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**Doctoral Thesis**

## Understanding Interest Rate Volatility

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UNDERSTANDING INTEREST RATE VOLATILITY

Desi Volker

# UNDERSTANDING INTEREST RATE VOLATILITY

The PhD School of Economics and Management

PhD Series 27.2016

**CBS**  COPENHAGEN BUSINESS SCHOOL  
HANDELSHØJSKOLEN

PhD Series 27-2016

# Understanding Interest Rate Volatility

Desi Volker <sup>1</sup>

Ph.D. Dissertation

Supervisor: Carsten Sørensen

PhD School of Economics and Management

Copenhagen Business School

July 5, 2016

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## *Preface*

This thesis is the result of my Ph.D. studies at the Department of Finance of the Copenhagen Business School. It consists of three essays covering topics related to the term structure of interest rates, monetary policy and interest rate volatility. The first essay, “Monetary Policy Uncertainty and Interest Rates”, examines the role of monetary policy uncertainty on the term structure of interest rates. The second essay, “A Regime-Switching Affine Term Structure Model with Stochastic Volatility” (co-authored with Sebastian Fux), investigates the ability of the class of regime switching models with and without stochastic volatility to capture the main stylized features of U.S. interest rates. The third essay, “Variance Risk Premia in the Interest Rate Swap Market”, investigates the time-series and cross-sectional properties of the compensation demanded for holding interest rate variance risk. The essays are self-contained and can be read independently. There is however a common thread in the themes covered as all essays focus on the understanding of interest rate volatility, its time-variation and main determinants.

## *Acknowledgements*

I would like to extend my sincerest gratitude and appreciation to Lasse Heje Pedersen. This thesis has benefited immensely from his comments and suggestions and his continuous support and encouragement have been invaluable. I am particularly grateful to him for generously employing his Elite Forsk Award research grant to partly finance my Ph.D. position. I would like to thank my advisor, Carsten Sørensen, for his guidance during the last years of my Ph.D. studies. His availability to discuss issues in depth and his pragmatic feedback have contributed significantly in improving the thesis. I am also very grateful to David Lando for his support and advice throughout the years.

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I am grateful to Monika Piazzesi for hosting me at Stanford University and for her kind support. I benefited greatly from discussions with her.

Lastly, I would like to thank my husband and family for their unconditional love.

Desi Volker

Copenhagen, August 17, 2016

## Summary

### *English Summary*

#### *Essay I: Monetary Policy Uncertainty and Interest Rates*

In this paper I analyze the impact of the uncertainty that surrounds the future path of monetary policy on the term structure of interest rates. Overall uncertainty has been shown to be an important driver of the variation in asset prices. One component of overall uncertainty, the uncertainty about the stance of the central bank, is bound to have a non negligible effect on interest rates, their volatilities and the premium that investors demand for holding interest rate risk. In this paper I provide evidence that this impact is statistically and economically significant. Monetary policy uncertainty is not directly observable and to proxy for it I use the cross-sectional dispersion in one-year-ahead forecasts of the fed funds rate from a large survey dataset. I assess the effect of monetary policy uncertainty on interest rate dynamics with regression analysis and in the context of a dynamic no-arbitrage term structure model. The evidence suggests that monetary policy uncertainty is an important contributor to the variation in conditional yield volatilities. The effect is stronger for shorter maturities of up to one year and it dissipates thereafter. This differential effect across maturities implies a slope effect on the volatility term structure. Secondly, monetary policy uncertainty is a priced source of risk and explains part of the variation in expected excess returns at short horizons. Lastly, monetary policy uncertainty has a negligible effect on the level of interest rates, being almost unspanned by the cross-section of yields. These findings have relevant policy implications and suggest that central banks should closely monitor not only the first moments of investor expectations about policy-relevant variables but also their higher moments.

#### *Essay II: A Regime-Switching, Affine Term Structure Model with Stochastic Volatility*

This essay develops and analyzes a regime-switching affine term structure model with a stochastic volatility feature. The increased complexity of introducing regime switches in terms of bond pricing and most importantly in terms of estimation has driven most of the literature to focus on Gaussian specifications of the state variable



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dynamics. We contribute to the literature by analyzing the whole class of maximally-affine regime-switching term structure models. More precisely, we evaluate the performance of the stochastic volatility models relative to the Gaussian model. We find evidence that regime-switching models with stochastic volatility approximate the observed yields more accurate than their Gaussian counterparts. Additionally, we also show that regime-switching affine term structure models with stochastic volatility successfully match some of the most important stylized facts of observed U.S. yield data.

*Essay III: Variance Risk Premia in the Interest Rate Swap Market*

In this paper I investigate the term structure of the compensation demanded for holding variance risk in the interest rate swap market. This compensation, the variance risk premium, is defined as the difference between expected and risk neutral interest rate variances. I use Black-implied Swaption volatilities of various terms and tenors as a proxy for risk neutral volatilities. Since expected realized variances are not observable, a model within the GARCH family is used to estimate the parameters of the volatility process, then conditioning on each period's information set, forecasts are made at horizons corresponding to the terms of the implied volatilities. Analyzing the time-series properties of variance risk premia and investigating the determinants of its variation yields interesting results. The compensation for volatility risk is highly correlated across terms and tenors and it has been negative on average, with brief periods where it switches sign. Investors are willing to pay a premium during normal times in order to insure against high realized volatility during periods of market turmoil. The process fluctuates around two distinct regimes, one with high (negative) level and high dispersion and the second with a nearly zero level and little dispersion. These regimes correspond to periods where the interest rate level is respectively low / high. The main determinants of the variation in variance risk premia are, as expected, the interest level and realized volatility, which explain most of its variation. Other measures, such as credit spreads, swap spreads, the interest rate slope and the stock market volatility index are significant predictors.

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*Dansk Resumé*

*Essay I: Renter og usikkerhed om pengepolitikken*

I denne artikel analyserer jeg, hvordan usikkerhed omkring pengepolitikken påvirker rentestrukturen. Det er vist, at usikkerhed generelt er en vigtig kilde til variation af aktivpriser. Et komponent af denne usikkerhed, usikkerhed omkring centralbankens standpunkt, vil nødvendigvis have en ikke negligerbar effekt på renter, volatiliteten i disse og risikopræmien investorer kræver for at holde renterisiko. I denne artikel præsenterer jeg bevis på, at sådanne effekter er både statistisk og økonomisk signifikante. Da usikkerhed om pengepolitikken ikke er direkte observerbar, bruger jeg som proxy variabel tværsnitsvariationen i et årlige prædiktions af den korte rente baseret på et stort spørgeskemadataset. Jeg evaluerer effekten af usikkerhed omkring pengepolitikken ved hjælp af regressionsanalyser og ved hjælp af en arbitrage fri rentestrukturmodel. Resultaterne indikerer, at usikkerhed omkring pengepolitikken bidrager væsentligt til de betingende rentevolatiliteter. Effekten er stærkest for løbetider op til et år, hvorefter effekten aftager. Denne forskel på tværs af løbetider medfører en hældningseffekt på volatilitetens termin struktur. Derudover er usikkerhed omkring pengepolitikken en prisfastsat risikofaktor, der forklarer dele af variationen i det forventede merafkast på kort sigt. Afslutningsvis har usikkerhed omkring pengepolitikken en negligerbar effekt på det generelle renteniveau, eftersom den næsten er ikke udspændt af rentestrukturen. Disse resultater har relevante implikationer for politik, og de antyder at centralbanker burde overvåge ikke blot det første moment af investorers forventninger til politiske beslutninger men også momenterne af højere orden.

*Essay II: Affine rentestruktur model med regime spring*

Det essay omhandler rentestrukturmodeller, hvor vi udvikler en affine rentestrukturmodel med regime spring og stokastisk volatilitetsfunktion. Den øgede kompleksitet med at indføre regime spring i form af obligationsprisindekstignelse og vigtigst i form af estimering har drevet det meste af litteraturen hvor der fokuseres på Gaussian specifikationer for dynamikken for “state” variabelen. Vi bidrager til litteraturen ved

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at analysere hele klassen af affine rentestrukturmodeller med regime spring. Vi evaluerer resultaterne af de stokastiske volatilitetsmodeller i forhold til den Gaussiske model. Vi finder beviser for, at regime spring modeller med stokastisk volatilitet approksimerer de observerede renter mere præcist end den Gaussiske model. Derudover viser vi også, at regime spring Affine rentestrukturmodeller med stokastisk volatilitet matcher nogle af de vigtigste fakta for observerede amerikanske rentedata

*Essay III: Risikopræmier på varians i rente swap markeder*

I denne artikel analyserer jeg terminstrukturen i variansrisikopræmien i rente swap markedet. Variansrisikopræmien er defineret som forskellen på forventede og implicerede rentevarianser. Jeg bruger implicitte Swaption volatiliteter for forskellige løbetider og tenors. Eftersom forventede realiserede volatiliteter ikke er observerbare, bruger jeg en GARCH model til at estimere parametrene i volatilitetsprocessen. På baggrund af det til enhver tids gældende informationsset, dannes prædiktioner på horisonter, der svarer til løbetiderne i de implicerede volatiliteter. Analyse af tidsrækkeegenskaberne af variansrisikopræmien og undersøgelse af determinanterne af dennes variation leder til spændende resultater. Kompensationen for variansrisiko er højt korreleret på tværs af løbetider og tenors, og den har i gennemsnit været negativ, med korte perioder hvor den skifter fortegn. Investorer er villige til at betale en merpris i normale tider for at forsikre sig mod høj realiseret volatilitet i perioder med markedstumult. Processen fluktuerer rundt om to regimer, en med høj (negativ) niveau og høj dispersion og en anden med et niveau tæt på nul og ingen dispersion. Disse regimer svarer til perioder, hvor renten er henholdsvis lav eller høj. Hoveddeterminanter for variation i variansrisikopræmien er som forventet renteniveauet og tidligere volatilitet, hvilke forklarer hoveddelen af variationen. Andre mål såsom kredit spænd, swap spreads, hældningen på rentestrukturen og aktiemarkedets volatilitetsindex har signifikant prædiktionskraft.

## Introduction

This thesis consists of three essays covering topics related to the term structure of interest rates, its relation to monetary policy and interest rate volatility. The term structure of interest rates plays an important role in shaping growth prospects for the real economy and contains relevant information for other asset prices. Appropriately modeling and understanding the behavior of interest rates and their volatilities is therefore of great interest.

Investors' perception of uncertainty about macroeconomic and policy fundamentals can play a significant role in determining asset price fluctuations (Baker, Bloom, and Davis (2013), Wright (2011), Cieslak and Povala (2015), Creal and Wu (2014), Ulrich (2012)). An important source of uncertainty regards the central banks' monetary policy stance, as indicated by the heightened anticipation ahead of statements from the central bank. With my first essay, I contribute to the existing literature by providing evidence in support of a link between uncertainty about the central bank's policy tool and interest rate dynamics. I measure monetary policy uncertainty as the cross-sectional dispersion in one-year-ahead forecasts of the federal funds rate from a large survey. I find that monetary policy uncertainty contributes to the variation in interest rate volatilities and it mainly affects short maturities. Due to the differential impact on long and short maturities, it has a slope effect on the volatility curve. Monetary policy uncertainty does not directly affect the cross-section of yields, however it explains part of the variation in risk premia over short horizons. While this may seem puzzling at first, it can be explained by the opposite effects on risk premia and expected future short rates. The paper also contributes to the literature on stochastic volatility affine term structure models. While existing literature has mostly focused on the ability of affine models to fit second moments of yields, without providing insights on the fundamental economic risk factors, this paper assumes an observable and economically interpretable driving factor for volatility. Furthermore the paper contributes to the discussion on whether volatility risk is spanned by the yield curve.

The second essay analyzes the properties of regime-switching affine term structure models with stochastic volatility. Economic theory suggests that monetary policy

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does not only affect the short end but the entire yield curve, since movements in the short rate affect longer maturity yields by altering investor expectations of future bond prices. From an economic perspective, it is hence intuitively appealing to allow the yield curve to depend on different macro-economic regimes. There is a large literature suggesting that interest rates are better described by a regime-switching process (Hamilton (1988), Gray (1996), Garcia and Perron (1996), Ang and Bekaert (2002a), Ang and Bekaert (2002b), Naik and Lee (1997), Evans (1998), Landén (2000), Bansal and Zhou (2002)). In the recent years more sophisticated regime-switching models in an affine term structure framework have been developed (Ang, Bekaert, and Wei (2007) and Dai, Singleton, and Yang (2007)), however due to the increased complexity in terms of bond pricing and estimation most of the literature has focused on Gaussian models. With this paper we contribute to the existing literature by analyzing the whole class of maximally-affine regime-switching term structure models with and without stochastic volatility. We evaluate their relative performance in terms of goodness-of-fit to historical yields as well as in terms of replicating some of the stylized facts of observed U.S. yield data. Our results provide some evidence that regime-switching stochastic volatility models are better equipped for fitting historical yield dynamics compared to the regime-switching Gaussian model as well as to the single regime models.

In the third essay I measure and study the behavior of variance risk premia in the interest rate swap markets. Swaption implied volatilities are on average higher than expected realized volatilities, embedding a premium associated with interest rate volatility risk. The literature on equity variance risk premia is very large, while that on fixed-income variance risk premia is small but growing quickly (Fornari (2010), Mele and Obayashi (2013), Choi, Mueller, and Vedolin (2015), Mueller, Vedolin, and Zhou (2011), Mele, Obayashi, and Shalen (2015)). I contribute to the existing literature by analyzing the properties of variance risk premia in different interest rate environments. I study the time series and cross-sectional features of variance risk premia and try to pin down the main variables that drive its variation over time. Higher moments of the distribution of interest rates can play an important role for variance risk premia. Given prospects for a protracted near zero interest rate environment, the last two decades can help shed a light on their behavior in various interest rate regimes. In particular, in a near zero rate environment the

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distribution of swap rates follows a lognormal rather than a normal distribution. Assuming non-negative rates (assumption that has been challenged in recent years), when rates are near zero, a larger probability is associated with an interest rate increase than it would if rates were normally distributed. This right fat tail in the distribution can have important consequences for variance risk premia. The results presented in the paper show that variance risk premia are negative, time-varying and economically significant. They tend to rise in absolute terms, in periods of market turmoil, where uncertainty about the economy and/or investor risk aversion is high. The results suggest that the term structure of variance risk premia differs in a low rate environment compared to normal times. Furthermore the frequency and severity of episodes where the premium switches sign is larger in the low rate regime.

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# Essay 1

## Monetary Policy Uncertainty and Interest Rates

Desi Volker <sup>1</sup>

### Abstract

This paper studies the effect of the uncertainty surrounding future monetary policy on interest rates across bond maturities. I measure uncertainty as the cross-sectional dispersion in one-year-ahead fed funds rate forecasts from survey data. Within a flexible dynamic term structure model with observable and latent factors, I provide evidence of a link between uncertainty and interest rate dynamics. I show that monetary policy uncertainty (i) is an important contributor to the variation in conditional yield volatilities; (ii) has a slope effect on the volatility term structure; (iii) is a priced risk that affects expected excess returns; (iv) is almost unspanned by the cross-section of yields; (v) has potential policy implications.

**JEL Classification:** G12, E43, E52

**Keywords:** Monetary Policy Uncertainty, Interest Rate Volatility, Affine Term Structure Model, MCMC

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## 1.1 Introduction

Monetary policy actions are an important driver of interest rate variation. By controlling the federal funds rate, the Federal Reserve can affect short term interest rates, and by influencing market expectations about future short rates it can affect rates at longer maturities. While the importance of expectations of policy variables has been documented extensively, the uncertainty surrounding these expectations can also have significant implications for interest rate dynamics. Consider for example reactions to announcements by Fed Chairwoman Janet Yellen that the Fed “could” raise rates in 2015. Uncertainty regarding the timing and pace of potential interest rate raises has contributed to large swings in bond prices, with large sell-offs in Treasuries and a rally in credit markets during the past months, raising renewed fears of a “taper-tantrum” episode similar to that of June 2013.<sup>2</sup>

This paper analyzes the effect of monetary policy uncertainty on the term structure of interest rates, the variation in conditional yield volatilities and bond risk premia. I proxy monetary policy uncertainty with the cross-sectional dispersion in one-year-ahead forecasts of the federal funds rate from a large survey dataset and find that it plays a significant role up to the horizon the agents are forecasting. The results show that monetary policy uncertainty: (i) contributes to the variation in the second moments of yields (ii) has a slope effect on the volatility curve; (iii) is a priced source of risk and explains part of the variation in expected excess returns at short horizons; (iv) is almost unspanned by the cross-section of yields; (v) has potential policy implications.

I identify the importance of monetary policy uncertainty on interest rate fluctuations both through regression analysis and in the context of a dynamic model. A no-arbitrage affine term structure model provides a theoretical framework for describing the joint behavior of interest rates at all maturities and allows for a quantitative analysis of their response to a monetary policy uncertainty shock. I consider a stochastic volatility term structure model within the maximally-affine class of [Dai and Singleton \(2000\)](#), with latent risk factors and a noisy version of the observable

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<sup>2</sup>See [Yellen \(2015\)](#) and <http://www.federalreserve.gov/FOMCpresconf>. For coverage in the financial press see: [ft.com/uncertainty](#), [economist.com/uncertainty](#), [ft.com](#), [bloomberg.com/yellen-tames-bond-traders](#), [brookings.edu/yellen/uncertainty](#). The “taper-tantrum” episode refers to bond market reactions to an announcement by Chairman Bernanke in the summer of 2013 that the Fed was considering turning off one of their QE programs conditional on continuing good economic news, which resulted in a dramatic increase in bond yields and a surge in their volatility.

policy variable. I examine how the contribution of the latent factors to interest rate conditional volatility and risk premia changes when the observable policy variable is introduced. I assume a flexible specification for the market prices of risk that allows for time varying and state-dependent risk premia and priced yield and uncertainty risks. Finally, the model allows for a two-way feedback between the observable policy variable and interest rates.

Regression analysis shows that monetary policy uncertainty has significant explanatory power for conditional volatilities, with t-statistics ranging from 6 to 9 for maturities up to 2 years and adjusted  $R^2$ 's ranging from 7% to 18%. The effect slowly dissipates for longer maturities implying that monetary policy uncertainty has a slope effect on the term structure of interest rate volatilities. Regressing the principal components of volatility on monetary policy uncertainty confirms the strong slope effect. The statistical significance and economic magnitudes of the coefficients persist when controlling for yield curve factors. The level, slope and curvature of the yield curve account for only 40% of the variation in monetary policy uncertainty. The explanatory power of monetary policy uncertainty for conditional volatility is robust to the addition of a number of relevant variables as controls in the regression, such as other uncertainty measures, credit spreads, swap spreads, as well as the volatility of inflation and real activity.

In the model, a one standard deviation shock to monetary policy uncertainty is associated with an increase of three quarters of a standard deviation in the conditional volatility of the three month rate. The estimated response of volatility to uncertainty shocks then slowly declines with maturity, with the effect on the 10 year rate dropping to one sixth of a standard deviation. Compared to the benchmark yields-only model  $A_1(3)$  of [Dai and Singleton \(2000\)](#), the performance of the model with uncertainty as a risk factor in fitting proxies of true conditional volatility improves significantly for short maturities.<sup>3</sup> In particular, the correlations of model-implied conditional volatility and EGARCH(1,1) estimated volatility improve from 13% to 35% for the 6 months rate and from 14% to 33% for the 1 year rate. Similar results are found for monthly realized volatility, where for example the correlations for the 6 month rate improve from 5% to 20%.

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<sup>3</sup>Two standard proxies of “true” conditional volatility in the literature are EGARCH(1,1) estimates and realized volatility, computed as the sum of squared daily yield changes ([Collin-Dufresne and Goldstein, 2002](#); [Jacobs and Karoui, 2009](#); [Andersen and Benzoni, 2010](#))

Furthermore, including monetary policy uncertainty as a risk factor in the model helps to match the well-documented “snake-shape” at the short end of the unconditional volatility curve (Piazzesi, 2005).<sup>4</sup> While yields-only models are able to match the hump in unconditional volatilities, they display difficulty in matching the head of the snake. The results suggest that the very short end of the volatility curve can be pinned down not only by the information contained in observed policy instruments but also by forward looking policy-related variables.

Looking at model-implied risk premia, I find that monetary policy uncertainty puts a downward pressure on the compensation investors require for holding long term bonds. A one standard deviation shock in monetary policy uncertainty is associated with a 1.3% decrease in the instantaneous annualized excess return of the 5 year bond. This is substantial given that average instantaneous excess returns for the 5 year bond are 3.3%. In the model, the channel through which monetary policy uncertainty affects risk premia is the state-dependence of the market price of risk. Monetary policy uncertainty, being mainly a volatility-driving risk factor, demands a negative risk premium. The compensation for bearing interest rate volatility risk is typically negative, since high volatility states tend to coincide with bad states of the nature where the marginal utility of income/consumption is high.<sup>5</sup> Analyzing the model’s ability to predict holding period excess returns, I find that correlations of model-implied expected excess returns with observed expected excess returns at short horizons and short maturities improve compared to the benchmark yields-only model. For example for the 3-month holding period return of the 6 month rate, correlations improve from 26% to 32% and modified  $R^2$ -statistics from 7% to 10%. It is important to note that a risk factor can affect risk premia and at the same time have no effect on current interest rates. This can be achieved if the factor has opposite effects on expected future interest rates and risk premia (see Duffee (2011)). Finally, monetary policy uncertainty behaves as a weakly spanned volatility factor within the model framework, having a negligible effect on the cross-section of yields. A one standard deviation shock to monetary policy uncertainty results in a meager

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<sup>4</sup>Piazzesi (2005) shows that the standard deviation of yield changes as a function of maturity displays a “snake-shape”, with volatility being high for very short maturities, decreasing around the maturity of 3 months, increasing again up to the 2 year maturity and decreasing thereafter.

<sup>5</sup>The story is similar to that in the equity variance risk premia literature, where assets whose payoffs covary positively with volatility command a negative risk premium in equilibrium (Carr and Wu, 2009). The results are in line with Joslin (2007), who finds that volatility, incrementally to level, slope and curvature, is an important determinant of expected excess returns.

3.5 basis point impact on the three month rate and a 1 basis point impact on the 10 year rate. Without imposing any parameter restrictions, I obtain a similar effect to that in the Unspanned Stochastic Volatility (USV) class of models of [Collin-Dufresne and Goldstein \(2002\)](#), where by construction, the risk factor driving volatility does not affect the contemporaneous term structure. Monetary policy uncertainty can therefore be interpreted as a risk factor that helps to capture the short end of the volatility curve.

This paper's results on the implications of monetary policy uncertainty for short rate volatility and risk premia are of relevance from a policy perspective. In particular, evidence that monetary policy can affect interest rate volatility through the uncertainty channel is important in light of the recent focus on financial stability as a policy objective. Short rate volatility is a fundamental variable for market participants investment decisions, affecting portfolio selection, risk management and derivatives pricing. Furthermore, since policy makers are ultimately interested in the monetary policy transmission mechanism, understanding the impact uncertainty has on risk premia is critical. The results point to the importance of forward guidance as a vital policy tool for anchoring investor expectations of future interest rates and containing the uncertainty that inevitably surrounds them.<sup>6</sup>

### *Related Literature*

In recent years a growing number of studies has documented that uncertainty plays a significant role on asset price fluctuations.<sup>7</sup> [Baker, Bloom, and Davis \(2013\)](#), for example, find that their measure of economic policy uncertainty can explain part of the variation in stock price volatility, while [Wright \(2011\)](#) shows that inflation uncertainty affects bond risk premia. An important source of uncertainty lies with the central bank's future policy actions. The heightened anticipation in financial markets ahead of statements and speeches made by central bankers testifies to this. This paper contributes to the literature by providing evidence in support of a link between the uncertainty surrounding the future path of short rates and interest rate

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<sup>6</sup>Studies show that through clear and transparent communication strategies, monetary policy makers can, to some extent, be successful in this task ([Williams, 2011](#); [Woodford, 2012](#); [Bauer, 2012](#)).

