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Gradual Covering Location Problem with Stochastic Radius

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Gradual Covering Location Problem with Stochastic Radius

Mahdi Bashiri, Elaheh Chehrepak and Saeed Gomari

Abstract

In this paper, we consider the gradual covering problem when the coverage radius, is determined by a random variable with distinct distribution functions. In this model, it is assumed that the certain amount of coverage radius is not available and the potential coverage radius is used. Model will be solved using CPLEX method for different distribution functions. Then, the objective function values for the selected layout calculated by changing coverage radius between 100 randomly generated numbers with distinct distributions. The results are compared with the classical model of gradual covering. The results show that the proposed model will provide the desired results for a possible covering radius.

Keywords: covering radius, gradual covering, stochastic models, normal and uniform distributions

1. Introduction

One of the most common facility location problems is the covering problem. It's applications in the real world, especially in emergency services makes researchers enthusiastic to research in this field. In covering problems customers often receive services or goods based on the distance between facility and customer. For example in a distribution network a demand point covers when is in a certain distance of a distribution center. This certain distance is called coverage radius. The purpose of these kind of problems is to determine the optimal location and number of facilities in order to service all customers or prepare maximum coverage for maximum number of customers with a predetermined number of facilities and the lowest possible cost. The first case is called the Set Covering Location Problem (SCLP) and the second one is called Maximum Covering Location Problem (MCLP) [1]. One important assumption in covering problems is zero-one covering. It means that a demand point is covered just inside of the coverage radius and is not covered outside of it [2]. In classical covering problems generally the assumption of zero-one covering is not applicable in most of real cases. This defect can be omitted with gradual covering approach which is done through defining of a partial covering function or covering rate function. Lots of applications have been found for these problems; for example in a distribution network, allocated products to each distribution center by the producer can be computed according to a gradual covering. Rate of satisfaction of post office customers can be considered as another example. The customers will be satisfied with a certain distance, though after that their satisfaction will gradually be decreased. Other applications for physical cases are location of warning sirens for emergency services, telecommunication towers, and internet access points. According to recent studies, in this paper we want to resolve the gradual covering problem in discrete place with a stochastic gradual coverage function that uses stochastic radius with specified distribution. The nature of this coverage function is mathematical expectation and we will show the advantages of this function compare to traditional coverage function

that called decay function. We solved the problem with some related parameters and considered in uniform, normal and exponential distributions of covering radiuses. We applied CPLEX solver to solve it which is an efficient method for discrete problems. The paper has been organized as following: in the next section, the literature of related approaches is reviewed. Then in section 3, the problem and structure of the model is discussed. After that the model is solved with both fixed and stochastic radius modes in discrete space and results are compared. Then in section 4 analyses of parameters are reported. Finally the conclusions are mentioned in the last section.

2. Literature review

The history of covering models is very rich and great. So we preferred to concentrate on a part that is more related to our approaches in this study. In this part we propose to introduce the studies have been done in the same fields that involves two different approaches. In order to review the literature of subject, it's necessary to review two related approaches separately.

2.1 Gradual covering

In 1983, Church & Roberts [3] introduced the gradual covering for the first time. They distributed a discrete model with a step-wise function. In this type of research for each facility, two types of coverage radius is defined; a radius with full coverage with r index and the other with partial coverage and index of R . Each demand inside the radius r ($d_{ij} \leq r$) will be fully satisfied. Demands between two radiuses ($r < d_{ij} \leq R$) will be satisfied partially. Demands out of radius R ($d_{ij} > R$) do not receive services at all [4]. This gradual covering calculates with a cover function which is positive and non-ascending called partial covering function or covering rate function. The proposed cover function produces values between zero and one according to distance between facility and demand point [4]. Berman & Krass [5] considered a network version of gradual covering problem and offered effective formulation and heuristic approaches. The model has been

