelihood estimators for large dynamic factor models developed by Doz, Giannone, and Reichlin (2012). The data for the NFCI starts in January 1973, which we use as starting point for our empirical investigation. We average the weekly NFCI data within the quarter to obtain a quarterly NFCI series.

Figure 3 shows the time series of QoQ real GDP growth, and the financial conditions index, NFCI, together with recession shadings. Real GDP growth is lower and financial conditions tighten during recessions, so that economic and financial conditions are (negatively) correlated. Adrian et al. (2019) document that, although the correlation is stronger during recessions, economic and financial conditions are correlated during normal times as well.

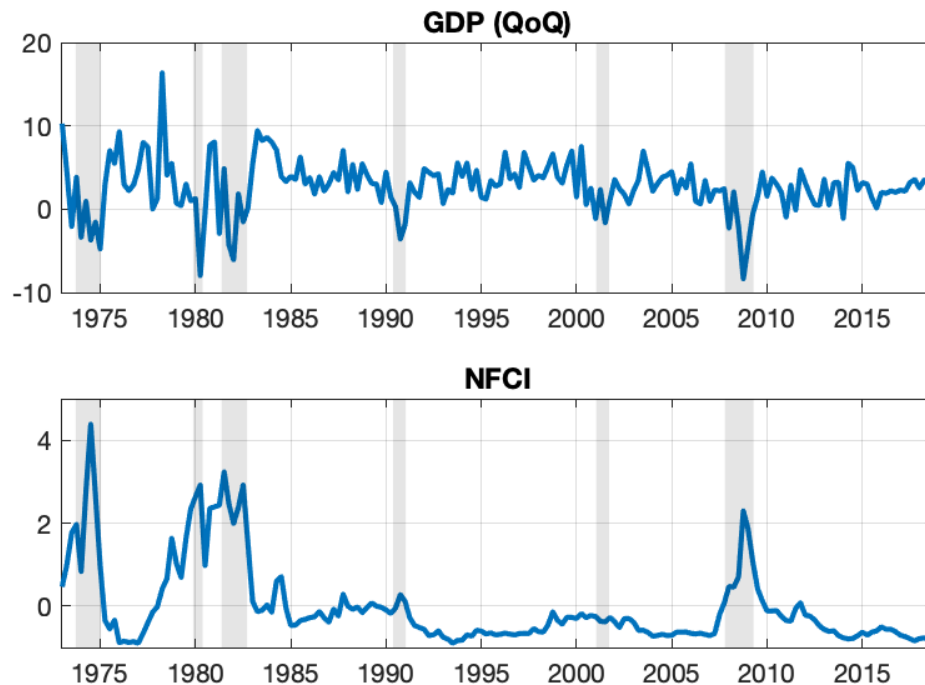
## 4 Joint distribution of economic and financial activity

We begin by examining the joint distribution of economic and financial conditions. We show that tight financial conditions lead to the emergence of multimodality in the joint

---

<sup>3</sup>The NFCI is computed by the Federal Reserve Bank of Chicago, available [here](#), respectively.

<sup>4</sup>The list of indicators is provided [here](#).



**Figure 3. Economic and financial conditions.** The figure shows the time series of QoQ real GDP growth and the financial conditions index, NFCI, together with NBER recession shadings.

distribution: the economy can either continue in a good economic conditions, loose financial conditions state or transition to an adverse economic conditions, tight financial conditions state.

## 4.1 Time series evolution of the joint distribution

We estimate the joint distribution of economic and financial conditions following the procedure in Section 3, using one lag of CFNAI and NFCI as the conditioning variables. Figure 4 plots the estimated one-quarter-ahead marginal (Figure 4a) and joint (Figure 4b) distributions over time. Consider first the marginal distributions of economic and financial conditions. As we documented in Adrian et al. (2019), the upper quantiles of the conditional distribution of economic conditions are stable over time, while the lower quantiles vary with



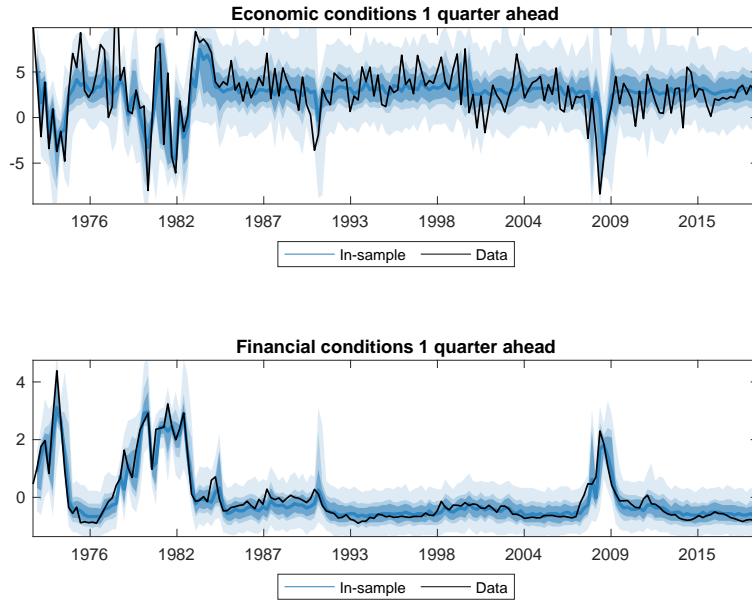
the state of the business cycle, as a function of financial conditions.<sup>5</sup> This feature arises as the conditional mean and the conditional variance of economic conditions are strongly forecasted by financial conditions: when financial conditions are easy, economic growth is forecast to be high, and economic risk is low. The negative correlation between the conditional first and second moment generates such asymmetrical behavior of the upper and lower quantiles on the conditional distribution. Turning to financial conditions, we see that the one-quarter-ahead marginal distribution of financial conditions is much more symmetric: changes in the upper quantiles of the distribution are mimicked by changes in the lower quantiles, so that an increase in the median of the distribution translates into a parallel shift of the distribution.

The novel result in this paper is the joint conditional distribution of real GDP growth and financial conditions, plotted in Figure 4b. More specifically, for each quarter in the sample, Figure 4b presents the contour plot of the one-quarter-ahead joint distribution of real GDP growth and financial conditions. Contour plots shaped like symmetric disks correspond to (joint) Gaussian distributions: the conditional distribution is equally uncertain about both improvements and deteriorations to both economic and financial conditions. Figure 4b shows that this is the “normal” mode of the one-quarter-ahead joint distribution of economic and financial conditions. Though the mode of the distribution moves around over time, during “good” times, the joint distribution looks close to being symmetric around that mode.

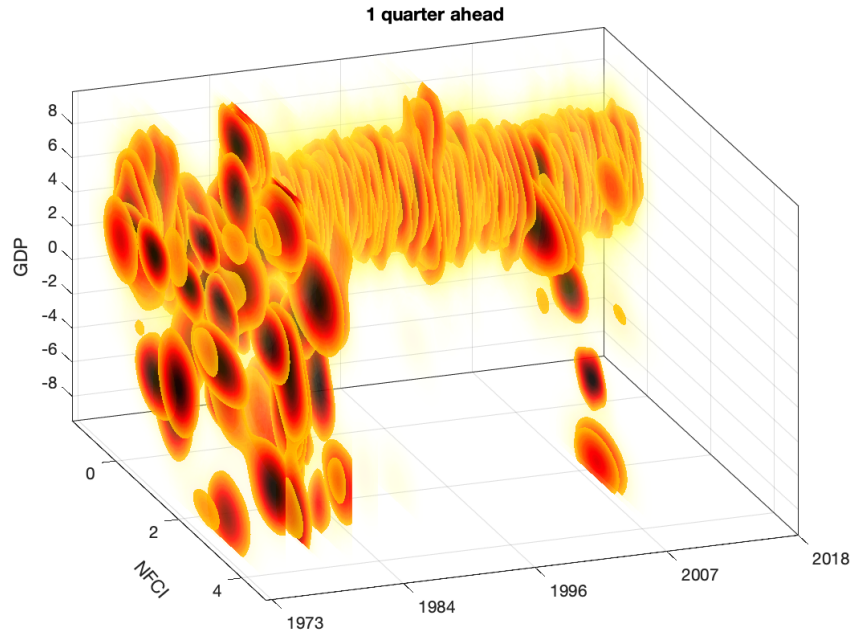
Instead, if we focus on periods of tight financial conditions, the shape of the joint distribution changes radically and additional modes of the distribution emerge. At the one quarter horizon, multimodality emerges in the early 1980s during the Volcker disinflation episode, in 1987 around the stock market crash, in 1998 around the LTCM crisis, and starting in

---

<sup>5</sup>Across data and methodologies, many papers have confirmed the stability of upside growth risk and the variability of its downside risk, often referred to as “vulnerable growth.” These results have been validated with quantile regression (Adrian, Grinberg, Liang, and Malik, 2018), at higher frequencies (Ferrara, Mogliani, and Sahuc, 2019), conditionally on shocks (Loria, Matthes, and Zhang, 2018), with VAR with stochastic volatility affecting the conditional mean (Carriero, Clark, and Marcellino, 2018a; Caldara, Scotti, and Zhong, 2019), with Markov switching models (Doz, Ferrara, and Pionnier, 2019), in labor markets (Kiley, 2018), and in housing markets (Valckx, Deghi, Katagiri, Khadarina, and Shahid, 2019).



(a) Marginal: One quarter ahead



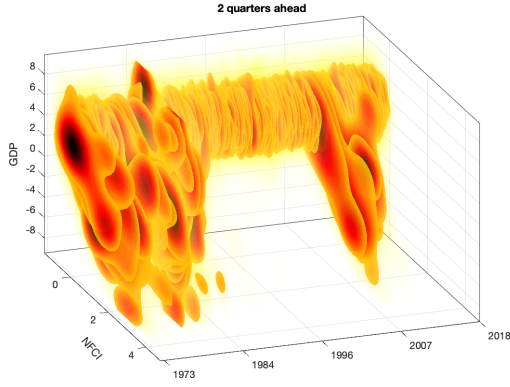
(b) Joint: One quarter ahead

**Figure 4. Density forecasts.** The figure shows the marginal (top) and joint distributions (bottom) for one quarter ahead forecasts. In Figure 4a, the blue shades give the 68%, and 95% quantile bands, while the solid blue line gives the median. Figure 4b presents the contour plots of the joint distribution in every quarter in the sample, with darker shades of red correspond to higher probability densities.

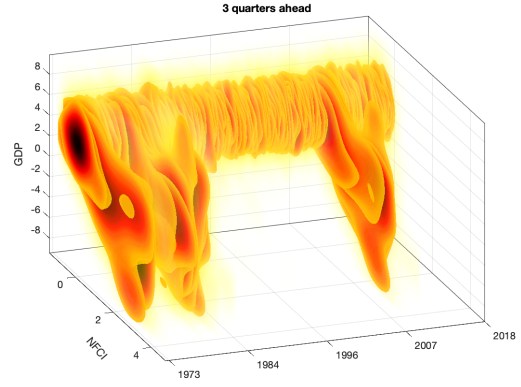
late 2007 for the great financial crisis. That is, when current financial conditions are tighter than normal, the estimated conditional distribution elongates and additional modes emerge. In the “good” mode, the economy returns to a good economic conditions, loose financial conditions state. In the “bad” mode(s), the economy transitions to a bad economic conditions, tight financial conditions state. When tight financial conditions are widespread in the economy, such as during the Volcker disinflation episode and during the great financial crisis, the additional modes are particularly adverse, and periods of multimodality last for multiple quarters. When tight financial conditions are more contained, such as during the 1987 stock market crash and during the LTCM crisis, the secondary modes are more moderate and the economy resolves quicker to the good mode.

