

Ben-Haim, Yakov; Akram, Q. Farooq; Eitrheim, Øyvind

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Monetary Policy Under Uncertainty: Min-Max vs Robust-Satisficing Strategies

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by

Yakov Ben-Haim, Q. Farooq Akram and Øyvind Eitrheim

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Monetary policy under uncertainty: Min-max vs robust-satisficing strategies*

Yakov Ben-Haim[†] Q. Farooq Akram[‡] Øyvind Eitrheim[§]

November 28, 2007

Abstract

We study monetary policy under uncertainty. A policy which ameliorates a worst case may differ from a policy which maximizes robustness and satisfies the performance. The former strategy is min-maxing and the latter strategy is robust-satisficing. We show an “observational equivalence” between robust-satisficing and min-maxing. However, there remains a “behavioral difference” between robust-satisficing and min-maxing. Policy makers often wish to respect specified bounds on target variables. The robust-satisficing policy can be more (and is never less) robust, and hence more dependable, than the min-max policy. We illustrate this in an empirical example where monetary policy making amounts to selecting the coefficients of a Taylor-type interest rate rule, subject to uncertainty in the persistence of shocks to inflation.

Keywords. Knightian uncertainty, robustness, info-gap decision theory, monetary policy, min-max policy, robust-satisficing policy.

JEL Classification. E52, E58.

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[†]Yitzhak Moda'i Chair in Technology and Economics, Technion—Israel Institute of Technology, Haifa 32000 Israel, yakov@technion.ac.il. Tel: +972-4-829-3262. Fax: +972-4-829-5711. Corresponding author.

[‡]Research Department, Norges Bank, Box 1179 Sentrum, N-0107 Oslo, Norway. farooq.akram@norges-bank.no.

[§]Research Department, Norges Bank, Box 1179 Sentrum, N-0107 Oslo, Norway. oyvind.eitrheim@norges-bank.no.

1 Introduction

We employ the robust-satisficing approach to derive monetary policy response under parameter uncertainty. This approach bases decision making on two main premises. First, uncertainty about some relevant aspect is of the Knightian kind, which cannot be modeled probabilistically (Knight 1921). And second, the decision maker aims to attain performance at some satisfactory level rather than at the peak level (Simon 1959 and 1979 and references therein).

The robust-satisficing policy maximizes robustness at a given level of acceptable performance (Ben-Haim 2006). Robustness is measured as the extent of deviation from a decision maker's underlying premises at which the performance will not deteriorate beyond some acceptable level. The performance can, however, deteriorate beyond the acceptable level for larger, unspecified, deviations from the underlying premises. This is in contrast with the robust control approach that seeks to limit the maximum potential loss and hence requires specification of a worst case; see e.g. Hansen and Sargent (2001).

The robust-satisficing approach quantifies the trade-off between the degree of robustness and the level of acceptable performance. Robustness of a policy can be raised by lowering one's aspirations regarding its performance and accepting a higher level of loss (Ben-Haim 2006).

The robust-satisficing approach is attractive when a decision maker wishes to keep losses within specified bounds. Sometimes a policy maker's credibility depends on performing within a specified range of outcomes, for instance keeping inflation within specified limits. A policy which ameliorates a worst case may be quite unreliable in achieving the specified performance requirements, and may be costly in terms of forsaken performance in the normal course of events; cf. Cogley and Sargent (2005).

The robust-satisficing approach is quite general and can be employed to derive decisions under various kinds of uncertainties separately or jointly.¹ However, this approach has so far not been employed rigorously in monetary policy analysis, though it has been illustrated within the monetary policy context (Akram *et al* 2006).

Studies of monetary policy decisions under uncertainty are mainly based on the Bayesian and the robust control approaches; see e.g. Hansen and Sargent (2001), Onatski and Williams (2003),

¹The robust-satisficing approach has been previously applied to a wide variety of decision problems with Knightian uncertainty, including financial risk assessment (Ben-Haim 2005); environmental regulation (Stranlund and Ben-Haim 2007); search behavior in animal foraging (Carmel and Ben-Haim 2005); policy decisions in marine reserve design (Halpern *et al* 2006); natural resource conservation decisions (Moilanen and Wintle 2006); inspection decisions by port authorities to detect terrorist weapons (Moffitt *et al* 2005) and to detect invasive species (Moffitt *et al* 2007); technological fault diagnosis (Pierce *et al* 2006) and testing (Vinot *et al* 2005); and project management (Regev *et al* 2006).

Levin and Williams (2003), Levin *et al* (2003), Leitemo and Söderström (2005), Rustem *et al* (2005) and Tetlow and von zur Muehlen (2001). The Bayesian approach requires that one select a probability distribution on the uncertain aspect of the decision problem, e.g. model parameters. This enables one to choose an expected-loss-minimizing policy. The robust control approach (also referred to here as min-max) suggests designing policies to perform relatively well in worst-case scenarios, i.e. when the underlying premises turn out to be false in the most unfortunate way. This approach enables one to limit the maximum potential loss. Accordingly, a fictitious malevolent agent who represents a policy maker’s worst fears concerning mis-specification is introduced into the optimization problem and motivates the policy maker to minimize the loss function under the worst-case scenario.² The level of uncertainty can be regulated by adjusting the resources available to the malevolent agent. Alternatively, Rustem *et al* (2005) recognize that the uncertain “range needs to be specified by the policy maker” in terms of the potential deterioration of the performance.

The main innovation of this paper is the comparison of min-max with robust-satisficing policy-selection strategies. Our comparison has two elements. First, we identify a mathematical equivalence between these strategies. Second, despite this equivalence between the strategies, there is a fundamental motivational distinction between them. Very different policy choices are made, depending on whether the goal is to minimize a maximal loss (min-max) or to maximize the range of uncertain contingencies within which the outcome is acceptable (robust-satisficing). We argue that policy makers who face explicit outcome requirements will tend to prefer the robust-satisficing approach.

The min-max strategy requires the policy maker to specify, before selecting a policy, the worst possible realization of the uncertain quantities. The min-max policy is the policy for which the loss under the worst-case realization is minimal (relative to the other policies considered).

The robust-satisficing policy requires the policy maker to specify, before selecting a policy, the greatest acceptable loss (without specifying a worst case of the uncertain quantities). The robust-satisficing policy is the policy for which no more than acceptable loss occurs for the greatest range of realizations of the uncertain quantities.

The equivalence between min-max and robust-satisficing policies results because any choice of

²Knightian uncertainty can be either structured or non-structured (Tetlow and von zur Muehlen 2004). Under structured Knightian uncertainty, the true values of one or more specified parameters of the model are supposed to be bounded between known extreme values. Under unstructured Knightian uncertainty, however, few restrictions are placed on the uncertain aspects of a model. The numerical example considered in this paper can therefore be classified as structured Knightian uncertainty.

the worst case (required by min-max) corresponds to a choice of greatest acceptable loss (required by robust-satisficing). There is, therefore, a numerical or graphical equivalence between these policies. We will refer to this as the “observational equivalence” between robust-satisficing and min-maxing, since it implies that a modeller can always describe the behavior of a policy maker, either a min-maxer or a robust-satisficer, with either strategy.

