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Myths and Facts About the Alleged Over-Pricing of U.S. Real Estate. Evidence from Multi-Factor Asset Pricing Models of REIT Returns

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Myths and Facts about the Alleged Over-Pricing of U.S. Real Estate

Evidence from Multi-Factor Asset Pricing Models of REIT Returns*

Massimo Guidolin[†], Francesco Ravazzolo[‡] and Andrea Donato Tortora[§]

This version: December 2011

Abstract

This paper uses a multi-factor pricing model with time-varying risk exposures and premia to examine whether the 2003-2006 period has been characterized, as often claimed by a number of commentators and policymakers, by a substantial missprcing of publicly traded real estate assets (REITs). The estimation approach relies on Bayesian methods to model the latent process followed by risk exposures and idiosynchratic volatility. Our application to monthly, 1979-2009 U.S. data for stock, bond, and REIT returns shows that both market and real consumption growth risks are priced throughout the sample by the cross-section of asset returns. There is weak evidence at best of structural misspricing of REIT valuations during the 2003-2006 sample.

Key words: REIT returns, Bayesian estimation, Structural instability, Stochastic volatility, Linear factor models.

JEL codes: G11, C53.

1. Introduction

Countless researchers, policy-makers, and commentators have recently taken as a fact that the 2003-2006 period was allegedly marked by massive and systematic over-pricing of U.S. real estate, including public real estate vehicles, such as REITs.[1] The sudden swing in the market to re-absorb such a mispricing would have been at the root of the "Great Financial Crisis" of 2007-2009 (henceforth, GFC). Yet, despite casual evidence of excesses and poor practices in the housing and mortgage

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[1]A few commentators have used the term "bubble" to refer to such a state of large and ever growing over-pricing, followed by a sudden decline, between 2007 and 2009. See e.g., Shiller (2009). In our paper we will refrain from using the technical notion of bubble as this would require the adoption of specific pricing frameworks and testing methodologies (see e.g., Scott, 1990) that are less general than the ones we pursue in our paper.

industries, real estate finance research has yet to document the existence and magnitude of the mispricing of the (spot) real estate asset class as a whole, in the period 2003-2006.

In our paper, we extend the methodologies and results in the literature on multi-factor, ICAPM-style models (see e.g., Ling and Naranjo, 1997, and Karolyi and Sanders, 1998) to investigate whether there is any evidence of systematic over-pricing of various categories of REITs in an asset pricing framework that is simultaneously estimated to price a wide range of equity and bond portfolios. Our extension is based on a Bayesian style, Gibbs sampling estimation approach that allows us to obtain the joint estimates of (the posterior distribution of) risk exposures and of risk premia in a single-step, that preserves consistency and strives to avoid well-known limitations of the standard, two-step Fama-MacBeth approach.[2]

When the multi-factor framework is generalized to include a range of standard macroeconomic factors (the excess return on the value-weighted market portfolio; the credit risk premium; the term premium; the unexpected inflation rate; the rate of growth of industrial production; the rate of growth of real personal consumption; the 1-month real T-bill rate) that are assumed to drive the stochastic discount factor in a linear fashion, we find no evidence of the alleged systematic overpricing of the REIT asset class over the 2003-2006 period. The overpricing of REITs as an asset class would have been stronger and more persistent in the late 1980s and early 1990s than in the recent years. The major episodes of mispricing have concerned instead a few equity sectors (such as high tech stocks) and speculative-grade bonds. Yet, we find some evidence of systematic over-pricing of one sub-class of REITs, the mortgage-based instruments, that appear to have been grossly and significantly over-priced between 2001 and 2004. This evidence is consistent with the notion that persistent mispricing would not really have been pervasive in the spot real estate market, and that the crisis would have originated more from the poor quality of lending standards than from the presence of obvious upward biases in prices, what Hendershott, Hendershott, and Shilling (2010) have recently defined the "mortgage finance bubble".[3]

The paper is based on three main building blocks. First, using a novel empirical approach we estimate a standard multifactor asset pricing model (MFAPM, see e.g., Cochrane, 2005) in which the proposed risk factors consist of shocks to observable macroeconomic variables that appear to be commonly tracked by researchers, policy-makers, and the press (e.g., aggregate market returns, the rate of growth of industrial production, inflation news, the spread between long- and short-term nominal rates, etc.). Going back to the seminal paper by Chen, Roll and Ross (1986) there is an ever expanding literature that has worked with such a class of models; Ferson and Harvey (1991) extended the early work on MFAPMs to incorporate the case of time-varying risk premia and betas. In general terms, a MFAPM has a very simple structure: the risk premium on any

[2]See Jacquier and Polson (2010) for a review of applications of Bayesian econometrics in finance.

[3]Of course, we do not mean to deny the fact that at the micro-economic level, poor lending practices in the U.S. residential housing sector may have decreased the quality of existing mortgage pools during 2003-2006. Our goal is to assess to what extent such biases have generated empirical evidence of systematic, aggregate misspricing in REIT portfolios that are widely diversified across properties and types of properties (as our data also include commercial real estate properties).

asset or portfolio is decomposed as the sum of a certain number (K) of products between risk exposures (also called betas) to each of the factors and the associated unit price of the risk factor, common across all assets. The difference at each point in time between actual, realized excess returns and the risk premium implied by the model is called residual or idiosyncratic risk. Second, our paper uses data on publicly traded stock, bond, and real estate securities (or traded funds invested in these securities), instead of focussing on only one of these asset classes. Therefore our paper relates to a vast literature that has examined the empirical performance of MFAPMs across asset classes. For instance, Chan, Hendershott and Sanders (1990) have shown that MFAPMs that include predetermined macroeconomic factors explain a significant proportion of the variation in (equity) real estate investment trusts (henceforth REITs) returns. Karolyi and Sanders (1998) have extended this evidence and allowed for time-varying risk premia and betas. In a way, our paper contributes both to the real estate literature that has investigated the economic determinants of securitized real estate returns (see, among the others, Devaney, 2001) and to the body of works that has examined the linkages and pricing differences between REIT returns and those of stocks and bonds (e.g., Clayton and MacKinnon, 2003, and Serrano and Hoesli, 2007).

Third, we model both factor sensitivities and idiosyncratic volatility as latent stochastic processes within a Bayesian framework by means of the mixture innovation approach as in Giordani and Kohn (2008). Furthermore, we estimate the sequence of risk premia following Ouysse and Kohn (2010) to try and deal with problems caused by the use of generated regressors.[4] We show that this approach helps reduce the extent of variations in estimated factor exposures and risk premia. The estimation strategy adopted in this paper is based on two steps (see also Guidolin, Ravazzolo, and Tortora, 2010, GRT):

- Time variation in risk exposures and premia is explicitly modelled as a break-point process; the parameters of interest (βs and log-volatilities) are constant unless a break-point variable (κ) takes a unit value, in which case the parameters are allowed to jump to a new level, as a result of a normally distributed shock; the break-point variable κ takes a value of one, signalling the occurrence of a jump, with some probability (π) which is itself estimable; finally, the breaks themselves are latent, so that data ought to be used also to make inferences on the dates and magnitudes of the breaks.
- The model is estimated using a Bayesian approach that is numerically convenient and, as usual, allows a researcher to feed her own priors on the quantities of interest in the estimation problem.

A number of recent papers have approached issues similar to ours, measuring the size and persistence of mispricings in the real estate (spot) market using "behavioral" approaches that do not

[4] Other, frequentist approaches have been pursued in the literature. For instance, Ling and Naranjo (1997) use nonlinear multivariate techniques to estimate a system of equations with cross- and within-equation restrictions. This fixed-coefficient method eliminates the generated regressors problems, although the risk sensitivities and risk premia are constrained to be constant. In our paper we remove this restruction by adopting a Bayesian estimation approach. That real estate abnormal performances may be spuriously due to unspanned time-variation in risk exposure has been known since Glascock (1991).

(solely) rely on the specification or estimation of structural (no-arbitrage) asset pricing frameworks that isolate a list of priced risk factors. Recent examples are Lai and Van Order (2010), Lin, Rahman and Yung (2009), and Pavlov and Wachter (2011). Lai and Van Order (2010) ask whether the post-1999 behavior of house price growth in the U.S. can be characterized as a bubble relative to fundamentals, where a bubble is defined as a regime shift in which house prices deviate from their fundamentals defined as the present value of expected future rents. A regime switching framework shows evidence of momentum in the deviations from fundamentals throughout the period and that momentum increased especially after 2003, a period that was associated with big changes in markets, such as the rise of the subprime market (see Coleman, LaCour-Little, and Vandell, 2008), subprime securitization and a decline in short-term interest rates. Similarly, Pavlov and Wachter (2011) establish a theoretical link between the availability of aggressive mortgage lending instruments and real estate prices. They use cross-sectional, county-level data to compare house price dynamics across regions with different concentrations of aggressive mortgage instruments: areas with high concentrations of aggressive lending instruments experience larger price run-ups during rising markets, and deeper crashes during down markets. Although these papers feature a link between real estate valuations and fundamentals, their framework does not fully impose the no-arbitrage constraints of a multi-factor asset pricing model. Moreover, their notion of mispricing is simply defined with reference to the U.S. housing market, while our empirical results stem from a multivariate analysis that considers also stocks and bonds, besides REITs. Although their paper is not explicitly targeted to examine mispricings during the GFC, Lin et al. (2009) report interesting results for the effects of investors' sentiment on REIT returns. They explore whether investor sentiment is a significant force when the Fama-French factors and the risk and spread variables are present in a factor model for excess returns. They find that REIT returns are related to investor sentiment: when investors are optimistic (pessimistic), REIT returns become higher (lower). Papers like Lin et al.'s need to be taken into account because they represent an important, behavioral alternative to our efforts in this paper, i.e., the issue of REIT mispricings may also be tackled using flexible models that impose less structure than what we are about to do in our paper.

Section 2 outlines the theoretical MFAPM and the Bayesian estimation strategy. The Section also presents a few standard (variance) ratios used to evaluate the "economic" fit of MFAPMs. Section 3 describes the data. Section 4 reports the main estimation results concerning risk exposures and risk premia. Section 5 gets to the core of our economic question and asks whether, how, and when there is evidence of mispricing in our asset menu, with particular emphasis on REITs. We report both mispricing measures that rely on estimates of risk premia and, because such a conditioning has been shown to be taken with caution in the recent literature, measures that do not, as these are based only in the decomposition of excess returns between risk factors and idiosyncratic variance. In Section 5, we also present related evidence obtained from more traditional, two-pass frequentist approaches in the spirit of Fama and MacBeth (1973). The concluding section summarizes our findings.

2. **Research Design and Methodology**

2.1. *The Asset Pricing Framework*

A MFAPM posits a linear relationship between asset returns and a set of macroeconomic factors that are assumed to capture business cycle effects on beliefs and/or preferences (as summarized by a linear stochastic discount factor with time-varying properties) and hence on risk premia. These macroeconomic factors are typically the market portfolio (i.e., aggregate wealth) returns, the default spread on corporate bond yields, the term spread implied by the riskless (Treasury) yield curve, and the rate of growth of industrial production (see Chen, Roll and Ross, 1986, and Liu and Mei, 1992). If we call the process of the (shocks to) macroeconomic risk factors $f_{j,t}$ $(j = 1, ..., K)$ and $x_{i,t}$ the period *excess* return on asset or portfolio $i = 1, ..., N$, then a typical MFAPM can be written as:

$$x_{i,t} = \beta_{i0,t} + \sum_{j=1}^{K} \beta_{ij,t} f_{j,t} + \epsilon_{i,t}, \tag{1}$$

where it is customary to assume that $E[\epsilon_{i,t}] = E[f_{j,t}] = E[\epsilon_{i,t} f_{j,t}] = 0$ for all $i = 1, ..., N$ and $j = 1, ..., K$. The $x_{i,t}$ are returns in excess of the risk-free rate proxied by the 1-month T-bill rate. The advantage of MFAPMs such as (1) consists of the fact that a number of systematic factors $K << N$ may efficiently capture relatively large portions of the variability in asset returns. Even though the notation $\beta_{ij,t}$ emphasizes that the factor loadings are in principle time-varying, such patterns of time variation are in general left unspecified at a theoretical level.

One problem with (1) is the difficulty of interpreting $\beta_{i0,t}$ (often called the "Jensen's alpha") when some (or all) the risk factors are not traded portfolios. Although analyses that use (1) to understand and decompose realized excess returns may still be implemented, unless all the factors are themselves tradable portfolios, it is impossible to interpret any non-zero $\beta_{i0,t}$ as an abnormal return on asset i "left on the table" after all risks ($f_{j,t}$, $j = 1, ..., K$) and risk exposures ($\beta_{ij,t}$, $j = 1, ..., K$) have been taken into account. If some of the factors are not replicated and replaced by traded portfolios, there may be an important difference between the theoretical alpha that the model uncovers, and the actual alpha that an investor may achieve by trading assets on the basis of the MFAPM. To eliminate such a possibility, we follow the literature (see e.g., Ferson and Korajczyk, 1995) and proceed as follows. When an economic risk factor is measured in the form of an excess return, such as the U.S. market portfolio, real T-bill rates, term structure spreads, and default spread variables, we use the excess return directly as a mimicking portfolio; Shanken (1992) has argued that this approach delivers the most efficient estimates of the risk premiums. When a factor is not an excess return, such as industrial production growth, unexpected inflation, and real consumption growth, we construct the corresponding mimicking portfolios by estimating time-series regressions of individual portfolio returns on M economic variables and lagged instruments (see the Appendix for additional details). Using the residuals of such regressions to form an estimate of the $N \times N$ (conditional) idiosyncratic covariance matrix, $\mathbf{V}_t$, we form in each month of our sample the factor-mimicking portfolios for each

of the $K' \leq K$ factors for which these are needed by finding weights $\mathbf{w}_{j,t}$ $(j = 1, ..., K')$ that solve

$$\min_{\mathbf{w}_{j,t}} \mathbf{w}'_{j,t}\mathbf{V}_t\mathbf{w}_{j,t} \quad \text{s.t. (i) } \mathbf{w}'_{j,t}\mathbf{B}_{[j],t} = \mathbf{0}; \text{ (ii) } \mathbf{w}'_{j,t}\mathbf{1}_N = 1, \tag{2}$$

where $\mathbf{B}_{[j],t}$ is the $N \times (M-1)$ matrix that excludes the jth row from the $N \times M$ matrix of slope coefficient estimates $\mathbf{B}_t$ obtained by regressing returns data on the N portfolios on the M factors and instruments. The jth mimicking portfolio is then formed from individual stock returns, using the vector of weights, $\mathbf{w}_{j,t}$.[5]

In the conditional version of Merton's (1973) ICAPM, the expected excess return (risk premium) on asset i over the interval $[t-1, t]$ may be related to its "betas" (i.e., factor loadings measuring the exposure of asset i to each of the priced, systematic risk factors) and the associated unit risk premia (i.e., average return compensations for unit exposure to risk):

$$E[x_{i,t}|\mathbf{Z}_{t-1}] = \lambda_0(\mathbf{Z}_{t-1}) + \sum_{j=1}^{K} \beta_{ij,t}\lambda_j(\mathbf{Z}_{t-1}), \tag{3}$$

where both the betas and the risk premia are conditional on the information publicly available at time $t-1$, here summarized by the $M \times 1$ vector of instruments $\mathbf{Z}_{t-1}$.

2.2. *Standard Estimation Approach*

(1) and (3) describe a general conditional pricing framework that is known to hold under a variety of alternative assumptions.[6] However, a range of methodologies have been proposed to tackle three related tasks which affect the empirical performance of (1)-(3): (i) how many factors ought to be selected, i.e., picking an appropriate value for K; (ii) given K, devising a methodology to rank competing factors and selecting those that best fit the data; (iii) estimating the factor loadings $\{\beta_{ij,t}\}$ and the risk premia λ_{jt}. These tasks are logically distinct from the formulation of the asset pricing framework and—albeit their optimal implementation affects our ability to answer any asset pricing questions—they have an exquisite statistical nature.

In this paper we follow the mainstream empirical finance literature (see e.g., Chen, Roll and Ross, 1986) as far (i)-(ii) are concerned—which means that we pre-select both K and which specific macroeconomic risk factors are considered, in the light of the existing literature—and introduce a novel econometric approach with regard to task (iii). However, it is useful to briefly describe an alternative, benchmark estimation approach. This is the classical, two-stage procedure *à* la Fama and MacBeth (1973) also used by Ferson and Harvey (1991). In the first stage, for each of the assets, the factor betas are estimated using time-series regressions from historical excess returns on

[5] The conditional beta of the jth mimicking portfolio on the jth economic factor may change as $\mathbf{B}_t$ and $\mathbf{V}_t$ change over time. However, such mimicking portfolios are typically adjusted to have constant factor betas by combining them with T-bills so that the combined portfolio has a beta equal to the time-series average of the betas that are produced by the constrained optimization. We provide additional details on our Bayesian implementation of this procedure in the Appendix.

