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An equilibrium model of credit rating agencies

Steinar Holden, Gisle James Natvik and Adrien Vigier

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An Equilibrium Model of Credit Rating Agencies

Steinar Holden* Gisle James Natvik† Adrien Vigier‡

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Abstract

We develop a model of credit rating agencies (CRAs) based on reputation concerns. Ratings affect investors' choice and, thereby, also issuers' access to funding and default risk. We show that – in equilibrium – the informational content of credit ratings is inferior to that of CRAs' private information. We find that CRAs have a pro-cyclical impact on default risk: in a liquidity boom CRAs help resolve investors' coordination problem, and lower the probability of default; in a liquidity crunch CRAs raise the probability of default. Furthermore, rating standards tend to be pro-cyclical, while biased CRA-incentives will ultimately be self-defeating.

Keywords: Credit rating agencies, global games, coordination failure

JEL: *G24, G33, D82, C72*

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1 Introduction

In the wake of the 2008 financial crisis, credit rating agencies (CRAs) have faced heavy fire from a variety of actors and commentators. It has been argued that CRAs were unable to detect the vulnerabilities in mortgage backed securities and other new financial products (U.S. Permanent Subcommittee on Investigations, 2010), and that they failed to anticipate fiscal distress in several advanced and emerging market economies. However, CRAs have also been accused of downgrading countries' debt with no clear deterioration in fundamentals to justify it. For example, in a joint letter 6 May 2010, Chancellor Angela Merkel and President Nicolas Sarkozy write that "The decision of a rating agency to downgrade the rating of the Greek debt even before the authorities' programme and amount of the support package were known must make us ponder the rating agencies' role in propagating crises." On top of this, a widely held suspicion has been that the current business model where CRAs are paid by issuers leads to so-called "rating inflation"(Pagano and Volpin, 2010). The U.S. Permanent Subcommittee on Investigations (2010) conclude that "... credit rating agencies allowed Wall Street to impact their analysis. ... And they did it for money." CRAs, on the other hand, argue that reputation concerns ensure that they are firmly disciplined to provide reliable information.[1]

The debate on CRAs raises a number of important and interesting questions. How do reputation concerns influence CRAs' rating strategy? Do CRAs ultimately increase the risk that funding difficulties force firms or sovereigns into default? Or do they lower it? From the reasoning of Merkel and Sarkozy's above, one may argue that an issuer bias on the part of CRAs would help to avoid undesirable equilibria. But that reasoning rests on the questionable assumption that investors ignore CRAs' structural biases. To address the above questions, we therefore examine the behavior and impact of CRAs in equilibrium, in the sense that credit ratings, investors' choice, and the performance of rated projects are jointly determined.

We develop an equilibrium model of credit rating agencies, in which reputation concerns penalize CRAs for making inaccurate predictions and reward them for making accurate ones. Our paper builds on Morris and Shin's (2004) model of debt roll-over. Here, rational investors (lenders) choose whether or not to refinance a borrower with an uncertain capacity to repay.[2] The project succeeds if sufficiently many investors roll over their loan, and fails otherwise, with a resulting loss to investors. A crucial coordination problem then prevails. We examine the effect of CRAs in this setting: Do CRAs help resolve investors' coordination problem, or

[1] Illustrative for CRAs weight on reputation is the following quote by the former VP of Moody's, Thomas McGuire , taken from Manso (2011):"What's driving us is primarily the issue of preserving our track record. That's our bread and butter."

[2] The borrower could be a sovereign state.

do they worsen it?

We first show that when CRAs are motivated by reputation concerns and their ratings affect default risk, the transmission of information from CRAs to investors is impeded. The intuition is that CRAs have incentives to exploit the potential self-fulfilling effect of credit ratings. Suppose for instance a CRA observes a clear risk of default, but also a non-negligible possibility that default will be avoided. Transmitting this precise, rather negative, information to investors still leaves open the possibility that the project might succeed. But if success indeed materializes, the CRA will seem to have made a poor assessment. In contrast, exaggerating the risk of default may trigger default for sure, and thus warrant the rating given by the CRA, with accordingly high reputational rewards as consequence. Of course, in equilibrium, rational investors understand CRAs' behavior and, as a result, discount ratings accordingly. The upshot is a severe limitation on the transmission of information from CRAs to investors: In equilibrium, the informational content of credits ratings is inferior to that of CRAs' private information.

We next examine CRAs' impact in equilibrium. Our main result establishes that CRAs have a pro-cyclical impact: they tend to amplify underlying market conditions. In a liquidity boom CRAs help resolve investors' (mild) coordination problem, and lower the (already low) probability of default. In a liquidity crunch, on the other hand, CRAs worsen investors' (acute) coordination problem, and raise the (already high) probability of default. The intuition is as follows. Consider the scenario of a liquidity crunch. Investors then have good alternative use for their money and are therefore inclined to liquidate. Hence, the "marginal" investors who in spite of the liquidity crunch are considering whether or not to roll debt over must have rather optimistic beliefs. As a result, "good news" from the CRA largely overlaps with those investors' private information, and therefore has little effect on their decisions. In contrast, "bad news" provides new and substantial information for the marginal investors, who therefore might be motivated to liquidate and, thereby, trigger default. In a liquidity boom on the other hand, investors have little alternative use for their money, and are therefore readily willing to roll over their loans. In this case, "marginal" investors hold rather pessimistic beliefs. Hence, "bad news" from the CRA largely overlaps with those investors' private information and has little effect. "Good news" on the other hand provides new and substantial information for marginal investors, who as a result might decide to roll over and, thereby, prevent default.

We find in addition that it is not only the impact of CRAs that is pro-cyclical, but also the rating standards they follow: When liquidity is abundant, it becomes more attractive for CRAs to give good ratings. One reason for this is straightforward, as high liquidity means that it is easy to roll debt over. The other reason is more subtle: As good ratings are more

influential than bad ratings, as explained above, a good rating is also more likely to look correct ex post. Thus, a CRA concerned about reputation will be more inclined to give a good rating when liquidity is abundant. This implies that favorable market conditions may be a cause of the rating inflation phenomenon. Conversely, in periods where liquidity is tight, bad ratings might become more attractive for CRAs.

We next take up the issue of CRA biases. As indicated earlier, a common concern is that CRAs may be biased towards good ratings, since their ratings are financed by issuers. We capture this possibility by allowing CRAs to prefer avoiding default: if the debtor defaults, he or she is less likely to ask the agency for a new rating at a later occasion. Alternatively, CRAs may have a cautious bias, in the sense of being excessively averse to mistakenly guide investors into financing projects that ultimately fail. We establish that, in equilibrium, any bias on the part of CRAs is ultimately self-defeating: in particular, a bias in favor of issuers increases the resulting risk of default. The intuition is as follows. If CRAs prefer to avoid default, then in equilibrium all investors know that CRAs are inclined to give good ratings rather than bad. "Good news" from the CRA is therefore discounted by investors and, as a result, does not affect default risk. Instead, it is only on the rare occasions when the CRA announces "bad news" that it affects investors. Thus, paradoxically, when the CRA wants to avoid default, it is, if anything, only able to trigger it. Of course, by symmetry, any bias in the opposite direction will tend to lower the risk of default. This suggests that cautious biases may be socially preferable, a remark we further discuss in section 4, together with other policy implications of our model.

A large empirical literature examines how credit ratings influence financial markets. Since the effect of ratings on investor behavior is endogenous in our model, it is compelling to ask how our predictions actually fare against some of this empirical research. In particular, our results establish that ratings tend to provide coarse information and affect investor behavior asymmetrically, so that some rating events impact the probability of default while others do not. These predictions seem to be largely consistent with empirical regularities documented elsewhere. We elaborate on these points in section 4.

Our paper is, to the best of our knowledge, the first one to simultaneously: (i) account for strategic behavior on the part of CRAs, (ii) allow for the possibility that ratings might affect the performance of the rated objects, and (iii) endogenize investors' response to credit ratings.

The closest studies to ours are Mariano (2012) and Manso (2011). While both papers incorporate (i) and (ii) in the former list, investors behave mechanically in these models.[3] The

[3]Mariano (2012) studies a setting where CRAs come in two types, distinguished by the precision of their

difference is naturally key. For instance, in sharp contrast to our paper, Manso (2011) argues in favor of issuer biases on the part of CRAs as a means to prevent CRAs from giving negative self-fulfilling ratings. That paper and ours' thus present complementary views, which should be weighted contextually, according to the perceived importance of regulatory restrictions regarding investors' behavior. If recent regulatory proposals which aim to down-play the role of credit ratings are passed (see discussion below), our argument against an issuer bias becomes more important.

Mathis, McAndrews and Rochet (2009), Bolton et. al (2012) and Bar-Isaac and Shapiro (2012) all share with our paper points (i) and (iii) above, to some extent.[4] Neither paper, however, considers the possibility that ratings may affect outcomes.

Earlier theoretical studies on CRAs on the other hand disregard strategic behavior on the part of CRAs, assuming at the outset that reputation concerns guarantee the efficient transmission of CRAs' private information to investors. Of these papers, Boot, Milbourn and Schmeits (2006) and Carlson and Hale (2006) are nevertheless related to our analysis since in both papers ratings affect outcomes through investors' coordination problem.[5] Skreta and Veldkamp (2009), Sangiorgi, Sokobin and Spatt (2009), and Faure-Grimaud, Peyrache and Quesada (2009) focus instead on the role of competition among rating agencies and ratings shopping by issuers.[6]

Our paper applies tools developed in the global games literature. An extensive survey of

information, and where a low-quality agency may choose to ignore private information, and instead issue ratings that conform to public opinion, in an attempt to conceal its true type. Manso (2011) focuses on the "feedback" effect between ratings and borrowers' survival probability, and shows how the default decision of a firm and the rating decision by a CRA become strategic complements due to the combination of feedback effects and reputation concerns. The key difference between these studies and ours is that they postulate exogenously that ratings affect projects' funding costs.