However, that equivalence can be illusory from the perspective of policy selection. A robust-satisficing policy maker might agree with the min-max analyst on the choice of the worst possible realization. However, if the policy maker’s choice of greatest acceptable loss is not the dual value of this worst case, then their policy choices can differ. Furthermore, the range of uncertain contingencies which result in acceptable loss is never smaller (and usually larger) with the robust-satisficing policy than with the min-max policy. We will refer to this as the “behavioral difference” between robust-satisficing and min-maxing.

To contrast robust-satisficing policy with the min-maxing policy, we employ the robust-satisficing approach to derive monetary policy response when there is uncertainty about the degree of persistence in the supply shock. Our analysis is based on the aggregated model of the US economy estimated by Rudebusch and Svensson (1999). Monetary policy is characterized by a simple Taylor-type interest rate rule, where the decision parameters are the response coefficients associated with inflation and output gaps as well as degree of interest rate smoothing; see Taylor (1999). The empirical analysis is mainly intended to illustrate the robust-satisficing approach and highlight when its policy implications differ from those of the robust control approach, and when they agree. The empirical analysis suggests that the policy implications of the two approaches can differ substantially if one seeks to maximize the robustness at relatively low levels of acceptable loss.

The paper is organized as follows. Section 2 formulates the robust-satisficing and min-maxing strategies for policy selection, and discusses their basic properties in a very general formulation. The observational equivalence is stated in proposition 3 of section 2.4, which is followed by a discussion of the behavioral difference between robust-satisficing and min-maxing. Section 3 employs the robust-satisficing approach to derive robust monetary policy in response to uncertain persistence in the supply shock. A concluding discussion appears in section 4. Proofs and derivations appear in the appendices.

2 Robust-Satisficing and Min-Maxing

In section 2.4 we formalize, very generally, the “observational equivalence” between min-maxing and robust-satisficing, and the “behavioral difference” between these strategies. Sections 2.1–2.3 lay the foundations.

2.1 Decisions, Uncertainties and the Loss Function

The policy maker will make decisions which we denote by Ω . These decisions may be choices of policy variables such as the parameters of a Taylor rule, or specific interest rates.

The policy maker’s decisions are based on models and data. Probability distributions may be part of these models. However, these models and data, including the probabilistic elements, may be uncertain or incomplete or erroneous in various ways. The uncertainty may be either “structured” or “unstructured” in the sense of Tetlow and von zur Muehlen (2001). The uncertainty may be in the size or structure of a set of entities, such as uncertain sets of vectors or functions. For instance, the values of some of the model parameters may be imprecise. The forms of supply and demand curves, or Philips curves, may be uncertain. The shapes of the tails of the probability distributions may be unknown. There may be unknown variables missing from the model.

In the following we use θ to denote any specific realization of the uncertain elements, which may be specific parameter values, functions, missing variables or relations, probability distributions, or sets of such entities. The assumed or estimated value of θ is denoted $\tilde{\theta}$. The uncertainty in θ is represented by an info-gap model of uncertainty (Ben-Haim 2006), which is an unbounded family of nested sets, $\mathcal{U}(\ell, \tilde{\theta})$, of possible θ -values. At any level of uncertainty, ℓ , the set $\mathcal{U}(\ell, \tilde{\theta})$ contains possible realizations of θ . As ℓ gets larger, the sets become more inclusive. The info-gap model expresses the decision maker’s beliefs about uncertain variation of θ around $\tilde{\theta}$.

Info-gap models obey two axioms:

$$\text{Contraction:} \quad \mathcal{U}(0, \tilde{\theta}) = \{\tilde{\theta}\} \tag{1}$$

$$\text{Nesting:} \quad \ell < \ell' \quad \text{implies} \quad \mathcal{U}(\ell, \tilde{\theta}) \subseteq \mathcal{U}(\ell', \tilde{\theta}) \tag{2}$$

The contraction axiom asserts that $\tilde{\theta}$ is the only possibility when there is no uncertainty ($\ell = 0$). The nesting axiom asserts that the range of possible realizations increases as the level of uncertainty increases. Info-gap models entail no probabilistic information and thus are one possible quantification of Knightian uncertainty.

The loss resulting from decision Ω , if the uncertain elements take the values θ , is $L(\Omega, \theta)$. The loss may be an expectation, or a deterministic model calculation.³ The satisficing policy maker desires low loss, and can accept loss no greater than the critical value L_c :

$$L(\Omega, \theta) \leq L_c \quad (3)$$

We treat L_c as a parameter which can be chosen small or large, so the satisficing requirement in eq.(3) includes minimizing the loss as a special case.

2.2 Decision Strategies: Robust-Satisficing, Conditional Optimization, and Min-Maxing

We will consider three types of decision strategies for choosing the decision, Ω , from within a set, R , of feasible decisions.

To define the *robust-satisficing decision* we must first define the robustness function. The robustness of decision Ω , with the satisficing requirement L_c of eq.(3), is the greatest level of uncertainty ℓ up to which all realizations of θ result in loss no greater than L_c :

$$\widehat{\ell}(\Omega, L_c) \equiv \max \left\{ \ell : \left(\max_{\theta \in \mathcal{U}(\ell, \tilde{\theta})} L(\Omega, \theta) \right) \leq L_c \right\} \quad (4)$$

The robust-satisficing decision maximizes the robustness and satisfies the loss at the value L_c , without specifying a limit on the level of uncertainty:

$$\Omega_s(L_c) \equiv \arg \max_{\Omega \in R} \widehat{\ell}(\Omega, L_c) \quad (5)$$

Conditional optimization is the decision, Ω_b , which minimizes the loss based on the assumed or estimated value of θ , which is $\tilde{\theta}$:

$$\Omega_b \equiv \arg \min_{\Omega \in R} L(\Omega, \tilde{\theta}) \quad (6)$$

The *min-max decision* minimizes the maximum loss based on an estimate, ℓ_m , of the greatest level of uncertainty:

$$\Omega_m(\ell_m) \equiv \arg \min_{\Omega \in R} \max_{\theta \in \mathcal{U}(\ell_m, \tilde{\theta})} L(\Omega, \theta) \quad (7)$$

³Our discussion can be readily extended to multiple loss functions.

2.3 Basic Properties of the Decision Strategies

These decision strategies—robust-satisficing $\Omega_s(L_c)$, conditional optimizing Ω_b , and min-maxing $\Omega_m(\ell_m)$ —have several basic properties which we now identify. The proof of proposition 1 appears in (thm. 1 and cor. 1/1, Ben-Haim 2000), and derives essentially from the nesting axiom. The proof of proposition 2 derives immediately from the contraction axiom and will not be elaborated. Proposition 3 (in section 2.4) is proven in appendix A.