[6] For instance, standard arguments in Cochrane (2005) show that (1)- (3) holds when the stochastic discount factor can be written as an exact linear function of the systematic risk factors f_1, f_2, ..., f_K.

the assets and economic factors. That is, for month t, we estimate the regression in (1) using the previous sixty months (ranging from $t-60$ to $t-1$) in order to obtain estimates for the betas, $\hat{\beta}^{60}_{ij,t-1}$. This time-series regression is updated each month. In the second stage, we estimate a cross-sectional regression, for each month, using ex-post realized excess returns

$$x_{i,t} = \lambda_{0,t} + \sum_{j=1}^{K} \lambda_{j,t}\hat{\beta}^{60}_{ij,t-1} + \zeta_{i,t} \qquad i = 1, ..., N, \tag{4}$$

(and for each $t = 61, ..., T-1$). In (4) $\lambda_{0,t}$ is the zero-beta (abnormal) excess return and the $\lambda_{j,t}$s are proxies for the factor risk premiums for each month, $j = 1, ..., K$, *that are common across assets.*

2.3. *A New Bayesian Estimation Approach*

Although widely used in the applied finance literature, the classical two-stage Fama-MacBeth approach has a number of obvious drawbacks. First, the second stage multivariate regression in (4) suffers from obvious generated regressor (error-in-measurement) problems as the first-stage rolling window beta estimates $\hat{\beta}^{60}_{ij,t-1}$ are used as regressors on the right-hand side. For instance, Ang and Chen (2007) have stressed that when the cross-sectional estimates of the betas $\hat{\beta}^{60}_{ij,t-1}$ co-vary with the underlying but unknown risk premia, (4) may yield biased and inconsistent estimates of the risk premia themselves. Unfortunately, this covariation is extremely likely to occur in practice: for instance, the asset pricing literature reflects a presumption that during business cycle downturns both the quantity of risk (the betas) and the unit risk prices would increase, because recessions are characterized by higher systematic uncertainty and by lower "risk appetite" (e.g., in a Campbell and Cochrane's, 1999, habit-formation framework). Second, the need to estimate (1)- (3) in two distinct stages that use rolling windows to capture parameter instability is not only *ad hoc* but also inefficient because the lack of specific parametric forms makes it testing for time-variation awkward and dependent on hard-to-justify choices concerning the window length, the selection of constant or decaying rolling weights, etc. Because it would be difficult to try and assess the magnitude and persistence of mispricings in REIT portfolios when their very instability is capture in completely ad-hoc ways and risk premia may be poorly estimated (see Section 5.4), in this paper we adopt a different strategy that has recently appeared in the asset pricing literature.

Clearly, both issues are tackled by any full-information estimation method that avoids using estimates of the first-stage betas as if these were observed variables constant in repeated samples and that would take into account the existence of time-varying factor loadings and idiosyncratic variance in specific parametric forms. This is what our Bayesian, time-varying beta, stochastic volatility (henceforth BTVBSV) approach accomplishes. Stochastic, time-varying betas have been recently found to be crucial ingredients of conditional asset pricing, in the sense that there is growing evidence that careful modelling of the dynamics in factor exposures may provide a decisive contribution to solve the typical anomalies associated with unconditional implementations of multi-factor models. For instance, Jostova and Philipov (2005) find that in the typical Fama and MacBeth's style exercise,

the CAPM is rejected with rolling OLS beta estimates while the opposite verdict emerges when they allow for stochastic variation (in the form of a simple AR(1) process) in the conditional CAPM betas. Similarly, Ang and Chen (2007) show that the persistence in betas help explain the book-to-market effect in the cross section of stock returns. In practice, we specify the relationship between excess returns and factors and the time-varying dynamics in factor loadings and idiosyncratic volatility in the following state-space form

$$\begin{aligned} x_{i,t} &= \beta_{i0,t} + \sum_{j=1}^{K} \beta_{ij,t} f_{j,t} + \sigma_{i,t}\epsilon_{i,t} \\ \beta_{ij,t} &= \beta_{ij,t-1} + \kappa_{1ij,t}\eta_{ij,t} \qquad j = 0, ..., K, \\ \ln(\sigma^2_{i,t}) &= \ln(\sigma^2_{i,t-1}) + \kappa_{2i,t}\upsilon_{i,t} \qquad i = 1, ..., N, \end{aligned} \tag{5}$$

where $\boldsymbol{\epsilon}_t \equiv (\epsilon_{1,t}, \epsilon_{2,t}, ..., \epsilon_{N,t})' \sim N(0, \mathbf{I}_N)$, $\boldsymbol{\eta}_{i,t} \equiv (\eta_{i0,t}, \eta_{i1,t}, ..., \eta_{iK,t}, \upsilon_{i,t})' \sim N(0, \mathbf{Q}_i)$ with $\mathbf{Q}_i$ a diagonal matrix characterized by the parameters q^2_{i0}, q^2_{i1}, ..., q^2_{iK}, $q^2_{i\upsilon}$. Stochastic variations (breaks) in the level of both the beta coefficients and of the idiosyncratic variance $\sigma^2_{i,t}$ are introduced and modelled through a mixture innovation approach as in Ravazzolo, Paap, van Dijk and Franses (2007) and Giordani and Kohn (2008). The latent binary random variables $\kappa_{1ij,t}$ and $\kappa_{2i,t}$ are used to capture the presence of random shifts in betas and/or idiosyncratic variance and—for the sake of simplicity—these are assumed to be uncorrelated among one another (i.e., across assets as well as factors) and over time.

This specification is very flexible as it allows for both constant and time-varying parameters. When $\kappa_{1ij,\tau} = \kappa_{2i,\tau} = 0$ for some time τ, then (5) reduces to (1) when the factor loadings and the quantity of idiosyncratic risk are assumed to be constant, as $\beta_{ij,\tau} = \beta_{ij,\tau-1}$ and $\ln \sigma^2_{i,\tau} = \ln \sigma^2_{i,\tau-1}$. However, when $\kappa_{1ij,\tau} = 1$ and/or $\kappa_{2i,\tau} = 1$, then a break hits either beta or idiosyncratic variance or both, according to the random walks $\beta_{ij,\tau} = \beta_{ij,\tau-1} + \eta_{ij,\tau}$ and $\ln(\sigma^2_{i,\tau}) = \ln(\sigma^2_{i,\tau-1}) + \upsilon_{i,t}$ (or $\sigma^2_{i,\tau} = \sigma^2_{i,\tau-1}\exp(\upsilon_{i,\tau})$). Note that because when a break affects the betas and/or variances, the random shift is measured by variables collected in $\boldsymbol{\eta}_{i,t}$, we can also interpret $\mathbf{Q}_i$ not only as a standard measure of the covariance matrix of the random breaks in $\boldsymbol{\eta}_{i,t}$, but also of the size of such breaks: a large q^2_j means for instance that—whenever $\beta_{ij,t}$ is hit by a break—such a shift is more likely to be large (in absolute value). Finally, while $\kappa_{1ij,\tau} = 0$ for all τs implies that (5) is a traditional linear factor model, when $\kappa_{1ij,\tau} \neq 0$ is allowed, (5) turns into a nonlinear asset pricing framework, where nonlinearities are captured by the stochastic time variation in risk exposures. Even though (5) marks a considerable generalization of (1), this occurs under specific assumptions on functional forms and parametric distributions for the shift variables $\kappa_{1ij,\tau}$ and $\kappa_{2i,\tau}$ in (5). All results presented in this paper should therefore be considered as a product of these specific choices, even though these all go in the direction of adding flexibility to standard MFAPM analysis.

We estimate (5) using a Bayesian approach, which is in fact the only numerically feasible estimation method for a model with the features of our BTVBSV framework.[7] Realistic values for the different prior distributions obviously depend on the problem at hand. In general, we use weak

[7]For instance, in classical MLE framework it would be hard to separately identify the stochastic shifts represented by

priors, excluding the size of the breaks $\mathbf{Q}_i$ and the probabilities $\Pr(\kappa_{1ij,\tau} = 1)$ and $\Pr(\kappa_{2i,\tau} = 1)$ for which our priors are informative. All other priors imply that the posteriors tend to be centered around their maximum likelihood estimates which eases comparisons with traditional methods to performed later on.[8] Once estimates *of the posterior densities* for unknown coefficients are obtained, we also implement a further, built-in estimation pass by estimating, for each month, the following cross-sectional multivariate regression:

$$x_{i,t} = \lambda_{0,t} + \sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1} + e_{i,t} \qquad i = 1, ..., N, \tag{6}$$

where $e_{i,t} \sim N(0, \sigma_t^2)$ and $\beta_{ij,t|t-1}$ measures the *expected* time t sensitivity of asset i to factor j, based on all information available at time $t-1$. $\beta_{ij,t|t-1}$ is carefully constructed for the purposes of our investigation: it is obtained by taking the lagged value from the updating step of the Kalman filter (see the Appendix for details) and simulating the occurrence of future breaks and the shock magnitude from the appropriate posteriors. This is the exact analog of the logic that advised Ferson and Harvey (1991) to estimate (4) using one-month lagged values of $\hat{\beta}^{60}_{ij,t-1}$: time t excess return on asset i should be determined by investors with reference only to information available up to time $t-1$ but keeping into account all features of the model (5) known up to time $t-1$. Even though our Bayesian estimation approach is still articulated on two steps, second pass estimation is performed similarly to Ouysse and Kohn (2010) to overcome the notorious error-in-variables problem that plagues traditional empirical MFAPMs in small samples. In fact, to avoid generated regressor problems in the most resolute form, for each time t we avoid collapsing the posterior density of the factor loadings $\beta_{ij,t|t-1}$ to a single value (e.g., their mean or median) and use instead the entire posterior for the betas (also see Cosemans et al., 2011). In practice, we draw a large number of times from such a posterior across all N assets and for each draw we estimate a multivariate cross-sectional regression to obtain a corresponding (implicit) draw for the risk premia (see Geweke and Zhou, 1996). Finally, note that (6) imposes tight cross-sectional restrictions because the unit risk premia coefficients, $\lambda_{j,t}$ $j = 1, ..., K$, are uniquely defined across all assets in the estimation menu, $i = 1, ..., N$.

2.4. *Decomposition Tests*

We use the posterior densities of the time series of factor loadings and risk premia to perform a number of economic tests that allow us to assess whether our asset pricing framework may explain an adequate percentage of excess asset returns. (6) decomposes excess asset returns in a component related to risk, represented by the term $\sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}$ plus a residual $\lambda_{0,t} + e_{i,t}$. In principle, a

the variables $\kappa_{1ij,t}$ and $\kappa_{2i,t}$ from the continuous shocks in $\eta_{ij,t}$ and $\upsilon_{i,t}$. In a Bayesian framework, proposing plausible priors informed by economic principles greatly helps to deal with these issues.

[8] These priors are commonly referred to as uninformative or "flat". However, the Appedix briefly summarizes results obtained using more informative priors and show that these have a negligible impact on our findings for the questions of interest.

multi-factor model is as good as the implied percentage of total variation in excess returns explained by its first component, $\sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}$. However, here we should recall that even though (6) refers to excess returns, it remains a statistical implementation of the framework in (1). This implies that in practice it may be naive to expect that $\sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}$ be able to explain much of the variability in excess returns. A more sensible goal seems to be that $\sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}$ ought to at least explain the *predictable* variation in excess returns. We therefore follow earlier literature, such as Karolyi and Sanders (1998), and adopt the following approach. First, the excess return on each asset is regressed onto a set of M instrumental variables that proxy for available information at time $t-1$, $\mathbf{Z}_{t-1}$,

$$x_{i,t} = \theta_{i0} + \sum_{m=1}^{M} \theta_{im} Z_{m,t-1} + \xi_{i,t}, \tag{7}$$

to compute the sample variance of fitted values,

$$Var[P(x_{it}|\mathbf{Z}_{t-1})] \equiv Var\left[\hat{\theta}_{i0} + \sum_{m=1}^{M} \hat{\theta}_{im} Z_{m,t-1}\right], \tag{8}$$

where the notation $P(x_{it}|\mathbf{Z}_{t-1})$ means "linear projection" of x_{it} on a set of instruments, $\mathbf{Z}_{t-1}$. Second, for each asset $i = 1, ..., N$, a time series of fitted (posterior) risk compensations, $\sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}$, is regressed onto the instrumental variables,

$$\sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1} = \theta'_{i0} + \sum_{m=1}^{M} \theta'_{im} Z_{m,t-1} + \xi'_{i,t} \tag{9}$$

to compute the sample variance of fitted risk compensations:

$$Var\left[P\left(\sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right)\right] \equiv Var\left[\hat{\theta}'_{i0} + \sum_{m=1}^{M} \hat{\theta}'_{im} Z_{m,t-1}\right]. \tag{10}$$

The predictable component of excess returns in (7) not captured by the model is then the sample variance of the fitted values from the regression of the residuals $\hat{\xi}_{i,t}$ on the instruments:

$$Var\left[\hat{\xi}_{i,t}\right] = Var\left[P\left(x_{i,t} - \sum_{m=1}^{M} \hat{\theta}_{im} Z_{m,t-1}|\mathbf{Z}_{t-1}\right)\right]. \tag{11}$$

At this point, it is informative to compute and report two variance ratios, commonly called $VR1$ and $VR2$, after Ferson and Harvey (1991):

$$VR1 \equiv \frac{Var\left[P\left(\sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right)\right]}{Var[P(x_{it}|\mathbf{Z}_{t-1})]} > 0 \tag{12}$$

$$VR2 \equiv \frac{Var\left[P\left(x_{i,t} - \sum_{m=1}^{M} \hat{\theta}_{im} Z_{m,t-1}|\mathbf{Z}_{t-1}\right)\right]}{Var[P(x_{it}|\mathbf{Z}_{t-1})]} > 0. \tag{13}$$

VR1 should be equal to 1 if the multi-factor model is correctly specified, which means that all the predictable variation in excess returns is captured by variation in risk compensations; at the same

time, VR2 should be equal to zero if the multi-factor model is correctly specified.[9] Notice that $VR1 = 1$ does not imply that $VR2 = 0$ and viceversa, because

$$Var[P(x_{it}|\mathbf{Z}_{t-1})] \neq Var\left[P\left(\sum_{j=1}^{K}\hat{\lambda}_{j,t}\hat{\beta}_{ij,t|t-1}|\mathbf{Z}_{t-1}\right)\right] + Var\left[P\left(r_{i,t} - \sum_{m=1}^{M}\hat{\theta}_{im}Z_{m,t-1}|\mathbf{Z}_{t-1}\right)\right]. \tag{14}$$

Finally, the predictable variation of returns due to the multi-factor model is further decomposed into the components imputed to each of the individual systematic risk factors, by computing the factoring of $Var[P(\sum_{j=1}^{K}\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1})]$ as

$$\sum_{j=1}^{K} Var\left[P\left(\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right)\right] + \sum_{j=1}^{K}\sum_{k=1}^{K} Cov[P\left(\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right), P\left(\lambda_{k,t}\beta_{ik,t|t-1}|\mathbf{Z}_{t-1}\right)] \tag{15}$$

and tabulating $Var\left[P\left(\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right)\right]$ for $j = 1, ..., K$ as well as the residual factor $\sum_{j=1}^{K}\sum_{k=1}^{K} Cov[$ $P\left(\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right), P\left(\lambda_{k,t}\beta_{ik,t|t-1}|\mathbf{Z}_{t-1}\right)]$ to pick up any interaction terms. Note that because of the existence of the latter term, the equality

$$\sum_{j=1}^{K}\frac{Var\left[P\left(\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right)\right]}{Var\left[P\left(\sum_{j=1}^{K}\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right)\right]} = 1 \tag{16}$$

fails to hold, i.e., the sum of the K risk compensations should not equal the total predictable variation from the asset pricing model because of the covariance among individual risk compensations.[10]

3. Data and Summary Statistics

Our paper is based on a large number of monthly time series (30) sampled over the period 1980:01 - 2010:12 for a total of 372 observations per series.[11] The series belong to three main categories. The first group, "Portfolio Returns", includes several asset classes like stocks, bonds and real estate, organized in portfolios, a procedure that is useful to tame the noise caused by non-diversifiable risk.[12] The stocks are publicly traded firms listed on the NYSE, AMEX and Nasdaq (from CRSP) and sorted according to two criteria. First, we form 10 industry portfolios by sorting firms according to their four-digit SIC code. Second, we form 10 additional portfolios by sorting (at the end of every year, and recursively updating this sorting in every year in our sample period) NYSE, AMEX and Nasdaq

[9] When these decomposition tests are implemented using the estimation outputs obtained from our BTVBSV framework, we preserve consistency with our Bayesian framework: drawing from the joint posterior densities of the factor loadings $\beta_{ij,t|t-1}$ and the implied risk premia $\lambda_{j,t}$, $i = 1, ..., N$, $j = 1, ..., K$, and $t = 1, ..., T$, and holding the instruments fixed over time, it is possible to compute VR1 and VR2 in correspondence to each of such draws.

[10] The fact that in (1) the risk factors are assumed to be orthogonal does not imply that their time-varying total risk compensations ($\lambda_{j,t}\beta_{ij,t|t-1}$ for $j = 1, ..., K$) should be orthogonal.

[11] Data for a longer 1972:01-2010:12 are in fact available. However, in a portion of our estimation experiments, we use a 5-year, 1972:02-1979:12 period to compute the priors that investors were likely to hold as of the beginning of 1980.

[12] An alternative approach to improve the precision of security-specific beta estimates is to use the shrinkage technique proposed by Vasicek (1973). This method uses the cross-sectional mean and variance of betas as prior information and, as recently shown by Cosemans et al. (2011), may be profitably extended to time-varying beta frameworks.

stocks according to their size, as measured by aggregate market value of the company's equity.[13] Using industry and size-sorted criteria to form spread portfolios of stocks to trade-off "spread" and reduction of idiosyncratic risk due to portfolio formation, is typical in the empirical finance literature (see e.g., Dittmar, 2002). Moreover, industry- and size-sorting criteria are sufficiently unrelated to make it plausible that industry- and size-sorted equity portfolios may contain different and non-overlapping information on the underlying factors and risk premia.

Data on long- (10-year) and medium-term (5-year) government bond *returns* are from Ibbotson and available from CRSP. Data on junk bond returns are approximated from Moody's (10-to-20 year maturity) Baa average corporate bond yields and converted into return data using Shiller's (1979) approximation formula and assuming a coupon rate equal to the average sample yield.[14] Finally, data on REIT total returns come from the North American Real Estate Investment Trust (NAREIT) Association and consists of data on three major categories of tax-qualified REITs, i.e. equity, mortgage, and hybrid equity/mortgage REITs using breakdowns common in the literature. All excess return series are computed as the difference between total returns and 1-month T-bill returns, as usual.