analyzed in discrete space and in network with a non-ascending general decay function by Berman et al. [4]. The planar version with linear function has been discussed by Drezner et al. [6]. In 2004, O. Karasakal & E. Krasakal [7] studied gradual covering model and named it partial cover. Eiselt & Marianov [8] considered gradual covering as a set covering problem. They considered quality of service as decision criteria. They although formulated model in order to maximize the minimum probabilistic cover. They showed gradual covering models have more flexibility than standard MCLP. Different kinds of functions are proposed for covering. The most common function is linear cover function. For instance, problem in the planar with linear cover function has discussed in Drezner et al. [6]. They considered cover between R and r linear and changed it to the weber problem forcing a special cost structure. They considered servicing cost according to the covering decay function as well. Between these mentioned radiuses the cost is increased linearly. Then they analyzed the problem and solved it by using branch and bound method. Eiselt & Marianov [8] drew different kinds of cover functions. Church & Roberts [3] and Berman & Krass [5] used cover function stepwise with break points $D1$, $D2$, and $D3$ for explaining different levels of coverage. Pirkul & Schilling [9], Araz et al. [10] used quality of service function that is equal to cost of cover function of Drezner et al. [6]. Berman et al. [4] introduced formation of cover function which is not concave or convex. Introducing a gradual decline of cover, as like as fuzzy membership functions in fuzzy sets theory, these functions present a quantity of service quality in which zero indicates no service and one indicates full coverage. Pirkul & Schilling [9] optimized linear convex combination of coverage and weighted distance, which leads to an objective with a piecewise linear, linearly sloping function. They solved the problem by Lagrangian relaxation approach.

2.2 Stochastic problems

Stochastic approach in various issues is considered when problems in the real world contain uncertain parameters. In those location problems, several parameters including stochastic demand have been discussed so far. In 2004, Hwang [11] studied a special case of stochastic set covering problem for ameliorating and deteriorating facilities and determined minimum number of storage facilities between a discrete set of sites. So that the probability of each customer being covered is not less than a critical value. Then they formulated and solved the problem using integer programming. In 2010, Drezner et.al [12] presented a model in which inner and outer radiuses used in the gradual coverage were considered as random variables. Gradual covering models taking these assumptions present more realistic depiction of actual behavior in many situations. In 2011, Berman & Wang [13] discussed the gradual covering problem (GCP) when the weights of demand points are not deterministic and their probability distributions are unknown. They found the “minmax regret” location that minimizes the worst-case coverage loss and showed that under some conditions, the problem is equivalent to known location problems (e.g. the minmax regret median problem). In 2011, Berman et al. [14] analyzed the gradual covering location problem on a network with uncertain demand and in single facility state. They assumed two radii for each node and considered demand weights as discrete stochastic variables. They presented a model which locates facilities in order to maximize the probability of covered demand be greater or equal to a pre-determined threshold. In 2013, Amiri et al. [15] used a stochastic multi-objective programming under uncertainty for emergency services. In their study demands, purchase costs and transportation costs considered as uncertain parameters. In addition, the model considers uncertainty for locations where demand may increase and the risk of equipment damage in the event of a disaster relief centers exists.

In classic gradual covering model, full and partial coverage radius is considered to be fixed. But in reality, it is possible that the extent of covering radius for facilities be unknown because of environment conditions. In these cases,

covering problem could be considered in a random mode. Berman & Krass [5] suggested that deterministic cover from closer facilities assumed certain and definite. Drezner & Wesolowsky [16] and Drezner & Drezner [17] interpreted partial coverage as probability of coverage and based on this assumption, calculated combinational coverage when probabilities are independent. In 2008, Berman et al. [18] offered a covering problem in which covering radius of a facility is controlled by a decision maker and the cost of achieving to a certain covering radius is a uniform function of distance in which the cost of placing a facility depends on the distance between facility and the demand point. They considered both discrete and planar versions of the problem for solving the problem of covering all demand points with minimum cost through finding number and locations of facilities and optimal coverage radius for each facility by heuristic algorithms. In 2010, Drezner et al. [12] discussed the gradual covering radius when stochastic radiuses and individual coverage are examined. In their model it is assumed that coverage radiuses have probability distribution functions and the amount of coverage in certain distance d is calculated as a mathematical expectation of coverage radius distributions. The model solved in planar case by BTST (big triangle small triangle) algorithm which is an effective approach for many planar location models. Similar models also have been presented in [4, 5] which uses shortest distance between facilities and demand points on a network. In [5] it is assumed that the gradual covering function is a decreasing step wise function of the distance. This assumption cannot solve the problem of discontinuities well. In [4] the gradual covering expressed as a decreasing general function which is not necessarily linear. In 2013, Drezner et al. [19] discussed a cooperative gradual covering problem in the discrete and deterministic space. They assumed that received coverage from each facility is a stochastic variable with a normal distribution.