<sup>7</sup>([David and Veronesi, 2004](#); [Bekaert, Engstrom, and Xing, 2009](#); [Bloom, 2009, 2013](#); [David and Veronesi, 2013](#); [Jurado, Ludvigson, and Ng, 2013](#)).

dynamics. Uncertainty about future short rates is a distinct source of risk and displays significant differences with both the economic policy uncertainty index and inflation uncertainty.<sup>8</sup>

The paper is more closely related to recent work finding a link between uncertainty and interest rate volatility. [Cieslak and Povala \(2015\)](#) estimate jointly yields, their realized covariances and implied volatilities and find that their model implied short-end volatility factor comoves with monetary policy uncertainty. [Creal and Wu \(2014\)](#) introduce a discrete-time affine term structure model with unpriced stochastic volatility and interpret one of their volatility factors as a policy uncertainty factor. This paper differs from these studies in that I examine the role of monetary policy uncertainty on the yield curve directly by using an exogenous measure of uncertainty as a risk factor in the estimation. In line with their interpretation I find that monetary policy uncertainty affects mainly the short end of the volatility curve. Another related paper is [Ulrich \(2012\)](#), who in a general equilibrium context finds that overall uncertainty, combined macroeconomic and inflation uncertainty, can be an important driver of interest rate volatility. In contrast to his analysis, I focus on the specific role of the uncertainty surrounding the future path of monetary policy, as captured by the cross-sectional dispersion in forecasts of the Federal Reserve's policy instrument, the federal funds rate.<sup>9</sup>

More broadly the paper is related to the literature on stochastic volatility affine term structure models. The literature has mostly focused on the ability of affine models to fit second moments of yield dynamics, without providing insights on the fundamental economic risk factors. This paper contributes to the literature by providing an observable and economically interpretable driving factor for volatility. Furthermore the paper contributes to the discussion on whether volatility risk is spanned by the yield curve. [Collin-Dufresne and Goldstein \(2002\)](#) and [Collin-Dufresne, Goldstein, and Jones \(2009\)](#) present evidence that bonds do not hedge volatility risk and argue for a new class of models with unspanned stochastic volatility (USV). Looking at daily realized volatilities from high-frequency data [Andersen and Benzoni \(2010\)](#) provide

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<sup>8</sup>For an exposition of the time-series of inflation uncertainty and short rate uncertainty see Figure 1.8 in the Appendix. The economic policy uncertainty index captures only a small part of the variation in short rate uncertainty, with a correlation of 13%.

<sup>9</sup>Since the cross-sectional forecast dispersion has been used empirically both as a measure of uncertainty and more properly of disagreement, the paper is also related to the literature on disagreement and the yield curve ([Buraschi and Whelan \(2012\)](#), [Buraschi, Carnelli, and Whelan \(2013\)](#)).

further supporting evidence for unspanned volatility risk.<sup>10</sup> [Thompson \(2008\)](#) and [Joslin \(2007\)](#) show that the knife-edge restrictions in USV models are rejected in the data. I find evidence in support of a weakly spanned volatility factor, however I do not impose USV restrictions.

Shedding a positive light on the ability of latent factor affine term structure models to fit volatilities, [Jacobs and Karoui \(2009\)](#) find that the poor results found in other studies are due to their specific sample and that correlations of model-implied volatility with EGARCH volatility can be as high as 75%. The results in this paper show correlations of similar magnitudes for mid/long maturity rates. However, I document that the very short end of the volatility curve is not well captured by standard affine models. Introducing monetary policy uncertainty improves the fit to the short end considerably, nevertheless correlations remain low compared to those at longer maturities.

The paper is also closely related to [Joslin \(2007\)](#), who shows that a latent volatility risk factor can be weakly spanned by the yield curve if it does not affect risk-neutral expectations of future interest rates. He finds furthermore that such volatility factor can explain part of the variation in risk premia. I find that monetary policy uncertainty behaves in a similar way, having important effects on the physical-dynamics of bond yields, i.e. on volatility and risk premia, but not on their risk-neutral dynamics. Other related literature examines the role of observable variables in improving the ability of standard-affine models to fit the time series of yields. Following [Ang and Piazzesi \(2003\)](#), several papers have explored the importance of the information content in observable variables in capturing the variation in interest rates. A number of studies incorporate monetary policy variables in dynamic term structure models. [Kim and Orphanides \(2005\)](#) and [Chun \(2010\)](#) point to the importance of expectations of policy variables in improving the fit of the models to observed data, while [Piazzesi \(2005\)](#) finds that introducing observable monetary policy-related variables, such as the fed funds target rate, helps to pin down both first and second moments of the short end of the yield curve.

Finally the paper is related to the literature on unspanned risks. [Joslin, Priebsch, and Singleton \(2014\)](#), introduce a model with latent and macroeconomic variables,

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<sup>10</sup>[Bikbov and Chernov \(2004\)](#) point to the importance of options data in identifying stochastic volatility factors, while [Thompson \(2008\)](#) finds that affine models miss-specify the conditional variance of short maturity rates

where the macro variables play an important role in the time-variation of risk premia but do not affect the cross-section of yields (“unspanned risks”). Similarly, [Duffee \(2011\)](#) develops a model where, due to parameter restrictions under the  $\mathbb{Q}$ -measure, one of the latent risk factors does not affect the cross-section of yields but only yield dynamics. This paper relates to the macro-finance literature with unspanned risks, in that my observable policy variable affects risk premia while being only weakly spanned by the yield curve. I do not impose, however, the knife-edge restrictions that break spanning.<sup>11</sup>

The remainder of the paper is organized as follows. Section [1.3](#) presents some stylized facts about the data and explores the explanatory power of monetary policy uncertainty for yield volatilities. Section [1.4](#) introduces the model specification and describes the estimation methodology, while Section [1.5](#) presents the model results. Finally, section [1.6](#) provides concluding remarks.

## 1.2 Interest rate volatility

### 1.2.1 Data description

The data used in this paper comprises US interest rates, survey data and a number of macroeconomic and financial variables used as controls. I use US Treasury yield data sampled at a daily frequency for the maturities of 3, 6 and 9 months as well as 1 to 5, 7 and 10 years, covering the period January 1988 to April 2011. The data is taken from [Gürkaynak, Sack, and Wright \(2007\)](#). To obtain data at the monthly frequency I use end of month values.

Secondly, I use data from the Blue Chip Financial Forecasts survey. The survey is conducted monthly and asks a panel of around 50 professional economists at leading financial institutions to provide their forecasts on a number of financial and macroeconomic variables at various horizons. See [Chun \(2010\)](#) for more details on the survey data and the construction of constant maturity forecast horizons. My variable of interest is the federal funds rate at the one-year-ahead forecast horizon. In particular, I take the cross-sectional standard deviation of the forecast panel data each month as

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<sup>11</sup>[Bauer and Rudebusch \(2015\)](#) find that these restrictions are rejected in the data and document that the presence of small measurement errors in observed yields is sufficient to break the exact theoretical spanning condition in model-implied yields.



a measure of monthly uncertainty at the one-year ahead horizon. The forecast data is available for the sample January 1988 to April 2011.

Lastly I use a number of macroeconomic and financial variables, the CBOE volatility index VIX, Moody’s Seasoned Baa corporate bond yield spread, the Consumer Price Index from the St. Louis Fed economic data (FRED), the Chicago Fed National Activity Index (CFNAI) and the Economic Policy Uncertainty index of [Baker, Bloom, and Davis \(2013\)](#).

### *1.2.2 Stylized facts about interest rate volatility*

Volatility is not an observable variable and to proxy for it I use two standard measures, EGARCH(1,1) estimates and realized volatility.<sup>12</sup> EGARCH(1,1) estimates are computed on yield changes, while monthly realized volatility is measured as the sum of squared daily yield changes. Summary statistics and plots of monthly conditional yield volatilities are reported in the Appendix. The time series of conditional yield volatilities across maturities highlight the following features. Firstly, yield conditional volatilities display considerable time-variation and the most pronounced spike in the sample coincides with the recent financial crisis episode of 2007. Secondly, conditional volatilities are highly correlated across maturities. Thirdly, there is a clear multi-factor structure dynamics for the term structure of conditional yield volatilities with yields at shorter maturities behaving quite differently from those at longer maturities. Shorter maturity rates have a lower level of conditional volatility and a higher volatility of volatility. Longer maturity rates, of two to ten years, are highly covarying and display very similar features in terms of average estimated conditional volatilities and volatilities of volatility. Furthermore, mid maturities display negligible positive skewness and have a kurtosis of less than three while short maturities (up to one year) and the longest maturity in the sample (the ten-years bond) display longer right tails and are highly leptokurtic, with high peaks and fat tails. Conditional yield volatilities are very highly correlated for consequent maturities, however they fall significantly for maturities further apart.

Figure 1.1 displays the term structure of unconditional volatilities (top panel) and the term structure of the volatility of volatility (bottom panel) for EGARCH(1,1)

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<sup>12</sup>The results in this paper are robust to using a GARCH specification for volatility instead.

estimates and realized volatility.

[Insert Figure 1.1 here.]

The term structure of unconditional volatilities displays the typical “snake-shape” documented by [Piazzesi \(2005\)](#), with the volatility of the 3 month rate being high, declining around the 6 month maturity, increasing at mid maturities and declining again thereafter with a hump around the 3 year rate. The very short end of the volatility curve seems therefore to be driven by a different factor. Principal component analysis confirms that at least two factors are necessary for capturing the whole volatility curve (results shown in the Appendix). The first two factors capture more than 90% of the variation in the cross-section of yield volatilities. Given this evidence any term structure model aiming to fit second moments of yields should allow for multiple volatility risk factors.

The clear positive correlation between interest rate volatilities and the level of interest rates observed before the 90’s, breaks down during the sample considered here, with correlations switching sign from positive to negative around the 2 year maturity. [David and Veronesi \(2013\)](#) show that the correlation between Treasury bond volatilities and yields is time-varying and can switch sign, with the variation being driven by changes in market participants’ beliefs about the state of the economy and the monetary policy stance.

In reduced form dynamic term structure models with stochastic volatility, the set of factors that drives the cross-section of yields, drives also the second moments of yields. In one factor models, by definition, the short rate and it’s volatility are perfectly correlated. In multi-factor models, there is a trade off in contemporaneously fitting the cross-section of yields and yield volatilities. In estimation the cross-section is given more weight and the latent volatility factor is typically not easily identified. In the  $A_1(3)$  model of the maximally affine class of [Dai and Singleton \(2000\)](#) for example, the latent factor entering the conditional volatility of the state variables, and hence of the yields, is highly correlated with the first principal component of yields. In order for the model to fit volatilities well, conditional volatilities across different maturities must be highly correlated with the level of the yield curve. While this is the case for mid maturities in the data, it clearly doesn’t hold for the short

end of the curve. While the contemporaneous yield curve can explain most of the variation in interest rate volatilities at mid maturities, with adjusted  $R^2$  ranging from 30% to 60%, it displays difficulties fitting the short end (results are reported in the Appendix).

One or more additional factors are necessary for capturing the dynamics of short term yield volatilities. To understand exactly how many additional factors over the yield curve level, slope and curvature, are needed to fit yield volatilities it is indicative to look at the amount of residual variation in volatilities captured by the residual's PC's. One additional factor explains the vast majority (80%) of the residual variation in yield volatilities, suggesting that a model with four risk factors (the level, slope, curvature and one additional factor) is a good starting point.

In the context of latent factor models, there is significant evidence that in order to correctly identify volatility, a time-series econometric approach should be taken. [Thompson \(2008\)](#) shows that when volatility is backed out from the cross-section of yields as it is when models are estimated with standard techniques (i.e. maximum likelihood), affine models do not pass a series of specification tests. This is mainly driven by the inability of these models to capture the variation in the short end of the volatility curve.

One approach for a time-series identification strategy is that taken by [Collin-Dufresne and Goldstein \(2002\)](#) and [Collin-Dufresne, Goldstein, and Jones \(2009\)](#), where parameter restrictions under the risk neutral measure are introduced to insure that the volatility factor does not affect the cross-section of yields. There is some evidence however that the knife-edge restrictions enforced in these models are rejected in the data (see e.g. [Thompson \(2008\)](#), [Joslin \(2007\)](#)).

Another approach is that of introducing in the estimation information from volatility sensitive instruments such as interest rate options, caps and floors (see e.g. [Bikbov and Chernov \(2004\)](#), [Almeida, Graveline, and Joslin \(2011\)](#)). While models that include interest rate options in the estimation outperform standard models in fitting the variation in conditional volatilities, they suffer from the same lack of interpretability of the factors by not providing any insight into the fundamental economic drivers of volatility.

Pinning down one of the volatility factors to an observable and economically justified variable, can be an alternative avenue that mitigates the identification problem

and allows for a clear economic interpretation of the driving underlying risk factor. Given the inability of the contemporaneous yield curve to capture the variation in conditional volatilities at short maturities, the question then follows, what observable variable can contain relevant information for the short end of the volatility curve? The evidence in Piazzesi (2005) points to policy-related variables.

### 1.3 Monetary policy uncertainty and interest rate volatility

In what follows, I explore whether uncertainty about the future path of monetary policy contains useful information about interest rate volatilities, in excess of the yield curve level, slope and curvature.

Monetary policy actions are to a large degree reflected in the movements of the Fed's main policy instrument, the federal funds rate. Expectations of monetary policy stance are not directly observable, however they can be inferred either from market prices of traded assets, or obtained by surveying professional forecasters. Traded fed funds futures and options on these futures reflect respectively market expectations about future interest rates and the market uncertainty surrounding these expectations. Options on federal funds futures have been traded since a relatively short period, therefore I rely on survey forecasts to gauge monetary policy uncertainty. I use the cross-sectional standard deviation of federal funds rate forecasts. The cross-sectional standard deviation of survey forecasts more accurately reflects the disagreement among forecasters about their expectations, however it is widely used in the literature as a measure of uncertainty (Cukierman and Wachtel (1979), Kim and Orphanides (2005), Wright (2011), Christensen and Kwan (2014)). This assumption is inconsequential to the extent that disagreement can capture the aggregate level of individual forecasters' uncertainty. A number of studies suggest that this is the case and that measures of forecaster disagreement are highly correlated to measures of uncertainty (Rich, Raymond, and Butler (1992), Batchelor and Dua (1996), Giordani and Söderlind (2003))<sup>13</sup>.

Summary statistics and time series of the cross-sectional mean of one year ahead forecasts of the federal funds rate, as well as their cross-sectional standard deviation are reported in the Appendix. Monetary policy uncertainty displays considerable

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<sup>13</sup>However the evidence is mixed (Lahiri and Teigland (1987), D'Amico and Orphanides (2008), Rich and Tracy (2010))

time-variation. In the period between 1988 and 1990 it declined sharply, tracking the fall in interest rates and reflecting the decrease in inflation expectations and increased credibility of the Fed following the Volcker disinflation era. From the 90's onwards the strong link with the level of interest rates becomes significantly less pronounced. Figure 1.2 plots the one month ahead conditional yield volatility for the one year rate along with monetary policy uncertainty, as measured by the cross-sectional standard deviation of fed funds rate forecasts.

[Insert Figure 1.2 here.]

Monetary policy uncertainty closely tracks the variation in conditional yield volatilities with an unconditional correlation of 0.43. Table 1.1 shows that the level, slope and curvature of the yield curve explain circa 40% of the variation in monetary policy uncertainty. A significant portion of the information contained in uncertainty is therefore not captured by the yield curve and could potentially be useful for fitting volatilities. In contrast most (almost 97%) of the information contained in the expectations about the fed funds rates is already captured by the term structure of interest rates. I proceed to explore the explanatory power of monetary policy uncertainty for conditional yield volatilities across maturities.

[Insert Table 1.1 and 1.2 here.]

Panel A in table 1.2, shows that monetary policy uncertainty is a significant driver of conditional yield volatilities at short maturities, with t-stats ranging from 6 to 9 and  $R^2$ 's from 7.2% to 18.3% for maturities up to 2 years. The significance dissipates for longer maturities. Panel B in table 1.2 shows that the significance persists for conditional volatilities at short maturities also after controlling for the level, slope and curvature factors, with t-stats ranging from 2.41 to 4.95. Table 1.3 panel A, shows regression results of the principal components of conditional volatilities on monetary policy uncertainty and the yield curve factors.

[Insert Table 1.3 here.]

Monetary policy uncertainty has a slope effect on the term structure of volatilities, it drives up volatilities at the short end while it does not affect longer maturities. Furthermore monetary policy uncertainty has a curvature effect on the volatility curve, with the effect at mid maturities being milder. The significance still holds when a number of other additional controls are included in the regression, such as the 10 year swap spread, estimated volatilities of inflation and real activity, Moody's BAA credit spread on 10 year corporate bonds, the policy uncertainty measure of [Baker, Bloom, and Davis \(2013\)](#) and VIX. Similar results are obtained when realized monthly volatility is considered instead.

Furthermore, monetary policy uncertainty has significant explanatory power for swap-  
 tion implied volatilities for short maturity rates attesting that it impacts not only the volatility dynamics under the physical measure but also risk-neutral expectations of volatility (results not reported for brevity). The results suggest that monetary policy is a significant driver of conditional volatilities at the short end of the curve and contains information not spanned by the yield curve.

For a more comprehensive analysis of the impact of monetary policy uncertainty on interest rates I estimate a dynamic term structure model with latent variables and monetary policy uncertainty as risk factors. A no-arbitrage framework provides a consistent description of the joint dynamics of interest rates across maturities. By introducing monetary policy uncertainty as an additional source of risk I can assess its contribution to the time-variation in conditional yield volatilities in excess of the yield curve factors. Furthermore, by specifying the dynamics of the yields both under the historical and the risk-neutral measure, it is possible to assess the impact of monetary policy uncertainty on the compensation that investors require for holding interest rate risk.

#### 1.4 Model with monetary policy uncertainty as a risk factor

I consider a 4-factor model within the maximally affine class of [Dai and Singleton \(2000\)](#) with latent and observable factors.<sup>14</sup> The short rate is given as an affine

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<sup>14</sup>I denote the model  $A_2^M(4)$ , reflecting the fact that it belongs to the maximally affine class, has four factors in total (three latent and one observable) and allows for two of the factors to drive volatility. The  $M$  superscript stands for the observable macro or policy variable.

function of a  $4 \times 1$  vector of state variables  $X_t$ :

$$r_t = \delta_0 + \delta'_X X_t, \quad (\text{A-1})$$

where  $\delta_0$  is a scalar and  $\delta_X \in \mathbb{R}^4$  a vector of loadings on the risk factors. The vector of state variables  $X_t = [V_t \ Z_t]'$  is composed of a factor  $V_t$  that is observed with noise and three latent variables  $Z_t$ . Under the risk-neutral measure the dynamics of the state variables follows a square root diffusion process:

$$\begin{bmatrix} dV_t \\ dZ_t \end{bmatrix} = \left( \begin{bmatrix} K_{0V}^{\mathbb{Q}} \\ K_{0Z}^{\mathbb{Q}} \end{bmatrix} - \begin{bmatrix} K_V^{\mathbb{Q}} & K_{VZ}^{\mathbb{Q}} \\ K_{ZV}^{\mathbb{Q}} & K_{ZZ}^{\mathbb{Q}} \end{bmatrix} \begin{bmatrix} V_t \\ Z_t \end{bmatrix} \right) dt + \Sigma_X \sqrt{S_{X,t}} dW_{X,t}^{\mathbb{Q}}, \quad (\text{A-2})$$

where  $W_{X,t}^{\mathbb{Q}}$  is a vector of independent Brownian motions,  $K_{0X}^{\mathbb{Q}} \in \mathbb{R}^4$ ,  $K_X^{\mathbb{Q}} \in \mathbb{R}^{4 \times 4}$ ,  $\Sigma_X \in \mathbb{R}^{4 \times 4}$  and the variance-covariance matrix  $S_{X,t}$  is diagonal with elements given by

$$[S_{X,t}]_{i,i} = \alpha_i + \beta'_i X_t. \quad (\text{A-3})$$

Parameter restrictions insuring no arbitrage and that the dynamics of the latent state variables is well defined are given in [Dai and Singleton \(2000\)](#). I do not impose any additional restrictions. This specification allows for closed form solutions for bond prices. [Duffie and Kan \(1996\)](#) show that modeled bond prices are an exponentially affine function of the state variables:

$$P(t, \tau) = e^{A^*(\tau) - B^*(\tau)' X_t}. \quad (\text{A-4})$$

where  $A^*$  is a  $\tau \times 1$  vector and  $B^*$  a  $\tau \times N$  matrix, with  $\tau$  denoting the vector of the selected yield maturities. The yield loadings on the state variables  $A^*(\tau)$  and  $B^*(\tau)$  solve the following ordinary differential equations

$$\frac{dA^*(\tau)}{d\tau} = -K_{0X}^{\mathbb{Q}} B^*(\tau) + \frac{1}{2} \sum_{i=1}^N \left[ \Sigma_X' B^*(\tau) \right]_i^2 \alpha_i - \delta_0, \quad (\text{A-5})$$

$$\frac{dB^*(\tau)}{d\tau} = -K_X^{\mathbb{Q}} B^*(\tau) - \frac{1}{2} \sum_{i=1}^N \left[ \Sigma_X' B^*(\tau) \right]_i^2 \beta_i + \delta_X \quad (\text{A-6})$$

with initial conditions  $A^*(0) = 0$  and  $B^*(0) = 0$ .

The continuously compounded yield for the bond with price  $P(t, \tau)$  is then given by

$$Y(t, \tau) = A(\tau) + B(\tau) X_t, \quad (\text{A-7})$$

with  $A(\tau) = -A^*(\tau)/\tau$  and  $B(\tau) = B^*(\tau)'/\tau$ .

The observable factor  $V_t$  is assumed to be observed with noise:

$$V_t^O = \hat{V}_t + \nu_t, \quad (\text{A-8})$$

where  $\nu_t \sim N(0, \sigma_{V_t}^2)$ . The presence of the noise implies that the observable factor is not fully spanned by bond yields. I allow for a two-way feedback, so that past values of the observable factor affect future values of both the latent variables and itself and conversely past values of the latent variables affect future values of the observable factor. The stochastic discount factor under these assumptions is given as follows:

$$\frac{dM_t}{M_t} = -r_t dt - \Lambda'_{X,t} dW_{X,t}^{\mathbb{P}}, \quad (\text{A-9})$$

where  $W_{X,t}^{\mathbb{P}}$  are Brownian motions under the physical measure  $\mathbb{P}$ . The market prices of risk follow the extended affine specification of [Cheridito, Filipović, and Kimmel \(2007\)](#):

$$\Lambda_{X,t} = \left( \sqrt{S_{X,t}} \right)^{-1} \left( \lambda_0 + \lambda'_X X_t \right), \quad (\text{A-10})$$

where  $\lambda_0$  is a  $N \times 1$  vector and  $\lambda_X$  is a  $N \times N$  matrix. This specification allows more flexibility for the market prices of risk of the factors affecting the volatility of yields (which in this case are  $V_t$  and  $Z_t^1$ ) without further restricting the market prices of risk of the remaining factors. The limitation comes from the parameter restrictions in  $S_{X,t}$ , which are there to insure that the volatility factors stay strictly positive and that arbitrage opportunities are precluded. With the assumed market prices of risk, under the physical measure the state variables follow the dynamics:

$$dX_t = \left\{ (K_{0X}^{\mathbb{Q}} + \lambda_0) - (K_X^{\mathbb{Q}} - \lambda_X) X_t \right\} dt + \Sigma_X \sqrt{S_{X,t}} dW_{X,t}^{\mathbb{P}}. \quad (\text{A-11})$$

I use three latent state variables  $Z_t$  to fit the cross section of yields, in line with the results from the vast existing empirical literature ([Litterman and Scheinkman, 1991](#)). Apart from the observable factor  $V_t$ , I allow only one of the latent state



variables to affect yield volatility (i.e.,  $S_{X,t}$  is a function of  $V_t$  and  $Z_t^1$  only), while the other factors are conditionally Gaussian. The choice is driven from the trade-off between fitting unconditional yield volatilities which heavily relies on the flexibility of the correlations among the risk factors and matching the conditional volatilities of yields.

Given the affine structure of the model the conditional variance of a yield with maturity  $\tau$  is an affine function of the latent and observable state variables:

$$\text{Var}_t(Y_{t+1}^\tau) = B^{\tau'} \text{Var}(X_{t+1}|X_t) B^\tau + \text{Var}(\epsilon_{Y,t+1}^\tau), \quad (\text{A-12})$$

where  $\epsilon_{Y,t+1}^\tau$  is the measurement error of the yield with maturity  $\tau$ .

