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When Preferences for a Stable Interest Rate Become Self-Defeating

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WORKING PAPER

When preferences for a stable interest rate become self-defeating

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When Preferences for a Stable Interest Rate Become Self-Defeating*

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Abstract

Monetary policy makers often seem to have preferences for a stable interest rate, in addition to stable inflation and output. In this paper we investigate the implications of having an interest rate level term in the loss function when the policymaker lacks commitment technology. We show that preferences for interest rate stability may lead to equilibrium indeterminacy. But even when determinacy is achieved, such preferences can become self-defeating, in the meaning of generating a less stable interest rate than in the case without preferences for interest rate stability. Aiming to stabilize the real interest rate instead of the nominal rate is more robust, as it always gives determinacy and also tends to give a more stable nominal interest rate than when the policymaker aims to stabilize the nominal rate.

Keywords: Monetary policy, Discretion, Interest rate stability

JEL codes: E52, E58.

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1 Introduction

Monetary policymakers' preferences are commonly modeled in terms of a quadratic loss function that penalizes deviations of inflation from the target and deviations of output from its potential. A welfare theoretical foundation for such policy preferences is provided by Woodford (2003a) within a simple New Keynesian framework. In addition to stability in inflation and the output gap, monetary policymakers seem to have preferences for a stable interest rate, either in terms of interest rate smoothing or as stability around a certain level. In this paper we explore how preferences for a stable *interest rate level* might be self-defeating, in the meaning of leading to a less stable interest rate than if the policymaker did not have such preferences. Thus, we consider a loss function that includes the term $(i_t - i^*)^2$, where i_t is the short-term nominal interest rate and i^* is the desired level.

It is not clear that stability in the interest rate level should be a separate objective of monetary policy. However, there are both theoretical arguments for, and practical evidence of, such policy preferences. Woodford (2003a) motivates the term $(i_t - i^*)^2$ in the loss function by Friedman's (1969) argument that high interest rates imply a welfare cost associated with transactions. If this deadweight loss is a convex function of the distortion, it implies that it is desirable to reduce not only the level but also the variability of the interest rate. In addition, the risk of hitting the zero lower bound (ZLB) and end up in a liquidity trap increases with the variance of the interest rate.¹ The ZLB constraint implies that the desired level i^* should be positive, although the transaction cost argument implies an optimal nominal rate of zero. Balancing these arguments implies a slightly positive i^* .² An interest rate level term in the loss function also has empirical support. Ilbas (2011) estimates the preferences of the US Federal Reserve based on the Smets and Wouters (2007) model by imposing a loss function rather than a simple rule, and she finds a significant weight on the interest rate level term in the loss function. Moreover, based on his first-hand experience as a member of the Riksbank's Executive Board, Lars Svensson (2011) provides evidence for such policy preferences. Referring to his peer Board members' arguments for the monetary policy tightening in June/July 2010, Svensson uses the term "the normalization argument", which says that the risk of financial instability is higher if the interest rate is far from a "normal level". Svensson (2011, p.18) writes that "*such arguments imply that, for given forecasts of inflation and resource utilization, more normal interest rate levels are preferred. It is like having an additional term $(i_t - i^*)^2$ in*

¹The ZLB-constraint argument for the term $\gamma(i_t - i^*)^2$ is also applied in Levine, MacAdam and Pearlman (2008).

²Representing the monetary policy objective by a loss function with an interest rate level term is not uncommon in the monetary policy literature. Examples include Giannoni (2014), Debortoli, Kim, Linde and Nunes (2015) and Taylor and Williams (2010).

the loss function[...]"'. Thus, even if Svensson himself is critical to such arguments, preferences for interest rate stability appear to have been present among Board members at the Riksbank. A policy preference for avoiding an interest rate that is far from its normal level can also find support in the "Austrian school", which argues that this may result in misalignments in the capital stock.

Further evidence of such preferences can be obtained by considering Norges Bank (the central bank of Norway) which in their Monetary Policy Reports has published the loss function used for deriving their economic forecasts, including the forecast for the policy rate. In Norges Bank's loss function, the interest rate level term entered, and it was motivated by the same line of reasoning as the "normalization argument" referred to by Lars Svensson.³

The ZLB argument for having the term $(i_t - i^*)^2$ in the loss function has become more relevant in recent years, as many central banks have been constrained by the ZLB. Can the ZLB be avoided simply by cutting rates more moderately? This question fostered some debate in the media when the former president of the European Central bank, Claude Trichet, in January 2009 indicated that the interest rate would not be reduced further from its 2 percent level at the next meeting, and at the same time emphasized the importance of avoiding a liquidity trap. The confusion about Trichet's remarks is illustrated by the following citation from *The Economist*⁴: "*Some ECB rate-setters seem to suggest that a liquidity trap can be avoided simply by not reducing interest rates. Many economists fear the opposite: if policy is kept too tight and deflation takes hold it will become harder to induce spending by cutting rates.*" As we show in this paper, the answer to the question depends on whether the central bank is able to commit, and on how persistent the shocks that require a low interest rate are. If the central bank is not able to commit, and the shocks hitting the economy are sufficiently persistent, the fear of hitting the ZLB may become self-fulfilling.⁵

In this paper we do not discuss whether the true welfare loss function should include the term $(i_t - i^*)^2$ or not. Rather, we analyze the implications - and potential pitfalls - of having preferences for interest rate stability. The potential pitfalls are related to the time-inconsistency problem in monetary policy. Specifically, we show that a preference for interest rate stability may in fact lead to the opposite outcome if the monetary policymaker is not able

³See Evjen and Kloster (2012) for a description of the loss function used by Norges Bank and a motivation for the terms entering it.

⁴*The Economist*, 29 January, 2009.

⁵A branch of the literature about the ZLB considers a multiplicity problem related to non-linearity of policy implied by the ZLB, see e.g. Benhabib, Schmitt-Grohe and Uribe (2001), Alstadheim and Henderson (2006) and Armenter (2013). In this paper, in order to focus solely on the implications of preferences for a stable interest rate, we do not impose the ZLB restriction. However, we conjecture that the problem of self-defeating preferences will be more severe with the ZLB.

to commit and therefore follows a time-consistent policy.

What is driving this result is the persistence in the shocks. In order to help intuition, we may consider a negative demand shock. Since the central bank is not willing to fully neutralize the demand shock because of the interest rate stability objective, the output gap becomes negative, which gives a fall in inflation. If the demand shock is short-lived, the effect on expected future inflation is small, and the interest rate does not need to be lowered much further. However, if the shock is persistent, the output gap is expected to remain negative for a considerable period of time, and inflation expectations will fall much more. To counteract the effect of lower inflation expectations on the real interest rate, the central bank must decrease the nominal rate further. If the shock is sufficiently persistent, the reluctance to lower the interest rate sufficiently forces the central bank to reduce the rate more than what would be needed in the first place to neutralize the demand shock. A preference for interest rate stability could therefore be self-defeating. If the central bank is able to commit, it will (credibly) promise to conduct an expansionary policy in the future, and this prevents a fall in inflation expectations and the need to lower the interest rate to offset this. Thus, under commitment a preference for interest rate stability always results in a more stable interest rate.

It is well known that if the central bank fails to respond sufficiently to inflation, the rational expectations equilibrium can become indeterminate, which may lead to self-fulfilling destabilizing expectations ("sun spots"). If the policymaker has a preference for a stable interest rate, she might respond insufficiently to inflationary pressures. Obviously, this is a potential pitfall of having such preferences. In this paper we do also consider the indeterminacy issue, but the focus is on the properties of the *determinate* equilibrium. Thus, we shall show that even if there is a unique stable rational expectations equilibrium, preferences for interest rate stability can be self-defeating.

