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Research Report

The Golden Interest Rule: Robust Simple Interest Rate Rules for the Norwegian Economy

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Staff Memo

The golden interest rule

Maria Brunborg Hoen, Monetary Policy

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The Golden Interest Rule

Robust simple interest rate rules for the Norwegian economy



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Master thesis for the degree Master of Economic Theory and Econometrics

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This master thesis is part of a research project together with Pehlin Ilbas, Øistein Røisland, Tommy Sveen and Mathis Mæhlum on robust monetary policy in the research department of Norges Bank's Monetary Policy Department. The research project has the objective to further explore the method of robustifying against model uncertainty by modifying a standard loss function with a small weight on a robust simple interest rate rule (Ilbas et al. 2012), and develop such robust rules tailored to the Norwegian economy. The latter is the goal of this master thesis, whereas Mathis Mæhlum extends the work on the modified loss function to Norwegian models.

I have worked for two years as an assistant on the research project, first one year contributing to Ilbas et al. 2012, and then one year on the analyses presented in this master thesis. I am most grateful to Norges Bank for letting me work on the project and for the financial support. I deeply thank my supervisor Tommy Sveen for invaluable guidance, help and instructive contributions throughout my whole period in Norges Bank. I also thank Øistein Røisland for his many and highly useful comments, and my fellow student and colleague Mathis Mæhlum for fruitful discussions, inspiration and pleasant company. A big thank goes to the other staff in Norges Bank, who has helped me in some way or another, in particular Leif Brubakk, Martin Seneca, Junior Maih, Bjørn Naug, Ørjan Robstad, Kenneth Paulsen, and Klaus Blomli and Karsten Molværsmyr for rescuing my knocked-out computer, and those I might have forgotten to mention. At last I would like to thank my family and friends for support, and my father Helge Brunborg and Hannes Peinl for proofreading.

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1 Summary and Introduction

Norges Bank has been delegated the task of contributing to a steady growth of the Norwegian economy by keeping inflation low and stable. The goal is to keep the yearly growth in consumer prices at 2.5 percent in the medium run, balancing the inflation path with the overall capacity utilization whenever there is a conflict between the two (Norges Bank 2011), which corresponds to flexible inflation targeting. The main instrument for doing this is the sight deposit rate (foliorenten) on commercial banks' overnight deposits in Norges Bank, which affects money market rates through banks' marginal funding costs.

The policy rate decisions are based on analyses done with NEMO (the Norwegian Economy Model), as well as on general assessments about the state of the economy. NEMO is a large New Keynesian DSGE model representing the Norwegian mainland economy in a simplified and stylized manner. Uncertainty about the true structure of the economy is always an issue, and it is by no means certain that NEMO is the best possible description of it. A way to accommodate this uncertainty is to set the interest rate in a *robust* manner, meaning that monetary policy achieves a satisfactory level of macroeconomic stabilization also when the economy is highly different from the NEMO economy. Such policies that work well across a wide range of structural models are *robust to model uncertainty*.¹

However, robustness comes at the cost of optimality. Optimal policy is fine-tuned to the dynamics of a specific model, and does not perform well for completely different economies, as intuition suggests and many analyses show (e.g. Mathis Mæhlum 2012). A policy reaction function utilizing less model-specific information performs better on average across models, but naturally worse in each separate model, yet not substantially worse, as my results show. Such a restricted information rule is called a *simple interest rate rule* and is said to be robust if it performs well across a large variety of models.

Simple rules are commonly used by central banks in the conduct of monetary policy, of which there is thorough proof. Janet Yellen in the Board of Governors of the Federal Reserve System indicated that she uses "the Taylor rule" (see Chapter 2.3 for a description of the rule) to provide her with "a rough sense of whether or not the funds rate is at a reasonable level" (FOMC transcripts, January 31-February 1, 1995). The popularity of simple interest rate rules is due to their applicability and the way they are intuitive and communicative to the general public, but most of all their robustness properties.

Norges Bank also uses simple rules to cross-check the policy derived from NEMO, among other the above-mentioned Taylor rule. The actual performance of these rules in the Norwegian economy has not been properly investigated, and little work has been done on robust

¹See for example Hammond (2012) and Svensson (2000) for details.

simple rules.² A large share of the international literature has however found that the Taylor rule performs quite poorly, and in particular it appears not to be hard-hitting enough (Rudebusch and Svensson 1998). This calls for a better robust rule for the Norwegian economy, which is the goal of my work to find. By analyzing the properties and performance of a number of different policy rules, I seek a rule tailored to the Norwegian economy that yields a satisfactory outcome in a variety of models.

I use five different models for the Norwegian economy, with NEMO as the benchmark model: two extensions of NEMO — "Credit NEMO" with a credit market incorporated, and a backward-looking version "Policy NEMO" — as well as a small open model economy, the "Leitemo-Gali-Monacelli" (LGM) model (Leitemo 2006), and a slightly modified version of the macroeconometric model "Norwegian Aggregated Model" (NAM) by Bårdsen and Nymoen (2009), which is a model in "the Norwegian tradition" similar to KVARTS used by the Statistics Norway and RIMINI previously used by Norges Bank. NAM differs substantially from the other four, and drives much of the results in this thesis, which gives sound justifications for including it in the set of models. All of the models prescribe highly divergent reaction functions for the nominal interest rate, and therefore constitute a good and wide-ranging base for robustness analyses.

I mostly base my work on the paper by Taylor and Wieland (2009) on robust simple interest rate rules. As they do, I first find the optimal coefficients in three specifications of a simple interest rate rule in each separate model: one rule where the nominal interest rate responds only to inflation fluctuations and the output gap; one where the lagged interest rate is added in order to allow for more gradual adjustments, and one with the lagged output gap as well.

The optimization procedure is done in MatLab with the software Dynare³ and a search algorithm developed Junior Maih⁴ for internal use in Norges Bank.

My results show that since the optimized rules are fine-tuned to the dynamics of the rule-generating model, they yield remarkably less stability in the competing models. In particular miserable is the rule with only two variables from the LGM model applied in NAM, generating instability. There is also a strong conflict between the prescribed three variable rules in CN and NAM.

²There has, of course, been some research on robust monetary policy in the Norwegian economy, of which the paper by Akram and Nymoen (2009) is a good representation. The authors investigate the consequences of trading off the empirical validity of a model against other desirable properties. Basing policy on a suite of models or assuming the model uncertainty to be higher than justified, can potentially create large losses. Because the three models in the paper entail widely different monetary policy responses to shocks, conducting robust monetary policy in the face of model uncertainty is quite costly.

³Free download available from <http://www.dynare.org>.

⁴Thanks to Junior Maih for provision of coding for the solution algorithm.

In order to improve the achievement of the simple rules, I look at "Bayesian rules" where an *average* of the outcomes in the models is optimized in order to find the interest rate rule that best stabilizes this "model-mean". The Bayesian rules are more robust as they reduce the variability in each model compared to the first-best rule from another model. They also perform well in the models that they have not been optimized over, NEMO and Policy NEMO.

Robustness properties are evaluated using the relative increase in loss in a model stemming from the interchange of two policy rules translated into variability of inflation, a measure called *Implied Inflation variability Premium (IIP)* (Kuester and Wieland 2010). My results show a large dispersion of IIPs in the models, with the Bayesian rules naturally generating the lowest average IIP. In particular low IIPs are yielded by the rules where the relative importance of NAM in the optimization is tuned down. I find that rules that respond to last period's rate, inflation deviations from target and the current output gap is the most robust class of rules.

Another robustness tool used is *fault tolerance (FT)* (Levin and Williams 2003), where the relative increase in loss resulting from gradually varying the optimal value of a coefficient in a policy rule is displayed in a graph. FT is both used to measure to which extent a rule is robust towards small changes in the parameters, and how much a model is tolerant to different policies. The three-parameter interest rate rules is proved to be the most robust, and are hence the best to insure against model uncertainty with. NEMO and Policy NEMO are the most fault tolerant models, closely followed by CN, and NAM is the least tolerant.

At the end of the thesis I present what I refer to as "the Golden Interest Rule" (GIR), which is the rule that do best on average across the models. Through evaluating IIPs and FT of the different optimized rules, I found the properties of the best-performing rules, and tried several combinations of the parameter values that appeared to be optimal. GIR is as expected, a three-parameter rule that allows for a moderate degree of interest rate smoothing, yet not too much due to NAM. The long run responses to economic disturbances are substantially above the original Taylor rule, in particular three to inflation deviations from target and one and a half to GDP movements from trend. These strong reactions give sufficient stabilization of the real economy in all models considered.

In the first chapter I explain some of the theoretical background for my thesis. Then I describe the models, followed by a brief discussion of their transmission mechanisms of monetary policy. Chapter 5 is about the method used. Results from the optimization of simple rules is then presented, and robustness results follows in Chapter 7. In Chapter 8 I propose the "Golden Interest Rule". I end with some concluding remarks, before references and appendices with results left out of the main text.

2 Theory

In this chapter I first discuss some features of inflation targeting and its rationale, before I describe some of the theoretical background of it. I then explain the use of simple instrument rules versus conducting optimal monetary policy, and eventually present a way to insure against model uncertainty.

2.1 Inflation targeting

Maintaining stability and growth in GDP is one of the main goals of all macroeconomic policy. When output grows at a steady pace, firms change their production gradually, with few fires and hires, such that workers experience little variation in their labour supply. High unemployment is harmful for individuals and for the society as a whole, and being unemployed has costs beyond the pure economic inefficiencies resulting from resources not being fully exploited. Monetary policy seeks to establish the desired stability through nominal variables, in particular inflation. A stable inflation and credible monetary policy anchors inflation expectations, which is crucial in wage and price setting and thus in the economic variables directly affecting households.

Norway is a flexible inflation targeting regime, which is commonly believed to be a way of conducting monetary policy that stabilizes inflation and the real economy in a satisfactory manner. 27 countries are today considered fully inflation targeting regimes, and many other are at the outset of becoming one. Inflation targeting is a framework for interest rate setting that is characterized by a pre-eminent, explicit target for the inflation rate in the medium run, i.e. a specified number that the CPI or whichever inflation measure is used, should return to after disturbances to the economy, at a horizon not too far into the future, usually around 1-3 years.

The regime is *flexible* when the monetary authority is not only concerned about stabilizing inflation around its target, but also about the real economy, i.e. having a direct preference for ensuring a stable growth of GDP and a normal resource utilization, beyond the effects working through the inflation. The case of a *strict* inflation targeter is someone who solely cares about inflation variability, and is sometimes referred to as a pure "inflation nutter" (King 1997).

Monetary policy in an inflation targeting regime is conducted by a politically and legally independent central bank. The central bank uses macroeconomic models and judgements of extra-model information about the state of the economy to make conditional forecasts of the economic variables on which the interest rate decision is based. Because policy uses *forecasts* of the key variables, inflation targeting could well be termed "Inflation-forecast targeting"

(Svensson 2000), as in the case of Norges Bank.

In most inflation targeting central banks the forecasts, as well as predictions of the intended future interest rate path and further macroeconomic assessments are regularly released in monetary policy reports. Norges Bank publishes such reports three times a year (e.g. Norges Bank 2011, Norges Bank 2012). Policy is state-contingent such that it has the flexibility to respond to short run fluctuations from the path towards the inflation goal, and these deviations must be justified in the reports.

In an inflation targeting regime the approach to monetary policy is candid and open, and it is easy for the public to understand the rationale for the interest rate decisions, since the aim is clear (returning to the inflation target in the medium run) and the underlying foundation is thoroughly documented in the reports. When the policy reaction function is comprehensive and systematic, agents' expectations about future interest rate and inflation developments are kept stable and consistent with the goal. Actual inflation is then maintained at the target level, in the absence of macroeconomic disturbances, because expectations is a central driver of it. There is indeed evidence of reduced volatility in short-term market interest rates on the days that Norges Bank publishes the interest rate decision, suggesting that precise communication of policy intentions improves agents' understanding of the reaction pattern (Holmsen et al. 2008). Transparency about all goals and aspects of monetary policy is crucial, as strongly argued by Woodford (2005).

The preferences of an inflation targeting central bank is modelled as a loss function where the arguments are the variability of the variables that it seeks to stabilize, usually GDP and consumer prices. In this regard, inflation targeting can be interpreted as optimizing behaviour by the central bank, where it commits to the policy prescribed by the minimization of the objective.

A large literature exists on whether strict or flexible inflation targeting, or other monetary policy regimes, is the most appropriate. Svensson (2000) concludes that a flexible inflation targeting regime results in less variability in other macroeconomic variables than the CPI by reacting more gradually and to a longer horizon of the inflation forecasts than strict inflation targeting. It leads to considerable stabilization of GDP, as well as of the real exchange rate since the effects of the exchange rate movements on other variables than inflation are included in the interest rate setting through the arguments in the policy objective. Effects further into the future than the direct exchange rate effect stemming from lower imported inflation are taken into account. This makes the policy approach particularly suitable for open economies like the Norwegian.

Hence, the flexible inflation targeting regime applied in Norway appears to be sensible. Woodford even recommended that the Federal Reserve adopts this monetary policy strat-

egy: "A forecast-targeting procedure similar to that of the Norges Bank could plausibly be introduced as a framework intended to ensure that policy conforms to the mandates of the Federal Reserve Act and to make this conformity more evident to Congress and to the public." (Woodford 2007, p. 22-23)

2.2 A role for monetary policy

New Keynesian Dynamic Stochastic General Equilibrium (DSGE)⁵ models have been commonly adapted by central banks all around the world as the main analytical tool, like NEMO in Norges Bank. They are macroeconomic models based on microeconomic principles, and consist of equations describing economic relations derived from first-order conditions of intertemporal problems facing consumers and firms. The explicit modelling of rational optimizing decision makers' preferences makes it possible to analyze economic effects and welfare implications of policy changes. The structural nature of the models prevents them from being vulnerable to the Lucas' critique (Lucas 1976), which says that agents change their behavior in reaction to policy changes and therefore alter the macroeconomic relations. New Keynesian models are able to explain aggregate economic phenomena, such as economic growth and business cycles. They also fit data well and have satisfying forecasting properties.

New Keynesian theory builds on Real Business Cycle models, but have incorporated elements from the Keynesian framework, thereby the name New Keynesian. By including nominal rigidities in the form of price and wage stickiness, monetary policy has an effect on the real economy in the short run. This is in contrast to the Real Business Cycle theory where it is at best superfluous, and may indeed be counterproductive because business cycles are only efficient responses to productivity shocks (Galí 2008). The nominal rigidities are commonly modelled using the canonical framework of staggered price setting laid out by Calvo (1983). In this framework, profit maximizing firms with identical technology produce a differentiated final good and set prices as a markup on the market clearing price that would have been the result in a completely competitive market. Nevertheless, they can not adjust prices as often as they desire due to constraints on the frequency of price revisions. There is only a certain fraction of the firms that are allowed to update their prices every period, resulting in sluggish responses of prices to shocks. The optimizing firms are drawn completely at random such that the expected (or average) duration of a price equals the inverse of the share of firms "winning the lottery" each period.

⁵The term DSGE models has in modern macroeconomics been closely (albeit, somewhat falsely) connected to New Keynesianism. New Keynesian models are indeed dynamic and stochastic models that provide a general equilibrium solution, but the converse is not necessarily true. Many dynamic stochastic general equilibrium models are not New Keynesian, for example the model NAM described below.

There are also other ways of modelling price sluggishness, and the method proposed by Rotemberg (1982) is used in NEMO. It differs from the Calvo pricing scheme in that *all* firms set prices optimally every period based on complete information. Revising prices is costly, and price setters must be forward-looking and base current prices on expectations as well as on last period's prices. The adjustment costs are modelled as quadratic deviations between the prices in two successive periods. At the aggregate level and up to a first order approximation, Rotemberg and Calvo pricing schemes are however equal (Lombardo and Vestin 2007).

Due to sticky prices there are two sources of inefficiencies in the canonical New Keynesian economy, namely fluctuations in the *average* output gap of firms, and price dispersion *between* firms. The output gap is the difference between the actual production level and the efficient level. The latter is the production level that would have been realized if prices were fully flexible, such that the marginal rate of substitution (MRS) is equal to the marginal rate of transformation (MRT).

When there is monopolistic competition in a market, the markup on the prices set by firms creates a wedge between MRS and MRT. This inefficiency can however be completely offset by a proper subsidy. But even with such an optimal subsidy in place, production will in general differ from the efficient level because prices and thus real wages do not fully adjust to changes in the marginal product of labour as shocks hit the economy. This leads to a suboptimal allocation of labour inputs and aggregate production level, which give monetary policy a motive for stabilizing the economy through responding to movements in the output gap. If, for example, a temporary positive demand shock occurs, production increases, and in consequence marginal costs go up and therefore also prices. But since prices cannot adjust instantaneously to the new marginal costs, monetary policy should increase the interest rate as a response to the positive output gap. A higher nominal interest rate increases real rates, thereby reducing demand and moving the economy faster back to steady state. This lowers the variability in the output gap and inflation.

Price dispersion between firms due to staggered price setting results in inflation because re-optimizing firms choose a price that differs from the average price in the last period. When firms with equal production technology face different output prices, they produce at different output levels. This leads to an inefficient *goods combination*, given that the consumers have a "taste for variety"⁶. The dispersion of production levels can be alleviated if monetary policy seeks to stabilize price inflation, because firms would then have no incentives to change prices and hence produce at the same level.

⁶Preferences for variation in consumption is ensured through a constant elasticity of substitution function for the aggregation of the consumption good.

In the most basic New Keynesian model with no *real* rigidities, complete inflation stabilization is optimal because no fluctuations in prices automatically stabilize output as well. This is sometimes referred to as "the divine coincidence", namely that the trade-off between inflation and output gap stabilization is absent when monetary policy stabilizes the marginal cost of firms at a level consistent with their desired markup, given the prices in place. No firms change their prices if they expect the policy to last forever, so the output gap remains closed and inflation stays at the target level.

However, the presence of imperfect competition in labour markets and sticky nominal wages renders pure *price* inflation stabilization as suboptimal. Differences in wages across workers lead to an inefficient allocation of labour, making production efficiency improvements possible. When wages respond slowly to macroeconomic distortions, the central bank should seek to stabilize a balance between price inflation, wage inflation and output gap movements, since they all lead to inefficient resource utilization. Despite that, optimal policy can well be approximated by stabilizing a weighted average of wage and price inflation (in addition to the output gap), where the weights depend on relative price and wage stickiness (Galí 2008). This leaves (averaged) inflation and output gap as the two arguments entering the central bank's loss function.

In the staggered price-setting framework above, the slow response of prices and wages to economic disturbances is the central explanation for business cycle fluctuations. The resulting price dynamics creates a stabilizing role for monetary policy of the real economy beyond nominal variables. Movements in the policy rate lead to altered inflation expectations, but since agents know that prices and wages do not adjust immediately in proportion to the interest rate change, the real interest rate will be affected, and thus consumption and investments, which in turn affects employment and production.

Albeit, in the long run all variables return to their steady state values or natural levels because wages and prices fully adjust, and monetary policy becomes neutral. Consequently, it is the presence of sticky prices and wages that makes efficient responses to shocks and welfare enhancing macroeconomic policies possible.

Even though nominal rigidities *theoretically* gives monetary policy a stabilizing role, it is, however, not obvious that it does so in the real world. But as Galí (2008) summarizes, there is ample evidence of both nominal rigidities and monetary non-neutrality from empirical studies and estimations. Significant effects of monetary policy on real variables have been found, which supports the theory that monetary policy can alleviate economic inefficiencies that arises with nominal rigidities, in contrast to the ideas of the RBC advocates.⁷

⁷The effects of monetary policy on real variables are of course difficult to identify: changes in the policy rate are usually entirely or partly *results* of changes in the variables, and the causality thus goes in the opposite direction of the one to be estimated.

A low and stable inflation rate is also desirable for other reasons than the theoretical efficiency arguments in the New Keynesian models. A small positive inflation rate renders real wage adjustments possible without adjusting nominal wages. It appears self-evident that frequent nominal wage increases accompanied by equal increases in the consumer prices are preferred by workers to a constant nominal and real wage (due to the "illusion of money" (Fisher 1928)). A positive inflation target also helps to prevent the nominal interest rate from hitting the "zero lower bound"⁸, in which communication about future policy intentions becomes especially important. By credibly committing to higher inflation rates in the future and thereby influencing expectations about future consumer prices, the central bank stimulates the economy today and thus mitigates the effects of the lower bound.