In sum, Figure 4 shows that the joint distribution of economic and financial conditions exhibits distinctly non-Gaussian behavior during periods of tight financial conditions. As we showed in Adrian et al. (2019), a univariate model for the evolution of real GDP growth, even a non-parametric one, does not capture the multimodality in the distribution of real GDP growth that arises during periods of tight financial conditions. Conditioning the evolution of real GDP growth on the tightness of financial conditions is thus crucial in uncovering the time-varying multimodality we document in this paper. That is, financial conditions Granger-cause real GDP growth in a distributional sense.

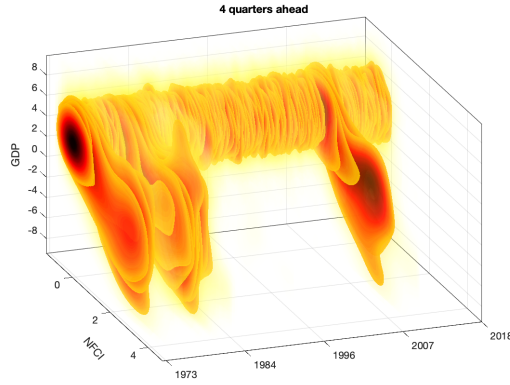
Figure 5 demonstrates that this non-Gaussian at the one-quarter horizon persists at longer forecast horizons. The tightening of financial conditions during the Volcker disinflation episode – the tightest financial conditions in our sample – generates pronounced non-Gaussianity in the joint distribution of economic and financial conditions up to four quarters out. During the great financial crisis, the forecasting distribution is non-Gaussian up to three quarters out; corresponding to the intuition that more pronounced and widespread tightenings of financial conditions generate more persistent periods of multimodality, the non-Gaussianity after the 1987 stock market crash and the LTCM crisis get resolved within two quarters.



(a) Two quarters ahead

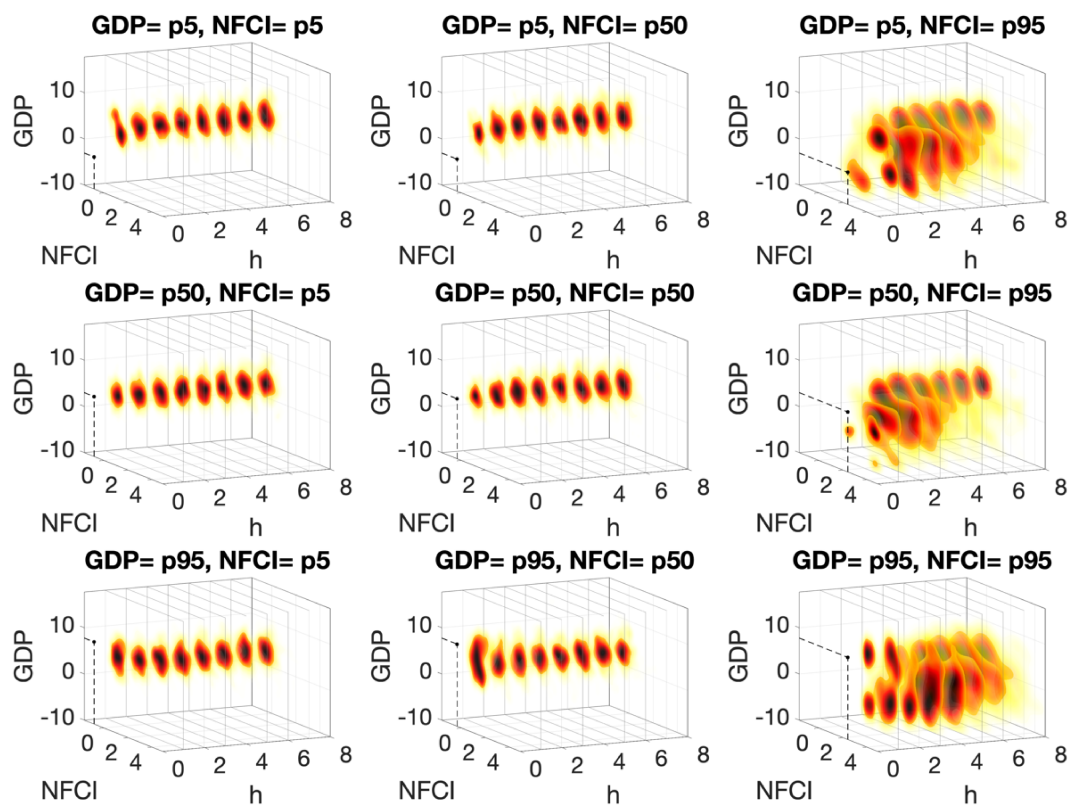


(b) Three quarters ahead



(c) Four quarters ahead

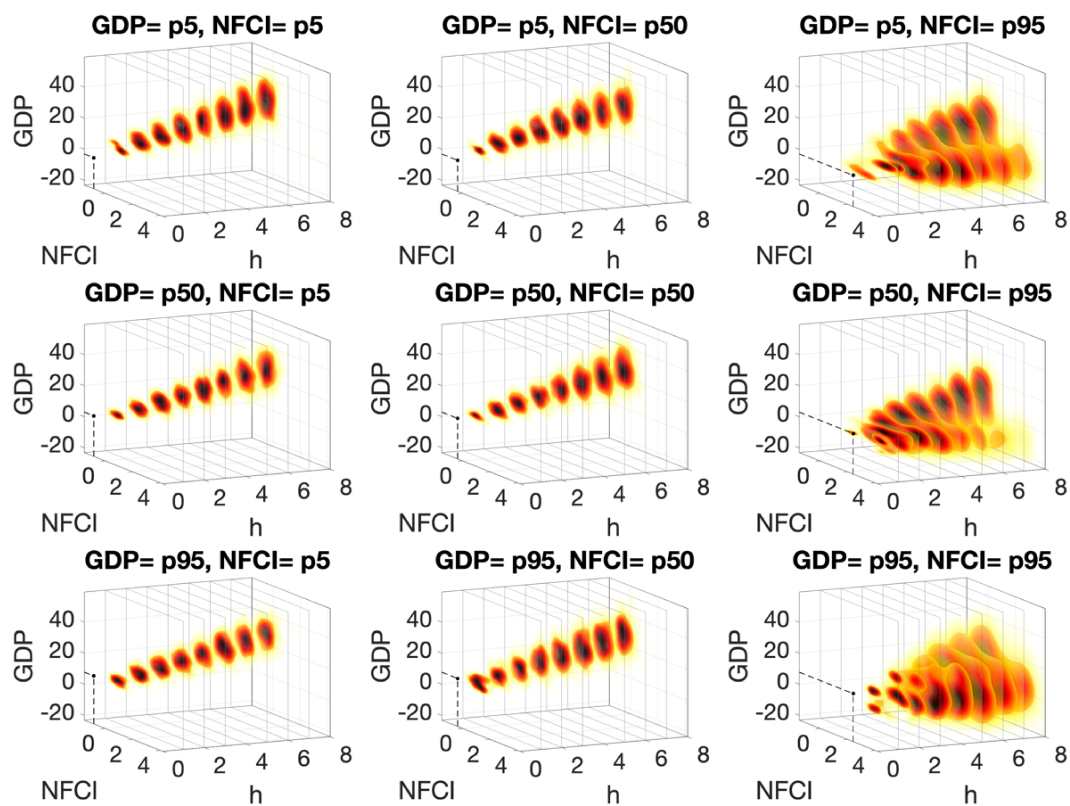
**Figure 5. Joint distributions across horizons.** The figure shows the joint distribution for two, three and four quarter ahead forecasts, as contour plots of the joint distribution in every quarter in the sample, with darker shades of red correspond to higher probability densities.



**Figure 6. Inspecting the mechanism.** The figure displays the evolution of joint QoQ real GDP growth and NFCI forecasts by varying the initial conditions (5 percentile, 50 percentile, and 95 percentile) for up to eight quarters ahead. Darker shades of red highlight regions of higher probability densities.

Figure 6 illustrates this mechanism by plotting the joint distribution across horizons up to two years ahead (eight quarters ahead), conditional on real GDP growth realization at the bottom fifth percentile, the median, and the top fifth percentile (moving across rows) and on financial conditions realization at the bottom fifth percentile, the median, and the top fifth percentile (moving across columns). The figure shows that, when financial conditions are tight (top fifth percentile), multimodality emerges and persists at even longer horizons regardless of the realization of real GDP growth. Even if real GDP growth is in the top fifth percentile of historical realizations, if financial conditions are in the tightest fifth percentile, the economy is fragile and additional modes are possible even two years out. In contrast, if real GDP growth is in the bottom fifth percentile of historical realizations, the distribution is non-Gaussian but unimodal but the non-Gaussianity disappears at horizons longer than two quarters, suggesting that the transmission mechanism between financial and economic conditions is asymmetric. If exceptionally tight financial conditions coincide with exceptionally adverse economic conditions, so that both real GDP growth and NFCI are in their respective worst fifth quantile, the one-quarter-ahead distribution is unimodal but concentrated in the bad mode, and multimodality is present from two quarters out.

Figure 7 shows that small non-Gaussianities in the distribution of QoQ real GDP growth cumulate into large non-Gaussianities for real GDP growth over multiple quarters. That is, even if the  $h$ -quarter ahead joint distribution of QoQ real GDP growth and financial conditions is roughly Gaussian, the joint distribution of  $h$ -quarter real GDP growth and financial conditions can still have multiple modes if the one-quarter-ahead is multimodal: although the distribution resolves eventually to the “good” state, how long the resolution takes can have profound implications on the level of GDP growth. Thus, macrofinancial linkages are at the heart of multimodality, which can manifest in two ways. If current financial conditions are tight but economic conditions are not particularly adverse, the predicted short- and medium-horizon distributions assign non-negligible probability both to the possibility of financial conditions loosening and economic conditions improving, and to the possibility



**Figure 7. Distribution of cumulative growth.** The figure displays the evolution of joint cumulative real GDP growth and NFCI forecasts by varying the initial conditions (5 percentile, 50 percentile, and 95 percentile) for up to eight quarters ahead. Darker shades of red highlight regions of higher probability densities.

of financial conditions tightening further and economic conditions declining. If, on the other hand, current financial conditions are tight and economic conditions are adverse, the short-run forecast keeps the conditional distribution concentrated at the bad mode but allows for the possibility of returning to the looser financial conditions, improved economic conditions state in the longer run.

## 4.2 Case study: Great Recession

Instead of catalyzing recovery, the financial system is working against recovery.[...] This is a dangerous dynamic, and we need to arrest it. [...] We believe that action has to be sustained until recovery is firmly established. In the United States in the 30s, Japan in the 90s, and in other cases around the world, previous crises lasted longer and caused greater damage because governments applied the brakes too early. We cannot make that mistake.

