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ESSAYS ON MARKET DESIGN

Yun Liu

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Essays on Market Design

Yun Liu

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Yun Liu
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ABSTRACT (ENGLISH)

This Ph.D. thesis is composed of four independent research papers in the field of Market Design. It begins with a general introduction for all four papers and ends with a brief conclusion. In this thesis, I study the impact of heterogeneous market participants on allocation outcomes in different market mechanisms; in addition, how to design alternative mechanisms that can more effectively allocate scarce resources with diverse economic and social goals.

Chapter 1 studies the impact of affirmative action policies in the context of school choice. It addresses the following two questions: what are the causes of possible perverse consequence of affirmative action policies, and when the designer can effectively implement affirmative actions without unsatisfactory outcomes. Using the *minority reserve* policy in the *student optimal stable mechanism* as an example, I show that two acyclicity conditions, *type-specific acyclicity* and *strongly type-specific acyclicity*, are crucial for effective affirmative action policies. However, these two cycle conditions are almost impossible to be satisfied in any finite market in practice. Given the limitation of the *point-wise* effectiveness in finite markets, I further illustrate that the minority reserve policy is *approximately* effective in the sense that the probability of a random market containing type-specific cycles converges to zero when the copies of schools grow to infinite.

Chapter 2 addresses the question of how ex ante asymmetry affects bidders' equilibrium strategies in two popular multi-unit auction rules: *uniform-price auction* (UPA) and *discriminatory-price auction* (DPA). I characterize the set of asymmetric monotone Bayes–Nash equilibria in a simple multi-unit auction game in which two units of a homogeneous object are auctioned among a set of bidders. I argue that bidders' strategic behavior essentially comes from their diverse market positions (i.e., the winning probability and the probability of deciding the market-clearing price). That is, if a bidder has a relatively strong market position, she has less incentive to shade her bid for the

second unit in a UPA, whereas in a DPA, weaker bidders tend to bid more aggressively on both of two units. Following Chapter 2, Chapter 3 further analyzes and contrasts bidders' collusion incentives at the ex ante stage. My results indicate that the UPA is more vulnerable to collusion than the DPA in term of the *expected per-member payoff* and the *core-stability*.

In the last chapter, I show that a variant of the Vickrey-Clarke-Groves auction, Ausubel's *clinching auction*, is vulnerable to collusion in the sense that it always has a non-empty core. I further discuss an isomorphism relation between *group strategy-proofness* and *non-bossiness* in allocation, and the incompatibility between efficient allocation and non-bossiness in finite auction markets.

RESUMÉ (DANSK)

Denne Ph.d. afhandling består af fire uafhængige forskningsartikler inden for Market Design. Den begynder med en generel introduktion af alle fire artikler og slutter med en kort konklusion. I denne afhandling undersøger jeg heterogene markedsdeltagers påvirkning af tildelingsudfaldet for forskellige markedsmekanismer; desuden undersøger jeg, hvorledes det er muligt at designe alternative mekanismer, der mere effektivt kan afsætte knappe ressourcer med forskellige økonomiske og sociale mål.

Kapitel 1 undersøger virkningen af positive særbehandlingspolitikker i forbindelse med et faggruppevalg. Den behandler følgende to spørgsmål: hvad er årsagerne til sådanne unaturlige konsekvenser, og hvorledes kan designeren effektivt gennemføre en positiv særbehandlingspolitik uden utilfredsstillende resultater. Ved brug af *mindretals forbeholdspolitik* i *levens optimale og stabile mekanisme* som et eksempel, påviser jeg, at to acykliske betingelser, *skrive-specifik acyklisitet* og *stærkt skrive-specifik acyklisitet*, er afgørende for effektive positive særbehandlingspolitikker. Disse to cyklusbetingelser er i praksis næsten umulige at få opfyldt i ethvert begrænset marked. I betragtning af den *punktvise* begrænsning af effektivitet i begrænsede markeder, illustrerer jeg yderligere, at mindretals forbeholdspolitik er *næsten* effektiv i den forstand, at sandsynligheden for, at et tilfældigt marked indeholder skrive-specifikke cykler nærmer sig nul, når kopier af skolerne vokser til det uendelige.

Kapitel 2 omhandler spørgsmålet om, hvordan forudgående asymmetri påvirker tilbudsgivernes ligevægtsstrategier i to populære multi-enheds auktionsregler: *ensartet pris auktion* (*uniform-price auction*, UPA) og *diskriminerende pris auktion* (*discriminatory-price auction*, DPA). Jeg karakteriserer først et sæt af asymmetriske monotone Bayes-Nash ligevægte i en enkel multi-enheds auktion, hvor to enheder af et homogent objekt bortauktioneres blandt sæt af tilbudsgivere og argumenterer for, at tilbudsgivernes strategiske adfærd væsentligst kommer fra deres forskellige markedsposition (dvs. den vindende

sandsynlighed og sandsynligheden for at fastlægge markedets slutpris). Det vil sige, at hvis en tilbudsgiver har en forholdsvis stærk markedspostion, har denne et mindre incitament til at skjule sit bud over for en anden enhed i UPA'en, mens der for svagere tilbudsgivere i en DPA er tendens til at byde mere aggressivt på begge af to enheder. Efter kapitel 2, kapitel 3 yderligere analyser og kontraster tilbudsgiveres incitament for aftalt spil på forudgående trin. Mine resultater viser, at UPA er mere sårbar over for aftalt spil end DPA på grund af det forventede payoff per medlem og grundstabiliteten i det forudgående trin.

I det sidste kapitel viser jeg, at en variant af Vickrey-Clarke-Groves auktioner, Ausubel's *clinchng auktionen*, er sårbar over for aftalt spil i den forstand, at den altid har en ikke-tom kerne. Jeg drøfter yderligere en isomorfisk relation mellem koncernens *strategibeskyttelse* og *ikke-dominans* i allokeringen, og uforeneligheden mellem en effektiv allokering og ikke-dominans i begrænsede auktionsmarkeder.

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INTRODUCTION

Market design is an emerging field in the past few decades with wide practical successes. Its initial motivation is to provide feasible solutions to improve extant market mechanisms or create new markets conforming to different social and economic objectives. Like the relation of physics to engineering, compared with the traditional scope of mechanism design theory, market design problems demand more attentions to the details encountered in practice.¹

In my Ph.D. thesis, I study the impact of heterogeneous market participants on allocation outcomes in different market mechanisms; in addition, how to design alternative mechanisms that can more effectively allocate scarce resources with diverse economic and social goals.² Two particular kinds of heterogeneity are studied in this thesis. One comes from players' exogenous differences, such as market incumbents and socioeconomic-privileged groups with inherited competitive advantages. The other kind of heterogeneity is induced by players' coalitional strategic behavior, i.e., several players may have incentives to misreport their valuations in a coordinated way, and split the coalitional gains among themselves.

Chapter 1 (entitled “On Effective Affirmative Action in School Choice”) studies the impact of affirmative action policies in the context of school choice. The purpose of affirmative action in school choice is to create a more equal and diverse social environment, i.e., granting students from disadvantaged social groups preferential treatments in

¹ A mechanism design problem is a specification of a message space for each individual and an outcome function that maps vectors of messages into social decisions and transfers. A market design problem focuses on implementing mechanisms into particular real-world markets. I classify those markets allowing monetary transfer as the *auction design* problem, and those markets without a price signal as the *matching design* problem. For example, governments use open market auctions to allocate radio spectrum, timber, electricity, and natural gas involving hundreds of billions of dollars worldwide (Milgrom, 2004, Krishna, 2009); for matching, noticeable applications include entry level labor market, school choice, paired kidney exchanges, among others (Roth and Sotomayor, 1990, Roth, 2008, Kojima, 2015).

² e.g., allocation efficiency, revenue optimality, budget balance, strategy-proofness, collusion-proofness, envy-free, among others.

school admission decisions to maintain racial, ethnic or socioeconomic balance. Recent evidences from both academia and practice, however, indicate that implementing affirmative action policies in school choice problems may induce substantial welfare loss on the purported beneficiaries (Kojima, 2012, Hafalir et al., 2013, Ehlers et al., 2014, Fragiadakis and Troyan, 2015). Using the *minority reserve* policy (Hafalir et al., 2013) in the *student optimal stable mechanism* (SOSM)(Gale and Shapley, 1962, Abdulkadiroğlu, 2005) as an example, this paper addresses the following two questions: what are the causes of such perverse consequence, and when the designer can effectively implement affirmative action policies without unsatisfied outcomes.

The minimal requirement of an effective affirmative action is that it should not make at least one minority student strictly worse off, while leaves all the rest minority students weakly worse off. I first show that a variant of the Ergin-acyclicity structure (Ergin, 2002), *type-specific acyclicity*, is necessary and sufficient to guarantee this minimal effectiveness criterion in a stable matching mechanism. Next, I introduce a more demanding effectiveness criterion which requires implementing a (stronger) affirmative action does not harm any minority students. I show that a stable mechanism makes no minority students strictly worse off if and only if the matching market is *strongly type-specific acyclic*. These two findings clearly reveal the source of perverse affirmation actions in school choice, which also imply that such adverse effects are not coincidences but rather a fundamental property concealed in the market structures. I then response to the second question such that when the designer can effectively implement affirmative action policies without unsatisfied outcomes. My results imply that the real-world school choice markets are almost impossible to be neither type-specific acyclic nor strongly type-specific acyclic.

Given the limitation of the *point-wise* effectiveness in finite markets, I further illustrate that the minority reserve policy is *approximately* effective in the sense that the probability of a random market containing type-specific cycles converges to zero as the copies of schools grow to infinite. At the policy level, these results suggest that instead of discriminating majority students through affirmative actions, i.e., exchanging the welfare gain of some minority students from impairing other students, an alternative policy practice to rebalance education opportunities is to increase the supply of high-quality schools.

Chapter 2 (entitled “Multi-unit Auction with Ex Ante Asymmetric Bidders: Uniform

vs Discriminatory”) addresses the question of how *ex ante* asymmetry affects bidders’ equilibrium strategies in two popular multi-unit auction rules: *uniform-price auction* (UPA) and *discriminatory-price auction* (DPA). Partly because of their intrinsic analytic complexity, most existing literature of multi-unit auctions is restricted to the symmetric environment in which all bidders have identical value distributions (Engelbrecht-Wiggans and Kahn, 1998a,b, Chakraborty, 2006, McAdams, 2006, Bresky, 2008). Symmetry gives a proper abstraction of the complex market environment when there are many small bidders. However, in circumstances with only a handful of qualified bidders (e.g., procurement auctions), asymmetry may become a more reasonable assumption.

This paper studies an auction market in which two units of an identical and indivisible good are sold to a set of *ex ante asymmetric* bidders, each with diminishing marginal values for the successive units. I say a bidder is *stronger* in the sense that she is more likely to have higher values for both units of the good than a *weaker* bidder, and vice versa. Such a feature is captured by imposing a standard conditional stochastic dominance property to bidders’ value distributions.³ I argue that bidders’ distinct strategic behavior essentially comes from their diverse market positions (i.e., the winning probability and the probability of deciding the market-clearing price). Instead of deriving a system of differential first-order conditions for the DPA and the UPA, which quickly becomes intractable given its multi-dimensional nature, I identify the comparative statics of equilibrium sets between two asymmetric bidders through the changes of their relative market positions. In brief, my results show that if a bidder has a relatively strong market position, she has less incentive to shade her bid for the second unit in a UPA; whereas in a DPA, weaker bidders tend to bid more aggressively on both of two units.

Following Chapter 2, Chapter 3 (entitled “Ex Ante Coalition in Multi-unit Auctions”) further investigates and contrasts bidders’ collusion incentives in the UPA and the DPA at the *ex ante* stage. I am interested in which of the two auction mechanisms is more likely to boost collusion incentives by investigating bidders’ *ex ante* formation of coalitions as *bidding rings* (Marshall et al., 1994, Waehrer, 1999, Bajari, 2001, Kim and Che, 2004, Biran and Forges, 2011).⁴

³ See, for example, Lebrun (1999), Waehrer (1999), Maskin and Riley (2000) and Cantillon (2008), which have used this property to study asymmetric single-unit auctions.

⁴ A bidding ring is composed of a group of bidders whom agree to collude together in order to gain

I first contrast bidders' collusion incentives from the perspective of the *expected per-member payoff*. I argue that the UPA is more vulnerable to collusion than the DPA as each bidder's expected payoff is unanimously increasing (resp. decreasing) in the UPA (resp. DPA) with the size of the coalition she belongs to. However, higher expected per-member payoffs still cannot prevent bidders' *joint* incentives to deviate from their current coalitions. I further shows that regardless of the sizes of their current coalitions in the UPA, no subgroups of bidders would like to collectively deviate from their current coalition once it is formed; by contrast, except for the grand coalition, all bidders would prefer staying in a smaller coalition to their current current coalitions in the DPA. These results contribute to the literature by providing new evidences in the choice between the UPA and the DPA apart from comparing their revenue difference, which also offer new insights into the regulation of anti-competitive behavior in auction markets beyond the single-unit case.

Chapter 4 (entitle "Stable Coalition in Multi-item Auctions") illustrates the coalition formation processes in a variant of the Vickrey-Clarke-Groves auction, the Ausubel's *clinching auction* (Ausubel, 2004). Compared with the commonly used simultaneous sealed-bid auctions in markets with multiple objectives (e.g., the uniform-price auction and the discriminatory-price auction), the clinching auction offers an open ascending-bid alternative with a clear improvement in allocation efficiency while maintaining simplicity to perform in practice.

The primary motive of this paper is to explore whether colluders can cooperatively facilitate a feasible revenue division scheme among themselves in auction markets with multiple *non-identical* objects.⁵ I first show that the clinching auction is vulnerable to collusion in the sense that it always has a non-empty core, i.e., all colluders perceive higher returns from staying in the current coalition compared to all other alternatives. Under a mild *super-additive* assumption of the coalition gains, the *grand coalition* containing all bidders will eventually be formed in equilibrium regardless of the former divisions of sub-

higher surplus by depressing competition in the grand auction.

⁵ Notice that different from the multi-unit auctions with more than one unit of a homogeneous good on sale, the objects are not necessarily identical to each other in multi-item auctions. Some prominent real-life examples include selling advertisement slots for search engines, FCC spectrum auctions, among others.

coalition groups.⁶ Thus, although the clinching auction guarantees efficient allocations as the VCG auction, caution should be exercised when applying it in markets where secret coalitions are highly suspicious. I further argue that a *non-bossy* condition (Satterthwaite and Sonnenschein, 1981) is crucial to such vulnerability, and illustrate the intrinsic tension among efficiency, truthfulness, and non-bossiness in auction mechanisms.

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⁶ Super-additivity implies that the joint gain of two merged coalition groups should be no less than the sum of each coalition group, i.e., for two coalition groups A and B , $v(A + B) \geq v(A) + v(B)$.

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1. ON EFFECTIVE AFFIRMATIVE ACTION IN SCHOOL CHOICE

Yun Liu*

Abstract: Recent evidence, from both academia and practice, indicates that implementing affirmative action policies in school choice problems may induce substantial welfare losses on the intended beneficiaries. This paper addresses the following two questions: what are the causes of such perverse consequences, and when we can effectively implement affirmative action policies without unsatisfactory outcomes. Using the *minority reserve* policy in the *student optimal stable mechanism* as an example, I show that two acyclicity conditions, *type-specific acyclicity* and *strongly type-specific acyclicity*, are crucial for effective affirmative action policies. I also illustrate how restrictive these two acyclicity conditions are, and the intrinsic difficulty of embedding diversity goals into stable mechanisms. Under some regularity conditions, I demonstrate that the minority reserve policy is *approximately* effective in the sense that the market is type-specific acyclic with a high probability when the number of schools is sufficiently large.

JEL Classification: C78; D61; I20

Keywords: school choice, affirmative action, deferred acceptance, type-specific acyclicity, strongly type-specific acyclicity, large market.

* Department of Economics, Copenhagen Business School, Porcelænshaven 16A, DK-2000 Frederiksberg. Email: yliueco@gmail.com. I am deeply indebted to my supervisor Peter Bogetoft for his continuous support and guidance. I thank Anette Boom, Jens Leth Hougaard, Peter Bro Miltersen, Olivier Tercieux for very helpful comments, as well as audiences at the New Trends in Mechanism Design Workshop 2013 (Aarhus), DGPE Workshop 2013 (Eggerup), AMES 2014 (Taipei), CMES 2014 (Xiamen), EMES 2014 (Toulouse), 8th International Symposium on Algorithmic Game Theory (SAGT 2015). Financial support from the Center for research in the Foundations of Electronic Markets (CFEM), supported by the Danish Council for Strategic Research, is gratefully acknowledged. All errors are mine.

1.1 Introduction

This paper studies the impact of affirmative action policies in the context of school choice.¹ Albeit controversial, the purpose of affirmative action in school choice is to create a more equal and diverse environment, i.e., granting students from disadvantaged social groups preferential treatments in school admission decisions to maintain racial, ethnic or socioeconomic balance.

One popular design in practice is the quota-based affirmative action (*majority quota*, henceforth) (Abdulkadiroğlu, 2005), which sets a maximum number less than the school’s capacity to majority students and leaves the difference to minority students (i.e., the policy-targeted student type).² However, Kojima (2012) reports that majority quota may actually hurt every minority student. Evidence from the real world also raises suspicion towards the legitimacy of majority quota.³ Recently, Hafalir et al. (2013) propose an alternative policy design, the reserve-based affirmative action (*minority reserve*, henceforth), which gives minority students preferential treatment up to the reserves. Hafalir et al. (2013) indicate that in term of students’ welfare, minority reserve is a better candidate over its quota-based counterpart.

Although Kojima (2012) and Hafalir et al. (2013) have adequately compared the welfare effects among different affirmative action designs, it remains unclear what the exact

¹ Traditionally, children were assigned to a public school in their immediate neighborhood. However, as wealthy families move to the neighborhoods close to schools with better qualities, such neighborhood-based school assignment may eventually led to socioeconomically segregations. Parents without such means have to send their children to their assigned neighborhood schools, regardless of the quality or other appropriateness of those schools. As a result of these concerns, school choice policies are implemented to grant parents the opportunity to choose the school their child will attend. Abdulkadiroğlu and Sönmez (2003) seminally reconstruct the school choice problem from a mechanism design perspective. They illustrate that some mechanisms used in practice had shortcomings, and propose two celebrated algorithms: the *student optimal stable mechanism* based on the deferred acceptance algorithm (Gale and Shapley, 1962), and the *top trading cycles mechanism* based on (Shapley and Scarf, 1974). See Roth and Sotomayor (1990), Roth (2008) and Sönmez and Ünver (2011) for more dedicated reviews of this problem.

² For simplicity, we call the policy-targeted student type as *minority* student, and all the other student types as *majority* student. However, the distinction between the majority type and the minority type does not depend on race or other single social-economic status; meanwhile, the number of minority students is not necessarily less than majority students.

³ For example, a parent in Louisville (KY) sued the school district after her kid was rejected by a school because of racial classification. “There was room at the school. There were plenty of empty seats. This was a racial quota” (<http://goo.gl/VA8PkK>). A more recent sue case is about the admissions policies of University of Texas, where an applicant claims that many minority students who were admitted had lower grades and test scores than she did (<http://goo.gl/7ASDVk>).

causes of such perverse consequence are. Moreover, I am also curious about whether and when we can effectively implement affirmative action policies without unsatisfactory outcomes. This paper addresses these two concerns through a detailed scrutiny of the minority reserve policy in the *student optimal stable mechanism* (SOSM) (Abdulkadiroğlu and Sönmez, 2003). The popularity of SOSM emerges from two aspects: (i) in theory, it produces the most desirable matching outcome among all *stable* mechanisms for students,⁴ and is *strategy-proof* for students (Roth, 1984);⁵ (ii) it is also relatively easier to be understood by policy makers and market participants (i.e., students and schools).

1.1.1 Main Results

The minimal requirement of an effective affirmative action is that a (stronger) affirmative action should not make some minorities match with their less preferred schools, while leaving other minorities indifferent compared to their previous matching without the (stronger) affirmative action.⁶ I first show that a variant of the acyclicity structure (Ergin, 2002), *type-specific acyclicity*, is necessary and sufficient to guarantee this minimal effectiveness criterion in a stable mechanism (Theorem 1.1).⁷ I then introduce a more demanding effectiveness criterion which requires that implementing a (stronger) affirmative action does not harm any minority students.⁸ I show that a stable mechanism makes no minority students strictly worse off if and only if there is no *quasi type-specific cycle* in the priority orders of schools over students (Theorem 1.2). Theorem 1.1 and Theorem 1.2 clearly reveal the source of perverse affirmative actions in school choice. In addition, these two results also indicate that the adverse effects—as illustrated by the examples in

⁴ A matching mechanism is stable if there are no individual players (i.e., students or schools) will prefer to be unmatched, or a pair of players who prefer to be matched with each other to their current assignments.

⁵ A matching mechanism is strategy-proof if no students have incentive to deviate from reporting their true preference orders.

⁶ Kojima (2012) employs this weak welfare condition to analyze the majority quota policy, and names it as *respect the spirit of quota-based affirmative action*.

⁷ Ergin (2002) says that a *priority structure* (which comprises a pair of schools' priorities and their corresponding capacities) is *acyclic*, if it never gives rise to situations where a player can block a potential settlement between any other two players without affecting her own position. See the formal definition as well as discussions of its relation with my two type-specific acyclicity conditions in Section 1.3.1.

⁸ Balinski and Sönmez (1999) say that a matching mechanism *respects improvements* if a student is never strictly worse off when her priority ranking is improved in some schools while the relative rankings among other students are unchanged. I extend Balinski and Sönmez (1999)'s notion to incorporate the analysis of students with different types. See the formal definition in Section 1.2.2.

Kojima (2012) and simulations in Hafalir et al. (2013)—are not coincidences, but rather a fundamental property concealed in the priority structures.

Theorem 1.3 addresses my second question, when we can effectively implement affirmative action policies without unsatisfactory outcomes. I show that priority structures in practice are very unlikely to be neither type-specific acyclic nor strongly type-specific acyclic. This finding suggests that even if helping disadvantaged social groups is deemed desirable for the society, caution should be exercised when applying affirmative action to rebalance education opportunities among different social groups. I further link a matching problem to a directed graph, where each student represents a vertex and the ranking of two adjacent students in each school’s priority as an edge. I argue that the presence of various cycle conditions in most extant mechanisms essentially describe the paths (i.e., a sequence of edges) and cycles (if a path has the same initial and terminal vertex) inherited in schools’ diverse priority orders. With the almost inevitable presence of paths in most real-life priority structures, the room left for effective affirmative actions through a simple amendment of extant mechanisms may be limited.

Given the limitation of the *point-wise* effectiveness in finite matching markets, I further illustrate that the minority reserve policy is *approximately* effective in the sense that the probability of a random market containing type-specific cycles converges to zero when the copies of schools grow to infinite (Theorem 1.4). Thus, instead of discriminating majority students through affirmative actions (i.e., exchanging the welfare gain of some minority students from impairing other students), an alternative policy practice to rebalance education opportunities is to increase the supply of high-quality schools.

Last, although this paper exclusively focuses on the implementation of minority reserve policy in SOSM, my type-specific notions can serve as a benchmark to analyze the performance of affirmative actions in other matching mechanisms. In addition, because my goal is to reveal the source of perverse affirmative action policies, two student types are sufficient to depict the effect of inter-type rejection chains. Results in this paper can be seamlessly developed into affirmative action policies with more than two types of students.

1.1.2 Related Literature

Incorporating diversity concerns into school choice mechanisms have drawn some attention in recent years. Besides the literature mentioned previously, Ehlers et al. (2014) propose an alternative mechanism to accommodate affirmative action with both maximum and minimum quota and cases when quotas are either hard or soft. Erdil and Kumano (2012) study a class of allocation rules that allow schools to have indifferent priorities over the same type of students. Echenique and Yenmez (2012) axiomatize a class of substitutable priority rules that allow schools to express preferences for diversity. However, none of these works have clearly answered the two questions I addressed in this paper. In addition, Braun et al. (2014) and Klijn et al. (2016) contrast the performance of minority reserve policy and majority quota in laboratories. Other papers study real-world implementations of affirmative action include the German university admissions system (Westkamp, 2013), and the study of Brazilian public federal universities (Aygün and Bo, 2013), among others.

The literature on market design in large markets has been growing rapidly in the past the decade. The two papers that are mostly close to my setting is Kojima and Pathak (2009) and Kojima et al. (2013). Kojima and Pathak (2009) define a *rejection chain algorithm* which begins from a school's strategic rejection of a student to initiate a chain of subsequent rejection and acceptance, and finally receive a more desirable student to apply the manipulator. They show that as the size of the market becomes large, such chain effect (initiated by a school's strategic rejection of a student) is unlikely to return a more desirable student to that school. Therefore, schools expect to match with the same set of students with a high probability. Kojima et al. (2013) further extend the model to analyze the National Resident Matching Program with two types of doctors, single and couple. Another distinct literature strand considers large matching markets with randomization, which enables the analysis of ordinal preferences by assuming a continuum economy as the limit case. See, for example, Che and Kojima (2010) and Che and Tercieux (2015), among others.

Doğan (2016) independently studies a similar problem as this paper and reaches some similar conclusions. In particular, he gives an analogous cycle structure to elaborate the ineffective implementation of minority reserve policy in SOSM, which corresponds to my type-specific cycle notion. However, there are several major differences. First,

the constructions of cycle structures are quite different. While Dogan characterizes a cycle through a chain of direct rejections of students from their original matched schools, my type-specific notion treats the presence of a cycle as the results of inter-type rejection chains after an auxiliary split procedure of schools' capacities based on their reserve seats. Second, in addition to respecting the spirit of a stronger minority reserve policy, I also introduce another welfare criterion, respecting the improvement of a stronger minority reserve, which requires that the improvement of some minorities' welfare should not be based on the welfare loss of some other minorities. Although Dogan's amendment of SOSM with minority reserve respects the spirit of a stronger minority reserve, it is not compatible with my second welfare criterion. Last, Dogan's mechanism also arises the strategic concern from students side,⁹ which largely obscures the true effectiveness of an affirmative action policy. My discussions of approximate effectiveness in large markets (Section 1.4) may offer an alternative theoretical remedy to such strategic concern, which I believe also has stand-alone value.

The rest of the paper proceeds as follows. Section 1.2 sets up the model and introduces the SOSM with minority reserve. Section 1.3 presents the two acyclicity conditions and their relations with possible welfare loss. Section 1.4 further discusses the approximate effective affirmative action in large market. Section 1.5 concludes the paper. All proofs are clustered in Appendix 1.6 and 1.7.

1.2 Model

1.2.1 Preliminary Definitions

Let there be a set of students S , $|S| \geq 3$,¹⁰ and a set of schools C , $|C| \geq 2$. There are two types of students, *majority* and *minority*. S are partitioned into two sets depend on their types. Denote S^M as the set of majority students, and S^m as the set of minority students, $S = S^M \cup S^m$ and $S^M \cap S^m = \emptyset$. Each student $s \in S$ has a strict *preference* order P_s over the set of schools and being unmatched (denoted by s), that is complete, transitive, and

⁹ i.e., students can benefit from misreporting their preferences. Since Dogan's mechanism is based on the idea of Kesten (2010), in order to mitigate the welfare losses for minorities, it bears the cost of losing strategy-proofness for students in complete information environments as is the case in Kesten (2010).