In this paper we model gradual covering location problem with different type of distribution functions for radiuses using Drezner et al. [12] suggested covering function. To have a better conclusion about related works, previous studies have been summarized as Table1.

Author's name	Specification			
Church et al. [3]	Stochastic radius□	Planar□	Discrete■	Gradual■
Church et al. [20]	Stochastic radius□	Planar□	Discrete□	Gradual■
Pirkul et al. [9]	Stochastic radius□	Planar□	Discrete■	Gradual■
Drezner et al. [16]	Stochastic radius□	Planar■	Discrete□	Gradual■
Berman et al. [2]	Stochastic radius□	Planar□	Discrete□	Gradual■
Drezner et al. [6]	Stochastic radius□	Planar□	Discrete□	Gradual■
Karaskal et al. [7]	Stochastic radius□	Planar□	Discrete■	Gradual■
Araz et al. [10]	Stochastic radius□	Planar□	Discrete□	Gradual■
Eiselt et al. [8]	Stochastic radius□	Planar□	Discrete□	Gradual■
Berman et al. [21]	Stochastic radius□	Planar□	Discrete□	Gradual■
Drezner et al. [12]	Stochastic radius■	Planar■	Discrete□	Gradual■
Berman et al. [2]	Stochastic radius■	Planar□	Discrete□	Gradual■
Berman et al. [22]	Stochastic radius□	Planar□	Discrete■	Gradual□
Berman et al. [13]	Stochastic radius□	Planar□	Discrete□	Gradual■
Drezner et al. [19]	Stochastic radius□	Planar□	Discrete■	Gradual■
The present paper	Stochastic radius■	Planar□	Discrete■	Gradual■

Tab. 1: The summary of previous studies with different characteristics

3. Problem statement

Consider a set of N facilities is provided. In the maximum covering location problem, we look for a set of P facilities ($P \in N$) so that the total facility coverage should be maximized. If the coverage radius for each facility supposed to be a random variable with a specific distribution, therefore, total coverage received from a facility (i.e., physical signal strength and the light radius emitted from facility) should be calculated per the average of received coverage.

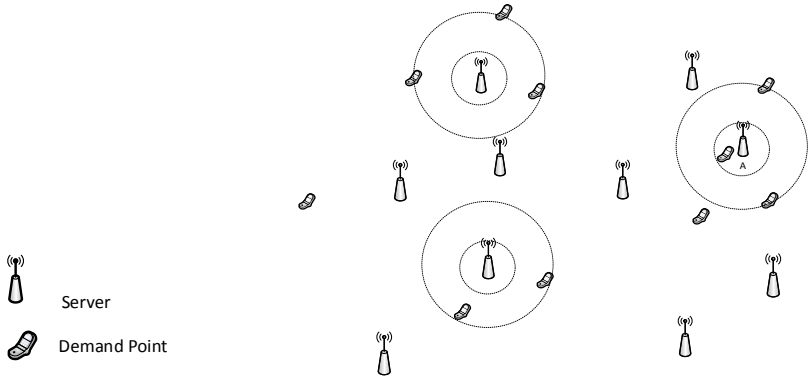


Fig. 1: The layout situation of a telecom service network

Figure 1 schematically shows the layout situation of a telecom service network. In that layout, three candidates out of the 10 candidates are selected for locating of telecommunication towers. Each telecommunication antenna provides maximum possible signal to the radius r and then signal strength is reduced gradually to the radius R . The following figure shows a network in situation where covering radiuses are changing with a probability distribution.