Proposition 1 *Performance trades-off against robustness, both at a fixed decision, Ω , and at the robust-satisficing decision $\Omega_s(L_c)$, if $L(\Omega, \theta)$ is uniformly continuous in θ .*

At fixed decision Ω :

$$L_c < L'_c \quad \text{implies} \quad \widehat{\ell}(\Omega, L_c) \leq \widehat{\ell}(\Omega, L'_c) \quad (8)$$

At the robust-satisficing decision $\Omega_s(L_c)$:

$$L_c < L'_c \quad \text{implies} \quad \widehat{\ell}[\Omega_s(L_c), L_c] \leq \widehat{\ell}[\Omega_s(L'_c), L'_c] \quad (9)$$

Better performance (lower loss L_c), entails lower robustness $\widehat{\ell}(\Omega, L_c)$. Relation (8) asserts that this holds at any fixed decision such as the conditional-optimum decision Ω_b or the min-max decision $\Omega_m(\ell_m)$ in which ℓ_m is fixed as L_c and $\widehat{\ell}[\Omega_m(\ell_m), L_c]$ vary. Relation (9) asserts that this trade-off also holds for the robust-satisficing $\Omega_s(L_c)$ which may vary as L_c varies.

Proposition 2 *Conditional-estimate aspirations have no robustness. For any decision Ω for which $L(\Omega, \theta)$ is not a local maximum at $\tilde{\theta}$:*

$$L_c = L(\Omega, \tilde{\theta}) \quad \text{implies} \quad \widehat{\ell}(\Omega, L_c) = 0 \quad (10)$$

The condition that “ $L(\Omega, \theta)$ is not a local maximum at $\tilde{\theta}$ ” means that outcomes could be worse than the conditional-estimated outcome, $L(\Omega, \tilde{\theta})$. If the outcomes cannot be worse than $L(\Omega, \tilde{\theta})$, then uncertainty is strictly favorable and entails only the possibility of better-than-anticipated outcomes.

For any decision, Ω , the resulting loss conditioned on the estimate $\tilde{\theta}$ is denoted $L(\Omega, \tilde{\theta})$. Proposition 2 asserts that, for any decision Ω , aspiring to loss as low as $L(\Omega, \tilde{\theta})$ has no robustness to error or uncertainty in $\tilde{\theta}$. Since this is true for *any* Ω , it is also true for each of the decision-strategies in eqs.(5), (6) and (7).

Propositions 1 and 2 can be illustrated schematically as in fig. 1. Proposition 1 asserts that a plot of $\widehat{\ell}(\Omega, L_c)$ vs. L_c will be a monotonically increasing curve, where Ω is either constant or equal to $\Omega_s(L_c)$. Proposition 2 asserts that the plot of $\widehat{\ell}(\Omega, L_c)$ will intersect the L_c -axis at the anticipated loss, $L_c = L(\Omega, \widehat{\theta})$.

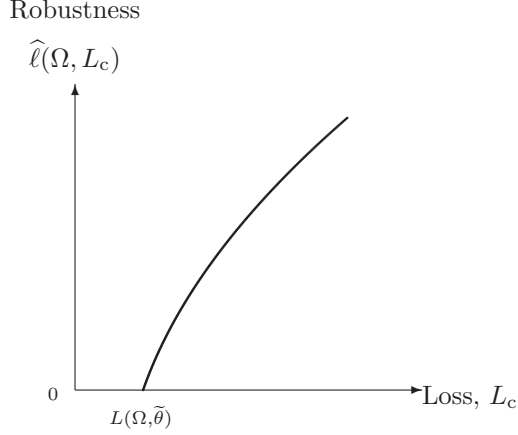


Figure 1: Robustness curve.

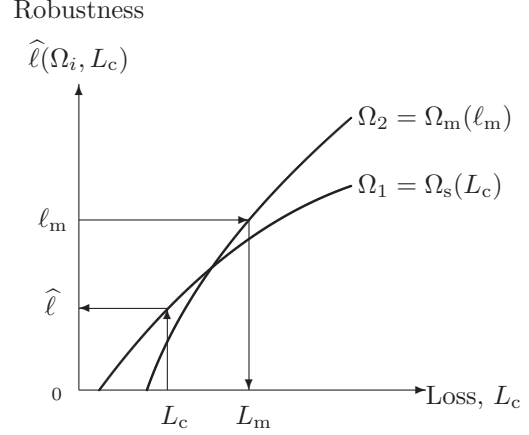


Figure 2: Crossing robustness curves.

The final property we need to discuss is that *robustness curves can cross*, as illustrated in fig. 2. Policy Ω_1 is more robust than policy Ω_2 at low loss and low robustness, and less robust at high loss and high robustness. Monotonic trade-off (proposition 1) and the crossing of robustness curves underlie the observational equivalence, and the behavioral difference, between robust-satisficing and min-maxing, as we now explain intuitively.

Consider first the behavioral difference. Fig. 2 compares two policies. If the level of uncertainty is known to be ℓ_m , then Ω_2 is the min-max policy choice (between these two options), because its maximal loss, L_m , is less than for policy Ω_1 . However, suppose the policy maker is required to aim at loss no greater than L_c , where $L_c < L_m$, and L_c is also less than the value at which the robustness curves cross. Then Ω_1 is the robust-satisficing policy (between these two options) since its robustness at L_c , $\widehat{\ell}$, exceeds the robustness of Ω_2 at L_c , meaning that Ω_1 will achieve loss no greater than L_c for a wider range of contingencies than Ω_2 .

The observational equivalence results as follows. When the policy maker chooses Ω_1 because it is the robust-satisficing decision at requirement L_c , the modeller can alternatively explain this as min-maxing by positing that the policy maker believes the level of uncertainty is $\widehat{\ell}$. With this belief, Ω_1 is the min-max policy.

In the following subsection we formalize this discussion.

It is worth noting that greater robustness for satisfying the performance requirements does not necessarily imply greater probability for doing so. That is, when we say that Ω_1 will achieve loss no

greater than L_c for a wider range of contingencies than Ω_2 , we are not asserting that either range of contingencies is a subset of the other. However, there are situations in which greater robustness does imply greater probability, as studied by Ben-Haim (2007).