¹⁰ Throughout, $|\cdot|$ denotes the cardinality of a set.

antisymmetric. All students prefer to be matched with some school instead of themselves, $c P_s s$, for all $s \in S$. Each school $c \in C$ has a total capacity of q_c seats, $q_c \geq 1$, and a strict *priority* order \succ_c over the set of students which is complete, transitive, and antisymmetric. Student s is unacceptable by a school if $e \succ_c s$, where e represents an empty seat in school c .¹¹ Denote the upper contour set of \succ_c at student s as $U_c(s) = \{s' \in S | s' \succ_c s\}$.

A *market* is a tuple $\Gamma = (S, C, P, \succ, q)$, where $P = (P_i)_{i \in S}$, $\succ = (\succ_c)_{c \in C}$ and $q = (q_c)_{c \in C}$. Denote $P_{-i} = (P_j)_{j \in S \setminus i}$ and $\succ_{-c} = (\succ_{c'})_{c' \in C \setminus c}$. For a given Γ , assume that all components, except the vector of students' preference orders P , is commonly known.¹² We call the priority order and capacity pair (\succ, q) as a *priority structure*.

A *matching* μ is a mapping from $S \cup C$ to the subsets of $S \cup C$ such that, for all $s \in S$ and $c \in C$:

1. $\mu(s) \in C \cup \{s\}$;
2. $\mu(c) \subseteq S$ and $|\mu(c)| \leq q_c$;
3. $\mu(s) = c$ if and only if $s \in \mu(c)$.

That is, a matching specifies the school where each student is assigned or matched with herself, and the set of students assigned to each school. Given two matchings μ and μ' , μ *Pareto dominates* μ' if (i) $\mu(s) P_s \mu'(s)$ for at least one $s \in S$, and (ii) $\mu(s) R_s \mu'(s)$ for all $s \in S$, where R_s represents two matched schools are equally good for s . A matching μ is *Pareto efficient* if it is not Pareto dominated by any another matchings.

A matching μ is *individually rational* if for each student $s \in S$, $\mu(s) P_s s$, and for each $c \in C$, (i) $|\mu(c)| \leq q_c$ and (ii) $s \succ_c e$ for every $s \in \mu(c)$. A matching μ is *blocked* by a pair of student s and school c if s strictly prefers c to $\mu(s)$ and either (i) c strictly prefers s to some $s' \in \mu(c)$, or (ii) $|\mu(c)| < q_c$ and s is acceptable to c .¹³ A matching is *stable* if it is individually rational and unblocked by a pair of (s, c) .

A *mechanism* f is a function that produces a matching $f(\Gamma)$ for each market Γ .¹⁴ We

¹¹ i.e., school c prefers to reserve an empty seat instead of accepting s .

¹² That is, only students are strategic players in the school choice problem, which is different from the school admission problem, where schools' priority orders are also private information.

¹³ In other words, the student s in the pair prefers school c over his assignment in μ , and school c prefers s either because it has a vacant seat or s is more preferred than another student assigned to c under μ .

¹⁴ I sometimes use $f(\Gamma)$ and μ interchangeably to represent the matching outcome in market Γ , if no confusion arises.

say a mechanism is efficient if there is no matching that Pareto dominates $f(\Gamma)$ for any Γ . Similarly, a stable mechanism is a mechanism that yields a stable matching with respect to reported preferences for every market.

1.2.2 Reserve-based Affirmative Action

A market Γ implements a minority reserve policy when some schools are required to reserve some of their seats to minority students. In particular, if the number of tentatively accepted minorities is less than a school's reserved seats, then all minority students are more preferred to all majority students in that school, while the ranking of each student remains unchanged within her own type.¹⁵

Since the set of students is fixed, I rewrite the market as $\Gamma = (C, P, \succ, (q, r^m))$, where r^m is the corresponding vector of minority reserves for each school c . Market $\tilde{\Gamma} = (C, P, \succ, (q, \tilde{r}^m))$ is said to have a *stronger* minority reserve than Γ , if the total capacity q of each school keeps unchanged, but $\tilde{r}_c^m \geq r_c^m$ for every $c \in C$, and $\tilde{r}_c^m > r_c^m$ for some $c \in C$.

Affirmative action policies intend to improve the matches of minority students, sometimes at the expense of majority students. I thus need some additional *type-specific* criteria to evaluate the welfare impact of implementing different affirmative action policies. Given two matchings μ and μ' , μ *Pareto dominates* μ' *for minorities* if (i) $\mu(s)R_s\mu'(s)$ for all $s \in S^m$, and (ii) $\mu(s)P_s\mu'(s)$ for at least one $s \in S^m$.

Individual rationality is not affected by the presence of minority reserve. A matching μ is *blocked* by a pair of student s and school c with minority reserve, if s strictly prefers c to $\mu(s)$ and either $|\mu(c)| < q_c$ and s is acceptable to c , or

1. if $s \in S^m$, c strictly prefers s to some $s' \in \mu(c)$;
2. if $s \in S^M$ and $|\mu(c) \cap S^m| > r^m$, c strictly prefers s to some $s' \in \mu(c)$;
3. if $s \in S^M$ and $|\mu(c) \cap S^m| \leq r^m$, c strictly prefers s to some $s' \in \mu(c) \cap S^M$.

Condition (1) describes a situation where a pair of school of student (c, s) forms a blocking pair because s is a minority student and c prefers s to some tentatively matched

¹⁵ One distinctive feature of minority reserve is that it is not as rigid as the majority quota. If there are not enough minority students to fill the reserves, majority students are still acceptable up to this school's capacity.

students in c . In condition (2), whereas blocking happens because s is a majority student, the number of minority students in c exceeds minority reserves and c prefers s to some students in c . Finally, in condition (3), (c, s) has a blocking pair because s is a majority student, the number of minority students in c does not exceed minority reserves, but c prefers s to some majority students in c . A matching is *stable* if it is individually rational and unblocked by a pair of (s, c) with minority reserve.

Hafalir et al. (2013) compose the following mechanism to accommodate the SOSM with minority reserve (SOSM-R henceforth):

- Step 1: Start from the matching where no student is matched. Each student i applies to her first-choice school. Each school c first accepts up to r_c^m minorities with the highest priorities if there are enough minority students on the waiting list. Then it accepts students from the remaining applications with the highest priorities until its capacity is filled or the applicants are exhausted. The rest (if any) are rejected.

⋮

- Step n : Each student i who was rejected in Step $(n - 1)$ applies to her next highest choice (if any). Each school c considers these students and students who are tentatively held from the previous step together. c first accepts up to r_c^m minorities with the highest priorities if there are enough minority students on the waiting list. Then it accepts students from the remaining applications with the highest priorities until its capacity is filled or the applicants are exhausted. The rest (if any) are rejected.

The algorithm terminates either when every student is matched to a school or every unmatched student has been rejected by every acceptable school. The algorithm always terminates in a finite number of steps. Denote the new mechanism, SOSM-R, by f^R , and its outcome under market Γ by $f^R(\Gamma)$.¹⁶

Example 1.1: Consider the following market $\Gamma = (C, P, \succ, (q, r^m))$. Let $C = \{c_1, c_2\}$ and $S = \{s_1, s_2, s_3\}$, $S^m = \{s_2, s_3\}$ and $S^M = \{s_1\}$. The priority orders and (type-specific)

¹⁶Note that SOSM-R is a special case of SOSM. When no school has reserved seats, SOSM-R is equivalent to SOSM.

capacities of schools are

$$\succ_{c_l}: s_1, s_2, s_3 \quad (q_{c_l}, r_{c_l}^m) = (1, 0) \quad l = 1, 2$$

Students preference orders are

$$P_{s_i}: c_1, c_2 \quad i = 1, 3$$

$$P_{s_2}: c_2, c_1$$

The matching produced by SOSM (or equivalently, SOSM-R without affirmative action) is

$$f^{GS}(\Gamma) = \begin{pmatrix} c_1 & c_2 \\ s_1 & s_2 \end{pmatrix}$$

which leaves s_3 unmatched. If implement a (stronger) minority reserve policy $\tilde{q}_{c_1} = (q_{c_1}, \tilde{r}_{c_1}^m) = (1, 1)$, while c_2 is unaffected, SOSM-R produces

$$f^R(\tilde{\Gamma}) = \begin{pmatrix} c_1 & c_2 \\ s_2 & s_1 \end{pmatrix}$$

Obviously, the stronger affirmative action with $\tilde{r}_{c_1}^m = 1$ causes both the minority student s_2 and the majority student s_1 strictly worse off compare to the previous outcome without affirmative action, while the other minority student s_3 is indifferent before and after implementing $\tilde{r}_{c_1}^m$. The matching outcome $f^R(\tilde{\Gamma})$ is Pareto dominated by $f^{GS}(\Gamma)$ for minorities.

Since the purpose of affirmative action policy is to improve students' welfare from the policy-targeted type (i.e., minority student in this paper), the Pareto dominated outcome as is the case in Example 1.1 should be avoided. I introduce the following two welfare criteria to evaluate the effectiveness of a stronger minority reserve policy.

Definition 1.1: A mechanism f respects the spirit of a stronger minority reserve \tilde{r}^m , if for any given pair of markets Γ and $\tilde{\Gamma}$ such that $\tilde{\Gamma}$ has a stronger minority reserve than Γ , no matching $f(\tilde{\Gamma})$ is Pareto dominated by $f(\Gamma)$ for minorities.

Definition 1.1 implies that implementing a stronger minority reserve policy should never make some minority students strictly worse off, while leaving the rest of the minority students indifferent. This idea is introduced by Kojima (2012) to study the performance of majority quota policy, which serves as the minimum welfare requirement in this paper.

Definition 1.2: A mechanism f respects the improvement of a stronger minority reserve \tilde{r}^m , if for any given pair of markets Γ and $\tilde{\Gamma}$ such that $\tilde{\Gamma}$ has a stronger minority reserve than Γ , no minority student is strictly worse off in $f(\tilde{\Gamma})$ than in $f(\Gamma)$.

Definition 1.2 requires that the possible welfare improvement of some minority students should not be based on the welfare loss of any other minorities. It provides a stronger welfare criterion compare to the preceding one, which also generalizes the *respect of improvements* condition (Balinski and Sönmez, 1999) to matching markets with different student types.

Remark 1.1: If a mechanism respects improvements, then it also respects the improvement of a stronger minority reserve. In addition, if a mechanism respects the improvement of a stronger minority reserve, then it also respects the spirit of a stronger minority reserve.

The next step is to introduce the following modified market which produces the same matching as the original market with SOSM-R. In a market $(C, P, \succ, (q, r^m))$, split each school c with capacity q_c and minority reserve r_c^m into two corresponding sub-schools, *original sub-school* (c^o) and *reserve sub-school* (c^r). Let C^m be the set of schools with both c^o and c^r . c^o has a capacity of $q_c - r_c^m$ and maintains the original priority order \succ_c .¹⁷ c^r has a capacity of r_c^m and its new priority \succ_c^r is

$$\succ_c^r \equiv \begin{cases} s \succ_c s' & \text{if } s, s' \in S^m \\ s \succ_c s' & \text{if } s, s' \in S^M \\ s \succ_c^r s' & \text{if } s \in S^m, s' \in S^M \end{cases}$$

c^r keeps the same pointwise priority orders as school c in the original market for all majority students and all minority students respectively, but prefers all minorities to any

¹⁷ If a school c is not affected by minority reserve, then c^o is equivalent to c after the split procedure.

majorities. For each student, if $c_1 P_s c_2$ in the original market, her preference in the new market is

$$\begin{aligned} c_1^r P'_s c_1^o P'_s c_2^r P'_s c_2^o & \quad \forall s \in S^m \\ c_1^o P'_s c_1^r P'_s c_2^o P'_s c_2^r & \quad \forall s \in S^M \end{aligned}$$

That is, (i) I preserve the same preference orders over schools in the new market, and assume that (ii.a) each minority student prefers the reserve sub-schools (c^r) over the original sub-schools (c^o); whereas (ii.b) each majority prefers c^o over c^r .

Denote the new market as $\Gamma^m = (C^m, P', (\succ^o, \succ^r), (q, r^m))$, where $\succ^x = (\succ_c^x)_{c \in C}$, $x = o, r$, and its matching outcome through SOSM is $f^{GS}(\Gamma^m)$. Let $((\succ^o, \succ^r), (q, r^m))$ be the corresponding priority structure of (\succ, q) in Γ^m .

Claim 1.1: For each market $\Gamma = (C, P, \succ, (q, r^m))$ and its corresponding $\Gamma^m = (C^m, P', (\succ^o, \succ^r), (q, r^m))$, $f^R(\Gamma) = f^{GS}(\Gamma^m)$.

Hafalir et al. (2013) give a similar split procedure and indicate that SOSM generates the same matching outcome in Γ^m as the SOSM-R in Γ . The only difference is that Hafalir et al. (2013) let all students first apply to the reserve sub-school c^r , whereas I assume majority students prefer the original sub-school c^o to the reserve sub-school c^r in each school c . As I maintain the relative rankings of each original school in Γ^m while all students are only tentatively accepted in each corresponding sub-schools after the split procedures, a different application order (between c^o and c^r) within each c will not change the final outcome.

1.3 Two Acyclicity Conditions

Although Hafalir et al. (2013) imply a clear welfare improvement of SOSM-R for minority students over embedding majority quota with SOSM, SOSM-R may still produce ineffective outcomes such as the case of Example 1.1. My first task is to understand the cause of such adverse effects on minority students.

Definition 1.3: Given a priority structure $(\succ, (q, r^m))$ and its corresponding $((\succ^o, \succ^r), (q, r^m))$, a *type-specific cycle* is constituted of $k + 1$ distinct schools c_0, c_1, \dots, c_k , and $k + 2$ distinct students s_i, s_j, s_k, s_l where $s_i, s_k \in S^m$, $s_j \in S^M$ and $s_l = \{s_1, s_2, \dots, s_{k-1}\} \in S$, $k \geq 1$, if the following two conditions are satisfied:

(C) Cycle condition: $s_k \succ_{c_0}^r s_i \succ_{c_0}^r s_j \succ_{c_1}^x s_1 \succ_{c_2}^x s_2 \dots s_{k-1} \succ_{c_k}^o s_k$, such that $x = o$ if $s_l \in S^m$, and $x = r$ if $s_l \in S^M$, $l = \{1, \dots, k - 1\}$.

(S) Scarcity condition: There exist $k + 1$ disjoint sets of students $S_{c_0}, S_{c_1}, \dots, S_{c_k} \subset S \setminus \{s_i, s_j, s_k, s_1, s_2, \dots, s_{k-1}\}$, such that $|S_{c_0}| = q_{c_0} - 1$, $|S_{c_l}| = q_{c_l} - 1$, $S_{c_0} \subset U_{c_0}^r(s_j) \cup U_{c_0}^o(s_i)$, $S_{c_l} \subset U_{c_l}^o(s_l) \cup U_{c_l}^r(s_l)$, $l = \{1, \dots, k - 1\}$. $U_c^x(s) = \{s' \in S \mid s' \succ_c^x s\}$, $x = o, r$.

$((\succ^o, \succ^r), (q, r^m))$ is *type-specific acyclic* if it has no type-specific cycles.

Condition (C) indicates a chain of rejections and acceptances with a group of distinct schools and students which is initiated by a majority student (s_j) and is terminated by a minority student (s_k) whom applies to the initial school rejected the majority student. Condition (S) excludes the situation that students are exhausted before filling up all seats.¹⁸

Lemma 1.1: For a market $\Gamma = (C, P, \succ, (q, r^m))$ and its corresponding $\Gamma^m = (C^m, P', (\succ^o, \succ^r), (q, r^m))$, let μ and $\tilde{\mu}$ be the matching outcomes of SOSM-R before and after a stronger affirmative action policy \tilde{r}^m . If $\tilde{\mu}(s)$ is Pareto dominated by $\mu(s)$ for all $s \in S^m$, then $\tilde{\mu}(s)$ is Pareto dominated by $\mu(s)$ for all $s \in S$.

Proof. See Appendix 1.6.1. □

Lemma 1.1 tells us that in cases when no minorities benefit from a (stronger) affirmative action $\tilde{\mu}$, then all majorities also prefer the previous matching outcome without $\tilde{\mu}$. With Lemma 1.1, I am now ready to show my first main result.

Theorem 1.1: Given a priority structure $(\succ, (q, r^m))$ and its corresponding $((\succ^o, \succ^r), (q, r^m))$, a stable matching mechanism respects the spirit of a stronger minority reserve \tilde{r}^m , if and only if $((\succ^o, \succ^r), (q, \tilde{r}^m))$ is type-specific acyclic.

¹⁸ If there is a school left with empty seats, the chain of rejections will be terminated (without rejecting another student) once some students rejected by other schools apply to this school.

Proof. See Appendix 1.6.2. □

Theorem 1.1 clearly reveals that ineffective affirmative action policies are due to the presence of type-specific cycles. I use Example 1.1 to outline the proof. For the “only if” part, since both of the two schools only have one available seat while they both prefer the majority student s_1 to the other two minority students s_2 and s_3 before the (stronger) minority reserve policy $\tilde{r}_{c_1}^m = 1$, the outcome that assigns s_1 to c_1 and s_2 to c_2 is Pareto efficient. If implementing $\tilde{r}_{c_1}^m = 1$, I have $c_1 \equiv c_1^r$ and the two minorities become more preferred to s_1 , i.e., $s_2 \succ_{c_1}^r s_3 \succ_{c_1}^r s_1$. c_2 is unaffected and with the priority order $s_1 \succ_{c_2}^o s_2 \succ_{c_2}^o s_3$. The rejection of s_1 from c_1^r initiates a chain reaction which causes s_2 to be rejected by c_2^o and s_3 's rejection from c_1^r . We can easily see the presence of a type-specific cycle with two schools and three students, $s_2 \succ_{c_1}^r s_3 \succ_{c_1}^r s_1 \succ_{c_2}^o s_2$, while Condition (S) is trivially satisfied because $q_{c_1} = q_{c_2} = 1$. The proof of the “if” part essentially generalizes the case of Example 1.1 by assuming that if a stable mechanism (e.g., SOSM-R) does not respect the spirit of minority reserve, we can always construct at least one type-specific cycle for a given priority structure.

Since some of the minorities may still be strictly worse off even if a priority structure contains no type-specific cycles, I am curious about when a matching mechanism can ensure that no minority is harmed by a stronger affirmative action.

Definition 1.4: Given a priority structure $(\succ, (q, r^m))$ and its corresponding $((\succ^o, \succ^r), (q, r^m))$, a *quasi type-specific cycle* is constituted of two distinct schools c, c' and three distinct students s_i, s_j, s_k where $s_i \in S$, $s_k \in S^m$, and $s_j \in S^M$, if the following two conditions are satisfied

(C') Cycle condition: $s_i \succ_c^r s_j \succ_{c'}^o s_k$.

(S') Scarcity condition: There exist two disjoint sets of students $S_c, S_{c'} \setminus \{s_i, s_j, s_k\}$, such that $|S_c| = q_c - 1$, $|S_{c'}| = q_{c'} - 1$, $S_c \subset U_c^r(s_j) \cup U_c^o(s_j)$, $S_{c'} \subset U_{c'}^r(s_k) \cup U_{c'}^o(s_k)$. $U_c^x(s) = \{s' \in S \mid s' \succ_c^x s\}$, $x = o, r$.

$((\succ^o, \succ^r), (q, r^m))$ is *strongly type-specific acyclic* if it has no quasi type-specific cycles.

The construction of a quasi type-specific cycle is analogous to the type-specific cycle. However, compare to its counterpart in Definition 1.3, Condition (C') permits the presence

of a much weaker cycle for a given priority structure. This makes the strongly type-specific acyclicity even more difficult to satisfy.

Remark 1.2: If $(\succ, (q, r^m))$ has a type-specific cycle, then it has a quasi type-specific cycle.

Lemma 1.2: For a market $\Gamma = (C, P, \succ, (q, r^m))$ and its corresponding $\Gamma^m = (C^m, P', (\succ^o, \succ^r), (q, r^m))$, let μ and $\tilde{\mu}$ be the matching outcomes of SOSM-R before and after a stronger affirmative action policy \tilde{r}^m . If there is at least one $s \in S^m$ who is strictly worse off in $\tilde{\mu}$ than in μ , then there must have at least one majority student who is strictly worse off in $\tilde{\mu}$ than in μ .

Proof. See Appendix 1.6.3. □

Compared with Lemma 1.1, Lemma 1.2 allows situations where some minorities may benefit from a stronger affirmative action policy.

Theorem 1.2: Given a priority structure $(\succ, (q, r^m))$ and its corresponding $((\succ^o, \succ^r), (q, r^m))$, a stable matching mechanism respects the improvement of a stronger minority reserve \tilde{r}^m , if and only if $((\succ^o, \succ^r), (q, \tilde{r}^m))$ is strongly type-specific acyclic.

Proof. See Appendix 1.6.4. □

Because for a given priority structure, strongly type-specific acyclicity is more confined than the type-specific acyclicity condition, Theorem 1.2 verifies my intuition that it is even more difficult to make no minority students worse off after a (stronger) affirmative action. Theorem 1.1 and Theorem 1.2 clearly demonstrate that the perverse consequence of affirmative actions on the purported beneficiaries does not happen occasionally, instead it is a fundamental phenomenon concealed in schools' priority orders.

The following result gives a quite negative response to my second question—when we can effectively implement affirmative action policies without unsatisfactory outcomes.

Theorem 1.3: Given a priority structure $(\succ, (q, r^m))$ and its corresponding $((\succ^o, \succ^r), (q, r^m))$, suppose that $|S^m| \geq 2$, $|S^M| \geq 1$, and for any two schools $c, c' \in C$, $q_c + q_{c'} \leq |S^m|$. Let s_j be a majority student.

(i) $((\succ^o, \succ^r), (q, r^m))$ is type-specific acyclic, only if there is no more than one minority student has lower priority than s_j in two different schools.

(ii) $((\succ^o, \succ^r), (q, r^m))$ is strongly type-specific acyclic, only if no minority students has lower priority than s_j in all schools.

Proof. See Appendix 1.6.5. □

In practice, it is almost impossible to find markets where almost all schools rank almost all minorities higher than each majority, let alone where each minority is ranked higher than each majority in all schools. Theorem 1.3 shows how restrictive the two type-specific acyclicity conditions are, and the difficulty to effectively incorporate diversity goals into school choice problems.

1.3.1 Relations with Other Acyclic Conditions

Ergin (2002) characterizes the efficient SOSM (with no diversity concerns) by the following condition.

Definition 1.5: (Ergin, 2002) Given a priority structure (\succ, q) , a *Ergin-cycle* is constituted of two distinct schools $c, c' \in C$ and three distinct students $s_i, s_j, s_k \in S$, if the following two conditions are satisfied

(i) $s_k \succ_c s_i \succ_c s_j \succ_{c'} s_k$.

(ii) There exist two disjoint sets of students $S_c, S_{c'} \setminus \{s_i, s_j, s_k\}$, such that $|S_c| = q_c - 1$, $|S_{c'}| = q_{c'} - 1$, $S_c \subset U_c(s_i)$, $S_{c'} \subset U_{c'}(s_k)$. $U_c(s) = \{s' \in S \mid s' \succ_c s\}$.

(\succ, q) is Ergin-acyclic if it has no Ergin-cycles.

My type-specific acyclicity generalizes Ergin's characterization into markets with different student types. Because I only require no cycles across types but do not restrict cycles with students from the same type,¹⁹ priority structures that are type-specific acyclic may still contain Ergin-cycles.

Remark 1.3: If $((\succ^o, \succ^r), (q, r^m))$ has a type-specific cycle, then it has a Ergin-cycle.

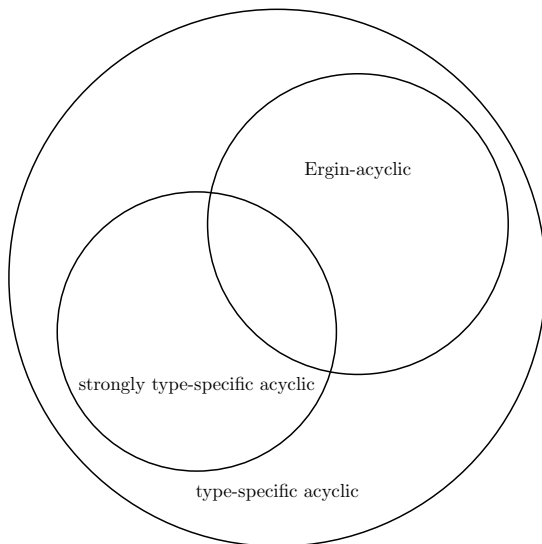


Fig. 1.1: Venn diagram of the three acyclic conditions

Various acyclicity conditions have been developed in other popular matching mechanisms since Ergin’s seminal work.²⁰ If considering a priority structure as a directed graph (where each student represents a *vertex* and the ranking of two adjacent students in each school’s priority as an *edge*), we can see that different cycle conditions essentially depict the paths (i.e., a sequence of edges) and cycles (if a path has the same initial and terminal vertex) inherited in schools’ diverse priority orders. Therefore, an unsophisticated implementation of affirmative actions will result in arbitrary changes of some schools’ priority orders which may disentangle some existent paths,²¹ but it may also create new paths with subsequent welfare losses to all students involved in the paths. With the almost

¹⁹ The reason is obvious, since my purpose is to investigate whether affirmative action policies will cause welfare loss on the type of minority students. If a cycle only involves students from the same type, implementing a (stronger) affirmative action will not change the matching outcome.

²⁰ Kesten (2006) shows that the deferred acceptance mechanism and the top trading cycle mechanism are equivalent if and only if the priority is *Kesten-acyclic*. Haeringer and Klijn (2009) further indicate that Ergin-acyclicity is a necessary and sufficient condition for Nash implementation of the stable correspondence. Kumano (2013) shows that Boston mechanism is stable and strategy-proof at the same time if and only if the priority is *Kumano-acyclic*.

²¹ For instance, Example 1 of Kojima (2012) illustrates a situation where a stronger majority quota policy benefits all students, including the majority students, in SOSM.

inevitable presence of paths in most real-life priority structures, a simple amendment of extant mechanisms may not be able to achieve desirable diversity goals in school choice.

1.4 Approximately Effective Affirmative Action in Large Market

Comparing approximate performances among different algorithms has a long tradition in computer science, which also draws interests from economists, especially in the field of market design, in recent years.²² Using SOSM-R as an example, Theorem 1.3 shows a quite disappointing result for effective implementations of affirmative action in finite market setting. I am curious about whether we can achieve a certain level of approximate effectiveness when the number of players are sufficiently large.²³

Recall Claim 1.1, I know that after splitting each school c with quota q_c and minority reserve r_c^m into two corresponding sub-schools, the *original sub-school* (c^o) and the *reserve sub-school* (c^r), running SOSM in the auxiliary market Γ^m generates the same matching outcome as the SOSM-R in the original market Γ . I first introduce a sequential version of the SOSM-R, denoted by *Sequential SOSM-R*, which still generates the same outcomes as the SOSM-R in market Γ . However, as minority students and majority students are added separately into the Sequential SOSM-R, this auxiliary procedure helps us clearly disentangle the possible rejection chains initiated from the two types of students.

