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#### Abstract

Suggested Citation: Jensen, Mads Vestergaard (2016) : Financial Frictions: Implications for Early Option Exercise and Realized Volatility, PhD Series, No. 46.2016, ISBN 9788793483576 , Copenhagen Business School (CBS), Frederiksberg, https://hdl.handle.net/10398/9386


This Version is available at:
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ISSN 0906-6934


# Financial Frictions: Implications for Early Option Exercise and Realized Volatility 

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Financial Frictions:
Implications for Early Option Exercise and Realized Volatility

1st edition 2016
PhD Series 46.2016
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ISSN 0906-6934
Print ISBN: 978-87-93483-56-9
Online ISBN: 978-87-93483-57-6


#### Abstract

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## Preface

This PhD thesis comprises three chapters, all of which deal with financial frictions, especially short-sale costs. The first two chapters also concern early exercise of equity call options. Although chapter two builds on results from chapter one, each chapter can be read independently.

I would like to acknowledge support from the FRIC Center for Financial Frictions (grant no. DNRF102), the European Research Council (ERC grant no. 312417), the Elite Research Prize awarded to Lasse Heje Pedersen, and the Danish Agency for Science, Technology and Innovation. Many people deserve thanks. First, my colleagues at the Department of Finance at Copenhagen Business School provided invaluable comments, input, and conversations, though my primary supervisor, Lasse Heje Pedersen, deserves special recognition for his keen academic insights, kind encouragement, and constant willingness to discuss new ideas. Working with him has been an enormous privilege. I am also grateful to my secondary supervisor, Jesper Lund, for his help and contributions. I would also like to thank my fellow PhD students for their ongoing reflections and important coffee breaks. Finally, I would like to express my gratitude to family and friends for their support throughout my studies.

## Chapter Summaries in English and Danish

This section contains English and Danish summaries of the three chapters in the thesis.

## Chapter 1: Early Option Exercise: Never Say Never

English summary. An equity call option grants the owner the right, but not the obligation, to buy a specific stock at a specific price no later than a specific expiration date. A classic result by Merton (1973) is that, in the absence of frictions, an equity call option should never be exercised early, only at expiration or just before the stock pays a dividend. Similarly, a convertible bond should never be converted early (Brennan and Schwartz, 1977; Ingersoll, 1977). We show that when frictions are severe enough, these rules breaks down, both theoretically and empirically. Not just any friction, however, makes early exercise optimal. Option and stock transaction costs alone cannot make early exercise optimal, nor can funding costs and short-sale costs only affecting the option owner. However, once option transaction costs are combined with funding and/or short-sale costs, early exercise can be optimal.

We extend the classic Black-Scholes-Merton model to include both funding costs and shortsale costs. Applying this model to data on stock prices, short-sale costs, and funding costs makes it possible to estimate when early exercise can be optimal. Data on call option exercises allow us to test if the estimates match observed behavior. The model is able to explain $66-84 \%$ of all early exercise decisions in our data, depending on how input variables are estimated. Statistical tests in the form of regressions indicate that the model estimates have explanatory power for early option exercise. We also document a number of early conversions of convertible bonds, especially among bonds on stocks with high short-sale costs.

## Kapitel 1: Førtidig optionsudnyttelse: Aldrig sig aldrig

Dansk resumé. En købsoption på en aktie giver dens ejer retten, men ikke pligten til at købe en bestemt aktie til en bestemt pris senest en bestemt dato. Et klassisk resultat af Merton (1973) siger, at i fravær af friktioner skal man aldrig udnytte sin option førtidigt, kun ved udløb eller lige før aktien betaler udbytte. Tilsvarende skal en konvertibel obligation aldrig konverteres førtidigt (Brennan and Schwartz, 1977; Ingersoll, 1977). Vi viser, at når friktionerne er strenge nok, bryder denne regel sammen, både teoretisk og empirisk. Men ikke enhver friktion kan gøre førtidig udnyttelse optimal. Options- og aktietransaktionsomkostninger alene kan ikke gøre førtidig udnyttelse optimal. Det samme gælder for finansieringsomkostninger eller kortsalgsomkostninger, der alene vedrører optionsejeren. Men hvis optionstransaktionsomkostninger er kombineret med finansierings- og/eller kortsalgsomkostninger, kan førtidig udnyttelse være optimal.

Vi udvider den klassiske Black-Scholes-Merton-model til at inkludere både finansierings- og kortsalgsomkostninger. Ved at anvende denne model på data for aktiekurser, kortsalgsomkostninger og finansieringsomkostninger er det muligt at estimere, hvornår førtidig udnyttelse kan være optimal. Data for købsoptionsudnyttelser gør det muligt at undersøge, om estimaterne afspejler den observerede adfærd. Modellen er i stand til at forklare $66-84 \%$ af alle førtidige udnyttelser i vores data, afhængigt af hvordan inputvariablene er estimerede. Statistiske test i form af regressioner viser at modelestimaterne har forklaringskraft for førtidig optionsudnyttelse. Vi påviser desuden et antal førtidige konverteringer af konvertible obligationer, især blandt obligationer på aktier med høje kortsalgsomkostninger.

## Chapter 2: Early Option Exercise Predicts Stock Returns

English summary. If a call option owner wants to reduce exposure to the underlying stock, this might lead to early option exercise. In case the wish to reduce stock exposure is based on private information, the exercise will be informative about future stock returns. This chapter shows that after early exercise, the underlying stocks do in fact underperform relative to the rest of the market, consistent with private information leading to early exercise.

Exercising a call option and selling the obtained stock reduces the stock exposure. If a wish
to reduce stock exposure confronts the option owner with the choice of either exercising early or starting to sell the stock short, high costs related to short selling can make early exercise optimal. Shorting can be costly both due to short-sale costs and the funding costs associated with increased margin requirements. When private information is the reason for reducing stock exposure and hence choosing early exercise, the exercise will be a negative predictor of stock returns.

I perform various tests to determine the empirical link between early exercise and consecutive stock returns, including: event studies; tests of returns for strategies selling stocks after early exercise, and regressions of returns on indicators for previous early exercises. The results show that early exercise is a negative predictor of stock returns, which is consistent with private information among option owners leading to early exercise. According to my findings the information content of early exercise does not significantly depend on whether the exercise is performed by a customer of a broker, firm, or market maker.

## Kapitel 2: Førtidig optionsudnyttelse forudsiger aktieafkast

Dansk resumé. Hvis en købsoptionsejer ønsker at reducere sin eksponering mod den underliggende aktie, kan det føre til førtidig optionsudnyttelse. I tilfælde af at grundlaget for ønsket om at reducere aktieeksponeringen er privat information, vil optionsudnyttelsen være informativ om fremtidige aktieafkast. Kapitel 2 viser, at underliggende aktier rent faktisk underpræsterer i forhold til resten af markedet, hvilket er i overensstemmelse med, at privat information fører til førtidig udnyttelse.

At udnytte en købsoption og sælge den opnåede aktie reducerer aktieeksponeringen. Hvis et $\varnothing$ nske om at reducere aktieeksponeringen stiller optionsejeren over for valget mellem enten at udnytte førtidigt eller at begynde at sælge aktien kort, kan høje omkostninger, der relaterer sig til kortsalg, gøre førtidig udnyttelse optimal. Kortsalg kan være omkostningsfyldt både på grund af kortsalgsomkostninger og på grund af finansieringsomkostninger i forbindelse med krav om forøget sikkerhedsstillelse. Når privat information er årsagen til at reducere aktieeksponeringen og dermed valget om førtidsudnyttelse, vil optionsudnyttelsen være en negativ retningsgiver for aktieafkast.

Jeg udfører forskellige undersøgelser for at fastslå den empiriske forbindelse mellem førtidig udnyttelse og efterfølgende aktieafkast, inklusive: eventstudier; test af afkast for strategier, der sælger aktier efter førtidig udnyttelse; regressioner af afkast på indikatorer for forudgående førtidige udnyttelser. Undersøgelserne viser, at førtidig udnyttelse er en negativ retningsgiver for aktieafkast, hvilket er konsistent med, at privat information blandt optionsejere fører til førtidig udnyttelse. Ifølge mine resultater afhænger informationsindholdet i førtidig udnyttelse ikke signifikant af, om udnyttelsen er foretaget af en kunde hos en mægler, en virksomhed eller en prisstiller.

## Chapter 3: Short-Sale Costs Predict Volatility

English summary. Short selling allows investors to borrow and sell stocks they do not own and to earn a profit if stocks decrease in value. In the shorting market, borrowers and lenders negotiate the fee that the borrower must pay to borrow a particular stock, i.e., the short-sale costs. An increased demand for short selling a stock can reflect an increased disagreement among market participants about the value of the stock. The increased disagreement can also induce higher volatility in future stock returns.

As a new test of models of differences of opinions, we study how shorting markets interact with stock volatility. We identify positive and negative demand shifts for shorting stocks by examining simultaneous changes in short-sale costs and short interest. Consistent with an increase in differences of opinions, a positive demand shift predicts, on average, a 2.8 percentage points higher volatility over the next quarter. Similarly, an average negative demand shift predicts a 3.9 percentage points lower volatility.

We perform event studies to investigate how the volatilities of stocks behave both before and after supply and demand shifts. The studies show that average volatilities of stocks hit by supply or demand shifts are higher than for other stocks, and volatilities peak around the time of shifts. Furthermore, we find that future volatility is positively related to both increases and the level of short-sale costs and short interest.

## Kapitel 3: Kortsalgsomkostninger forudsiger volatilitet

Dansk resumé. Kortsalg tillader investorer at låne og sælge aktier, de ikke ejer, og tjene profit, hvis aktier falder i værdi. I kortsalgsmarkedet forhandler lånere og udlånere den afgift, som låneren skal betale for at låne aktien, dvs. kortsalgsomkostningerne. En øget efterspørgsel på kortsalg af en aktie kan afspejle en forøget uenighed blandt markedsdeltagerne om værdien af aktien. Den forøgede uenighed kan også føre til højere volatilitet i fremtidige aktieafkast.

Som en ny test af modeller med uenighed undersøger vi, hvordan kortsalgsmarkeder interagerer med aktievolatilitet. Vi identificerer positive og negative efterspørgselsforskydninger for kortsalg af aktier ved at finde samtidige ændringer i kortsalgsomkostninger og andelen af udestående aktier, der er udlånt. I overensstemmelse med en forøget uenighed forudsiger en positiv efterspørgselsforskydning i gennemsnit en 2.8 procentpoint højere volatilitet over det næste kvartal. Tilsvarende forudsiger en negativ efterspørgselsforskydning en 3.9 procentpoint lavere volatilitet.

Vi udfører eventstudier for at undersøge, hvordan aktievolatiliteterne opfører sig både før og efter udbuds- og efterspørgselsforskydninger. Studierne viser, at den gennemsnitlige volatilitet for aktier ramt af udbuds- eller efterspørgselsforskydninger er højere end for andre aktier, og at volatiliteterne topper omkring forskydningstidspunktet. Endvidere viser vi, at fremtidig volatilitet er positivt relateret til både stigninger i og niveau af kortsalgsomkostninger samt hvor mange aktier, der er solgt kort.

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## Introduction

A call option on a stock is a common and widely used derivative. On an average trading day in 2015, more than 800,000 such options traded on the Chicago Board Options Exchange, the largest options exchange in the United States. Each option grants its owner the right to buy 100 of a specific stock at a pre-specified price, no later than a pre-specified date. For example, an option can grant the right to buy 100 General Electric shares for USD 31 each no later than October 21, 2016. An interesting issue is determining when an option is optimally exercised. Merton (1973) shows that in a world without frictions, a call option should never be exercised early, but only at expiration or just before the underlying stock pays a dividend. Chapter one of this thesis shows that sufficiently severe frictions can make early exercise optimal. Short-sale costs especially represent an important driver of early exercise. Chapter two shows that when option owners exercise early, it predicts stock returns, consistent with option owners acting on private information. Chapter three does not include options but shows that demand shifts in the shorting market for stocks predict the volatility of the affected stocks, which is consistent with increases in differences of opinions among market participants.

Chapter one (with Lasse Heje Pedersen) deals with a classic result by Merton (1973), who states that, in the absence of frictions, early exercise is never optimal. In a similar result, Brennan and Schwartz (1977) and Ingersoll (1977) show that a convertible bond should never be converted early. We show that frictions can in fact make early exercise optimal and we provide empirical evidence of early exercise induced by frictions. In particular, we consider the following frictions: short-sale costs, funding costs, and transaction costs.

The existing literature already links option prices to the aforementioned frictions. For example,

Christoffersen, Goyenko, Jacobs, and Karoui (2011) show that equity option prices are affected by transaction costs, while (Brenner, Eldor, and Hauser, 2001) provide a similar result for currency options. Both Ofek, Richardson, and Whitelaw (2004) and Avellaneda and Lipkin (2009) link short-sale costs to equity option prices, with the latter mentioning that short-sale costs can lead to early exercise. Another topic of various studies (Bergman, 1995; Santa-Clara and Saretto, 2009; Leippold and $\mathrm{Su}, 2015$ ) is the connection between funding constraints and option prices. Frazzini and Pedersen (2012) show how embedded leverage is a valuable component of options, while Piterbarg (2010) and Karatzas and Kou (1998) provide a link between interest-rate spreads and option prices. The literature also links convertible bond prices to financial frictions (Mitchell, Pedersen, and Pulvino, 2007; Agarwal, Fung, Naik, and Loon, 2011). Chapter one complements the literature on how frictions affect option prices by showing that frictions also lead to option exercises.

Regarding option exercise, various papers document irrational early exercise decisions (Gay, Kolb, and Yung, 1989; Diz and Finucane, 1993; Overdahl and Martin, 1994; Finucane, 1997; Poteshman and Serbin, 2003), while Pool, Stoll, and Whaleyn (2008) provide evidence of irrational failures of exercise for call options and Barraclough and Whaley (2012) do so for put options. Finally, Battalio, Figlewski, and Neal (2015) document early exercise of call options with options bid prices that are below the intrinsic value, which is a necessary condition for optimal early exercise. We show how frictions help explain why the option prices can be so low.

The chapter contains the following steps to show how early exercise is induced by frictions, both theoretically and empirically. We provide the theoretical result that transaction costs alone cannot justify early exercise, nor can short-sale costs and funding costs that only affect the option owner but leave other market participants unconstrained. If, however, the option cannot be sold above intrinsic value and funding costs and/or short-sale costs affect the option owner, early exercise can be optimal.

Next, we extend the classic Black-Scholes-Merton model (Black and Scholes, 1973; Merton, 1973) for option pricing to include funding costs and short-sale costs, allowing us to give quantitative estimates of when early exercise can be optimal by combining large data sets. Using Fama-

MacBeth regressions, we show that frictions have explanatory power for early exercises. Our model explains $66-84 \%$ of all early exercises, depending on how input variables are estimated. We also use the temporary short-sale ban in September 2008-October 2008 of selected stocks as a natural experiment. A difference-in-differences test shows that options written on stocks for which short selling is banned are exercised with a higher propensity than other options, consistent with our theoretical prediction. We also document early conversions of convertible bonds, especially among bonds on stocks with high short-sale costs.

Chapter two shows that early exercise of call options predicts stock returns negatively, consistent with option owners exercising based on private information. This chapter builds on the insight from chapter one that early exercise can be optimal if frictions are sufficiently severe. In particular, it utilizes the model from chapter one to show how a wish to reduce stock exposure can lead to early exercise. If reducing stock exposure by selling stock requires the option owner to start shorting the stock, frictions related to shorting can make it optimal to exercise the option early and sell the stock instead. If the wish to reduce stock is based on private information, early exercises will predict future stock returns negatively. The chapter provides empirical evidence that early exercise is a negative predictor of stock returns.

A vast literature exists on private information and option markets. Black (1975) conjectured that traders with private information are more prone to trade options due to embedded leverage. To name a few others, Easley, O'Hara, and Srinivas (1998) derive an equilibrium in which option trades are informative about future stock prices, and provide consistent empirical evidence. Based on observations of buyer- and seller-initiated trades, Pan and Poteshman (2006) show how option trades predict stock returns. This result is closely related to my results, with the main difference being that I focus on option exercises instead of option trades. Kacperczyk and Pagnotta (2016) use data from cases on insider trading to show that option-based measures for private information are stronger than stock-based measures. Johnson and So (2012) provide evidence that the option to stock volume ratio is informative about future returns. Fodor, Krieger, and Doran (2011) show that recent changes in the call-to-put open-interest ratio have predictive power regarding equity returns.

To my knowledge, chapter two distinguishes itself from the previous literature by linking early
exercise to future stock returns and private information. The private information content of early exercise is detected by lower abnormal returns of the underlying stocks in the days after early exercise. I identify abnormal stock returns based on both a one-factor capital asset pricing model and a four-factor model with the three Fama-French factors (excess market returns, small market capitalization minus big, and high book-to-market ratio minus low) and the momentum factor. Returns on day of exercise and the day after potentially reflect public information that induced the early exercise. To avoid endogeneity issues, I use abnormal returns from two trading days after early exercise and later as evidence of private information.

Event studies show that the cumulated four-factor abnormal stock return for two to ten trading days after early exercise is significantly negative, consistent with option owners exercising based on private information. The corresponding one-factor abnormal return is negative, but insignificant. The four-factor abnormal return stays significantly negative, even after adding a conservative (i.e., likely upward-biased) measure for short-sale costs. This is consistent with negative information giving rise to early exercise. Trading strategies based on shorting stocks after an option written on them has been exercised early also indicate private information. For the strategy in which stocks are shorted two trading days after early exercise and the position hedged based on ex ante four-factor loadings, a positive four-factor daily alpha of $0.05 \%$ is observed with a $t$-statistic of 3.38. The yearly Shape ratio is 1.26 . I also perform Fama-MacBeth regressions and show that early exercise two and three trading days prior are negatively related to abnormal returns of the underlying stock.

I also test if the private information content of the average early exercise depends on which type of agent (customer of broker, firm, or market maker) exercises, though without finding statistically significant differences. In addition, I show that the results are driven by stocks with high or low market capitalization only. Furthermore, I find that stock reversal cannot explain the abnormal negative returns after early exercise. The empirical results provide evidence that early exercise predicts stock returns because option owners exercise early based on private information.

In the last chapter (with Christian Skov Jensen), we investigate the relationship between the shorting market and stock volatility. The main prediction of the paper connects two strings of the existing literature: (i) increases in differences of opinions lead to a positive demand shift in the
shorting market (Duffie, Gârleanu, and Pedersen, 2002), and (ii) higher differences of opinions lead to higher stock volatility (Andrei, Carlin, and Hasler, 2014; Chang, Cheng, and Yu, 2007). We combine these insights into the prediction that a positive demand shift in the shorting market will reflect higher future predicted volatility for the affected stock.

The rich data set from Data Explorers allows us to identify both supply and demand shifts in the shorting market. Inspired by Cohen, Diether, and Malloy (2007), we identify, e.g., a positive demand shift when we observe a simultaneous increase in short-sale costs and short interest. We can then match supply and demand shifts with stock volatility from OptionMetrics and investigate their relationship. In our regression results, we find evidence consistent with the prediction that positive (negative) demand shifts are positive (negative) predictors of volatility, reflecting that an increase (decrease) in differences of opinions can lead to future higher (lower) volatility.

Various papers link short-sale constraints and volatility. Bai, Chang, and Wang (2006), for instance, predict that short-sale constraints increase stock volatility when the information asymmetry is high. Avellaneda and Lipkin (2009) show how short-sale costs and constraints can lead to short sellers being forced to close their short positions and to increased volatility. Chang et al. (2007) find empirically that volatility of the individual stock increases when short selling for the stock is allowed. Based on yearly estimates of stock lending supply and stock volatility, Saffi and Sigurdsson (2011) find that high short-sale cost levels are linked to high levels of stock volatility. Diether, Lee, and Werner (2009) find that the fraction of stock volume that can be attributed to short selling is positively related to contemporaneous volatility.

Differences of opinions and short-sale constraints are considered by Miller (1977), who shows how short-sale constraints and differences of opinions can result in overpricing because limited stock supply yields an equilibrium price where the marginal investor is more optimistic than average. Diamond and Verrecchia (1987), who predict skewness in returns of stocks with short-sale constraints, state that the overpricing is not sustainable in equilibrium because investors will realize the effects identified by Miller (1977) and adjust accordingly. Boehme, Danielsen, and Sorescu (2006) present empirical results supporting the hypothesis that stocks with both high short-sale constraints and high differences of opinions are overvalued.

Finally, volatility and differences of opinions are also linked in the literature. In a model with differences of opinions, Dumas, Kurshev, and Uppal (2009) show how overconfidence can lead to increased stock volatility, while Karl B. Diether (2002) show empirically that dispersion in analysts' earnings forecasts is positively related to stock volatilities over the past period. Carlin, Longstaff, and Matoba (2014) find that increased disagreement among Wall Street mortgage dealers about prepayment speed is followed by higher levels of mortgage return volatility.

Our contribution is to link supply and demand shifts in the shorting market, changes in the short-sale costs and short interest, and levels in short-sale costs and short interest to future stock volatility. Based on daily data from 2006 to 2012, we obtain results consistent with changes in differences of opinions driving changes in volatility and shorting markets.

Regression results show that positive demand shifts predict significantly higher stock volatility. Likewise, negative demand shifts are significant negative predictors of volatility. To address the concern that our results are driven by extraordinary circumstances during the financial crisis of 2007-2009, we test the prediction for three sub-periods: pre-crisis, crisis, and post-crisis. For all three sub-periods, we find that positive or negative demand shifts are significant predictors of volatility. Event studies show that positive and negative supply and demand shifts are associated with contemporaneous higher volatility of the affected stock, both relative to the same stock at other times and other stocks. We also find that changes in short-sale costs have predictive power for volatility, consistent with short-sale costs being related to differences of opinions (D'Avolio, 2002) and, thereby, volatility. Finally, we establish that the level of short-sale costs and level of short interest are positively related to future volatility. Based on daily data, our results are similar to those obtained by Saffi and Sigurdsson (2011), who use yearly estimates.

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## Chapter 1

# Early Option Exercise: Never Say Never 

## With Lasse Heje Pedersen.

Published in Journal of Financial Economics: 121 (2016) 278-299.


#### Abstract

A classic result by Merton (1973) is that, except just before expiration or dividend payments, one should never exercise a call option and never convert a convertible bond. We show theoretically that this result is overturned when investors face frictions. Early option exercise can be optimal when it reduces short-sale costs, transaction costs, or funding costs. We provide consistent empirical evidence, documenting billions of dollars of early exercise for options and convertible bonds using unique data on actual exercise decisions and frictions. Our model can explain as much as $98 \%$ of early exercises by market makers and $67 \%$ by customers.


### 1.1 Introduction: Never exercise a call and never convert a convertible?

One of the classic laws of financial economics is that equity call options should never be exercised, except at expiration or just before dividend payments (Merton, 1973) and, similarly, convertible bonds should not be converted early (Brennan and Schwartz, 1977; Ingersoll, 1977a). Stock lending fees are similar to dividends and can therefore give rise to early exercise as is commonly
understood. ${ }^{1}$ The fact that the financial friction of lending fees can lead to early exercise raises several broader questions: Which financial frictions lead to early exercise? When should we expect to observe friction-driven early exercise? Do customers of brokers, market makers, and other investors actually exercise early? Are actual lending fees and other financial frictions large enough to drive significant early exercise decisions? If so, to what extent are actual early exercise decisions driven by actual financial frictions?

We seek to address these questions theoretically and empirically. First, we show that early exercise can be optimal when agents face short-sale costs, transaction costs, or funding costs, and we characterize both a lower and upper bound for the optimal exercise policy under such financial frictions. Second, we show empirically that investors indeed exercise equity options early and convert convertibles when facing these frictions, using unique data on actual exercise and conversion decisions.

To understand our result, first recall the famous arbitrage argument of Merton (1973): Rather than exercising a call option and receive the stock price $S$ less the strike price $X$, an investor is better off shorting the stock, putting the discounted value of $X$ in the money market, and possibly exercising the option at expiration - or selling the option to another agent who can do so. However, this arbitrage argument can break down when shorting is costly or agents face transaction costs or funding costs.

We introduce these financial frictions in a model. We first show that Merton's no-exercise rule holds even with "mild" frictions, meaning either (i) when short-sale costs and funding costs are small (even if transaction costs are large), or (ii) when transaction costs are small and the option price is above the intrinsic value (which can be driven by other agents facing low shorting and funding costs). However, we show that early exercise is in fact optimal when frictions are more severe such that the option price net of transaction costs is below the intrinsic value and the option owner faces sufficiently high shorting and/or funding costs.

Finally, we show how the effects of financial frictions can be quantified in a continuous-time model in which the parameters can be directly calibrated to match the data. Indeed, exercise is

[^1]justified when the stock price is above a lower exercise boundary, which we derive. The exercise boundary is decreasing in short-sale costs, margin requirements, and funding costs. In other words, exercise happens earlier (i.e., for lower stock prices) with larger short-sale costs, larger margin requirements, and larger funding costs.

To intuitively understand our model and to illustrate its clear quantitative implications, consider the example of options written on the iShares Silver Trust stock (the largest early exercise day in our sample of options on non-dividend-paying stocks). Fig. 1.1 shows the stock price of iShares Trust and the lower exercise boundary that we derive based on the short-sale cost (or "stock lending fee") and funding costs that we observe in our data. While exercise is never optimal before expiration when there are no frictions, we see that the exercise boundary is finite due to the observed financial frictions. Furthermore, we see that investors actually exercise shortly after the stock price crosses our model-implied lower exercise boundary.

This illustrative example provides evidence consistent with our model, but does it reflect a broader empirical phenomenon? To address this question, we collect and combine several large data sets. For equity options, we merge databases on option prices and transaction costs (OptionMetrics), stock prices and corporate events (Center for Research in Security Prices (CRSP)), short-sale costs (Data Explorers), proxies for funding costs, and actual option exercises (from the Options Clearing Corporation). Focusing only on options on non-dividend-paying stocks, we find that 1.8 billion option contracts are exercised early (i.e., before Merton's rule) in the time period from 2003 to 2010, representing a total exercise value of $\$ 36.3$ billion. Of course, the amount of exercises before Merton's rule would be larger if we included dividend-paying stocks, but for clarity we restrict attention to the most obvious violations.

Consistent with our theory's qualitative implications, we find that early exercise is more likely when (i) the short-sale costs for the underlying stock are higher, (ii) the option's transaction costs are higher, (iii) the option is more in-the-money, and (iv) the option has shorter time to expiration. These results are highly statistically significant due to the large amounts of data. Moreover, our data allow us to identify exercises for each of three types of agents: customers, market makers, and proprietary traders. We find that each type of agent exercises options early, including the

Call option on iShares Trust, Strike: 14, Expiry: 2010-01-16


Figure 1.1. Early exercise of call options before expiration: iShares Trust. The upper panel shows the daily closing price of iShares Trust stock (Silver) and the model-implied lower exercise boundary based on the following parameters: The risk-free rate is the Fed funds rate, the volatility is estimated as the 60-day historical volatility, the short-sale fee is from Data Explorers, the funding cost is the LIBOR-OIS spread, and the assumed margin requirements are $100 \%$ for the option and $50 \%$ for the stock. The lower panel shows the open interest and early exercise of the option. Shortly after the stock price is above the exercise boundary, $84 \%$ of the open interest is exercised in one day. Early exercises are also observed the following days. The closing bid-price of the option is below closing bid-price of the stock minus strike price in periods with gray background.
professional market makers and proprietary traders, and that each type is more likely to do so when frictions are severe, consistent with our theory of rational exercise.

We also test the quantitative implications of the model more directly. For each option that is exercised early, we estimate the lower exercise boundary by solving our model-implied partial differential equation (PDE) based on the observed frictions. We find that $66-84 \%$ of all early exercise decisions in our data happen when the stock price is above the model-implied exercise boundary, depending on how input variables are estimated (and even higher if we exclude corporate events). The behavior of market makers is most consistent with our model (their exercise decisions coincide with the model-implied prediction in $86-98 \%$ of the cases), while customers of brokers make the most exercise decisions that we cannot explain and proprietary traders are in between, consistent with the idea that market makers are the most sophisticated agents facing the lowest frictions while customers of brokers face the highest frictions.

Furthermore, using logit and probit regressions, we find that real-world investors are more likely to exercise early when the stock price is above the model-implied exercise boundary. This consistent evidence is both statistically and economically significant: The estimated probability that an option contract is exercised early, cumulated over a 20 -trading-day period in which the stock price is above the boundary, is $20.7 \%$ (21.8\%) based on logit (probit) regressions. The corresponding probability when the stock price is below the boundary is $0.4 \%$ ( $0.4 \%$ ), a large difference across these two model-implied cases. The numbers also indicate that far from all options are exercised immediately when the stock price goes above our model-implied lower boundary (which is not surprising given that it is a lower bound).

We also entertain alternative potential reasons for early exercise and examine the issue of causality. Indeed, we find some early exercises that are not explained by our estimated model, and, some of the largest of those are related to corporate events. Therefore, we repeat our analysis in the subsample where corporate events are excluded and find similar results. To test for causality, we consider the natural experiment of the short-sale ban of certain stocks in 2008 and conduct a difference-in-differences analysis that supports the idea that short-sale frictions lead to early exercise. Indeed, early exercise rose for options on affected stocks during the ban period relative to
unaffected options.
For convertible bonds, we combine data on equities and short-sale costs with the Mergent Fixed Income Securities Database (FISD) on convertible bond features and actual conversions. We find 25.4 million early conversions, representing an equity value of $\$ 7.7$ billion at conversion. The early conversion rates for convertible bonds is increasing in the short-sale cost of the stock and in the moneyness of the convertible bond, again consistent with our theory, but we note that this data set is smaller and subject to potential errors and inaccuracies.

Our paper complements the large literature following Black and Scholes (1973) and Merton (1973). Option prices have been found to be puzzlingly expensive (Longstaff, 1995; Bates, 2000, 2003; Jackwerth, 2000; Ni, 2009; Constantinides, Jackwerth, and Perrakis, 2009) and several papers explain this based on frictions: Option prices are driven by demand pressure (Bollen and Whaley, 2004; Gârleanu, Pedersen, and Poteshman, 2009), are affected by transaction costs (Brenner, Eldor, and Hauser, 2001; Christoffersen, Goyenko, Jacobs, and Karoui, 2011), short-sale costs (Ofek, Richardson, and Whitelaw, 2004; Avellaneda and Lipkin, 2009), funding constraints (Bergman, 1995; Santa-Clara and Saretto, 2009; Leippold and Su, 2015), embedded leverage (Frazzini and Pedersen, 2012), and interest-rate spreads and other portfolio constraints (Karatzas and Kou, 1998; Piterbarg, 2010). We complement the literature on how frictions affect option prices by showing that frictions also affect option exercises.

Turning to the literature on option exercise, several papers document irrational early exercise decisions (Gay, Kolb, and Yung, 1989; Diz and Finucane, 1993; Overdahl and Martin, 1994; Finucane, 1997; Poteshman and Serbin, 2003), ${ }^{2}$ irrational failures of exercise of call options (Pool, Stoll, and Whaley, 2008) and put options (Barraclough and Whaley, 2012), and irrational delivery decisions (Gay and Manaster, 1986). We complement these findings by linking early exercise decisions to financial frictions, both theoretically and empirically, and by drawing a parallel to convertible bonds. Early exercise therefore exists both for rational and irrational reasons. While Poteshman and Serbin (2003) find that customers sometimes irrationally exercise early, we find

[^2]that market makers and firm proprietary traders also frequently exercise early and that most early exercises appear to be linked to financial frictions. Battalio, Figlewski, and Neal (2015) also find that option bid prices can be below intrinsic value, which is a necessary condition for optimal early exercise, and our model helps explain why the option price can be this low.

Regarding convertible bonds, the literature has linked their prices to financial frictions (Mitchell, Pedersen, and Pulvino, 2007; Agarwal, Fung, Naik, and Loon, 2011) and examined whether the companies call these bonds too late (often convertible bonds are also callable, see the literature following Ingersoll (1977b)), while we study early conversions by the owners of the convertible bonds due to financial frictions.

In summary, we characterize how frictions can lead to optimal early exercise of call options and conversion of convertibles, and we provide extensive empirical evidence consistent with our predictions. These findings overturn one of the fundamental laws of finance, providing another example that the basic workings of financial markets are affected by financial frictions with broader implications for economics.

### 1.2 Theory

We are interested in studying when it is optimal to exercise an American call option early, that is, during times other than expiration and days before ex-dividend days of the underlying stock. Such rational early exercises must be driven by frictions since they violate Merton's rule. We first consider a simple model to illustrate how early exercise can be optimal for an investor who is long an option (Section 1.2.1) and next present a continuous-time model with testable quantitative predictions for early exercise (Section 1.2.2).

### 1.2.1 When is early exercise optimal?

Consider an economy with three securities that all are traded at times 0 and 1: a risk-free security with interest rate $r^{f}>0$, a non-dividend-paying stock, and an American call option with strike price $X>0$ that expires at time $t=1$. The stock price at time $t$ is denoted $S_{t}$ and the option price
$C_{t}$. The stock price $S_{1}$ at time 1 can take values in $[0, \infty)$ and is naturally unknown at time 0 . The final payoff of the option is $C_{1}=\max \left(S_{1}-X, 0\right)$.

All agents are rational, wealth-maximizing price takers, subject to financial frictions. Agent $i$ faces a proportional stock transaction cost of $\lambda^{i, S} \in[0,1]$ per dollar stock sold. Furthermore, agent $i$ faces a proportional transaction cost of $\lambda^{i, C} \in[0,1]$ per dollar option sold. If agent $i$ sells the stock short at time $t=0$, agent $i$ incurs a proportional securities-lending fee of $S_{0} L^{i}$ at time $1, L^{i} \geq 0$. If $i$ is long the stock, agent $i$ can lend out the stock and receive a proportional securities-lending fee of $S_{0} l^{i}$ at time 1 , where $l^{i} \in\left[0, L^{i}\right]$. Agent $i$ also faces a funding cost of $F^{i}(x, y)$ at time 0 if the agent chooses to hold a value of $x \in \mathbb{R}$ of the stock and $y \in \mathbb{R}$ of the option. This funding cost could be due to an opportunity cost associated with binding capital requirement. Naturally, the funding cost is zero if the agent takes a zero position, $F^{i}(0,0)=0$, and increasing in the absolute sizes of $x$ and $y .{ }^{3}$

We are interested in whether early exercise can be optimal. We therefore analyze whether a strategy is "dominated." Inspired by Merton (1973), we say that a strategy is dominated if there exists another strategy that generates at least as high cash flows in each time period and in every state of nature, and a strictly higher cash flow in some possible state. Further, early exercise is defined as being dominated if any possible strategy that includes early exercise is dominated. We assume that there exists no pure arbitrage net of transaction costs because such a strategy would trivially dominate all other strategies (or, said differently, all strategies are either non-dominated or dominated by a non-dominated strategy).

We first show that, under certain "mild" frictions, early exercise is always dominated. This result extends Merton's classic no-early-exercise rule and shows that the rule is robust to certain frictions. All proofs are in the Appendix.

Proposition 1 (No Exercise with "Mild" Frictions).

## Early exercise is dominated for an agent ithat has:

i. zero short-sale and funding costs, i.e., $L^{i}=F^{i}=0$ (regardless of all transaction costs); or

[^3]ii. a sale revenue of the option above the intrinsic value, $C_{0}\left(1-\lambda^{i, C}\right)>S_{0}-X$. A sufficient condition for this high sale revenue is that agent i has zero option transaction costs, $\lambda^{i, C}=0$, and the existence of another type of agents $j$ with zero short-sale costs, funding costs, and stock transaction costs, $L^{j}=F^{j}=\lambda^{j, S}=0$.

The first part of this proposition states that transaction costs alone cannot justify rational early exercise. The reasoning behind this is as follows: When the option is exercised it is either to get the underlying stock or to get cash. In the case in which the option holder wants cash, exercising early and immediately selling the stock is dominated by hedging the option position through shortselling of the underlying stock and investing in the risk-free security. The transaction cost from selling the stock after early exercise and from selling the stock short are the same so positive transaction costs of the stock cannot in themselves make early exercise optimal.

In the case in which the option holder wants stock, early exercise is dominated by holding on to the option, exercising later, and investing the strike price discounted back one period, $\frac{X}{1+r^{f}}$, in the risk-free asset. Thereby the investor will still get the stock, but on top of that earn interest from the risk-free asset. This strategy does not involve any direct trading with the stock and, hence, is not affected by stock transaction costs. (Note that these two alternative strategies do not involve option transactions and hence dominate early exercise even with high option transaction costs.)

The second part of the proposition states that early exercise is also dominated if the option owner's net proceeds from selling the option exceeds intrinsic value. In this case, the owner is better off by selling the option than by exercising early. If there is a type of agents, $j$, who faces no short-sale costs, no funding costs, and no stock transaction costs then these agents value the option at strictly more than its intrinsic value (as explained above). Therefore, the option holder $i$ prefers selling to $j$ over exercising early if no option transaction costs apply.

While it is important to recognize that frictions need not break Merton's rule, we next show that Merton's rule indeed breaks down when frictions are severe enough. Specifically, a combination of short-sale costs and transaction costs can make early exercise optimal.

Proposition 2 (Rational Early Exercise with "Severe" Frictions).
Consider an agent $i$ who is long a call option which is in-the-money taking stock transaction costs
into account, $S_{0}\left(1-\lambda^{i, S}\right)>X$. Early exercise is not dominated for $i$ if the revenue of selling the option is low, $C_{0}\left(1-\lambda^{i, C}\right) \leq S_{0}\left(1-\lambda^{i, S}\right)-X$ and one of the following holds:
a. the short-sale costs, $L^{i}$, is large enough or
b. the funding costs, $F^{i}$, is large enough.

The condition $C_{0}\left(1-\lambda^{i, C}\right) \leq S_{0}\left(1-\lambda^{i, S}\right)-X$ is satisfied if the option transaction cost $\lambda^{i, C}$ is large enough and/or the option price is low enough.

To understand the intuition behind how early exercise can be optimal, consider an option owner who wants cash now (with no risk of negative cash flows at time 1). Such an agent can either (i) sell the option, (ii) hedge it, or (iii) exercise early. Option (i) is not attractive (relative to early exercise) if the sale revenue after transaction costs is low. Further, option (ii) is also not attractive if the funding costs or short-sale costs (or those in combination) make hedging very costly. Therefore, option (iii), early exercise, can be optimal.

Note that a low option price can itself be a result of frictions. For instance, the option price is expected to be low if all agents face high short-sale costs and can earn lending fees from being long stocks as we explore further in the next section.

### 1.2.2 Quantifying early exercise: Exercise boundaries and comparative statics

We next consider a model that is realistic and tractable enough that we can use its quantitative implications in our empirical analysis. We solve for a lower bound of the optimal exercise boundary in a continuous-time model in which all parameters have clear empirical counterparts. The exercise boundary is the critical value of the stock price above which exercise is optimal - so we can examine empirically whether people actually exercise when the stock price is above the lower boundary.

The model solution also allows us to derive interesting comparative statics, showing how the exercise decision depends on short-sale costs, funding costs, and margin requirements. To accomplish these quantitative results, we must assume that the stock has no transaction costs. Clearly,
stocks have much lower transaction costs than options in the real world and we primarily included stock transaction costs in the previous sections to show that they are not the main driver of early exercise (Proposition 1).