Remarks introducing the Capital Assistance Program for the U.S. financial system,  
Treasury Secretary Geithner, February 10, 2009.<sup>6</sup>

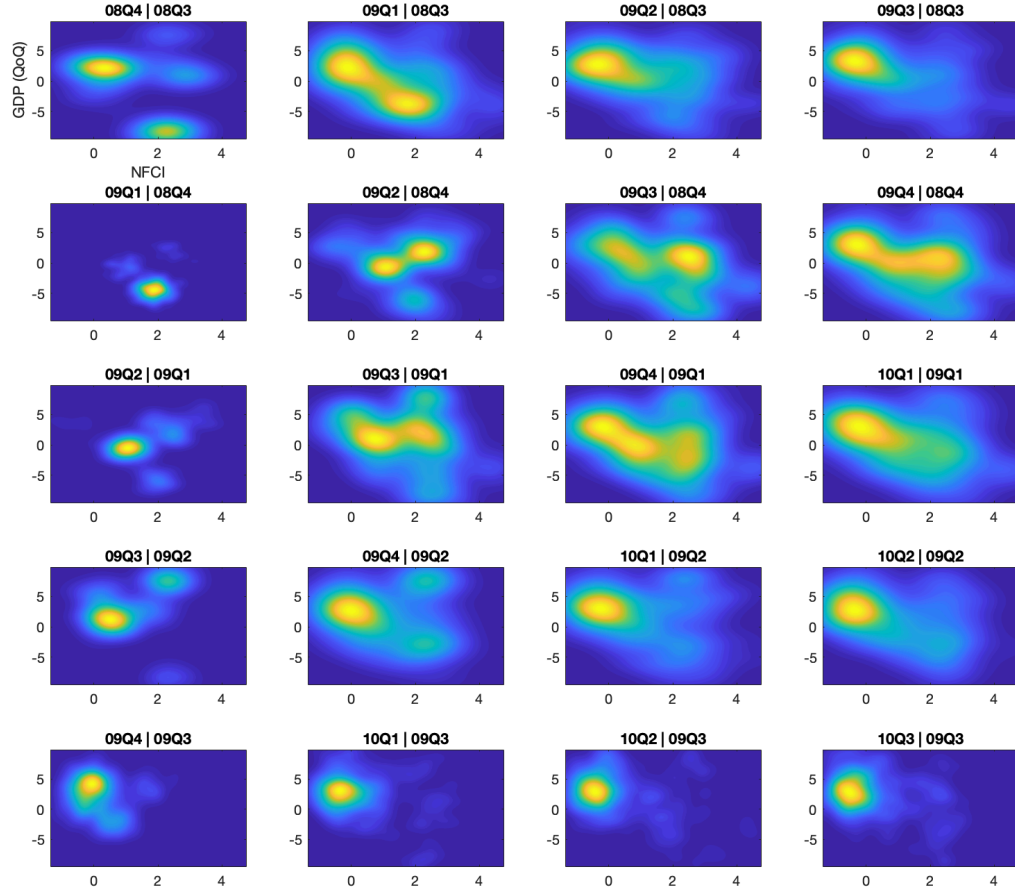
In his remarks introducing the Capital Assistance Program, then-Secretary Geithner pointed to the possibility that absence of decisive policy action could lead to a negative outcome for financial conditions with persistent adverse economic consequences. Figure 8 illustrates how these dynamics played out during the Great Recession in the joint distribution of real GDP growth and financial conditions, with different columns corresponding to forecast horizons from one (leftmost column) to four (rightmost column) quarters, and different rows corresponding to different conditioning information from Q3 2008 (top row) to Q3 2009 (bottom row).

Consider first the four distributions predicted using information as of Q3 2008, plotted in the top row of Figure 8. At the one quarter horizon, the predicted distribution has two easily

---

<sup>6</sup>See <https://www.treasury.gov/press-center/press-releases/Pages/tg18.aspx>





**Figure 8. Joint distribution of economic and financial conditions during the Great Recession.** Contour plots of 1–4 quarter-ahead density forecasts of real GDP growth and NFCI during the financial crisis. Brighter colors indicate great probability.

distinguishable modes: a “good” mode, with higher growth and looser financial conditions, and a “bad” mode, with lower growth and tighter financial conditions. Importantly, the predicted distribution as of Q3 2008 assigns greater probability mass to the bad mode, and the good mode has close to zero real GDP growth and roughly average tightness of financial conditions (NFCI around 0). As we extend the horizon of the forecast, the predicted distribution assigns progressively more weight to the good mode, so that the multimodality clearly visible at the one and two quarter horizons becomes more muddled at the three and four quarter horizon, and the good outcome becomes the prevalent mode.

So how do the predicted distributions change as we move through the crisis? Focusing on the one-quarter-ahead distributions plotted in the left most column, we see that the outlook gets worse before it gets better: while the distributions forecasted as of Q4 2008 and Q1 2009 still exhibit multimodality, the good mode all but disappears. The resolution of the joint dynamics of economic and financial conditions to the good mode starts in Q2 2009 with the re-emergence of the good secondary mode to the distribution, and the full normalization to a unimodal joint distribution is completed in Q3 2009. The longer-horizon predicted distributions follow the same pattern. The two- and three-quarter-ahead distributions concentrate less mass on the good mode in Q4 2008 and Q1 2009 but start to resolve to the unimodal distribution in Q2 2009. The four-quarter-ahead distribution shows the possibility for additional modes emerging through Q2 2009 but also resolves to the unimodal norm in Q3 2009.

In sum, Figure 8 suggests that although the forecast distribution always resolves back to the good outcome at longer forecast horizons, the transition to that state can be long and arduous. The longer the economy takes to resolve to the good mode, the greater the potential losses to the level of economic activity. We examine cross-country differences in how quickly the economy resolves to the good mode in the next section.

## 5 Density impulse response (DIR)

We turn now to the construction of counterfactual predictive densities in the spirit of Gallant et al. (1993). Comparing the (counterfactual) evolution of the conditional density after a perturbation of the one-period-ahead predictive density to the baseline evolution allows us to compute the *density* impulse response function (DIR), tracking how the entire joint distribution of economic and financial conditions responds dynamically to an initial shock. DIRs can be used in a variety of settings, from evaluating the potential policy effects to constructing dynamically consistent stress testing scenarios. In the present paper, we focus on the former application, comparing the effectiveness of improving financial and economic conditions during the financial crisis.

### 5.1 Constructing DIRs

Recall that the basic building block of our term structure of conditional joint distributions is the one-step-ahead conditional joint distribution,  $\hat{p}(y_t|x_t)$ . Generically, DIRs are constructed by perturbing this initial distribution, creating, say,  $\hat{\rho}(y_t|x_t)$ , and then following the procedure in Section 3.2 under the baseline model to propagate this initial disturbance across horizons to generate  $\{\hat{\rho}(y_{t+h}|x_t)\}_{h=0}^H$ :

**Algorithm 3.** *Generating density impulse responses.*

*The counterfactual distributional term structure  $\{\hat{\rho}(y_{t+h}|x_t)\}_{h=0}^H$  is generated by drawing Monte Carlo paths as:*

1. *Draw  $v_t$  from the desired perturbed one-period-ahead distribution  $\hat{\rho}(y_t|x_t)$ .*
2. *Conditional on the draw  $v_t$ , draw the two-period-ahead realization  $v_{t+1}$  from the baseline distribution  $\hat{p}(y_{t+1}|v_t)$ .*
3. *Repeat Step 2 to horizon  $H$ .*

The density impulse response (DIR) of distribution  $\hat{p}(y_t|x_t)$  to the initial disturbance  $\hat{\rho}(y_t|x_t)$  is then the difference between  $\{\hat{p}(y_{t+h}|x_t)\}_{h=0}^H$  and  $\{\hat{\rho}(y_{t+h}|x_t)\}_{h=0}^H$  for every  $h = 0, \dots, H$ .

That is, conditional on draws one-period-ahead, both the baseline and the counterfactual are performed in the same manner; the only difference is the one-period-ahead joint distribution. Importantly, since DIRs are constructed by perturbing the *conditional* joint distributions, the resultant DIRs are state-dependent: a DIR conditional on a normal state of the economy will look different from a stress-period DIR.

To keep the analogy with traditional VARs, in this paper, we examine DIRs constructed by perturbing the univariate conditional distributions. Denote by  $e_t$  the economic conditions in quarter  $t$  and by  $f_t$  the financial conditions in quarter  $t$ , so that  $y_t = (e_t, f_t)'$  and  $x_t = (e_{t-1}, f_{t-1})'$ . Then we can factor the conditional joint distribution function  $\hat{p}(y_t|x_t)$  as

$$\hat{p}(y_t|x_t) = \underbrace{\hat{p}_{m,e}(e_t|x_t)}_{\text{Marginal over } e} \underbrace{\hat{p}_{c,f}(f_t|e_t, x_t)}_{\text{Conditional over } f} = \underbrace{\hat{p}_{c,e}(e_t|f_t, x_t)}_{\text{Conditional over } e} \underbrace{\hat{p}_{m,f}(f_t|x_t)}_{\text{Marginal over } f}.$$

We define the DIR to financial conditions as the difference between the baseline estimated distribution and the counterfactual distribution constructed by perturbing the conditional distribution over financial conditions while keeping the marginal distribution over economic conditions fixed:

$$\hat{\rho}_f(y_t|x_t) \equiv \hat{p}_{m,e}(e_t|x_t) \hat{\rho}_{c,f}(f_t|e_t, x_t).$$

The DIR to financial conditions thus examines how the joint distribution of economic and financial conditions evolves in response to a shock that does not have a contemporaneous effect on the marginal distribution of economic conditions. The analogous traditional VAR object is the impulse response function to a shock to financial conditions in a VAR with Cholesky-identified shocks, where the financial conditions are ordered second.

Similarly, the DIR to economic conditions is the difference between the baseline estimated

distribution and the counterfactual distribution constructed by perturbing the conditional distribution over economic conditions while keeping the marginal distribution over financial conditions fixed:

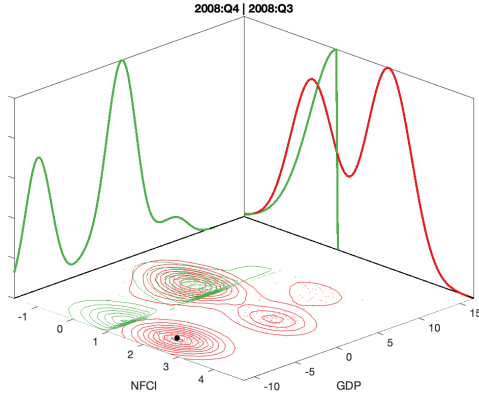
$$\hat{\rho}_e(y_t|x_t) \equiv \hat{\rho}_{c,e}(e_t|f_t, x_t) \hat{p}_{m,f}(f_t|x_t).$$

This is the distributional equivalent to an impulse response function to a shock to economic conditions in a VAR with Cholesky-identified shocks, where the economic conditions are ordered second.

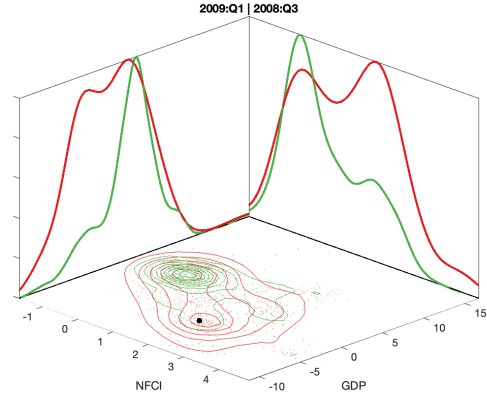
## 5.2 DIR to financial conditions

We begin by examining the in-sample DIR to financial conditions during the financial crisis. More specifically, we perturb the conditional distribution of financial conditions as of Q3 2008, moving some of the mass from the upper tail of the distribution (corresponding to tight financial conditions) to the center of the distribution, thus making “bad” financial conditions outcomes less likely. This DIR thus answers the question: If, in Q3 2008, policy were able to limit the possibility of extreme tightening of financial conditions during Q4 2008 without affecting the distribution of possible economic outcomes in the same quarter, what would the predicted joint distribution for the remaining quarters of the financial crisis have looked like?