It is also well known since the seminal work by Kydland and Prescott (1977) that the inability of policymakers to make credible commitments can give bad outcomes compared to the outcomes under commitment. Our result that a discretionary policy may produce an undesirable outcome is therefore neither new nor surprising. However, what is not well known is the particular bias introduced by the interest rate term in the loss function. Typically, under optimal policy - with or without commitment - increasing the weight on one target variable in the loss function tends to make that particular variable more stable, at the cost of higher variability in one or more of the other variables in the loss function. Under discretion, the cost in terms of higher variability in other variables can be so high that the policymaker would have been better off by having a smaller weight than his true preferences.⁶ When the policymaker has preferences for a stable

⁶This gives, for example, a case for delegating policy to a policymaker with less weight

interest rate, increasing the weight on interest rate stability may actually *increase* the variability of the interest rate *itself*.

This feature of discretionary policy seems to have gone unrecognized in the literature, with one notable exception: Woodford (1999) identified this property of discretionary policy in a working paper, which to our knowledge is the only reference to it in the literature. Woodford referred to it as a "discretion trap", and showed within a simple model that it could give an extreme excess welfare loss. The potential implications of such preferences stand in sharp contrast to Woodford's results on preferences for interest rate *smoothing*. Given the potentially disastrous effects of preferences for a stable interest rate *level*, the lack of attention in the literature is somewhat surprising. Possibly, the reason is that the "discretion trap" was regarded as a special case which applies under extreme or unrealistic parameter values only. However, this is an empirical question, and in this paper we shall therefore investigate the empirical relevance of the "discretion trap" by considering the Smets and Wouters (2007) model of the US economy. We find that a preference for interest rate level stability increases the variance of the interest rate level (as well as the variance of output) under discretion. This suggests that the "discretion trap" is indeed a case that cannot be dismissed.⁷

The paper is organized as follows: In Section 2, we analyze the effects of an interest rate stability objective and provide analytical results that extend and generalize some of the results in Woodford (1999). In Section 3 a preference for stabilizing the real interest rate is discussed. Section 4 considers discretionary policy in the Smets and Wouters (2007) model and compares the results to the analytical results in Section 2. Section 5 concludes.

2 Theoretical framework

2.1 A simple analytical model

In order to derive analytical solutions, we first consider the following simple canonical New Keynesian model:

$$y_t = E_t y_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - r_t^*), \quad (1)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + u_t. \quad (2)$$

on the output gap, as shown by Rogoff (1985) and Clarida, Galí and Gertler (1999).

⁷Discretionary policy may also result in other bad equilibria. Blake and Kirsanova (2012) show that in many models, discretionary policy may result in multiple equilibria, where one cannot rule out that the economy will end up in a 'bad' equilibrium with high volatility - an "expectation trap". The "discretion trap" in our model is not related to the "expectation trap" in Blake and Kirsanova's model, since in our simple benchmark model, there is only one equilibrium.

y_t is the output gap, i_t is the one-period nominal interest rate, and π_t is the rate of inflation. The first equation is a linearization of the Euler equation representing optimal intertemporal consumption decisions, where σ is the inverse of the intertemporal rate of substitution. r_t^* is the neutral real interest rate, i.e. the real interest rate which would have prevailed in an economy without any nominal rigidities. Equation (2) is the New Keynesian Phillips curve, which can be derived from optimal price setting under monopolistic competition and price rigidity. $\kappa > 0$ depends on the deep parameters of the model, including the probability that the representative firm will change its output price. u_t is a 'mark-up shock' stemming from e.g., stochastic fluctuations in firms' market power. We assume that r_t^* and u_t follow AR(1)-processes:

$$\begin{aligned} r_t^* &= \rho_r r_{t-1}^* + \varepsilon_t^r, \\ u_t &= \rho_u u_{t-1} + \varepsilon_t^u, \end{aligned}$$

where $0 \leq \rho_j < 1$ and $\varepsilon_t^j \sim i.i.d.(0, \sigma_j^2)$, $j = r, u$. The central bank's loss function is

$$E_0(1 - \beta) \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda y_t^2 + \gamma i_t^2], \quad (3)$$

where the interest rate term is motivated in the introduction and γ is the relative weight on interest rate variability. In order to keep the analytical solutions simpler, we have, with no loss of generality, set the inflation target and the desired level of the interest rate to zero.⁸

Assuming the central bank lacks a commitment technology, the central bank sets the policy instrument i_t to minimize the loss function (3), treating private agents' expectations as exogenous.⁹ The first-order condition under discretion is:¹⁰

$$-\sigma^{-1} \kappa \pi_t - \sigma^{-1} \lambda y_t + \gamma i_t = 0. \quad (4)$$

⁸Alternatively, one may interpret π_t and i_t as deviations of inflation and the interest rate from their respective desired levels.

⁹With only inflation and the output gap in the loss function, one could interpret the real interest rate as the *de facto* policy instrument, i.e. that the central bank sets the nominal rate to achieve the level of the real rate that satisfies the first-order condition. This interpretation also applies to the case with the real interest rate in the loss function, as analyzed below.

¹⁰As discussed by e.g. Svensson (2010), section 4, the discretionary solution may be derived with a Lagrangian approach rather than with recursive methods in this simple case of a model without endogenous state variables. Also, as emphasized by e.g. Blake and Kirsanova (2012), section 2.1, the multiplicity problem associated with discretionary policy is not present in this simple case. Thus, below we discuss the traditional nominal indeterminacy problem associated with a too weak response to inflation deviations from target, rather than any particular indeterminacy problem associated with discretionary policy.

Before analyzing the solution, we first consider whether the equilibrium is determinate. The following Proposition summarizes the results. (All proofs are found in the Appendix).

Proposition 1 *The discretionary equilibrium when the central bank minimizes (3) is determinate if and only if*

$$\gamma < \frac{\lambda(1 - \beta) + \kappa^2}{\sigma\kappa}. \quad (5)$$

The intuition is that if the central bank finds it costly to adjust the interest rate, it will not respond sufficiently to inflationary pressure to satisfy the Taylor principle.¹¹ Equilibrium indeterminacy is considered a potentially serious problem, as self-fulfilling expectations ('sun spots') might lead to large and inefficient fluctuations in inflation and output. So far the empirical literature on determinacy has focused on whether the estimated coefficients on inflation and output in Taylor-type rules have been sufficiently large to satisfy the Taylor principle.¹² However, as pointed out by Jensen (2011), such estimated rules do not tell anything about determinacy if monetary policy is based on minimization of a loss function rather than commitment to an interest rate rule. Therefore, a topic for future empirical research on determinacy is to estimate the preferences (weights in the loss function) of the central bank, along with the model, to investigate whether condition (5) is satisfied.

A potential equilibrium indeterminacy resulting from preferences for interest rate stability is not the main topic of this paper, and in the following we assume that (5) is satisfied so that the equilibrium is determinate. This choice of attention does not reflect that we consider equilibrium indeterminacy unimportant. One problem with considering specific solutions under indeterminacy is that there is no consensus on which particular equilibrium within the infinite set of possible equilibria that is most relevant or most likely to be realized.¹³ Because of the arbitrariness of choosing particular solutions under indeterminacy, we focus on cases where determinacy prevails in this paper. The equilibrium is then characterized by the following proposition:

¹¹Interestingly, in a similar model, but with a ZLB restriction, Armenter (2013) finds that condition (5), but with the inequality turned, must be satisfied in order to rule out the "bad" equilibrium. See also the literature referenced in footnote 5. The other side of the coin is that avoiding the expectations trap introduces a different trap, namely potential sun-spot equilibria.