The optimal exercise decision is closely connected to the rational valuation of American options in the context of financial frictions. Hence, we seek to joint solve for the value of the option and the optimal exercise decision. We start in the classic Black-Scholes-Merton framework, in which agents can invest in a risk-free money-market rate of $r^{f}>0$ and a stock with price process $S$ given by:

$$
\begin{equation*}
\mathrm{d} S(t)=S(t) \mu \mathrm{d} t+S(t) \sigma \mathrm{d} W(t) \tag{1.1}
\end{equation*}
$$

where $\mu$ is the drift, $\sigma$ is the volatility, and $W$ is a Brownian motion. The stock can be traded without cost, but we consider the following financial frictions.

First, agents face short-sale costs, modeled based on standard market practices: To sell the stock short, an agent must borrow the share and leave the short-sale proceeds as collateral. Agent $i$ 's short-sale account must have an amount of cash equal to $S(t)$, which earns the interest rate $r^{f}-L^{i}$ (called the "rebate rate"). The fact that the rebate rate is below the money-market rate reflects an (implicit) continuous short-sale cost of $L^{i}$ (called the "rebate rate specialness"). The securities lender - the owner of the share - holds the cash and must pay a continuous interest of $r^{f}-l^{i}$. Since he can invest the cash in the money market, this corresponds to a continuous securities-lending income of $l^{i} \in\left[0, L^{i}\right]$. We allow that the securities-lending fees depend on the agent $i$, and that lender earns less than the short-seller pays ( $l^{i}<L^{i}$ ) since the difference is lost to intermediaries (custodians and brokers) and search costs and delays. ${ }^{4}$

The second friction that we consider is funding costs. In particular, there exists a wedge $\psi^{i} \geq 0$ between the agent's cost of capital and the risk-free rate. The agent's margin account earns the risk-free money-market rate, $r^{f}>0$, while the cost of capital is $r^{f}+\psi^{i}$ in the sense that using his own equity for a risk-free investment is associated with an opportunity cost of $r^{f}+\psi^{i}$. Such a

[^4]capital cost can arise from costly equity financing and from a binding capital constraint. ${ }^{5}$ The cash in the agent's margin account must be at least $K^{i}(x, y)$, depending on the number of stocks $x \in \mathbb{R}$ and options $y \in \mathbb{R}_{+}$.

Based on these assumptions about the stock dynamics and agent frictions, we seek to determine an option owner's optimal exercise policy. Consider an owner $i$ of an American call option with expiration $T$ and strike price $X$ and his option valuation $C^{i}$. The option value is assumed to be a $C^{1,2}$ function of time $t$ and the stock price $S$ so we apply Itô's lemma to write the option price dynamics as:

$$
\begin{equation*}
\mathrm{d} C^{i}(t)=\left(C_{t}^{i}+\frac{1}{2} \sigma^{2} S^{2} C_{S S}^{i}\right) \mathrm{d} t+C_{S}^{i} \mathrm{~d} S(t) \tag{1.2}
\end{equation*}
$$

where subscripts denote derivatives (e.g., $C_{S S}$ is the second order derivative of the option value with respect to the stock price $S$ ), and we assume the natural condition that $C_{S}^{i} \geq 0$ for all $i$. To derive bounds on the optimal exercise policy for any agent, we consider the strategies of two hypothetical "extreme" agents, $\underline{i}$ and $\bar{i} \psi$. First, hypothetical agent $\underline{i}$ has the most strict frictions, leading to the lowest exercise boundary, $\underline{B}$, and the lowest option valuation, $\underline{C}$. To accomplish this lower bound, agent $\underline{i}$ is always short the stock, has the highest funding $\operatorname{cost}\left(\psi^{\underline{i}}:=\max _{i} \psi^{i}\right)$, the highest short-sale cost $\left(L^{i}:=\max _{i} L^{i}\right)$, and must have cash in his margin account equal to

$$
\begin{equation*}
K^{i}(x, y)=m^{i, S} S|x|+\left(m^{i, C}-1\right) \underline{C y} \tag{1.3}
\end{equation*}
$$

where $m^{i . S}, m^{i . C} \in[0,1]$ are margin requirements (and we recall that this margin account is in addition to the proceeds in the short-sale account). Given that the agent also owns options worth $\underline{C y}$, this expression corresponds to a margin equity of $m^{i, S} S|x|+m^{i, C} C y$. Hence, $m^{i, S}$ is the margin requirement for the stock and $m^{i, C}$ is the margin requirement for the option. The required amount on the margin account approximates the real-world margin requirements in a way that is tractable enough for our analytical results. The real-world margin requirements differ across exchanges and market participants and are very complex, see, e.g., Chicago Board Options Exchange (2000). All

[^5]other agents $i$ have looser margin requirements in the sense that
\[

$$
\begin{align*}
& K^{i}\left(x_{2}, y\right)-K^{i}\left(x_{1}, y\right) \leq m^{i, S}\left(x_{1}-x_{2}\right) S \text { for } x_{2} \leq x_{1} \text { and } \forall y \in \mathbb{R}_{+}  \tag{1.4}\\
& K^{i}\left(x, y_{2}\right)-K^{i}\left(x, y_{1}\right) \leq\left(m^{i, C}-1\right)\left(y_{2}-y_{1}\right) \underline{C} \text { for } y_{2} \geq y_{1} \geq 0 \text { and } \forall x \in \mathbb{R} .
\end{align*}
$$
\]

The first condition says that a decrease in the number of stocks held increases the required margin cash at least as much for agent $\underline{i}$ as for $i$. Likewise, the second condition says that an increase in the number of options increases the required margin cash at least as much for agent $\underline{i}$ as for $i$.

To focus on the exercise strategy, we assume that agents cannot sell the option at or above $\underline{C}$. This assumption can be viewed as a large option transaction cost or as a result of low equilibrium option prices arising from other agents facing the same frictions.

Consider the portfolio dynamics of buying one (additional) option at price $\underline{C}$, hedging by selling (additional) $\underline{C}_{S}$ shares of the stock, and fully financing the strategy based on margin loans and the use of equity capital. The value of this fully financed strategy evolves as according to:

$$
\begin{align*}
& \left(\underline{C}_{t}+\frac{1}{2} \sigma^{2} S^{2} \underline{C}_{S S}\right) \mathrm{d} t+\underline{C}_{S} \mathrm{~d} S(t)-\left(1-m^{i, C}\right) \underline{C}^{f} \mathrm{~d} t-m^{i, C} \underline{C}\left(r^{f}+\psi^{\underline{i}}\right) \mathrm{d} t \\
& -\underline{C}_{S} \mathrm{~d} S(t)+\underline{C}_{S} S\left(m^{i, S} r^{f}-m^{i, S}\left(r^{f}+\psi^{\underline{i}}\right)+\left(r^{f}-L^{\underline{i}}\right)\right) \mathrm{d} t . \tag{1.5}
\end{align*}
$$

Let us carefully explain each of the terms in this central expression. The first two terms simply represent the dynamics of the option (as seen in Eq. (1.2)). The next two terms represent the funding of the option. Specifically, $\left(1-m^{i \cdot C}\right) \underline{C}$ can be borrowed against the option at the money-market funding cost $r^{f}$. The remaining option value, the margin requirement $m^{i, C} \underline{C}$, must be financed as equity at a rate of $r^{f}+\psi \underline{\underline{i}}$. The remaining terms stem from the stock position and its financing. The first of these terms is the dynamics of the stock position, given the number $\underline{C}_{S}$ of shares sold. The last three terms capture the various financing costs. The stock sold short to hedge the option increases the required amount in the margin account by $\underline{C}_{S} S m^{i, S}$ which earns the interest $r^{f}$. This amount must be financed as equity at the rate $r^{f}+\psi \underline{i}$. Agent $\underline{i}$ must deposit the cash from the stock sold short, $\underline{C}_{S} S$, on a short-sales account earning interest $r^{f}-L^{i}$.

We are ready to state the free boundary problem for the option value and the exercise boundary
$\underline{B}(T-t)$, which depends on the time to expiration $T-t$. First, the stock position is chosen to offset the risk of the option, so the stochastic terms involving $\mathrm{d} W(t)$ cancel out in (1.5). Second, as the portfolio is fully financed and the change in value is deterministic, the drift must also be zero, which yields the following PDE:

$$
\begin{equation*}
\underline{C}_{t}+\frac{1}{2} \sigma^{2} S^{2} \underline{C}_{S S}-\left(r^{f}+m^{i, C} \psi^{-}\right) \underline{C}+\underline{C}_{S} S\left(r^{f}-m^{i, S} \psi^{\underline{i}}-L^{i}\right)=0, \tag{1.6}
\end{equation*}
$$

for all stock prices $S<\underline{B}(T-t)$. Whenever $S(t) \geq \underline{B}(T-t)$, the option is exercised and the following boundary conditions ensure that the problem is well-posed (Merton, 1973):

$$
\begin{array}{rlrl}
\underline{C}(T, S) & =\max (S-X, 0) & & \\
\underline{C}(t, 0) & =0 & & t<T  \tag{1.7}\\
\underline{C}(t, S) & =S-X & S \geq \underline{B}(T-t), t<T \\
\underline{C}_{S}(t, \underline{B}(T-t)) & =1 & t<T .
\end{array}
$$

The first condition is the standard boundary condition for the value of the option at the expiration date $T$. The second condition expresses that, if the stock is worthless, so is the option. The third condition imposes that the value of the option is equal to its intrinsic value at and above the exercise boundary. The fourth condition is the high-contact boundary condition (smooth-pasting condition), stating that the delta of the option goes to one as the stock price goes to the exercise boundary (both from above and below), which ensures that the exercise boundary maximizes the option value (Kim, 1990). We note that the exercise boundary can take the value $\infty$ if early exercise is not optimal for any stock price at time $t$.

Likewise, to derive an upper exercise bound $\bar{B}^{\psi}$, we define a hypothetical agent $\bar{i}^{\psi}$ in a slightly more complex way: This agent is long the stock, has the lowest lending fee, and has extreme margin requirement among agents with a given level of funding $\operatorname{cost} \psi$ as specified in the Appendix. We derive the PDE for this agent's valuation $\bar{C}^{\psi}$ and exercise strategy:

$$
\begin{equation*}
\bar{C}_{t}^{\psi}+\frac{1}{2} \sigma^{2} S^{2} \bar{C}_{S S}^{\psi}-\left(r^{f}+m^{\bar{i}, C} \psi\right) \bar{C}^{\psi}+\bar{C}_{S}^{\psi} S\left(r^{f}+m^{\bar{i} \psi, S} \psi-\bar{l}^{i^{\psi}}\right)=0, \quad S<\bar{B}(T-t) \tag{1.8}
\end{equation*}
$$

subject to the same boundary conditions as (1.7) for $\bar{B}$ and $\bar{C}$. We see that the free boundary problems are mathematically equivalent to that arising from the pricing of an American call option in a Black-Scholes-Merton (BSM) model with a modified interest rate and a continuous dividend yield. Specifically, in (1.6), the role of the interest rate is played by $r^{f}+m^{i . C} \psi^{\underline{i}}$ and the role of the dividend yield is played by $L^{i}+\psi^{i}\left(m^{i, C}+m^{i, S}\right)$ and similarly for (1.8).

To understand the intuition for this equivalence, note first that the implied interest rate corresponds to a "weighted average cost of capital" for arbitrage trades, WACC $=r^{f}+m^{i}{ }^{i \cdot C} \psi^{\underline{i}}$, since a fraction $1-m^{i, C}$ of the capital can be borrowed at the rate $r^{f}$ while the remaining part $m^{i, C}$ of the capital must be financed with equity with opportunity $\operatorname{cost} r^{f}+\psi^{i}$. In other words, the opportunity cost of buying an option is $W A C C \times C$, corresponding to the opportunity cost of the risk-free rate times the option price in the standard BSM model. The implied "dividend yield" is the opportunity cost of exercising the option later rather than now. In the standard BSM model, this opportunity cost is the dividend income you gain by exercising. In our framework, the opportunity cost is the saved short-sale cost $L^{\underline{i}}$ plus the extra earning $\psi^{\underline{i}}$ on the total amount of capital that is freed up by exercising, $m^{i \cdot C}+m^{i \cdot S}$.

The equivalence with the BSM model with dividends means that our model can be solved by traditional numerical methods for American options as we do in our empirical analysis. The solution includes the option values and optimal exercise boundaries. The following proposition partly relies on results from Merton (1973), Kim (1990), and Dewynne, Howison, Rupf, and Wilmott (1993).

Proposition 3 (Lower and Upper Exercise Boundaries).
(i) Any option owner i has a value of the option in $\left[\underline{C}, \bar{C}^{\psi^{i}}\right]$; exercise is dominated if $S(t)<\underline{B}(T-t)$ and failing to exercise is dominated when $S(t)>\bar{B}^{\psi^{i}}(T-t)$.
(ii) If $L^{i}+\psi^{i}\left(m^{i, C}+m^{i, S}\right)>0$, then the lower exercise boundary, $\underline{B}$, is finite for all $t \leq T$ (i.e., early exercise for agent $\underline{i}$ ); otherwise, $\underline{B}$ is infinite for $t<T$.
(iii) Similarly, if $l^{i^{\psi}}+\psi\left(m^{i \psi}, C-m^{i \psi, S}\right)>0$, then the upper exercise boundary $\bar{B}^{\psi}(T-t)$ is finite for all $t \leq T$ (i.e., early exercise for agent $\bar{i}^{\psi}$ ); otherwise, $\bar{B}^{\psi}$ is infinite for $t<T$.

Proposition 3 provides general results that apply to any agent about when early exercise is dominated and when it is dominated not to exercise early. In the empirical section, we derive the exercise bounds by numerically solving the PDEs, and we focus on the lower bound, $\underline{B}$, for several reasons: First, and most importantly, we are interested in whether the observed exercise decisions can be rationally justified and this is the case when the stock price is above the lower boundary. Second, the lower boundary applies to all agents, independent of $\psi$. Third, the lower boundary depends on the lending fee $L$ which we observe (while we don't have data on the part $l$ that accrues to the owner).

It can be generally shown that the lower exercise boundary $\underline{B}$ is weakly decreasing in the short-sale cost, funding costs, option margin requirements, and stock margin requirements. ${ }^{6}$ These properties of the optimal exercise strategy are seen in the numerical example illustrated in Fig. 1.2.

In particular, we compute the exercise boundary for variations over the following realistic basecase parameters for an agent who is long a call option and shorts the stock: the risk-free rate is $r^{f}=2 \%$, the volatility is $\sigma=40 \%$, the funding cost is $\psi^{i}=1 \%$, the short-sale cost is $L^{i}=1 \%$, the margin requirement for the stock is $m^{i, S}=50 \%$, and margin requirement for the option is $m^{i \cdot C}=100 \%$. For each set of parameters, we solve the PDE (1.6) and (1.7) using the CrankNicolson finite-difference method and derive the corresponding optimal exercise boundary.

In each of the four panels, the time to expiration $T-t$ is on the $x$-axis and the scaled exercise boundary is on the $y$-axis. Specifically, we scale the exercise boundary $\underline{B}(T-t)$ by the strike price $X$, which yields an intuitive number. For example, a value of $\underline{B}(T-t) / X=1.6$ means that early exercise is optimal when the stock price is at least $60 \%$ in the money. Said differently, early exercise happens when $S(t) \geq \underline{B}(T-t)$ or, equivalently, when $S(t) / X \geq \underline{B}(T-t) / X$, and the latter scaled measure is more intuitive. Clearly, a lower exercise boundary corresponds to an earlier optimal exercise decision (i.e., for lower moneyness of the stock price).

The top left panel illustrates how higher short-sale costs correspond to a lower boundary, implying earlier optimal exercise. This result is natural as short-sale costs make it costly to hedge the option. The top right panel shows that higher funding costs also make it optimal to exercise earlier,

[^6]


Exercise boundaries for different option margin requirements



Figure 1.2. The lower exercise boundary with frictions: Comparative statics. This figure shows theoretical lower exercise boundaries for equity options with frictions for an agent who is short stock. The boundary is a solution to the PDE (1.6)-(1.7) for a stock that pays no dividend. Each graph varies the parameters around a base-case cost where the risk-free rate is $r^{f}=2 \%$, the lending fee is $L^{i}=1 \%$, the funding cost is $\psi^{i}=1 \%$, the volatility is $\sigma=40 \%$, and margin requirements are $m^{i, S}=50 \%$ for the stock and $m^{i, C}=100 \%$ for the option. Early exercise is seen to be increasing in lending fees, funding costs, option margin requirements, and decreasing in volatility and time to expiration.
namely, to free up capital. The bottom left panel shows that a higher option margin requirement encourages an earlier exercise. Finally, the bottom right panel shows that early exercise is delayed with higher volatility. To see why, recall that a higher volatility increases the value of optionality, therefore making it less attractive to exercise early.

In all the graphs we see that the optimal exercise boundary decreases in the time to expiration. In fact, it is a general result that the exercise boundary must be weakly decreasing in time to expiration. To understand this result, note that, if it is optimal to exercise a longer-dated option, then it must also be optimal to exercise a shorter-dated one (since you give up less optionality).

Finally, the figures provide quantitative insights into when we should expect early exercise due to frictions. In the base-case, early exercise is optimal when the stock is $67 \%$ in-the-money three months before expiration and $27 \%$ in-the-money ten days before expiration-hence, even if the option is bought at-the-money six months before expiration, the stock price has a realistic chance of crossing the boundary.

We next turn to the empirical analysis, where we also implement our model for each option in a large data set and analyze whether the real-world exercise decisions occur when the stock price is above the exercise boundary that we calculate.

### 1.3 Data and preliminary analysis

This section describes our data sources, provides summary statistics, and outlines our empirical methodology. We start with the data and then turn to the summary statistics, which already show large amounts of early exercises and early conversions both in terms of number of contracts and in terms of dollar value.

### 1.3.1 Data

Our study combines a number of very large data sets as described in Table 1.1. For equity options, we combine the OptionMetrics database on U.S. option prices and option bid-ask spreads with the CRSP tape of U.S. equity prices and corporate events. We use data on the cost of short-selling
stocks from Data Explorers, focusing on their Daily Cost of Borrow Score (DCBS), which is an integer from 1 to 10 with 1 indicating a low cost of shorting and 10 indicating a high one.

We analyze actual exercise behavior using data originally from the Options Clearing Corporation (OCC). ${ }^{7}$ These data contain the number of contract exercises for each option series each day. The daily exercises can be separated into three groups of market participants, namely, exercises done by customers of brokers (e.g., retail customers and hedge funds), market makers, and firm proprietary traders. The option exercise data run from July 2001 to and including August 2010. The data are missing in the months of November 2001, January and July 2002, and January 2006.

We use the Mergent FISD on convertible bonds. This database provides time and amount of conversions together with total outstanding amount for convertible bonds. Finally, we use data on equity issues and mergers and acquisitions from Thomson One. This data set includes issue dates and settlement dates for equity issues and announcement dates, effective dates, and withdrawals dates for both acquiror and target in mergers and acquisitions.

### 1.3.2 Sample selection

To identify option exercises that clearly violate Merton's rule, we focus on early exercise of options on stocks that do not pay dividends. In particular, we exclude any option series if OptionMetrics reports a nonzero forecast of future dividends during any day of the life time of the option. Further, we exclude observations of option series on the day of expiry to focus on early exercise. Lastly, we exclude options that do not follow the standard practice of having an expiration date on the day after the third Friday in a given month (this excludes only a tiny fraction).

The data from OptionMetrics are merged with the exercise data from OCC on date, ticker, option ticker, strike price, and expiry date. For each option series, we further merge the data with that of the corresponding stock from CRSP and Data Explorers based on CUSIP.

We further clean the data in a number of ways. We exclude any option series (i) where the underlying according to CRSP has ex-date for a distribution event (split, dividend payment, exchanges, reorganizations, etc.) within the observed life time of the option series; (ii) where the

[^7]
## Table 1.1

Data sources.
This table shows the data sources used in our study, the variables that we use, the start and end date of each data source, the number of securities, and the number of observations (which is the number of rows in the data).

| Data set | Data | Start date | End date | Number of call options/ convertible bond series | Number of underlying securities | Number of observations |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CRSP ${ }^{1}$ | Dividends, prices, corporate events | 30-08-1985 | 31-12-2011 |  | 23,597 | 18,314,652 |
| OCC Exercises ${ }^{2}$ | Exercises of equity options | 01-07-2001 | 31-08-2010 | 821,052 | 5,727 | 7,852,739 |


| OptionMetrics | Option prices, open <br> interests, volatilities, <br> expected future dividends | $01-01-1996$ | $31-01-2012$ | $3,949,199$ | 7,509 | $355,259,334$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | $19-06-2002$ | $03-12-2012$ | 41,188 | $55,139,348$ |  |


| Mergent FISD ${ }^{4}$ | Convertible bond features <br> and conversions | $30-08-1985$ | $31-12-2011$ | 4,501 | 1,721 | 14,144 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |


| Bloomberg | LIBOR-OIS spreads | 02-01-1990 | 22-01-2013 | 8,620 |
| :--- | :--- | :--- | :--- | :--- |


| Thomson One ${ }^{5}$ | Equity Issues, mergers and <br> acquisitions | $12-01-1963$ | $17-09-2015$ | 402,101 | 439,456 |
| :--- | :--- | :--- | :--- | :--- | :--- |

[^8]underlying at some point according to OptionMetrics was expected to have an ex-dividend date within the observed life time of the option series; (iii) which has records with different strike prices for the same series, different underlying identifiers (Security ID or CUSIP), or different expiry dates (indicating data errors or changes in the contract); (iv) which has no data available on the last trading day before expiry (indicating some possible outside event); (v) which has several records for the same day in OptionMetrics; (vi) for which another option series with same underlying stock ticker, option ticker (root of option symbol for old option symbol and first part of symbol for new Options Symbology Initiative (OSI) symbol), strike price, and expiry date observed on the same day exists in the data; (vii) which has settlement special, e.g., AM-settlement; or (viii) which for some day during its observed life time has no matching observation in the CRSP data for the underlying stock.

We note that the OCC data only have records of exercises, meaning that option series that never experienced an exercise (before or at expiry) are not part of that sample. Hence, by requiring a match between OptionMetrics and the OCC data, our sample only includes options that were exercised at some point. An alternative approach is to include our entire OptionMetrics sample and assume that option series missing in the OCC data were never exercised. Since we do not know whether the OCC data are complete, neither approach is perfect. To resolve this, we focus on whether the observed early option exercises can be explained by financial frictions, not whether people fail to exercise when they should. ${ }^{8}$

Convertible bond data are acquired through Mergent FISD. Our sample only includes convertible bonds that at some point in time were converted (including at a dividend, at maturity, or as a response to a call). If Mergent FISD has not been able to identify the exact day of a conversion they set the date to the end of the quarter or even fiscal year (the latter seems to be the case only rarely). This makes it difficult to identify whether the conversion happened on the day before exdividend or not in these cases. To avoid problems related to these issues, we only include bonds where the underlying did not have any distribution events (including dividend payments) during the sample period, using data of distribution events from CRSP. We also exclude bonds where the underlying is first observed in CRSP more than one day later than the offering date of the bond.

[^9]Furthermore, we exclude bonds that at some point had an exchange offer or a tender offer, or where the underlying is not either a common stock or an American depository stock. If Mergent FISD data have no maturity date or conversion price of the bond, then it is also excluded. The original conversion prices of the bonds are recorded in FISD and through the cumulative adjustment factors provided by CRSP they can be updated to reflect any changes, e.g., due to stock splits. ${ }^{9}$ We only include observations from days on which bond holders had the right to convert early. If CRSP data for the underlying are missing starting at some point in time, we exclude the observations from five days before this happened and onwards, to avoid inclusion of conversions related to some kind of exogenous corporate event. If the day of the initial observation of the bond in Mergent FISD is after the offering day of the bond, we exclude observations before this initial observation. If the bond has been partly or fully called at some point in time, then we exclude all observations which are less than three months earlier or three months later to this event. This measure is taken to avoid inclusion of conversions that are a response to a call from the issuer (and hence not early conversion initiated by the bond holder), though it is not guaranteed that we catch all such events. Likewise, if the record shows any reorganization or exchange of the bond, observations from three months before this event and onwards are excluded.

### 1.3.3 Summary statistics

Table 1.2 provides summary statistics for our final sample. We see a substantial amount of early exercises and conversions. Panel A reports the total number of early exercises of equity options, that is, exercises that violate Merton's rule. Our data contain 1,806 million early exercises, representing a total exercise value of $\$ 36.3$ billion or a total intrinsic value of $\$ 22.8$ billion. Naturally, the exercises are concentrated among in-the-money options, especially deep-in-the-money options. Our data do contain a small fraction of exercised out-of-the-money options, which could be due to measurement error or investors exercising to save transaction costs when they want the actual stock. Measurement error may occur for instance when options are exercised during the day and we measure the moneyness based on the end-of-day price.

[^10]
## Table 1.2

Summary statistics.
This table summarizes the number of early exercises and early conversions in the sample used in our study. It reports the number of contracts exercised and bonds converted. The total numbers are broken into categories of Moneyness, Expiration, and Agent type performing the exercise. Options are defined as "out of the money" if the closing stock price is below the strike price, "in the money" if the stock price is $0-25 \%$ above the strike price, and "deep in the money" if the stock price is more than $25 \%$ higher than the strike price. For convertible bonds the definition is parallel with conversion price used instead of strike price. Option exercise data are from 2001-2010 and convertible bond data are from 1985-2011 as seen in Table 1.

## Panel A: Early exercises of equity call options in sample

|  | Exercises <br> (number of <br> contracts, millions) | Exercises <br> (value of strike, USD <br> millions) | Exercises <br> (intrinsic value, USD <br> millions) |
| :--- | ---: | ---: | ---: |
| All | 1,806 | 36,250 | 22,811 |
| By moneyness |  |  |  |
| Out of the money | 2 | 38 | -1 |
| In the money | 1,324 | 14,170 | 1,942 |
| Deep in the money |  | 22,042 | 20,870 |
| By time to expiration | 1,720 |  |  |
| Less than 3 months | 72 | 35,422 | 21,199 |
| Between 3 and 9 months | 13 | 690 | 1,006 |
| More than 9 months |  | 137 | 605 |
| By agent type | 808 | 16,007 |  |
| Customer | 81 | 1,955 | 6,338 |
| Firm | 916 | 18,288 | 998 |
| Market maker |  |  | 15,475 |

Panel B: Early conversions of convertible bonds in sample

|  | Conversions <br> (number of bonds, <br> millions) | Conversions <br> (principal amount, <br> USD millions) | Conversions <br> (value of stock, USD <br> millions) |
| :--- | ---: | ---: | ---: |
| All | 25.4 | 5,655 | 7,732 |
| By moneyness |  |  |  |
| Out of the money | 3.8 | 2,063 | 1,255 |
| In the money | 15.4 | 1,001 | 1,127 |
| Deep in the money | 6.3 | 2,591 | 5,350 |

Table 1.2 also shows that early exercises are concentrated among shorter-term options. This finding is consistent with our theory, since short-term options have less optionality, but it could also be driven by the simple fact that there is a larger open interest of such options. Our formal empirical tests therefore consider the number of exercises as a fraction of the open interest.

The final part of Panel A in Table 1.2 shows that all types of agents exercise early for billions of dollars. Customers of brokers exercise early with a total strike price of $\$ 16.0$ billion, firm proprietary traders for a total of about $\$ 2.0$ billion, and market makers for a total of $\$ 18.3$ billion.

Panel B in Table 1.2 reports the total number of early conversions of convertible bonds. In our data, 25 million bonds are converted early, corresponding to about $\$ 5.7$ billion worth of principal or $\$ 7.7$ billion of equity value. A few conversions of out-of-the-money bonds are seen, which relates to the definition of moneyness applied. A convertible bond is considered out-of-the-money if the price of the underlying stock is less than conversion price, in-the-money if stock price is up to $25 \%$ above conversion price, and deep-in-the-money if stock price is more than $25 \%$ above conversion price. Conversion price is defined by the principal amount of bond that must be converted to get one share of stock. Our definition of moneyness is not perfect: Indeed, the market value of a nonconvertible bond can deviate some from the face value, so it might actually be attractive to convert even when it is out-of-the-money according to the above definition. While this adds noise to our analysis, we see no reason that it would drive our conclusions.

Fig. 1.3 shows the evolution of relative share of early option exercise over time for the three different agent types observed. The picture is dominated by Customers (which includes retail customers and hedge funds) and Market makers. Interestingly, the share of early exercise for the professional Market makers has increased over time. We next discuss how we use these exercise and conversion data to test our theoretical predictions.

### 1.3.4 Variables of interest and methodology

We are interested in the fraction of options that are exercised and how this relates to our theoretical predictions. For each day $t$ and each option series $i$ in our sample, we define the options that are

## Share of early exercises by agent type



Figure 1.3. Share of early exercises by agent type over time. This figure shows how the monthly relative share of the number of total early exercises are distributed among the three agent types: Customers (retail customers and hedge funds), Firms (proprietary traders), and Market makers. We see early exercises for all three groups and an increasing share from Market makers over time.
exercised $E X$ as a fraction of the open interest $O I$ on the close of the day before.

$$
\begin{equation*}
\left.E X_{t}^{i}=\frac{\text { \#exercised options }{ }_{t}^{i}}{\max \left\{\text { OI }_{t-1}^{i},\right. \text { \#exercised options }}{ }_{t}^{i}\right\} . \tag{1.9}
\end{equation*}
$$

We take the maximum in the denominator to ensure that $E X$ is between zero and one, including the rare instances when the number of exercises is greater than the open interest the day before (which must be due to options that are bought and exercised on the same day or data errors). Similarly, in our logit and probit regressions, we compare the number of exercises to the number of "trials" given by $\max \left\{O I_{t-1}^{i}\right.$, \#exercised options $\left.{ }_{t}^{i}\right\}$.

For each daily observation of an option series, we measure the option transaction costs as the
relative bid-ask spread constructed in the following way:

$$
\begin{equation*}
\operatorname{TCOST}_{t}^{i}=\frac{\text { ask price }^{s(i)}-\text { bid price }^{s(i)}}{\text { ask price } \left.^{s(i)}+\text { bid price }^{s(i)}\right) / 2} . \tag{1.10}
\end{equation*}
$$

Here, the superscript $i$ denotes the option series and $s(i)$ is the corresponding at-the-money option series with the same underlying stock and expiration, defined as the series with the smallest absolute difference between stock price and strike price (where $s(i)$ can be equal to $i$ itself). We use the at-the-money option instead of the option itself to avoid endogeneity issues. The possibility of exercising the option early will itself affect bid and ask prices, especially for deep-in-the-money options. The bid price will in such cases often go below, but not much below, the intrinsic value. As a result, the bid-ask is endogenous and can be affected by the ìprice floorî close to the intrinsic value. Our focus is to test how the general level of transaction costs of an option series affects early exercise in the first place.

We measure the short-sale fee, $L$, as follows. For each stock and each date, we observe its Daily Cost of Borrow Score (DCBS), which is an integer score from from one to ten. For the rare cases where different DCBSs are observed for the same stock on the same day, we use the average DCBS rounded to the nearest integer. We map this DCBS to a short-sale fee level by using the median among all observations with this DCBS that have data on both their DCBS and their fee level. In the analysis based on short-sale fees, we only include stock-dates with non-missing DCBS.

The model-implied optimal exercise boundary $\underline{B}^{j}(T-t)$ for any option $j$ on day $t$ is computed as follows. We numerically solve the PDE problem (1.6)-(1.7) that takes frictions into account using the observed characteristics on any date $t$ using the Crank-Nicolson finite-difference method. The stock volatility $\sigma$ is set as the 60 -day average historical volatility. We use the historical volatility as an objective measure of risk and note that we cannot use the standard Black-Scholes-Merton implied volatility for two reasons: (i) in many of the interesting cases when options are exercised early, option prices don't satisfy Merton's lower bound and, therefore, the Black-Scholes-Merton implied volatility cannot be computed; and, more broadly, (ii) the Black-Scholes-Merton implied volatility does not take frictions into account, so it should be a biased estimate of volatility when frictions are severe. For the risk-free rate $r^{f}$, we use the Fed Funds rate, the margin requirement
of the stock is set to $50 \%$, the margin requirement of the option is set to $100 \%$, the short-sale costs are as defined above, and the funding cost $\psi$ is set as the LIBOR-OIS interest-rate spread based on Gârleanu and Pedersen (2011), that is, the spread between the London Interbank Offered Rate and the overnight indexed swap rate. We measure the stock price $S$ as the closing price on the given day.

Similarly to the exercise measure $E X$ for equity options, we are interested in the daily converted bonds as a fraction of the total outstanding amount. We define $\operatorname{CONV}_{t}$ as amount converted on day $t$ divided by the sum of amount converted and outstanding amount after the conversion. (Equivalently, the denominator is the amount outstanding before the conversion.)

$$
\begin{equation*}
\operatorname{CONV}_{t}^{i}=\frac{\text { Amount converted }{ }_{t}^{i}}{\text { Amount converted }_{t}^{i}+\text { Amount outstanding after conversion }_{t}^{i}} . \tag{1.11}
\end{equation*}
$$

### 1.4 Empirical results: Never exercise a call option?

We turn to formally testing the link between early option exercises and financial frictions. We first sort the exercises by short-sale costs and transaction costs to analyze the connection between early exercises and financial frictions in a simple way. Next, we test the model more directly by considering whether the stock price is above or below the model-implied lower exercise boundary at the time of exercise. Finally, we use multivariate logit and probit regressions to analyze how the propensity to exercise can be explained by the model and how it depends on the joint effects of a number of option characteristics.

Table 1.3 and Fig. 1.4 show how the fraction of early option exercises $E X$ (defined in (1.9) above) varies with short-sale costs. We see that the fraction of early exercise decisions increases monotonically in the short-sale cost, consistent with our model. Among options with minimal short-sale costs, the fraction of options exercised early is only $0.17 \%$, while among options with the highest short-sale costs, the fraction exercised is above $4 \%$. As seen in the table, the difference in these two extreme groups is highly statistically significant.

Table 1.3 and Fig. 1.4 further consider the exercises broken down by moneyness. Naturally, there are virtually no out-of-the-money options that are exercised (which would clearly be irra-

## Table 1.3

Early exercise of equity options by short-sale costs.
This table shows the average number of early option exercises as a fraction of open interest on the previous day for options sorted on the short-sale cost of the underlying equity. The table further classifies options by their moneyness, expiration, and agent type. Options are defined as "out of the money" if the closing stock price is below the strike price, "in the money" if stock price is $0-25 \%$ above strike price, and "deep in the money" if stock price is more than $25 \%$ higher than strike price. We note that the number of exercises for each agent type is reported as a fraction of the total open interest since our data do not include open interest by agent type. Standard errors are reported in parentheses. The data cover 2003-2010 as seen in Table 1.

|  | $\begin{gathered} \hline 1 \\ \text { Low } \\ \text { cost of } \\ \text { shorting } \end{gathered}$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | High cost of shorting |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| All | $\begin{array}{r} 0.17 \% \\ (0.00 \%) \\ \hline \end{array}$ | $\begin{gathered} 0.31 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.44 \% \\ (0.01 \%) \end{gathered}$ | $\begin{gathered} 0.58 \% \\ (0.01 \%) \end{gathered}$ | $\begin{gathered} 0.85 \% \\ (0.02 \%) \end{gathered}$ | $\begin{gathered} 1.21 \% \\ (0.02 \%) \end{gathered}$ | $\begin{gathered} 1.74 \% \\ (0.03 \%) \end{gathered}$ | $\begin{array}{r} 2.57 \% \\ (0.04 \%) \\ \hline \end{array}$ | $\begin{array}{r} 3.05 \% \\ (0.05 \%) \\ \hline \end{array}$ | $\begin{array}{r} 4.28 \% \\ (0.07 \%) \end{array}$ | $\begin{array}{r} 4.12 \% \\ (0.07 \%) \\ \hline \end{array}$ |
| By moneyness |  |  |  |  |  |  |  |  |  |  |  |
| Out of the money | $\begin{gathered} 0.00 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.00 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.00 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.00 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.00 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.00 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.00 \% \\ (0.00 \%) \end{gathered}$ | $\begin{array}{r} 0.00 \% \\ (0.00 \%) \end{array}$ | $\begin{gathered} 0.00 \% \\ (0.00 \%) \end{gathered}$ | $\begin{array}{r} 0.02 \% \\ (0.01 \%) \end{array}$ | $\begin{array}{r} 0.02 \% \\ (0.01 \%) \end{array}$ |
| In the money | $\begin{gathered} 0.10 \% \\ (0.00 \%) \end{gathered}$ | $\begin{array}{r} 0.18 \% \\ (0.00 \%) \end{array}$ | $\begin{gathered} 0.24 \% \\ (0.01 \%) \end{gathered}$ | $\begin{gathered} 0.28 \% \\ (0.01 \%) \end{gathered}$ | $\begin{gathered} 0.31 \% \\ (0.02 \%) \end{gathered}$ | $\begin{gathered} 0.32 \% \\ (0.02 \%) \end{gathered}$ | $\begin{gathered} 0.38 \% \\ (0.02 \%) \end{gathered}$ | $\begin{array}{r} 0.41 \% \\ (0.03 \%) \end{array}$ | $\begin{gathered} 0.50 \% \\ (0.03 \%) \end{gathered}$ | $\begin{gathered} 0.63 \% \\ (0.05 \%) \end{gathered}$ | $\begin{gathered} 0.53 \% \\ (0.05 \%) \end{gathered}$ |
| Deep in the money | $\begin{array}{r} 0.31 \% \\ (0.00 \%) \\ \hline \end{array}$ | $\begin{array}{r} 0.55 \% \\ (0.01 \%) \\ \hline \end{array}$ | $\begin{array}{r} 0.78 \% \\ (0.01 \%) \\ \hline \end{array}$ | $\begin{array}{r} 1.06 \% \\ (0.02 \%) \\ \hline \end{array}$ | $\begin{array}{r} 1.51 \% \\ (0.03 \%) \\ \hline \end{array}$ | $\begin{array}{r} 2.23 \% \\ (0.04 \%) \\ \hline \end{array}$ | $\begin{array}{r} 3.13 \% \\ (0.05 \%) \\ \hline \end{array}$ | $\begin{array}{r} 4.52 \% \\ (0.08 \%) \\ \hline \end{array}$ | $\begin{array}{r} 5.75 \% \\ (0.09 \%) \\ \hline \end{array}$ | $\begin{array}{r} 8.44 \% \\ (0.14 \%) \\ \hline \end{array}$ | $\begin{array}{r} 8.13 \% \\ (0.14 \%) \\ \hline \end{array}$ |
| By time to expiration |  |  |  |  |  |  |  |  |  |  |  |
| Less than 3 months | $\begin{gathered} 0.28 \% \\ (0.00 \%) \end{gathered}$ | $\begin{array}{r} 0.51 \% \\ (0.01 \%) \end{array}$ | $\begin{gathered} 0.71 \% \\ (0.01 \%) \end{gathered}$ | $\begin{gathered} 0.99 \% \\ (0.02 \%) \end{gathered}$ | $\begin{array}{r} 1.49 \% \\ (0.03 \%) \end{array}$ | $\begin{array}{r} 2.10 \% \\ (0.04 \%) \end{array}$ | $\begin{array}{r} 2.97 \% \\ (0.05 \%) \end{array}$ | $\begin{array}{r} 4.51 \% \\ (0.08 \%) \end{array}$ | $\begin{array}{r} 5.32 \% \\ (0.09 \%) \end{array}$ | $\begin{array}{r} 7.58 \% \\ (0.14 \%) \end{array}$ | $\begin{array}{r} 7.29 \% \\ (0.14 \%) \end{array}$ |
| Between 3 and 9 months | $\begin{gathered} 0.02 \% \\ (0.00 \%) \end{gathered}$ | $\begin{array}{r} 0.05 \% \\ (0.00 \%) \end{array}$ | $\begin{gathered} 0.09 \% \\ (0.01 \%) \end{gathered}$ | $\begin{gathered} 0.10 \% \\ (0.01 \%) \end{gathered}$ | $\begin{gathered} 0.21 \% \\ (0.01 \%) \end{gathered}$ | $\begin{gathered} 0.29 \% \\ (0.02 \%) \end{gathered}$ | $\begin{gathered} 0.48 \% \\ (0.02 \%) \end{gathered}$ | $\begin{array}{r} 0.61 \% \\ (0.03 \%) \end{array}$ | $\begin{gathered} 0.88 \% \\ (0.04 \%) \end{gathered}$ | $\begin{array}{r} 1.15 \% \\ (0.07 \%) \end{array}$ | $\begin{array}{r} 1.13 \% \\ (0.07 \%) \end{array}$ |
| More than 9 months | $\begin{array}{r} 0.01 \% \\ (0.00 \%) \\ \hline \end{array}$ | $\begin{array}{r} 0.03 \% \\ (0.00 \%) \end{array}$ | $\begin{array}{r} 0.07 \% \\ (0.01 \%) \end{array}$ | $\begin{array}{r} 0.08 \% \\ (0.01 \%) \end{array}$ | $\begin{array}{r} 0.10 \% \\ (0.02 \%) \end{array}$ | $\begin{array}{r} 0.14 \% \\ (0.03 \%) \end{array}$ | $\begin{array}{r} 0.23 \% \\ (0.03 \%) \\ \hline \end{array}$ | $\begin{array}{r} 0.53 \% \\ (0.05 \%) \end{array}$ | $\begin{array}{r} 0.40 \% \\ (0.05 \%) \end{array}$ | $\begin{array}{r} 0.29 \% \\ (0.05 \%) \end{array}$ | $\begin{array}{r} 0.28 \% \\ (0.05 \%) \end{array}$ |
| By agent type |  |  |  |  |  |  |  |  |  |  |  |
| Customer | 0.11\% | 0.19\% | 0.24\% | 0.32\% | 0.43\% | 0.56\% | 0.76\% | 0.96\% | 1.24\% | 1.76\% | 1.65\% |
|  | (0.00\%) | (0.00\%) | (0.00\%) | (0.01\%) | (0.01\%) | (0.02\%) | (0.02\%) | (0.02\%) | (0.03\%) | (0.04\%) | (0.04\%) |
| Firm | 0.01\% | 0.02\% | 0.02\% | 0.03\% | 0.06\% | 0.07\% | 0.11\% | 0.13\% | 0.17\% | 0.17\% | 0.16\% |
|  | (0.00\%) | (0.00\%) | (0.00\%) | (0.00\%) | (0.00\%) | (0.00\%) | (0.01\%) | (0.01\%) | (0.01\%) | (0.01\%) | (0.01\%) |
| Market maker | 0.05\% | 0.11\% | 0.18\% | 0.23\% | 0.37\% | 0.58\% | 0.86\% | 1.48\% | 1.64\% | 2.36\% | 2.31\% |
|  | (0.00\%) | (0.00\%) | (0.00\%) | (0.01\%) | (0.01\%) | (0.02\%) | (0.02\%) | (0.03\%) | (0.03\%) | (0.05\%) | (0.05\%) |



Figure 1.4. Actual early exercise of equity options for varying short-sale costs. This figure shows the empirical fraction of equity options exercised early as a function of the short-sale costs, 2003-2010. Panel A groups the data by the moneyness of the option, Panel B by expiration, and Panel C by agent type. Consistent with our theory, early exercise is increasing in short-sale costs, increasing in moneyness, decreasing in time to expiration, and the exercise pattern is prevalent for all agent types including professional market makers and firm proprietary traders.