2.4 Min-Max and Robust-Satisficing: Equivalence and Difference

Before introducing our main result, we first mention a useful way of evaluating the robustness. Let $\mu(\ell, \Omega)$ denote the inner maximum in the definition of the robustness in eq.(4), that is, $\max_{\theta} L(\Omega, \theta)$. By the nesting axiom, $\mu(\ell, \Omega)$ increases monotonically as ℓ increases. By definition, the robustness, $\widehat{\ell}(\Omega, L_c)$, is the greatest value of ℓ for which $\mu(\ell, \Omega) \leq L_c$. Therefore, by monotonicity, the robustness is the greatest value of ℓ satisfying $\mu(\ell, \Omega) = L_c$. Hence a plot of $\mu(\ell, \Omega)$ vs. ℓ is identical to a plot of L_c vs. $\widehat{\ell}(\Omega, L_c)$. In other words, $\mu(\ell, \Omega)$ vs. ℓ is the inverse of $\widehat{\ell}(\Omega, L_c)$ vs. L_c . It is readily shown that this also holds for the min-max and the robust-satisficing decisions, $\Omega_m(\ell)$ and $\Omega_s(L_c)$.

Proposition 3 *The robust-satisficing and min-maxing policies, $\Omega_s(L_c)$ and $\Omega_m(\ell_m)$, are identical for appropriate choices of the parameters L_c and ℓ_m .*

Given: $\mu[\ell, \Omega_m(\ell)]$ and $\mu[\ell, \Omega_s(L_c)]$ are strictly monotonically increasing in ℓ .

Then:

- For any ℓ_m for which $\Omega_m(\ell_m)$ exists, there is an L_c such that:

$$\Omega_s(L_c) = \Omega_m(\ell_m) \quad (11)$$

- For any L_c for which $\Omega_s(L_c)$ exists, there is an ℓ_m such that:

$$\Omega_m(\ell_m) = \Omega_s(L_c) \quad (12)$$

What proposition 3 does *not* assert is that a policy maker will be indifferent between the min-maxing and the robust-satisficing decision strategies. We reason as follows.

First of all, if the policy maker does not know, or have an estimate of, the greatest level of uncertainty, then the min-max strategy cannot be implemented, though robust-satisficing is still feasible.

But suppose that the policy maker agrees that the greatest level of uncertainty is ℓ_m . The corresponding min-max policy is $\Omega_m(\ell_m)$. Let L_1 be the loss at which, according to eq.(11) of proposition 3, the robust-satisficing and min-maxing policies are identical, so $\Omega_s(L_1) = \Omega_m(\ell_m)$. (L_1 is the maximum loss which can accrue to policy $\Omega_m(\ell_m)$ at uncertainty ℓ_m .) However, suppose

that the policy maker aspires to a lower level of loss, $L_0 < L_1$, and let $\Omega_s(L_0)$ be the corresponding robust-satisficing policy. If $\Omega_s(L_0) \neq \Omega_s(L_1)$ then the policy maker will achieve aspiration L_0 with policy $\Omega_s(L_0)$ for a larger set of contingencies than with policy $\Omega_s(L_1)$. Hence policy $\Omega_s(L_0)$ could be preferred. We will encounter an example in section 3.4. This is the *behavioral difference* between robust-satisficing and min-maxing.

What eq.(12) of proposition 3 *does* assert is that any robust-satisficing decision $\Omega_s(L_c)$ is identical to, and can be modelled as if it were, a min-max decision, $\Omega_m(\ell_m)$. This only requires the modeller to posit that the policy maker believes the level of uncertainty is ℓ_m . This is not problematic for the modeller if the policy maker's beliefs about uncertainty are private, though the policy maker, if asked, might disagree. This is the *observational equivalence* between min-max and robust-satisficing.

3 Example

In this section we illustrate the preceding general discussion with a simple example. In particular, we illustrate the observational equivalence and the behavioral difference between the min-max and robust-satisficing strategies for policy selection.

3.1 Monetary Policy Objectives and Instrument

We assume that monetary policy authorities have a standard quadratic loss function, which can be presented in terms of variance of the inflation, output gap, and interest-rate stability:

$$L = V(\pi - \pi^*) + \lambda V(y) + \phi V(dr), \quad (13)$$

where $V(\cdot)$ denotes the unconditional variance of its argument; π^* is the inflation target; λ denotes the authority's preference for stabilization of the output-gap relative to that for the inflation gap, $\pi - \pi^*$; and ϕ is the relative preference for interest-rate stability.

We characterize monetary policy response by a simple interest rate rule:

$$r_t = \omega_r r_{t-1} + (1 - \omega_r)[r^* + \pi^* + \omega_\pi(\pi_t - \pi^*) + \omega_y y_t], \quad (14)$$

where ω 's are constant coefficients representing the interest rate response to the lagged interest rate, the inflation gap and the output gap. r^* denotes the steady state value of the real interest

rate. The inflation target and the steady state real interest rate sum to the steady state nominal interest rate.

Monetary policy is implemented with the parameters, $\Omega = (\omega_r, \omega_\pi, \omega_y)$, in the interest rate rule eq.(14), in the face of Knightian uncertainty regarding specific parameters, θ , of an economic model. It is useful to express the loss function, eq.(13), as an explicit function of Ω and θ , as $L(\Omega, \theta)$.

The optimal policy, in the absence of Knightian parameter uncertainty regarding θ , can be formally defined as in eq.(6):

$$\Omega_b(\theta) = \arg \min_{\Omega} L(\Omega, \theta). \quad (15)$$

That is, if we knew θ , we could calculate the optimal policy, $\Omega_b(\theta)$. This optimal policy will depend on the degree of concern for stability in the real economy and of the interest rate, expressed by λ and ϕ .

We evaluate a policy in a given state relative to the optimal policy in that state. Therefore, we employ the relative loss function defined as:

$$dL(\Omega, \theta) \equiv \frac{L(\Omega, \theta) - L[\Omega_b(\theta), \theta]}{L[\Omega_b(\theta), \theta]}. \quad (16)$$

$L(\Omega, \theta)$ denotes the level of loss by choosing Ω conditional on a specific θ -value, while $L[\Omega_b(\theta), \theta]$ expresses the loss under optimal policy given the same specific θ -value. It follows that $dL(\Omega, \theta) > 0$ for $\Omega \neq \Omega_b(\theta)$ while $dL(\Omega, \theta) = 0$ when $\Omega = \Omega_b(\theta)$, assuming the loss function has a unique minimum. The relative loss, eq.(16) for a particular policy Ω , makes it possible to separate the policy's contribution to the performance (loss level) from that of the realized value of the parameter.

3.2 Robust Satisficing Policy

The robustness of policy Ω , for satisfying the relative loss requirement $dL(\Omega, \theta) \leq dL_c$, is $\widehat{\ell}(\Omega, dL_c)$, defined as in eq.(4). The robust-satisficing policy, $\Omega_s(dL_c)$, maximizes the robustness and satisfices the relative loss at the value dL_c , from among a set R of available policies, as in eq.(5).