[4]Mathis et. al (2009) explore the occurrence of reputation cycles, where CRAs accumulate a reputation for truth-telling, only to exploit this reputation when it is strong enough. Bolton et. al (2012) study competition among CRAs and ratings shopping by issuers, and derive conditions for when agencies behave truthfully and opportunistically. Bar-Isaac and Shapiro (2012) study how ratings quality varies over the business cycle when CRAs must use wages to attract competent employees, and errors are punished by investors.

[5]Boot et. al (2006) emphasize the monitoring role of CRAs in credit watch procedures and the importance of credit ratings for institutional investors. Multiple equilibria are possible because issuers choose risk-strategies contingent on their funding costs, while ratings can serve to coordinate markets since institutional investors are *required* to follow them. The approach of Carlson and Hale (2006) is more similar to ours, in that they also use global games tools to study CRAs and a roll-over problem. They show that CRAs may induce multiple equilibria by making public information more precise, and numerically assess how ratings may influence equilibria when they remain unique. CRA incentives are here ignored, and it is assumed that CRAs provide all the information they possess to the market.

[6]Skreta and Veldkamp (2009) and Sangiorgi, Sokobin and Spatt (2009) show that noisier information leads to rating shopping. Grimaud, Peyrache and Quesada (2009) find that issuers may suppress ratings that are too noisy.

this literature is given by Morris and Shin (2003). Other applications of global games that lie particularly close to ours, in addition to Carlson and Hale (2006) mentioned above, are Morris and Shin (2006, 2004) and Corsetti, Guimarães and Roubini (2006) who all investigate roll-over problems.

The paper is organized as follows. Section 2 develops the model. Section 3 presents our results. In section 4 we discuss how our results relate to the rich empirical evidence available on credit ratings, and policy implications of our findings. Section 5 concludes. All proofs are contained in the appendix.

2 Model

We build on Morris and Shin's (2004) model of the roll-over problem facing a project that relies on short term financing. Our main extension to this framework is to add a strategic credit rating agency (CRA). A continuum of investors are financing a firm's project (or a country's debt). The mass of investors is 1. The project is financed with a conventional debt contract specifying a final-period payoff V in case of success, and 0 in case of failure. To focus the analysis it is further assumed that success or failure is entirely determined by the project's ability to meet short-term claims, so that project failure is identified with default.

At an interim stage, investors have an opportunity to liquidate the asset. We let v denote investors' opportunity cost of rolling over at that stage. Hence, v will be high in a situation where alternative high yield assets are abundant, or in a liquidity squeeze where investors have strong immediate needs for cash. Conversely, v will be low if alternative investments opportunities are scarce, or liquidity is abundant. Let $\lambda = v/V$. In order to fix ideas, we will throughout this text interpret λ as an indicator of investors' liquidity needs.

The ability to meet short-term claims, and thereby avoid default, is summarized by a variable θ, which is unknown to creditors. One may think of θ as the project's cash, liquid assets, or access to alternative credit lines other than the debt market. Let l indicate the mass of investors liquidating at the interim stage. The project is able to meet short-term claims if and only if $\theta \geq l$. Thus, in particular, if $\theta > 1$ the project never defaults, even if all creditors liquidate prematurely. Likewise, the project defaults for sure if $\theta < 0$. However, if $\theta \in [0, 1]$, the project may or may not default, depending on investors' behavior. A coordination problem then prevails: Each investor would gain if all were to roll debt over, but no investor would gain from being the only creditor to roll over.[7] We assume that θ is uniformly distributed in

[7]The coordination problem here is therefore similar to the bank run problem of Diamond and Dybvig (1983). In particular, multiple equilibria exist under common knowledge of θ.

$\Delta = [\underline{\delta}, \overline{\delta}]$, where $\underline{\delta} << 0$, and $\overline{\delta} >> 1$, ensuring that all relevant θ values arise with positive probability. Let I denote a a binary variable taking value 1 in case of success and 0 in case of default. Then:

$$I = 1 \Leftrightarrow \theta \geq l \tag{1}$$

While θ is unobserved, each investor receives a private signal x prior to the interim stage, with uniform distribution on $[\theta - \beta, \theta + \beta]$. All signals are independent, conditional on θ.

The CRA receives information in the form of a private signal y, uniformly distributed on $[\theta - \alpha, \theta + \alpha]$. The CRA conveys its information in the form of a rating that is announced to all investors. A rating mechanism consists of a partition $r(\Delta)$ of Δ into intervals and a map $r : \Delta \to r(\Delta)$, such that the CRA publicly announces having observed $y \in r(y)$ (so that $r(y)$ is the "rating" of the CRA when it observes y). If the CRA always tells the truth, formally $y \in r(y)$, $\forall y \in \Delta$, we will say that the rating mechanism r is a *consistent* rating mechanism. In particular we let r_0 denote the trivial consistent rating mechanism given by $r_0(\Delta) = \{\Delta\}$. Thus, a CRA following rating mechanism r_0 never brings any information to the table, and from investors' viewpoint is consequently equivalent to no CRA. A class of consistent rating mechanisms that will be important in the analysis of our model is the class of *threshold* rating mechanisms, where the rating of the CRA indicates whether or not the signal y was above a threshold t. Formally, let $t \in \Delta$, $R_t^- = [\underline{\delta}, t]$, and $R_t^+ = [t, \overline{\delta}]$, and let r_t denote the consistent rating mechanism given by $r_t(\Delta) = \{R_t^-, R_t^+\}$.

Identifying CRA ratings with their actual informational content is both analytically and expositionally convenient. The connection to actual ratings, as provided by Moody's for example, is as follows. A CRA gathers information and derives a picture of the fundamentals, *Baa* say. This corresponds to observing y in our model. Suppose now profit maximizing considerations induce the CRA to communicate *Aaa* instead - and assume that it would also do so after observing *Aa* or *A*. This state of affairs is known by investors, who therefore interpret Moody's communication of *Aaa* as an indication that the CRA really observed anything between *Baa* to *Aaa*. In other words, the information conveyed by the rating *Aaa* is an interval containing all of *Baa*, *A*, *Aa*, and *Aaa*. This paper simply identifies ratings with the actual information they convey and thus, in particular, allows ratings to be intervals as well as isolated "points".

We wish to study how ratings are affected by the desire to give advice that is validated ex post. To this end, we impose a payoff structure for the CRA, denoted Π, that directly captures this incentive. There are two ways to be correct in this environment: the CRA may give a good rating to a successful project, or it may give a bad rating to a project that defaults. For the time being, we treat these two alternatives in a symmetric way, and assume that a

CRA predicting default ($I = 0$) receives payoff 1 if default materializes and 0 otherwise. In a similar way, a CRA predicting success ($I = 1$) receives payoff 1 if success materializes and 0 otherwise. By contrast, a CRA failing to take a clear stance will receive an intermediate payoff for both outcomes. Let R denote an arbitrary rating (we will throughout this text use upper case R to denote actual ratings and lower case r to denote rating mechanisms). Formally, a function $m(.)$ taking values in $[0, 1]$ determines payoffs so that:

$$\Pi(R, I) = m(R)I + (1 - m(R))(1 - I) \tag{2}$$

We assume that given ratings $R = [\underline{r}, \overline{r}]$ and $R' = [\underline{r}', \overline{r}']$, then: $\overline{r} \leq \underline{r}' \Rightarrow m(R) \leq m(R')$. In other words, if the whole interval of R' is above the whole interval of R and success materializes, then a CRA is rewarded more if it announced R' than if it announced R. Conversely, if default materializes, then a CRA is rewarded more if it announced R than if it announced R'. We further assume for simplification that given rating $R = [\underline{r}, \overline{r}]$, $R \neq \Delta$, then: $\overline{r} = \overline{\delta} \Rightarrow m(R) = 1$, while $\underline{r} = \underline{\delta} \Rightarrow m(R) = 0$. [8]

A rating mechanism r is said to be an *incentive compatible* if and only if the CRA maximizes its expected profits by sticking to it, i.e. if and only if $\Pi^e(r(y), y) \geq \Pi^e(r(y'), y)$, $\forall y, y' \in \Delta$, where $\Pi^e(R, y)$ indicates the expected value of Π given rating R and private CRA signal y.

Within the period when roll-over decisions are made, timing is as follows. First the CRA receives its private signal y, and announces its rating $r(y)$. This rating is observed by all investors. Thereafter each investor receives his own private signal x, and chooses whether to roll over or liquidate.

Considering the game played among investors separately will prove useful analytically in the overall determination of equilibria. We define this game as follows:

Definition 1 *The investment game given rating $R = [\underline{r}, \overline{r}]$ is the game played among investors maximizing expected profits after observing rating R and updating beliefs as follows:*

1. *If $x - \beta > \overline{r} + \alpha$, then assign probability 1 to $\theta = \overline{r} + \alpha$.*
2. *If $x + \beta < \underline{r} - \alpha$, then assign probability 1 to $\theta = \underline{r} - \alpha$.*
3. *If $x - \beta \leq \overline{r} + \alpha$ and $x + \beta \geq \underline{r} - \alpha$, then assume θ uniformly distributed on $[x - \beta, x + \beta] \cap [\underline{r} - \alpha, \overline{r} + \alpha]$.*

[8] Note that if the signal of the CRA is close to one of the bounds, say the lower bound $\underline{\delta}$, the outcome is default with certainty, as we assume that $\underline{\delta} + \alpha < 0$. The assumption implies that if the CRA gives a rating that includes the lower bounds (except if it gives the null rating), i.e. a rating that does not rule out a signal which implies default with certainty, then the CRA is only rewarded if default actually occurs.