## Table 1.4

Early exercise of equity options by transaction costs.
This table shows the average number of early option exercises as a fraction of open interest on the previous day for options sorted on the transaction costs. Transaction costs for options are measured daily as the bid-ask spread divided by the mid price for the at-the-money option with same underlying, expiration, and observation day. Observations are grouped into quintiles by transaction costs. The table further classifies options by their moneyness, expiration, and agent type. Options are defined as "out of the money" if the closing stock price is below the strike price, "in the money" if stock price is $0-25 \%$ above strike price, and "deep in the money" if stock price is more than $25 \%$ larger than strike price. We note that the number of exercises for each agent type is reported as a fraction of the total open interest since our data do not include open interest by agent type. Standard errors are reported in parentheses. The data cover 2001-2010 as seen in Table 1.

|  | $\begin{gathered} 1 \\ \text { Low } \\ \text { T-cost } \end{gathered}$ | 2 | 3 | 4 | 5 <br> High <br> T-cost | 5-1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| All | $\begin{gathered} 0.13 \% \\ (0.00 \%) \end{gathered}$ | $\begin{array}{r} 0.16 \% \\ (0.00 \%) \end{array}$ | $\begin{array}{r} 0.21 \% \\ (0.00 \%) \\ \hline \end{array}$ | $\begin{array}{r} 0.27 \% \\ (0.00 \%) \\ \hline \end{array}$ | $\begin{array}{r} 0.50 \% \\ (0.00 \%) \end{array}$ | $\begin{array}{r} 0.37 \% \\ (0.00 \%) \end{array}$ |
| By moneyness |  |  |  |  |  |  |
| Out of the money | $\begin{gathered} 0.00 \% \\ (0.00 \%) \end{gathered}$ | $\begin{array}{r} 0.00 \% \\ (0.00 \%) \end{array}$ | $\begin{gathered} 0.00 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.00 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.00 \% \\ (0.00 \%) \end{gathered}$ | $\begin{array}{r} 0.00 \% \\ (0.00 \%) \end{array}$ |
| In the money | $\begin{gathered} 0.02 \% \\ \text { (0.00\%) } \end{gathered}$ | $\begin{gathered} 0.03 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.05 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.11 \% \\ (0.00 \%) \end{gathered}$ | $\begin{array}{r} 0.35 \% \\ (0.00 \%) \end{array}$ | $\begin{gathered} 0.33 \% \\ (0.00 \%) \end{gathered}$ |
| Deep in the money | $\begin{gathered} 0.25 \% \\ (0.00 \%) \end{gathered}$ | $\begin{array}{r} 0.34 \% \\ (0.00 \%) \end{array}$ | $\begin{gathered} 0.47 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.62 \% \\ (0.01 \%) \end{gathered}$ | $\begin{array}{r} 0.98 \% \\ (0.01 \%) \end{array}$ | $\begin{array}{r} 0.73 \% \\ (0.01 \%) \\ \hline \end{array}$ |
| By time to expiration |  |  |  |  |  |  |
| Less than 3 months | $\begin{gathered} 0.29 \% \\ (0.00 \%) \end{gathered}$ | $\begin{array}{r} 0.31 \% \\ (0.00 \%) \end{array}$ | $\begin{gathered} 0.36 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.42 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.62 \% \\ (0.00 \%) \end{gathered}$ | $\begin{array}{r} 0.33 \% \\ (0.00 \%) \end{array}$ |
| Between 3 and 9 months | $\begin{gathered} 0.04 \% \\ (0.00 \%) \end{gathered}$ | $\begin{array}{r} 0.04 \% \\ (0.00 \%) \end{array}$ | $\begin{gathered} 0.04 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.05 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.07 \% \\ (0.00 \%) \end{gathered}$ | $\begin{array}{r} 0.03 \% \\ (0.00 \%) \end{array}$ |
| More than 9 months | $\begin{array}{r} 0.02 \% \\ (0.00 \%) \\ \hline \end{array}$ | $\begin{array}{r} 0.02 \% \\ \text { (0.00\%) } \end{array}$ | $\begin{array}{r} 0.04 \% \\ (0.00 \%) \\ \hline \end{array}$ | $\begin{array}{r} 0.04 \% \\ (0.00 \%) \\ \hline \end{array}$ | $\begin{array}{r} 0.03 \% \\ (0.00 \%) \\ \hline \end{array}$ | $\begin{array}{r} 0.00 \% \\ (0.00 \%) \end{array}$ |
| By agent type |  |  |  |  |  |  |
| Customer | $\begin{array}{r} 0.06 \% \\ \text { (0.00\%) } \end{array}$ | $\begin{array}{r} 0.08 \% \\ (0.00 \%) \end{array}$ | $\begin{gathered} 0.11 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.16 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.35 \% \\ (0.00 \%) \end{gathered}$ | $\begin{array}{r} 0.30 \% \\ (0.00 \%) \end{array}$ |
| Firm | $\begin{gathered} 0.01 \% \\ (0.00 \%) \end{gathered}$ | $\begin{array}{r} 0.01 \% \\ (0.00 \%) \end{array}$ | $\begin{gathered} 0.01 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.01 \% \\ (0.00 \%) \end{gathered}$ | $\begin{gathered} 0.02 \% \\ (0.00 \%) \end{gathered}$ | $\begin{array}{r} 0.02 \% \\ (0.00 \%) \end{array}$ |
| Market maker | $\begin{gathered} 0.07 \% \\ (0.00 \%) \end{gathered}$ | $\begin{array}{r} 0.07 \% \\ (0.00 \%) \\ \hline \end{array}$ | $\begin{array}{r} 0.08 \% \\ (0.00 \%) \\ \hline \end{array}$ | $\begin{array}{r} 0.10 \% \\ (0.00 \%) \\ \hline \end{array}$ | $\begin{array}{r} 0.12 \% \\ (0.00 \%) \\ \hline \end{array}$ | $\begin{array}{r} 0.05 \% \\ (0.00 \%) \\ \hline \end{array}$ |



Figure 1.5. Actual early exercise of equity options for varying transaction costs. This figure shows the empirical fraction of equity options exercised early as a function of transaction costs, 2001-2010. The daily data on option series are divided in quintiles based on their transaction costs, measured each day as the bid-ask spread divided by the mid price for the corresponding at-the-money option series with the same expiration and underlying stock. Panel A groups the data by the moneyness of the option, Panel B by expiration, and Panel C by agent type. Consistent with our theory, early exercise is increasing in transaction costs, increasing in moneyness, and decreasing in time to expiration.
tional or due to measurement error), and the exercises are concentrated among deep-in-the-money options, as we would expect. Splitting the data by expiration, the table and figure show that the exercises are more frequent for shorter-term options. This is consistent with our theory as the benefits of postponing exercising is smaller (smaller optionality) for shorter-term options. Again, we see that the option exercises increase in short-sale costs within expiration group.

Lastly, we split the data by agent types, that is, across customers of brokers (retail customers and hedge funds), market makers, and firm proprietary traders. We see that all types of agents exercise early and more so when the short-sale costs are higher. We note that the absolute magnitude of the numbers should not be compared across groups for the following reason: Our data do not contain open interest by agent type, so we measure the number of exercises by each agent type as a fraction of the total open interest. Hence, the fraction of exercises by firm proprietary traders may be low simply because this agent type trades few options relative to the total open interest. In any case, the pattern of an increasing propensity to exercise as short-sale costs increase is consistent with our theory.

Table 1.4 and Fig. 1.5 show how the fraction of early exercises varies with the transaction costs. We measure transaction costs as the relative bid-ask spread of the at-the-money with the same expiration (TCOST) as defined in (1.10) above. We assign all observations an integer score from 1 to 5 (low to high transaction costs) based on this measure, using the full sample quintile breakpoints. Further, observations are classified in three groups: out-of-the-money (with stock price below strike price), in-the-money options (with stock price up to $25 \%$ above strike price), and deep-in-the-money options (with stock price more than $25 \%$ above strike price).

We see that the fraction of options exercised increases monotonically with the transaction costs. This pattern holds overall, for each moneyness group, and for each type of agent. The absolute of numbers in Table 1.4 are smaller on average than the numbers in Table 1.3. This is because $81 \%$ of the data have a short-sale cost code (DCBS) equal to 1 (as classified by Data Explorers) and, as expected, the fraction of exercises is low in this group as seen in Table 1.3. In Table 1.4, our groups by transaction costs are more balanced.

In summary, consistent with our model's qualitative predictions, we have seen that option ex-
ercises increase in short-sale costs and transaction costs and that these patterns tend to hold within groups sorted by moneyness, expiration, and agent types. Next, we seek to test our model's quantitative predictions. In particular, for each exercised option, we compare the model-implied lower exercise boundary to the high of the stock price at the day of the exercise as seen in Table 1.5. The table reports the fraction of exercises consistent with our model for each agent type and for all agents (in each row of the table). We look at three different samples: full sample (Panel A and Panel D), a sample excluding corporate events (Panel B), and a sample including only options with more than nine months to expiration (Panel C). Panel D differs from Panel A by setting funding costs to zero instead of the LIBOR-OIS spread when computing the exercise boundary. Each column of the table corresponds to a specific set of assumptions underlying the model, with increasingly conservative assumptions going from left to right. In the left-most column, we estimate each stock's volatility based on the 60 -day realized volatility. Given that volatility is mean-reverting, the next column uses a lower estimate of future volatility, namely, the minimum of the current 60-day volatility and its median in the OptionMetrics sample (June 1 2001-January 31 2012). The third column uses both the conservative volatility estimate and a conservative estimate of short-sale costs, namely, the 90th percentile of short-sale costs within each group (rather than the median observed cost). Finally, the fourth column has the most conservative boundary, which is $90 \%$ of the estimated boundary from column three.

For all cases of model input assumptions, we see that the majority of option exercises happen when the stock price is above the model-implied lower exercise boundary. The fraction of exercises consistent with our model is highest for market makers, which could be because these agents face the lowest financial frictions and are the most active market participants. Naturally, the fraction of exercises consistent with the model increases in the columns with more conservative assumptions (by construction). The model can explain $98 \%$ of the market makers' exercises with the most conservative assumptions, a very large fraction in light of the remaining noise.

Table 1.5 Panel B consistently reports higher numbers than Panel A, which shows that excluding corporate events disproportionately excludes early exercises not explained by the model. Section 1.4.2 elaborates on this result.

## Table 1.5

Early exercise of equity options: Model-implied lower exercise boundary.
This table shows the fraction of the observed early exercise decisions that can be rationalized by our model for our full sample (Panel A), a sample excluding corporate events (Panel B), a sample of options with more than 9 months to expiration (Panel C), and the full sample with funding costs set to zero (Panel D), 2003-2010. An early exercise can be rationalized by our model when the daily high of the stock price, $S$, is above the model-implied estimated lower exercise boundary, $\underline{B}$. Here, $\underline{B}$ is computed as the solution to the PDE (1.6)-(1.7) based on the estimated volatility, short-sale cost, and funding cost. The table reports this fraction for all agents and by agent type (across the rows) and for four different implementations of the model (across the columns). The first column estimates the volatility as the 60-day historical volatility and the short-sale cost as the median short-sale cost among stock loans with this DCBS score. The second column uses a more conservative (i.e., lower) estimate of expected future volatility. The third column also uses a conservative (i.e., higher) estimate of short-sale costs, namely, the 90th percentile among stock loans of this type. The fourth column has the most conservative boundary, which is $90 \%$ of the estimated boundary from column three.

|  | Exercises w/ S>B | Exercises w/ S>ㅡㅗ, conservative volatility estimate | Exercises w/ S>ㅡㅗ, conservative volatility and short-sale cost estimates | Exercises w/ S>B, conservative boundary |
| :---: | :---: | :---: | :---: | :---: |
| Panel A: Early exercises: full sample |  |  |  |  |
| All | 65.8\% | 75.1\% | 79.4\% | 84.2\% |
| By agent type |  |  |  |  |
| Customer | 41.3\% | 51.3\% | 58.2\% | 66.8\% |
| Firm | 64.2\% | 76.6\% | 79.7\% | 84.2\% |
| Market maker | 86.0\% | 94.4\% | 96.7\% | 98.4\% |
| Panel B: Early exercises: excluding corporate events |  |  |  |  |
| All | 69.4\% | 77.3\% | 81.8\% | 86.4\% |
| By agent type |  |  |  |  |
| Customer | 45.0\% | 54.0\% | 61.3\% | 70.0\% |
| Firm | 70.4\% | 80.6\% | 83.9\% | 87.8\% |
| Market maker | 88.1\% | 94.9\% | 97.3\% | 98.9\% |
| Panel C: Early exercises: more than 9 months to expiration |  |  |  |  |
| All | 82.9\% | 88.4\% | 89.8\% | 90.1\% |
| By agent type |  |  |  |  |
| Customer | 53.0\% | 65.3\% | 69.2\% | 70.3\% |
| Firm | 72.9\% | 90.7\% | 91.1\% | 91.3\% |
| Market maker | 98.3\% | 99.5\% | 99.7\% | 99.7\% |
| Panel D: Early exercises: full sample with zero funding cost |  |  |  |  |
| All | 61.4\% | 71.8\% | 78.0\% | 83.2\% |
| By agent type |  |  |  |  |
| Customer | 35.8\% | 46.3\% | 55.8\% | 64.8\% |
| Firm | 60.2\% | 73.8\% | 78.5\% | 83.0\% |
| Market maker | 82.5\% | 92.5\% | 96.1\% | 98.2\% |

Table 1.5 Panel C focuses on the exercises that happen with more than nine months to expiration. This sample has a larger fraction of exercises (relative to the full sample) that are in line with the estimated model. For market makers, the fraction of exercises consistent with the model are as high as $98.3 \%$ to $99.7 \%$, depending on the model inputs.

Table 1.5 Panel D reports that less exercises are explained when funding costs are set to zero. Nevertheless, the model continues to perform reasonably well with zero funding costs, which illustrates the importance of short-sale costs.

Next, we study the propensity to exercise in logit and probit regression settings. To do so, we need in principle to compute the model-implied lower exercise boundary for each type of option and each date, including days when no exercises are observed. The very large amount of data combined by the numerical complexity in solving our model's PDE makes such a complete analysis unfeasible. To address this issue, we look at a subsample only consisting of one day per month, namely, 17 days before option expiry (which is the day after the third Friday in every month). The subsample analysis is sufficient to obtain statistically significant results and we have confirmed that our model independent results hold up in the full sample (i.e., by regressing the propensity to exercise on characteristics such as short-sale costs). For each of the selected 80 dates, we compute the optimal exercise boundary for each option by solving the PDE problem (1.6)-(1.7) that takes frictions into account.

If we run a pooled logit (or probit) regression using all options on all of the selected dates, then we get highly significant results consistent with our model (not reported). However, such standard errors would be heavily downward biased since investors usually exercise many options simultaneously, generating a strong correlation across options on a given date. To address this correlation issue, we proceed as follows. First, we run a logit (or probit) regression for each subsample of three dates (the last subsample has only two dates). This generates an estimated vector of parameters, $\hat{\theta}_{s}$, for each subsample $s$. Second, we estimate the full-sample parameters and their standard errors based on the insight of Fama and MacBeth (1973) that each parameter can be viewed as sampled from parameters' distribution. In particular, we estimate the full-sample parameters as the sample average, $\hat{\theta}=1 / 27 \sum_{s} \hat{\theta}_{s}$, and the standard errors based on the sample standard de-
viation corrected for possible auto-correlation using Newey-West correction with automatic lag selection with a Bartlett-kernel, Newey and West (1987, 1994). This estimation method is relatively immune to cross-sectional correlation in option exercises and assumes that any time-series correlation is captured by the Newey-West correction.

Table 1.6 reports the results, where Panel A is the logit regressions and Panel B is probit. The first regression specification simply considers how the propensity to exercise depends on the indicator that the stock price is above the lower exercise boundary ( $S>\underline{B}$ ). We see that the estimated coefficient for $S>\underline{B}$ is positive, consistent with our model, and the effect is highly statistically significant. The estimated probability of exercise on a day with $S>\underline{B}$ is $1 /(1+\exp (8.54-4.09))=1.2 \%$, while estimated probability on other days is $1 /(1+\exp (8.54-0))=0.02 \%$. Cumulated over 20 trading days in a simple way, these probabilities correspond to aggregate exercise probabilities of $1-(1-1.2 \%)^{20}=20.7 \%$ when the stock price is above the exercise boundary and $1-(1-0.02 \%)^{20}=0.4 \%$ otherwise. We see that the exercise behavior is very different depending on whether the stock price is above or below the exercise boundary.

The second specification shows a similar result, but where we allow the exercise probability to increase in the distance between the stock price and the boundary using the variable $(S-\underline{B})^{+} / \underline{B}$. The third specification shows that having a bid price for the option below the intrinsic value is also a significant predictor of early option exercise, which is also consistent with our model (and a basic trade-off between selling the option vs. exercising it). Of course, the option bid price is an endogenous variable, so this regression does not address the deeper question of why option prices can be so low that early exercise can make sense. The first specification based on our model is not subject to the same issue since the exercise boundary is computed based on the observed financial frictions.

The fourth specification includes all these variables jointly. We see that all three variables remain positive and statistically significant, both with logit and probit. The fifth specification also includes a number of control variables. Both of the model-implied variables continue to be positive, and $1_{(S>\underline{B})}$ remains highly significant in all specifications. Several of the other variables

Table 1.6
Early exercise of equity options: Regression analysis.
This table shows logit (Panel A) and probit (Panel B) regressions for the determinants of early option exercises, 2003-2010. The dependent variable is one for every option contract that is exercised early and zero for every contract outstanding at end of the previous day that is not exercised early on a given day. The independent variables are as follows. The first variable is the indicator that the stock closing price $S$ is above the model-implied lower exercise boundary, $\underline{B}$, and the second variable is $(S-\underline{B})^{+} / \underline{B}$. The third variable is the indicator that the option's closing bid price, $C_{b i d}$, is below the intrinsic value given the strike price, $X$. Moneyness is $S / X$. Short-sale cost is the DCBS score, where a higher score indicates a higher cost. Bid-ask spread is the relative bid-ask spread for the option closest to at-the-money with same underlying and expiry date. Time to expiration is in 100 days. Historical volatility is estimated based on the stock returns of the previous 60 days. The LIBOR-OIS spread is in basis points. We report estimated $t$-statistics in parentheses based on standard errors that account for cross-sectional and time-series correlations using the method of Fama and MacBeth (1973). First, the regressions are estimated in each 3-month subsample. Second, the full-sample parameters are estimated as the sample means of the estimates and the standard errors are estimated based on the sample standard deviation of parameter estimates across subsamples, using the Newey-West correction.
Panel A: Logit

| Independent variables | (1) | (2) | (3) | (4) | (5) | (6) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1_{(S>B)}$ | 4.09 |  |  | 1.84 | 1.19 | 1.06 |
|  | (13.42) |  |  | (9.84) | (6.06) | (4.09) |
| $[(S-\underline{B}) / \underline{B}]^{+}$ |  | 2.13 |  | 0.45 | 0.39 | 0.40 |
|  |  | (5.98) |  | (2.72) | (1.30) | (1.37) |
| $1_{(\text {C_bid }<\text { S-X) }}$ |  |  | 5.23 | 4.46 | 3.92 | 3.96 |
|  |  |  | (20.74) | (17.93) | (23.47) | (29.31) |
| Moneyness |  |  |  |  | 0.03 | 0.05 |
|  |  |  |  |  | (0.32) | (0.65) |
| Short-sale cost score |  |  |  |  | 0.33 | 0.34 |
|  |  |  |  |  | (10.25) | (9.36) |
| Bid-ask spread |  |  |  |  | 0.26 | 0.25 |
|  |  |  |  |  | (3.19) | (2.86) |
| Time to expiration |  |  |  |  | -1.61 | -1.61 |
|  |  |  |  |  | (-6.20) | (-5.99) |
| Historical volatility |  |  |  |  | 0.73 | 0.71 |
|  |  |  |  |  | (5.50) | (5.36) |
| LIBOR-OIS spread |  |  |  |  | 0.25 | 0.25 |
|  |  |  |  |  | (0.99) | (1.00) |
| Intercept | -8.54 | -7.75 | -10.63 | -10.70 | -12.89 | -12.87 |
|  | (-81.10) | (-36.70) | (-53.54) | (-50.09) | (-6.25) | (-6.22) |
| Funding cost | LIBOR-OIS | LIBOR-OIS | LIBOR-OIS | LIBOR-OIS | LIBOR-OIS | Zero |
| Method | Logit | Logit | Logit | Logit | Logit | Logit |

## Panel B: Probit

| Independent variables | (1) | (2) | (3) | (4) | (5) | (6) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1_{(S>B)}$ | 1.29 |  |  | 0.62 | 0.42 | 0.38 |
|  | (11.71) |  |  | (9.54) | (5.91) | (4.15) |
| $[(S-\underline{B}) / \underline{B}]^{+}$ |  | 0.91 |  | 0.24 | 0.20 | 0.21 |
|  |  | (5.81) |  | (2.89) | (1.78) | (1.96) |
| $1_{(\text {(C_bid<S-K) }}$ |  |  | 1.45 | 1.22 | 1.15 | 1.16 |
|  |  |  | (17.04) | (17.85) | (18.13) | (21.07) |
| Moneyness |  |  |  |  | 0.03 | 0.04 |
|  |  |  |  |  | (0.82) | (1.51) |
| Short-sale cost score |  |  |  |  | 0.13 | 0.13 |
|  |  |  |  |  | (9.88) | (9.08) |
| Bid-ask spread |  |  |  |  | 0.12 | 0.11 |
|  |  |  |  |  | (6.44) | (5.73) |
| Time to expiration |  |  |  |  | -0.57 | -0.57 |
|  |  |  |  |  | (-6.00) | (-5.75) |
| Historical volatility |  |  |  |  | 0.24 | 0.24 |
|  |  |  |  |  | (6.71) | (6.59) |
| LIBOR-OIS spread |  |  |  |  | 0.06 | 0.06 |
|  |  |  |  |  | (0.79) | (0.79) |
| Intercept | -3.54 | -3.35 | -4.06 | -4.10 | -4.73 | -4.71 |
|  | (-128.80) | (-58.35) | (-85.96) | (-79.33) | (-7.19) | (-7.17) |
| Funding cost | LIBOR-OIS | LIBOR-OIS | LIBOR-OIS | LIBOR-OIS | LIBOR-OIS | Zero |
| Method | Probit | Probit | Probit | Probit | Probit | Probit |

are also significant, suggesting that the precise probability of exercise is a complicated function of the observable data. We expect differences across investors in terms of the frictions that they face, the frequency with which they observe the markets, and their estimate of future volatility. While some investors' exercise decision may be captured by the distance between the stock price $S$ and the boundary $\underline{B}$ that we calculate, other investors may exercise based on their individualspecific frictions, and this idiosyncratic variation may be partly picked up by the control variables. Of course, economic models of individual behavior are always less precise than those of prices (which aggregate individual noise) and, thus, these imperfections echo those in the literature on individual investors' refinancing of mortgage bonds. Still, our results support that early exercise decisions are driven by frictions in most cases, especially for market makers.

Lastly, we compute the exercise boundary for zero funding cost and repeat the analysis with the full set of control variables. This is done since using the LIBOR-OIS spread is an imperfect measure of funding costs, which does not vary in the cross-section. Our results continue to hold qualitatively in this specification, indicating the robustness of our results and that short-sale
frictions are important drivers.

### 1.4.1 A natural experiment: The short-sale ban of 2008

To further test the theory that early option exercises are caused by, and not only correlated with, frictions we consider exercise behavior during the short-sale ban in 2008. After the market closed on September 18, 2008, the U.S. Securities and Exchange Commission (SEC) introduced an emergency order that banned short-sale of certain stocks (SEC Release No. 34-58592). Boehmer, Jones, and Zhang (2013) provide a useful summary of the time line of the events. Initially, the ban applied to a list of 797 tickers for financial stocks (c.f. SEC's emergency order). On Sunday, September 21, SEC sent out an amendment to the emergency order (SEC Release No. 34-58611) by which the exchanges became responsible for adding and deleting stocks to the list of banned stocks such that the ban would include all financial firms, leading to around 200 additions and a few deletions.

The short sale ban serves as a shock to the short-sale frictions. Of course, the fall of 2008 was a volatile period with many events, but the fact that the ban only applied to certain firms allows us to control general events in this time period. Specifically, we compare the exercise behavior before and after the ban for, respectively, affected and unaffected firms (a so-called difference-indifferences approach often used to study causality). Our theory predicts that options on affected stocks should experience an increase in early exercise relative to unaffected stocks.

For this natural experiment, we obtain data from Nasdaq allowing us to identify every stock that was affected by the ban. We create an indicator variable for each option series taking the value one for options written on Affected stocks. Further, we create a Ban period indicator variable for the period when the ban was in effect (September 19 to October 8, both included).

To consider the effects of the ban on exercise behavior, we include these indicators and, importantly, their interaction in the logit and probit regressions. To estimate standard errors, we modify the Fama-MacBeth procedure used in Table 1.6 as follows. Focusing on September to October 2008, we randomly divide the sample into ten equal-sized subsamples of option-series/days, each large enough to include several observations for all four combinations of stocks being financial/nonfinancial and during/outside the ban period. We compute the model-implied exercise

## Table 1.7

Early exercise of equity options: Short-sale ban difference-in-differences.
This table shows logit and probit regressions that study whether the short-sale ban caused a rise in early option exercises for affected stocks. The short-sale ban applied to US financial stocks and serves as a natural experiment. The dependent variable is one for every option contract exercised early and zero for every contract outstanding at end of the previous day not exercised early. The "Ban period" and "Affected stock" are dummy variables for the ban period (Sep. 19 to Oct. 8, 2008) and the affected stocks, respectively. Using a difference-in-differences analysis, the key variable in the interaction: The positive, statistically significant "Affected stock $\times$ Ban period" supports that the short-sale ban increased early exercise for options written on affected stocks relative to options on unaffected stocks. $t$-statistics are in parentheses.

| Independent variables | $(1)$ | $(2)$ | $(3)$ | $(4)$ |
| :--- | ---: | ---: | ---: | ---: |
| $1_{(S>B)}$ | 1.43 | 1.49 | 0.54 | 0.58 |
|  | $(11.21)$ | $(8.88)$ | $(14.09)$ | $(11.79)$ |
| [S-B $^{\prime} /$ B $\left.^{+}\right]^{+}$ | 1.09 | 1.32 | 0.58 | 0.67 |
| $1_{\left(\mathrm{c} \_ \text {bid<s-x) }\right.}$ | $(4.66)$ | $(4.98)$ | $(5.83)$ | $(5.81)$ |
| Moneyness | 3.96 | 4.24 | 1.38 | 1.52 |
|  | $(9.64)$ | $(9.18)$ | $(10.46)$ | $(9.62)$ |
| Aug 08 short-sale cost score | -0.16 | -0.32 | -0.11 | -0.17 |
|  | $(-0.96)$ | $(-1.63)$ | $(-1.75)$ | $(-2.19)$ |
| Bid-ask spread | 0.31 | 0.31 | 0.12 | 0.12 |
|  | $(13.41)$ | $(13.22)$ | $(11.90)$ | $(11.64)$ |
| Time to expiration | 0.22 | 0.15 | 0.13 | 0.09 |
|  | $(1.05)$ | $(0.92)$ | $(1.46)$ | $(1.26)$ |
| Historical volatility | -1.58 | -1.77 | -0.59 | -0.66 |
|  | $(-9.95)$ | $(-8.30)$ | $(-9.75)$ | $(-8.06)$ |
| Affected stock | -0.34 | -0.28 | -0.08 | -0.06 |
|  | $(-2.30)$ | $(-1.90)$ | $(-1.51)$ | $(-1.21)$ |
| Ban period | 0.60 | 0.74 | 0.26 | 0.31 |
|  | $(2.04)$ | $(2.58)$ | $(1.88)$ | $(2.40)$ |
| Affected stock $\times$ Ban period | 0.49 |  | 0.17 |  |
|  | $(3.12)$ |  | $(2.87)$ |  |
| LIBOR-OIS spread | 1.19 | 1.21 | 0.54 | 0.51 |
|  | $(2.95)$ | $(3.16)$ | $(3.34)$ | $(3.44)$ |
| Intercept | 0.00 |  | 0.00 |  |
| Date fixed effect | $(-1.21)$ |  | $(-1.39)$ |  |
| Method | Yes | No | Yes | No |

boundary based on the ex ante short-sale costs measured at the end of August 2008 (and we also use this ex ante short-sale cost as a control variable). We use this ex ante short-sale cost to avoid having our results be driven by confounding effects due to the ban's effect on the securities-lending market.

Table 1.7 reports the results of the regressions. The key coefficient is the coefficient on Affected stock $\times$ Ban period. The fact that this coefficient is positive supports that early exercise of options on banned stocks increased during the ban period, relative to early exercise of options on nonbanned stocks. This serves as evidence that increased short-sale frictions causes an increase in early exercises. We find this result both with probit and logit regressions and both with and without date fixed effects.

### 1.4.2 Alternative reasons for early exercise

Lastly, we consider alternative potential drivers of early exercise such as corporate events. To begin the search for alternative explanations, we first consider the top five observations where market makers exercise options when the stock price is below the most conservative boundary, ranked on the number of contracts exercised on a given day for a given series (recall that market makers exercise is consistent with the model in $98.4 \%$ of the cases as seen in Table 1.5.A, so we study the top five cases among the remaining $1.6 \%$ ).

Interestingly, three of the five cases are associated with corporate events. ${ }^{10}$ Corporate events

[^11]could be important for two reasons: first, they could simply affect the parameters of the model, e.g., lower the volatility, which would lead us to mis-estimate the exercise boundary. More worryingly, corporate events could drive early exercise for reasons unrelated to the model. To address these concerns, we have repeated our analysis in the subsample that excludes all option-days when the underlying stock experiences a corporate event according to the Thomson One database. In particular, we exclude data around equity issues (from 30 days before to 50 days after or settlement date, whichever comes first) and around mergers and acquisitions for both the target stocks and the acquiror (from five days before the announcement until the offer is effective or withdrawn, or, if no end date is observed, until 180 days after the announcement).

Table 1.5.B shows that our model explains the early exercise behavior better in the subsample that excludes corporate events. Also, we have repeated our logit/probit regressions in the subsample that excludes corporate events, and the results do not change qualitatively (available upon request from the authors). Hence, it appears that corporate events could create noise as an alternative driver of early exercise, but it does not appear to be driving our results (as an omitted variable).

In addition to equity issuance and mergers and acquisitions, using shares for voting could be a consideration. However, rather than exercising an option, shares can in principle be obtained in the securities-lending market, and, therefore, the model should capture this effect via the lending fees and short-sales costs. Further, Christoffersen, Geczy, Musto, and Reed (2007) find that "the average vote sells for zero," suggesting that this is not a major driver of our results. Also, we expect that a number of the important voting events are excluded by the filter applied above.

A final reason for early exercise could be that investors are not fully rational as shown by Gay and Manaster (1986) and Poteshman and Serbin (2003). This could help explain why some options are exercised even when the stock price is below the model-implied boundary. However, irrationality does not appear to be the whole picture as seen from the model's explanatory power and the natural experiment of Section 1.4.1. Further, the fact that early exercise is also observed by market makers who are professional investors likely to make rational decisions also suggests that a number of early exercise decisions are driven by frictions.

Gay et al. (1989) address transaction costs and the opportunity to exercise after the closing of the futures market. They study options where exercise can be optimal even without frictions so, in their case, transactions cost and market closure can change the optimal exercise time. We show that transaction costs and closed stock exchanges alone are not sufficient to rationalize early exercise of call options although transaction costs do play a role in combination with other frictions (cf. Proposition 1 and Proposition 2). ${ }^{11}$

In summary, there could be alternative reasons for early exercise such as corporate events, but, within the limits of any empirical study, our results suggest that frictions constitute a separate driver of early exercise.

### 1.5 Empirical results: Never convert a convertible?

We next consider how early conversions of convertible bonds are related to financial frictions. Table 1.8 reports our results. We see that early conversions are more frequent among companies with large short-sale costs for the equity, consistent with the theory.

The table breaks down the conversions by the moneyness of the convertible bonds. We consider a convertible bond to be out-of-the-money if the price of the underlying stock is less than conversion price, in-the-money if stock price is up to $25 \%$ above conversion price, and deep-in-the-money if stock price is more than $25 \%$ above conversion price. As expected, we see that conversions are concentrated among in-the-money and deep-in-the-money convertibles. We see a few conversions of out-of-the-money bonds, which relates to the definition of moneyness discussed in Section 1.3.2.

More importantly, we see that conversion rates increase monotonically in short-sale costs for deep-in-the-money convertibles, providing further evidence consistent with the theory. However, we note that the difference across groups is not statistically significant due to the small and noisy data set.

[^12]
## Table 1.8

Early conversion of convertible bonds.
This table shows the average amount of early conversion as a fraction of the outstanding amount for convertible bonds sorted on the short-sale costs of the underlying equity, 2003-2011. The observations are grouped into Low, Medium, and High cost of shorting based on the DCBS score from Data Explorers. "Low" cost reflects a DCBS of 1, "Medium" a score of $2-5$, and "High" 6-10. The table further classifies convertible bonds by their moneyness. Convertible bonds are defined as "out of the money" when stock price is below conversion price, "in the money" when stock price is at or up to $25 \%$ above the conversion price, and "deep in the money" when stock price is more than $25 \%$ above conversion price. The conversion price defines the amount of face value of bond that must be converted to obtain one share of the underlying stock. Standard errors are reported in parentheses.

|  | 1 <br> Low cost of <br> shorting | 2 | High cost of <br> shorting | $3-1$ |
| :---: | ---: | ---: | ---: | ---: |
| Panel A: All | $0.09 \%$ | $0.08 \%$ | $0.16 \%$ | $0.07 \%$ |
| Panel B: By moneyness | $(0.02 \%)$ | $(0.04 \%)$ | $(0.08 \%)$ | $(0.09 \%)$ |
| Out of the money |  |  |  |  |
|  | $0.03 \%$ | $0.00 \%$ | $0.00 \%$ | $-0.03 \%$ |
| In the money | $(0.02 \%)$ | $(0.00 \%)$ | $(0.00 \%)$ | $(0.02 \%)$ |
|  | $0.09 \%$ | $0.08 \%$ | $0.16 \%$ | $0.07 \%$ |
| Deep in the money | $(0.05 \%)$ | $(0.08 \%)$ | $(0.13 \%)$ | $(0.13 \%)$ |
|  | $0.17 \%$ | $0.19 \%$ | $0.36 \%$ | $0.19 \%$ |
|  | $(0.04 \%)$ | $(0.10 \%)$ | $(0.22 \%)$ | $(0.23 \%)$ |

### 1.6 Conclusion: Never say never again

A classic rule in financial economics states that, except just before expiration or dividend payments, one should never exercise a call option and never convert a convertible bond. This rule is ubiquitous in option theory and taught in most introductory finance classes. We show that this rule breaks down - theoretically and empirically - when financial frictions are introduced, just as frictions break the Modigliani-Miller Theorem, the Law of One Price, and other classic rules in financial economics.