#### 1.4.1 Bayesian Estimation with Markov Chain Monte Carlo

In estimating the model, I assume that all bond yields are observed with a measurement error:

$$y_t(\tau) = A(\tau) + B(\tau)X_t + \epsilon_{Y,t}(\tau) \quad \text{for } \tau \in \{1, \dots, n\}, \quad (\text{A-13})$$

where  $\epsilon_{Y,t}(\tau) \sim i.i.d. \mathcal{N}(0, \sigma_{Y,\tau}^2)$ . For simplicity, and without loss of generality I will assume that the observation errors for yields at all maturities have the same variance  $\sigma_{Y,\tau}^2 = \sigma_Y^2$ . I assume furthermore that the factor  $V_t$  is also observed with some measurement error

$$V_t^O = V_t + \epsilon_{V,t} \quad \text{with} \quad \epsilon_{V,t}(\tau) \sim \mathcal{N}(0, \sigma_V^2). \quad (\text{A-14})$$

I estimate the model using Markov Chain Monte Carlo (MCMC). The problem concerns extracting information regarding the  $N$  latent state variables  $\{X_t^j\}_{t=1, \dots, T}^{j=1, \dots, N}$  and the model parameters  $\Theta$  from the observable series of yields  $\{Y_t^\tau\}_{t=1, \dots, T}^{\tau=1, \dots, n}$  and the observable state variable  $\{V_t\}_{t=1, \dots, T}$ . The parameter space is given by:

$$\Theta = \{K_{0X}^{\mathbb{Q}}, K_x^{\mathbb{Q}}, \delta_0, \delta_X, \lambda_0, \lambda_X, \alpha, \beta, \sigma_Y^2, \sigma_V^2.\} \quad (\text{A-15})$$

The solution to the problem is summarized by the posterior distribution of the parameters and latent variables given the observed data  $p(\Theta, X|Y, V)$ . Characterizing the

joint posterior distribution however is complicated. By the Hammersley-Clifford theorem, sampling from the joint posterior distribution is equivalent to sampling from the complete set of conditional distributions  $p(\Theta|X, Y, V)$  and  $p(X|\Theta, Y, V)$ . The MCMC algorithm generates a sequence of random variables sampled from the conditional distributions. The sequence is a Markov Chain with a distribution converging to the target distribution. If the conditional distribution is known in closed form, I draw parameters using the Gibbs sampler, otherwise I use Metropolis-Hastings algorithms. Metropolis-Hastings is carried in two steps, firstly I sample a candidate draw from a proposal density and then I either accept or reject the draw based on a prespecified acceptance criteria. The marginal posterior mean from the Markov Chain for each parameter will then represent that parameter's posterior estimate. A detailed exposition of the conditional distributions is provided in the Appendix.

## 1.5 Model results

In this section I use the no-arbitrage model results to analyze the impact of monetary policy uncertainty on yield dynamics. In particular I first discuss the time-series implications of the parameter estimates for monetary policy uncertainty and its effect on yields. Then I turn to looking at the effect uncertainty has on conditional yield volatilities and analyze the model's ability to fit their variation compared to a standard benchmark model. I follow by examining the model's fit to unconditional volatilities and their ability to capture the snake-shape and hump documented in the data. Lastly I examine the impact of uncertainty on model implied risk premia and the model's ability to predict holding period expected excess returns.

### 1.5.1 *Parameter Estimates*

The model's parameter estimates and the its fit to the cross-section of yields are reported in the Appendix. The estimates are roughly consistent with the existing literature on multi factor affine models. The model fits observed yields well with root mean squared errors ranging from 2 to 5 basis points. Average pricing errors are in the order of less than one basis point. The persistence of the variables can be inferred from the eigenvalues of the speed of mean reversion matrix  $K_X$ . Monetary policy uncertainty turns out to be quickly mean reverting with a half life of shocks of

7 months. The other volatility factor  $X_{1,t}$  reverts to the mean very quickly as well, while the two conditionally Gaussian factors  $X_{2,t}$  and  $X_{3,t}$  are very persistent, with half life of shocks of 16 years and 6 years.

Standard interpretation of latent risk factors in the no-arbitrage framework has been to associate them with the level, slope and curvature of the yield curve ([Litterman and Scheinkman, 1991](#)). In order to verify whether this association holds in the estimated model, I look at the yield curve response to a one standard deviation shock in each of the state variables. The top panel in [Figure 1.3](#) shows the response to a one standard deviation shock in each of the two volatility factors, while the bottom panel that of a shock to the conditionally Gaussian factors.

[Insert [Figure 1.3](#) here.]

The latent factors follow approximately the standard interpretation. The two conditionally Gaussian factors  $X_{3,t}$  and  $X_{2,t}$ , have a level and slope effect respectively, while the second volatility factor  $X_{1,t}$  has a curvature effect, with a stronger impact on mid maturity yields. Interestingly, while I do not impose any parameter restrictions, monetary policy uncertainty seems to have a negligible effect on the yield curve. A one standard deviation shock in monetary policy uncertainty is associated on average with a 1.5 bps decrease in yields. Given that the magnitude of yields is in the order of %'s this is inconsequential, making monetary policy uncertainty behave as a weekly spanned risk factor. In the Unspanned Stochastic Volatility (USV) class of models of [Collin-Dufresne and Goldstein \(2002\)](#) and [Collin-Dufresne, Goldstein, and Jones \(2009\)](#) the two channels through which volatility affects long rates, short rate expectations and convexity, exactly offset each other. This is obtained through the introduction of stringent parameter restrictions on the speed of mean reversion matrix. Under such restrictions, one of the state variables affects only conditional yield volatilities without affecting the contemporaneous term structure. Monetary policy uncertainty in this model behaves in a similar way, affecting the physical world dynamics of yields but not their risk-neutral dynamics, however here the convexity and risk-neutral expectations effects do not exactly cancel each other out. These results are in line with [Joslin \(2007\)](#), who shows that a volatility risk factor can be weekly spanned by the yield curve, to the extent that it is largely uncorrelated with short

rate expectations.

### 1.5.2 Fit of conditional volatilities

The results from the regression analysis in section 1.3 suggest that monetary policy uncertainty has significant explanatory power for conditional yield volatilities at short maturities. Will these results be corroborated in the model framework? To answer this question I start by examining the impact of a shock to uncertainty regarding the future path of monetary policy on the term structure of conditional yield volatilities. I follow by assessing the model's performance in fitting short term conditional volatilities and the unconditional volatility curve relative to the best-performing yields-only model in the maximally affine class.

While there are several ways to compute model-implied conditional yield volatilities, I follow the recent literature and use the analytical expressions:<sup>15</sup>

$$\text{Var}_t(Y_{t+1}^\tau) = B^{\tau'} \text{Var}(X_{t+1}|X_t) B^{\tau'} + \text{Var}(\epsilon_{Y,t+1}^\tau),$$

where  $\epsilon_{Y,t+1}^\tau$  denotes the measurement error of the yield with maturity  $\tau$  and  $B^\tau = B^*(\tau)'/\tau$ . Figure 1.4 displays the response of conditional volatilities over the maturity spectrum to a one standard deviation shock in each of the risk factors .

[Insert Figure 1.4 here.]

Monetary policy uncertainty turns out to play a significant role in the variation of short term conditional volatilities. A one standard deviation shock in monetary policy uncertainty is associated with an increase of 2.1 bps per month or three quarters of a standard deviation in the conditional volatility of the three month rate. The six months and one year rate will increase by 1.9 bps per month or two thirds of a standard deviation and 1.5 bps per month or two fifths of a standard deviation respectively. The effect becomes increasingly smaller for longer maturities, with a one standard deviation shock in monetary policy uncertainty associated with an increase

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<sup>15</sup> A few other studies use instead the reprojection method of [Gallant and Tauchen \(1998\)](#). [Jacobs and Karoui \(2009\)](#) argue in favor of using the analytical expressions, since the reprojection technique can yield counterfactual implications for the homoschedastic model  $A_0(3)$

of 1 bps per month, or one sixth of a standard deviation in the conditional volatility of the 10 year rate.

A standard metric used in the literature for assessing the model’s ability to fit conditional volatilities is the correlation of model-implied volatility with proxies of true conditional volatility. I use EGARCH(1,1) estimates and realized monthly conditional volatility obtained from intra-month (daily) data as proxies for the “true” volatility. The fit of model-implied conditional volatilities and EGARCH(1,1) estimates of volatility is shown in the Appendix. The proposed  $A_2^M(4)$  model with monetary policy uncertainty as a risk factor, tracks fairly well the movements in mid maturity rates, however displays difficulties in fitting the large swings in volatility at shorter maturities as it appears to be too smooth. This is however to be expected as most models in the affine class fall short of matching the high volatility of volatility in short term rates. In order to assess whether the introduction of monetary policy uncertainty brings relevant information, in excess of that already contained in the contemporaneous yield curve, for capturing the variation in conditional volatilities I look at the model’s performance relative to the best performing yields-only model within the maximally affine class.<sup>16</sup>

[Insert Table 1.4 here.]

Table 1.4 shows regressions of EGARCH(1,1) estimates of conditional volatility on model implied volatility for the proposed model with monetary policy uncertainty as an observable risk factor (Panel A) and for the standard yields-only model  $A_1(3)$  of Dai and Singleton (2000) (Panel B). Correlations between model-implied conditional volatilities and EGARCH(1,1) estimates are significantly higher at short maturities for the model with monetary policy uncertainty compared to the yields-only model, increasing from 0.19, 0.13 and 0.14 to 0.31, 0.35 and 0.33 for maturities of 3 months, 6 months and 1 year respectively. For longer maturities the yields-only model performs better.

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<sup>16</sup>Dai and Singleton (2003), and Jacobs and Karoui (2009) find that this is the model with one volatility factor.

Table 1.5 shows the same results for realized volatility, with correlations improving significantly for short maturity bonds. The correlations increase from 0.01, 0.05 and 0.11 to 0.19, 0.20 and 0.21 for maturities of 3 months, 6 months and 1 year respectively. The model with monetary policy uncertainty continues to perform slightly better along all of the maturity spectrum.

[Insert Table 1.5 here.]

One may assume that the improvement is to be expected, given that we are comparing a model with four risk factors, two of which affect volatility to one with three risk factors, one of which affects volatility. This, however, is not the case. In the class of affine models with stochastic volatility, models with multiple volatility factors underperform the model with one volatility factor if additional information is not provided to identify volatility, such as prices of volatility-sensitive instruments like options, caps and floors. To test this I estimate a yields-only model with four factors, two of which drive volatility and I do not impose any additional parameter restrictions. The results confirm that it does worse in fitting volatilities than the yields-only  $A_1(3)$  model or the proposed model with monetary policy uncertainty (results are not shown here for brevity). Similarly, estimating the model with another economic variable instead of monetary policy uncertainty, such as inflation uncertainty or real activity uncertainty underperforms the model with monetary policy uncertainty in fitting conditional volatilities. Interestingly the improvement in the fit of conditional volatilities that comes from the inclusion of monetary policy uncertainty as a volatility risk factor is similar to that in models that jointly price bonds and options (see results in Almeida, Graveline, and Joslin (2011)). These models however, do not provide any economic intuition about the factors driving volatility. A model that aims to capture the variation in conditional volatilities across the maturity spectrum should necessarily display small correlations of conditional yield volatilities for maturities further apart. EGARCH volatilities and realized volatilities display similar patterns in the correlations across maturities. While the correlation is typically very high for maturities that are close by, it drops significantly for maturities further apart (results reported in the Appendix. The yields-only model  $A_1(3)$  has perfectly correlated conditional volatilities across maturities, due to the fact that it

allows for only one of the latent risk factors to drive volatility. The model with monetary policy uncertainty as a risk factor,  $A_2^M(4)$ , captures the declining correlations for maturities further apart, however it is not able to replicate the full extent of the large divergence that is observed in the data. Compared to yields-only models the introduction of monetary policy uncertainty helps to capture this feature without increasing the number of volatility factors. For example correlations are slightly smaller than those in the  $A_3(3)$  model, i.e. the model with all three of the latent variables driving volatility.<sup>17</sup> The  $A_3(3)$  model strongly underperforms in other dimensions, due to its constrained correlations among the risk factors.

Due to their affine structure, standard models with latent factors imply that the variation in conditional yield volatilities comes from the cross-section of yields, as represented by the level, the slope and the curvature factors. To gain further insight on why the model with monetary policy uncertainty captures better the short end of the volatility curve, I look at correlations between model-implied conditional volatilities for different maturities and the level, slope and curvature factors, and compare them to the correlations of EGARCH volatilities and realized volatilities with these factors. Table 1.6 reports correlations of conditional volatilities with the level, slope and curvature factors as well as with monetary policy uncertainty.

[Insert Table 1.6.]

EGARCH volatility and realized volatility display positive correlations with the level of interest rates at short maturities of up to 1 year and negative correlations for longer maturities.<sup>18</sup> The standard yields-only model  $A_1(3)$ , by construction, has constant correlations across maturities with any of the factors, since it allows for only one factor to drive volatility. It displays a pronounced negative correlation of  $-0.65$  with the level factor, which explains in part why this model falls short of capturing the short end of the volatility curve. The  $-0.65$  correlation is similar to that displayed by EGARCH at mid maturities, which is why its fit at mid and long maturities is

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<sup>17</sup>Jacobs and Karoui (2009) find that the  $A_3(3)$  model fits this feature best among the yields-only models in the maximally affine class.

<sup>18</sup>Cieslak and Povala (2015) find using high frequency data that the correlations of realized volatilities with the level factor display a hump.

quite good. The pattern of correlations of the model with monetary policy uncertainty,  $A_2^M(4)$ , with the level factor across maturities tracks much more closely the correlations observed for EGARCH and realized volatility. Turning to the slope and curvature factors, all measures of conditional volatility display positive correlations with them. Both the  $A_2^M(4)$  and the benchmark  $A_1(3)$  model display significantly higher correlations with the slope and curvature than that displayed by EGARCH and realized volatility. Lastly, I examine correlations of conditional volatility with monetary policy uncertainty. EGARCH volatility shows high correlations ranging from 0.27 to 0.43 for short maturities, implying that monetary policy uncertainty is an important driver of volatility at the short end. The results are similar for realized volatility. Model-implied conditional volatilities for the  $A_2^M(4)$  display very large correlations with monetary policy uncertainty at the short end of the curve, ranging from 0.6 to 0.9. This confirms that within the model, monetary policy uncertainty drives out the latent volatility factor in fitting short maturities. The  $A_1(3)$  model is also positively correlated with uncertainty, capturing the fraction of uncertainty that is spanned by the yield curve. Overall these results suggest that observable policy variables can have important informational content for short term conditional volatilities.

#### *Unconditional Volatilities*

Including monetary policy uncertainty as a risk factor in the model helps to match the snake-shape and hump of the volatility curve. Model-implied volatility curves can display a hump if the correlation among some of the state variables is negative or if the loadings on the state variables  $B^T$  are hump-shaped (Dai and Singleton, 2000, 2003). Figure 1.5 displays simulated unconditional volatility curves for the proposed macro-finance model and the benchmark yields-only model  $A_1(3)$ , along with two-standard deviation confidence bounds.

[Insert Figure 1.5 here.]

I have treated the converged MCMC parameter estimates as true population parameters and used them to simulate 1000 time-series of yields with the same length as that of the historical data. I have considered a larger number of maturities, in order



to explore the behavior of volatilities at the very short end, where I do not have observable data. The average standard deviation of monthly yield changes for the simulated yield series represents the simulated model-implied unconditional volatilities. The proposed model with monetary policy uncertainty as a volatility risk factor, captures the snake-shape documented by Piazzesi (2005) at short maturities of less than 6 months, while the benchmark  $A_1(3)$  model does not. This is in line with the finding in Piazzesi (2005) that policy variables can help to pin down the short end of the volatility curve.

### 1.5.3 Risk premia and predictability of excess returns

Having analyzed the impact of monetary policy uncertainty in fitting conditional yield volatilities I turn to examine whether it can play a role in explaining risk premia. Risk averse investors require a risk premium for holding long maturity bonds, in order to compensate them for the interest rate risk inherent in these securities. The required compensation will depend on both the perceived quantity of risk and the associated price of risk. The quantity of risk arguably reflects how volatile bond prices are expected to be and the uncertainty surrounding these expectations. Potential factors influencing it can be: uncertainty regarding the future path of monetary policy, uncertainty regarding future inflation rates, uncertainty surrounding expectations of future real activity and other macroeconomic fundamentals both at the country and global level among other factors. The market price of risk, determined by the degree of investor risk aversion, is influenced by a large number of factors such as, business cycles, liquidity considerations, behavioral biases etc.

In the model framework I assess the impact of monetary policy uncertainty on instantaneous excess returns, then turn to analyze the model's ability to predict holding period excess returns at the three month horizon. Given that monetary policy uncertainty is a quickly mean-reverting factor, I expect the effect to be mostly present at short holding period horizons. Due to the Markov structure of the model, the no-arbitrage condition gives the following dynamics for bond prices:

$$\frac{dP(t, \tau)}{P(t, \tau)} = (r_t + \eta_t^\tau)dt + V_t^\tau dWt, \quad (\text{A-16})$$

where

$$\eta_t^\tau = -B^*(\tau)' \Sigma S_{X,t}^{1/2} \times \Lambda_t \quad (\text{A-17})$$

denotes the instantaneous excess return on a  $\tau$ -maturity zero-coupon bond. The fact that monetary policy uncertainty plays a significant role in determining interest rate volatilities at short maturities, implies that it will affect risk premia through the quantity of risk channel. However, given the extended affine specification for the market price of risk, conditional volatilities  $S_{X,t}^{1/2}$  will affect risk premia also through the price of risk channel with an exactly offsetting magnitude. The variation in instantaneous risk premia will therefore come only through the state vector  $X_t$ , preserving the affine structure of the Gaussian case:

$$\eta_t^\tau = -B^*(\tau)' \Sigma (\lambda_0 + \lambda'_X X_t). \quad (\text{A-18})$$

The parameter estimates  $\lambda_0$  and  $\lambda_X$  indicate that the two conditionally Gaussian factors  $X_{2,t}$  and  $X_{3,t}$  have the largest impact on the market prices of risk. Monetary policy uncertainty is also a priced risk factor and has a significant impact on the market prices of risk of the two conditionally Gaussian factors. Figure 1.6 plots the average effect of a one standard deviation shock in each of the risk factors on instantaneous risk premia for different maturities.

[Insert Figure 1.6 here.]

A one standard deviation shock in monetary policy uncertainty is associated with a 1.3% decrease in the instantaneous excess return of the 5 year rate from an average of 3.3%. The other risk factors affect risk premia to a similar extent. While these effects can seem quite large, it is important to notice that the risk factors are correlated. Models with stochastic volatility have a smaller flexibility in fitting risk premia compared to the Gaussian case, since the matrix  $\lambda_X$  is constrained by the admissibility restrictions which impose that the state variables that drive volatility stay positive. There is a trade-off between fitting bond excess returns and volatilities, and the benefit of time-varying volatility comes at the cost of diminished flexibility in fitting risk

premia.<sup>19</sup> Having two volatility factors in the proposed model therefore implies less flexibility. Given that the purpose of the analysis conducted here is to examine the effect of monetary policy uncertainty on interest rates this is impertinent.

#### *Holding Period Excess Returns*

How well do model-implied risk premia predict excess log holding period returns for treasury yields? Observed  $w$ -horizon realized log holding period excess returns are given as the excess return from buying an  $n$ -year bond at time  $t$  and selling it as an  $n - w$  year bond at time  $t + w$ :

$$\begin{aligned} rx_{t,t+w}^n &= p_{t+w}^{n-w} - p_t^n - w y_t^w \\ &= -(n-w) y_{t+w}^{n-w} + n y_t^n - w y_t^w. \end{aligned} \tag{A-19}$$

Model-implied expected excess returns can be calculated as:

$$E_t[rx_{t,t+w}^n] = -(n-w) \left\{ A^{n-w} + B^{n-w} E_t[X_{t+w}] \right\} + n \left\{ A^n + B^n X_t \right\} - w \left\{ A^w + B^w X_t \right\},$$

where  $A^\tau = -A^*(\tau)/\tau$  and  $B^\tau = B^*(\tau)'/\tau$ , and the expectation of the state variables  $w$ -periods ahead is given by:

$$E_t[X_{t+w}] = (I_N - e^{-K^{\mathbb{P}} w}) \theta^{\mathbb{P}} + e^{-K^{\mathbb{P}} w} X_t,$$

with  $\theta^{\mathbb{P}} = (K_X^{\mathbb{Q}} - \lambda_X)^{-1} (K_{0X}^{\mathbb{Q}} + \lambda_0)$ . Figure 1.7 plots one-year holding period excess returns, model-implied expected excess returns from the model with monetary uncertainty as a risk factor  $A_2^M(4)$ , as well as the return predicting factor (CP) of Cochrane and Piazzesi (2005) computed with 5 and 3 forward rates. Model-implied expected excess returns follow closely the CP factor computed with 3 forward rates.

[Insert Figure 1.7 here.]

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<sup>19</sup>Recent advances in the literature have shown that this trade-off can be mitigated to some extent (Feldhütter, Heyerdahl-Larsen, and Illieditsch (2015)).

Following the literature, I use the following modified  $R^2$  statistic, to assess the model's goodness of fit to holding period excess returns:

$$R^2 = 1 - \frac{\text{mean}\left\{\left(rx_{t,t+w}^n - E_t[rx_{t,t+w}^n]\right)^2\right\}}{\text{var}(rx_{t,t+w}^n)}. \quad (\text{A-20})$$

Table 1.7 reports results from regressions of observed 3-month holding period excess returns on model-implied expected excess returns.

[Insert Table 1.7 here.]

Panel A displays results for the proposed model  $A_2^M(4)$  with monetary policy uncertainty as a risk factor, while Panel B those for the standard yields-only model  $A_1(3)$ . Panel C in Table 1.7 reports the fit of the CP return predicting factor computed from three forward rates  $f^{0 \rightarrow 1}$ ,  $f^{2 \rightarrow 3}$  and  $f^{4 \rightarrow 5}$ , as a baseline for the predictability assessment. I use only three forward rates, instead of five as in [Cochrane and Piazzesi \(2005\)](#), in order to mitigate the almost perfect multicollinearity that would arise. This is due to the fact that the data used in this analysis is that of [Gürkaynak, Sack, and Wright \(2007\)](#), computed using the [Svensson \(1994\)](#) method. [Cochrane and Piazzesi \(2005\)](#) use instead the unsmoothed Fama-Bliss yield data. They show that the use of smoothed vs. unsmoothed data, has implications for the ability of forward rates to forecast excess returns, as the removal of measurement errors comes at the cost of lessening the forecasting power.

The results show that modified  $R^2$  statistics and correlations improve at shorter maturities, when monetary policy uncertainty is included as a risk factor in the model. The fact that the improvement is mostly at short maturities and dissipates at longer ones is in line with the fact that monetary policy uncertainty is a quickly mean-reverting factor. The generally weak predictability observed for both the benchmark model and the proposed model comes from the sample in consideration as well as the data used. In particular due to the recent financial crisis episode also very powerful forecasting factors such as the CP factor display a reduced performance in predicting excess returns ([Sekkel, 2011](#)). Overall the evidence presented in this section suggests that monetary policy uncertainty is a priced risk factor that can help to predict holding period excess returns at short horizons and short maturities. Its small persistence

implies that the effect at longer holding period returns will decline with the horizon.

## 1.6 Conclusion

This paper investigates the role that monetary policy uncertainty plays in interest rate dynamics. In the framework of a stochastic volatility dynamic term structure model with latent and observable variables, I study the extent to which monetary policy uncertainty can explain the variation in interest rates, their conditional volatilities across the maturity spectrum and risk premia.

Monetary policy uncertainty has significant explanatory power for short-term conditional second moments of yields. The explanatory power declines with maturity implying that monetary policy uncertainty has a slope effect on the volatility curve. While I do not impose any parameter restrictions, I find that monetary policy uncertainty behaves as a weakly-spanned volatility factor, having a negligible effect on the cross-section of yields. Furthermore, monetary policy uncertainty demands a negative risk premium since it is a volatility risk factor and therefore is high in states of the world where the marginal utility of income is high. It is quickly mean-reverting and mainly affects short horizon bond risk premia. A risk factor can affect risk premia and yet have no effect on the cross-section of yields. This happens if the factor has opposing effects on short rate expectations and risk premia.

