Boutselis, Petros; McNaught, Ken

Conference Paper

Finite-Time Horizon Logistics Decision Making Problems: Consideration of a Wider Set of Factors

Finite-Time Horizon Logistics Decision Making Problems: Consideration of a Wider Set of Factors

Petros Boutselis and Ken McNaught

Abstract

The newsvendor's problem (NVP) formulation is applied to many logistics problems in which the principal decision is the level of inventory which should be ordered to meet stochastic demand during a finite time-horizon. This type of decision makes demand the central variable to be examined and since the time horizon is finite, there is variable risk throughout the period. While the NVP formulation is applicable to many areas (e.g. retail business, service booking, investment in health-insurance, humanitarian aid, defence inventory for operations), modelling and research into the factors affecting demand and its uncertainty has been conducted mainly where the goal is to increase demand (e.g. price, rebate, substitutability). This paper describes ongoing work on modelling demand within the NVP framework where little prior specific demand information exists and uncertainty plays a crucial role. The suggested approach is to model demand and its uncertainty using other causally related, case-specific factors by applying Bayesian inference. Initial work in progress on a case study is outlined. In future the approach will be tested in several case studies and will adopt the innovative approach of Sherbrooke (2004) and Cohen et al (1990) for its validation, through which the model's outputs along with the real life demand data are provided as inputs to a simulation and the results compared. Thus the simulation's final output is the evaluation measure. The future expected benefit from this work is to offer decision makers an intuitive demand modelling tool within an NVP framework where modelling uncertainty is of great importance and past demand data are scarce.

Keywords: newsvendor, bayesian, risk, validation

1. Introduction

The newsvendor problem (also commonly known as the newsboy or single period problem) is one of the classical problems in operations management and has been extensively studied since the pioneering effort of Edgeworth (1888). A recent review of the area is provided by Qin et al (2011). The main question that it seeks to answer is how much of one or more types of commodity a ”newsvendor” should order, as an effort to deal with some unknown/uncertain and in some cases even risky future demand, given that the time horizon that he/she expects demand for it is finite[6]. If the newsvendor orders too much, the left-over items are usually assumed to have low or even zero residual value, while if the newsvendor orders too little, there is an opportunity cost associated with lost sales. Even though its name seems to limit its applicability to the case of a professional newspaper salesman under the dilemma of how many papers to order for the following day, its area of application is much wider and the lost sales component may be replaced, for example, with a more general shortage penalty. A list of existing and possible future areas of application includes the following:

[6] This paper differentiates the use of the newsvendor’s type of problems from what Nahmias (1989, pp 233) suggests who expands its applications to incorporate infinite periods. The main decision making issue that arises when the time period is considered “finite” is not just that it is a one-off case. On the contrary, it can include more than one period in between. The suggested idea of “finite” is that it is not practically possible to use any of the leftover inventories for subsequent periods after the “finite” planning one. Regarding the intention for “no inventory left”, it includes the cases in which all leftover inventory can be used but for a less profitable gain than if it was actually used within the examined period. Even though, the same idea of the newsvendor’s finite-period planning could be applied in a repetitive, infinite notion known as “myopic policies” (see for example Powell (2010), the problems dealt with by the newsvendor’s formulation are different due to the fact that after the period of interest (which in certain cases can be of unknown length) the periods which follow cannot be used to make-up for opportunity costs by keeping some of the left-over inventory. The examples provided will make this notion clearer.

1 Seasonal / fashion goods
2 Advanced booking in services such as hotels, airplanes, etc. In such cases the commodity that is managed/ordered is the number of places in the airplane or in the hotel and the finite period is the duration of the flight or the hotel season.
3 Amount of insurance money to invest for health etc. In this case the uncertain demand concerns future health issues and the type of commodity to decide upon in advance is the amount of insurance money that has to be invested. This demand will be realized not within a repetitive, infinite manner. A point to stress in this example is that past data on the demand do exist but only for similar cases.
4 Selecting spare parts for a product at the end of its production life, i.e. before it becomes obsolete[7]. Here, the uncertain demand is the number of spare parts that have to be produced, not as was done in the past when the system that used them had a long operational life and thus inventory could be held to cover future demands and backorders could be issued for unfulfilled ones, but for a shorter, finite period since the production line will stop.
5 Deciding on the water reservoir level on an island that is isolated and waits for replenishment for the summer period. Obviously the commodity that has to be decided upon is the amount of water to be stored, given that it has a shelf-life shorter than the next examined period. The demand can be known with adequate precision from past / historical data, however, extreme weather forecasts or a new differentiation in the relative tank capacities could ask for a new formulation of how this demand could be realized.

[7] As Khouja (1999) stresses "…, the reduction in product life cycles brought about by technological advances makes the SPP (Single-Period Problem ie the newsvendor's one) more relevant", and obsolescence costs and management is an issue of great importance to large organizations like the MOD.

6 Large military operations “Last Order”, i.e. how much to order when a decision to withdraw from an operation has been taken. In these cases decisions on ordering any amount of a certain commodity have to be taken wisely since whatever is transported out, unless used, will also have to be transported back.

7 Humanitarian aid in multiple places that have suffered disasters. As Chakravarty (2014) stresses, such cases include an increased risk since prior values of lead times and supply chain efficiency may have changed due to distractions to the supporting infrastructure. In his work Chakravarty (2014) studies scenarios that deal with limited resources and develops a model in which an initial capacity based on demand forecast is built and further capacity is added when additional information on demand is available. The suggestion in this case is that such efforts could benefit from an NVP formulation.