In this paper we will consider the set R , of available policies, to be those policies which are optimal with respect to one from among n specific realizations of the model parameters, $\theta_1, \dots, \theta_n$. That is:

$$R = \{\Omega_b(\theta_j), j = 1, \dots, n\} \quad (17)$$

The info-gap model, $\mathcal{U}(\ell, \Omega)$, for uncertainty in θ , is defined in appendix B. Notice that we specify the centerpoint of the info-gap model in terms of the policy, Ω , rather than in terms of a specific value of θ as we have done earlier. The axioms of nesting and contraction still hold.

We will deal with the uncertainty of a single parameter, θ , which, in our subsequent empirical example, is a shock-persistence. In this case one can understand the info-gap model intuitively as follows. At level of uncertainty ℓ , the uncertainty set $\mathcal{U}(\ell, \Omega)$ is an interval of θ -values of length ℓ . Specifically, it is the interval of length ℓ for which the relative losses $dL(\Omega, \theta)$ at the extremes of the interval are equal to each other. This does not require the loss function to be symmetric or convex; only weakly convex as defined in appendix B.

3.3 Model

We characterize the economy by the well known aggregate model of the US economy developed by Svensson and Rudebusch (1999):

$$\pi_t = 0.7\pi_{t-1} - 0.1\pi_{t-1} + 0.28\pi_{t-3} + 0.12\pi_{t-4} + 0.14y_{t-1} + u_{\pi,t}, \quad (18)$$

$$y_t = 1.16y_{t-1} - 0.25y_{t-2} - 0.1(\bar{r}_{t-1} - \bar{\pi}_{t-1}) + u_{y,t}. \quad (19)$$

Here, π_t is the quarterly inflation rate, y is the output gap, while $\bar{\pi}$ and \bar{r} are smoothed values of quarterly inflation rate and the nominal interest rate, respectively. Precisely, $\bar{\pi}_t = \frac{1}{4} \sum_{i=0}^3 \pi_{t-i}$ while $\bar{r}_t = \frac{1}{4} \sum_{i=0}^3 r_{t-i}$. Finally, $u_{\pi,t}$ and $u_{y,t}$ are unobservable variables representing supply and demand shocks, respectively.

We assume that both of these shocks follow AR(1) processes:

$$u_{\pi,t} = \rho_{\pi} u_{\pi,t-1} + \varepsilon_{\pi,t}, \quad (20)$$

$$u_{y,t} = \rho_y u_{y,t-1} + \varepsilon_{y,t}, \quad (21)$$

where ρ_{π} and ρ_y are constant parameters representing persistence in the supply and demand shocks, respectively. We assume that $\rho_{\pi} \in [0, 1)$ and $\rho_y \in [0, 1)$. The ε 's are assumed to be IID-shocks.

It would be interesting to examine how uncertainty regarding the parameters of this model, (18)–(21), will affect monetary policy selection of Ω , when the policy aim is satisficing rather than minimizing the loss. In the next subsection we study the effect of Knightian uncertainty in the persistence in the supply shock, ρ_{π} . In particular, we will illustrate the difference between the

min-max and the robust-satisficing policies.

3.4 Results: Uncertain Inflation Persistence

In this section we discuss empirical results which illustrate the application of propositions 1–3. We discuss the observational equivalence between robust-satisficing and min-maxing, as well as the behavioral difference between these strategies which is important for the policy maker.

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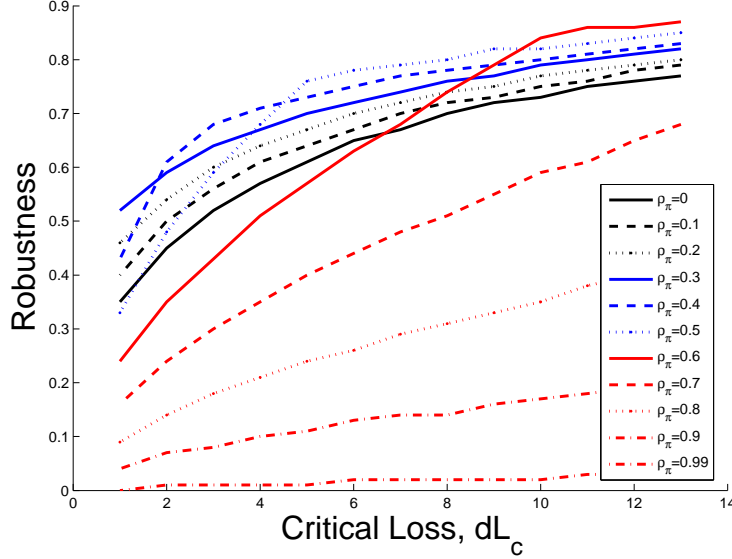


Figure 3: Robustness, $\hat{\ell}[\Omega_b(\rho_\pi), dL_c]$, vs. critical loss, dL_c , for various policy choices $\Omega_b(\rho_\pi)$.

Robustness calculations. Fig. 3 shows numerical evaluation of the robustness function, $\hat{\ell}[\Omega_b(\rho_\pi), dL_c]$, for uncertainty in the inflation persistence ρ_π , with no output shocks or other uncertainties, for various policies $\Omega_b(\rho_\pi)$. Thus $\theta = \rho_\pi$ in this example. ρ_π , by its definition, is constrained to take values in the unit interval $[0, 1)$, which is therefore the range of robustness values. Similar results are obtained when considering uncertainty in the persistence of output shocks, and are not discussed.

Each curve in the figure is evaluated for a different policy rule, $\Omega_b(\rho_\pi)$, defined as follows. $\Omega_b(\rho_\pi)$ is the policy rule which minimizes the loss, $L(\Omega, \rho_\pi)$, if the inflation persistence equals ρ_π . That is, $\Omega_b(\rho_\pi)$ is the conditional-optimization strategy defined in eqs.(6) and (15).

For fixed $\Omega_b(\rho_\pi)$, the relative loss $dL[\Omega_b(\rho_\pi), \rho'_\pi]$ is evaluated as ρ'_π varies from 0 to 0.99 in steps of 0.01. Each point on the robustness curve is the fraction of ρ'_π -values for which the requirement $dL(\Omega_b(\rho_\pi), \rho'_\pi) \leq dL_c$ is satisfied. This fraction is a discrete approximation to the robustness, $\hat{\ell}(\Omega, dL_c)$, which (as explained in appendix B) is the length of the interval of tolerable ρ_π -value.

Proposition 1. The positive slopes of the robustness curves in fig. 3 express the trade-off between robustness and performance. This illustrates proposition 1: the robustness increases as the performance deteriorates (relative loss increases).

Proposition 2. The relative loss of the best-estimate rule vanishes: $dL[\Omega_b(\rho_\pi), \rho_\pi] = 0$. Proposition 2 asserts that the robustness for this aspiration, $dL_c = 0$, is zero, which is not illustrated in fig. 3.