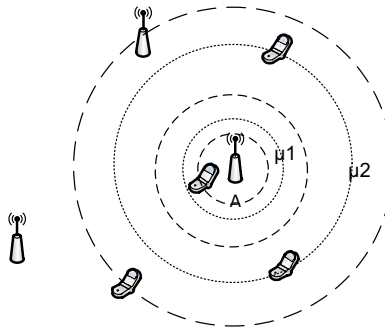


Fig. 2: A candidate tower with stochastic covering radius

As can be seen in Figure 2, in this case the demand placed anywhere within a distance between covering radiuses r and R , has a certain probability.

For example suppose that the signal power is changed as a random variable. Drezner et al. [12] suggested covering function as the expected coverage per various radii. In the gradual covering function, the amount of covering between r and R generally depends on the distance. According to [12] the coverage received from a facility can be calculated as follows:

$$cover(d) = \begin{cases} 1 & d \leq r \\ \frac{R-d}{R-r} & r \leq d \leq R \\ 0 & d \geq R \end{cases} \quad (1)$$

Therefore, the cover at distance d , and for distribution function of cover radii $c(d)$ is:

$$c(d) = E(cover(d)) = \iint_{0 \leq r \leq R \leq \infty} cover(d) f_{r,R}(y, z) dz dy \quad (2)$$

So, if we assume that there is no dependency between the covering radius distribution functions $f_{r,R}(y, z) = \phi_r(y) \phi_R(z)$ (Assuming full independence is somewhat inaccurate). Considering the coverage function, equation (1), equation (2) would be as follows:

$$c(d) = \int_d^\infty \int_0^\infty 1 \times \phi_r(y) \phi_R(z) dz dy + \int_0^d \int_d^\infty \frac{z-d}{z-y} \phi_r(y) \phi_R(z) dz dy \int_d^\infty \int_0^\infty 0 \times \phi_r(y) \phi_R(z) dz dy = P_r(r \geq d) + \int_0^d \int_d^\infty \frac{z-d}{z-y} \phi_r(y) \phi_R(z) dz dy \quad (3)$$

Equation (3) is the function suggested by Drezner [12] to calculate cover. Hereinafter, the equation (1) will be called 'decay function'.

3.1 Notation

Let:

- i Index for the set of candidate facilities for locating
- j Index for the set of demand points (customers)
- r Inner covering radius. The facility inside of mentioned radius can be fully covered
- R Outer covering radius. The facility outside of mentioned radius will be uncovered
- $C(d_{ij})$ The proportion of coverage which facility i prepares for customer j in distance d_{ij}

W_j	The demand in point j
g_i	The cost of locating facility i
S	The adjustment factor to equal the cost unit and demand weight
X_i	Is one if facility locates on site i and zero otherwise
Y_{ij}	Is one if point j receives some services from i and zero otherwise

3.2 Model formulation

Given the above assumptions, the model is as follows:

$$\max z = S \sum_{j=1}^n \sum_{i=1}^n W_j \cdot c(d_{ij}) \cdot y_{ij} - \sum_{i=1}^n g_i x_i \quad (4)$$

$$y_{ij} \leq x_i \quad \forall i, j \quad (5)$$

$$\sum_{i=1}^n y_{ij} \leq 1 \quad \forall j \quad (6)$$

$$\sum_{i=1}^n x_i \leq p \quad (7)$$

$$x_i, y_{ij} \in \{0,1\} \quad \forall i, j \quad (8)$$

Objective function is composed of two parts. In the first part, the benefit of covering of all demand points is calculated and in the second part, the costs associated with the facility location are put forward. The objective function looks for maximizing net benefits. In this type of objective function, coverage radiuses with certain distribution functions are involved and unlike other cover functions, possible value of cover is calculated through the mathematical expectation. In all gradual covering models with cover function, value $c(ij)$ is considered proportional to the distance of facility to service. Constraint (5) implies that each demand point can only be established with the covered facility. Constraint (6) each facility will be assigned only once to each demand point. Constraint (7) indicates that the number of located facilities is certain. Constraint (8) expresses the variables are binary.

3.3 Numeric solution of the model

In this section, the model for covering radius with various distribution functions will be solved, and then the results will be analyzed.