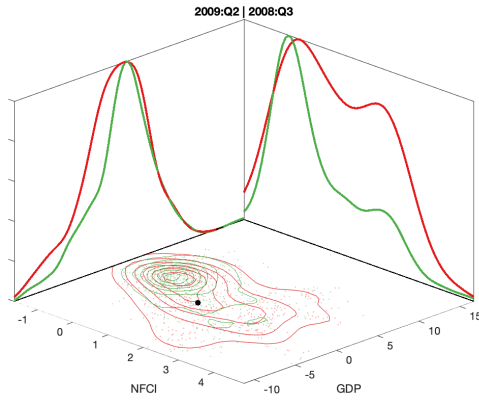
Figure 9 plots the baseline (in red) and counterfactual (in green) distributions one through four quarters ahead. Figure 9a shows that, at the one quarter ahead horizon, removing the upper tail of the financial conditions distribution is insufficient to remove the multimodality: the counterfactual distribution still allows for the possibility of an adverse economic conditions, tight financial conditions outcome in Q4 2008, though, by construction, the tightness of financial conditions in that mode is more mild than in the bad mode under the baseline distribution. The multimodality is resolved (to the good mode) at two quarters ahead, as



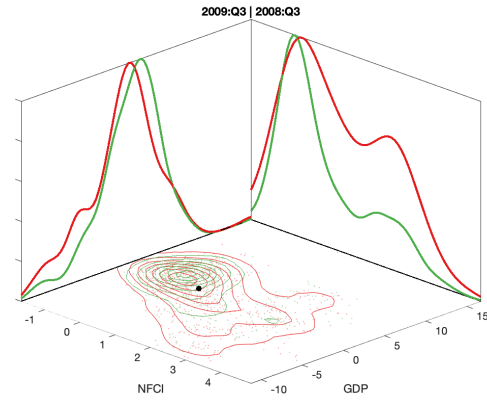
(a) One quarter ahead



(b) Two quarters ahead



(c) Three quarters ahead



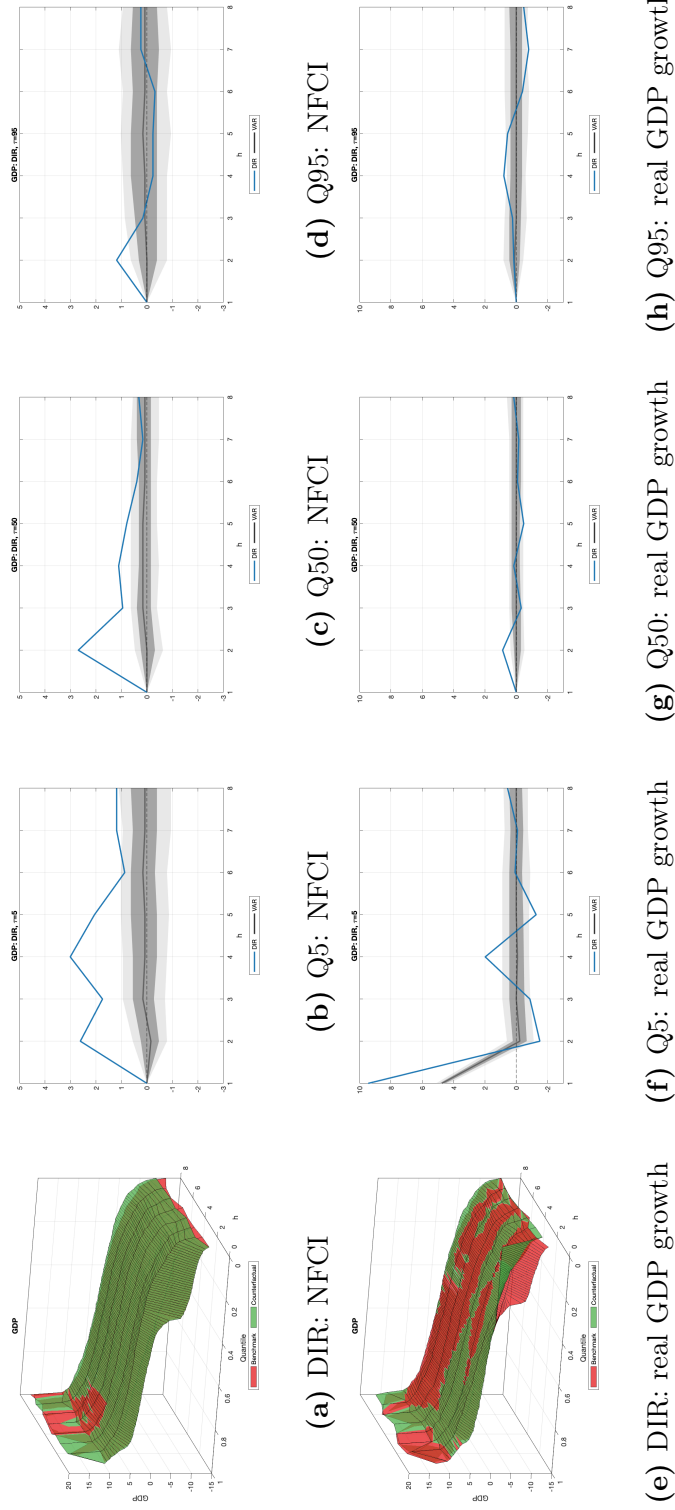
(d) Four quarters ahead

**Figure 9. Baseline and counterfactual distributions for a financial conditions DIR.** The figure plots how the joint distribution of economic and financial conditions changes in response to a shock to the conditional distribution of financial conditions in Q4 2008| Q3 2008 without a contemporaneous shock to the marginal distribution of economic conditions. The subfigures show the counterfactual impact of the perturbation 1 – 4 quarters ahead. Baseline distribution plotted in red; counterfactual in green, with the marginal distributions for real GDP growth and NFI shown on the sides and the joint density as contour plots in the center of each panel.

shown in Figure 9b. Thus, removing the possibility of extremely tight realizations of financial conditions in Q4 2008 is sufficient to eliminate the possibility of the low economic activity and tight financial conditions in Q1 2009. Propagating this further, we see from Figures 9c – 9d that the counterfactual distributions three and four quarters ahead also exhibit thinner “bad” tails for both economic and financial conditions, where the “bad” outcomes are in the left tail for economic conditions but in the right tail for financial conditions.

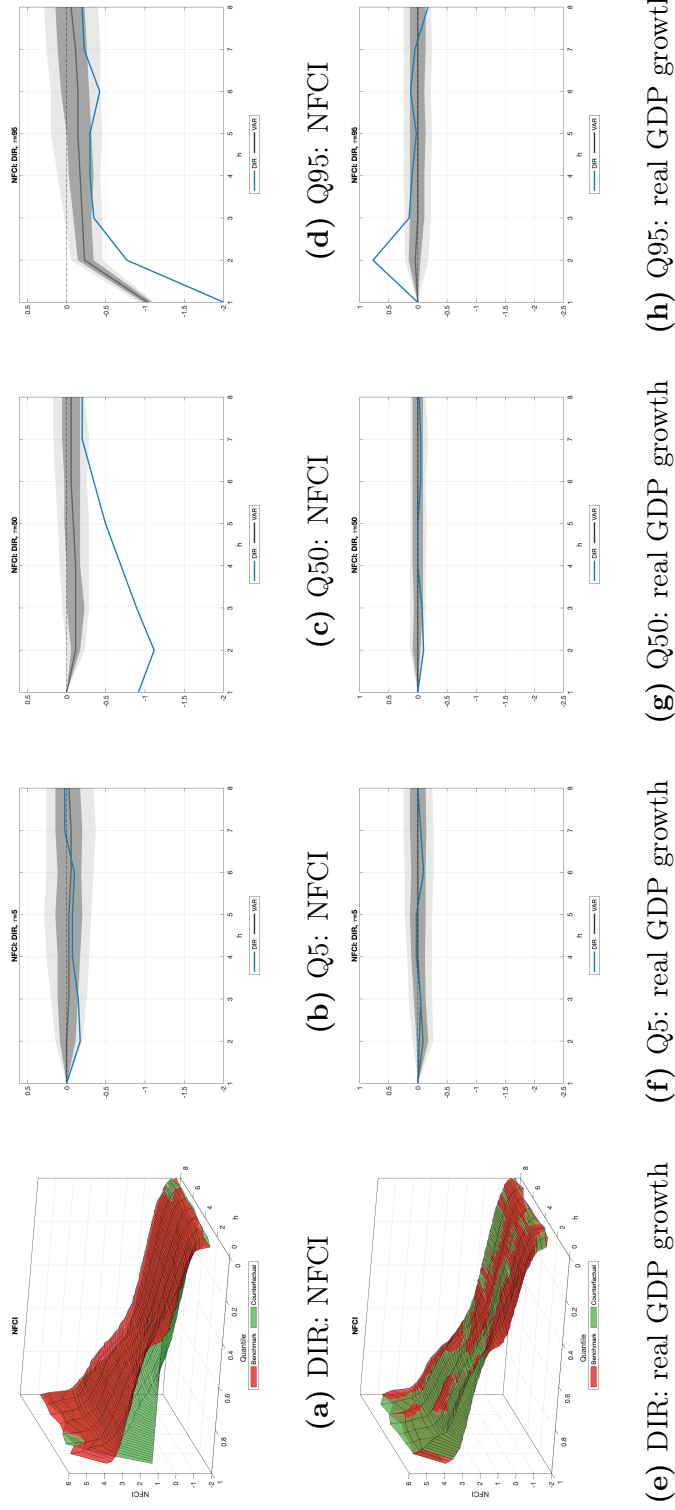
The top row of Figure 10 plots the difference between the counterfactual and the baseline distributions in the quantile distribution functions – the DIR to financial conditions – at horizons up to 8 quarters for real GDP growth and the top row of Figure 11 that of financial conditions, together with 95% confidence bands (in grey) constructed under the assumption a linear relationship between economic and financial conditions. DIRs that fall outside of the confidence bands indicate non-linear propagation of the initial shock to the distribution of financial conditions. For both economic and financial conditions, the impulse response of the respective adverse tail is well outside the confidence band at all horizons. That is, the thinning of the left tail of the distribution of economic conditions and the thinning the right tail of distribution of financial conditions multiple quarters in response to an initial improvement in the distribution of financial conditions are meaningfully different from what would be predicted by a linear model.

The top row of Figure 12 then shows that the improvements in the left-tail of the QoQ real GDP growth cumulate into even bigger and persistent improvements in the left-tail and median of cumulative real GDP growth. Hence, improving the distribution of financial conditions as of Q3 2008 reduces downside risk to both QoQ and cumulative real GDP growth. Nonlinearities in macro-financial linkages thus matter greatly for the conditional distribution of real outcomes.

