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Working Paper

Global trends in interest rates

Staff Report, No. 866

Provided in Cooperation with:

Federal Reserve Bank of New York

Suggested Citation: Del Negro, Marco; Giannone, Domenico; Giannoni, Marc Paolo; Tambalotti, Andrea (2018) : Global trends in interest rates, Staff Report, No. 866, Federal Reserve Bank of New York, New York, NY

This Version is available at:

<https://hdl.handle.net/10419/210718>

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Federal Reserve Bank of New York
Staff Reports

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JEL classification: E43, E44, F31, G12

Abstract

The trend in the world real interest rate for safe and liquid assets fluctuated close to 2 percent for more than a century, but has dropped significantly over the past three decades. This decline has been common among advanced economies, as trends in real interest rates across countries have converged over this period. It was driven by an increase in the convenience yield for safety and liquidity and by lower global economic growth.

Key words: world interest rate, convenience yield, interest rate parity, VAR with common trends

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To view the authors' disclosure statements, visit
https://www.newyorkfed.org/research/staff_reports/sr866.html.

1 Introduction

Ten years after the most acute phase of the financial crisis, the world economy remains mired in a low-interest-rate environment. At the time of this writing, the nominal yields on ten-year government bonds are below 3 percent in the United States, a bit above 1 percent in the U.K., around 40 basis points in Germany, and essentially zero in Japan. How unusual is this situation from a historical perspective? What role do global factors play in depressing interest rates? Does this phenomenon reflect only headwinds still emanating from the global financial crisis that will eventually dissipate, or is it connected to secular developments that partly predate the Great Recession?

To address these questions, we study the joint dynamics of short- and long-term interest rates, inflation, and consumption for seven now-advanced economies since 1870. We do so through a flexible time-series model—a vector autoregression (VAR) with common trends. This econometric tool allows us to use economic theory to model and interpret the long-run relationships across variables, while remaining agnostic on whether these restrictions hold at other frequencies. For example, the absence of arbitrage opportunities in the long-run implies that we can interpret the estimated common trend in real interest rates across countries as the trend in the world real interest rate. The same theoretical framework also suggests a decomposition of this trend into some of its potential drivers, such as global consumption growth.

The interest rates in our data set are on either government securities or close substitutes, which are relatively safe and liquid compared to other privately issued assets. Therefore, we allow the convenience yield for safety and liquidity offered by these “safe” assets to play a role in driving the international cross section of returns. To measure this convenience yield, the empirical analysis also includes Moody’s Baa corporate bond yield for the United States, as in Krishnamurthy and Vissing-Jorgensen (2012). Under certain assumptions, which we will discuss, this information is sufficient to capture the long-run effect of convenience on the world interest rate. This approach is similar to the one we pursued in Del Negro et al. (2017), though that paper focused only on U.S. data over a much shorter sample.

Four main results emerge from our empirical analysis. First, the estimated trend in the world real interest rate is stable around values a bit below 2 percent through the 1940s. It rises gradually after World War II, to a peak close to 2.5 percent around 1980, but it has been declining ever since, dipping to about 0.5 percent in 2016, the last available year of data. The exact level of this trend is surrounded by substantial uncertainty, but the drop over the

last few decades is precisely estimated. A decline of this magnitude is unprecedented in our sample. It did not even occur during the Great Depression in the 1930s.

Second, the trend in the world interest rate since the late 1970s essentially coincides with that of the U.S. In other words, the U.S. trend *is* the global trend over the past four decades. In fact, this has been increasingly the case for almost all other countries in our sample: idiosyncratic trends have been vanishing since the late 1970s. This convergence in cross-country interest rates is arguably the result of growing integration in international asset markets.¹

Third, the trend decline in the world real interest rate over the last few decades is driven to a significant extent by an increase in convenience yields, which points to a growing imbalance between the global demand for safety and liquidity and its supply. This contribution is especially concentrated in the period since the mid-1990s, supporting the view that the Asian financial crisis of 1997 and the Russian default in 1998, with the ensuing collapse of LTCM, were key turning points in the emergence of global imbalances (e.g., Bernanke, 2005; Bernanke et al., 2011; Caballero and Krishnamurthy, 2009; Caballero, 2010; Caballero and Farhi, 2014; Caballero et al., 2015; Gourinchas and Rey, 2016; Hall, 2016; Caballero et al., 2017; Caballero, 2018).

Fourth, a global decline in the growth rate of per-capita consumption, possibly linked to demographic shifts, is a further notable factor pushing global real rates lower. Its contribution is comparable in magnitude to that of the convenience yield since 1980, but only about half as important over the last twenty years (and less precisely estimated).

An important implication of these findings is that the persistent macroeconomic headwinds emanating from the financial crisis, including the effects of the extraordinary policies that were put in place to combat it, are far from being the only cause of the low-interest-rate environment. Longer-standing secular forces connected with a decline in economic growth since the early 1980s and the rise of convenience yields since the late 1990s also appear to be crucial culprits, even though these trends might have been exacerbated by the crisis. In fact, Caballero (2018) observes that the causality might run the other way, from the global safe asset imbalances that emerged in the late 1990s, depressing safe returns, to the financial fragility and macroeconomic turbulence that have afflicted the world economy since then.

¹Our finding of a “global trend” in real interest rates overshadowing country-specific trends presents interesting similarities with the findings of recent literature that emphasizes the emergence of a “global cycle”—or global factor—explaining a very large share of the variance in returns of risky assets around the world (e.g., Miranda-Agrippino and Rey, 2015; Gerko and Rey, 2017).

Furthermore, the global nature of the drivers of low interest rates limits the extent to which national policies can address the problem.

This paper is connected to several strands of the literature. The steady decline in real interest rates over the past few decades has been at the center of the academic and policy debate at least since the mid-2000s, when Bernanke (2005) suggested that a global saving glut might be holding down interest rates around the world. Following the Great Recession and slow subsequent recovery, secular stagnation became one of the most popular explanations for this phenomenon, as argued most prominently by Summers (2014).² In an attempt to shed light on this debate, a number of empirical papers have investigated the drivers of secular movements in real interest rates. Lunsford and West (2017), for instance, study long time-series for the United States. They find that demographic factors are robustly correlated with real interest rates, while productivity growth is not.³

A concept that has proved useful in the analysis of the causes of low interest rates is the natural real rate of interest, or r^* , which Laubach and Williams (2016) define as “the real short-term interest rate consistent with the economy operating at its full potential once transitory shocks to aggregate supply or demand have abated.” Estimates of r^* therefore focus on the underlying, secular drivers of the movements in interest rates, abstracting from shorter-term influences such as those related to the stance of monetary policy. In Del Negro et al. (2017), we discuss in some detail the theoretical and empirical connections between the concept of r^* proposed by Laubach and Williams (2003), the one obtained in DSGE models with nominal rigidities, and the type of low-frequency movements in real interest rates that are the focus of this paper.⁴

Compared to Del Negro et al. (2017), where we restrict attention to U.S. data since the 1960s, this paper significantly widens the scope of the analysis by including data from seven advanced economies dating back to 1870. Aside from generating evidence on the trends in interest rates for a wider set of countries, this paper identifies an explicitly global component in the secular movements of international interest rates, creating a direct connection with

²Eggertsson et al. (2017) provide a quantitative evaluation of the factors leading to secular stagnation in a calibrated overlapping-generations model.

³Other recent work stressing the role of demographics in the movements of interest rates includes Aksoy et al. (2015), Favero et al. (2016), Carvalho et al. (2016), Gagnon et al. (2016), and Ferrero et al. (2017).

⁴Recent work estimating variants of r^* with data from the United States includes Justiniano and Primiceri (2010), Barsky et al. (2014), Curdia et al. (2015), Kiley (2015), Lubik and Matthes (2015), Pescatori and Turunen (2015), Christensen and Rudebusch (2016), Crump et al. (2016), Johannsen and Mertens (2016), and Koenig and Armen (2017). Yellen (2015) and Williams (2018) discuss its monetary policy implications.

several hypotheses on the origin of the low-interest-rate environment based on worldwide forces, such as Bernanke’s saving glut hypothesis.

Holston et al. (2017) estimate r^* using international data, in their case for the U.S., Canada, the euro area, and the U.K. since 1961. Different from our approach, though, their estimates do not account explicitly for the global dimension of r^* , treating each country as a closed economy.⁵ The work of Hamilton et al. (2016) and Borio et al. (2017) is also related to ours, as they study the potential drivers of r^* using data from the 1800s for a number of countries. However, neither of these papers explicitly takes into account the comovement in interest rates across countries, which is the focus of our analysis.⁶ Gourinchas and Rey (2018) take a more explicitly global perspective in studying the connection between real returns, consumption, and wealth over long spans of time. They apply a present-value approach to data for the aggregate of the G4 economies, which they treat as a closed world economy.

Jordà et al. (2017a) broaden the historical analysis to include the rate of return on stocks and housing together with those on short- and long-term bonds for 16 countries. They measure global trends using decadal moving averages, which are then averaged across countries. As we do, they find that global safe interest rates have been declining since the mid-1980s, while risky returns have been roughly stable over this period, consistent with the safety trap hypothesis. However, they identify even more pronounced declines in the safe rate in the period comprising the two world wars. This result contrasts with our finding that the trend in the world real interest rate was relatively stable over this period. The source of this discrepancy is that, unlike moving averages, our trend-cycle decomposition attributes much of the decline in rates in the interwar period to cyclical fluctuations. We argue that decadal moving averages conflate trends with cyclical variations. A further difference between the two approaches is that we use no-arbitrage arguments to explicitly define the trend in the world real interest rate as the common component of the trends in real rates across countries, rather than as a simple cross-sectional average.⁷

The theoretical framework described in Section 3, from which we deduce restrictions on the long-run behavior of global real rates and their drivers, is based on the vast literature

⁵Neri and Gerali (2017) estimate and compare the natural rate of interest in two closed-economy DSGE models for the U.S. and the euro area.

⁶Hamilton et al. (2016) extract trends country by country. Borio et al. (2017) run a panel regression of long-term interest rates across countries on a number of possible drivers, but their approach ignores dynamics and hence the distinction between trend and cycle as well as their comovement across countries.

⁷King and Low (2014a) also define the “world” real interest rate more informally as a weighted average across countries, as did Barro and Sala-i Martin (1990) in a much earlier paper.

in international economics on the connection between interest rates, inflation, and exchange rates across countries. This work has mostly focused on testing interest rate and purchasing power parity conditions, and on investigating the possible sources of their failures. Engel (2014) surveys the part of this literature that deals with the relationship (or lack thereof) between exchange rates and interest rate differentials—the so-called uncovered interest rate parity (UIP). Unlike our paper, most of this literature estimates this relationship without distinguishing between fluctuations at different frequencies, even though assumptions on the long-run behavior of some of the variables are often maintained in the analysis. Engel (2016) is a prominent recent example of this approach. He uses a vector auto-correction model (VECM) that embeds long-run restrictions to study possible sources of deviation from UIP.

A related strand of literature has focused on the long-run relationship between exchange rates, prices, and productivity, following Froot and Rogoff (1995). More recent work in this vein is by Chong et al. (2010) who model the joint dynamics of nominal interest rates, inflation, exchange rates, and economic growth for a panel of 21 countries spanning data since 1973. While their results focus on the cointegration between the *level* of the real exchange rate and relative productivity between countries, which they interpret as evidence of a Balassa-Samuelson effect, their findings are consistent with our maintained assumption that the *growth rate* of the real exchange rate is stationary, even though its level might not be.⁸

Finally, we contribute to the fast-growing literature on the role of convenience yields in depressing the returns of safe assets (e.g., Krishnamurthy and Vissing-Jorgensen, 2012; Greenwood and Vayanos, 2014; Greenwood et al., 2015; Nagel, 2016). Several recent papers highlight the role of convenience yields in the international context. Valchev (2017) presents a general equilibrium model linking international convenience yields with monetary and fiscal policy. The resulting endogenous fluctuations in convenience yields can account for the predictability of excess returns of foreign over domestic bonds at various horizons first discovered by Engel (2016).⁹ Jiang et al. (2018) discuss the role of various international

⁸Also related is the work on pass-through of exchange rate movements into prices as summarized by Burstein and Gopinath (2014), as well as the time-series literature that applies factor analysis to international data sets for inflation (e.g., Ciccarelli and Mojon, 2010) or interest rates (e.g., Diebold et al., 2008; Coroneo et al., 2018; Nikolaou and Modugno, 2009). Chin et al. (2018) study long-term international interest rate comovement in the context of a DSGE model.

⁹Itskhoki and Mukhin (2017) similarly stress the role of international asset demand shocks in driving the exchange rate disconnect and the observed UIP violations. Those shocks, which they microfound in a general

investor preferences for the safety and liquidity of bonds issued by different countries in driving potential violations of covered interest rate parity. They show that the convenience yield that foreign investors derive from holding U.S. Treasuries accounts for up to 25 percent of the quarterly movements in the dollar exchange rate over the past thirty years.¹⁰ Du et al. (2017) measure the difference in the convenience yields between U.S. Treasuries and the bonds of other sovereigns, which they call the U.S. Treasury premium. They find that, at medium- and long-term maturities, this premium rose significantly during the financial crisis, but has since disappeared. At shorter maturities, to the contrary, the premium remains positive around levels similar to those observed before the financial crisis.

Our results are closely in line with this literature in stressing the important role of convenience yields in driving international returns. They are complementary because we focus on the contribution of these factors in driving the international comovement of interest rates at low frequencies, and over a much longer span of time. Therefore, we can explicitly address the question of how the global dimension of the demand for safety and liquidity has shaped the secular decline in real rates around the world over the past few decades, which also allows us to make contact with the large literature on r^* discussed above.

The rest of the paper proceeds as follows. Section 2 describes the VAR with common trends that we use in the empirical analysis. Section 3 lays out the theory from which we derive long-run restrictions for the joint behavior of real interest rates across countries. This theory also provides us with a well-defined notion of a global interest rate trend. Section 4 presents the empirical results. Section 5 concludes.

2 Econometric Framework

The empirical model is a VAR with common trends. It is essentially the same as that used in Del Negro et al. (2017) down to the specification and parameterization of the priors. It is a state-space model estimated with Bayesian methods, which we use to perform a multivariate

equilibrium model with limits to arbitrage, can also be thought of as reflecting asymmetric preferences for the safety of bonds from different countries.

¹⁰Du et al. (2018) extensively document that significant failures of CIP have persisted since the financial crisis. They attribute them to imbalances in the demand for saving and investment across currencies, interacted with the increased cost of financial intermediation.

trend-cycle decomposition.¹¹ Its measurement equation is given by

$$y_t = \Lambda \bar{y}_t + \tilde{y}_t, \quad (1)$$

where y_t is an $n \times 1$ vector of observables, \bar{y}_t is a $\tau \times 1$ vector of trends, and \tilde{y}_t is an $n \times 1$ vector of stationary components. $\Lambda(\lambda)$ is a $n \times \tau$ matrix of loadings, which is restricted using economic theory and depends on the vector of parameters λ . If the number of common trends τ is smaller than the number of variables n , then the observables are cointegrated (in this case, the decomposition between trends and cycles in (1) coincides with the Stock and Watson (1988) representation of a cointegrated system). We do not confine ourselves to this case and also consider situations where $\tau > n$, which arise since we have both global and country-specific trends.

Both \bar{y}_t and \tilde{y}_t are latent and evolve according to a random walk and a VAR, respectively:

$$\bar{y}_t = \bar{y}_{t-1} + e_t, \quad (2)$$

$$\Phi(L)\tilde{y}_t = \varepsilon_t, \quad (3)$$

where $\Phi(L) = I - \sum_{l=1}^p \Phi_l L^l$ and the Φ_l 's are $n \times n$ matrices. The $(\tau + n) \times 1$ vector of shocks $(e_t', \varepsilon_t')'$ is independently and identically distributed according to

$$\begin{pmatrix} e_t \\ \varepsilon_t \end{pmatrix} \sim \mathcal{N} \left(\begin{pmatrix} 0_\tau \\ 0_n \end{pmatrix}, \begin{pmatrix} \Sigma_e & 0 \\ 0 & \Sigma_\varepsilon \end{pmatrix} \right), \quad (4)$$

with the Σ 's being conforming positive definite matrices, and where $\mathcal{N}(\cdot, \cdot)$ denotes the multivariate Gaussian distribution.¹² Equations (2) and (3) represent the transition equations in the state-space model. The initial conditions \bar{y}_0 and $\tilde{y}_{0:-p+1} = (\tilde{y}'_0, \dots, \tilde{y}'_{-p+1})'$ are distributed according to

$$\bar{y}_0 \sim \mathcal{N}(\underline{y}_0, \underline{V}_0), \quad \tilde{y}_{0:-p+1} \sim \mathcal{N}(0, V(\Phi, \Sigma_\varepsilon)), \quad (5)$$

where $V(\Phi, \Sigma_\varepsilon)$ is the unconditional variance of $\tilde{y}_{0:-p+1}$ implied by (3). Very importantly for this application, the procedure straightforwardly accommodates missing observations and can be scaled up to VARs of relatively large dimensions. Section A in the Appendix

¹¹This is essentially the VAR model of Villani (2009), except that his deterministic trend is replaced by the stochastic trend (2). Very related approaches have been used by Kozicki and Tinsley (2012), Crump et al. (2016), Johansen and Mertens (2016), and Hasenzagl et al. (2017).

¹²The shocks affecting the trend and the cycle are assumed to be orthogonal. In the parlance of Watson (1986), our model features an ‘‘independent trend/cycle decomposition’’.

describes the Gibbs sampler, which takes advantage of the Durbin and Koopman (2002) “fast simulation smoother.” All results are based on 10,000 simulations, of which we discard the first 5,000 as burn-in draws.

The priors for the VAR coefficients $\Phi = (\Phi_1, \dots, \Phi_p)'$ and the covariance matrices Σ_ε and Σ_e have standard form, namely

$$p(\varphi|\Sigma_\varepsilon) = \mathcal{N}(\text{vec}(\underline{\Phi}), \Sigma_\varepsilon \otimes \underline{\Omega})\mathcal{I}(\varphi), \quad p(\Sigma_\varepsilon) = \mathcal{IW}(\kappa_\varepsilon, (\kappa_\varepsilon + n + 1)\underline{\Sigma}_\varepsilon),$$

$$p(\Sigma_e) = \mathcal{IW}(\kappa_e, (\kappa_e + \tau + 1)\underline{\Sigma}_e), \quad (6)$$

where $\varphi = \text{vec}(\Phi)$, $\mathcal{I}(\varphi)$ is an indicator function which is equal to zero if the VAR is explosive (some of the roots of $\Phi(L)$ are less than one) and to one otherwise, $\mathcal{IW}(\kappa, (\kappa + n + 1)\underline{\Sigma})$ denotes the inverse Wishart distribution with mode $\underline{\Sigma}$ and κ degrees of freedom. The prior for λ is given by $p(\lambda)$, the product of independent Gaussian distributions for each element of the vector λ .

The prior for the VAR parameters φ is a standard Minnesota prior with the hyperparameter for the overall tightness equal to the commonly used value of 0.2 (see Giannone et al., 2015), except that of course the prior for the “own-lag” parameter is centered at zero rather than one, as we are describing stationary processes. The prior for the covariance Σ_ε of the innovations to the cycles \tilde{y}_t , is a relatively diffuse inverse Wishart distribution with just enough degrees of freedom ($\kappa_\varepsilon = n + 2$) to have a well-defined prior mean, which is set to be a diagonal matrix. The square root of these diagonal elements is set to 2, except for the inflation cycle. Its prior mean is set to 4, to reflect the belief that nominal cycles might be more volatile than the other cycles.¹³

We use a conservative prior implying limited time variations of the trends—the same approach taken in Del Negro et al. (2017). Specifically, we set the prior for Σ_e , the variance-covariance matrix of the innovations to all (common and country-specific) trends \bar{y}_t , to have a mode equal to a diagonal matrix with elements equal to 1/100 for all the real trends. This prior implies that the expected change in the trend over one century has a standard deviation equal to 1 percent (all the variables are measured in percentage points). In different models, we will decompose the trend of the world real interest rate into subcomponents, such as the convenience yield and the stochastic discount factor. We set the prior such that the

¹³This is the only difference relative to Del Negro et al. (2017) in terms of prior specification, where the square roots of the diagonal elements were half the size used here. This difference is driven by the need to accommodate the higher cyclical volatility in the interwar period.

standard deviation of the innovation of the total is always the same and, at each level of disaggregation, all subcomponents contribute equally. For the inflation trends, we use a value equal to $1/50$ —which implies that expected change in the trend over half a century has a standard deviation equal to one. In addition, these priors are quite tight, as we set the degree of freedom $\kappa_e = 100$.

Turning to the initial conditions, for the world trends we use the same priors as in Del Negro et al. (2017). The expected values are calibrated to target expected initial values of 0.50 for the real rate, 2 for inflation, 1 for the term spread, 1 for the convenience yield, and 1.5 for consumption growth. The standard deviation for the initial conditions is set to 2 for the world inflation trend and 1 for all the others. The initial conditions for the country-specific trends have mean zero and standard deviations equal to half the value of the corresponding world counterparts.

3 Some Theory

This section introduces a simple theoretical framework that guides the specification of the long-run relationships imposed on the time-series models estimated in Section 4. This framework is based on standard asset-pricing ideas, as captured by a set of international Euler equations augmented to allow for the presence of convenience yield factors. These factors reflect the money-like convenience services offered by assets with special safety and liquidity characteristics—safe assets, for short—such as U.S. Treasury bonds. In equilibrium, the willingness of investors to pay for these services gives rise to a wedge between the return on safe assets on the one hand and that on securities with the same pecuniary payoffs, but no such special attributes, on the other. In the international context, the presence of convenience yield differentials between assets denominated in different currencies, and/or originating in different economies, also gives rise to deviations from the usual interest rate parity conditions, as recently discussed by Valchev (2017), Jiang et al. (2018), and Engel (2014).