When equilibrium inflation is positive, the nominal interest rate supporting this equilibrium will also be positive, because it is the *real* interest that affects economic agents' decisions, and depending on the size of this equilibrium real rate, there will be more room for monetary stimuli in economic downturns.

Yet, high inflation rates could create macroeconomic instabilities by causing large and varying inflation expectations, rent-seeking and irrational behavior, uncertainty about the conduct of monetary policy, and undermine the role of money, etc. *Some* inflation is preferable, but there is no consensus about the optimal level. The goal of Norges Bank is to keep the growth in the consumer price index in the medium run at 2.5 percent, but other central banks have different inflation targets, most of them below the Norwegian. The European Central Bank, Sveriges Riksbank and Bank of England all have goals of 2 percent. A 2 percent inflation target is commonly viewed to be sufficient to avoid significant macroeconomic instabilities of the type explained above (Taylor and Williams 2010).

2.3 Simple interest rate rules

"Why does the Bank make things so complicated? Why doesn't it just follow the Taylor rule?" [Interruption by a distinguished macro economist at an American university, when Lars E. O. Svensson was presenting Bank of Sweden's approach to inflation targeting.] (Svensson 2000: p. 1)

Setting the policy rate is a demanding engagement. There are often several conflicting considerations in the evaluation procedure, and it is important to have a consistent and comprehensive way of doing it. There are two main approaches to the conduct of monetary policy: Optimal policy or simple instrument rules. Optimal policy refers to when the central

⁸A negative nominal interest rate is obviously not possible for more than very short periods of time, as it could lead to a collapse of the bank sector if all agents withdrew their deposits.

bank sets the policy rate in order to minimize a specific objective, like Equation 2 below. Simple instrument rules are, as the name suggests, easy rules of thumb to be followed by the monetary authority. The simple rules consist of a few selected variables that the nominal interest rate should respond to fluctuations in, and can hence be implemented without further knowledge about the economy's functioning or the realization of shocks. The rules are thus robust to all kinds of uncertainties, in particular model uncertainty, which is also the main advantage of such rules. In addition, they make monetary policy transparent and predictable due to their simple structure.

The original and famous "*Taylor rule*" (Taylor 1993) is a classical example of a simple interest rate rule. The rule dictates that the nominal interest rate moves 1.5 percentage point when inflation deviates one percentage point from the inflation target and 0.5 percentage point when output deviates one percentage from its trend or potential level:

$$r_t^T = 1.5\pi_t + 0.5y_t, \quad (1)$$

where π_t is year-on-year inflation and y_t is the output gap, and the targets for inflation and output are set to zero for simplicity.

As highlighted by Taylor, the crucial feature of the interest rate feedback rule is that it dictates monetary authorities to "lean against the wind" in the sense that if the output gap is positive or inflation rises above target, the nominal interest rate is increased in order to dampen economic overheating.

The principle that the nominal interest rate should react more than one-to-one to inflationary movements in order to avoid indeterminacy of the price level, has become known as "The Taylor principle" (Woodford 2001). This principle should always be followed when instrument rules are used to set the interest rate. For example, for an increase in demand without any change in economic fundamentals (e.g. a sunspot shock), production and marginal costs increase, creating inflationary pressure. If the nominal interest rate did not respond enough to the rise in inflation, the real interest rate would actually go *down* and the effects on the economy would be the opposite of the intended, justifying the initial increase in consumption. Therefore, monetary policy must be designed so that this type of self-fulfilling prophecies cannot happen. Taylor (1993) and others (e.g. Clarida et al. 2000) argues that the Fed did not follow the Taylor principle before Volcker was appointed as Chairman for the Federal Reserve in 1979, and that this is the reason for the great inflation and macroeconomic instability in the USA in the 1960s and 1970s.

Responding to inflation and output solely though, without taking all other available information into consideration, may seem too simple and sub-optimal. This is indeed the case, but the two variables are such good indicators of the state of the economy that reacting to

movements in them yields sufficient macroeconomic stabilization. Taking the robustness and transparency qualities of simple policy rules into account, they seem like a good alternative to optimal monetary policy.

It has been shown that simple rules do describe actual monetary policy in most countries quite well and fit data in a satisfying manner (Kuester and Wieland 2010), which was actually the main goal of rule (1). Taylor's famous paper was meant as a contribution to the discussion of optimal monetary policy with discretion versus using a time-invariant policy rule. The proposed rule is able to explain the Federal funds rate movements remarkably well from the 1970s to 1992. Thus, the policy conducted by the Fed could be approximated by such a simple rule. It was, however, not intended as a rule for the actual conduct of monetary policy, and it is unlikely that policy makers literally follow the simple rules. But they anyhow work well as a benchmark for assessing monetary policy.

2.4 Optimal monetary policy

Inflation targeting could well be described as optimizing behaviour of the central bank, in the sense that it takes all relevant information into account and responds with the policy rate to shocks in order to minimize the expected value of some objective. The central bank commits to the prescribed policy for all future periods, and acts as if it made the commitment a long time ago by treating all periods equal. This is referred to as conducting optimal monetary policy in a timeless perspective.⁹

The objective function for most inflation targeting central banks consists of the unconditional variance of inflation, output gap and change in the nominal interest rate:

$$\mathcal{L}_t = E_t[(\pi_t - \pi^*)^2 + \lambda^y(y_t - y^*)^2 + \lambda^r(r_t - r_{t-1})^2], \quad (2)$$

where π^* is the inflation target, y^* is output target or trend, and r_t is the nominal interest rate. E_t is the mathematical unconditional expectation operator, and λ^y and λ^r represent the central bank's relative preferences for output gap stabilization and interest rate smoothing, respectively.

The arguments entering the loss function and the relative weights assigned each of them is frequently debated in the literature, but most seem to agree that some concerns for output and interest rate smoothing should be included in addition to inflation. As argued above,

⁹I refer to the loss generated by optimal policy as "Ramsey loss" in order to distinguish it from "optimal simple rule loss", even though the *policies* are not the same. The difference is that Ramsey policy exploits the gain in the first period after a shock by promising to overshoot the inflation target in future periods and thereby reduce current inflation due to the forward-looking behaviour of price setters. The losses are however the same when they are calculated from theoretical variances and the discount factor of the central bank is unity, such that all periods are treated equal, which is what I use.

the preferences of the monetary authority should be to maintain low inflation variability and a steady output growth in order to secure a stable economic environment, and it appears natural to set the policy rate in a manner that minimizes deviations of inflation and GDP away from target or trend.

Including the output gap has been found to be necessary in order to attain *robust* monetary policy (Levin and Williams 2003), which is the goal of this thesis. Consequently, excluding output from the loss function appears to be counterproductive.¹⁰

By including the variability of the change in the nominal interest rate in the loss function, the central bank ensures that the interest rate is moved gradually and with caution, which is preferable when there is general uncertainty about the structure of the economy and the effects of monetary policy (Levin et al. 1999). It appears intuitively correct that economic agents prefer predictability in the *direction* of movements of the nominal interest rate, rather than frequent small fluctuations around a constant level, which could be the case if the *level* of the interest rate entered as an argument¹¹. Such frequent adjustments in alternate directions may cause policy makers to appear uninformed and whimsical, and undermine the role of monetary policy as an anchor for nominal variables. As explained in Holmsen et al. (2008), using the first difference of the policy rate delivers reasonable paths for the nominal interest rate: "paths that do not look unacceptable to the policy maker at first glance". In addition, Svensson (2000) finds that the case of strict CPI targeting does not converge unless a small weight on interest rate smoothing is added to the loss function.

Further, interest rate smoothing¹² leads to higher macroeconomic stabilization by providing a better control over long term rates which are the main driving forces of inflation expectations, actual inflation and production. When movements in the policy rate are expected to be long-lived, as in the case of gradualism in interest rate setting, the effect on long term rates is significantly larger. A smooth interest rate path makes real variables move in an even fashion, preventing undesirable jumps in consumption and labour. Besides, substantial and sudden changes in the interest rate amplify financial market volatilities, which may lead to instabilities in the real economy. Taylor and Wieland (2009) and many others include the change in the nominal interest rate in the monetary policy objective. Because

¹⁰Taylor and Wieland (2009) and Kuester and Wieland (2008) find, however, that the insurance against model uncertainty can be done at a relatively low cost even if the objective function does not contain any output stabilization concern.

¹¹It is, of course, possible to include both the level of and change in the nominal interest rate. This is in fact done in Norges Bank's most recent monetary policy report (Norges Bank 2012), in order to mitigate the risk of a buildup of financial imbalances, where the deviations from a "normal" interest rate level is one of the arguments.

¹²I shall use the term "interest rate smoothing" to refer to a policy reaction function where there is a high degree of persistence in the nominal interest rate, i.e. that the coefficient on the lagged interest rate is large, and not as a policy that leads to a smooth interest rate path of other reasons.

low interest rate volatility is found in data, preferences for interest rate smoothing by policy makers appears to be justified empirically (Levin et al. 1999).

For all these reasons it is wise to conduct monetary policy with caution and "stodginess", it being due to preferences for interest smoothing in a loss function or used in a monetary policy reaction function.

2.5 Robustifying monetary policy

In order to avoid disastrous outcomes and reduce the risk of total failure of monetary policy, but without switching from optimal policy to a simple interest rate rule, it is possible to robustify optimal policy by extending the loss function with a fourth argument, namely deviations from the interest rate implied by a simple instrument rule (Ilbas et al. 2012):

$$\mathcal{L}_t = (1 - \theta)((\pi_t - \pi^*)^2 + \lambda^y(y_t - y^*)^2 + \lambda^r(r_t - r_{t-1})^2) + \theta(r_t - r_t^{SR})^2, \quad (3)$$

where r_t^{SR} is the nominal interest rate that would have been set if the central bank followed the simple rule SR. This rule would be a type of rule like Equation 1, which does not perform outstandingly in any single model, but does *on average* quite well and thus serves as an insurance against model uncertainty.

Optimal policy and simple instrument rules are combined in a framework that robustifies monetary policy. As argued in Ilbas et al. (2012), this modified loss function is realistic, flexible and sensible. Central banks can easily adopt to it and there are no limits to which or how many simple rules that can be included. The last term in (3) could easily be substituted or modified when new knowledge about robust simple instrument rules is obtained or as experiences about which rules best serve the robustness purposes develop. The authors further argue that simple rules are not realistic since central banks rarely follow them in a strict manner. Conducting optimal monetary policy gives no room for simple rules which are proved to be a lot more robust. Thus, a combination of the two monetary policy approaches appears to be a good solution.

Robustness is a big topic within monetary policy research, and there are numerous papers discussing different ways of insuring against mis-specifications, and model and parameter uncertainties. However, the method developed by Ilbas et al. (2012) is as far as we know the only approach combining optimal policy with a simple interest rate rule. The large literature on parameter uncertainty and model mis-specification supplies ways of robustifying when there are uncertainties about details of a specific model, but not if the true economy is completely different. As Levin and Williams (2003, p. 958) notes:

"...the results suggest that simply designing a rule to be robust in the neigh-

borhood of a given reference model does *not* ensure that the rule will perform robustly in competing reference models."

Therefore, by extending the loss function with a small weight on deviations from a simple interest rate rule, the central bank insures against uncertainties of several forms, without sacrificing much. In this thesis I search for the best simple rule(s) to modify the loss function of Norges Bank with. The rule(s) I end up with should in particular do well in the models that Norges Bank seeks to insure against disastrous outcomes in, i.e. NAM, as optimal NEMO policy yields instability in this model (Mathis Mæhlum 2012).

3 Models

Following is a description of the models used in this thesis. Since it is part of a larger project on which my fellow student Mathis Mæhlum also works, we have cooperated on this chapter. He has written the parts describing the LGM model and NAM, and I have written about the NEMO models. He does however not consider "Policy NEMO", so this model is only included in my analysis. I hence use four main models, but extend to five whenever appropriate. Credit NEMO, NAM and the LGM model are those I refer to as "reference models" because they are the models I compare NEMO with. The models differ in many important aspects, although four are New Keynesian and all are DSGE models, albeit with highly different specifications.

Lower case letters represent the log-deviation from a variable's steady state value, except from growth rates and interest rates.

3.1 NEMO

The Norwegian Economy Model, NEMO (Brubakk et al. 2006), is a New Keynesian DSGE model used by Norges Bank for policy evaluation and forecasting . It is a model of a small open economy consisting of two countries, home and foreign, interpreted as Norway and its trading partners, and two sectors, one producing intermediate goods and one producing a single final good. The model economy is a representation of the Norwegian mainland economy, with the petroleum sector entering as an exogenous process for oil investments. The foreign economy is modelled symmetrically to the home economy, but enter as exogenous variables, such that Norway has no influence on its trading partners.

All variables in NEMO are detrended with a common stochastic growth trend. We use a first-order Taylor approximation of the model.

The economy consists of a continuum of infinitely lived households that are divided into two types, savers and spenders, who both supply labour services to the intermediate goods sector. The share slc of spenders are rule of thumb consumers and spend their total labour income every period, whereas the share $(1 - slc)$ of savers have access to a credit market and choose consumption and saving plans that maximize expected utility over the lifetime subject to a budget constraint, which leads to the following Euler equation:

$$c_t^{sa} = f_{191}E_t c_{t+1}^{sa} + f_{192}c_{t-1}^{sa} - f_{193}E_t \{r_t - \pi_{t+1}\} - f_{194}\pi_t^Z + f_{195}z_t^U, \quad (4a)$$

where c_t^{sa} is consumption done by savers, $E_t(r_t - \pi_{t+1})$ the real interest rate, π_t^Z a shock to the growth trend and z_t^U a preference shock raising the marginal utility of consumption relative to leisure. Savers are forward-looking and wish to smooth consumption over time, and due to habit persistence, current consumption also depends on last period's consumption. A temporary rise in growth reduces the value of (detrended) consumption and households thereby postpone consumption.

Forward-looking households invest in domestic and foreign bonds, receive all dividends from firms, pay lump sum taxes and set nominal wages taking firms' labour demand into account. They have some degree of monopoly power in the labour market such that the resulting wages are above the competitive wages, whereas spenders receive the average wage rate of the savers and simply supply the amount of labour demanded from them at this wage. There are quadratic costs of adjusting wages that makes wage growth, π_t^w , respond sluggishly to shocks, which thus depends on past and future wage growth, deviations of the actual wage from the optimal wage (equal to the marginal rate of substitution between consumption and leisure), $(w_t - mrs_t)$, and the degree of bargaining power represented by the substitution elasticity between labour inputs, ω_t :

$$\pi_t^W = \frac{\beta}{1 + \beta}E_t \pi_{t+1}^W + \frac{1}{1 + \beta}\pi_{t-1}^W - f_{231}(w_t - mrs_t) - f_{232}\omega_t. \quad (4b)$$

Production of the final good, A, is done using a combination of imported and domestically produced intermediates, respectively M and Q, with the shares given by the degree of "home bias", i.e. the relative preferences for input factors produced in the home economy. The final good is used for consumption, C, capital investments in the intermediate sector, I, government spending, G, and oil investments, IOIL. The only source of imports in the economy are the imported intermediate goods, T*, and exports consist purely of domestically produced intermediate goods, M*.

In the intermediate goods sector, monopolistically competitive firms produce differentiated goods t_t , utilizing capital services, $\bar{k}_t = u_t + k_{t-1} - \pi_t^Z$, and labour in a Constant

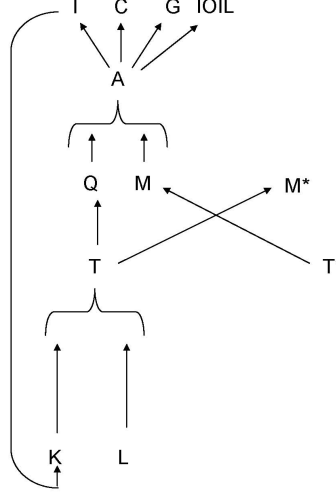


Figure 1: The production structure of NEMO. The figure is taken from Brubakk et al. (2006).

Elasticity of Substitution (CES) production function:

$$t_t = f_{61}(l_t + z_t^L) + f_{62}\bar{k}_t, \quad (4c)$$

where z_t^L is a labour augmenting productivity shock that temporary increases the level of production, and drives the exogenous total factor productivity.

The amount of capital services depends on the capital stock and the utilization rate, whereas the stock itself is determined by investments done one period earlier and capital depreciation. There are convex adjustment costs of changing both the level of the investment to capital ratio, $(inv_t - k_{t-1})$, and the rate of change in this ratio, which together with variable capital utilization and habit persistence are the real rigidities in NEMO. The investment to capital ratio is thus a slowly moving variable that reacts positively to increases in the expected real return to capital, $E_t r_{t+1}^K$, and negatively to the expected real interest rate since it reduces the discounted value of the return. A somewhat simplified version of the investment Euler equation:

$$inv_t - k_{t-1} = f_{111}(inv_{t-1} - k_{t-2}) + f_{112}E_t \{inv_{t+1} - k_t\} - f_{113}E_t \{(r_t - \pi_{t+1}) - f_{114}r_{t+1}^K\} + shock_t^{inv}. \quad (4d)$$

Intermediate firms set prices as a markup above the competitive price, and prices respond sluggishly to shocks due to convex adjustment costs à la Rotemberg (1982). Inflation on intermediate goods, π_t^Q , increases with real marginal costs and decreases with a cost push shock represented by the substitution elasticity between the domestically produced intermediate goods, θ_t^H , by the following Phillips curve:

$$\pi_t^Q = \frac{\beta}{(1+\beta)} E_t \pi_{t+1}^Q + \frac{1}{(1+\beta)} \pi_{t-1}^Q + f_{131}(mc_t - p_t^Q) - f_{132} \theta_t^H. \quad (4e)$$

Prices on the exported factor inputs are set in the local currency at the destination where they are sold, and evolves in a similar way to domestic intermediate prices. Imported inflation has a corresponding Phillips curve, meaning that foreign intermediate good producing firms set domestic prices in an identical way to domestic firms.

The real exchange rate, s_t , is governed by a version of the standard uncovered interest rate parity (UIP) condition, stating that in optimum the expected return on domestic and foreign bonds must be equal, with an exogenous risk premium, z_t^B , of which a positive realization means that the return to foreign bonds relative to domestic bonds increases, i.e. that foreigners demand a higher real return for a given exchange rate:

$$s_t = f_{201} E_t s_{t+1} - E_t \{r_t - \pi_{t+1}\} + E_t \{r_t^* - \pi_{t+1}^*\} + z_t^B. \quad (4f)$$

The government purchases final goods financed through a lump-sum tax, invests in the petroleum sector and sets the short term nominal interest rate. Government spending and oil investments are exogenous variables. The other exogenous variables include domestic shock processes and all the foreign variables except export prices (Norwegian import prices) and are modelled as AR(1) processes with a white noise shock:

$$z_t = \lambda z_{t-1} + \varepsilon_t. \quad (4g)$$

The model is closed by assuming market clearing to ensure that demand equals supply for the final good, intermediate goods, labour, and domestic bonds.

We use the estimated version of NEMO as in the Monetary Policy Report no. 3/2011 (Norges Bank 2011).

3.1.1 Policy NEMO

As mentioned in the introduction, I also use a second version of NEMO that is the exact version found in the Monetary Policy Report no. 3/2011 (Norges Bank 2011), which I refer to as Policy NEMO (PN for short). The only difference between Policy NEMO and benchmark NEMO is that a quarter of the price setters are rule of thumb price setters and three quarters use pure indexation, which makes the model more persistent. Total intermediate goods inflation is a weighted average of the prices set by the rational and the rule of thumb pricers. In all other aspects the two models are identical. The benchmark model is the *estimated*

version of NEMO, whereas the version used for policy analyses in Norges Bank¹³ includes the extra price sluggishness in order to fit data in more satisfying manner.

Since policy makers in Norges Bank obviously has some belief in both of these NEMO versions, I include Policy NEMO for robustness checks, but leave out results from the optimization of rules in this model.

3.2 Credit NEMO

Credit NEMO (CN for short) is another extension of the benchmark NEMO, with a credit market explicitly modelled as a separate sector producing houses (Brubakk and Natvik 2010). It builds on the models by Kiyotaki and Moore (1997), Iacoviello (2005) and Iacoviello and Neri (2008) in which credit markets are included in otherwise standard DSGE models in order to incorporate effects from asset prices and credit constraints to the real economy. The housing sector in CN is endogenous, in contrast to a fixed real estate amount in Iacoviello (2005), such that housing investments and production becomes an extra driving force of business cycles. In addition to the shocks in NEMO there are three housing shocks (housing demand, housing productivity and loan-to-value ratio) that contribute noticeably to the variability in CN.