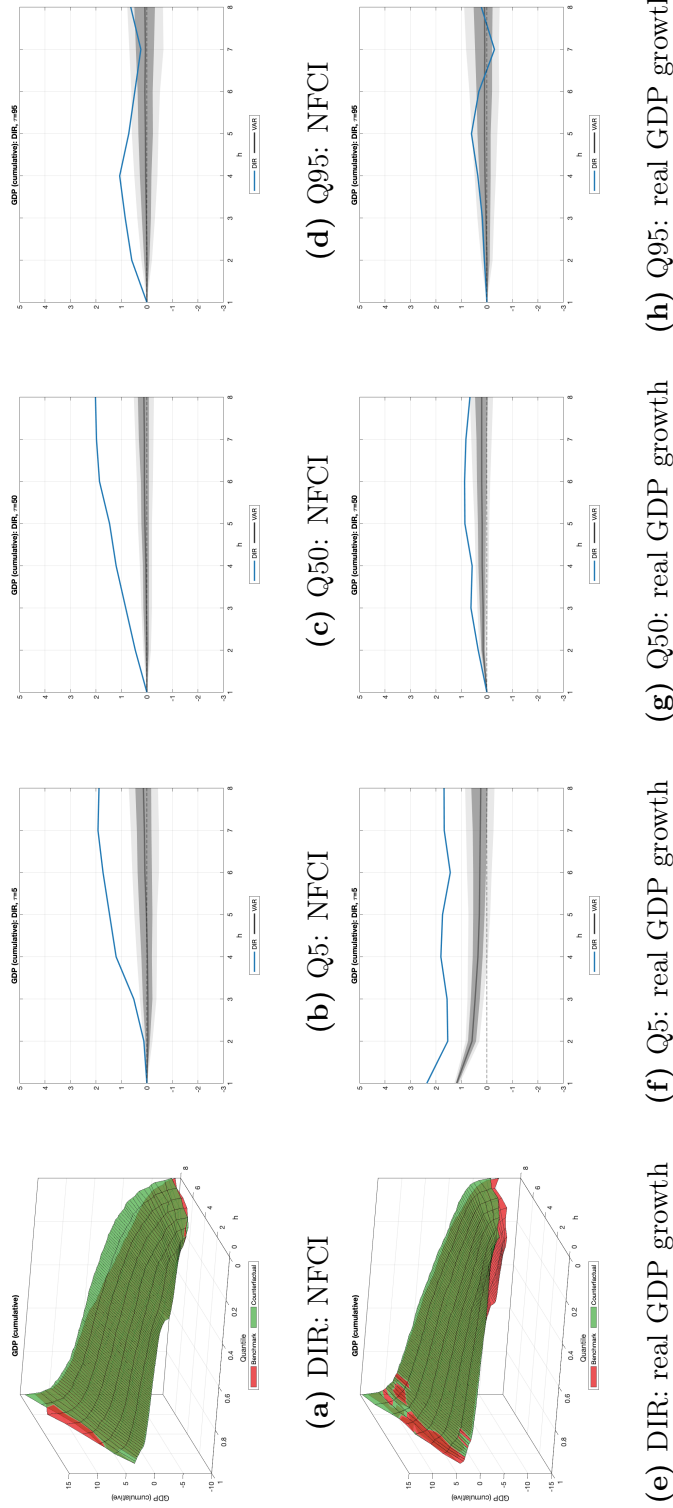


**Figure 10. DIR of real GDP growth.** This figure plots the counterfactual and the baseline distributions in the quantile distribution functions at horizons up to 8 quarters for real GDP growth to a financial conditions (top row) and economic conditions (bottom row) shock as of Q3 2008, together with the implied difference across quantiles. 95% confidence bands (in gray) constructed under the assumption a linear relationship between economic and financial conditions.

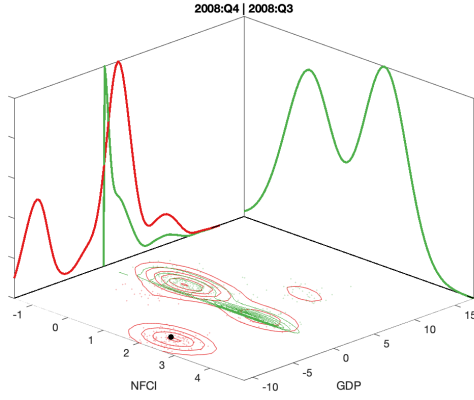




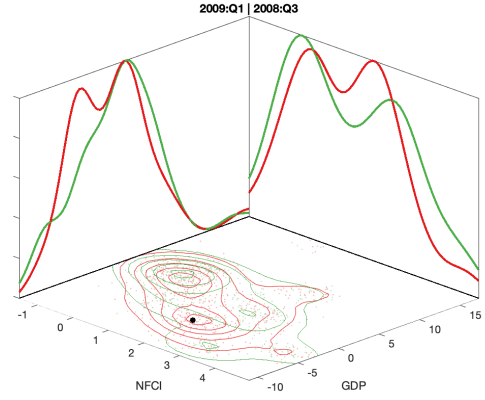
**Figure 11. DIR of financial conditions.** This figure plots the counterfactual and the baseline distributions in the quantile distribution functions at horizons up to 8 quarters for financial conditions to a financial conditions (top row) and economic conditions (bottom row) shock as of Q3 2008, together with the implied difference across quantiles. 95% confidence bands (in gray) constructed under the assumption a linear relationship between economic and financial conditions.



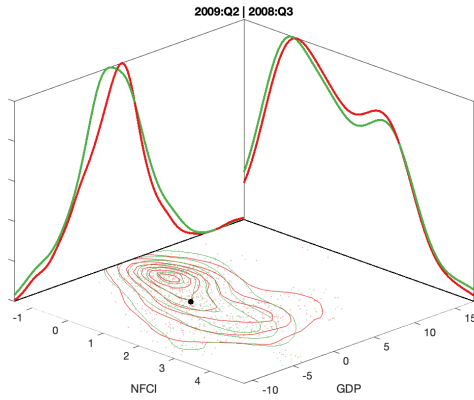
**Figure 12. DIR of cumulative real GDP growth.** This figure plots the counterfactual and the baseline distributions in the quantile distribution functions at horizons up to 8 quarters for cumulative real GDP growth to a financial conditions (top row) and economic conditions (bottom row) shock as of Q3 2008, together with the implied difference across quantiles. 95% confidence bands (in gray) constructed under the assumption a linear relationship between economic and financial conditions.



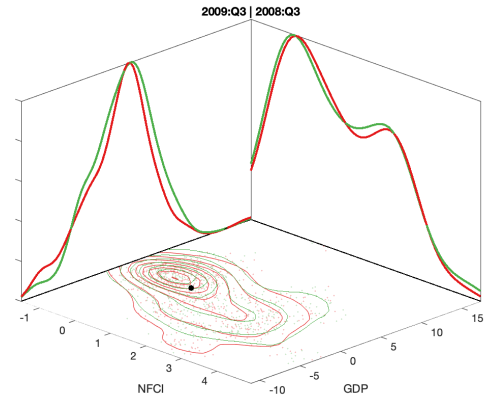
(a) One quarter ahead



(b) Two quarters ahead



(c) Three quarters ahead



(d) Four quarters ahead

**Figure 13. Baseline and counterfactual distributions for a economic conditions DIR.** The figure plots how the joint distribution of economic and financial conditions changes in response to a shock to the conditional distribution of economic conditions in Q4 2008| Q3 2008 without a contemporaneous shock to the marginal distribution of financial conditions. The subfigures show the counterfactual impact of the perturbation 1 – 4 quarters ahead. Baseline distribution plotted in red; counterfactual in green, with the marginal distributions for real GDP growth and NFCI shown on the sides and the joint density as contour plots in the center of each panel.

### 5.3 DIR to economic conditions

Consider now the in-sample DIR to economic conditions during the financial crisis. Analogously to the DIR to financial conditions, we perturb the conditional distribution of economic conditions as of Q3 2008, moving some of the mass from the lower tail of the distribution (corresponding to adverse economic conditions) to the center of the distribution, thus making “bad” economic conditions outcomes less likely. This DIR thus answers the question: If, in Q3 2008, policy were able to limit the possibility of extremely adverse economic conditions during Q4 2008 without affecting the distribution of possible financial outcomes in the same quarter, what would the predicted joint distribution for the remaining quarters of the financial crisis have looked like?

Figure 13 plots the baseline (in red) and counterfactual (in green) distributions one through four quarters ahead. As with the DIR to financial conditions, Figure 13a shows that, at the one quarter ahead horizon, removing the lower tail of the real GDP growth distribution is insufficient to remove the multimodality: the counterfactual distribution still allows for the possibility of low GDP growth, tight financial conditions outcome in Q4 2008, though, by construction, real GDP growth in that mode is more positive than in the bad mode under the baseline distribution. The multimodality persists at the remaining horizons, as shown in Figures 13b–13d. Moreover, the two-quarter-ahead distribution of financial conditions under the alternative has somewhat fatter tails than under the baseline distribution; at three and four quarters ahead, the baseline and the counterfactual joint distributions are nearly identical. Thus, removing the possibility of negative real GDP growth in Q4 2008 without a corresponding improvement in the distribution of financial conditions improves the distribution of real GDP growth in the short run, but at the cost of potentially tighter financial conditions and without large long-run effects.

The bottom row of Figure 10 plots the difference between the counterfactual and the baseline distributions in the quantile distribution functions – the DIR to economic condi-

tions – at horizons up to 8 quarters for real GDP growth, and the bottom row of Figure 11 that of financial conditions, together with 95% confidence bands (in grey) constructed under the assumption a linear relationship between economic and financial conditions. DIRs that fall outside of the confidence bands indicate non-linear propagation of the initial shock to the distribution of financial conditions. While the improvement in the left tail of real GDP growth is economically meaningful in the short run, changes in the distribution of financial conditions are virtually non-existent and the entire response of the cumulative real GDP growth is due to only improvements in the distribution in the first two quarters (bottom row of Figure 12). Thus, the transmission of shocks to the distribution of real GDP growth is very different from the transmission of shocks to the distribution of financial conditions, indicating that asymmetries in macro-financial and financial-macro linkages are crucial to understanding the term structure of the joint distribution of economic and financial conditions.

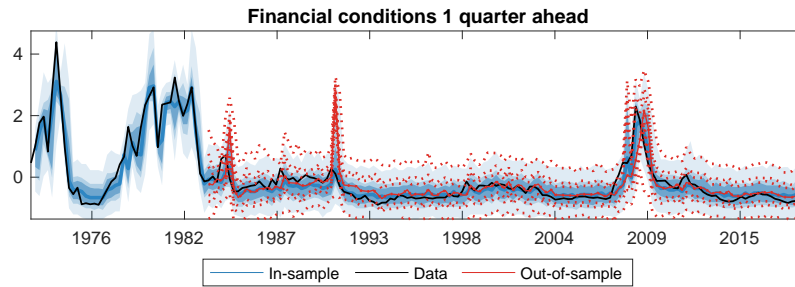
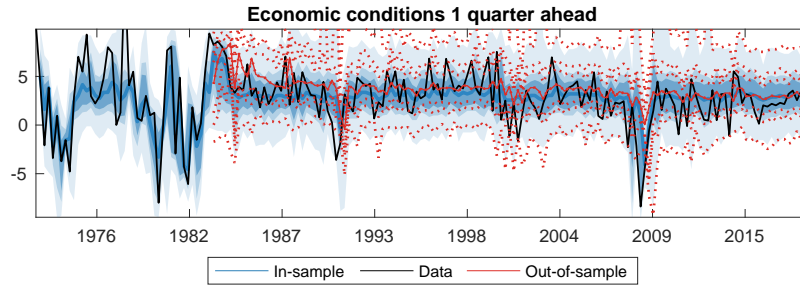
## 6 Out-of-sample evidence and alternative estimators

In this section, we evaluate the out-of-sample performance of the non-parametric estimate of the joint distribution of real GDP growth and financial conditions. We backtest the model by replicating the analysis that an economist would have done by using the proposed methodology in “real time”, with the caveat that we use final revised data only as real-time data for NFCI are only available for the recent past. We produce predictive distributions using an expanding window, starting with the estimation sample that ranges from Q1 1973 to Q3 1982. We perform two types of out-of-sample analysis. First, we show that the out-of-sample distributions are similar to the in-sample distributions, in general and during the financial crisis in particular. Second, we evaluate the out-of-sample accuracy and calibration of the density forecasts relative to two VAR alternatives, analyzing the predictive score and the probability integral transform (PIT).