3.1 International Arbitrage with Convenience Yields

Consider investors based in two different economies—say, the U.S. and the EU for concreteness—trading safe and liquid one-period bonds denominated in dollars (\$) and in euros (€). Call

$R_t^\$$ the net nominal yield on the former—a three-month U.S. Treasury bill. If M_{t+1}^{US} is the marginal rate of substitution between consumption today and tomorrow for a U.S. investor—the U.S. stochastic discount factor (SDF)—and $P_t^\$$ is the dollar price of that consumption, the pricing equation for the bill is

$$E_t \left[M_{t+1}^{US} (1 + CY_{t+1}^{US}) (1 + CY_{t+1}^\$) (1 + R_t^\$) \frac{P_t^\$}{P_{t+1}^\$} \right] = 1. \quad (7)$$

The term $(1 + CY_{t+1}^{US})(1 + CY_{t+1}^\$)$ represents the convenience yield associated with U.S. Treasuries from the perspective of a U.S. investor. We model this convenience yield as having two components. The first one, denoted by $1 + CY_{t+1}^{US}$, stems from the way U.S. investors evaluate the money-like convenience services provided by any asset (hence the US superscript). As such, this discount is independent of the currency and the country in which the asset is issued. If U.S. investors have a special motive for holding safe assets in their portfolios, perhaps because of regulatory requirements that uniquely apply to them, the demand generated by these requirements will tend to depress the returns of those securities, regardless of their origin.

The second source of convenience, denoted by $1 + CY_t^\$$, stems from asset-specific characteristics, such as the currency of denomination (hence the $\$$ superscript), and it is independent from who holds the asset. If international investors gain utility from holding U.S. Treasuries, perhaps because they value dollar liquidity or because they put special faith in the U.S. government's ability to repay its obligations, Treasuries will trade at a premium compared to similar assets originating elsewhere.

The equation that captures how U.S. investors price safe and liquid European bonds denominated in euros helps further clarify the distinction between the two sources of convenience we have in mind. This equation is

$$E_t \left[M_{t+1}^{US} (1 + CY_{t+1}^{US}) (1 + CY_{t+1}^\epsilon) (1 + R_t^\epsilon) \frac{S_{t+1}}{S_t} \frac{P_t^\$}{P_{t+1}^\$} \right] = 1, \quad (8)$$

where R_t^ϵ is the net return on the safe and liquid euro bond and S_{t+1} is the nominal exchange rate that converts that euro return into dollars, so that an increase in S represents a dollar depreciation. We assume that the convenience of European safe assets also has two components. The first one is the same as that found in U.S. Treasuries, since arbitrage implies that the same marginal investor is applying her taste for safety and liquidity ($1 + CY_{t+1}^{US}$) to both assets. The second component ($1 + CY_{t+1}^\epsilon$) instead is specific to the euro bond. It captures

the fact that the convenience services offered by this bond to any investor are likely to be different from those of U.S. Treasuries.¹⁴

If the joint second moments of the variables that enter the Euler equations have no trend, we can generate useful long run restrictions from these equations by focusing on a first-order approximation. This approximation results in a modified uncovered interest rate parity (UIP) condition of the form

$$R_t^{\$} = R_t^{\text{€}} + E_t [\Delta s_{t+1}] + cy_t^{\text{€}} - cy_t^{\$}, \quad (9)$$

where $s_t \equiv \log S_t$ is the log nominal exchange rate and $cy_t^i \equiv E_t \log(1 + CY_{t+1}^i)$ is the expected net convenience yield for either country or currency i .¹⁵ Although our empirical strategy relies on this linear approximation to hold in the long-run, we make no assumptions on its accuracy at any other frequency. This equation shows that the standard UIP in terms of safe rate of returns does not hold when euro and dollar assets generate different levels of convenience for investors. An increase in the relative convenience of dollar assets depresses their rate of return compared to that of euro assets, even if the dollar is not expected to appreciate. Equivalently, the dollar appreciates on impact if dollar assets become more desirable, even if the safe interest rate differential remains unchanged. On the contrary, if all safe assets were created equal, their interest rates would be lower than those on comparable assets that do not offer the same money-like services, but this effect would be symmetric across countries and currencies, therefore preserving UIP.

We can also write the first-order approximation of the two pricing equations for dollar and euro assets as

$$R_t^{\$} - E_t [\pi_{t+1}^{\$}] = m_t^{US} - cy_t^{US} - cy_t^{\$} \quad (10)$$

$$R_t^{\text{€}} - E_t [\pi_{t+1}^{\text{€}}] + E_t [\Delta q_{t+1}] = m_t^{US} - cy_t^{US} - cy_t^{\text{€}}, \quad (11)$$

where $\pi_t^i \equiv \log(P_{t+1}^i/P_t^i)$ is inflation, $m_t^i \equiv -E_t [\log M_{t+1}^i]$ is the negative of the expected growth rate of marginal utility, or the return on bonds that do not provide convenience

¹⁴This decomposition of the convenience yield into independent investor and asset-specific components is fairly flexible, but it does rule out the possibility that EU and U.S. investors might value the money-like services of \$ and € safe assets differently. For such an asymmetric preference to be sustained in equilibrium, though, U.S. and EU investors would need to have other offsetting reasons to hold both assets in their portfolios, such as different exposures to exchange rate risk, as discussed by Jiang et al. (2018). We do not consider this possibility in our subsequent analysis because we are mainly interested in the long run implications of the theory.

¹⁵We are also using the approximation $R \approx \log(1 + R)$.

services, and $q_t \equiv \ln(S_t P_t^\epsilon / P_t^\$)$ is the log real exchange rate. These equations highlight the factor structure in real interest rates implied by international arbitrage, once those returns are expressed in common consumption units. The fact that the same marginal investor prices both bonds implies that their returns share a common component, given by $m_t^{US} - cy_t^{US}$. In addition, the presence of asset-specific convenience yields introduces a wedge between the two interest rates in the form of an idiosyncratic component. This factor structure forms the basis for our empirical analysis of the trends in global interest rates in the next section.

Equations (10) and (11) focus on asset/currency/country-specific convenience yields as sources of UIP violations. A vast literature has documented failures of UIP stemming from many other deviations from the very simple assumptions that lead to these equations. The most notable among these assumptions is the effective risk neutrality of investors implied by the log-linear approximation of the Euler equations. If we moved to higher-order approximations, heteroskedasticity in the joint distribution of the marginal utility of consumption, the convenience yields, and the exchange rate would result in the addition of several terms to equations (10) and (11), reflecting the presence of time-varying risk. Some of these terms, such as the covariance between the SDF and the exchange rate, would be country-specific and hence give rise to deviations from UIP. When looking at trends in the data, we do not take movements in these second moments into account, since we assume them to be stationary. If they are not, however, their effect will still be captured by the country-specific convenience yields.

So far, we have assumed that U.S. investors are the marginal asset buyers in our economy. What would happen if the EU investors priced them instead? In that case, the Euler equations would become

$$\begin{aligned} R_t^\$ - E_t[\pi_{t+1}^\$] - E_t[\Delta q_{t+1}] &= m_t^{EU} - cy_t^{EU} - cy_t^\$, \\ R_t^\epsilon - E_t[\pi_{t+1}^\epsilon] &= m_t^{EU} - cy_t^{EU} - cy_t^\epsilon, \end{aligned}$$

which yield the same modified UIP condition as before, but also the further restriction

$$m_t^{US} - cy_t^{US} = m_t^{EU} - cy_t^{EU} + E_t[\Delta q_{t+1}]. \quad (12)$$

Adding the pricing equations for the assets that do not offer any convenience service, we also obtain

$$m_t^{US} = m_t^{EU} + E_t[\Delta q_{t+1}],$$

which together with equation (12) implies $cy_t^{US} = cy_t^{EU}$.

These restrictions have an intuitive interpretation. International arbitrage implies the existence of a unique stochastic discount factor that prices all assets once their returns are expressed in common consumption units and adjusted for the convenience yields that they carry. With risk neutrality, which is implied by the first-order approximation to the Euler equations considered above, the global discount factor can be represented interchangeably as the growth rate of the marginal utility of either U.S. or EU investors. Therefore, the answer to the question posed in the previous paragraph is that, under our maintained assumptions, the “identity” of the investors that price the assets is irrelevant in the long run.¹⁶

An important implication of the existence of this global SDF is that only common factors across countries matter for adjusted returns. In the consumption-based asset-pricing model discussed in Section 4.3, this consideration in turn implies that only global growth can be a priced factor. Allowing for country-specific loadings on this common factor—or for country-specific trends in consumption in the return equations, as it might be tempting to do if approaching the problem from a purely statistical perspective—would imply a violation of arbitrage in the long run.

3.2 Long-Run Implications

We conclude this section by explicitly writing the factor model for the trends in nominal interest rates across countries implied by equations (10) and (11), together with the restrictions on the EU and U.S. stochastic discount factors that we just derived. Denoting trends with upper bars, those restrictions imply

$$\bar{m}_t^w - \bar{c}y_t^w \equiv \bar{m}_t^{US} - \bar{c}y_t^{US} = \bar{m}_t^{EU} - \bar{c}y_t^{EU}, \quad (13)$$

where \bar{m}_t^w is the trend in the “world” stochastic discount factor and $\bar{c}y^w = \bar{c}y^{US} = \bar{c}y^{EU}$ is the common convenience premium applied by international arbitrageurs to safe assets. Here, we impose the standard assumption that the *growth rate* in the real exchange rate has no trend, or $\overline{\Delta q}_t = 0$. This restriction is implied by, but is weaker than, purchasing power parity in the long run ($\bar{q}_t = 1$ in our notation).¹⁷

¹⁶Jiang et al. (2018) discuss some of the frictions that would invalidate this irrelevance result and their implications for UIP.

¹⁷Most of the literature studying trends in real returns across countries (e.g., Hamilton et al., 2016; Borio et al., 2017) implicitly makes this assumption. Without it, those trends would not be directly comparable, since they would in part reflect secular changes in the value of the consumption bundles in which they are quoted. In Section B.8 of the Online Appendix we provide independent evidence that this assumption holds

Given this definition of the world stochastic discount factor and convenience yield, we can write

$$\bar{R}_{i,t} = \bar{\pi}_{i,t} + \bar{m}_t^w - \bar{cy}_t^w - \bar{cy}_t^i, \quad (14)$$

where $\bar{R}_{i,t}$ is the trend in the nominal interest rate of country i (expressed in terms of that country's currency), $\bar{\pi}_{i,t}$ is the trend in that country's inflation rate, and \bar{cy}_t^i is the trend in the country/currency-specific convenience yield ($\bar{cy}_t^{\$}$ and $\bar{cy}_t^{\text{€}}$ in the previous section). This variable has an i superscript because it represents an idiosyncratic factor in the cross section of real interest rates. Note that, even if segmentation in international asset markets prevented EU investors from engaging in cross-border arbitrage, invalidating (13), equation (14) would maintain its structure as long as U.S. investors can trade the two assets. In that case, the common component in interest rates would reflect the preferences of U.S. investors, as in equations (10) and (11). This is the model for the low-frequency comovement of interest rates that we estimate in the next section.

4 Empirical Results

This section discusses our estimates of the global trends in real interest rates and the models that we use to decompose and interpret some of their drivers. These decompositions rely on the simple economic theory based on international arbitrage described in the previous section. Since we are focusing on trends, we only impose the theoretical restrictions in the long run, making no economic assumptions on short-run dynamics. Technically, we take a stance on the structure of the matrix Λ that determines how different trends enter the measurement equations (1), but we leave the VAR matrices in (3) unconstrained. An advantage of this approach is that the restrictions we impose are fairly uncontroversial in the long run, but might easily be violated at other frequencies. In any case, readers who are skeptical of these restrictions can still interpret our results as a trend-cycle decomposition obtained from a (mildly restricted) reduced-form model and skip the interpretation of the empirical objects in terms of the economic quantities discussed in Section 3.

The basic building block of our analysis is the long-run relationship

$$\bar{R}_{i,t} - \bar{\pi}_{i,t} = \bar{r}_t^w + \bar{r}_t^i, \quad (15)$$

true in our sample. There we present the results of the model described in Section 2 applied to real exchange rate changes for all countries relative to the United States. We show that there is no evidence of either common (Figure A24) or country-specific (Figure A23) trends.

where $\bar{R}_{i,t}$ and $\bar{\pi}_{i,t}$ are trends in nominal short-term yields and inflation in country i , \bar{r}_t^w is the trend in the short-term world real interest rate, and \bar{r}_t^i is a country-specific trend. This relationship rewrites (14) to highlight that the trend in the world real interest rate \bar{r}_t^w reflects trends in the discount factor of the marginal international investor, $\bar{m}_t^w - \bar{c}y_t^w$, including her taste for safety and liquidity. In light of Section 3, we interpret this idiosyncratic trend \bar{r}_t^i as capturing the different degrees of safety and liquidity of the government paper issued by the various countries in our sample, namely the fact that U.S. Treasuries and German bunds are generally considered a safe haven, while Italian government securities may not be considered as such (these are the $\bar{c}y_t^i$ terms in equation (14)). However, these country-specific factors could be interpreted more generally as capturing any long-run deviation from UIP, regardless of its source.

Section 4.1 presents estimates of the different components of the nominal interest rate in (15), \bar{r}_t^w , \bar{r}_t^i , and $\bar{\pi}_{i,t}$, from a baseline model based on data on inflation and nominal yields on short- and long-term government securities across countries. Since most of these assets are considered both safe and liquid, we interpret \bar{r}_t^w as the global trend for safe and liquid returns. Section 4.2 further decomposes \bar{r}_t^w into the part that we attribute to the global convenience yield for safety and liquidity, $\bar{c}y_t^w$, and a worldwide stochastic discount factor \bar{m}_t^w , using yields on “unsafe” and “illiquid” U.S. securities. Finally, Section 4.3 adds data on consumption growth to split the stochastic discount factor \bar{m}_t^w into a component due to global growth, which we call \bar{g}_t^w , and a residual component unrelated to it.

Our data come from the Jordà-Schularick-Taylor Macrohistory Database, which is described in Jordà et al. (2017b).¹⁸ In particular, we use annual data from seven advanced countries (Canada, Germany, France, Italy, Japan, the U.K., and the U.S.) on short-term and long-term interest rates, consumer prices (the database contains an index, which we log-difference to obtain inflation), and real consumption per capita.¹⁹ We augment this data set with annual averages of Moody’s Baa corporate bond yield for the U.S., which is available from FRED dating back to 1919. The short-term rates are from government bills or money market instruments, while the long-term rates are all from government bonds. For instance, the long-term yield for the U.S. coincides with the 10-year Treasury constant maturity rate obtained from FRED and going back to 1954. Therefore, we consider all these yields as

¹⁸We are very grateful to Oscar Jordà, Moritz Schularick, and Alan Taylor for making their data publicly available.

¹⁹The time series of interest rates and inflation contain large outliers in the period around the world wars, which are not very informative on the secular trends we are interested in. Therefore, we treat all observations above 30 percentage points in absolute value as if they were missing data.

reflecting “safe and liquid” returns, because government paper is generally more liquid than its privately issued equivalents, since it tends to be traded heavily, and is safer, since it is backed by taxation. Of course, the degree of safety and liquidity of government securities varies across countries, and we account for that in our analysis.

In all specifications we use one lag in the VAR, both because we use annual data and for computational feasibility.

4.1 A Baseline Model of the World Real Interest Rate

This section presents estimates of the trend in the world real interest rate from a baseline model with data on the nominal yields of short-term ($R_{i,t}$) and long-term ($R_{i,t}^L$) government (or closely related) securities and inflation ($\pi_{i,t}$) in each of the seven countries in our sample. Having three observable variables per country, we extract three trends—in inflation, the level of interest rates, and the spread between long and short maturity rates. Moreover, the cross section of countries allows us to separate a common, or “world,” component and a country-specific component in each of these trends. Next, we describe the trend equations that embed the restrictions discussed in the theory section, and provide a complete description of the matrix Λ in equation (1) for each of the models we estimate.

We already discussed the model for the trends in short-term real rates in equation (15). Intuitively, we split trends in real returns into a common component and an idiosyncratic component specific to each country. With observations on seven cross-sectional units, we could also estimate country-specific loadings on the common component. However, Section 3 discussed how international arbitrage implies that these loadings must be one because, in the long run, the free movement of capital across countries will equalize the return on assets with the same characteristics. This common component in the returns induced by arbitrage is therefore the textbook world real interest rate. As a further check on the no-arbitrage restrictions embedded in equation (15), however, we will also present a model that does allow for country-specific loadings on the world real interest rate. None of these loadings is significantly different from one.

Moving now to the model for the trends in long rates, it is convenient to express it in terms of its implications for the term spread, namely the difference between the trends in long and short rates: $\bar{R}_{i,t}^L - \bar{R}_{i,t}$. Although much of the literature models this spread as stationary, we want our analysis to be robust to the possible failure of this assumption.

This approach is dictated both by the longer time series that we are modeling, where low-frequency movements in the slope of the yield curve might be more evident, as well as by our focus on secular movements. Under no arbitrage, a trend component in the spread between long and short-term yields must originate from a trend in the term premium. As pointed out by our discussant Carlo Favero, allowing for such a trend is in slight contradiction with our maintained assumption that risk premia are stationary. Our view is that the trend in the term spread is simply a convenient econometric device to model low-frequency movements in the term premium, even if these are not literally a random walk.

As we do for the level of rates, we model the trend in the term spread as the sum of a common and a country-specific component:

$$\overline{R}_{i,t}^L - \overline{R}_{i,t} = \overline{ts}_t^w + \overline{ts}_t^i. \quad (16)$$

Both \overline{ts}_t^w and \overline{ts}_t^i are assumed exogenous. In Del Negro et al. (2017), we also considered a specification in which the inflation trend affects the term spread, but found the results little affected by this more general specification. Therefore, we do not consider it here.²⁰

We are restricting the loadings on the world trend in the term spread to be the same across countries for the same reasons discussed above in relation to the level of rates. International arbitrage implies convergence of interest rates at all maturities, as long as we are comparing identical assets. Therefore, we can interpret \overline{ts}_t^w as the trend in the slope factor in the SDF of the marginal global investor. To the extent that this factor is an important driver of term premia, \overline{ts}_t^w could thus also capture potential low-frequency variation in the riskiness of long-term bonds. At the same time, we allow for deviations from this common pricing of term spreads to reflect possible cross-country differences in maturity for the long-term bonds in our sample, as well as relative differences in safety and liquidity between long and short-term bonds in different countries. These and other potential deviations from perfect arbitrage are captured by the term \overline{ts}_t^i .

Similarly, we decompose the trends in inflation in each country ($\overline{\pi}_{i,t}$) into a common and a country-specific component as

$$\overline{\pi}_{i,t} = \lambda_i^\pi \overline{\pi}_t^w + \overline{\pi}_t^i. \quad (17)$$

In this case, we do allow different countries to load differently on the global inflation trend $\overline{\pi}_t^w$ through the coefficients λ_i^π , since there is no economic force equivalent to no arbitrage

²⁰This model of the trends in short and long-term rates implies that the latter are the sum of the former and of the trend in the term spread. Therefore, the trend in the world real interest rate \overline{r}_t^w should be interpreted as having a short-term maturity.

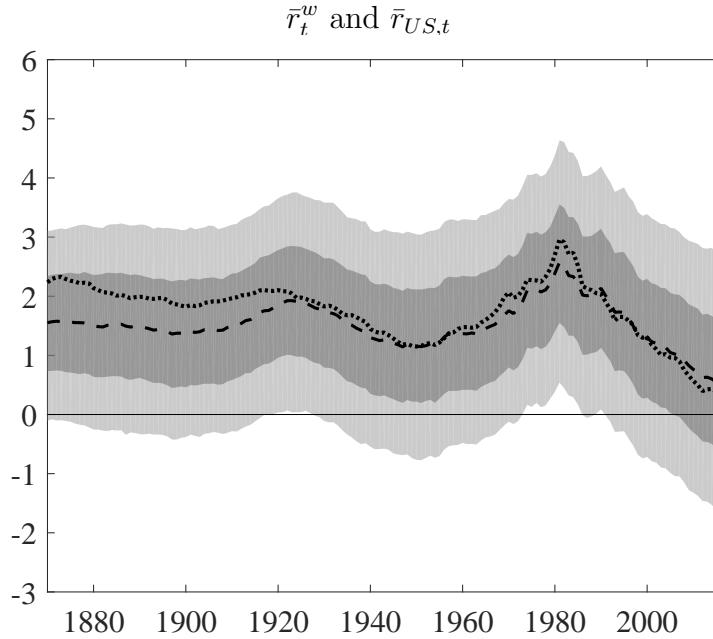
enforcing the convergence of monetary policies and any other potential long-run determinant of inflation across countries.²¹ This assumed structure of the inflation trends is the low-frequency analog of the global inflation factor model estimated by Ciccarelli and Mojon (2010).

Summarizing, the first model we estimate is

$$\begin{aligned} R_{i,t} &= \bar{r}_t^w + \bar{r}_t^i + \lambda_i^\pi \bar{\pi}_t^w + \bar{\pi}_t^i + \tilde{R}_{i,t}, \\ R_{i,t}^L &= \bar{r}_t^w + \bar{r}_t^i + \bar{t}S_t^w + \bar{t}S_t^i + \lambda_i^\pi \bar{\pi}_t^w + \bar{\pi}_t^i + \tilde{R}_{i,t}^L, \\ \pi_{i,t} &= \lambda_i^\pi \bar{\pi}_t^w + \bar{\pi}_t^i + \tilde{\pi}_{i,t}, \end{aligned} \quad (18)$$

for $i = 1, \dots, n$. The system is estimated jointly for all seven countries in the sample (so $n = 21$ and $\tau = 24$, as we have both global and country-specific trends). Note that we do not impose cointegration in either real interest or inflation rates across countries as the number of trends is larger than the number of variables (for both inflation and real rates, we have both global and country-specific trends).