We use a range of macroeconomic variables as standard proxies for the systematic, economy-wide risk factors potentially priced in asset returns. Lagged values of these risk factors are also used as "instruments" when relevant in our methodology, our logic being that all these variables belonged to the information set of the investors when they made their portfolio decisions. In practice, we employ seven factors (as in Ling and Naranjo, 1997): the excess return on a wide, value-weighted market portfolio that includes all stocks traded on the NYSE, AMEX, and Nasdaq (from CRSP); the credit risk premium measured as the difference between Baa Moody's *yields* and yields on 10-year government bonds; the change in the term premium, the difference between 5-year and 1-month Treasury yields;[15] the rate of growth of (seasonally adjusted) industrial production; the rate of growth of (seasonally adjusted) real personal consumption growth; the 1-month real T-bill rate of return computed as the difference between the 1-month T-bill nominal return and realized CPI inflation rate (not seasonally adjusted), and the unexpected inflation rate, computed as the residual of a simple ARIMA(0,1,1) model applied to (seasonally adjusted) CPI inflation.[16]

Table 1 presents summary statistics for the time series under investigation. Because we benchmark a portion of our results to earlier papers that have used data for the 1980s and early 1990s, such as Karolyi and Sanders (1998), to favor comparisons, Table 1 presents summary statistics for

[13] Data on size- and industry-sorted portfolios are available from Ken French's web site at http://mba.tuck.dartmouth.edu/ pages/faculty/ ken.french/data_library.html.

[14] Approximated returns from this formula are correlated with actual, Baa rating bracket returns (from Bloomberg) over recent years (2005-2010), with a correlation in excess of 0.8.

[15] Data on 1-month T-bill, 10-year and 5-year government bond *yields* are from FREDII® at the Federal Reserve Bank of St. Louis.

[16] The trailing, 12-month dividend yield on all stocks traded on the NYSE, AMEX, and Nasdaq (computed from CRSP data) is also used as an instrument in some of the exercises. However, it is not used as priced factor because it only relates to stock cash distributions and differs from REITs' cap rates.

two different sub-samples, 1980:01 - 1992:12 and 1993:01 - 2010:12.[17] In particular, the table reports sample means, medians, standard deviations, and the resulting Sharpe ratios (computed with reference to 1-month T-bill returns). None of the summary statistics in Table 1 is surprising. Most industry portfolios and all cap-sorted portfolios have mean returns between 11 and 14% per year in the overall sample period. Moreover, for all stock portfolios (but one, energy stocks) median returns are substantially higher than mean returns, a clear indication of asymmetric return distributions. Volatilities tend to be between 15 and 25 percent in annualized terms; small stock portfolios are more volatile than large stocks, while the most volatile industries are high tech and durable goods. As a result, most Sharpe ratios are in the 0.1-0.16 range (on a monthly basis), with very few outliers such as high tech, durable goods (with ratios below 0.1) and non-durable goods with a Sharpe ratio of 0.19. There is nothing abnormal to report with reference to returns on 5- and 10-year government bonds, apart from their stunning Sharpe ratios in excess of most stock portfolios, due to the fact that our sample is dominated by the disinflation and declining interest rates of the early 1980s. The summary performance statistics for real estate portfolios contain instead some unexpected results. While equity REITs are characterized by means (13% per year), volatility (18%), and a Sharpe ratio (0.14) directly comparable to those of stocks (for instance, the value-weighted CRSP portfolio has a mean return of 12%, volatility of 16%, and a Sharpe ratio of 0.13), mortgage and hybrid REITs have produced much lower mean returns (around 5-6% per year) but display volatilities in excess of long-term bonds, with resulting Sharpe ratios close to zero. However, because for most of our sample the overall REIT portfolio, NAREIT composite, is dominated by equity REITs, the result is that the corresponding Sharpe ratios are generally close to those of the stock market indices.

The second group of statistics in Table 1 concerns the shorter 1980-1992 sample. The summary statistics are indeed rather close to those reported by Karolyi and Sanders' (1998) with reference to a 1983-1992 period. However, the differences between sub-periods are considerable. For instance, the post-1992 age has been a rather disappointing period for stocks, and this emerges independently of the portfolio sorting criterion employed, with the only exception of small and high tech stocks. Even though a few volatilities are lower in the post-1992 periods than in the earlier sub-sample, the generalized decline in mean stock returns implies lower Sharpe ratios in the 1993-2010 period for most portfolios. This is also reflected in the statistics concerning the market portfolio, which has recently yielded lower mean returns (9.8% vs. 12.5%), imposed higher volatility on investors (16% vs. 14%), with a Sharpe ratio of 0.11 vs. 0.18 in the 1980-1992 period. However, over the same recent sample, the Sharpe ratio for NAREIT composite has jumped to 0.13 from 0.09 in the earlier period, and a solid contribution is given by mortgage REITs which had negative Sharpe ratios in the 1980s and switched to positive ratios in the past 20 years. Finally, some instability characterizes data for long-term bonds returns: these display significantly more volatility in the post-1992 period

[17] 1993 is also the date of an important tax reform Act that has entitled REITs to look through pension funds and count the number of participants with the result of favoring institutional investment without jeopardizing the trust's tax-favored status. As a result, in the 1990s the REIT market expanded considerably and became much more dominated by institutional investors (see e.g., Ling, and Ryngaert, 1997).

but also higher mean returns. However, their Sharpe ratios decline from an exceptionally high range of 0.23-0.36 over 1980-1992 to 0.17-0.23 in the post-1992 sample.

4. **Empirical Results**

4.1. *Factor Loadings*

In Sections 4.1-4.2 we report empirical estimates obtained for the case in which all factors are tradable, which implies that a few of the assumed factors has been replaced by a corresponding factor-mimicking portfolio. Figures 1-3 show medians and 90% Bayesian credibility intervals computed from the posterior densities of the loadings $\beta_{ij,t}$ over time from the BTVBSV model.[18] A time t, the 90% credibility interval is characterized by the 5th and 95th percentiles of the posterior density of $\beta_{ij,t}$. Figure 1 is key to this paper because it shows results for the NAREIT Composite Index (left-most column of plots) and for the NAREIT portfolio components (right-most column, equity, on the left scale, and mortgage and hybrid REITs, on the right scale) for each of the seven factors listed in Section 3. While most posterior median recursive estimates of the loadings are smooth and with hardly visible changes, a few exceptions are visible. Similarly, while for many of the factors the 90% intervals often include zero—which may be loosely interpreted as meaning that the posteriors attach a non-negligible probability to a zero or small loading on the factor under investigation—notable exceptions may be found in which portfolios appear to be significantly exposed to risks. Both NAREIT composite and its components appear to be significantly exposed to market risk, with a beta that has somewhat increased over time (especially during the 1990s), for instance from approximately 0.6 to 0.8 in the case of the overall REIT index. As one would expect in the light of the literature, all real estate portfolios have "defensive" market betas that do not exceed 1 (even though by the end of our sample, the upper bound of the confidence bands often includes 1), and equity REITs show betas that exceeds those of mortgage REITs. Only mortgage REITs show a significant and relatively high, positive and stable exposure to the credit risk premium factor (around 1.2), which means that when default risks are increasingly priced in corporate bond yields, the risk premium required of mortgage REITs increases as well. On the contrary, equity REITs and, as a reflection, the overall NAREIT index fails to show an economically significant or precisely measured exposure to the credit risk factor. REITs have instead an economically large negative exposure to changes in the slope of the yield curve, and this is mostly due to the exposure of mortgage and hybrid REITs (see e.g., Peterson and Hsieh, 1997). The negative sign is expected because when the long-end of the yield curve moves above the short-term segment, this presumably translates into higher mortgage rates and negative excess real estate returns. Finally, with minor exceptions noted below, the aggregate REIT portfolio does not display precisely estimated exposures to unexpected inflation, the rate of

[18] Note that pinning down the "statistical significance" of coefficients (betas or lambdas) on the basis of 90% credibility intervals represents a rather stringent criterion because the Bayesian posterior density will reflect not only the uncertainty on the individual coefficient but also the overall uncertainty on the entire model (e.g., the uncertainty on structural instability of all the coefficients), see e.g., the discussion in Uno et al. (2005).

growth of IP, real consumption growth, or the real T-bill rate. The only exceptions are the betas on unexpected inflation, IP growth, real personal consumption growth, and the real short-term rate, which—in the case of equity REITs—are positive for the two first factors and negative for the real short rate, and large for equity REITs.

It is interesting to notice that while the BTVBSV model, that allows for explicit modelling of exposure instability over time, yields essentially flat time series of $\beta_{ij,t}$ posterior medians for most factors (e.g., credit risk, change in the term premium, real consumption growth, and the real T-bill rate), some important exceptions exist in which structural instability is captured and estimated, which may have first-order effects for the economic implications of the model. There is evidence of a gradual and steep increase in the market beta exposure of most REIT portfolios between the early 1980s and 2003; the same applies to the exposure to inflation risk, which grows over time for both equity REITs and the composite portfolio. All in all, Figure 1 gives evidence that publicly traded real estate portfolios are significantly exposed to global market, yield curve, real business cycle (especially through IP growth) and real riskless short-term rate risks with the expected signs.[19]

Figure 2 presents the same type of information as Figure 1 does, but with reference to 4 selected stock industry portfolios and 5 of the 7 factors only. To make the plots readable, we have omitted beta posterior densities for 6 residual industries but results were qualitatively similar to those plotted here and these are available from the Authors upon request. In this case, we briefly comment across factors. Many (but not all) industries (including those not plotted in Figure 2) are significantly exposed to market beta risk, and hardly any significant patterns of time variation emerge. Surprisingly, in the case of industries we find quite a few portfolios for which the market beta is either positive but imprecisely estimated and modest, or even negative (but also in this case, imprecisely estimated). As it shall become clear later on, this depends on the fact that for the industry portfolios that display such features, it is other real business factors (such as the term structure one) that capture the general association with market behavior. Very few portfolios have significant exposure to the credit risk factor (durables and retail shops appear to be the exceptions, probably through an asset-backed securities market linkage), both in statistical and economic terms. Also stock portfolios load considerably more on changes in the riskless term structure factor than they do on the default risk factor. Their riskless yield curve loadings are very stable over time. However, while a few industries imply positive and large beta loadings (e.g., high tech stocks and manufacturing), which is consistent with the slope of the term structure representing a business cycle indicator, other industries load negatively on this factor with betas that are large in absolute value (e.g., energy and utilities). This is sensible because the former group collects industries that are cyclical and the latter industries that are typically anti-cyclical. A similar comment extends to the industry betas concerning unexpected inflation and short-term rate factors: the posterior densities for the betas yield credibility intervals that often fail to include zero but the sign of the median posteriors are heterogeneous. In general,

[19] However, should we interpret the term premium to be a business cycle indicator—in the sense that a higher (lower) term premium signals an improvement (deterioration) of business cycle conditions, see e.g. Estrella and Hardouvelis (1991)—then a negative exposure of REITs to this factor may be puzzling.

industry portfolios yield small betas on IP and real consumption growth (not reported here but available on request), with posterior densities that tend to always attach a substantial probability on coefficients close to zero.

Figure 3 reports in the first column of plots results on beta posteriors for three size-sorted stock portfolios that can be taken to span the range of portfolios used in estimation (these are the first, fifth, and tenth deciles, dubbed small-, medium-, and large-size stocks, respectively), and in the second column results on beta posteriors for the three fixed income portfolios. As far as the size-sorted portfolios are concerned, we find evidence consistent with the size premium puzzle in the empirical finance literature: market betas do not vary much across portfolios and in fact medium- and small-caps have considerably *lower* betas than other portfolios have, with negative and rather narrow 90% credibility regions in the case of medium-caps. There is a lot of interesting time variation in large cap betas with a visible dip (to below 0.8) during the 1998-2000 period, while the market betas of small and medium capitalization stocks are essentially driftless. The other two factors that seem to explain size-sorted equity returns with precisely estimated coefficients are the real short-term rate and unexpected inflation (amid considerable time-variation) and term spread changes (for small caps), and real consumption growth (for both large and small caps). In particular, small and medium caps have relatively large, positive betas (with posterior 90% credibility bands that do not include zero) on the term spread risk factor and negative and large betas on the short-term real rate factor. Because a variety of papers (e.g., Fama and French, 1989, and Stock and Watson, 2003) have argued that a surging term structure premium is a predictor of economic expansions, this is a sensible finding because less diversified and smaller companies are likely to be more sensitive to business cycle dynamics (see Perez-Quiros and Timmermann, 2000). For the same reason, large caps do not seem to have exposure to the term structure factor and have a positive and precisely estimated exposure to real rate risk. As stressed by many papers (e.g., see Ang, Piazzesi and Wei, 2006), it may be advisable to use not only the slope of the yield curve but also some measure of level—for instance as measured by the T-bill rate—to capture the dynamics of the business cycle.

Similarly to stock industry portfolios, Figure 3 shows that all bond portfolios have a negative and rather significant exposure to market risk once the six additional macroeconomic factors are controlled for.[20] The posterior credibility regions for market betas are all tight but include zero, with posterior medians that over time span the range [-1.5, -0.5], which are rather large betas. All bond portfolios display positive (and large, in the case of corporate bonds) betas on the default risk factor. It is interesting that 10-year government bond risk premia may increase when the credit risk premium increases, although this may relate more to using this factor as a business cycle indicator than to the credit quality of the U.S. government; consistently with this intuition, we notice that the beta of 5-year Treasuries is small. All bond risk premia have negative exposures to the slope of the yield curve factor; these betas seem to be large and with a posterior distribution clearly tilted

[20] In Figure 3, we report posterior beta results only for 4 factors out of 7. Complete results are available from the Authors upon request.

away from zero especially in the case of 10-year government bonds, which signals a flight-to-quality effects. For similar reasons—because real rates increase in the expansionary stages of the business cycle—all bond portfolios are positively and massively exposed to real rate risk, with betas that are large in the case of long-term Treasuries and long-term corporate bonds. Treasury bonds, especially the long-term ones, have a negative and precisely estimated exposure to unexpected inflation, which is sensible because government securities are notoriously exposed to inflationary shocks. Finally, the BTVBSV model allows us to infer considerable instability in the betas of all Treasuries vs. market, IP growth, and real consumption growth risks, with rather heterogeneous trends.

As already stressed in GRT (2010), an overview of the plots in Figures 1-3 reveals that the Bayesian estimates of the loadings are often smooth, even though exceptions have been noted. In fact, GRT emphasize that even though (5) formally allows the $\beta_{ij,t}$ to be subject to jumps, as a result of the realization of a latent binary random variable, the resulting posterior densities are often smoother than what one could retrieve using a naïve rolling window scheme, following for instance Karolyi and Sanders (1998). Interestingly, this smoothness mimics exactly what many earlier papers have imposed by assuming near unit root processes ($\beta_{ij,t} = \phi_{ij}\beta_{ij,t-1} + u_{ij,t}$) with small variance of the shocks, but emerges endogenously, as suggested by the data, which means that occasional large jumps in exposure and/or high volatility of the corresponding process may be accommodated. Similarly, in unreported results we find that there are very few neat trends in the posterior medians of annualized idiosyncratic volatility, σ_{it}, as the posterior volatility estimates have tails so thick that it is always possible to notice movements in the median idiosyncratic volatility that may easily confused with sample (as well as coefficient, given the Bayesian nature of the analysis) random variation.

4.2. *Risk Premia*

Table 2 shows results on the posterior densities for the time series of risk premia estimates $\{\hat{\lambda}_{j,t}\}$ ($j = 1, ..., K$) across factors (these are instead uniquely estimated across all assets).[21] The table reports both summary statistics for the full sample as well as for three sub-samples, 1980-1992 and 1993-2010 for the reasons explained in Section 3, and 2007:07 - 2009:06 identified as the 24 months most affected by the recent GFC (see Guidolin and Tam, 2010, for issues related to the dating of the GFC in the U.S.). The table shows results that have to be interpreted with great caution. If one applies standard (but frequentist) statistical inference to the time series of *mean* posterior estimates of the risk premia $\{\hat{\lambda}_{j,t}\}$—which however assumes normality of the resulting posterior distributions—and computes standard t-tests of the null of zero risk premia, then we have interesting

[21] Plots are available from the Authors upon request. However, risk premia are sufficiently variable over time that in this case plots are not particularly revealing, especially because the size of the 90% confidence bands is rather volatile. Because these risk premia estimates are to be used in Section 5.4 to produce measures of portfolio misspricing that do not depend on our procedure of construction of factor mimicking portfolios, in this Section we report risk premia estimates for the case in which factors are not replaced by their mimicking portfolios. Risk premia estimates for the case matching Sections 4.1-4.2 are available upon request from the Authors but these are qualitatively similar to those presented here.

evidence in favor of two priced risk factors in the cross-section of U.S. asset returns: both market and real consumption growth risks appear to be priced, in the full as well as in the sub-samples investigated in this paper. Market risk carries a mean posterior price of 1.20% per month with a classical p-value of essentially zero; consumption growth risk carries a mean posterior price of 0.64% per month, again with a p-value of zero. While the finding of a significantly priced market factor may be not surprising, the result that also typical macroeconomic, real consumption growth risk is priced is consistent with earlier evidence centered on real estate data (see e.g., Ling and Naranjo, 1997). This result is robust across the sub-samples, even though with only 24 observations the GFC period assigns a large (2.30%) and precisely estimated price only to real consumption growth risk, which may be interpreted as the recession being priced in the assets under examination. In the full sample and the early 1980-1992 sample, we also obtain evidence of abnormal pricing in the sense that the time series of intercept mean posterior terms $\{\hat{\lambda}_{0,t}\}$ is non-negligible in economic terms (e.g., 0.39% return per month that does not seem to come from any of the assumed risk factors) and precisely estimated. However, the more recent, modern REIT era offers a slightly more "benign" view on the performance of our model because the mean posteriors of the $\hat{\lambda}_{0,t}$ stop being precisely estimated, while there is also some evidence of inflation and real T-bill risks being priced.