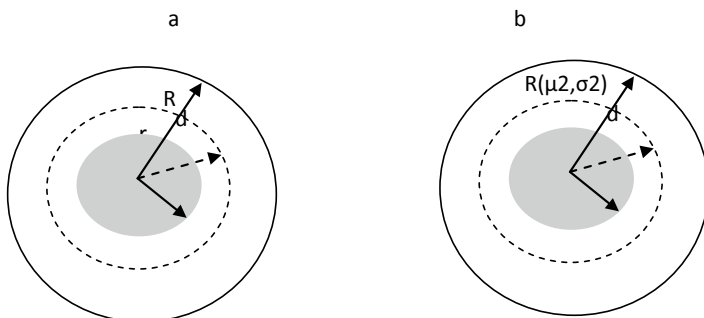


Fig. 3: Coverage area with fixed and stochastic radiuses

In Figure 3, the shaded area indicating that the region is fully covered. Figure 3-a reveals the general state of gradual covering problem. Figure 3-b coverage radiuses have certain distribution functions. d is the distance between each demand point and the server facility.

To check the applicability and validation of the proposed model, random data for the covering radius were generated in three states. In the first state data were generated according to the uniform distribution while in the second and third states, data were generated according to the bivariate normal and exponential distributions respectively. Generated data have been plotted in figure 4.

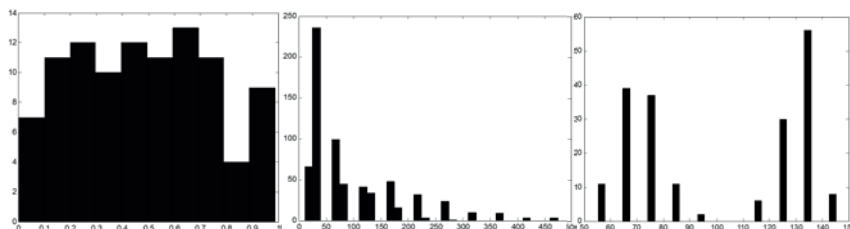


Fig. 4: Distribution function plots for randomly generated data with different distribution functions

The problem was solved twice. In the first one the problem was solved with specific covering radius which is one of the generated data and is called decay model, then in the second one the problem with expected coverage is solved which is called mathematical expectation. According to the network structure extracted by two models, the objective function values for all 100 random generated covering radiuses were computed according to two different network structures. The total differences between the two calculated objective function values (deviation) are calculated which is reported in Table 2. In the Table 2, μ_1 is the average of distribution of coverage radius r , σ_1 is the variance of distribution of coverage radius r . μ_2 is the average of distribution of coverage radius R , σ_2 is the variance of distribution of covering radius R . β is the parameter of exponential distribution. r and R are the inner and outer coverage radiuses at a fixed radius model. ρ is the correlation coefficient of normal distribution and deviation is the total difference in objective function values in 100 times of calculation. These differences are calculated through subtracting the cover function value of mathematical expectation from decay cover function, so positive values indicate better cover function values of mathematical expectation. The percentage indicates the contribution of positive differences out of total amount of differences between positive and negative values.

<i>Distribution</i>	μ_1	σ_1	μ_2	σ_2	ρ	β	r	R	<i>Deviation</i>	<i>Percentage</i>
<i>Uniform</i> $a_1=a_2=\mu_1-\sigma_1$ $=\mu_2-\sigma_2$ $b_1=b_2=\mu_1+\sigma_1$ $=\mu_2+\sigma_2$	100	30	100	30	-	-	70	130	6550.813	80 %
<i>Normal</i> <i>(dependent variables)</i>	70	30	130	50	-	-	70	130	26592.85	100 %
<i>Normal</i> <i>(independent)</i>	70	30	130	50	0.6	-	70	130	19894.57	100 %

<i>Distribution</i>	μ_1	σ_1	μ_2	σ_2	ρ	β	r	R	<i>Deviation</i>	<i>Percentage</i>
<i>Exponential</i>	-	-	-	-	-	100	70	130	960.720	64 %

Tab. 2: Performance comparison of decay and proposed model in a stochastic covering radius environment

Results in Table 2 shows that in the state of uncertainty and with stochastic covering radius, using estimated fixed radiuses amounts cannot be efficient enough. Moreover the proposed model performs better than the classic model in an uncertain environment. In the next part we are seeking to show how important parameters can effect on the problem. We do experiments with two more common normal and uniform distributions.