The housing sector in CN uses the final good as input, and has a lower productivity growth than the rest of the economy, which is consistent with the observed upwards trend in the relative price of housing. All variables are detrended with their respective long run growth rates. The housing stock depreciates over time and is increased by new investments. House prices evolve according to the productivities in the housing and intermediate goods sector, to the level of and change in the investments to housing stock ratio, and a housing investment shock.

Households exhibit habits in housing consumption and the housing services enter directly into their utility function. They are divided into two groups, patient and impatient, where the latter are credit constrained and by assumption only borrow a given share of the value of their housing stock (Iacoviello 2005). This loan-to-value ratio is exogenously given and set to 0.9.¹⁴ Impatient households earn labour income and borrow from the patient households. Only patient households have access to a foreign bonds market where they can borrow to finance consumption, housing services and lending to impatient households. Borrowing is

¹³The model Norges Bank uses is constantly under revision, with adjustments and further improvements done whenever new information arrive or additional knowledge about the functioning of the economy is acquired. NEMO therefore changes slightly from Report to Report.

¹⁴Until recently Norwegian house buyers had to self-finance minimum 10 percent of the price, such that a 90 percent loan-to-value ratio seems reasonable. It has however been increased to 15 percent in order to dampen the fast increase in Norwegian housing prices.

in zero net supply, and the total stock of housing is divided between impatient and patient households, with shares equal to the income shares.

The intermediate sector is modelled as in the standard NEMO, but with two types of labour, patient and impatient workers, and total labour input is a Cobb-Douglas function of the hours worked by the two types. Intermediate firms choose prices and factor inputs in order to maximize expected cash flow, and households choose consumption, housing services, wages and borrowing in order to maximize expected utility given the budget constraint, and the collateral constraint for impatient households.

By relaxing the assumption of homogeneity among households and incorporating a channel from balance sheet positions to agents' decisions, CN is able to capture a financial accelerator effect in which shocks that influence house prices are amplified and propagated through the effects on consumption and housing demand. Two mechanisms lead to this financial accelerator, one wealth effect through higher consumption when asset prices increases, and one indirect balance sheet effect. The latter is a result of higher value of the accessible credit of impatient households, which drives up their demand for housing services and consumption. The two effects are further reinforced, creating the financial accelerator.

3.3 LGM

3.3.1 Description of the model

The Leitimo-Gali-Monacelli (LGM) model is an open economy small-scale New Keynesian DSGE model stemming from the work of Galí and Monacelli (2005) and Monacelli (2006). Our version is closer to the one developed and estimated by Leitimo (2006). It shares many features with the canonical New Keynesian model for open economies (Galí and Monacelli 2005; Galí 2008), but it includes more realistic open economy aspects by allowing for incomplete pass-through of exchange rate movements to import prices. This creates a source of frictions in addition to the standard ones in the canonical model, and it is more consistent with data (Monacelli 2006, p. 1048). In addition, the model allows both expected future inflation and previous periods' inflation to determine inflation and output today.

The core of the model consists of four equations: two Phillips curves for domestic and imported inflation, respectively, an IS curve governing output gap movements, and an equation for the real exchange rate. The domestic economy is populated by a representative agent who chooses consumption, savings and labour supply in order to maximize discounted utility given the budget constraint. There are complete international markets for state contingent assets, such that consumers in all countries can invest in the same assets. This assumption pins down the relationship between domestic consumption, foreign consumption and the terms of

trade. The household consumes aggregates of two types of goods: domestic and imported. The domestic good is in turn an aggregate of a continuum of goods, each produced by a monopolistic firm that wants to set the price as a markup over marginal costs in order to maximize discounted profits. However, prices are set in the Calvo (1983) manner. This leads to some price stickiness, as firms are not able to translate marginal cost changes into price changes without a delay.

While in NEMO foreign exporters set prices for their products in Norwegian currency (local currency pricing), imported intermediary goods in LGM are priced by a separate, domestic import sector that takes prices on the world market as given and then set the domestic currency price in a similar way to domestic producers. These firms need to take into account that when prices are sluggish, exchange rate movements lead to deviations of the world price (in domestic currency units) from local market prices. This difference is called the law of one price (LOP) gap, given by

$$\begin{aligned}\psi_t^F &= e_t + p_t^* - p_t^F = e_t + p_t^* - p_t - (1 - \gamma) [p_t^F - p_t^H] \\ &= q_t - (1 - \gamma)s_t,\end{aligned}\tag{5a}$$

where e_t is the nominal exchange rate, p_t^* is the world price in foreign currency, p_t^F is the imported goods price (in domestic currency), p_t^H is the price of the domestically produced good, q_t is the real exchange rate, γ is the share of imported inflation in CPI inflation, and $s_t = p_t^F - p_t^H$ is the terms of trade. When ψ_t^F is large, inflation rises as importers seek to raise local prices in order to get them in line with the price they face in the world market. Due to price-setting frictions, the LOP gap will not be closed instantly, and this leads to incomplete short run pass-through.

In order to make the model more realistic, we do some changes to the core structure outlined above. First, we follow Leitemo (2006) in allowing for a more gradual adjustment of prices and output. This can be explained by information and implementation lags due to e.g. rule of thumb pricing and habit formation in consumption. We allow for four lags of inflation in the two Phillips curves, and two lags of the output gap in the dynamic IS equation.

Second, we depart from Leitemo's specification by allowing for a more gradual development of the real exchange rate than what is given by a standard uncovered interest rate parity (UIP) condition. Instead of being a pure forward-looking variable, the real exchange rate depends partly on the expectations of next quarter's rate and partly on the previous quarter's rate. It follows the equation

$$q_t = (1 - \alpha)E_t q_{t+1} + \alpha q_{t-1} - \beta(r_{q,t} - E_t \pi_{q,t+1}) + (r_{q,t}^* - E_t \pi_{q,t+1}^*) + \tau_t,\tag{5b}$$

where $r_{q,t}^*$ is the foreign nominal interest rate, τ_t is a shock, and all variables are in quarterly terms.

Third, the forward component of the Phillips curves consists of expectations of only next period's inflation rate, not the whole year ahead. This is in line with both Monacelli's (Monacelli 2006) specification and the canonical representation from the literature (e.g. Galí 2008). However, the decisions are subject to a one quarter implementation lag, meaning that the previous quarter's expectations of future variables determine this quarter's variables.

We calibrate the share of imported inflation in CPI inflation to $\gamma = 0.4$, which is higher than the values used by Leitemo (2006). There are two reasons for this change. First, the Norwegian economy is more open than the British, which means that imported goods constitute a larger fraction of total consumption and production. Second, the value 0.4 corresponds roughly to the share of imported intermediate goods in production of the final good in NEMO¹⁵.

Foreign variables — the interest rate, inflation and the output gap — are modelled as in NEMO, using estimated AR(1) processes for each variable. Since we want the foreign economy to be identical across models, we keep the parameter values for the persistence coefficients from NEMO, but estimate the variance of the white noise shocks. For estimation purposes (but not for later simulations), we close the model by specifying a simple interest rate rule that includes current inflation, the current output gap, and one lag of the interest rate.

3.3.2 Estimation

The model is estimated as a system using Bayesian methods. This allows us to incorporate prior information regarding the parameter values and in this way avoid the "absurd" values that can result from maximum likelihood estimation when the model is mis-specified (An and Schorfheide 2007). By weighting the likelihood function by a prior density, information that is not contained in the sample used for estimation can be included in the estimation process.

The Bayesian framework means that we must specify prior probability distributions that reflect our beliefs about the parameters to be estimated. As prior mean values we use the estimates that Leitemo (2005) obtains with data from the United Kingdom. We specify normal distributions for most parameters, but use the beta distribution for those constrained to lie between zero and one. We estimate eight Gaussian shocks (error terms) and use the inverse gamma distribution, which restricts them to be positive, as prior.

¹⁵Furthermore, our calibration corresponds to that which Monacelli (2006) finds to be reasonable for a small open economy.

The monetary policy rule is a three-parameter rule that includes inflation, the output gap and the lagged interest rate. It has the form

$$r_t = \rho r_{t-1} + \phi_\pi \pi_t + \phi_y y_t. \quad (5c)$$

The prior means in this equation are based on the standard Taylor rule, but we include a considerable degree of interest rate smoothing ($\rho = 0.75$). As priors for the standard deviations of shocks in the AR(1) processes for foreign variables, we use the beta distribution with means equal to the values used in NEMO. The parameters on the lags of inflation in the equations for imported and domestic inflation are restricted to sum to one, i.e.

$$\sum_{j=1}^4 \alpha_j = \sum_{j=1}^4 \chi_j = 1.$$

We use eight data series for the period 1993 : Q4 – 2011 : Q2, which is the period used for estimating the benchmark version of NEMO. All data is observed at a quarterly frequency and have been obtained from Norges Bank’s Datawarehouse. The eight data series used for estimation are reported in Appendix A.2.3, Table 10. These are for the most part the same as those used for estimation of NEMO. We transform the observable variables in a way that is consistent with the facts that the model variables are log-linearized around the steady state, and that there is no long-run growth in the model. To create the output gap from the series for GDP per capita, we use the Hodrick-Prescott (HP) filter with a smoothing parameter $\lambda = 16000$. This is ten times the value originally proposed and most commonly used for US quarterly data (Hodrick and Prescott 1997). The reason for choosing this value is that it creates a smoother trend and thus more volatile cycles, thought to fit the Norwegian economy better. We also use this filtering for the real exchange rate, as we find a clear downward trend in this variable throughout the data period. Such detrending makes the observable variables consistent with the model. In addition, all variables are demeaned prior to estimation.

The model is estimated in Dynare. First we obtain an approximation of the mode of the posterior distribution. Then we construct a Gaussian approximation of this distribution around the mode using a Metropolis-Hastings Markov Chain Monte Carlo optimization routine. The routine makes 500.000 draws from the distributions - half of which are discarded - and runs two parallel chains. We use the mean of these distributions as point estimates of the parameters. Priors and results of the estimation are reported in Appendix A.2.3, Table 11.

3.4 NAM

The Norwegian aggregated model (NAM) is a quarterly macroeconometric model developed specifically for the Norwegian economy by Bårdsen and Nymoen (2001), Bårdsen, Jansen, and Nymoen (2003), and Bårdsen (2005). The version used in this thesis is the one documented in Bårdsen and Nymoen (2009). As opposed to the other models we consider, it does not assume that the economy is a system in general equilibrium and there are no forward-looking rational agents modelled. Instead, different parts of the economy are modelled separately, relying partly on theory and partly on data to identify the relevant variables in each Equation. The model is formulated in error correction form. First, starting from a general vector autoregression, a cointegrating relationship between variables in levels is identified as a long-run steady state. Then the short-run dynamic structure is estimated, using the long-run relationships as error correction terms. When the system is out of equilibrium, i.e. when the long-run relationship between endogenous variables does not hold, this cointegrating term will make sure that the relevant variables move back towards their long-run values. The model can be written on the form:

$$\Delta y_t = \alpha + \sum_{i=1}^j \Gamma_i y_{t-i} + \sum_{i=1}^k \Pi_i \Delta y_{t-i} + u_t, \quad (6)$$

where y_t is a vector of (logged) endogenous variables, α is a vector of constants, Γ_i and Π_i are parameter matrices $\forall i$, and u_t is a vector of error terms. Here the second term on the right hand side is the error correction term, which in each equation describes a cointegrating relationship between the left hand side variable and a linear combination of other variables. The short run dynamics is described by lags of differenced variables.

The model consists of equations for the wage, prices, productivity, output, unemployment, household credit, money market interest rates, and the nominal exchange rate. Wages are modelled in a Nash bargaining framework meant to capture the high degree of coordination in Norwegian wage setting. In the long run nominal wages will move one-for-one with the general price level and productivity, and it will also depend to some extent on the unemployment rate. Domestic prices are set by firms engaged in monopolistic competition. Thus the general price level will in the long run depend on wages relative to productivity, as well as imported prices. Long run equilibrium unemployment is determined by the growth of the real wage, as well as the real interest rate and output. The long run behavior of the nominal exchange rate is derived assuming that expected depreciation depends on deviations of the exchange rate from its long-run value, and that there is a constant long run risk premium in the foreign exchange market. Movements in relative real interest rates do not lead to one-for-one changes in the real exchange rate.

Total production is in the long run determined to a large extent by government demand, which in the original system is exogenous and will be assumed constant in our model (see below). In addition, depreciations of the real exchange rate and decreases in the real interest rate both affect output positively in the long run. In the short run, output growth is significantly affected by its own lag, changes to government expenditures and changes in real credit. The latter effect may be due to frictions in the credit market. The growth of real credit is in turn determined in the long run by the growth of output and - to a smaller extent - by interest rate differentials. Since output affects credit and *vice versa*, there is a simple financial accelerator mechanism at work. Labour productivity depends in the long run both on real wages, the unemployment rate and a linear trend. In the short run it is affected by the change in real wages.

Most of NAM is estimated equation-by-equation using OLS, but the wage and price block is estimated as a system with full information maximum likelihood. Identification of the system is achieved by means of theoretical and ad hoc overidentifying restrictions on the short run dynamics. Seasonal dummies are added for better fit. The original model's long-run growth is driven by neutral technological progress, approximated by the linear trend in labour productivity. Simulations show that the model induces a constant (apart from seasonal variations) growth rate of output, nominal wages and prices, and constant unemployment rate and nominal exchange rate in steady state (Bårdsen and Nymoen 2009, p. 879-883).

In order to make numerical simulations of the model tractable by making also nominal variables stationary, we remove all trends and constant terms so that all variables are zero in steady state. The original model can be viewed as a log-linearization. Under this interpretation, the variables in our modified model will be interpreted as deviations of the actual (logged) variables from either a deterministic balanced growth path (for some variables, such as the output gap and productivity) or constant steady state values (for other variables, including the inflation rate and the unemployment rate). This corresponds roughly to the log-linearization used to make NEMO and Leitemo stationary, and we will thus interpret the relevant variables in the same way across models.

NAM contains several exogenous variables, including the oil price and foreign variables such as consumer prices and interest rates. This poses a problem for our simulations. Instead of assuming dynamic processes for all these variables, we set the domestic exogenous variables equal to zero (their steady state values) in all periods. This is clearly unrealistic, and it means that the total variation in the endogenous variables will be smaller than what is observed in the data. However, we do not want to change the original model dynamics in any important ways by adding new equations, and thus this approach is the most convenient for our purposes. As for the foreign variables, we tried to model these in the same way as

in NEMO, but the AR(1) process for foreign inflation created stability-problems in NAM, leading to infinite variance of several important variables, including the domestic inflation rate. For this reason, we let foreign inflation be constant, but model the foreign interest rate as in the other models.

Because Dynare has problems in dealing with models in which some variables have infinite variance - which is the case for the nominal prices in NAM - we use a stationarized version when calculating optimal policy rules. In this version, growth rates of price variables and the cointegrating relationships are defined as new variables.

4 Transmission mechanisms of monetary policy

Generally in a small open economy like the Norwegian, changes in the policy rate works on inflation through three main channels: the demand channel, the exchange rate and the expectation channel.¹⁶ When the nominal interest rate increases, the real interest rate also increases due to sticky prices and wages. A higher real interest rate reduces investments done by firms through lower profits and availability of credit; it decreases households' demand through a higher relative price of current consumption to future and lower income of households with negative net savings. This reduces production and prices set by firms due to lower marginal costs, slowing down price growth by the *demand channel*.

Increased Norwegian interest rates raises the attractiveness of investments in NOK and drives up the price, leading to an appreciation. A lower exchange rate, meaning a higher value of NOK, increases imports and reduces exports, which lowers production and inflationary pressure in domestic firms, adding to the demand channel. Total inflation is dampened by reduced prices (in domestic currency) on imported goods, both directly through the effect of lower imported inflation on consumer goods and indirectly through prices set by domestic producers utilizing imported input factors. These two effects constitute the (direct and indirect) *exchange rate channel* to inflation.

The *expectation channel* works through the effects of all forward-looking variables on current inflation by changing agents' expectations about future economic developments. When firms expect smaller price increases because of higher real interest rates, they set lower prices today due to adjustment costs. Firms decrease investments when future return to capital is expected to be lower, thereby reducing wage pressure, and hence production costs and inflation. Households reduce current demand when they expect future consumption to be lower, due to consumption smoothing, contributing to the fall in inflation.

Expectations are also very important in the determination of the exchange rate, since this

¹⁶See <http://www.norges-bank.no/en/price-stability/inflation/effect-of-interest-rate-changes/>.

is a forward-looking variable. Expectations give monetary policy a way of influencing inflation that works with a shorter lag than the demand channel, since forward-looking variables respond immediately to policy changes.

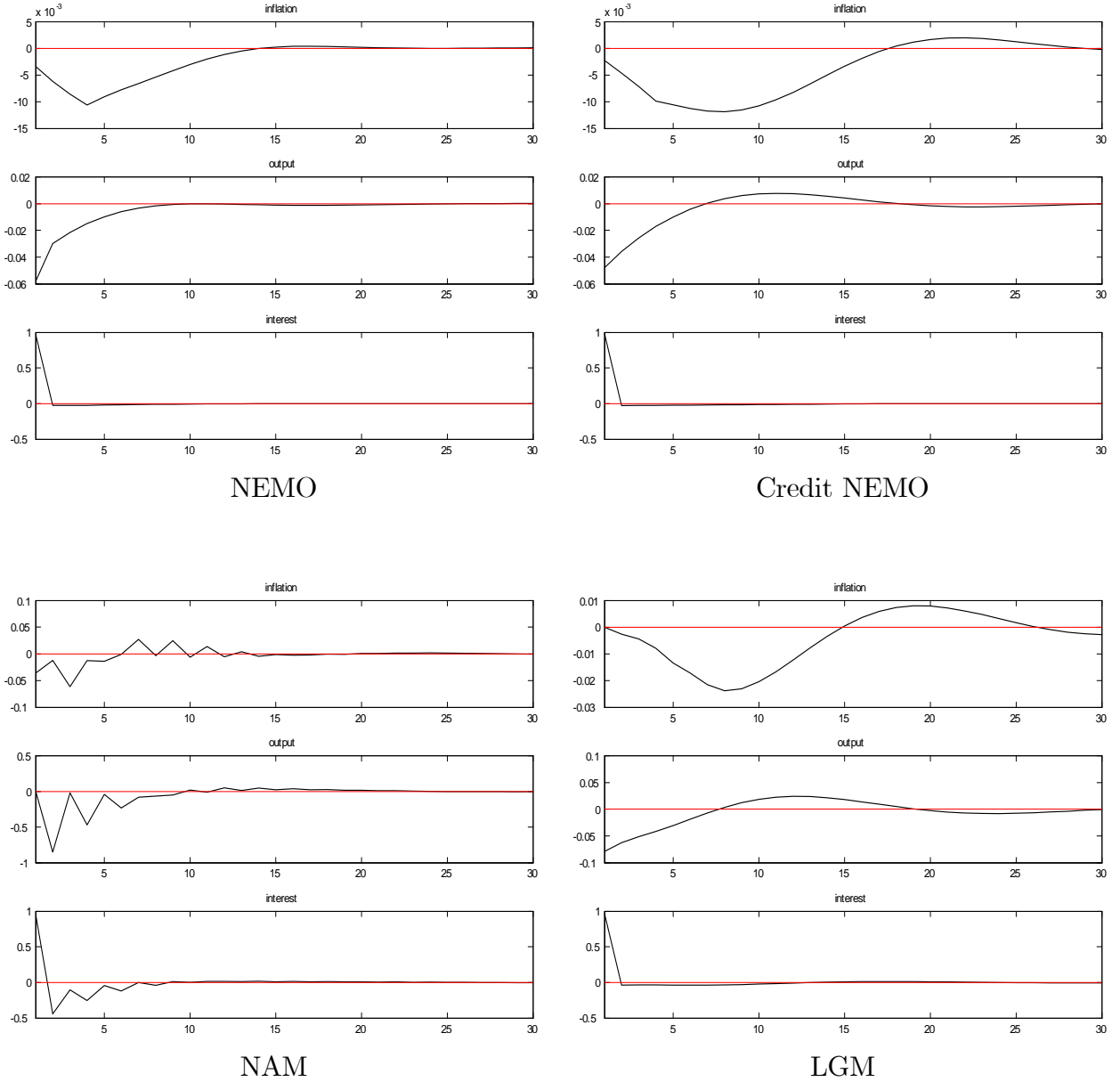


Figure 2: Impulse responses (in percent) of year-on-year inflation, the output gap and the quarterly annualized nominal interest rate to a one percentage point temporary increase in the latter when monetary policy follows the original Taylor rule. Periods along the horizontal axis are quarters.