The evidence turns inconclusive if one tries to use (averages over time of) 90% Bayesian credibility intervals built using posterior densities from the model. The reason is that without any exceptions, all these densities attach a non-negligible probability to zero or small risk premia on the factors. For instance, over the full sample, the 90% interval for the market risk premium goes from -3.36% per month to 3.76% per month. Although the median of the posterior is a sizeable and sensible 1.45% per month, an (unreported) density plot reveals that almost 30% of the probability mass can be assigned to zero and negative risk premia.[22] On the one hand, this leads to the puzzling conclusion that—at least under our weak priors—the data fail to give strong indications in favor of any of the assumed macroeconomic risk factors being priced. On the other hand, it is comforting to see that this does not occur through probability mass being shifted out of the risk factors and into the posterior density of the mispricing indicator $\hat{\lambda}_{0,t}$, as also the 90% credibility interval for the intercept is wide and attaches substantial posterior mass to the absence of mispricing (e.g., spanning -3.15% to 3.94% in the full sample).

How sensible are these posterior estimates of the risk premia associated to the assumed risk factors? On the one hand, some of the posterior means reported above may seem unusually large (e.g., a posterior mean for the market of 1.20% per month implies an annualized risk premium on an asset of unit beta of 14.4%) and ask for a more systematic assessment of how reasonable the estimates may be. On the other hand, we should keep in mind that under (5), both the portfolio-specific risk exposures and the unit prices of risk are themselves time-varying, so that simply visualizing the annualized risk premium on unit exposure mimicking portfolio may at best yield a partial view on

[22] Large differences in results derived from means vs. quantiles of the posterior densities of the risk premia coefficients are possible because the posterior densities have a highly-non normal, asymmetric shape.

the economic implications of our estimates. We have therefore proceed to use the time series of posterior medians for both the portfolio-specific factor risk exposures, $\{\beta_{ij,t}\}$ for $i = 1, ..., N$ and $j = 0, ..., K$, and of the risk premia, $\{\lambda_{j,t}\}$ for $j = 0, ..., K$, to systematically compute the posterior median of the excess returns implied by the MFAPM,

$$x_{i,t\to t+1}^{MFAPM} = \lambda_{0,t+1} + \sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t+1|t}. \tag{17}$$

Although we refrain from a complete tabulation of the implied averages over our sample period of median posteriors of $x_{i,t\to t+1}^{MFAPM}$ (results are available upon request; see also Section 5.4), the results are generally sensible. For instance, the annualized time average of median posterior excess returns for the NAREIT composite portfolio is 2.44%, while the resulting average risk premium estimates are 4.32%, 2.28%, and 1.21% per year for equity REITs, mortgage REITs, and hybrid REITs, respectively. In the case of industry portfolios, the implied risk premia range from 3.22% for utilities to 6.25% for manufacturing. In the case of size-sorted equity portfolio, the risk premia estimates range from 3.96% for large capitalization stocks to 5.93% for the smallest stocks. Finally, the implied risk premia for the three bond portfolios are 1.82%, 1.27%, and 0.76% per year for long-term Treasuries, medium-term Treasuries, and high yield corporate bonds, respectively. If anything, these model-implied estimates of risk premia seem on the low end of the commonly accepted spectrum and one may wonder how such findings are compatible with the rather large mean posterior estimates for factor-specific risk premia in Table 2. Let's consider for instance the NAREIT composite index. The annualized 3.44% reported above may be decomposed in the following median contributions by each of the single factors, in annualized terms: +5.66% from market exposure, -1.24% from credit premium exposure, -2.74% from yield curve exposure, +0.16% from unexpected inflation exposure, -0.20% from IP growth risk, -3.39% from real consumption growth exposure, and +0.18% from real rate risk.[23]

Even though our Bayesian design has endeavored to escape all issues with spurious measurement error induced by the fact that the regressors used in the "second step" are generated, a great deal of caution must be exercised when interpreting these posterior densities for risk premia and the implied summary statistics.[24] For instance, as recently discussed by Lewellen, Nagel and Shanken (2010), factors that are specified in the MFAPM but that in fact fail to explain the variation in returns (i.e., when the MFAPM is misspecified because it includes factors that in reality are not priced) may often command very significant (but misleading) prices of risk. Ang, Liu, and Schwarz (2010) demonstrate that an approach based on relatively large portfolios such as ours results in large efficiency losses in cross-sectional tests of asset pricing models. In particular, they show that while creating portfolios reduces estimation error in betas, standard errors of risk premia estimates are larger due to the smaller spread in betas. This advises us to rely more on the first step of our estimation program to try and tease out the economic implications of our MFAPM for our key questions, which is exactly

[23] The difference between the sum of these contributions and the 3.44% estimate, comes from the average median λ_0.

[24] We thank Peter Schotman for drawing our attention on these residual issues and concerns.

what Section 5.3 does.

5. **Economic Implications**

So far our discussion has focussed on the statistical performance of the model with emphasis on whether there was evidence of either the $\lambda_{i,t}$s or the $\beta_{ij,t}$s coefficients being "different from zero". However, our primary goal does not consist of the estimation of a MFAPM; instead, we have proposed such a framework to ask whether and by how much available U.S. data contain any evidence of gross and persistent mispricing of REITs. Before proceeding to these tasks in Sections 5.3 and 5.4, in Section 5.1 and 5.2 we briefly discuss further economic implications of the estimates presented so far. The intuition is that it is only after showing that our MFAPM yields sensible economic insights and possesses a good explanatory power, that one can reliably use it as a tool to interpret the evidence of mispricing in the data. In Sections 5.1 and 5.2, the information at time $t-1$ ($\mathbf{Z}_{t-1}$) is proxied by the instrumental variables listed in Section 3. While Section 5.3 does not rely on our estimates of factor risk premia, Section 5.4 does.

5.1. *Variance Ratios*

The first two columns of Table 3 present posterior medians of (normalized) $VR1$ and $VR2$ obtained from (5) for each of the 27 portfolios. The normalization is performed by dividing the posterior medians by the variance of the underlying excess return series. Variance ratio results are encouraging. Under a VR1 perspective, we can claim that approximately 43% of the predictable variation in excess returns is captured by the MFAPM. However such percentages vary considerably across different assets. They are relatively high, also in relation to what is typically reported in the literature, for real estate portfolios, with several indices well in excess of 50%. VR1 also tends to be large for bond portfolios. The explanatory performance of the model is instead much less impressive when it comes to equity portfolios: although medium- and large-size portfolio are relatively well explained by the MFAPM (but VR1 for the tenth size decile is an average 0.4), VR1 tends to be considerably smaller for small caps. There is also considerable heterogeneity in the case of industry portfolios, with some good VR1 indices close to 50% (like in the case of high tech and telecommunications), and other more disappointing indices for other industries (e.g., retail and utilities).[25] Because $VR1 + VR2 = 1$ does not hold, the finding of good VR1 ratios fails to imply that the VR2 ratios are as close to zero as much as we would want. In 11 portfolios out of 27 we at least find that VR2 is below 0.5. VR2 is indeed uniformly below 0.5 for all the REIT and fixed income portfolios. All in all, under both the VR1 and VR2, we find evidence of appreciable performance of the model.

[25] Unreported results for the case in which the factors and not their mimicking portfolios are employed reveal much higher VR1 ratios. For instance, in the case of size-sorted portfolios, $VR1$ always exceeds 70% and it averages close to 87%, which is impressive. However, for REIT portfolios we obtain VR1 statistics around 50% which are similar to ones reported in Table 3. The additional parameter uncertainty induced by the need to estimate the structure of the mimicking portfolios is likely responsible for this observed deterioration that however hardly affects the fit of the model in the case of REITs.

5.2. *Decomposing Predictable Variation*

Table 3 shows that the predictable variation in excess stock returns is mostly explained by the market risk factor: with three exceptions (the smallest capitalization portfolio, durable goods and health stocks), all the ratios $Var[P(\lambda_{MKT,t}\beta_{iMKT,t|t-1}|\ \mathbf{Z}_{t-1})]\ /Var[P(\sum_{j=1}^{7}\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1})]$ concerning stocks exceed 0.5 with peaks in excess of 1 for a number of industries as well as large-cap portfolios.[26] However, the market factor explains little or nothing of the predictable variation in excess bond returns. REIT portfolios stand in between, with a contribution of the market risk factor between 0.43 (for equity REITs) and 0.91 (for hybrids). As far as stocks are concerned, the next most important factor contributions come from the credit risk premium (especially for industry portfolios) and to some extent, the real short term rate, although the heterogeneity across portfolios is pronounced. In the case of bond portfolios, many other factors—e.g., default and yield curve slope risks, and to some extent also business cycle factors such as IP growth and the real short term rate—different from the market give contributions to explain the predictable variation in excess returns. For instance, the (large) predictable portion of junk corporate excess returns seems to be mostly driven by default risk (representing a likely flight-to-quality effect during economic downturns), the IP growth factor, and the real short term rate. Once more, real estate assets fall in between with important contributions from the credit risk factor as well as the market.

5.3. *The Size and Persistence of Mispricings*

Figure 4 reports posterior median estimates of $\beta_{i0,t}$ for the REIT and bond portfolios and for 7 sample stock portfolios. As usual, complete results for all stock portfolios are available from the Authors. In an ICAPM interpretation of (1), when all factors are tradeable, and under the null of correct specification, the time series $\{\beta_{i0,t}\}$ gives indications on time-varying mispricing. If $x_{i,t}$ represents returns in excess of the riskless rate and the K factors have been correctly specified, $\beta_{i0,t} \neq 0$ represents evidence of non-zero excess returns for a portfolio i with zero exposures to the K risk factors, which implies the existence of an arbitrage opportunity and it is inconsistent with first principles (e.g., non-satiation). In the finance literature, estimates of quantities like $\beta_{i0,t}$ are often named Jensen's "alphas" and interpreted as measures of abnormal (excess) returns, where their abnormality refers to the principle that only systematic risk should be priced in equilibrium. Starting with REIT portfolios, which are our main object of interest, the plots show no evidence of strong or precisely estimated mispricing at the aggregate, composite level: the corresponding $\beta_{i0,t}$ initially implies a positive mispricing of risk exposures in the early 1980s; subsequently, the mispricing declines and it is almost completely re-absorbed by 2004 and actually turns negative after 2005. However, the associated 90% credibility interval includes zero throughout the entire sample period, similarly to the early findings by Chan et al. (1990). This means that, if anything, over part of the 2003-2006

[26] As explained in Section 2.4, these ratios may exceed 100% because $Var[P(\sum_{j=1}^{7}\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1})]$ will also reflect the contribution of covariance terms between factor terms. In fact, in Table 3 the only two contributions exceeding 100% are obtained in the presence of sizably negative covariance contributions.

period, REITs as an asset class have yielded excess returns that were lower than what they should have given, when their risk exposures are taken into account; however, the measure of such mispricing is subject to considerable uncertainty, coming from both the sampling that underlies the data and the coefficients of the model. Interestingly, the right-most panel of Figure 5 shows that most of this mispricing comes not from equity REITs but from mortgage REITs, with $\beta_{i0,t}$ oscillating between -2 and -3 percent per month in the case of mortgage REITs and 90% credibility bands that are at most as large as to include -1%, but never zero. This mispricing becomes particularly strong after 2001 and more recently during 2006-2009, which is hardly surprising.[27] In the case of mortgage REITs, if there ever was mispricing of this portfolio, this occurred in the sense of yielding excess returns *inferior* to what they should have been, conditioning on their risk exposures and the estimated prices of risk.

However, it may still remain unclear how these results for the posterior distribution of the REIT $\beta_{i0,t}$s may map into our motivating question: were REITs over-priced in the period preceding the GFC? The answer is that while in general REITs were never over-priced—and this conclusion strongly applies to equity REITs—mortgage REITs and to some extent hybrid REITs may have been structurally over-priced. This derives from the fact that when looking at excess realized returns, a negative sequence of $\beta_{i0,t}$ over time implies *under-pricing of risk*—because historical excess returns have been lower than what their risk exposures imply—and *over-pricing of the asset* to which excess returns refer to, because a negative $\beta_{i0,t}$ at time t implies a low expected excess returns and hence (for instance, in a simple risk-adjusted present value model) over-pricing of the asset at time t.[28] Such an over-pricing of mortgage REITs has become strong after 2001; it has gotten gradually worse between 2003 and 2007 and only after 2008 it seems to have stabilized, although our data report no evidence of the mispricing being on its way to be gradually re-absorbed. To given an idea of the dimension of the mispricing, a mortgage REIT $\beta_{i0,t}$ of -3.1% per month as of December 2010 may correspond to enormous over-pricing of the underlying assets, because it implies that its cash flows may be discounted at a rate that is almost 40% per year lower than it should actually be. Yet, the posterior median of the REIT class as a whole marked a -0.5% as of December 2010 that does not point to any massive mispricing, while the 95% upper confidence band of 1.4% may even cast some doubts on the very notion that any further pricing correction is to be expected, on the basis of our MFAPM. In fact, as of December 2010, equity REITs as an asset class may have been slightly under-priced.

Figure 4 also reports the dynamics of posterior medians and 90% credibility intervals for a few selected stock portfolios. Although these plots do not concern REITs, they are crucial to the economic implications of this paper. The three panels for stock portfolios show that the BTVBSV model is not forced to produce for all assets the same type of implications concerning the posteriors of $\beta_{i0,t}$. For instance, energy stocks seem to have systematically yielded excess returns that are too high when

[27] Interestingly, for a comparable sample, also Peterson and Hsieh (1997) report large and statistically significant negative alphas (hence, over-pricing) of mortgage REITs, although in a linear factor model that is based on a 5-factor extension of the classical Fama-French framework.

[28] The two perspectives are logically consistent because under-pricing the risk exposures of an asset implies that the asset will be over-priced and will fail to yield adequate rates of capital gain over time.

compared to their risk exposures, so that their $\beta_{i0,t}$ has a posterior median that is positive, large, and mildly increasing to reach an enormous level of 2.5% per month around the end of December 2010; their confidence bands fail to include zero, even though the lower band is often close to zero. To the contrary, durable stocks imply posterior densities that are strongly shifted to the left of zero, indicating massive over-pricing of the underlying stocks as even the 95% upper bound takes values below -0.5% per month for most of our sample. Consistently with the bulk of the empirical finance literature, small capitalization stocks appear to have been over-priced in the light of our model while there is no evidence of mispricing of large caps.[29] Finally, all fixed income portfolios under analysis appear not have been persistently mispriced during our sample period, even though in the case of Baa corporate bonds, some evidence of over-pricing surfaces after 2000.

5.4. *Does the Methodology Matter? Selected Fama-MacBeth Evidence on REIT Mispricing*

As in most applications of asset pricing methods, we would like our key economic insights to be as robust as possible to variations of the research design and of estimation methods. In particular, in spite of our efforts in Section 2 to discuss the potential advantages of a Bayesian implementation to deal with issues concerning parameter uncertainty and generated regressor problems, it is important to show whether (and if not, how) our key findings concerning the mispricing of REITs are robust to adopting the simpler, traditional two-pass estimation approach described in Section 2.2.[30] GRT (2010) report extensively on results from the Fama-MacBeth's approach using similar data and samples. They stress that many time series of rolling window betas turn out to be extremely volatile, almost erratic, and that in many situations it is hard to provide any intuition for their sign or dynamics. Interestingly, our Bayesian estimates of the loadings in Section 4.1 are considerably smoother than the classical, rolling window ones: even though (5) formally allows the $\beta_{ij,t}$ to be subject to jumps over time, as a result of the realization of a latent binary random variable, the resulting posterior densities are actually smooth. In this subsection, we take a more focussed perspective and ask what a simple Fama-MacBeth approach implies for mispricings.

Figure 5 starts by reporting complete results on the "Jensen's alphas" ($\beta_{i0,t}$) of the REIT portfolios and a few selected stock and bond portfolios. To provide some qualitative intuition for the type of results that a Fama-MacBeth approach would yield for factor exposures, Figure 5 also plots time-varying market betas. In particular, the first two plots compare Jensen's alphas and market betas from the Bayesian and from the traditional two-step methods with reference to the NAREIT composite portfolio. Fama-MacBeth 90% confidence intervals are computed using standard t-student

[29] More generally, out of 20 portfolios, in 5 cases we find $\beta_{i0,t}$s with a posterior median that is uniformly positive over our entire sample period (this occurs for non durables, energy, telecommunication, health, and utility stocks), although there are other 9 portfolios (durables, high tech, retail, size deciles 1-6 stocks) for which the posterior means of the abnormal returns are negative. In the remaining 6 cases, the mean $\beta_{i0,t}$ changes sign over time.

[30] For the sake of comparison, the methods described in Section 2.2 are augmented by a GARCH(1,1) model for shocks to the linear factor model which wants to match the fact that our BTVSVM implementation has also exploited a stochastic volatility component to the purpose of identifying dynamics in risk exposures and risk premia. Complete results concerning a 60-month rolling window Fama-MacBeth implementation of our model are available upon request.

based confidence intervals. Consistently with the results in GRT (2010), Figure 5 emphasizes that the two-step $\hat{\beta}_{i0,t}$ behaves erratically and displays wide confidence intervals that in most of our sample (there are only 47 months out of 372 that make an exception to this statement; in 33 months the alpha is significantly negative at a 10% size, in the remaining 14, it is significantly positive at a 10% size) include a zero abnormal return. In particular, there is no evidence of an accurately estimated mispricing for REITs after 2002 and through 2010. One can also verify that in roughly two-thirds of our sample, the Bayesian 90% credibility region is completely contained inside the frequentist 90% confidence interval. Even though the average over time of the two-step $\hat{\beta}_{i0,t}$ estimates is positive (0.85% per month) and therefore displays a sign opposite to what found in our Bayesian analysis, the facts commented above along with its p-value of approximately 0.77 confirm that there is little evidence of a large or persistent mispricing of REITs as an asset class independently of the estimation approach adopted. Specific results concerning equity, mortgage, and hybrid REIT portfolios in the right-most plot (to be compared to one in Figure 1) appear to be even more erratic than the overall findings for the NAREIT composite index: only in 10-15% of the months on our sample the 90% confidence bands fail to include a zero abnormal return; using standard test procedures in the Fama-MacBeth literature, we obtain p-values for $\hat{\beta}_{i0}$ that range between 0.74 and 0.79 that cast considerable doubt on the reliability of the average point estimates of -0.6% to 0.8% per month. All in all, we conclude that the specific estimation approach does not seem to matter when it comes to judge the mispricing of publicly traded real estate vehicles, in the sense that in both cases the alpha estimates are imprecise and attach substantial probability mass to the absence of abnormal returns. However, while in the BTVBSV case, this occurs within a framework in which posterior uncertainty is realistic, a Fama-MacBeth approach ends producing estimates that erratically span a huge [-16%, +13%] monthly interval.