Part 1 and 2 pin down investors' out-of-equilibrium beliefs, i.e. whenever information provided by the CRA is inconsistent with their own private information.[9] Part 3 approximates the standard Bayesian updating procedure by assuming uniform posterior distributions throughout, and guarantees tractability.[10]

We assume throughout that $\alpha > \beta > \frac{1}{2}$, and that $\frac{1}{2\beta+1} < \lambda < 1 - \frac{1}{2\beta+1}$. The first assumption imposes limited information on the part of the CRA and preserves uniqueness of equilibrium, as usual in global games with public information.[11] The second assumption is related, and ensures existence and uniqueness of the equilibrium.

3 Results

3.1 Preliminary analysis

Our model is conveniently analyzed by first examining the investment game in isolation, abstracting from strategic considerations on the part of the CRA. We here follow Morris and Shin (2003, 2004), among others.

As usual in global games, equilibrium in the investment game is characterized by two threshold values. First, there is a threshold value for investors' signal, denoted x^*, such that investors observing $x < x^*$ liquidate while investors observing $x > x^*$ roll over. Second, there is a threshold value for the fundamental, denoted θ^*, such that default occurs if and only if $\theta < \theta^*$. Start by considering x^*. The marginal investor, who observes x^*, is indifferent between liquidating for a sure gain v and rolling over to receive a payoff V with probability $\mathbb{P}(\theta > \theta^* | x^*, R)$. The indifference equation of the marginal investor is therefore

$$\mathbb{P}(\theta > \theta^* | x^*, R) = v/V = \lambda \tag{3}$$

Consider now θ^*. Since investors are symmetric, the mass of investors choosing to liquidate

[9] The obvious alternative is to set probability 1 to $\theta = x - \beta$ if $x - \beta > \sup R$, and probability 1 to $\theta = x + \beta$ if $x + \beta < \inf R$. However, this assumption implies that there would be no individual noise if $x - \beta > \sup R$, possibly leading to multiple equilibria in the investment game. This defeats the purpose of using global games to obtain a unique equilibrium, see Carlson and Van Damme (1993) and Morris and Shin (2000) for thorough discussions of the issue.

[10] Notice that Part 3 coincides with Bayesian updating whenever $\underline{r} = \overline{r}$. In general however, the probability density of θ conditional on $y \in R$ tapers off at the edges of its domain. If $[\underline{r} + \alpha \leq \overline{r} - \alpha]$ for instance, then the conditional distribution is uniform on that interval and tapers of on $[\underline{r} - \alpha, \underline{r} + \alpha] \bigcup [\overline{r} - \alpha, \overline{r} + \alpha]$.

[11] See e.g. Morris and Shin (2003). In our model there is no market-determined interest rate from which agents can extract information. Should such a price be observed, equilibrium uniqueness requires that there is sufficient noise in the price process, for instance due to uncertainty regarding the supply of debt or noise traders. See Hellwig, Mukherji and Tsyvinski (2006) for a discussion in the context of currency crises.

is equal to the probability that a single investor receives a signal below the threshold value x^*. By (1), the project thus succeeds if and only if $\mathbb{P}(x < x^*|\theta) \leq \theta$, yielding:

$$\theta^* = \mathbb{P}(x < x^*|\theta = \theta^*) \tag{4}$$

To illustrate how the model works, let us first consider the equilibrium in the case where the rating provides no information. In this case an investor who observes x assumes that θ is uniformly distributed on $[x - \beta, x + \beta]$. Using the uniform distribution, the indifference equation of the marginal investor (3) now reads

$$\mathbb{P}(\theta > \theta^*|\theta \in [x^* - \beta, x^* + \beta]) = 1 - \frac{\theta^* - x^* + \beta}{2\beta} = \lambda \tag{5}$$

or

$$x^* = \theta^* - \beta + 2\beta\lambda \tag{6}$$

Similarly, equation (4) gives

$$\mathbb{P}(x < x^*|\theta = \theta^*) = \frac{x^* - \theta^* + \beta}{2\beta} = \theta^* \tag{7}$$

or

$$\theta^* = \frac{x^* + \beta}{2\beta + 1} \tag{8}$$

Equations (6) and (8) can together be solved, yielding threshold values that characterize the equilibrium without a CRA:

$$\theta^* = \lambda \tag{9}$$

$$x^* = \lambda(2\beta + 1) - \beta \tag{10}$$

Now consider the effect of a given rating. (For now we take the rating as exogenous. Strategic considerations on the part of the CRA are taken up in the next section.) Note first that the rating will not affect how the critical value for θ is determined, so equation (4) still prevails. However, the rating will affect the beliefs of the marginal investor. The indifference equation of the marginal investor is now

$$\mathbb{P}(\theta > \theta^*|\theta \in [x^* - \beta, x^* + \beta] \cap [\underline{r} - \alpha, \overline{r} + \alpha]) = \lambda \tag{11}$$

Comparing the indifference equations without and with a CRA, equations (5) and (11),

we observe that if the information in the rating encompasses the beliefs of the marginal investor, i.e. if the interval $[\underline{r}-\alpha, \overline{r}+\alpha]$ covers the whole support of the marginal investor's θ-distribution, which is $[x^*-\beta, x^*+\beta]$, or, using (10), $[\lambda(2\beta+1)-2\beta, \lambda(2\beta+1)]$, the distribution will not be affected, and hence the marginal investor's behavior will not be affected either. Thus, the rating will not influence the equilibrium. However, if the information in the rating restricts the beliefs of the marginal investor, his beliefs and thus also his behavior will be affected. A sufficiently negative rating, where the upper bound of the rating plus noise is below the upper bound of the marginal investor's θ-distribution, i.e. $\overline{r}+\alpha < \lambda(2\beta+1)$, will make the marginal investor more pessimistic about θ, inducing him to liquidate. The threshold x^* will increase, implying that θ^* also increases, i.e. a default is more likely. Similarly, if the lower bound of the rating minus noise is above the lower bound of the marginal investor's θ-distribution, i.e. if $\underline{r}-\alpha > \lambda(2\beta+1)-2\beta$, the rating will have a positive impact on the marginal investor's beliefs. The marginal investor will become more optimistic about θ. Now, x^* will decrease, causing θ^* to decrease in turn, and making default less likely.

If the rating is sufficiently pessimistic, all investors will be negatively affected, and the rating will trigger default unless $\theta > 1$. Likewise, a sufficiently optimistic rating fully resolves the coordination problem among investors: all investors then roll over and default is avoided provided that $\theta > 0$. The following lemma records these important observations for the coming analysis of our model (proof in appendix).

Lemma 1 *A unique equilibrium of the investment game exists for each rating R. The unique equilibrium is characterized by the threshold $\theta^*(R) \in [0,1]$ such that: $I = 1 \Leftrightarrow \theta \geq \theta^*(R)$.*

In particular, ratings affect θ^* as follows:

1. $\theta^*(\Delta) = \lambda$
2. $\overline{r}+\alpha < (2\beta+1)\lambda \;\Leftrightarrow\; \theta^*(R) > \lambda$
3. $\underline{r}-\alpha > (2\beta+1)\lambda - 2\beta \;\Leftrightarrow\; \theta^*(R) < \lambda$
4. $\overline{r}+\alpha < 1 \Leftrightarrow \theta^*(R) = 1$
5. $\underline{r}-\alpha > 0 \Leftrightarrow \theta^*(R) = 0$
6. $R < R' \Rightarrow \theta^*(R) \geq \theta^*(R')$

Part 1 of the Lemma observes that if the rating is fully non-informative then it does not affect the outcome of the game and the critical θ-value remains equal to λ. Part 2 says that if the upper bound of the interval implied by the rating, $\overline{r}+\alpha$, is below the lower bound of

the interval implied by the signal of the marginal investor, $(2\beta + 1)\lambda$, the rating makes the marginal investor more pessimistic, thus increasing the critical θ-value. Similarly, part 3 says that a sufficiently positive rating makes the marginal investor more optimistic, thus decreasing the critical θ-value. Part 4 and 5 record the limit of these two effects.

Suppose there are no CRAs and an independent observer receives an unbiased signal of θ. By lemma 1 that observer will interpret as good news any signal above λ, and will interpret as bad news any signal below λ. On the other hand, a CRA following the threshold rating mechanism r_t announces good news for any signal above t and bad news for any signal below t. These observations motivate the following definitions: we shall say that a threshold rating mechanism r_t exhibits *positive rating inflation* if $t < \lambda$, that is, if the CRA will give a positive rating for some values of the signal which an independent observer will interpret as bad news. Correspondingly, we will refer to it as *negative rating inflation* if $t > \lambda$, that is, when there are signals which the observer will view as good news, and yet for which the CRA gives a negative rating.

3.2 Irreducible, consistent, and incentive compatible rating mechanisms

In this section we move on to consider the effect of CRA's optimal choice of rating mechanism. We are interested in truthful ratings, in line with our assumption of rational investors. We will therefore restrict attention to rating mechanisms that are consistent, as defined above. Furthermore, to avoid irrelevant multiplicity, we must ensure that rating mechanisms which for all relevant purposes are identical, are not distinguished formally. This is the idea behind the concept of an irreducible rating mechanism, which we now define in detail.

To this end, note first that the sole transmission channel from CRA ratings to default outcomes passes through investors' coordination problem. In particular, if two rating mechanisms r and r' always induce identical θ-thresholds, $\theta^*(r'(y)) = \theta^*(r(y))$, $\forall y \in \Delta$, then they are effectively the same. Note also that two rating mechanisms may effectively be the same even if we can find a y such that $\theta^*(r'(y)) \neq \theta^*(r(y))$, if the difference in rating only happens in situations where the rating does not affect outcome, i.e. if the two rating mechanisms induce identical default outcomes with probability 1. Take for example, $r = r_0$ and $r' = r_t$ where $t < \lambda - \alpha$. The rating mechanisms differ with respect to the information they provide, and by Lemma 1: $1 = \theta^*(r'(y)) \neq \theta^*(r(y)) = \lambda \forall y \in R_t^-$. However, this difference does not affect the probability of success, as $\mathbb{P}(I = 1|r(y), y) = \mathbb{P}(I = 1|r'(y), y) = 0$, $\forall y \in R_t^-$. The reason is that r' only provides new information when $y \in R_t^-$, but in this case fundamentals

are already so weak that default would have occurred in any case, independently of the CRA's contribution.