Our theory shows that early exercise of options can be rational in light of financial frictions and, indeed, we would expect early exercise to occur. Consistent with our theory, the empirical propensity to exercise equity options is increasing in the short-sale costs, transaction costs, and moneyness, and decreasing in the time to expiration. We find that options are exercised by customers of brokers, market makers, and firm proprietary traders and, for each group, exercises are more prevalent when the financial frictions are more severe. Our model further implies that it can be optimal to convert a convertible bond. We document a number of early conversions of convertible bonds, especially among stocks with high short-sale costs.

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## Chapter 2

## Early Option Exercise Predicts Stock

## Returns


#### Abstract

In a world with frictions, a call option owner's wish to reduce stock exposure can induce early exercise. If the wish to reduce exposure is based on negative information about the stock, then early exercise provides a negative signal for future stock returns. Consistent with this idea, I find empirically that early exercise predicts stock returns negatively. The information content of early exercise is not found to significantly depend on whether a customer of broker, a firm, or a market maker performs the exercise.


### 2.1 Introduction

Do any market participants have private information? If so, how is that information brought to the market place? These fundamental questions arise in economics in general and finance in particular. This paper shows that option owners possess private information and act on it in the form of exercising call options early. Also, no significant differences are found in the information content of early exercises by customers of brokers, firms, and market makers. To identify the private information content of early exercises, the paper looks at the abnormal returns of the underlying stocks in the days after options written on them are exercised early. Consistent with early exercise being due to negative information about the stock among the option owners, these abnormal returns
are negative.
Frictions can justify exercising call options early, i.e., before expiration and at times when the stock pays no dividends (Avellaneda and Lipkin, 2009; Jensen and Pedersen, 2016). Jensen and Pedersen (2016) provide the theoretical framework for the predictions made in this paper. If the option owner is short the underlying stock, then the incentive to exercise is larger than if the agent is long the stock. When selling a stock from a long position the seller forgoes potential income from lending out the stock. When selling stock from a zero or negative position, the seller incurs a short-sale cost. Both lending fees and short-sale costs can make early exercise attractive if the option cannot be sold above intrinsic value. The stock obtained upon early exercise can generate income by being lent out or reduce costs by covering a short position. However, the short-sale cost often is larger than the lending fee due to costs paid to intermediaries and search costs (Duffie, Gârleanu, and Pedersen, 2002). Thus, an option owner who is short the underlying stock has a larger incentive to exercise early than an option owner who is long the underlying stock.

To reduce exposure to the underlying stock due to private information, an option owner could sell the underlying stock. If selling off the stock leads to the stock position becoming zero, or if the stock position is zero in the first place, early exercise increases in attractiveness if further reductions in stock exposure are wanted and the short-sale cost is larger than the lending fee. Now, instead of forgoing the lending fee, a larger short-sale cost must be paid. The discontinuity in the marginal cost of selling stock when the stock position is zero results in a shift in the incentive to exercise early. Hence, if option owners want to reduce stock exposure due to private information, early exercise can be informative about future stock returns.

Just like short-sale costs, funding costs can provide a link between private information and early exercise. Suppose the interest rate earned on the margin account is inferior to the risk-free return an option owner can obtain elsewhere. Then, posting margin becomes costly. Selling stock increases (reduces) margin requirements if the option owner is short (long) stock. Similar to the positive spread between short-sale costs and lending fees, a larger incentive exists to exercise early if the option owner is short the underlying stock relative to long. Early exercise can be induced if the option owner wants to reduce stock exposure based on private information. Instead of starting
to sell the stock short, the option owner can choose to exercise early and sell the obtained stock. This way, margin requirements and, thus, funding costs are reduced. Regardless of the channel being short-sale costs, funding costs, or a combination of both, negative private information can lead to early exercise.

Black (1975) conjectures that informed traders choose to trade in the option market due to embedded leverage. Easley, O'Hara, and Srinivas (1998) support this idea, deriving a pooling equilibrium for the option market and the market for the underlying stock in which option trades are informative about future stock prices. Their empirical findings, as well as those of Kacperczyk and Pagnotta (2016), are that option trades contain private information.

The private information content of early exercise is detected by lower abnormal returns of the underlying stocks in the days after early exercise. Because an exercise observed on a specific day might have happened after the stock exchange closed that day, endogeneity issues arise with looking at the stock return one trading day after the early exercise. Hence, the focus is on the abnormal return from two trading days up to ten trading days after. Abnormal returns are calculated based on both a one-factor capital asset pricing model (CAPM) and a four-factor model, both with ex ante estimated betas. The four factors are the three Fama-French factors (excess market returns (MKT), small market capitalization minus big (SMB), and high book-to-market ratio minus low (HML)) and the momentum factor (up minus down (UMD)).

The abnormal returns are observed in event time with events being the 164,846 unique stockdays with early exercises. The cumulated four-factor abnormal return for two to ten trading days after early exercise is significantly negative consistent with option owners exercising based on private information. The one-factor abnormal return is negative, but insignificant. The four-factor abnormal return stays significantly negative even after adding a conservative (i.e., likely upwardbiased) measure for short-sale cost. The negative abnormal return indicates that shorting the stock after early exercise and hedging the exposures to the four factors is attractive, consistent with negative information giving rise to early exercise.

To further test for private information, I construct calendar time strategies for the sample period from 2003 to 2010. Stocks are shorted after options written on them are exercised early, and the
exposures to the market factor or the four factors are hedged based on ex ante betas. For the strategy in which stocks are shorted two trading days after early exercise and the position hedged based on ex ante four-factor loadings, a positive four-factor daily alpha of $0.05 \%$ is observed with a $t$-statistic of 3.38. The yearly Shape ratio is 1.26 . After subtracting a conservative measure for short-sale cost, the Sharpe ratio is 0.76 . The attractive returns cannot be obtained in reality because market participants cannot observe option exercises. But the attractive returns are evidence of private information leading to early exercise.

The relationship between negative private information and early exercises is also validated using Fama-MacBeth regressions in which the dependent variable is abnormal return of the underlying stock and the independent variables are dummy variables for early exercises on the previous trading days. Also in this setup, I find significant under-performance of stocks two and three trading days after early exercise without seeing stock prices rebounding on consecutive days. This pattern holds when abnormal returns are calculated based on both a one-factor CAPM and the four-factor model.

Whether a customer of a broker, a firm, or a market maker performs an early exercise can be determined. Utilizing this fact, three separate Fama-MacBeth regressions are run, in which the indicator variable is one only if the given agent type exercised early. The regressions do not result in significant differences between the three types. The only type for which the loading on the indicator variable is significantly negative is customer of a broker. This is not surprising given that the number of stock-days with early exercise for customers is more than double the number for firms and market makers combined.

The stock-days with early exercise can also be sorted into groups based on the number of contracts exercised. The stock-days are sorted into three groups: stock-days for which one to ten contracts were exercised, stock-days with 11-100 exercises, and stock-days with more than 100 exercises. Dummy variables are created for each group. The dummy is one if the number of contracts exercised is in the given interval on the stock-day and zero otherwise. Then, a FamaMacBeth regression is performed for each group. The dummy variables for the group with 11-100 contracts exercised have the most economically and statistically significant estimates for early
exercise two and three trading days ago. The differences between the three groups is, however, not statistically significant.

Robustness checks show that the results are not driven only by stocks with low or high market capitalization. Also, option exercises are sorted by moneyness and time to expiration. The trend seems to be that exercises of options with a longer time to expiration are stronger predictors of consecutive returns. This is consistent with option owners requiring a stronger private signal to compensate for the higher optionality lost through exercise. Finally, it is shown that stock reversal cannot explain the abnormal calendar time returns of the strategies used to test for private information.

Several other papers explore the private information content of different derivative measures. Pan and Poteshman (2006) provide evidence of informed trading in the put and call option markets using data allowing them to directly distinguish between buyer- and seller-initiated trades. Prior to days with important news, option trades predict future underlying stock returns, as shown by Amin and Lee (1997) for earnings announcements and Cao, Chen, and Griffin (2005) for announcements of takeovers. Several other papers explore the relationship between information, trades, and prices for options and underlying stocks. Stephan and Whaley (1990) provide evidence from intra-day data that stock price changes lead option price changes, and Chan, Chung, and Fong (2002) provide evidence consistent with informed traders being active in the stock market, but not in the option market. Johnson and So (2012) show that the option to stock volume ratio is informative about future returns, and Fodor, Krieger, and Doran (2011) find that large changes in open interest have predictive power for future stock returns, with their results being stronger for call options than for put options. ${ }^{1}$ My paper, to my knowledge, distinguishes itself from the previous literature by examining how early exercise decisions (instead of trades or quotes) may reflect private information about the value of the underlying stock.

[^13]
### 2.2 Theoretical motivation

This section provides the theoretical motivation for the idea that private information among option holders would make early exercise a negative predictor for future stock returns. Classic theory by Merton (1973) says that one should exercise call options only on the day of expiration or on the day before an ex-dividend date. Jensen and Pedersen (2016) show that this rule breaks down, both theoretically and empirically, when frictions are severe enough. Jensen and Pedersen (2016) provide the theoretical framework that yields the mechanism of my paper, namely, that early exercise can be the optimal way to reduce exposure to the underlying stock. They introduce frictions in the form of funding and short-sale costs of the underlying stock as well as option transaction costs. Their theory provides the insight that it might be dominated to exercise early in some states of the world and dominated not to exercise early in others if the option cannot be sold at or above intrinsic value. ${ }^{2}$ Yet, the theory also leaves room for states in which the optimal decision depends on the agent's exposure to the underlying stock. If the agent is long stock, the critical stock price at which the option is optimally exercised is the same or higher than if the agent is short stock. In other words, an upper and a lower exercise boundary exist for the option, leaving a channel through which private information can be conveyed. For an option owner who has a positive holding of the underlying stock the non-dominated way to reduce stock exposure when the stock price is between the two boundaries is to sell stock. This dominates exercising the option early. But when the option owner no longer holds any of the underlying stock, and if the option cannot be sold above intrinsic value, a non-dominated way to reduce stock exposure is by exercising the option early and selling the underlying stock. This early exercise strategy dominates reducing stock exposure by selling short the underlying stock. Only after exercising the option position and selling the delivered stocks, selling short stock is not dominated.

The difference between the upper and lower exercise boundary exists due to the difference between being short and long stock. When the cost of borrowing and selling a stock short is larger

[^14]than the fee obtainable from holding and lending out the stock, the upper and lower boundary are different from each other. Also, funding costs can introduce the wedge between the boundaries. When being long (short) stock, an extra stock increases (reduces) the margin requirement. An increase (reduction) of the margin requirement is a cost (gain) to an agent with positive funding costs, i.e., an agent who has alternative risk-free investment opportunities that generate a higher return than the margin account. Fig. 2.1 illustrates a lower boundary that describes the optimal exercise policy for an agent who is short stock, and an upper boundary that outlines the optimal exercise policy for an agent who is long stock. In between is the region $F$ in which the optimal exercise policy depends on the agent's position in the underlying stock. If the agent owns zero of the underlying stock, exercising early and selling the obtained stock is not dominated if the option cannot be sold above intrinsic value. The lower exercise boundary in Fig. 2.1 is based on the modified Black-Scholes-Merton partial differential equation (PDE) derived by Jensen and Pedersen (2016), here simplified to the case of one agent $i$ with no funding costs:
\[

$$
\begin{equation*}
C_{t}+\frac{1}{2} \sigma^{2} S^{2} C_{S S}-r^{f} C+C_{S} S\left(r^{f}-L^{i}\right)=0, \tag{2.1}
\end{equation*}
$$

\]

where $\sigma=30 \%$ is the volatility of the underlying stock, $r^{f}=2 \%$ is the risk-free rate, and $L^{i}=2 \%$ is the proportional continuous short-sale cost for agent $i$, all in yearly terms. $S$ is the stock price, $C$ is the option value, and subscripts denote derivatives with respect to the given variable. The PDE with the following boundary conditions is a well-posed free-boundary problem (Merton, 1973):

$$
\begin{array}{rlr}
C(T, S) & =\max (S-X, 0) & \\
C(t, 0) & =0 & t<T  \tag{2.2}\\
C(t, S) & =S-X & S \geq B(T-t), t<T \\
C_{S}(t, B(T-t)) & =1 & t<T,
\end{array}
$$

where $B(\tau)$ is the optimal exercise boundary, with $\tau$ being time to expiration. The solution provides the boundary and option value and is derived using a finite difference Crank-Nicolson method. The upper bound is derived equivalently, the only difference being substituting the short-sale cost
$L^{i}=2 \%$ with the stock lending fee $1 \%$.


Figure 2.1. Lower and upper boundaries for optimal early exercise. When the stock price is in region $G$, exercising the option early is never optimal. If the option cannot be sold at or above intrinsic value, not exercising the option early when the stock price is in region $E$ is never optimal. In region $F$, it is optimal to exercise early for an agent who is short the underlying stock, and it is not optimal for an agent who is long the underlying stock. The boundaries are derived as solutions to the free boundary problem in Eqs. (2.1)-(2.2) with the following parameters: strike price $X=10$, volatility $\sigma=30 \%$, and risk-free rate $r^{f}=2 \%$. The boundaries differ through $L^{i}$, which for the lower bound represents a short-sale cost of $2 \%$ and for the upper boundary represents a stock lending fee of $1 \%$. Region $F$ consists of the states in which a change in desired exposure to the stock by the option owner may optimally lead to early exercise.

In this case, the funding cost is zero, which means that the positive spread between lending fee and short-sale cost yields the entire difference between the exercise boundaries. Above the upper boundary, the agent is better off exercising early even when long stock if the option can be sold only below intrinsic value. The reason for this is that the earned lending fee from one extra stock exceeds the cost of exercising early, which consists of both lost optionality and forgone interest on the strike price $X$. Below the lower boundary, the immediate cash flow from hedging the option by shorting the stock is higher than the intrinsic value of the option, implying that it is dominated to exercise early. In between, the immediate cash flow from hedging the option by shorting the stock is smaller than the intrinsic value, meaning that exercising early is optimal if the option cannot be
sold above intrinsic value and the agent is short stock. But if the option is hedged by reducing a long stock position, the immediate cash flow exceeds the intrinsic value, meaning that an agent long stock should not exercise the option early. If an option owner holds no stocks and cannot sell the option at or above its intrinsic value, and the stock price is in the region $F$ between the boundaries, then the option owner should exercise early and sell the delivered stock if he wants to reduce the positive exposure to the underlying stock that the option entails. If the option owner is satisfied with the positive exposure that the option entails, he should not exercise early. That the early exercise decision depends on whether the option owner wants to reduce stock exposure or not provides the mechanism through which early exercise can convey private information.

If an option owner changes the desired exposure to the stock when the stock price is between the lower and upper bound for early exercise, early exercise can result if the change is negative. Such a change may reflect private information that the stock from this point will under-perform in the future. It could also happen due to a change in risk preferences by the option owner. Exercises not motivated by private information add noise to the information signal of early exercise in the empirical analyses.

Early exercise also may optimally happen at the two boundaries. An option owner who is short stock and does not want to change exposure should optimally exercise the option at the lower boundary if the option cannot be sold for the intrinsic value or more. The early exercise could signal negative, private information about the stock, as it reflects that the option holder has a negative stock position. However, this signal is conjectured to be weaker than the signal from exercises in between the boundaries for two reasons. First, the choice of portfolio might not reflect private information, but risk preferences, e.g., a wish to hedge the option. In that case, the exercises just add noise to the analysis. Second, the time of the exercise does not reflect the time of change of perspective on the stock, but the time when the stock price reached the boundary.

Likewise, for exercises at the upper boundary, the wish to keep the long exposure to the stock might reflect positive, private information, but it might also just reflect risk preferences. Again, the signal is conjectured to be relatively weak because the timing of the exercise is not related to the private information. Further, per construction, the stock price at some point could be above
the lower boundary without it ever going above the upper boundary, while the opposite never happens. In such cases, no early exercises at the upper boundary, which might contain private positive information about the stock, are observed, and exercises at the lower boundary, which might contain private negative information about the stock, are possible. The upper boundary might very well be far from the lower boundary, leaving a large room for early exercises coinciding with the emerging of a wish to reduce exposure to the underlying stock. In the realistic case in which the option owner has a positive short-sale cost but earns nothing from lending out the stock and has no funding costs, the upper boundary would be infinite and the lower boundary would be finite. Overall, the conjecture is that if early exercises reflect private information, the negative signal dominates on average.

The existence of such a region between the boundaries does not depend on the modified Black-Scholes-Merton model being true. The model simply makes possible to illustrate an example. As long as frictions result in the net cost of shorting stock being larger than the net cost of selling off stock from a long position, the driver that allows early exercise to convey private information is in place.

Further, an interaction between private information among option owners and bid price of the option can strengthen early exercise as a negative predictor of early exercise. Private information is optimally reflected in early exercise only when the bid price of the option is not above its intrinsic value. Otherwise, the option owner would be better off selling the option. At the same time, a high level of private information in the option market can itself drive down the bid price (see, e.g., Glosten and Milgrom (1985)). The intuition is that, in equilibrium, market makers protect themselves against informed traders by a bid-ask spread, a spread that all else equal is increasing in the average content of private information in the trades.

### 2.3 Data and methodology

This section describes the data sources, explains the sample selection, provides summary statistics, and presents different measures for abnormal returns.

### 2.3.1 Data sources

Data derive from five main sources: (1) data on option exercises from Options Clearing Corporation (OCC), (2) data on daily stock returns and distribution events from the Center for Research in Security Prices (CRSP), (3) daily option data and information on expected future dividends for stocks from OptionMetrics, (4) data on short-sale cost for stocks from DataExplorers, and (5) daily returns of the three Fama-French factors and the momentum factor are downloaded from Kenneth French's data library (KFDL). ${ }^{3}$

The OCC data with observations of option exercises contain the number of contract exercises for each option series each day. The daily exercises can be separated into three groups of market participants: exercises done by customers of brokers (e.g., retail customers and hedge funds), firm proprietary traders, and market makers. The option exercise data run from July 2001 to August 2010. The data are missing in the months of November 2001, January and July 2002, and January 2006. The daily option data from OptionMetrics include a unique option identification, date of expiration, underlying stock, and an indicator for options with special settlement. The daily data from CRSP contain distribution events with ex-dividend dates and the daily stock returns. The data on short-sale costs from DataExplorers provide an integer Daily Cost of Borrow Score (DCBS) from one to ten for a stock identified by CUSIP for a given period, with one indicating a low short-sale cost, and ten a high cost.

### 2.3.2 Sample selection

The approach used by Jensen and Pedersen (2016) is replicated to create a sample of early exercises of call options. Option series are excluded if OptionMetrics reports a nonzero forecast of future dividends during any day of the lifetime of the option. Further, observations of option series are excluded on the day of expiry to focus on early exercise. Lastly, options that do not follow the standard practice of having an expiration date on the day after the third Friday in a given month are excluded (this excludes only a tiny fraction). The data from OptionMetrics are merged with the

[^15]exercise data from OCC on date, ticker, option ticker, strike price, and expiry date. Excluded are any option series for which the underlying stock according to CRSP has ex-date for a distribution event (split, dividend payment, exchanges, reorganizations, etc.) within the observed lifetime of the option series; for which the underlying stock at some point according to OptionMetrics was expected to have an ex-dividend date within the observed lifetime of the option series; for which records exist with different strike prices for the same series, different underlying identifiers (Security ID or CUSIP), or different expiry dates (indicating data errors or changes in the contract); for which no data are available on the last trading day before expiry (indicating some possible outside event); for which several records are for the same day in OptionMetrics; for which another option series exists in the data with the same underlying stock ticker, option ticker (root of option symbol for old option symbol and first part of symbol for new Options Symbology Initiative (OSI) symbol), strike price, and expiry date observed on the same day; for which settlement special, e.g., AM-settlement, has been established; or for which some day during its observed lifetime no matching observation has been made in the CRSP data for the underlying stock.

Short-sale costs are relevant to include when performing empirical tests for stock returns. The short-sale costs are observed in the data from DataExplorers using the Daily Cost of Borrow Score (DCBS) which is first observed on October 22, 2003. The focus of the study is on returns up to ten trading days after an early exercise, so the sample is restricted to include exercise observations only from ten trading days before October 22, 2003. Thus, the first day in the sample is October 8, 2003.

### 2.3.3 Summary statistics

Table 2.1 provides summary statistics for the final sample of early exercises. Panel A reports the number of contracts exercised early and shows that most of the early exercises are performed by either customers of brokers or market makers. Panel B shows that in total 3,706 different stocks experience being the underlying stock of an early exercise in the sample. Almost all of the affected stocks, 3,646 , experience being the underlying stock of an option exercised by the customer of a broker. Panel C summarizes the number of stock-days with early exercise. For every early exercise,
the underlying stock and the date of the exercise are combined. The total set of these combinations with duplicates removed make up the set of stock-days with early exercise. The set of stock-days can be observed both for all early exercises and for each of the three agent types. Stock-days with early exercise are particularly relevant for the empirical study, as they are the events that potentially reveal private information among option holders. In total, 164,846 stock-days have early exercise, a number large enough to make comprehensive empirical tests. Agents who are registered as customers of brokers have the most stock-days with early exercise, and almost half of all stock-days with early exercise have ten or fewer contracts exercised early.

### 2.3.4 Calculation of abnormal returns and trading strategy returns

Daily returns for individual stocks from CRSP are used to estimate both one-factor and four-factor betas. For every stock at the end of every month, the betas are estimated based on observed daily returns for the previous year if more than 20 daily returns are observed for this period. The KFDL contains daily data for excess market returns (MKT.RF which I call MKT in this paper), the small-minus-big (SMB) factor, the high-minus-low (HML) factor, the momentum (UMD) factor, and the risk-free return (RF). The excess stock return for an individual stock is observed by subtracting the risk-free return from the observed stock return from CRSP. By ordinary least sqaures (OLS) regression, the one- and four-factor betas are then estimated. The daily abnormal returns of the individual stocks can now be estimated by subtracting from the stock return the expected return in the one- and four-factor model, respectively, using the estimated betas from the previous month. Flipping the sign of the abnormal returns, they can also be thought of as one factor-hedged and four factor-hedged returns of a short trading strategy, respectively, because the returns can be realized by shorting the stock and hedging the position according to the ex ante estimated betas.

A possible endogeneity issue arises from short-sale costs. These costs are a driver of early exercise (Jensen and Pedersen, 2016) and they are associated with lower stock returns (Jones and Lamont, 2002; Ofek, Richardson, and Whitelaw, 2004). Therefore, short-sale costs are taken into account in several of the empirical analyses. To estimate the short-sale cost of the individual stock, data from DataExplorers are used. The data contain a DCBS for individual stocks, which is an

## Table 2.1

Summary statistics for early exercises.
This table shows summary statistics for the sample of early exercises. The sample starts October 8, 2003, and ends August 31, 2010, with data missing for January 2006. Filters are applied to exclude exercises at option expiration or related to dividends. Panel A shows the total number of contracts exercised early, and the number is broken down by agent type: customers of brokers, firms, and market makers. Panel B presents the number of different underlying stocks, identified using PERMNO from the Center for Research in Security Prices, for which an option at some point is exercised early. Panel C summarizes the stock-days, i.e., the set of unique combinations of underlying stock and date that experience an early exercise. Both the total number and the numbers for the three agent types are reported. The stock-days with early exercise are also grouped according to the number of contracts exercised on the given day for the given stock. For Panel B and Panel C, the number for "All" is smaller than the sum of the numbers for the three agent types because more types of agents may exercise options with the same underlying stock on the same day.

|  | All | Customer | Firm | Market maker |
| :---: | :---: | :---: | :---: | :---: |
| Panel A: Number of early exercises |  |  |  |  |
| Contracts (thousands) | 17,516 | 7,670 | 742 | 9,104 |
| Panel B: Number of underlying stocks affected by early exercise |  |  |  |  |
| Stocks | 3,706 | 3,646 | 1,960 | 2,085 |
| Panel C: Number of stock-days with early exercise |  |  |  |  |
| Total | 164,846 | 138,299 | 11,708 | 39,105 |
| By number of contracts exercised |  |  |  |  |
| 1-10 | 82,241 | 76,090 | 7,155 | 12,996 |
| 11-100 | 61,774 | 50,131 | 3,655 | 16,825 |
| > 100 | 20,831 | 12,078 | 898 | 9,284 |

integer score from one to ten, going from low to high short-sale costs. For a small fraction of the sample, the same stock has two or more different DCBSs on the same date. For these cases, the average is taken and rounded to the nearest integer.

To be able to match a DCBS with a short-sale fee, the subsample for which these fees are observed is utilized. For each of the ten DCBSs, the 90th percentile of the short-sale cost is found. Whenever only the DCBS, but no actual cost, is observed, the 90th percentile fee is used as a conservative estimate. The fee is used if directly observed. If more than one fee is observed for the same day and stock, the highest of the observed fees is used as a conservative (i.e., likely upward-biased) estimate. $L_{t}^{i}$ denotes the conservative estimate of the short-sale fee. The unit for this short-sale cost is the percentage of the stock value that must be paid per year. To measure the cost of shorting the stock from the previous trading day $t^{-}$to the current trading day $t$, let $M_{t}$ be defined by

$$
\begin{equation*}
M_{t}^{i}:=\left(1+L_{t^{-}}^{i} \frac{\frac{1-t^{-}}{365}}{365},\right. \tag{2.3}
\end{equation*}
$$

where $t-t^{-}$is the number of calendar days between day $t^{-}$and $t$. In the rest of the paper, when a return is reported as including short-sale cost, $M_{t}^{i}$ has been added to the return from day $t^{-}$to day $t$, be it the raw observed stock return or the abnormal return. Thus, negative abnormal returns with short-sale costs can be thought of as positive gains if the stock is shorted and the exposures to the relevant factors are hedged.

### 2.4 Empirical results: How exercise predicts returns

Three main methods are used to test the predictions for performance of the stock after early exercise: (1) event studies of the abnormal returns of the underlying stock after early exercise, (2) calendar time returns of trading strategies based on previous early exercises, and (3) Fama-MacBeth style regressions with abnormal stock returns as the dependent variable and dummy variables for previous early exercises as independent variables.

### 2.4.1 Event studies of abnormal returns after early exercise

Table 2.2 shows the average daily abnormal one-factor and four-factor returns of stocks in event time after options written on them have been exercised early. Both abnormal returns are reported with and without short-sale costs. The daily returns are measured from stock exchange close the previous day to stock exchange close the present day. Not surprisingly, the average abnormal returns on the day of early exercise are positive and highly significant. An increase in stock price might be what triggers the early exercise, because the incentive to exercise early is increasing in the moneyness of the option. For the first trading day after the day of early exercise, an endogeneity issue remains, as an option exercise on a given day could have happened after the close of the stock exchange that day. If information relevant to the stock price becomes public after the stock exchange closed, the option exercise may reflect the content of this information, not private information. However, if the option owner exercises the option after the stock exchange closed on a given trading day and sells the stock immediately, price pressure on the stock may be the result, bringing reduced return the next trading day. Further, the early exercise might induce trading in the underlying stock by the counterpart to whom the exercise is assigned. The net result of these effects is not obvious, but using the abnormal return of the first trading day after exercise as evidence of the early exercise revealing private information is hard. Table 2.2 shows that the returns on this day on average are insignificant for three out of four measures, and they are significantly negative when using abnormal four-factor returns including short-sale costs.

The most natural single day to look at to identify the possible private information content of the early exercise is two trading days after the early exercise. This day is the closest to the early exercise that is not subject to the endogeneity of possible early exercise after the close of the stock exchange. And the price impact of the option owner selling the stock after obtaining it is present only if the option owner waits one day extra before selling the stock. This would be hard to justify if the motivation for exercising early is to reduce the stock exposure. ${ }^{4}$ Table 2.2 shows that the four-

[^16]
## Table 2.2

Abnormal returns of underlying stock after early exercises in event time.
This table shows means and $t$-statistics (in parentheses) for abnormal returns of stocks for a varying number of trading days after an option with the stock as underlying has been exercised early (event time). The abnormal returns are calculated using a one-factor capital asset pricing model and a four-factor model. The four factors are the three Fama-French factors (excess market returns (MKT), small market capitalization minus big (SMB), and high book-to-market ratio minus low (HML)) and the momentum factor (up minus down (UMD)). Short-sale costs are added to both to reflect the abnormal returns for an agent who follows a short strategy and incurs such costs. The short-sale costs are estimated based on an observed Daily Cost of Borrow Score (DCBS) from DataExplorers. Missing observations for DCBS mean that the samples with and without short-sale costs are not identical. The last four rows show the means and $t$-statistics for the cumulative sums of abnormal returns over a range of days and include only observations for which the abnormal returns are observed for all days within the range. Table 2.1 contains sample descriptive characteristics.

| Event day | One-factor alpha | One-factor alpha including short-sale cost | Four-factor alpha | Four-factor alpha including short-sale cost |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 1.14\% | 1.15\% | 1.08\% | 1.09\% |
|  | (89.25) | (89.96) | (85.08) | (85.79) |
| 1 | -0.01\% | 0.01\% | -0.03\% | -0.01\% |
|  | (-1.08) | (1.06) | (-3.81) | (-1.70) |
| 2 | -0.04\% | -0.02\% | -0.05\% | -0.03\% |
|  | (-4.50) | (-1.81) | (-5.58) | (-3.05) |
| 3 | -0.02\% | 0.00\% | -0.04\% | -0.02\% |
|  | (-2.53) | (-0.53) | (-4.74) | (-2.69) |
| 4 | 0.01\% | 0.02\% | -0.01\% | 0.00\% |
|  | (0.93) | (2.94) | (-1.66) | (0.24) |
| 5 | 0.00\% | 0.02\% | -0.03\% | -0.01\% |
|  | (-0.24) | (2.25) | (-3.60) | (-1.09) |
| 6 | 0.01\% | 0.03\% | -0.03\% | -0.01\% |
|  | (1.25) | (3.12) | (-3.41) | (-1.46) |
| 7 | 0.02\% | 0.03\% | -0.03\% | -0.01\% |
|  | (2.07) | (4.18) | (-3.76) | (-1.30) |
| 8 | 0.00\% | 0.02\% | -0.03\% | -0.01\% |
|  | (0.23) | (2.78) | (-4.16) | (-1.46) |
| 9 | -0.01\% | 0.01\% | -0.03\% | -0.02\% |
|  | (-0.67) | (1.28) | (-4.22) | (-2.01) |
| 10 | 0.01\% | 0.03\% | -0.02\% | 0.00\% |
|  | (0.96) | (3.25) | (-2.69) | (-0.32) |
| 2-10 | -0.02\% | 0.16\% | -0.27\% | -0.10\% |
|  | (-0.67) | (6.51) | (-11.39) | (-4.05) |
| 3-10 | 0.02\% | 0.18\% | -0.20\% | -0.08\% |
|  | (0.95) | (7.49) | (-10.05) | (-3.28) |

Accumulated returns of one factor-hedged short strategy in event time


Figure 2.2. Cumulative returns for trading strategies in event time. This figure shows the average of the cumulative sum of daily returns for different short strategies in event time. Events are the stock-days for which one or more options written on the stock were exercised early. Two strategies are illustrated. In top panel, after event, the stock is shorted and the market exposure is hedged based on ex ante estimated betas in a one-factor capital asset pricing model. In bottom panel, the stock is shorted and the exposures to the three Fama-French factors (excess market returns (MKT), small market capitalization minus big (SMB), and high book-to-market ratio minus low (HML)) and the momentum factor (up minus down (UMD)) are hedged based on ex ante estimated betas. The cumulative sum of returns is reported both with and without short-sale costs. The dotted lines are the average, plus or minus two standard deviations.
factor abnormal returns two trading days after the early exercise are significantly negative even when including the short-sale cost. The one-factor abnormal return is also significantly negative, while only borderline significant after including short-sale costs. This is evidence that private information is contained in the previous early exercises.

A concern could be that the negative, abnormal return is due to price pressure stemming from the previous exercises. Maybe an option owner has a large option relative to the liquidity of the underlying stock and therefore gradually offloads it by exercising some days in a row and selling the shares in the market. This would lead to correlation between early exercises over time and thus create correlation between exercises two trading days ago and negative price impact today. To address this, the abnormal returns for up to ten trading days after early exercise are included in the empirical analyses. If price impact is an issue, prices would be expected to recover after a temporary drop.

The event study reports abnormal returns for each trading day up to ten trading days after the early exercise. It also reports the cumulated sum of abnormal return (using a simple sum) for both two to ten days and three to ten days after the early exercise. Big differences emerge between the one-factor and four-factor abnormal returns. While the one-factor returns without short-sale costs are not significantly different from zero for either two to ten days or three to ten days after early exercise, the four-factor abnormal returns are strongly significantly negative. Once short-sale costs are included, the one-factor abnormal returns become strongly positive, and the four-factor abnormal returns stay significantly negative. The same pattern can be seen in Fig. 2.2, in which abnormal returns are accumulated for the trading days after the early exercise for a hedged short strategy, leading to an opposite sign. This difference reflects the overall pattern in the sample in which both the abnormal one-factor returns and abnormal four-factor returns are significantly positive, with the former being larger than the latter. The cumulative return has to overcome these positive trends to get an overall negative return. This issue is addressed by the Fama-MacBeth style regressions which also are suitable to reveal potential price impacts. The main source for the difference between the one-factor and four-factor abnormal returns is the SMB factor, i.e., the difference in performance of firms with small and big market capitalization.

### 2.4.2 Calendar time returns of trading strategies based on early exercise

Is the private information content of the early exercises valuable enough to trade on? Table 2.3 reports the alphas and exposures of different trading strategies in which the underlying stock is shorted after early exercises. Panel A considers strategies in which stocks are shorted if an option written on them was exercised two trading days ago. The stocks shorted on a given day are equally weighted. Three different hedges are used: (1) the stocks are shorted without any hedges and the proceeds placed in the risk-free asset, (2) the stocks are shorted and the market exposures are hedged using ex ante estimated betas from a one-factor CAPM, and (3) the stocks are shorted and the exposures to the three Fama-French factors and the momentum factor are hedged using ex ante estimated betas. For each of the three strategies, the returns are reported both with and without short-sale costs. The average returns of the strategies can be observed, as well as the ex post alphas and betas of the strategies in both a one- and a four-factor model. Daily Sharpe ratios are converted to yearly Sharpe ratios by multiplying by $\sqrt{N}$, with $N=251.67$ being the average number of trading days per year in the sample. Panel B reports the same numbers for which the stocks are shorted on the third trading day after early exercises, and Panel C reports the strategy in which stocks are shorted from two to ten trading days after an early exercise of an option written on the stock.