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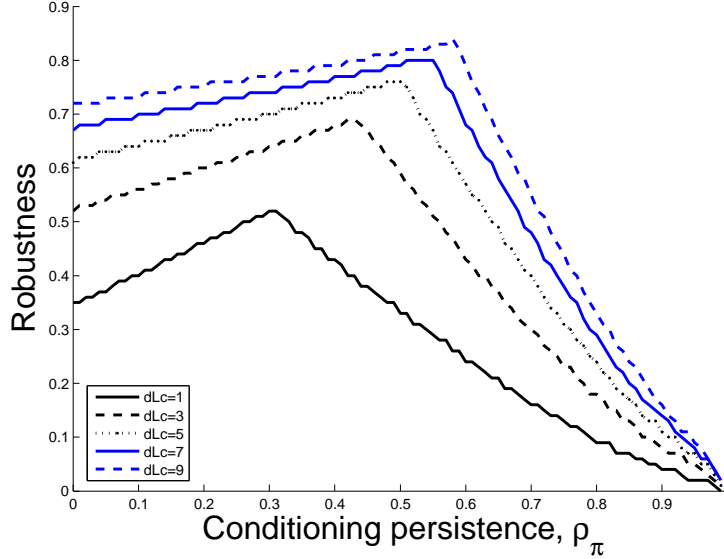


Figure 4: Robustness, $\widehat{\ell}[\Omega_b(\rho_\pi), dL_c]$, vs. conditioning persistence, ρ_π , for various critical losses dL_c . Illustrating the robust-satisficing policy rule, $\Omega_s(dL_c)$. $dL_c = 1, 3, 5, 7$ and 9% , bottom to top curve.

Robust-satisficing policy rule. Finally, before considering proposition 3 and the observational equivalence and behavioral difference, we illustrate the robust-satisficing policy, $\Omega_s(dL_c)$, defined in eq.(5) and explained also in section 3.2. $\Omega_s(dL_c)$ is the policy which satisfies the relative loss at dL_c for the greatest level of uncertainty. This is illustrated in fig. 4, which shows the robustness vs. the inflation-persistence upon which the available policies are conditioned, for various critical losses dL_c . Consider the bottom curve, which is $\widehat{\ell}[\Omega_b(\rho_\pi), dL_c]$ vs. ρ_π for $dL_c = 1\%$. The robustness is maximum at $\rho_\pi = 0.31$, indicating that the robust-satisficing policy is $\Omega_b(0.31)$. For other values of dL_c , (moving up among the curves) the maximum robustness occurs for $\rho_\pi = 0.43, 0.49, 0.53$ and 0.58 .

Min-max policy rule. The min-max policy rule, $\Omega_m(\ell_m)$ defined in eq.(7), is the vector of policy-rule coefficients which minimizes the maximum loss, on the info-gap uncertainty set at level

of uncertainty ℓ_m . The maximum relative loss for policy Ω , at level of uncertainty ℓ_m , is:

$$\max_{\rho_\pi \in \mathcal{U}(\ell_m, \Omega)} dL(\Omega, \rho_\pi) \quad (22)$$

The min-max policy at level of uncertainty ℓ_m minimizes this maximum loss, as in eq.(7):

$$\Omega_m(\ell_m) = \arg \min_{\Omega \in R} \max_{\rho_\pi \in \mathcal{U}(\ell_m, \Omega)} dL(\Omega, \rho_\pi) \quad (23)$$

where R is the set of available policies in eq.(17).

Proposition 3. Proposition 3 establishes an equivalence between the min-max and robust-satisficing strategies. Either strategy can be used to describe behavior of an agent who is using the other strategy. That is, proposition 3 entails an *observational equivalence* between these strategies for policy formulation. However, as explained immediately after proposition 3 in section 2.4, this does not imply that a policy maker would be indifferent between the two strategies. On the contrary, there is an important *behavioral difference* between the two strategies.

Eq.(11) of the proposition asserts that, for any level of uncertainty ℓ_m for which there is a min-max policy, there exists a dL_c for which this is also a robust-satisficing policy. Eq.(12) of proposition 3 asserts that, for any dL_c for which there is a robust-satisficing policy, $\Omega_s(dL_c)$, there is a level of uncertainty ℓ_m for which this is also a min-max policy.

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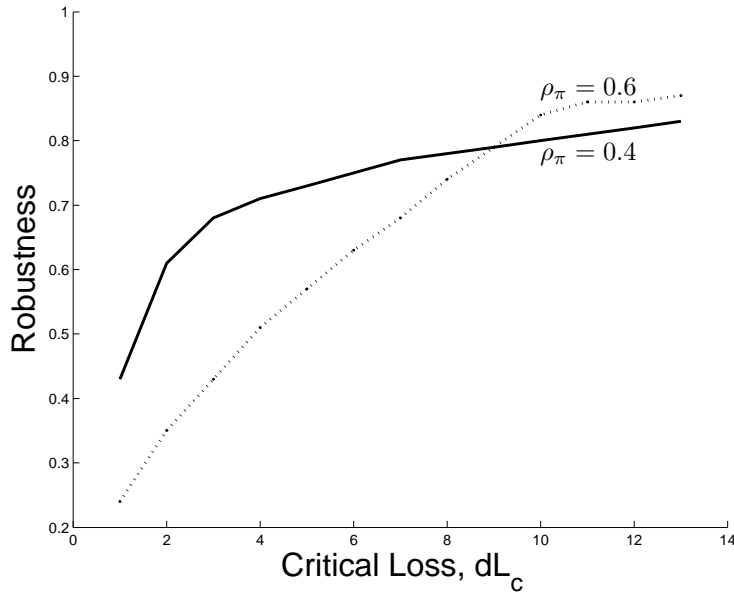


Figure 5: Robustness, $\hat{\ell}[\Omega_b(\rho_\pi), dL_c]$, vs. critical loss, dL_c , for policy choices $\Omega_b(\rho_\pi)$ two different ρ_π 's. Illustrating distinction between min-max and robust-satisficing policies. Two curves reproduced from fig. 3.

These assertions were illustrated schematically in fig. 2, and the same phenomena are manifested

in fig. 3. For instance, suppose the level of uncertainty is known to be $\ell_m = 0.85$. From fig. 3 (see also fig. 5) we find that the policy whose maximal loss is minimal is $\Omega_b(0.6)$, and that the min-max loss is 10.5%. This is also the robust-satisficing policy when the critical loss is $dL_c = 10.5\%$. However, suppose the policy maker is required to keep the loss below 3%, even if the level of uncertainty is acknowledged to be $\ell_m = 0.85$. From figs. 3 and 5 we see that the most robust policy for keeping the loss below 3% is $\Omega_b(0.4)$, for which the robustness is 0.68. $\Omega_b(0.4)$ is the robust-satisficing policy. But it is also the min-max policy if the level of uncertainty is $\ell_m = 0.68$. The robustness of the min-max policy $\Omega_b(0.6)$, when satisficing the loss at 3%, is only 0.43, which is substantially less robust than the robust-satisficing policy $\Omega_b(0.4)$. Consequently, the policy maker may prefer the robust-satisficing policy, $\Omega_b(0.4)$, over the min-max policy, $\Omega_b(0.6)$, even while acknowledging that loss in excess of 3% can occur.