The paper contributes to the macro-finance literature by providing empirical evidence in support of a link between yield dynamics and a fundamental policy-related risk factor. Standard affine models, due to the latent nature of their assumed risk factors, do not provide insight on the type of the shocks that drive yield dynamics or their relation to fundamental economic variables. Understanding which underlying factors drive short term interest rate volatility is important, given its role in market participants investment and savings decisions. Risk premia on the other hand are important from a policy perspective, since they affect the monetary policy transmission from the very short to mid and long term rates.

The findings in this paper suggest that monetary policy can affect interest rates not only through the first moments of policy expectations but higher moments as well. They point to the importance of forward guidance as an effective policy tool in influencing investor expectations of future short rates and containing the perceived

uncertainty around future interest rates. More robust measures of uncertainty and at higher frequency can be useful for understanding movements in bond markets not explained by standard fundamentals.

## 1.7 Tables

**Table 1.1**  
**Regressions of Monetary Policy Expectations and Uncertainty on Yield PC's**

The table reports regression results of monetary policy expectations and uncertainty on the principal components of yields. This gives an indication of the fraction of the variation in expectations and uncertainty that is not spanned by the yield curve.  $t$ -statistics are shown in parenthesis, below the reported estimated coefficients. The standard errors are corrected for autocorrelation and heteroskedasticity using the [Hansen and Hodrick \(1983\)](#) GMM correction. The data covers the period January 1988 - April 2011. All variables are standardized.

|                              | Expectations      | Uncertainty    |
|------------------------------|-------------------|----------------|
| Level                        | 0.97<br>[82.25]   | 0.41<br>[7.14] |
| Slope                        | -0.12<br>[-12.82] | 0.33<br>[8.26] |
| Curv                         | 0.05<br>[4.66 ]   | 0.39<br>[7.54] |
| <b>Adj. <math>R^2</math></b> | 96.80             | 41.76          |

**Table 1.2**  
**Regressions of Conditional Volatilities on Monetary Policy Uncertainty**

The table reports results of regressions of EGARCH(1,1) estimates of conditional volatilities for maturities of 3, 6 and 9 months and 1 to 5, 7 and 10 years on monetary policy uncertainty ( $U^{MP}$ ) and on the first three principal components of yields (capturing the level, slope and curvature of the yield curve):

$$\sigma_{t,t+1}^2(\tau) = \alpha + \beta \times U_t^{MP} + \sum_{i=1}^3 \gamma_i \times PC_{i,t}^{yields} + \eta_t(\tau)$$

$t$ -statistics are shown in parenthesis, below the reported estimated coefficients. The standard errors are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. The data covers the period January 1988 to April 2011. All variables are standardized.

| Monetary Policy Uncertainty                |        |        |        |        |         |         |          |          |          |         |
|--|--------|--------|--------|--------|---------|---------|----------|----------|----------|---------|
|  | 3 m    | 6 m    | 9 m    | 1 y    | 2 y     | 3 y     | 4 y      | 5 y      | 7 y      | 10 y    |
| $U^{MP}$                                   | 0.27   | 0.38   | 0.41   | 0.43   | 0.35    | 0.17    | 0.01     | -0.08    | -0.16    | 0.13    |
|  | [6.03] | [8.71] | [9.16] | [9.46] | [5.87]  | [2.78]  | [0.11]   | [-1.38]  | [-2.91]  | [2.70]  |
| Adj. $R^2$                                 | 7.20   | 13.88  | 16.17  | 18.25  | 12.17   | 2.59    | -0.36    | 0.22     | 2.05     | 1.28    |
| Monetary Policy Uncertainty and Yield PC's |        |        |        |        |         |         |          |          |          |         |
|  | 3 m    | 6 m    | 9 m    | 1 y    | 2 y     | 3 y     | 4 y      | 5 y      | 7 y      | 10 y    |
| $U^{MP}$                                   | 0.13   | 0.19   | 0.22   | 0.24   | 0.15    | -0.01   | -0.09    | -0.11    | -0.16    | 0.13    |
|  | [2.41] | [3.82] | [4.52] | [4.95] | [2.91]  | [-0.11] | [-2.56]  | [-2.86]  | [-3.35]  | [2.31]  |
| Level                                      | 0.06   | 0.18   | 0.21   | 0.19   | -0.11   | -0.35   | -0.52    | -0.57    | -0.49    | -0.21   |
|  | [0.98] | [3.05] | [3.61] | [3.17] | [-1.81] | [-7.06] | [-13.96] | [-15.29] | [-10.87] | [-2.87] |
| Slope                                      | 0.13   | 0.16   | 0.17   | 0.20   | 0.30    | 0.37    | 0.43     | 0.37     | 0.22     | 0.08    |
|  | [2.72] | [3.22] | [3.36] | [4.00] | [7.08]  | [9.63]  | [12.01]  | [9.70]   | [4.95]   | [1.78]  |
| Curv                                       | 0.29   | 0.29   | 0.26   | 0.29   | 0.49    | 0.51    | 0.38     | 0.30     | 0.23     | 0.23    |
|  | [4.61] | [4.75] | [4.17] | [4.70] | [10.01] | [11.85] | [9.72]   | [7.48]   | [4.93]   | [4.77]  |
| Adj. $R^2$                                 | 10.58  | 16.23  | 17.49  | 19.85  | 35.09   | 51.14   | 61.61    | 56.84    | 36.23    | 11.15   |
|  |        |        |        |        |         |         |          |          |          |         |



**Table 1.3**  
**Regressions of Principal Components of Conditional Volatilities on Monetary Policy Uncertainty**

The table reports results of regressions of principal components of EGARCH(1,1) estimates of conditional volatilities on monetary policy uncertainty ( $U^{MP}$ ), the level, slope and curvature of the yield curve and other explanatory variables captured in the vector  $\Phi_{i,t}$ : (the 5 year swap spread, estimated volatilities computed for CPI inflation ( $\sigma_{t-1,t}^{CPI}$ ) and the real activity measure CFNAI ( $\sigma_{t-1,t}^{CFNAI}$ ), Moody's corporate bond yield spread on the 10 year rate ( $BAA$ ), the Economic Policy Uncertainty index of [Baker, Bloom, and Davis \(2013\)](#) ( $U^{BBB}$ ) and the CBOE volatility index VIX):

$$\sigma_{t,t+1}^{PC}(\tau) = \alpha + \beta \times U_t^{MP} + \sum_{i=1}^3 \gamma_i \times PC_{i,t}^{yields} + \sum_{i=1}^M \phi_i \times \Phi_{i,t} + \eta_t(\tau)$$

$t$ -statistics are shown in parenthesis, below the reported estimated coefficients. The standard errors are corrected for autocorrelation and heteroskedasticity using the [Hansen and Hodrick \(1983\)](#) GMM correction. The data covers the period January 1988 to April 2011. All variables are standardized.

|                          | Panel A                  |                          |                         | Panel B                  |                          |                         |
|--------------------------|--------------------------|--------------------------|-------------------------|--------------------------|--------------------------|-------------------------|
|                          | $\sigma_{t,t+1}^{Level}$ | $\sigma_{t,t+1}^{Slope}$ | $\sigma_{t,t+1}^{Curv}$ | $\sigma_{t,t+1}^{Level}$ | $\sigma_{t,t+1}^{Slope}$ | $\sigma_{t,t+1}^{Curv}$ |
| $\bar{U}^{MP}$           | 0.18<br>[3.69]           | -0.19<br>[-5.21]         | -0.17<br>[-3.23]        | 0.20<br>[3.70]           | -0.10<br>[-2.37]         | -0.08<br>[-1.15]        |
| Level                    | 0.14<br>[2.33]           | -0.59<br>[-16.04]        | -0.15<br>[-2.64]        | 0.51<br>[5.11]           | -0.48<br>[-4.92]         | 0.08<br>[0.59]          |
| Slope                    | 0.15<br>[2.90]           | 0.24<br>[6.53]           | -0.14<br>[-2.56]        | 0.13<br>[1.85]           | 0.03<br>[0.73]           | -0.22<br>[-2.39]        |
| Curv                     | 0.36<br>[6.38]           | 0.38<br>[8.56]           | -0.02<br>[-0.24]        | 0.13<br>[1.62]           | 0.43<br>[7.11]           | -0.08<br>[-0.88]        |
| Swap Spread              | -                        | -                        | -                       | 0.01<br>[0.06]           | -0.28<br>[-3.60]         | 0.08<br>[0.63]          |
| $\sigma_{t-1,t}^{CPI}$   | -                        | -                        | -                       | -0.11<br>[-1.65]         | -0.54<br>[-8.72]         | 0.12<br>[1.33]          |
| $\sigma_{t-1,t}^{CFNAI}$ | -                        | -                        | -                       | -0.21<br>[-2.61]         | 0.11<br>[1.73]           | 0.08<br>[0.98]          |
| BAA                      | -                        | -                        | -                       | 0.78<br>[7.72]           | 0.19<br>[1.92]           | 0.03<br>[0.18]          |
| $U^{BBB}$                | -                        | -                        | -                       | 0.11<br>[0.90]           | 0.01<br>[0.17]           | 0.28<br>[2.21]          |
| VIX                      | -                        | -                        | -                       | -0.09<br>[-1.27]         | -0.12<br>[-1.42]         | -0.11<br>[-1.11]        |
| Adj. $R^2$               | 19.49                    | 57.34                    | 5.59                    | 31.31                    | 68.88                    | 6.16                    |

**Table 1.4**  
**Regressions of EGARCH(1,1) estimated volatility on model-implied conditional volatilities**

The table reports results of regressions of EGARCH(1,1) estimates of one-month ahead conditional volatilities on model-implied one-month ahead conditional volatilities across maturities for the proposed model  $A_2^M(4)$ , with monetary policy uncertainty as a risk factor (Panel A) and for the benchmark model  $A_1(3)$  of Dai and Singleton (2000) (Panel B) :

$$\sigma_{t,t+1}^{EGARCH}(\tau) = \alpha + \beta \times \sigma_{t,t+1}^{Model}(\tau) + \eta_t(\tau)$$

$t$ -statistics are shown in parenthesis, below the reported estimated coefficients. The standard errors are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. The data covers the period January 1988 to April 2011.

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| Model $A_2^M(4)$ with monetary policy uncertainty |                  |                  |                |                  |                  |                 |                 |                  |                  |
|---|------------------|------------------|----------------|------------------|------------------|-----------------|-----------------|------------------|------------------|
|   | 3 m              | 6 m              | 1 y            | 2 y              | 3 y              | 4 y             | 5 y             | 7 y              | 10 y             |
| $\alpha$  | -0.37<br>[-0.12] | -5.33<br>[-1.62] | 4.50<br>[1.58] | 13.83<br>[12.59] | 12.43<br>[12.50] | 9.88<br>[9.53]  | 9.92<br>[8.70]  | 13.77<br>[10.50] | 21.38<br>[21.25] |
| $\beta$   | 0.93<br>[6.68]   | 1.11<br>[7.38]   | 0.80<br>[6.41] | 0.52<br>[11.98]  | 0.59<br>[16.60]  | 0.67<br>[18.74] | 0.65<br>[16.46] | 0.47<br>[10.58]  | 0.20<br>[5.27]   |
| Adj- $R^2$  | 0.10             | 0.12             | 0.10           | 0.32             | 0.49             | 0.54            | 0.46            | 0.25             | 0.10             |
| Correlation                                       | 0.31             | 0.35             | 0.33           | 0.57             | 0.70             | 0.74            | 0.68            | 0.51             | 0.31             |

| Yields-only model $A_1(3)$ |                |                 |                 |                  |                  |                 |                 |                 |                  |
|----------------------------|----------------|-----------------|-----------------|------------------|------------------|-----------------|-----------------|-----------------|------------------|
|                            | 3 m            | 6 m             | 1 y             | 2 y              | 3 y              | 4 y             | 5 y             | 7 y             | 10 y             |
| $\alpha$                   | 8.78<br>[2.38] | 12.21<br>[3.18] | 16.78<br>[5.13] | 15.48<br>[12.01] | 11.77<br>[10.91] | 7.66<br>[7.40]  | 6.84<br>[5.96]  | 10.21<br>[7.23] | 20.64<br>[16.65] |
| $\beta$                    | 0.61<br>[3.23] | 0.38<br>[2.11]  | 0.30<br>[2.24]  | 0.42<br>[9.30]   | 0.56<br>[15.83]  | 0.68<br>[20.54] | 0.70<br>[18.75] | 0.59<br>[12.17] | 0.23<br>[4.88]   |
| Adj- $R^2$                 | 0.03           | 0.01            | 0.02            | 0.25             | 0.51             | 0.63            | 0.57            | 0.34            | 0.09             |
| Correlation                | 0.19           | 0.13            | 0.14            | 0.51             | 0.71             | 0.80            | 0.75            | 0.59            | 0.31             |

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**Table 1.5**  
**Regressions of realized volatility on model-implied conditional volatilities**

The table reports results of regressions of one-month ahead realized volatilities, computed as the sum of squared daily yield changes, on model-implied one-month ahead conditional volatilities across maturities for the proposed model  $A_2^M(4)$ , with monetary policy uncertainty as a risk factor (Panel A) and for the benchmark model  $A_1(3)$  of Dai and Singleton (2000) (Panel B) :

$$\sigma_{t,t+1}^{Realized}(\tau) = \alpha + \beta \times \sigma_{t,t+1}^{Model}(\tau) + \eta_t(\tau)$$

$t$ -statistics are shown in parenthesis, below the reported estimated coefficients. The standard errors are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. The data covers the period January 1988 to April 2011.

| Model $A_2^M(4)$ with monetary policy uncertainty |                 |                  |                 |                 |                |                |                |                |                |
|---|-----------------|------------------|-----------------|-----------------|----------------|----------------|----------------|----------------|----------------|
|   | 3 m             | 6 m              | 1 y             | 2 y             | 3 y            | 4 y            | 5 y            | 7 y            | 10 y           |
| Intercept   | 0.07<br>[0.01]  | -0.49<br>[-0.11] | 4.96<br>[1.35]  | 7.89<br>[2.59]  | 7.82<br>[2.71] | 7.18<br>[2.55] | 6.86<br>[2.48] | 7.11<br>[2.68] | 8.04<br>[3.24] |
| Slope   | 0.88<br>[3.79]  | 0.78<br>[4.09]   | 0.62<br>[3.93]  | 0.63<br>[5.32]  | 0.68<br>[6.32] | 0.72<br>[6.92] | 0.73<br>[7.18] | 0.70<br>[7.20] | 0.63<br>[7.01] |
| Adj- $R^2$  | 0.03            | 0.04             | 0.04            | 0.08            | 0.11           | 0.14           | 0.15           | 0.16           | 0.16           |
| Correlation                                       | 0.19            | 0.20             | 0.21            | 0.29            | 0.34           | 0.38           | 0.40           | 0.41           | 0.40           |
| Yields-only model $A_1(3)$                        |                 |                  |                 |                 |                |                |                |                |                |
|   | 3 m             | 6 m              | 1 y             | 2 y             | 3 y            | 4 y            | 5 y            | 7 y            | 10 y           |
| Intercept   | 19.92<br>[3.20] | 13.95<br>[3.10]  | 13.51<br>[3.53] | 11.01<br>[3.30] | 8.93<br>[2.80] | 7.07<br>[2.27] | 5.85<br>[1.90] | 4.99<br>[1.64] | 5.47<br>[1.83] |
| Slope   | 0.05<br>[0.16]  | 0.17<br>[0.79]   | 0.27<br>[1.70]  | 0.47<br>[4.04]  | 0.58<br>[5.47] | 0.66<br>[6.38] | 0.71<br>[6.85] | 0.75<br>[7.00] | 0.75<br>[6.71] |
| Adj- $R^2$  | 0.00            | 0.00             | 0.01            | 0.06            | 0.10           | 0.13           | 0.15           | 0.17           | 0.16           |
| Correlation                                       | 0.01            | 0.05             | 0.11            | 0.24            | 0.32           | 0.37           | 0.39           | 0.41           | 0.40           |
|   |                 |                  |                 |                 |                |                |                |                |                |

**Table 1.6**  
**Correlations between conditional volatilities and yield curve factors**

The table reports correlations of conditional volatilities across maturities with the level (Panel A), slope (Panel B) and curvature (Panel C) of the yield curve as well as for monetary policy uncertainty. Conditional volatilities comprise EGARCH(1,1) estimates, realized monthly volatility and model-implied volatilities from the proposed  $A_2^M(4)$  with monetary policy uncertainty as a volatility risk factor and the benchmark model  $A_1(3)$  of Dai and Singleton (2000). The data covers the period January 1988 to April 2011. \*\* and \* indicate that the correlation is not significant at the 10% and 5% level respectively.

|              | Level Factor                       |         |        |        |       |        |         |        |        |
|--------------|------------------------------------|---------|--------|--------|-------|--------|---------|--------|--------|
|              | 3 m                                | 6 m     | 1 y    | 2 y    | 3 y   | 4 y    | 5 y     | 7 y    | 10 y   |
| EGARCH(1,1)  | 0.06**                             | 0.18    | 0.19   | -0.11* | -0.34 | -0.51  | -0.57   | -0.49  | -0.22  |
| Realized Vol | 0.18                               | 0.16    | 0.07** | -0.10* | -0.20 | -0.26  | -0.29   | -0.32  | -0.29  |
| $A_1(3)$     | -0.65                              | -0.65   | -0.65  | -0.65  | -0.65 | -0.65  | -0.65   | -0.65  | -0.65  |
| $A_2^M(4)$   | 0.04**                             | -0.07** | -0.29  | -0.42  | -0.45 | -0.46  | -0.46   | -0.48  | -0.49  |
|              | Slope Factor                       |         |        |        |       |        |         |        |        |
|              | 3 m                                | 6 m     | 1 y    | 2 y    | 3 y   | 4 y    | 5 y     | 7 y    | 10 y   |
| EGARCH(1,1)  | 0.13                               | 0.15    | 0.18   | 0.29   | 0.37  | 0.45   | 0.39    | 0.24   | 0.09** |
| Realized Vol | 0.06**                             | 0.05**  | 0.10** | 0.14   | 0.16  | 0.18   | 0.19    | 0.21   | 0.22   |
| $A_1(3)$     | 0.55                               | 0.55    | 0.55   | 0.55   | 0.55  | 0.55   | 0.55    | 0.55   | 0.55   |
| $A_2^M(4)$   | 0.56                               | 0.60    | 0.65   | 0.66   | 0.65  | 0.65   | 0.65    | 0.65   | 0.65   |
|              | Curvature Factor                   |         |        |        |       |        |         |        |        |
|              | 3 m                                | 6 m     | 1 y    | 2 y    | 3 y   | 4 y    | 5 y     | 7 y    | 10 y   |
| EGARCH(1,1)  | 0.28                               | 0.29    | 0.28   | 0.49   | 0.51  | 0.39   | 0.31    | 0.23   | 0.23   |
| Realized Vol | 0.16                               | 0.21    | 0.15   | 0.16   | 0.17  | 0.18   | 0.18    | 0.17   | 0.18   |
| $A_1(3)$     | 0.52                               | 0.52    | 0.52   | 0.52   | 0.52  | 0.52   | 0.52    | 0.52   | 0.52   |
| $A_2^M(4)$   | 0.55                               | 0.58    | 0.59   | 0.58   | 0.57  | 0.57   | 0.56    | 0.56   | 0.55   |
|              | Monetary Policy Uncertainty Factor |         |        |        |       |        |         |        |        |
|              | 3 m                                | 6 m     | 1 y    | 2 y    | 3 y   | 4 y    | 5 y     | 7 y    | 10 y   |
| EGARCH(1,1)  | 0.27                               | 0.37    | 0.43   | 0.35   | 0.17  | 0.01** | -0.07** | -0.15  | 0.13   |
| Realized Vol | 0.22                               | 0.23    | 0.25   | 0.22   | 0.18  | 0.14   | 0.11*   | 0.07** | 0.07** |
| $A_1(3)$     | 0.11*                              | 0.11*   | 0.11*  | 0.11*  | 0.11* | 0.11*  | 0.11*   | 0.11*  | 0.11*  |
| $A_2^M(4)$   | 0.87                               | 0.81    | 0.60   | 0.44   | 0.40  | 0.39   | 0.38    | 0.36   | 0.33   |

**Table 1.7**  
**Regressions of 3-month holding period excess returns on model-implied expected holding period excess return**

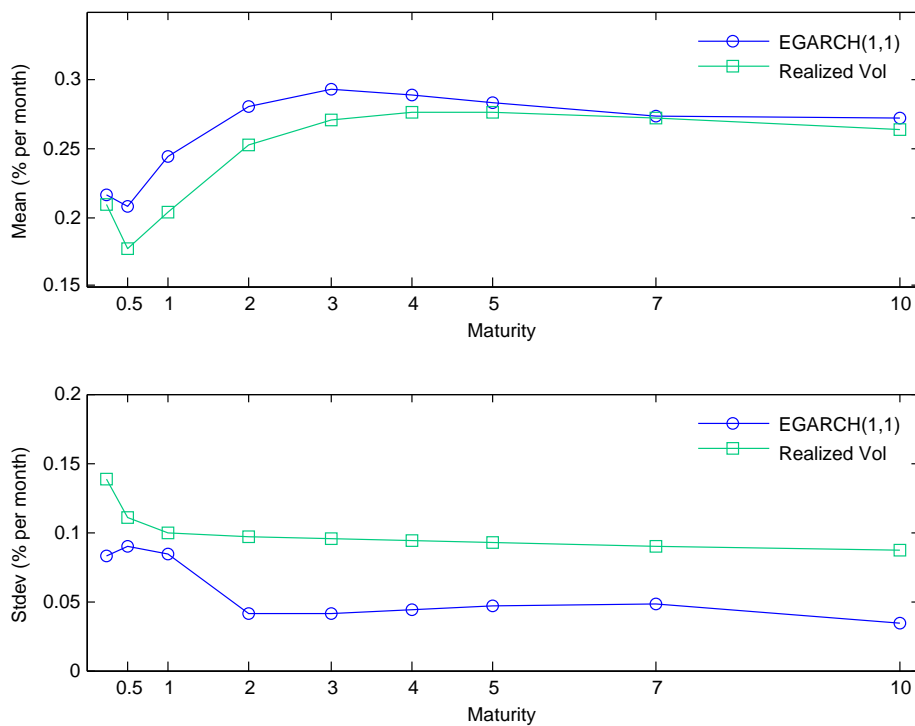
The table reports results of regressions of 3-month holding period excess returns on model-implied expected excess returns for the same horizon for the proposed model  $A_2^M(4)$  with monetary policy uncertainty as a risk factor (Panel A) and for the benchmark model  $A_1(3)$  of Dai and Singleton (2000) (Panel B), as well as for the return predicting factor computed using three forward rates  $f^{0 \rightarrow 1}$ ,  $f^{2 \rightarrow 3}$  and  $f^{4 \rightarrow 5}$ :

$$rx_{t,t+3/12}^n = \alpha + \beta \times E_t[rx_{t,t+3/12}^n] + \eta_t(\tau)$$

$t$ -statistics are shown in parenthesis, below the reported estimated coefficients. The standard errors are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. The data covers the period January 1988 to April 2011.