Figure 1: Trends in Global and U.S. Real Rates: 1870-2016, Baseline Model



Note: The dashed black line shows the posterior median of \bar{r}_t^w and the shaded areas show the 68 and 95 percent posterior coverage intervals. The dotted black line shows the posterior median of $\bar{r}_{US,t} = \bar{r}_t^w + \bar{r}_t^{US}$.

²¹Whenever we introduce loadings λ , we use as a prior the product of independent Gaussian distributions with mean 1 and standard deviation 0.5.

4.1.1 Results

Figure 1 plots the posterior median of the trend in the world real interest rate \bar{r}_t^w (dashed line), together with its 68 and 95 percent posterior coverage intervals, as well as the posterior median of the trend in the U.S. real rate $\bar{r}_{US,t}$ (dotted line). This figure delivers the first two important results of the paper. First, \bar{r}_t^w fluctuated around 1.5 percent for about a century, but it has been on a steady decline over the past few decades, dropping almost 200 basis points. Second, \bar{r}_t^w and \bar{r}_t^{US} essentially coincide over the last century: the world trend is the U.S. trend.

More in detail, our estimates indicate that \bar{r}_t^w fluctuated in a fairly narrow range around 1.5 percent through the post-World War II period, reaching a peak close to 2 percent just before the Great Depression and a trough a bit above 1 percent in the early 1950s. From then it rose steadily through the early 1980s, when it touched 2.5 percent. It has been on a steady decline since, plunging to about 50 basis points by the end of the sample, below its previous minimum. The uncertainty on the level of the trend at any point in time is large, so the point estimates should be interpreted with caution. However, the decline over the past few decades is statistically significant, as shown in the top panel of Table 1. This decline has totaled close to 2 percentage points since 1980, with more than 150 basis points of it occurring since 1990, and more than 1 percentage point over the last 20 years of the sample. The 90 percent posterior coverage intervals for the estimated declines over the three periods, which are in parentheses, all exclude zero. In fact, the posterior probability that the decline is positive is greater than 95 percent across the board, as indicated by the stars. Table A1 in the Appendix offers even more detail on the posterior distribution of these declines, but the message is clear: the declines are large and highly statistically significant.

The dotted line in Figure 1 shows that \bar{r}_t^w and $\bar{r}_{US,t}$, which is the sum of \bar{r}_t^w and the country-specific trend \bar{r}_t^{US} , are very close since the 1920s. This implies that \bar{r}_t^{US} has been small. However, the U.S. overall trend has fallen more than the world interest rate since 1980 and it has been below it since the late 1990s, indicating that the country-specific component \bar{r}_t^{US} has been negative since then and growing. This evidence suggests that U.S. government bonds have enjoyed a larger convenience yield than those of other sovereigns over the past three decades.

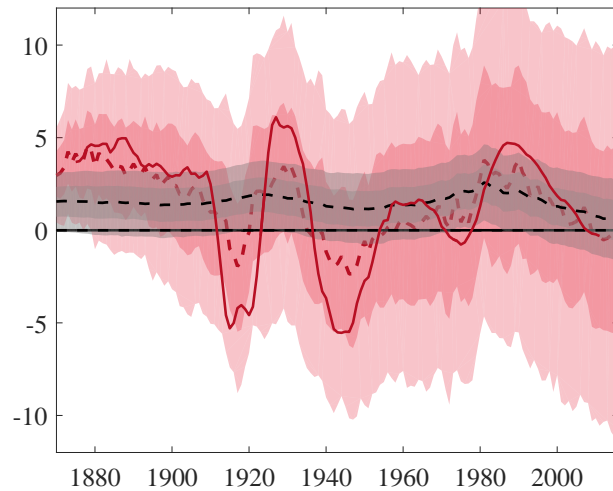
Intuitively, one would think that the trend world rate would be more or less a moving average of the cross-country average of the real rates of interest (see, for example, King and Low, 2014b). Figure 2 compares the global trend in the real rate shown in Figure 1

Table 1: Change in \bar{r}_t^w and Its Components

	1980-2016	1990-2016	1997-2016
Baseline Model			
\bar{r}_t^w	-1.88*** (-3.24, -0.61)	-1.64*** (-2.84, -0.47)	-1.03** (-1.96, -0.09)
Convenience Yield Model			
\bar{r}_t^w	-1.71*** (-2.94, -0.55)	-1.73*** (-2.84, -0.67)	-1.13*** (-2.01, -0.26)
$-\overline{cy}_t^w$	-0.82** (-1.60, -0.05)	-0.97*** (-1.65, -0.24)	-0.66** (-1.25, -0.06)
\overline{m}_t^w	-0.90 (-1.89, 0.08)	-0.78* (-1.66, 0.09)	-0.48 (-1.17, 0.22)
Consumption Model			
\bar{r}_t^w	-1.93*** (-3.18, -0.69)	-1.94*** (-3.10, -0.82)	-1.22*** (-2.18, -0.29)
$-\overline{cy}_t^w$	-0.71* (-1.51, 0.11)	-0.92*** (-1.67, -0.19)	-0.65** (-1.25, -0.02)
\overline{g}_t^w	-0.74** (-1.50, -0.03)	-0.61* (-1.28, 0.06)	-0.35 (-0.88, 0.19)
$\overline{\beta}_t^w$	-0.47 (-1.21, 0.31)	-0.42 (-1.07, 0.25)	-0.24 (-0.78, 0.30)

Note: For a variable x , where $x = \{\bar{r}_t^w, \overline{m}_t^w, -\overline{cy}_t^w, \overline{g}_t^w, \overline{\beta}_t^w\}$ depending on the model, the table shows the posterior median of $\Delta x = x_{2016} - x_{t_0}$ for t_0 being equal to 1980 (left column), 1990 (middle column), and 1997 (middle column), and the 90 percent posterior coverage interval for Δx (in parenthesis). The stars next to the posterior median indicate that $Pr\{x \leq 0\}$ is greater or equal to 97.5 (***), 95 (**), or 90 (*) percent, where $Pr\{\}$ is the posterior probability.

Figure 2: Trends in Global Real Rates Under Alternative Priors for the Standard Deviation of Innovations to the Trend and Decadal Moving Averages



Note: The dashed black line is the posterior median and the shaded gray areas are the 68 and 95 percent posterior coverage intervals for r_t^w in the baseline model. The dashed red line is the posterior median and the shaded red areas are the 68 and 95 percent posterior coverage intervals for r_t^w obtained when centering the prior for the variance of innovations to the trend equal to 1 as opposed to 1/100 as in the baseline specification. The solid red line is the decadal moving average of the cross-sectional average of ex-post real rates across all countries (specifically, at any point in time the moving average is constructed by taking for each country the average ex-post short-term real rate of return over the previous five years, the current year, and the following five years, and then taking a cross-sectional average of the resulting objects).

(dashed black line with gray bands) with the cross-country average of the decadal moving average for each country in our sample (solid red line). Clearly, the moving average approach yields a measure of the global trend in the real rate that is much more volatile than what we find. For instance, it reaches even lower levels than today in the second and fourth decade of the 20th century, around WWI and WWII, and rises to almost 5 percent in the 1980s. Why these differences? We argue that the moving-average approach provides a good description of the trend for a researcher having a prior belief that this trend can fluctuate substantially over a short period. Our own beliefs are quite conservative, as they reflect skepticism about the possibility that secular trends could fluctuate wildly from one year to the next: at the prior mode, the expected change in the trend has a standard deviation equal to one over a one-century horizon (see Section 2). By shortening this horizon, one allows the trend to capture more of the short-term fluctuations in the data. The dashed red line in Figure 2 shows that the trend obtained from the cross-sectional average of moving averages can be rationalized by a prior belief that this horizon is as short as one year—which means attributing a substantial fraction of cyclical fluctuations in the real rate to the trend.²² The

²²Even using a one-year horizon, our estimates are not as low as the moving average around the two world wars, partly because these very low values are due to outliers associated with hyperinflations that our trend

cost associated with having such a loose belief of what represents a trend is apparent from the large posterior uncertainty associated with these estimates: The 50 percent posterior coverage intervals for \bar{r}_t^w are as large as 10 percentage points in 2016! Interestingly, our results based on the conservative prior are not very sensitive to modifications of the prior that cut the horizon from one century to a half-century or a quarter of a century, or even a decade—as shown in Figure A3 of the Online Appendix—as long as one does not use extreme values such as one year. Summing up, our conclusion that the decline in the global trend in interest rates over the past two decades is unprecedented is quite robust to the prior view of what constitutes a trend, as long as one does not conflate trends with cyclical variations.

The left panel of Figure 3 presents our estimates of \bar{r}_t^w in the context of some of the information that we use to extract this trend, namely the ex-post real short-term interest rates ($R_{i,t} - \pi_{i,t}$) for all the countries in the sample. Although these rates fluctuated wildly in the first part of the sample, the movements in \bar{r}_t^w do capture some evident common patterns in the data at least back to the 1930s. Real rates fell in the 1930s and 1940s, as well as in the 1970s with high inflation, although the model interprets the latter movement as mostly cyclical. Most notably, real rates have been falling closely together since the 1980s, dragging the world trend down with them. Partly due to our conservative prior on the amount of variation in the trends, the model interprets a good part of this decline as cyclical. Yet, the persistent comovement of real rates over the past four decades is evident to the naked eye. Ultimately, this low-frequency comovement is what drives the estimated decline in the trend.²³

The pronounced fluctuations in real rates in the first part of the sample, and especially during the two world wars, highlighted in Figure 3, raise the concern that our VAR with constant volatilities might be misspecified. Even if the trends are homoskedastic, as we have argued they are likely to be, their estimates might be affected by ignoring a possible break in the volatilities of the innovations to the stationary VAR (3).²⁴ To address this concern, we also estimated the baseline model on a shorter sample starting in 1950. As shown in Figure A4 in the Appendix, the trend estimates are very similar to those obtained with the longer

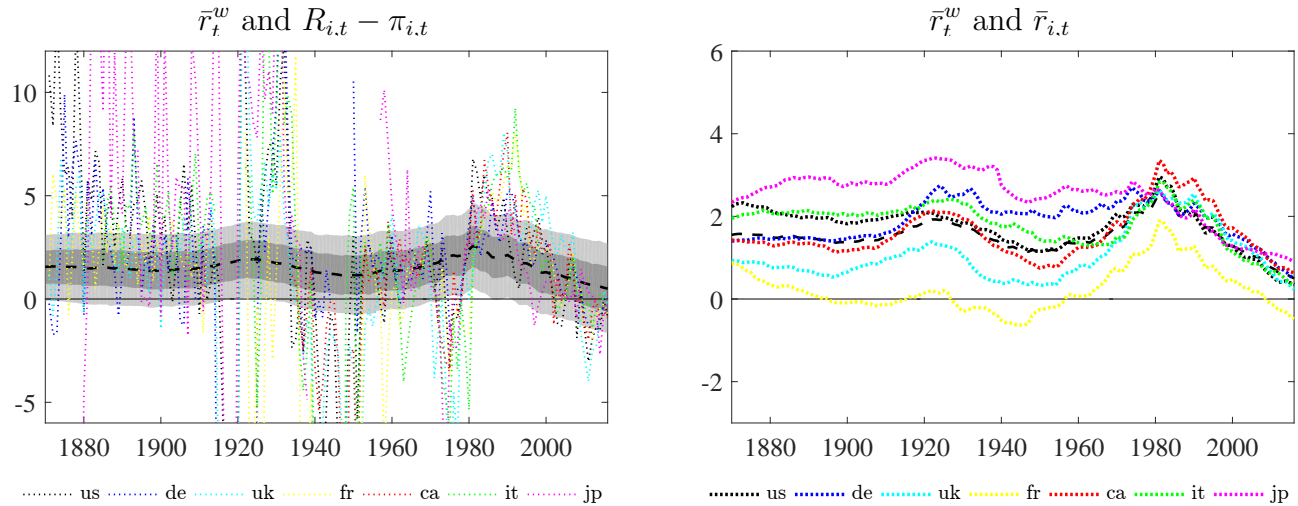
refuses to fit.

²³Section B.4 in the Appendix shows the results of the baseline model with a looser prior: we use 50 instead of 100 degrees of freedom. Those results are even stronger than those shown in this section in terms of the size of the post-1980 decline in \bar{r}_t^w .

²⁴A model with stochastic volatility in the innovations to the VAR describing the cyclical components, similar to that estimated by Johansen and Mertens (2016), is arguably appropriate for this sample. We leave this interesting extension to future research.

sample.

Figure 3: Trends and Observables for Short-Term Real Rates, Baseline Model



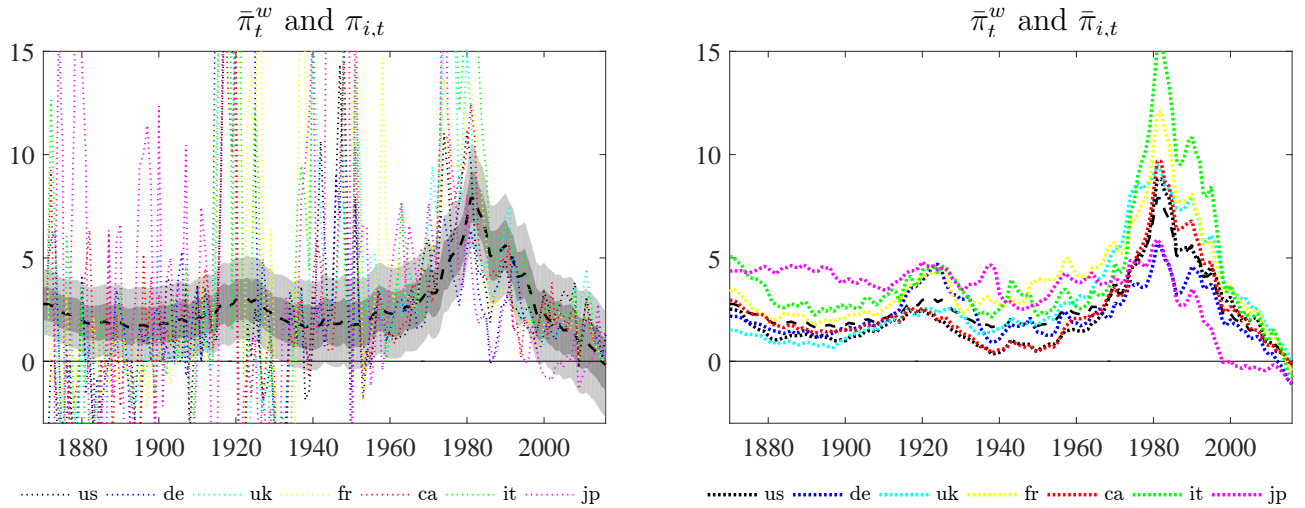
Note: The left panel shows $R_{i,t} - \pi_{i,t}$ for each country i (dotted lines; see legend), together with the trend \bar{r}_t^w (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{r}_{i,t} = \bar{r}_t^w + \bar{r}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend \bar{r}_t^w (dashed black line).

The right panel of Figure 3 displays the trend in the world real interest rate (black dashed line) and the overall trend $\bar{r}_{i,t} = \bar{r}_t^w + \bar{r}_t^i$ for each country in the sample (dotted lines). Two interesting facts emerge from this figure. First, the country-specific components have shrunk noticeably since the late 1970s, bringing the trends much closer together and also to the world real interest rate. This long-run convergence in rates of return arguably reflects the increased liberalization of capital movements over the past fifty years, which should move global capital markets closer to the perfect arbitrage ideal.²⁵ Second, to the extent that we want to interpret \bar{r}_t^i as indicative of a country-specific convenience yield, U.K. government paper yielded greater convenience than U.S. Treasuries for the first century of the sample, but this ranking has been reversed over the last fifty years (see Gourinchas and Rey, 2014). One puzzling feature of the figure is the trend for France, which is estimated to be below

²⁵Bekaert and Mehli (2017) propose a measure of global financial market integration that can be computed back to 1885. They find that it follows a “swoosh.” It was relatively high before the First World War, it fell in the interwar period, and it has been on a steady rise since around 1950. See also Chapter 3 of the IMF World Economic Outlook from April 2014 for a discussion of the connection between financial market integration and the decline in world real interest rates over the past few decades. We should note that our assumption that the country-specific trends are random walks, if taken literally, implies that this situation of convergence is unlikely to persist. However, the fact that our trends are very slow moving (the variance of their innovations is low) also implies that it may take a long time for trends in real rates to diverge again.

all others for the entire sample. Figure A2 in the Appendix compares the estimates of the country-specific trend \bar{r}_t^i for each country with their closest observed counterpart, the ex-post real rates $R_{i,t} - \pi_{i,t}$ in deviation from the cross-country average $\frac{1}{n} \sum_{i=1}^n (R_{i,t} - \pi_{i,t})$. This comparison suggests there are no major discrepancies between the estimated idiosyncratic trends and the low-frequency movements in the observables. One exception is France in the past twenty years, where the country-specific trend is clearly lower than the data.

Figure 4: Trends and Observables for Inflation, Baseline Model



Note: The left panel shows $\pi_{i,t}$ for each country i (dotted lines, see legend), together with the trend $\bar{\pi}_t^w$ (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{\pi}_{i,t} = \lambda_i^{\pi} \bar{\pi}_t^w + \bar{\pi}_t^i$ for each country i (dotted lines, see legend), together with the posterior median of the trend $\bar{\pi}_t^w$ (dashed black line).

Our data set does not include direct observations on real interest rates. Therefore, our estimates of inflation trends are a crucial input in the extraction of the low-frequency component of global real returns. In fact, the inflation trends that we compute are of independent interest because they characterize secular movements in inflation across countries and the extent to which these reflect global rather than country-specific forces. In addition, they represent a useful reality check on the ability of our econometric tools to separate trend from cycle.

The left panel of Figure 4 reports one such check by comparing the estimated global trend in inflation $\bar{\pi}_t^w$ with the observed inflation rates of all the countries in the sample. Similar to what we observed for real interest rates, inflation has become an increasingly global phenomenon, at least since World War II. Inflation rates in all countries were low in

the 1950s, rose in the 1960s and 1970s, and then fell in the last forty years. The results indicate that the global trend in inflation is at an all-time low.

The right panel of Figure 4 displays the world inflation trend $\bar{\pi}_t^w$ (black dashed line) along with the trend $\bar{\pi}_{i,t} = \lambda_i^\pi \bar{\pi}_t^w + \bar{\pi}_t^i$ for each country in the sample (dotted lines). Although the trends, like the raw data, tend to move more closely together in the second half of the sample, this convergence is less pronounced and uniform over time than for real returns, with notable country-specific idiosyncrasies. For instance, Italy is, not surprisingly, the country where the inflation trend reaches higher at around 1980, followed by France. Germany's trend, on the contrary, barely touches 5 percent at its peak and is consistently toward the bottom of the distribution. Japan's trend, which was often the highest in the first century of data, has been well below all others since the burst of its real estate bubble and the ensuing struggles with deflation and the zero lower bound. Except for Japan, all other inflation trends have been extremely close since the 1990s, heading together toward deflationary territory since the global financial crisis. In part, this recent convergence probably reflects the long-run effects on inflation of the adoption of a common monetary policy by Germany, France, and Italy after the introduction of the euro.

The last trend we discuss is that capturing low-frequency movements in term spreads. Figure A1 in the Appendix displays the estimated global trend $\bar{t}s_t^w$ together with observations on the term spread $R_{i,t}^L - R_{i,t}$ for each country i (dotted lines) on the left, as well as the estimated trends for each country $\bar{t}s_{i,t}$ on the right. These results demonstrate that the slope of the yield curve does fluctuate at low frequencies and that much of these fluctuations are common across countries, especially over the last few decades.

We close this section by discussing an empirical specification that relaxes the arbitrage condition (15). We do so by letting real returns in each country load on the common factor with a potentially different coefficient, as in

$$\bar{R}_{i,t} - \bar{\pi}_{i,t} = \lambda_i^r \bar{r}_t^w + \bar{r}_t^i. \quad (19)$$

Table A2 in the Appendix reports the estimated loadings under this specification. None of them is significantly different from one.²⁶ Moreover, Figure A12 shows that the estimates of \bar{r}_t^w and $\bar{r}_{US,t}$ produced by the unconstrained model are very close to those reported above, at least from the 1940s onward. We do not consider these results a particularly stringent test of perfect international arbitrage in the long run, although they are consistent with it.

²⁶More specifically, all the 90 percent posterior credible intervals include zero. For the U.K., the 68 percent credible interval is below one, while it is above one for the U.S. (barely) and Japan.

Their relevant implication for our empirical strategy is that the restricted model, which has the great advantage of being easily interpretable, is not at odds with the data. Therefore, we will maintain the no arbitrage restrictions in the empirical specifications explored in the remaining sections.