I provide a brief description of the Sequential SOSM-R here, and defer the formal definition in Appendix 1.7.

- Loop 1: Run the SOSM for a sub-market composed of all schools, minority students and possibly matched majorities from Loop 2 (if any). Each minority retains her relative ranking of all schools as in the original SOSM-R, and first applies to the reserve sub-school (c^r) of her most favorable school c . Each c^r school accepts as many as applicants

²² Loosely, the idea is to show that for some desirable properties that are unattainable (or incompatible) in finite market (i.e., with a small amount of schools and students in the context of school choice), it is able to retrieve their approximate counterparts in *large markets* (where the number of market participants goes to infinity).

²³ Note that although (countably) infinite largely serves as a theoretical upper bound for the sake of computing convergent rate (or proving the existence of approximate equilibria), many real world matching markets do have a large number of applicants and institutions. For instance, in the National Resident Matching Program (NRMP), the number of hospital programs is between 3,000 and 4,000 and the number of students is over 20,000 each year. In the New York public school choice program, there are about 500 schools and over 90,000 students per year.

to fill up its empty seats. If there is still some minority applicants on the waiting list, c^r first rejects an equivalent number of majorities matched from Loop 2 (if any), accepts the rest applicants with the highest priorities until its capacity is filled or the applicants are exhausted. All rejected minorities then applies to the corresponding original sub-school c^o of c . Each c^o accepts up to $q_c - r_c^m$ applicants with the highest priorities and rejects the rest. If the minority gets rejected again, she applies to her next highest choice of school c (if any) accordingly. Keep all rejected majorities from either c^r or c^o unmatched until Loop 1 terminates and add to the applicants in Loop 2. Loop 1 stops until no rejection occurs and tentative matching at that step is finalized.

- Loop 2: One by one, run the SOSM for a sub-market of all unmatched applicants from Loop 1, all original sub-schools (c^o) and only schools still have empty seats (or matched with some majorities) in the set of reserve sub-schools (c^r) from Loop 1. First place each unmatched majority to c^o of her most favorable school c . Each c^o accepts up to $q_c - r_c^m$ applicants with the highest priorities from either types. All rejected majorities apply to the corresponding reserve sub-school c^r of c . Each c^r school (with empty seats or matched with some majorities from Loop 1) accepts the set of majorities with the highest priorities until its empty seats is filled up, replaces some less preferred majorities matched in Loop 1, and rejects the rest.²⁴ If the majority gets rejected again, she applies to her next highest choice of school c (if any) accordingly. Keep all rejected minorities from any c^o unmatched until Loop 2 terminates, and add back to Loop 1. Loop 2 stops until no rejection occurs and tentative matching at that step is finalized.

The Sequential SOSM-R algorithm terminates either when every student is matched to a school or every unmatched student has been rejected by every acceptable school. It terminates in a finite number of steps, and will produce the same matches as the original SOSM-R. Denote its outcome, under market Γ^m by $f^{SE}(\Gamma^m)$.

Before going further, I first use Example 1.1 to illustrate how the Sequential SOSM-R works with the (stronger) minority reserve $\tilde{q}_{c_1} = (q_{c_1}, \tilde{r}_{c_1}^m) = (1, 1)$. In the Sequential SOSM-R, I first split the two schools c_1 and c_2 into their corresponding (c_1^o, c_1^r) and

²⁴ i.e. a majority student will be accepted in any c^r only if there is an empty seat or she is more preferred than a tentatively matched majority; no minorities are allowed to be rejected from c^r in Loop 2.

(c_2^o, c_2^r) . (i) Initiate Loop 1, the minority student s_2 first applies to c_2^r while s_3 to c_1^r , given $q_{c_1} = |c_1^r| = 1$ (with $|c_1^o| = 0$) and $q_{c_2} = |c_2^o| = 1$ (with $|c_2^r| = 0$). s_2 and s_3 are tentatively accepted by c_2^o and c_1^r respectively. Loop 1 stops. (ii) Initiate Loop 2, the majority students s_1 applies to c_1^r directly (as c_1^o has no capacity given $q_{c_1} = |c_1^r| = 1$), and is rejected given $s_3 \succ_{c_1}^r s_1$. Next, s_1 applies to c_2^o . As $s_1 \succ_{c_2}^o s_2$, the minority student s_2 previously matched with c_2^o from Loop 1 get rejected, and is kept unmatched until Loop 2 stops. (iii) Initiate Loop 1 again, s_2 now applies to c_1^r . Given $s_2 \succ_{c_1}^r s_3$ while $s_2, s_3 \in S^m$, s_3 gets rejected. s_3 then applies to c_2^o and gets rejected again. The Sequential SOSM-R terminates. Clearly, $f^R(\Gamma) = f^{SE}(\Gamma^m)$.

In order to analyze the convergence process in large matching markets, I need to consider a sequence of markets of different sizes. I first extend my notation of the market tuple Γ to incorporate the uncertainty when adding additional students and schools into the market. A *random market* is a tuple $\Gamma = ((S^m, S), C, P, \succ, (q, r^m), k, \mathcal{P})$, where S^m is the subset of minority students from the set of students S , k is a positive integer and $\mathcal{P} = (p_c)_{c \in C}$ is a probability distribution on C , with $p_c > 0$ for each $c \in C$. For simplicity, I assume that minorities and majorities have similar favors for schools, i.e., all students generate their preferences from the same probability distribution of schools.²⁵

Each random market induces a market by randomly generated preferences of each student s as follows (Immorlica and Mahdian, 2005):²⁶

- Step 1: Select a school independently from the distribution \mathcal{P} . List this school as the top ranked school of student s .
- ⋮
- Step $t \leq k$: Select a school independently from \mathcal{P} which has not been drawn from steps 1 to step $t - 1$. List this school as the t^{th} most preferred school of student s .

Student s only lists these k schools as her preference order. For each realization of student preferences, a market with perfect information is obtained.²⁷ A sequence of

²⁵ My main result (Theorem 1.4) will not change even if schools are drawn by students with different patterns. However, the result may give a different convergence rate.

²⁶ Terminologies used in this section can also be found in Kojima and Pathak (2009), Kojima et al. (2013). Also, see Knuth et al. (1990) for an earlier intellectual contribution.

²⁷ One important assumption is that student preferences are drawn independently from one another, and the way in which each student's preference order is drawn also follows a particular procedure. Again, for simplicity, I only consider the above procedure with distribution \mathcal{P} to generate preferences.

random markets is denoted by $(\hat{\Gamma}^1, \hat{\Gamma}^2, \dots)$, where $\hat{\Gamma}^n = ((S^{m,n}, S^n), C^n, (q^n, r^{m,n}), \succ_{C^n}, k^n, \mathcal{P}^n)$ is a random market in which $|C^n|$ is the number of schools, and $|r^{m,n}|$ is the number of seats reserved for minorities.²⁸

Definition 1.6: A sequence of random markets $(\hat{\Gamma}^1, \hat{\Gamma}^2, \dots)$ is *regular*, if there exist $\lambda > 0$, $a \in [0, \frac{1}{2})$, $b > 0$, $r \geq 1$, and positive integers k and \bar{q} , such that for all n ,

1. $k_n = k$,
2. $q_c \leq \bar{q}$ for all $c \in C^n$,
3. $|S^{m,n}| \leq \lambda n$, $|r^m| \leq bn^a$
4. $\frac{p_c}{p_{c'}} \in [\frac{1}{r}, r]$ for all $c, c' \in C^n$,
5. every $s \in S^n$ is acceptable to c at any realization of preferences for c at \mathcal{P}^n .

Condition (1) assumes that the length of students' preferences does not increase with the market size. Condition (2) requires that the capacity of each school is bounded across schools and markets. Condition (3) requires that the number of minority students does not grow much faster than the number of schools. Moreover, the number of seats reserved for minority students grows at a slower rate of $\mathcal{O}(n^a)$ where $a \in [0, \frac{1}{2})$.²⁹ Condition (4) requires that the popularity of different schools (as measured by the probability of being selected by students as acceptable) does not vary too much. Condition (5) requires schools to find any student acceptable, but priority orders are otherwise arbitrary.

Given all these preparations, the following result gives my main argument under the large market setting, which states that the SOSM-R is *very likely* to respect the spirit of a stronger minority reserve when the number of schools is sufficiently large.

Theorem 1.4: Consider a regular sequence of random markets. There exists n_0 such that the SOSM-R approximately respects the spirit of a stronger minority reserve \tilde{r}^m for any market in that sequence with more than n_0 schools.

²⁸ In this section, superscripts are used for the types of each single school c after the splitting process in Γ^m , the number of schools present in the sequence of random markets, and the types of students. These notations will be relabeled in Appendix 1.7.

²⁹ Also, I assume that the number of majority students grows at a same rate as minorities, but such assumption is irrelevant for my main result.

I give the intuition of my proof here and leave its details to Appendix 1.4. First, notice that when there is a large number of schools presenting in the market, after Loop 1, many of them either are not listed by any minorities, or even if some minorities have already applied to these schools but they are not required to implement minority reserves. Next, consider at each instance in Loop 2 when majorities are added to the Sequential SOSM-R, according to Lemma 1.3, it is very unlikely for the current applicants (i.e., majority students) submit to a school which has been applied by any minorities (and implemented with minority reserve at the same time) in Loop 1. Therefore, I show that when the market becomes sufficiently large while the number of seats reserved for minorities are not growing too fast, it is unlikely to have minorities rejected by some majorities in Loop 2. As the type-specific cycle essentially characterizes a chain of rejections and acceptances, when the market is sufficiently large, a rejection chain which returns a current matched minority student to the initial school c_0 and makes each minority student involved in this chain strictly worse off becomes unlikely to happen.³⁰

However, even though a priority structure is unlikely to contain any type-specific cycles when the number of schools is large, as long as some minority students have lower rankings than any majority students in some random schools, I still cannot safely eliminate the possible presence of quasi type-specific cycles.

1.5 Conclusion

This paper proposes two welfare criteria to evaluate the effective implementation of affirmative action policies in school choice problems. I characterize two type-specific acyclicity conditions in the SOSM-R (Hafalir et al., 2013) and demonstrate their respective (material) equivalence with the two welfare criteria in stable matching mechanisms. I further show that type-specific cycles will gradually vanish with the increase of the market size. At the policy level, my results suggest that instead of discriminating majority students through affirmative actions, i.e., exchanging the welfare gain of some minority students from impairing other students, an alternative policy practice to rebalance education op-

³⁰ Kojima et al. (2013) use similar arguments to show that in large markets, because a strategic rejection of a female doctor will not return another more preferred male applicant to this hospital with a high probability, truth-telling thus becomes an approximate equilibrium from the hospital side under the doctor-proposing deferred acceptance algorithm.

portunities is to increase the supply of high-quality schools.

Last, since in general we can treat the effective implementation of affirmative actions as a market design problem with different types of players and type-specific capacity constraints, I believe a thorough analysis of affirmative action also preserves general theoretical interests that are not limited in school choice problems.

1.6 Appendix for Chapter 1: Proofs of Finite Market Results

Notations: (the following notations are used throughout the proofs)

Let μ be the matching by SOSM-R in a random market $\Gamma = (C, P, \succ, (q, r^m))$ and $\tilde{\mu}$ be the matching outcome after a stronger affirmative action \tilde{r}^m , $\tilde{r}^m > r^m$. Denote the market after \tilde{r}^m by $\tilde{\Gamma} = (C, P, \succ, (q, \tilde{r}^m))$. $\Gamma^m = (C^m, P', (\succ^o, \succ^r), (q, r^m))$ and $\tilde{\Gamma}^m = (C^m, P', (\succ^o, \succ^r), (q, \tilde{r}^m))$ are the two respective markets of Γ and $\tilde{\Gamma}$ after splitting each school into the original sub-school (c^o) and the reserve sub-school (c^r).

1.6.1 Proof of Lemma 1.1

I prove the Lemma by contradiction. Suppose that if $\tilde{\mu}(s)$ is Pareto dominated by $\mu(s)$ for all $s \in S^m$, there has at least one majority student $s_0 \in S^M$ who prefers $\tilde{\mu}$ to μ in $\tilde{\Gamma}^m$, $\tilde{\mu}(s_0)P_{s_0}\mu(s_0)$. Let $\tilde{\mu}(s_0) = c_1$ and $\mu(s_0) = c_0$. Since s_0 is a majority student, she is rejected by a reserve sub-school. Because s_0 prefers c_1 to c_0 , she must have been rejected by c_1 in Γ^m (which leads to the matching μ), at an earlier step before s_0 applies to c_0 . Denote the step when s_0 is rejected by c_1 in Γ^m step l of the SOSM algorithm. At that step, c_1 must have exhausted its capacity, $|\mu(c_1)| = q_{c_1}$, and $s \succ_{c_1}^x s_0$, $x = o, r$, for all s tentatively accepted by c_1 at step l . Since $\tilde{\mu}(s_0) = c_1$, there must have another student, denote by s_1 , such that s_1 is tentatively accepted by c_1 at step l (in Γ^m) but matches with another school in $\tilde{\Gamma}^m$. Recall that at step l , s_0 is rejected by c_1 , it implies that $s_1 \succ_{c_1}^r s_0$.

I first show that s_1 must be a majority student who has applied to c_1 at a step earlier than l in Γ^m . Otherwise, if $s_1 \in S^m$, because $\tilde{\mu}$ is Pareto dominated by μ for all $s \in S^m$, while $\mu(s_1) \neq \tilde{\mu}(s_1)$, it implies that $\mu(s_1)P_{s_1}\tilde{\mu}(s_1)$. Also, recall that s_1 is tentatively accepted by c_1 before her final match in Γ^m , $c_1R_{s_1}\mu(s_1)$, I have $c_1P_{s_1}\tilde{\mu}(s_1)$. Since the stronger affirmative action \tilde{r}^m only increases the capacity of some c^r , (s_1, c_1) forms a

blocking pair in $\tilde{\Gamma}^m$, which contradicts the stability of $\tilde{\mu}$ (with minority reserve). Thus, $s_1 \in S^M$. Obviously, s_1 applies to c_1 at a step earlier than l .

Next, since $\tilde{\mu}(s_1) \neq c_1$, while $s_1 \succ_{c_1}^r s_0$ and $s_0, s_1 \in S^M$, it implies that $\tilde{\mu}(s_1)P_{s_1}c_1$. Otherwise, (s_1, c_1) is a blocking pair in $\tilde{\Gamma}^m$. Combine with $c_1R_{s_1}\mu(s_1)$, I have $\tilde{\mu}(s_1)P_{s_1}\mu(s_1)$. Denote $\tilde{\mu}(s_1) = c_2$. Recall that in Γ^m , s_1 applies to c_1 before step l . Without loss of generality, denote this step by $l - 1$. I can repeat the proceeding arguments for s_0 and s_1 , and construct a set of l majority students who are all better-off in $\tilde{\Gamma}^m$. That is, $\tilde{\mu}(s_i)P_{s_i}c_iR_{s_i}\mu(s_i)$, $i = \{0, \dots, l - 1\}$, $s_i \in S^M$. c_i belongs to a set of l schools in which for each s_i she is tentatively accepted at step $l - i$. In particular, let step 1 be the step that initiates the matching in market Γ^m when s_{l-1} applies to c_{l-1} . Because s_{l-1} applies to c_{l-1} at the first step, it implies that $c_{l-1}P_{s_{l-1}}c$, for all $c \in C \setminus c_{l-1}$. Recall that $c_{l-1} \neq \tilde{\mu}(s_{l-1})$, which contradicts to $\tilde{\mu}(s_{l-1})P_{s_{l-1}}c_{l-1}$. \square

1.6.2 Proof of Theorem 1.1

(i) *Type-specific acyclicity* \implies *Respect the spirit of reserve-based affirmative action*. I prove the contrapositive, such that if $\mu(s)R_s\tilde{\mu}(s)$ for all $s \in S^m$, and $\mu(s)P_s\tilde{\mu}(s)$ for at least one $s \in S^m$, there must contain a type-specific cycle with at least two schools and three students.

Lemma 1.1 indicates that if $\tilde{\mu}(s)$ is Pareto dominated by $\mu(s)$ for all $s \in S^m$, then there has at least one $s' \in S^M$, $\mu(s')P_{s'}\tilde{\mu}(s')$. Denote $\tilde{S} = \{s \in S \mid \mu(s)P_s\tilde{\mu}(s)\}$ be the set of students strictly prefer the matching μ . Because $\mu(s)R_s\tilde{\mu}(s)$ for all $s \in S \setminus \tilde{S}$, for those who are not strictly worse off after implementing \tilde{r}^m , they are matched with the same school under μ , i.e., $S \setminus \tilde{S} = \{s \in S \mid \mu(s) = \tilde{\mu}(s)\}$.

Choose a set of students from \tilde{S} , $\tilde{S}' \subseteq \tilde{S}$, such that for all $s \in \tilde{S}'$, $\tilde{\mu}(s) \neq s$. \tilde{S}' is nonempty. Otherwise, there has at least one minority student $s \in \tilde{S}$ and $\tilde{\mu}(s) = s$, such that s and $\mu(s)$ forms a blocking pair after the stronger affirmative action \tilde{r}^m . Further, \tilde{S}' contains at least one minority student and one majority student. Because if all $s' \in S^M \cap \tilde{S}$, $\tilde{\mu}(s') = s'$, then for some $s \in S^m \cap \tilde{S}$, s and $\mu(s)$ forms a blocking pair after \tilde{r}^m .

Without loss of generality, denote $s_j \in S^M \cap \tilde{S}'$ who is *directly* affected by \tilde{r}^m ,³¹

³¹ A majority student s who is directly affected by a stronger affirmative action \tilde{r}^m in the sense that if $\mu(s) = c$ and $\tilde{\mu}(s) \neq c$, $r_c^m < \tilde{r}_c^m$, then there is a minority student s' such that $\mu(s') \neq c$, $cP_{s'}\mu(s')$, and s' is tentatively accepted by c at the step when s is rejected. Further, by $r_c^m < \tilde{r}_c^m$, I know that $\mu(s) = c$,

$\mu(s_j) = c_0$. Since $c_0 P_{s_j} \tilde{\mu}(s_j)$, and $\tilde{\mu}$ is stable (Hafalir et al., 2013), this implies that $|\tilde{\mu}(c_0)| = q_{c_0}$, $|\tilde{\mu}(c_0) \cap S^m| = \tilde{r}_{c_0}^m$, and for all s who are tentatively accepted by c_0^x , $s \succ_{c_0}^x s_j$, $x = o, r$. Since s_j is a majority student who is directly affected by the stronger affirmative action \tilde{r}^m , there has a minority student tentatively accepted by c_0 , denote by s_i , such that $c_0 P_{s_i} \mu(s_i)$ and $s_j \succ_{c_0}^o s_i$. It implies that $s_i \in c_0^r$ in $\tilde{\Gamma}^m$ (because of $s_i \succ_{c_0}^r s_j$). Otherwise, (s_j, c_0) forms a blocking pair. However, $\tilde{\mu}(s_i) \neq c_0$ by assumption (otherwise $\tilde{\mu}(s_i) P_{s_i} \mu(s_i)$). Since s_i cannot be rejected by a majority student from c_0^r , there must have another minority student, denote by s_k , such that $s_k \in S^m \cap \tilde{S}'$, $s_k \in \tilde{\mu}(c_0) \setminus \mu(c_0)$ and $s_k \succ_{c_0}^r s_i$. Thus, I have

$$s_k \succ_{c_0}^r s_i \succ_{c_0}^r s_j, \quad s_i, s_k \in S^m, s_j \in S^M \quad (1.1)$$

Denote $\mu(s_k) = c_k$. Because $c_k P_{s_k} c_0$ and $\tilde{\mu}$ is stable, it implies that $|\tilde{\mu}(c_k)| = q_{c_k}$, and there exists a student in \tilde{S}' , denote by s_{k-1} , such that

$$s_{k-1} \in \tilde{\mu}(c_k) \setminus \mu(c_k), \quad s_{k-1} \succ_{c_k}^o s_k \quad (1.2)$$

Otherwise, (s_k, c_k) forms a blocking pair in $\tilde{\Gamma}^m$. Apply similar arguments of s_{k-1}, s_k and c_0, c_k for each student in \tilde{S}' repeatedly. Because the set of students in \tilde{S}' are finite, let $\{s_0, s_1, \dots, s_{k-2}, s_{k-1}\} \in \tilde{S}' \setminus \{s_k\}$, I can construct a finite sequence of schools $c_1, c_2, \dots, c_{k-1}, c_k$ such that for each $l = \{0, 1, 2, \dots, k-1\}$

$$s_l \in \tilde{\mu}(c_{l+1}) \setminus \mu(c_{l+1}), \quad \mu(s_l) = c_l, \quad c_l P_{s_l} c_{l+1} \quad (1.3)$$

$$|\tilde{\mu}(c_l)| = q_{c_l}, \quad s \succ_{c_l}^x s_l, \quad x = o, r, \quad \text{for each } s \in \tilde{\mu}(c_l) \quad (1.4)$$

In particular, I have

$s \succ_c^o s'$, and s is tentatively accepted by c^r (before s' applies to c) after the stronger affirmative action. The set of majority students that are directly affected by \tilde{r}^m is nonempty; otherwise, $\mu(\Gamma^m) = \mu(\tilde{\Gamma}^m)$.

$$\left\{ \begin{array}{ll} s_l \succ_{c_{l+1}}^o s_{l+1} & \text{if } s_{l+1} \in S^m \\ s_l \succ_{c_{l+1}}^r s_{l+1} & \text{if } s_{l+1} \in S^M, \quad l = \{0, 1, 2, \dots, k-1\} \end{array} \right. \quad (1.5)$$

It is not difficult to see that $s_0 \equiv s_j$ by preceding arguments. Combining (1.1) and (1.5) gives us the cycle condition. The scarcity condition is satisfied by (1.2), (1.3) and (1.4), and the stability of SOSM.

(ii) *Respect the spirit of reserve-based affirmative action \implies Type-specific acyclicity.* Suppose that $\tilde{\Gamma}^m$ has a type-specific cycle, I use a counter-example to show that the stronger minority reserve policy \tilde{r}^m will cause all minorities worse off.

Recall Example 1.1 that after implementing $\tilde{r}_{c_1}^m = 1$, the three students $s_1 \in S^M$ and $s_2, s_3 \in S^m$, and the two schools $\{c_1, c_2\}$, constitute a type-specific cycle: Condition (C) is given by $s_2 \succ_{c_1}^r s_3 \succ_{c_1}^r s_1 \succ_{c_2}^o s_2$, Condition (S) is trivially satisfied because $q_{c_l} = 1$, $l = 1, 2$. The matching outcome after the stronger affirmative action $\tilde{r}_{c_1}^m$ is $\mu(s_1) = c_2$ and $\mu(s_2) = c_1$. Compared with the corresponding matching before \tilde{r}^m : $\mu(s_1) = c_1$ and $\mu(s_2) = c_2$, it is obviously that s_2 is strictly worse off after $\tilde{r}_{c_1}^m$ while s_3 is indifferent. \square

1.6.3 Proof of Lemma 1.2

I prove the Lemma by contradiction. Suppose at least one of the minority students is strictly worse off in $\tilde{\Gamma}^m$ compare to Γ^m , but no majority students are strictly worse off after implementing \tilde{r}^m . Let \tilde{S}^m be the set of minority students who are strictly worse off after \tilde{r}^m , $\mu(s)P_s\tilde{\mu}(s)$, for all $s \in \tilde{S}^m \subset S^m$. And for all $s' \in S^m \setminus \tilde{S}^m$, either $\tilde{\mu}(s')R_{s'}\mu(s')$ or $\tilde{\mu}(s')P_{s'}\mu(s')$.

Suppose that a minority student, denote by s_0 , is strictly worse off in $\tilde{\Gamma}^m$ compare to Γ^m . Let $\mu(s_0) = c_1$. Since $c_1P_{s_0}\tilde{\mu}(s_0)$, the capacity of c_1 is full at the step when s_0 is rejected by c_1 in $\tilde{\Gamma}^m$, there is another student, say s_1 , such that s_1 is tentatively accepted by c_1 when s_0 is rejected. Denote the step when s_1 applies to c_1 (or equivalently, s_0 is rejected by c_1) in $\tilde{\Gamma}^m$ be step l of the SOSM algorithm.

I first show that if s_1 is a majority student, then there have a group of minority students, denote by \tilde{S}_1^m , who are strictly worse off in $\tilde{\Gamma}^m$ compare to Γ^m , i.e., $\tilde{S}_1^m \in \tilde{S}^m$, and apply to c_1 at a step earlier than l in $\tilde{\Gamma}^m$. Since $s_1 \in S^M$, and no majority students

are strictly worse off after implementing \tilde{r}^m by assumption, I know that either $c_1 P_{s_1} \mu(s_1)$ or $c_1 R_{s_1} \mu(s_1)$. Recall that $c_1 P_{s_0} \tilde{\mu}(s_0)$, $s_0 \in S^m$, and all minorities have higher priorities than any majorities in all reserve sub-schools c^r , I know that $s_1 \succ_{c_1}^o s_0$, $s_0 \in \mu(c_1^r)$ but $s_0 \in \tilde{\mu}(c_1^o)$. Otherwise (s_0, c_1) would form a blocking pair in $\tilde{\Gamma}^m$. Therefore, there must have a group of minority students, denote by \tilde{S}_1^m , who apply to and are tentatively accepted by c_1 at a step earlier than l (when c_1 rejects s_0) in $\tilde{\Gamma}^m$, but do not apply to c_1 in Γ^m . $s \succ_{c_1}^o s_0$ for all $s \in \tilde{S}_1^m$,³² but $\mu(s) P_s c_1$ for all $s \in \tilde{S}_1^m$. Otherwise, (s, c_1) are blocking pairs in Γ^m for all $s \in \tilde{S}_1^m$. Without losing of generality, denote the least preferred student in \tilde{S}_1^m be s_2 (if $|\tilde{S}_1^m| = 1$, then $\tilde{S}_1^m \equiv s_2$), such that $s \succ_{c_1}^o s_2 \succ_{c_1}^o s_0$ for all $s \in \tilde{S}_1^m \setminus s_2$.

If s_1 is a minority student, I know that s_1 must be strictly worse off in $\tilde{\Gamma}^m$ compare to Γ^m , $\mu(s_1) P_{s_1} c_1$. Otherwise, (s_1, c_1) forms a blocking pair in Γ^m . Thus, $s_1 \in \tilde{S}^m$, and s_1 is rejected by $\mu(s_1)$ at a step earlier than l by another student, denote by \dot{s} . Since s_0 is a random minority student who is strictly worse off after \tilde{r}^m , I can equivalently treat s_1 as s_0 when $s_1 \in S^m$. Therefore, (i) if $\dot{s} \in S^m$, repeat the same arguments in this paragraph, I know that $\dot{s} \in \tilde{S}^m$, rewrite \dot{s} as s_2 ; (ii) if $\dot{s} \in S^M$, apply the arguments in the previous paragraph (i.e., equivalently treat \dot{s} as s_1 when $s_1 \in S^M$), and I have another set of minority students, denote by \tilde{S}_2^m , who are strictly worse off in $\tilde{\Gamma}^m$ compare to Γ^m , write the least preferred minority student in \tilde{S}_2^m as s_2 .