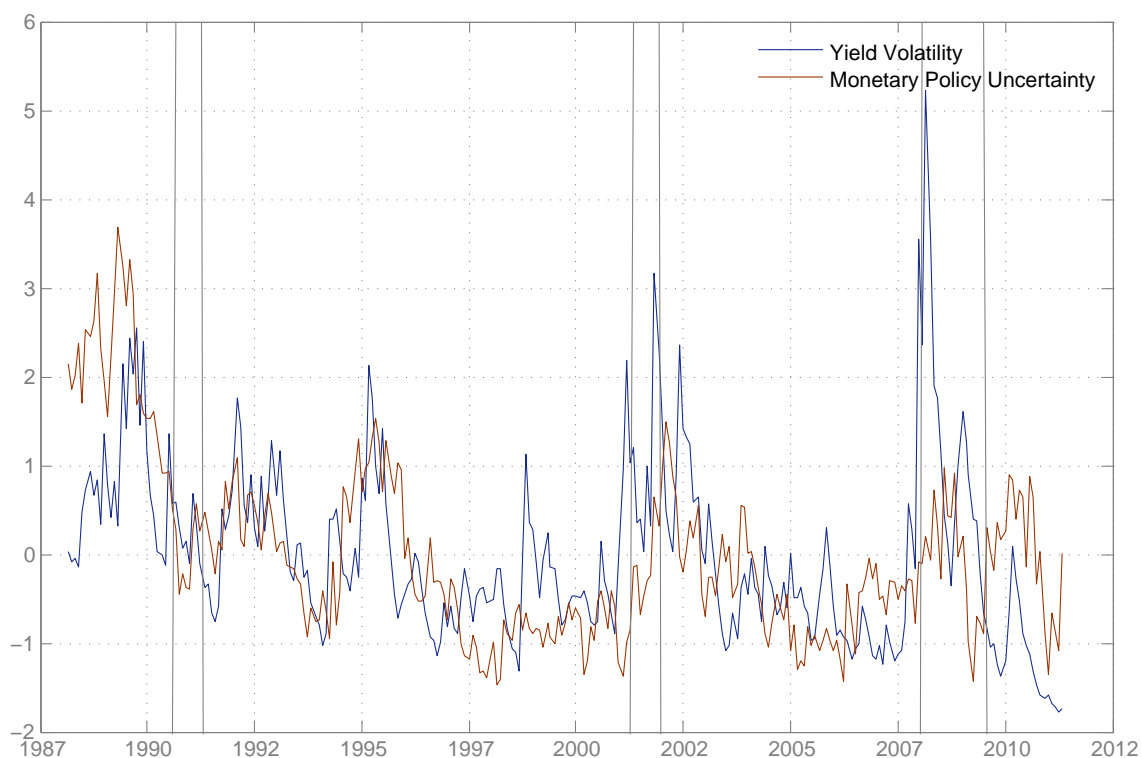
| <b>Model <math>A_2^M(4)</math> with monetary policy uncertainty</b> |                  |                  |                  |                |                |                |                |
|---|------------------|------------------|------------------|----------------|----------------|----------------|----------------|
|   | <b>6 m</b>       | <b>9 m</b>       | <b>1 y</b>       | <b>2 y</b>     | <b>3 y</b>     | <b>4 y</b>     | <b>5 y</b>     |
| $\alpha \times 10^3$  | -0.28<br>[-0.54] | -0.28<br>[-0.25] | -0.08<br>[-0.04] | 1.44<br>[0.32] | 2.82<br>[0.42] | 3.75<br>[0.44] | 4.33<br>[0.43] |
| $\beta$   | 1.21<br>[5.31]   | 1.15<br>[4.63]   | 1.10<br>[4.13]   | 0.99<br>[3.19] | 0.96<br>[2.99] | 0.96<br>[3.01] | 0.97<br>[3.10] |
| $Mod R^2$   | 0.10             | 0.08             | 0.06             | 0.04           | 0.03           | 0.03           | 0.03           |
| Correlation   | 0.32             | 0.28             | 0.25             | 0.18           | 0.17           | 0.16           | 0.17           |
| <b>Yields-only model <math>A_1(3)</math></b>                        |                  |                  |                  |                |                |                |                |
|   | <b>6 m</b>       | <b>9 m</b>       | <b>1 y</b>       | <b>2 y</b>     | <b>3 y</b>     | <b>4 y</b>     | <b>5 y</b>     |
| $\alpha \times 10^3$  | 0.53<br>[1.05]   | 1.07<br>[1.00]   | 1.59<br>[0.94]   | 3.14<br>[0.72] | 3.44<br>[0.50] | 2.64<br>[0.30] | 1.08<br>[0.10] |
| $\beta$   | 0.85<br>[3.83]   | 0.85<br>[3.52]   | 0.85<br>[3.27]   | 0.89<br>[2.86] | 0.96<br>[2.86] | 1.04<br>[3.01] | 1.12<br>[3.22] |
| $Mod R^2$   | 0.07             | 0.05             | 0.05             | 0.03           | 0.03           | 0.03           | 0.04           |
| Correlation   | 0.26             | 0.23             | 0.21             | 0.18           | 0.17           | 0.18           | 0.19           |
| <b>Return-predicting factor (<math>f1, f3, f5</math>)</b>           |                  |                  |                  |                |                |                |                |
|   | <b>6 m</b>       | <b>9 m</b>       | <b>1 y</b>       | <b>2 y</b>     | <b>3 y</b>     | <b>4 y</b>     | <b>5 y</b>     |
| $\beta$   | 0.14<br>[4.04]   | 0.27<br>[3.71]   | 0.39<br>[3.45]   | 0.85<br>[2.95] | 1.31<br>[2.93] | 1.78<br>[3.09] | 2.27<br>[3.31] |
| $Mod R^2$   | 0.07             | 0.06             | 0.05             | 0.04           | 0.03           | 0.03           | 0.04           |
| the Correlation   | 0.29             | 0.25             | 0.22             | 0.18           | 0.17           | 0.19           | 0.20           |

## 1.8 Figures



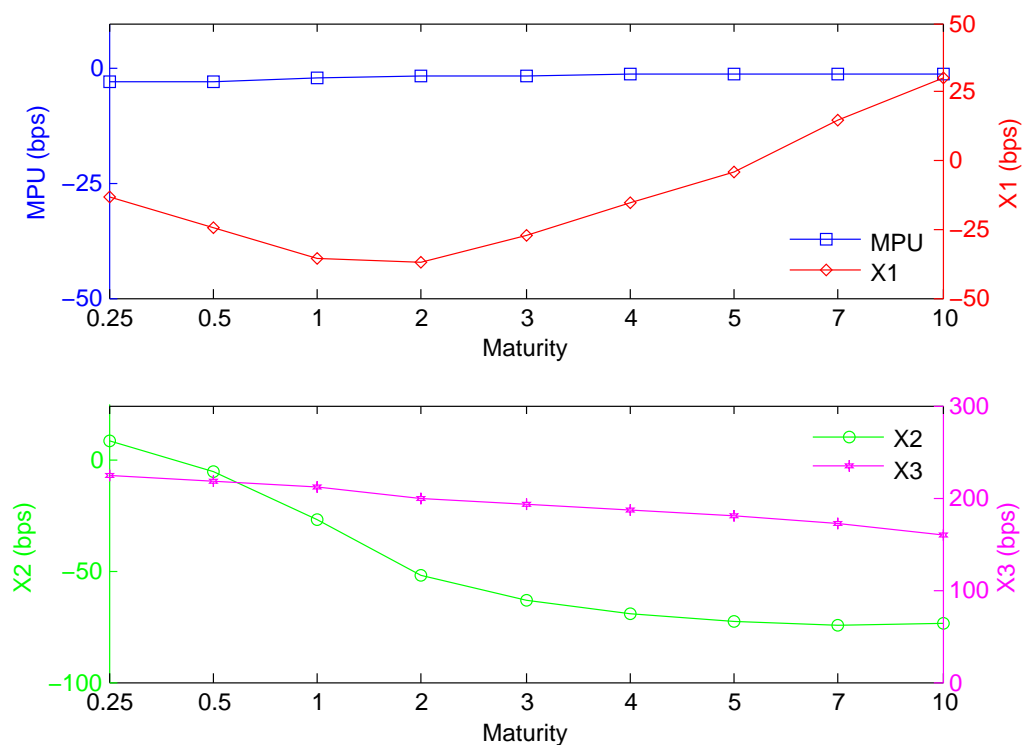
**Figure 1.1. Yield Volatility Curve**

This figure plots the first two moments of EGARCH(1,1) estimates of conditional yield volatility and realized monthly yield volatility. The top figure plots the average level of volatility for each maturity, while the bottom figure plots their standard deviation. The Treasury yield data used is that of [Gürkaynak, Sack, and Wright \(2007\)](#) and covers the period January 1988 to April 2011.



**Figure 1.2. Conditional volatility and monetary policy uncertainty**

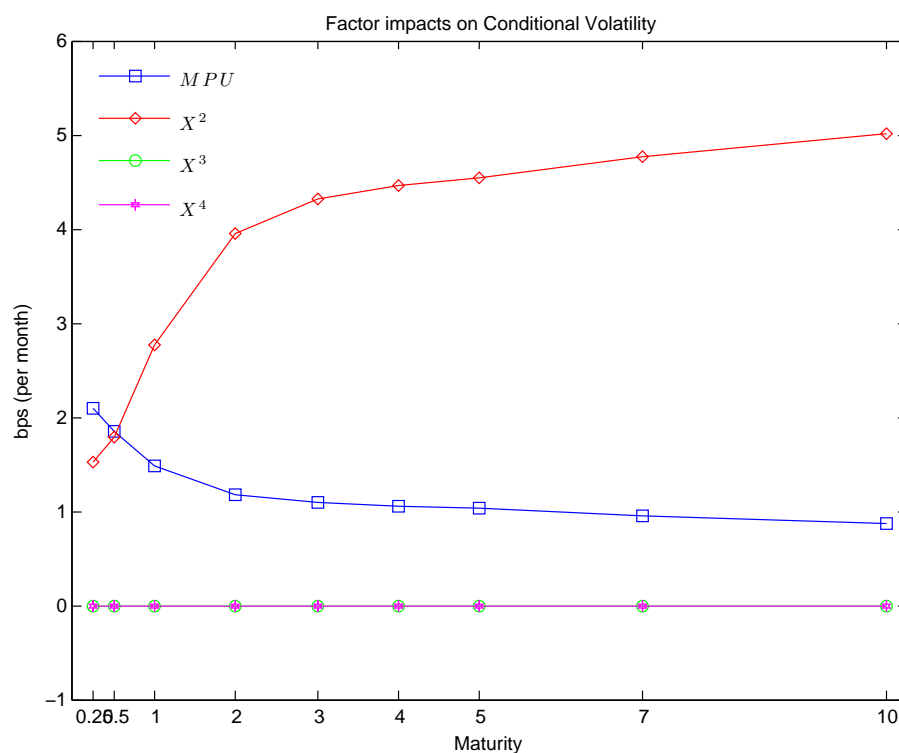
This figure plots EGARCH(1,1) estimated conditional volatility for the 1 year rate and monetary policy uncertainty as proxied by the cross-sectional dispersion in one-year ahead federal funds rate forecasts from the Blue Chip survey. Variables are standardized. The data covers the period January 1988 to April 2011.



**Figure 1.3. Factor impacts on the cross-section of yields**

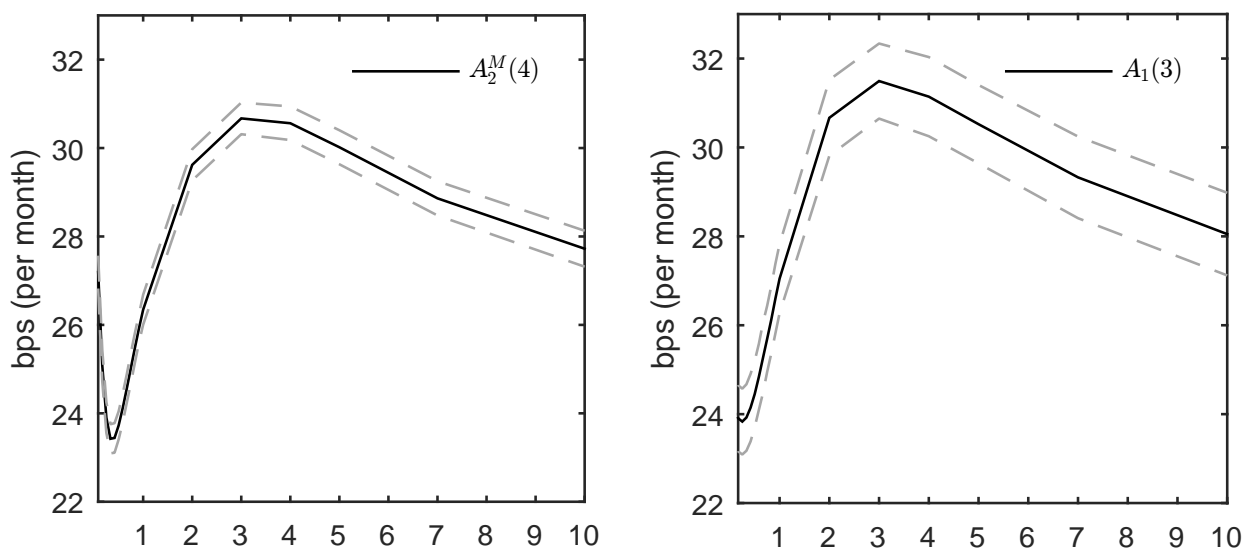
This figure plots the impact on the yield curve of a one standard deviation shock to each of the risk factors. The top plot (left axis), displays the impact of a one standard deviation shock to monetary policy uncertainty (in blue) while on the right axis the impact to the other latent volatility risk factor (in red). The bottom plot, displays the impact of the two latent conditionally Gaussian factors. Figures are in basis points (bps) per year. The data covers the period January 1988 to April 2011.





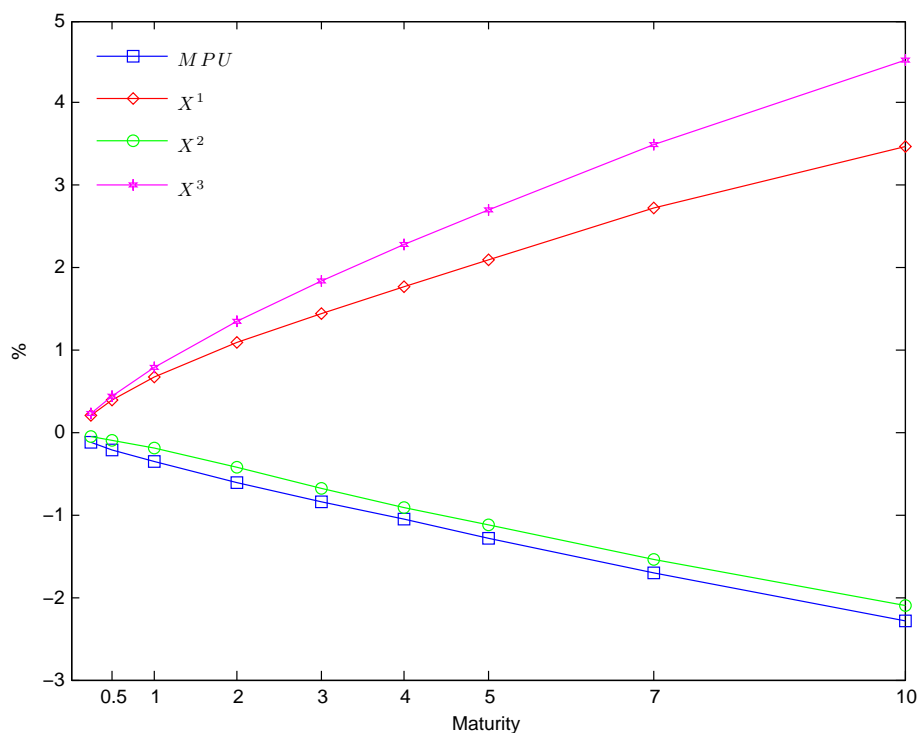
**Figure 1.4. Factor impacts on conditional yield volatility**

This figure plots the impact on the conditional volatility curve of a one standard deviation shock to each of the risk factors. The impact of a one standard deviation shock to monetary policy uncertainty  $MPU$  is depicted in blue. Figures are in basis points (bps) per month. The data covers the period January 1988 to April 2011.



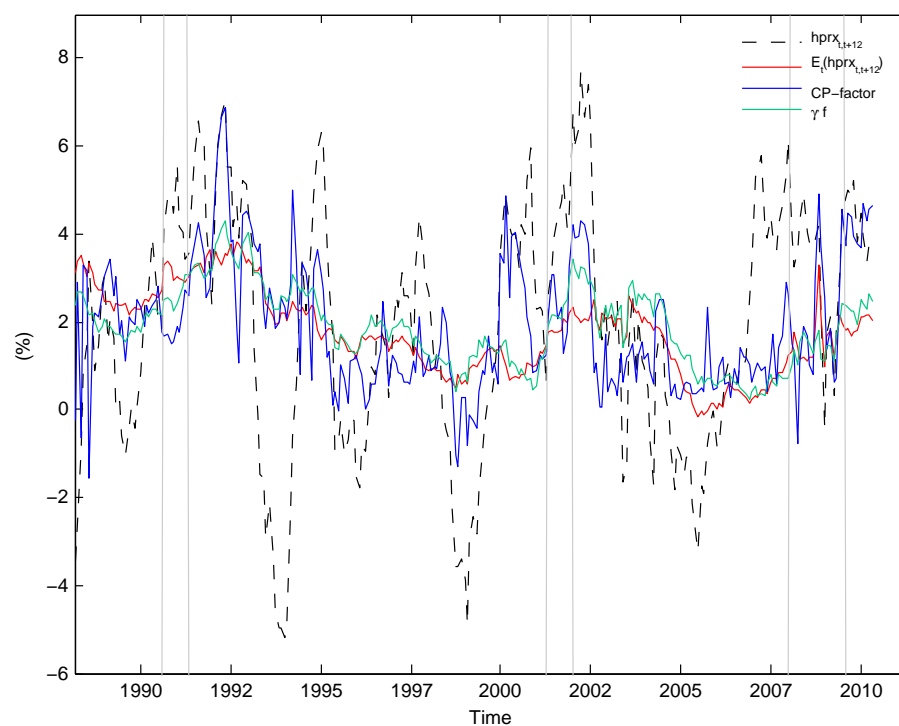
**Figure 1.5. Unconditional volatilities**

The figure displays simulated model-implied unconditional yield volatilities along with two standard deviation confidence bounds for the proposed model  $A_2^M(4)$  with monetary policy uncertainty as a risk factor and for the yields-only model  $A_1(3)$  of [Dai and Singleton \(2000\)](#). For the simulations I have treated the converged MCMC parameter estimates as true population parameters and simulated 1000 time-series of yields with the same length as that of the historical data. Figures are in basis points (bps) per month. The data covers the period January 1988 to April 2011.



**Figure 1.6. Factor impacts on instantaneous risk premia**

This figure plots the impact on instantaneous risk premia of a one standard deviation shock to each of the risk factors. The impact of a one standard deviation shock to monetary policy uncertainty  $MPU$  is given in blue. Figures are in percent per year. The data covers the period January 1988 to April 2011.



**Figure 1.7. Average one year holding period excess returns**

This figure plots one-year holding period excess returns, model-implied expected excess returns from the proposed  $A_2^M(4)$  model, as well as the return predicting factor of [Cochrane and Piazzesi \(2005\)](#) computed with 5 and 3 forward rates. The Treasury yield data used is the [Gürkaynak, Sack, and Wright \(2007\)](#) and covers the period January 1988 to April 2011. The areas within the grey lines denote NBER recession periods.

## 1.9 Appendix: Estimation with MCMC

### *Conditional distributions of parameters, state variables and yields*

Given initial assumptions for the parameters and latent state variables I sample parameters from:

$$p(\Theta_i | \Theta_{\setminus \Theta_i}, X, V, Y) \propto p(Y | \Theta, X, V) p(X | \Theta, V) p(V | \Theta, X) p(\Theta_i | \Theta_{\setminus \Theta_i}) \quad (\text{A-21})$$

and the latent state variables from:

$$\begin{aligned} p(X_t | X_{\setminus t}, \Theta, Y, V) &\propto p(X_t | X_{t-1}, X_{t+1}, \Theta, Y, V) \\ &\propto p(Y_t | X_t, V_t, \Theta) p(X_t | X_{t-1}, V_{t-1}, \Theta) p(X_{t+1} | X_t, V_t, \Theta) p(V_t | V_{t-1}, \Theta). \end{aligned}$$

I approximate the continuous time specification of the model with an Euler scheme and for the estimation consider this discretized version of the model over the time interval  $\Delta t = 1/12$  of a year for the latent state variables, reflecting the fact that the yield data is monthly. Letting  $\Sigma_X = I_N$ , the vector of latent variables follows between time  $t$  and  $t+1$  the dynamics:

$$X_{t+1} - X_t = \left\{ (K_{0,X}^{\mathbb{Q}} + \lambda_0) - (K_X^{\mathbb{Q}} - \lambda_X) X_t \right\} \Delta t + \sqrt{S_{X,t} \Delta t} \varepsilon_{t+1}, \quad (\text{A-22})$$

where  $\varepsilon_{t+1} \sim \mathcal{N}(0, 1)$ . A brief exposition of the conditional distributions used to sample the individual parameters and latent variables follows below. The conditional distribution of the latent state variables is given as follows:

$$p(X | \Theta) = \left( \prod_{t=1}^T p(X_t | X_{t-1}, V_{t-1}, \Theta) \right) p(X_0). \quad (\text{A-23})$$

Collecting the drift terms  $\mu_{X,t}^{\mathbb{P}} := (K_{0,X}^{\mathbb{Q}} + \lambda_0) - (K_X^{\mathbb{Q}} - \lambda_X) X_t$ , I can rewrite the discretized dynamics of  $X_t$  as:

$$\Delta X_t = \mu_{X,t}^{\mathbb{P}} \Delta t + \sqrt{S_{X,t} \Delta t} \varepsilon_{t+1} \quad (\text{A-24})$$

and define:

$$\begin{aligned} \hat{\varepsilon}_t &= \Delta X_t - \mu_{X,t-1}^{\mathbb{P}} \Delta t \\ \Phi_{X,t-1} &= S_{X,t-1} \Delta t, \end{aligned}$$

The conditional distribution of the latent state variables will then be given by:

$$p(X | \Theta) \propto \left( \prod_{t=1}^T \Phi_{X,t-1}^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} \hat{\varepsilon}_t \Phi_{X,t-1}^{-1} \hat{\varepsilon}_t \right\} \right) p(X_0). \quad (\text{A-25})$$

Turning to the observable factor, since I assume that  $V_t^0$  is observed with some measurement error:

$$V_t^O = V_t + \epsilon_{V,t} \quad \text{with} \quad \epsilon_{V,t}(\tau) \sim \mathcal{N}(0, \sigma_V^2) \quad (\text{A-26})$$

it follows that:

$$p(V^O | X, \Theta) \propto \sigma^{-T} \exp \left\{ -\frac{1}{2\sigma_V^2} \sum_{t=1}^T \epsilon'_{V,t} \epsilon_{V,t} \right\}. \quad (\text{A-27})$$

While for the yield data the conditional distribution given the state variables and parameters is given by:

$$p(Y | X, \Theta) = \prod_{\tau=1}^n \prod_{t=1}^T p(Y_t | X_t, \Theta). \quad (\text{A-28})$$

Since the observed zero coupon bond yields are given as the model-implied yields plus an observation error:

$$y_t(\tau) = A(\tau) + B(\tau)X_t + \epsilon_{Y,t}(\tau) \quad \text{for } \tau \in \{1, \dots, n\}, \quad (\text{A-29})$$

where  $\epsilon_t(\tau) \sim i.i.d. \mathcal{N}(0, \sigma^2)$ , the conditional distribution of the yield data is:

$$p(Y|X, \Theta) \propto \sigma^{-nT} \exp \left\{ -\frac{1}{2\sigma^2} \sum_{t=1}^T \epsilon'_{Y,t} \epsilon_{Y,t} \right\}. \quad (\text{A-30})$$

**1.10 Appendix: Tables**

**Table 1.8**  
**Descriptive Statistics of Monthly Yields and Conditional Volatilities**

The table reports descriptive statistics for yields and their monthly conditional volatilities. EGARCH(1,1) estimates of monthly conditional volatility are computed on yield changes. Monthly realized volatilities are computed as the sum of squared daily yield changes. The yield data used is that of [Gürkaynak, Sack, and Wright \(2007\)](#) and covers the period January 1988 to April 2011.