4 Conclusions

This paper is a theoretical and empirical study of the relation between info-gap robust-satisficing and min-max. We develop, both theoretically and with empirical demonstration, two concepts: (1) observational equivalence and (2) behavioral difference between robust-satisficing and min-maxing. *Observational equivalence* is the assertion that a modeller can always describe the behavior of a policy maker, either a min-maxer or a robust-satisficer, with either strategy. However, that equivalence can be illusory from the perspective of policy selection. *Behavioral difference* is the assertion that a robust-satisficing policy maker might agree with the min-max analyst on the choice of the worst possible realization, and still disagree on policy selection. We formulate and study the observational equivalence and behavioral difference theoretically, and demonstrate their occurrence with a commonly employed, though simple, econometric model of the US economy. The present article clarifies the important distinction between min-maxing and robust-satisficing policy selection strategies.

If the level of loss which is acceptable to the decision maker is no less than the maximum loss under a min-max approach, the two approaches will lead to the same policy decision. However, if the decision maker is prohibited from losing as much as the min-max value, then the robust-satisficing policy will differ from the min-max policy. Furthermore, the robust-satisficing policy will be at least as robust (and usually more so) than the min-max policy. True, by reducing the budget of a fictitious evil agent, a min-max policy can be equated to the robust-satisficing policy.

However, this amounts effectively to reducing the level of uncertainty or modifying the worst case scenario, which the policy maker may be unable or unwilling to do.

We have employed the info-gap robust-satisficing approach to derive satisficing monetary policy responses in the face of Knightian uncertainty about a key parameter, persistence in the supply shock. We have found that for relatively high level of acceptable loss, the robust satisficing policy suggests conditioning on relatively high degree of persistence. This is in line with the policy suggested by min-max. However, for lower levels of acceptable loss, we find that conditioning on a relatively low degree of persistence would offer higher robustness, i.e. loss below the required level is guaranteed for a larger range of parameter values, than all alternative policies including the min-max policy. Accordingly, a policy maker who prefers to deviate little from whatever is the optimal level of loss in an unknown state (degree of persistence), would be advised to base policy on a low degree of persistence, while accepting the risk that in some states, the loss will exceed that implied by the min-max policy.

We have dealt with uncertainty in one dimension in this paper to easily interpret and highlight the difference between policies based on the robust-satisficing and min-max approaches. An extension of the robust satisficing approach to uncertainty in several dimensions is straightforward, theoretically and empirically, and is currently being implemented.

A Appendix: Proof of Proposition 3

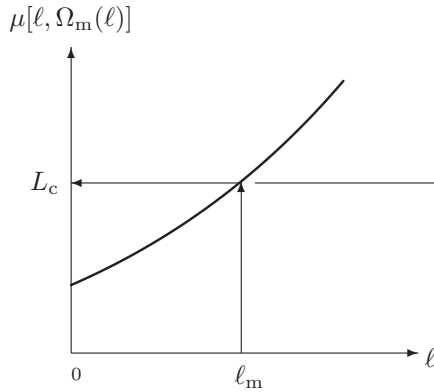


Figure 6: Inverse of the min-max robustness curve.

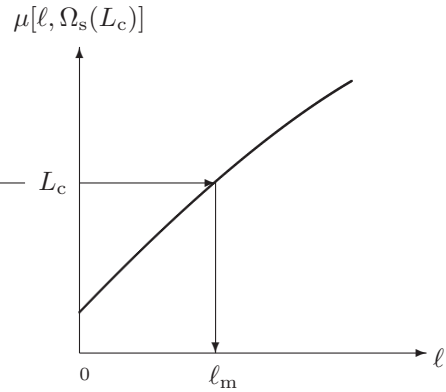


Figure 7: Inverse of the robust-satisficing robustness curve.

Recall from section 2.4 that $\mu[\ell, \Omega_m(\ell)]$ and $\mu[\ell, \Omega_s(L_c)]$ both necessarily increase monotonically as ℓ increases. The proposition assumes that this increase is *strictly* monotonic, as illustrated in figs. 6 and 7. Let R_s denote the set of all robust-satisficing decisions, $\Omega_s(L_c)$, for all L_c values.

Likewise, let R_m denote all min-max decisions $\Omega_m(\ell_m)$ for all ℓ_m -values.

Proof of eq.(11). Eq.(11) is equivalent to the assertion that $R_m \subseteq R_s$, which we will prove. For any value of ℓ_m , let $\Omega_m(\ell_m)$ be the min-max decision. As illustrated in fig. 6, let L_c be the unique⁴ value specified by:

$$\max_{\theta \in \mathcal{U}(\ell_m, \tilde{\theta})} L[\Omega_m(\ell_m), \theta] = L_c \quad (24)$$

The lefthand side of eq.(24) is $\mu[\ell_m, \Omega_m(\ell_m)]$, which is the inverse of $\widehat{\ell}[\Omega_m(\ell_m), L_c]$. Thus eq.(24) implies that the robustness of $\Omega_m(\ell_m)$, at aspiration L_c , is:

$$\widehat{\ell}[\Omega_m(\ell_m), L_c] = \ell_m \quad (25)$$

Now, for the value of L_c in eq.(24), $\Omega_s(L_c)$ denotes a robust-satisficing decision. From the definition of robustness, eq.(4), this implies:

$$\max_{\theta \in \mathcal{U}(\ell_m, \tilde{\theta})} L[\Omega_s(L_c), \theta] \leq L_c \quad (26)$$

$\Omega_m(\ell_m)$ is the min-max decision so its loss at uncertainty ℓ_m cannot exceed the loss of $\Omega_s(L_c)$ at uncertainty ℓ_m . Thus strict inequality in (26) contradicts the definition of $\Omega_m(\ell_m)$ in eq.(7), hence (26) must be an equality:

$$\mu[\ell_m, \Omega_s(L_c)] = L_c \quad (27)$$

$\mu[\ell_m, \Omega_s(L_c)]$ is the inverse of $\widehat{\ell}[\Omega_s(L_c), L_c]$, so eq.(27) implies:

$$\widehat{\ell}[\Omega_s(L_c), L_c] = \ell_m \quad (28)$$

Comparing the equality in eq.(25) with eq.(28) we see that $\Omega_m(\ell_m)$ is a robust-satisficing decision at aspiration L_c . We have proven that any min-max decision is also a robust-satisficing decision: $R_m \subseteq R_s$.

Proof of eq.(12). Eq.(12) is equivalent to the assertion that $R_s \subseteq R_m$, which we will prove.