4.1.2 Trends in Real Interest Rates and Demographics

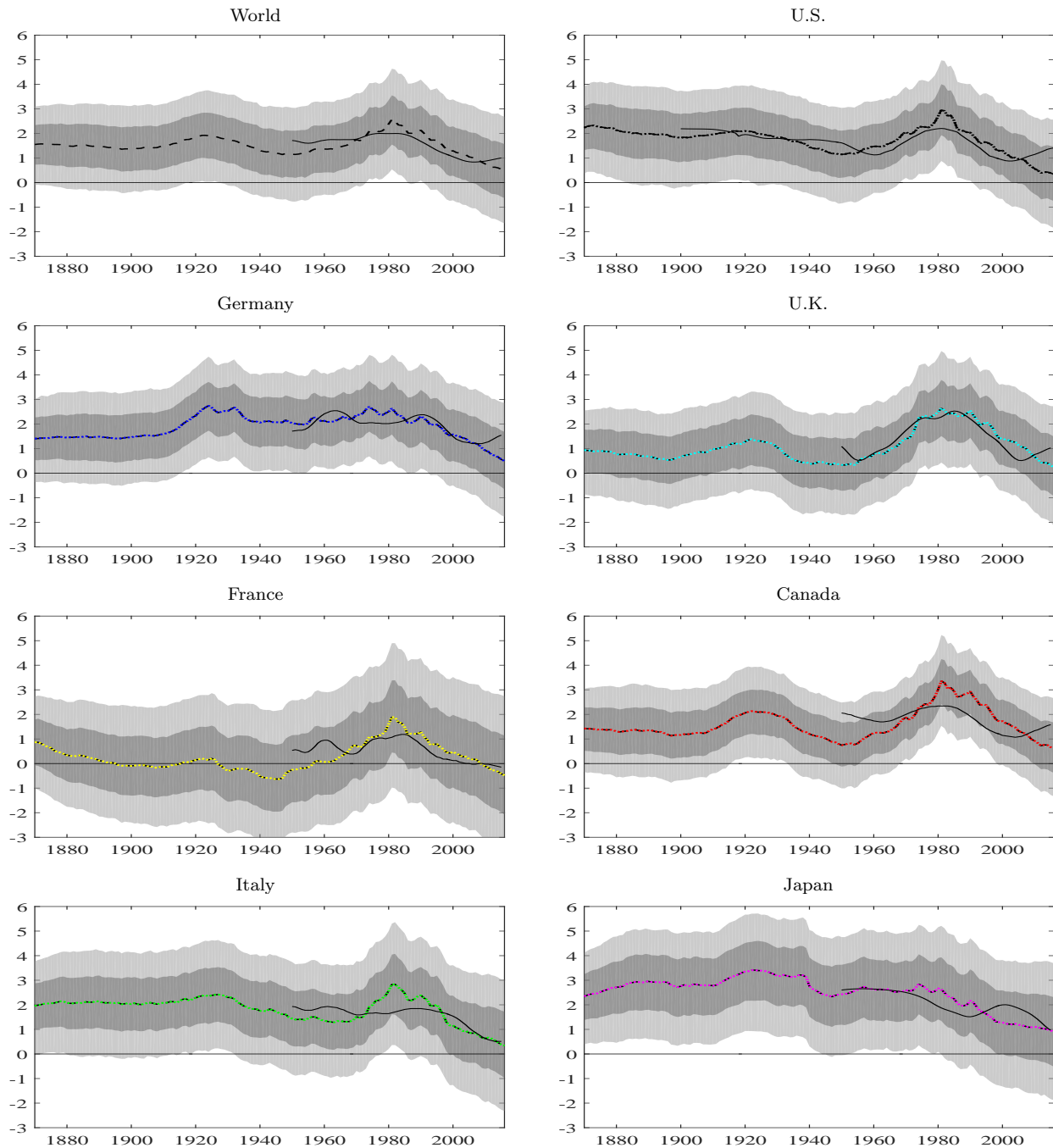
The aim of the rest of this paper is to identify some of the potential drivers of the trend in the world real interest rate. A growing literature finds a connection between demographics and low-frequency movements in interest rates (e.g., Aksoy et al., 2015; Carvalho et al., 2016; Favero et al., 2016; Gagnon et al., 2016; Eggertsson et al., 2017; Ferrero et al., 2017). The general idea is that the balance between different cohorts in the population affects the overall supply of savings. Middle-aged individuals tend to save and hence provide funds to the rest of the economy, while the young and the old tend to dissave and demand funds. As a result, the real interest rate that balances the overall supply of savings with the demand for investment is affected by the relative size of these cohorts. Favero et al. (2016) find that one variable effectively summarizes this connection between demographic composition and real interest rates—the ratio between middle-aged (those between 40 and 49) and young individuals (those between 20 and 29), or MY, as suggested by the overlapping-generations model of Geanakoplos et al. (2004).

Figure 5 shows that this variable indeed comoves with our estimates of the trend real interest rate at the global level and in most of the seven countries that we consider, although the fit varies substantially depending on the period and the country.²⁷ The panels show again our baseline estimates of the trend in the world and each country’s real interest rate (displayed in the right panel of Figure 3), along with the fitted value of a regression of that trend on MY.²⁸ In the United States, for which we have a longer sample, MY fits both the mild secular decline in the real interest rate between 1900 and 1950 and the hump that peaks

²⁷For the United States, we build MY with data from the Census Bureau, as in Favero et al. (2016). For the other countries, the data come from the UN World Population Statistics, which are available only since 1950. As the counterpart to the world real interest rate, we construct a “global” MY as the ratio between the total populations of middle-aged and young in the seven countries in the sample.

²⁸These regressions are run separately for the world economy and for each country. The R^2 in these regressions ranges from 26 to 81 percent (in the U.K.), while the regression coefficients are between -5.3 and -1.7. These results are consistent with the visual impression that the fit of this relationship is close overall, but also variable from country to country.

Figure 5: MY regression



Note: The upper-left panel reproduces the estimates of \bar{r}_t^w , and the remaining panels show the estimates of $\bar{r}_{i,t}$ for each country together with the fitted values of the regression of $\bar{r}_{i,t}$ on a constant and MY for each country. Similarly, the solid black line in the upper-left panel shows the fitted values of the regression of \bar{r}_t^w on a constant and a “world” MY, which is constructed as the ratio of the total middle-aged population to the total young population in all countries considered here. In all panels the dashed line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

around 1980. In fact, MY captures this hump remarkably well in all the countries where one is visible, with the possible exception of Italy. From this (admittedly only illustrative) evidence, we conclude that demographic factors, as captured by the variable MY, are plausible drivers

of secular movements in real interest rates.

However, these demographic factors turn upward (or remain flat) starting around 2000 in all countries except Italy and Japan. In fact, the recent evolution of the MY ratio suggests that demographic factors have been putting *upward* pressure on interest rates since the turn of the century. Therefore, demographics do not appear to be behind the decline in the trend real interest rate that has taken place over the past two decades.

As explained in Section 3, the trend in the real interest rate is driven both by movements in the stochastic discount factor and in the convenience yield. According to the theories outlined above, demographic factors are primarily reflected in the stochastic discount factor. Instead of detailing the interaction between demographics, economic growth, and the stochastic discount factor—which we leave for future research—we now proceed with disentangling fluctuations in the stochastic discount factor and in the convenience yield.

4.2 The Role of Convenience Yields

The baseline model described in the previous section used just enough information to identify this trend and distinguish it from trends in inflation and the term premium. In what follows, we will bring more information into the estimation that will allow us to decompose the overall trend in the world return on safe assets into some of its fundamental components. A key component that we consider is the convenience yield that distinguishes widely traded government bonds from comparable assets that are less liquid and safe. Going back to equation (14), this distinction is based on the relationship

$$\bar{r}_t^w = \bar{m}_t^w - \bar{c}y_t^w, \quad (20)$$

which splits \bar{r}_t^w into the trend of the stochastic discount factor of the marginal world investor, \bar{m}_t^w , and the low-frequency component of her taste for safety and liquidity, the global convenience yield $\bar{c}y_t^w$. Under the maintained no-arbitrage assumption, only one SDF and one convenience yield factor—those of the marginal international investor—are relevant to pin down the trend in the world interest rate. Therefore, one extra observable is sufficient to separately identify these two components, given the identification of \bar{r}_t^w that we had already achieved in the baseline model.

The key observable that gives us this identification is the yield on U.S. Baa corporate bonds, as computed by Moody's. Building on the work of Krishnamurthy and Vissing-Jorgensen (2012), we assume that these corporate bonds are both less safe and less liquid

than U.S. Treasuries of roughly equivalent maturity. As in Del Negro et al. (2017), we take this assumption to the extreme and postulate that these bonds have no convenience yield whatsoever. Therefore, the long-run component of the spread between the Baa yields and those on Treasuries is

$$\overline{R}_{US,t}^{Baa} - \overline{R}_{US,t}^L = \overline{cy}_t^w + \overline{cy}_t^{US}. \quad (21)$$

The assumption that Baa securities have no convenience at all is conservative, in the sense that its violation would result in an *underestimation* of the size of the convenience yield trend, as discussed in Del Negro et al. (2017). Indeed, Baa securities are not completely illiquid, and there are certainly less safe assets. Therefore, they are likely to earn some fraction of the convenience yield of Treasuries. If that is the case, the Baa spread will move less than one-to-one with the convenience yield, thus providing a lower bound on its true size.²⁹

The trend in the U.S. Baa spread provides observations on the *sum* of \overline{cy}_t^w and \overline{cy}_t^{US} , as shown in equation (22). How do we separate the two with no information on the returns of illiquid/unsafe securities in other countries? The answer is that, under the assumption that deviations from UIP are due to country-specific convenience yields \overline{cy}_t^i , the estimates of the idiosyncratic component of the trend in the U.S. real interest rate from the baseline model already give us a time series for \overline{cy}_t^{US} . Given this estimate, the U.S. Baa spread is enough to identify \overline{cy}_t^w .³⁰

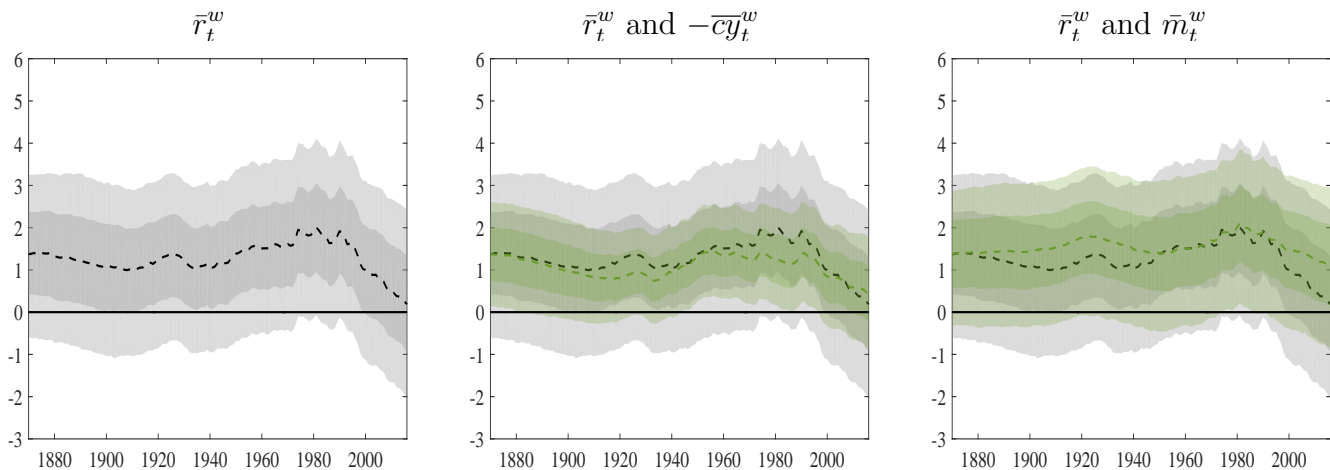
In summary, the second model that we estimate is the same as (18), but now includes the two common factors \overline{m}_t^w and \overline{cy}_t^w , as in (20). Those factors are identified by adding $R_{US,t}^{Baa}$ to the list of observables, according to the equation

$$\overline{R}_{US,t}^{Baa} = \overline{m}_t^w + \overline{ts}_t^w + \overline{ts}_t^i + \lambda_i^\pi \overline{\pi}_t^w + \overline{\pi}_t^i + \tilde{R}_{US,t}^{Baa}. \quad (22)$$

In this model $n = 22$ and $\tau = 25$.

²⁹Equation (22) ignores trends in the default rate as a potential determinant of the Baa spread. This is because Del Negro et al. (2017) document that, at least in the last forty years, default rates for U.S. Baa corporate bonds trended down, based on data from Gilchrist and Zakrajsek (2012). Therefore, accounting for the contribution of this trend to the spread would lead to even larger estimates of the convenience yield.

³⁰Adding a time-series for yields on corporate or similar bonds outside of the U.S. to our dataset would be an interesting extension of our work, as it would provide a more direct measure of convenience yields in other countries, thus allowing an explicit distinction between this and other potential sources of UIP violation on safe returns. This approach would be tantamount to studying long-run UIP violations using returns on unsafe/illiquid assets, rather than government bonds.

Figure 6: \bar{r}_t^w , $\bar{c}y_t^w$, and \bar{m}_t^w 

Note: In all three panels, the dashed black line is the posterior median and the shaded gray areas are the 68 and 95 percent posterior coverage intervals for \bar{r}_t^w . The dashed green line is the posterior median and the shaded green areas are the 68 and 95 percent posterior coverage intervals for $-\bar{c}y_t^w$ (middle panel) and \bar{m}_t^w (right panel).

4.2.1 Results

Figure 6 shows the trend in the world real interest rate \bar{r}_t^w from the model that includes Baa yields, as well as its decomposition between the trend in the stochastic discount factor of international investors \bar{m}_t^w and the trend in the global convenience yield $\bar{c}y_t^w$. The estimates of \bar{r}_t^w are reproduced in all three panels; the levels of $-\bar{c}y_t^w$ in the middle panel and of \bar{m}_t^w in the rightmost one are normalized to coincide with the posterior median of \bar{r}_t^w at the beginning of the sample, so as to provide a visual sense of the contributions of each factor to the secular fluctuations in the world interest rate.

This figure delivers the third result of the paper. Low-frequency movements in the global convenience yield are a key driver of the trend in the world real interest rate and especially of its pronounced decline over the past few decades. To a certain extent, this conclusion was already implied by our previous result that the trend in the world real interest rate is very close to that of the safe and liquid return in the U.S. This evidence, together with the finding in Del Negro et al. (2017) that the decline in the U.S. return since the late 1990s is driven in large part by an increase in its convenience yield, already delivers the result qualitatively. Figure 6 and Table 1 formalize and quantify this informal conclusion in the context of our global model.³¹

³¹Figure A13 in the Appendix shows that the estimates of $\bar{r}_{US,t}$ from the model with the Baa spread are consistent with those from the baseline model. The fact that they are not identical is not surprising, since the former uses one more piece of information to estimate the U.S. trend. In both models, $\bar{r}_{US,t}$ and \bar{r}_t^w fall

More specifically, the table shows that the trend in the global convenience yield accounts for about half of the secular decline in the world real interest rate since 1980, which in this model totals 171 basis points, and close to 60 percent of the more than 1 percent decline since 1997. These contributions are surrounded by sizable uncertainty, as is the estimate of the overall trend, but they are significantly different from zero at all three horizons. This evidence is therefore consistent with the view expressed by Caballero and Krishnamurthy (2009), Caballero (2010), Caballero et al. (2016), and Bernanke et al. (2011), among others, that the increased global demand for safe assets since the Asian crisis of 1997 has played a crucial role in driving interest rates lower across the world.³²

The right panel of Figure 6 shows that the global stochastic discount factor \bar{r}_t^w has also played an important role in the fall of the world real interest rate over the past decades. Table 1 reports that the SDF has declined by about the same amount as the convenience yield since 1980, but by less since 1990 and 1997. Moreover, the changes in the SDF are less precisely estimated than those of the global convenience yield, as clearly illustrated by the wider posterior probability bands in the right panel of Figure 6. As a result, the 90 percent posterior probability intervals for the declines over the three periods that we are considering all include zero.³³ What forces lie behind the estimated decline in the global stochastic discount factor? The next section takes up this question by bringing to the table information on consumption growth, as suggested by asset pricing theory.

closely together since the 1980s, with $\bar{r}_{US,t}$ declining a bit more toward the end of the sample. In addition, Figures A14 and A15 show that the estimated country-specific trends in real rates and in inflation from the model that includes the Baa spread are very similar to those from the baseline model shown in Figures 3 and 4.

³²Figure A25 in the Appendix plots our estimates of the convenience yield for the U.S. together with the Baa-Treasury spread. It shows that the estimated convenience yield captures very well the low-frequency movements in the spread, in particular its secular increase since the mid-1990s. Figure A26 shows spreads for five other countries (the spreads for Germany, France, and Italy are obtained from Gilchrist and Mojon (2018)), together with the estimates of $\bar{c}y_{i,t}$. While these spreads are generally available only for the past twenty years, the figure shows that, during this period, the spreads increased on average by 50 to 100 basis points, roughly consistent with the increase in $\bar{c}y_{i,t}$. Note that while the *change* in the spreads is in line with our estimates, the *level* is not. This is likely due to the fact that composition of the corporate bond indexes in terms of credit ratings, liquidity, and maturity is different relative to the U.S.

³³Table A1 in the Appendix provides more details on the posterior distribution of the decline in the world real interest rate and in the factors that drive it.

4.3 A Model with Consumption

The models that we have discussed so far treat the trend in the stochastic discount factor \overline{m}_t^w as an unobservable variable. In the specification considered in Section 4.2, this was estimated as the common factor among short- and long-term yields to all securities across the world, regardless of their safety and liquidity characteristics. In this section, we push the decomposition of the world real interest rate one step further.

We use data on per capita consumption growth to identify a component of \overline{m}_t^w connected to the global trend in consumption growth. We denote this trend as \overline{g}_t^w . A connection between the stochastic discount factor and some function of consumption growth forms the basis of most macro-finance asset-pricing theories, even if the empirical relevance of the resulting relationship between economic growth and rates of return has often been questioned. In light of the mixed evidence on the extent to which interest rates and consumption growth actually correlate even at low frequencies (see, e.g., Hamilton et al., 2016; Lunsford and West, 2017), our proposed model of the stochastic discount does not connect the two very tightly, allowing for other factors to shift that relationship. In particular, we assume that

$$\overline{m}_t^w = \overline{g}_t^w + \overline{\beta}_t^w, \quad (23)$$

where \overline{g}_t^w is the trend in consumption growth that is common across countries, while $\overline{\beta}_t^w$ captures trends in the SDF that are unrelated to consumption.

We extract the global consumption trend \overline{g}_t^w from real interest rates (using equation (26)) and from the low-frequency dynamics of consumption growth across countries, which evolve according to the equation

$$\overline{\Delta c}_{i,t} = \overline{g}_t^w + \overline{\gamma}_t^w + \overline{\gamma}_t^i. \quad (24)$$

Here, we allow for additional trends in consumption growth ($\overline{\gamma}_t^w$ and $\overline{\gamma}_t^i$) that are unrelated to the stochastic discount factor and that therefore do not affect real interest rates (do not enter (26)). These trends represent one more degree of freedom in the posited relationship between growth and returns, which allow the empirical model to account for the potentially tenuous connection between the two. Following the approach that we adopted for inflation and the term premiums, we assume that one of these trends, $\overline{\gamma}_t^w$, is common across countries, while another one, $\overline{\gamma}_t^i$, is idiosyncratic. This specification levers the cross-sectional information contained in our international data set to generalize a similar model that we estimated in Del Negro et al. (2017) on data for the United States. One possible interpretation of

the $\bar{\gamma}$ trends is that they reflect the growth rate of consumption of households that are hand-to-mouth or liquidity constrained (Campbell and Mankiw, 1989; Kaplan and Violante, 2014) and/or are excluded from international asset markets, as in the limited-participation literature (e.g., Vissing-Jorgensen, 2002).

From an econometric perspective, we could also identify a country-specific trend in consumption growth, say, \bar{g}_t^i , which would enter the expression for the stochastic discount factor. We restrict this term to zero because otherwise the stochastic discount factor would no longer be unique, violating the maintained assumption of perfect international arbitrage discussed in Section 3.

In sum, the third model we estimate includes consumption growth across countries as an observable

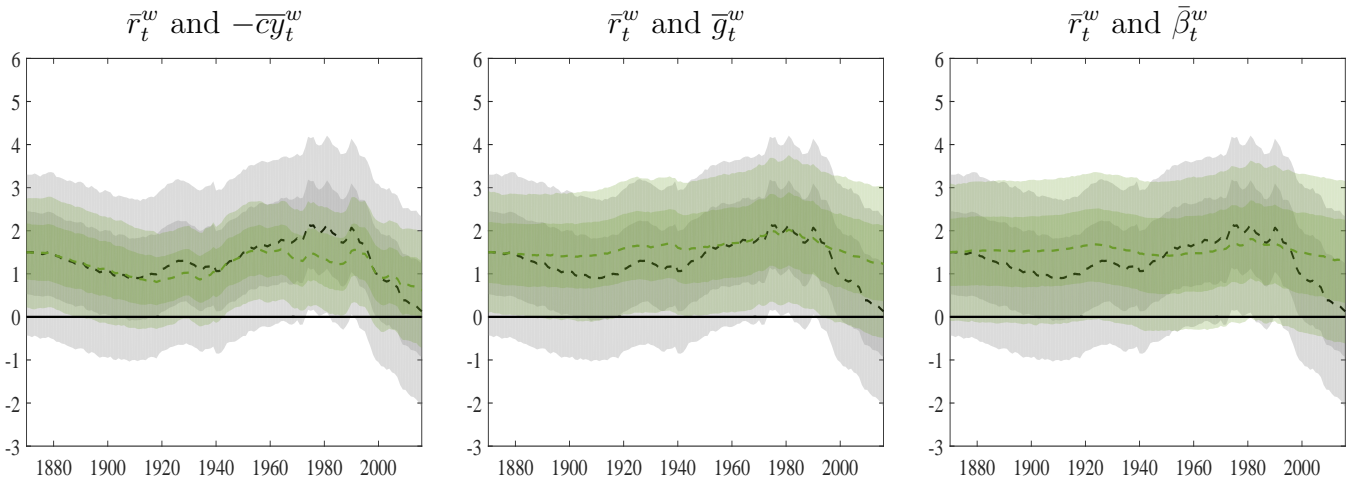
$$\Delta c_{i,t} = \bar{g}_t^w + \bar{\gamma}_t^w + \bar{\gamma}_t^i + \widetilde{\Delta c}_{i,t}, \quad (25)$$

for $i = 1, \dots, n$, in addition to the variables in (18) and (22). Moreover, the safe world real interest rate in this system is decomposed as

$$\bar{r}_t^w = \bar{g}_t^w + \bar{\beta}_t^w - \bar{c}y_t^w. \quad (26)$$

Therefore, this specification includes four global trends: one that is common to all yields and consumption growth rates across countries (\bar{g}_t^w), one that is common only to yields ($\bar{\beta}_t^w$), one that is common only to the yields for liquid assets ($\bar{c}y_t^w$), and finally one that is common only to consumption growth rates ($\bar{\gamma}_t^w$). In this model, $n = 29$ and $\tau = 34$.