¹²The estimation of interest rate rule parameters is subject to substantial identification problems, see Leeper and Zha (2001) and Cochrane (2011).

¹³The 'minimal state variable' (MSV) solution suggested by McCallum (1983) is a common benchmark solution under indeterminacy. See McCallum (1999) for a comparison between the MSV solution and other suggested solutions in the literature. More recent suggested solutions include the 'continuity solution' and the 'orthogonality solution' by Lubik and Schorfheide (2003).

Proposition 2 *If (5) is satisfied, the unique discretionary solution is*

$$z_t = a_r^z r_t^* + a_u^z u_t, \quad z = \pi, y, i,$$

where

$$a_{r^*}^\pi = \frac{\gamma \kappa \sigma}{D_r} > 0, \quad (6)$$

$$a_u^\pi = \frac{\lambda + \gamma \sigma^2 (1 - \rho_u)}{D_u} > 0,$$

$$a_{r^*}^y = \frac{\gamma \sigma (1 - \beta \rho_r)}{D_r} > 0, \quad (7)$$

$$a_u^y = -\frac{\kappa - \sigma \rho_u \gamma}{D_u},$$

$$a_{r^*}^i = \frac{\lambda (1 - \beta \rho_r) + \kappa^2}{D_r} > 0, \quad (8)$$

$$a_u^i = \frac{\lambda \rho_u + \kappa \sigma (1 - \rho_u)}{D_u} > 0,$$

where

$$D_j = \lambda (1 - \beta \rho_j) + \kappa^2 + \sigma^2 \gamma ((1 - \rho_j)(1 - \beta \rho_j) - \kappa \sigma^{-1} \rho_j), \quad j = r, u.$$

The responses of the endogenous variables to the exogenous shocks then have "normal" signs. For example, the interest rate increases when the neutral real interest rate increases. The only ambiguous sign is the response in output to a cost-push shock. If the persistence of the shock is sufficiently large, a positive cost-push shock can have an expansionary effect on output. The reason is that when the shock is persistent, future expected inflation is high, which results in lower real interest rates.

Generally, one would expect that when the central bank has preferences for interest rate stability, it would use the interest rate less aggressively. In other words, one would expect that $\text{var}(i_t)$ would fall as γ increases. This is obviously the case under optimal policy with commitment. However, if the central bank lacks a commitment technology, $\text{var}(i_t)$ may in fact increase in γ . Thus, preferences for interest rate stability may be self-defeating. The following Corollary summarizes sufficient conditions for this to be the case.

Proposition 3 *If (5) is satisfied, sufficient conditions for $\text{var}(i_t)$ to be increasing in γ are*

$$\rho_j > \rho^*, \quad j = r^*, u,$$

where

$$\rho^* = \frac{1}{2\sigma\beta} \left(\kappa + \sigma(1 + \beta) - \sqrt{\kappa^2 + \sigma^2(1 - \beta)^2 + 2\kappa\sigma(1 + \beta)} \right). \quad (9)$$

The necessary conditions are more likely to be satisfied when shocks are highly persistent. Why does a preference for interest rate stability lead to a more variable interest rate if the persistence of the shocks is sufficiently high? In order to help intuition, we may consider a negative shock to the neutral real interest rate. Because of the cost of adjusting the interest rate, the central bank wants to respond less than one-for-one to a fall in the real interest rate. This leads to a positive real interest rate gap $\tilde{r}_t \equiv i_t - E_t \pi_{t+1} - r_t^*$ and thereby a negative output gap. If the persistence is small, so that the fall in r_t^* and thereby the rise in the real interest rate gap is expected to be short-lived, the negative effect on output and inflation is small, as can be seen from the forward solution of the IS curve;

$$y_t = -\sigma^{-1} E_t \sum_{j=0}^{\infty} \tilde{r}_{t+j}. \quad (10)$$

Since the fall in the output gap is small, inflation expectations do not fall much, and the need to lower the policy rate to offset the effect on the real interest rate of a fall in inflation expectations is small. If the persistence of the shock is high, however, the real interest rate gap is expected to remain positive for a considerable period of time, which from (10) leads to a larger fall in y_t and thereby a larger fall in inflation and inflation expectations. The central bank must then decrease the policy rate *more* than the fall in r_t^* to offset the increase in $\sum_{j=0}^{\infty} \tilde{r}_{t+j}$ due to a fall in $E_t \pi_{t+1}$. This is the case which Woodford (1999) refers to as the "discretion trap", and he shows that it can lead to a disastrous outcome, in terms of an extremely high expected welfare loss compared with the loss under commitment. Note that if the central bank has a commitment technology, it will avoid the large fall in inflation expectations and thereby the need for a very low policy rate today due to a credible promise to create a positive output gap in the future.

If both shocks have sufficiently small persistence so that the inequalities in (9) are turned around, $\text{var}(i_t)$ is decreasing in γ . However, if (9) is satisfied for only one of the shocks, the relationship between $\text{var}(i_t)$ and γ might become non-monotonic:

Corollary 4 *If $\rho_j > \rho^*$ for either $j = r$ or $j = u$, cases where $\text{var}(i_t)$ is a non-monotonic function of γ exist. In such cases, $\text{var}(i_t)$ is decreasing in γ for 'small' values of γ and increasing in γ for 'large' values of γ .*

2.2 A numerical illustration

Whether preferences for interest rate stability can be self-defeating depends on the values of the structural parameters $\rho_r, \rho_u, \kappa, \sigma$.¹⁴ To give a first indication of whether this could be a realistic case or only applies for unreasonable parameter values, we consider the estimated parameter values in Lubik and Schorfheide (2004), which are given in Table 1.

Table 1: Parameter values (post 1982) in Lubik and Schorfheide (2004)

	Mean	Conf Interval
σ	1.86	[1.04, 2.64]
κ	0.58	[0.27, 0.89]
ρ_r	0.83	[0.77, 0.93]
ρ_u	0.85	[0.77, 0.93]
β	0.99	[.991, .995]

Since the degree of persistence in the shocks plays a key role, we first consider how persistent the shocks must be in order to give rise to self-defeating preferences. Inserting the mean values for σ , κ and β from Table 1 into (9) gives $\rho^* = 0.58$. Note that the 90 percent confidence intervals for ρ_j for both shocks are above this critical value. Lower values of κ and higher values of σ make ρ^* larger. If we pick the lower limit on the 90 percent interval for κ and the higher limit for σ , i.e., $\kappa = 0.27$ and $\sigma = 2.64$, we find that $\rho^* = 0.73$. This value is still below the 90 percent confidence intervals for ρ_r and ρ_u . Thus, based on the estimated parameter values in Lubik and Schorfheide (2004), the sufficient conditions for self-defeating preferences (or the "discretion trap") are satisfied for all reasonable draws of parameter values.

Figure 1 shows how the standard deviations of i_t , π_t and y_t vary with the weight γ on interest rate stability within the region of determinacy. In this numerical example, the model is calibrated with the mean parameter values in Table 1 and the estimated standard deviations of shocks from Lubik and Schorfheide (2004) taking into account the estimated correlation between ε_t^r and ε_t^u .¹⁵ Moreover, we have assumed that $\lambda = 0.25$, which reflects Janet Yellen's (2012) parameterization of the 'dual mandate'.¹⁶ However, as seen

¹⁴ β has no key role, and we will in the following set $\beta = 0.99$, which is common in the literature (with a quarterly calibration).