In order to visualize the effects of monetary policy on the key variables, Figure 2 displays the impulse response functions (IRF) to a one period one percent increase in the nominal

interest rate in period $t=0$ of each of the four main models when the original Taylor rule is applied: $r_t = 1.5\pi_t + 0.5y_t$.

In all models inflation decreases less than output, and is more persistent, although the magnitude and length of the disturbance varies substantially across the models. Output in NEMO falls immediately nearly 0.06 percent below trend and rises gradually for two years until it reaches the steady state growth path. Inflation reaches the minimum after one year and then returns to target after three and a half years after the nominal interest rate shock. Due to nominal rigidities, a higher nominal interest rate leads to a higher real interest rate, reducing investments, consumption and the exchange rate. The lower demand and investments reduces production, leading to a downward pressure on prices and nominal wages through lower marginal costs. Real wages fall as well, further contributing to the demand channel of monetary policy, but it is dampened by reduced utilization of capital causing marginal costs to rise. The exchange rate channel works through the effect of a real appreciation following the higher interest rate on lower prices of imported intermediaries and higher export prices. Both imports and exports are reduced — the former because of reduced demand and investments despite the lower import prices causing a substitution towards foreign inputs. Lower demand will in turn lead to lower export prices, and exports pick up again after the initial decrease. The expectation channel is present through the effects on expected future marginal costs.

CN has relatively similar responses to a monetary policy shock as NEMO, although the variables move in smoother way and the effects are slower and last longer. The persistence of the disturbance is due to the financial accelerator. The additional *credit channel* present in this model is due to the fact that a higher real interest rate leads to lower house prices through lower demand and investments, reducing the collateral value of the credit constrained households and thereby dampening their demand. Lower housing prices also reduces consumption of the patient households, further dampening GDP growth.

In NAM the variables have more high frequent variation than in the other models, but the duration of the disturbance to the variables is roughly the same as in NEMO, albeit a more hump-shaped and larger effect. An increase in the nominal interest rate leads to an immediate nominal and real appreciation of the NOK which affects domestic prices and wages through decreased import prices. This is the direct exchange rate channel to inflation, but there is also an indirect effect in the demand channel through lower competitiveness of exporting firms following an appreciation, and therefore lower GDP and higher unemployment. Due to lower inflation, the increase in the nominal interest rate is translated into an even bigger increase in the real interest rate¹⁷ which affects production with a lag and unemployment

¹⁷The real interest rate in NAM is defined as the difference between the nominal interest rate and *current*

with two lags, contributing to the demand channel. A third channel is the *credit channel* working through the effects of the interest rate on output due to reduced availability of real credit. In NAM there is no expectation channel since the model is purely backward-looking.

The Leitemo-Gali-Monacelli model appears in Figure 2 to be the most persistent model, but this is just an artifact of the policy rule used, which is not hard-hitting enough to stabilize the economy as good as in the other three models. Due to the rule, output falls substantially more, and moves slowly in dampened cycles back to the long run equilibrium. The monetary policy shock immediately reduces demand and production, which lowers domestic inflation through the demand channel with a lag. The expectation channel works through expected future inflation that lowers current inflation, and the complicated lag structure causes GDP and consumer prices to move in a hump-shaped fashion. A higher nominal interest rate decreases the exchange rate, creating a negative law-of-one-price gap, *ceteris paribus*. But the reduced terms of trade reduces the gap because importing firms lowers prices on imported goods, further reducing the output gap and inflation.

5 Method

The program used for the technical part of the analysis is MatLab with the software Dynare. Dynare solves the models using a first order approximation around (a zero) steady state and calculates the theoretical variances of the endogenous variables. In order to find the first-best policy reaction function of a specified form, the variances of the variables entering the objective function is minimized. The size of the shocks in the models are given by their standard deviation, and the resulting value of the objective function, or loss, is the sum of the unconditional variances of the included variables.

The estimated size of the standard deviations of identical shocks could be different across models because it depends on the other shocks included in the estimation. If a model has few shocks, each of them will play a significant role in explaining the variability in the data, but as the number of shocks increases, the relative importance of each shock decreases. The standard deviations determines the size of the loss, as more variability in the exogenous variables in the simulations naturally leads to larger fluctuations in the endogenous variables. This can indeed be seen in the results, where NEMO yields losses of a magnitude far above the other models, and NAM quite a lot below. CN is the model with the greatest number of shocks (17), but has the second largest loss conducting optimal monetary policy. NEMO has 15 shocks, NAM 11, and the LGM model only 7 estimated shocks. It may seem that the shocks included in NAM have less explanatory power than the shocks in the other models,

inflation: $\rho_t = i_t - \pi_t$.

and therefore estimated to be smaller, but as explained in Chapter 3.4 about NAM, the small loss is partly due to the fact that we have removed some of the original exogenous variables, in particular government spending, which is an important driver of short run fluctuations in GDP. We have, of course, also removed the interest rate shock in all models, and since the models are estimated with different numbers of shocks, the variability explained by the excluded shock differs. Hence, the removal of it could possibly explain some of the difference in the size of losses.

Below is a presentation of the simple interest rate rules in this thesis, both first-best rules and Bayesian rules, and an explanation of how they are derived. Bayesian rules are relevant for my analyses because they are found to be more robust than simple rules optimized within a single model. The chapter ends with a discussion of the different loss function specifications.

5.1 Optimal simple rules

To find the first-best simple interest rate rules in a model, the following loss function¹⁸

$$\mathcal{L}_t = E_t[(\pi_t - \pi^*)^2 + \lambda^y(y_t - y^*)^2 + \lambda^r(r_t - r_{t-1})^2]$$

is minimized in order to find the coefficients ϕ^r , ϕ^π , ϕ^y and ϕ^{y-1} in the following equations:

$$r_t = \phi^\pi \pi_t + \phi^y y_t, \quad (\text{Rule 2})$$

$$r_t = \phi^r r_{t-1} + (1 - \phi^r)(\phi^\pi \pi_t + \phi^y y_t), \text{ or} \quad (\text{Rule 3})$$

$$r_t = \phi^r r_{t-1} + (1 - \phi^r)(\phi^\pi \pi_t + \phi^y y_t + \phi^{y-1} y_{t-1}). \quad (\text{Rule 4})$$

Here π_t is the current year-on-year inflation rate and y_t is the output gap. The resulting value of the loss function is what I refer to as "osr loss" (optimal simple rule loss), denoted by \mathcal{L}^{osr} , and the rules as the two-, three- and four-parameter interest rate rules, respectively. Rule 2 is of the same form as the classical Taylor rule, capturing the important aspect of central banks' tendency to "lean against the wind", but not allowing for responses to further information. Rule 3 incorporates gradualism in the adjustments of the policy rate, as this is found in data and therefore appears to better represent actual monetary policy than Rule 2, as in Levin et al. (1999). Following Taylor and Wieland (2009), I include Rule 4, where more lagged information is used for monetary policy decisions. I also consider two variations

¹⁸The loss function is the same as for optimal monetary policy, but is minimized subject to the simple policy rule. In optimal policy the unconditional variances are minimized by reacting in a highly complex manner with the interest rate to economic disturbances, whereas it is restricted to respond only to a limited set of variables in the above optimization.

of Rule 3: One where the lagged interest rate has been replaced by the lagged output gap, and one where it has been replaced by lagged inflation.

The values ϕ^π and ϕ^y represent the long-run response of the nominal interest rate to deviations in inflation and output from trend or target, i.e. when the smoothing of the interest rate is completed, and are thus the "true" responses. They are the values I report in the tables, if nothing else is stated. The short run coefficients are attained by multiplying the bracket in front of the long run rule with the coefficients in this rule, and indicate how much the central bank reacts in the same period as deviations is observed.

The rationale for not including forward-looking variables, only contemporaneous and lagged variables in the interest rate rules, is that the former are highly model dependent and should not be used in robustness analyses. Basing the policy decision in a model on a future variable naturally implies the use of a forecast within the model, and transferring a rule with a model-specific forecast to another model in order to analyze the rule's performance would not be fully valid since the variable that the rule responds to would be different across the models. Also, a forward-looking rule may be less robust due to the fact that it uses more model-specific information (Brubakk and Natvik 2010).

Contemporaneous values of inflation and output gap are, of course, difficult to observe (in particular the latter which may not be revealed until many years later when the trend has been properly estimated) and must be estimated. If the models are used to find the values, the same argument as for future variables would hold true. But in Norges Bank they use SAM (System of Aggregated Models)¹⁹ for nowcasts, so real time observations are done outside the model, and can hence be utilized in an instrument rule.

5.2 Bayesian rules

The optimal Bayesian interest rate rules is found by combining the three competing models²⁰ Credit NEMO, the Leitemo-Gali-Monacelli model and NAM into one model script, and minimizing a weighted sum of the losses generated in them when using the rule found to be optimal for the "model average":

¹⁹SAM has since 2008 been used to produce contemporaneous and short run CPI and GDP forecasts, and has significantly improved forecasting in Norges Bank (Jore 2012). It consists of 167 different models, including NEMO and a slightly different version of NAM as the only two structural models. But their importance in the nowcasts are minor due to the large number of models, so the use of current variables is still valid.

²⁰NEMO is not included because it would lead to a "false" picture of the performance of the Bayesian rules in this model, as the rule would do artificially well by construction. The evaluation of a rule based on only the three alternative models is more clear-cut for robustness purposes.

$$\mathcal{L}_t^{Bayesian,abs} = \lambda^{NAM} \mathcal{L}_t^{NAM} + \lambda^{LGM} \mathcal{L}_t^{LGM} + \lambda^{CN} \mathcal{L}_t^{CN}. \quad (\text{Bayesian "absolute loss" function})$$

Here λ^{NAM} , λ^{LGM} and λ^{CN} are the probabilities assigned by the monetary authority to each of the models, and \mathcal{L}_t^{NAM} , \mathcal{L}_t^{LGM} and \mathcal{L}_t^{CN} the resulting loss from applying the optimized Bayesian rule in the separate models.

The form of the Bayesian loss function is clearly a subject for discussion. First of all, it is not obvious that the *sum* of the separate loss functions is the proper form, it could just as well be a quadratic or some other function. Apart from that, the relative weights depend on both the subjective beliefs about the probability of the different models and on how the losses of these models are scaled. The losses of each model could either enter with their absolute size (in *level*) or be normalized in some way in order to have a common reference for the performance of the policy rule in the optimization, since they are of such different magnitude, as already discussed. Because NAM yields significantly smaller losses than the two other models for all policies, using absolute losses could be viewed as "unfair" to this model, which is seen from the optimized rules in Chapter 6.2. It is natural to question whether the losses are the "true" ones when we have removed some variability in the models and changed a few of the exogenous processes. They may not have the same interpretation and are thus inappropriate as arguments in the objective function, $\mathcal{L}_t^{Bayesian,abs}$. But on the other hand, the losses are supposed to be expressions of the actual harm caused by macroeconomic fluctuations, and it could thus be argued that they correctly represent the preferences of the central bank.

Another approach is to weight the Bayesian objective with the loss in the separate models generated by the models' first-best simple rules of the same form as the Bayesian rule to be found, $\mathcal{L}^{Model,osr}$, to reflect how well the Bayesian rules do relative to the models' own rules. NAM now receives relatively more importance in the optimization, and the Bayesian rule using this osr-loss-weighted objective is more similar to the rules from NAM than with the absolute size of the losses.²¹

$$\mathcal{L}_t^{Bayesian,osr} = \lambda^{NAM} \frac{\mathcal{L}_t^{NAM}}{\mathcal{L}^{NAM,osr}} + \lambda^{LGM} \frac{\mathcal{L}_t^{LGM}}{\mathcal{L}^{LGM,osr}} + \lambda^{CN} \frac{\mathcal{L}_t^{CN}}{\mathcal{L}^{CN,osr}}. \quad (\text{Bayesian osr loss})$$

I have looked at four different combinations of relative weighting of the models: First

²¹ A third alternative would be to scale the Bayesian objective with the individual Ramsey losses, reflecting concerns about the damage of using a Bayesian rule instead of conducting optimal monetary policy in each model. As will be seen, the difference between Ramsey losses and osr losses are roughly of the same magnitude across the models. Using osr losses appears to be more intuitive since I am looking at the use of simple interest rate rules, and I therefore present results with the osr losses only.

$\lambda^{NAM} = \lambda^{LGM} = \lambda^{CN} = 1/3$ and then $1/6$ for two of the models and $2/3$ for the last, for each of the three possible combinations. Optimized coefficients are reported in the next chapter.

This type of rule is commonly referred to as a "Bayesian rule" in the literature, although the term is somewhat misleading as it rather is a "model averaging" rule than "Bayesian" in the original meaning. The weights assigned to each model are fixed "priors" and are not updated using Bayes' rule when policy makers acquire additional knowledge about the probability distribution of the models. Assuming that beliefs are never updated is highly debatable. It is natural to believe that new information about the structure of the economy will affect the priors, and the rules should in consequence be changed.²² Nevertheless, the Bayesian rules can be interpreted as an upper bound of the insurance possibilities, when the central bank commits to the prescribed policy and never updates the relevance of the competing models.

5.3 Different loss functions

The variables entering the loss function are the unconditional variances of yearly inflation, output deviations relative to trend and the change in the nominal interest rate. As the models are quarterly, the prescribed interest rate rules are also in quarterly terms and had to be converted into yearly terms, either year-on-year or annualized. Year-on-year inflation is less noisy and appears to be a better variable for policy making than quarterly annualized inflation which includes a lot of short run fluctuations that are impossible and undesirable to mitigate²³. The optimal coefficients in the simple interest rate rules increase when quarterly inflation is used, since the central bank then needs to react more aggressively. The goal of monetary policy is after all to ensure macroeconomic stabilization, so responding to large shocks that moves the economy substantially away from the trend, and not to short run fluctuations, seems natural. I therefore choose year-on-year inflation in the objective functions and simple interest rate rules, in line with the existing literature (e.g. Taylor and Wieland 2009, Taylor 1993, Levin et al. 1999).

I consider three variations of the loss function in Equation 2, which I refer to as \mathcal{L}_1 , \mathcal{L}_2 and \mathcal{L}_3 . The first is the benchmark loss function that I present throughout the thesis. The

²²Thanks to Lars Svensson for pointing this out to me.

²³Looking at the autocorrelation coefficients confirms this. All models have a high degree of persistence in yearly inflation when optimal monetary policy is conducted, around 0.8-0.9, but the correlation between two succeeding quarters is substantially lower. In NAM the autocorrelation coefficient decreases from 0.85 for yearly to negligible 0.0014 for quarterly, and in the LGM model from 0.8 to 0.3. The decreases are large in Credit NEMO and NEMO as well, 0.11 and 0.29, respectively. Thus, quarterly inflation fluctuates more randomly and is harder to stabilize than year-on-year inflation in the set of models.

two others have been used mainly for cross-checking, and the results from them are placed in the appendix.

Relative weights	\mathcal{L}_1	\mathcal{L}_2	\mathcal{L}_3
λ^π	1	1	1
λ^y	0.5	1.5	0.5
λ^r	0.25	0.25	0.1

Table 1: The three different loss function specifications considered.

The weight on inflation is normalized to unity in all three loss functions, such that the other weights are relative to inflationary concerns. Policy makers' preferences are of course hard to model, and the weights assigned to the different arguments in the loss function may seem a bit *ad hoc*. As already argued, assigning inflation, GDP and interest stabilization some weight is preferred by an inflation targeting central bank. I therefore do not consider the case of a pure "inflation nutter".

The benchmark, \mathcal{L}_1 , uses the weights from Norges Bank's Monetary Policy Report No.3/2011 (Norges Bank 2011) with half of the concern for fluctuations in output and a quarter for nominal interest rate movements relative to the preferences for a stable inflation rate. This corresponds to the preferences of a relatively flexible inflation targeting central bank.

In the second loss function, \mathcal{L}_2 , the relative concern for closing the output gap is higher than for reaching the inflation target — it has been increased from 0.5 to 1.5, with the other weights as in \mathcal{L}_1 . If the central bank doubts the close connection between inflation and output stabilization, and since the output gap represents the real economy that it actually cares about, it could attach a higher weight on the latter.

In the first Monetary Policy Report of 2012, No. 1/2012 (Norges Bank 2012), the weights in the loss function are changed in the direction of \mathcal{L}_2 from No. 3/2011. Output stabilization concerns are increased to 0.75 with the motivation that it better ensures financial stability. It is argued that since financial imbalances often build up in periods with high capacity utilization, i.e. when there is a large deviation in production from the trend, increasing the weight of the output gap can reduce the risk of such imbalances. I have cross-checked some results with this loss function, but kept \mathcal{L}_2 as the main function since it is a more extreme version and thus better serves the intended robustness purposes.

The third loss function, \mathcal{L}_3 , barely contains any concern for partial adjustments of the interest rate *per se*, but since the central bank prefers a balanced evolution of all endogenous variables, changing the policy rate gradually may be reasonable anyhow. If the economy returns faster and stays closer to the steady state for large adjustments in the nominal interest

rate, the central bank will adjust the policy instrument accordingly, since its variability is paid little attention to.

At the outset of the analysis, I tried with another version of \mathcal{L}_3 where λ'' was set to zero, which I refer to as "the modified \mathcal{L}_3 ". The optimized rules derived from this loss function were so extreme that the robustness check felt useless and the consequent policy implications were unrealistic. These are similar results to those of Svensson (2000) referred to in Chapter 2.4. The only model that gave "reasonable" rules was NAM, where interest rate smoothing is hardly an issue due to the backward-looking nature of the model.

In the LGM model it was not possible to find optimal coefficients with the modified \mathcal{L}_3 , as they reached the bounds of the searching area no matter how large I made it. In particular the output gap coefficient was large, even larger than the one on inflation. This indicates that the only reason to pursue gradualism in the policy rate adjustments in this model is that the central bank has explicit preferences for such behaviour. Complete stabilization is possible if the nominal interest rate is allowed to react sufficiently to shocks, but it is hard to imagine that any central bank would apply such aggressive rules. Most importantly, for rules of this magnitude the log linearization of the models no longer holds as an approximation. Consequently, I added a small weight on interest rate smoothing in the third loss function.

6 Results

In this chapter I present the optimal simple rules resulting from the optimization of each individual loss function and the Bayesian objectives. There are several forms of the Bayesian rules, and in order to simplify the reading of the results, they are given "code names" in Table 2. I also repeat the specifications of the three rules afterwards.

Relative weights	Absolute loss			Osr loss		
	Rule 2	Rule 3	Rule 4	Rule 2	Rule 3	Rule 4
$\lambda^{NAM} = \lambda^{LGM} = \lambda^{CN} = 1/3$	B2abs	B3abs	B4abs	B2osr	B3osr	B4osr
$\lambda^{NAM} = \lambda^{LGM} = 1/6, \lambda^{CN} = 4/6$	B2aC	B3aC	B4aC	B2oC	B3oC	B4oC
$\lambda^{NAM} = \lambda^{CN} = 1/6, \lambda^{LGM} = 4/6$	B2aL	B3aL	B4aL	B2oL	B3oL	B4oL
$\lambda^{LGM} = \lambda^{CN} = 1/6, \lambda^{NAM} = 4/6$	B2aN	B3aN	B4aN	B2oN	B3oN	B4oN

Table 2: Bayesian rules. "Absolute loss" refers to a rule where the level of losses in each model is minimized, whereas with "osr loss" the losses enter relative to what the first-best rule of the same specification as the Bayesian rule generates in the separate models. The last capital letter indicates which model is attached the highest probability in the Bayesian objective function. No capital letter means equal relative weights.

$$r_t = \phi^\pi \pi_t + \phi^y y_t \quad (\text{Rule 2})$$

$$r_t = \phi^r r_{t-1} + (1 - \phi^r)(\phi^\pi \pi_t + \phi^y y_t) \quad (\text{Rule 3})$$

$$r_t = \phi^r r_{t-1} + (1 - \phi^r)(\phi^\pi \pi_t + \phi^y y_t + \phi^{y-1} y_{t-1}) \quad (\text{Rule 4})$$

6.1 Optimal simple rules

Table 3 shows the first-best simple interest rate rules from each model with the benchmark loss function, \mathcal{L}_1 . As can be seen from the table, the optimal coefficients vary to a large extent across the models, but in all rules except those of NAM and the output gap coefficient of Rule 2 in NEMO, the values are significantly larger than in the rule proposed by Taylor (1993). This holds true for the other loss functions considered as well (see Appendix B.1, Table 12, 13 and 14). Rudebusch and Svensson (1998) consider optimal interest rate rules of the same form as in Table 3 in a small model for the U.S. economy, and report coefficients of roughly the same size, albeit somewhat smaller interest rate smoothing parameters. In particular they find values substantially above the original Taylor rule (above 2 on inflation and above 1 on output) in five different loss function specifications. Other authors have also argued that the coefficients in the Taylor rule should be higher than 1.5 and 0.5, e.g. Ball (1997).