Figure 7: \bar{r}_t^w , $\bar{c}y_t^w$, \bar{g}_t^w , and $\bar{\beta}_t^w$



Note: In all three panels, the dashed black line is the posterior median and the shaded gray areas are the 68 and 95 percent posterior coverage intervals for \bar{r}_t^w . The dashed green line is the posterior median and the shaded green areas are the 68 and 95 percent posterior coverage intervals for $-\bar{c}y_t^w$ (middle panel) and \bar{m}_t (right panel).

4.3.1 Results

Figure 7 shows the trend in the world real interest rate estimated from the model with consumption and its decomposition into global trends in the (negative of the) convenience yield $-\overline{cy}_t^w$, the part of consumption growth that prices assets \overline{g}_t^w , and the residual component of the stochastic discount factor unrelated to consumption $\overline{\beta}_t^w$. As in Figure 6, the series are all normalized so that their posterior medians coincide at the beginning of the sample.

The left panel of Figure 7, as well as Table 1, show that the estimated trend in the world real interest rate and the convenience yield trend are very similar to those obtained using the models of Sections 4.1 and 4.2. The middle panel shows that the global decline in consumption growth, possibly related to the demographic changes discussed in Section 4.1.2, also plays an important role in bringing down \overline{r}_t^w . The contribution of this factor to the recent trend decline in the world real interest rate is about 75 basis points from 1980, and 60 basis points from 1990. These median estimates are less precise than those of the convenience yield, but the posterior probability that the global consumption growth factor did decline after 1980 is above 95 percent, although it becomes lower after 1990.

The right panel of Figure 7 shows there is not much left to explain in the secular decline in the world safe return once the convenience yield and consumption growth are accounted for. The residual pricing factor $\overline{\beta}_t^w$ is roughly flat throughout the sample and its mild decline after 1980 is never statistically significant.³⁴

5 Conclusions

Ten years after the most acute phase of the global financial crisis, interest rates remain at or near historically low levels for many countries. We studied the secular drivers of this low-interest-rate environment through the lens of a vector autoregression with common trends, using historical data from seven countries dating back to 1870. We found that the trend in the world safe real interest rate, which was roughly stable at a bit below 2 percent for more than a hundred years, has dropped significantly over the past three decades. This global trend, which we identified as the common component in the low-frequency movements of the real yields on safe and liquid assets (government bonds or close substitutes) in the seven economies in our sample, closely resembles the trends for all advanced economies, including

³⁴Figures A18, A19, and A20 reproduce the results shown in Figures 1, 3, and 4 of Section 4.1 for the consumption model.

the United States, in the recent period. We find that country-specific trends have all but vanished since the 1970s.

This secular decline in global real rates is driven primarily by an increase in the premium that international investors are willing to pay to hold safe and liquid assets, as well as by lower economic growth around the world. The latter trend has been putting downward pressure on real rates since around 1980, possibly linked to demographic shifts, while the former emerged in the late 1990s. This timing points to the scarcity of safe assets in the context of a global saving glut as a fundamental secular force behind the low-interest-rate environment.

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Online Appendix

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A Gibbs Sampler for VARs with Common Trends

Let us use the notation $x_{i:j}$ to denote the sequence $\{x_i, \dots, x_j\}$ for a generic variable x_t . The Gibbs sampler is structured according to the following blocks:

1. $\bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$
 - (a) $\lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$
 - (b) $\bar{y}_{0:T}, \tilde{y}_{-p+1:T} | \lambda, \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$
2. $\varphi, \Sigma_\varepsilon, \Sigma_e | \bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$
 - (a) $\Sigma_\varepsilon, \Sigma_e | \bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$
 - (b) $\varphi | \Sigma_\varepsilon, \Sigma_e, \bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$

Details of each step follow:

1. $\bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$

This is given by the product of the marginal posterior distribution of λ (conditional on the other parameters) times the distribution of $\bar{y}_{0:T}, \tilde{y}_{-p+1:T}$ conditional on λ (and the other parameters).

- (a) $\lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$

The marginal posterior distribution of λ (conditional on the other parameters) is given by

$$p(\lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}) \propto L(y_{1:T} | \lambda, \varphi, \Sigma_\varepsilon, \Sigma_e) p(\lambda),$$

where $L(y_{1:T} | \lambda, \varphi, \Sigma_\varepsilon, \Sigma_e)$ is the likelihood obtained from the Kalman filter applied to the state space system (1) through (5). $p(\lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T})$ does not have a known form so we will use a Metropolis-Hastings step.

(b) $\bar{y}_{0:T}, \tilde{y}_{-p+1:T} | \lambda, \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$

Given λ and the other parameters of the state space model we can use Durbin and Koopman (2002)'s simulation smoother to obtain draws for the latent states $\bar{y}_{0:T}$ and $\tilde{y}_{-p+1:T}$. Note that in addition to $\bar{y}_{1:T}$ and $\tilde{y}_{1:T}$ we also need to draw the initial conditions \bar{y}_0 and $\tilde{y}_{-p+1:0}$ in order to estimate the parameters of (3) and (2) in the next Gibbs sampler step.

Note that missing observations do not present any difficulty in terms of carrying out this step: if the vector y_{t_0} has some missing elements, the corresponding rows of the observation equation (1) are simply deleted for $t = t_0$.

2. $\varphi, \Sigma_\varepsilon, \Sigma_e | \bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$

This step is straightforward because for given $\bar{y}_{0:T}$ and $\tilde{y}_{-p+1:T}$ equations (2) and (3) are standard VARs where in case of (2) we actually know the autoregressive matrices. The posterior distribution of Σ_e is given by

$$p(\Sigma_e | \bar{y}_{0:T}) = \mathcal{IW}(\underline{\Sigma}_e + \hat{S}_e, \kappa_e + T)$$

where $\hat{S}_e = \sum_{t=1}^T (\bar{y}_t - \bar{y}_{t-1})(\bar{y}_t - \bar{y}_{t-1})'$. The posterior distribution of φ and Σ_ε is given by

$$p(\Sigma_\varepsilon | \tilde{y}_{0:T}) = \mathcal{IW}(\underline{\Sigma}_\varepsilon + \hat{S}_\varepsilon, \kappa_\varepsilon + T),$$

$$p(\varphi | \Sigma_\varepsilon, \tilde{y}_{0:T}) = \mathcal{N} \left(\text{vec}(\hat{\Phi}), \Sigma_\varepsilon \otimes \left(\sum_{t=1}^T \tilde{x}_t \tilde{x}_t' + \underline{\Omega}^{-1} \right)^{-1} \right),$$

where $\tilde{x}_t = (\tilde{y}'_{t-1}, \dots, \tilde{y}'_{t-p})'$ collects the VAR regressors,

$$\hat{\Phi} = \left(\sum_{t=1}^T \tilde{x}_t \tilde{x}_t' + \underline{\Omega}^{-1} \right)^{-1} \left(\sum_{t=1}^T \tilde{x}_t \tilde{y}_t' + \underline{\Omega}^{-1} \underline{\Phi} \right), \quad \hat{S}_\varepsilon = \sum_{t=1}^T \hat{\varepsilon}_t \hat{\varepsilon}_t' + (\hat{\Phi} - \underline{\Phi})' \underline{\Omega}^{-1} (\hat{\Phi} - \underline{\Phi}),$$

and $\hat{\varepsilon}_t = \tilde{y}_t - \hat{\Phi}' \tilde{x}_t$ are the VAR residuals.

B Additional Tables and Figures

Table A1: Change in \bar{r}_t^w and Its Components – Additional Details

	1980-2016	1990-2016	1997-2016
Baseline Model			
	−1.88	−1.64	−1.03
	[−2.68, −1.11]	[−2.38, −0.93]	[−1.59, −0.45]
\bar{r}_t^w	(−3.24, −0.61)	(−2.84, −0.47)	(−1.96, −0.09)
	⟨−3.47, −0.38⟩	⟨−3.03, −0.25⟩	⟨−2.18, 0.07⟩
	{0.99}	{0.99}	{0.96}
Convenience Yield Model			
	−1.71	−1.73	−1.13
	[−2.45, −1.00]	[−2.39, −1.10]	[−1.67, −0.61]
\bar{r}_t^w	(−2.94, −0.55)	(−2.84, −0.67)	(−2.01, −0.26)
	⟨−3.16, −0.33⟩	⟨−3.08, −0.48⟩	⟨−2.19, −0.11⟩
	{0.99}	{1.00}	{0.99}
	−0.82	−0.97	−0.66
	[−1.29, −0.35]	[−1.38, −0.54]	[−1.02, −0.30]
$-\bar{c}y_t^w$	(−1.60, −0.05)	(−1.65, −0.24)	(−1.25, −0.06)
	⟨−1.74, 0.12⟩	⟨−1.79, −0.08⟩	⟨−1.36, 0.06⟩
	{0.96}	{0.98}	{0.96}
	−0.90	−0.78	−0.48
	[−1.48, −0.31]	[−1.32, −0.25]	[−0.91, −0.06]
\bar{m}_t^w	(−1.89, 0.08)	(−1.66, 0.09)	(−1.17, 0.22)
	⟨−2.07, 0.28⟩	⟨−1.83, 0.26⟩	⟨−1.34, 0.35⟩
	{0.94}	{0.93}	{0.87}

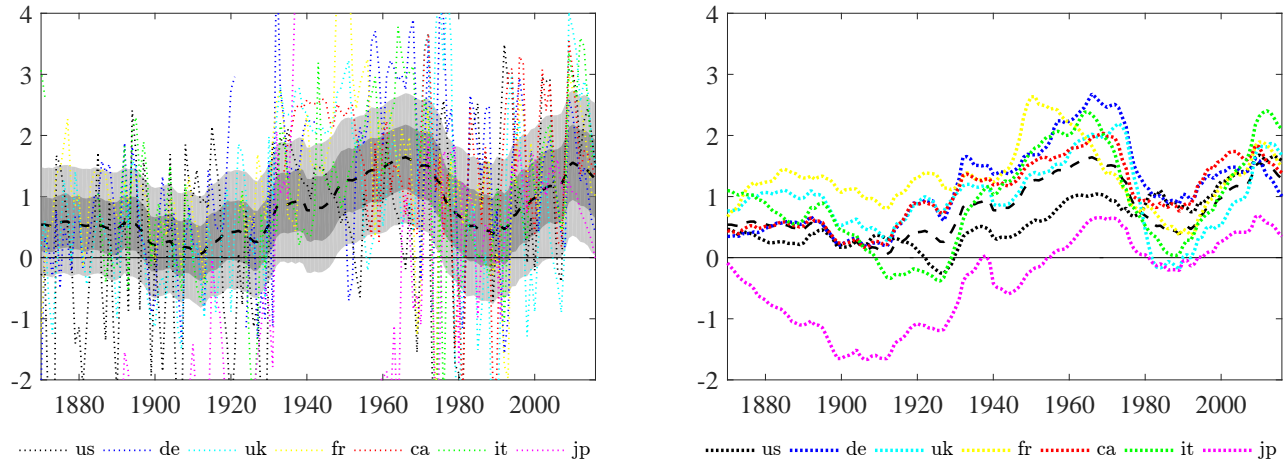
Consumption Model

	-1.93	-1.94	-1.22
	[-2.66, -1.16]	[-2.63, -1.25]	[-1.78, -0.65]
\bar{r}_t^w	(-3.18, -0.69)	(-3.10, -0.82)	(-2.18, -0.29)
	\langle -3.44, -0.39 \rangle	\langle -3.35, -0.60 \rangle	\langle -2.38, -0.11 \rangle
	{0.99}	{1.00}	{0.98}
	-0.71	-0.92	-0.65
	[-1.20, -0.22]	[-1.37, -0.47]	[-1.01, -0.26]
$-\bar{c}y_t^w$	(-1.51, 0.11)	(-1.67, -0.19)	(-1.25, -0.02)
	\langle -1.68, 0.27 \rangle	\langle -1.80, -0.03 \rangle	\langle -1.37, 0.10 \rangle
	{0.92}	{0.98}	{0.96}
	-1.22	-1.02	-0.58
	[-1.83, -0.60]	[-1.57, -0.48]	[-1.05, -0.15]
\bar{m}_t^w	(-2.23, -0.19)	(-1.95, -0.12)	(-1.33, 0.16)
	\langle -2.45, 0.03 \rangle	\langle -2.16, 0.10 \rangle	\langle -1.48, 0.31 \rangle
	{0.97}	{0.97}	{0.90}
	-0.74	-0.61	-0.35
	[-1.20, -0.31]	[-1.00, -0.21]	[-0.66, -0.03]
\bar{g}_t^w	(-1.50, -0.03)	(-1.28, 0.06)	(-0.88, 0.19)
	\langle -1.66, 0.10 \rangle	\langle -1.42, 0.17 \rangle	\langle -0.99, 0.29 \rangle
	{0.96}	{0.93}	{0.86}
	-0.47	-0.42	-0.24
	[-0.91, -0.01]	[-0.82, -0.02]	[-0.56, 0.08]
$\bar{\beta}_t^w$	(-1.21, 0.31)	(-1.07, 0.25)	(-0.78, 0.30)
	\langle -1.35, 0.46 \rangle	\langle -1.22, 0.38 \rangle	\langle -0.89, 0.44 \rangle
	{0.84}	{0.85}	{0.77}
	-1.84	-1.31	-0.72
	[-2.53, -1.14]	[-2.00, -0.66]	[-1.31, -0.14]
$\bar{\Delta}c_t^w$	(-2.99, -0.67)	(-2.41, -0.21)	(-1.71, 0.25)
	\langle -3.20, -0.43 \rangle	\langle -2.63, 0.03 \rangle	\langle -1.91, 0.46 \rangle
	{0.99}	{0.97}	{0.89}

Note: For a variable x , where $x = \{\bar{r}_t^w, \bar{m}_t^w, -\bar{c}y_t^w, \bar{g}_t^w, \bar{\beta}_t^w, \bar{\Delta}c_t^w\}$ depending on the model, the table shows the posterior median of $\Delta x = x_{2016} - x_{t_0}$ for t_0 being equal to 1980 (left column), 1990 (middle column), and 1997 (middle column), and the 68 ([.,.]), 90 ((.,.)), and 95 68 (\langle .,.\rangle) percent posterior coverage interval for Δx (in parenthesis). The figures in curly brackets ({.}) show $Pr\{x \leq 0\}$.

B.1 Baseline Model (Section 4.1) – Additional Results

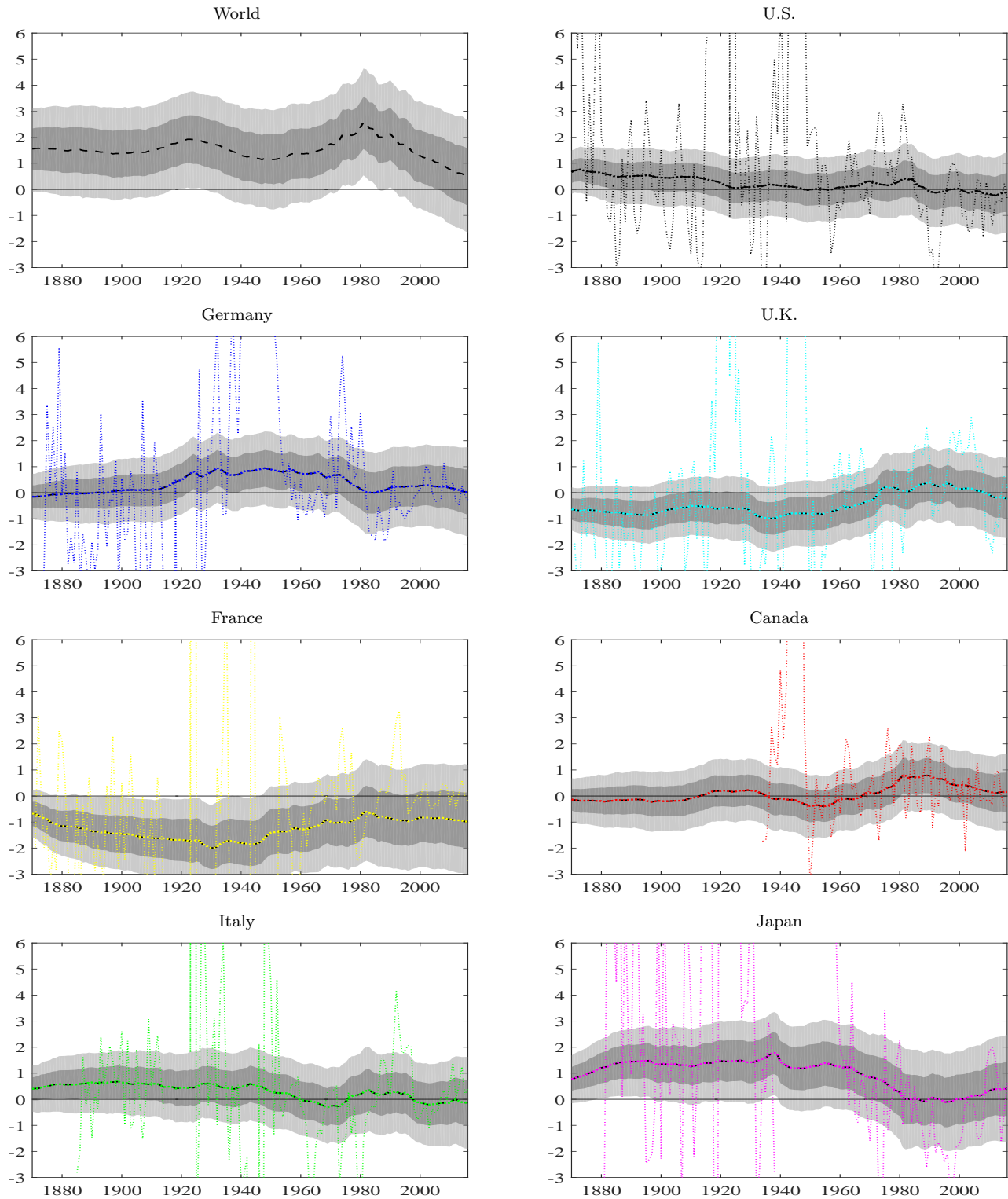
Figure A1: Trends and Observables for Term Spreads, Baseline Model
 \bar{ts}_t^w and $R_{i,t}^L - R_{i,t}$ \bar{ts}_t^w and $\bar{ts}_{i,t}$



Note: The left panel shows $R_{i,t}^L - R_{i,t}$ for each country i (dotted lines; see legend), together with the trend \bar{ts}_t^w (the dashed-and-dotted black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{ts}_{i,t} = \bar{ts}_t^w + \bar{ts}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend \bar{ts}_t^w (dashed-and-dotted black line).