The results show that the hedged strategies are attractive, especially the four factor-hedged return that even after including short-sale costs gets as high as 0.76 for two trading days after exercise (Panel A) with significant positive alpha in all three models. This is evidence of private information leading option owners to early exercise. The four factor-hedged shorting strategy for two to ten days after exercise (Panel C) has significant positive alpha without short-sale costs and insignificant, positive alpha with short-sale costs. Both with and without short-sale costs, the loading on SMB is significantly negative and the loading on UMD is significantly positive. Notably, the mean is insignificant in calendar time ( $t$-statistic 0.86 , c.f. Table 2.3, Panel C, constant, four factors with short-sale cost), and the event time return is significant ( $t$-statistic -4.05 , c.f. Table 2.2, four-factor alpha including short-sale cost). Calendar time returns differ from event time returns because the number of stocks held in the portfolio varies over time, meaning that stocks

Table 2.3
Return and factor exposures in calendar time.
This table shows the abnormal returns of trading strategies in which stocks are shorted if an option written on the stock has been exercised early two trading days ago (Panel A), three trading days ago (Panel B), or at least two but no more than ten trading days ago (Panel C). The stocks shorted on a given day are equally weighted. Three strategies are reported: the stocks are shorted and the proceeds invested in the risk-free asset (excess stock return), the market exposure of the short stock position is hedged (one factor), and the exposures to the three Fama-French factors (excess market returns (MKT), small market capitalization minus big (SMB), and high book-to-market ratio minus low (HML)) and the momentum factor (up minus down (UMD)) of the short stock position are hedged (four factor). The returns after subtracting short-sale costs are also reported. The columns show the ex post average return (mean), ex post alpha and beta for market exposure (one factor), and ex post alpha and betas in a four-factor model (four factors) estimated by ordinary least squares regressions. Yearly Sharpe ratios are reported in the final column. $t$-statistics are in parentheses.

| Strategy | Constant | One factor |  | Four factors |  |  |  |  | Sharpe ratio |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | Alpha | MKT | Alpha | MKT | SMB | HML | UMD |  |
| Panel A: Two days after early exercise |  |  |  |  |  |  |  |  |  |
| Short stock | $\begin{gathered} 0.02 \% \\ (0.49) \end{gathered}$ | $\begin{gathered} 0.04 \% \\ (1.86) \end{gathered}$ | $\begin{array}{r} -1.15 \\ (-84.20) \end{array}$ | $\begin{gathered} 0.04 \% \\ (2.41) \end{gathered}$ | $\begin{array}{r} -1.10 \\ (-80.84) \end{array}$ | $\begin{array}{r} -0.70 \\ (-25.93) \end{array}$ | $\begin{array}{r} -0.03 \\ (-0.93) \end{array}$ | $\begin{array}{r} 0.02 \\ (1.27) \end{array}$ |  |
| Short stock with short sale cost | $\begin{gathered} 0.02 \% \\ (0.37) \end{gathered}$ | $\begin{array}{r} 0.03 \% \\ (1.56) \end{array}$ | $\begin{array}{r} -1.12 \\ (-82.63) \end{array}$ | $\begin{array}{r} 0.03 \% \\ (2.04) \end{array}$ | $\begin{array}{r} -1.08 \\ (-79.55) \end{array}$ | $\begin{array}{r} -0.70 \\ (-26.01) \end{array}$ | $\begin{array}{r} -0.02 \\ (-0.80) \end{array}$ | $\begin{array}{r} 0.02 \\ (1.01) \end{array}$ |  |
| Short stock one factor hedge | $\begin{array}{r} 0.04 \% \\ (2.19) \end{array}$ | $\begin{array}{r} 0.04 \% \\ (2.17) \end{array}$ | $\begin{array}{r} 0.05 \\ (3.61) \end{array}$ | $\begin{array}{r} 0.04 \% \\ (2.74) \end{array}$ | $\begin{gathered} 0.11 \\ (8.30) \end{gathered}$ | $\begin{array}{r} -0.64 \\ (-24.11) \end{array}$ | $\begin{gathered} -0.05 \\ (-1.80) \end{gathered}$ | $\begin{array}{r} 0.06 \\ (3.68) \end{array}$ | 0.84 |
| Short stock one factor hedge with short sale cost | $\begin{array}{r} 0.02 \% \\ (1.14) \end{array}$ | $\begin{array}{r} 0.02 \% \\ (1.11) \end{array}$ | $\begin{array}{r} 0.05 \\ (3.56) \end{array}$ | $\begin{array}{r} 0.02 \% \\ (1.52) \end{array}$ | $\begin{array}{r} 0.12 \\ (8.42) \end{array}$ | $\begin{array}{r} -0.65 \\ (-24.04) \end{array}$ | $\begin{array}{r} -0.05 \\ (-1.68) \end{array}$ | $\begin{array}{r} 0.07 \\ (4.19) \end{array}$ | 0.44 |
| Short stock four factor hedge | $\begin{array}{r} 0.05 \% \\ (3.28) \end{array}$ | $\begin{gathered} 0.05 \% \\ (3.27) \end{gathered}$ | $\begin{array}{r} 0.01 \\ (1.34) \end{array}$ | $\begin{array}{r} 0.05 \% \\ (3.38) \end{array}$ | $\begin{array}{r} 0.03 \\ (2.64) \end{array}$ | $\begin{aligned} & -0.01 \\ & (-0.48) \end{aligned}$ | $\begin{array}{r} -0.11 \\ (-3.86) \end{array}$ | $\begin{gathered} -0.01 \\ (-0.91) \end{gathered}$ | 1.26 |
| Short stock four factor hedge with short sale cost | $\begin{array}{r} 0.03 \% \\ (1.98) \end{array}$ | $\begin{gathered} 0.03 \% \\ (1.97) \end{gathered}$ | $\begin{array}{r} 0.01 \\ (1.28) \end{array}$ | $\begin{array}{r} 0.03 \% \\ (2.06) \end{array}$ | $\begin{array}{r} 0.03 \\ (2.49) \end{array}$ | $\begin{array}{r} -0.01 \\ (-0.46) \end{array}$ | $\begin{gathered} -0.11 \\ (-3.60) \end{gathered}$ | $\begin{gathered} -0.01 \\ (-0.84) \end{gathered}$ | 0.76 |
| Panel B: Three days after early exercise |  |  |  |  |  |  |  |  |  |
| Short stock | $\begin{array}{r} 0.01 \% \\ (0.30) \end{array}$ | $\begin{array}{r} 0.03 \% \\ (1.43) \end{array}$ | $\begin{array}{r} -1.19 \\ (-84.26) \end{array}$ | $\begin{gathered} 0.02 \% \\ (0.89) \end{gathered}$ | $\begin{array}{r} -1.14 \\ (-85.91) \end{array}$ | $\begin{array}{r} -0.74 \\ (-26.64) \end{array}$ | $\begin{array}{r} 0.92 \\ (0.41) \end{array}$ | $\begin{array}{r} 0.03 \\ (1.80) \end{array}$ |  |
| Short stock with short sale cost | $\begin{gathered} 0.00 \% \\ (0.04) \end{gathered}$ | $\begin{gathered} 0.02 \% \\ (0.84) \end{gathered}$ | $\begin{array}{r} -1.15 \\ (-81.95) \end{array}$ | $\begin{gathered} 0.02 \% \\ (0.95) \end{gathered}$ | $\begin{array}{r} -1.11 \\ (-83.74) \end{array}$ | $\begin{array}{r} -0.73 \\ (-26.59) \end{array}$ | $\begin{gathered} -0.59 \\ (-0.26) \end{gathered}$ | $\begin{array}{r} 0.02 \\ (1.15) \end{array}$ |  |
| Short stock one factor hedge | $\begin{array}{r} 0.03 \% \\ (1.41) \end{array}$ | $\begin{gathered} 0.03 \% \\ (1.41) \end{gathered}$ | $\begin{array}{r} 0.01 \\ (0.87) \end{array}$ | $\begin{array}{r} 0.02 \% \\ (0.82) \end{array}$ | $\begin{array}{r} 0.07 \\ (5.31) \end{array}$ | $\begin{array}{r} -0.68 \\ (-24.53) \end{array}$ | $\begin{array}{r} 1.00 \\ (0.44) \end{array}$ | $\begin{array}{r} 0.07 \\ (4.67) \end{array}$ | 0.54 |
| Short stock one factor hedge with short sale cost | $\begin{gathered} 0.01 \% \\ (0.51) \end{gathered}$ | $\begin{array}{r} 0.01 \% \\ (0.50) \end{array}$ | $\begin{array}{r} 0.01 \\ (0.66) \end{array}$ | $\begin{gathered} 0.00 \% \\ (0.09) \end{gathered}$ | $\begin{gathered} 0.07 \\ (5.41) \end{gathered}$ | $\begin{array}{r} -0.69 \\ (-24.55) \end{array}$ | $\begin{array}{r} 1.19 \\ (0.52) \end{array}$ | $\begin{array}{r} 0.09 \\ (5.38) \end{array}$ | 0.20 |
| Short stock four factor hedge | $\begin{gathered} 0.04 \% \\ (2.31) \end{gathered}$ | $\begin{gathered} 0.04 \% \\ (2.33) \end{gathered}$ | $\begin{array}{r} -0.02 \\ (-1.43) \end{array}$ | $\begin{gathered} 0.05 \% \\ (1.87) \end{gathered}$ | $\begin{array}{r} -0.01 \\ (-0.50) \end{array}$ | $\begin{array}{r} -0.05 \\ (-1.93) \end{array}$ | $\begin{array}{r} -1.05 \\ (-0.49) \end{array}$ | $\begin{array}{r} 0.02 \\ (1.36) \end{array}$ | 0.89 |
| Short stock four factor hedge with short sale cost | $\begin{gathered} 0.02 \% \\ (1.18) \end{gathered}$ | $\begin{array}{r} 0.02 \% \\ (1.20) \end{array}$ | $\begin{array}{r} -0.02 \\ (-1.56) \end{array}$ | $\begin{array}{r} 0.03 \% \\ (1.02) \end{array}$ | $\begin{array}{r} -0.01 \\ (-0.55) \end{array}$ | $\begin{array}{r} -0.05 \\ (-1.78) \end{array}$ | $\begin{array}{r} -0.69 \\ (-0.32) \end{array}$ | $\begin{array}{r} 0.02 \\ (1.53) \end{array}$ | 0.45 |
| Panel C: Two to ten days after early exercise |  |  |  |  |  |  |  |  |  |
| Short stock | $\begin{array}{r} -0.01 \% \\ (-0.22) \end{array}$ | $\begin{array}{r} 0.01 \% \\ (0.38) \end{array}$ | $\begin{array}{r} -1.21 \\ (-120.00) \end{array}$ | $\begin{array}{r} 0.00 \% \\ (0.23) \end{array}$ | $\begin{array}{r} -1.17 \\ (-146.60) \end{array}$ | $\begin{array}{r} -0.69 \\ (-41.78) \end{array}$ | $\begin{array}{r} 0.57 \\ (0.42) \end{array}$ | $\begin{array}{r} 0.03 \\ (3.56) \end{array}$ |  |
| Short stock with short sale cost | $\begin{array}{r} -0.02 \% \\ (-0.40) \end{array}$ | $\begin{aligned} & 0.00 \% \\ & (-0.17) \end{aligned}$ | $\begin{array}{r} -1.20 \\ (-118.62) \end{array}$ | $\begin{gathered} 0.01 \% \\ (0.51) \end{gathered}$ | $\begin{array}{r} -1.16 \\ (-143.98) \end{array}$ | $\begin{array}{r} -0.69 \\ (-41.02) \end{array}$ | $\begin{array}{r} -0.81 \\ (-0.59) \end{array}$ | $\begin{array}{r} 0.03 \\ (2.63) \end{array}$ |  |
| Short stock one factor hedge | $\begin{array}{r} 0.01 \% \\ (0.77) \end{array}$ | $\begin{array}{r} 0.01 \% \\ (0.77) \end{array}$ | $\begin{gathered} -0.01 \\ (-0.70) \end{gathered}$ | $\begin{array}{r} 0.01 \% \\ (0.61) \end{array}$ | $\begin{array}{r} 0.05 \\ (6.11) \end{array}$ | $\begin{array}{r} -0.63 \\ (-35.79) \end{array}$ | $\begin{array}{r} 0.44 \\ (0.30) \end{array}$ | $\begin{array}{r} 0.08 \\ (7.99) \end{array}$ | 0.29 |
| Short stock one factor hedge with short sale cost | $\begin{aligned} & 0.00 \% \\ & (-0.04) \end{aligned}$ | $\begin{aligned} & 0.00 \% \\ & (-0.03) \end{aligned}$ | $\begin{gathered} -0.01 \\ (-0.62) \end{gathered}$ | $\begin{aligned} & 0.00 \% \\ & (-0.13) \end{aligned}$ | $\begin{array}{r} 0.06 \\ (6.70) \end{array}$ | $\begin{array}{r} -0.64 \\ (-36.50) \end{array}$ | $\begin{gathered} 0.57 \\ (0.39) \end{gathered}$ | $\begin{array}{r} 0.09 \\ (8.97) \end{array}$ | -0.02 |
| Short stock four factor hedge | $\begin{array}{r} 0.02 \% \\ (2.11) \end{array}$ | $\begin{array}{r} 0.02 \% \\ (2.15) \end{array}$ | $\begin{gathered} -0.03 \\ (-4.07) \end{gathered}$ | $\begin{gathered} 0.04 \% \\ (2.31) \end{gathered}$ | $\begin{gathered} -0.01 \\ (-1.56) \end{gathered}$ | $\begin{gathered} -0.04 \\ (-2.56) \end{gathered}$ | $\begin{array}{r} -1.61 \\ (-1.18) \end{array}$ | $\begin{array}{r} 0.04 \\ (4.24) \end{array}$ | 0.81 |
| Short stock four factor hedge with short sale cost | $\begin{array}{r} 0.01 \% \\ (0.86) \\ \hline \end{array}$ | $\begin{array}{r} 0.01 \% \\ (0.89) \\ \hline \end{array}$ | $\begin{array}{r} -0.03 \\ (-3.92) \end{array}$ | $\begin{array}{r} 0.02 \% \\ (1.45) \\ \hline \end{array}$ | $\begin{array}{r} -0.01 \\ (-1.29) \end{array}$ | $\begin{array}{r} -0.04 \\ (-2.71) \end{array}$ | $\begin{gathered} -1.50 \\ (-1.11) \end{gathered}$ | $\begin{array}{r} 0.04 \\ (4.53) \end{array}$ | 0.33 |

shorted in times with few events get a relatively higher weight. As a trend, the number of stocks with early exercise is increasing over time in the sample.

The main difference between the one-factor and four-factor abnormal returns is the loading on the SMB factor, i.e., the factor representing the long small firms short big firms portfolio. Apparently, the short-strategy involves shorting stocks that on average are highly exposed to this factor. After ex ante hedging for the four factors, including the SMB factor, the ex post loading on the factor gets close to zero, being insignificant both two and three trading days after early exercise.

The cumulated sums of four-factor abnormal returns over calendar time are illustrated in Fig. 2.3. In the first part of the sample, the strategy with short-sale costs can even temporarily be above the return without short-sale costs because excluding observations with missing short-sale costs makes the sample slightly smaller. Over time, the differences become clearer and short-sale costs matter increasingly as they are accumulated.

Fig. 2.3 also illustrates the daily number of stocks that experience early exercise. The trend appears to be an increase over time and what seems to be a strong seasonality. This is not surprising given that early exercise is decreasing in time to expiration of the option and that all options expiring the same month expire the same day, namely, the day after the third Friday in the month.

The returns for the strategies in which the stock is shorted between two and ten trading days after early exercises are lower than the returns when the stocks are shorted only two or three trading days after early exercise. This reflects that the stocks are equal-weighted and that the relatively larger abnormal gains two and three trading days after early exercise are watered down by the relatively lower gains four to ten trading days after early exercise. A natural question would be whether the returns four to ten trading days after early exercise are lower, not relative to the returns two and three trading days after early exercise but to the rest of the overall sample.

### 2.4.3 Fama-MacBeth regressions of returns on previous early exercises

Until now, event time and calendar time studies have been used to identify how early exercise predicts stock returns. To measure abnormal stock returns, a one-factor CAPM and a four-factor model (the three Fama-French factors and the momentum factor) have been estimated. Although


Figure 2.3. Cumulative returns for trading strategies in calendar time. This figure shows the cumulative sum of daily returns of trading strategies in which stocks are shorted if options written on them have been exercised early two trading days ago (top panel), three trading days ago (mid panel), or at least two but no more than ten trading days ago (bottom panel). The stocks shorted on a given day are equally weighted. The strategies hedge the exposures of the short stock position to the three Fama-French factors (MKT, SMB, and HML) and the momentum factor (UMD). The cumulative sums of returns both before and after subtracting short-sale costs are illustrated. For each date in the sample, the number of stocks that experience early exercise of an option written on them (i.e., number of events) is illustrated with a gray bar.
these models do capture much of the variation in the returns, both the one- and four-factor abnormal returns are significantly positive on average, with the former being larger than the latter. Therefore, the strategies considered so far might not fully capture the predictive power of early exercise for future stock returns.

The Fama-MacBeth regression setup presented in this subsection takes care of this problem in at least two ways. First, it compares the abnormal returns after early exercise not to zero, but to the abnormal returns of the rest of the stock universe from CRSP. Thus, a potential general trend not captured by the one- and four-factor models does not bias the result as long as it has, within the same month on average, the same effect on the stocks that have recently experienced early exercise and those which have not. Second, the Fama-MacBeth procedure allows for changing deviations in the abnormal returns over time and corrects for possible serial correlation in these deviations. While one-factor and four-factor abnormal returns are significantly positive in a classic calculation of the $t$-statistics, both become insignificant in the Fama-MacBeth regression setup. The Fama-MacBeth setup is flexible enough to capture the serial correlation of the deviations from the models to a point where the deviations from zero become insignificant. Thus, if early exercise is still a significant predictor of future stock returns within this setup, it provides important evidence.

The Fama-MacBeth regressions are performed in the following way. The abnormal returns for all stocks in the CRSP data are subdivided into groups consisting of returns for each of the months from October 2003 to August 2010, except January 2006 because of missing option exercise data. Then, for each of the 82 months, an OLS regression is run with one- or four-factor abnormal returns as the dependent variable and one or more dummy variables for early exercises on previous trading days as independent variables. A fixed effect for DCBS, the integer score from one to ten for shortsale cost, is included to take into account that returns and early exercise may both correlate with short-sale costs. The subregressions result in 82 estimates for each dummy variable, one for each subregression. The average of these estimates is then the estimate for the overall Fama-MacBeth regression. The $t$-statistic for each dummy is the sample standard deviation of the 82 estimates corrected for possible autocorrelation using Newey-West correction with automatic lag selection with a Bartlett kernel (Newey and West, 1987, 1994).

The results of the regressions, reported in Table 2.4, are similar for the dependent variable being one-factor abnormal return (Panel A) and four-factor abnormal return (Panel B). As expected, a highly significant positive abnormal return is seen for stocks for which an option written on them have been exercised early the same day. On days with early exercise, one trading day before a significant, negative abnormal return is observed. The same holds for stocks for which an option written on them has been exercised early two or three trading days ago. The predictive power of early exercises on days even further back are all insignificant, except one. However, they all have a negative sign in the regressions with only one dummy variable. This is consistent with early exercise predicting lower future stock returns due to private information, not price impact being the driver. On the days after the days with significantly lower returns, the returns do not bounce back and recover what was lost the previous days. This can also be seen in the regression with a dummy variable for early exercise for each of the previous two to ten trading days. Abnormal returns are still negative two and three trading days after early exercise. That underlying stocks do not bounce back on consecutive days is evidence that the negative abnormal returns two and three trading days after early exercise is not just a temporary price impact from stocks being sold.

### 2.5 The effect of agent type, size, and time to expiration

So far, the focus has been on the set of stock-days with early exercise without considering further details of the exercises. This section looks at how the return of the underlying stock after early exercise varies in different dimensions: the agent type exercising, the number of contracts exercised for the given stock on the given day, the market capitalization of the underlying stock, and a sort on both moneyness and time to expiration for the option at time of exercise. Finally, the calendar time returns presented in Subsection 2.4.2 are controlled for stock reversal using the reversal factor constructed by Nagel (2012).

Table 2.4
Abnormal returns after early exercise: Regression analysis.
This table shows the results of Fama-MacBeth style regressions with the dependent variable being abnormal return from a one-factor capital asset pricing model (Panel A) and a four-factor model (Panel B). The four factors are the three Fama-French factors (excess market returns (MKT), small market capitalization minus big (SMB), and high book-to-market ratio minus low (HML)) and the momentum factor (up minus down (UMD)). The independent variables are dummy variables equal to one if early exercise is observed for options written on the stock the given number of trading days before. For each of the 12 Fama-MacBeth regressions, a subregression is performed for each month from October 2003 to August 2010 (excluding January 2006 because of missing exercise data). The averages of each of the time series of the estimates are reported in percentages, and the $t$-statistics for the time series corrected for autocorrelation are reported in parentheses. DCBS (Daily Cost of Borrow Score) is an integer score from one to ten reflecting the cost of shorting the stock and has been included as a fixed effect in all subregressions.


### 2.5.1 Return after early exercise by type of agent exercising

The data allow for the identification of the following three types of agents performing the early exercise: customers of brokers, firms, and market makers. Does the predictive power of early exercise over future stock returns depend on what type of agent is exercising? Table 2.5 shows the results of Fama-MacBeth style regressions of four-factor abnormal returns with dummy variables for previous early exercises by agent type. Panel A shows the results of the regression for which only exercises by customers of brokers are included. Panel B shows the results for firms; Panel C, for market makers. For customers, significant negative abnormal returns on days with early exercise two and three trading days before are observed, both in the regressions with one dummy variable and the regression with a dummy variable for each day two to ten days prior. The same pattern is not observed for firms and market makers. One explanation for this finding could be that customers have many more stock-days with early exercise than firms or market makers, c.f. Table 2.1. It is not a trend that the estimates are closer to zero for firms and market makers than for customers, which supports that the lack of significance could relate to lower statistical power because of less data.

I test whether a real difference exists between the three groups. For each subregression in Table 2.5 , specification (3) (one dummy for early exercise two trading days ago), the market maker dummy estimate is subtracted from the corresponding customer dummy estimate. This leads to a time series with 82 observations of the difference. Taking the average of this time series and estimating its sample standard deviation corrected for autocorrelation using Newey-West correction results in a $t$-statistic of 0.23 for the difference, i.e., insignificant. Repeating the procedure for the difference between customers and firms, the $t$-statistic is -0.02 . For the difference between firms and market makers, it is 0.13 . In other words, the average prediction for abnormal stock return two trading days after early exercise does not depend significantly on the type of agent exercising early, at least in this sample. This result is robust to using one-factor abnormal returns instead of four-factor abnormal returns as the dependent variable. It indicates that private information is not restricted to only one of the groups.

The tests of trading strategies in calendar time can also be performed separately for early ex-

Table 2.5
Abnormal returns after early exercises by agent type: Regression analysis.
This table shows the results of Fama-MacBeth style regressions with the dependent variable being abnormal return from a four-factor model, i.e., the three Fama-French factors (excess market returns (MKT), small market capitalization minus big (SMB), and high book-to-market ratio minus low (HML)) and the momentum factor (up minus down (UMD)). The independent variables are dummy variables equal to one if early exercise is observed for options written on the stock the given number of trading days before, and the exercise is performed by a customer (Panel A), firm (Panel B), or market maker (Panel C). For each of the 12 Fama-MacBeth regressions in a panel, a subregression is performed for each month from October 2003 to August 2010 (excluding January 2006 because of missing exercise data). The averages of each of the time series of the estimates are reported in percentages, and the $t$-statistics for the time series corrected for autocorrelation are reported in parentheses. DCBS (Daily Cost of Borrow Score) is an integer score from one to ten reflecting the cost of shorting the stock and has been included as a fixed effect in all subregressions.



ercises for each of the three types of agents. The cumulative sums of returns for such strategies are illustrated in Fig. 2.4. The top panel illustrates the cumulative sum of one-factor abnormal returns two trading days after early exercise. The bottom panel shows the corresponding cumulative four-factor abnormal returns. In the final part of the sample, the returns of the strategies based on exercises by customers and the one based on exercises by market makers are highly correlated. One reason for this is overlap between the stock-days with early exercise for customers and the stock-days for market makers.

For all three types of agents, a positive cumulated sum of abnormal return at the end of the sample period is evident and the cumulated sum of returns is larger for the four factor-hedged strategy than for the one factor-hedged strategy. No obvious trend difference between the returns of the three strategies emerges, as variability makes the cumulative sum of returns fluctuate. Not surprisingly, the fluctuations are largest for firms, the agent type with the least stock-days with early exercise, and smallest for customers, the agent type with the most stock-days with early exercise. The return of the strategy based on early exercises by market makers stabilizes in the last part of the sample period, which is also the time period with the highest density of stock-days with early exercise for this agent type.

Cumulative one factor-hedged return with early exercise two trading days ago


Cumulative four factor-hedged return with early exercise two trading days ago


Figure 2.4. Cumulative returns for trading strategies in calendar time by agent type. This figure shows the cumulative sum of returns of trading strategies in which stocks are shorted if an option written on the stock has been exercised early two trading days ago by a customer, firm, or market maker. The stocks shorted on a given day are equally weighted. Two strategies are reported. In the top panel, the market exposure of the short stock position is hedged using ex ante estimated betas (one factor-hedged). In the bottom panel, the exposures to the three Fama-French factors (excess market returns (MKT), small market capitalization minus big (SMB), and high book-to-market ratio minus low (HML)) and the momentum factor (up minus down (UMD)) of the short stock position are hedged using ex ante betas (four factor-hedged). Short-sale costs are not subtracted from the returns.

### 2.5.2 Return after early exercise by number of contracts exercised

The set of stock-days with early exercises can be sorted based on the number of contracts exercised for the given underlying stock on the given date. This way, what is tested is whether the number of contracts exercised is correlated with the future return of the underlying stock and, thus, the level of private information. Stock-days are divided into three groups: stock-days with one to ten option contracts exercised early, stock-days with 11-100 option contracts exercised early, and stock-days with more than 100 option contracts exercised early. Table 2.1 shows the number of stock-days in each group.

For each of the three groups, Fama-MacBeth regressions are performed with four-factor abnormal return as the dependent variable. The independent variables are dummy variables equal to one only if the number of contracts exercised corresponds to the given group. The results are in Table 2.6. The estimated effects of early exercise two and three trading days ago are negative for all three groups, but significantly negative only in some of the cases. Abnormal returns on stock-days with 11-100 contracts exercised two and three trading days ago are significantly negative (Panel B). For the group with one to ten exercises (Panel A), a significant negative impact is evident for early exercise these days in the regressions with only one dummy variable, but it is no longer significant on a $5 \%$ level once all the dummies for early exercise two to ten trading days ago are included. For the group with more than 100 early exercises, the estimates are not significantly different from zero. The lack of significance may be attributed to the smaller number of stock-days with more than 100 early exercises, c.f. Table 2.1. As for different agent types, I test for a difference in the abnormal return two days after early exercise depending on the number of contracts exercised. No significant differences between the three groups are found. However, if one-factor abnormal returns are used instead of four-factor abnormal returns, the abnormal returns two trading days after 11-100 contracts are exercised are significantly lower than when only one to ten contracts were exercised. Going from four-factor to one-factor abnormal returns, the $t$-statistic changes from -1.74 to -2.55 , which indicates a difference. The evidence is, however, not conclusive, because the result is significant only for one of the two proposed measures for abnormal returns.

Table 2.6
Abnormal returns after early exercise by size of exercise: Regression analysis.
This table shows the results of Fama-MacBeth style regressions with the dependent variable being abnormal return from a four-factor model, i.e., the three Fama-French factors (excess market returns (MKT), small market capitalization minus big (SMB), and high book-to-market ratio minus low (HML)) and the momentum factor (up minus down (UMD)). The independent variables are dummy variables equal to one if early exercise is observed for options written on the stock the given number of trading days before, and the number of contracts exercised is one to ten (Panel A), 11-100 (Panel B), and more than 100 (Panel C). For each of the 12 Fama-MacBeth regressions in a panel, a subregression is performed for each month from October 2003 to August 2010 (excluding January 2006 because of missing exercise data). The averages of each of the time series of the estimates are reported in percentages, and the $t$-statistics for the time series corrected for autocorrelation are reported in parentheses. DCBS (Daily Cost of Borrow Score) is an integer score from one to ten reflecting the cost of shorting the stock and has been included as a fixed effect in all subregressions.

| Independent variable | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A: One to ten contracts exercised |  |  |  |  |  |  |  |  |  |  |  |  |
| Early exercise |  |  |  |  |  |  |  |  |  |  |  |  |
| Same day | $\begin{gathered} 0.884 \% \\ (15.09) \end{gathered}$ |  |  |  |  |  |  |  |  |  |  |  |
| One day before |  | $\begin{array}{r} -0.037 \% \\ (-1.96) \end{array}$ |  |  |  |  |  |  |  |  |  |  |
| Two days before |  |  | $\begin{array}{r} -0.038 \% \\ (-1.97) \end{array}$ |  |  |  |  |  |  |  |  | $\begin{array}{r} -0.028 \% \\ (-1.60) \end{array}$ |
| Three days before |  |  |  | $\begin{array}{r} -0.042 \% \\ (-2.04) \end{array}$ |  |  |  |  |  |  |  | $\begin{array}{r} -0.032 \% \\ -(1.73) \end{array}$ |
| Four days before |  |  |  |  | $\begin{array}{r} -0.014 \% \\ (-0.73) \end{array}$ |  |  |  |  |  |  | $\begin{array}{r} -0.001 \% \\ (-0.05) \end{array}$ |
| Five days before |  |  |  |  |  | $\begin{array}{r} -0.027 \% \\ (-1.76) \end{array}$ |  |  |  |  |  | $\begin{array}{r} -0.015 \% \\ (-1.15) \end{array}$ |
| Six days before |  |  |  |  |  |  | $\begin{array}{r} -0.027 \% \\ (-1.40) \end{array}$ |  |  |  |  | $\begin{array}{r} -0.014 \% \\ (-0.77) \end{array}$ |
| Seven days before |  |  |  |  |  |  |  | $\begin{array}{r} -0.029 \% \\ (-1.70) \end{array}$ |  |  |  | $\begin{array}{r} -0.018 \% \\ (-0.96) \end{array}$ |
| Eight days before |  |  |  |  |  |  |  |  | $\begin{array}{r} -0.035 \% \\ (-3.06) \end{array}$ |  |  | $\begin{array}{r} -0.025 \% \\ (-2.57) \end{array}$ |
| Nine days before |  |  |  |  |  |  |  |  |  | $\begin{array}{r} -0.025 \% \\ (-1.37) \end{array}$ |  | $\begin{array}{r} -0.014 \% \\ (-0.85) \end{array}$ |
| Ten days before |  |  |  |  |  |  |  |  |  |  | $\begin{array}{r} -0.014 \% \\ (-1.07) \end{array}$ | $\begin{array}{r} -0.003 \% \\ (-0.30) \end{array}$ |
| DCBS fixed effects | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |




### 2.5.3 Return after early exercise by market capitalization of underlying stock

So far, this paper documents that early exercises predict future returns of the underlying stock. But one might be concerned that the results are driven by something else, e.g., unobserved illiquidity of the underlying stock. As a robustness check, the Fama-MacBeth regressions are repeated on two disjointed, restricted samples based on the market capitalization of the underlying stock.

To construct the restricted samples, I use the monthly median market equity breakpoints for New York Stock Exchange stocks. ${ }^{5}$ Each stock in the sample is then assigned accordingly as either low or high market capitalization. The abnormal stock returns are grouped into low or high market capitalization based on which group the stock belonged to at the end of the month two months prior. By going two months back in time, I avoid endogeneity issues between the return, the previous early exercises (that goes up to ten trading days back in time), and the market capitalization used to sort the stock. The Fama-MacBeth regressions are then performed for each of the two subsamples of abnormal returns.

The results of the Fama-MacBeth regressions are presented in Table 2.7. The results for stocks with low market capitalization (Panel A) are similar to those with high market capitalization (Panel B). For stocks with low market capitalization, early exercise significantly predicts lower abnormal

[^17]stock returns two and three trading days later. Once controlling for early exercises for each of the days two to ten trading days ago, the indicator for early exercise three trading days ago remains significant on a 5\% level and the indicator for early exercise two trading days ago becomes borderline significant with at $t$-statistic of -1.96 . The returns do not show any signs of bouncing back afterward.

For stocks with high market capitalization (Panel B), a significant, negative effect of early exercise is evident two trading days ago, both with and without controls for early exercises prior to this time. The estimates resemble those for stocks with low market capitalization. The indicator for early exercise three trading days ago is negative and borderline significant when not controlling for exercises at other times. Once controlling for early exercises prior to this, the estimate does not remain significant, however. Both for low and high market capitalization stocks, no systematic positive trend is observed for the loadings on indicators for early exercise four to ten trading days ago. A systematic positive trend could have indicated that stock prices bounce back after a price impact from stocks sold. Generally, the results are consistent with the early exercises reflecting private information among the option owners exercising early.

### 2.5.4 Return after early exercise by moneyness and time to expiration

Does the moneyness and time to expiration of the exercised option matter for the prediction of stock return? This subsection presents results of Fama-MacBeth regressions in which option exercises are sorted on moneyness and time to expiration. The median moneyness for all unique optiondays with early exercise is observed. Option-days are assigned to the group for high moneyness if they have a moneyness larger than this median and to the group for low moneyness otherwise. The option exercises are similarly sorted on time to expiration. Combining these sorts creates four groups.

For the Fama-MacBeth regressions, the dependent variable is the four-factor abnormal return. The independent variables are dummy variables set to one if an option written on the stock was exercised a given number of trading days before and zero otherwise. Only option exercises for the relevant combination of moneyness and time to expiration can lead to the dummy variables

## Table 2.7

Abnormal returns after early exercise, restricted samples by market capitalization: Regression analysis.
This table shows the results of Fama-MacBeth style regressions with the dependent variable being abnormal return from a four-factor model, i.e., the three Fama-French factors (excess market returns (MKT), small market capitalization minus big (SMB), and high book-to-market ratio minus low (HML)) and the momentum factor (up minus down (UMD)). The independent variables are dummy variables equal to one if early exercise is observed for options written on the stock the given number of trading days before. The regressions are performed on two restricted samples based on market capitalization of the stock. At the end of each month, all stocks with a market capitalization above (below) the median market capitalization for NYSE companies are assigned to the bucket for high (low) market capitalization. The abnormal returns for a stock in a month are then assigned to either the high or the low market capitalization group based on which of the buckets the stock belonged to at the end of the month two months earlier (to avoid endogeneity issues). The results for stocks with low and high market capitalization are shown in Panels A and B, respectively. For each of the 12 Fama-MacBeth regressions in a panel, a subregression is performed for each month from November 2003 to August 2010 (excluding January 2006 because of missing exercise data). The averages of each of the time series of the estimates are reported in percentages, and the $t$-statistics for the time series corrected for autocorrelation are reported in parentheses. DCBS (Daily Cost of Borrow Score) is an integer score from one to ten reflecting the cost of shorting the stock and has been included as a fixed effect in all subregressions.


being one. DCBS is included as a fixed effect in all the regressions. Subregressions are performed monthly. However, no subregressions in the Fama-MacBeth procedure are performed for October 2003. The return data start October 22, 2003, with the first upcoming option expiration date being November 22, 2003. Hence, the model cannot be estimated for option exercises with short time to expiration for October 2003. Therefore, the month is excluded from all four Fama-MacBeth regressions.

Panel A of Table 2.8 reports the results for early exercises with low moneyness and short time to expiration. These exercises do not have a statistically significant effect on consecutive stock returns. A possible explanation is that most of these exercises are unrelated to private information but are performed by option owners who already hedge their option position by shorting the stock. They optimally exercise their options once the stock price reaches the lower boundary, c.f. Section 2.2 and Fig. 2.1. These exercises happen at the lower boundary, and those induced by changes in the agent's preference for the underlying stock due to private information happen between the boundaries. All else equal, the exercises with strongest private information content would be expected to happen at higher moneyness levels. A counteracting effect could be that it might take a stronger private signal to give up more optionality when exercising early. All else equal, optionality is decreasing in moneyness (when in the money) and increasing in time to expiration.

Panel B reports the results for abnormal returns after early exercises of options with low moneyness and long time to expiration. Here, more negative signs are on the indicator variables, consistent with the idea that giving up more optionality requires a stronger private signal. However, only the coefficients for early exercise five and six trading days ago are significant, and only when not including other indicator variables in the regression.

Panel C shows the results for exercise of options with high moneyness and short time to expiration. Here, the abnormal return two trading days after early exercise is significantly lower, both with and without including indicators for early exercise three to ten trading days prior. This is consistent with early exercises induced by private information being more prevalent among options with higher moneyness. Again, results are stronger when looking at options with longer time to expiration (Panel D). Abnormal returns are significantly negative not only on days with early

Table 2.8
Abnormal returns after early exercise by moneyness and time to expiration: Regression analysis.
This table shows the results of Fama-MacBeth style regressions with the dependent variable being abnormal return from a four-factor model, i.e., the three Fama-French factors (excess market returns (MKT), small market capitalization minus big (SMB), and high book-to-market ratio minus low (HML)) and the momentum factor (up minus down (UMD)). The independent variables are dummy variables equal to one if early exercise is observed for options written on the stock the given number of trading days before. Panel A reports the results for low moneyness and short time to expiration, i.e., the dummy variables can be equal to one only if the exercised option had moneyness at or below median moneyness and time to expiration is at or below the median time to expiration of all options exercised. The results for low moneyness and long time to expiration are in Panel B; for high moneyness and short time to expiration, in Panel C; and for high moneyness and long time to expiration, in Panel D. For each of the 12 Fama-MacBeth regressions in a panel, a subregression is performed for each month from November 2003 to August 2010 (excluding January 2006 because of missing exercise data). The averages of each of the time series of the estimates are reported in percentages, and the $t$-statistics for the time series corrected for autocorrelation are reported in parentheses. DCBS (Daily Cost of Borrow Score) is an integer score from one to ten reflecting the cost of shorting the stock and has been included as a fixed effect in all subregressions.

| Independent variable | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A: Low moneyness and short time to expiration |  |  |  |  |  |  |  |  |  |  |  |  |
| Early exercise |  |  |  |  |  |  |  |  |  |  |  |  |
| Same day | $\begin{gathered} 0.908 \% \\ (18.56) \end{gathered}$ |  |  |  |  |  |  |  |  |  |  |  |
| One day before |  | $\begin{array}{r} -0.014 \% \\ (-0.61) \end{array}$ |  |  |  |  |  |  |  |  |  |  |
| Two days before |  |  | $\begin{array}{r} -0.017 \% \\ (-0.66) \end{array}$ |  |  |  |  |  |  |  |  | $\begin{array}{r} -0.019 \% \\ (-0.98) \end{array}$ |
| Three days before |  |  |  | $\begin{array}{r} -0.001 \% \\ (-0.02) \end{array}$ |  |  |  |  |  |  |  | $\begin{array}{r} -0.001 \% \\ -(0.03) \end{array}$ |
| Four days before |  |  |  |  | $\begin{array}{r} 0.029 \% \\ (1.13) \end{array}$ |  |  |  |  |  |  | $\begin{array}{r} 0.042 \% \\ (1.94) \end{array}$ |
| Five days before |  |  |  |  |  | $\begin{array}{r} -0.008 \% \\ (-0.37) \end{array}$ |  |  |  |  |  | $\begin{array}{r} -0.004 \% \\ (-0.20) \end{array}$ |
| Six days before |  |  |  |  |  |  | $\begin{array}{r} -0.020 \% \\ (-0.86) \end{array}$ |  |  |  |  | $\begin{array}{r} -0.017 \% \\ (-0.81) \end{array}$ |
| Seven days before |  |  |  |  |  |  |  | $\begin{array}{r} -0.006 \% \\ (-0.22) \end{array}$ |  |  |  | $\begin{array}{r} 0.005 \% \\ (0.21) \end{array}$ |
| Eight days before |  |  |  |  |  |  |  |  | $\begin{array}{r} -0.015 \% \\ (-0.68) \end{array}$ |  |  | $\begin{array}{r} -0.013 \% \\ (-1.36) \end{array}$ |
| Nine days before |  |  |  |  |  |  |  |  |  | $\begin{array}{r} -0.006 \% \\ (-0.20) \end{array}$ |  | $\begin{array}{r} -0.011 \% \\ (-0.40) \end{array}$ |
| Ten days before |  |  |  |  |  |  |  |  |  |  | $\begin{array}{r} 0.010 \% \\ (0.41) \end{array}$ | $\begin{array}{r} 0.007 \% \\ (0.26) \end{array}$ |
| DCBS fixed effects | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |



Panel C: High moneyness and short time to expiration

| Same day | $\begin{gathered} 1.188 \% \\ (17.66) \end{gathered}$ |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| One day before |  | $\begin{array}{r} -0.104 \% \\ (-2.63 \end{array}$ |  |  |  |  |  |  |  |  |  |  |
| Two days before |  |  | $\begin{array}{r} -0.088 \% \\ (-2.61 \end{array}$ |  |  |  |  |  |  |  |  | $\begin{gathered} -0.090 \% \\ (-2.99) \end{gathered}$ |
| Three days before |  |  |  | $\begin{array}{r} -0.029 \% \\ (-0.67) \end{array}$ |  |  |  |  |  |  |  | $\begin{array}{r} -0.020 \% \\ -(0.52) \end{array}$ |
| Four days before |  |  |  |  | $\begin{array}{r} 0.018 \\ 10.42 \end{array}$ |  |  |  |  |  |  | $\begin{array}{r} 0.034 \% \\ (0.82) \end{array}$ |
| Five days before |  |  |  |  |  | $\begin{array}{r} 0.019 \% \\ (0.60) \end{array}$ |  |  |  |  |  | $\begin{array}{r} 0.027 \% \\ (0.87) \end{array}$ |
| Six days before |  |  |  |  |  |  | $\begin{array}{r} 0.024 \% \\ (0.57 \end{array}$ |  |  |  |  | $\begin{array}{r} 0.039 \% \\ (0.93) \end{array}$ |
| Seven days before |  |  |  |  |  |  |  | $\begin{array}{r} -0.014 \% \\ (-0.32) \end{array}$ |  |  |  | $\begin{array}{r} -0.001 \% \\ (-0.01) \end{array}$ |
| Eight days before |  |  |  |  |  |  |  |  | $\begin{array}{r} 0.004 \% \\ (0.06 \end{array}$ |  |  | $\begin{array}{r} 0.013 \% \\ (0.21) \end{array}$ |
| Nine days before |  |  |  |  |  |  |  |  |  | $\begin{array}{r} 0.009 \% \\ (0.31) \end{array}$ |  | $\begin{array}{r} 0.001 \% \\ (0.05) \end{array}$ |
| Ten days before |  |  |  |  |  |  |  |  |  |  | $\begin{array}{r} -0.045 \% \\ (-1.36) \end{array}$ | $\begin{array}{r} -0.071 \% \\ (-1.92) \end{array}$ |
| DCBS fixed effects | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |


exercise two trading days prior, but also on days with early exercise three trading days prior. The significance is robust to including indicators for early exercise prior to that. This is consistent with longer time to expiration requiring stronger private signals to make early exercise attractive, as more optionality is sacrificed by the option owner. While the analysis is somewhat crude, as it does not control for all relevant factors (e.g., volatility and interest rates), the results indicate that exercises of options with longer time to expiration give stronger predictions for stock returns. This is consistent with the idea that stronger private information is needed to compensate for lost optionality.