¹⁵The estimated the correlation coefficient is $\rho_{ru} = 0.36$. The standard deviations of the white noise part of the shocks are $sd(\varepsilon_t^r) = \sigma \times 0.18 = 0.3348$ and $sd(\varepsilon_t^u) = \kappa \times 0.64 = 0.3712$.

¹⁶Yellen (2012) suggests a unit weight on the unemployment gap in the loss function and an Okun's law coefficient of approximately 2.0.

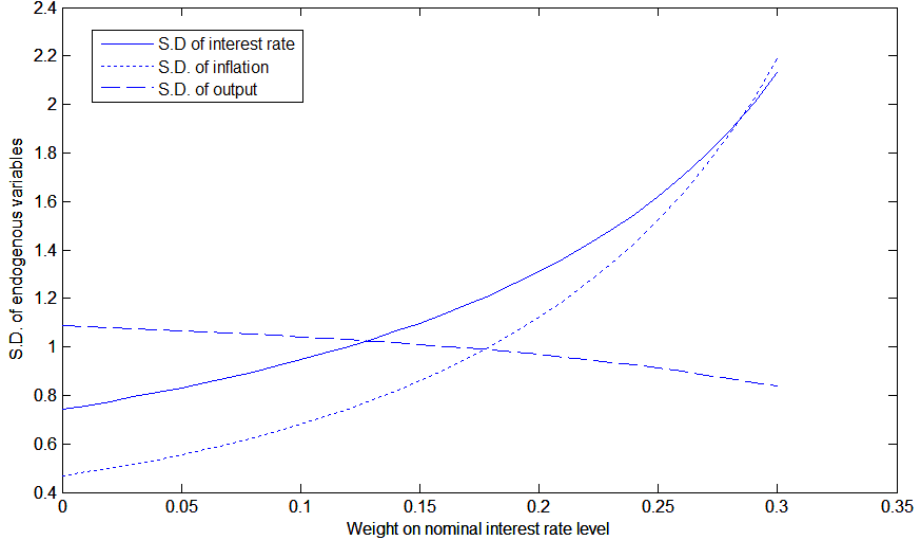


Figure 1: S.D. of endogenous variables with Dual Mandate loss function in small NK model. Increasing weight on nominal interest rate level.

from Proposition 3, whether a preference of interest rate stability is self-defeating or not is independent of λ . We see that the variability of the interest rate increases exponentially as γ becomes larger. The same is the case for the standard deviation of inflation. This is a general result when (9) is satisfied, as seen from (2).

2.3 Optimal delegation

A commonly suggested institutional response to the time-inconsistency problem is optimal delegation. This is the approach used in the seminal work by Rogoff (1985), who showed that the discretionary solution can be improved if the government delegates monetary policy to a (weight-) 'conservative' central banker. In principle, it is always possible to achieve a solution identical to the optimal solution under commitment if there are no restrictions on the "preferences" of potential central bankers. Through "reverse engineering", one may design a loss function such that the first-order conditions for optimal discretionary policy become identical to the first-order conditions for optimal policy under commitment.¹⁷ In the model considered here, such unrestricted (first-best) optimal delegation would imply that the loss function of an optimal central banker would include history-dependent target variables that track the law of motion of the Lagrange multipliers associated with the constraints (1) and (2) in the minimization problem. We shall

¹⁷See e.g. Røisland (2001).

not consider optimal delegation in this broad sense of adding or substituting other variables like the price-level¹⁸, nominal GDP¹⁹ or the change in the interest rate²⁰, but rather restrict the analysis to the case where the only preference-parameters that the principal can choose from are the weights λ and γ in the loss function (3). One advantage of this is that we can derive analytical results. Woodford (2003b) analyzed optimal delegation, but considered the case with an interest rate smoothing objective in addition to an interest rate level term, so that analytical solutions were not attainable except in a limiting special case. Since we only consider delegation through altering the weights in the loss function, the solution will lack the history-dependence which characterizes optimal policy under commitment. One may interpret this as improving the discretionary solution without altering the whole monetary policy regime. For simplicity, we shall follow Giannoni (2014) and focus on the case where $\rho_r = \rho_u = \rho$. The result on optimal delegation is summarized in the following proposition:

Proposition 5 *If the preferences of the government are represented by (3), monetary policy should be delegated to a central banker with the following weights in her loss function:*

$$\begin{aligned}\tilde{\lambda} &= \lambda(1 - \beta\rho) \\ \tilde{\gamma} &= \gamma((1 - \rho)(1 - \beta\rho) - \kappa\sigma^{-1}\rho).\end{aligned}\tag{11}$$

Two interesting features emerge. First, the optimal weight $\tilde{\lambda}$ under discretion is lower than the weight λ in the loss function of the government (welfare loss function). The solution for the optimal weight is identical to the optimal weight in the case without interest rate variability in the loss function, as shown by Clarida, Gali and Gertler (1999). Second, the optimal weight $\tilde{\gamma}$ on interest rate variability is negative if $\rho > \rho^*$, where ρ^* is defined in (9). Thus, in the case when preferences for interest rate stability are self-defeating, it would be optimal for the government to delegate monetary policy to a policymaker who *values* interest rate variability. Woodford (2003b) also found that it is optimal to delegate policy to a policymaker with a negative weight on interest rate stability in the limiting case with $\kappa = 0$, but with an interest-rate smoothing term in addition to an interest-rate level term in the loss function. The reason for the negative level weight in Woodford's specification is, however, different from ours, as the negative level weight in his model should counteract some of the implied policy attenuation caused by the positive optimal weight on interest rate smoothing.

¹⁸Vestin (2006).

¹⁹Jensen (2002).

²⁰Woodford (1999) and (2003b).

3 Stabilizing the real interest rate

We have so far focused on nominal interest rate stability, since this is what is common in the literature and seems to capture policymakers' concerns. The arguments for interest rate stability provided by Woodford (2003a) are related to the nominal interest rate. However, if the underlying reason for the preference for interest stability is its relationship with financial stability or an efficient capital allocation ("Austrian school" arguments), it is not clear that policymakers should be concerned about stability in the *nominal* interest rate. Unless people have money illusion or they face binding nominal financial constraints, what determines agents' economic decisions should, according to theory, be the *real* interest rate. It is therefore somewhat surprising that the concerns for deviations from a "normal" level of the interest rate among policymakers and in the public debate appear to refer to the nominal, as opposed to the real, interest rate. Suppose instead that the policymaker dislikes variability in the real interest rate, and that her preferences are represented by the following loss function:

$$E_0(1 - \beta) \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda y_t^2 + \gamma r_t^2], \quad (12)$$

where $r_t = i_t - E_t \pi_{t+1}$. The first-order condition under discretion now becomes

$$-\sigma^{-1} \kappa \pi_t - \sigma^{-1} \lambda y_t + \gamma r_t = 0. \quad (13)$$

Solving the model yields the following proposition:

Proposition 6 *The discretionary equilibrium when the central bank minimizes (12) is always determinate, and the solution is*

$$z_t = c_{r^*}^z r_t^* + c_u^z u_t, \quad z = \pi, y, i, r$$

where

$$c_{r^*}^\pi = \frac{\gamma \kappa \sigma}{G_r} > 0, \quad (14)$$

$$c_u^\pi = \frac{\lambda + \gamma \sigma^2 (1 - \rho_u)}{G_u} > 0,$$

$$c_{r^*}^y = \frac{\gamma \sigma (1 - \beta \rho_r)}{G_r} > 0, \quad (15)$$

$$c_u^y = -\frac{\kappa}{G_u},$$

$$c_{r^*}^i = \frac{\lambda (1 - \beta \rho_r) + \kappa^2 + \gamma \kappa \sigma \rho_r}{G_r} > 0, \quad (16)$$

$$c_u^i = \frac{\lambda \rho_u + \kappa \sigma (1 - \rho_u) + \gamma \sigma^2 \rho_u (1 - \rho_u)}{G_u} > 0,$$

$$\begin{aligned} c_{r^*}^r &= \frac{\lambda(1 - \beta\rho_r) + \kappa^2}{G_r} > 0, \\ c_u^r &= \frac{\kappa\sigma(1 - \rho_u)}{G_u} > 0, \end{aligned} \tag{17}$$

where

$$G_j = \lambda(1 - \beta\rho_j) + \kappa^2 + \sigma^2\gamma(1 - \rho_j)(1 - \beta\rho_j), \quad j = r, u.$$

We are now able to investigate how policy preferences for a stable real interest rate affect the variability of the real interest rate and the nominal rate respectively. The following corollary summarizes the results:

Corollary 7 (i) $\text{var}(r_t)$ is always decreasing in γ . (ii) Sufficient conditions for $\text{var}(i_t)$ to be increasing in γ are

$$\rho_j > \rho^*, \quad j = r^*, u.$$

Thus, we have the "normal" result that placing weight on real interest rate stability in the loss function results in a more stable real interest rate under discretion. Whether a preference for a stable real interest rate also leads to a more stable *nominal* interest rate depends on the persistence of the shocks. We see that the case where it leads to a more volatile nominal interest rate occurs at exactly the same level of persistence as when the nominal rate enters the loss function. Therefore, replacing the nominal interest rate by the real rate in the loss function does not let the central bank escape the 'discretion trap' of excessive nominal interest rate volatility. However, an interesting question is whether the 'discretion trap' becomes more or less severe under real interest rate stabilization. By comparing (16) with (8) we find the following result:

Corollary 8

$$c_j^i \leq a_j^i \quad \text{if } \rho_j \geq \rho^*.$$

Thus, if the persistence in both the shocks are above the threshold ρ^* , a loss function that penalizes variability in the real interest rate implies lower variability also in the nominal rate than a loss function that penalizes variability in the nominal rate. The result is illustrated in Figure 2, where we have used the parameter values from Section 2.2, except that we have assumed that $\rho_r = \rho_u = \rho$ and $\gamma = 0.1$. Note that for $\rho < \rho^*$, the standard deviation of i_t is only moderately higher under real interest rate stabilization than under nominal interest rate stabilization, while the difference in

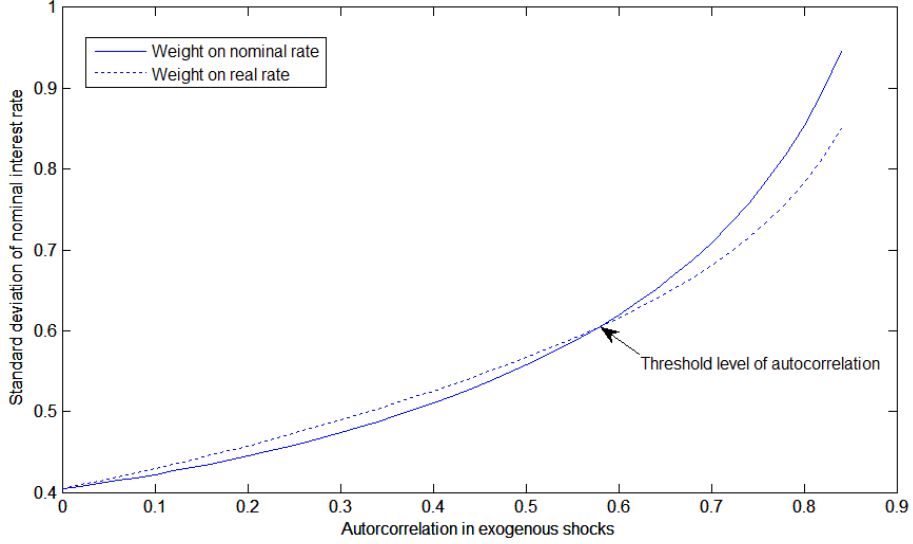


Figure 2: The relationship between autocorrelation of shocks and the S.D. of nominal rates. Weight in loss function on real and nominal interest rate, respectively, is equal to 0.1.

standard deviations becomes more significant when ρ increases beyond ρ^* . This suggests that if the monetary policy maker has preferences for a stable interest rate, it seems more robust to focus on the real interest rate rather than the nominal rate: Weight on the real rate always gives a more stable real rate, and when self-defeating preferences kick in - above the threshold - also the nominal rate is more stable with weight on the real rate. In addition, by including the real, as opposed to the nominal, interest rate in the loss function one avoids a potential equilibrium indeterminacy.

4 A medium-scale DSGE model

In order to assess the empirical relevance of potentially self-defeating preferences for interest rate stability, we consider the Smets and Wouters (2007) (SW hereafter) model for the US economy.²¹ The reason for choosing this model is that it has the key properties of the canonical New Keynesian model

²¹We have downloaded the code for the Smets and Wouters (2007) model from the “Macroeconomic Model Database” established by a project headed by Volker Wieland, see Wieland et. al (2012).

from Section 2, such as nominal rigidities and forward-looking agents, but it has a richer, and thus more realistic, structure. For example, the model includes real rigidities such as habit formation, investment adjustment costs, fixed costs in production and variable capital utilization. The model includes a number of shocks that can be given structural interpretations, and the dynamic properties of these shocks are estimated along with the rest of the model. Since the SW model is well known, we refer to the original paper for details.

The SW model is estimated under the assumption that the central bank followed a Taylor-type rule. Since we consider optimal policy, i.e. minimizing a loss function, we will assume that the model is sufficiently robust to the Lucas critique so that we can treat the obtained parameters as estimates of the true structural parameters, and thus we may analyze alternative policy specifications.

Based on the results in Section 2, the persistence of shocks are key parameters regarding the potential self-defeating feature of interest rate stabilization. Excluding the monetary policy shock that appears in the (here omitted) Taylor rule, there are six structural shocks in the SW model, characterized by the standard deviations and AR(1)-process parameters reported in Table 2. Note that there is a relatively high persistence in most of the shocks, which could potentially imply that preferences for interest rate stability may be self-defeating.

Table 2: Shock processes in the Smets and Wouters (2007) model:

Shock	S.D.	Autocorr.
Total Factor Prod. shock	0.45	0.95
Risk premium shock	0.23	0.22
Gov. Spending shock	0.53	0.97
Inv. spec. tech. shock	0.45	0.71
Price markup shock	0.14	0.89
Wage markup shock	0.24	0.96

To characterize monetary policy, we will use the following loss function, with the weights specified in Table 3:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda y_t^2 + \gamma i_t^2 + \delta (i_t - i_{t-1})^2].$$

Table 3: Weights in the monetary policymaker's loss function:

Loss function	π_t^2	y_t^2	i_t^2	$(i_t - i_{t-1})^2$
		λ	γ	δ
Dual mandate	1	0.250	range	0
	1	0.250	range	1
DKLN (2014)	1	1.042	range	0
	1	1.110	range	1