These observations motivate the following definitions. Given a rating mechanism r let $P[r]$ denote the function $\Delta \to [0,1]$:

$$P[r](y) = \mathbb{P}(I = 1|r(y), y) \tag{12}$$

Thus, $P[r](y)$ is the probability of success under rating mechanism r, as a function of the signal received by the CRA, y. We naturally think of $P[r] - P[r_0]$, that is, the difference relative to a situation without a CRA, as a measure of the effective impact of the rating mechanism r. In particular, $P[r] - P[r_0] = 0$ indicates that the rating mechanism r is without any impact, since it does not affect the probability of default, irrespective of the value of y. If on the other hand $P[r](y) - P[r_0] \geq 0$, $\forall y \in \Delta$, and $P[r](y) - P[r_0](y) > 0$ for some $y \in \Delta$, then the existence of the CRA unambiguously increases the occurrence of success. We will use the notation $P[r] > P[r_0]$ to signify this scenario, while $P[r] < P[r_0]$ indicates the alternative scenario in which the existence of the CRA unambiguously increases the occurrence of default.

Definition 2 *A rating mechanism r is* reducible *if either (a) $r \neq r_0$ and $P[r] = P[r_0]$, or (b) we can find a rating mechanism r' such that:*

1. $\#r'(\Delta) < \#r(\Delta)$
2. $\forall R' \in r'(\Delta)$, $\exists \Gamma \subset r(\Delta) : R' = \cup_{R \in \Gamma} R$
3. $\theta^*(r'(y)) = \theta^*(r(y))$, $\forall y \in \Delta$

Thus, a rating mechanism is reducible if either (a) it does not affect the probability of success $P[r] = P[r_0]$, or (b) it can be replaced by a simpler rating mechanism, with fewer distinct ratings, but which nevertheless induces identical outcomes.

Let ICIC denote the set of irreducible, consistent, and incentive compatible rating mechanisms. ICIC rating mechanisms have several attractive features. First, any ICIC rating mechanism entails a Perfect Bayesian Equilibrium between investors and the CRA, subject to the approximation that in updating their beliefs investors use uniform distributions throughout, see Definition 1. Second, to any Perfect Bayesian Equilibrium corresponds an ICIC rating mechanism. Third, while Perfect Bayesian Equilibria may involve irrelevant multiplicity, this is not true of ICIC mechanisms, since we specifically crafted our definition of irreducibility to circumvent the irrelevant multiplicity of Perfect Bayesian Equilibria. ICIC rating mechanisms

are thus precisely the rating mechanisms we seek in order to describe the overall equilibrium behavior of our model.

We devote the rest of this section to the study of ICIC rating mechanisms. In this setting: (i) investors understand the behavior of the CRA, and maximize expected returns given ratings' informational content; (ii) the CRA internalizes investors' response to credit ratings and chooses ratings which maximize expected profits.

Our first result is a pivotal step in the analysis. We show that – in equilibrium – the informational content of CRA reports is much poorer than that corresponding to CRAs' private information, effectively consisting of coarse and polarized reports of the form "good news" and "bad news". The intuition is simple and can be conveyed by an illustrative example. Suppose that the CRA gives an accurate rating which implies that the probability of default is 0.5. Can this happen in equilibrium? No, because this rating involves a probability of 0.5 of being incorrect ex post, and the CRA can do better. If the CRA instead gives a more negative rating, it will increase the probability of default, and thus also increase the probability of being correct ex post. Likewise, if the CRA gives a more positive rating, this will decrease the probability of default, and again increase the likelihood of being found correct ex post. In equilibrium the CRA will choose one of the two extremes, giving either a positive or a negative rating.[12] Hence, any ICIC rating mechanism is either r_0 or belongs to $\{r_t\}_{t\in\Delta}$. The following proposition summarizes this idea.

Proposition 1 *Any irreducible and consistent incentive compatible rating mechanism is either the empty rating r_0, or it is a threshold rating $\{r_t\}_{t\in\Delta}$. Formally, $ICIC \subset r_0 \cup \{r_t\}_{t\in\Delta}$.*

Thus, under our assumptions, the only ICIC rating mechanisms are either entirely non-informative, or they take the form of a threshold rating, which only says whether the signal of the CRA is above or below a given threshold.

3.3 The equilibrium impact of credit ratings

Having established how CRAs will choose to assign ratings, we can now turn to the main question of interest: How do credit ratings affect outcomes in equilibrium?

[12] This situation mirrors that of a gambler whose bets positively influence outcomes, so that the higher is the bet on A, the higher is the probability that A occurs. Let, to illustrate, $s \in [0,1]$ denote the bet placed on A, and $1-s$ that placed on $\rceil A$. The expected gain $g(s)$ is $s\mathbb{P}(A|s)+(1-s)(1-\mathbb{P}(A|s)) = (1-s)-(1-2s)\mathbb{P}(A|s)$, and $g'(s) = (2\mathbb{P}(A|s)-1) + 2s\big(d\mathbb{P}(A|s)/ds\big) > 2\mathbb{P}(A|s)-1$. In particular, if $s<1$ and $\mathbb{P}(A|s) \geq 1/2$, then $g(1) > g(s)$ and the gambler is strictly better off placing all chips on A. Similarly, if $s>0$ and $\mathbb{P}(A|s) \leq 1/2$, then $g(0) > g(s)$ and the gambler is strictly better off placing no chips on A at all. Either way, $s \in (0,1)$ is never optimal, just as no strategic CRA can ever report R if there exists R' and R'' such that $R' < R < R''$.

Our next proposition addresses the question and establishes that, in equilibrium, CRAs have a pro-cyclical impact: they tend to amplify underlying market conditions. In a liquidity boom CRAs help resolve investors' (mild) coordination problem, and lower the (already low) probability of default. In a liquidity crunch, on the other hand, CRAs worsen investors' (acute) coordination problem, and raise the (already high) probability of default.

Proposition 2 *In equilibrium, credit ratings amplify underlying market conditions.*

1. If $\lambda < 1 - \frac{\alpha}{2\beta}$ (liquidity boom) then *ICIC* $= \{r_0, r_{t^*}\}$ *for some* $t^* \in \Delta$. *In this case* $P[r_{t^*}] - P[r_0] > 0$ *for* $y \in (t^*, \lambda+\alpha)$, *while* $P[r_{t^*}] - P[r_0] = 0$ *for* $y < t^*$ *and for* $y > \lambda+\alpha$. *Moreover,* r_{t^*} *exhibits positive rating inflation.*

2. If $1 - \frac{\alpha}{2\beta} < \lambda < \frac{\alpha}{2\beta}$ then*:* *ICIC* $= \{r_0\}$.

3. If $\lambda > \frac{\alpha}{2\beta}$ (liquidity crunch) then *ICIC* $= \{r_0, r_{t^*}\}$ *for some* $t^* \in \Delta$. *In this case* $P[r_{t^*}] - P[r_0] < 0$ *for* $y \in (\lambda - \alpha, t^*)$, *while* $P[r_{t^*}] - P[r_0] = 0$ *for* $y < \lambda - \alpha$ *and for* $y > t^*$. *Moreover,* r_{t^*} *exhibits negative rating inflation.*

Part 1 states that for sufficiently low values of λ (a liquidity boom), the only informative ICIC rating mechanism is a threshold rating mechanisms which exhibits rating inflation, and which lowers the probability of default for some values of the CRA signal (and never raises it). Likewise, part 3 states that for sufficiently large values of λ (a liquidity crunch), the only informative ICIC rating mechanism is a threshold rating mechanism which exhibits negative rating inflation, and which raises the probability of default for some values of y (and never lowers it). The non-informative rating mechanism r_0 is always an ICIC, and for intermediate values of λ (part 2), it is also the only ICIC rating mechanism.

The intuition is the following. In a liquidity crunch (λ high) investors have good alternative use for their money, so only investors with optimistic beliefs regarding the project will contemplate refinancing it. The marginal investor, in particular, must hold optimistic information regarding fundamentals. "Good news" from the CRA then overlaps with his own information. Thus, a good rating will leave θ^* equal to the value it would take if no CRA existed. "Bad news", on the other hand, provides new and substantial information on the basis of which to revise beliefs, as shown in connection with Lemma 1 above. A bad report from the CRA will raise θ^*, thereby raising default risk. Thus, in a liquidity crunch CRAs' impact tends to be unfavorable.[13]

[13] With uniform distributions the asymmetric effect is taken to the extreme in the sense that a rating which overlaps with the beliefs of the marginal investor will have no effect on his posterior beliefs. With more general

Conversely, in a liquidity boom (λ low) investors have less immediate use for their money, so even investors with pessimistic beliefs will consider rolling over. The marginal investor, in particular, will tend to hold pessimistic information regarding fundamentals. "Bad news" from the CRA will then overlap with his own information, leaving θ^* unchanged. In contrast, "good news" will provide new and substantial information on the basis of which to revise beliefs, leading θ^* to fall. So in a liquidity boom CRAs' impact tends to be favorable.