### 2.5.5 Calendar time returns controlling for stock reversal

As a robustness test, I test whether the results are driven by stock reversal. A concern could be that, if stock reversal is strong, a rise in the stock price could induce early exercise followed by a drop in stock price, which is not related to private information but to the cost of providing liquidity in the market. This concern has already been addressed by showing that early exercise is a negative predictor of abnormal returns for stocks with both low and high market capitalization. A further test would be to control for a reversal factor such as the one constructed by Nagel (2012). His factor is based on strategies in which stocks are bought if they under-performed recently and sold if they over-performed. Five reversal strategies are constructed. For each strategy, stocks are weighted proportional to the negative of market-adjusted returns one to five days prior. The reversal factor return on a given day is the average return of these five strategies. ${ }^{6}$

To test if early exercise predicts stock returns due to stock reversal instead of private information, the calendar time regressions from Section 3.4, Subsection 2.4.2 are rerun, this time including the reversal factor on the right-hand side. The results of the regressions are in Table 2.9. For the trading strategy in which stocks are shorted two trading days after early exercise (Panel A), a positive loading is evident on the reversal factor. However, this loading is insignificant once the strategy hedges its exposure to the four factors. The alpha declines relative to the results in Table 2.3 but remains significant for the four factor-hedge without short-sale costs. For the strategies in which

[^18]Table 2.9
Return and factor exposures in calendar time with reversal factor included.
This table shows the abnormal returns of trading strategies in which stocks are shorted if an option written on the stock has been exercised early two trading days ago (Panel A), three trading days ago (Panel B), or at least two but no more than ten trading days ago (Panel C). The stocks shorted on a given day are equally weighted. Three strategies are reported: the stocks are shorted and the proceeds invested in the risk-free asset (excess stock return), the market exposure of the short stock position is hedged (one factor), and the exposures to the three Fama-French factors (excess market returns (MKT), small market capitalization minus big (SMB), and high book-to-market ratio minus low (HML)) and the momentum factor (up minus down (UMD)) of the short stock position are hedged (four factor). The returns after subtracting short-sale costs are also reported. The columns show the ex post alpha and betas in a five-factor model: the three Fama-French factors (MKT, SMB, HML), the momentum factor (UMD), and the reversal factor (REV) from Nagel (2012). The estimates are based on ordinary least squares regressions with $t$-statistics reported in parentheses.

| Strategy | Alpha | MKT | SMB | HML | UMD | REV |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Panel A: Two days after early exercise |  |  |  |  |  |  |
| Short stock | $0.01 \%$ |  |  |  |  |  |
|  | $(0.66)$ | $(-79.39)$ | $(-25.82)$ | $(-0.88)$ | $(1.10)$ | $(3.65)$ |
| Short stock with short sale cost | $0.00 \%$ | -1.09 | -0.69 | -0.02 | 0.01 | 0.14 |
|  | $(0.05)$ | $(-78.42)$ | $(-25.91)$ | $(-0.74)$ | $(0.81)$ | $(4.32)$ |
| Short stock one factor hedge | $0.02 \%$ | 0.10 | -0.64 | -0.05 | 0.06 | 0.08 |
|  | $(1.44)$ | $(7.40)$ | $(-23.99)$ | $(-1.77)$ | $(3.56)$ | $(2.51)$ |
| Short stock one factor hedge with short sale cost | $0.01 \%$ | 0.11 | -0.65 | -0.05 | 0.07 | 0.08 |
|  | $(0.36)$ | $(7.53)$ | $(-23.93)$ | $(-1.65)$ | $(4.08)$ | $(2.42)$ |
| Short stock four factor hedge | $0.04 \%$ | 0.03 | -0.01 | -0.11 | -0.02 | 0.04 |
|  | $(2.58)$ | $(2.25)$ | $(-0.42)$ | $(-3.84)$ | $(-0.97)$ | $(1.17)$ |
| Short stock four factor hedge with short sale cost | $0.02 \%$ | 0.03 | -0.01 | -0.11 | -0.01 | 0.04 |
|  | $(1.40)$ | $(2.12)$ | $(-0.40)$ | $(-3.58)$ | $(-0.89)$ | $(1.13)$ |

Panel B: Three days after early exercise

| Short stock | 0.02\% | -1.14 | -0.74 | 0.92 | 0.03 | 0.00 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (0.82) | (-82.70) | (-26.59) | (0.40) | (1.79) | (-0.00) |
| Short stock with short sale cost | 0.02\% | -1.11 | -0.73 | -0.45 | 0.02 | 0.01 |
|  | (0.74) | (-80.70) | (-26.52) | (-0.20) | (1.12) | (0.34) |
| Short stock one factor hedge | 0.04\% | 0.08 | -0.68 | 0.43 | 0.08 | -0.05 |
|  | (1.32) | (5.50) | (-24.58) | (0.19) | (4.75) | (-1.44) |
| Short stock one factor hedge with short sale cost | 0.02\% | 0.08 | -0.69 | 0.65 | 0.09 | -0.05 |
|  | (0.63) | (5.58) | (-24.59) | (0.28) | (5.46) | (-1.38) |
| Short stock four factor hedge | 0.07\% | 0.00 | -0.05 | -2.08 | 0.02 | -0.09 |
|  | (2.83) | (0.28) | (-2.08) | (-0.96) | (1.55) | (-2.81) |
| Short stock four factor hedge with short sale cost | 0.05\% | 0.00 | -0.05 | -1.67 | 0.03 | -0.09 |
|  | (1.97) | (0.17) | (-1.92) | (-0.76) | (1.70) | (-2.60) |

Panel C: Two to ten days after early exercise

| Short stock | 0.03\% | -1.16 | -0.70 | -0.37 | 0.04 | -0.08 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1.80) | (-140.71) | (-42.12) | (-0.27) | (3.84) | (-4.01) |
| Short stock with short sale cost | 0.03\% | -1.15 | -0.69 | -1.72 | 0.03 | -0.08 |
|  | (1.99) | (-138.17) | (-41.33) | (-1.24) | (2.90) | (-3.85) |
| Short stock one factor hedge | 0.04\% | 0.06 | -0.64 | -0.70 | 0.08 | -0.10 |
|  | (2.38) | (7.16) | (-36.21) | (-0.48) | (8.33) | (-4.60) |
| Short stock one factor hedge with short sale cost | 0.03\% | 0.07 | -0.65 | -0.57 | 0.09 | -0.10 |
|  | (1.71) | (7.74) | (-36.92) | (-0.39) | (9.31) | (-4.62) |
| Short stock four factor hedge | 0.08\% | 0.00 | -0.05 | -3.07 | 0.04 | -0.13 |
|  | (4.64) | (0.18) | (-2.94) | (-2.24) | (4.70) | (-6.32) |
| Short stock four factor hedge with short sale cost | 0.06\% | 0.00 | -0.05 | -2.95 | 0.05 | -0.13 |
|  | (3.82) | (0.43) | (-3.08) | (-2.16) | (4.99) | (-6.26) |

stocks are shorted three trading days after early exercise (Panel B) and two to ten days after early exercise, the result is a negative loading on the reversal factor. In the latter case, the loading is significantly negative for all strategies: non-hedged, one factor-hedged, and four factor-hedged. This negative loading is associated with an increase in the alphas of the strategies. For example, for the strategy in which stocks are shorted two to ten trading days after early exercise, four-factor exposures are hedged ex ante, and short-sale costs are included, the $t$-statistic of alpha goes from 0.86 to 3.82 (c.f. bottom of Panel C). The alpha itself increases from $0.01 \%$ to $0.06 \%$. These results are inconsistent with reversal being the omitted variable driving the result that early exercise predicts stock returns.

### 2.6 Conclusion

I provide a theoretical foundation for how private information among option owners can make early exercise a negative predictor of future stock returns. Consistent with the existence of private information among option owners, stocks do in fact have lower abnormal returns after early exercises. I combine data sources for call option exercises with sources for stock returns and short-sale costs to test the predictions. Consistent with private information inducing early exercise, trading strategies that short the underlying stock after early exercises of options are shown to yield attractive risk-adjusted returns. Event studies show that stocks after early exercise have significantly negative cumulative abnormal returns over the next two to ten trading days after early exercise, using a four-factor model to estimate abnormal returns. The result is also negative, albeit insignificant, using a one-factor CAPM.

For every exercise, the type of agent exercising early is determined: customer of a broker, firm, or market maker. I do not find significant differences of the predictive power over future returns of the underlying stock between exercises performed by the three agent types, indicating that private information is not restricted to only one group. Looking at the number of option contracts exercised early and the future stock returns, no robust relationship can be established between them. I show, as a robustness check, that the results do not vanish when running the tests on restricted samples based on the level of market capitalization of the underlying stock. Furthermore, stock reversal is
not found to drive the results.
Overall, the results serve as evidence that private information among option owners leads to early exercise decisions and that early exercises related to private information are not concentrated among any of the three groups of agents: customers of brokers, firms, or market makers.

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## Chapter 3

## Short-Sale Costs Predict Volatility

With Christian Skov Jensen.


#### Abstract

As a new test of models of differences of opinions, we study how shorting markets interact with equity volatility. We identify positive (negative) demand shifts for shorting of stocks by simultaneous increases (decreases) in short-sale costs and short interest. Consistent with an increase in differences of opinions, a positive (negative) demand shift, on average, predicts a volatility that is 2.8 (3.9) percentage points higher (lower) over the next quarter. Event studies show that average volatilities of stocks hit by supply or demand shifts are higher than for other stocks, and volatilities peak around the time of shifts. We also find that future volatility increases with both increases and the level of short-sale costs and short interest.


### 3.1 Introduction

This paper presents results consistent with increases in differences of opinions leading to both increased stock volatility and positive demand shifts in the shorting market. Duffie, Gârleanu, and Pedersen (2002) show how an increase in differences of opinions increases demand for shorting the stock. At the same time, Andrei, Carlin, and Hasler (2014) provide a model in which higher differences of opinions cause higher stock volatility. Furthermore, Chang, Cheng, and Yu (2007) directly use stock volatility as a proxy for differences of opinions. Combining these results leads
to the prediction that a positive demand shift in the shorting market predicts higher volatility for the affected stock. We provide evidence consistent with this prediction.

We use daily data on short-sale costs and quantities for the period 2006-2012 to identify shifts in demand and supply for borrowing stocks and link the shifts to changes in volatilities. The procedure for identifying the shifts resembles the one used by Cohen, Diether, and Malloy (2007), where a positive (negative) shift in demand is observed when both short-sale costs and short interest increase (decrease). Likewise, a positive (negative) shift in supply is identified whenever short-sale costs decrease (increase) and short interest increases (decreases) simultaneously. We find that a positive (negative) demand shift in the shorting market predicts a 2.80 (3.90) percentage points higher (lower) volatility over the next quarter. The average annualized volatility over a quarter is $48.2 \%$. For this volatility level, 2.80 percentage points is an increase of $5.8 \%$. We also test the link between supply and demand shifts and consecutive volatility for three sub-periods: before, during, and after the financial crisis of 2007-2009. For all three periods, we provide evidence consistent with our prediction.

Event studies show that supply and demand shifts are associated with higher stock volatility, both relative to other stocks and to the same stock at other times. Also, many shifts happen when the average volatility in the overall stock market is high. Furthermore, we find that the level of as well as the increase in short-sale costs and short interest are positive predictors of volatilities.

Various papers study the relationship between short-sale constraints and differences of opinions. Miller (1977) shows how differences of opinions combined with short-sale constraints can drive up asset prices and reduce subsequent returns as the marginal investor becomes more optimistic than average. Harrison and Kreps (1978) show that in markets for which short selling is not possible and agents assign different subjective probabilities to the same events, agents buy stocks above their own subjective present value of future dividends. The option to sell the stock in the future increases the stock value today due to incomplete markets and the impossibility of short selling. Along the same lines, Scheinkman and Xiong (2003) present an equilibrium in which overconfidence leads to higher valuations and bubbles. Short-sale costs and heterogeneity among investors do not lead to overvaluation in the Diamond and Verrecchia (1987) model, where the
agents' information level differs. The agents realize the effects of short-sale constraints and adjust their behavior such that prices are unbiased in equilibrium. A left-skewed distribution of returns, however, is predicted on days where information is released. Short-sale costs slow down the adjustment to private bad news more than private good news, generating an asymmetric distribution. Hong and Stein (2003) construct a model with differences in both information and opinions. Shortsale constraints mean that prices sometimes only reflect the valuation of optimists. The optimists are aware of this, as is a group of arbitrageurs. As prices decline, pessimists will at some point start buying. A decline that does not lead pessimists to buy will reveal that pessimists assign an even lower value to the stock based on private information. This revelation is informative and drives prices further down, providing a mechanism for market crashes in equilibrium. Boehme, Danielsen, and Sorescu (2006) proxy for belief dispersion by idiosyncratic volatility of stock returns and combine it with data on short-sale fees obtained from one broker for almost two years. They test and confirm the hypothesis from Miller (1977) that stocks with both high short-sale constraints and high differences of opinions are overvalued. D'Avolio (2002) utilizes an 18-month data sample to show that short-sale constraints are increasing in the difference of opinions. He argues that short-sale costs increase with the difference in valuations between stock owners who do not lend their shares and other agents, i.e., short sellers and stock owners who lend their shares.

Differences of opinions can also be linked to stock volatility. Andrei et al. (2014) build a model with a single stock in which differences of opinions on the length of the business cycle lead to higher stock volatility. Dumas, Kurshev, and Uppal (2009) show in a model with differences of opinions how overconfidence can lead to increased stock volatility, while Shalen (1993) constructs a model in which dispersion of beliefs related to private information increases return volatility. Stock volatility is used as a proxy for differences of opinions by Chang et al. (2007). Karl B. Diether (2002) show empirically that dispersion in analysts' earnings forecasts is positively related to stock volatilities over the past period. Frankel and Froot (1990) report that dispersion in expectations for exchange rates among forecasters Granger-causes higher exchange rate volatility. They also show that volatility Granger-causes dispersion, although they argue that this may, in part, be spurious. Carlin, Longstaff, and Matoba (2014) find that an increase in disagreement among Wall

Street mortgage dealers about prepayment speed is followed by higher levels of mortgage return volatility.

Other papers look at the relationship between short-sale constraints and stock volatility. Bai, Chang, and Wang (2006) predict that short-sale constraints increase stock volatility when the information asymmetry among market participants is high. They also identify an opposing effect, namely, that short-sale constraints can reduce volatility through a reduction in fluctuations of aggregate demand. Cao, Zhang, and Zhou (2007) also present ambiguous predictions. Avellaneda and Lipkin (2009) model how short-sale costs and constraints can lead to short sellers being forced to close their short positions and increased stock volatility. Empirically, Chang et al. (2007) find that allowing short selling of individual stocks increases the volatility of these stocks. Using yearly estimates of stock lending supply and stock volatility, Saffi and Sigurdsson (2011) find that high short-sale cost levels are linked to high levels of stock volatility. Diether, Lee, and Werner (2009) observe the daily size of short sales and the total trade volume. They proxy intraday volatility by dividing the difference between the stock's high and low price during the day with the high price. Using this measure, they find that the daily number of short sales relative to total volume is positively related to contemporaneous volatility which they associate with short sellers being risk-bearers during periods of uncertainty. They link the uncertainty to asymmetric information instead of differences of opinions because contemporaneous stock spreads are positively related to short selling activity, however only borderline significant. Henry and McKenzie (2006) analyze a small sample of 21 stocks traded on the Hong Kong Stock Exchange from 1994-2001. They find that stock volatility is higher after a period during which the stock was sold short.

We complement the literature by empirically linking demand and supply shifts in the shorting market to the volatilities of the stocks. Our large data set, which includes daily observations, allows us to study how both short-sale costs and short interest relate to volatilities. Examining 2006-2012 makes it possible to test the prediction under various market conditions.

### 3.2 Data and sample characteristics

To test our predictions, we combine data sources and apply sample selection criteria. This section describes our data sources and how the sample for the empirical tests is constructed. Finally, we provide a variety of summary statistics.

### 3.2.1 Data and sample selection

The following two main data sources were combined to reach the results in this paper: a data set from Data Explorers containing observations for short-sale costs and short interest, and an OptionMetrics data set containing realized volatilities of stocks at different horizons.

The sample is constructed in the following way: Data Explorers data are observed on a daily frequency from July 3, 2006 to December 3, 2012. For a given stock and time interval, Data Explorers report a Daily Cost of Borrow Score (DCBS), i.e., an integer score from one to ten, for which a higher score indicates a higher cost of borrowing the stock. It also includes a Data Explorers Short Score (DSS), which is an integer score from zero to five based upon the total number of borrowed securities net of double counting as a percentage of the shares outstanding. Zero is a proxy for low short interest, five is high. The vast majority of observations for a given stock have a time interval of either one or three calendar days. Three days is natural due to weekends, but holidays can make the interval even longer. We exclude observations with an interval longer than six days because this kind of interval seems to indicate that the data are not updated from the previous trading date. We only include observations for which a CUSIP of at least eight digits is observed.

These data are then merged with data from OptionMetrics matching on observation date and CUSIP. OptionMetrics data include daily observations of realized historical volatilities over different horizons for individual stocks. In the few cases for which various DCBSs or DSSs are observed for an observation in OptionMetrics, we use the average of DCBS and DSS rounded to the nearest integer. To obtain the future realized volatility for a given horizon, say 30 calendar days, we find the first observation date in OptionMetrics at least 30 days into the future for the given stock and look up the historical realized volatility. As a result, our data set contains both past and future
realized volatilities.
We study changes in both short-sale costs and short interest. As a control in our empirical setup, we use lagged DCBS and lagged DSS. For each observation, we look up the DCBS and DSS for the previous day for the given stock. If the previous day is more than six days prior, we set the lag to missing. We also observe the realized past volatility two days prior. This is important since we want to see how volatility changes with short-sale costs when controlling for historical volatility. Going back two days ensures that the historical volatility is not estimated based on the return on the day for which we observe the change in DCBS or DSS. From this combined data set, we exclude observations for which we do not observe DCBS and DSS, both current and lagged. Realized past volatilities are at a $10,30,60$, and 91 day horizon, both with and without two-day lag, and future realized volatilities are on a 10, 30, 60, and 91 day horizon.

### 3.2.2 Summary statistics

Table 3.1 reports summary statistics for the sample sorted into buckets based on DCBS and DSS. Panel A reports the number of stock dates for each of the combinations and totals. A majority of the stock dates have a DCBS equal to one. This reflects that most stocks are not expensive to short. Panel B shows consecutive 30-day volatilities, which, as a trend, have a positive relationship with DSS and DCBS.

One prediction in this paper relates to demand shifts. Supply and demand shifts are identified by combined changes in DSS and DCBS. Table 3.2 summarizes a number of observations and lists average subsequent volatilities for the stock dates for each combination of DSS and DCBS (up, down and stable). We see that more than $90 \%$ of the stock dates have no change in either DSS or DCBS. The data set, however, is large enough to obtain more than 13,000 observations for which both DSS and DCBS change. These observations are relatively evenly distributed between the four categories of positive and negative supply and demand shifts. Each of the four groups has a higher average volatility than the full sample.

Fig. 3.1 illustrates the distribution of the four types of supply and demand shifts over time. All of the shifts spike during the peak of the financial crisis in late 2008. Table 3.3 reports the top five
days for each type of shift. Three of the top five dates for negative demand shifts are one trading day after a top five day for positive demand shifts. And three out of the top five dates for positive supply shifts are one day after a top five negative supply shift. This could indicate that many shifts, at least during the most heated times of the financial crisis, are responses to recent shifts in the opposite direction.

Table 3.1
Summary statistics: Levels.
This table shows, for each combination of Daily Cost of Borrow Score (DCBS) and Data Explorers Short Score (DSS), the number of stock dates (Panel A), and the average annualized volatilities for the consecutive 30 calendar days (Panel B). DCBS is an integer score from 1-10 for which a higher score indicates higher short-sale costs. DSS is an integer score from $0-5$ for which a higher score indicates higher short interest relative to shares outstanding. Data contain daily observations from July 3, 2006 to December 3, 2012.

## Panel A: Number of stock days

|  | DSS 0 | DSS 1 | DSS 2 | DSS 3 | DSS 4 | DSS 5 | Total |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| DCBS 1 | 649,117 | $1,556,169$ | 738,360 | 303,578 | 128,845 | 84,874 | $3,460,943$ |
| DCBS 2 | 58,519 | 80,990 | 60,814 | 39,770 | 24,655 | 30,860 | 295,608 |
| DCBS 3 | 31,532 | 43,657 | 28,490 | 19,908 | 13,090 | 20,329 | 15,7006 |
| DCBS 4 | 15,425 | 25,111 | 18,914 | 13,330 | 8,644 | 14,767 | 96,191 |
| DCBS 5 | 6,510 | 12,907 | 11,695 | 7,254 | 5,107 | 8,248 | 51,721 |
| DCBS 6 | 3,567 | 11,170 | 9,573 | 6,768 | 4,570 | 6,851 | 42,499 |
| DCBS 7 | 2,729 | 10,927 | 10,428 | 6,426 | 4,321 | 6,318 | 41,149 |
| DCBS 8 | 1,132 | 6,574 | 6,195 | 3,623 | 2,462 | 3,229 | 23,215 |
| DCBS 9 | 902 | 6,673 | 5,944 | 3,797 | 2,278 | 3,962 | 23,556 |
| DCBS 10 | 1,088 | 9,132 | 7,465 | 4,846 | 2,721 | 3,099 | 28,351 |
| Total | 770,521 | $1,763,310$ | 897,878 | 409,300 | 196,693 | 18,537 | $4,220,239$ |

Panel B: Average annualized volatilities for the consecutive 30 calendar days

|  | DSS 0 | DSS 1 | DSS 2 | DSS 3 | DSS 4 | DSS 5 | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | ---: |
| DCBS 1 | $35.9 \%$ | $42.3 \%$ | $46.2 \%$ | $48.0 \%$ | $48.7 \%$ | $47.4 \%$ | $42.8 \%$ |
| DCBS 2 | $42.7 \%$ | $57.7 \%$ | $62.1 \%$ | $61.2 \%$ | $62.3 \%$ | $57.5 \%$ | $56.5 \%$ |
| DCBS 3 | $42.4 \%$ | $59.3 \%$ | $65.9 \%$ | $65.7 \%$ | $69.2 \%$ | $60.9 \%$ | $58.9 \%$ |
| DCBS 4 | $48.2 \%$ | $63.8 \%$ | $69.9 \%$ | $68.7 \%$ | $73.1 \%$ | $64.6 \%$ | $64.1 \%$ |
| DCBS 5 | $54.0 \%$ | $65.9 \%$ | $76.4 \%$ | $73.8 \%$ | $73.9 \%$ | $69.1 \%$ | $69.2 \%$ |
| DCBS 6 | $59.1 \%$ | $69.6 \%$ | $79.7 \%$ | $81.3 \%$ | $76.1 \%$ | $78.8 \%$ | $75.1 \%$ |
| DCBS 7 | $60.1 \%$ | $72.5 \%$ | $80.4 \%$ | $88.1 \%$ | $83.2 \%$ | $75.7 \%$ | $77.7 \%$ |
| DCBS 8 | $55.4 \%$ | $75.5 \%$ | $80.9 \%$ | $83.5 \%$ | $82.6 \%$ | $73.1 \%$ | $77.6 \%$ |
| DCBS 9 | $66.4 \%$ | $77.4 \%$ | $80.2 \%$ | $85.0 \%$ | $86.9 \%$ | $80.7 \%$ | $80.4 \%$ |
| DCBS 10 | $74.9 \%$ | $75.4 \%$ | $88.7 \%$ | $93.6 \%$ | $100.8 \%$ | $92.5 \%$ | $86.3 \%$ |
| Total | $37.4 \%$ | $44.7 \%$ | $50.4 \%$ | $53.7 \%$ | $56.5 \%$ | $57.1 \%$ | $46.5 \%$ |

Number of positive demand shifts


Number of negative demand shifts


Number of positive supply shifts


Number of negative supply shifts


Figure 3.1. Frequency of supply and demand shifts over time. This figure shows the number of daily positive and negative supply and demand shifts. That is, the number of firms on a given day that experiences one of the events of a simultaneous: increase in short-sale costs and short interest (positive demand); a decrease in short-sale costs and short interest (negative demand); an increase in short-sale costs and a decrease in short interest (negative supply); a decrease in short-sale costs and an increase in short interest (positive supply). Note that a firm can experience multiple different supply and demand shifts on a given day. Our data show that the firms in this figure experienced at least one of these shifts.

Table 3.2
Summary statistics: Changes.
This table shows combinations of changes in Daily Cost of Borrow Score (DCBS) and Data Explorers Short Score (DSS). For each combination, it reports the number of stock dates (Panel A) and the average annualized volatilities for the consecutive 30 calendar days (Panel B). DCBS is an integer score from 1-10 for which a higher score indicates higher short-sale costs. DSS is an integer score from $0-5$ for which a higher score indicates higher short interest relative to shares outstanding. Data contain daily observations from July 3, 2006 to December 3, 2012.

## Panel A: Number of stock days

|  | DSS down | DSS stable | DSS up | Total |
| :--- | ---: | ---: | ---: | ---: |
| DCBS down | 3,579 | 97,366 | 3,182 | 104,127 |
| DCBS stable | 85,981 | $3,841,174$ | 84,987 | $4,012,142$ |
| DCBS up | 2,800 | 97,089 | 4,081 | 103,970 |
| Total | 92,360 | $4,035,629$ | 92,250 | $4,220,239$ |

Panel B: Average annualized volatilities for the consecutive 30 calendar days

|  | DSS down | DSS stable | DSS up | Total |
| :--- | ---: | ---: | ---: | ---: |
| DCBS down | $68.4 \%$ | $63.7 \%$ | $62.3 \%$ | $63.8 \%$ |
| DCBS stable | $48.5 \%$ | $45.5 \%$ | $47.8 \%$ | $45.6 \%$ |
| DCBS up | $69.5 \%$ | $64.4 \%$ | $64.5 \%$ | $64.6 \%$ |
| Total | $49.9 \%$ | $46.3 \%$ | $49.0 \%$ | $46.5 \%$ |

### 3.3 Predictions

Duffie et al. (2002) show that short-sale costs increase as differences of opinions increase among market participants about future development of the stock. Furthermore, they also show that the number of short sellers increases as differences of opinions increase, which implies an increase in the short interest. ${ }^{1}$ We identify a simultaneous increase in both short-sale costs and the short interest as a positive demand shift in the shorting market. Likewise, we identify negative demand shifts for which we expect the opposite relationship to volatility. Positive demand shifts can arise for other reasons than an increase in differences of opinions, which is why we do not expect a perfect relationship between the two. Our prediction is based on differences of opinions but other sources, e.g., asymmetric information, might lead to similar predictions.

Bai et al. (2006) predict that short-sale constraints increase stock volatility when the information asymmetry among market participants is high. Diether et al. (2009) attribute high level of short

[^19]
## Table 3.3

Days with most supply and demand shifts.
This table reports the top five days for number of shifts for positive (negative) supply and demand shifts. Most of the days occur during the peak of the financial crisis of 2008. March 2008 was also characterized by financial turmoil as Bear Stearns was taken over by JP Morgan Chase on March 16, 2008. Interestingly, three out of the top five dates for negative demand shocks are one trade after a top five date for positive demand shocks. Likewise, three out of top five positive supply shocks are one day after a top five date for negative supply shocks.

| Date | Number of shifts |
| :--- | :---: |
| Panel A: Positive demand shifts |  |
| March $20^{\text {th }}, 2008$ | 131 |
| September $30^{\text {th }}, 2008$ | 39 |
| November $1^{\text {st }}, 2007$ | 29 |
| October $7^{\text {th }}, 2008$ | 28 |
| October $3^{\text {rd }}, 2008$ | 18 |
|  |  |
| Panel B: Negative demand shifts | 70 |
| October 10 $0^{\text {th }}, 2008$ | 60 |
| October $1^{\text {st }}, 2008$ | 54 |
| October $8^{\text {th }}, 2008$ | 40 |
| March 24 $4^{\text {th }}, 2008$ | 34 |
| September $16^{\text {th }}, 2008$ |  |
| Panel C: Positive supply shifts | 52 |
| September $17^{\text {th }}, 2008$ | 41 |
| October $1^{\text {st }}, 2008$ | 29 |
| October 8 ${ }^{\text {th }}, 2008$ | 26 |
| March 24 ${ }^{\text {th }}, 2008$ | 23 |
| March 18 ${ }^{\text {th }}, 2008$ | 34 |
| Panel D: Negative supply shifts | 107 |
| September $15^{\text {th }}, 2008$ | 87 |
| September 30 |  |
| October $7^{\text {th }}, 2008$ | 65 |
| September $29^{\text {th }}, 2008$ | 34 |
| March $17^{\text {th }}, 2008$ |  |

selling on days with high differences between the high price and the low price of the stock to short sellers being risk-bearers during periods of increased uncertainty and they link the uncertainty to asymmetric information. Avellaneda and Lipkin (2009) include the risk of stock borrowers being forced to deliver the shares back in a model implying higher volatilities for stocks which are hard to locate and expensive to borrow. These other sources are alternative explanations for our results, but also leave room for differences of opinions being a driver.

In addition to our main prediction, we test the relationship between stock volatility and positive and negative supply shifts. Supply shifts can be related to volatility through counterparty risk: a higher future volatility of the stock can result in a higher risk for the lender of the stock. The risk that a stock borrower who sells the stock short cannot deliver the stock when requested increases if the stock value has risen rapidly in value, which is more likely with higher volatility. If potential stock lenders' anticipated volatility increases, they can become unwilling to lend out their shares, resulting in a negative supply shift. If potential stock lenders' anticipated stock volatility decreases, it can lead to a positive supply shift.

Furthermore, we look into the empirical relationship between changes of either short-sale costs (DCBS) or short interest (DSS) without the other one changing. Finally, we test if the level of short-sale costs (DCBS) and the level of short-interest (DSS) predict volatility. For the levels, we expect to see results similar to those obtained by Saffi and Sigurdsson (2011), who find a positive relationship between volatility and lending supply, as well as between volatility and short-sale costs.

### 3.4 Supply and demand shifts

In this section, we study the empirical link between stock volatilities and supply and demand shifts in the shorting market. Our main prediction is that positive (negative) demand shifts in the shorting market predict higher (lower) consecutive volatility of the affected stocks. We perform tests using regressions on the entire sample, event studies, and regressions on three sub-periods: before, during, and after the financial crisis of 2007-2009.

### 3.4.1 Regressions

In this subsection, we perform cross-sectional regressions to test how supply and demand shifts affect volatility. We test the relationship between stock volatility and the supply and demand shifts by regressing future realized volatility on our variable 'Shift' which captures the effect associated with the four different possible supply and demand shifts. The dependent variable in our regression setup represents realized volatility over the next $10,30,60$, and 91 calendar days. 'Shift' is a categorical variable with five levels, the four combinations of positive and negative supply and demand shifts, and zero if no shift occurred. That is, 'Shift ${ }_{t}^{\mathrm{t}}$ ' takes on the category 'negative demand' if both short-sale costs and short interest decrease from day $t-1$ to $t$ for stock $i$ and, correspondingly, for the other three levels. Notice this implies that a firm in our model can only be subject to, at most, one of the four shifts on any given day. Using only price and quantity data, we cannot disentangle if multiple shifts occurred on the same day, e.g., simultaneous negative demand and positive supply shifts. If, however, we observe, for example, that both price and short interest go down on a single day, then we know that at least a negative demand shift occurred (Cohen et al., 2007). With this in mind, we set up the regression:

$$
\begin{align*}
\sigma_{t+1, t+s}^{i}= & \operatorname{Shift}_{t}^{i}+\beta_{1} \sigma_{t-11, t-2}^{i}+\beta_{2} \sigma_{t-31, t-2}^{i}+\beta_{3} \sigma_{t-61, t-2}^{i}+\beta_{4} \sigma_{t-92, t-2}^{i}  \tag{3.1}\\
& +\mathrm{DCBS}_{t-1}^{i}+\mathrm{DSS}_{t-1}^{i}+\mathrm{DFE}_{t-1}+\varepsilon_{t+1, t+s}^{i}
\end{align*}
$$

The future volatility $\sigma_{t+1, t+s}^{i}$ is the earliest estimated volatility for the stock that does not include date $t$. In most cases, this means that it is from $t+1$ to $t+s$, but weekends, holidays, and missing observations, in some cases, lead to an estimate that is based on later days. The historical volatilities are observed on the latest observation day of the stock prior to $t-1$. Because we want to test how volatilities change after the shift when controlling for volatilities before the shift, neither past nor future volatility estimates are based on the returns of day $t-1$ and $t$. We control for historical volatilities lagged $10,30,60$, and 91 calendar days. To make sure our results are not driven by clustering of shifts around periods with particularly high or particularly low volatilities, we add date fixed effects (DFE). We wish to control for the overall consecutive realized volatility in the
stock market. ${ }^{2}$ Furthermore, to make sure that our results are not a consequence of the level of short-sale costs and short interest, we control for the one-day lagged level of DCBS and DSS as fixed effects. The reference level is when neither DCBS nor DSS changed. In other words, we omit cases for which precisely one of short-sale costs or short interest changed because these are ambiguous in our setup; i.e., since we cannot determine which shifts occurred, they are excluded. This is consistent with the methodology that Cohen et al. (2007) use.

Table 3.4 shows the results of the regressions described by (3.1), including test statistics corrected for autocorrelation and clustered on firm level. For all horizons, a positive demand shift is a significant positive predictor of future volatility. The estimate increases with the horizon from 1.7 percentage points for ten-day future volatility to 2.8 percentage points for 91 days. This is consistent with an increase in differences of opinions leading to both higher stock volatility and a positive demand shift in the shorting market. The average annualized quarterly volatility in the sample is $48.2 \%$. For this volatility level, a 2.8 percentage points increase corresponds to $5.8 \%$ higher volatility over the next quarter. The increasing positive predictive power of a positive demand shift with the horizon supports that demand shifts have long-term effects on volatility. Negative demand shifts are significantly negative predictors for future volatility. This is consistent with a reduction in differences of opinions. The effect is, in absolute terms, economically larger than for positive demand shifts.

For supply shifts, the effects are smaller. Negative supply shifts are insignificant in predicting volatility. Positive supply shifts are significant negative predictors of volatility over the next 60 and 91 days, but insignificant for shorter horizons. The result for positive supply shifts aligns with the idea that a lower expected future volatility, all else being equal, makes potential lenders more willing to lend due to reduced counterparty risk.

### 3.4.2 Event studies

The regressions discussed in the previous section test how supply and demand shifts affect volatility while controlling for date fixed effects, historical volatilities, lagged DCBS, and lagged DSS. In this

[^20]Table 3.4
Do demand and supply shifts in the shorting market predict stock volatility?
This table shows the results of the cross-sectional regression:

$$
\begin{aligned}
\sigma_{t+1, t+s}^{i}= & \operatorname{Shift}_{t}^{i}+\beta_{1} \sigma_{t-11, t-2}^{i}+\beta_{2} \sigma_{t-31, t-2}^{i}+\beta_{3} \sigma_{t-61, t-2}^{i}+\beta_{4} \sigma_{t-92, t-2}^{i} \\
& +\mathrm{DCBS}_{t-1}^{i}+\mathrm{DSS}_{t-1}^{i}+\mathrm{DFE}_{t-1}+\varepsilon_{t+1, t+s}^{i}
\end{aligned}
$$

where $t$ is date and $i$ is stock. The dependent variable, $\sigma_{t+1, t+s}^{i}$, is future realized stock volatility from day $t+1$ to $t+s$, for varying horizons, $s$. The categorical variable, 'Shift ${ }_{t}{ }^{\prime}$, can take five levels: the four combinations of positive and negative supply and demand shifts, and zero if no shift occurred. Shifts are identified by simultaneous changes in Daily Cost of Borrow Score (DCBS) and Data Explorers Short Score (DSS) from $t-1$ to $t$. DCBS is a measure of short-sale costs and has ten levels (1-10), where a higher score indicates higher costs. DSS is a measure of short interest and has six levels (0-5), where a higher score indicates higher short interest. We control for one-day lagged DCBS and DSS as fixed effects, date-fixed effects, and past realized volatilities on different horizons. All volatilities are annualized. We cluster standard deviations by firm. $t$-statistics corrected for autocorrelation are in parentheses. Significance levels of $10 \%, 5 \%$, and $1 \%$ are shown with $*$, **, and $* * *$, respectively. Observations for which exactly one of DCBS and DSS changed from $t-1$ to $t$ are excluded. Data are from July 3, 2006 to December 3, 2012.

| Shift type | 10 day vol | 30 day vol | 60 day vol | 91 day vol |
| :--- | :---: | :---: | :---: | :---: |
| Positive demand | $0.017^{* * *}$ | $0.018^{* * *}$ | $0.023^{* * *}$ | $0.028^{* * *}$ |
|  | $(2.07)$ | $(2.88)$ | $(3.61)$ | $(4.13)$ |
| Negative demand | $-0.021^{* *}$ | $-0.040^{* * *}$ | $-0.042^{* * *}$ | $-0.039^{* * *}$ |
|  | $(-2.54)$ | $(-6.14)$ | $(-6.62)$ | $(-6.10)$ |
| Positive supply | -0.011 | -0.009 | $-0.018^{* * *}$ | $-0.019^{* * *}$ |
|  | $(-1.44)$ | $(-1.31)$ | $(-2.75)$ | $(-2.90)$ |
| Negative supply | -0.007 | -0.001 | -0.005 | -0.000 |
|  | $(-0.85)$ | $(-0.06)$ | $(-0.59)$ | $(-0.03)$ |
| 10 day historical volatility | $0.095^{* * *}$ | $0.082^{* * *}$ | $0.074^{* * *}$ | $0.071^{* * *}$ |
|  | $(16.47)$ | $(14.31)$ | $(12.75)$ | $(12.40)$ |
| 30 day historical volatility | $0.071^{* * *}$ | $0.070^{* * *}$ | $0.059^{* * *}$ | $0.070^{* * *}$ |
|  | $(9.91)$ | $(9.71)$ | $(8.38)$ | $(10.97)$ |
| 60 day historical volatility | -0.007 | -0.023 | 0.018 | 0.017 |
|  | $(-0.21)$ | $(-0.58)$ | $(0.44)$ | $(0.41)$ |
| 91 day historical volatility | $0.434^{* * *}$ | $0.484^{* * *}$ | $0.461^{* * *}$ | $0.450^{* * *}$ |
|  | $(19.28)$ | $(19.80)$ | $(19.71)$ | $(19.68)$ |
| Adj. $R^{2}$ | 0.447 | 0.548 | 0.582 | 0.602 |
| No obs. | $3,854,816$ | $3,854,816$ | $3,854,816$ | $3,854,816$ |

subsection, we show in event time, how the level of volatilities varies around supply and demand shifts in the shorting market. Each supply and demand shift can be seen as an event. As in the previous sections, supply and demand shifts are identified when both DCBS and DSS change from one day to the next. We use realized volatilities over periods of ten calendar days obtained from OptionMetrics, generating observations every ten calendar days in event time. Realized volatilities are only reported by OptionMetrics on trading days, making technical adjustments necessary. ${ }^{3}$

Fig. 3.2 presents the results for demand shifts, while Fig. 3.3 shows supply shifts. Error bars of plus and minus two standard deviations of the mean estimates are also illustrated. The illustrations in Fig. 1 and 2 include an average of volatilities of all stocks. For each date in the OptionMetrics database, we observe the average equity volatility. Next, for each event date, we look up the average equity volatility, say ten days prior, and then average these volatilities.

For all types of shifts, the average volatilities of stocks experiencing a shift are consistently higher than the average volatilities of all stocks. Stocks with higher volatilities seem more prone to supply and demand shifts in the shorting market. Another trend for all four types of shifts is the quite strong correlations in event time between the average volatility of affected stocks and all stocks.