Hence, if there is one minority student, s_0 , who is strictly worse off in $\tilde{\Gamma}^m$ compare to Γ^m , there must have another minority student, s_2 , who is also strictly worse off in $\tilde{\Gamma}^m$ and is rejected by $\mu(s_2)$ at a step earlier than l in $\tilde{\Gamma}^m$. Repeat the preceding arguments I can construct a set of l minority students, denote by \tilde{S}_l , such that $c_{i+1} P_{s_i} c_i$, $i = \{1, \dots, l\}$, $s_i \in \tilde{S}_l \subset \tilde{S}^m$, where $c_{i+1} = \mu(s_i)$, and c_i belongs to a set of l schools in which s_i is tentatively accepted by c_i at step $l - i + 1$ of the SOSM algorithm in $\tilde{\Gamma}^m$. In particular, $s_l \in \tilde{S}_l$ applies to and is tentatively accepted by c_l at step 1. It implies that $c_l P_{s_l} c$, for all $c \in C \setminus c_l$, recall $\mu(s_l) \neq c_l$, which contradicts to $\mu(s_l) P_{s_l} c_l$. \square

³² i.e., all minority students belong to \tilde{S}_1^m have higher priorities in c_1 than s_0 in both the reserve sub-school and the original sub-school (c_1^o). Recall that the point-wise priorities among the minorities do not change in both kinds of sub-schools.

1.6.4 Proof of Theorem 1.2

(i) *Strongly type-specific acyclicity* \implies *Respect the improvement of reserve-based affirmative action*. Suppose if $\mu(s)P_s\tilde{\mu}(s)$ for at least one $s \in S^m$, I show that $\tilde{\Gamma}^m$ must have a quasi type-specific cycle with two schools and three students.

Lemma 1.2 implies that if $\mu(s)P_s\tilde{\mu}(s)$ for at least one $s \in S^m$, then $\mu(s')P_{s'}\tilde{\mu}(s')$ for at least one $s' \in S^M$. Denote s_0 be a minority student who is strictly worse off after the stronger affirmative action \tilde{r}^m . Let $\mu(s_0) = c_0$, and step k be the step of the SOSM algorithm when s_0 is rejected by c_0 in $\tilde{\Gamma}^m$. Without loss of generality, I can construct a set of $k - 1$ students, $\mathbf{s}_1 = \{s_1, s_2, \dots, s_{k-1}\} \in S$, such that $\mu(s_l)P_{s_l}\tilde{\mu}(s_l) \neq s_l$, $\mu(s_l) = c_l$, $l = \{1, \dots, k - 1\}$, $k \geq 2$. Let $k - l$ be the step when s_l is rejected by $\mu(s_l)$ in $\tilde{\Gamma}^m$. s_l applies to c_{l-1} at step $k - l + 1$. In particular, I have s_1 is rejected by $\mu(s_1) = c_1$ at step $k - 1$ and applies to c_0 at step k . Thus,

(i.a.) if all students in \mathbf{s}_1 except s_{k-1} are minorities. By my construction of \mathbf{s}_1 , s_{k-1} is rejected by $\mu(s_{k-1}) = c_{k-1}$ at step 1 of the SOSM algorithm in $\tilde{\Gamma}^m$, and applies to c_{k-2} in the next step. Obviously, s_{k-1} is *directly* affected by \tilde{r}^m (recall Footnote 31), and $s_{k-1} \in S^M$. Thus, there must have another minority student, denote by \dot{s} , $\dot{s} \in S^m \setminus \mathbf{s}_1$, who prefers c_{k-1} to all the rest schools but is rejected by c_{k-1} in Γ^m (i.e., before the stronger affirmative action \tilde{r}^m). That is, $c_{k-1}P_{\dot{s}}c$, for all $c \in C \setminus c_{k-1}$, $s_{k-1} \succ_{c_{k-1}}^o \dot{s}$ but $\dot{s} \succ_{c_{k-1}}^r s_{k-1}$. Otherwise, (s_{k-1}, c_{k-1}) forms a blocking pair in $\tilde{\Gamma}^m$. In addition, I know that s_{k-2} is rejected by $\mu(s_{k-2}) = c_{k-2}$ at step 2, when s_{k-1} applies to c_{k-2} . As $s_{k-1} \in S^M$ and $s_{k-2} \in S^m$, I have $s_{k-1} \succ_{c_{k-2}}^o s_{k-2}$. Thus, $\dot{s} \succ_{c_{k-1}}^r s_{k-1} \succ_{c_{k-2}}^o s_{k-2}$.

(i.b.) if $s_1 \in S^m$, and there is at least one student in \mathbf{s}_1 besides s_{k-1} is a majority student. Let s_l be a minority student in $\mathbf{s}_1 \setminus \{s_1\}$, who is rejected from $\mu(s_l) = c_l$ in $\tilde{\Gamma}^m$ when a majority student in \mathbf{s}_1 applies to c_l . Denote this majority student s_{l-1} and $\mu(s_{l-1}) = c_{l-1}$. Thus, $s_{l-1} \succ_{c_l}^o s_l$ (a minority student can be rejected by a majority student only from an original sub-school). By my construction of \mathbf{s}_1 , there is another student $s_{l-2} \in \mathbf{s}_1$, who is tentatively accepted by c_{l-1} at the step when s_{l-1} is rejected by c_{l-1} . Thus, $s_{l-2} \succ_{c_{l-1}}^r s_{l-1}$ (a majority student can be rejected by another student only from a reserve sub-school). With $s_{l-2} \in S$, $s_{l-1} \in S^M$, and $s_l \in S^m$, I have $s_{l-2} \succ_{c_{l-1}}^r s_{l-1} \succ_{c_l}^o s_l$.

(i.c.) if $s_1 \in S^M$. Since $s_0 \in S^m$, c_0 rejects s_0 at step k of the SOSM algorithm when s_1 applies to c_0 , I have $s_1 \succ_{c_0}^o s_0$. Similar to the previous cases, by my construction of \mathbf{s}_1 ,

s_1 is rejected by $\mu(s_1) = c_1$ at step $k - 1$ when $s_2 \in \mathbf{s}_1$ applies and is tentatively accepted by c_1 . Thus, $s_2 \succ_{c_1}^r s_1$, and I have $s_2 \succ_{c_1}^r s_1 \succ_{c_0}^o s_0$, with $s_2 \in S$, $s_1 \in S^M$ and $s_0 \in S^m$.

Condition (S') is trivially satisfied through the preceding arguments and the stability of SOSM in all three cases.

(ii) *Respect the improvement of reserve-based affirmative action \implies Strongly type-specific acyclicity.* Suppose that $\tilde{\Gamma}^m$ has a quasi type-specific cycle, I argue that there is at least one minority student strictly worse off after implementing the stronger minority reserve policy \tilde{r}^m . Remark 1.2 implies that if $(\succ, (q, r^m))$ has a type-specific cycle, then it has a quasi type-specific cycle. Example 1.1 used in the Proof of Theorem 1.1 (Appendix 1.6.4), which constructs a type-specific cycle and leaves s_2 strictly worse off after \tilde{r}^m , suffices for my purpose. \square

1.6.5 Proof of Theorem 1.3

For a given Γ , let $|S^m| = m$ and s_j be a random majority student. Choose two schools $c, c' \in C$ and relabel the minority students with the lowest priority and second lowest priority in c as i_{m-1} and i_m , and in c' as k_{m-1} and k_m respectively. I prove the contrapositive for both of the two parts.

Part (i) Suppose that s_j ranks higher than two different minority students in c^o and c'^o , I will show that Γ contains a type-specific cycle.

Case (i.a.) $i_m \neq k_m$. Because $s \succ_c^r i_m$, for all $s \in S^m \setminus \{i_m\}$, I have $k_m \succ_c^r i_m$, and there are other $m - 2$ minority students who have higher priority than k_m in c (recall that the priority order are unchanged among the minorities within c^r and c'^o). As I assume $q_c + q_{c'} \leq m$, it implies that $q_c - 1 \leq m - 2$. Thus, I can find a set of $q_c - 1$ minority students who have higher priority than i_m in c from $S^m \setminus \{i_m, k_m\}$, denote by S_c . For school c' , because $m - 2 - (q_c - 1) \geq m - 2 - (m - q_{c'} - 1) = q_{c'} - 1$, I can find a set of minority students that are distinct from i_m, k_m and S_c who are ranked higher than k_m in c' , denote by $S_{c'}$. Condition (C) is satisfied by $k_m \succ_c^r i_m \succ_c^r s_j \succ_c^o k_m$, S_c and $S_{c'}$ suffices Condition (S).

Case (i.b.) $i_m = k_m$. Without loss of generality, suppose that $s_j \succ_{c'}^o k_{m-1}$. Since there are $m - 2$ minority students who have higher priority than k_{m-1} in c' , with similar argument in Case (i.a.), I can find a set of $q_{c'} - 1$ minority students that are distinct

from k_m, k_{m-1} , denote by $S_{c'}$, and a set of $q_c - 1$ minority students that are distinct from k_m, k_{m-1} and $S_{c'}$, denote by S_c . Condition (C) is satisfied by $k_{m-1} \succ_c^r i_m \succ_c^r s_j \succ_c^o k_{m-1}$, S_c and $S_{c'}$ suffices Condition (S).

Part (ii) I have already shown in Part (i) that when s_j ranks higher than two different minority students in two schools, there is a type-specific cycle. Recall Remark 1.2, if $(\succ, (q, r^m))$ has a type-specific cycle, then it has a quasi type-specific cycle. Thus, I only need to discuss the situation when there is only one minority student ranked lower than s_j in one (original sub-)school. Without loss of generality, suppose that $s_j \succ_c^o k_m$. **Case (ii.a.)** $i_m \neq k_m$, since $i_m \succ_c^r s_j$, i_m, s_j and k_m suffice Condition (C'). **Case (ii.b.)** $i_m = k_m$, Condition (C') is given by $i_{m-1} \succ_c^r s_j \succ_c^o k_m$. Condition (S') is satisfied in both of the two cases with the same arguments in (i.a.). \square

1.7 Appendix for Chapter 1: Proofs of Large Market Results

The proof involves a few steps. In brief, I first show that there is a large number of schools that are not listed on any minorities' preference orders at the end of Loop 1 of the Sequential SOSM-R. Then, under the regularity conditions (Definition 1.6), I show that the probability that no majority students apply to such schools converges to one when the number of schools are sufficiently large.

Define the Stochastic Sequential SOSM-R (Algorithm 1.)

Notations Use A_s (and D_s) to record schools that $s(m)$ (and $s(M)$) has already drawn from \mathcal{P}^n (respectively). When $|A_s| = k$ is reached, A_s is the set of schools acceptable to s . Also, B^i (and E^j) to represent the set of rejected minorities (and majorities) from Loop 2 (and Loop 1, respectively).

1. Initialization: Let $l(m) = 1$ and $l(M) = 1$. For every $s(m) \in S(m)$ ($s(M) \in S(M)$, respectively), let $A_s = \emptyset$ ($D_s = \emptyset$, respectively). Order all majorities and minorities in their respective arbitrarily fixed manner. Set $B^0 = \emptyset$, $E^0 = \emptyset$, $i = 0$.
2. Loop 1:

- (a) If $B^i = \emptyset$, then go to Step (2b). Otherwise, pick some minority $s(m)$ in B^i , let $B^{i+1} = B^i \setminus s(m)$, increment i by one and go to Step (2c).
- (b) Choose the applicant:
- i. If $l(m) \leq |S(m)|$, then let $s(m)$ be the $l(m)^{\text{th}}$ student and increment $l(m)$ by one.
 - ii. If not, then go to Step (3).
- (c) Choosing the applied:
- i. If $|A_s| \geq k$, then return to Step (2a).
 - ii. If not, select c randomly from distribution \mathcal{P}^n until $c \notin A_s$, and add c to A_s . Split each c listed by any students into two corresponding sub-schools, original sub-school ($c(o)$) and reserve sub-school ($c(m)$), according to the process defined in Section 1.2.2, and adapt the preferences of schools and students correspondingly.
- (d) Acceptance and/or rejection:
- i. Each $s(m)$ first applies to the reserve sub-school ($c(m)$) of her most favorable school c . If $c(m)$ prefers each of its current mates to $s(m)$ and there is no empty seat, $c(m)$ rejects $s(m)$. $s(m)$ then applies to the corresponding original sub-school $c(o)$ of c . If $c(o)$ prefers each of its current mates to $s(m)$ and there is no empty seat, $c(o)$ rejects $s(m)$. Go back to Step (2c).
 - ii. If $c(m)$ has no empty seat but it prefers $s(m)$ to one of its current mates, then $c(m)$ rejects the least preferred student tentatively accepted. If the rejected student is a majority, add her to E^j , and go to Step (2a). If the rejected student is a minority, let this student be $s(m)$. Let her apply to the corresponding original sub-school $c(o)$ of c .
 - A. If $c(o)$ prefers each of its current mates to $s(m)$ and there is no empty seat, then $c(o)$ rejects $s(m)$, and go back to Step (2c).
 - B. If $c(o)$ prefers $s(m)$ to one of her current mates, $s(m)$ is accepted. If the rejected student is a majority, add her to E^j , and go to Step (2a). If the rejected student is a minority, let this student be $s(m)$, and go back to Step (2c).

- (iii.) If $c(m)$ prefers each of its current mates to $s(m)$ and there is no empty seat, then $c(m)$ rejects $s(m)$. If the corresponding $c(o)$ of c also has no empty seat but it prefers $s(m)$ to one of its current mates, then $c(o)$ rejects the least preferred student tentatively accepted. If the rejected student is a majority, add her to E^j , and go to Step (2a). If the rejected student is a minority, let this student be $s(m)$ and go back to Step (2c).
- (iv.) If either $c(m)$ or its corresponding $c(o)$ has an empty seat, then $s(m)$ is tentatively accepted. Go back to Step (2a).

3. Loop 2:

- (a) If $E^j = \emptyset$ but $B^i \neq \emptyset$. Go to Step (2)
- (b) If E^j and B^i are both empty. Go to Step (3d).
- (c) Otherwise, pick some minority $s(M)$ in E^j , let $E^{j+1} = E^j \setminus s(M)$, increment j by one and go to Step (3e).
- (d) Choose the applicant:
 - i. If $l(M) \leq |S(M)|$, then let $s(M)$ be the $l(M)^{th}$ student and increment $l(M)$ by one.
 - ii. If not, then terminate the algorithm.
- (e) Choosing the applied:
 - i. If $|D_s| \geq k$, then return to Step (3).
 - ii. If not, select c randomly from distribution \mathcal{P}^n until $c \notin D_s$, and add c to D_s . Split each c listed by any students into two corresponding sub-schools, the minority-favoring reserve ($c(m)$) and the original ($c(o)$), according to the process defined in Section 1.2.2, and adapt the preferences of schools and students correspondingly.
- (f) Acceptance and/or rejection (“Round j ”):
 - i. Each $s(M)$ first applies to the original school ($c(o)$) of her most favorable school c . If $c(o)$ prefers each of its current mates to $s(M)$ and there is no empty seat, $c(o)$ rejects $s(M)$. $s(M)$ then applies to the corresponding

- reserve sub-school $c(m)$ of c . If $c(m)$ prefers each of its current mates to $S(M)$ and there is no empty seat, then $c(m)$ rejects $s(M)$. Go back to Step (3e).
- ii. If $c(o)$ has no empty seat but it prefers $s(M)$ to one of its current mates, then $c(o)$ rejects the least preferred student tentatively accepted. If the rejected student is a minority, add her to B^i and go to Step (3c). If the rejected student is a majority, let this student be $s(M)$. Let her apply to the corresponding reserve sub-school $c(m)$ of c .
- A. If $c(m)$ prefers each of its current mates to $S(M)$ and there is no empty seat, then $c(m)$ rejects $s(M)$. Go back to Step (3e).
- B. If $c(m)$ prefers $s(M)$ to one of its current matched majority, $s(M)$ is accepted, and let the rejected majority student be $s(M)$, go to Step (3e).
- iii. If $c(o)$ prefers each of its current mates to $s(M)$ and there is no empty seat, then $c(o)$ rejects $s(M)$. If the corresponding $c(m)$ of c also has no empty seat but it prefers $s(m)$ to one of current matched majority, then $c(m)$ rejects the least preferred majority tentatively accepted. Let the rejected majority student be $s(M)$, go to Step (3e).
- iv. If either $c(o)$ or its corresponding $c(m)$ has an empty seat, then $s(M)$ is tentatively accepted. Go back to Step (3c).

Step 2: The market is type-specific acyclic with a high probability

Denote V_n be a random set of schools that are either not listed in any minorities' preference orders at the end of Loop 1 of the Sequential SOSM-R, or listed by some minority students but are not required to implement affirmative actions. Let $X_n = |V_n|$ be a random variable counts the number of schools in V_n .³³ I first state the following result which provides a lower bound of X_n at the beginning of Loop 2. Since it is almost identical to Lemma 2 of Kojima et al. (2013), the proof is omitted.

³³ I denote a random variable and its realization by the same letter, if no confusion arises.

Lemma 1.3: For any $n > 4k$

$$E[X_n] \geq \frac{n}{2} e^{-16\lambda k}$$

Kojima et al. (2013) write their result based on the set of schools not listed by any minority students, denote by Y_n , and prove that $E[|Y_n|] \geq \frac{n}{2} e^{-16\lambda k}$ (i.e. a large set of schools not listed by any minority students). Since I denote X_n to include all schools in Y_n and an additional set of schools that even have been listed by some minorities but without seats reserved for minorities (i.e. the capacity of its sub-school $c(m)$ is zero). Clearly, $E[X_n] \geq E[|Y_n|]$.

Let $Pr(\hat{\Gamma}^{n,tsc})$ be the probability that the corresponding priority structure in a random market $\hat{\Gamma}^n$ is type-specific acyclic. Also, let $\bar{R} = bn^a$ be the upper bound on the number of seats reserved for minority students in the random market $\hat{\Gamma}^n$. The following lemma states that when market is sufficiently large (and conditional on $X_n > \frac{E[X_n]}{2}$), it becomes type-specific acyclic with a high probability.

Lemma 1.4: For any sufficiently large n ,

$$Pr\left(\hat{\Gamma}^{n,tsc} \mid X_n > \frac{E[X_n]}{2}\right) \geq \left(1 - \frac{\bar{R}}{E[X_n]/4r}\right)^{\bar{R}} \quad (1.6)$$

if the conditioning event has a strictly positive probability.

Proof. First, note that there are at most \bar{R} seats reserved for minorities, which also implies the maximum number of schools implemented with minority reserve policy (i.e. allocate one minority reserved seat to one school). Let C_1 be the set of schools implemented with minority reserve policy and are tentatively matched to one minority student in its $r(m)$ at the end of Loop 1. Recall the condition (4) of Definition 1.6, which can be rewritten as

$$\sum_{c \in C_1} p_c \leq r\bar{R} \cdot \min_{c \in C} \{p_c\}$$

Also, denote C_2 as a set of schools belong to X_n . Obviously,

$$\sum_{c \in C_2} p_c \geq X_n \cdot \min_{c \in C} \{p_c\}$$

I am interested in computing the probability that in Round 1 of Step (3) of Algorithm 1. That is the probability when a majority student applies to some school not in C_1 , which is bounded below by:

$$1 - \frac{\sum_{c \in C_1} p_c}{\sum_{c \in C_2} p_c + \sum_{c \in C_1} p_c} \geq 1 - \frac{\bar{R}}{\frac{X_n}{r} + \bar{R}} > 1 - \frac{\bar{R}}{\frac{E[X_n]/2}{r} + \bar{R}}$$

Now assume that in all Rounds $1, \dots, j-1$, no majority matches to schools in C_1 . Then there are still at least $X_n - (j-1)$ schools which are either not listed by any minorities, or matched with some minorities but without minority reserved seat(s). This follows since at most $j-1$ schools have had their seats filled in Rounds $1, \dots, j-1$ from the set of schools in V_n . Similar to the above procedures, I can compute that in Round j , the probability that the Sequential SOSM-R produces the same match before and after implementing a (stronger) affirmative action policy is at least,

$$1 - \frac{\bar{R}}{\frac{X_n - (j-1)}{r} + \bar{R}} > 1 - \frac{\bar{R}}{\frac{E[X_n]/2 - (j-1)}{r} + \bar{R}}$$

Since there are at most \bar{R} minorities can be replaced by majorities from their minority reserved seats ex ante, the probability that Algorithm 1 produces a matching without initiating a rejection chain after a (stronger) affirmative action policy (conditional on $X_n > \frac{E[X_n]}{2}$) is at least,

$$\prod_{j=1}^{\bar{R}} \left(1 - \frac{\bar{R}}{\frac{E[X_n]/2 - (j-1)}{r} + \bar{R}} \right) \geq \left(1 - \frac{\bar{R}}{\frac{E[X_n]/2 - (\bar{R}-1)}{r} + \bar{R}} \right)^{\bar{R}} \geq \left(1 - \frac{\bar{R}}{E[X_n]/4r} \right)^{\bar{R}}$$

where the first inequality follows as $j \leq \bar{R}$, $j \in \{1, \dots, \bar{R}\}$. The second inequality holds since $E[X_n]/2 - \bar{R} + 1 \geq E[X_n]/4 > 0$, which follows from Lemma 1.3 and the assumption that n is sufficiently large. \square

The last step is to show that the unconditional type-specific acyclic probability converges to one as the market becomes large, which can be verified through the following inequalities.

$$\begin{aligned}
Pr\left(\hat{\Gamma}^{n,tsc}\right) &\geq Pr\left(X_n > \frac{E[X_n]}{2}\right) \cdot \left(1 - \frac{\bar{R}}{E[X_n]/4r}\right)^{\bar{R}} \\
&\geq \left(1 - \frac{4}{E[X_n]}\right) \cdot \left(1 - \frac{\bar{R}}{E[X_n]/4r}\right)^{\bar{R}} \\
&\geq \left(1 - \frac{8e^{16\lambda k}}{n}\right) \cdot \left(1 - \frac{8r\bar{R}e^{16\lambda k}}{n}\right)^{\bar{R}}
\end{aligned}$$

The first inequality is given by Equation (1.6) (of Lemma 1.4). The second inequality follows the result by Kojima et al. (2013),³⁴ and the last inequality is given by Lemma 1.3.

For the two items of the last line, it is obvious that the first item converges to one as $n \rightarrow \infty$. For the second item, recall that there exists $b > 0$, such that $\bar{R} < bn^a$, for any n (condition (4) of Definition 1.6). Therefore,

$$\left(1 - \frac{8r\bar{R}e^{16\lambda k}}{n}\right)^{\bar{R}} > \left(1 - \frac{8rbn^ae^{16\lambda k}}{n}\right)^{bn^a} = \left(1 - \frac{8rbe^{16\lambda k}}{n^{1-a}}\right)^{n^{1-a}bn^{2a-1}} \geq (e^{8rbe^{-16\lambda k}})^{bn^{2a-1}}$$

where the last inequality follows as $(1 - \frac{\beta}{x})^x \geq e^{-\beta}$, when $\beta, x > 0$. Since I assume $a \in [0, \frac{1}{2})$, the term n^{2a-1} converges to zero as $n \rightarrow \infty$. Thus, $(e^{8rbe^{-16\lambda k}})^{bn^{2a-1}}$ converges to one as $n \rightarrow \infty$. This completes the proof of Theorem 1.4, given the (material) equivalence between type-specific acyclicity and respecting the spirit of a stronger minority reserve (Lemma 1.1). \square

³⁴ In short, first $Pr[X_n \leq \frac{E[X_n]}{2}] \leq Pr[X_n \leq \frac{E[X_n]}{2}] + Pr[X_n \geq \frac{3E[X_n]}{2}] = Pr[|X_n - E[X_n]| \geq \frac{E[X_n]}{2}] \leq \frac{Var[X_n]}{(E[X_n]/2)^2}$, where the first inequality is by the fact that any probability is non-negative and less than or equal to one, and the second inequality is given by the Chebychev inequality. Next, use the result $Var[X_n] \leq E[X_n]$ by Immorlica and Mahdian (2005), I get $Pr[X_n \leq \frac{E[X_n]}{2}] \leq \frac{4}{E[X_n]}$.

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2. MULTI-UNIT AUCTIONS WITH EX ANTE ASYMMETRIC BIDDERS: UNIFORM VS DISCRIMINATORY

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Abstract: This paper studies how ex ante differences in bidders' values affect their behavior in two standard multi-unit auction formats, *uniform-price auction* (UPA) and *discriminatory-price auction* (DPA). I characterize the set of asymmetric monotone Bayes–Nash equilibria in a simple multi-unit auction game in which two units of a homogeneous object are auctioned among a set of bidders with independent private values. I show that if a bidder possesses a stronger market position, she has less incentive to shade her bid for the second unit in a UPA, whereas in a DPA, weaker bidders tend to bid more aggressively on both of two units.

JEL Classification: D44

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2.1 Introduction

Auction markets with goods (assets) worth trillions of dollars are held every year around the world. Some noticeable examples include treasury bill, electricity, spectrum, oil

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drilling rights, and mineral rights. In most of these markets, the seller supplies more than one unit of goods, while bidders can also submit different prices for each unit on sale and there can be more than one winner.¹ Partly because of their intrinsic analytic complexity, most extant literature of multi-unit auctions is restricted to the symmetric environment in which all bidders have the same valuation distribution. Symmetry gives a proper abstraction of the complex market environment when there are many small bidders. However, in circumstances with only a handful of qualified participants (e.g., procurement auctions), asymmetry may be a more reasonable assumption. For instance, in whole electricity markets, market incumbents are more likely to enjoy a competition advantage over newcomers through their lower marginal production costs.

In this paper, I am interested in understanding how *ex ante* differences in bidders' distributions of valuations affect their behavior in two popular simultaneous sealed-bid multi-unit auction formats, *discriminatory-price auction* (henceforth DPA, also known as *pay-as-bid auction*) and *uniform-price auction* (henceforth UPA). In both auction formats, bidders submit bidding schedules that specify prices for different units. The seller then aggregates all submitted schedules to determine the market-clearing price, and winning bidders are allocated units for which their bids exceed the market-clearing price. These two formats differ in terms of payment rules: all winning bids are filled at the market-clearing price in the UPA, whereas in the DPA bidders pay their own bids for each of their winning units.

I study an auction market in which two units of an identical and indivisible good are sold to a set of *ex ante asymmetric* bidders, each with diminishing marginal values for the successive units. A bidder is *stronger* in the sense that she is more likely to have higher values for both units of the good than a *weaker* bidder. Such a feature is captured by imposing a standard stochastic dominance property to bidders' value distributions (see, for example, Lebrun (1999), Waehrer (1999), Maskin and Riley (2000), and Cantillon (2008), who have used this property to study asymmetric single-unit auctions).²

¹ In this paper, I only discuss auctions with multiple copies of a homogeneous good, i.e., multi-unit auction. Auctions with heterogeneous goods are normally called *multi-item auctions* (See Chapter 4), or *combinatorial auction* if bids for packages are allowed.

² To my knowledge, the only exception to the assumption of first-order stochastic dominance (or stronger) is Kirkegaard (2009). Instead of analyzing the system of differential equations that determines bidding strategies, Kirkegaard studies asymmetric first-price auctions by comparing the ratio of bidders' (endogenous) payoffs to the ratio of their (exogenous) distribution functions.