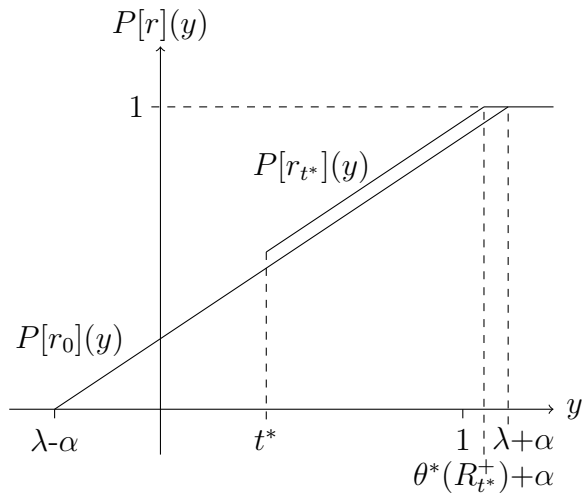
The rating inflation result reflects the asymmetry discussed above. Consider a situation in which the CRA observes signal $y = \lambda$. By lemma 1, if ratings were non-informative , the probability of default occurring would be 1/2, irrespective of the rating given by the CRA. However, as discussed above, if λ is low then a good rating affects investors' behavior, while a bad rating does not. Thus, by giving a good rating the CRA increases the probability of being correct more than it does by giving a bad rating. It follows that a CRA observing $y = \lambda$, where λ is low, will choose to give a good rating, implying that t^* is below λ. Similarly, if λ is high, then bad ratings affect investors' behavior while good ratings do not. So the CRA can increase the probability of being correct more by giving a bad rating than by giving a good rating. This provides incentives to give bad ratings, implying that t^* is above λ.

The right panel of Figure 1 illustrates the situation for high values of λ (liquidity crunch). If the CRA signal is good, $y > t^*$, the rating will be positive and it will have no effect on the probability of success. However, for lower values of the signal, $\lambda - \alpha < y < t^*$, the rating will be negative, and this will reduce the probability of success, $P[r_{t^*}] < P[r_0]$. For even lower values of y, the project will default irrespective of whether there is a rating or not. The left panel illustrates the case with a low value of λ (liquidity boom). If the CRA signal is bad, $y < t^*$, the rating will be negative and it will have no effect on the probability of success. However, for higher values of the signal, $t^* < y < \lambda + \alpha$, the rating will be positive, and this will increase the probability of success, $P[r_{t^*}] > P[r_0]$. For even higher values of y, the project will succeed irrespective of whether there is a rating or not.

distribution functions there will be an effect. However, the asymmetric impact of CRA reports in Proposition 2, that in a liquidity crunch the impact of "good news" from the CRA is much less than the impact of "bad news", and conversely in a liquidity boom, is robust to much more general distribution functions.

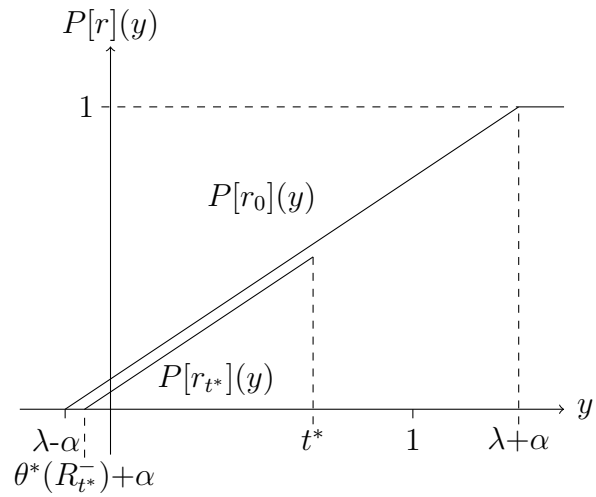
Figure 1: Credit ratings in a liquidity boom and a liquidity crunch

a) Liquidity boom, $\lambda < 1 - \frac{\alpha}{2\beta}$



For high CRA signals ($t^* < y < \lambda + \alpha$) $P[r_{t^*}] > P[r_0]$, so the rating will increase the probability of project success.

b) Liquidity crunch, $\lambda > \frac{\alpha}{2\beta}$



For low CRA signals ($\lambda - \alpha < y < t^*$) $P[r_{t^*}] < P[r_0]$, so the rating will reduce the probability of project success.

A numerical example provided in the appendix further illustrates the workings of Proposition 2.

3.4 CRA incentives and the effect of ratings

In this section we consider the effect of the CRAs' incentives on default outcomes. This relates to a recent debate where it is argued that CRAs are biased towards positive ratings, since their ratings are paid for by the issuers (see e.g. Pagano and Volpin, 2010). We capture this idea by allowing the CRA to prefer a positive outcome for the project: if the debtor defaults, he or she is less likely to ask the rating agency for a rating at a later occasion. However, one may also argue that CRAs should be more concerned about correctly predicting defaults, as a default after all is the less frequent and more serious outcome. These opposing concerns imply that CRAs in principle could be biased in either direction.

We will in this section limit our attention to threshold rating mechanisms, i.e. rating mechanisms r_t, $t \in \Delta$. Furthermore, as suggested above, we allow the payoff from correctly predicting success to differ from the payoff from correctly predicting default, and set:

$$\Pi(R_t^+, I) = \rho_1 I \tag{13}$$

$$\Pi(R_t^-, I) = \rho_0(1 - I) \tag{14}$$

A CRA predicting success receives payoff ρ_1 in case of success and 0 otherwise, while a CRA predicting default receives payoff ρ_0 in case of default and 0 otherwise. A CRA exhibiting $\rho_1 > \rho_0$ is thus biased towards success; one possibility is $\rho_1 = \rho_0 + \phi$, where $\phi > 0$ represents a fee paid by issuers in case of success (whether now or in the form of future business). In contrast, $\rho_1 < \rho_0$ indicates a bias towards default, i.e. a cautious or negative bias, which may reflect that the CRA expects to lose more – reputation-wise – from luring investors into faulty projects than from luring them away from successful ones.

Knowing that a CRA may affect the project outcome, one might expect that the result would be that the CRA pushes the outcome in the desired direction. Thus, one might expect that a CRA which prefers a successful outcome is more inclined to report a positive signal, and thus is able to coordinate investors towards success. However, this intuition does not prevail in equilibrium. While we find that CRAs with a positive bias tend to give more positive ratings, they also tend to raise the probability of default. In contrast, CRAs with a negative bias tend to lower the probability of default.

Proposition 3 *CRA biases*

1. *If $\frac{\rho_0}{\rho_1} < \frac{\frac{\beta}{\alpha}\lambda}{1-\frac{\beta}{\alpha}\lambda}$ ("positive bias") then there exists a unique $t^* \in \Delta$ such that $r_{t^*} \in ICIC$. In this case $P[r_{t^*}] - P[r_0] < 0$ for $y \in (\lambda - \alpha, t^*)$, while $P[r_{t^*}] - P[r_0] = 0$ for $y < \lambda - \alpha$ and for $y > t^*$.*

2. *If $\frac{\rho_0}{\rho_1} > \frac{1-\frac{\beta}{\alpha}(1-\lambda)}{\frac{\beta}{\alpha}(1-\lambda)}$ ("negative bias") then there exists a unique $t^* \in \Delta$ such that $r_{t^*} \in ICIC$. In this case $P[r_{t^*}] - P[r_0] > 0$ for $y \in (t^*, \lambda + \alpha)$, while $P[r_{t^*}] - P[r_0] = 0$ for $y < t^*$ and for $y > \lambda + \alpha$.*

3. *t^* increases with $\frac{\rho_0}{\rho_1}$. In particular if ρ_0/ρ_1 is sufficently low, then the unique threshold rating mechanism exhibits positive rating inflation, while the unique threshold rating mechanism exhibits negative rating inflation if ρ_0/ρ_1 is sufficently high.*

Part 1 states that if $\frac{\rho_0}{\rho_1}$ is sufficiently low, the unique threshold rating mechanism never lowers the probability of default and, for some values of the CRA signal, strictly raises it. Correspondingly, part 2 states that if $\frac{\rho_0}{\rho_1}$ is sufficiently high, the unique threshold rating mechanism never raises the probability of default and, for some values of the CRA signal, strictly lowers it. It follows in other words from proposition 3 that any bias on the part of CRAs is self-defeating: a positive bias makes default more likely, while a negative bias makes default less likely. While at first counterintuitive, the underlying idea is simple. For example,

a CRA with a positive bias naturally exhibits optimism. But, in equilibrium, that fact is known to investors who therefore accordingly (i) discount "good news" from the CRA and (ii) mark-up "bad news". As a result good ratings have no impact on investors' behavior. By contrast, bad ratings, given in spite of the CRA's inclination, tend to impact investors' behavior strongly.

Observe finally that increasing ρ_1 has two opposing consequences. By part 3 of the proposition increasing ρ_1 lowers t^*. Bad ratings consequently imply worse news for investors, which in turn strengthens the negative impact that bad ratings have on default risk. On the other hand it follows from part 1 that the range over which this negative impact can be felt, namely $y \in (\lambda - \alpha, t^*)$, diminishes accordingly. Whether the overall effect raises or lowers average default risk depends. In particular, t^* converges to $\lambda - \alpha$ as $\frac{\rho_0}{\rho_1}$ converges to zero. Thus, in this extreme scenario, bad ratings are only given to firms who would have defaulted with probability one anyway.

4 Discussion

We here relate our findings to the empirical literature on credit ratings, and discuss policy implications.

4.1 Relation to Empirical Evidence

Our model provides three empirical predictions that may be cast against evidence provided elsewhere. Proposition 1 implies that ratings at most indicate whether a project is "good" or "bad". Hence, our first prediction is that ratings provide only coarse information. Our second prediction follows from Proposition 2: Both ratings' impact and standards are pro-cyclical. Finally, both Proposition 2 and Proposition 3 reveal that in our model it is never the case that both good and bad ratings matter. Either only bad ratings matter, or only good ratings matter, or none of the two matter. Hence, our third prediction is that the effect of credit ratings is asymmetric.