8 The load of a shipment within a supply chain. The amounts and mixtures of commodities loaded each time are usually not completely known in advance and have to be either consolidated in full loads and thus result in accumulating inventory costs and increased lead times for some of the items as they wait in the warehouses to be gathered, or depart in half-loads and thus increase transport costs. For some commodities it is space[8] allocation that has to be decided in advance before the commodities “arrive” and place a “demand” for some space. What has to be decided is how much space will be allocated and when for a finite period of time that the actual demand for that space will be realized.

9 The “hedge contracts” that energy providers use in order to insure against the risk of unknown demand.

[8] The commodities provide data which are prone to change in number and type, while space (or space per value to differentiate among commodities which will compete for the same space area) is a more generic variable.

2. Modelling demand in the newsvendor's problem

Demand is the main problem for the newsvendor. He/she has to predict or even affect its value in order to be able to optimize his/her set objectives.

2.1 The importance of modelling demand uncertainty

In the classical infinite-horizon inventory systems in which demand needs to be modelled like the continuous review, or the periodic review ones, the length of the risk period is more or less controllable by the manager. This risk period is considered as the period during which the decision maker wants the amount on hand and the amount ordered to be *enough*[9] so that the commodity does not run-out until the arrival of that next order. It is a period of risk because during that, any demand fluctuations cannot be faced immediately by a new arrival but by only just hoping that what is on hand and what is expected to arrive soon will be able to cope with these fluctuations. In inventory systems like the Continuous Review, or the Periodic Review, this risk period affects the inventory planning and the level of inventory kept has to be increased by a safety stock. This stock can be subsequently adjusted according to new different demand data or decisions on budget or service levels. Therefore, in these policies there is the opportunity to improve unsatisfactory performance by applying adjustments.

In the case of the newsvendor's problem, the whole period from the time that the order is placed until the time that the problem is over is a period of risk. Even when there are cases in which some small corrections[10] can be made on the way until the end of the examined period the same risks/dilemmas hold for the newsvendor: to order neither too little nor too much since in either case he/she

[9] This notion of "enough" is the one used to define *the service level* for the infinite inventory management systems. However, in newsvendor type problems it cannot be considered as such. This is because the actual idea of a safety stock is one of the things that the optimum newsvendor's solution wants to avoid, ie not to have left-overs.

[10] Even though the sequential filling of the inventory has many practical applications it is not a common research subject. For relative work and discussion see Murray and Silver (1966) and Khouja (1999).

will have unwanted opportunity costs. Furthermore, when the examined period is over there is no opportunity to make up for unsatisfactory performance. Consequently, in the newsvendor's settings demand uncertainty should be considered for the whole period of the problem[11].

As Porteus (2002) notices, what needs to be minimized is the combination of the holding and shortage costs, ie what will be provoked by the *variation of the demand* below or above its expected "deterministic" value.

2.2 Models of the newsvendor's demand problems

What has previously been shown is that at least within the newsvendor's problem setting, uncertainty plays a vital role and therefore should be considered within any relevant model. The demand models used in the literature mainly fall into 6 categories:

1. Those that use functions in order to investigate the dependency of the demand on other factors, mainly the price and thus see how these factors can be used so that they can drive demand.
2. Those that model demand by considering scenarios of possible values but without considering any probability distribution for these values.
3. Those that model demand using certain probability distributions and calculate their respective parameters from past demand data or from Subject Matter Experts (SMEs).
4. Those that model demand using certain probability distributions and calculate their respective parameters using Bayesian inference from "fresh" data.
5. Those that model the values of demand using uncertainty theory.
6. Those that model the values of demand using fuzzy logic.

[11] For another comprehensive discussion on misleading managerial results/decisions when not incorporating uncertainty in the demand see Shih (1979, p. 688).

2.2.1 Using functions to model demand

Functions have been used to model and investigate the dependency of demand on other variables, mainly the price. Within a functional form, the uncertainty has been considered in the literature in either an "additive", "multiplicative", or "additive-multiplicative" modelling approach[12]:

a. (Polatoglu 1991) and (Qin, et al. 2011)refer to the previous work of (Mills 1959) who expresses demand D(p,ε) by taking the sum of the expected demand as a function of the price d(p), plus a random variable ε which is independent, and has a zero expected value: $D(p,\varepsilon) = d(p) + \varepsilon$. The name convention used for this type of model is the *additive model*. However, as Polatoglu (1991) reasonably argues, in that way there is a strong assumption that the variance of the demand is constant and independent of the price. This is not always correct of course. It is reasonable that there were will be cases in which comparatively very low prices will have a different effect not only in the expected demand but also on its variance. Additionally, Polatoglu (1991) and Zabel (1970) emphasize the fact that the use of a function which relates demand to the expected price via a linear function with a negative coefficient, allows for negative values of the demand unless there is a further assumption on a lower limit to the values of the prices. On the same issue Zabel (1970) further emphasises the need to make sure that the probability of getting negative values due to the additive density function is set to zero.

b. A similar type of treatment of the demand is obtained by representing it with a *multiplicative model* (Qin et al. 2011; Chen, Yan and Yao 2004; Zabel 1970). The structural difference between this and the additive one

[12] Huang, Leng and Parlar (2013) have created a very comprehensive table for the used functional forms (Table 1) in which they include the Linear, Power, Hybrid, Exponential, Logarithmic and Logit deterministic demand functions. Furthermore, in their (Table 2) they have also included a list of the stochastic equivalent functions which are frequently applied in the newsvendor types of problem setting.

is that the random factor is now multiplied by the function of the expected price $D(p,\varepsilon) = d(p)\varepsilon$. The random variable is again independent of the price but is now multiplied by the main part and has a mean of 1. However, as is again reasonably stressed by Polatoglu (1991), in this way the variance of the demand is dependent on the square of the product of the random term and the expected value of the price and as a result as prices keep on rising, the variance decreases at a much faster rate.