	NEMO			NAM			LGM			CN		
Rule	2	3	4	2	3	4	2	3	4	2	3	4
ϕ^r		0.80	0.81		0.34	0.08		0.80	0.80		0.96	0.97
ϕ^π	1.88	3.87	3.96	1.12	1.16	1.14	2.19	4.46	4.42	3.08	22.17	29.90
ϕ^y	0.19	1.14	1.32	0.35	0.55	0.36	1.76	5.08	4.93	0.57	8.36	17.87
ϕ^{y-1}			-0.14			0.36			0.11			-6.94
\mathcal{L}	23.26	22.40	22.40	2.53	2.40	2.15	4.55	3.53	3.53	6.14	5.57	5.56
$\% \Delta \mathcal{L}^*$	12.1	8.0	8.0	33.0	25.7	12.6	46.9	13.9	13.9	23.2	11.9	11.5

Table 3: Optimal simple interest rate rules of each model, and the generated loss. At the bottom row is the percentage increase in loss relative to Ramsey loss.

As can be seen from Table 3, NEMO and the Leitemo-Gali-Monacelli model prefer quite similar reaction patterns for the nominal interest rate, except from the response to output fluctuations, to which LGM demands significantly stronger responses than NEMO. The output coefficient is in fact even larger than the inflation coefficient.

First-best policy rules from NAM and CN are quite the opposite of each other, with

NAM dictating only slight responses to deviations of output and inflation from target. The rules barely fulfill the Taylor principle, and there is hardly any smoothing of the interest rate path. Strong reactions to shocks are not necessary in NAM since they die out relatively fast regardless of policy, due to the equilibrium correction mechanisms. Because expectations do not enter in the model, the output gap is a function of historic interest rates only, and monetary policy effects the real economy sluggishly. In order to have a contemporaneous effect, it is optimal to respond to lagged output movements (or even better to *changes* in the output gap because the model is in difference form), which can be seen from the negative coefficient on lagged production in Equation 65 in Appendix A.3.2. This negative relation creates dampened oscillations back to the long run equilibrium.

Conducting optimal policy in NAM shows that output is little persistent and that the second order autocorrelation coefficient is markedly larger than the first order coefficient, indicating the gain from reacting to last period's movements. The third and fourth order autocorrelation coefficients are indeed negative, so striving to dampen current fluctuations might only enlarge the oscillations. Interest rate smoothing therefore gains nothing, and may in worst case lead to instability, as historic interest rates are poor indicators for the optimal rate today.

The optimal coefficient on the lagged interest rate is almost negligible in the four-parameter rule, but increases substantially when the lagged output gap is removed from the rule, indicating that the latter variable is successful in achieving sufficient stabilization. Without the possibility to react to y_{t-1} , policy uses the lagged interest rate as a substitute for the missing historic information.

The optimized coefficients in the three and four-parameter rules of CN are surprisingly large, but due to the high degree of interest rate smoothing, the actual short run reactions are of similar magnitudes as in the other models: The net coefficients attained by multiplying the inflation and output gap coefficients in Table 3 with $(1 - \phi^r)$ vary to a smaller extent across models than the long run coefficients. The parameters in CN's rules can be interpreted as a "threat" by the central bank sufficient to stabilize growth in consumer prices and GDP through the effect on agents' expectations about the future interest rate path, and hence about price and production developments. The highly inertial policy ensures that the economy reverts back to steady state before these long run responses actually occur. This holds true even for a loss function with no concern for gradualism in policy (the modified \mathcal{L}_3): The optimal interest rate smoothing parameter is 0.75 and 0.84 for Rule 3 and Rule 4, respectively, and the other coefficients are of roughly equal magnitude as with \mathcal{L}_1 . When interest rate smoothing is removed from the rule, the two-parameter rule is similar to those in the other models, although still quite aggressive.

In the last row of Table 3, the performance of the first-best rules relative to optimal policy is displayed, i.e. the relative increase in the loss generated by the simple rule, \mathcal{L}^{SR} , to the minimum loss, \mathcal{L}^* :

$$\% \Delta \mathcal{L}^* = \frac{\mathcal{L}^{SR} - \mathcal{L}^*}{\mathcal{L}^*}.$$

Following the simple guidelines for monetary policy is not as harmful as one would have thought, considering the limited information utilized. Most of the rules perform quite well, at least the two larger forms of rules. Rule 4 yields less than 14 percent higher loss than optimal policy in the LGM model, which is the worst among the four models.²⁴ Rule 3 is less successful, in particular in NAM, as expected. 14 percent higher loss is, of course, a significant number, yet still a reasonable price to pay for the huge robustness gain from simple policy rules.

It is a common result in the literature that simple instrument rules of the form in Table 3 do not substantially deteriorate the performance of monetary policy relative to optimal policy. Levin et al. (1999) uses four different structural models for the U.S. economy and find that rules where the first difference of the nominal interest rate reacts to the current output gap and the one-year average inflation deviations from target, i.e. a three-parameter rule with the interest rate smoothing restricted to unity, are more robust to model uncertainty than both smaller and larger rules. Rudebusch and Svensson (1998) concludes that simple instrument rules tend to perform quite well in the small empirical U.S. economy model considered, even the Taylor type of rules (i.e. Rule 2).

Looking at the size of the losses²⁵ generated by using the first-best simple instrument rules in the four models in Table 3, it becomes clear that the gain from including more information beyond the lagged interest rate is in most cases negligible. To highlight the magnitude of the differences, the relative increase in loss following the removal of one variable at a time from the policy reaction function is displayed in Table 4 for the three different loss functions. For all models except NAM, and all loss functions, the increase in loss never exceeds one percent changing from a four to a three-parameter rule, indicating that setting the policy rate on the basis of the contemporaneous inflation and output deviations in addition to allowing for a certain degree of gradualism, is sufficient. Because Rule 4 uses more model-specific information it runs the risk of being less robust, and it is also less comprehensive due to the more complicated reaction pattern of the central bank.

²⁴For \mathcal{L}_2 the highest numbers with Rule 4 are 16.7 per cent in NAM and 14.8 per cent in Credit NEMO. For the other models and the rules in \mathcal{L}_3 the increases are smaller.

²⁵The numbers have been multiplied by 10000 in order to simplify the exposition. This corresponds to multiplying all standard deviations by 100 in order to get them in percentage terms.

Model	\mathcal{L}_1		\mathcal{L}_2		\mathcal{L}_3	
	4 to 3	3 to 2	4 to 3	3 to 2	4 to 3	3 to 2
NEMO	0.003	3.8	0.45	5.51	0.0001	1.54
NAM	11.68	5.8	17.1	6.5	13.58	5.01
LGM	0.003	29.0	0.02	25.65	0.09	19.98
CN	0.30	10.2	0.95	12.40	0.77	7.35

Table 4: Percentage increase in loss from reducing the number of parameters of the optimal simple rules in each model for the three different loss function specifications.

Removing the lagged interest rate as well from a simple interest rate rule appears to cause more harm. The biggest increase in loss is in the LGM model of nearly 30 percent, but NAM experiences smaller increases compared to the removal of the lagged output gap. Naturally, the deterioration decreases as interest rate smoothing is attached less weight in the loss function, as seen from Table 4. But even with a very small weight, as in \mathcal{L}_3 , it is substantially better to adjust the interest rate gradually. In NAM this follows from the information contained in the lagged interest rate, and in the other three models it is because of the history dependence of policy.

Another point that should be highlighted are the generally small increases in NEMO compared to the other models. It seems that NEMO is relatively insensitive to the design of the monetary policy reaction function.

The small *average* gain from extending Rule 3 with the lagged output gap is remarkable in Table 4. Yet, including the fourth variable should be considered if the central bank's belief in NAM is sufficiently strong. The lagged output gap drives the performance of the simple instrument rules in NAM, which becomes even clearer from the evaluation of two alternative specifications of Rule 3 (results in Appendix B.2, Table 17):

$$r_t = \phi^\pi \pi_t + \phi^y y_t + \phi^{y-1} y_{t-1}, \quad (\text{Rule 3b})$$

$$r_t = \phi^\pi \pi_t + \phi^{\pi-1} \pi_{t-1} + \phi^y y_t. \quad (\text{Rule 3c})$$

Not surprisingly, removing the lagged interest rate from the four-parameter rule as in Rule 3b, increases the loss marginally in NAM, as the degree of interest rate smoothing is minimal and hence contributes little to the rule's performance. But the corresponding optimized rule for each of the other models performs a lot worse than both Rule 3 and Rule 4, and only slightly better than Rule 2.

Rule 3c does not seem to be superior to any rule, except from a small improvement in NEMO compared to Rule 3b. Allowing for a reaction to the lagged inflation is consequently

not worth considering if the central bank believes in the set of models used in this thesis, but rather to the historic output gap if NAM is attached a high probability.

For optimal simple instrument rules within each separate model, it appears that three-parameter rules are quite good. But as will be seen in Chapter 7.1, they do poorly in other models and are hence little robust. Instead Bayesian rules should be considered for robustness purposes, because they accommodate the conflict between the preferred policies in the models.

6.2 Bayesian rules

Below are interest rate rules prescribed by averaging over the outcomes of the three reference models, NAM, the LGM model and CN. Table 5 display the two, three and four-parameter optimized Bayesian rules with equal relative weights attached to the models, both for absolute and relative losses in the objective function. Again, the differences in loss between four- and three-parameter rules are relatively small (albeit somewhat larger than the difference between two and three for the "osr loss"-rules due to NAM), and the smaller rule appears to be sufficient for robustness purposes.

Coefficients	B2abs	B3abs	B4abs	B2osr	B3osr	B4osr
ϕ^r		0.66	0.53		0.63	0.40
ϕ^π	2.16	3.53	3.24	1.96	2.87	2.55
ϕ^y	0.65	1.98	1.24	0.59	1.68	0.84
ϕ^{y-1}			0.53			0.54
$\mathcal{L}^{Bayesian}$	5.571	4.697	4.626	1.299	1.245	1.172

Table 5: Optimal Bayesian interest rate rules with equal relative weighting of the three models.

The rules are less aggressive when osr losses are used to normalize the Bayesian loss function, since NAM then receives more importance because of its small losses that the Bayesian objective is scaled with. As already explained, the first-best rules from NAM are quite mild and little inertial, whereas the LGM model prefers aggressive rules. The conflict in the Bayesian optimization is mostly between these two models, and the optimal coefficients seem to be "averages" of the two individual rules. NAM is relatively more important in the optimization over the output gap coefficient, because larger values of this may lead to instability, as will be seen from the performance of the LGM-rules in NAM in Chapter 7.1.

Similar patterns with different *relative* weights on the three models can be seen in Table 6. When NAM enters in the objective function with four times the probability of the two other models, the coefficients in the Bayesian rule decreases substantially, and again, particularly

the coefficient on the lagged interest rate. The rules tailored more to CN and the LGM model are similar to each other, although the CN-rules in the first three columns have somewhat larger coefficients, except for the output gap coefficient.

Coefficients	B2oC	B3oC	B4oC	B2oL	B3oL	B4oL	B2oN	B3oN	B4oN
ϕ^r		0.65	0.50		0.73	0.59		0.52	0.24
ϕ^π	2.28	3.75	3.32	2.01	3.49	2.99	1.65	1.98	1.89
ϕ^y	0.59	1.78	1.04	0.74	2.82	1.57	0.48	1.04	0.52
ϕ^{y-1}			0.43			0.65			0.45
$\mathcal{L}^{Bayesian}$	1,185	1,192	1,168	1,326	1,176	1,049	1,244	1,210	1,157

Table 6: Different relative weights on the alternative models, using relative losses in the Bayesian loss function.

The Bayesian loss generated by B3oC is actually *larger* than B2oC, which is a consequence of the conflict between the preferred policy preferences in NAM and CN when the nominal interest rate responds to three variables.

NAM is the model driving most of the results. Without this model, i.e. averaging over the two other reference models only, the rules become more aggressive (see Appendix B.2, Table 16). Not surprisingly, they perform significantly better in all models but NAM, in which the loss is tripled relative to the first-best four-parameter rule. Switching to the three-parameter rule generated by LGM and CN, the improvement becomes substantial, however — the loss in NAM is "only" doubled. This result may appear surprising since the outcome in NAM is found to be better with Rule 4 than Rule 3. The superiority of the larger rule hinges crucially on a *positive* reaction to the lagged output gap. CN, on the other hand, strongly prefers the opposite, such that the optimized rule for the average of CN and LGM adopts this characteristic, and hence the performance in NAM is poor.

7 Robustness

”Simple monetary policy rules are designed to take account of only the most basic principle of monetary policy of leaning against the wind of inflation and output movements. Because they are not fine tuned to specific assumptions, they are more robust to mistaken assumptions.” (Taylor and Wieland 2009)

Next the robustness properties of some of the interest rate rules considered in this thesis is presented. I use two different tools for checking how well the rules perform in the set of models, *implied inflation variability premium (IIP)* and *fault tolerance*. Below I describe

both measures and how to derive them. In the Discussion section the results are related to the existing literature on robust monetary policy rules.

I start out by analyzing IIP in order to eliminate those rules that generate particularly high inflation variability in the models. These rules are not robust and not worth considering further. I then analyze the fault tolerance properties of the remaining rules. It is desirable to have a rule that is both robust in the sense of generating little fluctuations in consumer prices across models, and robust to changes in the parameter values. If a rule is fault tolerant, the policy maker does not cause too much harm if he "misses" on the optimal value of a coefficient. Even though I have found that four-parameter rules do not significantly outperform smaller rules in the separate models, they could still be more robust, and are therefore included in the analyses below.

7.1 Implied Inflation variability Premium

Implied Inflation variability Premium (IIP) is used to evaluate the performance of a policy rule relative to another rule, proposed by Kuester and Wieland (2010). It is defined as the percentage point (pp) increase in the standard deviation of the inflation rate for a given increase in absolute loss, keeping output and interest rate variability constant, i.e. the difference in loss caused by the interchange of two rules, from A to B, translated into inflation variation:

$$IIP^B = 100 \times [\sqrt{\mathcal{L}_t^B - \lambda^y var(y_t^A) - \lambda^r var(\Delta r_t^A)} - s.d.(\pi_t^A)].$$

For constant variance of y_t and Δr_t , the increase in loss generated by rule B relative to rule A will be equal to the relative increase in the variance of inflation, but since the variances of the former two variables in general differ between the rules, the variance dispersion is converted into variance of inflation in order to make the rules' achievements comparable. IIP is then the square root of this "altered inflation variance". For example, the optimized four-parameter rule in NEMO yields a loss of 22.40217, while the rule from NAM yields a loss in NEMO of 45.68388, which is an increase of 104 percentage. The standard deviation of the inflation must increase by 2.67 pp in order to yield the same increase in loss in NEMO switching from NEMO's to NAM's first-best rule.

IIP is measured in absolute terms and is thus easily comparable across models, whereas the *relative* increase in loss in a model depends on the size of this loss. Due to the large difference in losses in the models, the relative increase is a less "neutral" measure. IIP is directly interpretable in terms of economic consequences, namely how much more consumer prices will fluctuate if the central bank uses the wrong model.

Table 7 illustrates the dispersion of the preferred monetary policy in the five models. The rules from NAM generally creates the biggest increase in inflation variability across models, with an IIP in CN of 3.68 pp in the case of Rule 4 and 3.60 pp in LGM with Rule 2. This is a big price to pay in form of increased inflation fluctuations if the economy turns out to be best described by CN or the LGM model, but the central bank believes it is NAM. Even larger is the IIP of Rule 3 from CN applied to NAM, of a striking 7.14 pp. This outlier is due to the fact that NAM is little tolerant to inertial policy, and in particular when the possibility to respond to the lagged output gap is absent. The three-parameter rule from CN holds both of these properties and therefore yields a miserable outcome in NAM. The extreme parameter values excludes it from the set of candidate rules for insurance against model uncertainty, because no policy maker would apply such an aggressive rule.

NAM is intolerant to policies from the other models because it is self-stabilizing, so reacting with large changes in the policy rate to movements in the key variables might do more harm than good. The IIP to be paid if NAM is the "true" economy, and the central bank acts as if it were another model is thus relatively high. On average it is 1.17 for Rule 4, 2.42 for Rule 3 and "infinity" for Rule 2. The original Taylor rule, shown in the bottom row of Table 7, performs the best, because it is the rule with the smallest parameter values and therefore resembles most NAM's first-best policy. The other models also appear to be relatively tolerant to Taylor's policy compared to the optimized rules from the other models, particularly the LGM model.

Table 7 reveals that Rule 2 is significantly less robust than Rule 3, but more than Rule 4 in most of the cases, due to the smaller amount of "model-tailoring". The IIP increases on *average* a lot more between Rule 3 and Rule 2 than Rule 3 and Rule 4, and again, particularly in the LGM model. Rule 2 from the latter model actually creates instability in NAM because of the large output response, which emphasizes the conflict between the preferred policies of the two models. However, reducing the output coefficient with 0.5 removes the instability it creates in NAM, because the *relation* between the inflation and output gap coefficients then becomes more equal to NAM's Rule 2, albeit twice the size in magnitude. A large inflation response can somehow "outweigh" the output reaction such that a rule's performance in NAM improves, which is why the four- and two-parameter rule from CN are better than Rule 3.

Again, the conflict between the models' preferences for reaction patterns of the nominal interest rate becomes clear. In particular, it seems to be NAM versus the other models, as expected, due to this model's highly different features.

On average across all the rules, Policy NEMO has the lowest IIP of the five models. This model is the most robust with respect to the design of monetary policy, closely followed

Rule 4	<i>IIP [%$\Delta\mathcal{L}^*$]</i>				
	NEMO	NAM	LGM	CN	PN
NEMO	0 [8.0]	1.21 [193.2]	2.16 [355.5]	0.06 [14.7]	0.41 [28.8]
NAM	2.67 [120.3]	0 [12.6]	1.34 [191.4]	3.68 [473.3]	0.80 [111.7]
LGM	1.04 [43.5]	0.88 [128.1]	0 [13.9]	1.00 [83.6]	2.46 [42.0]
CN	0.06 [9.7]	1.86 [352.5]	1.71 [259.8]	0 [11.5]	0.83 [42.8]
Rule 3					
NEMO	0 [8.0]	1.21 [207.6]	2.26 [380.2]	0.06 [14.9]	0.38 [28.4]
NAM	2.07 [89.3]	0 [25.7]	1.92 [302.1]	2.50 [266.9]	0.78 [88.1]
LGM	1.03 [43.3]	0.82 [133.0]	0 [13.9]	1.00 [83.3]	1.93 [42.0]
CN	0.06 [9.8]	7.14 [3325.4]*	1.84 [287.3]	0 [11.9]	0.71 [39.6]
Rule 2					
NEMO	0 [12.1]	0.66 [112.8]	2.09 [384.7]	0.15 [31.4]	0.01 [17.2]
NAM	1.80 [79.8]	0 [33.0]	3.60 [806.6]	2.24 [238.2]	0.92 [77.3]
LGM	1.21 [54.1]	Instability*	0 [46.9]	1.09 [102.2]	1.69 [6.2]
CN	0.17 [74.3]	0.84 [142.8]	0.95 [165.6]	0 [23.2]	0.16 [21.5]
Taylor rule	0.66 [33.4]	0.15 [47.5]	0.92 [160.6]	0.70 [68.6]	0.51 [32.9]

Table 7: IIP: increase in the standard deviation of inflation changing between each model's optimized four-parameter rules. Percentage increase in loss relative to Ramsey loss is displayed in parentheses.

by NEMO. The least robust model is LGM, having the highest average IIP, disregarding the two "outlier" results (marked with an asterisk). LGM needs strong responses to output fluctuations, and since none of the other models entail similar properties, their rules lead to significantly less stability.