The second of row of plots in Figure 5 compares Fama-MacBeth and BTVBSV time-varying market betas for the NAREIT composite index. In this case the difference is striking because the Fama-MacBeth estimates have now confidence intervals of roughly the same width as the Bayesian ones, but the betas tend to locate themselves—amidst considerable oscillations, in fact in only 85% of the sample we have evidence of a non-zero market beta for the REIT class—around an average level of 0.5 which seems to lie below the posterior median estimated using Bayesian methods. The right-most plot in the second row shows instead Fama-MacBeth betas for the NAREIT components. Also in this case, the wide oscillations and confidence regions give a picture that, in our view, is considerably less reliable and, ultimately, less realistic than what we have already reported in Figure 1. The last row of plots in Figure 5 compares Fama-MacBeth and BTVBSV time-varying Jensen's alpha estimates for two sample non-REIT portfolios, i.e., large capitalization stocks and 10-year Treasury bonds. Although in both cases, the mispricing measures tend to be approximately located in the same region of the plots, the Fama-MacBeth confidence tend to be between five and twenty times wider than those obtained using Bayesian methods. These two portfolios were selected because, although in both cases the average estimated mispricings are rather small (0.73% for large caps and -0.87%

for Treasuries under the two-step approach), it is only adopting a Bayesian approach that explicitly captures instability in betas and idiosyncratic variance that delivers sufficiently precise posterior estimates that occasionally allow us to conclude that any of these portfolios were mispriced. On the opposite, a traditional Fama-MacBeth analysis would have led to very high p-values; for instance, the 90% confidence interval includes a zero mispricing in 351 months out of 372 in the case of large stocks, and in 298 months out of 372 in the case of 10-year Treasuries.

More generally, an inspection of the complete set of 5-year rolling window estimates obtained from Fama-MacBeth first stage regressions (unreported) reveals a rather odd result: most of the 27 portfolios used in our analysis display large and negative mean estimates of $\hat{\beta}_{i0}$. In Figure 5 this was the case for 10-year Treasuries and both mortgage and hybrid REIT portfolios. The means over the entire 1980-2010 sample range from 1% per month in the case of equity REITs to -4.9% in the case of high tech stocks. However, as already emphasized, all the time series estimates of the time-varying alphas are plagued by considerable estimation uncertainty so that the resulting p-values are rarely below 0.2. We view this set of implications from the standard Fama-MacBeth procedure is implausible: on average, over a long 30-year period, most of the 27 portfolios would have been significantly over-priced, and as a result their abnormal returns were negative: of course, a state of generalized, persistent and significant over-pricing of all publicly traded assets in the U.S. is as shocking as implausible, in the light of a widespread faith in the efficiency of U.S. capital markets. This makes the results in Figures 1-3 all the more realistic as the adoption of an estimation strategy that explicitly accounts for instability and breaks delivers stable and sensible estimates of both the priced components and of the mispricing.

5.5. *Did Markets Expect Too Low REIT Returns?*

As a last exercise aimed at testing whether our MFAPM may shed any light on the commonly held conjecture that real estate (hence, REITs) may have been massively over-priced during portions of the 1980s and more recently of the period 2003-2006, we have proceeded in the following way. First, using a projection model identical to (7) we have regressed the excess return on each asset onto a set of M instrumental variables that proxy for available information at time t, $\mathbf{Z}_t$, to compute a time series of conditional, time-varying expected excess returns as

$$x^{mkt}_{i,t\to t+1} = \theta_{i0} + \sum_{m=1}^{M} \theta_{im} Z_{m,t}, \tag{18}$$

where the notation $x^{mkt}_{i,t\to t+1}$ stresses that this represents the market expectation at time t for the excess return between t and $t+1$. The coefficients of this regression are estimated using Bayesian methods, which means that instead of obtaining only a point estimate of $x^{mkt}_{i,t\to t+1}$, in fact we can compute the posterior density of $x^{mkt}_{i,t\to t+1}$ over time.[31] Second, using the time series of posterior densities for both the portfolio-specific factor risk exposures and for the risk premia (see Sections 3.1 and 3.2), we

[31] Also in this case we employ uninformative priors.

compute the posterior density of the analogous, model-specific excess return series, as in (17). Because this test relies on posterior estimates for the risk premia and also as a way to provide a robustness check to results reported in Section 5.3, in this case we use the macroeconomic factors directly, instead of replicating them with mimicking portfolios. At this point our asset pricing framework implies that, given a riskless rate applicable on the interval $[t, t+1]$, when $x^{mkt}_{i,t\to t+1} > x^{MFAPM}_{i,t\to t+1}$ the market will be discounting cash flows from any asset or portfolio using a required (expected) rate higher than what our model implies: as a result, the portfolio will appear to have been under-priced by the market, compared to what the risk exposures for portfolio i and risk premia imply in our model. To the contrary, when $x^{mkt}_{i,t\to t+1} < x^{MFAPM}_{i,t\to t+1}$ then the market will be discounting cash flows from any asset or portfolio using a required rate lower than what our model implies: as a result, the portfolio will appear to have been over-priced by the market. As a result, if the conjecture that REITs may have been over-priced by the market over specific periods of time is correct, during these periods we ought to observe that, at least on average, $x^{mkt}_{i,t\to t+1} < x^{MFAPM}_{i,t\to t+1}$ or $x^{mkt}_{i,t\to t+1} - x^{MFAPM}_{i,t\to t+1} < 0$.

Figure 6 plots 5-year rolling window averages of the posterior median of the difference $x^{mkt}_{i,t\to t+1} - x^{MFAPM}_{i,t\to t+1}$ for the usual set of portfolios, starting with the 4 REIT portfolios. The rationale for using 5-year moving averages is that in practice the posterior density of $x^{mkt}_{i,t\to t+1} - x^{MFAPM}_{i,t\to t+1}$ turns out to be volatile over time, to the point that although its sub-sample moments do reveal useful information on the issues we are interested in, such variance interferes with the possibility to adequately visualize the results. To favor comparisons, all series are plotted with reference to an identical vertical scale. Figure 6 gives results that are largely consistent with Section 5.3: there is never strong evidence of over-pricing of REITs as an asset class but to the contrary there are signs of a large and growing under-pricing of REITs as a result of the steep market declines caused by the GFC between late 2007 and mid-2009 (the shaded period in the plots). While equity REITs show a similar behavior (also because they dominate the NAREIT composite index), mortgage REITs seem to have been prone to massive and persistent over-pricing, with peaks in the mid-1990s and again in 2004-2005. Although such over-pricing seem to deflate already during 2006, the GFC sweeps away the mispricing and by the end of our sample also mortgage REITs appear to be under-priced. As far stocks are concerned, most industry portfolios share a similar dynamics, while stocks seem to be the asset class subject to the most visible swings between over- and under-pricing. Most portfolios have been largely over-priced in 1985-1986 and between 1997 and 2001, in correspondence to what has been now dubbed the "dot-com" bubble; on the other hand, stock under-pricing prevailed in the late 1980s and early 1990s and ensues from the GFC, persisting at the end of our sample, in 2010. This oscillations extend to size-sorted portfolios, although the size of the mispricing in this case is modest, apart from the end of the sample, when all size-sorted portfolios appear to be under-valued. Finally, there is weaker evidence of mispricing in bond returns, although 5-year Treasuries have been severely over-priced between 2003 and 2004, when the U.S. yield curve turned downward sloping in the midst of a powerful economic expansion (the so-called "conundrum"). Interestingly, our portfolio of long-term Baa corporate bonds appears to have never been severely mispriced.

6. **Conclusions**

We have used a rich multi-factor asset pricing model with time-varying risk exposures and prices of risk to ask whether 30 years of monthly data on U.S. financial asset returns contain any evidence of persistent mispricing of publicly traded real estate vehicles (REITs) during the 2003-2006 period leading up to the GFC. Following Ouysse and Kohn (2010), we have implemented a novel Bayesian estimation approach in which both risk exposures and risk premia are explicitly modeled as following a time-varying process.

Using an analysis of the posterior densities of coefficients that ought to capture mispricing in the linear factor model, we report no evidence of strong or precisely estimated pricing anomalies at the aggregate, composite REIT portfolios level. Interestingly, most of this mispricing comes not from equity REITs but from mortgage REITs. In the latter case, it was particularly strong after 2001, which is somewhat consistent with commonly heard stories of a "bubble" in the mortgage-financed U.S. residential market. This implies some over-pricing of mortgage REITs that has gradually worsened between 2003 and 2007. As of the end of our sample (December 2010), mortgage REITs did appear to remain somewhat over-priced while equity REITs were fairly priced or even slightly under-priced. However, our model points to the existence of much more severe and persistent mispricing episodes occurring elsewhere in the U.S. market: a number of stock and bond portfolios seem to be subject to large swings in the mispricing measure. For instance, the risk to which Baa corporate bonds are exposed seem to have been massively under-priced—as a result, corporate bonds may have been largely over-priced—exactly over the period leading up to the GFC. In Section 5.5, we have performed a related experiment: using a regression of excess returns on a set of instrumental variables that proxy for available information to estimate market risk premia, we have compared such expectations with the time-varying risk premia implied by the MFAPM. When we analyze posterior medians for this difference in risk-adjusted discount rates, we find that there is never strong evidence of over-pricing of REITs as an asset class but to the contrary there are signs of large and growing under-pricing of REITs as a result of the steep market declines caused by the GFC. Mortgage REITs seem to have been prone to massive and persistent over-pricing, with peaks in the mid-1990s and again in 2004-2005.

Of course, it must be stressed that all these economic implications rely on an assumption that our model has been correctly specified. On the one hand, the wide span of macroeconomic factors and of asset classes used in the analysis and the flexible nature of the model, with explicitly time-varying risk exposures and premia, do make us hopeful in this sense. On the other hand, it would be interesting both to further fine-tune the standard, more traditional part of the model—such as the number of factors as well as their nature and definition (such as the REIT cap rate, see e.g., Liu and Mei, 1992, or the use of Fama-French factors in place of the macroeconomic variables employed in this paper, as in Peterson and Hsieh, 1997)—and at the same time to work on the specific structure and assumptions appearing in (5) to investigate whether our conclusions are robust to the details of the framework. For instance, our model for stochastic variation in betas might be fined-tuned to

accommodate a diffusive, continuous component besides the abrupt shifts currently featured in our analysis. Additionally, we have assumed that idiosyncratic shocks are uncorrelated across assets, as typically MFAPMs do. However, Kleibergen (2010) has argued that any factor structure that fails to be captured by the MFAPM and that remains in the time-series residuals may cause spurious effects in cross-sectional regressions used to estimate risk premia. Even though our key tests have not relied on risk premia estimates, it would be interesting to further fine tune the model to check whether the imposed restrictions actually hold. Up to this point, all the available evidence confirms this conjecture.

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Appendix

The assumption of $\boldsymbol{\epsilon}_t \equiv (\epsilon_{1,t}, \epsilon_{2,t}, ..., \epsilon_{N,t})' \sim N(0, \mathbf{I}_N)$ in (5) allows to estimate independently parameters for each asset $i = 1, ..., N$. For the sake of brevity, we rewrite the model in (5) for each i as

$$\begin{aligned} x_t &= \beta_{0,t} + \sum_{j=1}^{K} \beta_{j,t} f_{j,t} + \sigma_t \epsilon_t \\ \beta_{j,t} &= \beta_{j,t-1} + k_{j,t}\eta_{j,t} \qquad j = 0, ..., K \\ \ln(\sigma_t^2) &= \ln(\sigma_{t-1}^2) + k_{2,t}\upsilon_t \qquad i = 1, ..., N, \end{aligned} \tag{19}$$

where the subscription i has been omitted, $\boldsymbol{\epsilon}_t \sim N(0,1)$, $(\boldsymbol{\eta}_t, \upsilon_t)' \sim N(0, \mathbf{Q})$ with $\mathbf{Q}$ a diagonal matrix characterized by the parameters q_0^2, q_1^2, ..., q_K^2, q_υ^2, and $\boldsymbol{\kappa}_t \equiv (\kappa_{0,t}, ..., \kappa_{K,t}, k_{2,t})'$ is a $(2(K+2) \times 1)$ vector of unobserved uncorrelated 0/1 processes with $\Pr[\kappa_{j,t} = 1] = \pi_j$ for $j = 0, ..., K+1$ and $\Pr[k_{2,t} = 1] = \pi_{2k}$. The model parameters are the structural break probabilities $\boldsymbol{\pi} \equiv (\pi_0, ..., \pi_K, \pi_2)'$ and the vector of variances of the break magnitude $\mathbf{q}^2 \equiv (q_0^2, q_1^2, ..., q_K^2, q_\upsilon^2)$. They are collected in a $(2(K+1) \times 1)$ vector $\boldsymbol{\theta} \equiv (\boldsymbol{\pi}', (\mathbf{q}^2)')'$.

Independent conjugate priors are used to ease posterior simulation. Priors could differ across assets $i = 1, ..., N$. For the break probability we assume simple Beta distributions,

$$\pi_j \sim Beta(a_j, b_j) \qquad \pi_j^{k_2} \sim Beta(a_j^{k_2}, b_j^{k_2}), \tag{20}$$

where the hyperparameters a_j and b_j $(j = 0, ...K+1)$ reflect prior beliefs about the occurrence of breaks. For the variance parameters the inverted Gamma-2 prior is chosen,

$$q_j^2 \sim IG(\nu_j, \delta_j) \qquad q_\upsilon^2 \sim IG(\nu_\upsilon, \delta_\upsilon), \tag{21}$$

where ν_j expresses the strength of the prior mean.

For posterior simulation we run the Gibbs sampler in combination with the data augmentation technique by Tanner and Wong (1987). The latent variables $B = \{\beta_t\}_{t=1}^T$, $R = \{\sigma_t^2\}_{t=1}^T$, and $\mathcal{K} = \{\kappa_t\}_{t=1}^T$ are simulated alongside the model parameters, $\boldsymbol{\theta}$. Define $x = \{x_t\}_{t=1}^T$ and

$f = \left\{ \{f_{j,t}\}_{j=1}^{K} \right\}_{t=1}^{T} = \{f_t\}_{t=1}^{T}$, the complete data likelihood function is given by

$$\begin{aligned} p(x, B, \mathcal{K}, R|\boldsymbol{\theta}, f) &= \prod_{t=1}^{T} p(x_t|f_t, \beta_t, \sigma_t^2) \prod_{j=0}^{m} p(\beta_{jt}|\beta_{jt-1}, \kappa_{jt}, q_j^2) \times \\ & \quad p(\sigma_t^2|\sigma_{t-1}^2, k_{2,t}, q_\upsilon^2) \prod_{j=0}^{k} \pi_j^{k_{jt}} (1-\pi_j)^{1-k_{jt}} \pi_{2k}^{k_{2t}} (1-\pi_{2k})^{1-k_{2t}}. \end{aligned} \tag{22}$$

Combining the prior and the data likelihood, we obtain the posterior density

$$p(\theta, B, \mathcal{K}, R|x, f) \propto p(\theta) p(x, B, \mathcal{K}, R|\theta, f). \tag{23}$$

Defining $\mathcal{K}_\beta = \{\kappa_{0,t}, ..., \kappa_{K,t}\}_{t=1}^{T}$ and $\mathcal{K}_\sigma = \{k_{2t}\}_{t=1}^{T}$, the sampling scheme consists of the iterative steps:

1. Draw $\mathcal{K}_\beta$ conditional on $R, \mathcal{K}_\sigma, \boldsymbol{\theta}$, x and f.
2. Draw B conditional on $R, \mathcal{K}, \boldsymbol{\theta}$, x and f.
3. Draw $\mathcal{K}_\sigma$ conditional on $B, \mathcal{K}_\beta, \boldsymbol{\theta}$, x and f.
4. Draw R conditional on $B, \mathcal{K}, \boldsymbol{\theta}$, x and f.
5. Draw $\boldsymbol{\theta}$ conditional on $B, \mathcal{K}$, x and f.