Engelbrecht-Wiggans and Kahn (1998a,b) provide thorough analyses of the two multi-unit auction formats when the good is indivisible.³ In particular, Engelbrecht-Wiggans and Kahn (1998b) reveal the effect of *demand reduction* in the UPA, which reflects a bidder's strategic shading of all her bids except on the first unit.⁴ The presence of strategic demand reduction not only causes allocation inefficiencies, and consequently a lower expected revenue for the seller; more importantly the diverse levels of bid shading significantly complicate the analyses of equilibrium bidding strategies in the UPA when bidders hold private information. Even though the UPA rules is the analog of second-price auction beyond the single-unit case, in most cases we can only depict bidders' equilibrium strategies through a system of differential equations instead of having truthful reporting as their dominant strategies. Furthermore, the problems of multiplicity and non-monotonicity of equilibria are also prevalent in UPA. These theoretical challenges in analyzing auctions beyond the single-unit case have led to most progress in the multi-unit auction literature in the past decade coming from the empirical side, which aims to provide environment-specific revenue comparisons among different auction formats, especially between a DPA and a UPA.⁵

This paper contributes to the literature by providing new equilibria characterizations for the DPA and the UPA when bidders have different valuation distributions. In an asymmetric DPA, my results imply that a weaker bidder tends to bid more aggressively on both units compared with her relatively stronger competitors (Theorem 2.1). As the direct extension of the first-price auction into multi-unit cases, the asymmetric equilibrium strategies in the DPA echo an analogous pattern, as in the case of asymmetric first-price auctions (Lebrun, 1999, Maskin and Riley, 2000). I further argue that in the DPA, a stronger bidder is more likely to pool her two bids (i.e., submit the same bid for both of the two units, even if she values them differently) than a weaker bidder. By contrast, I find that in the UPA a stronger bidder tends to decrease the level of demand reduction

³ An alternative approach is to consider a *perfectly divisible* good for which each bidder submits a continuous demand (bid) function for a share of the good (Wilson, 1979, Back and Zender, 1993, Ausubel et al., 2014). Although the assumption of a perfectly divisible good gives undeniable analytic convenience for revenue ranking in different multi-unit auction formats, this approach explicitly avoids the multi-dimensional origin of multi-unit auctions by assuming a single-dimensional linear type for all bidders.

⁴ See also Noussair (1995) for an earlier contribution.

⁵ The empirical multi-unit auction literature is quite abundant; among others, interested readers can refer to Athey and Haile (2007), Hickman et al. (2012) for two excellent reviews of the related literature.

compared with a weaker bidder when both of their valuations for the second unit are above the corresponding threshold values for nonzero bids (Theorem 2.2). Because strategic demand reduction is likely to create both an inefficient allocation and lower revenues for the seller, the DPA seems to be a better candidate than the UPA when the effect of demand reduction is severe. My results, however, imply that the unsatisfactory effects of demand reduction in the UPA are partly remitted by the presence of asymmetric bidders.

The rest of the paper proceeds as follows. Section 2.2 introduces the model and assumptions. Section 2.3 and Section 2.4 provide characterization results of asymmetric UPA and DPA respectively. Section 2.5 concludes with discussions. All proofs are clustered in Appendix 2.6.

2.2 Model

Two indivisible and identical units of a good are auctioned among a set of risk neutral and payoff-maximizing bidders, $N = \{1, \dots, n\}$, $|N| \geq 2$.⁶ The seller's valuation to keep unsold units is zero, and all bidders bid for both units on sale. Each bidder has a pair of valuations $\mathbf{v}_i = (v_i^h, v_i^l)$, $i \in N$, for the two successive units. $v_i^h \geq v_i^l$ with probability one. \mathbf{v}_i is privately known and independently distributed across bidders according to a continuously differentiable distribution function $F_i(v_i^h, v_i^l)$ with support on $\mathbf{v}_i \in [0, \bar{v}] \subset \mathbb{R}_+^2$. Let $F_i^h(v_i^h)$ and $F_i^l(v_i^l)$ denote the two marginal distributions of F_i . The information of F_i and its two marginal distributions is common knowledge.

Bidders are *asymmetric* if $F_i(\mathbf{v}) \neq F_j(\mathbf{v})$ for some $i \neq j$ and for a non-zero measure of valuations \mathbf{v} . Therefore, the vector of bidders' valuation distributions $\mathbf{F} = (F_1, \dots, F_n)$ essentially describes the market environment; in particular, $\mathbf{F}_{-i} = (F_1, \dots, F_{i-1}, F_{i+1}, \dots, F_n)$ gives a characterization of the market competition faced by bidder i . As I want to understand how asymmetries affect bidders' optimal bidding strategies, one way of doing this is through a simple but important class of power distributions. Denote $F_i(\mathbf{v}_i) = \mathcal{F}(\mathbf{v}_i)^{\alpha_i}$, where \mathcal{F} is a continuously differentiable distribution function with support on $\mathbf{v}_i \in [0, \bar{v}] \subset \mathbb{R}_+^2$, and $\alpha_i \in \mathbb{R}_+$. We call the vector of real numbers $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_n)$ a *configuration* of an auction market with \mathbf{F} , and denote $\kappa_i = \sum_{j \in N \setminus i} \alpha_j$.

⁶ Throughout, $|\cdot|$ denotes the cardinality of a set.

Two popular simultaneous sealed-bid multi-unit auction formats are discussed in this paper, the *uniform-price auction* (UPA) and the *discriminatory-price auction* (DPA). In both auction formats, each bidder i submits a pair of two bids b_i^h and b_i^l for the first and second units of the good on sale respectively, $b_i^h \geq b_i^l$. Denote by $b_i^m(\mathbf{v}_i)$, $m = h, l$, the bidding strategy of bidder i for the m th unit. The equilibrium concept used in this paper is the usual Bayes-Nash equilibrium, and is simply referred to as the equilibrium. Within the independent private values information framework, we know there exists at least one pure strategy equilibrium when bidders are *ex ante asymmetric*.⁷

2.3 Bidding Behavior in Asymmetric Discriminatory-price Auctions

In a DPA, bidders pay the price they bid for each of the units they win. Write the bid b^m in the DPA as ϕ^m , $m = h, l$. From the perspective of a bidder i , there are $2(n-1)$ competing bids (with equal number of high bids and low bids, and may include zero bids), which are random variables that depend on her rivals' bidding strategies and their own value distributions. Denote c^1 and c^2 as the highest and second highest bids from all of i 's rivals. Since the auction involves two units for sale, the distributions of c^1 and c^2 will be particularly relevant to our analysis. Let G_i^1 denote the distribution of the highest competing bid c^1 , and G_i^2 the distribution of c^2 .

When bidder i submits a pair of bids (ϕ_i^h, ϕ_i^l) , her expected payoff is

$$\pi_i(\phi_i^h, \phi_i^l; v_i^h, v_i^l) = (v_i^h - \phi_i^h) G_i^2(\phi_i^h) + (v_i^l - \phi_i^l) G_i^1(\phi_i^l) \quad (2.1)$$

where $G_i^2(\phi_i^h)$ represents the probability that bidder i wins the first unit when her high bid ϕ_i^h is higher than the second highest competing bid c^2 ; she wins the second unit when her low bid ϕ_i^l is higher than the highest competing bid c^1 with probability $G_i^1(\phi_i^l)$.

Writing down the explicit equilibrium strategies in multi-unit auctions is notoriously difficult. Engelbrecht-Wiggans and Kahn (1998a) and Chakraborty (2006) have provided

⁷ Reny (1999) shows that a pure strategy equilibrium exists in a class of discontinuous games as long as no strategies create a discontinuous payoff decrease, which includes the asymmetric discriminatory-price auction case; Jackson and Swinkels (2005) further prove the existence of a monotone pure strategy equilibrium in a general class of auction games, which also includes the asymmetric uniform-price auction case.

some important equilibrium characterizations of the DPA via the first-order conditions. Within the *independent* (across bidders) and *private* value framework, their characterization results are applicable with my *ex ante asymmetry* assumption as long as each bidder's equilibrium strategies can be derived from maximizing Equation (2.1) with respect to the high bid ϕ_i^h and low bid ϕ_i^l respectively. However, instead of the identical first-order conditions for all bidders, we will have a system of equations for each bidder given their G_i^1 and G_i^2 . I employ the following of their results as the premises for my analysis.

Lemma 2.1: In any equilibrium of the two-unit discriminatory-price auction with independent private values, $i \in N$

- (i) $\pi_i(\phi_i^h, \phi_i^l; v_i^h, v_i^l)$ is differentiable for all relevant bids;
- (ii) $\phi_i^m(v_i^h, v_i^l)$, $m = h, l$ is weakly increasing with v_i^m , and $0 \leq \phi_i^m \leq v_i^m$, $m = h, l$;
- (iii) all equilibrium bids can be obtained by solving

$$\max_{\phi_i^h} (v_i^h - \phi_i^h) G_i^2(\phi_i^h) \quad \text{and} \quad \max_{\phi_i^l} (v_i^l - \phi_i^l) G_i^1(\phi_i^l) \quad (2.2)$$

Lemma 2.1 guarantees that all equilibrium strategies can be represented by the first-order conditions from the bidders' expected payoff maximization problem; in particular, part (iii) states that for each bidder her bids for the two units can be decided separately.

One important analytic advantage from my value distribution assumption $\mathcal{F}(\mathbf{v})^{\alpha_i}$ is the direct application of some standard stochastic dominance properties. I show that the marginal distribution of the highest competing bid G_i^1 and the marginal distribution of the second highest competing bid G_i^2 can both be ranked through the reverse hazard rate dominance for bidders with different κ .⁸

Lemma 2.2: Given a configuration $(\alpha_1, \dots, \alpha_n)$, if $\kappa_j \geq \kappa_i$, $i, j \in N$, then G_j^1 (and G_j^2 resp.) dominates G_i^1 (and G_i^2 resp.) in terms of the reverse hazard rate.

Proof. See Appendix 2.6.1. □

Recall that for each bidder i , I denote $\kappa_i = \sum_{j \in N \setminus i} \alpha_j$ as a measure of the aggregate competition from other bidders. Lemma 2.2 formalizes the intuition that when facing a

⁸ Given two distribution functions G_1 and G_2 , G_1 dominates G_2 in terms of the reverse hazard rate if, for all $x \in [0, \bar{x}]$, $\frac{g_1(x)}{G_1(x)} \geq \frac{g_2(x)}{G_2(x)}$.

lower level of market competition, i.e., a smaller κ , a bidder is more likely to win when submitting the same bid.

Theorem 2.1: Given a configuration $(\alpha_1, \dots, \alpha_n)$, if $\kappa_j \geq \kappa_i$, $i, j \in N$, then

- (i) $\phi_i^h(v^h) \leq \phi_j^h(v^h)$, for all $v^h \in [0, \bar{v}]$;
- (ii) $\phi_i^l(v^l) \leq \phi_j^l(v^l)$, for all $v^l \in [0, \bar{v}]$.

Proof. See Appendix 2.6.2. □

Part (i) (resp. part (ii)) of Theorem 2.1 implies that a bidder bids more aggressively with her high (resp. low) bid when she expects to face with stronger competitors in the market (i.e., a larger κ), regardless of her realized value of the other unit. Let a bidder facing with a smaller (resp. larger) κ as the *stronger* (resp. *weaker*) bidder, Theorem 2.1 reveals an analogous fashion of bid shading to the asymmetric first-price auction in which a weaker bidder bids more aggressively compared to her stronger competitors (Lebrun, 1999, Maskin and Riley, 2000).

As a bidder is more likely to further shade her high bid if she expects a modest level of market competition, Engelbrecht-Wiggans and Kahn (1998a) have shown that for each bidder, *separating* (i.e., $\phi^h \neq \phi^l$) and *pooling* (i.e., $\phi^h = \phi^l$) her two bids coexist with positive probability in equilibrium. In particular, there exists an *iso-bid line* in which a bidder is indifferent between pooling and separating her high and low bids made in equilibrium (see the dashed curve in Figure 2.1). Even though I cannot rule out the coexistence of separating and pooled bids in equilibrium, Theorem 2.1 implies that introducing bidders with heterogeneous value distributions alters the shape of the iso-bid line.

Remark 2.1: A bidder will increase (resp. decrease) the region of pooled bids when she expects less (resp. more) fierce competition from other bidders.

To see this, let (\hat{v}^h, \hat{v}^l) be a pair of values belonging to the iso-bid line, i.e., $\phi^h(\hat{v}^h, \hat{v}^l) = \phi^l(\hat{v}^h, \hat{v}^l)$, but $\phi^h(\hat{v}^h + \epsilon, \hat{v}^l) \neq \phi^l(\hat{v}^h + \epsilon, \hat{v}^l)$ and $\phi^h(\hat{v}^h, \hat{v}^l - \epsilon) \neq \phi^l(\hat{v}^h, \hat{v}^l - \epsilon)$. Equivalently, I can write $\phi = \hat{v}^h - BS^h = \hat{v}^l - BS^l$, where BS^m represents the level of bid shading, $m = h, l$. If the bidder faces less fierce competition from other bidders, i.e., $\hat{\kappa} \leq \kappa$, by

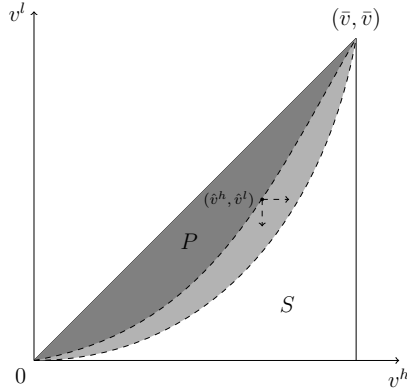


Fig. 2.1: Pooled (P) and separated (S) bids in an asymmetric DPA

Theorem 2.1 we know that $B\hat{S}^m(v^m) \geq BS^m(v^m)$, $v^m \in [0, \bar{v}]$, $m = h, l$. In addition, for a given v and a given κ , $BS^l(v) \leq BS^h(v)$ with probability one. To ensure \hat{v}^h remains on the iso-bid line, we need a higher valuation for the first unit $v^h > \hat{v}^h$. This implies a rightward shift of the iso-bid line. Similar arguments for \hat{v}^l suggest a downward shift of the iso-bid line.

2.4 Bidding Behavior in Asymmetric Uniform-price Auctions

In a UPA, all winning bidders pay the same price for each unit they win, where the *market-clearing price* is set at the highest losing bid. Let φ_i^h and φ_i^l be i 's bid for the first and second unit of the good, respectively. Similar to the DPA case, as each bidder is facing $2(n-1)$ competing bids, I denote by H_i the distribution from which the $2(n-1)$ competing bids are drawn, which reflects i 's belief of possible competition from a set of ex ante asymmetric competitors. Let c^1 and c^2 be the highest and second highest competing bids from all of i 's rivals. H_i^1 denotes the distribution of the highest competing bid c^1 with density h_i^1 , and H_i^2 the distribution of c^2 with density h_i^2 .

Given the strategies of all other bidders, the corresponding expected payoff of a bidder with value (v_i^h, v_i^l) and bids $(\varphi_i^h, \varphi_i^l)$ is

$$\begin{aligned}
\pi_i(\varphi_i^h, \varphi_i^l; v_i^h, v_i^l) &= (v_i^h + v_i^l)H_i^1(\varphi_i^l) - 2 \int_0^{\varphi_i^l} c^1 h_i^1(c^1) dc^1 \\
&+ v_i^h[H_i^2(\varphi_i^h) - H_i^1(\varphi_i^l)] - \varphi_i^l[H_i^2(\varphi_i^l) - H_i^1(\varphi_i^l)] - \int_{\varphi_i^l}^{\varphi_i^h} c^2 h_i^2(c^2) dc^2
\end{aligned} \tag{2.3}$$

where the first line describes i 's expected payoff when she wins both units, i.e., i 's low bid φ_i^l defeats all competing bids and win two units with probability $H_i^1(\varphi_i^l)$. The second line is the case in which she wins one unit with probability $H_i^2(\varphi_i^h) - H_i^1(\varphi_i^l)$. $H_i^2(\varphi_i^l) - H_i^1(\varphi_i^l)$ represents the probability that the market-clearing price is φ_i^l .

The difficulty of equilibrium characterization in the UPA comes not only from the lack of closed-form expressions of equilibrium strategies, as is the case with the DPA, but more importantly, from the possible discontinuities in bidding functions and the prevalent presence of equilibria multiplicity.⁹ Engelbrecht-Wiggans and Kahn (1998b) provide a relatively tractable method to characterize the equilibria of low bid φ_i^l in undominated strategies. I follow their approach and extend their insights to incorporate the case with ex ante asymmetric bidders.

Let $\gamma_i(v)$ be a weakly increasing function representing the low bid as a function of valuation, and write its inverse function as

$$\gamma_i^{-1}(\varphi_i^l) \equiv \sup\{x | \gamma_i(x) < \varphi_i^l\}$$

which gives the highest possible value for bidding below φ_i^l . Engelbrecht-Wiggans and Kahn (1998b) have shown that the set of undominated strategies in the UPA with independent private values involves submitting their true valuations for the first unit $\varphi_i^h(v_i^h) = v_i^h$, and bidding no greater than their valuations for the second unit $\varphi_i^l(v_i^l) \leq v_i^l$, $i \in N$. Replacing φ_i^h by v_i^h in Equation (2.3), I can differentiate $\pi_i(\varphi_i^h, \varphi_i^l; v_i^h, v_i^l)$ with respect to φ_i^l when $\varphi_i^l > 0$,

⁹ A series of examples in Engelbrecht-Wiggans and Kahn (1998b) clearly demonstrates the sensitivity of bidders' strategies to the changes of their value distributions which includes bidding functions with discontinuities. Ausubel et al. (2014) illustrate an example with two bidders and two units in which bidders always bid truthfully on the first unit; however, submitting truthful bid or zero bid on the second unit are both in equilibrium if the other bidder chooses the same behavior.

$$\frac{\partial \pi_i}{\partial \varphi_i^l} = (v_i^l - \varphi_i^l) h_i^1(\varphi_i^l) - [H_i^2(\varphi_i^l) - H_i^1(\varphi_i^l)] \quad (2.4)$$

Equation (2.4) gives a necessary condition for i 's optimal nonzero low bids after equating it to zero. Recall that H_i^1 and H_i^2 are the respective distributions of the highest two competing bids c^1 and c^2 conditional on κ_i . $H_i^1(\varphi_i^l)$ thus represents the probability of winning both units, which is the event that the realized values from all competing bidders are less than or equal to φ_i^l . Therefore, I can write $H_i^1(\varphi_i^l) = \prod_{j \in N \setminus i} F_j^h(\varphi_i^l)$, and the corresponding probability density function $h_i^1(\varphi_i^l) = \sum_j f_j^h(\varphi_i^l) \prod_{k \neq i, j} F_k^h(\varphi_i^l)$.

Accordingly, $H_i^2(\varphi_i^l)$ is the probability that the second highest competing bid c^2 is less than or equal to φ_i^l , which is the union of the following disjoint events: (i) φ_i^l beats the highest competing bid c^1 ; (ii) the high bids from $n - 2$ bidders are less than or equal to φ_i^l and one is greater than φ_i^l ,

$$H_i^2(\varphi_i^l) = \prod_{j \in N \setminus i} F_j^h(\varphi_i^l) + \sum_{j \in N \setminus i} \prod_{k \neq i, j} F_k^h(\varphi_i^l) (F_j^l(\gamma_i^{-1}(\varphi_i^l)) - F_j^h(\varphi_i^l)) \quad (2.5)$$

where the summation ranges over all possible bidders. I can now rewrite the right-hand side of Equation (2.4) as

$$\prod_{k \in N \setminus i, j} F_k^h(\varphi_i^l) \left[\sum_{j \neq i} f_j^h(\varphi_i^l) (v_i^l - \varphi_i^l) + F_j^h(\varphi_i^l) - F_j^l(v_i^l) \right]$$

Define

$$\Gamma_i(\varphi_i^l; v_i^l) = \int_0^{\varphi_i^l} \prod_{k \in N \setminus i, j} F_k^h(x) \left[\sum_{j \neq i} f_j^h(x) (v_i^l - x) + F_j^h(x) - F_j^l(v_i^l) \right] dx$$

and

$$C_i(v_i^l) = \operatorname{argmax}_{\varphi_i^l \in [0, \bar{v}]} \Gamma(\varphi_i^l; v_i^l)$$

$C_i(v)$ is an *increasing correspondence* if $v < \hat{v}$, $\varphi \in C_i(v)$ and $\hat{\varphi} \in C_i(\hat{v})$, we have $\varphi \leq \hat{\varphi}$. Engelbrecht-Wiggans and Kahn (1998b) have shown that if $C_i(v_i^l)$ is an increasing correspondence and $\varphi_i^l(v_i^l)$ is a selection from $C_i(v_i^l)$, then for each $v_i^l \in [0, \bar{v}]$, either $\varphi_i^l = 0$ or $\varphi_i^l \in C_i'(v_i^l)$, where C_i' is the set of solutions to

$$\sum_{j \neq i} f_j^h(\varphi_i^l)(v_i^l - \varphi_i^l) + F_j^h(\varphi_i^l) - F_j^l(v_i^l) = 0 \quad (2.6)$$

i.e., i 's optimal nonzero low bid must be an interior local maximum of Γ_i . As each bidder follows her low bid strategy $\gamma_i(v)$ in equilibrium which is weakly increasing by assumption, and obviously $\varphi_i^l(0) = 0$ and $\varphi_i^l(\bar{v}) = \bar{v}$,¹⁰ we know there is a unique threshold value of the nonzero bid $v_i^* \in [0, \bar{v}]$, such that for all $v_i^l \in [0, v_i^*)$, bidding zero for the second unit gives a higher expected payoff, i.e., $\Gamma(0; v_i^l) > \Gamma(\varphi_i^l; v_i^l)$; and for $v_i^l \in (v_i^*, \bar{v}]$, $\Gamma(0; v_i^l) < \Gamma(\varphi_i^l; v_i^l)$. The arguments upon this point suffice the following result.

Proposition 2.1: For a given configuration $(\alpha_1, \dots, \alpha_n)$, there is an equilibrium in the uniform-price auction with independent private values, such that

$$(\varphi_i^h(v_i^h), \varphi_i^l(v_i^l)) = \begin{cases} (v_i^h, 0) & \text{for } v_i^h \in [0, \bar{v}], v_i^l \in [0, v_i^*) \\ (v_i^h, \varphi_i^{l*}) & \text{for } v_i^h \in [0, \bar{v}], v_i^l \in (v_i^*, \bar{v}] \end{cases} \quad (2.7)$$

where $\varphi_i^{l*} \in C_i'(v_i^l)$, $i = 1, \dots, n$.

Proposition 2.1 characterizes the threshold value v_i^* for nonzero low bids, which also implies that the equilibrium bidding strategy for the second unit $\varphi_i^l(v_i^l)$, $i \in N$, comes from the real-number solutions of Equation (2.6) when $v_i^l \in (v_i^*, \bar{v}]$. Clearly, both v_i^* and $\varphi_i^l(v_i^l)$ are identical to all bidders when the market is ex ante *symmetric*, i.e., $F_i(\mathbf{v}) = F_j(\mathbf{v})$, $\forall i, j \in N$. However, as indicated by Equation (2.6), bidders will have different equilibrium bidding strategies for the second unit once I introduce asymmetries through the market configuration $(\alpha_1, \dots, \alpha_n)$, i.e., $F_i(\mathbf{v}) = \mathcal{F}(\mathbf{v})^{\alpha_i}$ and $\alpha_i \neq \alpha_j$, $\exists i, j \in N$. The experimental results from Engelbrecht-Wiggans et al. (2006) imply that the threshold value for nonzero low bids v_i^* monotonically decreases (but does not converge to zero) with the increase in the number of homogeneous bidders in the market. Given the assumption of a fixed set of bidders, instead of investigating the effect of new entrants, I am interested in how v_i^* and φ_i^{l*} will vary with our measure of the expected market competition κ_i , where $\kappa_i = \sum_{j \in N \setminus i} \alpha_j$.

¹⁰ When the realized value of the second unit is 0, all nonzero low bids are strictly dominated by bidding 0 on the second unit. When $v_i^l = \bar{v}$, it is with probability one that $\max_{j \neq i} v_j < \bar{v}$ given F_j is atomless for all $j \in N \setminus i$. Bidder i is certain to win both units when bidding truthfully, whereas any other low bid with a positive level of bid shading will reduce her winning probability and is strictly dominated.

Example 2.1: Consider an auction market with two asymmetric bidders $\{1, 2\}$ and two units of a homogeneous good. Let (α_1, α_2) be its configuration, $\alpha_1 \neq \alpha_2$, $\alpha_1 + \alpha_2 = 2$, $\alpha_i \in (0, 2)$, $i = 1, 2$. Each of the two bidders independently draws their valuations for the two units from

$$D_i(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ x^{\alpha_i} & \text{for } 0 \leq x \leq 1 \\ 1 & \text{for } 1 \leq x \end{cases}$$

with densities $d_i(x) = \alpha_i x^{\alpha_i - 1}$. Therefore, $F_i^h(x) = (D_i(x))^2$ and $F_i^l(x) = 2D_i(x) - (D_i(x))^2$ give bidder i 's respective marginal distributions of the first and second unit.

Case (i). $\alpha_1 \in (0, 1.5]$. $(v_i^h, 0)$, $i = 1, 2$ is the unique equilibrium, as there is no real-number solution to Equation (2.6).¹¹

Case (ii). $\alpha_1 \in (1.5, 2)$. That is, bidder 1 becomes *sufficiently* stronger than her competitor. I can solve $\varphi_i^l(v_i^l)$, $i = 1, 2$, via Equation (2.6), which gives

$$2\alpha_j(\varphi_i^l)^{2\alpha_j - 1}(v_i^l - \varphi_i^l) + (\varphi_i^l)^{2\alpha_j} - (2(v_i^l)^{\alpha_j} - (v_i^l)^{2\alpha_j}) = 0$$

Figure 2.2 depicts the numerical calculations for bidder 1's low bid functions when her competitor has $\alpha_2 = 0.4$ and $\alpha_2 = 0.1$. However, $(v_2^h, 0)$ is still the unique equilibrium for bidder 2. Clearly, bidder 1 bids more aggressively (i.e., reducing the difference between v_1^l and $\varphi_1^l(v_1^l)$ for each $v_1^l \in [0, 1]$), when her competitor turns to be even weaker (i.e., α_2 is decreased from 0.4 to 0.1).

For the threshold value of nonzero bid v^* , recall that in Case (i) both of the two bidders submit zero bids on the second unit, which implies that when $\alpha_1 \in (0, 1.5]$, the threshold value for nonzero bids v_i^* is at least 1 for $v_i \in [0, 1]$, $i = 1, 2$. When $\alpha_1 > 1.5$, v_1^* reduces to 0 given the monotonic increasing bid function from $\varphi_1^l(0) = 0$ to $\varphi_1^l(1) = 1$ for bidder 1; however, v_2^* is still at least 1, as $(v_2^h, 0)$ is the only equilibrium for bidder 2

¹¹ When the two bidders are identical in term of value distributions, i.e., $\alpha_1 = \alpha_2 = 1$. D_i is essentially uniformly distributed. The examples in Ausubel et al. (2014) have shown that bidding truthfully on the first unit whereas submitting zero bid on the second unit is the only equilibrium when $v_i^h > v_i^l$ with probability one. However, if the two bidders have constant values for the two units $v_i^h = v_i^l$, $i = 1, 2$, we further encounter the problem of multiple equilibria such that both the single-unit bid equilibrium $(v_i^h, 0)$ and the truthful bidding equilibrium (v_i, v_i) coexist with nonzero probability.