c. A combined additive-multiplicative model incorporates both notions in a function which has the form of $D(p) = d(p,Z) = \alpha(p)Z + \beta(p)$, (Young, 1978). In this generic form p is the price and Z is a random variable with price independent cdf, $\alpha(p)$ and $\beta(p)$ are decreasing functions of the price. Therefore, the additive model can be obtained by setting $\alpha(p) \equiv 1$, so the prices affect only $\beta(p)$, ie the location of the demand distribution, while the multiplicative model can be obtained by setting $\beta(p) \equiv 0$, which means that the prices influence the demand scale. However, as Kocabiyikoglu and Popescu (2011) point out, even in this general formation the additive-multiplicative models make restrictive assumptions that drive the newsvendor's problem results, such as a monotonic relationship between price and demand variability, while empirical studies show that such relationships may not hold and thus the theoretically formed functions often have a poor performance.

The above counter-intuitive elements in the modelling approaches to uncertain demand give further ground to criticism when they are examined and evaluated in the same problem areas. In a number of papers there has been consideration of:

- The newsvendor's selling price (for example Qin et al., 2011; Arcelus, Kumar and Srinivasan, 2005; Petruzzi and Dada, 1999; Polatoglu, 1991),
- The portion of the "supplier's price discount" to be passed from the newsvendor to the end customer (for example Qin et al., 2011; Petruzzi and Dada, 1999).

The suggested decisions relative to a deterministic model differ when the demand uncertainty is represented within an additive and within a multiplicative model.

In order to deal with these ambiguities Kocabiyikoglu and Popescu (2011) introduced a unified framework. Their model has the general form of $D(p) = d(p, Z)$, with p again being the price and Z a random variable with price independent cdf, which makes no assumptions regarding the effect of price on the variability of demand and furthermore can incorporate other functional forms of the relationship between demand and price, apart from the additive, multiplicative and additive-multiplicative ones, such as the attraction models (eg power, logit), willingness-to-pay (WTP), etc. Furthermore, they introduce another concept, the Lost Sales Rate (LSR) with which they try to capture a framework in which the study of decisions on price and inventory level can take place when the demand uncertainty has to be taken into consideration. They define LSR to be the complement of the demand CDF (and therefore the probability of lost sales). Additionally, LSR elasticity is defined with the use of both price and inventory level and thus, for a given quantity, it combines the sensitivity of lost sales given its building factors of inventory and price. Therefore, their significant contribution with regards to the problems previously mentioned is that they identify which combinations of price and inventory level lead to LSR elasticity being monotone.

However, questions on the root causes of the differences in the results originating from the different treatment of the demand uncertainty, ie through additive, multiplicative and additive-multiplicative functions remain, and the reasonable arguments of Polatoglu (1991), Zabel (1970), Kocabiyikoglu and Popescu (2011)

and other authors have not been adequately addressed, apart from stating that there should be specific restrictive assumptions applied in each case.

2.2.2 Using scenarios to model demand

In these cases demand is represented by considering scenarios with different possible values but without assigning any probability distributions.

This approach has been developed by Vairaktarakis (2000). He argues that reasonable probability distributions of demand can be almost impossible to determine in cases when the decision situation is unique and there is little or no historical data. Furthermore, he stresses that when probability distributions are used, the problem can become too difficult to solve and the assumptions that are then introduced to make it tractable can turn it into a "mediocre representation of reality".

Vairaktarakis's (2000) suggested approach to modeling uncertainty is through the different possible scenarios/values that the demand can take and then applying robust decision theory criteria to assist the decision maker. If demand is continuous then a range of values can be used instead. He then solves a constrained optimization problem to suggest the optimum order level.

One of the assumptions considered in this case is that possible demand values for each item are defined based on the understanding of the manager on "the sources that affect uncertainty (such as market and competitive environment)" (Vairaktarakis, 2000, p. 214). The author thus implicitly states that the elicited scenarios/values of demand will be considered and used under the assumption of different values of factors which have an effect on demand. However, as the functional models have shown, the mean demand can take different values according to the relevant factors which sometimes can vary a lot. Furthermore, the number of different factors to be considered can possibly make the elicitation of demand values a difficult task for the manager.

On the same issue Vairaktarakis (2000, p. 215) suggests that the values for the demand should only contain likely realizations unless the decision maker wants to hedge the operational risk against some extremely unlikely scenarios. This

assumption is inherent to this applied modelling approach since it does not imply any different weight/probability to the different possible scenarios/values and thus it would consider an equal weight to very unlikely scenarios/values. However, in cases where there is little prior experience, the decision maker may be reluctant to define which of the different values/scenarios are very unlikely.
At another point the author states: “stocking decisions for subsequent periods are independent of decisions made in previous periods” (Vairaktarakis, 2000 p 215). This assumption is not always realistic. Prior stocking decisions affect demand and obviously posterior stocking decisions as well, and have been studied extensively (Huang, Leng and Parlar, 2013). Furthermore, this assumption reduces the valuable opportunity offered to the decision maker to learn from past experience and it is not the way that people tend to work in practice (Cyert, DeGroot and Holt, 1978; Harpaz, Lee and Winkler, 1982).