The first step applies the efficient sampling algorithm of Gerlach, Carter and Kohn (2000), the main advantage being drawing $\kappa_{j,t}$ without conditioning on the states $\beta_{j,t}$, as Carter and Kohn (1994) instead do. The conditional posterior density for $\kappa_{\beta,t}$, $t = 1, .., T$ unconditional on B is:

$$\begin{aligned} p(\kappa_{\beta,t}|\mathcal{K}_{\beta,-t}, \mathcal{K}_\sigma, R, \boldsymbol{\theta}, r) &\propto p(r|\mathcal{K}_\beta, \mathcal{K}_\sigma, R, \boldsymbol{\theta}) p(\kappa_t|\mathcal{K}_{\beta,-t}, \boldsymbol{\theta}) \\ &\propto p(x^{t+1,T}|f^{t+1,T}, \mathcal{K}, R, \boldsymbol{\theta}) p(x_t|x_{1,t-1}, f_t, \kappa_{\beta,1,t-1}, R, \boldsymbol{\theta}, x) p(\kappa_{\beta,t}|\mathcal{K}_{\beta,-t}, \boldsymbol{\theta}). \end{aligned} \tag{24}$$

Gerlach, Carter and Kohn (2000) show how to evaluate the first two terms while the last one is obtained from the prior. When $\mathcal{K}_{\beta,t}$ and $\beta_{j,t}$ are highly dependent, the sampler of Carter and Kohn (1994) breaks down completely: the higher the correlation (dependence), the bigger the efficiency gain. The latent process for the betas is estimated by means of the forward-backward algorithm of Carter and Kohn (1994). $\mathcal{K}_\sigma$ and R are drawn in the same way as $\mathcal{K}_\beta$ and B. To do so we follow Kim, Shepard and Chib (1998) and approximate the log of a $\chi^2(1)$ distribution by means of a mixture of seven normals. In each iteration of the Gibbs sampler we simulate a component of the mixture distribution in order to get a conditional linear state space model for $\ln(\sigma_t^2)$. Finally, $\boldsymbol{\theta}$ is easily sampled as we use conjugate priors. We use a burn-in period of 1,000 and draw 5,000 observations storing every other of them to simulate the posterior distributions of parameters and latent variables. The resulting autocorrelations of the draws are very low.[32]

To estimate the cross section in (6) at each time t and for each draw of $B_{t|t-1} = (\beta_{1,t|t-1}, ..., \beta_{N,t|t-1})$ where each $\beta_{j,t|t-1}$ is a $((K+1) \times 1)$ vector and N is the total number of assets, we use natural conjugate priors. In particular,

$$p(\lambda, \sigma^2) = p(\lambda|\sigma^2) \times p(\sigma^2) \tag{25}$$

[32] In order to gain a rough idea of how well the chain mixes in our algorithm we follow Primiceri (2005) and check the autocorrelation function of the draws.

where

$$(\lambda|\sigma^2) \sim N(\underline{\lambda}, \sigma^2\underline{V}) \text{ and } (\sigma^2) \sim IG(\frac{\underline{\nu}}{2}\underline{s}^2, \frac{1}{2}\underline{\nu}) \tag{26}$$

Combining them with the data likelihood we obtain a joint posterior density with convenient analytical form. The resulting marginal posterior distributions are

$$(\lambda|x) \sim t(\overline{\lambda}, \overline{s}^2\overline{V}, \overline{\nu}) \qquad (\sigma^2|x) \sim IG(\frac{\overline{\nu}}{2}\overline{s}^2, \frac{1}{2}\overline{\nu}) \tag{27}$$

with

$$E(\lambda|x) = \overline{\lambda} \quad Var(\lambda|x) = \frac{\overline{\nu s}^2}{\overline{\nu} - 2}\overline{V} \quad E(\sigma^2|x) = \frac{\overline{\nu s}^2}{\overline{\nu} - 2} \quad Var(\sigma^2|x) = \frac{(\overline{\nu s}^2)^2}{(\overline{\nu} - 2)^2(\frac{\overline{\nu}}{2} - 2)} \tag{28}$$

where

$$\overline{V} = (\underline{V}^{-1} + ((B'^{-1})^{-1} \qquad \overline{\lambda} = (\underline{V}^{-1} + ((B'^{-1})^{-1}(\underline{V}^{-1}\underline{\lambda} + ((B'^{-1}\hat{\lambda}) \tag{29}$$

where $B = \left\{B_{t|t-1}\right\}_{t=2}^{T}$, $\overline{\nu} = \underline{\nu} + N$, and $\hat{\lambda}$ is the OLS estimate. Results were obtained under two different sets of priors. In the former case priors were non-informative ($\underline{\nu} = 0$ and $\underline{V}^{-1} = 0$) and exploit the well known Jeffreys' prior while in the latter case we impose some prior information. In more detail, we opted for a small amount of strength ($\underline{\nu} = 5$) supporting a prior view for premiums with zero mean and standard deviation equal to a twelfth of the maximum absolute return observed in the sample. When informative priors were used, we recorded a striking reduction in the variability of the estimated posterior distributions (as well as their medians) for the risk premia relative to the baseline case.[33] Finally, the prior residual variance is centered at about 10, a value that appeared in the higher range of the maximum likelihood estimates.

A final note goes to issues related with the estimation of factor-mimicking portfolios to replace the three non-traded factors that are featured in our empirical application (i.e., IP and real consumption growth, and unexpected inflation). In this case, the time-varying (conditional) idiosyncratic covariance matrix reflects the possibility of breaks in variances and at the same time it is able to "net out" from idiosyncratic variances the effects due to breaks in risk exposures (the betas). Moreover, the $N \times M$ matrix of time series slope coefficients $\mathbf{B}_t$ is obtained as the matrix of posterior medians from (5) so that also these coefficients take into account the presence of breaks. As a result, the vector of weights defining the mimicking portfolio of factor j ($j = 1, 2, 3$) has a zero time t exposure to median risk represented by the remaining 6 factors, but such a median exposure also takes into account the chances of instability in exposure occurring, which is consistent with the set up of (5); additionally, the conditional beta of the jth mimicking portfolio on the jth economic factor may change as the matrix of posterior median exposures and of idiosyncratic variances change over time, according to the process postulated in (5).

[33] In any event, the qualitative results and insights from Table 2 were intact, and the only important change was that the 90% credibility intervals become tighter. Both real consumption growth and market risks re-assert themselves as important drivers of the cross section of U.S. returns, even though during the initial samples other factors (term spread, unemployment, and the real short term rate) appear to have been priced.

Table 1

Summary Statistics for Financial and Macroeconomic Time Series Used in the Paper

The table reports summary statistics of stock, Treasury bond, corporate bond, and real estate portfolio returns. For all assets, the Sharpe ratio is computed with reference to 1-month T-bill rates. The 10 industry portfolios are obtained by sorting firms according to their four-digit SIC code. The 10 additional portfolios are obtained by sorting (at the end of every year) NYSE, AMEX and Nasdaq stocks according to the aggregate market value of the company's equity. Data on 10- and 5-year government bond returns are from Ibbotson/CRSP. Data on junk bond returns are approximated from Moody's (10-to-20 year maturity) Baa average corporate bond yields and converted into return data using Shiller's (1979) approximation. Data on REIT total returns are from NAREIT and concern three major categories of tax-qualified REITs, i.e. equity, mortgage, and hybrid equity/mortgage. The additional economic variables consist of excess returns on a value-weighted market portfolio, the credit risk premium (the difference between Baa Moody's yields and yields on 10-year government bonds), the change in the term premium (the difference between 5-year and 1-month Treasury yields), the rate of growth of (seasonally adjusted) industrial production, the rate of growth of (seasonally adjusted) real personal consumption growth, the 1-month real T-bill yield computed as the difference between 1-month nominal returns and realized CPI inflation rate, and the unexpected inflation rate computed as the residual of a simple ARIMA(0,1,1) model.

	1980 - 2010				1980 - 1992				1993-2010			
	Mean	Median	Std. Dev.	Sharpe Ratio	Mean	Median	Std. Dev.	Sharpe Ratio	Mean	Median	Std. Dev.	Sharpe Ratio
	10 Industry Portfolios, Value-Weighted (Source: CRSP, NYSE/AMEX/NASDAQ)											
Non Durable Goods	1.206	1.250	4.311	0.194	1.656	1.390	4.686	0.244	0.824	1.115	3.821	0.144
Durable Goods	0.951	1.205	6.884	0.085	1.144	1.045	5.753	0.110	0.824	1.275	7.247	0.076
Manufacturing	1.133	1.485	5.076	0.151	1.325	1.610	4.789	0.170	1.034	1.365	4.983	0.153
Energy	1.203	1.040	5.271	0.159	1.241	1.285	4.567	0.160	1.180	0.990	5.492	0.165
High Tech	0.986	1.310	7.296	0.085	1.034	1.155	5.708	0.092	1.135	1.445	7.902	0.109
Telecommunications	0.941	1.435	5.335	0.108	1.520	1.835	4.393	0.230	0.590	1.155	5.637	0.056
Shops and Retail	1.066	1.315	5.207	0.134	1.329	1.315	5.502	0.149	0.758	1.140	4.712	0.103
Health	1.075	1.220	4.824	0.147	1.485	1.420	5.202	0.187	0.823	1.060	4.424	0.124
Utilities	0.961	1.315	4.044	0.147	1.209	1.145	3.427	0.204	0.757	1.315	4.316	0.112
Other	0.942	1.470	5.277	0.109	1.311	1.830	4.731	0.169	0.738	1.375	5.324	0.087
	10 Size-Sorted Portfolios, Value-Weighted (Source: CRSP, NYSE/AMEX/NASDAQ)											
Decile 1	1.034	1.360	6.129	0.108	0.897	1.170	5.042	0.077	1.180	1.635	6.470	0.140
Decile 2	1.065	1.505	6.342	0.109	1.020	1.650	5.157	0.099	1.102	1.390	6.716	0.123
Decile 3	1.123	1.920	5.947	0.126	1.239	2.350	5.081	0.143	1.060	1.760	6.186	0.127
Decile 4	1.049	1.815	5.778	0.117	1.199	1.920	5.113	0.135	0.951	1.600	5.909	0.115
Decile 5	1.132	1.880	5.676	0.134	1.317	1.995	4.937	0.164	1.009	1.715	5.835	0.126
Decile 6	1.092	1.575	5.193	0.139	1.335	1.615	4.680	0.176	0.949	1.555	5.250	0.129
Decile 7	1.140	1.440	5.110	0.150	1.314	1.240	4.613	0.174	1.034	1.500	5.167	0.147
Decile 8	1.092	1.570	5.132	0.140	1.280	1.730	4.559	0.169	0.956	1.400	5.215	0.131
Decile 9	1.099	1.755	4.723	0.154	1.336	1.740	4.387	0.188	0.944	1.755	4.682	0.143
Decile 10	0.948	1.180	4.430	0.130	1.328	1.485	4.033	0.203	0.716	1.165	4.433	0.100
	Bond Returns (Source: FREDII, Ibbotson via CRSP)											
10-Year Treasury Notes	0.700	0.726	1.793	0.183	0.806	0.826	1.267	0.234	0.570	0.593	1.730	0.171
5-Year Treasury Notes	0.538	0.485	0.642	0.260	0.735	0.738	0.823	0.274	0.400	0.375	0.553	0.229
Baa Corporate Bonds (10-20)	1.020	1.075	2.863	0.227	1.078	1.148	1.582	0.359	0.742	0.928	2.739	0.171
1-month Treasury Bills	0.376	0.400	0.210	0.000	0.510	0.490	0.171	0.000	0.273	0.310	0.161	0.000
	Real Estate Returns (Source: NAREIT and Dow Jones)											
NAREIT - Composite	0.889	1.167	4.906	0.106	0.788	0.655	3.114	0.089	0.983	1.591	5.668	0.125
NAREIT - Equity TR	1.066	1.294	5.121	0.136	1.092	1.033	3.333	0.175	1.043	1.618	5.895	0.131
NAREIT - Mortgage TR	0.530	0.986	5.550	0.029	0.401	0.514	3.744	-0.029	0.713	1.652	6.404	0.069
NAREIT - Hybrid TR	0.398	0.907	5.619	0.005	0.690	0.563	3.679	0.049	0.332	1.277	6.387	0.009
	Economic Risk Variables (Source: FREDII and CRSP, Ibbotson)											
Excess Value-Weigted Market	0.589	1.045	4.566	0.129	0.740	1.030	4.127	0.179	0.520	1.155	4.595	0.113
Default Premium (annualized)	2.226	2.040	0.742	__	1.984	1.920	0.407	__	2.326	2.120	0.863	__
Change in Term Spread	0.000	0.000	0.058	__	-0.010	-0.025	0.728	__	0.000	-0.001	0.045	__
Unexpected Inflation	0.007	0.004	0.316	__	0.008	0.006	0.224	__	0.011	0.005	0.353	__
Industrial Production Growth	0.209	0.246	0.631	__	0.285	0.294	0.536	__	0.185	0.237	0.668	__
Real Pers. Consumption Growth	0.230	0.238	0.552	__	0.265	0.243	0.150	__	0.209	0.220	0.467	__
Real 1-month T-Bill Returns	0.134	0.144	0.341	__	0.219	0.217	0.239	__	0.071	0.104	0.366	__
	Instrumental Variables (Source: FREDII, CRSP, Ibbotson via CRSP)											
5Y Govt. Yield - 1m T-Bill (annual)	1.521	1.565	1.156	__	2.028	2.155	1.028	__	1.282	1.295	1.064	__
Yield Spread Baa - Aaa (annualized)	0.487	0.515	1.163	__	0.785	0.790	0.456	__	0.331	0.310	1.130	__
Dividend Yield (annualized)	2.580	2.424	1.016	__	3.417	3.315	0.640	__	1.924	1.909	0.481	__

Table 2

Summary Statistics for Second-Pass Bayesian Posterior Estimates of Risk Premia Coefficients

The table reports posterior estimates-based statistics (means and medians over time, with 5% and 95% credibility regions) for the unit risk premia (common across assets) associated to each of the factors in the asset pricing model:

$$x_{i,t} = \beta_{i0,t} + \sum_{j=1}^{K} \beta_{ij,t} f_{j,t} + \sigma_{i,t}\epsilon_{i,t}$$

$$\beta_{ij,t} = \beta_{ij,t-1} + \kappa_{1ij,t}\eta_{ij,t} \qquad j = 0, ..., K, \ln(\sigma^2_{i,t}) = \ln(\sigma^2_{i,t-1}) + \kappa_{2i,t}\upsilon_{i,t} \qquad i = 1, ..., N$$

$$x_{i,t} = \lambda_{0,t} + \sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1} + e_{i,t} \qquad i = 1, ..., N$$

The model is estimated using a Gibbs sampler in combination with the algorithm by Gerlach, Carter and Kohn (2000). The posterior density of the beta coefficients $\beta_{ij,t|t-1}$ on which the second, cross-sectional step conditions are obtained by taking the lagged value from the updating step of the Kalman filter and simulating the occurrence of future breaks and the shock magnitude from the appropriate posteriors.

	Average	Std. Error	t-stat	p-value	5% CI	Median	95% CI
			Full Sample (1980-2010)				
Intercept (Avg. cross-sectional abnormal returns)	0.393	2.644	2.866	**0.004**	-3.149	0.338	3.935
Market	1.198	4.974	4.640	**0.000**	-3.359	1.450	3.758
Default (credit) spread	0.045	2.139	0.408	0.683	-2.224	0.006	2.311
Term spread riskless yields	0.017	0.235	1.426	0.155	-0.290	0.016	0.322
(Unexpected) Inflation	0.072	1.075	1.291	0.197	-1.249	0.039	1.391
IP growth	-0.117	2.402	-0.935	0.351	-2.965	-0.089	2.736
Real consumption growth	0.639	2.781	4.426	**0.000**	-3.141	0.447	4.006
Real Treasury Bill	-0.056	0.836	-1.294	0.196	-0.969	-0.020	0.859
			1980-1992				
Intercept (Avg. cross-sectional abnormal returns)	0.497	2.234	2.873	**0.005**	-2.770	0.484	3.741
Market	1.101	4.736	3.004	**0.003**	-3.152	1.442	3.368
Default (credit) spread	0.009	1.668	0.067	0.945	-1.710	0.009	1.727
Term spread riskless yields	0.021	0.234	1.168	0.245	-0.232	0.017	0.273
(Unexpected) Inflation	-0.066	0.854	-0.995	0.321	-1.296	-0.067	1.164
IP growth	-0.052	2.028	-0.330	0.742	-2.303	-0.060	2.210
Real consumption growth	0.674	2.422	3.595	**0.001**	-1.853	0.649	2.768
Real Treasury Bill	0.037	0.694	0.687	0.493	-0.788	0.094	0.861
			1993-2010				
Intercept (Avg. cross-sectional abnormal returns)	0.309	2.940	1.501	0.135	-3.459	0.146	4.093
Market	1.278	5.170	3.530	**0.001**	-3.528	1.498	4.078
Default (credit) spread	0.075	2.463	0.437	0.662	-2.644	-0.017	2.790
Term spread riskless yields	0.014	0.237	0.866	0.387	-0.337	0.016	0.362
(Unexpected) Inflation	0.185	1.217	2.170	**0.031**	-1.211	0.137	1.577
IP growth	-0.170	2.674	-0.906	0.366	-3.506	-0.206	3.167
Real consumption growth	0.611	3.049	2.860	**0.005**	-4.195	0.279	5.019
Real Treasury Bill	-0.132	0.932	-2.029	**0.044**	-1.116	-0.063	0.856
			Great Financial Crisis (2007-2009)				
Intercept (Avg. cross-sectional abnormal returns)	-0.814	4.076	-0.979	0.338	-6.133	-0.934	4.550
Market	0.754	5.170	0.715	0.482	-5.545	0.412	5.022
Default (credit) spread	0.492	3.068	0.786	0.440	-4.254	0.612	5.263
Term spread riskless yields	0.022	0.316	0.344	0.734	-0.504	0.049	0.539
(Unexpected) Inflation	0.394	1.743	1.106	0.280	-1.984	0.075	2.724
IP growth	-0.536	4.901	-0.536	0.597	-6.216	-1.046	5.118
Real consumption growth	2.301	5.028	2.242	**0.035**	-7.039	1.636	10.704
Real Treasury Bill	-0.019	0.819	-0.112	0.912	-1.679	0.011	1.663

Table 3

Variance Ratio Coefficients and Predictable Variation Decompositions from Bayesian Factor Model with Instability in Risk Exposures and Idiosyncratic Variance

The table presents posterior medians for the full-sample ratios

$$VR1 \equiv \frac{Var\left[P\left(\sum_{j=1}^{K}\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right)\right]}{Var[P(x_{it}|\mathbf{Z}_{t-1})]} \qquad VR2 \equiv \frac{Var\left[P\left(x_{i,t}-\sum_{m=1}^{M}\hat{\theta}_{im}Z_{m,t-1}|\mathbf{Z}_{t-1}\right)\right]}{Var[P(x_{it}|\mathbf{Z}_{t-1})]} > 0$$

VR1 should be equal to 1 if the multi-factor model

$$x_{i,t} = \beta_{i0,t} + \sum_{j=1}^{K}\beta_{ij,t}f_{j,t} + \sigma_{i,t}\epsilon_{i,t} \quad \beta_{ij,t} = \beta_{ij,t-1} + \kappa_{1ij,t}\eta_{ij,t} \quad j = 0, ..., K, \quad \ln(\sigma_{i,t}^2) = \ln(\sigma_{i,t-1}^2) + \kappa_{2i,t}\upsilon_{i,t} \quad i = 1, ..., N$$

is correctly specified, which means that all the predictable variation in excess returns is captured by variation in risk compensations; VR2 should be equal to zero if the multi-factor model is correctly specified. However, VR1 + VR2 is not guaranteed to sum to one, because of the presence of non-zero covariance effects.