We use the 'dual mandate' loss function suggested by Yellen (2012) and applied in Section 2.2 above as a benchmark. The qualitative results, in particular whether preferences for interest rate stability are self-defeating or not, seem to be independent of the choice of λ also in the SW model. Debortoli, Kim, Linde and Nunes (DKLN hereafter) (2015) find that a loss function with a larger weight on the output gap than the one suggested by Yellen results in outcomes that come quite close to outcomes under optimal Ramsey policy in the SW model. We therefore also consider their "optimal" simple loss function, which has $\lambda = 1.042$. One should note that DKLN assume commitment, while we consider discretionary policy.²² The results for the two loss functions are illustrated in Figure 3.²³ Since monetary policy cannot use the expectations channels actively under discretion, the standard deviations for the variables of interest become significantly larger under discretion than the standard deviations that DKLN find under commitment. Comparing the outcome under the 'dual mandate' loss function ($\lambda = 0.25$) with the outcome under the "optimal" DKLN loss function illustrates the well-known result that there is a gain from having a smaller λ under discretion than the one in the true welfare loss function (i.e. a 'conservative' central bank). As seen from the figures, we find similar results for the SW model as for the simple analytical model in Section 2, i.e. that preferences for a stable interest rate become self-defeating, except for extremely small values of γ . Thus, self-defeating preferences for a stable interest rate seem to be empirically relevant. The range for γ in which a numerical solution is found when solving for optimal policy under discretion is, unfortunately, limited.²⁴

²²Whether central banks act under discretion or commitment is ultimately an empirical question, and empirical studies on this issue are ambiguous. Since adjusted ad hoc loss functions used under discretion may mimic commitment behavior, identification may be a challenge. Within a framework of "loose commitment" applied to the SW model, Debortoli and Lakdawala (2014) find support neither for full commitment nor for full discretion, but the estimated degree of commitment is large. Givens (2012) and Chen, Kirsanova and Leith (2013) on the other hand, find that US monetary policy is best described as being conducted under discretion.

²³In order to solve for optimal policy under discretion, we use the MATLAB toolkit derived by Debortoli, Maih and Nunes (2014).

²⁴Finding a numerical solution is less dependent on γ under commitment than under discretion.

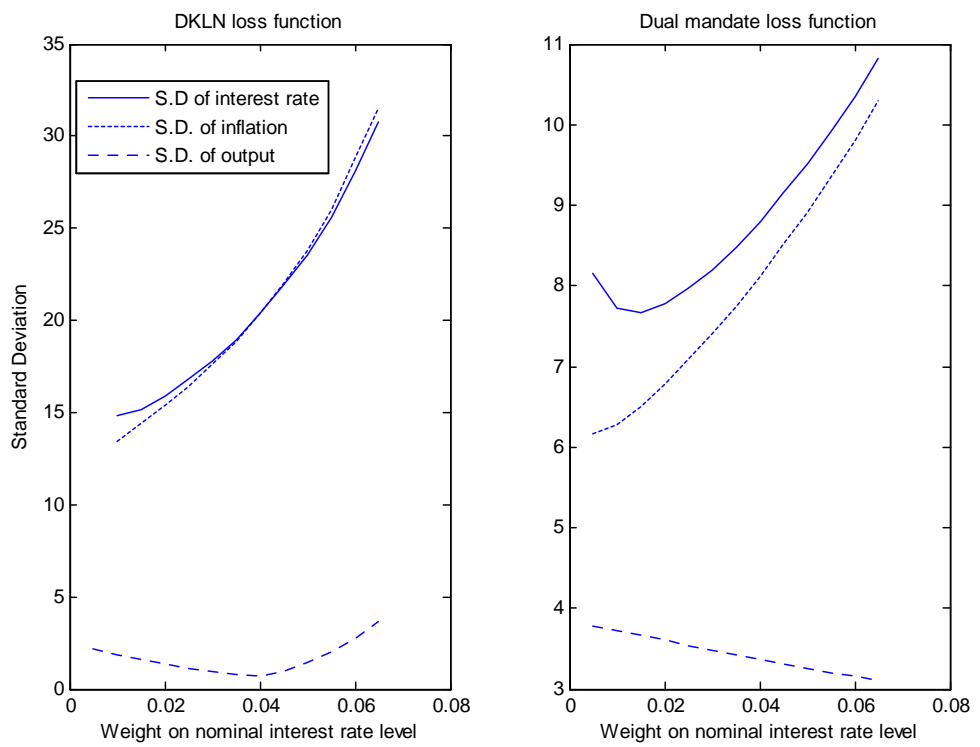


Figure 3: Standard Deviations of endogenous variables in the Smets-Wouters model. Increasing weight on the interest rate level.

The focus of this paper is preferences for a stable interest rate *level*, or reluctance to deviate too much from a level that is considered "normal". In addition to such preferences, which we motivated in the introduction, monetary policy-makers are often assumed to have a preference for gradualism in policy, i.e. interest rate smoothing. It is not possible to reach an analytical closed-form solution for discretionary policy with interest rate smoothing in the simple model.²⁵ But here, we also consider the case with an interest rate smoothing term in the loss function, in addition to the interest rate level term. There are two reasons for doing this: First, we want to investigate whether the inclusion of interest rate smoothing changes the qualitative results. Second, by adding an interest rate smoothing term, the range for the weight on the level term in which a numerical solution is found increases significantly. Thus, we shall add the term $\delta(i_t - i_{t-1})^2$ to the loss function, and use the same weight suggested by Yellen (2012) and used by DKLN, i.e. $\delta = 1$. DKLN find that the "optimal" weight on the output gap changes from $\lambda = 1.042$ to $\lambda = 1.11$ when the interest rate smoothing term is added. The results for the two loss functions - the "dual mandate" function and the "optimal" DKLN function with interest rate smoothing - are found in Figure 4. We see that the standard deviation of the policy rate increases in γ also when interest rate smoothing is added to the loss function. Thus, the existence of self-defeating preferences for a stable interest rate seems robust to whether interest rate smoothing is added. While preferences for a stable interest rate *level* can be self-defeating, we do not find any evidence for self-defeating preferences for interest rate *smoothing*.²⁶ This is not surprising, based on Woodford's result that preferences for interest rate smoothing in fact improves upon the discretionary solution.

We showed in Section 4 that preferences for stability in the *real* interest rate might also give a more stable *nominal* rate than when one has preferences for stability in the nominal rate. The effects of increasing the weight on the real interest rate, as opposed to the nominal rate, are represented by the solid lines in Figure 5. We see that also this result to a large extent carries over to the SW model. Preferences for real interest rate stability result in a more stable real interest rate (the result applies for all the three loss functions we consider), and it also leads to a more stable nominal rate than in the case with equal weight on the nominal rate. Our results therefore suggest that if policy-makers judge interest rate stability as advantageous, they should be concerned about stabilizing the real interest rate and not focus on the nominal rate.

²⁵See Woodford (2003b), Section 4.1.

²⁶We do not report these results, but can provide numerical results upon request.