As expected, NEMO is tolerant to CN policy, and vice versa, because they have similar core structure and main transmission mechanisms for monetary policy. This holds true for PN as well, even though it is significantly more backward-looking than the former two.

The values in parentheses in Table 7 show the percentage increases in loss relative to conducting optimal policy, in order to highlight the difference between the two measures. A high increase in loss in a model could either be due to the fact that simple rules generally perform poorly, or because the model is little tolerant to rules from other models. A high IIP can also be caused by two different factors: either due to general policy intolerance or because the new rule creates larger fluctuations in interest rate changes and/or the output gap relative to inflation fluctuations. When the increase in loss resulting from the interchange of two policy rules is translated into inflation variability, the IIP is higher if the relative variability in the other two variables caused by the new rule is larger than with the old rule.

The ordering of the rules' performances across models actually changes in some instances, but the supremacy of PN as the most tolerant model is strengthened. NEMO is however almost as tolerant as PN when the relative increase in loss is used as the robustness measure

instead of IIP. Rule 4 and rule 3 from the LGM model now performs better in PN than in NAM, and CN is no longer the model that is most tolerant to LGM’s Rule 2 — again it is PN. All rules perform best in this model, except Rule 4 and Rule 3 interchanged between NEMO and CN. The latter is a consequence of the high degree of interest rate smoothing in these rules, which is not desirable in PN due to the backward-lookingness of this model.

The reason for the change in the ranking of the models may be that LGM-rules are so aggressive that they significantly increase interest rate movements in order to stabilize inflation and GDP, but since Policy NEMO is so persistent, the dampened inflation fluctuations are small relative to the increased interest rate volatility.

The trade-off between outcomes in the different models displayed in Table 7 calls for more robust monetary policy. Using either of the first-best individual rules appears to be a bad idea if a positive probability is attached to all five models, as the rules could potentially lead to very high variances of key variables. Therefore, Bayesian rules should be considered. The trade-off between the outcomes in the models has already been taken into account, such that the resulting inflation variability premium is lower compared to the individual first-best rules transferred between models. Since NEMO and Policy NEMO are quite robust to policy specifications, the IIPs generated by the Bayesian rules are relatively low in these models as well. The improvements in outcome across all models from Table 7 to Table 8 is indeed striking.

Rule	<i>IIP [%$\Delta\mathcal{L}^*$]</i>				
	NEMO	NAM	LGM	CN	PN
B2abs	0.16 [17.0]	0.52 [92.7]	0.54 [106.8]	0.18 [33.2]	0.07 [18.9]
B2osr	0.21 [18.5]	0.39 [74.4]	0.60 [115.8]	0.26 [37.6]	0.11 [20.2]
B3abs	0.28 [16.6]	0.45 [75.7]	0.26 [39.0]	0.30 [29.3]	0.20 [23.0]
B3osr	0.41 [20.9]	0.35 [62.3]	0.25 [38.6]	0.42 [37.4]	0.29 [25.7]
B4abs	0.34 [18.7]	0.42 [57.6]	0.26 [39.3]	0.34 [31.7]	0.19 [21.6]
B4osr	0.48 [23.2]	0.27 [39.5]	0.29 [42.4]	0.47 [40.6]	0.26 [23.8]

Table 8: IIP: increase in the standard deviation of inflation changing from the optimized rule of each model to a Bayesian rule of the same specification. Relative increase to Ramsey loss are in parentheses.

Switching from a four- to a three-parameter Bayesian rule improves the outcome in all models except for NAM, and marginally PN, confirming the robustness superiority of smaller rules. In the LGM model these two rule specifications achieves roughly equally good outcomes, while in NEMO and CN the improvement is more pronounced. The three-parameter rule is more robust for the other loss functions considered as well. In NAM though, the IIP

increases from 0.27 to 0.35 pp between B4osr and B3osr, revealing that the larger rule is more tailored to this model. The increase in NAM is, however, not enough to balance the reduced IIPs in the other models.

The relative increases in loss between four- and three-parameter rules in NAM are at a much larger level than in the other models. If the central bank seeks to avoid large potential losses, four-parameter rules should be considered for robustness checks. But since the losses in NAM generally are very small, the percentage increase will be higher even though the absolute increase is of the same magnitude as in the other models. Since IIP is measured in absolute terms, it is more neutral, and therefore serves the purpose of analyzing robustness properties better.

Not surprisingly, the Bayesian rules optimized over relative losses perform better in NAM and worse in CN than those with the level of losses. The rules are about equally good in the LGM model, although absolute losses is best with the four- and two-parameter rules due to the conflict with NAM in the three-parameter rule. I therefore conclude that the Bayesian rules based on the relative outcome in the three models are more robust than those based on the absolute outcome.

If the central bank changes its beliefs about the likelihood of the three alternative models, the Bayesian rules with different relative weights could be considered instead. The performance of the three-parameter rules for the benchmark loss function is displayed in Table 9. The IIP is naturally smallest in the more important model, particularly in the LGM model. In NEMO the IIP is substantially lower for the rule with the highest weight on CN, which also performs quite well in the other two models. The conflict appears to be smaller inbetween NAM and the LGM model, than between the NEMO versions and these two models.

Rule	<i>IIP [%$\Delta\mathcal{L}^*$]</i>				
	NEMO	NAM	LGM	CN	PN
B3osr	0.41 [20.9]	0.35 [62.3]	0.25 [38.6]	0.42 [37.4]	0.29 [25.7]
B3oC	0.17 [13.1]	0.48 [80.0]	0.38 [52.5]	0.19 [22.6]	0.14 [21.1]
B3oL	0.65 [29.0]	0.52 [85.0]	0.07 [20.3]	0.63 [52.2]	0.48 [31.8]
B3oN	0.64 [28.5]	0.15 [40.2]	0.47 [63.2]	0.63 [52.7]	0.47 [31.5]

Table 9: IIP in the individual models generated by the Bayesian three-parameter rules with different relative weighting of the osr losses. The last letter in the name of the rule indicates which model is assigned the highest probability in the Bayesian objective. No capital letter means equal weights. Increases in loss relative to Ramsey loss is displayed in parantheses.

On average of the five models and the eight variations of the Bayesian rules (four with osr losses and four with absolute losses), the three-parameter rule yields lower IIPs than both four- and two-parameter rules. Since it performs best across the models, it is the most robust

rule. First-best rules of the separate models were shown to be little robust, and I therefore do not investigate them further.

7.1.1 Discussion

The results above confirm existing results from the literature on robust monetary policy in the face of model uncertainty. Levin et al. (1999) find that adding more variables than three does not significantly improve the performance of Bayesian rules in their four models of the U.S. economy. Bayesian rules are designed to the "average" behaviour of the models, so adding more variables in order to fine-tune the optimization to one model will not be fruitful because the models behave so differently. Removing the lagged interest rate as well, deteriorates the rule's performance. Interest rate smoothing is optimal in the simple interest rate rules, both because it yields smaller losses, and because it is normatively desirable to avoid large jumps in the interest rate. This is somewhat contrary to the results of Taylor and Wieland (2009), who analyze the robustness of first-best rules from three different models of the U.S. economy. The authors conclude that rules responding to only inflation and output are more robust than rules that in addition respond to either just the lagged interest rate or the lagged output as well. Yet, the optimal three-parameter rules are preferred within each model, and the gain from extending them with a fourth variable is minimal.

A well-known result in the literature is that policy inertia is favorable in forward-looking models. The performance of the inertial rules from CN, NEMO and the LGM model is poor in the backward-looking model NAM, equivalent to what is found by many authors, e.g. Taylor and Williams (2010) and Kuester and Wieland (2010). Backward-looking models lacks the expectation channel of monetary policy and interest rate smoothing is thus not necessary.

In addition, rules from backward-looking models may not be active enough to anchor expectations if agents indeed are forward-looking (Kuester and Wieland 2010): The three-parameter rules from NAM generate large IIPs in the other models, in particular in CN. Adalid et al. (2005), however, conclude that the optimized simple rules from backward-looking models perform better in models with forward-looking features than the opposite, contrary to my results about NAM (except the extreme Rule 3 from CN).

Akram and Nymoen (2009) find that the use of a suit of models for robustness purposes is not optimal when monetary policy is based on the average of the policy prescribed by three equally likely macroeconomic models for the Norwegian economy, of which one is more probable but the other two have more desirable properties. The generated loss in the most valid model is by far larger than with optimized rules utilizing sub-optimal policy horizons (which is the uncertainty measure used). Akram and Nymoen (2009) conclude that because

the performance of the "average rule" is miserable, empirical validity should not be traded off against other properties, such as consistency with economic theory or data, transparency, resemblance to other models commonly used, parsimony, etc. Averaging over *policies* and not *outcomes* in the models will necessarily deteriorate performance, since the behaviour of each model is not taken into account. The Bayesian rules in this thesis are not the average of the three individual first-best rules, as such a rule could indeed potentially lead to disastrous outcomes.

7.2 Fault tolerance

Fault tolerance is a tool used to analyze how robust a model is to changes in the monetary policy rule, for example due to the "trembling hand" of the policy maker, originating from Levin and Williams (2003). It is a widely used concept in engineering, and is defined as "the ability of a system or component to continue normal operation despite the presence of ... faults"(IEEE Standard Computer Dictionary 1990). By varying one coefficient in a rule at a time, we find how robust a certain model is to the specification of the policy rule. If the loss increases only slightly for large changes in a parameter, we say that the model is *fault tolerant with respect to this parameter*, and if the model is relatively insensitive to alterations of all parameters in the rule, the model is *fault tolerant* (Levin and Williams 2003). If, on the other hand, small deviations lead to a large increase, the model is *fault intolerant*. Fault tolerance is thus a way to display the degree of curvature in the loss function.

The advantage of fault tolerance as a robustness measure is that it renders evaluation of the same interest rate rule's performance in different models possible, and thus comparisons of the models' robustness properties. By breaking it down into each single parameter, one can easily "track" the causes of robustness. If one variable in the rule is more sensitive to variations in the associated parameter, the central bank should be extra careful setting its value.

Fault tolerance can be used both for models and for policy rules, as noted by Kuester and Wieland (2010). An obvious drawback about the method described above, is that it measures the robustness of one specific parameter, *ceteris paribus*, and is hence not properly comparable across models. As a parameter in a rule that is applied in another model than the rule-generating model is changed, the optimal responses to the other variables in that rule will change as well, and hence the fault tolerance properties of the model for that rule. Instead it is possible to measure how well an interest rate rule performs across a set of competing models by looking at the fault tolerance of the *policy* rather than the models. If perturbations in the policy parameters have only marginal effect on the performance across models, the rule is said to be fault tolerant. I therefore only consider the Bayesian rules'

performance in the models, and not the robustness properties of the models for the individual first-best policies.

7.2.1 Bayesian rules

Fault tolerance for the Bayesian rules are constructed in the following way: In each separate model I apply the instrument rule found by minimizing a weighted average of the outcomes in the three alternative models NAM, LGM and CN. I then fix the coefficients at their optimal values while gradually altering one coefficient at a time, plotting the increase in loss relative to the loss generated by the first-best individual rule of the same specification as the Bayesian rule along the vertical axis and the parameter values along the horizontal axis.²⁶

In Figure 3 the fault tolerance of the Bayesian three-parameter rule with equal weighting of the osr losses in each of the three reference models is showed. It is highly fault tolerant in the NEMO models, and somewhat less in the LGM model. NAM is the least tolerant model to this rule, at least with respect to the coefficients on the output gap and lagged interest rate. The inflation coefficient can however vary more in the LGM model than in NAM. Due to NAM, the output response is restricted to the close neighborhood of 1-2, and interest rate smoothing should be kept at the level of roughly 0.6, in order to insure against model uncertainty. The LGM model demands that inflation fluctuations must not result in too large movements in the nominal interest rate, but the Taylor principle must by all means be fulfilled. At best, a one percentage increase in the inflation rate should lead to a policy rate increase in the range of 2-4 percentage points. The other models are more fault tolerant with respect to the inflation responses, and for the three NEMO versions it could be increased significantly without causing much harm.

²⁶Plotting relative to Ramsey losses gave similar figures since all models have a roughly equal relation between the loss from optimal policy and the optimized simple rule.

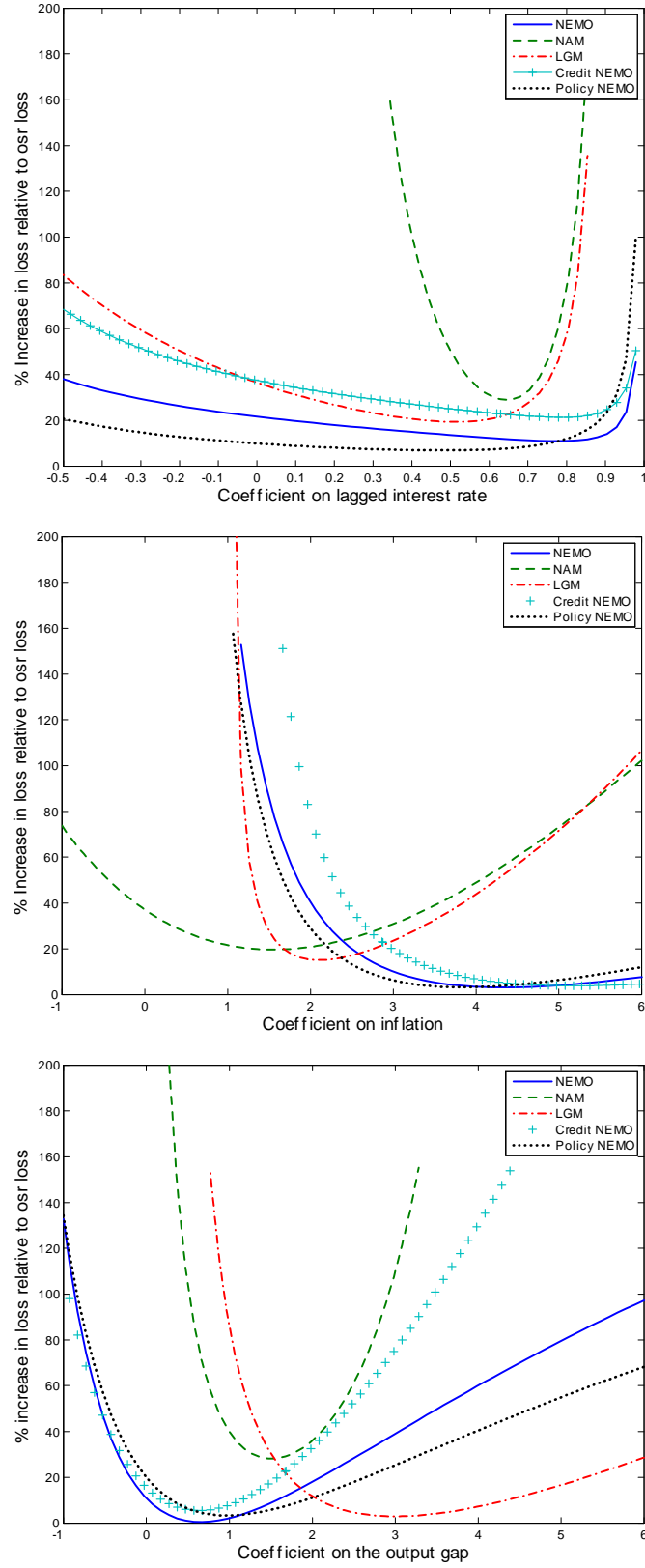


Figure 3: Fault tolerance of B3osr, the Bayesian three-parameter rule with flat prior and osr losses in the objective function (relative increase in loss to osr loss in each individual model).

As the IIP results showed, Policy NEMO is the most fault tolerant model, followed closely by NEMO and CN, even though the latter two, and in particular CN, permit larger increases in the inflation coefficient than PN. All three models are fault tolerant with respect to interest rate smoothing and the inflation response, but less for the output coefficient. They are remarkably more tolerant for increases than for decreases of the inflation parameter, where LGM and NAM are the reasons why this coefficient is "held down".

Fault tolerance graphs for the different Bayesian rules (see Appendix B.4, Figure 5 for B4osr) reveal that the models are more tolerant to perturbations in the inflation and output gap coefficients in three- than four-parameter rules, for both B3abs and B3osr. But they are less tolerant with respect to the interest rate coefficient, in particular NAM. Interest rate smoothing appears to be the crucial issue in the Bayesian three-parameter rules. The result is again related to the importance of NAM in the optimized policy. The rules generated with little weight on this model, as the absolute loss-rules and the three-parameter rules, are more robust. But they anyhow perform *relatively* well in NAM (30-40 percent higher loss than the first-best rule), so using them does not imply a complete exclusion of NAM as the best representation of the Norwegian economy.

8 "The Golden Interest Rule"

After having analyzed the properties of tens after tens of simple interest rate rules and their performance in the five different models, I started getting a picture of what a robust rule for the Norwegian economy would look like, given that the set of models I use cover a broad range of possible and realistic descriptions of the true economy. Some of the rules appear to do much better in all of the models than the rest. These rules are naturally those that have been optimized over the three reference models and not a single model. Due to the highly dispersed monetary policy preferences of the models, not all of the Bayesian rules do well on average. Some are actually quite poor, in particular the rules that only react to consumer prices and GDP. The rules where the conflict between two models become particularly pronounced, as in three-parameter rules in CN versus NAM and two-parameter rules in the LGM model versus NAM, perform bad as well.

The Bayesian rules with either flat priors or four times higher probability on Credit NEMO outperform the other rules, and above all those responding to current inflation and output gap and the lagged interest rate. On average with respect to IIP, the best rule of all optimized rules is B3oC, not surprisingly, since NEMO and PN are similar to CN and thus experience little deterioration of monetary policy using this rule. Evaluating percentage increase in loss instead of IIP, however, renders the Bayesian rule B3oN superior to any of the

rules other than those with flat priors. This rule performs so well relative to B3osr because the improvement in NAM is almost enough to outweigh the deterioration in the LGM model.

The rules found with the smallest relative weight assigned NAM, i.e. with absolute losses in the Bayesian objective function or a higher probability on one of the other two reference models, or both, yield the lowest average IIP (see Appendix B.3, Table 18). But considering NAM as a likely model, it would not be wise to use these rules for robustness purposes. They yield almost half a percentage point higher inflation variability compared to using the first-best three-parameter rule in NAM, which already gives an excess loss of 26% compared to optimal policy.

In order to find the parameter values in the "Golden Interest Rule" (GIR), I investigated the best performing rules described above and their IIP and fault tolerance properties. I first looked at the fault tolerance of a large number of rules in order to see the potential intervals within the coefficients in GIR would be. In all the graphs it seemed that 0.6 on the lagged interest rate never yields disastrous outcomes in NAM, or any other model, but larger values could do in some cases. More interest rate smoothing would be preferred by the other models (except PN), but increasing beyond 0.6 is not a good idea if the actual economy behaves exactly as NAM.

For the inflation coefficient I limited the search to three-parameter rules, as this is the size of the most robust rules. The coefficients cover a huge range, from a scarce fulfillment of the Taylor principle in the rules generated by NAM to immense sizes in CN and LGM for loss functions with less or no explicit gradualism preferences incorporated. Setting it equal to 1.5 as in the original Taylor rule, appears to be an insufficient response in order to achieve a desirable degree of economic stability. None of the fault tolerance graphs display significant worsening in any of the models for inflation responses in the neighborhood of 2-3, and increasing it to this level improves the average performance more than it deteriorates it in NAM. A larger value should indeed be used if the central bank preferences for an even interest rate path is lower, as in \mathcal{L}_3 , or stabilization of the output gap is higher, as in \mathcal{L}_2 , as these preferences give more aggressive rules than for \mathcal{L}_1 in all five models. Taking into account that Norges Bank recently increased the concern for closing the output gap, a more aggressive rule would definitely be appropriate.