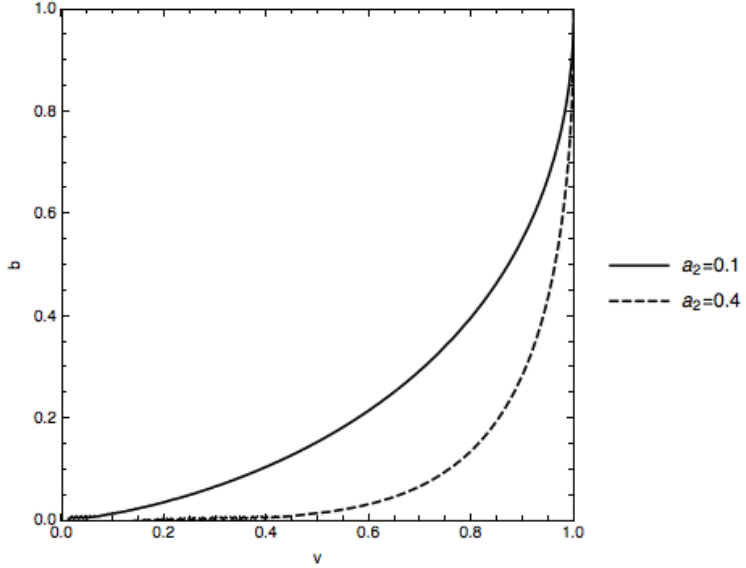


Fig. 2.2

in both cases.

If I fix $\sum_i \alpha_i$ in a given configuration and treat α_i as a measure of i 's own market position, a higher α_i thus implies a lower κ_i , i.e. a market with less aggressive competitors. Example 2.1 illustrates that when a bidder expects to possess a better market position with less intense competition from other bidders, she tends to lower her threshold value for nonzero low bids and submits higher bids for each realized value of the second unit. In the rest of this section, I will show that such asymmetric bidding pattern is generally valid in the UPA providing that the valuation distributions of any two different bidders can be stochastically ordered.

First, I introduce the following lemma to describe the stochastic dominance relations of H_i^1 the distribution of the highest competing bid and H_i^2 the distribution of the second highest competing bid, conditional on κ the measure of market competition.

Lemma 2.3: Given a configuration $(\alpha_1, \dots, \alpha_n)$, if $\kappa_j \geq \kappa_i$, $i, j \in N$, then H_j^1 (resp. H_j^2) dominates H_i^1 (resp. H_i^2) in terms of the hazard rate.

Similar to Lemma 2.2, Lemma 2.3 implies that a bidder expects to face higher (resp. lower) competing bids when she has a relatively weaker (resp. stronger) market position, i.e., a larger (resp. smaller) κ . To justify Lemma 2.3, I need to demonstrate that the stochastic dominance relations in bidders' valuation distributions can be converted to their corresponding pairs of H^1 and H^2 . From Proposition 2.1, we know that each bidder's high bid is separable from her low bid; in addition, the independent private values assumption implies that a bidder's two bids are independent from all her competing bids. Thus, for each bidder i I can treat her $2(n-1)$ competing bids are independently drawn from the distribution H_i . Because all bidders bid submit their true valuations for the high bids, and their low bids are either zero or come from the weakly increasing function $\gamma_i(v_i^l)$, $i \in N$, I can inverse each of her $2(n-1)$ competing bids to its corresponding values which is drawn from either F_j^h (for the first unit) or F_j^l (for the second unit), $j \in N \setminus i$. The rest of the proof follows the arguments as that of Lemma 2.2, and is omitted.

I am now ready to present the following equilibrium characterizations of the UPA with ex ante asymmetric bidders.

Theorem 2.2: Given a configuration $(\alpha_1, \dots, \alpha_n)$, if $\kappa_j \geq \kappa_i$, $i, j \in N$, then

- (i) $v_i^* \leq v_j^*$;
- (ii) $\varphi_i^l(v^l) \geq \varphi_j^l(v^l)$, for all $v^l \in [0, \bar{v}]$.

Proof. See Appendix 2.6.3. □

In words, Theorem 2.2 says that when bidder i has a relatively strong market position with less aggressive bidders (i.e., a lower κ), she tends to bid more aggressively on the second unit in the UPA in term of both a lower threshold value for nonzero bids (Part (i)), and a higher nonzero low bid for each realized value of the second unit (Part (ii)). Ausubel et al. (2014) argue that although demand reduction in the UPA brings a superficially lower level of market demand, and in most cases, results in an unsatisfied market-clearing price for the seller, such welfare loss is nevertheless offset by allowing smaller market participants more room to survive, which may further encourage competition and innovation. Theorem 2.2, however, points to the opposite situation: stronger bidders are also more likely to submit higher bids in the UPA and seize a larger market

share, whereas weaker bidders foresee their lower winning chances and are less likely to participate in the market, especially when participating requires non-negligible effort or monetary cost (e.g., research contests in the form of an all-pay auction). In addition, as I will discuss in Chapter 3, the asymmetric UPA also fosters bidders' incentives to form larger coalitions, which is clearly not conducive to promoting market competition.

2.5 Conclusion

This paper presents new equilibrium characterizations for asymmetric discriminatory-price and uniform-price auctions with privately informed bidders. I argue that bidders' distinct strategic behavior essentially comes from their diverse market positions (i.e., the winning probability and the probability of deciding the market-clearing price), which is measured by a configuration α of the differences in their valuation distributions. Instead of deriving a system of differential first-order conditions for the DPA and the UPA, which quickly becomes intractable given its multi-dimensional nature, I identify the comparative statics of equilibrium sets between two asymmetric bidders through the stochastic dominance relations in their valuation distributions.

Conceptually, we can treat multi-unit auctions as the simplest case of a multi-dimensional resource allocation problem. Thus, besides its practical relevance, having more comprehensive analyses of bidders' behavior in markets with multiple objects is a nontrivial question in auction theory. Here, I mention a few possible directions for future research. First, even if providing a general result of revenue rankings among different multi-unit auction formats is difficult, comparing revenues among markets with different degrees of bidders' asymmetry is still appealing, which may also contain fewer technical challenges.¹² Second, I have thus far only considered bidders' collusion incentives at the ex ante stage, which includes asymmetries that arise from mergers or joint bidding before bidders receive their private information. Analyzing bidders' interim collusion behavior in single-item auctions has been widely discussed in the existing literature. However, since each coalition will face different incentive compatibility and participation constraints from its members given their realized valuations of the two units, characterizing bidders' in-

¹² See, for example, Cantillon (2008) who shows that a higher degree of asymmetries among bidders reduces the seller's expected revenue in both first-price and second-price auctions.

terim collusive strategies in a multi-unit auction even with the conventional independent private values setting appears to be a technically nontrivial exercise. Last, it would also be interesting to examine my results through experimental evidences.¹³

2.6 Appendix for Chapter 2

2.6.1 Proof of Lemma 2.2

I use some established stochastic ordering results from Karlin and Rinott (1980) to complete the proof. First, note that from the perspective of each bidder i , she is facing $2(n-1)$ bids (with equal number of high bids and low bids, and may include zero bids). For bidder i , let $\mathcal{F}^h(v)^{\kappa_i}$ denote the joint distribution of the $n-1$ distributions of marginal valuations for the first unit from her competitors, $\kappa_i = \sum_{j \in N \setminus i} \alpha_j$. Recall that given $F_i^m(v) = \mathcal{F}^m(v)^{\alpha_i}$, if $\alpha_i \geq \alpha_j$, $i, j \in N$, then $F_i^m(v)$ likelihood ratio dominates $F_j^m(v)$, $m = h, l$, $v \in [0, \bar{v}]$. Thus, we know that for bidder i , $\mathcal{F}^h(v)^{\kappa_i}$ likelihood ratio dominates $\mathcal{F}^h(v)^{\kappa_{i'}}$, if $\kappa_i \geq \kappa_{i'}$. Similarly, the joint distribution of the $n-1$ distributions of marginal valuations for the second unit from her competitors $\mathcal{F}^l(v)^{\kappa_i}$ likelihood ratio dominates $\mathcal{F}^l(v)^{\kappa_{i'}}$, if $\kappa_i \geq \kappa_{i'}$. Proposition 3.3 of Karlin and Rinott (1980) states that the joint density of two densities with likelihood ratio dominance relation also satisfies likelihood ratio dominance. Thus, the joint distribution of $\mathcal{F}^h(v^m)^{\kappa_i}$ and $\mathcal{F}^l(v^m)^{\kappa_i}$, $v^m \in [0, \bar{v}]$, $m = h, l$, also satisfies likelihood ratio dominance property given different κ_i .

For bidder i , let G_i denote the distribution from which her $2(n-1)$ competing bids are drawn. That is, G_i reflects i 's belief of possible competition from a set of ex ante asymmetric rivals. Part (iii) of Lemma 2.1 implies that for each bidder her high bid is separable from her low bid in the DPA; in addition, one's two bids are also independent from the bids from other bidders given the independent private values assumption. Therefore, the $2(n-1)$ competing bids can be treated as independently drawn from G_i . Thus, I can inverse each competing bid back to its corresponding original valuation distribution. By Proposition 3.6 of Karlin and Rinott (1980) which shows that likelihood ratio dominance is preserved after a monotone transformation, we know that G_i also satisfies likelihood

¹³ To my knowledge, Engelbrecht-Wiggans et al. (2006) and Engelmann and Grimm (2009) are the two studies that are close to my setting.

ratio dominance property.

The corresponding likelihood ratio dominance property of the first and second order statistics of G_i , G_i^1 and G_i^2 , follows Proposition 3.2 of Karlin and Rinott (1980).¹⁴ And we know that likelihood ratio dominance implies reverse hazard rate dominance. \square

2.6.2 Proof of Theorem 2.1

Part (i) $\phi_i^h(v^h) \leq \phi_j^h(v^h)$, for all $v^h \in [0, \bar{v}]$. I will argue by contradiction. First, let $\lambda^h(\phi^h)$ denote the inverse of the high bid function, such that $\lambda^h(\phi^h(v^h)) = v^h$. Thus, suppose that $\phi_i^h(v^h) > \phi_j^h(v^h)$, for all $v^h \in [0, \bar{v}]$, we should have $\lambda_i^h(\phi^h) < \lambda_j^h(\phi^h)$.

Part (iii) of Lemma 2.2 implies that each bidder's equilibrium high bids and low bids can be separably derived from the respective first-order conditions of Equation (2.2). That is,

$$(v_i^h - \phi_i^h) g_i^2(\phi_i^h) = G_i^2(\phi_i^h) \quad \text{and} \quad (v_i^l - \phi_i^l) g_i^1(\phi_i^l) = G_i^1(\phi_i^l)$$

which gives us

$$\lambda_i^h(\phi_i^h) = \frac{G_i^2(\phi_i^h)}{g_i^2(\phi_i^h)} + \phi_i^h, \quad i \in N$$

for the equilibrium high bids. By Lemma 2.2, we know that if $\kappa_j \geq \kappa_i$, then $\frac{g_j^h(\phi^h)}{G_j^h(\phi^h)} \geq \frac{g_i^h(\phi^h)}{G_i^h(\phi^h)}$, $i, j \in N$. Thus,

$$\lambda_i^h(\phi^h) = \phi^h + \frac{G_i^2(\phi^h)}{g_i^2(\phi^h)} \geq \frac{G_j^2(\phi^h)}{g_j^2(\phi^h)} + \phi^h = \lambda_j^h(\phi^h)$$

which is a contradiction.

Part (ii) $\phi_i^l(v^l) \leq \phi_j^l(v^l)$, for all $v^l \in [0, \bar{v}]$. The proof follows the same arguments for part (i) with the corresponding first-order condition for the equilibrium low bids, and is thus omitted. \square

¹⁴ Here I follow the terminology and notation used in the auction literature, where the k th order statistics is the k th *highest* observation in a sequence of samples. In statistics, however, the k th order statistics is conventionally denoted as the k th *smallest* observation.

2.6.3 Proof of Theorem 2.2

Part (i) $v_i^* \leq v_j^*$. First, by Proposition 2.1 and Equation (2.4), the threshold value for nonzero bid v_i^* , $i \in N$, should satisfy that for any $\varphi_i^l > 0$,

$$(v_i^\varepsilon - \varphi_i^l) h_i^1(\varphi_i^l) - [H_i^2(\varphi_i^l) - H_i^1(\varphi_i^l)] < 0 \quad \text{and} \quad (v_i^* - \varphi_i^l) h_i^1(\varphi_i^l) - [H_i^2(\varphi_i^l) - H_i^1(\varphi_i^l)] = 0$$

where $v_i^\varepsilon < v_i^*$. Rearranging the equation gives us

$$v_i^* - \varphi_i^l = \frac{1 - H_i(\varphi_i^l)}{h_i(\varphi_i^l)}$$

Next, recall that I denote by H_i the distribution from which the $2(n-1)$ competing bids for bidder i are drawn, by H_i^1 the distribution of the first (highest) order statistic, and by H_i^2 the distribution of the second (highest) order statistic. For a given nonzero bid φ_i^l , we have

$$\begin{aligned} H_i^1(\varphi_i^l) &= H_i(\varphi_i^l)^{2(n-1)}, & h_i^1(\varphi_i^l) &= 2(n-1)H_i(\varphi_i^l)^{2n-3}h_i(\varphi_i^l) \\ H_i^2(\varphi_i^l) &= H_i(\varphi_i^l)^{2(n-1)} + 2(n-1)H_i(\varphi_i^l)^{2n-3}(1 - H_i(\varphi_i^l)) \end{aligned}$$

Thus,

$$\frac{H_i^2(\varphi_i^l) - H_i^1(\varphi_i^l)}{h_i^1(\varphi_i^l)} = \frac{1 - H_i(\varphi_i^l)}{h_i(\varphi_i^l)}$$

By Lemma 2.3, we know that if $\kappa_j \geq \kappa_i$, then $\frac{h_j(\varphi^l)}{1 - H_j(\varphi^l)} \leq \frac{h_i(\varphi^l)}{1 - H_i(\varphi^l)}$, $i, j \in N$, $\varphi^l > 0$. Thus,

$$v_i^* = \varphi^l + \frac{1 - H_i(\varphi^l)}{h_i(\varphi^l)} \leq \frac{1 - H_j(\varphi^l)}{h_j(\varphi^l)} + \varphi^l = v_j^*$$

as I need.

Part (ii) $\varphi_i^l(v^l) \geq \varphi_j^l(v^l)$, for all $v^l \in [0, \bar{v}]$. Suppose by contradiction that $\varphi_i^l(v^l) < \varphi_j^l(v^l)$, for all $v^l \in [0, \bar{v}]$. By Proposition 2.1, for all $\varphi^l > 0$, we should have $\gamma_i^{-1}(\varphi^l) > \gamma_j^{-1}(\varphi^l)$, where $\gamma_m^{-1}(\varphi^l)$, $m = i, j$, gives bidder m 's highest possible value for bidding below

φ^l .

By Equation (2.4), we have the necessary first-order condition for i 's optimal nonzero low bids:

$$(\gamma_i^{-1}(\varphi_i^l) - \varphi_i^l) h_i^1(\varphi_i^l) - [H_i^2(\varphi_i^l) - H_i^1(\varphi_i^l)] = 0, \quad i \in N$$

By Lemma 2.3, for all $\varphi^l > 0$, we have

$$\gamma_i^{-1}(\varphi^l) = \varphi^l + \frac{1 - H_i(\varphi^l)}{h_i(\varphi^l)} \leq \frac{1 - H_j(\varphi^l)}{h_j(\varphi^l)} + \varphi^l = \gamma_j^{-1}(\varphi^l)$$

which is a contradiction. □

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3. EX ANTE COALITION IN MULTI-UNIT AUCTIONS

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Abstract: Following Chapter 2, this paper further examines bidders' *ex ante* collusion incentives in a one-shot multi-unit auction game with incomplete information. I claim that the *uniform-price auction* (UPA) is more vulnerable to collusion than the *discriminatory-price auction* (DPA) in the sense that: 1) a bidder's expected payoff is always lower from a larger coalition in a DPA; 2) for a class of all possible distributions, all coalitions are core-stable in the UPA, whereas only the grand coalition has a nonempty core in the DPA. At the policy level, these results offer new insights into the regulation of anti-competitive behavior in auction markets beyond the single-unit case.

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3.1 Introduction

Following Chapter 2, this paper further investigates bidders' collusion incentives in the *uniform-price auction* (UPA) and the *discriminatory-price auction* (DPA). I am interested in which of the two auction mechanisms is more likely to boost collusion incentives by investigating bidders' ex ante formation of coalitions as *bidding rings* (Marshall et al.,

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1994, Waehrer, 1999, Bajari, 2001, Kim and Che, 2004, Biran and Forges, 2011).^{1,2} I begin the analysis from imposing an *a priori* given coalition structure upon all bidders, which partitions the set of bidders into separated groups before they receive their private information.³ Next, assume a *preauction knockout* (McAfee and McMillan, 1992) is run within each coalition before the grand auction. That is, only a representative bidder of each coalition will participate in the grand auction and bid accordingly; the rest members of each coalition submit irrelevant bids (or become inactive) in the grand auction.

Notice that even if bidders are *ex ante* symmetric, the presence of coalitions as bidding rings essentially introduces asymmetries between bidders who are members of a ring and those who are not, i.e., coalitions can be treated as prototypes of asymmetric bidders. As I have shown in Chapter 2, analyzing equilibrium behavior in multi-unit auctions becomes more problematic with asymmetric bidders. To circumvent possibly intractable technical issues, I simplify the analysis by assuming bidders within each ring are committed to truthfully communicate their value with other ring members.⁴ Each bidder, however, is allowed to switch the initial partition subset she belongs to before knowing her private value. Therefore, instead of analyzing bidders' *interim* collusive strategies, this paper investigates which auction mechanism is more vulnerable to collusions at the *ex ante* stage by rewarding bidders for forming larger coalitions.

I first contrast bidders' collusion incentives from the perspective of the *expected per-member payoff*. That is, because all bidders are identical before receiving their private values, each bidder will expect an equal portion of the total collusive gain from her coalition. A bidder's *ex ante* collusive decision (i.e., staying in or leaving her current

¹ A bidding ring is composed of a group of bidders whom agree to collude together in order to gain higher surplus by depressing competition in the grand auction.

² An alternative approach is to analyze the interim formation of bidding rings, which focuses on an *ex ante* given ring and explores optimal ways to organize the ring (Graham and Marshall, 1987, McAfee and McMillan, 1992, Marshall and Marx, 2007, Hendricks et al., 2008).

³ Such assumption is justified in cases where the auction markets contain both *incumbents* (i.e., bidding rings) and *newcomers* (i.e., individual bidders). For instance, in oil drilling (or mineral) rights auctions where newcomers (or small companies) only have blurred valuations of the goods, while joining a conglomerate (i.e., a ring) may help them receive a clearer valuation or reduce redundant overhead costs (e.g., machine tools, administration costs, logistics). Also, in government procurements, incumbents may have information advantages and consider to collude before the precise project specifications are published.

⁴ Marshall and Marx (2007) term such coalition mechanisms the *bid submission mechanisms*. Biran and Forges (2011) further prove that the bid submission mechanism is *ex ante* incentive-compatible and budget-balanced in a general Bayesian game with independent private values. As their setting clearly include the multi-unit auction case, I expect a similar incentive-compatible ring mechanism also exist in the setting as I discussed here.

coalition before receiving her private value) will be based on the difference of her expected per-member payoff between the current coalition and feasible outside options. Theorem 3.1 (in Section 3.3.1) implies that the UPA is more vulnerable to collusion than the DPA as each bidder's expected payoff is unanimously increasing (resp. decreasing) in the UPA (resp. DPA) with the size of the coalition she belongs to. However, higher expected per-member payoffs still cannot prevent bidders' *joint* incentives to deviate from their current coalitions. Proposition 3.1 (in Section 3.3.2) further shows that regardless of the sizes of their current coalitions in the UPA, no subgroups of bidders would like to collectively deviate from their current coalition once it is formed; by contrast, except for the grand coalition, all bidders would prefer staying in a smaller coalition to their current coalitions in the DPA. These two results contribute to the literature by providing new evidences in the choice between the UPA and the DPA apart from comparing their revenue difference. It also offers new insights into the regulation of anti-competitive behavior in auction markets beyond the single-unit case.

3.2 Model

3.2.1 Preliminary Definitions

Consider an auction game with two indivisible and identical units of a good, and a set of risk neutral and payoff-maximizing bidders labeled by i , $N = \{1, \dots, n\}$, $|N| \geq 2$.⁵ The seller's valuation to keep the unsold units is zero, and all bidders bid for both units on sale. Each bidder has a privately known diminishing value pair $\mathbf{v}_i = (v_i^h, v_i^l)$ for the two successive units, $\mathbf{v}_i \in [0, \bar{v}] \subset \mathbb{R}_+^2$, $v_i^h \geq v_i^l$ with probability one.

Denote $\mathcal{F}(\mathbf{v}_i)$ the joint distribution function of bidder i 's value pair \mathbf{v}_i . Let \mathcal{F} be atomless, the density f exists and is positive for all $\mathbf{v}_i \in [0, \bar{v}]^2$. Before knowing their private values, bidders are partitioned into a set of distinct subsets according to \mathcal{S} . All bidders belong to one and only one subset, no subset is empty, and the union of all subsets equals to N . Both the partition structure \mathcal{S} and the distribution function \mathcal{F} are common knowledge.

Assume within each $s \in \mathcal{S}$, bidders agree to share the information of their private

⁵ Throughout, $|\cdot|$ denotes the cardinality of a set.

valuation, and only the bidder with the highest realized value will submit nonzero bids in the grand auction. However, before knowing their private value, each bidder $i \in s$ is allowed to switch the partition subset $s \in \mathcal{S}$ she belongs to. We call a bidder with at least one nonzero bid as an *active* bidder, and all others who bid zero on both units as *inertia* bidders.⁶ Denote the bidding strategy of bidder i for the m th unit, $m = h, l$, as $b_i^m(\mathbf{v}_i)$. Each bidder i submits a pair of two bids b_i^h and b_i^l for the first and second units of the good respectively, $b_i^h \geq b_i^l$. Let c^1 and c^2 be the highest and second highest competing bids from i 's rivals outside her own partition subset s respectively. The equilibrium concept used in this paper is the usual (pure) Bayes-Nash equilibrium.

Section 3.2.2 introduces the *uniform-price auction* (UPA) and the *discriminatory-price auction* (DPA), which are exclusively discussed in this paper. The setting of these two auction formats is exactly the same as in Chapter 2.

3.2.2 Two Formats of Multi-unit Auction

In a DPA, bidders pay the price they bid for each of the units they win. Write the bid b^m in the DPA as ϕ^m , $m = h, l$. When bidder i submits a pair of bids (ϕ_i^h, ϕ_i^l) , her expected payoff is

$$\pi_i(\phi_i^h, \phi_i^l; v_i^h, v_i^l) = (v_i^h - \phi_i^h) G_i^2(\phi_i^h) + (v_i^l - \phi_i^l) G_i^1(\phi_i^l) \quad (3.1)$$

where G_i^1 is the marginal distribution of the highest competing bid c^1 , and G_i^2 is the marginal distribution of the second highest competing bid c^2 . Thus, $G_i^2(\phi_i^h)$ represents the probability that bidder i wins the first unit when her high bid ϕ_i^h is higher than the second highest competing bid c^2 ; she wins the second unit when her low bid ϕ_i^l is higher than the highest competing bid c^1 with probability $G_i^1(\phi_i^l)$.

In a UPA, all winning bidders pay the same price for each unit they win, where the market-clearing price is set at the highest losing bid. Write φ_i^h and φ_i^l as i 's bid for the first unit and second unit of the good respectively. Similar to the case of the DPA, let c^1 and c^2 be the highest and second highest bids from all of i 's rivals. Denote by H_i^1 be

⁶ Note that a bidder is always the active bidder of her subset, if she belongs to a singleton subset (i.e., $|s| = 1$).

the marginal distribution of the highest competing bid c^1 with density h_i^1 , and by H_i^2 the marginal distribution of c^2 with density h_i^2 . Given the strategies of all other bidders, the corresponding expected payoff of a bidder with value (v_i^h, v_i^l) and bids $(\varphi_i^h, \varphi_i^l)$ is

$$\begin{aligned} \pi_i(\varphi_i^h, \varphi_i^l; v_i^h, v_i^l) &= (v_i^h + v_i^l)H_i^1(\varphi_i^l) - 2 \int_0^{\varphi_i^l} c^1 h_i^1(c^1) dc^1 \\ &\quad + v_i^h [H_i^2(\varphi_i^h) - H_i^1(\varphi_i^l)] - \varphi_i^l [H_i^2(\varphi_i^l) - H_i^1(\varphi_i^l)] - \int_{\varphi_i^l}^{\varphi_i^h} c^2 h_i^2(c^2) dc^2 \end{aligned} \tag{3.2}$$

where the first line describes i 's expected payoff when she wins both units, i.e., i 's low bid φ_i^l defeats all competing bids and win two units with probability $H_i^1(\varphi_i^l)$. The second line is the case in which she wins one unit with probability $H_i^2(\varphi_i^h) - H_i^1(\varphi_i^l)$. $H_i^2(\varphi_i^l) - H_i^1(\varphi_i^l)$ represents the probability that the market-clearing price is φ_i^l .

3.3 Results

I first summarize and rephrase the multi-unit auction game through the following timeline.

- Time 0: Market is set up with: (i) two indivisible and identical units of a good; (ii) a non-strategic seller and $n \geq 2$ risk neutral and payoff-maximizing bidders; (iii) bidders have decreasing marginal value for the two units on sale; (iii) each bidder i has privately known diminishing marginal values $\mathbf{v}_i = (v_i^h, v_i^l)$ for the two successive units, which are drawn from a commonly known distribution $\mathcal{F}(\mathbf{v}_i)$, $i \in N$.
- Time 1: Before receiving their private values \mathbf{v} for the good, a partition structure \mathcal{S} is imposed on the set of bidders. Each partition subset contains $|s|$ bidders, $s \in \mathcal{S}$. Bidders are allowed to switch the partition subsets they belong to before receiving their private values. Only the initial \mathcal{S} is publicly observable.
- Time 2: After knowing \mathbf{v} , bidders within each partition subset are committed to form a coalition. The coalition acts as a bidding ring. That is, for each ring, a representative bidder participates in the grand auction (organized by the seller) and bids according to

the highest valuation pair reported within the ring; all the other ring members submit irrelevant bids. Assume no strategic behavior within each ring.

- Time 3: The seller collects all bids and chooses the highest two bids for assignment. Payments are decided by the chosen auction mechanism (UPA or DPA).