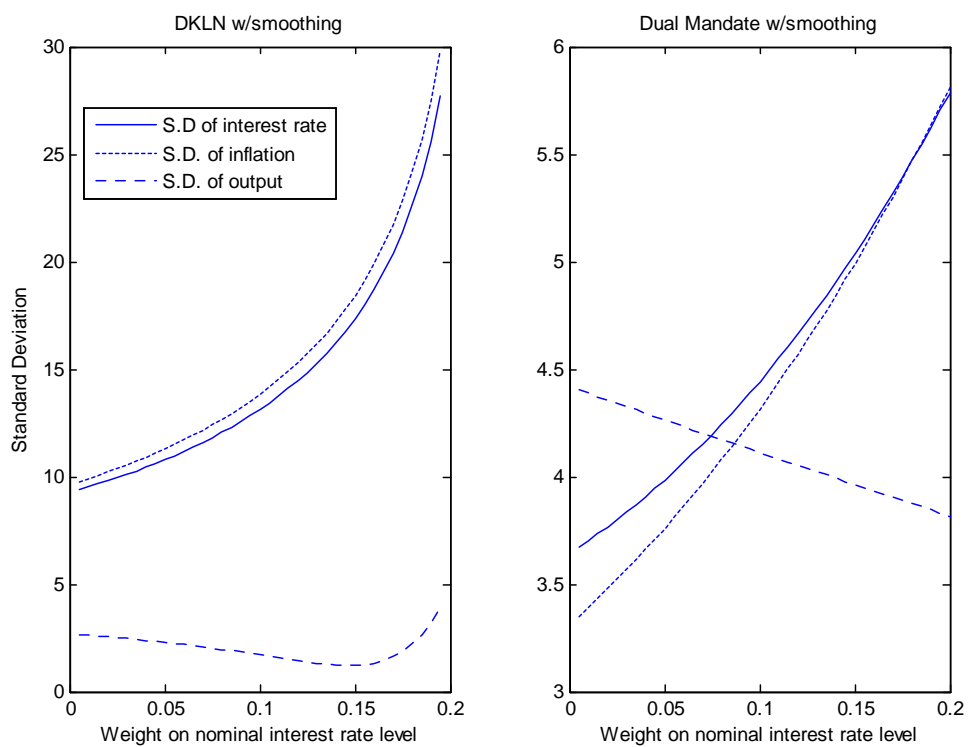


Figure 4: Standard Deviations of endogenous variables in the Smets-Wouters model, loss functions with interest rate smoothing. Increasing weight on the interest rate level.

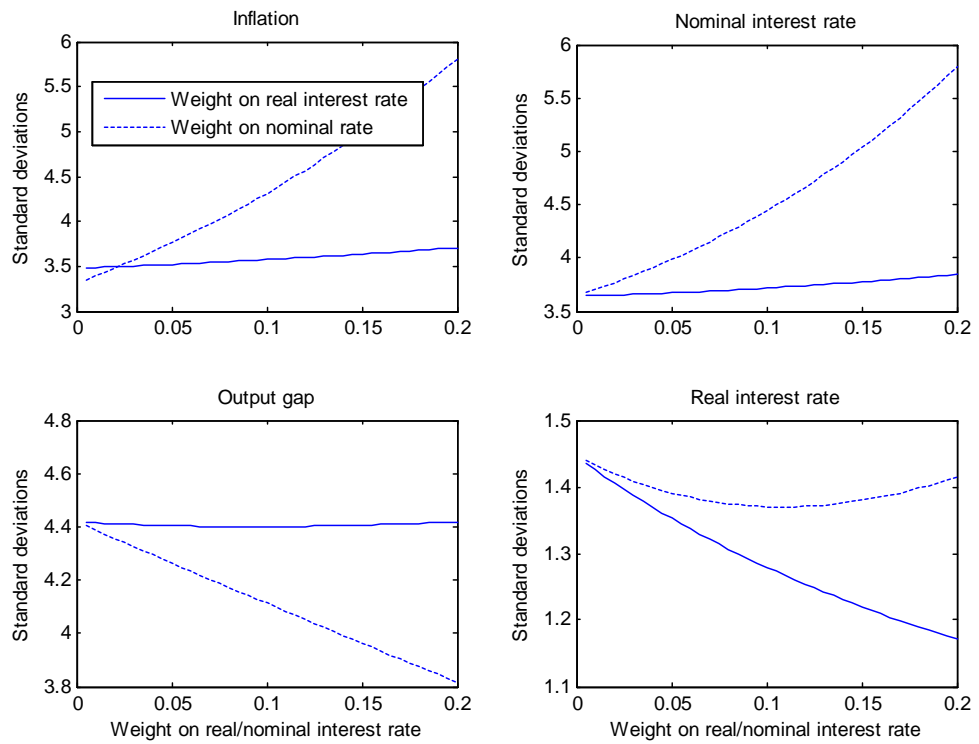


Figure 5: Standard deviations depending on weights on the real interest rate and nominal rate, respectively. Dual mandate loss function, Smets-Wouters model.

5 Conclusion

There is some evidence that monetary policymakers have concerns about large deviations of the interest rate from a "normal" level, and there are also theoretical arguments for including the interest rate level in the welfare loss function. We have considered the implications of having preferences for a stable interest rate when the policymaker is not able to commit and therefore conducts an optimal time-consistent (discretionary) policy. The persistence in the shocks is crucial for the outcome of such a policy, and if the persistence is above a certain threshold, preferences for interest rate stability are self-defeating and lead to higher interest rate variability than what would have been the outcome without such preferences. The self-defeating properties of such preferences from the simple canonical New Keynesian model are confirmed when considering the medium-scale estimated DSGE model for the US of Smets and Wouters (2007).

We have also considered preferences for a stable real, as opposed to a nominal, interest rate and showed that such preferences always imply a determinate equilibrium. Moreover, if the shock persistence exceeds the above-mentioned threshold, having preferences for a stable real interest rate also gives a more stable nominal rate than if the policymaker tries to stabilize the nominal rate.

The policy recommendations that can be drawn from this analysis is that central banks should avoid constraining the use of the interest rate if the goal is to avoid an interest rate level that is far from a "normal" level, for example motivated by financial stability considerations. If the policymaker is not able to credibly commit, the outcome may well be the opposite of what one aims for. It would then be better for the principal (society) to appoint a monetary policymaker that does not have preferences for interest rate stability, or even prefers interest rate variability, even if the principal has such preferences. If nominal interest rate stability is considered important, we have shown that it may be more robust to aim for a stable *real* interest rate rather than nominal interest rate stability.

Appendix

Proof of Proposition 1

Solving the first-order condition (4) with respect to the interest rate gives a "Taylor rule" of the form $i_t = a\pi_t + by_t$. As shown by Woodford (2001) and Bullard and Mitra (2002), determinacy of a Taylor rule in the above model requires that $a + \frac{1-\beta}{\kappa}b > 1$. Inserting the expressions for a and b from (4), i.e., $a = \frac{\kappa}{\sigma\gamma}$ and $b = \frac{\lambda}{\sigma\gamma}$, implies condition (5).

Proof of Proposition 2

Make use of (1) and (2) to eliminate y_t and i_t in (4). This gives the following second-order rational expectations difference equation in the inflation rate:

$$-\sigma^{-1}\kappa\pi_t - \sigma^{-1}\lambda\kappa^{-1}(\pi_t - \beta E_t\pi_{t+1} - u_t) + \gamma[-\sigma\kappa^{-1}\pi_t + (1 + \sigma\kappa^{-1}(1 + \beta))E_t\pi_{t+1} - \sigma\beta\kappa^{-1}E_t\pi_{t+2} + \sigma\kappa^{-1}(1 - \rho_u)u_t + r_t^*] = 0.$$

Insert the conjectured solution $\pi_t = a_r^\pi r_t^n + a_u^\pi u_t$ into the above equation and make use of $E_t\pi_{t+n} = a_r^\pi \rho_r^n r_t^n + a_u^\pi \rho_u^n u_t$. Solving for a_r^π and a_u^π gives (6), and we have from Proposition 1 that this solution is unique. The solution for y_t is found by inserting the solution for π_t into (2), and the solution for i_t is found by inserting the solutions for π_t and y_t into (1).