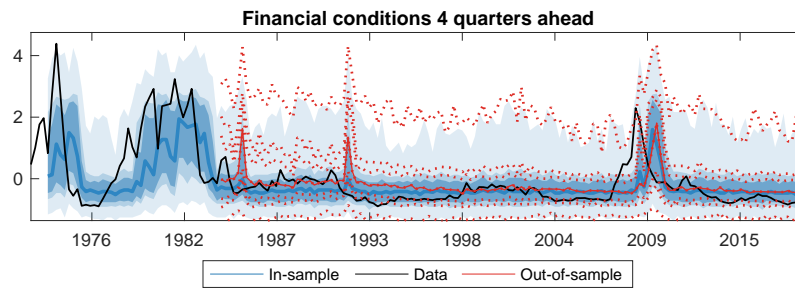
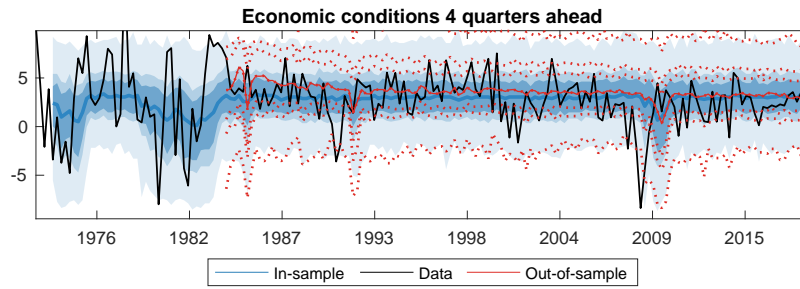
## 6.1 Out-of-sample evidence

We begin by comparing the in-sample and out-of-sample predicted distributions, plotted in Figure 14. The figure shows that the in-sample and out-of-sample estimates of the one- and four-quarter-ahead quantiles are virtually indistinguishable for both real GDP growth and financial conditions. The similarities are more striking as the Great Recession is a significant tail event that is not in the sample when estimating the out-of-sample joint distribution. When the out-of-sample quantiles do deviate noticeably from the in-sample estimates, as is the case for the four-quarters-ahead forecast of financial conditions in the 1980s, the out-of-sample forecast frequently assigns greater probability to more adverse outcomes. That is, when the recursive forecast is “wrong” relative to the full-sample one, the recursive forecast is more conservative.

To understand better the differences between the in-sample and out-of-sample forecasts, consider the out-of-sample forecasts during the Great Recession, plotted as dashed-red contours in Figure 15. Much like the in-sample forecast, the out-of-sample distribution during the Great Recession exhibits multimodality, even four quarters out. At the beginning of the recession (forecasts as of Q3 2008 and Q4 2008), the out-of-sample multimodality in the one-quarter-ahead distribution is relatively mild, in particularly missing how tight financial conditions could and would get during the financial crisis. The out-of-sample distribution does, however, take longer to normalize, both looking at longer horizons – even the four-quarter-ahead is clearly non-Gaussian – and looking at the one-quarter-ahead distribution in later quarters. Thus, even though the out-of-sample one-quarter-ahead distribution as of Q3 2008 was somewhat more benign than the in-sample distribution the possibility for extreme adverse outcomes was evidenced in longer horizon forecasts. Once the one-quarter-ahead distribution recognizes that possibility, it takes longer for the out-of-sample distribution to resolve to the good mode, suggesting that the real GDP losses during the Great Recession could have been even larger than observed in practice.

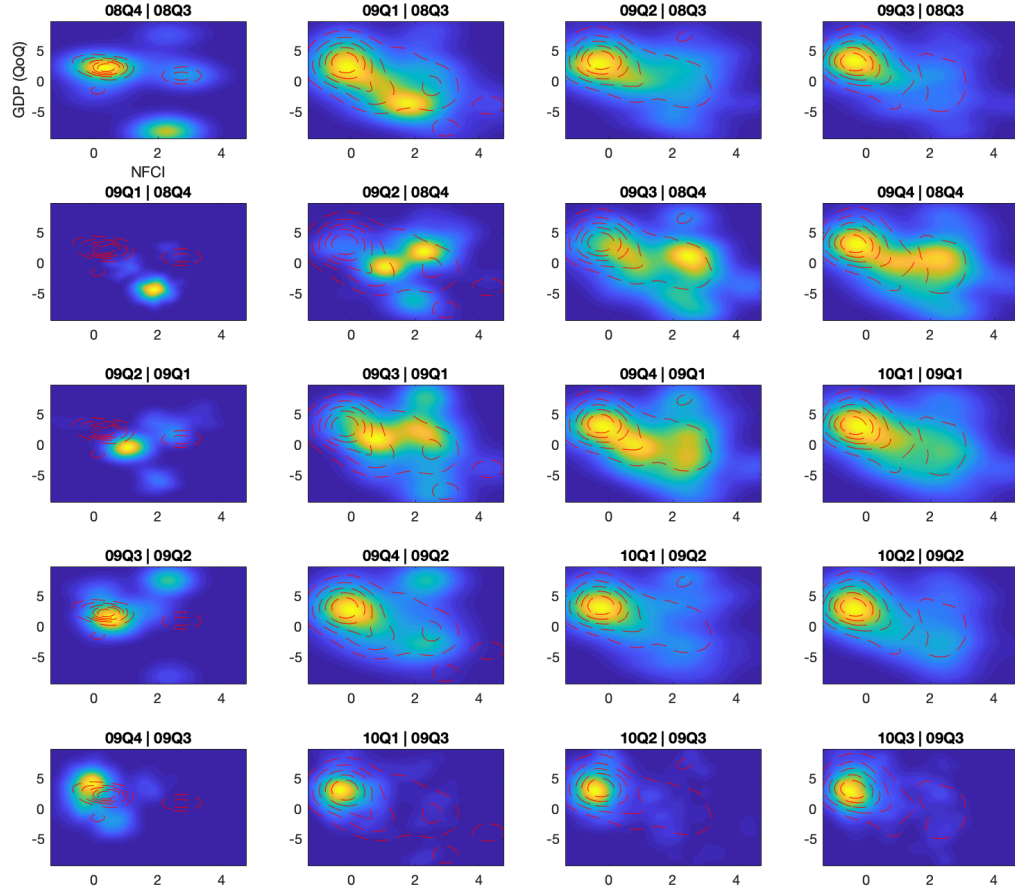


(a) One quarter ahead



(b) Four quarters ahead

**Figure 14. Out-of-sample quantiles.** The figure plots the marginal distributions of real GDP growth and financial conditions one- and four-quarters-ahead in-sample (blue shaded area) and out-of-sample (red dashed lines), together with the data realization. First out-of-sample forecast as of Q2 1982.



**Figure 15. Out-of-sample forecast of the Great Recession.** The figure reports in-sample joint densities (heatmap) with out-of-sample results overlayed (contours) during the Great Recession.



## 6.2 Alternative models

We now turn to evaluating the out-of-sample performance of our non-parametric estimator relative to two alternatives: a vector autoregression (VAR) with Gaussian errors and a VAR with non-parametric errors. More specifically, our non-parametric estimation method allows us to model the joint evolution of real GDP growth and NFCI as

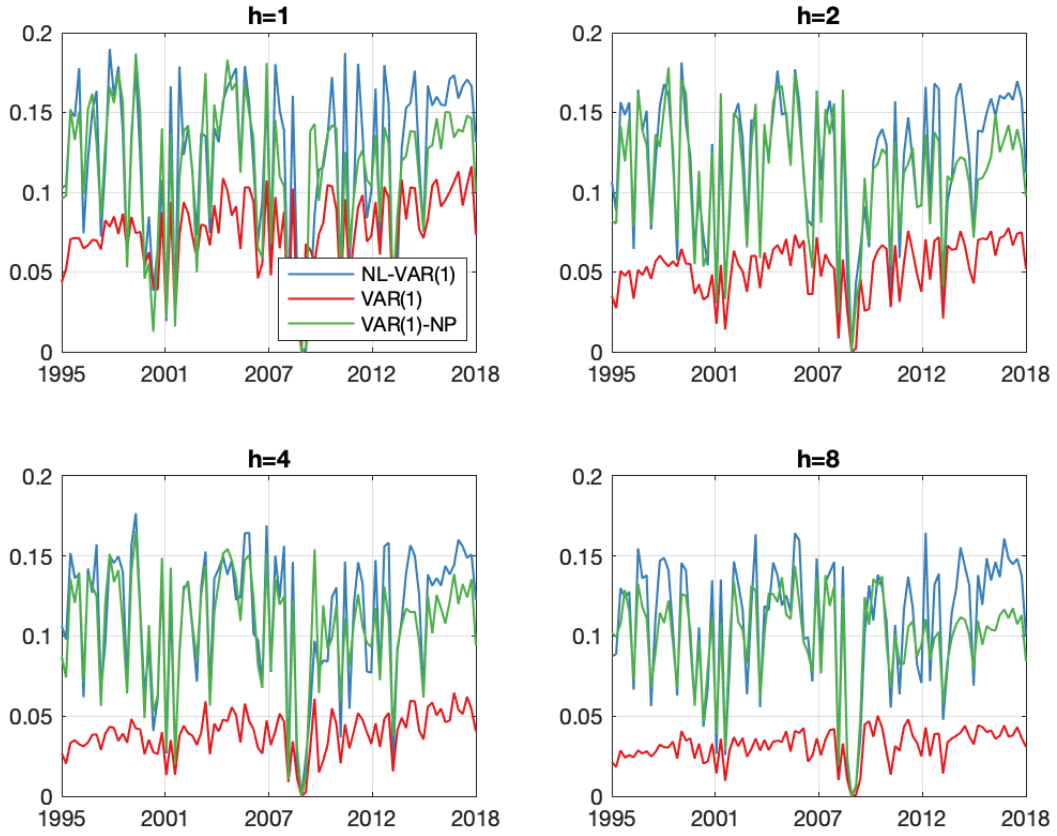
$$y_{t+1} = f(y_t, \epsilon_{t+1}),$$

with implied innovations  $\epsilon_{t+1}$  distributed according to  $f(y_t, \epsilon_{t+1}) \sim \hat{p}(y_t)$ . In Figures 16 – 17, we denote this model “NL-VAR(1)”, a non-linear first order vector autoregression. Our two alternatives both take the form

$$y_{t+1} = \rho y_t + \epsilon_{t+1}.$$

In the first alternative, a linear first order VAR with Gaussian errors,  $\epsilon_{t+1} \sim \mathcal{N}(0, \Sigma)$ ; we denote this model by “VAR(1)”. The second alternative maintains the assumption of linearity but does not impose a distributional assumption on the innovations  $\epsilon_{t+1}$ ; we denote this model by “VAR(1)-NP”. Comparing the predictions from the fully non-parametric model with those from the linear model with non-parametric errors allows us evaluate whether the multimodality we observe during periods of tightened financial conditions arises due to non-Gaussian innovations or due to non-linear transmission.