Empirical support for the coarseness of credit ratings can be found in the latter's performance as predictor of default probability. Hilscher and Wilson (2012) find that ratings are poor predictors of default probability, and conclude that "[ratings] are dominated by a simple default prediction model based on publicly available accounting and market based measures; they explain little of of the variation in default probability across firms.′ Bussire and Ristiniemi (2012) reach similar conclusions. Our analysis also implies that different ratings might in ef-

fect convey identical information to investors (say, for example, all ratings going from Baa to Aaa). These findings are consistent with Kiff, Nowak and Schumacher, (2012), as well as Goh and Ederington (1999), who find that most rating events do not move prices. At a less formal level, Pagano and Volpin (2010) provide substantial anecdotal evidence illustrating the coarseness of credit ratings.[14]

Pro-cyclical ratings standards are implied by the results in Ashcraft, Goldsmith-Pinkham and Vickery (2009). They find a progressive decline in rating standards around the MBS market peak between the start of 2005 and mid-2007, i.e. prior to the financial crisis when credit was cheap and in abundant supply. Furthermore, Griffin and Tang (2012) show that from 2003 to 2007, CRAs frequently made upward adjustments to the their models' assessments of rated objects, and that the extent of the adjustments increased substantially over the period. As one might expect, assets with larger adjustments initially experienced more severe downgrading later on.

That CRAs propagate underlying market conditions was suggested by Ferri, Liu and Stiglitz (1999), in the context of the East Asian crisis: "The results from our econometric model illustrate that rating agencies attached higher weights to their qualitative judgement than to the economic fundamentals [...], such behaviour may have helped to exacerbate the boom and bust cycle in East Asia.′. Related views have been held more recently by a number of politicians and independent observers in the context of the Eurozone crisis, as illustrated by the Merkel and Sarkozy quote in the introduction. Another example is Helmut Reisen (2010) Head of Reseach, OECD Development Centre who conclude that: "Unless sovereign ratings can be turned into proper early warning systems, they will continue to add to the instability of international capital flows, to make returns to investors more volatile than they need be, and to reduce the benefits of capital markets for recipient countries.′ The empirical literature is however somewhat divided on the issue. Reisen and von Maltzan (1999) suggest that CRAs affect markets in a more neutral manner, and claim that CRAs fail to utilize their potential to dampen market euphoria.

Asymmetric effects of credit ratings have been widely documented. The pattern was first emphasized by Holthausen and Leftwich (1986), while later studies finding asymmetries include Galil and Soffer (2011), Afonso, Furceri and Gomes (2012), Hull et. al (2004), Norden and Weber (2004). The common finding here is that it is primarily negative rating events that shift prices, whereas the effects of positive events are weak or non-existent. Our model

[14]Pagano and Volpin (2010) use coarse in the sense that the ratings provide much less details about the assets than what might be useful to assess the value and risk of the assets. Furthermore, the authors argue that the CRAs use too crude measures to assess the risk.

can explain such an asymmetry.[15] In particular, for a given state of the market, Proposition 3 implies that if CRAs are biased in favor of issuers, which is a widely held view (see e.g. Pagano and Volpin, 2010, and Bolton et.al., 2012), only bad ratings will matter. Likewise, Proposition 2 implies that even if CRAs are unbiased, the same asymmetry occurs if markets conditions are weak. Moreover, our theory also implies that asymmetries may in principle go in the opposite direction, with good ratings mattering more than bad ones. The study by Ismailescu and Kazemi (2010) is one of the few that finds such asymmetry. They consider sovereign debt of emerging economies. In light of our model, one might hypothesize that their findings are driven by a negative CRA bias for this specific asset class.

More generally, by tying asymmetries to specific sources, namely CRA incentives and underlying market conditions, our theory could provide useful guidance for future empirical work in the field. For instance, for assets where CRAs are more likely to be biased toward issuers, negative rating events should have a stronger effect than positive rating events. If ratings are unsolicited, positive rating events should matter relatively more.

4.2 Policy implications

According to our findings, the desire to be correct ex post will make ratings coarse. This contrasts sharply with the argument often heard from CRAs themselves, namely that reputation concerns disciplines them to provide informative ratings (see the references in Mathis et. al, 2009). Thus, our results support current policy initiatives in the U.S. and the EU aiming to reduce the importance of CRAs and improve the quality of ratings, see Securities and Exchange Commission, 2011; European Commission, 2011. Precisely which institutional changes to implement is not addressed by our model.

Our paper also contributes to the ongoing debate on how CRAs should be paid. A widely held concern is that existing remuneration schemes, in which CRAs typically are paid by issuers, cause too generous ratings.[16] To resolve this issue, one suggestion has been that investors – not issuers – should finance CRAs. Our Proposition 3 sides with this view, but from a different angle than the arguments that have been used so far: Credit ratings can serve as a welfare improving coordination device, but only if agencies have a sufficiently negatively biased incentive structure. Importantly, this implication directly contradicts what one might expect,

[15] Of course, given that our model is static, we cannot in the strict sense account for downgrades or upgrades. However, if a CRA has given a rating in the past then as time elapses this rating looses relevance. We therefore argue that by the time a new rating is announced, the situation may to some extent be assimilated to one where no previous rating existed.

[16] For an overview of the debate, see e.g. Mathis, McAndrews and Rochet (2009) and Pagano and Volpin (2010).

namely that a positive bias is desirable for CRAs to prevent inefficient liquidations. Manso (2011) makes this argument forcefully. The reason we conclude oppositely is that we endogenize the response of investors to credit ratings, whereas Manso treats it as given. Now, the latter argument may well be realistic for institutionally constrained investors, who are forced to sell once assets drop below investment grade. The relevance of our conclusion will depend on the role played by ratings in regulation, and will clearly increase if the aforementioned policy proposals are passed.

A final implication from our analysis regards the role of banning ratings in times of crisis. According to Proposition 2, this could make sense, since the coordinating effect of ratings is procyclical. However, the correct interpretation here is that it is a general ban of ratings, linked to publicly available information such as tight liquidity, that our results support. A selective ban on specific assets, or a banning policy which is not tied to public information, is unadvisable as it would provide new information which in itself might negatively affect investors. The relevance of this point is highlighted by the recent announcement by markets commissioner Michel Barnier, who stated that the EU would let the European Securities and Markets Authority (ESMA) have "the possibility of suspending temporarily the notation of a country for two months, if it considers the notation would aggravate or accelerate the instability or irrationality of the markets." [17] Such a discretionary, judgemental, and selective banning policy is not what our model supports. Should a banning policy be introduced, it must be rules based, contingent on publicly observable variables such as market liquidity indicators, and cover broad asset classes.

More generally, because CRAs will ultimately exacerbate booms and busts, our results give additional support to the many proposals for policies to prevent systemic risk taking.

5 Conclusion

Our findings indicate that reputation concerns on the part of credit rating agencies (CRAs) far from constitute a panacea to the industry's shortcomings. First, reputation concerns motivate CRAs to give ratings that push outcomes in one direction, with a view to vindicate ratings ex post. In equilibrium, when investors realize this effect, CRAs are only able to provide very coarse information, simply indicating whether rated objects are above or below some minimum standard. Second, reputation concerns induce CRAs to exacerbate underlying market

[17] http://economictimes.indiatimes.com/articleshow/10732356.cms

As yet this idea is not implemented - in a press release 15 November 2011, the EU commission states that "The possible suspension of sovereign ratings is a complex issue which we believe merits further consideration.

conditions. In situations where investors have limited need for liquidity, and it therefore is relatively unproblematic to refinance projects, CRAs further reduce coordination risk. On the other hand, in a liquidity squeeze where coordination is a severe problem, CRAs only make matters worse.

Furthermore, our analysis reveals that while it is explicitly desirable that CRAs stimulate investment, it need not be desirable that they are favorably inclined to issuers. On the contrary, when CRAs' effect is purely informational, their incentive structure should be conservative, since otherwise investors will just ignore positive ratings as reflecting CRAs' biased incentives. While in practice, this implication must be balanced against other concerns, it forcefully shows that a discussion of the appropriate incentive structure of CRAs cannot treat their impact on real outcomes as given.

Our analysis focuses solely on the information-providing role of CRA, and ignores their potential effects through regulation. Combining the two channels seems a fruitful avenue for further research.

References

Afonso, A., D. Furceri, and P. Gomes (2012): "Sovereign credit ratings and financial market linkages: Application to European data," *Journal of International Money and Finance*, 31, 606-638.

Ashcraft, Adam, P. Goldsmith-Pinkham, and J. Vickery (2010): "MBS ratings and the mortgage credit boom," Federal Reserve Bank of New York Staff Reports no. 449.

Bar-Isaac, H., and J. Shapiro (forthc.): "Ratings Quality over the Business Cycle," *Journal of Finance*, forthcoming.

Bolton, P., X. Freixas, and J. Shapiro (2012): "The Credit Ratings Game," *Journal of Finance*, 67, 85-112.

Boot, A. W. A., T. T. Milbourn, and A. Schmeits (2006): "Credit ratings as coordination mechanisms," *Review of Financial Studies*, 19, 81-118.

Bussire, M., and A. Ristiniemi (2012): "Credit ratings and debt crises," Banque de France Working Papers no. 396.

Carlson, M., and G. B. Hale (2006): "Rating agencies and sovereign debt rollover," *Topics in Macroeconomics*, 6, article 8.

Carlson, H., and E. Van Damme (1993): "Global Games and Equilibrium Selection," *Econometrica*, 61, 989-1018.

Corsetti, G., B. Guimarães, and N. Roubini (2006): "The tradeoff between an international lender of last resort to deal with liquidity crisis and moral hazard distortions: A model of the IMF's catalytic finance approach," *Journal of Monetary Economics*, 53, 441-471.