The signs follow from the fact that the denominators are positive if (5) is satisfied. To see this, note that the denominator can be negative if the last term $-\gamma\sigma\kappa\rho_i$ is sufficiently large (in absolute value). Multiply (5) by $\sigma\kappa\rho_i$, which gives $\gamma\sigma\kappa\rho_j < \lambda(1 - \beta)\rho_j + \kappa^2\rho_j$. Inserting the maximum admissible value $\gamma\sigma\kappa\rho_j = \lambda(1 - \beta)\rho_j + \kappa^2\rho_j$ into the denominators in (6)-(8) gives the following expression for the minimum admissible value for the denominators: $\lambda(1 - \rho_j) + \kappa^2(1 - \rho_j) + \sigma^2\gamma(1 - \rho_j)(1 - \beta\rho_j) > 0$, $j = r, u$.

Proof of Proposition 3

We have that $\frac{\partial \text{var}(i_t)}{\partial \gamma} = \frac{\sigma_r^2}{1 - \rho_r^2} \frac{\partial (a_r^i)^2}{\partial \gamma} + \frac{\sigma_u^2}{1 - \rho_u^2} \frac{\partial (a_u^i)^2}{\partial \gamma}$. Since $a_j^i > 0$ from Proposition 2, we see from (8) that the sign of $\frac{\partial (a_j^i)^2}{\partial \gamma}$ is equal to the sign of $\kappa\sigma^{-1}\rho_j - (1 - \rho_j)(1 - \beta\rho_j)$, which is positive for $\rho_j > \rho^*$ and negative for $\rho_j < \rho^*$.

Proof of Corollary 4

Denote $V = \text{var}(i_t) = (a_{r^*}^i)^2 \frac{\sigma_r^2}{1 - \rho_r^2} + (a_u^i)^2 \frac{\sigma_u^2}{1 - \rho_u^2}$. From the expressions for $a_{r^*}^i$ and a_u^i in (8), and by introducing appropriate symbol definitions, we can write $V = \left(\frac{f_i}{g_i + h_i\gamma}\right)^2 + \left(\frac{f_j}{g_j - h_j\gamma}\right)^2$, $i = r, u$, $j = r, u$, $i \neq j$. Let i

denote the shock that satisfies $\rho_i < \rho^*$ and j denote the shock that satisfies $\rho_j > \rho^*$. All the newly defined parameters are then positive. We have that $\frac{\partial V}{\partial \gamma} = -\frac{2h_i f_i^2}{(g_i + h_i \gamma)^3} + \frac{2h_j f_j^2}{(g_j - h_j \gamma)^3}$ and $\frac{\partial^2 V}{\partial \gamma^2} = \frac{6h_i^2 f_i^2}{(g_i + h_i \gamma)^4} + \frac{6h_j^2 f_j^2}{(g_j - h_j \gamma)^4} > 0$. Since $g_j - h_j \gamma > 0$ from Proposition 2 (see the proof above) and no restrictions are put on the relative sizes of σ_r^2 vs σ_u^2 that enter f_i and f_j , one cannot rule out the possibility that $\frac{\partial V}{\partial \gamma} = 0$ for a value of γ in its admissible range. Since $\frac{\partial^2 V}{\partial \gamma^2} > 0$, it follows that this point is a minimum for $var(i_t)$.

Proof of Proposition 5

Replacing λ and γ by $\tilde{\lambda}$ and $\tilde{\gamma}$ respectively in the first-order condition (4) and solving for i_t gives the following "Taylor rule": $i_t = a\pi_t + by_t$, where $a = \frac{\kappa}{\sigma\gamma}$ and $b = \frac{\lambda}{\sigma\gamma}$. Within an identical model, Giannoni (2014) shows that the optimal coefficients in a Taylor rule are $a^{opt} = \frac{\kappa}{\sigma\gamma((1-\rho)(1-\beta\rho)-\kappa\sigma^{-1}\rho)}$ and $b^{opt} = \frac{\lambda(1-\beta\rho)}{\sigma\gamma((1-\rho)(1-\beta\rho)-\kappa\sigma^{-1}\rho)}$. Choosing $\tilde{\lambda}$ and $\tilde{\gamma}$ so that $a = a^{opt}$ and $b = b^{opt}$ gives (11).

Proof of Proposition 6

Solving (13) for the nominal interest rate gives the following "Taylor rule":

$$i_t = E_t \pi_{t+1} + a\pi_t + by_t, \quad (18)$$

where $a = \kappa(\sigma\gamma)^{-1}$ and $b = \lambda(\sigma\gamma)^{-1}$. Inserting (18) into (1) gives, together with (2) a system that can be written as $E_t z_t = Az_t + e_t$, where $z_t = (\pi_t, y_t)'$, $e_t = (r_t^*, u_t)'$ and

$$A = \begin{pmatrix} \beta^{-1} & -\beta^{-1}\gamma \\ \sigma^{-1}a & \sigma^{-1}(\sigma + b) \end{pmatrix},$$

which has the eigenvalues

$$\theta_j = \frac{\sigma + \beta(b + \sigma) \pm \sqrt{b^2\beta^2 + 2b\sigma\beta^2 - 2b\sigma\beta + \sigma^2\beta^2 - 2\sigma^2\beta + \sigma^2 - 4a\kappa\sigma\beta}}{2(b + \sigma + a\kappa)},$$

$j = 1, 2$. The eigenvalues must lie outside the unit circle to have a determinate stationary solution. This is satisfied if $a + \frac{b(1-\beta)}{\kappa} > 0$. Since a and b are both positive, determinacy is always achieved. To derive the solutions for π_t, y_t, i_t and r_t , use (1) and (2) to eliminate y_t and i_t in (13). This gives the following second-order rational expectations difference equation in the inflation rate:

$$\begin{aligned} & -\sigma^{-1}\kappa\pi_t - \sigma^{-1}\lambda\kappa^{-1}(\pi_t - \beta E_t \pi_{t+1} - u_t) + \gamma(-\sigma\kappa^{-1}\pi_t \\ & + (\sigma\kappa^{-1}(1 + \beta))E_t \pi_{t+1} - \sigma\beta\kappa^{-1}E_t \pi_{t+2} + \sigma\kappa^{-1}(1 - \rho_u)u_t + r_t^*) = 0, \end{aligned} \quad (19)$$

which is solved as in the proof of Proposition 2.

Proof of Corollary 7

Result (i) follows directly from (17). To show (ii), note that $\frac{\partial \text{var}(i_t)}{\partial \gamma} = \frac{\sigma_r^2}{1-\rho_r^2} \frac{\partial (a_r^i)^2}{\partial \gamma} + \frac{\sigma_u^2}{1-\rho_u^2} \frac{\partial (a_u^i)^2}{\partial \gamma}$. We have that $\frac{\partial a_r^i}{\partial \gamma} = \frac{\sigma^2(\kappa^2 + \lambda(1-\beta\rho_r)(\kappa\sigma^{-1}\rho_j - (1-\rho_j)(1-\beta\rho_j))}{(\lambda(1-\beta\rho_r) + \kappa^2 + \sigma^2\gamma(1-\rho_r)(1-\beta\rho_r))^2}$, which has the same sign as $\kappa\sigma^{-1}\rho_j - (1-\rho_j)(1-\beta\rho_j)$ and is positive for $\rho_j > \rho^*$ and negative for $\rho_j < \rho^*$. We have that $\frac{\partial a_u^i}{\partial \gamma} = \frac{\kappa\sigma^3(1-\rho_u)(\kappa\sigma^{-1}\rho_j - (1-\rho_j)(1-\beta\rho_j))}{(\lambda(1-\beta\rho_r) + \kappa^2 + \sigma^2\gamma(1-\rho_r)(1-\beta\rho_r))^2}$, which is positive for $\rho_j > \rho^*$ and negative for $\rho_j < \rho^*$.

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