| <b>Panel A: Treasury Yields ( % per year)</b> |            |            |            |            |            |            |            |            |            |             |
|---|------------|------------|------------|------------|------------|------------|------------|------------|------------|-------------|
|   | <b>3 m</b> | <b>6 m</b> | <b>9 m</b> | <b>1 y</b> | <b>2 y</b> | <b>3 y</b> | <b>4 y</b> | <b>5 y</b> | <b>7 y</b> | <b>10 y</b> |
| <b>Mean</b>                                   | 4.08       | 4.16       | 4.24       | 4.32       | 4.58       | 4.81       | 5.02       | 5.20       | 5.52       | 5.87        |
| <b>St. Dev.</b>                               | 2.31       | 2.34       | 2.35       | 2.34       | 2.26       | 2.14       | 2.04       | 1.94       | 1.79       | 1.63        |
| <b>Skewness</b>                               | -0.05      | -0.07      | -0.09      | -0.10      | -0.12      | -0.09      | -0.03      | 0.04       | 0.18       | 0.32        |
| <b>Kurtosis</b>                               | 2.22       | 2.20       | 2.19       | 2.20       | 2.23       | 2.25       | 2.23       | 2.20       | 2.15       | 2.11        |

| <b>Panel B: Conditional Volatility Estimates (% per month)</b> |            |            |            |            |            |            |            |            |            |             |
|--|------------|------------|------------|------------|------------|------------|------------|------------|------------|-------------|
| <b>EGARCH(1,1) Volatility</b>                                  |            |            |            |            |            |            |            |            |            |             |
|  | <b>3 m</b> | <b>6 m</b> | <b>9 m</b> | <b>1 y</b> | <b>2 y</b> | <b>3 y</b> | <b>4 y</b> | <b>5 y</b> | <b>7 y</b> | <b>10 y</b> |
| <b>Mean</b>  | 0.22       | 0.21       | 0.23       | 0.25       | 0.28       | 0.29       | 0.29       | 0.28       | 0.27       | 0.27        |
| <b>St. Dev.</b>  | 0.08       | 0.09       | 0.10       | 0.08       | 0.04       | 0.04       | 0.04       | 0.05       | 0.05       | 0.03        |
| <b>Skewness</b>  | 2.35       | 1.71       | 1.52       | 1.30       | 0.70       | 0.13       | 0.06       | 0.15       | 0.28       | 2.92        |
| <b>Kurtosis</b>  | 11.71      | 8.20       | 7.14       | 6.05       | 3.18       | 2.38       | 2.25       | 2.27       | 2.44       | 19.06       |

| <b>Realized Volatility</b> |            |            |            |            |            |            |            |            |            |             |
|----------------------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|-------------|
|                            | <b>3 m</b> | <b>6 m</b> | <b>9 m</b> | <b>1 y</b> | <b>2 y</b> | <b>3 y</b> | <b>4 y</b> | <b>5 y</b> | <b>7 y</b> | <b>10 y</b> |
| <b>Mean</b>                | 0.21       | 0.18       | 0.19       | 0.20       | 0.25       | 0.27       | 0.28       | 0.28       | 0.27       | 0.26        |
| <b>St. Dev.</b>            | 0.14       | 0.11       | 0.10       | 0.10       | 0.10       | 0.09       | 0.09       | 0.09       | 0.09       | 0.09        |
| <b>Skewness</b>            | 3.42       | 3.10       | 2.27       | 1.89       | 1.60       | 1.33       | 1.12       | 1.01       | 1.00       | 1.13        |
| <b>Kurtosis</b>            | 22.27      | 19.30      | 12.39      | 10.25      | 8.55       | 6.77       | 5.48       | 4.76       | 4.49       | 4.70        |

**Table 1.9**

**Variation in conditional volatilities explained by the first  $k$  PC's**

The table reports the variation in conditional volatilities that is explained by the first  $k$  principal components of volatility for EGARCH(1,1) estimates of volatility and realized volatility. EGARCH(1,1) estimates of monthly conditional volatility are computed on yield changes. Monthly realized volatilities are computed as the sum of squared daily yield changes. The yield data used is that of [Gürkaynak, Sack, and Wright \(2007\)](#) and covers the period January 1988 to April 2011.

| $k^{th}$ PC <sup>Vola</sup> | 1 <sup>st</sup> | 2 <sup>nd</sup> | 3 <sup>rd</sup> | 4 <sup>th</sup> | 5 <sup>th</sup> |
|-----------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| EGARCH(1,1)                 | 68%             | 91%             | 95%             | 98%             | 99%             |
| Realized Vol                | 70%             | 92%             | 98%             | 99%             | 100%            |



**Table 1.10**  
**Interest Rate Volatility and Yields**

The table reports results of regressions of EGARCH(1,1) estimates of conditional volatilities for maturities of 3, 6 and 9 months and 1 to 5, 7 and 10 years on the first three principal components of yields (capturing the level, slope and curvature of the yield curve):

$$\sigma_{t,t+1}^2(\tau) = \alpha + \sum_{i=1}^3 \beta_i \times PC_{i,t}^{yields} + \eta_t(\tau)$$

$t$ -statistics are shown in parenthesis below the reported estimated coefficients. The standard errors are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. All variables are standardized. Panel B, reports the results of factor analysis on the residuals from the regression in Panel A, providing an indication of the number of factors needed to capture the variation in the residuals. The yield data used is that of Gürkaynak, Sack, and Wright (2007) and covers the period January 1988 to April 2011.

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| Panel A: Regressions of Conditional Volatilities on yield PC's |                |                |                |                |                  |                  |                   |                   |                   |                  |
|--|----------------|----------------|----------------|----------------|------------------|------------------|-------------------|-------------------|-------------------|------------------|
|  | 3 m            | 6 m            | 9 m            | 1 y            | 2 y              | 3 y              | 4 y               | 5 y               | 7 y               | 10 y             |
| Level  | 0.05<br>[0.91] | 0.18<br>[2.91] | 0.21<br>[3.40] | 0.19<br>[2.97] | -0.11<br>[-1.91] | -0.35<br>[-7.11] | -0.52<br>[-13.88] | -0.57<br>[-15.21] | -0.49<br>[-10.65] | -0.22<br>[-2.86] |
| Slope  | 0.13<br>[2.64] | 0.15<br>[3.10] | 0.16<br>[3.24] | 0.19<br>[3.84] | 0.30<br>[6.92]   | 0.37<br>[9.63]   | 0.44<br>[11.88]   | 0.38<br>[9.60]    | 0.23<br>[4.91]    | 0.08<br>[1.71]   |
| Curv   | 0.29<br>[4.44] | 0.29<br>[4.47] | 0.26<br>[3.87] | 0.28<br>[4.31] | 0.49<br>[9.47]   | 0.51<br>[11.89]  | 0.38<br>[9.96]    | 0.30<br>[7.71]    | 0.23<br>[5.14]    | 0.23<br>[4.61]   |
| <b>Adj. <math>R^2</math></b>                                   | 9.13           | 12.84          | 12.74          | 14.12          | 32.90            | 51.30            | 60.86             | 55.76             | 33.92             | 9.75             |

| Panel B: Percent of Residual Variation in Volatilities Explained by PC's<br>Extracted from the Regression's Error Terms |                 |                 |                 |                 |                 |                 |                 |                 |
|---|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|   | 1 <sup>st</sup> | 2 <sup>nd</sup> | 3 <sup>rd</sup> | 4 <sup>th</sup> | 5 <sup>th</sup> | 6 <sup>th</sup> | 7 <sup>th</sup> | 8 <sup>th</sup> |
| PC's<br>% exp.  | 81.60%          | 8.51%           | 6.11%           | 2.09%           | 1.09%           | 0.50%           | 0.04%           | 0.02%           |

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**Table 1.11**  
**Descriptive Statistics of Federal Funds Rate Forecasts**

The table reports descriptive statistics for the cross-sectional mean of one-year ahead forecasts of the federal funds rate (a proxy for monetary policy expectations), and the cross-sectional standard deviation of one-year ahead forecasts of the fed funds rate (a proxy for monetary policy uncertainty). The data covers the period January 1988-April 2011 and is taken from the Blue Chip Financial Forecasts Survey. Figures are in percent per year.

|                 | Cross-sectional<br>Mean | Cross-sectional<br>Dispersion |
|-----------------|-------------------------|-------------------------------|
| <b>Mean</b>     | 4.54                    | 0.49                          |
| <b>St. Dev.</b> | 2.06                    | 0.16                          |
| <b>Skewness</b> | -0.16                   | 1.24                          |
| <b>Kurtosis</b> | 2.46                    | 4.57                          |

**Table 1.12**  
**Correlations of yield volatility with uncertainty measures**

The table reports unconditional correlations of interest rate volatility for the one-year rate (Vol), monetary policy uncertainty (MPU), inflation uncertainty (IU) and the economic policy uncertainty (EPU) index of [Baker, Bloom, and Davis \(2013\)](#). The data covers the period January 1988 - April 2011.

|            | Vol | MPU  | IU   | EPU  |
|------------|-----|------|------|------|
| <b>Vol</b> | 1   | 0.43 | 0.22 | 0.14 |
| <b>MPU</b> | -   | 1    | 0.63 | 0.13 |
| <b>IU</b>  | -   | -    | 1    | 0.38 |
| <b>EPU</b> | -   | -    | -    | 1    |

**Table 1.13**  
**Model Parameter Estimates**

The table reports parameter estimates for the proposed  $A_2^M(4)$  model with monetary policy uncertainty as a risk factor. Below the parameter estimates are reported the 95% confidence bounds obtained from the parameters' posterior distribution estimated with MCMC.

|                | Parameters Estimates for the proposed model $A_2^M(4)$                           |                               |                             |                              |
|----------------|--|-------------------------------|-----------------------------|------------------------------|
| $\delta_0$     | 0.0384<br>(0.0375,0.0388)  |                               |                             |                              |
| $\delta_X$     | -0.0222<br>(-0.0231, -0.0215)  | 0.0001<br>(-0.0002,0.0005)    | 0.0004<br>( 0.0003,0.0005)  | 0.0030<br>(0.0029,0.0032)    |
| $\kappa\theta$ | 0.5225<br>(0.5005,0.5770)  | 1.6221<br>(1.3555, 2.0018)    | 0                           | 0                            |
| $\kappa$       | 1.0482<br>(0.9240,1.1942)  | -0.4735<br>(-0.5339,-0.4370)  | 0                           | 0                            |
|                | -0.1108<br>(-0.2370,-0.0369)   | 0.1473<br>(0.1109,0.1997)     | 0                           | 0                            |
|                | -4.6883<br>(-4.8081,-4.5562 )  | 1.5741<br>(1.5145,1.6237)     | 1.1005<br>(1.0279,1.2098)   | 0.2501<br>(0.1045,0.3497)    |
|                | -0.4444<br>(-0.5307,-0.3495)   | -1.0856<br>(-1.1973,-0.9904)  | 0.5841<br>(0.5205,0.6464)   | 0.1862<br>(0.0880,0.2550)    |
| $\lambda_0$    | 0.0044<br>(-0.0582,0.0647)   | 0.1774<br>(-0.8450,1.2694)    | -0.2762                     | 1.6142                       |
| $\lambda_X$    | -10.9015<br>(-11.3932,-10.3999)  | -0.4645<br>( -0.5227,-0.4256) | 0                           | 0                            |
|                | 0.0996<br>(-0.1689,0.4418)   | -0.2759<br>(-0.5839,0.0107)   | 0                           | 0                            |
|                | -15.3369<br>(-23.6613,-7.7370)   | -4.6756<br>(-7.8967,-2.1514)  | -0.1141<br>(-0.4424,0.2069) | -0.2633<br>(-0.6569,0.1280)  |
|                | 27.9830<br>(19.0313,34.7536)   | -4.3934<br>(-5.3406,-3.6571)  | 0.0819<br>(-0.0220,0.1839)  | -0.4099<br>(-0.5473,-0.2749) |
| $\alpha$       | 0  | 0                             | 1                           | 1                            |
| $\beta$        | 1  | 0                             | 0                           | 0                            |
|                | 0  | 1                             | 0                           | 0                            |
|                | 126.4497<br>(68.3884,189.2884)   | 15.2897<br>(8.5972,23.5940)   | 0                           | 0                            |
|                | 47.5127<br>(22.8416,76.5220)   | 0.4463<br>(0.1221,0.7725)     | 0                           | 0                            |
| $\sigma_Y$     | $1.869 \times 10^{-07}$<br>( $1.749 \times 10^{-07}$ , $1.996 \times 10^{-07}$ ) |                               |                             |                              |
| $\sigma_V$     | $1.070 \times 10^{-06}$<br>( $5.604 \times 10^{-12}$ , $3.968 \times 10^{-06}$ ) |                               |                             |                              |

**Table 1.14**  
**Model Fit for the proposed  $A_2^M(4)$  model**

Panel A in the table shows average pricing errors and root mean squared errors for the fit of model-implied yields to observed yields. Panel B shows (ordered) eigenvalues of the speed of mean reversion matrix under the risk neutral measure as well as the half-lives of shocks to the state variables.

| Panel A                              |      |       |       |      |      |       |       |       |       |
|--------------------------------------|------|-------|-------|------|------|-------|-------|-------|-------|
| Fit of model implied yields (in bps) |      |       |       |      |      |       |       |       |       |
|                                      | 3 m  | 6 m   | 1 y   | 2 y  | 3 y  | 4 y   | 5 y   | 7 y   | 10 y  |
| APE                                  | 0.42 | -0.40 | -0.53 | 0.23 | 0.12 | -0.60 | -1.20 | -1.22 | -1.92 |
| RMSE                                 | 5.16 | 2.95  | 5.52  | 1.98 | 2.29 | 3.24  | 3.26  | 2.04  | 4.74  |

| Panel B   |      |      |      |       |
|---|------|------|------|-------|
| Eigenvalues of $\kappa^Q$ (ordered) and Half-lives of shocks (in years) |      |      |      |       |
|   | 1.25 | 1.20 | 0.11 | 0.04  |
| Eigenvalue  | 1.25 | 1.20 | 0.11 | 0.04  |
| Half-Life   | 0.55 | 0.58 | 6.44 | 15.69 |

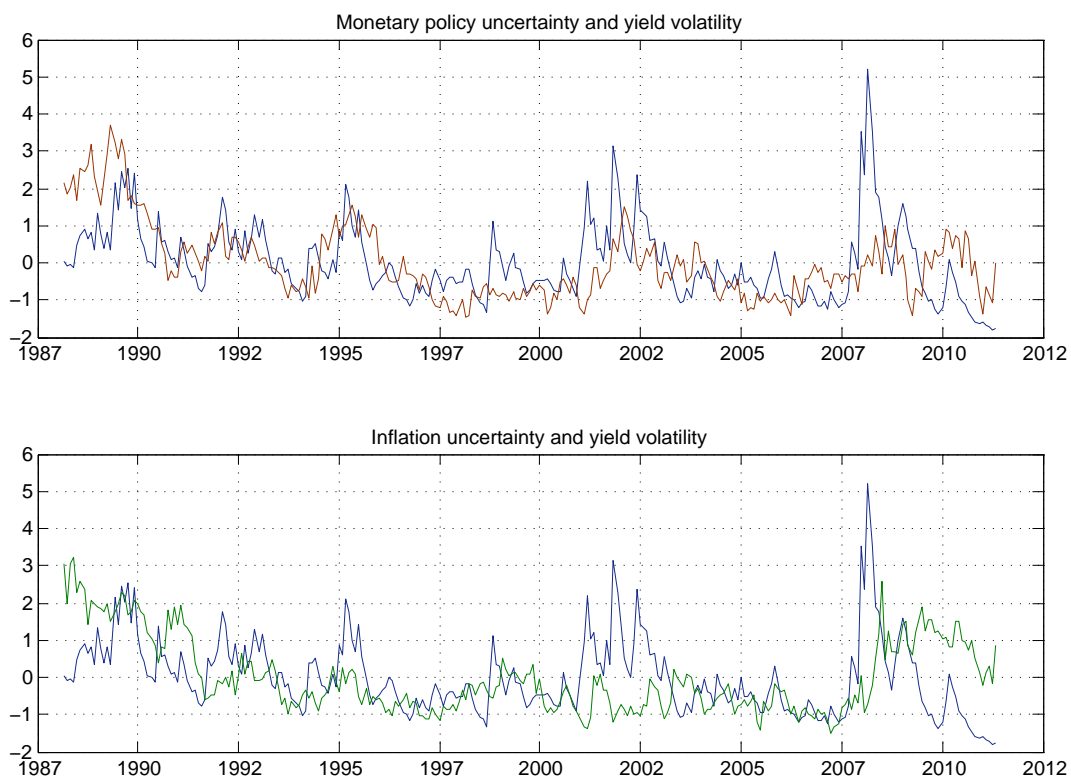
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**Table 1.15**  
**Unconditional Correlations of Conditional Volatilities**

The table reports unconditional correlations of conditional volatilities across maturities for EGARCH(1,1) estimates of volatility (panel A), realized monthly volatility (panel B) and model-implied conditional volatilities from the proposed  $A_2^M(4)$  with monetary policy uncertainty as a volatility risk factor. The data covers the period January 1988 to April 2011.

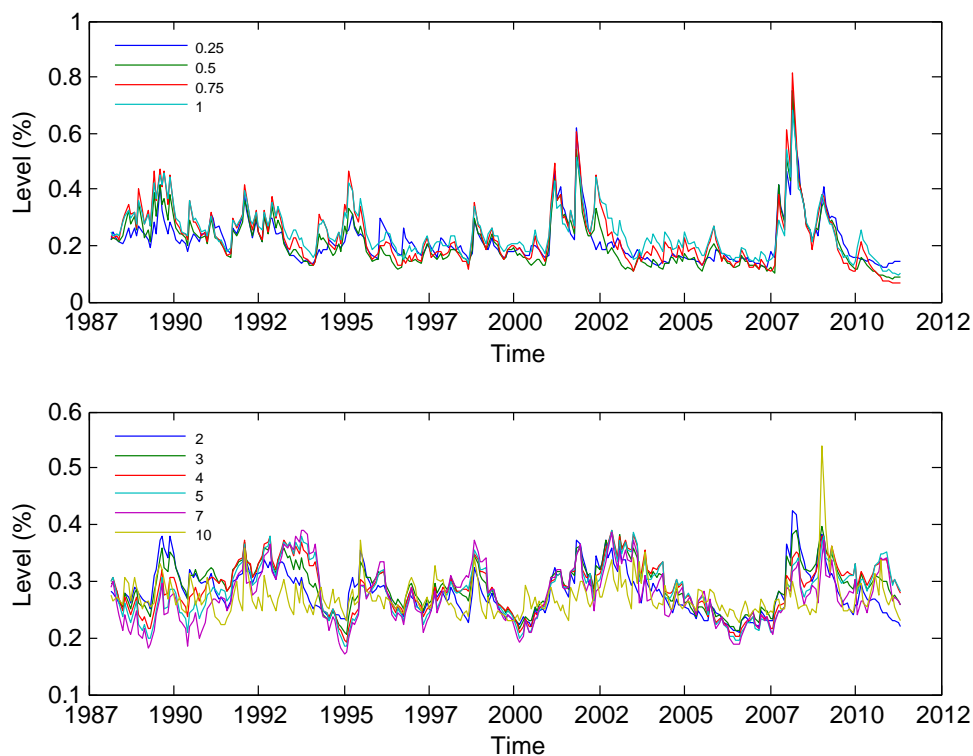
| EGARCH(1,1)         |     |        |        |        |        |        |
|---------------------|-----|--------|--------|--------|--------|--------|
|                     | 3 m | 6 m    | 1 y    | 2 y    | 5 y    | 10 y   |
| 3 m                 | 1   | 0.9232 | 0.8057 | 0.6849 | 0.2082 | 0.2544 |
| 6 m                 |     | 1      | 0.9256 | 0.7085 | 0.1291 | 0.2488 |
| 1 y                 |     |        | 1      | 0.7994 | 0.1608 | 0.3432 |
| 2 y                 |     |        |        | 1      | 0.6050 | 0.5237 |
| 5 y                 |     |        |        |        | 1      | 0.4321 |
| 10 y                |     |        |        |        |        | 1      |
| Realized Volatility |     |        |        |        |        |        |
|                     | 3 m | 6 m    | 1 y    | 2 y    | 5 y    | 10 y   |
| 3 m                 | 1   | 0.8715 | 0.6825 | 0.5008 | 0.3389 | 0.2070 |
| 6 m                 |     | 1      | 0.8900 | 0.6904 | 0.4860 | 0.2755 |
| 1 y                 |     |        | 1      | 0.9099 | 0.6724 | 0.4195 |
| 2 y                 |     |        |        | 1      | 0.8679 | 0.6225 |
| 5 y                 |     |        |        |        | 1      | 0.8847 |
| 10 y                |     |        |        |        |        | 1      |
| $A_2^M(4)$          |     |        |        |        |        |        |
|                     | 3 m | 6 m    | 1 y    | 2 y    | 5 y    | 10 y   |
| 3 m                 | 1   | 0.9919 | 0.9138 | 0.8188 | 0.7796 | 0.7451 |
| 6 m                 |     | 1      | 0.9580 | 0.8852 | 0.8529 | 0.8239 |
| 1 y                 |     |        | 1      | 0.9814 | 0.9667 | 0.9517 |
| 2 y                 |     |        |        | 1      | 0.9979 | 0.9929 |
| 5 y                 |     |        |        |        | 1      | 0.9986 |
| 10 y                |     |        |        |        |        | 1      |
|                     |     |        |        |        |        |        |

### 1.11 Appendix: Figures



**Figure 1.8. EGARCH(1,1) estimates of conditional yield volatility**

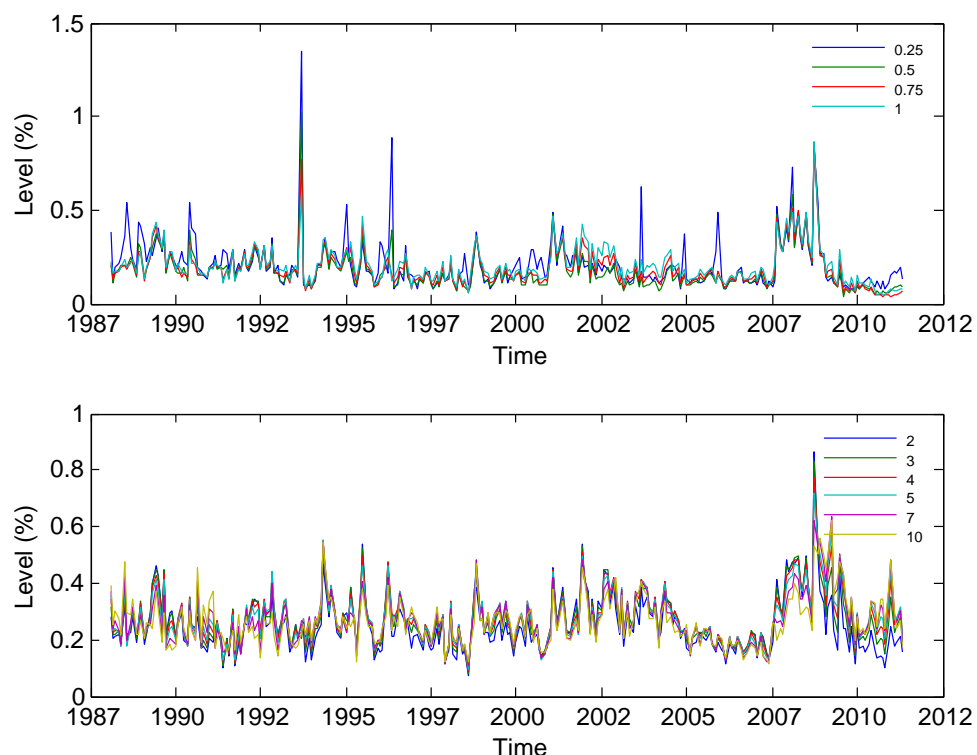
This figure plots the time series of conditional yield volatility for the one year rate along with monetary policy uncertainty (top panel) and inflation uncertainty (bottom panel). The Treasury yield data used is that of [Gürkaynak, Sack, and Wright \(2007\)](#) and covers the period January 1988 to April 2011.