4. Sensitivity analysis of parameters for both Normal and Uniform distributions

In this section, by changing the parameters of the normal and uniform distributions the effect of changes can be analyzed. It's important to remind that in this comparison, differences are calculated through subtracting the cover function value of mathematical expectation from decay cover function. The tested values have been tuned according to other input parameters such as distance between nodes to be more realistic. Each test case has been evaluated about 500 to 800 times.

4.1 Uniform distribution with a mean and variance change

In the following graphs vertical axis shows summation of differences between objective function with decay cover function and cover function of mathematical expectation.

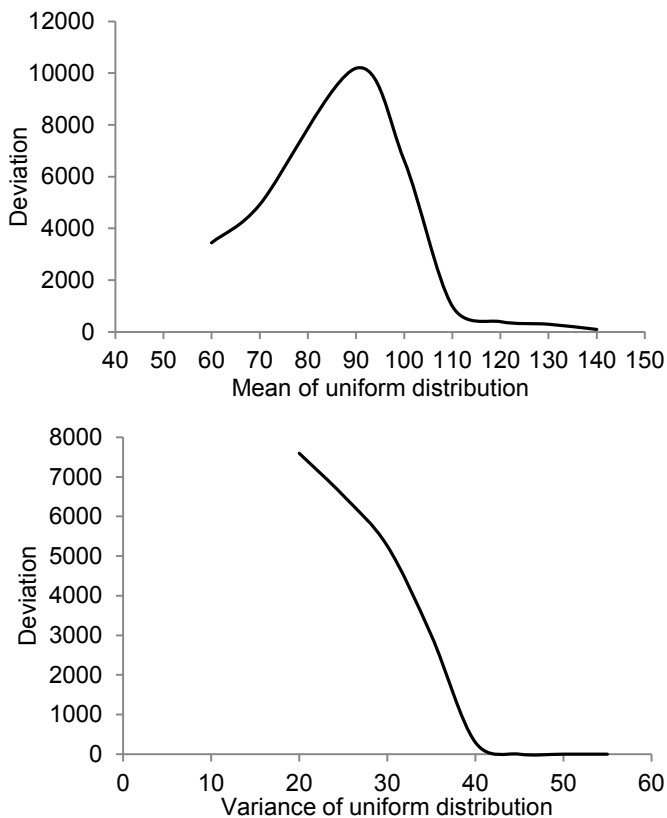


Fig. 5: Effect of changing mean and variance of radius with uniform distribution

Figure 5 shows deviation changes by increasing mean and variance of distribution. In the left hand side chart we increase both stochastic radius mean and the radiuses of decay model. Considering distance matrix, the maximum of distances between demand points and distribution centers is about 140. So according to following relation we expect sort of indifferentness does happen for deviation:

$$\text{Max}\{d(i,j)\} \leq b1 (=b2 = \mu1 + \sigma 1 = \mu2 + \sigma 2)$$

The trend as our expectation limits to zero too.

The lower chart shows a decreasing trend by increasing of variance. In this state we increase the variance of radiuses distribution in stochastic model and radiuses of decay model. It seems that the trend limits to zero about 140, witch with 100 for mean, it's happen about 40. To have a better conclusion we experimented the similar changing in the state that radiuses of decay model don't change. We fixed them on 70 and 130 and changed mean and variance of stochastic model. We expect after 70 and 130 see an increasing trend. As it appears in Figure 6 in upper part for changing of mean, there is some undulation around 100 where two means over lap and after that the trend grows up again. The analysis for variance has a little more complexity. As it seems in upper chart in Figure 6 before lower limit of decay model (it is 70) there is even negative amounts for deviation. Then we see an increasing trend with a fast slop. The summit of chart is the place that radiuses of both models overlap. After it, the trend goes down and about 140 for upper bound it starts to produce negative amounts again.