2.2.3 Using probability distributions to model demand

In these cases demand is modeled by the use of certain probability distributions, with parameters estimated from past demand data.
Extensive work has been done by Braden and Freimer (1991) who have worked on providing a selection of demand distributions for the newsvendor models. Their work is developed under the realization that the demand may not be perfectly known from the sales that follow. This is because sales can be a mixture of exact and left-censored observations of demand. The term “left-censored” is used to show that since there will be cases in which customers do not find the product because all of the previously ordered quantity has been sold-out and therefore the newsvendor’s estimate of the demand is that it is at least as much as his initial order. Braden and Freimer (1991) have shown that the exponential family of distributions is applicable to such type of mixed observations.
As was mentioned earlier and pointed out by Vairaktarakis (2000), this approach can suffer from difficulty due to the sometimes complicated demand probability functions, while it is quite difficult to elicit probabilities when there is little past data and experience. Regarding the latter, an additional drawback of the

approach is that it does not use data from other factors that have a causal relationship with the demand and may be easier to obtain. Such factors could be like the ones implied by Vairaktarakis (2000) who asked the experts to conceptually consider drivers affecting demand uncertainty related to the market and competition, an approach which on the other hand is used in the functional models.

2.2.4 Using probability distributions to model demand and Bayesian Inference to refine the probability parameters

Here, demand is modelled again by the use of certain probability distributions; however, their parameters θ_i follow prior distributions which are updated to posterior distributions using Bayesian inference from "fresh" data. The chosen probability families can include those suggested by Braden and Freimer (1991), while the initial parameters can be obtained from approaches like:

- Reliability analysis as suggested by Petrovic, Senborn and Vujosevic (1988),
- "Objective Bayes" suggested and practically tested by Sherbrooke (2004)[13]
- "Empirical Bayes" implemented by Scarf (1959) and Robbins (1964)

One of the early, very important works in this area was that of Murray and Silver (1966) who examined the optimum sequential production of fashion goods with the demand being unknown but updated by the use of Bayesian inference over time. A different application was developed by Cyert, DeGroot and Holt (1978) who studied an approach to reducing risk in investments. Their suggestion was to find how decision makers could make sequential investments that could gather information useful for subsequent decisions. The information they were looking

[13] It should be noticed that Petrovic, Senborn and Vujosevic (1988) and Sherbrooke (2004) refer to low demand and high unit price items and even though the suggested policies are for the infinite-time horizon problems this does not differentiate the suggested approach to the modelling of demand.

to obtain was the profit gained (or lost) and thus their respective prior probability distribution could be refined into a better informed posterior by the use of Bayesian Inference. Their work was continued by Harpaz, Lee and Winkler (1982) who again used Bayesian inference to model a perfectly competitive firm's learning from experience and thus reducing its uncertainty on demand.

The attractiveness of learning in facilitating decision making prompted several other researchers to work on this idea. Azoury (1985) used two finite horizon inventory models, one for consumable and one for reparable items and showed that solving the Bayesian problem by applying dynamic modelling, under certain conditions is no harder than if the distribution parameters were known. Eppen and IYer (1997) applied the same modelling approach for fashionable goods in order not only to optimize the amount of order but also to suggest how much to divert to the owned outlet stores. Their model is developed by the use of historical data and buyer judgement and is solved through stochastic dynamic programming. Furthermore, Berk, Gurler and Levine (2007) show that a two-moment Bayesian modelling of demand parameters distribution works adequately both in theoretical and operational applications.

However, the same issues apply as the ones referred to in the last category.

2.2.5 Using uncertainty theory to model demand

The users of this approach argue that when there are not enough sample data only the experts' belief degree of the underlying distribution can be used, and within that there can be considerable bias. In order to deal with experts' belief degree in modelling demand uncertainty Ding and Gao (2014) adopt and apply the Uncertainty Theory developed by Liu(2007)[14].

[14] This theory was further expanded by Liu (2009)

2.2.6 Using fuzzy logic to model demand

In this case uncertainty is modelled by considering demand values as fuzzy variables. One of the works in this area was that of Ji and Shao (2006) whose main goal was to model the combined decision making of the manufacturer and the retailers and apply a hybrid optimization based on fuzzy simulation along with a genetic algorithm to optimize the expected profit.

2.3 Factors that affect demand and have been used to model it

This paper has found great value in the seminal work of Huang, Leng and Parlar (2013) who have thoroughly examined not only the functions but also the factors that have been used in order to model demand and not only for the newsvendor's problem formulation, and in the work of Khouja (1999) who has introduced a similar taxonomy specifically for the newsvendor's problem. The factors examined are:

1 Prices set either by the newsvendor or his/her supplier
2 Rebate
3 Lead-time that affects the customer's level of satisfaction
4 Space presented to the customer, thus affecting his perception of product availability
5 Quality
6 Advertising

One point worth noticing is that the literature's orientation has mainly been on examining demand's dependency on factors which can increase it. This obviously is true if the newsvendor is a retailer or manufacturer and primarily wants to optimize monetary objectives. However, there are cases when it would be preferable if the demand is the smallest possible. A number of factors that are relevant in such cases could include:

1 In the case of deciding how much money to invest in an individual's health, the quality of the state's health care programs

2 In the case of deciding on the final quantity of military supplies to deliver before a withdrawal, the level of enemy military presence in an area
3 The weather forecast in problems where water has to be provided to an isolated place