	VR1	VR2	Market	Chg. Credit Risk Prem.	Chg. In Yield Curve Slope	Unexp. Inflation	IP Growth	Real Cons. Growth	Real T-Bill	Interaction Effects
					10 Industry Portfolios, Value-Weighted					
Non Durable Goods	0.29	0.69	0.97	0.17	0.00	0.05	0.09	0.03	0.02	-0.39
Durable Goods	0.22	0.67	0.40	0.12	0.00	0.02	0.07	0.01	0.01	0.33
Manufacturing	0.42	0.58	1.02	0.00	0.01	0.01	0.01	0.02	0.00	-0.10
Energy	0.31	0.62	0.79	0.13	0.02	0.10	0.08	0.02	0.03	-0.25
High Tech	0.52	0.50	0.81	0.01	0.07	0.19	0.02	0.01	0.37	-0.51
Telecommunications	0.50	0.49	0.86	0.18	0.36	0.36	0.02	0.27	1.58	-2.78
Shops and Retail	0.21	0.68	0.84	0.28	0.01	0.06	0.02	0.03	0.02	-0.33
Health	0.22	0.73	0.45	0.13	0.26	0.09	0.14	0.02	0.48	-0.68
Utilities	0.20	0.68	1.31	0.86	0.43	0.03	0.43	0.05	0.04	-2.44
Other	0.16	0.70	1.00	0.01	0.02	0.08	0.01	0.02	0.03	-0.20
					10 Size-Sorted Portfolios, Value-Weighted					
Decile 1	0.14	0.81	0.26	0.07	0.01	0.01	0.04	0.00	0.15	0.43
Decile 2	0.18	0.79	0.50	0.09	0.00	0.01	0.01	0.01	0.13	0.23
Decile 3	0.12	0.84	0.61	0.05	0.00	0.01	0.00	0.01	0.12	0.18
Decile 4	0.14	0.78	0.75	0.06	0.01	0.04	0.01	0.02	0.22	-0.15
Decile 5	0.16	0.86	0.86	0.02	0.02	0.02	0.01	0.02	0.11	-0.09
Decile 6	0.22	0.77	0.89	0.07	0.02	0.02	0.01	0.02	0.08	-0.13
Decile 7	0.96	0.10	0.81	0.02	0.01	0.03	0.01	0.02	0.07	0.02
Decile 8	0.92	0.11	0.94	0.02	0.03	0.04	0.02	0.03	0.09	-0.20
Decile 9	0.98	0.07	0.94	0.01	0.01	0.02	0.01	0.01	0.03	-0.04
Decile 10	0.40	0.62	1.10	0.01	0.01	0.02	0.01	0.02	0.05	-0.26
					Bond Returns					
10-Year Treasury Notes	0.68	0.35	0.07	0.74	0.24	0.03	0.47	0.07	0.25	-0.98
5-Year Treasury Notes	0.77	0.22	0.06	1.04	0.01	0.19	0.09	0.10	1.26	-1.90
Baa Corporate Bonds (10-20)	0.45	0.51	0.14	1.37	0.13	0.05	0.54	0.08	0.37	-1.82
					Real Estate Returns					
NAREIT - Composite	0.51	0.48	0.86	0.18	0.05	0.02	0.02	0.01	0.04	-0.28
NAREIT - Equity TR	0.60	0.41	0.43	0.04	0.00	0.02	0.04	0.04	0.14	0.22
NAREIT - Mortgage TR	0.56	0.46	0.76	0.53	0.08	0.04	0.03	0.03	0.22	-0.92
NAREIT - Hybrid TR	0.71	0.37	0.91	0.21	0.19	0.07	0.05	0.03	0.06	-0.70

Figure 1
Bayesian Posterior Medians and 90% Credibility Intervals for Beta Exposures: REIT Portfolios

For each of the 4 REIT portfolios under investigation and the assumed 7 factors in our asset pricing model,

$$x_{i,t} = \beta_{i0,t} + \sum_{j=1}^{K} \beta_{ij,t} f_{j,t} + \sigma_{i,t}\epsilon_{i,t}$$

$$\beta_{ij,t} = \beta_{ij,t-1} + \kappa_{1ij,t}\eta_{ij,t} \qquad j = 0, ..., K, \qquad \ln(\sigma^2_{i,t}) = \ln(\sigma^2_{i,t-1}) + \kappa_{2i,t}\upsilon_{i,t} \qquad i = 1, ..., N$$

we plot the posterior medians for time-varying beta coefficients and 5 and 95 percent posterior profiles estimated using a Gibbs sampler in combination with the algorithm by Gerlach, Carter and Kohn (2000).

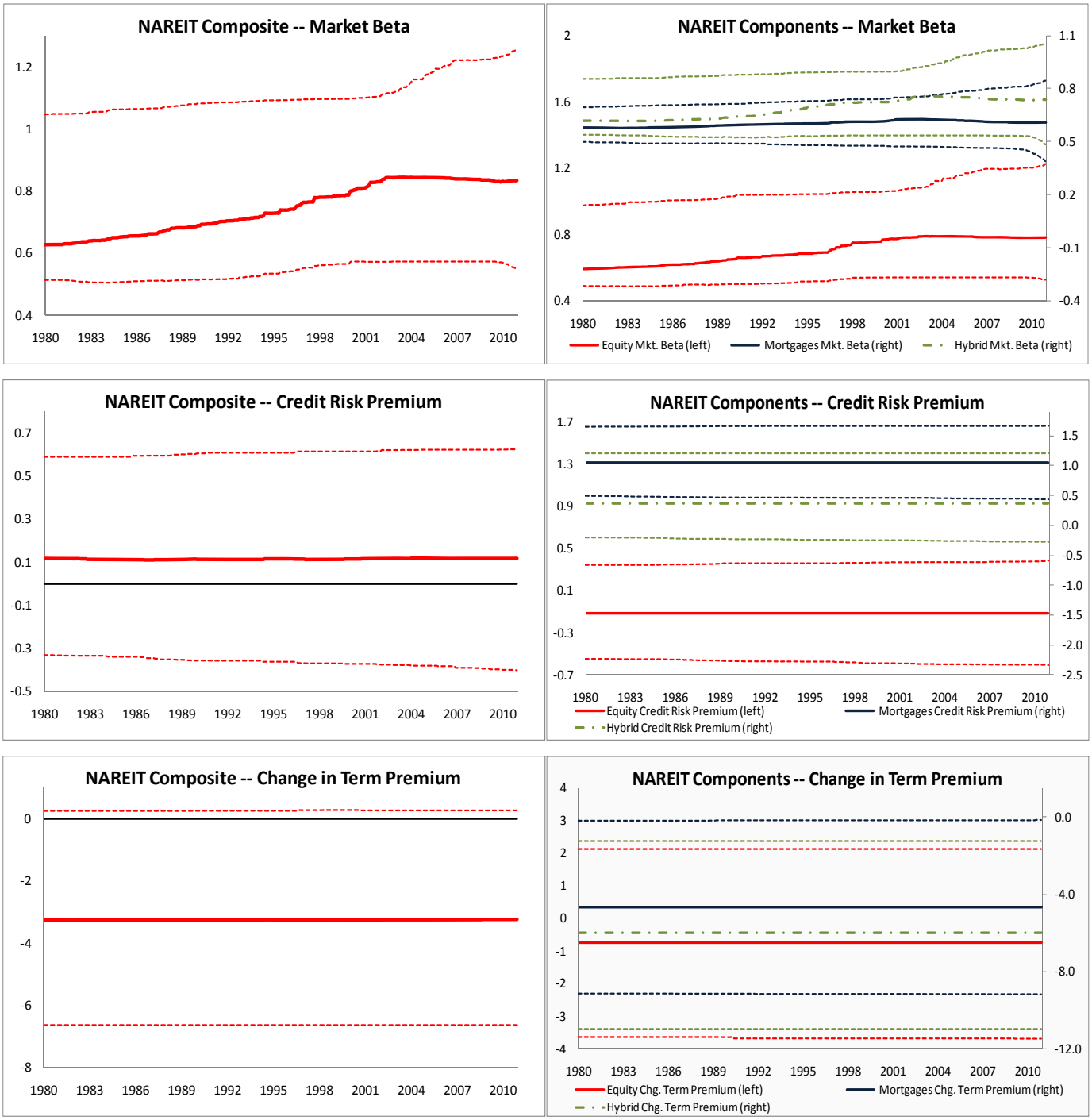


Figure 1 (continued)
Bayesian Posterior Medians and 90% Credibility Intervals for Beta Exposures: REIT Portfolios

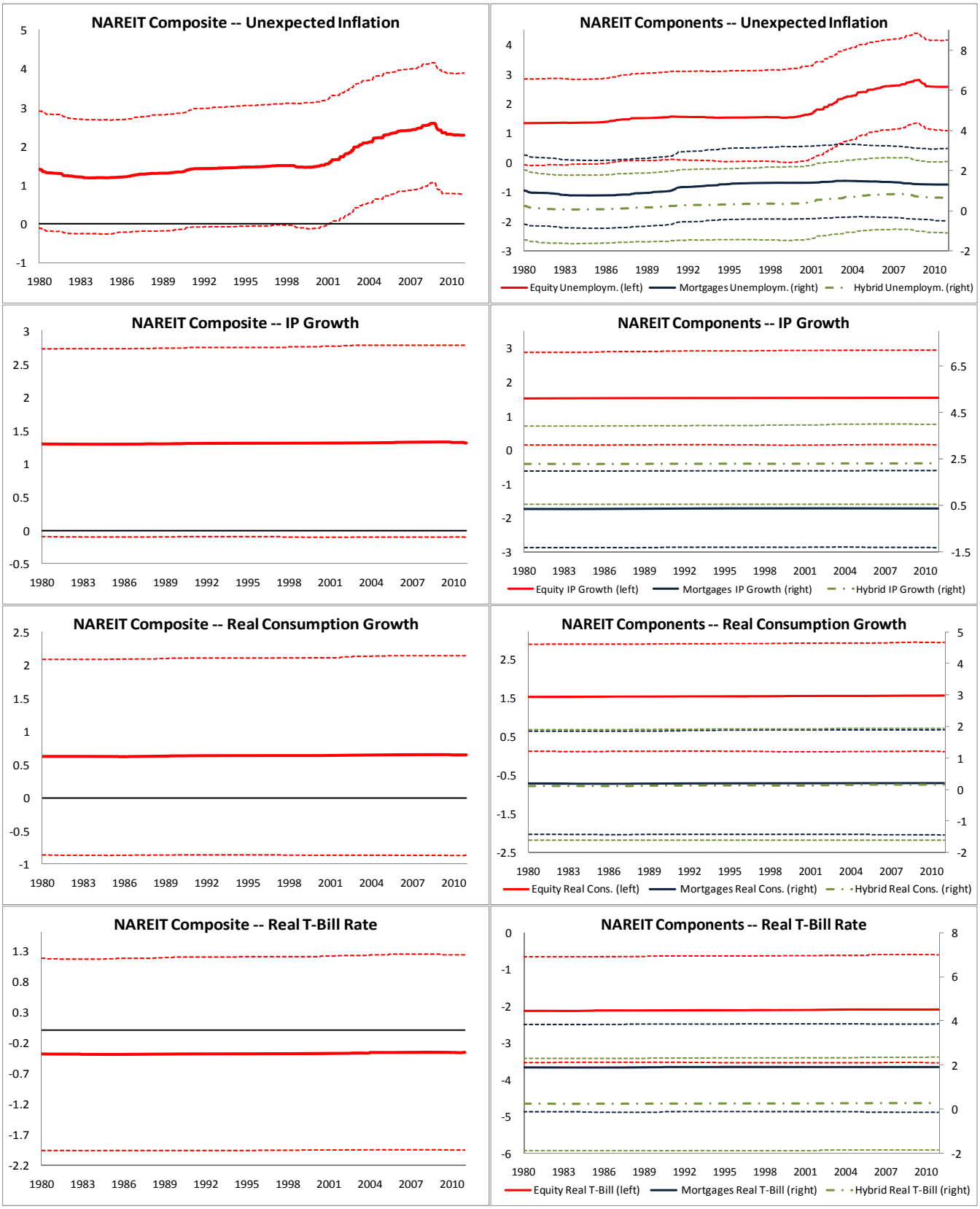


Figure 2

Bayesian Posterior Medians and 90% Credibility Intervals for Beta Exposures: Equity Industry Portfolios

For each of 4 industry-sorted equity portfolios and 4 among the assumed 7 factors in our asset pricing model,

$$x_{i,t} = \beta_{i0,t} + \sum_{j=1}^{K} \beta_{ij,t} f_{j,t} + \sigma_{i,t}\epsilon_{i,t} \quad \beta_{ij,t} = \beta_{ij,t-1} + \kappa_{1ij,t}\eta_{ij,t} \quad j = 0, ..., K, \quad \ln(\sigma^2_{i,t}) = \ln(\sigma^2_{i,t-1}) + \kappa_{2i,t}\upsilon_{i,t} \quad i = 1, ..., N$$

we plot the posterior medians for time-varying beta coefficients and 5 and 95 percent posterior profiles estimated using a Gibbs sampler in combination with the algorithm by Gerlach, Carter and Kohn (2000).

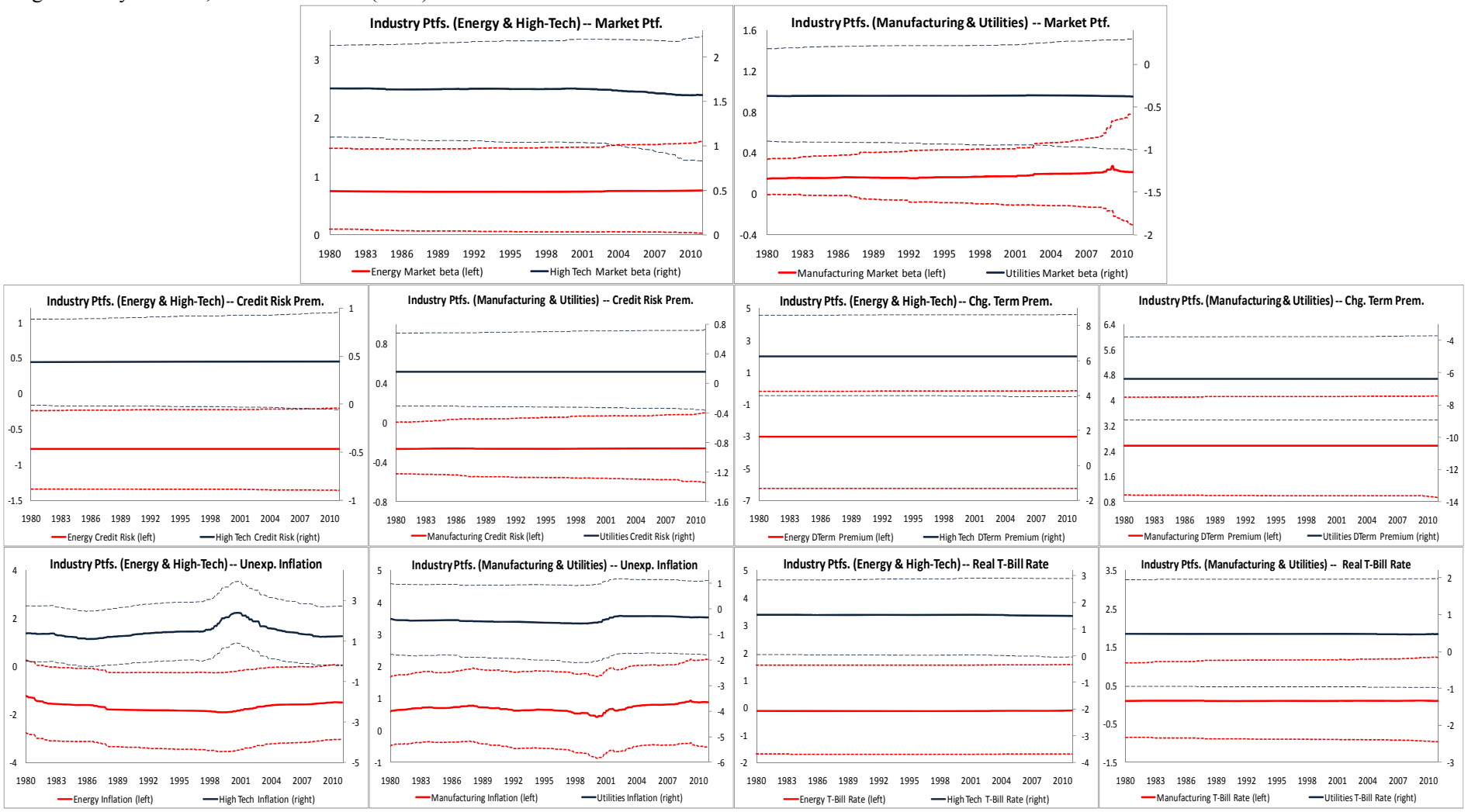


Figure 3

Bayesian Posterior Medians and 90% Credibility Intervals for Beta Exposures: Selected Size-Sorted Equity, Bond Portfolios, and Factors

For the 3 bond portfolios, 3 selected size-sorted equity portfolios, and the 3 of the 4 factors in the pricing model,

$$x_{i,t} = \beta_{i0,t} + \sum_{j=1}^{K} \beta_{ij,t} f_{j,t} + \sigma_{i,t}\epsilon_{i,t}$$

$$\beta_{ij,t} = \beta_{ij,t-1} + \kappa_{1ij,t}\eta_{ij,t} \qquad j = 0, ..., K, \ln(\sigma^2_{i,t}) = \ln(\sigma^2_{i,t-1}) + \kappa_{2i,t}\upsilon_{i,t} \qquad i = 1, ..., N$$

we plot the posterior medians for time-varying beta coefficients and 5 and 95 percent posterior profiles.