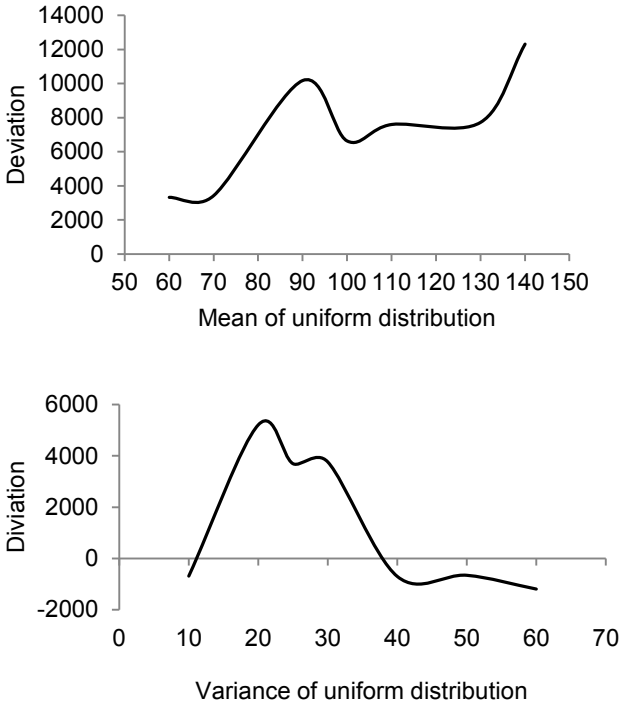


Fig. 6: Effect of changing mean and variance of radius with uniform distribution when the decay model radiuses don't change

4.2 Normal distribution with mean and variance change of outer radius/R

The charts in Figure 7 are results of changing both radiuses of classic model and stochastic radius model amounts. In upper chart when the outer radius for both modes increase, the deviation at first have an increase with a fast slop. After 100 we see very little changes in trend. It is the place where is closing to $\mu+z \alpha/2 \times \sigma$ which is upper bound in normal distribution. But generally with increasing outer radius we expect an increasing behavior, because the space of gradual coverage is getting bigger. Deviation changes chart reveals a decreasing trend with a

downfall in the middle. With a big variance for outer radius the probability of having very small coverage radius sustains. So it seems naturally we face a decreasing trend.

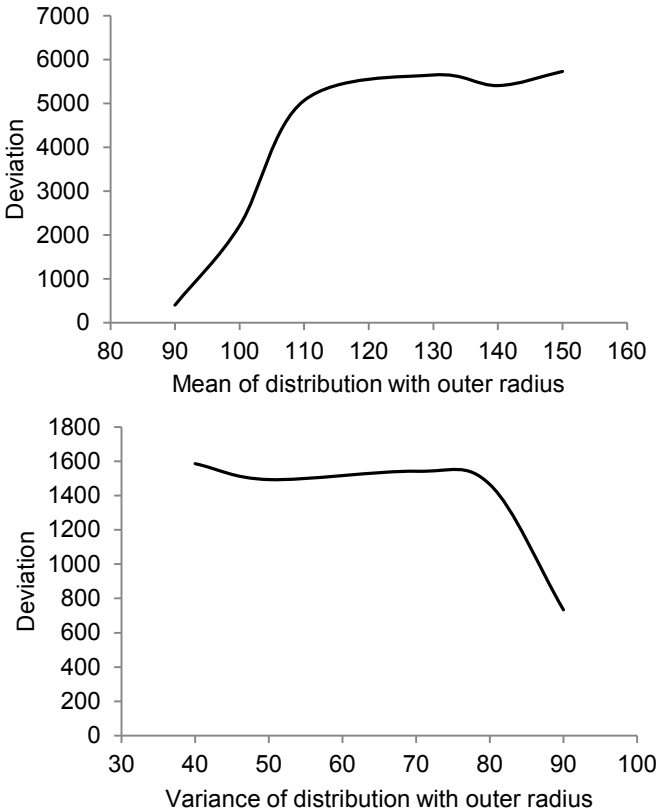


Fig. 7: Effect of changing mean and variance of radiuses with normal distribution

There is no discussable trend for inner radius changes, so we prefer to change parameters for both radiuses simultaneously.

4.3 Simultaneous changes in both the mean and variance within normal

In this part we examine simultance changing of mean and variance of both radiuses.