3. The application of Bayesian Networks to newsvendor problems

An important point regarding demand distributions has been raised by Sherbrooke (2004) in which he observes that the mean demand is "drifting" with time. The factors in each case that cause/impose this drift[15] can be analysed through engineering breakdown approaches using Subject Matter Experts' (SME) opinions. Consequently, there is an inherent gap in the approaches that use just probability distributions (3rd modelling approach) since they define them by the use of past data only. The same issue seems to exist in the use of uncertainty theory and fuzzy logic (5th and 6th modelling approaches). Vairaktarakis (2000) in his use of scenarios in the 2nd modelling approach asks the SMEs to consider the factors that define the values which demand can take but fails to account for the relative strengths of their effects and also prompts them to exclude very rare values of demand unless they want to hedge for them. On the other hand, Bayesian inference models (4th modelling approach) consider this drift effect but need fresh data on demand itself to do that, something that is rarely available in many of the newsvendor-type problems. Of all the previously discussed modelling approaches, only the functional ones take into consideration the main factors that define the context within which demand is considered. However, they also fail to adequately model the uncertainty and its effect on decision making which is of vital importance to newsvendor-type decisions. Furthermore, in order to work they need sufficient data which is

[15] E.g. the wear-out of the system, different environmental conditions, change in the system's use rate

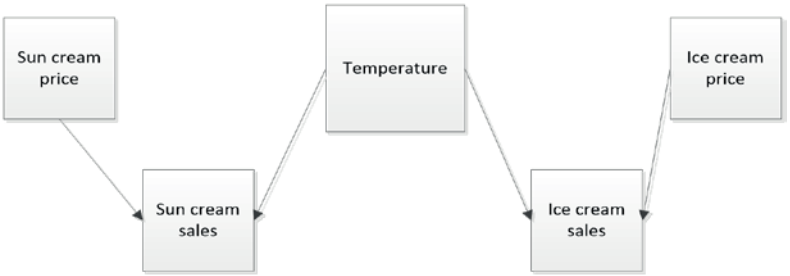
available mainly in market related applications but not in other areas within the newsvendor spectrum of problems.

What is suggested is to combine the consideration of demand uncertainty and the demand-defining context formed by causal factors, which are likely to take uncertain values as well. A natural way of doing this is to form a joint probability distribution of all the relevant variables. The modern probabilistic framework of Bayesian Networks (BNs) provides an efficient way of representing, building and manipulating such a distribution in order to perform inference of various types. Furthermore, based on our literature search, we believe that its application to newsvendor type problems is novel.

3.1 Requirements of a BN formulation

A BN (Pearl, 1998) is a directed acyclic graph (DAG) in which the nodes correspond to variables of interest in the modelled domain and arcs correspond to direct probabilistic dependencies. Absence of an arc between two nodes does not necessarily imply that they are completely independent but that dependence might be mediated by another variable, for example. It does imply that they are at least conditionally independent under some conditions. For example, a simple BN containing the five variables ‘daily ice cream sales’, ‘ice cream price’ ‘temperature’, ‘daily sun cream sales’ and ‘sun cream price’ is shown in Fig 1. Although the two sales variables are probabilistically dependent, there is no arc drawn between them since their dependence is a result of their common-cause parent variable, ‘temperature’ which does have a direct probabilistic dependence with each of them. We say that ‘ice cream sales’ and ‘sun cream sales’ are conditionally independent given temperature. In fact, the arcs from temperature to sales represent causal links but arcs in a BN do not have to be causal in general.

Fig. 1: Example of a simple Bayesian Network



The DAG for a BN, describing its qualitative structure and conditional independence assumptions, is most often elicited from a domain expert. Each node is also given a state space, which is often discrete for computational convenience. Truly continuous variables can be handled in various ways but are most often simply discretized. In order to operationalize the BN for quantitative modelling, however, we also need to assign marginal probability distributions to nodes which have no parents and conditional probability distributions to nodes which have parents. It is necessary to have a conditional distribution for every combination of a node's parent states. In some cases, functional relationships can be employed which eases this burden but in others we need to extract the various distributions one by one, making use of any existing past data and making up for gaps and deficiencies in that data through a process of elicitation with the relevant domain experts.

3.2 An example BN of a newsvendor type problem

Considering the case referred to in the introduction as military operations' "Last Order", suppose that the demand for a certain support item is expected to be directly affected by the following variables:

- "Equipment usage"
- "Operating hours"
- "Equipment losses"

In addition other variables will have an indirect effect and be included in the BN to make the model more complete and the probability distributions easier to specify and obtain from either data or SMEs.

It is obvious that the different values of these variables form different contexts within which demand can again take different distributions of values. Combining the likely demand distributions conditioned on the influencing variables with the prior distributions of the influencing variables themselves provides a coherent approach to modelling the demand. It also allows for the inclusion of related forecast variables in a straightforward fashion, e.g. relevant environmental forecasts such as temperature. The distinction between the forecast temperature which is known and the actual realised temperature which will be unknown when the order is placed is important in this type of model. As well as allowing the demand distribution to be updated when a new forecast is made, questions about the value of buying improved forecast information can also be addressed.

Based on the above example, a BN model could have the form shown in Fig 2. Since this paper is reporting on work in progress, our next task is to quantify this model by obtaining the required probability distributions which its structure suggests. For a specific equipment or commodity this will involve a combination of historical data analysis and elicitation from SMEs.