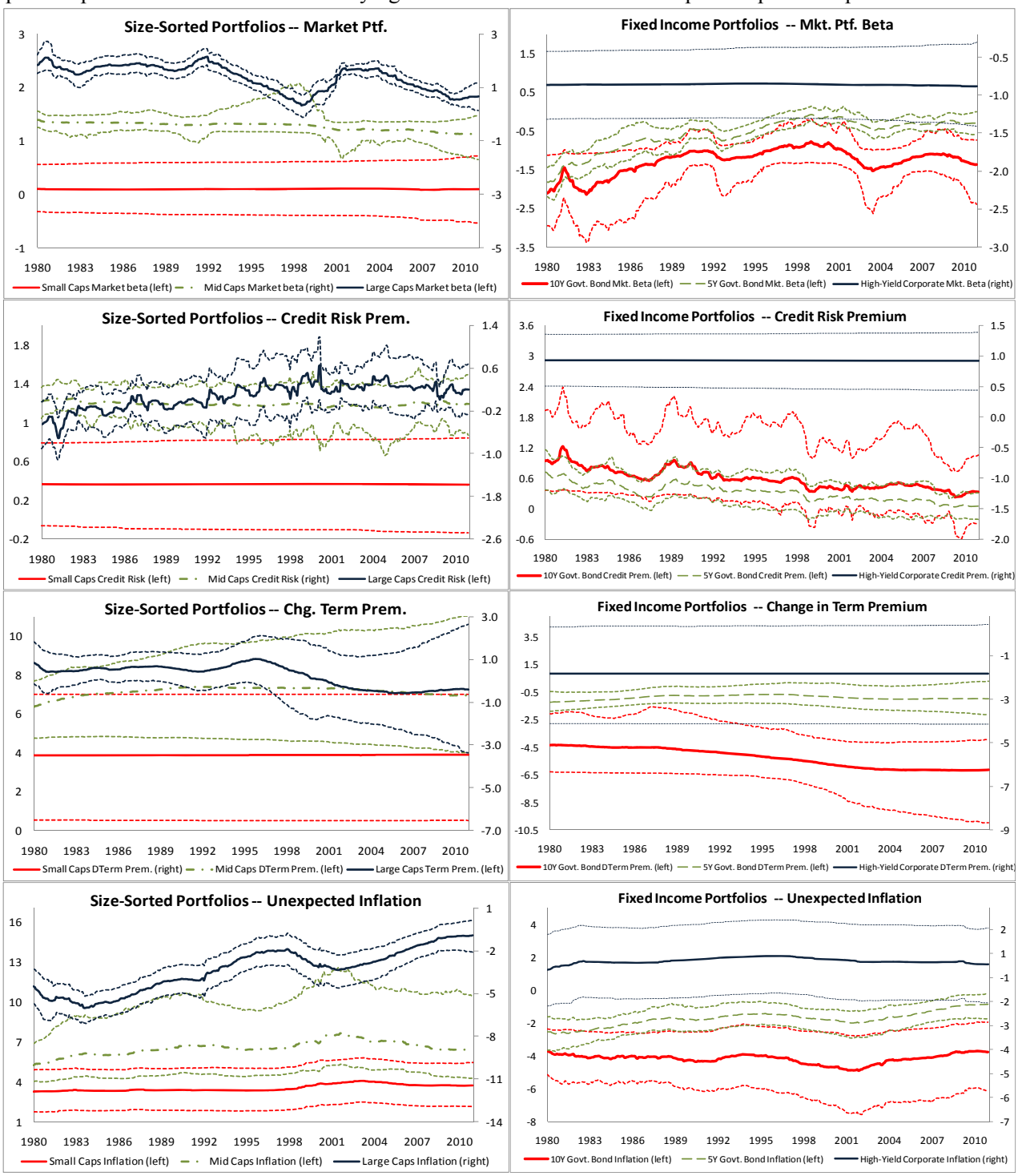


Figure 4

Bayesian Posterior Medians and 90% Credibility Intervals for Jensen's Alphas Estimates

For each of the 4 REIT portfolios, the 3 bond portfolios, 4 equity industry portfolios, and 3 selected size-sorted equity portfolios we plot the posterior medians and 95 percent posterior bands for time-varying "alpha", the abnormal time-varying return $\beta_{i0,t}$ obtained from the asset pricing model

$$x_{i,t} = \beta_{i0,t} + \sum_{j=1}^{K} \beta_{ij,t} f_{j,t} + \sigma_{i,t}\epsilon_{i,t}$$

$$\beta_{ij,t} = \beta_{ij,t-1} + \kappa_{1ij,t}\eta_{ij,t} \qquad j = 0, ..., K, \ln(\sigma^2_{i,t}) = \ln(\sigma^2_{i,t-1}) + \kappa_{2i,t}\upsilon_{i,t} \qquad i = 1, ..., N$$

using a Gibbs sampler in combination with the algorithm by Gerlach, Carter and Kohn (2000).

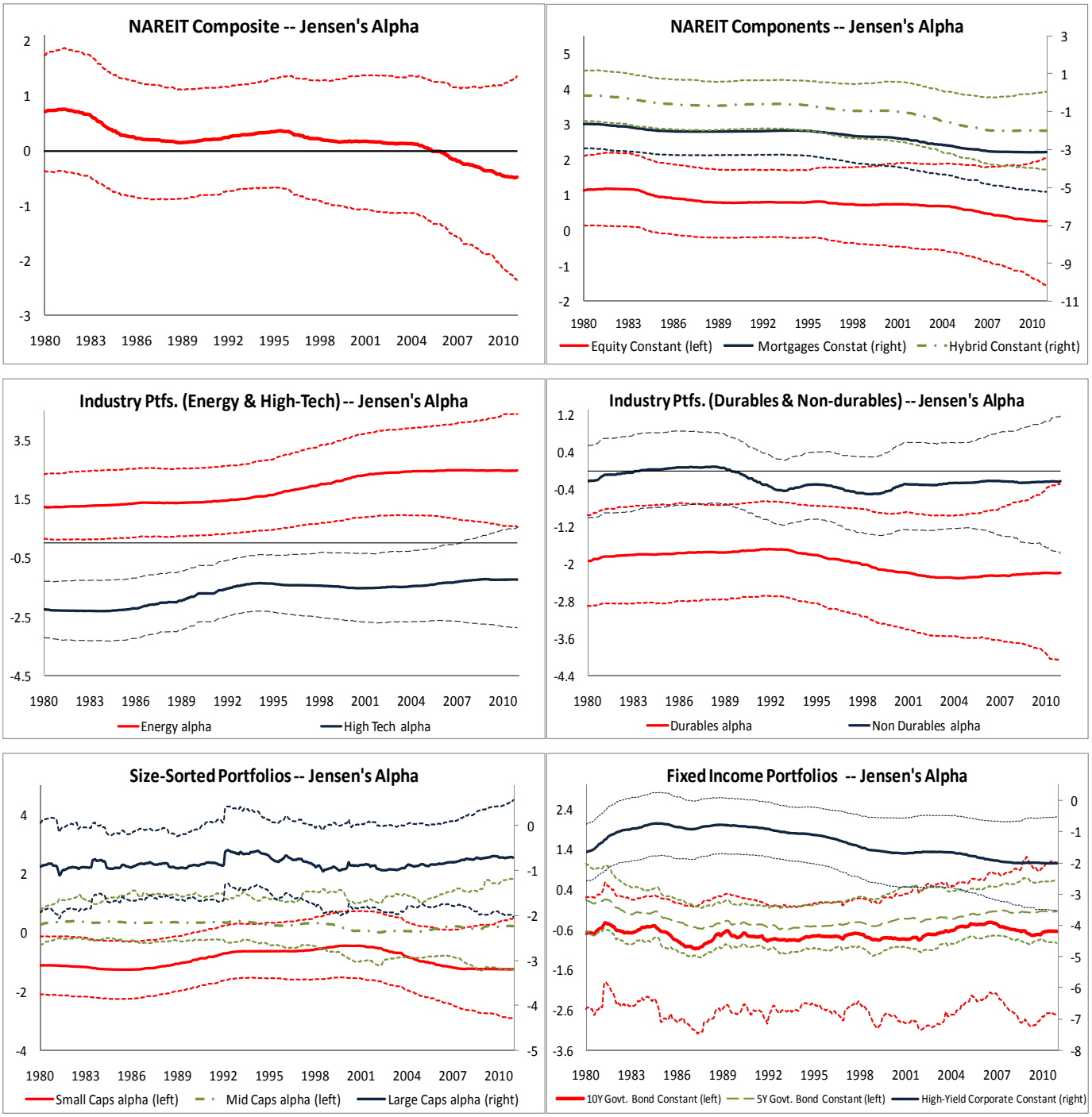


Figure 5

Comparing Bayesian Posterior Medians with Classical Two-Step Fama-MacBeth Estimates of Jensen's Alphas

For each of the 4 REIT portfolios, the 3 bond portfolios, and 2 selected equity portfolios, we plot point estimates and 90% confidence intervals and posterior medians with 90 percent posterior bands for time-varying "alpha", the abnormal time-varying return $\beta_{i0,t}$ obtained from a multi-factor asset pricing model with 7 priced macro-style risk factors.

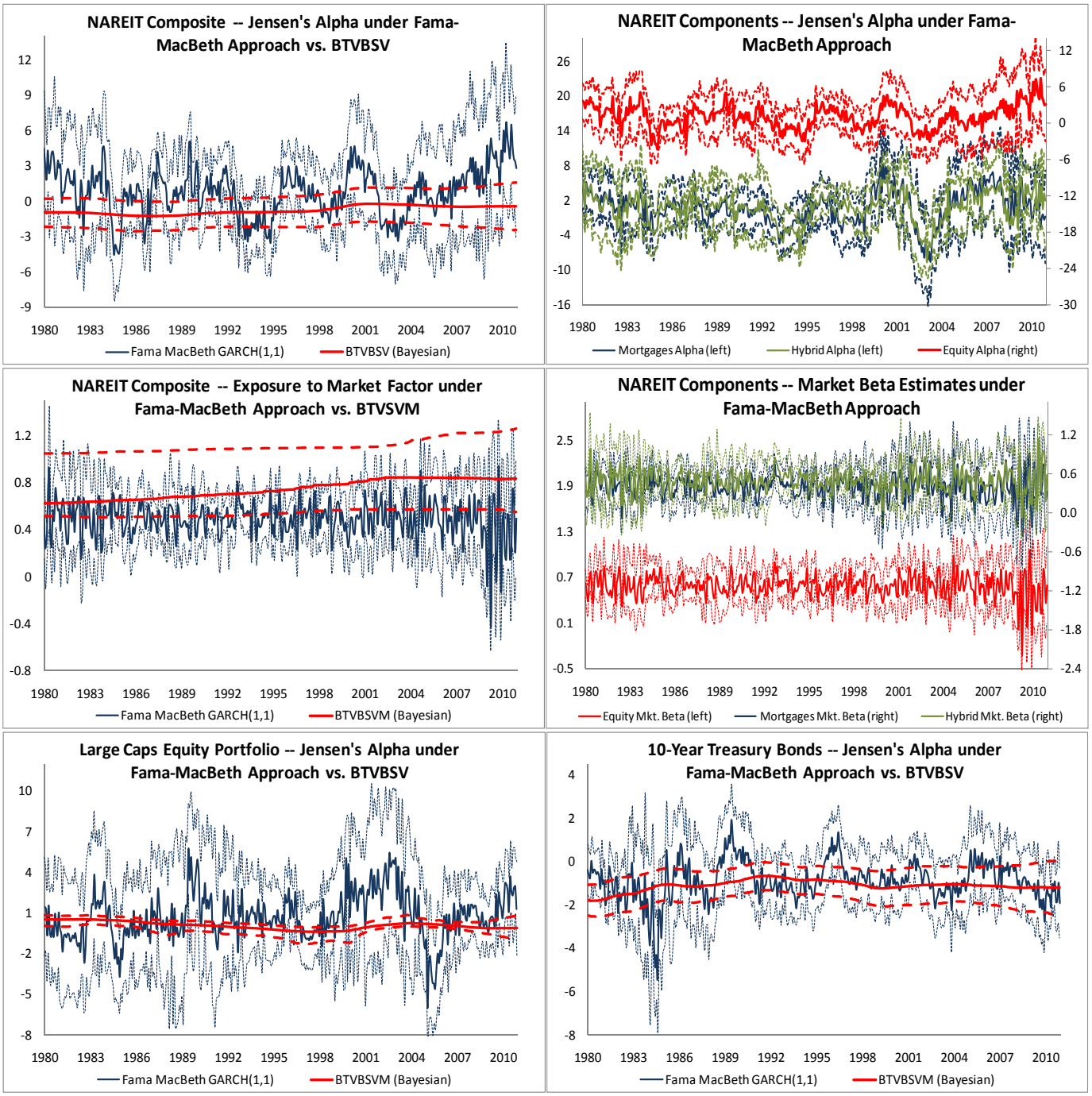
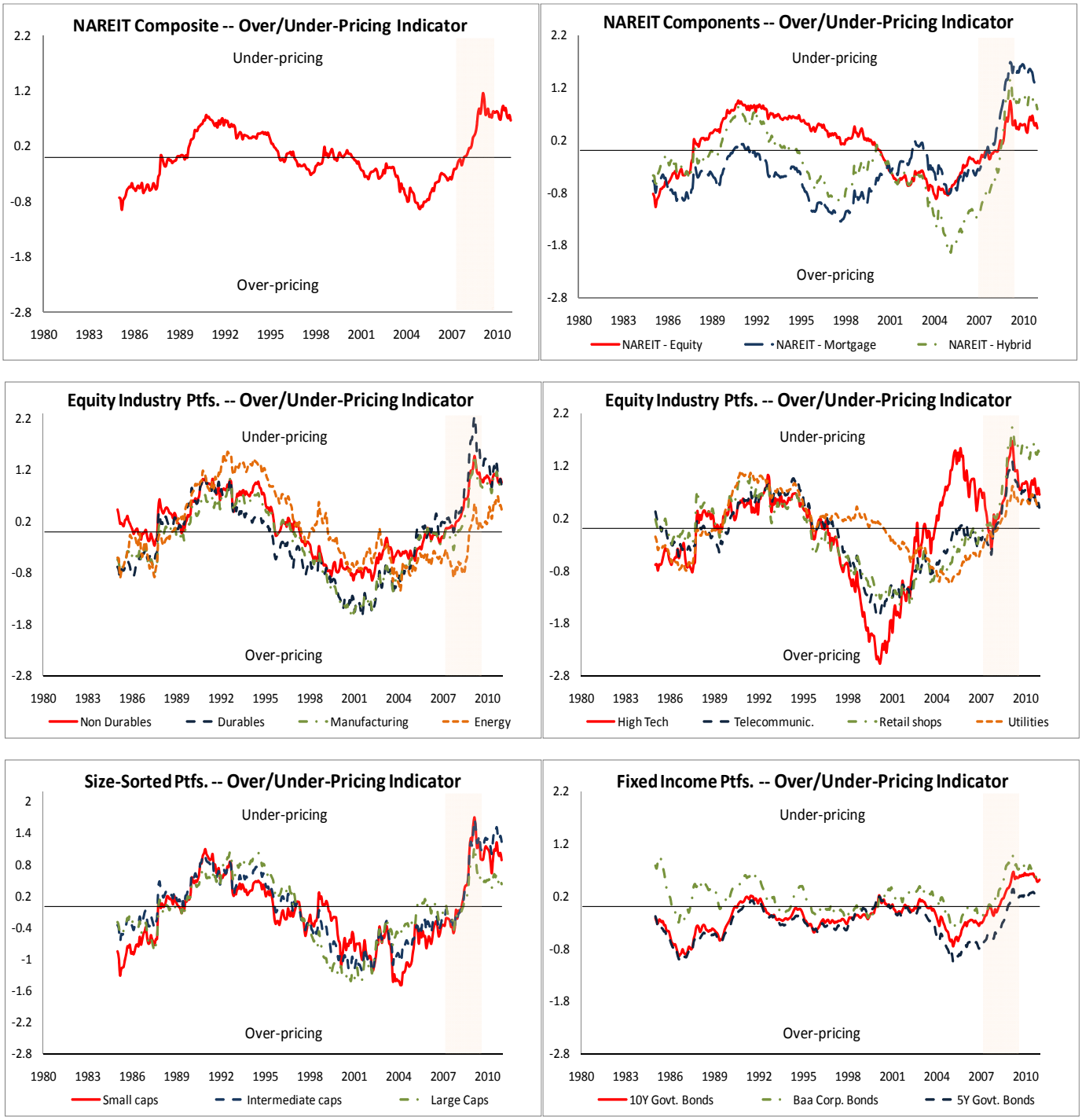


Figure 6

Difference Between Posterior Medians of Market- and Model-Implied Excess Asset Returns

With references to the 4 REIT portfolios, 3 fixed income portfolios, and 11 selected stock portfolios (among a total of 20), the plots report 5-year rolling windows averages of posterior medians of the mispricing indicator $x^{mkt}_{i,t\to t+1} - x^{MFAPM}_{i,t\to t+1}$ where $x^{mkt}_{i,t\to t+1}$ is the one-step ahead excess return on asset i obtained from a regression of realized excess returns onto a set of M instrumental variables that proxy for available information at time t, and $x^{MFAPM}_{i,t\to t+1}$ is computed from the posterior densities of portfolio-specific factor risk exposures and risk premia. If portfolio i is over-priced by the market at time t, then $x^{mkt}_{i,t\to t+1} - x^{MFAPM}_{i,t\to t+1} < 0$. The M instrumental variables consist of the excess return on the CRSP value-weighted market index, the term spread measured as the difference between 5-year Treasury notes and 1-month bill yields, the default spread measured as the difference between Moody's (10-20 year) Baa and Aaa corporate bond yields, and the 12-month trailing dividend yield on the CRPS index.