Volatilities are elevated just after, and especially just before, positive demand shifts, also by more than the overall market trend would suggest, c.f. upper part of Fig. 3.2. If increases in differences of opinions drive elevated volatilities both before and after the shift, it requires that increased differences of opinions drive up volatility before inducing the demand shift. An alternative mechanism could be that sudden uncertainty and unexpectedly high levels of volatility increase differences of opinions among market participants, inducing the positive demand shift.

The trend for negative demand shifts is also high volatilities just before the shift. When the

[^21]Realized volatilities in event time around positive demand shifts


Realized volatilities in event time around negative demand shifts


Figure 3.2. Event study for demand shifts. This figure shows two event studies on realized volatilities. The dots represent the average of realized volatility over a range of ten calendar days. The events are positive and negative demand shifts in the lending market for the stock. For comparison, the crosses show the average realized ten-day volatilities observed for all equities in the OptionMetrics database on the same dates. The error bars represent plus/minus two times the standard error of the mean.

Realized volatilities in event time around positive supply shifts


Realized volatilities in event time around negative supply shifts


Figure 3.3. Event study for supply shifts. This figure shows two event studies on realized volatilities. The dots represent the average of realized volatility over a range of ten calendar days. The events are positive and negative supply shifts in the lending market for the stock. For comparison, the crosses show the average realized ten-day volatilities observed for all equities in the OptionMetrics database on the same dates. The error bars represent plus/minus two times the standard error of the mean.
negative demand shift is a reversal of a recent positive demand shift, it is not surprising that high levels of volatilities occur just before the negative shift. The dates in Table 3.3 indicate that many shifts could be responses to recent shifts in the opposite directions. Three out of the top five dates for negative demand shifts (Panel B) are one trading day after a top five day for positive demand shifts (Panel A). Looking into the data, $20 \%$ of the negative demand shifts occur four calendar days or less after a positive demand shift for the same stock. Based on the event studies, it is hard to detect long-term effects of the demand shifts.

When comparing the event study results to the previous regression results, it is important to note that the regression results do not necessarily imply that volatilities increase (decrease) in absolute terms after positive (negative) demand shifts, but that the predicted volatilities are higher (lower) compared to predicted volatilities without the shift. The event studies reveal that both positive and negative supply and demand shifts are more common during times, when the stocks experience higher volatilities than usual. In addition, they show that shifts are also more common for stocks with higher volatilities than average.

Turning to the supply shifts illustrated in Fig. 3.3, some of the same trends found for demand shifts are evident. There is, especially for negative supply shifts, for example, a build-up of volatility before the shift, which reflects the overall market trend. The average of the last ten-day volatility before both positive and negative supply shifts is higher than the consecutive volatility, contrary to the market trend. Volatility building up before negative supply shifts and staying relatively high after is consistent with stock lenders reducing supply due to increased anticipated counterparty risk. Higher volatility increases the probability of extreme positive returns such that short sellers cannot deliver the stock back to the lender due to insufficient margins.

Average volatility around positive supply shifts is lower than for negative supply shifts. In cases where a positive supply shift is a response to lowered counterparty risk, we would expect to see a downward trend after the shift. Even though the volatility peaks just before the shift, the trend after the shift is not clear.

Supply shifts can also be linked to volatility by large trades. If stocks are traded between two parties with differences in willingness to lend shares, it can result in a supply shift. Gener-
ally, a positive correlation between volume and volatility has been documented (see, e.g., Karpoff (1987)). This correlation could lead to supply shifts being more prevalent in times of high volatility. Delayed settlement in the stock market can increase this effect for negative supply shifts. If a stock lender sells stocks, the seller cannot lend out the stock from the time of the transaction. The buyer will not be able to lend out the stock until after the trade has settled, often three days after the transaction date (Duffie et al., 2002).

### 3.4.3 Regressions in sub-periods

To investigate if our results are driven by extraordinary market circumstances in a small part of our sample we rerun the analyses for sub-periods. The concern arises because 19 out of 20 days in the four top fives for days with most demand and supply shifts are during times with extreme financial turmoil in 2008, c.f. Table 3.3. Also, during the fall of 2008, a ban on shorting of financial stocks was suddenly implemented and lifted again around three weeks after, see Boehmer, Jones, and Zhang (2013) for further details. To address the concern that our results stem from such special conditions we test our predictions for three subsamples, i.e., before, during and after the financial crisis. We repeat regressions of the form stated in (3.1) for three sub-periods: pre-crisis (July 3, 2006 to November 30, 2007); crisis (December 1, 2007 to June 30, 2009); and post-crisis (July 1, 2009 to December 3, 2012). Table 3.5 reports the results.

The supply and demand shifts are spread out across the sub-periods, with more than 600 shifts of each type in each sub-period. Positive demand shifts are estimated to be positive predictors of volatility for all sub-periods and all horizons. The estimates are statistically significant for longer horizons, for pre-crisis and crisis. Negative demand shifts are significantly negative predictors of volatility for all horizons and periods except ten-day future volatility during the crisis. Combined, the results for demand shifts are evidence that demand shifts predict volatility under different market conditions, i.e., consistent with differences of opinions driving both.

Both positive and negative supply shifts are insignificant during the crisis. Positive supply shifts are negative predictors of volatility before and after the crisis, and a majority of these estimates are significant on a 5\% level. Negative supply shifts vary, but two of the estimates before the crisis

Table 3.5
Do demand and supply shifts in the shorting market predict stock volatility in different periods?
This table shows the results of the cross-sectional regression:

$$
\begin{aligned}
\sigma_{t+1, t+s}^{i}= & \operatorname{Shift}_{t}^{i}+\beta_{1} \sigma_{t-11, t-2}^{i}+\beta_{2} \sigma_{t-31, t-2}^{i}+\beta_{3} \sigma_{t-61, t-2}^{i}+\beta_{4} \sigma_{t-92, t-2}^{i} \\
& +\operatorname{DCBS}_{t-1}^{i}+\mathrm{DSS}_{t-1}^{i}+\mathrm{DFE}_{t-1}+\varepsilon_{t+1, t+s}^{i}
\end{aligned}
$$

where $t$ is date and $i$ is stock. Regressions are performed on three sub-periods: pre-crisis (July 2006 to November 2007), crisis (December 2007 to June 2009), and post-crisis (July 1, 2009 to December 3, 2012). The dependent variable, $\sigma_{t+1, t+s}^{i}$, is future realized stock volatility from day $t+1$ to $t+s$, for varying horizons, $s$. The categorical variable, 'Shift ${ }_{t}^{i}$ ', can take five levels: the four combinations of positive and negative supply and demand shifts, and zero if no shift occurred. Shifts are identified by simultaneous changes in Daily Cost of Borrow Score (DCBS) and Data Explorers Short Score (DSS) from $t-1$ to $t$. DCBS is a measure of short-sale costs and has ten levels (1-10), where a higher score indicates higher costs. DSS is a measure of short interest and has six levels ( $0-5$ ), where a higher score indicates higher short interest relative to shares outstanding. We control for one-day lagged DCBS and DSS as fixed effects, date-fixed effects, and past realized volatilities on different horizons. All volatilities are annualized. We cluster standard deviations by firm. $t$-statistics corrected for autocorrelation are in parentheses. Significance levels of $10 \%, 5 \%$, and $1 \%$ are shown with ${ }^{*},{ }^{* *}$, and ${ }^{* * *}$, respectively. Observations for which exactly one of DCBS and DSS changed from $t-1$ to $t$ are excluded.

| Shift type | 10 days | 30 days | 60 days | 91 days | Number of shifts |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Panel A: Pre crisis |  |  |  |  |  |
| Positive demand | 0.011 | $0.022^{* *}$ | $0.024^{* * *}$ | $0.042^{* * *}$ | 1113 |
|  | $(1.27)$ | $(2.51)$ | $(2.93)$ | $(3.30)$ |  |
| Negative demand | $-0.040^{* * *}$ | $-0.044^{* * *}$ | $-0.046^{* * *}$ | $-0.032^{* * *}$ | 791 |
|  | $(-2.97)$ | $(-4.09)$ | $(-4.77)$ | $(-2.97)$ |  |
| Positive supply | $-0.020^{* *}$ | $-0.022^{* *}$ | $-0.021^{* *}$ | $-0.018^{*}$ | 905 |
|  | $(-2.00)$ | $(-2.46)$ | $(-2.05)$ | $(-1.86)$ |  |
| Negative supply | $-0.027^{* * *}$ | -0.014 | $-0.017^{*}$ | 0.003 | 665 |
|  | $(-3.06)$ | $(-1.41)$ | $(-1.89)$ | $(0.28)$ |  |
| 10 day historical volatility | $0.080^{* * *}$ | $0.073^{* * *}$ | $0.063^{* * *}$ | $0.064^{* * *}$ |  |
|  | $(12.14)$ | $(11.61)$ | $(11.82)$ | $(12.94)$ |  |
| 30 day historical volatility | $0.061^{* * *}$ | $0.054^{* * *}$ | $0.042^{* * *}$ | $0.062^{* * *}$ |  |
|  | $(4.74)$ | $(3.87)$ | $(3.55)$ | $(5.85)$ |  |
| 60 day historical volatility | -0.033 | -0.022 | $0.043^{* *}$ | 0.009 |  |
|  | $(-1.61)$ | $(-0.94)$ | $(2.25)$ | $(0.37)$ |  |
| 91 day historical volatility | $0.383^{* * *}$ | $0.410^{* * *}$ | $0.376^{* * *}$ | $0.400^{* * *}$ |  |
|  | $(10.68)$ | $(10.23)$ | $(10.61)$ | $(10.70)$ |  |
| Adj. $R^{2}$ | 0.258 | 0.344 | 0.379 | 0.408 |  |
| No obs. | 745,275 | 745,275 | 745,275 | 745,275 |  |


| Panel B: Crisis |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Positive demand | $\begin{aligned} & 0.018 \\ & (1.22) \end{aligned}$ | $\frac{0.022^{* *}}{(2.01)}$ | $\begin{gathered} 0.034^{* * *} \\ (3.11) \end{gathered}$ | $\begin{gathered} 0.034^{* * *} \\ (3.36) \end{gathered}$ | 1602 |
| Negative demand | $\begin{aligned} & 0.006 \\ & (0.40) \end{aligned}$ | $\begin{gathered} -0.027^{* *} \\ (-2.44) \end{gathered}$ | $\begin{gathered} -0.028^{* *} \\ (-2.49) \end{gathered}$ | $\begin{gathered} -0.029^{* * *} \\ (-2.69) \end{gathered}$ | 1576 |
| Positive supply | $\begin{aligned} & 0.000 \\ & (0.02) \end{aligned}$ | $\begin{aligned} & 0.012 \\ & (0.99) \end{aligned}$ | $\begin{aligned} & 0.002 \\ & (0.15) \end{aligned}$ | $\begin{aligned} & -0.005 \\ & (-0.46) \end{aligned}$ | 1232 |
| Negative supply | $\begin{aligned} & -0.004 \\ & (-0.27) \end{aligned}$ | $\begin{aligned} & 0.010 \\ & (0.66) \end{aligned}$ | $\begin{aligned} & 0.008 \\ & (0.60) \end{aligned}$ | $\begin{aligned} & 0.008 \\ & (0.57) \end{aligned}$ | 1305 |
| 10 day historical volatility | $\begin{gathered} 0.128^{* * *} \\ (15.11) \end{gathered}$ | $\begin{gathered} 0.101^{* * *} \\ (13.25) \end{gathered}$ | $\begin{gathered} 0.092^{* * *} \\ (12.76) \end{gathered}$ | $\begin{gathered} 0.085^{* * *} \\ (11.91) \end{gathered}$ |  |
| 30 day historical volatility | $\underset{(8.34)}{0.101^{* * *}}$ | $\begin{gathered} 0.106^{* * *} \\ (10.15) \end{gathered}$ | $\begin{gathered} 0.091 * * * \\ (10.61) \end{gathered}$ | $\begin{aligned} & 0.097^{* * *} \\ & (12.94) \end{aligned}$ |  |
| 60 day historical volatility | $\begin{gathered} 0.064^{* * *} \\ (2.82) \end{gathered}$ | $\begin{aligned} & 0.031 \\ & (1.34) \end{aligned}$ | $\begin{gathered} 0.050^{* *} \\ (2.27) \end{gathered}$ | $\underset{(2.60)}{0.056^{* * *}}$ |  |
| 91 day historical volatility | $\begin{aligned} & 0.391^{* * *} \\ & (25.79) \end{aligned}$ | $\begin{aligned} & 0.461^{* * *} \\ & (24.61) \end{aligned}$ | $\begin{gathered} 0.458^{* * *} \\ (24.22) \end{gathered}$ | $\begin{gathered} 0.445^{* * *} \\ (26.30) \end{gathered}$ |  |
| Adj. $R^{2}$ No obs. |  | $\begin{gathered} 0.572 \\ 890,446 \end{gathered}$ | $\begin{gathered} 0.597 \\ 890,446 \end{gathered}$ | $\begin{gathered} 0.604 \\ 890,446 \end{gathered}$ |  |
| Panel C: Post crisis |  |  |  |  |  |
| Positive demand | $\begin{aligned} & 0.023 \\ & (1.62) \end{aligned}$ | $\begin{aligned} & 0.012 \\ & (1.18) \end{aligned}$ | $\begin{aligned} & 0.010 \\ & (1.09) \end{aligned}$ | $\begin{aligned} & 0.011 \\ & (1.09) \end{aligned}$ | 1366 |
| Negative demand | $\begin{gathered} -0.047^{* * *} \\ (-4.47) \end{gathered}$ | $\begin{gathered} -0.061^{* * *} \\ (-6.47) \end{gathered}$ | $\begin{gathered} -0.066^{* * *} \\ (-7.38) \end{gathered}$ | $\begin{gathered} -0.067^{* * *} \\ (-7.17) \end{gathered}$ | 1212 |
| Positive supply | $\begin{aligned} & -0.011 \\ & (-0.77) \end{aligned}$ | $\begin{aligned} & -0.019 \\ & (-1.62) \end{aligned}$ | $\begin{gathered} -0.037^{* * *} \\ (-3.47) \end{gathered}$ | $\begin{gathered} -0.036^{* * *} \\ (-3.32) \end{gathered}$ | 1045 |
| Negative supply | $\begin{aligned} & -0.001 \\ & (-0.04) \end{aligned}$ | $\begin{aligned} & -0.010 \\ & (-0.63) \end{aligned}$ | $\begin{aligned} & -0.020 \\ & (-1.46) \end{aligned}$ | $\begin{aligned} & -0.020 \\ & (-1.57) \end{aligned}$ | 830 |
| 10 day historical volatility | $\begin{gathered} 0.066^{* * *} \\ (8.10) \end{gathered}$ | $\begin{gathered} 0.063^{* * *} \\ (7.38) \end{gathered}$ | ${ }_{(6.34)}^{0.057^{* * *}}$ | $\underset{(6.48)}{0.057^{* * *}}$ |  |
| 30 day historical volatility | $\underset{(4.48)}{0.044^{* * *}}$ | $\underset{(3.69)}{0.043^{* * *}}$ | $\underset{(2.86)}{0.035^{* * *}}$ | $\underset{(4.51)}{0.048^{* * *}}$ |  |
| 60 day historical volatility | $\begin{aligned} & -0.059 \\ & (-0.89) \end{aligned}$ | $\begin{aligned} & -0.069 \\ & (-0.94) \end{aligned}$ | $\begin{aligned} & -0.019 \\ & (-0.25) \end{aligned}$ | $\begin{aligned} & -0.016 \\ & (-0.21) \end{aligned}$ |  |
| 91 day historical volatility | $\begin{gathered} 0.465^{* * *} \\ (12.96) \end{gathered}$ | $\begin{gathered} 0.507^{* * *} \\ (13.30) \end{gathered}$ | $\begin{gathered} 0.474^{* * *} \\ (13.00) \end{gathered}$ | $\begin{gathered} 0.457^{* * *} \\ (12.72) \end{gathered}$ |  |
| Adj. $R^{2}$ No obs. | $\begin{gathered} 0.298 \\ 2,220,373 \\ \hline \end{gathered}$ | $\begin{gathered} 0.389 \\ 2,220,373 \\ \hline \end{gathered}$ | $\begin{gathered} 0.422 \\ 2,220,373 \\ \hline \end{gathered}$ | $\begin{gathered} 0.448 \\ 2,220,373 \\ \hline \end{gathered}$ |  |

are significantly negative, i.e., the same sign as positive supply shifts. The unclear results could relate to large trades and delayed settlement with no persistent predictive power for volatility. As mentioned, a large trade can matter if the buyer has a different willingness to lend stock than the seller. Delayed settlement matters, especially after large trades, since a traded share cannot be lent out before the trade settles.

### 3.5 Changes and levels

In the previous section, we have focused on supply and demand shifts in the shorting market. Our rich data set allows us to study how changes in either short-sale costs or short interest with the other one constant predict volatility. We also test if the level of short-sale costs and short interest has predictive power for future stock volatility. This section provides results for how such changes and levels predict stock volatility.

### 3.5.1 Changes in short-sale costs or short interest

In this section, we regress future realized volatility on a categorical variable indicating changes in either DCBS (measuring short-sale costs) or DSS (measuring short interest). We define a categorical variable, 'Change ${ }_{t}^{i}$, which takes five values, one for either DCBS or DCBS going up or down from $t-1$ to $t$, and zero if neither DCBS or DSS changed. We omit cases for which both DCBS and DSS changed. Like the regressions for demand and supply shifts in the stock lending market, we include controls for past volatility, level of DSS, level of DCBS and DFE. We want to purely test the marginal effect of short-sale costs and short interest and not their joint effect. This leads us to the regression:

$$
\begin{align*}
\sigma_{t+1, t+s}^{i}= & \text { Change }_{t}^{i}+\beta_{1} \sigma_{t-11, t-2}^{i}+\beta_{2} \sigma_{t-31, t-2}^{i}+\beta_{3} \sigma_{t-61, t-2}^{i}+\beta_{4} \sigma_{t-92, t-2}^{i}  \tag{3.2}\\
& +\mathrm{DCBS}_{t-1}^{i}+\mathrm{DSS}_{t-1}^{i}+\mathrm{DFE}_{t-1}+\varepsilon_{t+1, t+s}^{i}
\end{align*}
$$

Table 3.6 reports the results of (3.2). An increase in short-sale costs is a highly significant positive predictor for future volatility. A decrease is a significant negative predictor. The effect is
not exactly symmetric, with the effect of increases being largest in absolute terms. The estimates increase in absolute terms with the horizon, indicating that it is a long-term effect.

Table 3.6
Do changes in short-sale costs and short interest predict stock volatility?
This table shows the results of the cross-sectional regression:

$$
\begin{aligned}
\sigma_{t+1, t+s}^{i}= & \text { Change }_{t}^{i}+\beta_{1} \sigma_{t-11, t-2}^{i}+\beta_{2} \sigma_{t-31, t-2}^{i}+\beta_{3} \sigma_{t-61, t-2}^{i}+\beta_{4} \sigma_{t-92, t-2}^{i} \\
& +\mathrm{DCBS}_{t-1}^{i}+\mathrm{DSS}_{t-1}^{i}+\mathrm{DFE}_{t-1}+\varepsilon_{t+1, t+s}^{i}
\end{aligned}
$$

where $t$ is date and $i$ is stock. The dependent variable, $\sigma_{t+1, t+s}^{i}$, is future realized stock volatility from day $t+1$ to $t+s$, for varying horizons, $s$. The categorical variable, 'Change ${ }_{t}{ }^{i}$, can take five levels: four for either a decrease or increase in Daily Cost of Borrow Score (DCBS) and Data Explorers Short Score (DSS), and zero if no change occurred. Changes are identified by a change in either DCBS or DSS from $t-1$ to $t$. DCBS is a measure of short-sale costs and has ten levels ( $1-10$ ), where a higher score indicates higher costs. DSS is a measure of short interest and has six levels ( $0-5$ ), where a higher score indicates higher short interest relative to shares outstanding. We control for one-day lagged DCBS and DSS as fixed effects, date-fixed effects, and past realized volatilities on different horizons. All volatilities are annualized. We cluster standard deviations by firm. $t$-statistics corrected for autocorrelation are in parentheses. Significance levels of $10 \%, 5 \%$, and $1 \%$ are shown with $*, * *$, and $* * *$, respectively. Observations for which both DCBS and DSS changed from $t-1$ to $t$ are excluded. Data are from July 3, 2006 to December 3, 2012.

| Horizon | 10 days | 30 days | 60 days | 91 days |
| :--- | :---: | :---: | :---: | :---: |
| Increase in short-sale costs | $0.015^{* * *}$ | $0.018^{* * *}$ | $0.019^{* * *}$ | $0.020^{* * *}$ |
|  | $(6.83)$ | $(8.71)$ | $(8.80)$ | $(8.87)$ |
| Decrease in short-sale costs | $-0.010^{* * *}$ | $-0.012^{* * *}$ | $-0.013^{* * *}$ | $-0.014^{* * *}$ |
|  | $(-9.21)$ | $(-11.64)$ | $(-11.02)$ | $(-11.54)$ |
| Increase in short interest | $0.003^{* *}$ | $0.004^{* * *}$ | $0.004^{* * *}$ | $0.000^{* * *}$ |
|  | $(2.00)$ | $(2.78)$ | $(3.19)$ | $(3.56)$ |
| Decrease in short interest | $-0.011^{* * *}$ | $-0.013^{* * *}$ | $-0.012^{* * *}$ | $-0.013^{* * *}$ |
|  | $(-9.21)$ | $(-11.64)$ | $(-11.02)$ | $(-11.54)$ |
| 10 day historical volatility | $0.097^{* * *}$ | $0.083^{* * *}$ | $0.074^{* * *}$ | $0.071^{* * *}$ |
|  | $(19.46)$ | $(18.74)$ | $(15.83)$ | $(16.16)$ |
| 30 day historical volatility | $0.073^{* * *}$ | $0.071^{* * *}$ | $0.061^{* * *}$ | $0.070^{* * *}$ |
|  | $(10.60)$ | $(10.61)$ | $(9.91)$ | $(13.20)$ |
| 60 day historical volatility | -0.002 | -0.018 | 0.020 | 0.017 |
|  | $(-0.07)$ | $(-0.53)$ | $(0.57)$ | $(0.49)$ |
| 91 day historical volatility | $0.423^{* * *}$ | $0.474^{* * *}$ | $0.454^{* * *}$ | $0.446^{* * *}$ |
|  | $(21.54)$ | $(21.98)$ | $(22.00)$ | $(21.96)$ |
| Adj. $R^{2}$ | 0.450 | 0.551 | 0.586 | 0.605 |
| No obs. | $4,206,597$ | $4,206,597$ | $4,206,597$ | $4,206,597$ |

D'Avolio (2002) argues that high short-sale costs relate to differences of opinions. Stock owners not lending their shares must assign a different value to the stock relative to short sellers and to stock owners lending their shares and earning lending fees. The result is consistent with an
increase in differences of opinions causing both higher volatility and higher short-sale costs. For short interest, an increase is a significant positive predictor for future volatility, and a decrease is a significant negative predictor. Interestingly, the effect of a decrease is more significant and roughly three times as large as the effect of an increase in absolute terms.

Compared to the results for supply and demand shifts, we generally see higher levels of significance for the changes in short-sale costs or short interest. This can be attributed to the sample having more observations for shifts of either DCBS or DSS than for both of them at the same time, c.f. Table 3.2. The estimated effects of positive demand shifts are larger than for increases in short-sale costs or short interest alone for each of the horizons. In absolute terms, the effect of negative demand shifts dominates the effect of decreases in short-sale costs or short interest alone in a similar way.

### 3.5.2 Levels of short-sale costs and short interest

In this section, we test if the level of short-sale costs (DCBS) and short interest (DSS) has explanatory power for future volatility. DSS is an indicator for the relative level of short interest, i.e., relative to the number of shares outstanding. We regress future realized volatility on the level of short-sale costs and short interest, i.e., DCBS ${ }_{t}^{i}$ captures the time $t$ level of short-sale costs and $\mathrm{DSS}_{t}^{i}$ captures the time $t$ level of short interest for stock $i$. Both DCBS and DSS are integer scores, allowing us to estimate separate coefficients for each level. The regression is:

$$
\begin{align*}
\sigma_{t+1, t+s}^{i}= & \mathrm{DCBS}_{t}^{i}+\mathrm{DSS}_{t}^{i}+\beta_{1} \sigma_{t-11, t-2}^{i}+\beta_{2} \sigma_{t-29, t-1}^{i}  \tag{3.3}\\
& +\beta_{3} \sigma_{t-59, t}^{i}+\beta_{4} \sigma_{t-90, t}^{i}+\mathrm{DFE}_{t}+\varepsilon_{t+1, t+s}^{i}
\end{align*}
$$

Table 7 shows the results and includes regression results of specifications where some of the independent variables are dropped. The reference level is DCBS equal to one and DSS equal to zero.

Strikingly, all estimates for other levels of DCBS and DSS are positive and highly significant. The vast majority of observations have DCBS equal one, c.f. Table 3.1, which can be interpreted as the stocks that have short-sale costs close to zero. DCBS equal to one is the reference level, hence,

Table 3.7
Do short-sale costs and short interests predict stock volatility?
This table shows the results of the cross-sectional regression:
$\sigma_{t+1, t+s}^{i}=\operatorname{DCBS}_{t}^{i}+\operatorname{DSS}_{t}^{i}+\beta_{1} \sigma_{t-11, t-2}^{i}+\beta_{2} \sigma_{t-29, t-1}^{i}+\beta_{3} \sigma_{t-59, t}^{i}+\beta_{4} \sigma_{t-90, t}^{i}+\operatorname{DFE}_{t}+\varepsilon_{t+1, t+s}^{i}$
where $t$ is date and $i$ is stock. The dependent variable, $\sigma_{t+1, t+s}^{i}$, is future realized stock volatility from day $t+1$ to $t+s, s$ being 10 or 30 . Daily Cost of Borrow Score (DCBS) is a measure of short-sale costs and has ten levels (1-10), where a higher score indicates higher costs. Data Explorers Short Score (DSS) is a measure of short interest and has six levels ( $0-5$ ), where a higher score indicates higher short interest relative to shares outstanding. They are estimated as categorical variables with one as the reference level for DCBS and zero for DSS. Regressions leaving out either DCBS or DSS are also reported. We control for date-fixed effects and past realized volatilities on different horizons. All volatilities are annualized. We cluster standard deviations by firm. $t$-statistics corrected for autocorrelation are in parentheses. Significance levels of $10 \%, 5 \%$, and $1 \%$ are shown with $*,{ }^{* *}$, and ${ }^{* * *}$, respectively. A permutation test based on Spearman's rank correlation indicates whether DCBS and DSS are significant predictors for volatility. Data are from July 3, 2006 to December 3, 2012.

| Horizon | 10 days | 10 days | 10 days | 30 days | 30 days | 30 days |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DCBS 2 | $0.028^{* * *}$ |  | $0.027^{* * *}$ | $0.036^{* * *}$ |  | $0.035^{* * *}$ |
|  | (8.76) |  | (8.68) | (10.38) |  | (10.19) |
| DCBS 3 | $0.034^{* * *}$ |  | $0.034^{* * *}$ | $0.044^{* * *}$ |  | $0.043^{* * *}$ |
|  | (8.23) |  | (8.20) | (9.70) |  | (9.58) |
| DCBS 4 | 0.053 *** |  | $0.052^{* * *}$ | $0.070^{* * *}$ |  | $0.068^{* * *}$ |
|  | (9.89) |  | (9.66) | (11.41) |  | (11.08) |
| DCBS 5 | $0.063{ }^{* * *}$ |  | 0.060 *** | $0.083^{* * *}$ |  | $0.080^{* * *}$ |
|  | (9.35) |  | (9.06) | (10.83) |  | (10.46) |
| DCBS 6 | 0.081 *** |  | $0.077^{* * *}$ | $0.101^{* * *}$ |  | $0.096^{* * *}$ |
|  | (9.24) |  | (8.91) | (10.56) |  | (10.12) |
| DCBS 7 | $0.082^{* * *}$ |  | $0.078^{* * *}$ | $0.108^{* * *}$ |  | $0.102^{* * *}$ |
|  | (9.14) |  | (8.82) | (11.09) |  | (10.67) |
| DCBS 8 | $0.078^{* * *}$ |  | $0.073^{* * *}$ | $0.094^{* * *}$ |  | $0.089^{* * *}$ |
|  | (8.06) |  | (7.70) | (9.26) |  | (8.79) |
| DCBS 9 | 0.067 *** |  | $0.062^{* * *}$ | $0.093 * * *$ |  | $0.087^{* * *}$ |
|  | (6.25) |  | (5.87) | (7.35) |  | (6.92) |
| DCBS 10 | ${ }_{0} 0.102^{* * *}$ |  | $0.097^{* * *}$ | $\begin{aligned} & 0.128^{* * *} \end{aligned}$ |  | $0.122^{* * *}$ |
|  | (8.08) |  | (7.86) | (8.80) |  | ${ }_{(8.53)}$ |
| DSS 1 |  | $0.017^{* * *}$ | $0.019^{* * *}$ |  | $0.019^{* * *}$ | $0.022^{* * *}$ |
|  |  | (10.27) | (10.72) |  | (10.27) | (10.96) |
| DSS 2 |  | $0.028^{* * *}$ | $0.028^{* * *}$ |  | $0.033^{* * *}$ | $0.033^{* * *}$ |
|  |  | $(11.94)$ | (12.08) |  | $(12.09)$ | $(12.30)$ |
| DSS 3 |  | $0.037 * * *$ | $0.034^{* * *}$ |  | $0.044^{* * *}$ | $0.040^{* * *}$ |
|  |  | (11.92) | (11.88) |  | (12.72) | (12.57) |
| DSS 4 |  | $0.043^{* * *}$ | $0.037 * * *$ |  | $0.053^{* * *}$ | $0.045^{* * *}$ |
|  |  | (11.12) | (10.58) |  | (12.20) | (11.51) |
| DSS 5 |  | $0.048^{* * *}$ | $0.033^{* * *}$ |  | $0.060{ }^{* * *}$ | $0.041^{* * *}$ |
|  |  | (10.87) | (8.91) |  | (11.99) | (9.49) |
| 10 day historical volatility | 0.116*** | $0.116^{* * *}$ | 0.116*** | 0.094*** | $0.094^{* * *}$ | $0.093 * * *$ |
|  | (23.97) | (23.63) | (23.97) | (21.84) | (21.15) | (21.91) |
| 30 day historical volatility | $0.077^{* * *}$ | $0.077^{* * *}$ | $0.076{ }^{* * *}$ | $0.073^{* * *}$ | $0.073^{* * *}$ | $0.072^{* * *}$ |
|  | (11.17) | (11.10) | (11.14) | (10.18) | (9.99) | (10.16) |
| 60 day historical volatility | -0.004 | -0.006 | -0.004 | -0.021 | -0.023 | -0.021 |
|  | (-0.15) | (-0.18) | (-0.14) | (-0.63) | (-0.65) | (-0.63) |
| 91 day historical volatility | $0.421^{* * *}$ | $0.439^{* *}$ | $0.416^{* * *}$ | 0.480 *** | $0.503 * * *$ | $0.473^{* * *}$ |
|  | (21.39) | (21.04) | (21.45) | (21.94) | (21.87) | (21.99) |
| Adj. $R^{2}$ | 0.456 | 0.455 | 0.457 | 0.555 | 0.552 | 0.556 |
| No obs. Rank test DCBS (p-value) | $4,220,239$ 0.001 | 4,220,239 | $4,220,239$ 0.001 | $4,220,239$ 0.001 | 4,220,239 | $4,220,239$ 0.001 |
| Rank test DSS (p-value) |  | 0.003 | 0.06 |  | 0.003 | 0.02 |

the conclusion is that a stock being costly to short contains information on the future volatility of the stock. If the stock has a DCBS of ten the consecutive volatility is around 10 to 12 percentage points larger relative to a stock without short-sale costs. The results are not only statistically but also economically significant. Likewise, a DSS larger than zero has positive predictive power. If a large fraction of the shares outstanding are lend out, it is informative about the future volatility of the stock.

The statistical tests, reported by $t$-statistics, all test for differences to the reference level. To test whether the effects of levels of short-sale costs and short interest are increasing with DCBS and DSS we also perform permutation tests based on Spearman's rank correlation coefficients. ${ }^{4}$ P-values from the permutation tests are reported at the bottom of Table 3.7. Again, we get significant results, especially for DCBS. The permutation test has much less power for DSS than for DCBS given that six levels have 720 unique permutations only, while ten levels have more than three million unique permutations. Both short-sale costs and short interests predict higher volatilities. Our results are consistent with Saffi and Sigurdsson (2011), who find a positive relationship between volatility and both short-sale costs and lending supply based on yearly data. We provide further evidence from daily observations over a longer period. The regressions were also run with quantitatively identical results for a subset in which DCBS and DSS had not changed from the previous day; that is, we removed observations with shifts and changes on the previous day (not reported). These regressions indicate that the level alone is informative about future volatility, also when there is no contemporaneous shift or change in any of them.

A possible explanation for the predictive power of short-sale costs is that high short-sale costs reflect significant positive demand shocks further back in time. Stocks have positive net supply, implying that at any time a fixed number of stocks are not lent out. If short-sale costs are high, fees can be earned from lending the stock and competition should drive short-sale costs to zero. Duffie et al. (2002) model allows for positive short-sale costs due to frictions in the form of search costs. But over time, as stock borrowers and lenders are matched, short-sale costs vanish. Thus,

[^22]high short-sale costs must reflect recent shifts in the market. We already control, however, for past volatility, which is also affected by recent shifts; still, the level of short sales still has predictive power.

Furthermore, stocks with high short-sale costs could resemble stocks for which short sales are restricted, which potentially can increase volatility. Bai et al. (2006) predict that a high level of information asymmetry can create a positive link between short-sale constraints and stock volatility, even though their general result is that the link is ambiguous. Avellaneda and Lipkin (2009) show how stocks which are very hard to borrow can have higher volatility. Empirically, Chang et al. (2007) show that allowing the short sale of specific stocks increases their volatility.

### 3.6 Conclusion

We document that demand shifts in the shorting market are predictors for future volatility of the affected stock, consistent with increases in differences of opinions driving both demand shifts and increased volatility. Duffie et al. (2002) show how higher differences of opinions are associated with higher short-sale costs and higher short interest. Andrei et al. (2014) build a model in which higher differences of opinions yield higher stock volatility. Empirically, Carlin et al. (2014) find that disagreement about future prepayments predicts higher mortgage return volatility. Combining these relationships leads to the main prediction of this paper: positive (negative) demand shifts for lending a stock are positive (negative) predictors of the stock's volatility.

Daily data from 2006 to 2012 of indicators for both short-sale costs and short interest allow us to identify supply and demand shifts in the shorting market for individual stocks. Inspired by Cohen et al. (2007), we identify a shift if both short-sale costs and short interest change the same day. The pairs of simultaneous changes can be translated into positive (negative) supply or demand shifts.

Cross-sectional regressions with controls for past volatility at different horizons, short-sale costs, short interest, and date-fixed effects confirm that positive (negative) demand shocks predict higher (lower) volatility. A positive (negative) demand shift significantly predicts a 2.8 (3.9) percentage points higher (lower) consecutive volatility of the stock. Regressions on sub-periods
(before, during, and after the financial crisis) show that the effect cannot be contributed to a particular sub-period.

In an event study, we show that supply and demand shifts are generally associated with temporary higher volatility of the affected stocks. The shifts are more prevalent when overall market volatility is high. Also, we find that increases in short-sale costs and short interest are positive predictors for volatility. Finally, higher levels of short-sale costs and short interest predict higher volatility.

In summary, this paper links individual stock volatility to the shorting market. We find that positive demand shifts in the shorting market are a positive predictor for volatility of the affected stock. This is consistent with increases in differences of opinions causing both positive demand shifts as well as increased volatility.

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## Appendix for Chapter 1

## Proof of Proposition 1

If the agent wants cash and exercises to immediately sell the stock in the market, the net proceeds would be $S_{0}\left(1-\lambda^{i, S}\right)-X$. If the investor instead places $\frac{X}{1+r^{f}}$ in the risk-free asset and sells short the stock, the cash flow would now be $S_{0}\left(1-\lambda^{i, S}\right)-\frac{X}{1+r^{f}}$ (as there are no funding or short-sale costs). In the next period the option can be exercised paid by the money from the risk-free asset and the received stock can be used to close the short position. This has a net payment of zero. Thereby the cash flow of the strategy dominates the one created by exercising and selling the stock (since $\left.r^{f}>0\right)$.

If the investor wants the exposure to the stock and exercises to get the stock, the cash flow is $-X$. The alternative dominating strategy is to place $\frac{X}{1+r^{f}}$ in the risk-free asset and wait and exercise later. This costs less today (as $r^{f}>0$ ), and the stock purchase can be made cash flow neutral at time 1 by the money from the risk-free asset. No value is forgone by not owning the stock in the meantime as there is no lending fee, $l^{i}=0$, and no funding costs are incurred.

Consider (ii) where the sale revenue of the option is above the intrinsic value for agent $i$, $C_{0}\left(1-\lambda^{i, C}\right)>S_{0}-X$. If agent $i$ wants cash, exercising the option and selling the stock obviously gives less than selling the option. If agent $i$ wants stock, selling the option and buying the stock has a net cost of $S_{0}-C_{0}\left(1-\lambda^{i, C}\right)$ which is less than what it costs to gain the stock through exercise, $X$. Hence, no matter if agent $i$ wants stock or cash, early exercise is dominated.

A sufficient condition for $C_{0}\left(1-\lambda^{i, C}\right)>S_{0}-X$ is that agent $i$ faces no option transaction costs, $\lambda^{i, C}=0$, and the existence of another type of agents $j$ with zero short-sale costs, funding costs, and stock transaction costs. We have just shown above that for an investor without short-sale costs, funding costs or stock transaction costs the value of the option will be at least $S_{0}-\frac{X}{1+r^{f}}$. If such unconstrained agents $j$ exist, the option price must be at least $S_{0}-\frac{X}{1+r^{f}}$ to avoid arbitrage. If $\lambda^{i, C}=0$ we get: $C_{0}\left(1-\lambda^{i, C}\right)=C_{0} \geq S_{0}-\frac{X}{1+r^{f}}>S_{0}-X$.

## Proof of Proposition 2

This proof shows that there exists a strategy involving early exercise that is not dominated by any strategy not involving early exercise if frictions are large enough. We introduce the following notation for the initial portfolio held by an agent: The number of stocks, $\alpha_{0}$, the amount invested in the risk-free asset, $\beta_{0}$, and the number of options, $\gamma_{0}>0$. Similarly, we use the subscript 1 for the portfolio that the agent trades to (e.g., $\alpha_{1}$ is the number of stocks held by agent $i$ after trading).