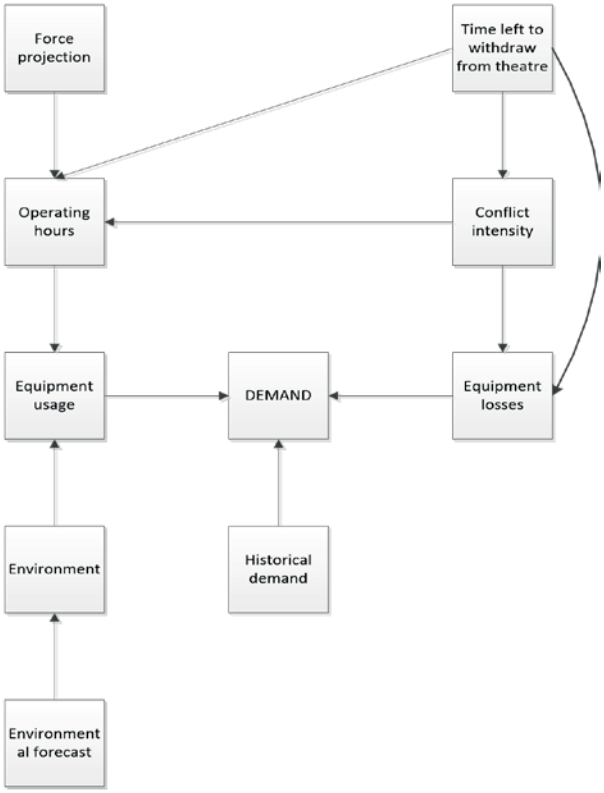


Fig. 2: Example of a Bayesian Network used to estimate demand and thus facilitate decision making in a military operations “Last Order” newsvendor-type problem

4. Approach to validation

An additional area of research that builds upon the results of demand prediction is the one documented by Ward, Chapman and Klein (1991) and further discussed in the seminal work of Khouja (1999). In both, the need is stressed to validate the different modelling results through practical studies. This need still exists as Huang, Leng and Parlar (2013) verify. Furthermore, Huang, Leng and

Parlar through their review on empirical studies they suggest the following measures of goodness of fit:

- The Standard Error of Estimate (SEE)
- The log-likelihood statistic
- Nonlinear Least Squares (NLS) with adjusted R2

However, these are still not what is implied by Khouja[16] since these measures are not intuitive to managers, and cannot be practically assessed.

A more practical and intuitive approach for managers is suggested and has been successfully applied by Sherbrooke (2004) and Cohen et al (1990). Through this approach future demand is initially estimated using a candidate demand model. The predictions are used in the inventory system to calculate the suggested level of spares. Then the actual demand which is known from past data (kept for the validation of the models) is used to calculate the attained availability through simulation. The best demand model is the one which gives the optimum measure of system performance when real data are used, thus, the measure of goodness of fit relates to the manager's objective itself. Furthermore, Sherbrooke (2004) also used as a complementary measure the estimated availability that the spares calculation model gave and then compared it to the "true"/attained availability to see if it was an optimistic estimate or not. Therefore, following this suggestion the following two measures of goodness of fit are suggested for research on practical cases:

1 The actual result that would be attained using the suggested method of modelling the demand within the context of each specific application of the model[17].

[16] Khouja (1999, p. 550) refers to the issue by saying: "Without some empirical work examining real life objectives of managers and the availability of information about demand, the practicality of these models cannot be assessed"

[17] For example if a Bayesian Network (BN) model of the demand was used in predicting the optimum space allocation in a transport problem, then the measure of the model's effectiveness would be the actual revenue acquired if the procedure would follow the suggestions of the model and the true demand was applied.

2 The predicted optimum estimate[18].

5. Conclusions

The newsvendor framework is applicable to a very wide problem spectrum. Several potential applications have been briefly described to give a sense of this. One of the major difficulties that its application poses is that compared to other infinite-time horizon management policies, consideration of demand uncertainty is of supreme importance to decision making. The traditional ways of modelling this demand have been outlined and some of the issues surrounding it discussed. In this paper, we propose the use of Bayesian networks to model demand in newsvendor type problems. This framework naturally permits consideration of a wider set of context-defining variables or factors which influence the demand. The usefulness of the BN approach is also enhanced in cases where little past demand data exists and decisions need to be taken by considering the relationship of demand with these context-defining variables. This paper is a report of work in progress and here we have only outlined the qualitative form of an example model relating to final supply of military equipment. The quantification of the model is the subject of ongoing work and will be reported in a future paper. According to our literature review the use of BNs in newsvendor type applications is novel.

Furthermore, in order to develop a BN, close cooperation of the analyst with the manager/SME is of fundamental importance in the procedure and therefore the validation approach has to be in accord with the decision maker's intuition. This is why the BN applications will be validated using practical approaches and measures of merit such as the ones suggested by Shrebrooke (2004) and Cohen et al (1990) that relate to the real effectiveness concerns of the decision makers.

[18] In the previous example this would be how much revenue was expected by the application of the model.

References

Arcelus, F., Kumar, S. & Srinivasan, G., 2005. Retailer's response to alternate manufacturer's incentives under a single-period, price-dependent, stochastic-demand framework. Decision Sciences, pp. 599-626.

Arcelus, F. & Srinivasan, G., 2003. Scanbacks and direct rebates: manufacturer's tools against forward buying. International Transactions in Operational Research, pp. 619-635.

Azoury, K. S., 1985. Bayes solution to dynamic inventory models under unknown demand distributiion. Management Science, pp. 1150-1160.

Baker, R. C. & Urban, T. L., 1988. A deterministic inventory system with an inventory-level-dependent demand rate. Journal of Operations Research, pp. 823-831.

Balakrishnan, A., Pangburn, M. & Stavrulaki, E., 2004. Stack them high, Let 'em fly": Lot-sizing policies when inventories stimulate demand. Management Science, pp. 630-644.

Berk, E., Gurler, U. & Levine, R. A., 2007. Bayesian demand updating in the lost sales newsvendor problem: A two-moment approximation. European Journal of Operational Research, pp. 256-281.

Bernstein, F. & Federgruen, A., 2005. Decentralized supply chains with competing retailers under demand uncertainty. Management Science, pp. 18-29.

Boyaci, T. & Ray, S., 2003. Product differentaition and capacity cost interaction in time and price sensitive markets. Manufacturing & Service Operations Management, pp. 18-36.