The response to output gap variations is by all means the coefficient that varies the most across the rules. As previously discussed, the LGM model achieves notably better outcomes for large reactions and NAM for small. Depending on the belief in NAM, the coefficient should be raised. CN parts from the other models with incredibly big adjustments of the nominal interest rate in response to macroeconomic fluctuations, and these rules would not be recommendable in view of robustness. Landing on a coefficient above 1, but below 2,

appears to be best on average across the different models.

For almost all of the optimal simple rules the coefficients are systematically larger than those in the original Taylor rule, for all sizes and specifications of central bank preferences, except the two-parameter rule in NEMO and the first-best rules in NAM, which are actually not too far from it. Since it is a common result in the literature that robust simple rules are more aggressive than the aforementioned rule (Rudebusch and Svensson 1998), it seems obvious that the rule(s) used for robustness checks of optimal policy by Norges Bank should have somewhat larger coefficients than today.

The optimized three-parameter rules that on average perform the best, have in common that the interest rate smoothing parameter is roughly 0.6, inflation response between 2.3 and 3.5 and output response around 1.5. Based on this together with the above evaluation of IIP and fault tolerance, I investigated several rules (see Appendix B.2, Table 15 for a limited selection of the "robust candidate rules") with parameter values within these intervals. In order to make the rule comprehensive and "saleable" to the policy makers, the coefficients are rounded to the nearest decimal.

The one rule that on average across the four main models and the three reference models yields the smallest IIP is as follows:

$$r_t = 0.6r_{t-1} + (1 - 0.6)(3\pi_t + 1.5y_t). \quad (\text{"The Golden Interest Rule"})$$

The rule is as envisioned more aggressive than the original Taylor rule. The large coefficients in this "golden interest rate rule" is naturally driven by the three versions of NEMO and the LGM model who all yield smaller losses for strong responses to macroeconomic fluctuations than NAM. It is however also the rule among all the rules considered in this thesis that yields the smallest IIP in NAM relative to NAM's first-best policy rule of the same form, except from the Taylor rule and the Bayesian rules where NAM is attached four times the probability of the other models. It yields 30 percent higher loss than the first-best Rule 3 in NAM, which is significantly less than any other simple rule. It hence performs relatively OK in the purely backward-looking model, in addition to being "taylored" well to the other four models, which exhibit various degrees of backward- and forward-lookingness.

GIR yields on average an increase in inflation variability relative to the first-best rule of each model of 0.339 pp (see Appendix B.3, Table 18), which is not an insuperable price to pay for the large insurance against model uncertainty it supplies.

Figure 4 shows that GIR is indeed fault tolerant, roughly as much as B3osr. The coefficients appear to be close to their optimal value in each model. I have tried with several perturbations (of one decimal at a time) of the rule, but the resulting average IIP rises in all candidate rules, although somewhat less across the three reference models than across

all five. It looks like the coefficient on inflation should be slightly reduced in order to improve the average outcome, but the decreased loss in NAM and LGM from reducing the inflation response to 2.8 (candidate rule 2 in Appendix B.2, Table 15), or lower, is more than outweighed by the increase in CN — even more so if the increase in NEMO and Policy NEMO is included as well. The same is however not the case for an increase in the output gap coefficient of 0.1 to 1.6 (candidate rule 5 in Appendix B.2, Table 15). Since a larger output response generally improves the performance of all simple rules in the LGM model, the decrease in loss is actually just enough to outperform GIR in the three reference models. Across the four main models however, GIR is still the best.

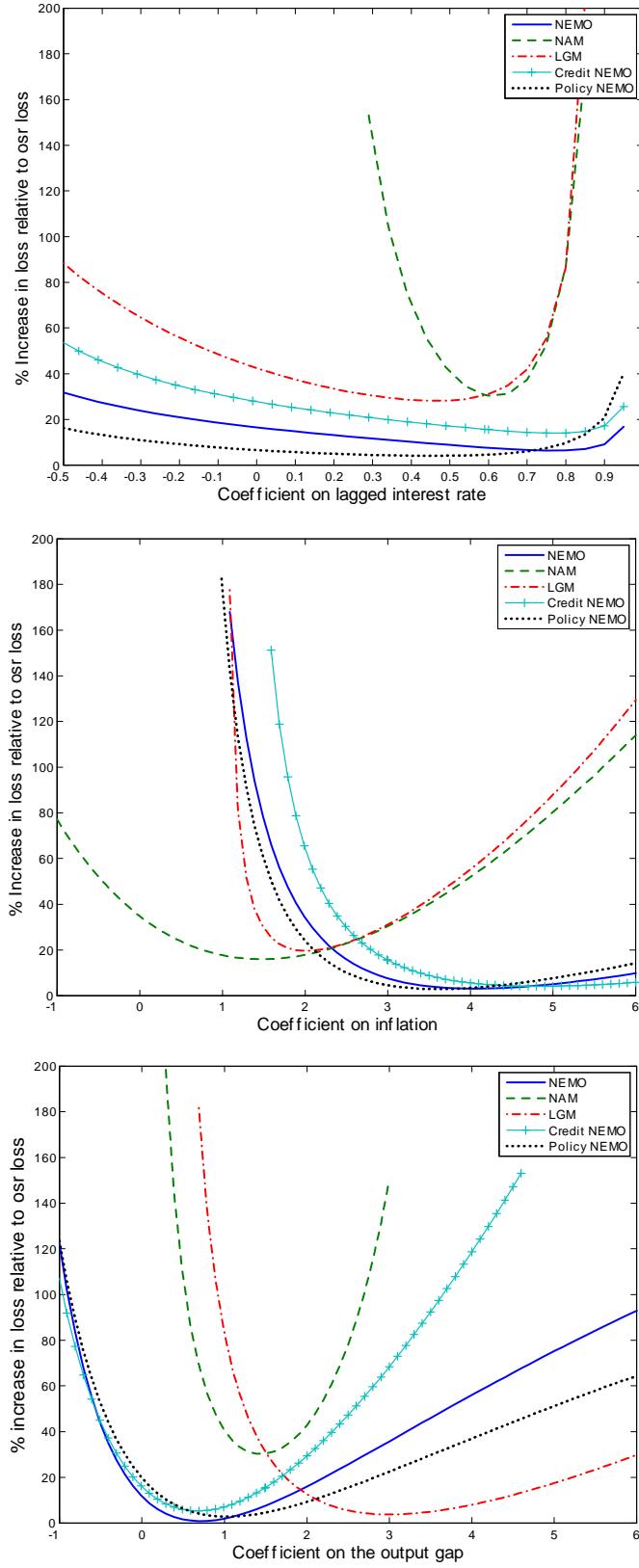


Figure 4: Fault tolerance of "The Golden Rule".

9 Concluding remarks

Uncertainty in various forms will always be an issue in monetary policy. The "true" effects of monetary policy and how the mechanisms in the actual economy function can never be precisely identified due to the impossibility of conducting macroeconomic experiments. The best solution is to conduct experiments in the closed environment of a model. But this just adds to the general uncertainty, since a model is a highly stylized and simplified representation of the economy, and will always be mis-specified in some way or another. Which model is best suited for "experimenting" with monetary policy will therefore remain unknown. But policy makers can use the knowledge obtained from research within the field to assess which of the available models are most propiarte. Because all models have strengths and weaknesses, using several different models may be a sensible approach.

Due to uncertainty, the members of a monetary policy committee may disagree about the specification of the (main) model used by the central bank. The model depends at all times on the staff doing the model revisions and on which research is given priority by the central bank. This could be accommodated by conducting robust monetary policy, either by using extra-model information, cross-checking optimal policy with a robust simple interest rate rule, or using a modified loss function of the form proposed by Ilbas et al. (2012). In both latter cases, the rule referred to as "The Golden Interest Rule" could be a good candidate for Norges Bank as a replacement of the Taylor rule, which in any case is suboptimal.

The main findings in my thesis is that rules optimized in a specific model do not perform well in other models, and are hence little robust. Bayesian rules where an average of the three reference models NAM, CN and the LGM model is optimized improves robustness significantly, as expected.

I find that rules with three variables are more robust than those with four, as they utilize less model-specific information. They are also more robust than two-parameter rules where interest rate smoothing is removed. Gradualism is preferable in interest rate setting due to uncertainty about the true structure of the economy, and monetary policy is conducted in a cautious manner with three-parameter rules.

Among the Bayesian rules, those with four times higher probability attached to CN naturally performs best on average across all models, since CN has similar structure and transmission mechanisms as NEMO and Policy NEMO. Apart from these rules, the three-parameter Bayesian rule based on absolute losses do quite well, because the weight attached to NAM is tuned down. However, all of the Bayesian rules lead to half a percentage point or more increase in the inflation variability compared to the first-best rules in NAM. If the rationale for using a robust rule is to avoid large movements in inflation in the face of model

uncertainty, the Bayesian rules should not be used when the belief in NAM is strong. Rather, the "Golden Interest Rule" (GIR) serves the purpose of a robust simple rule of thumb for cross-checking optimal monetary policy in Norway. For all the rules I have considered in my work, GIR yields the lowest average implied inflation variability premium (IIP) across the three reference models. It is however outperformed on average of all five models by rules that reduces variability in the NEMO models, such as the CN-rules. The reduction in IIP in these models are, however, from already low levels, but the increase in NAM and the LGM model are at higher levels. The large potential increase in consumer price fluctuations could be avoided by robustifying monetary policy with GIR. The rule does well on average, in addition to significantly reduce the IIP in NAM compared to the other Bayesian and "robust candidate" rules.

The analysis done in this thesis is by all means not complete, and there are numerous extensions and further robustness checks that could have been done. It would be interesting to look at other variables in the simple rule, in particular key foreign variables, e.g. the exchange rate or foreign interest rates. The latter is included in one of the simple rules that Norges Bank uses for robustness checks, and would thus be interesting to investigate further. The exchange rate is relevant for a small open economy like the Norwegian, in particular because the exporting sector plays a key role in the wage determination. Norges Bank often takes the exchange rate channel of monetary policy explicitly into consideration in the interest rate decision. In this respect, allowing for a direct response of the nominal interest rate to movements in foreign variables in a simple rule used for robustness purposes would indeed be relevant.

There is no consensus in the literature whether exchange rate rules improve the performance of monetary policy and are more robust than other simple interest rate rules. Many analyses (e.g. Galí 2008 and Dennis, Leitemo, and Sørderstrøm 2006) find that responding to exchange rate movements in a simple rule gains little, because the variable is highly correlated with the interest rate itself and with variables already included in the rule, namely inflation and GDP. However, the exchange rate is a forward-looking variable, so including it may improve the outcome compared to a "contemporaneous rule" in models where it is advantageous to respond to future developments (Dennis 2000).

Other possible extensions of my work would be the inclusion of the unemployment rate and the wage inflation. Wage inflation is not a variable in the LGM model, so the robustness analysis could not be executed for a rule including this variable. Targeting a low and stable unemployment rate is desirable as it may possibly represent the welfare of the households in a better way than GDP, but unemployment is not a variable in any of the NEMO versions or LGM neither. Reestimating the models to include these two variables is beyond the scope

of this thesis. Due to time and space constraints, I had to limit my analysis to rules utilizing the lagged interest rate, inflation, and the current and lagged output gap, which are standard rules in the literature. My analysis confirms that such rules are robust also for the Norwegian economy, but not necessarily the rules tailored to one single model. Bayesian rules are more robust, and in particular the "Golden Interest Rule" seems to be a propriate robustness insurance for monetary policy in Norway.

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Appendix

A Models

A.1 NEMO

A.1.1 List of variables

y_t	output gap
c_t	total consumption
c_t^{sa}	consumption savers
c_t^{sp}	consumption spenders
a_t	final good
t_t	intermediate production
q_t	domestically produced intermediates used in production of the final good
m_t	imported intermediates used in production of the final good
l_t	labour hours
k_t	capital stock
u_t	utilization rate of capital
inv_t	capital investments
inv_t^{oil}	oil investments
g_t	government spending
π_t	CPI inflation
π_t^Q	inflation on intermediates
π_t^M	inflation on imported intermediates
π_t^W	nominal wage inflation
p_t^Q	real price on domestic intermediate goods
p_t^M	real price on imported intermediate goods
w_t	real wage
mc_t	real marginal costs
mrs_t	marginal rate of substitution between consumption and leisure
r_t	nominal interest rate
r_t^K	real return to capital
b_t	domestic holdings of foreign bonds
ω_t	elasticity of substitution between labour inputs
θ_t^H	elasticity of substitution between domestically produced intermediates
θ_t^F	elasticity of substitution between foreign intermediates used in domestic production
ν_t	share of domestic intermediates used in domestic production
s_t	real exchange rate

y_t^*	foreign output gap
π_t^{M*}	inflation on exported intermediates
p_t^{M*}	real price on exported intermediates
m_t^*	exported intermediates
θ_t^{F*}	elasticity of substitution between domestic intermediates in foreign production
mc_t^*	foreign real marginal costs
r_t^*	foreign nominal interest rate
ν_t^*	share of foreign intermediates used in foreign production
π_t^Z	shock to the growth rate of technology
z_t^L	temporary labour augmenting productivity shock
z_t^I	shock to investments
z_t^B	risk premium shock
z_t^U	shock to consumption preferences
z_t^M	shock to imports

A.1.2 Model

Final goods

$$a_t = \nu q_t + (1 - \nu)m_t \quad (7)$$

$$q_t = a_t - p_t^Q \quad (8)$$

$$m_t = a_t - p_t^M \quad (9)$$

$$m_t^* = y_t^* - \mu^*[p_t^{M*} + f_{51}(p_t^M - s_t)] - \frac{\nu^*}{1 - \nu^*}\nu_t^* \quad (10)$$

Intermediate goods

Production

$$t_t = f_{61}(l_t + z_t^L) + f_{62}[u_t + k_{t-1} - \pi_t^Z] \quad (11)$$

$$k_t = \frac{inv}{k} inv_t + f_{81}(k_{t-1} - \pi_t^Z) \quad (12)$$

$$r_t^K = mc_t + \frac{1}{\xi}[t_t - (u_t + k_{t-1} - \pi_t^Z)] \quad (13)$$

$$mc_t = w_t + \frac{1}{\xi}(l_t - t_t) + \frac{1 - \xi}{\xi} z_t^L \quad (14)$$

$$inv_t + \pi_t^Z - k_{t-1} = f_{111}(inv_{t-1} + \pi_{t-1}^Z - k_{t-2}) + f_{112}E_t \{ inv_{t+1} + \pi_{t+1}^Z - k_t \} \quad (15)$$

$$- f_{113}E_t \{ (r_t - \pi_{t+1}) - f_{114}r_{t+1}^K \} - f_{115}E_t \{ \beta z_{t+1}^I - z_t^I \} \quad (16)$$

$$\phi^U u_t = r_t^K$$

Domestic prices

$$\pi_t^Q = \frac{\beta}{(1 + \beta)} E_t \pi_{t+1}^Q + \frac{1}{(1 + \beta)} \pi_{t-1}^Q + f_{131}(mc_t - p_t^Q) - f_{132}\theta_t^H \quad (17)$$

$$\pi_t^Q = \pi_t + p_t^Q - p_{t-1}^Q \quad (18)$$

Export prices

$$\pi_t^{M*} = \frac{\beta}{1 + \beta} E_t \pi_{t+1}^{M*} + \frac{1}{1 + \beta} \pi_{t-1}^{M*} + f_{151}(mc_t - p_t^{M*} - s_t) - f_{152}\theta_t^{F*} \quad (19)$$

$$\pi_t^{M*} = \pi_t^* + p_t^{M*} - p_{t-1}^{M*} \quad (20)$$

Import prices

$$\pi_t^M = \frac{\beta}{1 + \beta} E_t \pi_{t+1}^M + \frac{1}{1 + \beta} \pi_{t-1}^M + f_{171}(mc_t^* - p_t^M + s_t) - f_{172}\theta_t^F \quad (21)$$

$$\pi_t^M = \pi_t + p_t^M - p_{t-1}^M \quad (22)$$

Households

Forward-looking optimizers (savers)

$$c_t^{sa} = f_{191}E_t c_{t+1}^{sa} + f_{192}c_{t-1}^{sa} - f_{193}E_t \{ r_t - \pi_{t+1} \} - f_{194}\pi_t^Z + f_{195}z_t^U \quad (23)$$

$$s_t = f_{201}E_t s_{t+1} - E_t \{ r_t - \pi_{t+1} \} + E_t \{ r_t^* - \pi_{t+1}^* \} + z_t^B \quad (24)$$

$$mrs_t = \zeta l_t + f_{211}(\pi_t^Z c_t^{sa} - b^c c_{t-1}^{sa}) + f_{212}\pi_t^Z \quad (25)$$

Liquidity constrained rule of thumb consumers (spenders)

$$c_t^{sp} = w_t + l_t \quad (26)$$

Wage-setting

$$\pi_t^W = \frac{\beta}{1+\beta} E_t \pi_{t+1}^W + \frac{1}{1+\beta} \pi_{t-1}^W - f_{231}(w_t - mrs_t) - f_{232} \omega_t \quad (27)$$

$$\pi_t^W = \pi_t + w_t - w_{t-1} + \pi_t^Z \quad (28)$$

Market clearing

$$a_t = \frac{c}{a} c_t + \frac{inv}{a} inv_t + \frac{inv^{oil}}{a} inv_t^{oil} + \frac{g}{a} g_t \quad (29)$$

$$t_t = \frac{q}{t} q_t + f_{261} \frac{m^*}{t} m_t^* \quad (30)$$

$$c_t = slc \frac{c^{sp}}{c} c_t^{sp} + (1 - slc) \frac{c^{sa}}{c} c_t^{sa} \quad (31)$$

$$y_t = \frac{a}{y} a_t + \frac{x}{y} m_t^* - \frac{x}{y} [m_t + f_{291} m_t^* + z_t^M] \quad (32)$$

Foreign block

$$y_t^* = \lambda^{y^*} y_{t-1}^* + \varepsilon_t^{y^*} \quad (33)$$

$$\pi_t^* = \lambda^{\pi^*} \pi_{t-1}^* + \varepsilon_t^{\pi^*} \quad (34)$$

$$r_t^* = \lambda^{r^*} r_{t-1}^* + \varepsilon_t^{r^*} \quad (35)$$

$$mc_t^* = \lambda^{mc^*} mc_{t-1}^* + \varepsilon_t^{mc^*} \quad (36)$$

$$\nu_t^* = \lambda^{\nu^*} \nu_{t-1}^* + \varepsilon_t^{\nu^*} \quad (37)$$

Domestic shock processes

$$g_t = \lambda^G g_{t-1} + \varepsilon_t^G \quad (38)$$

$$inv_t^{oil} = \lambda^{oil} inv_{t-1}^{oil} + \varepsilon_t^{oil} \quad (39)$$

$$\pi_t^Z = \lambda^{\pi^Z} \pi_{t-1}^Z + \varepsilon_t^{\pi^Z} \quad (40)$$

$$z_t^U = \lambda^U z_{t-1}^U + \varepsilon_t^U \quad (41)$$

$$z_t^{inv} = \lambda^{inv} z_{t-1}^{inv} + \varepsilon_t^{inv} \quad (42)$$

$$z_t^M = \lambda^M z_{t-1}^M + \varepsilon_t^M \quad (43)$$

$$z_t^L = \lambda^L z_{t-1}^L + \varepsilon_t^L \quad (44)$$

$$\psi_t = \lambda^\psi \psi_{t-1} + \varepsilon_t^\psi \quad (45)$$

$$z_t^B = \lambda^B z_{t-1}^B + \varepsilon_t^B \quad (46)$$

$$\theta_t^H = \lambda^{\theta^H} \theta_{t-1}^H + \varepsilon_t^{\theta^H} \quad (47)$$

A.2 LGM

A.2.1 List of variables

π_t^H	quarterly annualized domestic goods inflation (GDP deflator)
π_t^F	quarterly annualized imported goods inflation
π_t	quarterly annualized CPI inflation
y_t	output gap
ψ_t^F	law-of-one-price gap
s_t	terms of trade
r_t	quarterly annualized nominal interest rate
$r_{q,t}$	quarterly nominal interest rate
q_t	quarterly real exchange rate
$r_{q,t}^*$	quarterly foreign nominal interest rate
$\pi_{q,t}^*$	quarterly foreign inflation rate
y_t^*	foreign output gap

A.2.2 Model

$$\pi_t^H = \mu_H E_{t-1} \pi_{t+1}^H + (1 - \mu_H) \sum_{j=1}^4 \alpha_j \pi_{t-j}^H + \kappa_x E_{t-1} x_t \quad (48)$$

$$+ \kappa_\psi E_{t-1} \psi_t^F + \epsilon_t \quad (49)$$

$$\pi_t^F = \mu_F E_{t-1} \pi_{t+1}^F + (1 - \mu_F) \sum_{j=1}^4 \chi_j \pi_{t-j}^F + \omega_\psi E_{t-1} \psi_t^F + v_t \quad (50)$$

$$y_t = \mu_x E_{t-1} y_{t+1} + (1 - \mu_x) [\eta y_{t-1} + (1 - \eta) y_{t-2}] - \chi(r_t - E_t \pi_{t+1}^H) + \varsigma E_{t-1} \Delta \psi_t^F + \phi E_{t-1} \Delta y_t^* + u_t \quad (51)$$

$$q_t = (1 - \alpha) E_t q_{t+1} + \alpha q_{t-1} - \beta(r_{q,t} - E_t \pi_{q,t+1}) + (r_{q,t}^* - E_t \pi_{q,t+1}^*) + \tau_t \quad (52)$$

$$\psi_t^F = q_t - (1 - \gamma) s_t \quad (53)$$

$$\Delta s_t \equiv \frac{1}{4} (\pi_t^F - \pi_t^H) \quad (54)$$

$$\pi_t = (1 - \gamma) \pi_t^H + \gamma \pi_t^F \quad (55)$$

$$\pi_{q,t}^* = \lambda^{\pi^*} \pi_{q,t-1}^* + \zeta_t^{\pi^*} \quad (56)$$

$$r_{q,t}^* = \lambda^{r^*} r_{q,t-1}^* + \zeta_t^{r^*} \quad (57)$$

$$y_t^* = \lambda^{y^*} y_{t-1}^* + \zeta_t^{y^*} \quad (58)$$

A.2.3 Estimation results

Series	Transformation	Model Variable
GDP mainland Norway per capita, seasonally adjusted	Log, HP-filtered with $\lambda = 16000$, demeaned	x
Quarterly core inflation (KPIJAE) Norway, seasonally adjusted	Annualized, demeaned	π^H
Quarterly core inflation imported goods, seasonally adjusted	Annualized, demeaned	π^F
3-month nominal money market interest rate Norway (NIBOR)	Annualized, demeaned	r
Trade weighted real exchange rate	Log, HP-filtered with $\lambda = 16000$, demeaned	q
World output gap, seasonally adjusted (from OECD)	Demeaned	y^*
3-month nominal money market interest rate main trading partners	Demeaned	r_q^*
Trade weighted quarterly inflation (KPI), seasonally adjusted	Demeaned	π_q^*

Table 10: List of data series used for estimation of LGM.