For any L_c , let $\Omega_s(L_c)$ be the robust-satisficing decision. As illustrated in fig. 7, let ℓ_m be the unique value defined by:

$$\mu[\ell_m, \Omega_s(L_c)] = L_c \quad (29)$$

⁴Uniqueness results from the assumption of *strict* monotonicity.

Consequently, because $\mu[\ell_m, \Omega_s(L_c)]$ is the inverse of $\widehat{\ell}[\Omega_s(L_c), L_c]$:

$$\widehat{\ell}[\Omega_s(L_c), L_c] = \ell_m \quad (30)$$

Now, for this ℓ_m , $\Omega_m(\ell_m)$ is a min-max decision. From eq.(29) and the definition of min-max decisions in eq.(7) we conclude:

$$\mu[\ell_m, \Omega_m(\ell_m)] \leq L_c \quad (31)$$

From this we conclude that:

$$\widehat{\ell}[\Omega_m(\ell_m), L_c] \geq \ell_m \quad (32)$$

$\Omega_m(\ell_m)$ cannot be more robust than the robust-satisficing decision $\Omega_s(L_c)$. So, in light of eq.(30), we conclude that eq.(32) must be a strict equality:

$$\widehat{\ell}[\Omega_m(\ell_m), L_c] = \ell_m \quad (33)$$

This implies that eq.(31) is also an equality:

$$\mu[\ell_m, \Omega_m(\ell_m)] = L_c \quad (34)$$

Comparing eqs.(29) and (34) we see that $\Omega_s(L_c)$ is a min-max decision. We have proven that any robust-satisficing decision is also a min-max decision: $R_s \subseteq R_m$. ■

B Appendix: Info-Gap Model of Uncertainty in Section 3

Here, we formulate the info-gap model, $\mathcal{U}(\ell, \Omega)$, for uncertainty in a single parameter θ . The analyst has complete Knightian uncertainty: the probability distribution of θ is unknown. One has no estimate, in any statistical sense, of the value of θ .⁵

Even though the true value of θ is not known, it is still useful to talk about values of θ that would motivate any particular choice of the parameters $\Omega = (\omega_r, \omega_\pi, \omega_y)$ of the policy rule. For instance, if we are considering a specific choice of Ω , one may ask: given our economic models, what should θ be in order to make this a good choice of Ω ? The value of θ which, were it the true

⁵This example could be extended to consider info-gap uncertainty in the probability distribution of θ . Examples of info-gap analysis of uncertainty probability distributions are found in Ben-Haim (2005a, 2006).

value, would justify a particular Ω , will be denoted $\tilde{\theta}(\Omega)$ and is defined implicitly in the relation:

$$\Omega = \Omega_b(\theta), \quad (35)$$

where $\Omega_b(\theta)$ is the loss-minimizing policy if the uncertain parameter equals θ , defined in eqs.(6) and (15). The solution of this relation, for θ , may not be unique, in which case we define $\tilde{\theta}(\Omega)$ as the lowest such solution:

$$\tilde{\theta}(\Omega) = \min \{ \theta : \Omega = \Omega_b(\theta) \}. \quad (36)$$

We now posit a “weak convexity” of the loss function. We assume that, for fixed Ω :

$$\frac{\partial [dL(\Omega, \theta)]}{\partial |\theta - \tilde{\theta}(\Omega)|} > 0. \quad (37)$$

That is, for a fixed Ω , the loss function $dL(\Omega, \theta)$ rises increasingly above the full-knowledge value, $dL[\Omega, \tilde{\theta}(\Omega)] = 0$, as θ deviates from $\tilde{\theta}(\Omega)$. Relation (37) does not assert that $dL(\Omega, \theta)$ is convex vs. θ , but only that it has a unique minimum vs. θ , for any given Ω . We posit that this weak convexity holds for $dL(\Omega, \theta)$.

The weak convexity property implies that, for any non-negative bound on the loss function, the corresponding set of θ -values is a simple interval, which we define as:

$$\mathcal{D}(x, \Omega) = \{ \theta \in \Theta : dL(\Omega, \theta) \leq x \}, \quad (38)$$

where Θ is the set of meaningful values for the parameter θ . For instance, if θ is a persistence, then $\Theta = [0, 1)$. Or, if θ is a model coefficient, then Θ might be the entire set of real numbers. For any level of loss x , the set of θ values for which the loss (with policy Ω) does not exceed x is the set $\mathcal{D}(x, \Omega)$. Because the loss function has the property of weak convexity, this set is an interval whose length we denote by $|\mathcal{D}(x, \Omega)|$. Since $dL(\Omega, \theta) = 0$ at its unique minimum, $\tilde{\theta}(\Omega)$, we note that $\mathcal{D}(0, \Omega) = \{\tilde{\theta}(\Omega)\}$.

The weak convexity property implies that the sets $\mathcal{D}(x, \Omega)$ are nested:

$$x < x' \quad \text{implies} \quad \mathcal{D}(x, \Omega) \subseteq \mathcal{D}(x', \Omega). \quad (39)$$

This states that any value, θ , whose loss is no less than x , also has loss no less than x' , when the same policy is used.

We use this concept to define an info-gap model for uncertainty in θ . For any contemplated policy parameters Ω , the info-gap model is the following family of nested sets of uncertain θ -values:

$$\mathcal{U}(\ell, \Omega) = \{\theta : \theta \in \mathcal{D}(x, \Omega), x \geq 0, |\mathcal{D}(x, \Omega)| \leq \ell\}, \quad \ell \geq 0. \quad (40)$$

At any level of uncertainty ℓ , the set $\mathcal{U}(\ell, \Omega)$ contains all values θ which belong to sets $\mathcal{D}(x, \Omega)$ no larger than ℓ , regardless of the loss value x . The level of uncertainty is unknown so ℓ can take any non-negative value. The info-gap model is a family of nested sets and obeys the axioms of contraction and nesting (Ben-Haim 2006).

In light of eq.(39) we see that, for any ℓ , there exists an $x(\ell)$ such that:

$$\mathcal{U}(\ell, \Omega) = \mathcal{D}(x(\ell), \Omega). \quad (41)$$

Combining eqs.(38) and (41) we see that the uncertainty set, evaluated at a level of uncertainty equal to the robustness, is:

$$\mathcal{U}[\widehat{\ell}(\Omega, dL_c), \Omega] = \mathcal{D}(dL_c, \Omega). \quad (42)$$

We emphasize that this info-gap model depends on a contemplated policy parameters Ω . The sets $\mathcal{U}(\ell, \Omega)$ are an expression of epistemic (rather than objective or ontological or aleatoric) uncertainty: if we use policy Ω , then the info-gap model contains all θ -values for which the loss will not exceed some specific value. We don't know which θ value will occur or the value of ℓ , so there is no known worst case (other than the limits on meaningful values of θ , such as unbounded persistence, expressed by the set Θ).

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Robust-satisficing policy