Consider an agent who wants as much cash as possible at time 0 without any negative cash flow in any possible state at time 1 . One way to do this is to exercise the option position early and close the positions in stock and risk-free asset. If stocks needs to be sold to close the position, stock transaction costs apply. However, in the event that more can be earned through lending out the stock and borrow the present value of the future earned lending fee than through selling the stock, this is done instead. If stock must be acquired this can be done either by buying stock or buying options and exercise immediately. The cheapest way is chosen. The cash-flow at time 0 of this "liquidating early exercise strategy" is:

$$
\mathrm{CF}=\left\{\begin{align*}
\left(\alpha_{0}+\gamma_{0}\right) S_{0}\left(1-\lambda^{S}\right)+\beta_{0}-\gamma_{0} X & \text { if } \alpha_{0}+\gamma_{0} \geq 0 \text { and } 1-\lambda^{S} \geq \frac{l^{i}}{1+r^{j}}  \tag{4}\\
\left(\alpha_{0}+\gamma_{0}\right) \frac{S_{0} 0^{i}}{1+r^{f}}+\beta_{0}-\gamma_{0} X & \text { if } \alpha_{0}+\gamma_{0} \geq 0 \text { and } 1-\lambda^{S}<\frac{l^{i}}{1+r^{f}} \\
\left(\alpha_{0}+\gamma_{0}\right) S_{0}+\beta_{0}-\gamma_{0} X & \text { if } \alpha_{0}+\gamma_{0}<0 \text { and } S_{0} \leq C_{0}+X \\
\left(\alpha_{0}+\gamma_{0}\right)\left(C_{0}+X\right)+\beta_{0}-\gamma_{0} X & \text { if } \alpha_{0}+\gamma_{0}<0 \text { and } S_{0}>C_{0}+X
\end{align*}\right.
$$

We want to show that this early exercise strategy is not dominated for agent $i$.
Any strategy without early exercise is identified by the allocation of wealth between the three assets. ${ }^{5}$ This allocation can be denoted by $\left(\alpha_{1}, \beta_{1}, \gamma_{1}\right)$ for the number of stocks, amount invested in risk-free asset, and number of options respectively. The cash-flow at time 0 for an arbitrary

[^23]strategy not involving early exercise can be described by the function $J_{1}$ where:
\[

$$
\begin{align*}
J_{1}\left(\alpha_{1}, \beta_{1}, \gamma_{1}\right):= & \left(\alpha_{0}-\alpha_{1}\right)\left(S_{0}-\mathbf{1}_{\left(\alpha_{0}-\alpha_{1}>0\right)} S_{0} \lambda^{S}\right)+\left(\beta_{0}-\beta_{1}\right)  \tag{5}\\
& +\left(\gamma_{0}-\gamma_{1}\right)\left(C_{0}-\mathbf{1}_{\left(\gamma_{0}-\gamma_{1}>0\right)} \lambda^{i, C} C_{0}\right)-F^{i}\left(\alpha_{1} S_{0}, \gamma_{1} C_{0}\right)
\end{align*}
$$
\]

In order for a strategy to dominate the liquidating early exercise strategy, the strategy must have non-negative values at time 1 in all possible states and at least as large a cash flow at time zero. To ensure a non-negative value at time 1 , for every stock agent $i$ is short, agent $i$ must also be long at least 1 option and have no less than $\frac{X+S_{0} L^{i}}{1+r^{i}}$ in the risk-free asset (the stock has unlimited upside so the option is needed to secure a non-negative payoff, and the cash is needed to pay the possible future exercise of the option and the short-sale fee of the stock). Similarly, for every option that agent $i$ is short, $i$ must be at least long 1 stock, and not use the risk-free asset to borrow more than $\frac{S_{0} l^{i}}{1+r^{f}}$ (where the stock is needed to be able to honor a possible future exercise of the option in all states and, since the stock could also turn out to be worthless at time 1, there cannot be borrowed more than $\frac{S_{0} l^{i}}{1+r r^{f}}$, the present value of the only secure income raised from lending out the stock in the period). The maximal cash-flow that can be obtained at time $t=0$ without early exercise and with non-negative value at time $t=1$ is therefore the solution to:

$$
\begin{align*}
& \max _{\alpha_{1}, \beta_{1}, \gamma_{1}} J_{1}\left(\alpha_{1}, \beta_{1}, \gamma_{1}\right)  \tag{6}\\
& \text { s.t. } \alpha_{1}+\gamma_{1} \geq 0 \tag{7}
\end{align*}
$$

$$
\begin{equation*}
\beta_{1}+\mathbf{1}_{\left(\alpha_{1}<0\right)} \alpha_{1} \frac{X+S_{0} L^{i}}{1+r^{f}}+\mathbf{1}_{\left(\alpha_{1}>0\right)} \alpha_{1} \frac{S_{0} l^{i}}{1+r^{f}} \geq 0 \tag{8}
\end{equation*}
$$

The early exercise strategy cannot be dominated by selling the option due to the premise of the proposition that doing so is less profitable $\left(C_{0}\left(1-\lambda^{i, C}\right) \leq S\left(1-\lambda^{i, S}\right)-X\right)$. Not selling the option corresponds to $\gamma_{1} \geq \gamma_{0}$. Combining this with (7) we get the inequality $\gamma_{1} \geq \max \left(-\alpha_{1}, \gamma_{0}\right)$. This inequality must bind since $J_{1}$ is decreasing in $\gamma_{1}$ for $\gamma_{1} \geq 0$. Likewise, $J_{1}$ is decreasing in $\beta_{1}$ which means that (8) must bind. We can now substitute both $\beta_{1}$ and $\gamma_{1}$ and reformulate the problem as:

$$
\begin{equation*}
\max _{\alpha_{1}} J_{2}\left(\alpha_{1}\right) \tag{9}
\end{equation*}
$$

where we define the function $J_{2}$ by:

$$
\begin{align*}
J_{2}\left(\alpha_{1}\right) & :=J_{1}\left(\alpha_{1},-\mathbf{1}_{\left(\alpha_{1}<0\right)} \alpha_{1} \frac{X+S_{0} L^{i}}{1+r^{f}}-\mathbf{1}_{\left(\alpha_{1}>0\right)} \alpha_{1} \frac{S_{0} l^{i}}{1+r^{f}}, \max \left(-\alpha_{1}, \gamma_{0}\right)\right) \\
& =\left(\alpha_{0}-\alpha_{1}\right)\left(S_{0}-\mathbf{1}_{\left(\alpha_{0}-\alpha_{1}>0\right)} S_{0} \lambda^{S}\right)+\beta_{0}+\mathbf{1}_{\left(\alpha_{1}<0\right)} \alpha_{1} \frac{X+S_{0} L^{i}}{1+r^{f}}  \tag{10}\\
& +\mathbf{1}_{\left(\alpha_{1}>0\right)} \alpha_{1} \frac{S_{0} l^{i}}{1+r^{f}}+\left(\gamma_{0}-\max \left(-\alpha_{1}, \gamma_{0}\right)\right) C_{0}-F^{i}\left(\alpha_{1} S_{0},-\max \left(-\alpha_{1}, \gamma_{0}\right) C_{0}\right)
\end{align*}
$$

We are ready to show part $a$. in the proposition under the condition that $L^{i}>\frac{X r^{f}}{S_{0}}$. We show that the optimal cash-flow (9) is smaller than the cash flow from the liquidating early exercise strategy (4), regardless of funding costs. Recall that the early exercise strategies incur no funding costs and funding costs for other strategies are non-negative so in the following we consider the case in which $F^{i}=0$. Then, $J_{2}$ is piece-wise linear in $\alpha_{1}$ with kinks at $\left\{-\gamma_{0}, \alpha_{0}, 0\right\}$. So a global maximum for the $J_{2}$ will either be at $-\gamma_{0}, \alpha_{0}, 0$ or at plus or minus infinity. We check the last first:

$$
\begin{array}{r}
\lim _{\alpha_{1} \rightarrow-\infty} J_{2}\left(\alpha_{1}\right)=\lim _{\alpha_{1} \rightarrow-\infty}\left[\left(\alpha_{0}-\alpha_{1}\right) S_{0}\left(1-\lambda^{i, S}\right)+\beta_{0}+\alpha_{1} \frac{X+S_{0} L^{i}}{1+r^{f}}+\left(\gamma_{0}+\alpha_{1}\right) C_{0}\right]  \tag{11}\\
=\alpha_{0} S_{0}\left(1-\lambda^{i, S}\right)+\beta_{0}+\gamma_{0} C_{0}+\lim _{\alpha_{1} \rightarrow-\infty}\left[\alpha_{1}\left[-S_{0}\left(1-\lambda^{i, S}\right)+\frac{X+S_{0} L^{i}}{1+r^{f}}+C_{0}\right]\right]
\end{array}
$$

It must hold that $C_{0}+X \geq S_{0}\left(1-\lambda^{i, S}\right)$, otherwise there would be an arbitrage strategy by buying options, exercise immediately and sell the stock. Given this and that $L^{i}>\frac{X_{r} f}{S_{0}}$ the entire expression (11) is diverging to $-\infty$. We next check $\alpha_{1} \rightarrow \infty$ :

$$
\begin{align*}
\lim _{\alpha_{1} \rightarrow \infty} J_{2}\left(\alpha_{1}\right) & =\lim _{\alpha_{1} \rightarrow \infty}\left[\left(\alpha_{0}-\alpha_{1}\right) S_{0}+\beta_{0}+\alpha_{1} \frac{S_{0} l^{i}}{1+r^{f}}\right]  \tag{12}\\
& =\alpha_{0} S_{0}+\beta_{0}+\lim _{\alpha_{1} \rightarrow \infty}\left[\alpha_{1}\left(-S_{0}+\frac{S_{0} i^{i}}{1+r^{f}}\right)\right]=-\infty
\end{align*}
$$

since no-arbitrage implies that $S_{0}>\frac{S_{0} i^{i}}{1+r^{\dagger}}$ (otherwise an arbitrage gain could be made through buying stocks and lending them out). We now evaluate the expression in $\alpha_{1}=\alpha_{0} \leq-\gamma_{0}$ :

$$
\begin{equation*}
J_{2}\left(\alpha_{0}\right)=\beta_{0}+\alpha_{0} \frac{X+S_{0} L^{i}}{1+r^{f}}+\left(\gamma_{0}+\alpha_{0}\right) C_{0}<C F \tag{13}
\end{equation*}
$$

using that $L^{i}>\bar{L}=\frac{X r^{f}}{S_{0}}$ and $\alpha_{0}+\gamma_{0} \leq 0$. Next, evaluate where $\alpha_{1}=\alpha_{0} \in\left(-\gamma_{0}, 0\right]$ :

$$
\begin{equation*}
J_{2}\left(\alpha_{0}\right)=\beta_{0}+\alpha_{0} \frac{X+S_{0} L^{i}}{1+r^{f}}<C F \tag{14}
\end{equation*}
$$

using that $\alpha_{0}+\gamma_{0}>0$ and $L^{i}>\bar{L}=\frac{X r^{f}}{S_{0}}$. Evaluating $J_{2}$ where $\alpha_{1}=\alpha_{0}>0$ :

$$
\begin{equation*}
J_{2}\left(\alpha_{0}\right)=\beta_{0}+\alpha_{0} \frac{S_{0} l^{i}}{1+r^{f}}<C F \tag{15}
\end{equation*}
$$

Next, we evaluate where $\alpha_{1}=0$ :

$$
\begin{equation*}
J_{2}(0)=\alpha_{0}\left(S_{0}-\mathbf{1}_{\left(\alpha_{0}>0\right)} S_{0} \lambda^{S}\right)+\beta_{0}<C F \tag{16}
\end{equation*}
$$

The final evaluation of the expression is where $\alpha_{1}=-\gamma_{0}$ :

$$
\begin{equation*}
J_{2}\left(-\gamma_{0}\right)=\left(\alpha_{0}+\gamma_{0}\right)\left(S_{0}-\mathbf{1}_{\left(\alpha_{0}+\gamma_{0}>0\right)} S_{0} \lambda^{i, S}\right)+\beta_{0}-\gamma_{0} \frac{X+S_{0} L^{i}}{1+r^{f}}<C F \tag{17}
\end{equation*}
$$

since $L^{i}>\bar{L}=\frac{X r^{f}}{S_{0}}$. Hence it has been proved that for sufficiently high short-sale costs, early exercise is not dominated regardless of funding costs.

Next we show that a similar result holds for sufficiently high funding costs. Specifically, we consider funding costs with $F^{i}(x, y) \geq \bar{F}(|x|+|y|)$ where $\bar{F}>\frac{r^{f} X}{\left(1+r^{f}\right)\left(S_{0}+C_{0}\right)}$. Under this condition, $J_{3}$ is greater than $J_{2}$ where:

$$
\begin{align*}
J_{3}\left(\alpha_{1}\right):= & \left(\alpha_{0}-\alpha_{1}\right)\left(S_{0}-\mathbf{1}_{\left(\alpha_{0}-\alpha_{1}>0\right)} S_{0} \lambda^{S}\right)+\beta_{0}+\mathbf{1}_{\left(\alpha_{1}<0\right)} \frac{X+S_{1} L^{i}}{1+r^{f}}+\mathbf{1}_{\left(\alpha_{1}>0\right)} \alpha_{1} \frac{S_{0} l^{i}}{1+r^{f}}  \tag{18}\\
& +\left(\gamma_{0}-\max \left(-\alpha_{1}, \gamma_{0}\right)\right) C_{0}-\bar{F}\left(\left|\alpha_{1} S_{0}\right|+\max \left(-\alpha_{1}, \gamma_{0}\right) C_{0}\right)
\end{align*}
$$

and we seek to show that $J_{3}$ is smaller than (4). Clearly, $J_{3}$ is piecewise linear with kinks in
$\left\{-\gamma_{0}, \alpha_{0}, 0\right\}$. We first consider the extremes:

$$
\begin{align*}
& \lim _{\alpha_{1} \rightarrow-\infty} J_{3}\left(\alpha_{1}\right) \\
= & \lim _{\alpha_{1} \rightarrow-\infty}\left(\alpha_{0}-\alpha_{1}\right) S_{0}\left(1-\lambda^{S}\right)+\beta_{0}+\alpha_{1} \frac{X+S_{0} L^{i}}{1+r^{f}}+\left(\gamma_{0}+\alpha_{1}\right) C_{0}+\alpha_{1} \bar{F}\left(S_{0}+C_{0}\right)  \tag{19}\\
= & \alpha_{0}+\beta_{0}+\gamma_{0} C_{0}+\lim _{\alpha_{1} \rightarrow-\infty}\left(\alpha_{1}\left[-S_{0}\left(1-\lambda^{S}\right)+\frac{X+S_{0} L^{i}}{1+r^{f}}+C_{0}+\bar{F}\left(S_{0}+C_{0}\right)\right]\right)=-\infty
\end{align*}
$$

since the expression in the squared brackets is positive since no-arbitrage implies that $S\left(1-\lambda^{S}\right) \leq$ $C+X$ (otherwise, an arbitrage strategy would be to buy options, exercise them immediately and sell the obtained stocks). Similarly:

$$
\begin{align*}
& \lim _{\alpha_{1} \rightarrow \infty} J_{3}\left(\alpha_{1}\right) \\
= & \lim _{\alpha_{1} \rightarrow \infty}\left(\alpha_{0}-\alpha_{1}\right) S_{0}+\beta_{0}+\alpha_{1} \frac{S_{0} l^{i}}{1+r^{f}}-\bar{F}\left(\alpha_{1} S_{0}+\gamma_{0} C_{0}\right)  \tag{20}\\
= & \alpha_{0}+\beta_{0}-\bar{F} \gamma_{0} C_{0}+\lim _{\alpha_{1} \rightarrow \infty}\left[\alpha_{1}\left[-S_{0}+\frac{S_{0} l^{i}}{1+r^{f}}-\bar{F} S_{0}\right]\right]=-\infty
\end{align*}
$$

since, as before, $S_{0}>\frac{S_{0} l^{i}}{1+r^{f}}$. Next, we evaluate $J_{3}$ where $\alpha_{1}=-\gamma_{0}$ :

$$
\begin{equation*}
J_{3}\left(-\gamma_{0}\right)=\left(\alpha_{0}+\gamma_{0}\right)\left(S_{0}-\mathbf{1}_{\left(\alpha_{0}+\gamma_{0}>0\right)} S_{0} \lambda^{S}\right)+\beta_{0}-\gamma_{0} \frac{X+S_{0} L^{i}}{1+r^{f}}-\bar{F}\left(\gamma_{0} S_{0}+\gamma_{0} C_{0}\right)<C F \tag{21}
\end{equation*}
$$

since $\bar{F}>\frac{r^{f} X}{\left(1+r^{f}\right)\left(S_{0}+C_{0}\right)}$. For $\alpha_{1}=0$ we get:

$$
\begin{equation*}
J_{3}(0)=\alpha_{0}\left(S_{0}-\mathbf{1}_{\left(\alpha_{0}>0\right)} S_{0} \lambda^{S}\right)+\beta_{0}-\bar{F} \gamma_{0} C_{0}<C F \tag{22}
\end{equation*}
$$

since $S\left(1-\lambda^{i, S}\right)-X>0$. Next, evaluate where $\alpha_{1}=\alpha_{0} \leq-\gamma_{0}$ :

$$
\begin{equation*}
J_{3}\left(\alpha_{0}\right)=\beta_{0}+\alpha_{0} \frac{X+S_{0} L^{i}}{1+r^{f}}+\left(\gamma_{0}+\alpha_{0}\right) C_{0}+\alpha_{0} \bar{F}\left(S_{0}+C_{0}\right)<C F \tag{23}
\end{equation*}
$$

Evaluating where $\alpha_{1}=\alpha_{0} \in\left(-\gamma_{0}, 0\right]$ :

$$
\begin{align*}
J_{3}\left(\alpha_{0}\right) & =\beta_{0}+\alpha_{0} \frac{X+S_{0} L^{i}}{1+r^{f}}-\bar{F}\left(-\alpha_{0} S_{0}+\gamma_{0} C_{0}\right)  \tag{24}\\
& =\beta_{0}+\alpha_{0} \frac{X+S_{0} L^{i}}{1+r^{f}}+\alpha_{0} \bar{F}\left(S_{0}+C_{0}\right)-\bar{F} C_{0}\left(\alpha_{0}+\gamma_{0}\right)<C F
\end{align*}
$$

Finally, we evaluate where $\alpha_{1}=\alpha_{0}>0$ :

$$
\begin{equation*}
J_{3}\left(\alpha_{0}\right)=\beta_{0}+\alpha_{0} \frac{S_{0} l^{i}}{1+r^{f}}-\bar{F}\left(\alpha_{0} S_{0}+\gamma_{0} C_{0}\right)<C F \tag{25}
\end{equation*}
$$

## Proof of Proposition 3

Both the problem for the lower boundary (1.6)-(1.7) and the problem for the upper boundary (1.8) are mathematical equivalent to problem of pricing American call options in the BSM model with continuous dividend yield, allowing us to utilize results established when proving (ii) and (iii).

Kim (1990) shows that the exercise boundary is increasing in time to expiration, and, in accordance with Merton (1973), identifies the closed-form limit value of the exercise boundary as time to expiration goes to infinity. For a positive dividend yield this limit value is finite, and hence the enitre boundary is finite as remarked by Dewynne et al. (1993). So if what corresponds to the dividend yield in our model (i.e., $L^{i}+\psi^{i}\left(m^{i, C}+m^{i, S}\right)$ in part $(i i)$ or $l^{\bar{i}}+\psi\left(m^{\bar{i}, C}-m^{\bar{i}, S}\right)$ in (iii)) is positive, the exercise boundary is finite. If what corresponds to the dividend yield is zero or negative, then Merton's lower bound still holds and early exercise is always dominated, implying infinite exercise boundaries for $t<T$.

Next, we prove $(i)$. If $S(t)<\underline{B}(T-t)$ then $\underline{C}>S(t)-X$ so it is dominated for agent $\underline{i}$ to exercise early and (1.6) must hold. We next consider any other agent $i$ who owns one option, showing that his valuation must be above $\underline{C}$ and, therefore, above the intrinsic value, leading us to conclude that exercise is dominated. Suppose that $i$ assigned the same value to the option as $\underline{i}$ and also hedged the option by selling off $\underline{C}_{S}$ stocks and financed the equity amount at $r^{f}+\psi^{i}$. The risk of the strategy
would be zero, leaving only a deterministic $\mathrm{d} t$-term times this value:

$$
\begin{align*}
& \underline{C}_{t}+\frac{1}{2} \sigma^{2} S^{2} \underline{C}_{S S}-\left[K^{i}\left(x-\underline{C}_{S}, 1\right)-K^{i}(x, 0)+\underline{C}\right]\left(r^{f}+\psi^{i}\right)  \tag{26}\\
& +\underline{C}_{S} S\left(r^{f}-\tilde{l}\right)+\left[K^{i}\left(x-\underline{C}_{S}, 1\right)-K^{i}(x, 0)\right] r^{f}
\end{align*}
$$

where $x \in \mathbb{R}$ is an arbitrary number of stocks held by $i$, and $\tilde{l}^{i}$ is equal to $L^{i}$ if $x \leq 0$, equal to $l^{i}$ if $x \geq \underline{C}_{S}$, and otherwise equal to $\frac{x}{\underline{C}_{S}} l^{i}+\frac{\underline{C}_{S}-x}{\underline{C}_{S}} L^{i}$ if $0<x<\underline{C}_{S}$. The final term $\left[K^{i}\left(x-\underline{C}_{S}, 1\right)-K^{i}(x, 0)\right]$ represents the increase in required amount on the margin account (relative to not holding and hedging one option), and we add $\underline{C}$ in the third term to capture the change in required equity (as the hedge must have the same cash flow as if the option was sold for $\underline{C}$ ). Because of (1.4), the drift (26) is greater or equal to:

$$
\begin{align*}
& \underline{C}_{t}+\frac{1}{2} \sigma^{2} S^{2} \underline{C}_{S S}-\left[m^{i, S} S \underline{C}_{S}+\left(m^{i, C}-1\right) \underline{C}\right] \psi^{i}-\underline{C}\left(r^{f}+\psi^{i}\right)+\underline{C}_{S} S\left(r^{f}-\tilde{l}\right) \\
= & \underline{C}_{t}+\frac{1}{2} \sigma^{2} S^{2} \underline{C}_{S S}-\left[m^{i, S} S \underline{C}_{S}+m^{i, C} \underline{C}\right] \psi^{i}-\underline{C} r^{f}+\underline{C}_{S} S\left(r^{f}-\tilde{l}^{i}\right)  \tag{27}\\
\geq & \underline{C}_{t}+\frac{1}{2} \sigma^{2} S^{2} \underline{C}_{S S}-\left[m^{i, S} S \underline{C}_{S}+m^{i, C} \underline{C}\right] \psi^{\underline{i}}-\underline{C} r^{f}+\underline{C}_{S} S\left(r^{f}-L^{\underline{i}}\right) \\
= & 0
\end{align*}
$$

Here, the inequality follows from $\psi^{i} \leq \psi^{\underline{i}}, L^{i} \leq L^{i}, L^{i} \geq l^{i} \geq 0, \psi^{i} \geq 0$, and $r^{f}>0$. The last equality follows from (1.6). This non-negative drift means that $i$ must assign a value to the option no smaller than agent $\underline{i}$ does, because the option could be hedged and generate a risk-free payment of at least $\underline{C}$ while for sure not give any negative cash flow in the future; i.e. $C^{i} \geq \underline{C}$. Hence, exercise is dominated for any $i$.

Turning to the upper bound part of (i), we first note that failing to exercise is dominated for $\bar{i}:=\bar{i} \bar{\psi}^{i}$ when the stock price is above the upper bound. We show that failing to exercise is not dominated for $i$ then the same is true for $\bar{i}$, implying that it is dominated for $i$ not to exercise whenever $S(t)>\bar{B}^{\psi^{i}}(T-t)$ (proof by contrapositive).

At any time for which it is not dominated for $i$ not to exercise early, $i$ can hedge the option
position by selling $C_{S}^{i}$ stocks, leaving a deterministic $\mathrm{d} t$ term that must be equal zero:

$$
\begin{align*}
& C_{t}^{i}+\frac{1}{2} \sigma^{2} S^{2} C_{S S}^{i}-\left[K^{i}\left(x-C_{S}^{i}, 1\right)-K^{i}(x, 0)+C^{i}\right]\left(r^{f}+\psi^{i}\right)  \tag{28}\\
& +C_{S}^{i} S\left(r^{f}-\tilde{l}^{i}\right)+\left[K^{i}\left(x-C_{S}^{i}, 1\right)-K^{i}(x, 0)\right] r^{f}=0
\end{align*}
$$

If $\bar{i}$ assigned the value $C^{i}$ to the option and also hedged an option by selling off $C_{S}^{i}$ stocks, then the corresponding drift for $\bar{i}$ can be seen to be non-negative using (30) below. This non-negative drift implies that $\bar{i}$ must assign a value to the option that is not immediately exercised no smaller than the value assigned by $i, \bar{C}^{\psi^{i}} \geq C^{i}$. Hence, it is not dominated for $\bar{i}$ not to exercise the option early when it is not dominated for $i$ not to exercise the option early.

## Derivation of PDE for agent $\bar{i} \psi$

For any $\psi \in \mathbb{R}_{+}$we consider agent $\bar{i}^{\psi}$ with $\psi^{i / \psi}=\psi$, who we show have the highest exercise boundary and option valuation among all agents with $\psi^{i}=\psi$. Agent $\bar{i}^{\psi}$ is always long stock. Let the function for the required amount on the margin account have the form

$$
\begin{equation*}
K^{i^{\psi}}(x, y)=m^{i^{\psi}, S} S|x|+\left(m^{i^{i}, C}-1\right) \bar{C}^{\psi} y \tag{29}
\end{equation*}
$$

where $x \in \mathbb{R}$ is the number of stocks held, $y \in \mathbb{R}_{+}$is the number of options held, and $\bar{C}^{\psi}$ is $\bar{i} \psi$ 's valuation of the option. Also, $m^{i \psi, S} \in[0,1]$ and $m^{i \psi, C} \in[0,1]$.

Let $\bar{l}^{i \psi}=\min _{i \in\left\{j: \psi^{j}=\psi\right\}} l^{i}$ and for $i \in\left\{j: \psi^{j}=\psi\right\}$ let

$$
\begin{align*}
& K^{i}\left(x_{2}, y\right)-K^{i}\left(x_{1}, y\right) \leq m^{i \psi, S}\left(x_{2}-x_{1}\right) S \text { for } x_{1} \leq x_{2} \text { and } \forall y \in \mathbb{R} \\
& K^{i}\left(x, y_{2}\right)-K^{i}\left(x, y_{1}\right) \geq\left(m^{i \psi}, C\right.  \tag{30}\\
& i)\left(y_{2}-y_{1}\right) \bar{C}^{\psi} \text { for } y_{2} \geq y_{1} \geq 0 \text { and } \forall x \in \mathbb{R}
\end{align*}
$$

The first line expresses that an increase in the number of stocks does not increase the required amount on the margin account for agent $i$ by more than it increases the required amount on the margin account for agent $\bar{i}^{\psi}$. The second line expresses that an increase in the number of options increases the amount $\bar{i}^{\psi}$ can borrow through the margin account by at least as much as it increases
the amount agent $i$ can borrow.
Next, we want to derive the PDE for agent $\bar{i}^{\psi}$ exercise boundary, $\bar{B}^{\psi}$, and option valuation, $\bar{C}^{\psi}$. Consider the portfolio dynamics for agent $\bar{i}^{\psi}$ of buying one (additional) option at price $\bar{C}^{\psi}$, hedging by selling (additional) $\bar{C}_{S}^{\psi}$ shares of the stock, and fully financing the strategy based on margin loans and the use of equity capital. The value of this fully-financed strategy evolves as according to:

$$
\begin{align*}
& \left(\bar{C}_{t}^{\psi}+\frac{1}{2} \sigma^{2} S^{2} \bar{C}_{S S}^{\psi}\right) \mathrm{d} t+\bar{C}_{S}^{\psi} \mathrm{d} S(t)-\left(1-m^{i^{i}, C}\right) \bar{C}^{\psi} r^{f} \mathrm{~d} t-m^{\bar{i}^{\psi}, C} \bar{C}^{\psi}\left(r^{f}+\psi\right) \mathrm{d} t  \tag{31}\\
& -\bar{C}_{S}^{\psi} \mathrm{d} S(t)+\bar{C}_{S}^{\psi} S\left(-m^{i \psi, S} r^{f}+m^{\bar{i}^{\psi}, S}\left(r^{f}+\psi\right)+\left(r^{f}-l^{\bar{l}^{\psi}}\right)\right) \mathrm{d} t
\end{align*}
$$

The first two terms simply represent the dynamics of the option (as seen in Eqn. (1.2)). The next two terms represent the funding of the option. Specifically, $\left(1-m^{i \psi}, C\right) \bar{C}^{\psi}$ can be borrowed against the option at the money-market funding cost $r^{f}$. The remaining option value, the margin requirement $m^{i \psi}, C_{C} \bar{C}^{\psi}$, must be financed as equity at a rate of $r^{f}+\psi \underline{\underline{i}}$.

The second line of (31) represents the terms stemming from the stock position and its financing. The first term is the stock dynamics, given the $\bar{C}_{S}^{\psi}$ number of shares sold. The last three terms capture the various financing costs. The stock sold to hedge the option decreases the long stock position thereby decreasing the required cash on the margin account by $\bar{C}_{S}^{\psi} S m^{i \psi}, S$ which earns the interest $r^{f}$. This cash can instead be invested at the rate $r^{f}+\psi$. Agent $\bar{i}^{\psi}$ reduces the cash that it borrows on the stock lending account by $\bar{C}_{S}^{\psi} S$ thereby saving the interest paid on this amount. The interest rate on the stock lending account is $r^{f}-l^{i \psi}$. The stochastic terms cancel out. The fully financed strategy with deterministic drift must have drift equal zero for the option valuation to be correct. This leads to the PDE stated in (1.8). The conditions follow from the fact that the option is an American call option and ensure that the solution will entail the exercise boundary that optimizes the option value, c.f. Merton (1973) and Kim (1990).

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A Semeiotic outline of Fundamental Signs, Significance-effects, Knowledge Profiling and their use in Knowledge Organization and Branding
2. Daniel Ronzani

When Bits Learn to Walk Don't Make Them Trip. Technological Innovation and the Role of Regulation by Law in Information Systems Research: the Case of Radio Frequency Identification (RFID)

## 2010

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Magten over livet og livet som magt Studier i den biopolitiske ambivalens


[^0]:    ${ }^{1}$ The author gratefully acknowledges support from the European Research Council (ERC grant no 312417), the Elite Research Prize awarded to Lasse Heje Pedersen, the FRIC Center for Financial Frictions (grant no. DNRF102), and the Danish Agency for Science, Technology and Innovation.

[^1]:    ${ }^{1}$ Avellaneda and Lipkin (2009) mention that lending fees can in principle lead to early exercise and, more broadly, the point may be common knowledge among option traders and researchers even if we did not find other references.

[^2]:    ${ }^{2}$ Gay et al. (1989) study futures options, for which early exercise can be optimal even without frictions. They also discuss transaction costs and after-hours exercise (where transaction costs in the underlying can be viewed as infinite), but, as we show in Proposition 1, transaction costs are not sufficient to justify early exercise for equity call options.

[^3]:    ${ }^{3}$ Stated mathematically, the funding cost function has the property that for $x_{2} \geq x_{1} \geq 0$ then $F^{i}\left(x_{2}, y\right) \geq F^{i}\left(x_{1}, y\right) \geq$ 0 for all $i$ and $y \in \mathbb{R}$. Similarly, if $x_{2} \leq x_{1} \leq 0$ then $F^{i}\left(x_{2}, y\right) \geq F^{i}\left(x_{1}, y\right) \geq 0$ for all $i$ and $y \in \mathbb{R}$, and similarly for the dependence on $y$.

[^4]:    ${ }^{4}$ The institutional details of short-selling and the over-the-counter securities-lending market are described in Duffie, Gârleanu, and Pedersen (2002) who also discuss why not all investors can immediately lend their shares in equilibrium.

[^5]:    ${ }^{5}$ See Gârleanu and Pedersen (2011) for an equilibrium model with binding margin requirements in which such implicit capital costs arise endogenously as $\psi^{i}$ is the Lagrange multiplier of the margin requirement.

[^6]:    ${ }^{6}$ Similarly, the upper boundary $\bar{B}{ }^{\psi}$ is weakly decreasing in lending fee and option margin requirement and increasing in stock margin requirement. Details are available from the authors upon request.

[^7]:    ${ }^{7}$ We are very grateful to Robert Whaley for providing these data.

[^8]:    ${ }^{1}$ CRSP data start in 1926, but we only use it when we have option and convertible bond data.
    ${ }^{2}$ Data for the months November 2001, January and July 2002, and January 2006 are missing.
    ${ }^{3}$ We focus on the Daily Cost of Borrow Score (DCBS), which is first observed from October 22, 2003.
    ${ }^{4}$ Mergent FISD has earlier bond observations, but this is the first date a convertible bond can be observed.
    ${ }^{5}$ The number of underlyings and observations for Thomson One are based on the subsample from 2003-01-01 to 2010-12-31.

[^9]:    ${ }^{8}$ Failure to exercise has been studied by Pool et al. (2008).

[^10]:    ${ }^{9}$ If, e.g., a stock is split in two, a convertible bond with this stock as underlying will have its conversion price halved at the same time.

[^11]:    ${ }^{10}$ The option-date with the highest number of unexplained early exercises by market makers is January 7, 2004 where they exercised 5, 957 option contracts written on Univision Communications Inc. with expiration January 17, 2004 and strike price 35.00 USD. The Thomson One database for corporate events shows that, on the same day as the exercises, Univision announced a Follow-On issue of the Univision stock of 600 million USD and that "the underwriter may engage in activities that stabilize, maintain or otherwise affect the price" of the stock. Such stabilizing activities can limit the expected downside of the stock and reduce the optionality value of in-the-money options, making early exercise induced by frictions more attractive. Also, hedging activities from lead underwriters could lead to a temporary increase in short-sale costs, enough to induce early exercise, but for a short enough period to not be observed in the short-sale cost data. The option-date with the second-highest number of unexplained early exercises by market makers is Monday, July 26, 2010 for options written on AmeriCredit with expiration August 21, 2010 and strike price 22.50 USD. Four days earlier, on July 22, an acquisition of AmeriCredit by General Motors was announced to take effect October 1, 2010 at the price of 24.50 USD per share. If the market expects the deal to go through for certain, the stock will act as a risk-free asset after the initial price adjustment. In fact, the volatility of the stock did become tiny until the successful acquisition. Given a vanishing volatility the observed frictions are large enough to justify early exercise by our model. Number five of the list is 2,190 option contracts exercised with Autonation Inc. as underlying stock on April 11, 2006. Autonation had an outstanding tender offer to buy back some of its own shares that expired April 12, 2006, which might have induced the early exercises.

[^12]:    ${ }^{11}$ Closed stock exchange corresponds to extreme stock transaction costs so that no positive value can be gained from selling the stock. Proposition 1 shows that stock transaction costs are not sufficient to drive early exercise.

[^13]:    ${ }^{1}$ For further references, see, e.g., Chowdhry and Nanda (1991), O'Hara (1995), Easley et al. (1998), and Vayanos and Wang (2013). Recent papers include Cao and Ou-Yang (2009), An, Ang, Bali, and Cakici (2014), Chang, Hsieh, and Lai (2009), Ahn, Kang, and Ryu (2008), Muravyev, Pearson, and Broussard (2013), Doran and Krieger (2010), Hu (2014), Choy and Wei (2012), Lin and Lu (2015), Ansi and Ben Ouda (2009), Hao, Lee, and Piqueira (2013), Ge, Lin, and Pearson (2016), Blasco, Corredor, and Santamaria (2010), Chen, Diltz, Huang, and Lung (2011), Valkanov, Yadav, and Zhang (2011), Sinha and Dong (2011), Kehrle and Puhan (2015), Battalio and Schultz (2006), Boehmer, Jones, and Zhang (2008), Xing, Zhang, and Zhao (2010), Cremers and Weinbaum (2010), and Hong and Yogo (2012).

[^14]:    ${ }^{2}$ Inspired by Merton (1973), a strategy $A_{1}$ is defined as dominating another strategy $A_{2}$ if the cash flow from $A_{1}$ in all states is at least as large as the cash flow from $A_{2}$ and with positive probability a state is realized in which the cash flow from $A_{1}$ exceeds the cash flow from $A_{2}$. A strategy is said to be dominated if there exists a strategy that dominates it.

[^15]:    ${ }^{3}$ I am very grateful to Robert Whaley for providing the OCC data on option exercises. For KFDL, see http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[^16]:    ${ }^{4}$ In some cases, regulation can prevent the option owner from selling the stock immediately after exercise. The option owner might be able to postpone the payment of the strike price until settlement, which is three trading days after exercise. Or the option owner might use unsettled funds to pay the strike price. In those cases, Regulation T might prevent the option owner from selling the stock until the strike price is paid or the unsettled funds settle. The U.S. Securities and Exchange Commission provides some educational guidelines for Regulation T. See, for cash accounts, which are more restricted than margin accounts, https://www.sec.gov/investor/alerts/cashaccounts.pdf. However, instead of exercising now and postponing payment into the future, the option owner would be better off postponing the exercise to the same time in the future. By doing so, the stock can be sold just as early, and optionality to not exercise is preserved which is valuable if the stock declines sufficiently in the meantime.

[^17]:    ${ }^{5}$ The breakpoints are downloaded from Kenneth French's data library:
    http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[^18]:    ${ }^{6}$ The reversal factor returns are downloaded from Stefan Nagel's website: https://sites.google.com/a/umich.edu/snagel/data.

[^19]:    ${ }^{1}$ Proposition 4 and Proposition 10 in Duffie et al. (2002) state the relevant results.

[^20]:    ${ }^{2}$ We could have used VIX instead, which would, however, only capture the conditional expected future volatility. Using date fixed effects, we control for both the expected and the unexpected future market volatility.

[^21]:    ${ }^{3}$ The way in which the sample is constructed means that there is always a volatility estimate for time $t-1$ based on returns $t-10$ to $t-1$. Prior to that, the optimal days to observe volatilities would be $t-11, t-21, \cdot$ We observe volatilities the closest possible days to the optimal days for the given stock. If two available observations are equally far from the optimal day, the earlier day is chosen. If no observations are available from five days before to four days after the optimal day, the observation is missing. After the event, if no volatility estimate is available for the stock at time $t+9$, we use the earliest later volatility estimate but no later than $t+14$. In the rare case that none is found, the value is missing. By not observing volatilities prior to $t+9$, we avoid using volatility estimates based on return days before the event as volatilities after the event. For consecutive days we observe volatilities as close as possible to the optimal days $(t+19, t+29, \cdots)$. If two available days are equally far from the optimal day, the later day is chosen. If no observation is available from four days before to five days after the optimal day, the observation is missing.

[^22]:    ${ }^{4}$ Under the null hypothesis, there is no relationship between future volatility and short-sale costs or future volatility and short interest. The permutation test calculates the correlation between the rank of, e.g., the DCBS levels, when sorted on their estimates and the perfect sort. The likelihood of obtaining such an extreme correlation from a random permutation is then calculated and a two-sided p -value determined.

[^23]:    ${ }^{5}$ Admittedly, the same portfolio can be obtained through several ways of trading. E.g. the number of stocks held can be reduced by 1 either by selling one stock or selling two stocks and buying one back immediately, corresponding to money burning when stock transaction costs are positive. For our purpose it is sufficient to consider strategies with the cheapest way to obtain the portfolio. If such strategies cannot dominate early exercise neither can strategies in which money is given up for nothing.