In Figure 8 the vertical axes don't have real numbers. It has omitted in order to shows the changes of both parameters with different values. In the upper chart both means of r and R radiuses have been increased, so the summation of variance between objective function with decay cover function and cover function of mathematical expectation has been increased too. It is obvious that with increasing mean of radiuses the coverage will get bigger and more demand points will cover. This event is more effective about stochastic radiuses, because variance of distribution can help too. So having an incremental trend is not out of mind. But there is a summit that shows a part of trend is under effect of inner radius changing.

In the lower chart, changing in trend is very smooth. When the variance of outer and inner radiuses change together, it is hard to say what exactly will happen. Because changing variance can cause to growing up the probability of having inner and outer radiuses. With all that, as trend shows, the resultant of these changes has a positive effect on deviation.

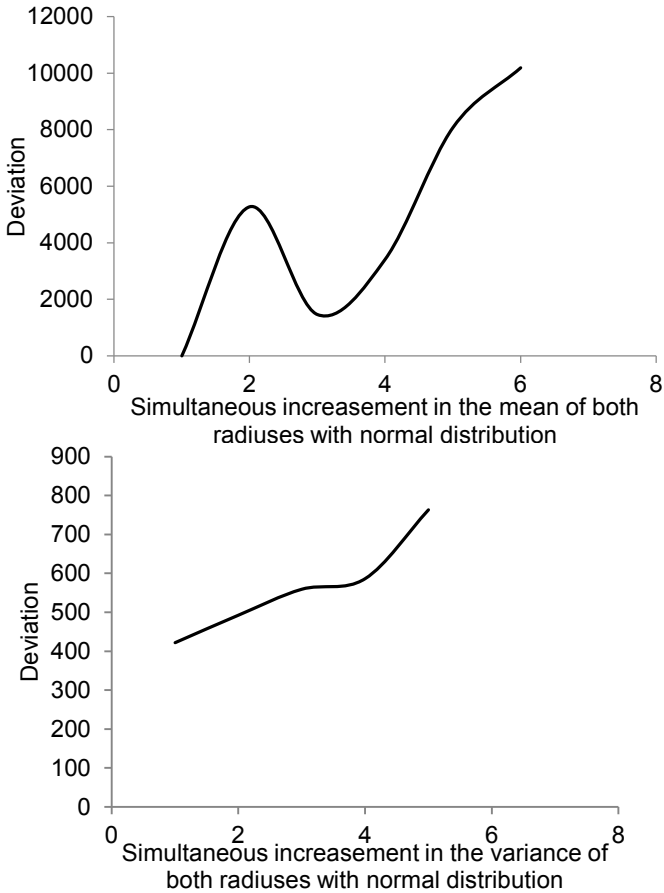


Fig. 8: Simultaneous changes in both the mean and variance within normal

4.4 Sensitivity analysis table

In Table 3 the observed results of parameter sensitivity are summarized. It is assumed that the testes parameter values are rising. Cells of the table represent the status of variance of objective function values regard to changing the

parameter. The term Incremental points to an increasing trend and Decline points to a decreasing trend.

Distribution Type	Mean of r & R	Variance of r & R	Mean of R (outer radius)	Variance of R (outer radius)
Uniform	Decline	Decline	-	-
Normal	Incremental	Incremental	Incremental	Decline

Tab. 3: Summary of different sensitivity analysis results

5. Conclusions

In this paper, we reviewed gradual cover models that have been proposed so far. Then, we studied a type of problem in which covering radius change randomly. Using Covering function proposed in [12] and with entering the cost parameters, we studied location problem in various modes of distribution functions and in discrete space. In the real word we face some uncertainty in problems and ignoring most of the states of problem is not reasonable. Covering location problem with stochastic radiuses has attended to all possible states of the radiuses with considering average of possible values. We produced different scenarios for radiuses and tried to give solution of both stochastic and deterministic models to calculate objective function values. According to reported results when the radiuses are not fixed, stochastic radius model is more efficient for the problem. In sensitivity analysis part we examined changes of parameters effect for normal and uniform cases, that are very common distributions of natural events behavior. The results show when the distribution mean of radiuses are big the stochastic radius model is more effective than the classic model. As a future research considering the hierarchical covering problem in stochastic environment is proposed.

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