Figure A2: Country-Specific Trends r_t^i and Observables, Baseline Model

$$\bar{r}_t^i \text{ and } R_{i,t} - \pi_{i,t} - \frac{1}{n} \sum_{i=1}^n (R_{i,t} - \pi_{i,t})$$

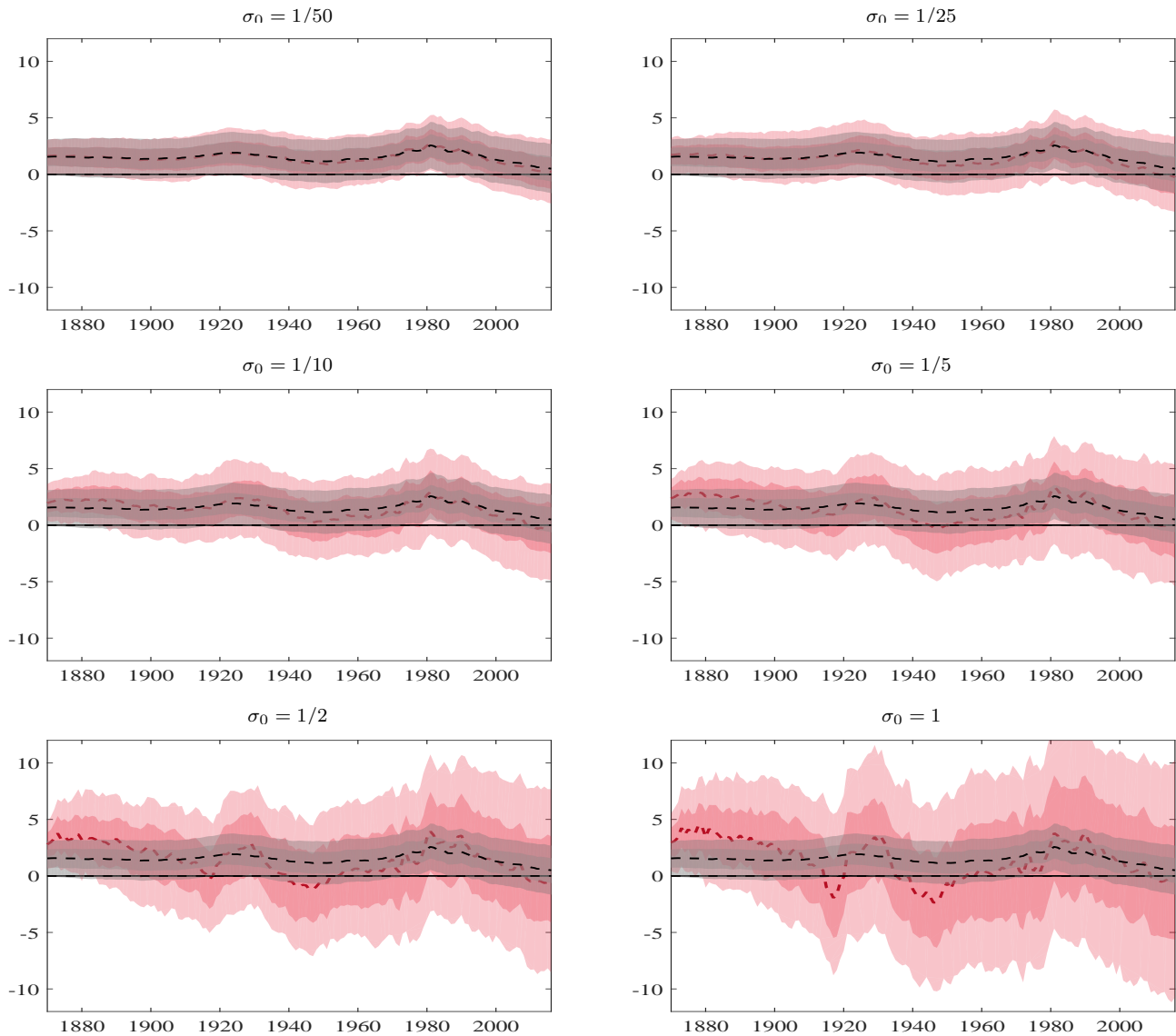


Note: The upper-left panel reproduces the estimates of \bar{r}_t^w , so as to put in context the magnitude of the fluctuations for the remaining panels, which show the estimates of the country-specific effect \bar{r}_t^i for each country together with the ex-post real rates $R_{i,t} - \pi_{i,t}$ in deviation from the average ex-post real rate across countries $\frac{1}{n} \sum_{i=1}^n (R_{i,t} - \pi_{i,t})$. In all panels, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

B.2 Alternative Priors for the Standard Deviation of Innovations to the Trend

Figure A3: Alternative Priors for the Standard Deviation of Innovations to the Trend,
Baseline Model

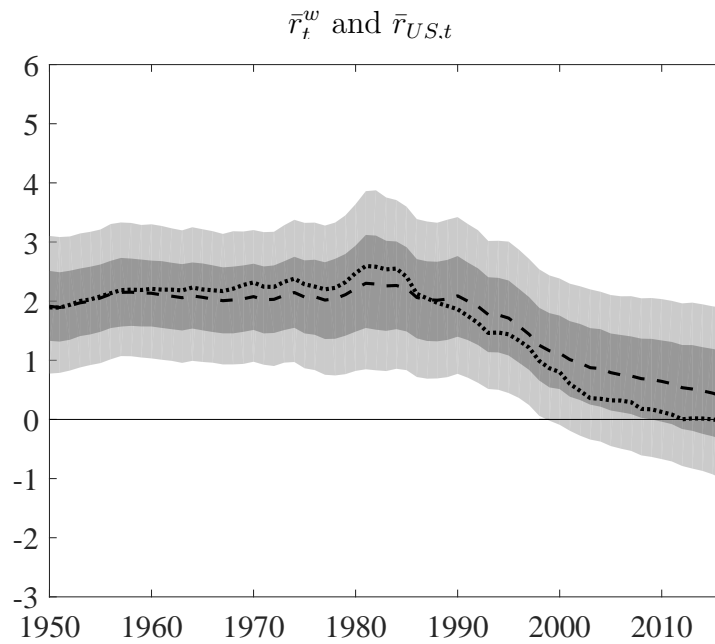
(prior for the variance of innovations to the trend centered at σ_0)



Note: The dashed black line is the posterior median and the shaded gray areas are the 68 and 95 percent posterior coverage intervals for r_t^w in the baseline model. The dashed red line is the posterior median and the shaded red areas are the 68 and 95 percent posterior coverage intervals for r_t^w obtained when centering the prior for the variance of innovations to the trend equal to $1/50$, $1/25$, $1/10$, $1/5$, $1/2$, and 1 as opposed to $1/100$ as in the baseline specification.

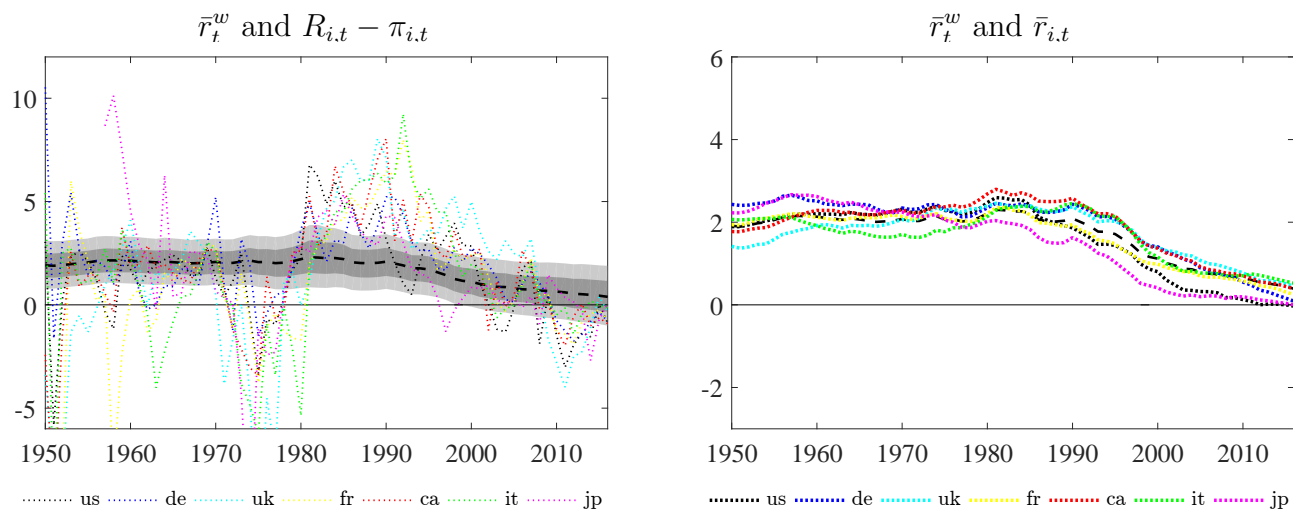
B.3 Baseline Model Estimated from 1950

Figure A4: Trends in Global and U.S. Real Rates: 1870-2016, Baseline Model Estimated from 1950



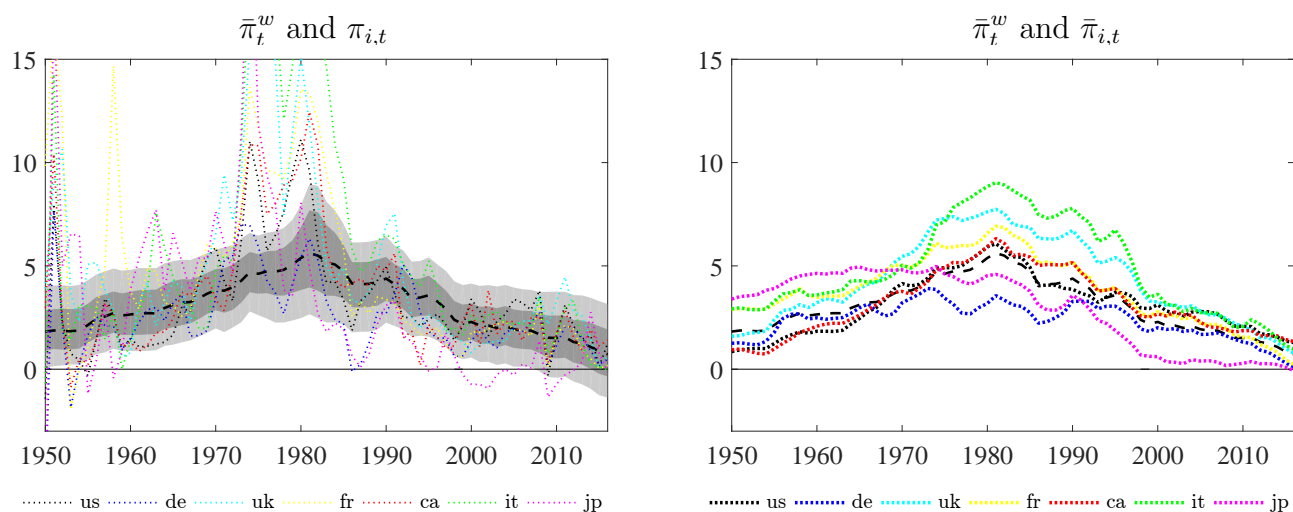
Note: The dashed black line shows the posterior median of \bar{r}_t^w and the shaded areas show the 68 and 95 percent posterior coverage intervals. The dotted black line shows the posterior median of $\bar{r}_{US,t} = \bar{r}_t^w + \bar{r}_t^{US}$.

Figure A5: Trends and Observables for Short-Term Real Rates, Baseline Model Estimated from 1950



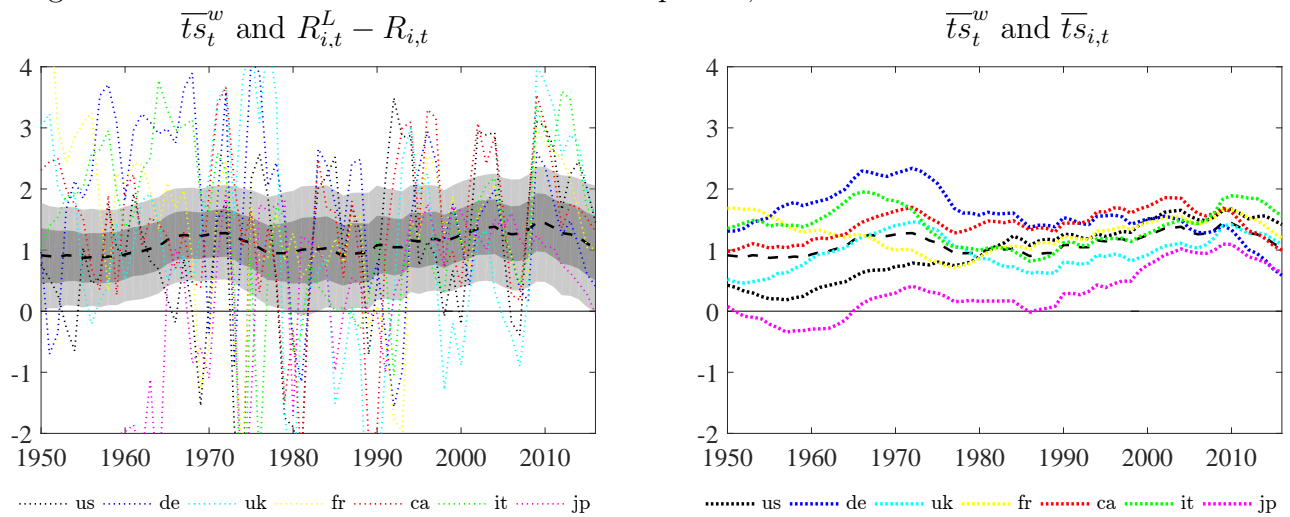
Note: The left panel shows $R_{i,t} - \pi_{i,t}$ for each country i (dotted lines; see legend), together with the trend \bar{r}_t^w (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{r}_{i,t} = \bar{r}_t^w + \bar{r}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend \bar{r}_t^w (dashed black line).

Figure A6: Trends and Observables for Inflation, Baseline Model Estimated from 1950



Note: The left panel shows $\pi_{i,t}$ for each country i (dotted lines; see legend), together with the trend $\bar{\pi}_t^w$ (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{\pi}_{i,t} = \lambda_i^{\pi} \bar{\pi}_t^w + \bar{\pi}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend $\bar{\pi}_t^w$ (dashed black line).

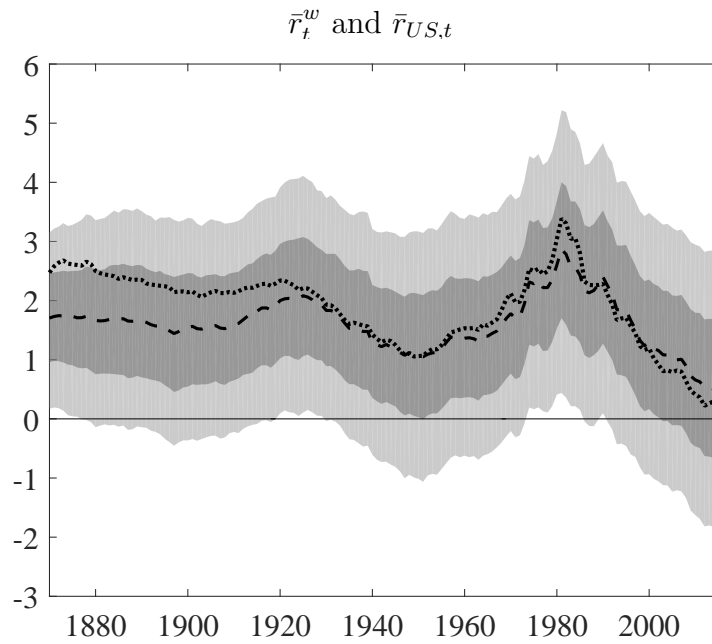
Figure A7: Trends and Observables for Term Spreads, Baseline Model Estimated from 1950



Note: The left panel shows $R_{i,t}^L - R_{i,t}$ for each country i (dotted lines; see legend), together with the trend \overline{ts}_t^w (the dashed-and-dotted black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\overline{ts}_{i,t} = \overline{ts}_t^w + \overline{ts}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend \overline{ts}_t^w (dashed-and-dotted black line).

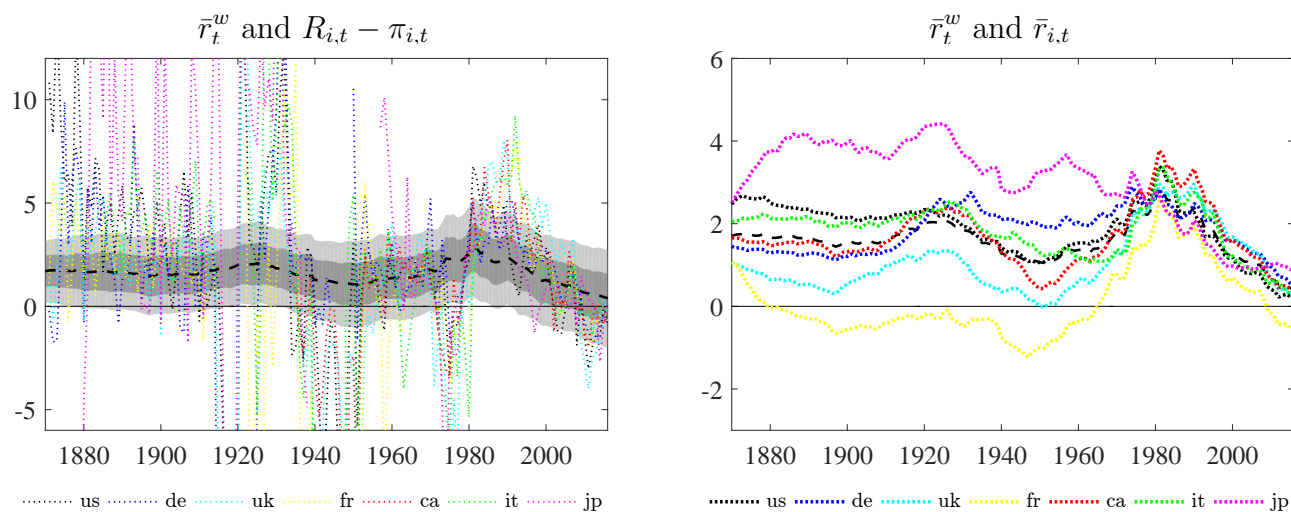
B.4 Baseline Model with 50 Degrees of Freedom

Figure A8: Trends in Global and U.S. Real Rates: 1870-2016, Baseline Model with 50 Degrees of Freedom



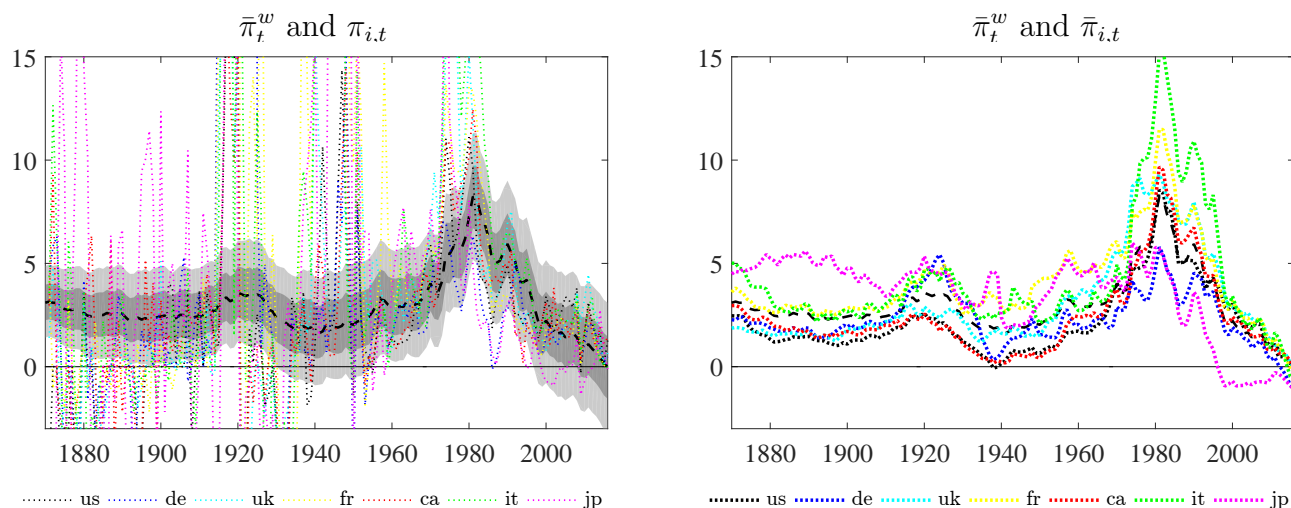
Note: The dashed black line shows the posterior median of \bar{r}_t^w and the shaded areas show the 68 and 95 percent posterior coverage intervals. The dotted black line shows the posterior median of $\bar{r}_{US,t} = \bar{r}_t^w + \bar{r}_t^{US}$.

Figure A9: Trends and Observables for Short-Term Real Rates, Baseline Model with 50 Degrees of Freedom



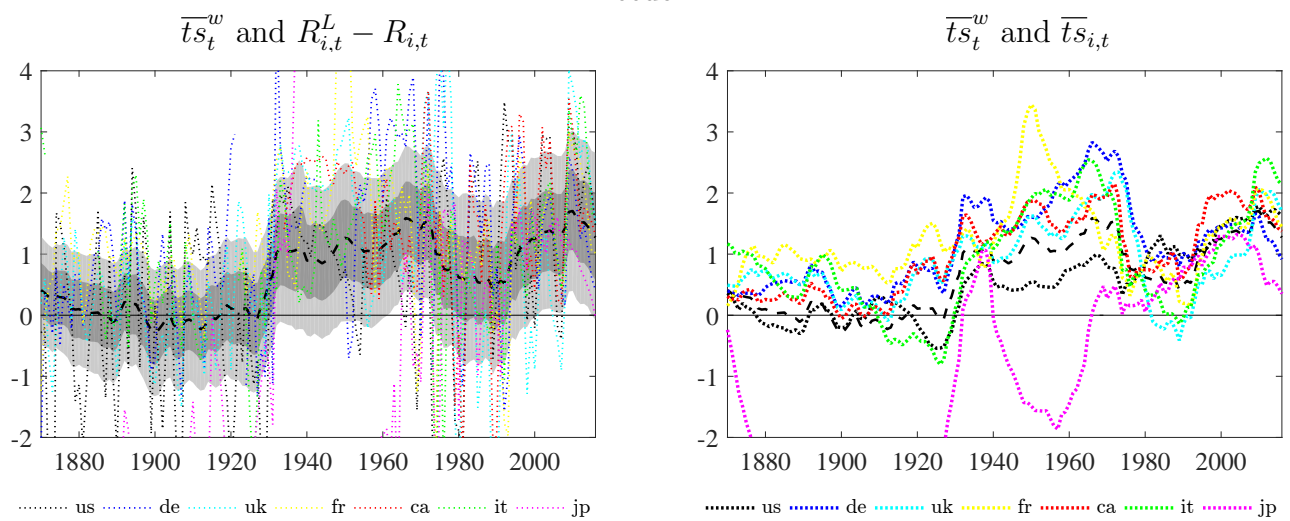
Note: The left panel shows $R_{i,t} - \pi_{i,t}$ for each country i (dotted lines; see legend), together with the trend \bar{r}_t^w (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{r}_{i,t} = \bar{r}_t^w + \bar{r}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend \bar{r}_t^w (dashed black line).

Figure A10: Trends and Observables for Inflation, Baseline Model with 50 Degrees of Freedom



Note: The left panel shows $\pi_{i,t}$ for each country i (dotted lines; see legend), together with the trend $\bar{\pi}_t^w$ (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{\pi}_{i,t} = \lambda_i^{\pi} \bar{\pi}_t^w + \bar{\pi}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend $\bar{\pi}_t^w$ (dashed black line).