**Figure 1.9. EGARCH(1,1) estimates of conditional yield volatility**

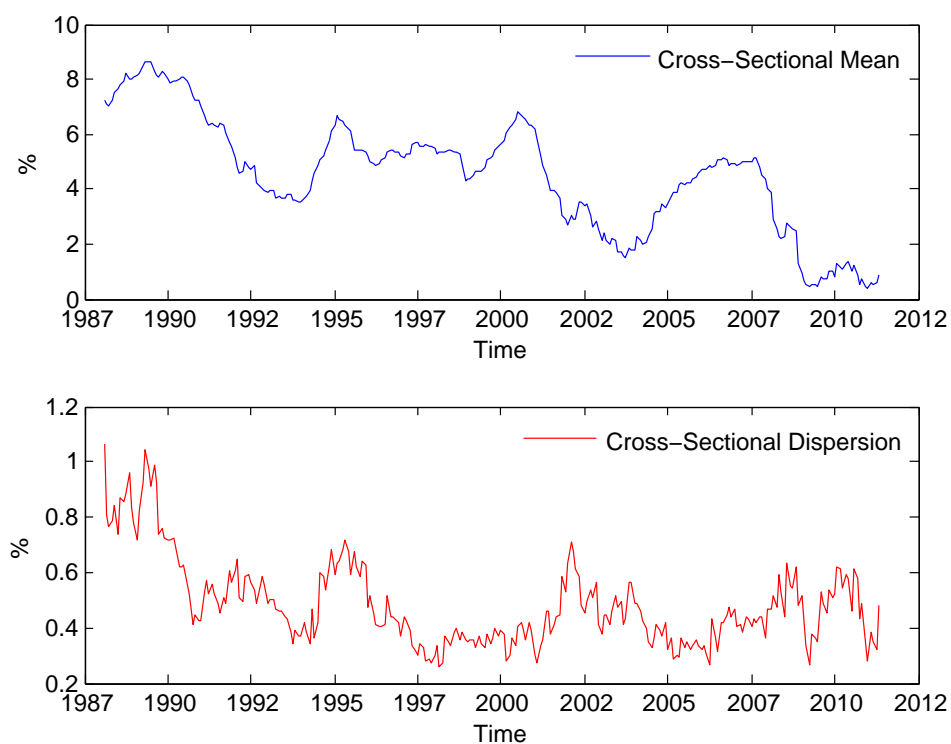
This figure plots EGARCH(1,1) estimates of monthly Treasury yield volatility computed on yield differences. The top figure plots volatility estimates for the maturities three months, six months, nine months and one year, while the bottom figure for maturities of two to five, seven and ten years. The Treasury yield data used is that of [Gürkaynak, Sack, and Wright \(2007\)](#) and covers the period January 1988 to April 2011.





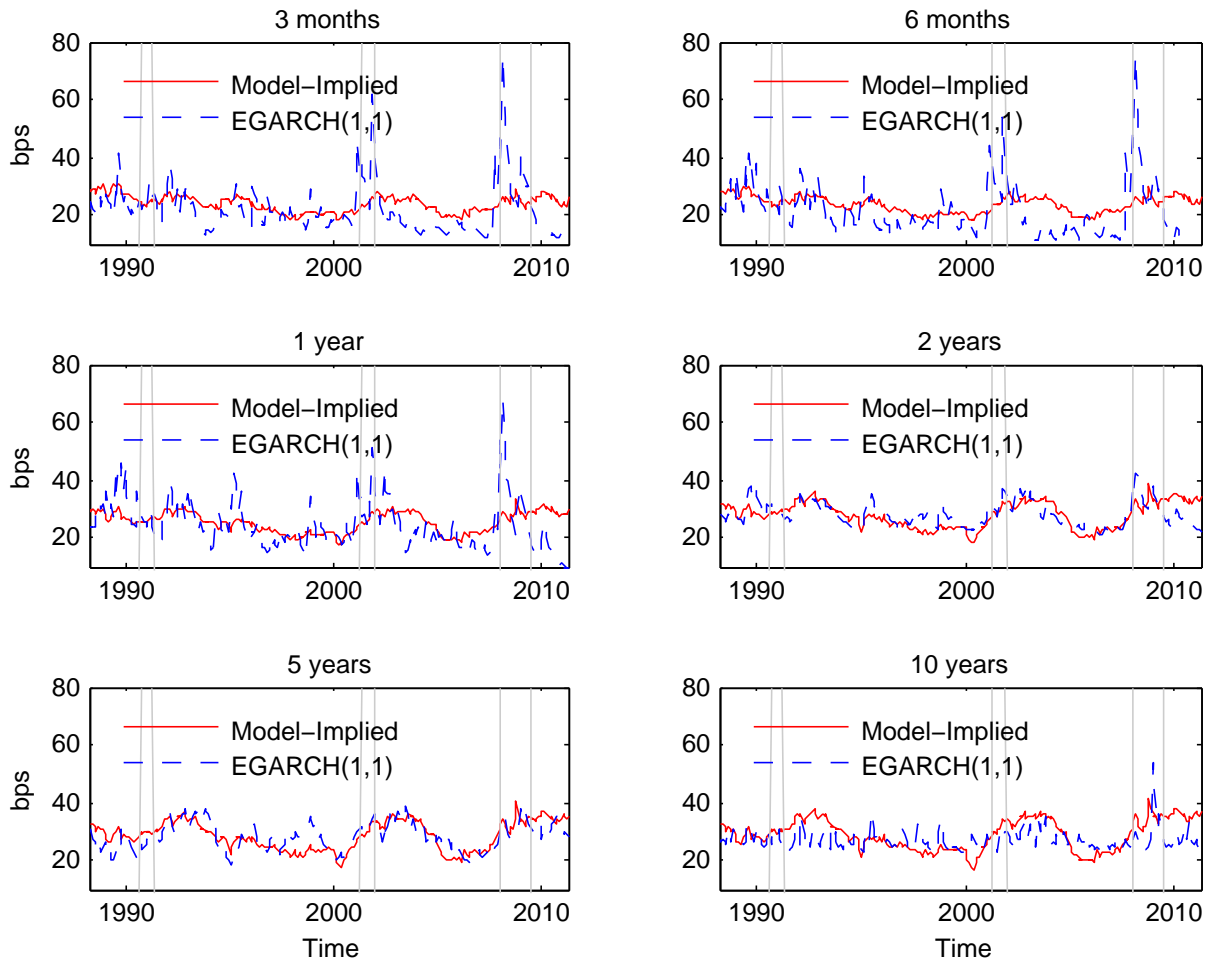
**Figure 1.10. Realized volatility**

This figure plots realized yield volatilities computed as the sum of squared daily yield changes within a month. The top figure plots volatility estimates for the maturities three months, six months, nine months and one year, while the bottom figure for maturities of two to five, seven and ten years. The Treasury yield data used is that of [Gürkaynak, Sack, and Wright \(2007\)](#) and covers the period January 1988 to April 2011.



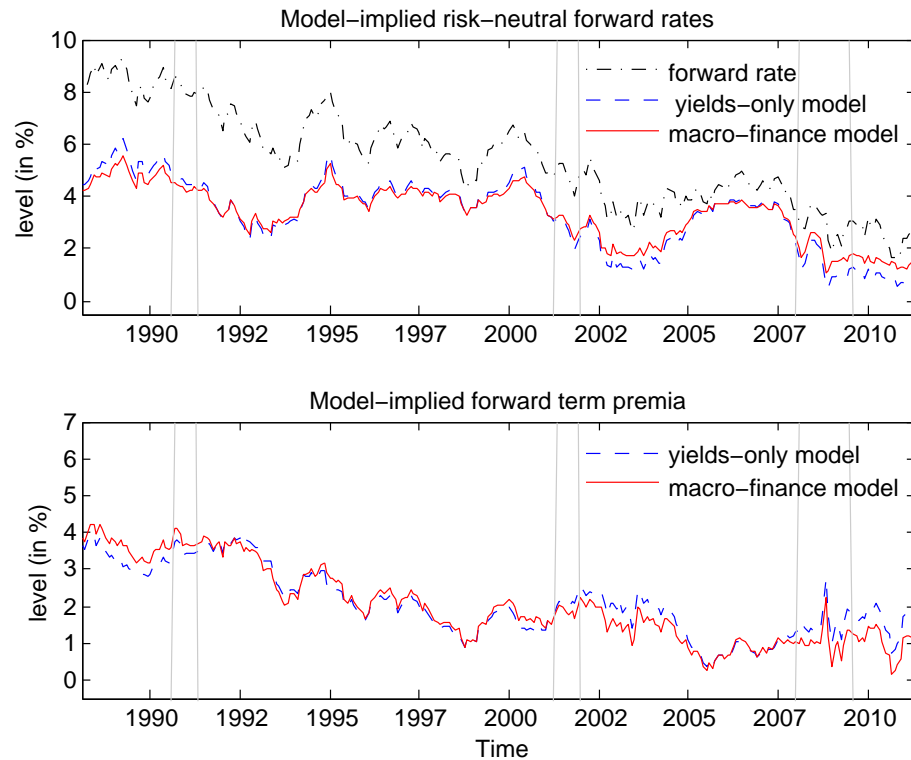
**Figure 1.11. Federal funds rate forecasts**

This figure plots the time series of the cross-sectional mean and dispersion of one year ahead forecasts of the Fed funds rate from the Blue Chip Financial Forecasts Survey. The data is sampled monthly and contains the period January 1988 to April 2011.



**Figure 1.12. Model-implied and EGARCH(1,1) conditional volatilities**

This figure plots the fit of model-implied conditional yield volatilities from the proposed model  $A_2^M(4)$  with monetary policy uncertainty as a risk factor (in red) to EGARCH(1,1) estimates of conditional yield volatility. Figures are in basis points (bps) per month. The data covers the period January 1988 to April 2011.



**Figure 1.13. Forward term premia, 1 year to 5 years**

This figure plots the decomposition of 1 year-to-5 years forward rates into model implied expected short rates (risk neutral forward rates) in the top panel for the two models (the proposed macro-finance model  $A_2^M(4)$  with monetary policy uncertainty as a risk factor as well as for the benchmark yields-only model  $A_1(3)$  of Dai and Singleton (2000)) and model-implied forward term premia (bottom panel). Shaded lines denote NBER recession periods. The data covers the period January 1988 to April 2011.

# Essay 2

## A Regime-Switching Affine Term Structure Model with Stochastic Volatility

Sebastian Fux and Desi Volker<sup>1</sup>

Working Paper

### Abstract

We develop and estimate a stochastic volatility regime-switching affine term structure model with state dependent transition probabilities and priced regime shift risk. We assume a flexible specification for the market prices of factor risks and time-varying market prices of regime shifts risk. We obtain closed form solutions for bond prices and estimate the model using MCMC. We assess the model's ability to match the time-series and cross-sectional properties of yields, as well as evaluate the role of the market prices of factor and regime risks in capturing the time variation in expected excess returns. We find that regime-switching models with stochastic volatility outperform their Gaussian counterparts and single-regime models in fitting the time-series properties of yield dynamics.

***JEL Classification:*** G12, E43, E52

***Keywords:*** Interest Rates, Regime Shifts, Stochastic Volatility

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## 2.1 Introduction

Monetary policy affects not only the short end but the entire yield curve, since movements in the short rate affect longer maturity yields by altering investor expectations of future bond prices. From an economic perspective, it is hence intuitively appealing to allow the yield curve to depend on different policy regimes. It is well documented in the literature that modeling the dynamics of the short rate as a regime-switching process is more appropriate in describing historical short rates (see, for example, [Hamilton \(1988\)](#), [Gray \(1996\)](#), [Garcia and Perron \(1996\)](#), [Ang and Bekaert \(2002a\)](#) and [Ang and Bekaert \(2002b\)](#)). In view of these findings, a number of papers followed by developing and analyzing interest rate models with regime switches, most notably [Naik and Lee \(1997\)](#), [Evans \(1998\)](#), [Landén \(2000\)](#) and [Bansal and Zhou \(2002\)](#), which confirmed that these models are better able in capturing the features of yield curve dynamics compared to their single-regime counterparts. In the recent years the literature has further moved on by analyzing regime-switching models in an affine term structure framework (we refer to e.g., [Ang, Bekaert, and Wei \(2007\)](#) and [Dai, Singleton, and Yang \(2007\)](#)). However, the increased complexity of introducing regime switches in terms of bond pricing and most importantly in terms of estimation has driven most of the literature to focus on Gaussian specifications of the state variable dynamics.

With this paper we contribute to the existing literature by analyzing the whole class of maximally-affine regime-switching term structure models, that is three-factor models with zero, one, two and three factors entering the volatility matrix. In line with the general definition of the single-regime class in [Dai and Singleton \(2000\)](#) the models are referred as  $A_0^{(RS)}(3)$ ,  $A_1^{(RS)}(3)$ ,  $A_2^{(RS)}(3)$ ,  $A_3^{(RS)}(3)$  where the subscript denotes the number of factors entering the volatility matrix and the superscript (RS) indicates regime-switching. We analyze the models performance in terms of overall goodness of fit as well as the ability to match some of the most important stylized facts of observed U.S. yield data. We examine the relative performance of the models along these lines and assess whether there is a benefit in moving firstly from a single-regime Gaussian model to a regime-switching Gaussian model, and secondly within the regime-switching class, moving from a Gaussian specification to stochastic-volatility specifications.

Our specification of the RS-ATSM's allows the intercept of the short rate and the market price of factor risk to be regime-dependent, enabling both the long run mean and the speed of mean reversion of the state variables to be regime-dependent under the physical measure. As indicated by [Bansal and Zhou \(2002\)](#) having a richer and regime-dependent specification of the market prices of factor risks is key for capturing the observed yield curve dynamics. With this specification of the RS-ATSM we are

still able to obtain analytical solutions for bond prices whilst allowing for considerable regime-dependence under the physical measure.

We generally would expect the models accounting for shifts in the economic regime to outperform their single-regime counterparts in terms of fitting historical yields. This effect is presumed to be larger for longer maturities, since during the life-span of longer maturity bonds the economy is more likely to be subject to changes in regimes. Our results provide some evidence that regime-switching stochastic volatility models are better equipped for fitting historical yield dynamics, compared to the regime-switching Gaussian model as well as to single-regime models. They display smaller variances of the measurement errors and generally smaller absolute average pricing errors, indicating that the yields implied by the RS-ATSM with stochastic volatility approximate the observed yields more closely. A model selection analysis using the Bayes factors confirms the above, indicating that the evidence provided by the data is in favor of RS-ATSM with stochastic volatility, the data-generating process of which seems more likely to give rise to the observed yields. Summarizing, we show evidence that affine term structure models with stochastic volatility (with one and two factors affecting volatility) display an improved ability to fit historical yields relative to both single-regime models and the regime-switching Gaussian model.

On a second step, we evaluate whether our preferred RS-ATSM models  $A_1^{(RS)}(3)$  and  $A_2^{(RS)}(3)$  are able to successfully match some of the most important stylized facts of U.S. yields. The main features of historical yields that we want our models to replicate are the predictability of bond returns (linear projections of changes in yields on the slope of the yield curve give negative fitted coefficients), the persistence and time-variability in conditional yield volatilities, as well as the term structure of the unconditional means.

The expectations hypothesis implies that excess returns are unpredictable. Conditional on current information, longer maturity yields are given as expected future short-rates plus a constant risk premium. Several empirical studies have shown that a significant portion of the variability in excess returns is forecastable and that the expectations hypothesis is violated. [Fama and Bliss \(1987\)](#) and [Campbell and Shiller \(1991\)](#) find that the slope of the yield curve has significant predictive power for excess returns, while [Cochrane and Piazzesi \(2005\)](#) find that a single factor, computed as a linear combination of forward rates, predicts an important part of the variation in excess returns, beyond the standard level, slope and curvature factors. In terms of matching these stylized facts of historical yield data, our results show an improvement of our preferred regime-switching stochastic volatility models over single-regime models. More precisely, within the single-regime class of models we find that the ability to capture the Campbell-Shiller regression coefficients decreases with the number

of factors that enter the volatility matrix of the latent factors, as documented in the previous literature (see, e.g., [Feldhütter \(2008\)](#)). In particular, within this class of models only the Gaussian model is able to replicate the sign and sizes of the coefficients. For the regime-switching models we find that now the  $A_1^{(RS)}(3)$  and  $A_2^{(RS)}(3)$  models, capture both the negative sign and the decreasing size with maturity of the Campbell-Shiller regression coefficients. Since sufficient variability and persistence in the market prices of risk is key in matching this feature, we conclude that the improvement of these models ability to replicate the failure of the expectations hypothesis is due to our specification of the market price of factor risk. In particular the variability in our extended-affine market price of risk comes both from its dependence in the risk factors (and their conditional volatility) and from the fact that its parameters ( $\lambda_0$  and  $\lambda_x$ ) are regime-dependent.

Another feature of the U.S. bond data is that the conditional volatility of yields displays significant time-variation and persistence (see, e.g., [Aït-Sahalia \(1996\)](#) and [Gallant and Tauchen \(1997\)](#)). Additionally, yield volatility is positively related to interest rates. A regression of squared yield changes on the level, slope and curvature of the U.S yield curve results in a positive coefficients associated with the level factor (see, e.g., [Brandt and Chapman \(2002\)](#) and [Piazzesi \(2010\)](#)). Within the class of RS-ATSM, we expect square root diffusion models to capture the higher moments of historical yield dynamics more closely than the single-regime counterparts. As for the Gaussian models, they preclude by definition time-varying conditional volatility. We find that RS-ATSM with stochastic volatility successfully capture the  $\beta$ -coefficient of a GARCH(1,1) model. The  $\beta$ -coefficient is around 0.8 and thus implying a rather strong persistence in the volatility of the yields. Furthermore, all specifications of the RS-ATSM with stochastic volatility are able to capture the level effect which showing positive regression coefficients when regressing model implied yield volatilities on the level factor.

Overall, this article shows that introducing regime-shifts in state-dependent volatility models narrows the gap between matching the cross-sectional and time-series properties of bond yields. We find evidence that RS-ATSM with stochastic volatility successfully describe historical yields while still being able to replicate important features of the U.S. yield curve.

The remainder of the paper is organized as follows. In [Section 2.2](#) we present the framework for our regime-switching affine term structure model. [Section 2.3](#) discusses the estimation methodology. [Section 2.4](#) presents the results and [Section 2.5](#) contains concluding remarks. An exposition of the technical details is supplied in the [Appendix](#).



## 2.2 Model Specification

In this section we present the formal set up of the regime-switching affine term structure model. We describe the model in its most general form, however, when estimating the model, we need to impose some restrictions which we explain in greater detail in Section 2.3. We begin by introducing the regime variable, proceed with a parameterization of the state variable dynamics under the risk neutral measure that allows analytical solutions for bond prices and terminate with the specification of the market prices of factor risk.

### 2.2.1 The Regime Variable

We assume a regime variable with discrete support  $k \in 1, 2, \dots, S^2$  and dynamics following a continuous-time Markov chain with infinitesimal matrix under the risk-neutral measure given by  $Q = \{q_{ij}\}_{i,j=1,\dots,S}$ . The intensity matrix is characterized by  $q_{ij} > 0, \forall i \neq j$  and  $q_{ii} < 0$ , such that  $q_{ii} = -\sum_{j=1}^S q_{ij}, \forall i$ . Hence the full transition rate matrix will be:

$$Q = \begin{bmatrix} -q_{11} & q_{12} & \dots & q_{1S} \\ q_{21} & -q_{22} & \dots & q_{2S} \\ \vdots & \vdots & \ddots & \vdots \\ q_{S1} & q_{S2} & \dots & -q_{SS} \end{bmatrix}$$

Over a small time interval  $\Delta_t$  the probability of staying in the same regime will be given by  $1 - q_{ii} \Delta_t$ . Thus, letting  $\Delta_t$  approach 0, we have that  $\lim_{\Delta_t \rightarrow 0^+} 1 - q_{ii} \Delta_t = 1$  implying that the probability of staying in the same regime approaches one over an infinite small time period. In a similar vein, the probability that the economy switches from regime  $i$  to regime  $j$  over a small time interval  $\Delta t$  is given by  $q_{ij} \Delta t$ . Thus, if  $\Delta_t$  approaches 0, we obtain that  $\lim_{\Delta_t \rightarrow 0^+} q_{ij} \Delta_t = 0$ , suggesting that the probability of a regime switch approaches zero over an infinite small time period.<sup>3</sup> Due to the Markov property the probability that the economy will be in a given regime in time  $t + 1$  depends only on the current regime and not on the entire history of the regime variable.

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<sup>2</sup>Theoretically there are no restrictions on how many regimes should be included in the analysis, however, for interpretational reasons we restrict our analysis to two regimes, as explained in Section 2.3.

<sup>3</sup>Over a time interval  $t$  the transition probability matrix is given by the exponential matrix  $\bar{Q} = e^{Q \cdot t}$ , which can be defined by means of a power series  $e^{Q \cdot t} = I + Q t + \frac{(Q t)^2}{2!} + \frac{(Q t)^3}{3!} + \dots$ , where  $I$  is the identity matrix. Over a small time interval we can ignore the quadratic and higher order terms and use the approximation  $\bar{Q} = I + Q \Delta t$ . For an introduction in continuous-time Markov Chains we refer to Karlin and Taylor (1975) and Lando (2004).

### 2.2.2 The Short Rate, the State Variables and Zero-Coupon Bond Pricing

In the absence of arbitrage opportunities the price of a zero-coupon bond at time  $t$  maturing at time  $T$  is given by:

$$P(t, T) = E_t^{\mathbb{Q}}[e^{-\int_t^T r_s ds}]$$

where the expectation is taken under the risk-neutral measure.

We specify the instantaneous short rate  $r_t$  to be an affine function of a vector of unobserved state variables  $X_t = (X_t^1, X_t^2, \dots, X_t^N)$

$$r_t = \delta_0^{(k)} + \sum_{i=1}^N \delta_i X_t = \delta_0^{(k)} + \delta_X' X_t$$

where  $k$  is an indicator for the regime. By allowing the constant term  $\delta_0^{(k)}$  to be regime-dependent we let the short rate's unconditional mean to vary across regimes. We restrict  $\delta_X$  to be regime-independent for analytical tractability.

The dynamics of the latent state variables is given by a mean-reversion square root diffusion process under  $\mathbb{Q}$ :

$$\begin{aligned} dX_t &= \kappa^{\mathbb{Q}} \left( \theta^{\mathbb{Q},(k)} - X_t \right) dt + \Sigma \sqrt{\sigma(X_t)} dW_t^{\mathbb{Q}} \\ &= \left( \kappa_0^{\mathbb{Q},(k)} - \kappa_1^{\mathbb{Q}} X_t \right) dt + \Sigma \sqrt{\sigma(X_t)} dW_t^{\mathbb{Q}} \end{aligned}$$

where  $dW_t^{\mathbb{Q}}$  is an  $N$ -dimensional vector of independent standard Brownian motions under the risk-neutral measure.  $\theta^{\mathbb{Q},(k)}$  is a regime-dependent vector representing the long-run mean of the state variables, while  $\kappa^{\mathbb{Q}}$  is the speed of mean reversion matrix. We keep  $\kappa_1^{\mathbb{Q}}$  constant across regimes in order to obtain closed-form solutions for bond prices.  $\kappa_0^{\mathbb{Q}}$  is a  $(N \times 1)$  vector for each regime while  $\kappa_1^{\mathbb{Q}}$  and  $\Sigma$  are  $(N \times N)$  matrices. As a novelty for RS-ATSM, we allow the volatility of the latent state variables to be state dependent which introduces conditional heteroskedasticity. In particular, the volatility matrix  $\sigma(X_t)$  is a diagonal matrix, with the  $i^{th}$  diagonal element given by  $[\sigma(X_t)]_{ii} = \alpha_i + \beta_i X_t$ , where  $\alpha_i \in \{0, 1\}$  and  $\beta_i$  is a  $N \times 1$  vector. [Dai and Singleton \(2000\)](#) classify models according to the number of state variables entering the volatility matrix  $\sqrt{\sigma(X_t)}$ . In their notation, an  $A_m(N)$  denotes a model with a total of  $N$  state variables, of which  $m$  enter the volatility matrix  $\sqrt{\sigma(X_t)}$ . In order for affine specifications to be admissible, restrictions must be imposed on the parameters to ensure positivity of the volatility matrix  $\sqrt{\sigma(X_t)}$ . [Dai and Singleton \(2000\)](#) provide the set of sufficient restrictions on the parameters of an  $A_m(N)$  model to assure admissibility.