We begin by comparing the sharpness of the out-of-sample forecasts under the three models, evaluated as the log-predictive score. Figure 16 plots the log-predictive score over time at the one-quarter-, two-quarter-, four-quarter-, and eight-quarter-ahead horizon. During good times, the fully non-parametric model out-performs the two linear alternatives at all forecast horizons, and the linear VAR with non-parametric errors outperforms the linear VAR with Gaussian errors. At the one-quarter-ahead horizon, the non-parametric model



**Figure 16. Forecast sharpness.** The figure reports scores for the joint predictive distribution of real GDP growth and NFCI at the one-quarter-, two-quarter-, four-quarter-, and eight-quarter-ahead horizon. Log-predictive-scores for the fully non-parametric model plotted in blue (“NL-VAR(1)”); for a linear VAR with Gaussian innovations in red (“VAR(1)”); for a linear VAR with non-parametric errors in green (“VAR(1)-NL”).

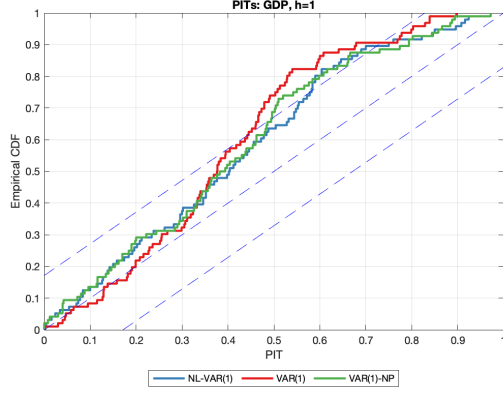
underperforms the alternatives during the financial crisis. Intuitively, the fit of the linear VAR, and especially the linear VAR with Gaussian errors, is driven by outliers. Thus, at the one-quarter-ahead horizon, the linear VAR fits the crisis relatively well. The non-parametric model, instead, recognizes that the financial crisis is an outlier in the data and thus assigns a relatively low probability to the outcomes that were observed during the crisis, lowering the log-predictive score. At longer horizons, the non-parametric model out-performs the alternatives even during crisis periods.

Consider next the calibration of the predictive distribution, evaluated through the probability integral transform (PIT). For each model and horizon, we compute the empirical cumulative distribution of the PITs, which measures the percentage of observations that are below any given quantile. In a perfectly calibrated model, the cumulative distribution of the PITs is a 45-degree line, so that the fraction of realizations below any given quantile of the predictive distribution is exactly equal to quantile probability; the closer the empirical cumulative distribution of the PITs is to the 45-degree line, the better calibrated the model is. Figure 17 plots the PITs for real GDP growth and NFCI, one and four quarters ahead for the three alternative models, together with Rossi and Sekhposyan (2017) 5% confidence bands.<sup>7</sup> For both variables and both forecast horizons, the empirical distribution of the PITs for the non-parametric model is well within the confidence bounds and generally closer to the 45-degree line than the alternatives. Thus, the non-parametric model is better calibrated than both linear VARs.

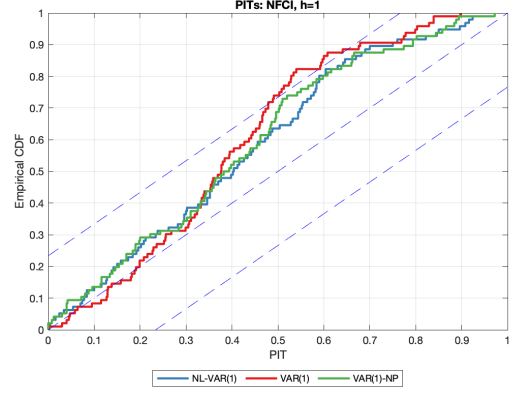
We conclude our exploration of alternative models by examining the joint distribution of real GDP growth and financial conditions during the Great Recession that is predicted by the linear VAR with non-parametric innovations, plotted in Figure 18. Unlike the in-sample (Figure 8) and the out-of-sample (Figure 15) forecasts of the non-parametric model, the

---

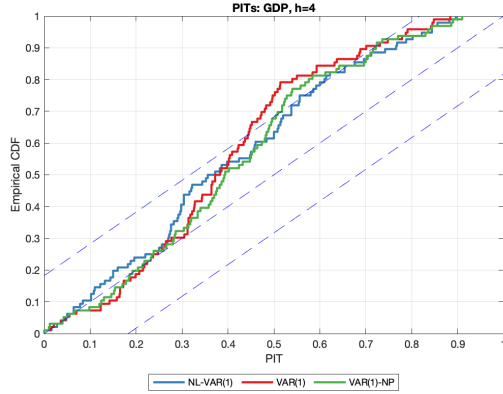
<sup>7</sup>The confidence bands should be taken as general guidance since they are derived for forecasts computed using a rolling scheme (with a constant size of the estimation sample) while we use an expanding estimation window. For the one-quarter-ahead forecasts, the bands are based on critical values derived under the null of uniformity and independence of the PIT. For the PITs of the four-quarter-ahead predictive distributions, bands are computed by bootstrapping under the assumption of uniformity only.



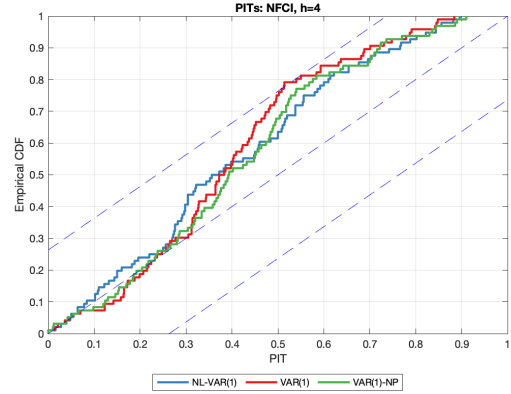
(a) Real GDP growth, one quarter ahead



(b) NFCI, one quarter ahead

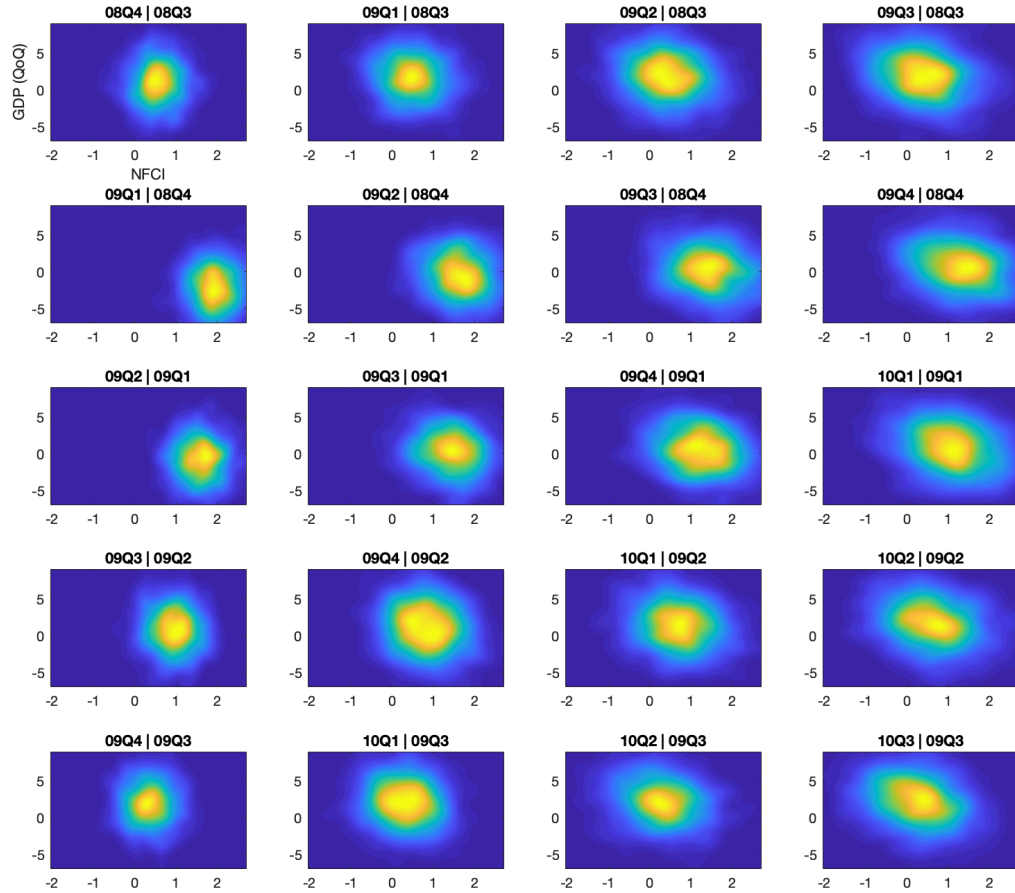


(c) Real GDP growth, one year ahead



(d) NFCI, four quarters ahead

**Figure 17. Calibration of the forecast.** The figure reports the empirical cumulative distribution of the marginal probability integral transform (PITs) from real GDP growth and NFCI one quarter and one year ahead. PITs for the fully non-parametric model plotted in blue (“NL-VAR(1)”); for a linear VAR with Gaussian innovations in red (“VAR(1)”); for a linear VAR with non-parametric errors in green (“VAR(1)-NL”); the 5% critical values as in Rossi and Sekhposyan (2017) plotted as dashed-blue lines.



**Figure 18. Forecasting the Great Recession with a VAR.** The figure reports joint density forecasts during the Great Recession for a single-lag VAR drawing non-parametrically from the residuals. Brighter colors correspond to higher probability.

joint distribution predicted by the linear VAR does not exhibit multimodality. Thus, the multimodality predicted by the non-parametric model cannot be explained by non-Gaussian innovations alone and non-linear amplification plays a crucial role in macro-financial dynamics.

## 7 Conclusion

Linear vector autoregressions (VARs) have long been a workhorse of empirical macroeconomic analysis, providing empirical targets for theoretical models and allowing the evaluation of counterfactual outcomes. In this paper, we describe a distributional equivalent to VARs, creating term structures of joint distributions of variables of interest by combining a flexible non-parametric approach to estimating one-step-ahead joint distribution with an efficient Monte Carlo approach. Our method is computationally straightforward to implement, quick to estimate, and appears well behaved both in- and out-of-sample, providing a fully nonparametric alternative to linear vector autoregressions. Instead of the impulse response functions of the VAR literature, we propose density impulse responses, building on the seminal work of Gallant et al. (1993).

We apply this methodology to estimate the joint distribution of economic and financial conditions in the U.S. Estimating the joint distribution allows us to uncover a novel feature of the data: though the conditional joint distribution is unimodal and approximately Gaussian during normal times, multimodality emerges when financial conditions are tight, regardless of how benevolent the economic conditions are. When the tightness of financial conditions is not extreme, the multimodality is resolved quickly to the good mode of the distribution. During periods of more extreme tightening of financial conditions, such as the 2007 – 2009 financial crisis and the Volcker disinflation of the early 1980s, the multimodality is present in the predictive distribution up to four quarters ahead. While the theoretical literature has long conjectured the possibility of multimodality in the joint distribution of economic and

financial conditions, due to either multiple equilibria or non-linear amplification mechanisms, to the best of our knowledge, our paper provides the first empirical evidence of periods of multimodality. Importantly, when the conditional distribution is multimodal, point forecasts are particularly misleading: by averaging between the two (or more) modes of the distribution, a point forecast underestimates both how good the good mode is and how bad the bad mode is.