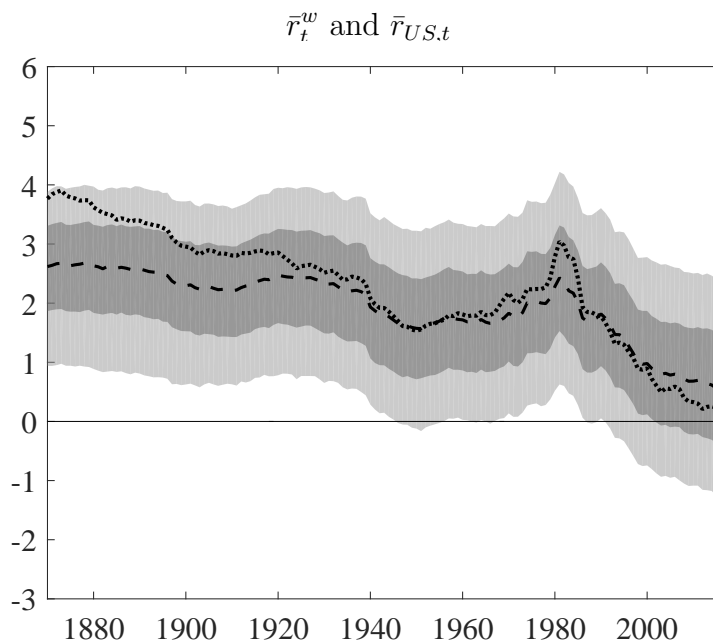
Figure A11: Trends and Observables for Term Spreads, Baseline Model with 50 Degrees of Freedom



Note: The left panel shows $R_{i,t}^L - R_{i,t}$ for each country i (dotted lines; see legend), together with the trend \bar{ts}_t^w (the dashed-and-dotted black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{ts}_{i,t} = \bar{ts}_t^w + \bar{ts}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend \bar{ts}_t^w (dashed-and-dotted black line).

B.5 Baseline Model – Unrestricted Version

Figure A12: Trends in Global and U.S. Real Rates: 1870-2016, Unrestricted Version of the Baseline Model (Section 4.1)



Note: The dashed black line shows the posterior median of \bar{r}_t^w and the shaded areas show the 68 and 95 percent posterior coverage intervals. The dotted black line shows the posterior median of $\bar{r}_{US,t} = \bar{r}_t^w + \bar{r}_t^{US}$.

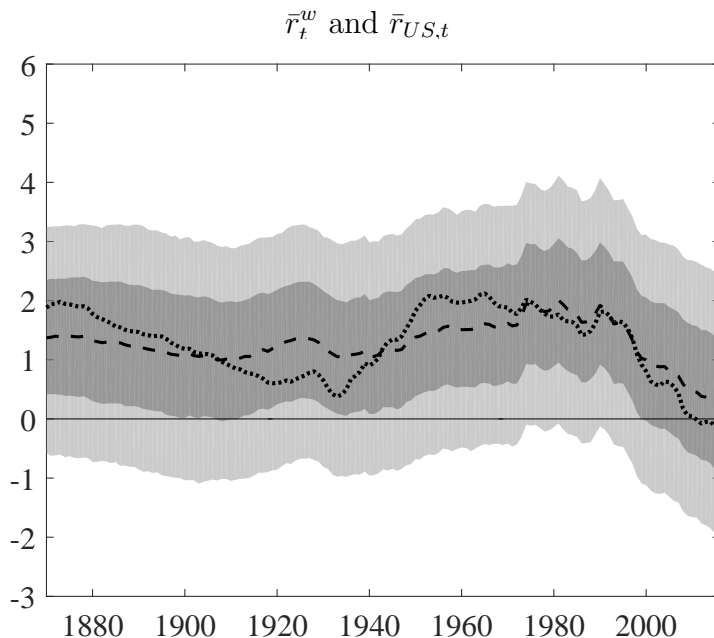
Table A2: Estimates of λ_i^r for the Unrestricted Version of the Baseline Model (Section 4.1)

us	1.23 [1.02, 1.45] $\langle 0.80, 1.68 \rangle$
de	0.86 [0.68, 1.07] $\langle 0.43, 1.36 \rangle$
uk	0.72 [0.50, 0.90] $\langle 0.32, 1.18 \rangle$
fr	0.85 [0.61, 1.08] $\langle 0.38, 1.35 \rangle$
ca	1.05 [0.84, 1.22] $\langle 0.67, 1.43 \rangle$
it	1.24 [0.94, 1.50] $\langle 0.68, 1.73 \rangle$
jp	1.37 [1.15, 1.53] $\langle 0.89, 1.75 \rangle$

Note: The table shows the posterior median of λ_i^r and the 68 and 95 percent posterior coverage intervals.

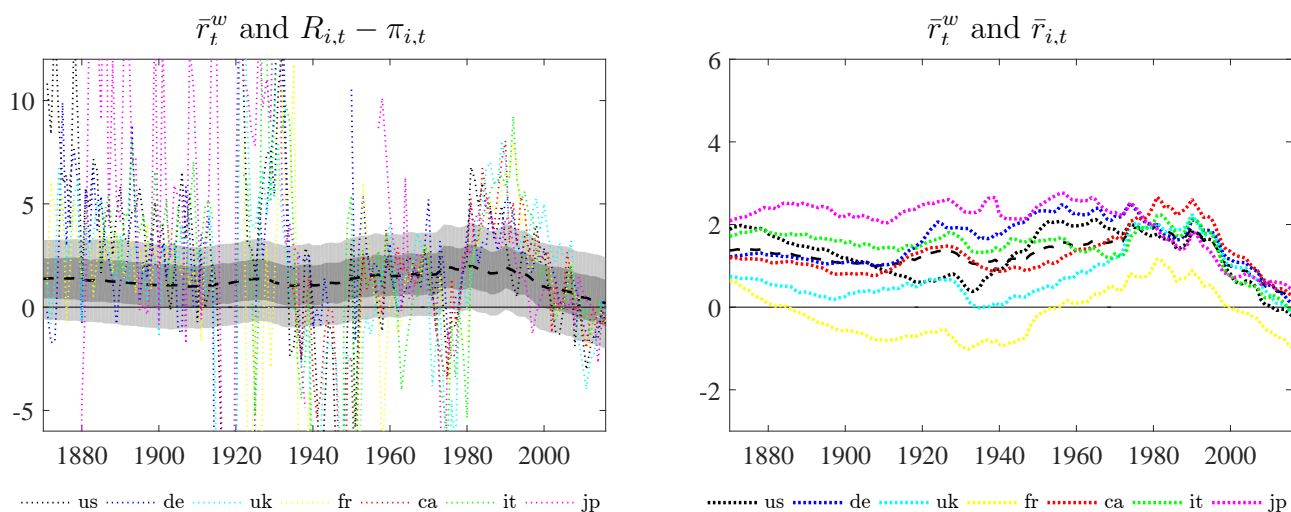
B.6 Convenience Yield Model (Section 4.2) – Additional Results

Figure A13: Trends in Global and U.S. Real Rates: 1870-2016, Convenience Yield Model



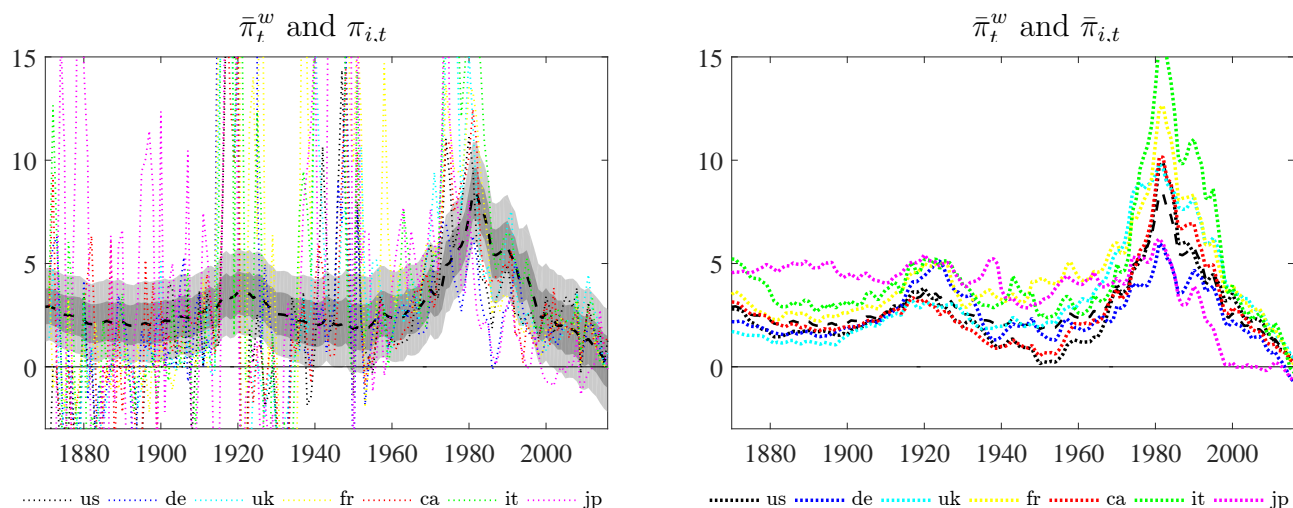
Note: The dashed black line shows the posterior median of \bar{r}_t^w and the shaded areas show the 68 and 95 percent posterior coverage intervals. The dotted black line shows the posterior median of $\bar{r}_{US,t} = \bar{r}_t^w + \bar{r}_t^{US}$.

Figure A14: Trends and Observables for Short-Term Real Rates, Convenience Yield Model



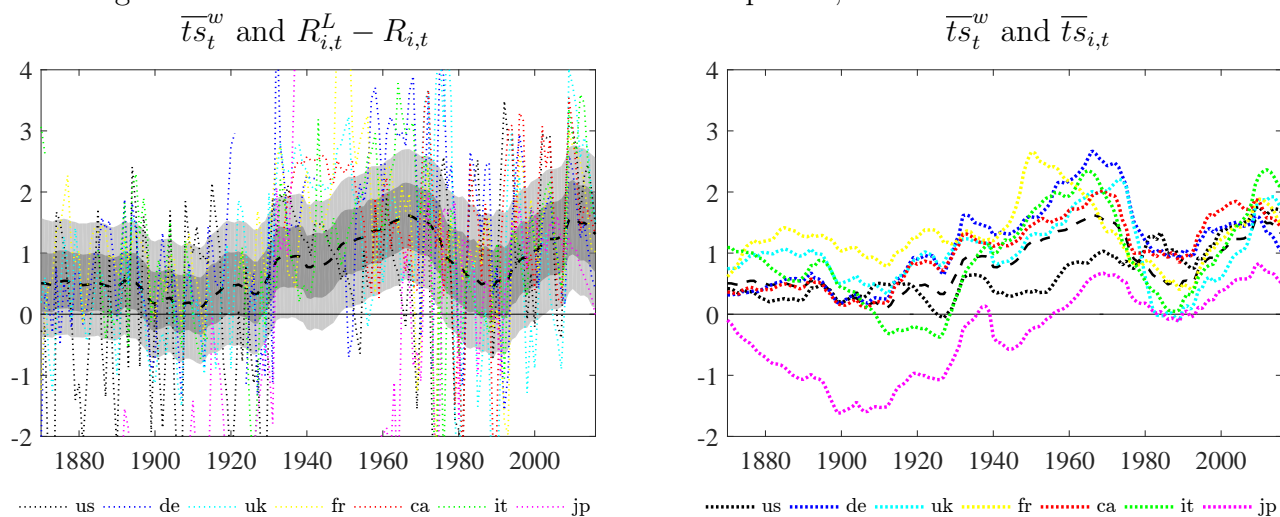
Note: The left panel shows $R_{i,t} - \pi_{i,t}$ for each country i (dotted lines; see legend), together with the trend \bar{r}_t^w (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{r}_{i,t} = \bar{r}_t^w + \bar{r}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend \bar{r}_t^w (dashed black line).

Figure A15: Trends and Observables for Inflation, Convenience Yield Model



Note: The left panel shows $\pi_{i,t}$ for each country i (dotted lines; see legend), together with the trend $\bar{\pi}_t^w$ (the dashed black line) shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{\pi}_{i,t} = \lambda_i^{\pi} \bar{\pi}_t^w + \bar{\pi}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend $\bar{\pi}_t^w$ (dashed black line).

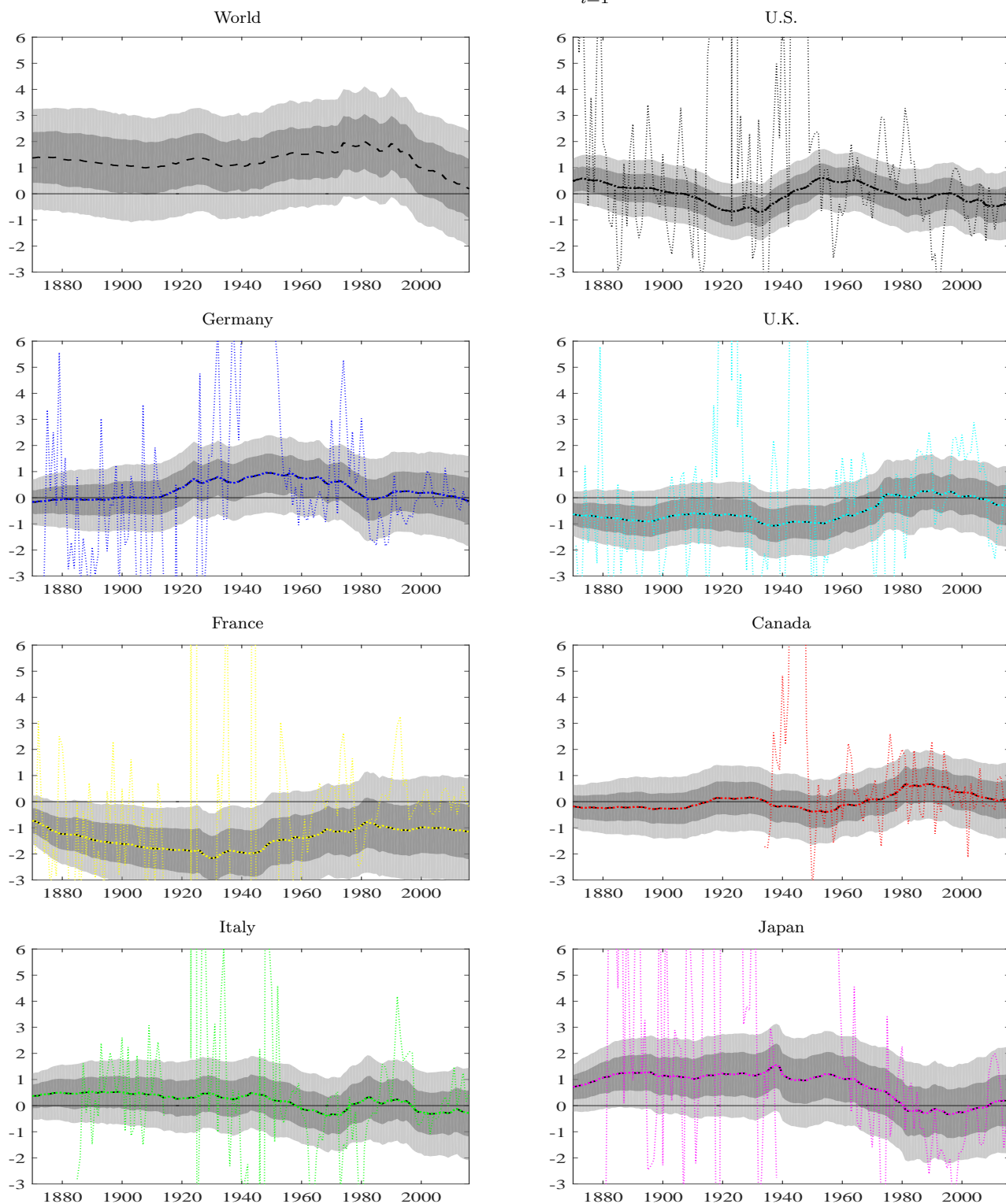
Figure A16: Trends and Observables for Term Spreads, Convenience Yield Model



Note: The left panel shows $R_{i,t}^L - R_{i,t}$ for each country i (dotted lines; see legend), together with the trend \bar{ts}_t^w (the dashed-and-dotted black line) shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{ts}_{i,t} = \bar{ts}_t^w + \bar{ts}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend \bar{ts}_t^w (dashed-and-dotted black line).

Figure A17: Country-Specific Trends \bar{r}_t^i and Observables, Convenience Yield Model

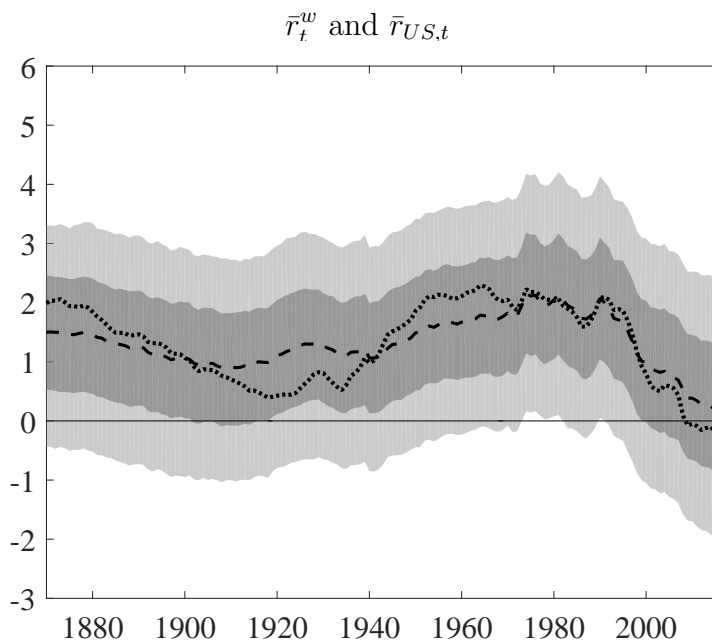
$$\bar{r}_t^i \text{ and } R_{i,t} - \pi_{i,t} - \frac{1}{n} \sum_{i=1}^n (R_{i,t} - \pi_{i,t})$$



Note: The upper-left panel reproduces the estimates of \bar{r}_t^w , so as to put in context the magnitude of the fluctuations for the remaining panels, which show the estimates of the country-specific effect \bar{r}_t^i for each country together with the ex-post real rates $R_{i,t} - \pi_{i,t}$ in deviation from the average ex-post real rate across countries $\frac{1}{n} \sum_{i=1}^n (R_{i,t} - \pi_{i,t})$. In all panels, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

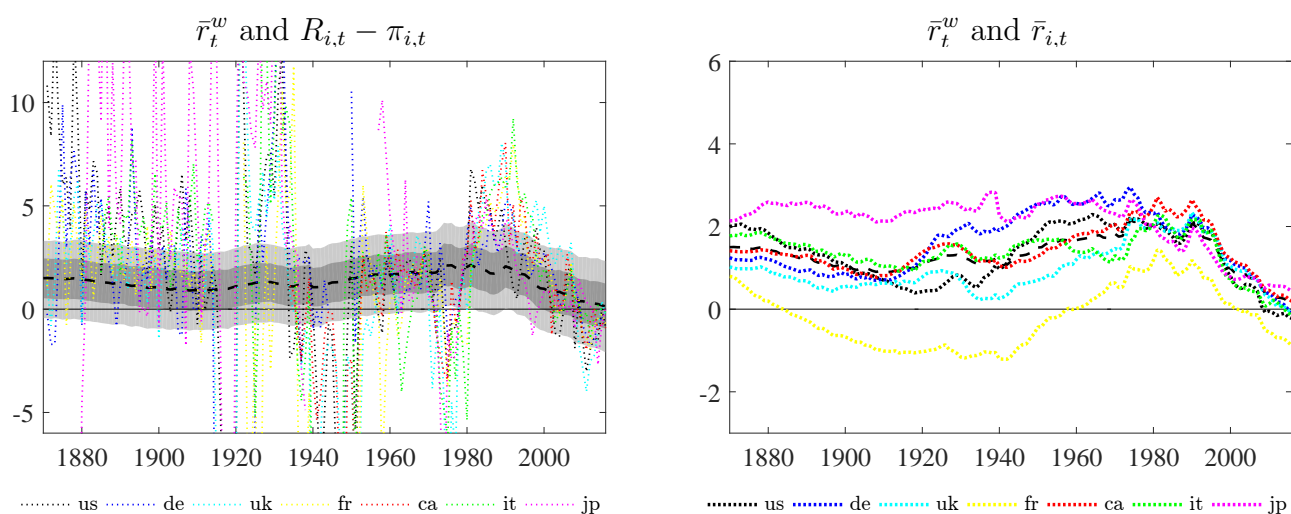
B.7 Consumption Model (Section 4.3) – Additional Results

Figure A18: Trends in Global and U.S. Real Rates: 1870-2016, Consumption Model



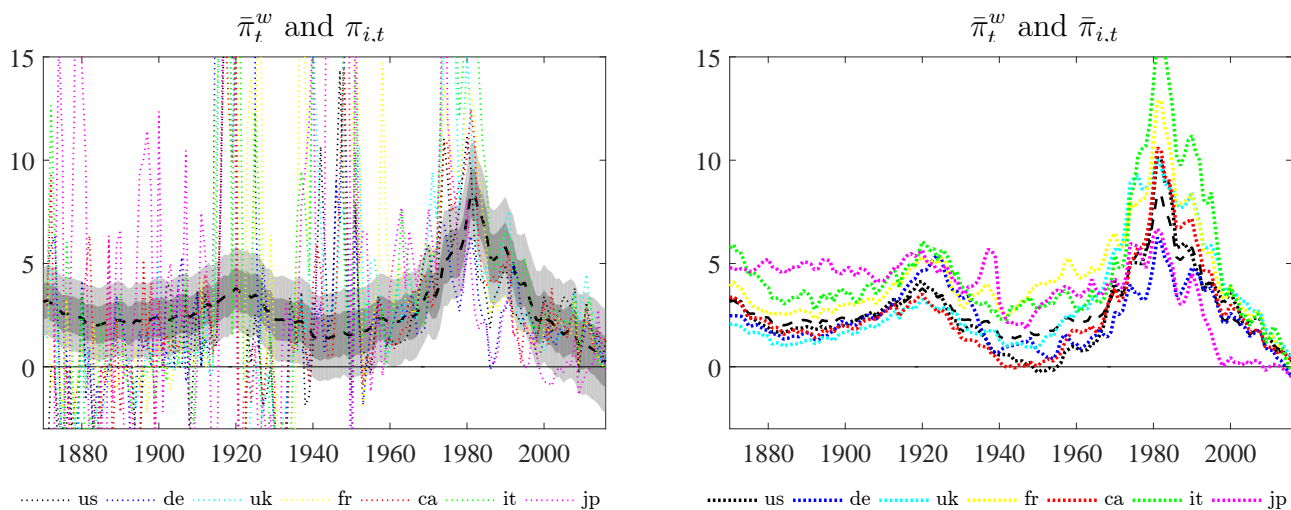
Note: The dashed black line shows the posterior median of \bar{r}_t^w and the shaded areas show the 68 and 95 percent posterior coverage intervals. The dotted black line shows the posterior median of $\bar{r}_{US,t} = \bar{r}_t^w + \bar{r}_t^{US}$.