Braden, D. J. & Freimer, M., 1991. Informational dynamics of censored observations. Management Science, pp. 1390-1404.

Chakravarty, A. K., 2014. Humanitarian reliefchain:Rapidresponseunderuncertainty. International Journal of Production Economics, pp. 146-157.

Chan, L., Shen, Z., Simchi-Levi, D. & Swann, J., 2004. Coordination of pricing and inventory decisions: A survey and classification. In: Handbook of quantitative supply chain analysis: Modelling and E-Business era. Dordrecht, The Netherlands: Kluwer Academic Publishers, pp. 335-392.

Chen, F. Y., Yan, H. & Yao, L., 2004. A newsvendor pricing game. IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans, pp. 450-456.

Chung, K. H., 1990. Risk in inventory models: The case of the newsboy problem - Optimality conditions. Journal of the Operational Research Society, pp. 173-176.

Cohen, M., Kamesam, P. V., Lee, H., Tekerian, A., 1990. Optimizer: IBM's multi-echelon inventory system for managing service logistics. Interfaces, pp. 65-82.

Corstjens, M. & Doyle, P., 1981. A model for optimizing retail space allocations. Management Science, pp. 822-833.

Corstjens, M. & Doyle, P., 1983. A dynamic model for strategically allocating retail space. Journal of the Operational Research Society, pp. 943-951.

Cyert, R. M., DeGroot, M. H. & Holt, C. A., 1978. Sequential investment decisions with bayesian learning. Management Science, pp. 712-718.

Dana, J. D. & Petruzzi, N. C. J., 2001. Note: The newsvendor model with endogenous demand. Management Science, pp. 1488-1497.

Datta, T. K. & Pal, A. K., 1990. A note on an inventory model with inventory level-dependent demand rate. Journal of Operational Research Society, pp. 971-975.

Desai, P. S. & Srinivasan, K., 1995. Demand signalling under unobservable effort in franchising: Linear and nonlinear price contracts. Management Science, pp. 1608-1623.

Ding, S. & Gao, Y., 2014. The (σ,S) policy for uncertain multi-product newsboy problem. Expert systems with applications, pp. 3769-3776.

Edgeworth, F. Y., 1888. The mathemetical theory of banking. Journal of the Royal Statistical Society, pp. 113-127.

Efron, B. & Morris, C., 1977. Stein's paradox in statistics. Scientific American, pp. 119-127.

Elmaghraby, W. & Keskinocak, P., 2003. Dynamic pricing in the presence ofinventory considerations: Research overview, current practices and future directions. Management Science, pp. 981-1000.

Eppen, G. D. & IYer, A. V., 1997. Improved fashion buying with bayesian updates. Operations Research, pp. 805-819.

Gans, N., 2000. Customer loyalty and supplier strategies for quality competition I: Customer loyalty. Philadelphia PA: The Wharton School, University of Pennsylvania.

Geng, Q. & Mallik, S., 2011. Joint mail-in rebate decisions in supply chains under demand uncertainty. Production and Operations Management, pp. 587-602.

Gerchak, Y. & Mossman, D., 1992. On the effect of demand randomness on inventories and costs. Operations Research, pp. 804-807.

Ghiani, G., Laporte, G. & Musmanno, R., 2004. Introduction to Logistics Systems Planning and Control. s.l.:John Wiley & Sons.

Hadley, G. & Whitin, T. M., 1963. Analysis of inventory systems. Englewood Cliffs, N.J.: Prentice-Hall, Inc.

Harpaz, G., Lee, W. Y. & Winkler, R. L., 1982. Learning, experimentation, and the optimal output decisions of a competitive firm. Management Science, pp. 589-603.

Hill, R. M., 1997. Applying bayesina methodology with uniform prior to the single period inventory model. European Journal of Operational Research, pp. 555-562.

Ho, T. & Zeng, Y.-S., 2004. Setting customer expectation in service delivery: An integrated marketing-operations perspective. Management Science, pp. 479-488.

Huang, J., Leng, M. & Parlar, M., 2013. Demand functions in decision modelling: A comprehensive survey and research directions. Decision Sciences, pp. 557-609.

Ji, X. & Shao, Z., 2006. Models and algorithm for bilevel newsboy problem with fuzzy demands and discounts. Applied Mathematics and Computation, pp. 163-174.

Johnson, G. E., 1964. Uncertainty and price theory: comment. Quarterly Journal of Economics, pp. 647-648.

Jucker, J. V. & Rosenblatt, M. J., 1985. Single-period inventory models with demand uncertainty and quantity discounts: behavioral implications and a new solution procedure. Naval Research Logistics Quarterly, pp. 537-550.

Karlin, S. & Carr, C., 1962. Prices and optimal inventory policy. In: Studies in applied probability and management science. Stanford CA: Stanford University Press, pp. 159-172 (Chapter 10).

Khouja, M., 1999. The single-period (news-vendor) problem: literature review and suggestions for future research. The International Journal of Management Science, pp. 537-553.

Kocabiyikoglu, A. & Popescu, I., 2011. An elasticity apporach to the newsvendor with price-sensitive demand. Operations research, pp. 301-312.

Lariviere, M. A. & Porteus, E. L., 1999. Stalking information: Bayesian inventory management with unobserved lost sales. Management Science, pp. 346-363.

Lau, A. H.-L. & Lau, H.-S., B 1988. The newsboy problem with price-dependent demand distribution. IIE Transactions, pp. 168-175.

Lau, A. H.-L. & Lau, H.-S., A 1988. Maximizing the probability of achieving a target profit in a two-product newsboy problem. Decision Sciences, pp. 392-408.