Over our sample period, the joint distribution always converges to a unimodal distribution concentrated at the good mode in the long run. We conjecture that this is due to the fact that public policy in the U.S. over this period has aggressively aimed to counteract negative economic outcomes. But the Great Depression of 1929 – 1933 might have been an example of insufficiently aggressive policy leading to a resolution of multimodality to a unimodal distribution concentrated at the “bad” mode. That is, we conjecture that multimodality of the distribution also appeared in 1929-33, and that “bad” policy led to equilibrium selection associated with persistently bad outcomes. While we leave a careful examination of this conjecture for future research, it is worth noting that the joint distribution during the financial crisis exhibits some of these features. The distribution as of Q4 2008 showed some evidence of resolving to the bad mode, with the good mode all but disappearing from the one-quarter-ahead distribution. Moreover, even if the joint distribution always resolves to the good mode, prolonged periods of multimodality can lead to substantial losses in the level of economic output along the resolution path.

Periodic multimodality arises organically in our non-parametric approach: rather than postulating that multimodality (or non-Gaussianity more generally) exists, we allow the data to inform us about the shape of the joint distribution over time. Out-of-sample evaluation demonstrates that the periodic multimodality we detect are a genuine and robust feature of macro-financial dynamics, a feature that is overlooked with commonly used linear methods or Gaussian models. Indeed, periodic multimodality is a feature that would be overlooked by most non-linear extensions to VARs. One would need, for example, a Markov-switching

VAR parametrized to have the probability of switching to the bad mode only sometimes be positive to generate the same features of the conditional joint distribution.

Our paper provides rich empirical evidence against which predictions of macro-financial models can be tested. Our results point to the importance of understanding the dynamics of the entire conditional distribution, rather than of the conditional point forecasts alone. Thus, instead of evaluating model fit relative to linear impulse response functions, models should be evaluated relative to *density* impulse response functions. The empirical evidence in this paper suggests that nonlinear shock propagation when financial conditions are tight, either due to occasionally binding financial constraints or due to the emergence of multiple equilibria, are an important of macro-financial dynamics.



# References

- ADRIAN, T. AND N. BOYARCHENKO (2012): “Intermediary Leverage Cycles and Financial Stability,” Staff Report N. 567, Federal Reserve Bank of New York.
- ADRIAN, T., N. BOYARCHENKO, AND D. GIANNONE (2019): “Vulnerable Growth,” *American Economic Review*, 109, 1263–1289.
- ADRIAN, T. AND F. DUARTE (2016): “Financial Vulnerability and Monetary Policy,” Staff Report N. 804, Federal Reserve Bank of New York.
- ADRIAN, T., F. GRINBERG, N. LIANG, AND S. MALIK (2018): “The Term Structure of Growth-at-Risk,” IMF Working Paper Series, IMF Working Paper Series 18/180.
- AIKMAN, D., A. LEHNERT, J. N. LIANG, AND M. MODUGNO (2016): “Financial Vulnerabilities, Macroeconomic Dynamics, and Monetary Policy,” Finance and Economics Discussion Series 2016–055, Board of Governors of the Federal Reserve System (US).
- ALTISSIMO, F. AND G. L. VIOLANTE (2001): “The non-linear dynamics of output and unemployment in the U.S,” *Journal of Applied Econometrics*, 16, 461–486.
- ARNOLD, B. C., E. CASTILLO, AND J. M. SARABIA (1999): *Conditional Specification of Statistical Models*, Springer.
- ARUOBA, S. B., L. BOCOLA, AND F. SCHORFHEIDE (2017): “Assessing DSGE model nonlinearities,” *Journal of Economic Dynamics and Control*, 83, 34–54.
- ASCARI, G. AND T. HABER (2019): “Sticky Prices and the Transmission Mechanism of Monetary Policy: A Minimal Test of New Keynesian Models,” Discussion Paper Series 869, University of Oxford Department of Economics.
- AUERBACH, A. J. AND Y. GORODNICHENKO (2013): “Fiscal Multipliers in Recession and Expansion,” in *Fiscal Policy after the Financial Crisis*, University of Chicago Press.
- BRAVE, S. AND A. BUTTERS (2012): “Diagnosing the Financial System: Financial Conditions and Financial Stress,” *International Journal of Central Banking*, 8, 191–239.
- BRUNNERMEIER, M. K. AND Y. SANNIKOV (2014): “A Macroeconomic Model with a Financial Sector,” *American Economic Review*, 104, 379–421.
- BRYANT, J. (1983): “A simple rational expectations Keynes-type model,” *The Quarterly Journal of Economics*, 98, 525–528.
- CAGGIANO, G., E. CASTELNUOVO, AND J. M. FIGUERES (2019): “Economic Policy Uncertainty Spillovers in Booms and Busts,” *Oxford Bulletin of Economics and Statistics*, 0.
- CALDARA, D., C. SCOTTI, AND M. ZHONG (2019): “Macroeconomic and Financial Risks: A Tale of Volatility,” Mimeo, Federal Reserve Board.
- CARRIERO, A., T. CLARK, AND M. MARCELLINO (2018a): “Capturing Macroeconomic Tail Risks with Bayesian Vector Autoregressions,” Mimeo, Queen Mary University of London, Bocconi University, and Federal Reserve Bank of Cleveland.
- CARRIERO, A., T. E. CLARK, AND M. MARCELLINO (2018b): “Measuring Uncertainty and Its Impact on the Economy,” *The Review of Economics and Statistics*, 100, 799–815.

- CHANG, Y., Y. CHOI, AND J. Y. PARK (2017): “A new approach to model regime switching,” *Journal of Econometrics*, 196, 127–143.
- COGLEY, T. AND T. J. SARGENT (2005): “Drift and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S,” *Review of Economic Dynamics*, 8, 262–302.
- COGLEY, T. AND A. M. SBORDONE (2008): “Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve,” *American Economic Review*, 98, 2101–2126.
- D’AGOSTINO, A., L. GAMBETTI, AND D. GIANNONE (2013): “Macroeconomic forecasting and structural change,” *Journal of Applied Econometrics*, 28, 82–101.
- DEL NEGRO, M. AND G. E. PRIMICERI (2015): “Time Varying Structural Vector Autoregressions and Monetary Policy: A Corrigendum,” *Review of Economic Studies*, 82, 1342–1345.
- DIAMOND, D. W. AND P. H. DYBVIK (1983): “Bank Runs, Deposit Insurance and Liquidity,” *Journal of Political Economy*, 93, 401–419.
- DIAMOND, P. A. (1982): “Aggregate Demand Management in Search Equilibrium,” *Journal of Political Economy*, 90, 881–894.
- DOZ, C., L. FERRARA, AND P.-A. PIONNIER (2019): “Business Cycle Dynamics after the Great Recession: An Extended Markov-Switching Dynamic Factor Model,” Mimeo, Paris School of Economics, Bank of France, and OECD.
- DOZ, C., D. GIANNONE, AND L. REICHLIN (2012): “A Quasi-Maximum Likelihood Approach for Large, Approximate Dynamic Factor Models,” *Review of Economics and Statistics*, 94, 1014–1024.
- FERNÁNDEZ-VILLAYERDE, J., S. HURTADO, AND G. NUÑO (2019): “Financial frictions and the wealth distribution,” Tech. rep., National Bureau of Economic Research, Working paper N. 26302.
- FERRARA, L., M. MOGLIANI, AND J.-G. SAHUC (2019): “Real-time high frequency monitoring of Growth-at-Risk,” Mimeo, SKEMA Business School and Bank of France.
- GALLANT, A. R., P. E. ROSSI, AND G. TAUCHEN (1993): “Nonlinear Dynamic Structures,” *Econometrica*, 61, 871–907.
- GERTLER, M. AND N. KIYOTAKI (2015): “Banking, liquidity, and bank runs in an infinite horizon economy,” *American Economic Review*, 105, 2011–43.
- GERTLER, M., N. KIYOTAKI, AND A. PRESTIPINO (2016): “Wholesale banking and bank runs in macroeconomic modeling of financial crises,” in *Handbook of Macroeconomics*, Elsevier, vol. 2, 1345–1425.
- HAMILTON, J. (1989): “A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle,” *Econometrica*, 57, 357–384.
- HE, Z. AND A. KRISHNAMURTHY (2013): “Intermediary Asset Pricing,” *American Economic Review*, 103, 732–770.
- HUBRICH, K. AND R. J. TETLOW (2015): “Financial stress and economic dynamics: The transmission of crises,” *Journal of Monetary Economics*, 70, 100–115.
- KILEY, M. (2018): “Unemployment Risk,” Finance and Economics Discussion Series, Board of Governors of the Federal Reserve System 2018–67.
- KILIAN, L. AND H. LUTKEPOHL (2018): *Structural Vector Autoregressive Analysis*, no. 9781107196575 in Cambridge Books, Cambridge University Press.

- KILIAN, L. AND M. P. TAYLOR (2003): “Why is it so difficult to beat the random walk forecast of exchange rates?” *Journal of International Economics*, 60, 85–107.
- KOENKER, R., S. LEORATO, AND F. PERACCHI (2013): “Distributional vs. Quantile Regression,” EIEF Working Papers Series 1329, Einaudi Institute for Economics and Finance (EIEF).
- LI, Q. AND J. RACINE (2007): *Nonparametric Econometrics: Theory and Practice*, Princeton University Press.
- LORIA, F., C. MATTHES, AND D. ZHANG (2018): “Assessing Macroeconomic Tail Risk,” Working Paper Series, Federal Reserve Bank of Richmond 19-10.
- MORRIS, S. AND H. S. SHIN (2000): “Rethinking Multiple Equilibria in Macroeconomic Modeling,” *NBER Macroeconomics Annual*, 15, 139–161.
- (2002): “Social Value of Public Information,” *American Economic Review*, 92, 1521–1534.
- MURPHY, K. M., A. SHLEIFER, AND R. W. VISHNY (1989): “Industrialization and the Big Push,” *Journal of Political Economy*, 97, 1003–1026.
- NORETS, A. AND D. PATI (2017): “Adaptive Bayesian estimation of conditional densities,” *Econometric Theory*, 33, 980–1012.
- PRIMICERI, G. (2005): “Time Varying Structural Vector Autoregressions and Monetary Policy,” *Review of Economic Studies*, 72, 821–852.
- ROSSI, B. AND T. SEKHPOSYAN (2017): “Alternative tests for correct specification of conditional predictive densities,” Working Paper, Texas A&M University.
- SILVERMAN, B. (1986): *Density Estimation for Statistics and Data Analysis*, no. 9780412246203 in Monographs on Statistics and Applied Probability, Chapman and Hall.
- SIMS, C. (2000): “Using a likelihood perspective to sharpen econometric discourse: Three examples,” *Journal of Econometrics*, 95, 443–462.
- SIMS, C. A. AND T. ZHA (2006): “Were There Regime Switches in U.S. Monetary Policy?” *American Economic Review*, 96, 54–81.
- TERASVIRTA, T. AND H. M. ANDERSON (1992): “Characterizing Nonlinearities in Business Cycles Using Smooth Transition Autoregressive Models,” *Journal of Applied Econometrics*, 7, 119–136.
- VALCKX, N., A. DEGHI, M. KATAGIRI, O. KHADARINA, AND S. SHAHID (2019): “Downside Risks to House Prices,” Global financial stability report: Vulnerabilities in a maturing credit cycle, International Monetary Fund.
- VAN DIJK, D., T. TERASVIRTA, AND P. H. FRANSES (2002): “Smooth Transition Autoregressive Models – A Survey Of Recent Developments,” *Econometric Reviews*, 21, 1–47.