Diamond, D. W., and P. H. Dybvig (1983): "Bank runs, deposit insurance, and liquidity," *Journal of Political Economy*, 91(3), 401-419.

Faure-Grimaud, A., E. Peyrache, and L. Quesada (2009): "The ownership of ratings," *The Rand Journal of Economics*, 40, 234-257.

Ferri, G, L. G. Liu, and J. E. Stiglitz (1999): "The procyclical role of rating agencies: Evidence from the East Asian crisis, " *Economic Notes by Banca Monte dei Paschi di Siena SpA*, 28, no 3, 335-255.

Galil, K., and G. Soffer (2011): "Good news, bad news and rating announcements: An empirical investigation," *Journal of Banking and Finance*, 35, 3101-3119.

Goh, J. C., and L. Ederington (1999): "Cross-sectional variation in the stock market reaction to bond rating changes," *The Quarterly Review of Economics and Finance*, 39, 101-112.

Griffin, J. M., and D. Y. Tang (2012): "Did subjectivity play a role in CDO Credit Raings?" *Journal of Finance*, 67, 1293-1328.

Hellwig, C., A. Mukherji, and A. Tsyvinski (2006): "Self-fulfilling currency crises: The role of interest rates," *American Economic Review*, 96(5), 1769-1787.

Hilscher, J., and M. I. Wilson (2012): "Credit Ratings and Credit Risk," mimeo.

Holthausen, R. W., and R. W. Leftwich (1986): "The effect of bond rating changes on common stock prices," *Journal of Financial Economics*, 17, 57-89.

Hull, J., M. Pedrescu, and A. White (2004): "The relationship between credit default swap spreads, bond yields and credit rating announcements," *Journal of Banking and Finance*, 28, 2789-2811.

Ismailescu, I., and H. Kazemi (2010): "The reaction of emerging market credit default swap spreads to sovereign credit rating changes," *Journal of Banking and Finance*, 34, 2861-2873.

Kiff, J., Nowak, S., and L. Schumacher (2012): "Are rating agencies powerful? An investigation into the impact and accuracy of sovereign ratings," IMF working paper 12/23.

Manso, G. (2011): "Feedback effects of credit ratings," mimeo.

Mariano, B. (2012): "Market power and reputational concerns in the ratings industry," *Journal of Banking and Finance*, 36, 1616-1626.

Mathis, J., J. McAndrews, and J.C. Rochet (2009): Rating the raters: Are reputation concerns powerful enough to discipline rating agencies?" *Journal of Monetary Economics,* 56, 657-674.

Morris, S., and H. S. Shin (2003): "Global Games: Theory and applications," in *Advances in Economics and Econometrics*, ed. by M. Dewatripont, L. Hansen and S. Turnovsky. Cambridge University Press.

Morris, S., and H. S. Shin (2004): "Coordination Risk and the Price of Debt," *European Economic Review,* 48(1), 133-53.

Morris, S., and H. S. Shin (2006): "Catalytic finance: When Does it Work?", *Journal of International Economics*, 70, 161-177.

Norden, L. and M. Weber (2004): "Informational efficiency of credit default swap and stock markets: The impact of credit rating announcements," *Journal of Banking and Finance*, 28, 2813-2843.

Pagano, M., and P. Volpin (2010): "Credit ratings failures and policy options," *Economic Policy*, April, 401-431.

Reisen, H., (2010): "Boom, bust and sovereign ratings: Lessons for the Eurozone from emerging-market ratings. " VOX 10 May 2010. http://www.voxeu.org/article/boom-bust-and-sovereign-ratings-lessons-eurozone-emerging-market-ratings

Reisen, H., and J. von Maltzan(1999): "Boom and bust and sovereign ratings "

International Finance, 2:2, 273-293.

Sangiorgio F., J. Sokobin, and C. Spatt (2009): "Credit -rating shopping, selection and equilibrium structure of ratings," mimeo.

Skreta, V., and L. Veldkamp (2009): "Ratings shopping and asset complexity: A theory of ratings inflation," *Journal of Monetary Economics*, 56, 678-695.

Appendix

Proof of Lemma 1: Say that an equilibrium of the investment game is interior if in this equilibrium an investor's behavior is contingent on his private signal x. Otherwise say that the equilibrium is a corner equilibrium.

As noted in the main text, any interior equilibrium is characterized by a pair (x^*, θ^*) satisfying equations (4) and (11), and repeated here:

$$\begin{cases} \theta^* = \mathbb{P}(x < x^* | \theta = \theta^*) \\ \mathbb{P}(\theta > \theta^* | \theta \in [x^* - \beta, x^* + \beta] \cap [\underline{r} - \alpha, \overline{r} + \alpha]) = \lambda \end{cases} \tag{15}$$

Note in particular that the second equation of the system defines a broken line in the (x^*, θ^*)-plane joining A, B, C, D where

$$A = (\underline{r} - \alpha - \beta, \underline{r} - \alpha)$$

$$B = (\underline{r} - \alpha + \beta, \underline{r} - \alpha + 2\beta(1 - \lambda))$$

$$C = (\overline{r} + \alpha - \beta, \overline{r} + \alpha - 2\beta\lambda)$$

$$D = (\overline{r} + \alpha + \beta, \overline{r} + \alpha)$$

By (15) an interior equilibrium thus exists if and only if (i) $\overline{r} + \alpha > 1$, and (ii) $\underline{r} - \alpha < 0$. If $\overline{r} + \alpha \leq 1$, a corner equilibrium exists in which all investors liquidate, irrespective of private signals. Similarly, if $\underline{r} - \alpha \geq 0$ then a corner equilibrium exists in which all investors roll over, irrespective of private signals. Hence an equilibrium always exists. Furthermore the equilibrium is unique since any time (i) and (ii) hold corner equilibria are precluded (investors receiving private signal above 1 must roll over while investors receiving private signal below 0 must liquidate).

Properties 1-6 follow from immediate computation using (15).

■

Proof of Proposition 1: Let $r \in ICIC$, $r \neq r_0$. Suppose $\#r(\Delta) \geq 3$ and let $i, j, k, l \in \Delta$ such that $r(\Delta) \supseteq \{(\underline{\delta}, i), (j, k), (l, \overline{\delta})\}$. To shorten notation, let respectively $R^- = (\underline{\delta}, i)$, $R = (j, k)$, and $R^+ = (l, \overline{\delta})$. Furthermore, $p(R, y) = \mathbb{P}(I = 1 | R, y)$.

Either $p(R, y) \geq 1/2$ for all $y \in R$ (case 1) or there exists $y \in R$ such that $p(R, y) < 1/2$ (case 2).

Case 1: Observe that $p(R, y) = 1$ for all $y \in R$ implies $\theta^*(R) = 0$. This in turn implies $\theta^*(R^+) = 0$, owing to Lemma 1. But then r can be reduced, which we assumed it cannot.

So there exists $y \in R$ with $1/2 \leq p(R,y) < 1$. Furthermore, by irreducibility of r: $\theta^*(R^+) < \theta^*(R)$. So there exists $y \in R$ with $1/2 \leq p(R,y) < p(R^+,y) \leq 1$. But then: (in the first equality we use that $\Pi^e(R,y) = p(R,y)m(R) + \Big(1 - p(R,y)\Big)\big(1 - m(R)\big)$, and $m(R^+) = 1$)

$$\begin{aligned}
\Pi^e(R^+,y) - \Pi^e(R,y) &= p(R^+,y) - p(R,y)m(R) - \Big(1 - p(R,y)\Big)\big(1 - m(R)\big) \\
&= \Big(p(R^+,y) - p(R,y) + p(R,y)\Big) - p(R,y)m(R) - \Big(1 - p(R,y)\Big)\big(1 - m(R)\big) \\
&= \Big(p(R^+,y) - p(R,y)\Big) + p(R,y)\big(1 - m(R)\big) - \Big(1 - p(R,y)\Big)\big(1 - m(R)\big) \\
&= \Big(p(R^+,y) - p(R,y)\Big) + \Big(2p(R,y) - 1\Big)\big(1 - m(R)\big) \\
&> 0 \qquad (16)
\end{aligned}$$

Where the last inequality contradicts incentive compatibility of r.

Case 2: $p(R,y) = 0$ for all $y \in R$ implies $\theta^*(R) = 1$, which in turn owing to Lemma 1 implies $\theta^*(R^-) = 1$. But then r can be reduced, which we assumed it cannot. So there exists $y \in R$ with $0 < p(R,y) < 1/2$. Furthermore, by irreducibility of r: $\theta^*(R^-) > \theta^*(R)$. So there exists $y \in R$ with $0 \leq p(R^-,y) < p(R,y) < 1/2$. But then:

$$\begin{aligned}
\Pi^e(R^-,y) - \Pi^e(R,y) &= \Big(1 - p(R^-,y)\Big) - p(R,y')m(R) - \Big(1 - p(R,y)\Big)\big(1 - m(R)\big) \\
&= \Big(1 - p(R^-,y) + p(R,y) - p(R,y)\Big) - p(R,y)m(R) - \Big(1 - p(R,y)\Big)\big(1 - m(R)\big) \\
&= \Big(p(R,y) - p(R^-,y)\Big) + \Big(1 - p(R,y)\Big)m(R) - p(R,y)m(R) \\
&= \Big(p(R,y) - p(R^-,y)\Big) + \Big(1 - 2p(R,y)\Big)m(R) > 0 \qquad (17)
\end{aligned}$$

Where the last inequality contradicts incentive compatibility of r.

■

Proof of Proposition 2: We know from Proposition 1 that $ICIC \subset r_0 \bigcup \{r_t\}_{t \in \mathbf{R}}$.