Lau, H.-S., 1980. The nesboy problem under alternative optimization objectives. Journal of the Operational Research Society, pp. 525-535.

Li, L., 1992. The role of inventory in delivery-time competition. Managmenet Science, pp. 182-197.

Little, J. D., 1966. A model of adaptive control of promotional spending. Operations Research, pp. 1075-1097.

Liu, B., 2007. Uncertainty theory (2nd ed). Berlin: Springer-Verlag.

Liu, B., 2009. Some research problems in uncertainty theory. Journal of Uncertain Systems, pp. 3-10.

Mills, E. S., 1959. Uncertainty and price theory. The Quarterly Journal of Economics, pp. 116-130.

Murray, G. R. & Silver, E. A., 1966. A Bayesian analysis of the style goods inventory problem. Management Science, pp. 785-797.

Nahmias, S., 1989. Inventory control subject to uncertain demand. In: Production and operations analysis. Boston, MA: Richard D. Irwin, pp. 178-179.

Netessine, S. & Rudi, N., 2003. Centralized and competive models with demand substitution. Operational Research, pp. 329-335.

Nguyen, D., 1985. An analysis of optimal advertising under uncertainty. Management Science, pp. 622-633.

Norland, R. J. E., 1980. Refinements in the Ismail-Louderback stochastic CVP model. Decision Sciences, pp. 562-572.

Pearl, J., 1988. Probabilistic reasoning in intelligent systems: Networks of plausible inference. San Mateo, CA: Morgan Kaufmann

Pekgun, P., Griffin, P. M. & Keskinocak, P., 2008. Coordination of marketing and production for price and leadtime decisions. IIE Transactions, pp. 12-30.

Petrovic, R., Senborn, A. & Vujosevic, M., 1988. A new adaptive algorithm for determination of stockd in spare parts inventory systems. Engineering Costs and Production Economics, pp. 405-410.

Petruzzi, N. C. & Dada, M., 1999. Pricing and the newsvendor problem: a review with extensions. Operations Research, pp. 183-194.

Polatoglu, H., 1991. Optimal order quantity and pricing decisions in single-period inventory systems. Inrernational Journal of Production Economics, pp. 175-185.

Popovic, J. B., 1987. Decision making on stock levels in cases of uncertain demand rate. European Journal of Operational Research, pp. 276-290.

Porteus, E. L., 2002. Foundations of stochastic inventory theory. Stanford, California: Stanford University Press.

Porteus, E. & Raz, G., 2006. A fractiles perspective to the joint price/quantity newsvendor model. Management Science, pp. 1764-1777.

Powell, W. B., 2010. Policies. In: Approximate dynamic programming: Solving the curse of dimensionality. New Jersey: John Wiley and Sons, p. Chapter 6.

Qin, Y. et al., 2011. The newsvendor problem: Review and directions for future research. European Journal of Operational Research, pp. 361-374.

Robbins, H., 1964. The Empirical Bayes approach to statistical decision problems. Annals of Mathematical Statistics, pp. 1-20.

Sankarasubramanian, E. & Kumaraswamy, S., 1983. Note on "Optimal ordering quantity to realize a pre-detrmined level of profit". Management Science, pp. 512-514.

Scarf, H. E., 1959. Bayes solution of the statistical inventory problem. The Annals of Mathematical Statistics, pp. 490-508.

Schwartz, B., 1966. A new approach to stockout penalties. Management Science, pp. B538-B518.

Sherbrooke, C. C., 2004. Optimal inventory modeling of systems. s.l.:Kluwer Academic Publishers.

Shih, W., 1979. A general decision model for Cost-Volume-Profit analysis under uncertainty. The Accounting Review, pp. 687-706.

So, K. C. & Song, J.-S., 1998. Price, delivery time guarantees and capacity selection. European Journal of Operational Research, pp. 28-49.

Taha, H., 2011. Operations research: An introduction. s.l.:Pearson Education Ltd.

Thakkar, R. B., Finley, D. R. & Liao, W. M., 1984. A stochastic demand CVP model with return on investment criterion. Contemporary Accounting Research, pp. 77-86.

Tyagi, R. K., 1999. A characterization of retail response to manufacturing trade deals. Journal of Marketing Research, pp. 510-516.

Vairaktarakis, G. L., 2000. Robust multi-item newsboy models with a budget constraint. International Journal of Production Economics, pp. 231-226.

Ward, S. C., Chapman, C. B. & Klein, J. H., 1991. Theoretical versus appled models: The newsboy problem. International Journal of Management Science, pp. 197-206.

Waters, D., 1989. A practical introduction to manegement science. s.l.:Pearson Education Ltd.

Whitin, T., 1955. Inventory control and price theory. Management Science, pp. 61-68.

Xu, X. & Hopp, W. J., 2009. Technical note — Price trends in a dynamic pricing model with heterogeneous customers: A martingale perspective. Operations Research, pp. 1298-1302.

Young, L., 1978. Price, inventory, and the structure of uncertain demand. New Zealand Operational Research Journal, pp. 157-177.

Zabel, E., 1970. Monopoly and uncertainty. Review of Economic Studies, pp. 205-219.

Zabel, E., 1986. Price smoothing end equilibrium in a monopolistic market. International Economic Review, pp. 349-363.

Zhao, W. & Zheng, Y.-S., 2000. Optimal dynamic pricing for perishable assets with nonhomogeneous demand. Management Science, pp. 375-388.

Zhao, X., Stecke, K. E. & Prasad, A., 2012. Lead time and price quotation mode selection: Uniform or differentiated?. Production and Operations Management, p. 177–193.