Because both the initial partition \mathcal{S} and the value distribution \mathcal{F} are common knowledge, for a coalition $s \in \mathcal{S}$, $\mathcal{F}(\mathbf{v})^s$ essentially reflects the belief of competing bidders outside s .⁷ Thus, even if bidders were ex ante symmetric within each coalition, the presence of coalitions of different sizes virtually introduces asymmetries among bidders. In addition to complicating bidders' equilibrium behavior (as I have shown in Chapter 2), a higher degree of market asymmetries will inevitably reduce the expected revenue to the seller. From the perspective of the seller, exacerbating market asymmetries through forming larger coalitions should nevertheless be avoided.

In this paper, I evaluate the vulnerability to collusion from the perspective of the *expected per-member payoff* (in Section 3.3.1), and the *core stability* (in Section 3.3.2).

3.3.1 Expected Per-member Payoff

From the perspective of bidder i , her *ex ante* expected payoff from staying in coalition s is

$$u_i = \frac{1}{|s|} \int_0^{\bar{v}} \int_0^{\bar{v}} \pi_s(b_s^h, b_s^l; v_s^h, v_s^l) dF_s(v_s^h, v_s^l) \quad (3.3)$$

where $\pi_s(b_s^h, b_s^l; v_s^h, v_s^l)$ is the expected payoff of coalition s at the ex ante stage when the representative bidder of coalition s has value (v_s^h, v_s^l) with the corresponding pair of bids (b_s^h, b_s^l) , and all other representative bidders of their respective coalitions follow their equilibrium bidding strategies. u_i is the *expected per-member payoff*, which determines i 's decision to remain with or leave her current coalition group. Denote u_i^{UPA} and u_j^{DPA} the expected per-member payoff in a UPA and a DPA respectively.

Theorem 3.1: Let \mathcal{S} be a partition of a set of ex ante symmetric bidders N , $|N| \geq 2$, $i \in s$ and $j \in s'$, $s, s' \in \mathcal{S}$. If $|s| \geq |s'|$, then $u_i^{DPA} \leq u_j^{DPA}$ and $u_i^{UPA} \geq u_j^{UPA}$.

⁷ With a slight abuse of notation, I use s to represent both a coalition set and its cardinality if no confusion arises.

Proof. See Appendix 3.5.1. □

Theorem 3.1 says that the expected per-member payoff is higher (resp. lower) when a bidder stays in a larger (resp. smaller) bidding coalition under the UPA (resp. DPA). In other words, the UPA encourages bidders to expand the size of their coalitions by absorbing those who belong to other smaller groups for the purpose of reducing the level of market competition. The DPA, however, reveals a different collusion mechanism in which bidders prefer to stay in a smaller coalition group rather than join a larger one. For example, in an auction market with three identical bidders, i.e., singleton coalitions, any two of the three bidders will intend to merge as a new coalition in a UPA, which gives them a higher level of ex ante expected payoff; in addition, the remaining singleton bidder also would like to join the larger coalition. By contrast, all three bidders prefer maintaining their current status to forming a larger coalition in a DPA. Therefore, the higher per-member payoffs from a smaller coalition in the DPA may further restrain the anti-competitive behavior in auction markets with heterogeneous participants.

In the next section, I further investigate bidders' *joint* incentives to deviate from their current coalitions.

3.3.2 Core Stability

In the language of cooperative games, my setting can be considered as a partition form game (Thrall and Lucas, 1963), which reflects the ex ante commitments of bidding coalitions given the underlying partition structure. Bidders are committed to sharing their private valuation with other members in the same coalition, submitting bids according to their collusive agreements and receiving compensations afterwards (Marshall and Marx, 2007, Biran and Forges, 2011).

I first extend my notation of the group expected payoff π to represent the expected payoff of the coalition s in a given partition \mathcal{S} when all coalitions behave according to the equilibrium $\sigma(\mathcal{S})$ (Ray and Vohra, 1997),⁸

$$\pi_{\sigma}(s; \mathcal{S}) = E \left[\sum_{i \in s} \pi_i(\tilde{v}_i, \sigma(\mathcal{S})(\tilde{v})) \right]$$

⁸ In the case of multiple equilibria, I fix a mapping σ associating a coalitional equilibrium $\sigma(\mathcal{S})$ with every \mathcal{S} , which is plausible in the context of auctions in which bidders are allowed to select their strategies.

where π_i is the share of i 's coalitional gains given \tilde{v}_i , a random realization of (v_i^h, v_i^l) , and the associated coalitional equilibrium $\sigma(\mathcal{S})(\tilde{v})$, $\tilde{v} = (\tilde{v}_k)_{k \in \mathcal{S}}$. Notice that for the two extreme cases, (i) when $|s| = 1$, i.e., the singleton coalition, $\pi_\sigma(i; \mathcal{S})$ is the expected payoff bidder i can obtain independently at the partition \mathcal{S} ; (ii) when $|s| = |N|$, i.e., the grand coalition, $\pi_\sigma(N; \mathcal{S})$ gives the Pareto optimal payoff of the \mathcal{S} .

Define the *characteristic function* of a coalition s as

$$v_\sigma^s(r) = \min_{s^c \in \mathcal{P}(s \setminus r)} \pi_\sigma(r; \{r, s^c, (s')_{s' \in \mathcal{S} \setminus s}\}), \quad r \subseteq s$$

where $\mathcal{P}(s \setminus r)$ is the set of all possible partitions of the remaining members in s without r . Therefore, $v_\sigma^s(r)$ reflects the most pessimistic expectation of the subgroup r once they deviate from s .⁹ A coalition s is *core-stable* (Biran and Forges, 2011), if

$$\sum_{i \in r} \pi_\sigma(i; \mathcal{S}) \geq v_\sigma^s(r), \quad \forall r \subseteq s$$

That is, no subgroup $r \subseteq s$ has the incentive to leave s together if no outside option would give r a higher possible coalitional gain than the sum of payoffs they can obtain from s independently.

Proposition 3.1: In an auction market satisfying the assumptions of Section 3.2, all coalitions are core-stable in the UPA, while only the grand coalition is core-stable in the DPA.

Proof. See Appendix 3.5.2. □

Proposition 3.1 further contrasts the vulnerability to collusion between the UPA and the DPA from a cooperative game perspective. It indicates that for any given coalition structure in the UPA, no subgroup members have incentives to jointly deviate from their current coalitions. By contrast, in the DPA, only the grand coalition which includes all bidders is able to provide a higher payoff to any possible subgroup members over their outside options. As the grand coalition is rarely observed in practice,¹⁰ Proposition 3.1

⁹ Here I implicitly assume the remaining members in s still form the complementary coalition and behave according to their corresponding equilibrium strategy in $\sigma(\mathcal{S})$, even if the subgroup bidders in r secede.

¹⁰ Some possible explanations include the problem of *interim* commitment, the communication cost among coalition members, and the penalty and deterrence from antitrust regulations and competition laws.

further supports my argument that the UPA is more vulnerable to collusion than the DPA in auction markets with private information.

3.4 Conclusion

This paper considers bidders' collusion incentives in a one-shot multi-unit auction game with incomplete information.¹¹ I argue that the UPA is more vulnerable to collusion than the DPA in term of the *expected per-member payoff* and the *core-stability* at the ex ante stage. Even though I only consider a simplified and highly stylized market environment, these observations still offer some nontrivial insights to the competition authorities when explicit collusion is a major concern.

3.5 Appendix for Chapter 3

3.5.1 Proof of Theorem 3.1

(i) $u_i^{DPA} \leq u_j^{DPA}$. By part (iii) of Lemma 2.1 in Chapter 2, I know that all bids can be made as separate bids in the DPA. Together with Equation (3.1) I first rewrite Equation (3.3) for coalition s in a DPA as

$$\begin{aligned}
 u_i^{DPA} &= \frac{1}{s} \int_0^{\bar{v}} \int_0^{\bar{v}} \pi_s(\phi_s^h, \phi_s^l; v^h, v^l) d\mathcal{F}(v^h, v^l)^s \\
 &= \underbrace{\int_0^{\bar{v}} (v - \phi_s^l(v)) G_s^1(\phi_s^l(v)) \mathcal{F}^l(v)^{s-1} f(v) dv}_{(a)} \\
 &\quad + \underbrace{\int_0^{\bar{v}} (v - \phi_s^h(v)) G_s^2(\phi_s^h(v)) \mathcal{F}^h(v)^{s-1} f(v) dv}_{(b)}
 \end{aligned}$$

where part (a) is the expected per-member payoff when a representative bidder in s wins the second unit when her low bid beats the highest competing bidder outside s , and

¹¹ Another strand of literature studies the case of *tacit collusions* (i.e., no monetary compensation within a coalition) in infinitely repeated auctions with complete information. See, for example, Fabra (2003).

part (b) is a representative bidder's expected payoff when she wins the first unit with her high bid. I can write part (a) as

$$\begin{aligned} & \int_0^{\bar{v}} (v - \phi_s^l(v)) G_s^1(\phi_s^l(v)) \mathcal{F}^l(v)^{s-1} f(v) dv \\ &= \int_0^{\bar{v}} (v - \phi_s^l(v)) \left[\prod_{k \in \mathcal{S} \setminus s, s'} p_k^h(\phi_s^l(v)) \mathcal{F}^h(\lambda_{s'}^h(\phi_s^l(v)))^{s'} \right] \mathcal{F}^l(v)^{s-1} f(v) dv \end{aligned} \quad (3.4)$$

where $p_k^h(\phi_s^l(v))$ is the probability that the high bid of a competing representative bidder outside s is below the low bid of the representative bidder of s ; and $\lambda_{s'}^h$ denotes the inverse of $\phi_{s'}^h$, such that $\lambda_{s'}^h(\phi_{s'}^h(v)) = v$.

For part (b), the event that $G_s^2(\phi_s^h)$ involves two components: ϕ_s^h beats the highest competing bid outside s (which is the same in part (a)), and ϕ_s^h is lower than one competing bid but higher than all the rest competing bids. Without loss of generality, let a bidder k' not in s' win the second unit.¹² Part (b) can be written as

$$\int_0^{\bar{v}} (v - \phi_s^h(v)) \mathbf{A} = \int_0^{\bar{v}} (v - \phi_s^h(v)) \mathbf{A}_1 + \int_0^{\bar{v}} (v - \phi_s^h(v)) \mathbf{A}_2 \quad (3.5)$$

with

$$\begin{aligned} \mathbf{A} &= G_s^2(\phi_s^h(v)) \mathcal{F}^h(v)^{s-1} f(v) dv \\ \mathbf{A}_1 &= \left[\prod_{k \in \mathcal{S} \setminus s, s'} p_k^h(\phi_s^h(v)) \mathcal{F}^h(\lambda_{s'}^h(\phi_s^h(v)))^{s'} \right] \mathcal{F}^h(v)^{s-1} f(v) dv \end{aligned}$$

¹² If there are more than two coalition groups, we can always choose the comparison group s' who does not win the second unit. However, for markets with only two coalitions, Equation (3.5) becomes

$$\begin{aligned} & \int_0^{\bar{v}} (v - \phi_s^h(v)) G_s^2(\phi_s^h(v)) \mathcal{F}^h(v)^{s-1} f(v) = \int_0^{\bar{v}} (v - \phi_s^h(v)) \mathcal{F}^h(\lambda_{s'}^h(\phi_s^h(v)))^{s'} \mathcal{F}^h(v)^{s-1} f(v) dv \\ & + \int_0^{\bar{v}} (v - \phi_s^h(v)) \left[\mathcal{F}^l(\lambda_{s'}^l(\phi_s^h(v)))^{s'} (1 - \mathcal{F}^h(\lambda_{s'}^h(\phi_s^h(v))))^{s'} \right] \mathcal{F}^h(v)^{s-1} f(v) dv \end{aligned}$$

where $\lambda_{s'}^l$ denotes the inverse of $\phi_{s'}^l$. All the following arguments are still valid, I thus omit the details.

$$\mathbf{A}_2 = \left[\sum_{k' \in \mathcal{S} \setminus s, s'} \prod_{k \in \mathcal{S} \setminus s, s', k'} p_k^h(\phi_s^h(v)) (1 - p_{k'}^h(\phi_s^h(v))) \mathcal{F}^h(\lambda_{s'}^h(\phi_s^h(v)))^{s'} \right] \mathcal{F}^h(v)^{s-1} f(v) dv$$

As I analyze bidders' incentives to deviate from her current coalition in term of the ex ante expected per-member payoff in UPA, I need to compare the payoff difference between a bidder i if she is in coalition s with another bidder j in s' . To make the comparison more clearly, I also write down the expected per-member payoff of coalition s'

$$\begin{aligned} u_j^{DPA} &= \frac{1}{s'} \int_0^{\bar{v}} \int_0^{\bar{v}} \pi_{s'}(\phi_{s'}^h, \phi_{s'}^l; v^h, v^l) d\mathcal{F}(v^h, v^l)^{s'} \\ &= \underbrace{\int_0^{\bar{v}} (v - \phi_{s'}^l(v)) G_{s'}^1(\phi_{s'}^l(v)) \mathcal{F}^l(v)^{s'-1} f(v) dv}_{(a')} \\ &\quad + \underbrace{\int_0^{\bar{v}} (v - \phi_{s'}^h(v)) G_{s'}^2(\phi_{s'}^h(v)) \mathcal{F}^h(v)^{s'-1} f(v) dv}_{(b')} \end{aligned}$$

where part (a') as

$$\begin{aligned} &\int_0^{\bar{v}} (v - \phi_{s'}^l(v)) G_{s'}^1(\phi_{s'}^l(v)) \mathcal{F}^l(v)^{s'-1} f(v) dv \\ &= \int_0^{\bar{v}} (v - \phi_{s'}^l(v)) \left[\prod_{k \in \mathcal{S} \setminus s, s'} p_k^l(\phi_{s'}^l(v)) \mathcal{F}^h(\lambda_s^l(\phi_{s'}^l(v)))^s \right] \mathcal{F}^l(v)^{s'-1} f(v) dv \end{aligned} \quad (3.6)$$

and part (b') as

$$\int_0^{\bar{v}} (v - \phi_{s'}^h(v)) \mathbf{B} = \int_0^{\bar{v}} (v - \phi_{s'}^h(v)) \mathbf{B}_1 + \int_0^{\bar{v}} (v - \phi_{s'}^h(v)) \mathbf{B}_2 \quad (3.7)$$

with

$$\mathbf{B} = G_{s'}^2(\phi_{s'}^h(v)) \mathcal{F}^h(v)^{s'-1} f(v) dv$$

$$\mathbf{B}_1 = \left[\prod_{k \in \mathcal{S} \setminus s, s'} p_k^h(\phi_{s'}^h(v)) \mathcal{F}^h(\lambda_s^h(\phi_{s'}^h(v)))^s \right] \mathcal{F}^h(v)^{s'-1} f(v) dv$$

$$\mathbf{B}_2 = \left[\sum_{k' \in \mathcal{S} \setminus s, s'} \prod_{k \in \mathcal{S} \setminus s, s', k'} p_k^h(\phi_{s'}^h(v)) (1 - p_{k'}^h(\phi_{s'}^h(v))) \mathcal{F}^h(\lambda_s^h(\phi_{s'}^h(v)))^s \right] \mathcal{F}^h(v)^{s'-1} f(v) dv$$

In order to show $u_j^{DPA} \geq u_i^{DPA}$, I can validate the following conditions

$$\begin{aligned} u_j^{DPA} &\equiv u_j^{DPA}((\phi_s^h, \phi_s^l), (\phi_{s'}^h, \phi_{s'}^l)) \geq u_j^{DPA}((\phi_s^h, \phi_s^l), (\phi_s^h, \phi_s^l)) \\ &\geq u_i^{DPA}((\phi_s^h, \phi_s^l), (\phi_s^h, \phi_s^l)) \\ &\geq u_i^{DPA}((\phi_s^h, \phi_s^l), (\phi_{s'}^h, \phi_{s'}^l)) \equiv u_i^{DPA} \end{aligned}$$

With a slight abuse of notation, I use $u_j^{DPA}((\phi_s^h, \phi_s^l), (\phi_{s'}^h, \phi_{s'}^l))$ to represent bidder j 's expected payoff when bidder i in coalition s bids according to the first component (ϕ_s^h, ϕ_s^l) and bidder j in coalition s' bids according to the second component $(\phi_{s'}^h, \phi_{s'}^l)$. Similar interpretations apply to $u_j^{DPA}((\phi_s^h, \phi_s^l), (\phi_s^h, \phi_s^l))$, $u_i^{DPA}((\phi_s^h, \phi_s^l), (\phi_s^h, \phi_s^l))$ and $u_i^{DPA}((\phi_s^h, \phi_s^l), (\phi_{s'}^h, \phi_{s'}^l))$ respectively. Thus, when a bidder in s consider moving to a different coalition, she assumes all the other bidders outside her current and target group follow their current equilibrium strategies, and only considers the effect of changing bids in her own coalition group s and the target group s' .

The first inequality holds is because $(\phi_{s'}^h, \phi_{s'}^l)$ is the equilibrium strategy for a representative bidder j in s' , which should be at least as good as some other bidding strategies. The second inequality comes from the fact that if the two representative bidders from s and s' submit the identical pair of bids, the expected per-member payoff is higher in a coalition with fewer members.

For the last inequality, first replace $\phi_{s'}^h$ with ϕ_s^h (i.e., change the inverse bid function $\lambda_{s'}^h(\phi^h)$ to $\lambda_s^h(\phi^h)$) in Equation (3.4) and (3.5). We can see that the only difference in

Equation (3.4) after substituting $(\phi_{s'}^h, \phi_{s'}^l)$ with (ϕ_s^h, ϕ_s^l) comes from $\mathcal{F}^h(\lambda_s^h(\phi_s^l(v)))$. Recall $\phi_i^l(v^h, v^l) \leq \phi_j^l(v^h, v^l)$ from Theorem 2.1, with $i \in s$ and $j \in s'$, we have $\mathcal{F}^h(\lambda_s^h(\phi_s^l(v))) \geq \mathcal{F}^h(\lambda_{s'}^h(\phi_{s'}^l(v)))$, for all $v \in [0, \bar{v}]$. Thus, i 's expected payoff from part (a) is higher when j bids according to (ϕ_s^h, ϕ_s^l) instead of $(\phi_{s'}^h, \phi_{s'}^l)$. Apply similar arguments to Equation (3.5), we can see that i 's expected payoff from part (b) is also higher when j bids according to (ϕ_s^h, ϕ_s^l) instead of $(\phi_{s'}^h, \phi_{s'}^l)$, which is given by $\mathcal{F}^h(v) \geq \mathcal{F}^h(\lambda_{s'}^h(\phi_{s'}^h(v)))$, for all $v \in [0, \bar{v}]$.

(ii) $u_i^{UPA} \geq u_j^{UPA}$. First, substitute v_i^h for φ_i^h and simplify Equation (3.2) after integration by parts as

$$\pi_s(\varphi_s^l; v_s^h, v_s^l) = (v_s^l - \varphi_s^l)H_s^1(\varphi_s^l) + 2 \int_0^{\varphi_s^l} H_s^1(c^1) dc^1 + \int_{\varphi_s^l}^{v_s^h} H_s^2(c^2) dc^2 \quad (3.8)$$

where π_s is the expected payoff of coalition s . As in Section 3.3, I abuse notation by using s to represent both a set of bidders and its cardinality, if no confusion arises. By Proposition 2.1 of Chapter 2, we know there is threshold value v^* of nonzero low bid. Substitute Equation (3.8) into Equation (3.3) and organize, we can write the expected per-member payoff in a UPA as

$$\begin{aligned} u_i^{UPA} &= \frac{1}{|s|} \int_0^{v^*} d\mathcal{F}^l(y)^s \int_y^{\bar{v}} d\mathcal{F}^h(x)^s \int_0^x H_s^2(c) dc + \frac{1}{|s|} \int_{v^*}^{\bar{v}} d\mathcal{F}^l(y)^s \int_y^{\bar{v}} \pi_s(\varphi_s^l; x, y) d\mathcal{F}^h(x)^s \\ &= \int_0^{v^*} \left[\int_y^{\bar{v}} \mathcal{F}^h(x)^{s-1} f(x) \int_0^x H_s^2(c) dc dx \right] d\mathcal{F}^l(y)^s \\ &\quad + \int_{v^*}^{\bar{v}} \mathcal{F}^l(y)^{s-1} f(y) \int_y^{\bar{v}} d\mathcal{F}^h(x)^s \left[(y - \varphi_s^l(y))H_s^1(\varphi_s^l(y)) + 2 \int_0^{\varphi_s^l(y)} H_s^1(c) dc \right] dy \\ &\quad + \int_{v^*}^{\bar{v}} \mathcal{F}^l(y)^{s-1} f(y) \left[\int_y^{\bar{v}} d\mathcal{F}^h(x)^s \int_{\varphi_s^l(y)}^x H_s^2(c) dc \right] dy \end{aligned} \quad (3.9)$$

where the second line is the case when v_s^l is lower than v^* ; that is, the coalition s wins one unit when v_s^h is greater than the second highest competing bid c^2 . c^2 is distributed according to H_s^2 . $\mathcal{F}^h(v)^s$ and $\mathcal{F}^l(v)^s$ are the distributions of marginal valuations for the first unit and the second unit of the representative bidder of s . The last two lines represent the expected per-member payoff when v_s^l is greater than the threshold of nonzero low bid. Similar to the DPA case, I compare the payoff difference between a bidder i in the coalition

s and a bidder j in s' . Write down the corresponding expression for u_j^{UPA}

$$\begin{aligned}
u_j^{UPA} &= \frac{1}{|s'|} \int_0^{v^{**}} d\mathcal{F}^l(y)^{s'} \int_y^{\bar{v}} d\mathcal{F}^h(x)^{s'} \int_0^x H_{s'}^2(c) dc + \frac{1}{|s'|} \int_{v^*}^{\bar{v}} d\mathcal{F}^l(y)^{s'} \int_y^{\bar{v}} \pi_{s'}(\varphi_{s'}^l; x, y) d\mathcal{F}^h(x)^{s'} \\
&= \int_0^{v^{**}} \left[\int_y^{\bar{v}} \mathcal{F}^h(x)^{s'-1} f(x) \int_0^x H_{s'}^2(c) dc dx \right] d\mathcal{F}^l(y)^{s'} \\
&\quad + \int_{v^{**}}^{\bar{v}} \mathcal{F}^l(y)^{s'-1} f(y) \int_y^{\bar{v}} d\mathcal{F}^h(x)^{s'} \left[(y - \varphi_{s'}^l(y)) H_{s'}^1(\varphi_{s'}^l(y)) + 2 \int_0^{\varphi_{s'}^l(y)} H_{s'}^1(c) dc \right] dy \\
&\quad + \int_{v^{**}}^{\bar{v}} \mathcal{F}^l(y)^{s'-1} f(y) \left[\int_y^{\bar{v}} d\mathcal{F}^h(x)^{s'} \int_{\varphi_{s'}^l(y)}^x H_{s'}^2(c) dc \right] dy \tag{3.10}
\end{aligned}$$

where v^{**} is the optimal threshold of nonzero low bid for coalition s' . Extend my notation to allow $u^{UPA}(\varphi_b^l, a; b)$, $a = v^*, v^{**}$ and $b = s, s'$, which represents the change of u with the nonzero bid threshold (v^* and v^{**}) and the coalition group (s and s'). To show $u_i(\varphi_s^l, v^*; s)^{UPA} \geq u_j^{UPA}(\varphi_{s'}^l, v^{**}; s')$, when $|s| \geq |s'|$, $i \in s$ and $j \in s'$, we can validate the following conditions

$$u_i^{UPA}(\varphi_s^l, v^*; s) \geq u_i^{UPA}(\varphi_{s'}^l, v^{**}; s) \geq u_j^{UPA}(\varphi_{s'}^l, v^{**}; s')$$

The first inequality follows because φ_s^l is the equilibrium bid for the representative bidder in s with (v^h, v^l) , and v^* is the corresponding optimal threshold of nonzero low bid. φ_s^l and v^* should be at least as good as bidding some alternative $\varphi_{s'}^l$ and v^{**} in coalition s . For the second inequality, since given a fix number of bidders N a larger s is equivalent to a smaller κ_s , by Lemma 2.3 of Chapter 2 I know that if $|s| \geq |s'|$ then H_s^1 (resp. H_s^2) dominates $H_{s'}^1$ (resp. $H_{s'}^2$) in terms of the hazard rate, which implies the first-order stochastic dominance $H_s^m(x) \geq H_{s'}^m(x)$, $m = 1, 2$. Replace φ_s^l and v^* in Equation (3.9) with $\varphi_{s'}^l$ and v^{**} respectively. Comparing each item in Equation (3.9) with its corresponding item in Equation (3.10), I can see that bidding strategy $\varphi_{s'}^l(v^h, v^l)$ will generate higher expected payoff given a smaller number of competing bidders for each $v^h, v^l \in [0, \bar{v}]$, which validates the second inequality. \square

3.5.2 Proof of Proposition 3.1

(UPA:) Since all bidders are ex ante symmetric within each coalition, to ensure that no subgroup would like to leave its current coalition, I only need to verify that $\frac{\pi_\sigma(s; \mathcal{S})}{|s|} \geq \frac{\pi_\sigma(s'; \mathcal{S})}{|s'|}$, for all $s' \subseteq s$. Recall Theorem 3.1, I have $u_i^{UPA} = \frac{\pi_\sigma(s; \mathcal{S})}{|s|}$, $i \in s$, and $u_i^{UPA} \geq u_j^{UPA}$, $i \in s$, $j \in s'$ and $s' \subseteq s$. The grand coalition case simply follows when setting $|s| = |N|$.

(DPA:) Also from Theorem 3.1, I have $u_i^{DPA} \leq u_j^{DPA}$, $i \in s$, $j \in s'$ and $s' \subseteq s$, which completes the statement that all partial coalitions are unstable in the DPA. To show the grand coalition is core-stable, I need to verify that $\frac{\pi_\sigma(N; \mathcal{S})}{|N|} \geq \frac{\pi_\sigma(s; \mathcal{S})}{|s|}$, for all $s \subseteq N$. First, because the grand coalition always offers the Pareto optimal payoff, I know that

$$\pi_\sigma(N; \mathcal{S}) \geq \pi_\sigma(s; \mathcal{S}) + \sum_{i \in N \setminus s} \pi_\sigma(i; \mathcal{S})$$

Also, Theorem 3.1 implies that

$$\pi_\sigma(i; \mathcal{S}) \geq \frac{\pi_\sigma(s; \mathcal{S})}{|s|}$$

where $\pi_\sigma(i; \mathcal{S})$ is i 's payoff when forming the singleton coalition and $2 \leq |s| \leq |N|$.

Combing the above two inequalities validates $\frac{\pi_\sigma(N; \mathcal{S})}{|N|} \geq \frac{\pi_\sigma(s; \mathcal{S})}{|s|}$ as I need. \square

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4. STABLE COALITION IN MULTI-ITEM AUCTIONS

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Abstract: This paper illustrates the coalition formation processes in the Ausubel's clinching auction. Compared with the commonly used simultaneous sealed-bid auctions in markets with multiple objectives (e.g., the uniform-price auction and the discriminatory-price auction), the clinching auction offers an open ascending-bid alternative with a clear improvement in allocation efficiency while maintaining simplicity to perform in practice. I first show how vulnerable to collusion the clinching auction is, in the sense that it always has a non-empty core. I further argue that a non-bossiness condition is crucial to such vulnerability, and briefly discuss the intrinsic tension among efficiency, truthfulness, and non-bossiness in auction mechanisms.