Note next that for any t, r_t is consistent. This is by definition. Moreover, note that r_t is incentive compatible if and only if $\Pi^e(R_t^+,t) = \Pi^e(R_t^-,t)$. This follows from the fact that $\Pi^e(R_t^+,y) = \mathbb{P}(I = 1|R_t^+,y)$ is non-decreasing in y, while $\Pi^e(R_t^-,y) = \big(1 - \mathbb{P}(I = 1|R_t^-,y)\big)$ is non-increasing in y. Let us thus examine the solution set of equation

$$\mathbb{P}(I = 1|R_t^+,t) = \big(1 - \mathbb{P}(I = 1|R_t^-,t)\big) \qquad (18)$$

By Lemma 1 the LHS of the equation is 1 for $t - \alpha > 0$ and 0 for $t + \alpha < \lambda$, while the RHS

of the equation is 1 for $t+\alpha<1$ and 0 for $t-\alpha>\lambda$. Moreover, the LHS of the equation is strictly increasing for $t\in[\lambda-\alpha,\alpha]$ and the RHS of the equation strictly decreasing for $t\in[1-\alpha,\lambda+\alpha]$. The two curves thus cross exactly once, and there is a unique t such that r_t is incentive compatible. Let t^* denote this unique t. This establishes that $ICIC\subset\{r_0,r_{t^*}\}$. Moreover r_{t^*} is consistent by definition and incentive compatible by construction. So we are only left to elicit the conditions under which r_{t^*} is irreducible, which we now undertake to do.

r_{t^*} can be reduced to r_0 if both rating mechanisms induce identical threshold θ-values, for all y. From Lemma 1 this will be avoided if and only if one of the following two conditions holds:

1. $t^*+\alpha<(2\beta+1)\lambda$ (in which case $\theta^*(R^-_{t^*})>\lambda$)
2. $t^*-\alpha>(2\beta+1)\lambda-2\beta$ (in which case $\theta^*(R^+_{t^*})<\lambda$)

Observe next that condition 1 is equivalent to

$$\mathbb{P}(I=1|R^+_t,y=t=(2\beta+1)\lambda-\alpha)>\big(1-\mathbb{P}(I=1|R^-_t,y=t=(2\beta+1)\lambda-\alpha)\big) \tag{19}$$

while condition 2 is equivalent to

$$\mathbb{P}(I=1|R^+_t,y=t=(2\beta+1)\lambda-2\beta+\alpha)<\big(1-\mathbb{P}(I=1|R^-_t,y=t=(2\beta+1)\lambda-2\beta+\alpha)\big) \tag{20}$$

Substituting in (19) using Lemma 1 gives

$$\frac{[(2\beta+1)\lambda-\alpha]+\alpha-\lambda}{2\alpha}>1-\frac{[(2\beta+1)\lambda-\alpha]+\alpha-\lambda}{2\alpha}$$

simplifying to

$$2\beta\lambda>\alpha$$

Substituting in (20) using Lemma 1 gives

$$\frac{[(2\beta+1)\lambda-2\beta+\alpha]+\alpha-\lambda}{2\alpha}<1-\frac{[(2\beta+1)\lambda-2\beta+\alpha]+\alpha-\lambda}{2\alpha}$$

simplifying to

$$2\beta\lambda<2\beta-\alpha$$

r_{t^*} can also be reduced to r_0 if $P[r_{t^*}]=P[r_0]$, so it remains to show that whenever either one of conditions 1 and 2 holds, then $P[r_{t^*}]\neq P[r_0]$. Suppose to fix ideas that condition 1

holds (the same reasoning applies if condition 2 holds instead of condition 1), so that $\theta^*(R^-_{t^*}) > \lambda = \theta^*(R^+_{t^*})$. Observe then that $P[r_{t^*}] = P[r_0]$ implies $\mathbb{P}(\lambda < \theta < \theta^*(R^-_{t^*})|y = t^*) = 0$. But then either $\mathbb{P}(I = 1|R^-_{t^*}, t^*) = \mathbb{P}(I = 1|R^+_{t^*}, t^*) = 0$ or $\mathbb{P}(I = 1|R^-_{t^*}, t^*) = \mathbb{P}(I = 1|R^+_{t^*}, t^*) = 1$. Either way, we obtain a contradiction to (18). This finishes to show that r_{t^*} is irreducible.

The rest of the proposition follows immediately from lemma 1, and the observation that

$$t^* = \frac{\theta^*(R^-_{t^*}) + \theta^*(R^+_{t^*})}{2} \tag{21}$$

■

Proof of Proposition 3: The proof follows the steps from Proposition 2, substituting (18) with

$$\rho_1 \mathbb{P}(I = 1|R^+_t, t) = \rho_0\big(1 - \mathbb{P}(I = 1|R^-_t, t)\big) \tag{22}$$

That t^* increases with $\frac{\rho_0}{\rho_1}$ is immediate from the above equation.

■

Example 1 *The following example aims to illustrate the workings of Proposition 2. Let $\alpha = \beta = 3/4$. We begin by characterizing equilibrium for $\lambda = 1/2$, with and without a CRA. Then we study the same two scenarios when $\lambda = 3/5$, i.e. a liquidity crunch. We will show that for $\lambda = 1/2$, the CRA plays no role and equilibria with and without CRA are identical, while for $\lambda = 3/5$ the CRA does affect equilibrium behavior, in the following way: "good news' from the CRA leaves investors' behavior unchanged, while "bad news' from the CRA worsens the coordination problem and raises θ^* over and above its value without the CRA.*

(a) Without CRA, $\lambda = 1/2$:

Equilibrium behavior without CRA is given from equations (9) and (10), i.e. $\theta^* = \lambda$ and $x^* = \lambda(2\beta + 1) - \beta$, which yield $\theta^* = 1/2$, $x^* = 1/2$.

(b) With CRA, $\lambda = 1/2$:

From Proposition 2, we know that the empty rating r_0 is the only ICIC. Thus, the outcome is as without a CRA, so from the analysis in a), we obtain $\theta^* = 1/2$, $x^* = 1/2$.

(c) Without CRA, $\lambda = 3/5$:

The same analysis as in (a) gives us $\theta^* = 3/5$, $x^* = 3/4$.

(d) With CRA, $\lambda = 3/5$:

We are here in case 3 of Proposition 2. We thus know that there exists t^* such that $r_{t^*} \in ICIC$, $r_{t^*} \neq r_0$. Let us compute this t^*. Of course t^*, $\theta^*(R^-_{t^*})$, $\theta^*(R^+_{t^*})$, $x^*(R^-_{t^*})$, and $x^*(R^+_{t^*})$ are jointly determined, so to compute one we must compute all. To shorten notation, in what follows we use $R^- = R^-_{t^*}$, $R^+ = R^+_{t^*}$, $\theta^- = \theta^*(R^-_{t^*})$, $\theta^+ = \theta^*(R^+_{t^*})$, $x^- = x^*(R^-_{t^*})$, $x^+ = x^*(R^+_{t^*})$.

Note to begin with that Proposition 2 and Lemma 1 directly imply $\theta^+ = \theta^*(\Delta) = \lambda$. Thus, if the CRA gives a good rating, it does not affect the beliefs of the marginal investor and consequently it has no impact on the critical θ-value. At $y = t^*$ the CRA must be indifferent between announcing R^- or R^+. This implies that

$\Pi^e(R^+, t^*) = P(\theta > \theta^+|t^*) = 1 - P(\theta > \theta^-|t^*) = \Pi^e(R^-, t)$, *i.e. that:*

$$\frac{t^* + \alpha - \theta^+}{2\alpha} = 1 - \frac{t^* + \alpha - \theta^-}{2\alpha} \tag{23}$$

which solves for

$$t^* = \frac{\theta^- + \theta^+}{2} = \frac{\theta^- + \lambda}{2} \tag{24}$$

We see from (24) that the CRA threshold t^* is equal to the average of the critical values θ^+ and θ^-. As we will see below, a negative rating R^- will push θ^- up, and this will also increase t^*.

As usual, we also have the system of simultaneous equations (4) and (11), now with the additional qualification that $[x^* - \beta, x^* + \beta] \cap [\underline{r} - \alpha, \overline{r} + \alpha] = [x^- - \beta, x^- + \beta] \cap [\underline{\delta}, t^* + \alpha] = [x^- - \beta, t^* + \alpha]$, so that:

$$\begin{cases} \theta^- = \mathbb{P}(x < x^-|\theta = \theta^-) \\ \mathbb{P}(\theta > \theta^-|\theta \in [x^- - \beta, t^* + \alpha]) = \lambda \end{cases} \tag{25}$$

Substituting (24) into (25) yields:

$$\begin{cases} \theta^- = \frac{x^- + 2\beta}{2\beta + 1} \\ \lambda = \frac{\frac{\theta^- + \lambda}{2} + \alpha - \theta^-}{\frac{\theta^- + \lambda}{2} + \alpha - (x^- - \beta)} \end{cases} \tag{26}$$

From (26) we first retrieve:

$$\theta^- = \frac{2\beta\lambda - (1 - \lambda)(\alpha + \frac{\lambda}{2})}{\lambda(2\beta + \frac{1}{2}) - \frac{1}{2}},$$

and subsequently t^* from (24) and x^- from the first equation in system (25). We find $\theta^- = 24/35$, $x^- = 27/28$, $\theta^+ = 3/5$, $x^+ = 3/4$ and $t^* = 9/14$.

Thus, if the CRA obtains a signal $y < t^* = 9/14$, it will give a bad rating, which constrains the beliefs of the marginal investor. The critical θ-value will increase from $\theta^* = \lambda = 3/5$ to $\theta^*(R^-_{t^*}) = 24/35$, thus increasing the probability that the project defaults.

Note also that $t^* < x^+$, which ensures that a positive rating from the CRA, indicating $y > t^*$, overlaps with the marginal investor's information in this case.