Parameter	Prior distribution	Prior mean	Prior st.dev.	Posterior mean	Posterior 95% confidence interval
μ_H	Normal	0.58	0.2	0.6299	0.5548 – 0.7051
α_o	Normal	−0.39	0.2	−0.4622	−0.5025 – −0.4135
α_1	Normal	0.22	0.2	0.0953	−0.0148 – 0.2255
α_2	Normal	0.72	0.2	0.7636	0.6829 – 0.8324
κ_x	Normal	0.28	0.1	0.1948	0.1608 – 0.2260
κ_ψ	Normal	0.04	0.02	0.0479	0.0415 – 0.0579
μ_f	Normal	0.78	0.2	0.6629	0.5962 – 0.7304
χ_0	Normal	1.11	0.2	1.0900	1.0619 – 1.1263
χ_1	Normal	0	0.2	0.0482	−0.0372 – 0.1403
χ_2	Normal	0	0.2	0.0419	−0.0443 – 0.1265
ω_{psi}	Normal	0.56	0.2	0.6198	0.5731 – 0.6617
μ_x	Normal	0.53	0.2	0.4037	0.3672 – 0.4459
η	Normal	1.36	0.2	1.1618	1.1117 – 1.1968
χ	Normal	0.07	0.02	0.0800	0.0718 – 0.0900
ς	Normal	0.11	0.05	0.0506	0.0372 – 0.0636
ϕ	Normal	0.25	0.1	0.2113	0.1783 – 0.2510
α	Beta	0.1	0.05	0.0740	0.0541 – 0.1024
β	Beta	0.9	0.05	0.9749	0.9614 – 0.9939
ρ	Beta	0.75	0.2	0.7055	0.6713 – 0.7350
ϕ_π	Normal	0.375	0.3	0.3848	0.3139 – 0.4568
ϕ_y	Normal	0.125	0.3	0.1988	0.1274 – 0.2556
σ_i	Inverse gamma	0.01	<i>inf</i>	0.0063	0.0054 – 0.0072
σ_{π_H}	Inverse gamma	0.02	<i>inf</i>	0.0132	0.0113 – 0.0150
σ_{π_F}	Inverse gamma	0.06	<i>inf</i>	0.0361	0.0302 – 0.0419
σ_x	Inverse gamma	0.004	<i>inf</i>	0.0078	0.0057 – 0.0101
σ_q	Inverse gamma	0.01	<i>inf</i>	0.0328	0.0280 – 0.0378
σ_{y*}	Inverse gamma	0.004	<i>inf</i>	0.0047	0.0040 – 0.0053
$\sigma_{\pi*}$	Inverse gamma	0.003	<i>inf</i>	0.0024	0.0021 – 0.0027
σ_{i*}	Inverse gamma	0.001	<i>inf</i>	0.0011	0.0010 – 0.0013

Table 11: Prior and posterior distributions for variables in the LGM model.

A.3 NAM

A.3.1 List of variables

All variables are for the Norwegian mainland economy

y_t	GDP output (total value added at market prices)
p_t	Consumer price index
v_t	Nominal exchange rate, trade-weighted
z_t	Productivity (value added per man hour)
$Trend_t$	Trend growth in productivity
u_t	Unemployment rate, registered
w_t	Nominal hourly wage costs
g_t	Government sector expenditure
$T1_t$	Payroll tax rate
po_t	Oil prices
pe_t	Electricity, fuel and lubricants prices in the CPI
l_t	Nominal credit volume
R_t	Money market interest rate (3 month euro-krone interest rate)
R_t^*	Foreign interest rate (ECU weighted effective interest rate on foreign bonds)
$R_{L,t}$	Average interest rate on bank loans
$R_{B,t}$	Yield on six year government bonds, quarterly average
$R_{B,t}^*$	Yield on long-term foreign bonds, NOK basket weighted
p_t^*	Consumer prices abroad, in foreign currency
pi_t	Price deflator of total imports
pi_t^*	Producer price index, trading partners

A.3.2 Model

In the simulations we keep oil prices, energy prices in CPI, government spending, taxes, productivity trend and all foreign variables but the nominal interest rate constant, and set constants equal to zero.

$$\begin{aligned}\Delta v_t = & -f_{11}\{(v_{t-1} + p_{t-1}^* - p_{t-1}) + f_{12}[(R_{t-1} - \pi_{t-1}) - (R_{t-1}^* - \pi_{t-1}^*)]\} \\ & -f_{13}\Delta(R_t - R_t^*) - f_{14}\Delta^2 po_{t-1} + \epsilon_t^v\end{aligned}\quad (59)$$

$$\begin{aligned}\Delta p i_t = & -f_{21}[(p i_{t-1} - v_{t-1} - p i_{t-1}^*) - f_{22}(p_{t-1} - v_{t-1} - p_{t-1}^*)] \\ & + f_{23}\Delta v_t + f_{25}\Delta p i_t^* + \epsilon_t^{pi}\end{aligned}\quad (60)$$

$$\begin{aligned}\Delta p_t = & -f_{31}[p_{t-3} - f_{32}(w_{t-1} - z_{t-1}) - f_{33}p i_{t-1}] - f_{34}\Delta z_t + f_{35}\Delta p_{t-2} \\ & + f_{36}\Delta p i_t + f_{37}\Delta p e_t + f_{38}\Delta y_{t-1} + \epsilon_t^p\end{aligned}\quad (61)$$

$$\begin{aligned}\Delta w_t = & -f_{41}[(w_{t-1} - p_{t-2} - z_{t-1}) + f_{42}u_{t-4}] + f_{43}\Delta p_t + f_{44}\Delta p_{t-1} \\ & - f_{45}(\Delta^2 u_{t-1} + \Delta u_{t-3}) + f_{46}\Delta T1_t + \epsilon_t^w\end{aligned}\quad (62)$$

$$\begin{aligned}\Delta z_t = & -f_{51}[z_{t-3} - f_{52}(w_{t-1} - p_{t-1}) - f_{53}Trend_t - f_{54}u_{t-2}] \\ & + f_{55}\Delta(w_t - p_t) - f_{56}\Delta^2 z_{t-1} + \epsilon_t^z\end{aligned}\quad (63)$$

$$\begin{aligned}\Delta u_t = & -f_{61}\{u_{t-1} - f_{62}\Delta(w_{t-2} - p_{t-2}) - f_{63}[(R_{L,t-2} - \pi_{t-2}) - 100\Delta_4 y_{t-2}]\} \\ & + f_{64}\Delta u_{t-1} - f_{65}\Delta u_{t-4} - f_{66}\Delta u_{t-5} + \epsilon_t^u\end{aligned}\quad (64)$$

$$\begin{aligned}\Delta y_t = & -f_{71}[y_{t-2} - f_{72}\Delta g_{t-1} - f_{73}(v_{t-1} + p_{t-1}^* - p_{t-1}) + f_{74}(R_{L,t-1} - \pi_{t-1})] \\ & - f_{75}\Delta y_{t-1} + f_{76}\Delta g_t + f_{77}\Delta(l_{t-1} - p_{t-1}) + \epsilon_t^y\end{aligned}\quad (65)$$

$$\begin{aligned}\Delta(l_t - p_t) = & -f_{81}[(l_{t-3} - p_{t-3}) - f_{82}y_{t-4} + f_{83}(R_{L,t-4} - R_{B,t-4})] \\ & + f_{84}\Delta^2 y_{t-2} + f_{85}\Delta^2(w_t - p_t) + \epsilon_t^{(l-p)}\end{aligned}\quad (66)$$

$$\Delta R_{L,t} = -f_{91}(R_{L,t-1} - f_{92}R_{B,t-1} - f_{93}R_{t-1}) + f_{94}\Delta R_t + \epsilon_t^{R,L}\quad (67)$$

$$\Delta R_{B,t} = -f_{101}(R_{B,t-1} - f_{102}R_{t-1} - f_{103}R_{B,t-1}^*) + f_{104}\Delta R_t + f_{105}\Delta R_t^* + \epsilon_t^{R,B}\quad (68)$$

$$\Delta x_t \equiv x_t - x_{t-1}\quad (69)$$

$$\pi_t \equiv \frac{p_t - p_{t-4}}{p_{t-4}}\quad (70)$$

B Results

B.1 Rules with alternative loss functions

Loss function	Coefficients	NEMO	NAM	LGM	CN
\mathcal{L}_2	ϕ^π	2.07	1.11	2.13	2.94
	ϕ^y	1.11	0.40	2.61	1.41
	$\% \Delta \mathcal{L}^*$	<i>12,5</i>	<i>45,4</i>	<i>41,8</i>	<i>30,3</i>
\mathcal{L}_3	ϕ^π	2.35	1.15	2.72	3.87
	ϕ^y	0.36	0.39	2.43	0.93
	$\% \Delta \mathcal{L}^*$	<i>9,0</i>	<i>34,5</i>	<i>35,1</i>	<i>21,6</i>

Table 12: Coefficients in the optimized two parameter rule with the alternative loss functions. Loss function 2 has a weight on output stabilization of 1.5, and Loss function 3 has a weight on interest rate smoothing of 0.1, but are in other aspects equal to the benchmark.

Loss function	Coefficients	NEMO	NAM	LGM	CN
\mathcal{L}_2	ϕ^r	1.05*	0.34	0.77	1.02*
	ϕ^π	0.64	1.14	4.24	1.04
	ϕ^y	0.82	0.63	7.05	0.86
	$\% \Delta \mathcal{L}^*$	<i>6,6</i>	<i>36,6</i>	<i>12,9</i>	<i>15,9</i>
\mathcal{L}_3	ϕ^r	0.66	0.33	0.79	0.95
	ϕ^π	3.55	1.20	5.99	23.94
	ϕ^y	0.95	0.62	6.90	9.55
	$\% \Delta \mathcal{L}^*$	<i>7,3</i>	<i>28,1</i>	<i>12,6</i>	<i>13,2</i>

Table 13: Coefficients in the optimized three-parameter rules with the alternative loss functions. Loss function 2 has a weight on output stabilization of 1.5, and Loss function 3 has a weight on interest rate smoothing of 0.1, but are in other aspects equal to the benchmark. Rules marked with an asterisk are displayed with short run (net) coefficients due to super inertia.

Loss function	Coefficients	NEMO	NAM	LGM	CN
\mathcal{L}_2	ϕ^r	1.08*	0.01	0.78	1.05*
	ϕ^π	0.64	1.14	4.33	0.80
	ϕ^y	1.41	0.37	7.57	1.31
	ϕ^{y-1}	-0.60	0.46	-0.40	-0.70
	$\% \Delta \mathcal{L}^*$	<i>6.2</i>	<i>16.7</i>	<i>12.8</i>	<i>14.8</i>
\mathcal{L}_3	ϕ^r	0.67	0.02	0.81	0.97
	ϕ^π	3.57	1.21	6.34	46.54
	ϕ^y	0.98	0.37	8.27	36.81
	ϕ^{y-1}	-0.02	0.45	-1.01	-19.27
	$\% \Delta \mathcal{L}^*$	<i>7.3</i>	<i>12.8</i>	<i>12.5</i>	<i>12.4</i>

Table 14: Coefficients in the optimized four-parameter rules with the alternative loss functions. Loss function 2 has a weight on output stabilization of 1.5, and Loss function 3 has a weight on interest rate smoothing of 0.1, but are in other aspects equal to the benchmark.

B.2 Alternative rules

Coefficients	Robust candidate rules (CR)							
	1= GIR	2	3	4	5	6	7	8
ϕ^r	0.6	0.6	0.6	0.6	0.6	0.7	0.7	0.7
ϕ^π	3.0	2.8	2.8	2.9	3.0	3.0	3.1	2.5
ϕ^y	1.5	1.5	1.6	1.6	1.6	1.6	1.6	1.5

Table 15: A limited selection of the different candidate rules for the best simple robust rule. Number 1, GIR, is the one referred to as "The Golden Interest Rule" in the main text.

Coefficients	Rule 2	Rule 3	Rule 4
ϕ^r		0.84	0.88
ϕ^π	2.80	8.04	10.17
ϕ^y	1.28	5.71	9.46
ϕ^{y-1}			-2.58
\mathcal{L}_t	1.126	1.135	1.026

Table 16: Bayesian rules optimized over Credit NEMO and the LGM model only, with equal weights and osr losses in the objective function.

Rule-generating model	<i>IIP</i> [% $\Delta\mathcal{L}^{osr}$]			
Rule 3b	NEMO	NAM	LGM	CN
NEMO	0[3.82]	0.79	2.09	0.17
NAM	2.96	0[-10.24]	1.40	3.96
LGM	1.34	0.88	0[19.88]	1.20
CN	0.23	0.79	0.85	0[8.99]
Rule 3c				
NEMO	0[2.64]	0.79	2.09	0.17
NAM	0.05	0[4.01]	1.40	3.96
LGM	1.34	0.88	0[25.03]	1.20
CN	0.23	0.79	0.85	0[9.04]

Table 17: IIP for the two alternative three-parameter rules, Rule 3b with inflation, current and lagged output as variables, and Rule 3c with current and lagged inflation and current output. In parentheses are the relative losses to the loss generated by the standard Rule 3.

B.3 IIP results

Rule	Average IIP of			IIP in			
	3 models	4 models	5 models	NEMO	NAM	LGM	CN
GIR	0.339	0.322	0.294	0.270	0.360	0.354	0.302
CR 5	0.338	0.334	0.309	0.321	0.370	0.300	0.343
CR 7	0.375	0.343	0.321	0.248	0.428	0.409	0.286
B3osr	0.342	0.360	0.347	0.414	0.348	0.254	0.424
B3abs	0.337	0.232	0.300	0.282	0.452	0.258	0.302
B3oC	0.353	0.307	0.274	0.168	0.483	0.382	0.194
B3oL	0.406	0.467	0.470	0.650	0.519	0.070	0.628

Table 18: Average IIP of all models, the four main models and the three reference models for a selection of the best performing rules.

B.4 Fault tolerance

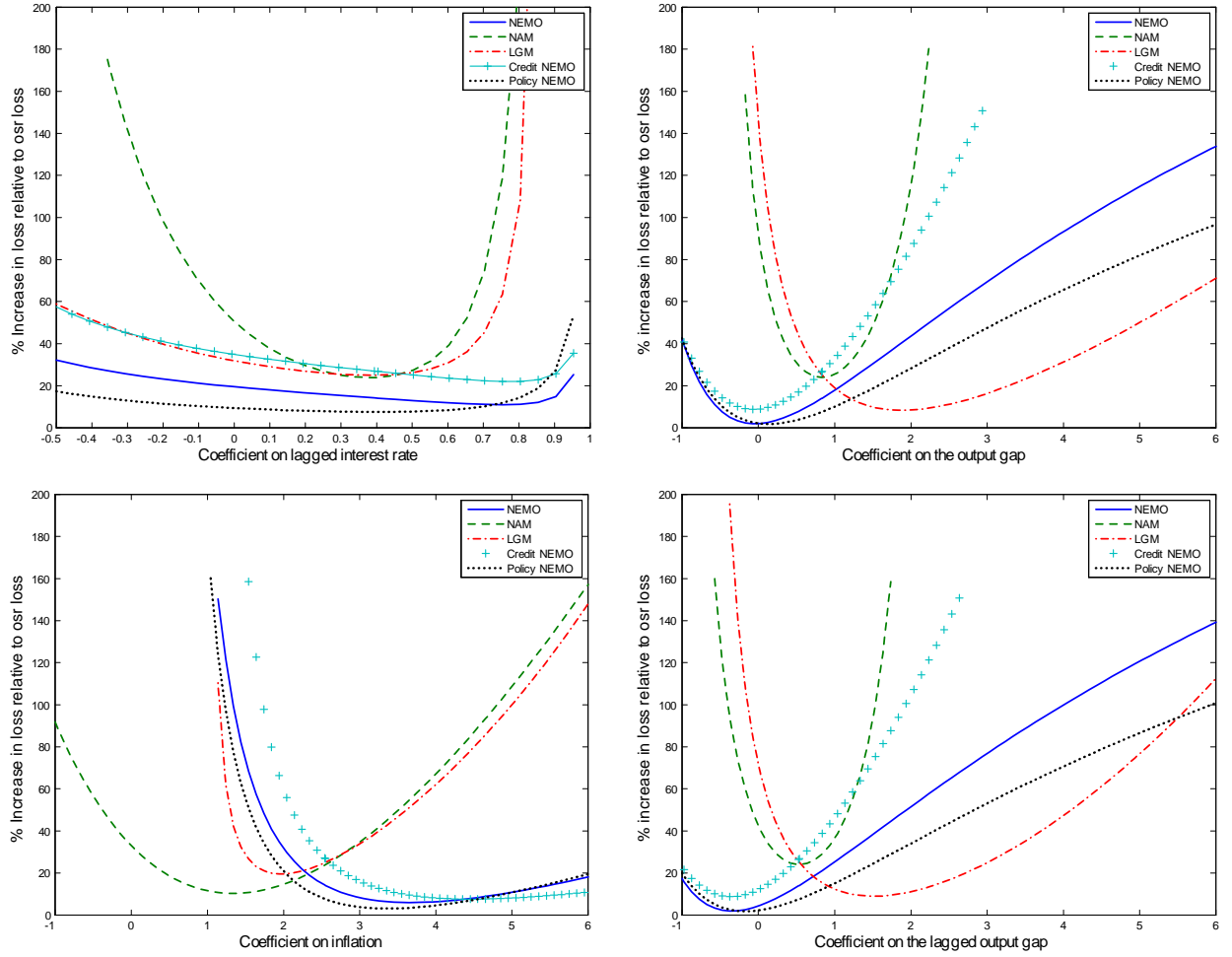


Figure 5: Fault tolerance of B4osr, the Bayesian four-parameter rule with flat prior and osr losses in the objective function (relative increase in loss to the first-best four-parameter rule in each model).