Figure A19: Trends and Observables for Short-Term Real Rates, Consumption Model



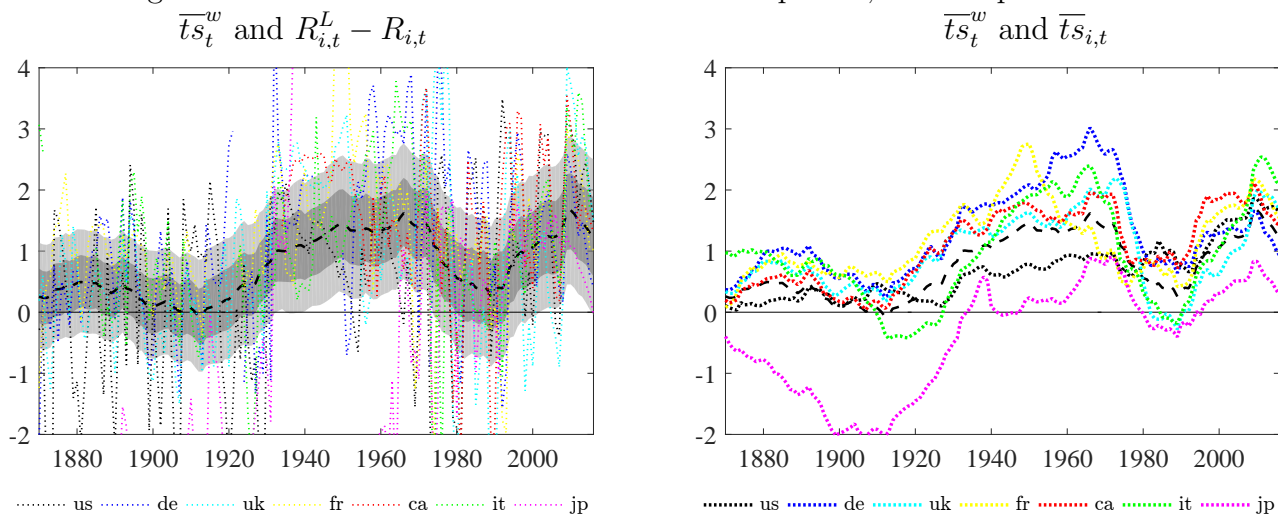
Note: The left panel shows $R_{i,t} - \pi_{i,t}$ for each country i (dotted lines; see legend), together with the trend \bar{r}_t^w (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{r}_{i,t} = \bar{r}_t^w + \bar{r}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend \bar{r}_t^w (dashed black line).

Figure A20: Trends and Observables for Inflation, Consumption Model



Note: The left panel shows $\pi_{i,t}$ for each country i (dotted lines; see legend), together with the trend $\bar{\pi}_t^w$ (the dashed black line) shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{\pi}_{i,t} = \lambda_i^{\pi} \bar{\pi}_t^w + \bar{\pi}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend $\bar{\pi}_t^w$ (dashed black line).

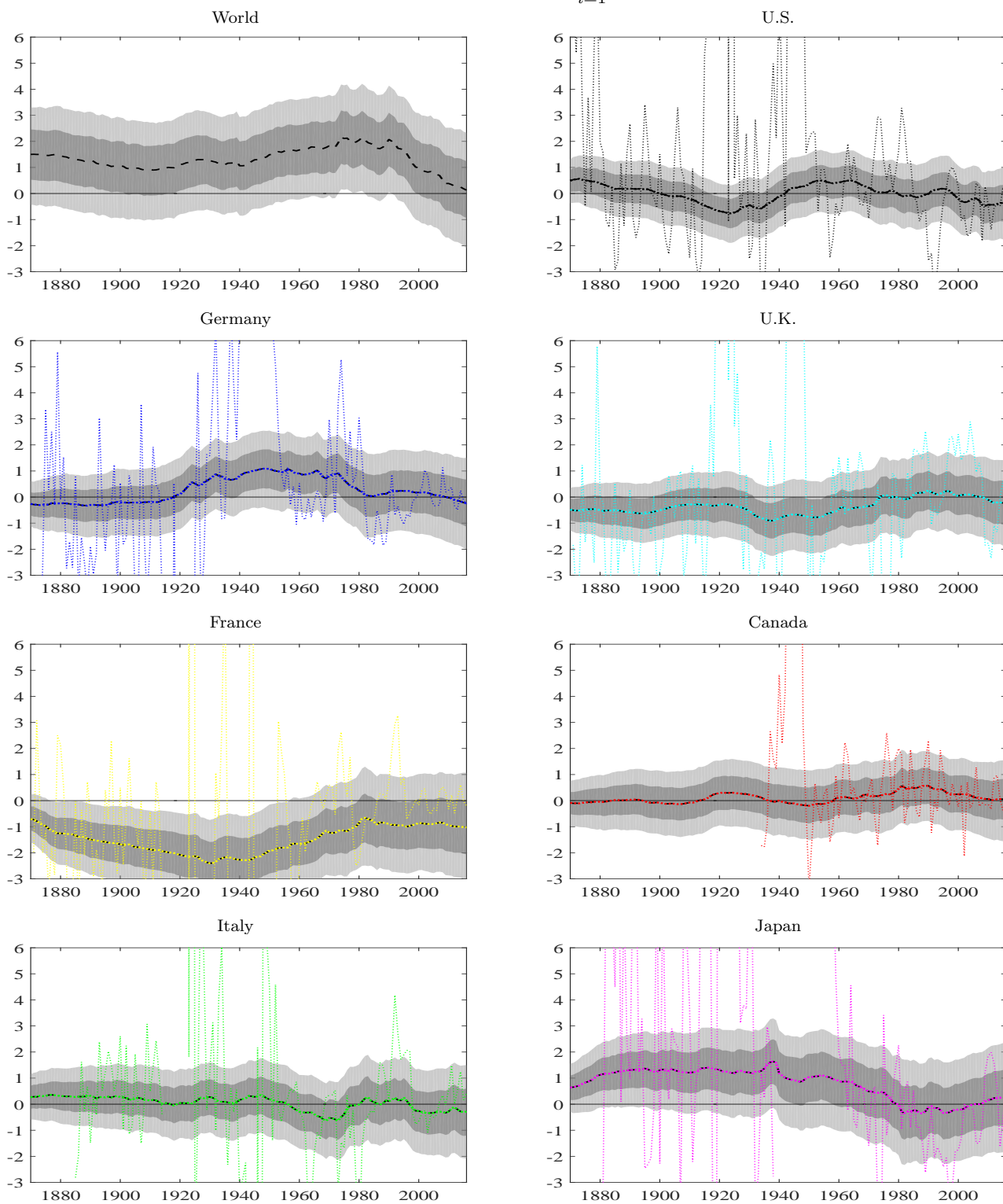
Figure A21: Trends and Observables for Term Spreads, Consumption Model



Note: The left panel shows $R_{i,t}^L - R_{i,t}$ for each country i (dotted lines; see legend), together with the trend \bar{ts}_t^w (the dashed-and-dotted black line) shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{ts}_{i,t} = \bar{ts}_t^w + \bar{ts}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend \bar{ts}_t^w (dashed-and-dotted black line).

Figure A22: Country-Specific Trends r_t^i and Observables, Consumption Model

$$\bar{r}_t^i \text{ and } R_{i,t} - \pi_{i,t} - \frac{1}{n} \sum_{i=1}^n (R_{i,t} - \pi_{i,t})$$



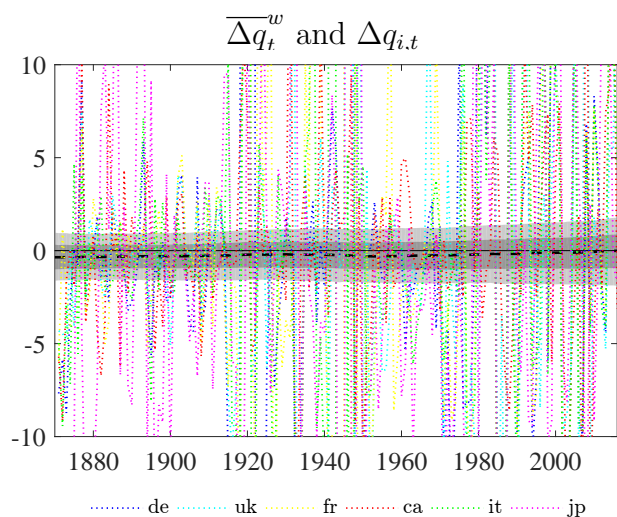
B.8 The VAR with Common Trends Applied to Changes in the Real Exchange Rate

In this section we present the results of the model

$$\Delta q_{i,t} = \overline{\Delta q}_t^w + \overline{\Delta q}_t^i + \widetilde{\Delta q}_{i,t}, \quad (\text{A-1})$$

for $i = 2, \dots, n$ ($i = 1$ is the U.S.), where $\Delta q_{i,t}$ is the first difference in the log real exchange rate in country i relative to the U.S., $\overline{\Delta q}_t^w$ captures trends in the real exchange rate changes relative to the U.S. for all countries, $\overline{\Delta q}_t^i$ captures country-specific trends, and $\widetilde{\Delta q}_{i,t}$ measures cyclical variations in the real exchange rate changes. The system is estimated jointly for all countries in the sample using the same priors as for the model described in section 4.1 of the paper.

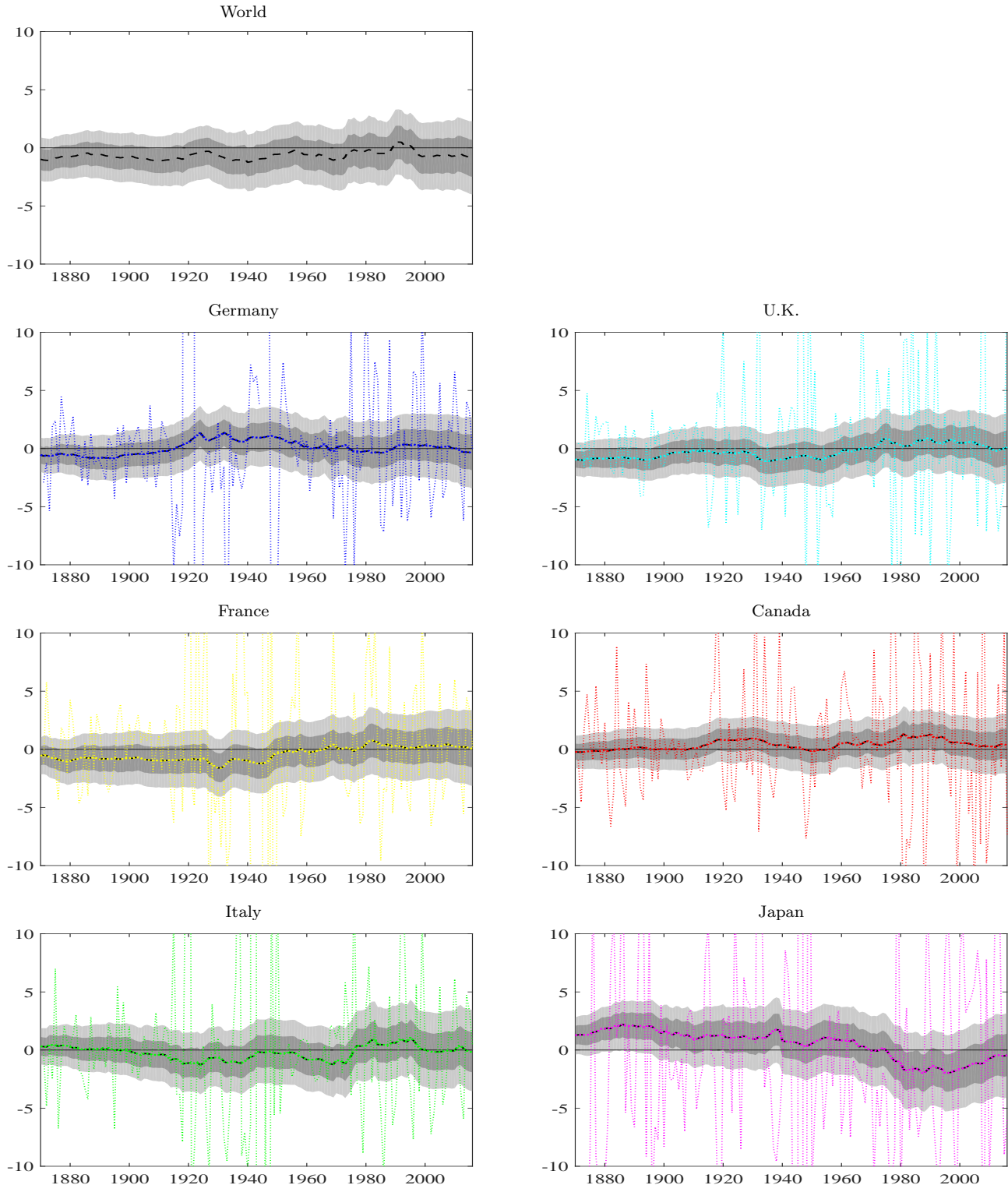
Figure A23: Common Trend and Observables for Changes in the Real Exchange Rate



Note: The panel shows $\Delta q_{i,t}$ for each country i (dotted lines; see legend), together with the trend $\overline{\Delta q}_t^w$ (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals).

Figure A24: Country-Specific Trends and Observables for Changes in the Real Exchange Rate, Exchange-Rate-Only Model

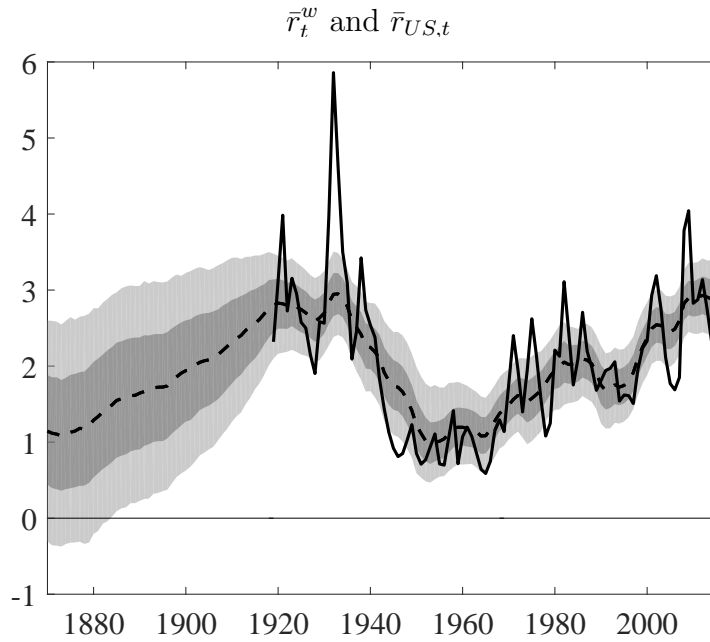
$$\bar{\Delta q}_t^i \text{ and } \Delta q_{i,t} - \frac{1}{n} \sum_{i=1}^n \Delta q_{i,t}$$



Note: The upper-left panel reproduces the estimates of $\bar{\Delta q}_t^w$, so as to put in context the magnitude of the fluctuations for the remaining panels, which show the estimates of the country-specific effect $\bar{\Delta q}_t^i$ for each country together with the ex-post real rates $\Delta q_{i,t}$ in deviation from the average ex-post real rate across countries $\frac{1}{n} \sum_{i=1}^n \Delta q_{i,t}$. In all panels, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

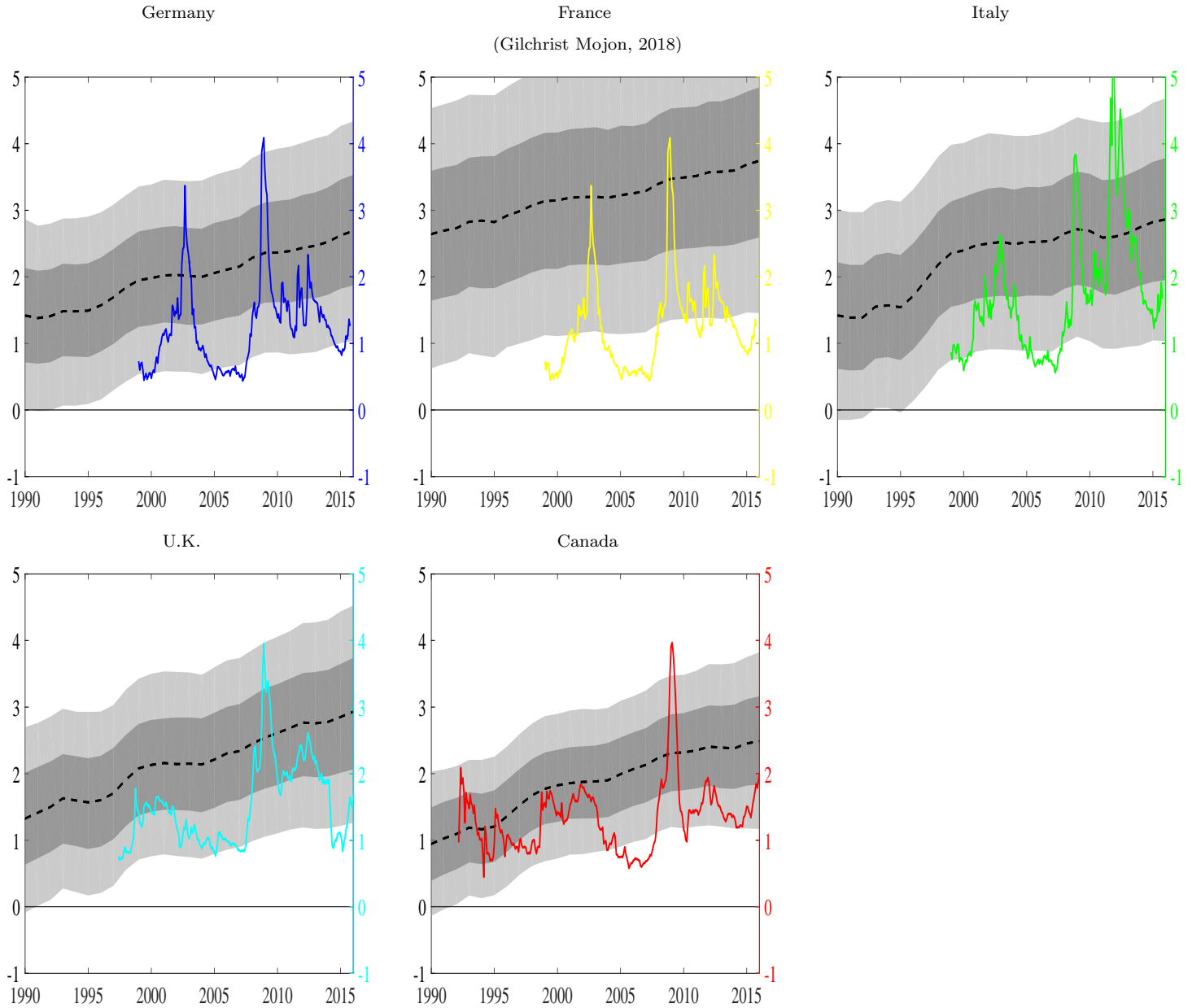
B.9 Spreads

Figure A25: Baa-Treasury Spread and Convenience Yield: US



Note: The dashed black line shows the posterior median of $\bar{c}y_{US,t} = \bar{c}y_t^w + \bar{c}y_t^{US}$ and the shaded areas show the 68 and 95 percent posterior coverage intervals. The solid black line shows Baa-Treasury spread.

Figure A26: Spreads



Note: In each panel, the dashed black line shows the posterior median of $\bar{c}y_{i,t} = \bar{c}y_t^w + \bar{c}y_t^i$ (convenience yield for country i) and the shaded areas show the 68 and 95 percent posterior coverage intervals. The solid lines show the spread between yields on corporate and government bonds. The spreads data for EMU countries (Germany, France, and Italy) are obtained from Gilchrist and Mojon (2018) and are computed with respect to German government bonds. U.K. corporate spreads are obtained from the Bank of England and are computed as the difference between corporate bond yields and the rate on 10-year government bonds. For Canada the spreads are obtained from the Federal Reserve Board's financial markets database and are the difference between corporate yields and five-year government bonds. We thank Ozge Akinci for providing us with these data.