JEL Classification: C78; I20

Keywords: multi-item auction, group strategy-proofness, core, non-bossiness.

4.1 Introduction

Market design is an emerging field in the past few decades with wide practical successes.¹

One key criterion of a good design is that players should have incentive to reveal their

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¹ For example, governments use open market auctions to allocate radio spectrum, timber, electricity, and natural gas involving hundreds of billions of dollars worldwide (Milgrom, 2004). In matching, noticeable applications include entry level labor market, house allocation, school choice, organ donation and exchange among others (Roth and Sotomayor, 1990, Roth, 2008).

true valuations. An assumption made in most literature in the area of efficient mechanism design, dating back to the Vickrey-Clarke-Groves (VCG) mechanism, is that selfish players do not collude with each other. However, even if a cautious design can ensure truthful bidding(s) from each individual bidder, it may still be ineffective in preventing their coalitional behavior, i.e., several players may agree to misreport their valuations in a coordinated way and split the gains from such manipulation.

Although the existing literature of collusion in auctions (and other pricing mechanisms) is far from rare, to my knowledge, not many well-established results have directly addressed coalitional behavior in auctions from the cooperative game aspect.² The primary motive of this paper is to explore whether colluders can cooperatively facilitate a feasible revenue division scheme among themselves in auction markets with multiple objects.³

At the current stage, I do not intend to propose a new multi-item auction format, but rather establishing my analysis based on the *clinching auction* (Ausubel, 2004) which essentially replicates the allocation of VCG mechanism in an ascending auction format with a relatively simple payment rule. I consider an auction game with multiple objects, and bidders with complete information. Bidders may collude with each other and jointly share the coalitional gains; in addition, transfers are not forbidden among colluders.⁴ I employ the conventional *core* notion as the solution concept, which characterizes the set of feasible division of coalition gains among the colluders.

My first result (Proposition 4.1) shows that the core is always non-empty in this game, i.e., all colluders perceive higher returns from staying in their current coalitions compared to all other alternatives). Under a mild *super-additive* assumption of the coalition gains, the *grand coalition* containing all bidders will eventually be formed in equilibrium regardless of the former divisions of sub-coalition groups.⁵ Thus, although the clinching auction

² Some exceptions include Ausubel and Baranov (2010), Bachrach (2010), Biran and Forges (2011), among others.

³ Notice that different from the multi-unit auctions with more than one unit of a homogeneous good on sale, the items are not necessarily identical to each other in multi-item auctions. Some prominent real-life examples include selling advertisement slots for search engines, FCC spectrum auctions, among others.

⁴ Although at first glance it may seem as a mere technical simplification by allowing transfers among colluders, detecting the concealed reciprocal agreements is nevertheless difficult in practice. Also, for those bidders who are unlikely to win in the grand auction, participating in a pre-auction collusion scheme will at least grant them the opportunity to receive compensation from the coalition.

⁵ Super-additivity implies that the joint gain of two merged coalition groups should be no less than

guarantees efficient allocations as the VCG auction, caution should be exercised when applying it in markets where secret coalitions are highly suspicious.⁶

I then explore the source of collusive incentives in auction games. A common collusion strategy is to cooperate reports among colluders (as I have discussed in Chapter 3), which will clearly mitigate the intensity of rivalry among bidders and eventually shift more surplus towards the side of bidders. The seller apparently wants to detect and deter possible coalitions through such joint manipulations of reports. Analogous to the strategy-proofness condition which requires truthful reporting from each individual bidder, I introduce the *group* strategy-proofness condition to address bidders' joint incentives of misreporting.⁷ Observation 1 indicates that an auction mechanism is group strategy-proof, if and only if it is *non-bossy* (Satterthwaite and Sonnenschein, 1981).⁸ One major challenge regarding the design of auction mechanisms satisfying the group strategy-proofness condition is the large number of extra incentive-compatibility constraints after introducing it into the seller's optimization problem.⁹ Given the relatively simple expression of the non-bossiness condition, Observation 1 may point out an alternative approach regarding the design of *collusion-proofness* mechanisms (i.e., mechanisms satisfying the group strategy-proofness condition) (Che and Kim, 2006, Pavlov, 2008, Che and Kim, 2009). Last, I briefly discuss the intrinsic tension among efficiency, truthfulness, and non-bossiness in auction mechanisms (Observation 2).

The rest of the paper proceeds as follows. Section 4.2 sets up the model and illustrates the Ausubel's clinching auction. Section 4.3 studies bidders' collusion incentives in the clinching auction. Section 4.4 further investigates the source of such vulnerability to collusion. Section 4.5 concludes the paper. Appendix 4.6 states the proof of Proposition

the sum of each coalition group, i.e., for two coalition groups A and B , $v(A + B) \geq v(A) + v(B)$.

⁶ Also, as the clinching auction always generates the same allocation outcome as the VCG auction does in a given market, it is plausible to suspect that other variants of the VCG mechanism in auction markets may also be vulnerable to similar coalitional schemes (Ausubel and Milgrom, 2006).

⁷ An auction mechanism is group strategy-proof if no subgroup of bidders can jointly manipulate their reports so that all of them weakly benefit from this manipulation, while at least one bidder in the subgroup strictly benefits.

⁸ The non-bossiness condition requires if a change of one's reported preference does not affect her own utility (i.e., the allocation and her payment to the mechanism), such change should not affect others' utilities.

⁹ For example, in an auction game with n ex ante symmetric bidders, the conventional strategy-proofness condition only requires one incentive-compatibility constraint. However, satisfying the group strategy-proofness condition will demand $(n - 1)$ ex ante incentive-compatibility constraints, and 2^{n-1} constraints at the interim stage.

4.1.

4.2 Model

4.2.1 Preliminary definitions

Consider a multi-item auction markets with X heterogeneous items and $|N|$ risk neutral and payoff-maximizing bidders. $N = \{1, \dots, n\}$ and N_{-i} is the set of all the players except i . The seller's valuation to keep unsold items is zero, and he needs to choose an auction mechanism for allocating the items and collecting payments from bidders. Let x be the vector of allocations which specifies the amount of items each bidder gets, and p be the vector of bidders' payments, Each bidder i has a type θ_i representing the vector of her preferences over X . She can choose to report $\theta'_i = s_i(\theta_i)$, where s_i represents her strategy. Denote $u_i(x_i, \theta_i)$ as bidder i 's valuation over her own type θ_i and her allocation outcome x_i chosen by the mechanism. Bidders have *quasi-linear* utility functions $\mu_i = u_i(x_i, \theta_i) - p_i$, where p_i is her payment to the mechanism if she receives x_i amount of the items. Denote the *marginal value* of the l 'th item to player i as $m_i(l) = u_i(l) - u_i(l - 1)$. Assume the marginal value of each additional items is non-increasing, $m_i(l) \geq m_i(l + 1)$.

A cooperative game (N, v) is composed of a set of players N , and a *characteristic function* $v : 2^N \rightarrow \mathbb{R}_+$ that indicates the total payoff these players can achieve together. A cooperative game is *monotonic* if for all coalitions $C' \subseteq C$, $v(C') \leq v(C)$, and is *convex* when for all coalitions $C', C \subseteq N$ I have $v(C') + v(C) \leq v(C' \cup C) + v(C' \cap C)$. The characteristic function only gives the total gains a coalition can achieve, but it does not define how these gains are distributed among its members. An *imputation* $(\eta_1, \dots, \eta_i)_{i \in C}$ is a division of coalition gains among $|C|$ colluders, where η_i is the payoff of i , and $\eta(C) = \sum_{i \in C} \eta_i$. The *core* of a cooperative game (N, v) is a set of payoff vectors:

$$Core(N, v) = \left\{ \eta \in \mathbb{R}^N : \sum_{i \in N} \eta_i = v(N), \sum_{i \in S} \eta_i \geq v(S), \forall S \subseteq N \right\}.$$

In words, the core is the set of feasible imputations (i.e., $\sum_{i \in N} \eta_i = v(N)$) which promises no coalition has a joint payoff (i.e., $v(S)$) greater than the sum of its members' payoffs (i.e., $\sum_{i \in S} \eta_i$).

4.2.2 The Ausubel's multi-item clinching auction

In the Ausubel's clinching auction, a price parameter gradually increases, and bidders keep decreasing their (discrete) demands for items with the price parameter. For bidder i , whenever the combined market demand from other bidders is strictly below the market supply, she can "clinch" one item at the current price. Therefore, each bidder's payment is the sum of the price parameters for each item she clinched throughout the auction process.

Formally, the auction keeps: i) an item counter for the number of items x_i bidder i has clinched, and a payment counter for these items p_i ; ii) a global price parameter β ; and iii) a global counter for the number of remaining items r , $r = X - \sum_{i \in N} x_i$. β keeps ascending as long as the total market demand $\sum_{i \in N} D_i(\beta)$ is larger than the total remaining supply r . If at some β the remaining supply r is larger than the *residual demand* $D_{-i}(\beta)$, $D_{-i}(\beta) = \sum_{j \in N \setminus i} D_j$, the mechanism allocates z item(s) to bidder i at price β , $z = r - D_{-i}(\beta)$, and records the current price parameter β times the number of clinched item(s) z to i 's payment counter p_i . The mechanism updates its parameters as follows: $x_i \leftarrow x_i + z$, $p_i \leftarrow p_i + \beta \cdot z$, $r \leftarrow r - z$. The clinching auction mechanism can be summarized as the following algorithm:

Algorithm 1. Ausubel's clinching auction

- (1) Initialization: $x_i \leftarrow 0$, $p_i \leftarrow 0$, $\beta \leftarrow 0$, $r \leftarrow X$.
- (2) When $\sum_{i \in N} D_i(\beta) > r$:
 - (a) If there exists a bidder i such that $D_{-i}(\beta) < r$, then allocate $z = r - D_{-i}(\beta)$ items to bidder i with payment $\beta \cdot z$. Update all running variables as $x_i \leftarrow x_i + z$, $p_i \leftarrow p_i + \beta \cdot z$, $r \leftarrow r - z$, and repeat.
 - (b) Otherwise, increase β , recompute the demands in Step (2.a.), and repeat.
- (3) When $\sum_{i \in N} D_i(\beta) = r$, allocate to each bidder his demand at β , and terminate.

After the termination, if $x_i > 0$, bidders pay the mechanism according to their payment counters p_i .

4.3 Vulnerability to Collusion

The Ausubel's design of multi-item auctions essentially replicates the allocations of the VCG mechanism in an ascending auction format, which guarantees truthful reports from each individual bidder, but with a relatively simple payment rule. However, like the VCG auction mechanism, it also suffers bidders' joint manipulations of their reports. I use the following example to illustrate how bidders' coalitional strategies can trim down the seller's revenue:

Example 4.1: (U.S. Nationwide Narrowband Auction) There are 5 identical licenses auctioned among 5 bidders. Each bidder is limited to bidding at most 3 licenses. Their preferences are:

utility license	a	b	c	d	e
first	123	75	125	85	45
second	113	5	125	65	25
third	103	3	49	7	5

(Case 1: no collusion) When no collusion is facilitated, the first license is “clinched” when the global price parameter β raises to 65, where the residual demand for bidder a is $D_{-a}(65) = 4$. Similarly, when $b = 75$, $D_{-a}(75) = 3$, and $D_{-c}(75) = 4$. That is, bidder a has clinched two items at price 65 and 75 respectively; meanwhile bidder c also clinches 1 units at 75. Continue the same fashion, when $b = 85$, the market clears ($\sum_{i \in N} D_i(\beta) = r$), bidder a and c win the rest two units and pay 85 for each. The total payment for a is $65+75+85$, and $75+85$ for c .

demand price	a	b	c	d	e
65	3	1	2	1	0
75	3	0	2	1	0
85	3	0	2	0	0

(Case 2: a partial collusion) Consider a partial coalition with bidder a , c and d , in which bidder a and c still report their true valuations as in Case 1, and bidder d agrees to report $\theta'_d = (65, 65, 7)$.

utility license	a	b	c	d	e
first	123	75	125	65	45
second	113	5	125	65	25
third	103	3	49	7	5

The allocations are the same as in Case 1, where bidder a receives three licenses and bidder c gets the other two licenses. However, the mechanism collects fewer payments from bidder a and bidder c compared with the situation when there is no collusion. That is, bidder a pays $65 + 65 + 75$ (v.s. $65 + 65 + 85$ in Case 1), and bidder c pays $75 + 75$ (v.s. $75 + 85$ in Case 1).

demand price	a	b	c	d	e
65	3	1	2	1	0
75	3	0	2	0	0

As monetary compensation is not forbidden (or simply cannot be observed) within the coalition, bidder d should be at least weakly better-off from manipulating her reports, even though she still does not win any item.

(Case 3: the grand coalition) Since bidder e does not win in any case, applying similar arguments as in Case 2, we can easily see that all bidders have incentives to collude together. Thus, bidder a and c can suppress bidding from b and d as $\theta'_b = \theta'_d = (0, 0, 0)$, and report $\theta'_a = (0 + \varepsilon, 0, 0)$ and $\theta'_c = (1, 0 + \varepsilon, 0)$ accordingly. Allocations do not change but now the seller makes (almost) zero revenue. In the next section, I will show that no subgroup bidders have incentives to deviate from the grand coalition once it is formed.

4.3.1 Non-empty core

I first present the formal definition of the collusion game. Denote the allocations under truthful report as $x^* = (x_1^*, \dots, x_n^*)$, and payments $p^* = (p_1^*, \dots, p_n^*)$. Let $C \subseteq N$ be a subgroup of bidders who may decide to collude. The utility of a coalition C when they bid truthfully is $\mu^*(C) = \sum_{i \in C} \mu_i = \sum_{i \in C} u_i(x_i^*) - p_i^*$.

If bidders in C decide to manipulate their reports, they can form a *simple* coalition such that it is able to reduce the total payments from its members but keep the same allocations as from truthful reporting, i.e., $p_i^C \leq p_i^*$ and $x_i^C = x_i^*$. The aggregate utility of the coalition is $\mu'(C) = \sum_{i \in C} u_i(x_i^*) - p_i^C$.

Definition 4.1: In an auction market satisfying the assumptions of Section 4.2.1, a *collusion game* $(C, v(C))$ is a certain subset of the grand coalition $(N, v(N))$, $C \subseteq N$, where $v(C) = \mu'(C)$.

I now turn to analyzing how the coalition can share its gains among the members. As monetary compensation within a coalition is not explicitly prohibited, the coalition members can negotiate and redistribute the coalitional gains among themselves regardless of the initial allocations of the items. A stable coalition can thus be formed in a way that guarantees no bickering among the colluders, i.e., no subgroup colluders perceive higher returns from any outside alternatives than their share of the current coalitional gains.

Proposition 4.1: The collusion game always has a non-empty core in the Ausubel's clinching auction.

Proof. See Appendix 4.6. □

Proposition 4.1 indicates that even though an auction mechanism (e.g., the Ausubel's clinching auction) is capable of preventing manipulation from each individual bidder and produces efficient allocation outcomes, bidders can still *jointly* affect the mechanism and make superficially low payments to the seller (e.g., the grand coalition case in Example 4.1).

Let θ_C be the vector of reports from each coalition member in C , and $\mu'_i(\theta_C; C)$ be i 's utility when her coalition jointly reports some θ_C other than their true types θ_C^* .

Analogous to the strategy-proofness condition targeting individual’s strategic reporting, I introduce the following condition to address players’ joint incentives of manipulation.

Definition 4.2: An auction mechanism is *group strategy-proof*, if $\forall \theta \in \Theta, \nexists C \subseteq N$, s.t. (i) $\mu'_i(\theta_C; C) \geq \mu_i^*(\theta_C^*; C), \forall i \in C$, and (ii) $\mu'_j(\theta_C; C) > \mu_j^*(\theta_C^*; C), \exists j \in C$.

That is, an auction mechanism is group strategy-proof if no subgroup of bidders can jointly manipulate their reports so that all of them weakly benefit from this manipulation, while at least one bidder in the subgroup strictly benefits. The group strategy-proofness condition characterizes the requirement for designing auctions that are able to prevent bidders’ coalitional incentives.¹⁰ However, satisfying the group strategy-proofness condition demands truthful reporting is not strictly dominated for all possible coalitions $C \subseteq N$. It thus introduces a large number of incentive-compatibility constraints into the seller’s objective function, which considerably complicates the design problem (see the illustration in footnote 9, page 93).

In the next section, I will present a condition that is isomorphic to the group strategy-proofness but with a relatively simple expression. I also briefly discuss the intrinsic conflicts over different design goals of auction mechanisms.

4.4 Non-bossiness

A property that has played an important role in developing the axiomatics of resource allocation problems is the so-called non-bossiness condition (Satterthwaite and Sonnenschein, 1981).¹¹ In brief, it says that whenever a change in a player’s preferences does not cause a change in her utility, it should not cause a change in others’ utilities. In other words, a player does not create externalities on others. Non-bossiness seems to be a mild and reasonable assumption as no one would like to be “bossed around”. In this section, I will discuss its relation with the group strategy-proofness condition and possible usages in designing collusion-proofness mechanisms.

¹⁰ Notice that if a mechanism is group strategy-proof, then it is clearly strategy-proof. However, the converse is not necessarily true. The question when strategy-proofness implies group strategy-proofness has been addressed by Le Breton and Zaporozhets (2009), Barberà et al. (2010), among others.

¹¹ The non-bossiness condition has widely appeared in the literature on incentive compatibility. See Thomson (2014) for a comprehensive survey.

Definition 4.3: An auction mechanism is *non-bossy*, if $\forall \theta \in \Theta, \forall i \in N, \mu_i(\theta'_i, \theta_{-i}) = \mu_i(\theta)$ implies $\mu(\theta'_i, \theta_{-i}) = \mu(\theta)$.

The following result presents an isomorphic relation between the group strategy-proofness condition and the non-bossiness condition in auction mechanisms. It also implies that bidders' coalitional incentives essentially come from the possible allocation externalities.

Observation 1: An auction mechanism is group strategy-proof if and only if it is non-bossy.

Similar results regarding the relation between non-bossiness and strategy-proofness can be found in the existing literature on other social choice rules.¹² Even though none of these results have directly addressed the usage of group strategy-proofness in the setting as I discussed here (i.e., an economy with multiple objects auctioning among players with quasi-linear utility functions), the proofs are quite similar. I thus omit the proof of Observation 1 for simplicity.

The following observation further states that the non-bossiness condition is not compatible with efficient allocations in finite auction markets, which reveals some intrinsic conflicts over different goals in auction design problems.

Observation 2: In any auction mechanism, if it has an efficient allocation rule, then it is always bossy.

Recall that the premise of an efficient allocation rule is truthful reporting of private types from each individual bidder (i.e., the allocation rule is strategy-proof). However, if one's utility (i.e., allocations and transfers) depends on her own report, she will report strategically as long as some non-truthful alternatives are profitable. As in any auction mechanisms, the allocations are decided either by each bidder's own report (e.g., the first-price auction in single-unit auction), or by others' reports (e.g., the second-price auction).¹³ All feasible strategy-proof allocation rules have to compute each bidder's allocation based on others' reports, i.e., they are bossy.

¹² See, for example, Svensson (1999), Pápai (2000) for matching mechanisms (without transfers), and Goswami et al. (2014) for general exchange economies (with transfers).

¹³ I exclude the *dictatorial* allocation rule from my discussions here.

4.5 Conclusion

This paper illustrates the coalition formation processes in a variant of the VCG auction, the Ausubel's clinching auction. I claim that the clinching auction is vulnerable to collusion in the sense that it always has a non-empty core. I further argue that a non-bossiness condition is crucial to such vulnerability. Given the incompatibility between efficiency and non-bossiness, designing a relatively efficient collusion-proofness auction mechanisms with multiple objects appears to be a nontrivial task for future research.

4.6 Appendix for Chapter 4

Proof of Proposition 4.1 I construct my proof based on two existing results: i) convexity of v is equivalent to $v(C' \cup \{i\}) - v(C') \leq v(C \cup \{i\}) - v(C)$, $\forall C' \subseteq C \subseteq N_{-i}$ (Driessen, 1988); ii) if a game is convex, it always has a non-empty core (Shapley, 1971). My task is therefore to show the collusion game is convex.

Notice that with non-increasing marginal values, the collusion game is monotonic, i.e. if $C' \subseteq C$ then $v(C') = \mu'(C') \leq \mu'(C) = v(C)$. From Definition 4.1, I have

$$\begin{aligned} v(C \cup \{i\}) - v(C) &= \mu'(C \cup \{i\}) - \mu'(C) \\ &= \sum_{i \in C \cup \{j\}} (u_i(x_i^*) - p_i^{C \cup \{j\}}) - \sum_{i \in C} (u_i(x_i^*) - p_i^C) \\ &= u_j(x_j) + \sum_{i \in C} p_i^C - \sum_{i \in C \cup \{j\}} p_i^{C \cup \{j\}} \end{aligned}$$

I need to show that $v(C' \cup \{i\}) - v(C') \leq v(C \cup \{i\}) - v(C)$, $\forall C' \subseteq C \subseteq N_{-i}$. This is equivalent to

$$u_j(x_j) + \sum_{i \in C'} p_i^{C'} - \sum_{i \in C' \cup \{j\}} p_i^{C' \cup \{j\}} \leq u_j(x_j) + \sum_{i \in C} p_i^C - \sum_{i \in C \cup \{j\}} p_i^{C \cup \{j\}}$$

Organize the above inequality, we get

$$\sum_{i \in C' \cup \{j\}} p_i^{C' \cup \{j\}} - \sum_{i \in C'} p_i^{C'} \geq \sum_{i \in C \cup \{j\}} p_i^{C \cup \{j\}} - \sum_{i \in C} p_i^C \quad (4.1)$$

That is, the payment change of C' when adding an additional colluder j is greater than the corresponding payment change in a larger collusion C , for all $C' \subseteq C$.

Since in an Ausubel's clinching auction, bidder i 's payment of the z 's item depends on the residual demand $D_{-i}(\beta)$, which will be different if i stays in different coalitions. I therefore need to analyze the payment difference for each possible case.

Denote the group of bidders that belong to C but not C' as B , $B = C \setminus C'$, and D be the group of "unaffected" bidders, $D = N \setminus \{C \cup j\}$. Let $\bar{m}_j(x)$ (resp. $\underline{m}_j(x)$) be j 's highest (resp. lowest) marginal value, and $\underline{b}^S(x)$ be the first losing bid from a subset of bidders S when a bidder outside S is able to clinch an item, $S \subseteq N$.

Case (i): $\bar{m}_j(x) \leq \underline{b}^D(x)$, the highest marginal value of j is lower than the first losing bid from the subset of bidders D . Then including j into C will not affect the right-hand-side (RHS) payment difference in Equation (4.1). Since the first losing bid of a bidder group cannot be lower than the one from its subgroup (i.e. $\underline{b}^D(x) \leq \underline{b}^{D \cup B}(x)$), given $C' \subseteq C$, the left-hand-side (LHS) is also unaffected. In this case, we get the equal sign in Equation (4.1).

Case (ii): $\underline{b}^D(x) \leq \bar{m}_j(x) \leq \underline{b}^{D \cup B}(x)$, the highest marginal value of j lies in between the first losing bid in D , and the first losing bid in the subset of bidders $D \cup B$. Similar to Case (i), including j will not affect the LHS payment difference in Equation (4.1). However, for the RHS, the coalition C can gain from lowering all $m_j(x)$ that are larger than $\underline{b}^D(x)$ to $\underline{b}^D(x) + \varepsilon$ after including j , regardless whether j 's smallest marginal value, $\underline{m}_j(x)$, is lower than $\underline{b}^D(x)$ or not. This cannot be the case without j . Thus, the payment difference $\sum_{i \in C \cup \{j\}} p_i^{C \cup \{j\}} - \sum_{i \in C} p_i^C \leq 0$, and Equation (4.1) holds.

Case (iii): $\underline{b}^{D \cup B}(x) \leq \bar{m}_j(x)$, the highest marginal value of j is larger than the first losing bid in $D \cup B$. For the LHS of Equation (4.1), similar to the argument for the payment difference of C in Case (ii), collusion $C' \cup \{j\}$ can suppress reports of all $m_j(x)$ that are larger than $\underline{b}^{D \cup B}(x)$ to $\underline{b}^{D \cup B}(x) + \varepsilon$ (no matter whether $\underline{m}_j(x)$ is lower than $\underline{b}^{D \cup B}(x)$ or not). Therefore, the payment difference of C from including j , is the number

of j 's marginal values that are larger than $\underline{b}^{D \cup B}(x)$. Similarly, the payment difference of C with and without j (i.e., the RHS of Equation (4.1)) is the number of j 's marginal values that are larger than $\underline{b}^D(x)$. Recall that $\underline{b}^D(x) \leq \underline{b}^{D \cup B}(x)$ and $C' \subseteq C$, it is clear that the absolute gain from the collusion $C \cup \{j\}$ (i.e., misreporting all $m_j(x) \geq \underline{b}^D(x)$ to $\underline{b}^D(x) + \varepsilon$) is larger than the smaller coalition $C' \cup \{j\}$ (i.e. misreporting all $m_j(x) \geq \underline{b}^{D \cup B}(x)$ to $\underline{b}^{D \cup B}(x) + \varepsilon$). Equation (4.1) holds as both sides of Equation (4.1) are now less than zero.

In sum, for any possible marginal value of bidder j , Equation (4.1) is always valid. This completes the proof. \square

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CONCLUSION

This thesis studies the impact of heterogeneous market participants on allocation outcomes in different market mechanisms; in addition, how to design alternative mechanisms that can more effectively allocate scarce resources with diverse economic and social goals.

Chapter 1 proposes two welfare criteria to evaluate the effective implementation of affirmative action policies in school choice problems. I characterize two type-specific acyclicity conditions in the *student optimal stable mechanism* with *minority reserve* policy, and demonstrate their respective (material) equivalence with the two welfare criteria in stable matching mechanisms. I further show that type-specific cycles will gradually vanish with the increase of the market size.

Chapter 2 studies how ex ante differences in bidders' values affect their behavior in two standard multi-unit auction formats, *uniform-price auction* (UPA) and *discriminatory-price auction* (DPA). I characterize the set of asymmetric monotone Bayes–Nash equilibria in a simple multi-unit auction game in which two units of a homogeneous object are auctioned among a set of bidders.

Following Chapter 2, Chapter 3 further investigates and contrasts bidders' collusion incentives in the UPA and the DPA when bidders have private information. I claim that the UPA is more vulnerable to collusion than the DPA in the sense that: 1) a bidder's expected payoff is always lower from a larger coalition in a DPA; 2) for a class of all possible distributions, all coalitions are core-stable in the UPA, whereas only the grand coalition has a nonempty core in the DPA.

Chapter 4 illustrates the coalition formation processes in a variant of the Vickrey–Clarke–Groves auction, the Ausubel's clinching auction, in auction markets with multiple heterogeneous objects. I claim that the clinching auction is vulnerable to collusion in the sense that it always has a non-empty core. I further argue that a *non-bossiness* condition is crucial to such vulnerability.

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