The price of a zero-coupon bond,  $P(t, \tau, X, k) = P(t, \tau, k)$ , where  $\tau = T - t$  denotes the time to maturity, satisfies the following partial differential-difference equation (PDDE):

$$\begin{aligned} & \frac{1}{2} Tr \left( \frac{\partial^2 P}{\partial X \partial X'} \Sigma \sigma(x_t) \Sigma' \right) + \frac{\partial P}{\partial X'} \left( \kappa \left( \theta^{(k)} - X_t \right) \right) - \frac{\partial P}{\partial \tau} - \\ & \left( \delta_0^{(k)} + \delta_X' X_t \right) P(\tau, X_t, k) + \sum_{j=1, j \neq k}^K Q_{k,j} \left( P(\tau, X_t, j) - P(\tau, X_t, k) \right) = 0 \end{aligned}$$

subject to the boundary condition  $P(t, 0, k) = 1$ .

Following [Duffie and Kan \(1996\)](#) we conjecture that the solution to the above PDDE is exponentially affine:

$$P(t, \tau, k) = e^{A(\tau, k) + B(\tau)' X_t}.$$

To verify our conjecture we substitute  $\frac{\partial P}{\partial \tau}$ ,  $\frac{\partial P}{\partial X'}$  and  $\frac{\partial^2 P}{\partial X \partial X'}$  in the PDDE and rearrange terms in order to get a system of ordinary differential equations (ODE's). The solution of the ODE's results in a vector  $B(\tau)$  and S scalars  $A(\tau, k)$ . In particular, the set of ODE's that define A and B is given as:<sup>4</sup>

$$\begin{aligned} \frac{dB(\tau)}{d\tau} &= \frac{1}{2} \sum_{i=1}^m [\Sigma' B(\tau)]_i^2 \beta_i - \kappa_1' B(\tau) - \delta_X \\ \frac{dA(\tau, k)}{d\tau} &= \frac{1}{2} \sum_{i=1}^m [\Sigma' B(\tau)]_i^2 \alpha_i + \kappa_0^{(k)'} B(\tau) - \delta_0^{(k)} + \sum_{j=1, j \neq k}^K q_{k,j} \left( e^{A(\tau, j) - A(\tau, k)} - 1 \right). \end{aligned}$$

The above set of ODE's is completely determined by the specification of the short rate and state variable dynamics under the risk neutral measure. We solve these ODE's numerically using the Runge-Kutta method, with initial conditions  $A(0) = 0$  and  $B_{N \times 1}(0) = 0$ .

The continuously compounded yields will then be given by:

$$Y(t, \tau, k) = A^*(\tau, k) + B^*(\tau) X_t$$

where  $A^*(\tau, k) = -\frac{A(\tau, k)}{\tau}$  and  $B^*(\tau) = -\frac{B(\tau)}{\tau}$

In order to use the closed-form solution for  $P(t, \tau, k) = \exp(A^*(\tau, k) + B^*(\tau)' X_t)$  in the empirical analysis, we need to know the distribution of  $X_t$  and  $P(t, \tau, k)$  under the historical probability measure  $\mathbb{P}$ . The most general specification of the market price of factor risk that preserves the affine structure of  $X_t$  under  $\mathbb{P}$  is the ‘‘extended’’

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<sup>4</sup>For a detailed derivation of the ODE's defining  $A(\tau, k)$  and  $B(\tau)$  we refer to [Appendix 2.8](#).

specification of [Cheridito, Filipović, and Kimmel \(2007\)](#). In particular,

$$\Lambda_t^{(k)} = (\lambda_0^{(k)} + \lambda_1^{(k)} X_t) \Sigma^{-1} \sqrt{\sigma(X_t)}^{-1}$$

where  $\lambda_0^{(k)}$  is a  $N \times 1$  vector and  $\lambda_1^{(k)}$  is a  $N \times N$  matrix which are both regime-dependent. Using the above market price of factor risk specification, we discretize the process for the latent factors applying the Euler method. For the change of measure we have:

$$dW_t^{\mathbb{Q}} = dW_t^{\mathbb{P}} + \Lambda_t^{(k)} dt$$

Thus, under the historical measure  $\mathbb{P}$  the latent factor process is given as:

$$\begin{aligned} dX_t &= \left( \kappa_0^{\mathbb{Q},(k)} - \kappa^{\mathbb{Q}} X_t \right) dt + \Lambda_t^{(k)} dt + \Sigma \sqrt{\sigma(X_t)} dW_t^{\mathbb{P}} \\ &= \left( \kappa_0^{\mathbb{P},(k)} - \kappa_1^{\mathbb{P},(k)} X_t \right) dt + \Sigma \sqrt{\sigma(X_t)} dW_t^{\mathbb{P}} \end{aligned}$$

where  $\kappa_0^{\mathbb{P},(k)} = \kappa_0^{\mathbb{Q},(k)} + \lambda_0^{(k)}$  and  $\kappa_1^{\mathbb{P}} = \kappa_1^{\mathbb{Q}} - \lambda_1^{(k)}$ . In order to obtain admissibility (in the sense of [Dai and Singleton \(2000\)](#)) we have restricted  $\Sigma$  to be an identity matrix.

### 2.3 Estimation Methodology

In this section, we discuss the MCMC algorithm for estimating the RS-ATSM. MCMC methods have been used in the term structure literature by [Eraker \(2001\)](#), [Scott \(2002\)](#), [Sanford and Martin \(2005\)](#), [Ang, Dong, and Piazzesi \(2007\)](#), [Feldhütter \(2008\)](#), [Li, Li, and Yu \(2011\)](#) among others.<sup>5</sup> MCMC methods are computationally more complex than Maximum Likelihood methods, however, they offer some advantages which we outlay below.

#### 2.3.1 Setting up the MCMC Algorithm

An empirical analysis of a regime-switching affine term structure model entails extracting information regarding model parameters, state variables and regimes conditional on observed yields (obtained from zero-coupon bond prices). To do so, we observe  $M$  yields ( $\tau \in 1, \dots, M$ , where  $\tau$  denotes the time to maturity) at time  $t = 1, \dots, T$ , which are stacked in the vector  $Y(t, \tau, k) = Y(t, 1, k, \dots, Y(t, M, k)$ . We assume that all actual yields are observed with an *i.i.d.* measurement error, i.e.

$$Y(t, \tau, k) = A^*(\tau, k) + B^*(\tau)' X_t + \epsilon_t. \tag{A-1}$$

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<sup>5</sup>[Casella and Robert \(2004\)](#) provide a thorough introduction in general Monte Carlo Methods while [Johannes and Polson \(2010\)](#) provide a survey of MCMC applications within financial econometrics.

The measurement errors are normally distributed such that  $\epsilon \sim N(0, H)$  where  $H = \sigma^2 I_M$ .

Most of the literature in term structure modelling relies on the assumption that at any point in time at least three yields (with three different maturities) are precisely observed. With the  $B(\tau)$  matrix being invertible, this allows for a one-to-one mapping from the observed yields to the state variables, which can hence be pinned down exactly. The obtained state variables can then be used to estimate the remaining yields, i.e., those observed with an error, and the dynamics of all yields over time. This assumption leads to tractable estimation of the model, such as with Maximum Likelihood. However, [Cochrane and Piazzesi \(2005\)](#) observe that the fact that we are only able to observe yields imprecisely might hinge on the Markov structure of the term structure and hence partially explain the inability of term-structure models to forecast future excess bond returns. [Duffee \(2011\)](#) notes that the existence of an observation error can potentially create partially hidden factors, where only part of the information regarding the factor can be found in the cross-section, so that models relying strictly on yield data will have difficulties in reliably fitting yield dynamics.

These facts motivated us to use a Bayesian approach which is less vulnerable to these issues than traditional maximum likelihood techniques. More precisely, MCMC methods enable us to relax the restrictive (and unrealistic) assumption of perfectly observed yields, so that we can allow all yields to be observed with an error. We assume that the observation error of the yields for any maturity has the same variance. The intuition behind this choice lies in the fact that the main sources of observation error are market imperfections which affect bond prices and risk premia and plain measurement error, all of which potentially affect bonds with different maturities in the same way.

The main objective of the estimation analysis is to make inference about the model parameters  $\Theta$ , the latent variables  $\mathbf{X} = \{X_t\}_{t=1}^T$  and the regime variables  $\mathbf{K} = \{k_t\}_{t=1}^T$  based on the observed yields  $\mathbf{Y} = \{Y_t^\tau\}_{t \in 1, \dots, T}^{\tau \in 1, \dots, M}$ .

Characterizing the joint posterior distribution,  $p(\Theta, \mathbf{K}, \mathbf{X} | \mathbf{Y})$ , is difficult due to its high dimension, the fact that the model is specified in continuous time while the yield data is observed discretely and since the state variables transition distributions are non-normal. Furthermore parameters enter the model as solutions to a system of ODE's (the A and B functions derived in the previous section). MCMC allows us to simultaneously estimate parameters, state variables and regimes for non-linear, non-Gaussian state space models as is our RS-ATSM and at the same time accounts for estimation risk and model specification uncertainty.

For interpretational reasons we restrict our analysis to two regimes, thus,  $k = 1, 2$ . Each of the regimes  $k$  is characterized by the following set of parameters:

$$\Theta = \left( \kappa_0^{\mathbb{Q},(k)}, \kappa_1^{\mathbb{Q}}, \delta_0^{(k)}, \delta_X, \lambda_0^{(k)}, \lambda_1^{(k)}, H, \text{ and } Q^{kj} \text{ for } k, j = 1, 2 \right).$$

In addition we also need to filter the regime of the underlying regime process  $\mathbf{K}$ , as well as the latent state variables  $\mathbf{X}$ . The numerical identification of this highly dimensional parameter space proves to be challenging. However, due to the flexibility of the Bayesian techniques we avoid imposing several parameter restrictions as e.g. in [Dai, Singleton, and Yang \(2007\)](#). The only restriction we impose in order to facilitate the estimation is that  $\kappa_0^{\mathbb{Q},(k)}$  is regime-independent, that is  $\kappa_0^{\mathbb{Q},(k)} = \kappa_0^{\mathbb{Q}}$ .

In order to be able to sample from the target distribution  $p(\Theta, \mathbf{K}, \mathbf{X} | \mathbf{Y})$ , we make use of two important results, the Bayes rule and the Hammersley-Clifford theorem.

By Bayes Rule we have:

$$\begin{aligned} p(\Theta, \mathbf{K}, \mathbf{X} | \mathbf{Y}) &\propto p(\mathbf{Y}, \mathbf{X}, \mathbf{K}, \Theta) \\ &= p(\mathbf{Y} | \mathbf{X}, \mathbf{K}, \Theta) p(\mathbf{X}, \mathbf{K} | \Theta) p(\Theta) \end{aligned}$$

where the conditional likelihood function of the yields is given by

$$\begin{aligned} p(\mathbf{Y} | \mathbf{X}, \mathbf{K}, \Theta) &= \prod_{\tau=1}^M \prod_{t=1}^T H_{\tau\tau}^{-\frac{1}{2}} \exp \left( -\frac{\left( Y(t, \tau) - \hat{Y}(t, \tau, k) \right)^2}{2H_{\tau\tau}} \right) \\ &= \frac{1}{\sigma^{-MT}} \exp \left( -\frac{1}{2\sigma^2} \sum_{t=1}^T \left( \epsilon_t^{k'} \epsilon_t^k \right) \right) \end{aligned}$$

where  $\epsilon_t^k = Y(t, \tau) - \hat{Y}(t, \tau, k)$ .

To derive the joint likelihood  $p(\mathbf{X}, \mathbf{K} | \Theta)$  we rely on a Euler discretization to approximate the continuous-time specification of the latent variable process resulting in the following discrete time process:

$$\Delta X_{t+1} = \mu_t^{\mathbb{P},(k)} \Delta t + \sqrt{\Delta t} \sigma(X_t) \varepsilon_{t+1}.$$

The drift under  $\mathbb{P}$  is given by  $\mu_t^{\mathbb{P},(k)} = \left( \kappa_0^{\mathbb{Q}} + \lambda_0^{(k)} \right) - \left( \kappa_1^{\mathbb{Q}} - \lambda_1^{(k)} \right) X_t$ , the measurement error is normally distributed  $\varepsilon_t \sim N(0, I_N)$  and  $\Delta t$  denotes the discrete time interval

between two subsequent observations. Thus, the joint density  $p(\mathbf{X}, \mathbf{K}|\Theta)$  is as

$$\begin{aligned} p(\mathbf{X}, \mathbf{K}|\Theta) &= \prod_{t=2}^T p(X_{t+1}|X_t, K_t) \exp(Q\Delta t)_{k_{t-1}, k_t} \\ &= \prod_{n=1}^N \left( \left( \prod_{t=2}^T \frac{1}{\sqrt{[\sigma(X_t)]_{nn}}} \right) \exp \left( -\frac{1}{2\Delta t} \sum_{t=1}^T \frac{[\Delta X_{t+1} - \mu_t^{\mathbb{P},(k)} \Delta t]_n^2}{[\sigma(X_t)]_{nn}} \right) \right) \\ &\quad \prod_{t=2}^T \exp(Q\Delta t)_{k_{t-1}, k_t}. \end{aligned}$$

MCMC is a method to obtain the joint distribution  $p(\Theta, \mathbf{K}, \mathbf{X}|\mathbf{Y})$  which is usually unknown and complex. The Hammersley-Clifford theorem (see [Hammersley and Clifford \(2012\)](#) and [Besag \(1974\)](#)) states that the joint posterior distribution is characterized by its complete set of conditional distributions:

$$p(\Theta, \mathbf{K}, \mathbf{X}|\mathbf{Y}) \iff p(\Theta|\mathbf{K}, \mathbf{X}, \mathbf{Y}), p(\mathbf{K}|\Theta, \mathbf{X}, \mathbf{Y}), p(\mathbf{X}|\Theta, \mathbf{K}, \mathbf{Y})$$

Given initial draws  $k^{(0)}$ ,  $X^{(0)}$  and  $\Theta^{(0)}$ , we draw  $k^{(n)} \sim p(k|X^{(n-1)}, \Theta^{(n-1)}, Y)$ ,  $X^{(n)} \sim p(X|k^{(n)}, \Theta^{(n-1)}, Y)$  and  $\Theta^{(n)} \sim p(\Theta|k^{(n)}, X^{(n)}, Y)$  and so on until we reach convergence. The sequence  $\{k^{(n)}, X^{(n)}, \Theta^{(n)}\}_{n=1}^N$  is a Markov Chain with distribution converging to the equilibrium distribution  $p(\Theta, \mathbf{K}, \mathbf{X}|\mathbf{Y})$ .

More specifically, at each iteration, we sample from the conditionals:

$$\begin{aligned} &p\left(\kappa_0^{\mathbb{Q},(k)} \mid \kappa_1^{\mathbb{Q}}, \delta_0^{(k)}, \delta_X, \lambda_0^{(k)}, \lambda_1^{(k)}, k, H, Q, X, Y\right) \\ &p\left(\kappa_1^{\mathbb{Q}} \mid \kappa_0^{\mathbb{Q},(k)}, \delta_0^{(k)}, \delta_X, \lambda_0^{(k)}, \lambda_1^{(k)}, k, H, Q, X, Y\right) \\ &\quad \vdots \\ &p\left(k \mid \kappa_0^{\mathbb{Q},(k)}, \kappa_1^{\mathbb{Q}}, \delta_0^{(k)}, \delta_X, \lambda_0^{(k)}, \lambda_1^{(k)}, H, Q, X, Y\right) \\ &p\left(X \mid \kappa_0^{\mathbb{Q},(k)}, \kappa_1^{\mathbb{Q}}, \delta_0^{(k)}, \delta_X, \lambda_0^{(k)}, \lambda_1^{(k)}, k, H, Q, Y\right) \end{aligned}$$

To sample new parameters, we rely on the Random-Walk Metropolis-Hastings (RW-MH) algorithm which is a two-step procedure that first samples a candidate draw from a chosen proposal distribution and then accepts or rejects the draw based on an acceptance criterion specified a priori. For example, we sample a new  $\delta_X$  as  $[\delta_X]^{n+1} = [\delta_X]^n + \gamma N(0, 1)$  where  $\gamma$  is used to calibrate the variance of the proposal distribution. In a second step we calculate the acceptance probability as:

$$\alpha = \min \left( 1, \frac{p([\delta_X]^{n+1}|\cdot)}{p([\delta_X]^n|\cdot)} \right).$$

In case that we are able to sample directly from the conditional distribution, we

make use of the Gibbs Sampler (GS). The Gibbs Sampling is a special case of the Metropolis-Hastings algorithm in which the proposal distributions exactly match the posterior conditional distributions and in which proposals are accepted with a probability of one.<sup>6</sup>

After having obtained  $\{K^{(n)}, X^{(n)}, \Theta^{(n)}\}_{n=1}^N$ , the point estimates of the parameters of interest will then be given as the marginal posterior means, that is

$$E(\Theta_i|Y) = \frac{1}{N} \sum_{n=1}^N \Theta_i^{(n)}.$$

Summing up, our hybrid MCMC algorithm looks as below:

$$\begin{aligned} p(k|X, Y, \Theta) &\sim \text{RW-MH} \\ p(X|k, Y, \Theta) &\sim \text{RW-MH} \\ p(\Theta^h|\Theta_{\setminus h}, X, k, Y) &\sim \text{RW-MH} \\ p(\sigma|Y) &\sim \text{GS}. \end{aligned}$$

Both the parameters and the latent factors are subject to constraints and if a draw violates a constraint it can be discarded (see [Gelfand, Smith, and Lee \(1992\)](#)). The efficiency of the RW-MH algorithm depends crucially on the variance of the proposal distribution. [Roberts, Gelman, and Gilks \(1997\)](#) and [Roberts and Rosenthal \(2001\)](#) show that for optimal convergence, we need to calibrate the variance such that roughly 25% of the newly sampled parameters are accepted. To calibrate these variances we run one million iterations where we evaluate the acceptance ratio after 100 iterations. The variance of the of the normal proposal are adjusted such that they yield acceptance ratios between 10% and 30%. This calibration sample is followed by burn-in period which consist of 700000 iterations. Finally, the estimation period consists of 300000 iterations where we keep every 100th iteration resulting in 3000 draws for inference.<sup>7</sup>

### 2.3.2 Yield Data

The empirical implementation of the MCMC algorithm relies on a set of monthly zero coupon Treasury yields obtained from the [Gürkaynak, Sack, and Wright \(2007\)](#) database, with time series November 1971 to January 2011.<sup>8</sup> The maturities included

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<sup>6</sup>We refer to [Chib and Greenberg \(1995\)](#) for introductory exposition of the Metropolis-Hastings algorithm and [Casella and George \(1992\)](#) for a detailed explanation of the Gibbs Sampler.

<sup>7</sup>For a complete description of the MCMC algorithm we refer to Appendix 2.9.

<sup>8</sup>The original data set available online at the Board of Governors of the Federal Reserve System, has a daily frequency. We have transformed the data to a monthly frequency by keeping the last day of each month as that months corresponding yield value.



in the estimation are one, three, five, seven, ten, twelve and fifteen years. Given the shorter available sample length for higher maturities, our choice in terms of the data used, is the result of an implicit trade-off between the length of the time series and the highest maturity included, both of relevance in a regime-switching set-up. We emphasize the importance of the sample period, which according to the National Bureau of Economic Research (NBER) is characterized by six recessions and includes the FED's monetary experiment in the 80's, providing a basis for different economic regimes to have potentially occurred. Secondly, relatively longer maturities allow for the possibility of regime changes to have occurred during their life-time, hence including them in the estimation might give rise to more robust results. In the next section we investigate how well regime-switching models fit historical yields and if they are able to match some of the features of observed U.S. yields.

## 2.4 Results

### 2.4.1 MCMC estimates

Table 2.1 presents the parameter estimates from the MCMC estimation for the single regime affine term structure models while regime-independent parameter estimates for the regime-switching model are shown in Table 2.2 and regime-dependent parameters are reported in Table 2.3. Parameter estimates are based on the mean of the MCMC estimation sample. The 2.5% and 97.5% quantile of the MCMC samples are reported in parenthesis.

INSERT TABLE 2.1 TO 2.3 ABOUT HERE

We begin our analysis by evaluating how well the different models are able to describe the conditional distribution of observed U.S. zero coupon bond yields. To assess the cross-sectional fit of the different models we look at several measures, starting with the variance of the measurement error in Equation A-1, proceeding with the average absolute pricing errors for each of these models and concluding with a model-comparison analysis performed with the Bayes Factor. We then move on to analyzing how well these models manage to match some of the most important features of observed U.S. zero coupon bond yield data, such as the relationship between the slope of the yield curve and expected excess returns, the matching of the unconditional first moment of yields as well as that of the shape and persistence of conditional volatilities of yield changes.

### 2.4.2 Model comparison

The first metric that we examine to compare the different model specifications is the measurement error of Equation A-1. Mikkelsen (2002) attributes the measurement error to data issues such as rounding errors, observational noise, different data sources, etc. but also to fact that the assumed model is only an approximation to the process that determines interests rates. Hence, the smaller the measurement error, the closer the approximation of observed yields by the model implied yields. In this paper, we focus on fitting a given term structure model to a given set of yields and thus, a small measurement error is taken as an indication of good fit of the term structure model to the actual yield data.

Table 2.4 reports the variance of the measurement error in basis points for all the estimated models.

INSERT TABLE 2.4 ABOUT HERE

The two models with the smallest variance of the measurement error are the  $A_1(3)^{(RS)}$  (where the superscript (RS) denotes regime-switching) and the  $A_2(3)^{(RS)}$  model, showing that RS-ATSM with stochastic volatility match the observed yields most accurately. We also find evidence that the  $A_3(3)$  model is outperformed by the  $A_1(3)$  and the  $A_2(3)$  model. This finding does not only hold for the models with a single regime but also for the regime-switching models and is well documented in e.g. Dai and Singleton (2000) where it is argued that the performance of the  $A_3(3)$  model deteriorates due to the restriction on the conditional correlation among the state variables.

#### *Pricing errors*

We proceed by evaluating the ability to match cross-sectional properties of the yields, that is, the ability of different model specifications to approximate the observed yield curve at any date during the sample period. For each maturity we calculate the absolute pricing error (APE( $\tau$ )), for  $\tau = \{1, 3, 5, 7, 10, 12, 15\}$  years, as below:

$$\text{APE}(\tau) = \frac{\sum_{t=1}^T |\hat{Y}(t, \tau) - Y(t, \tau)|}{T}.$$

where  $\hat{Y}(t, \tau)$  denotes simulated model implied yields and  $Y(t, \tau)$  denotes observed yields. To calculate the simulated model-implied yields for each date, we treat the parameter estimates of each MCMC draw after convergence has occurred as the true population parameters and simulate for each maturity a set of yields with the

same length as our observed yields sample. The simulated model implied yields for each maturity will then be given as the average over these sets of yields. Table 2.5 provides a summary statistics of the  $APE(\tau)$  for the affine term structure models we have considered.

INSERT TABLE 2.5 ABOUT HERE

Since pricing errors mainly arise due to model misspecification, generally the smaller the pricing error the lower is the likelihood that the model is misspecified. As shown in Table 2.5 , pricing errors decrease for models accounting for stochastic volatility as well as multiple regimes. Moving from single regime to multiple regime models seems to generate a significant decrease in average absolute pricing errors across all classes of models regardless of the number of factors affecting the volatility of the risk factors. Furthermore, a passage from the Gaussian regime-switching model to regime-switching models with time-varying conditional volatility decreases the pricing errors further.

In accordance with the evidence from the variance of the measurement error, the pricing errors show that the  $A_1^{(RS)}(3)$  model and the  $A_2^{(RS)}(3)$  model show a better fit to observed yields compared to single regime models as well as to the regime-switching Gaussian model. This subfamily of term structure models lies between the Gaussian model, that is the  $A_0^{(RS)}(3)$  model, and the correlated square-root diffusion, that is the  $A_3^{(RS)}(3)$  model. Dai and Singleton (2000) find that this subfamily of term structure models is superior.<sup>9</sup> Thus in the subsequent sections we follow their approach and analyze the performance of the  $A_1^{(RS)}(3)$  and  $A_2^{(RS)}(3)$  relative to the Gaussian model with either one regime or multiple regimes.

### *The Bayes factor*

In this section we turn to formally investigate the relative performance of the models to fit historical yields. A widely used means of model selection in the Bayesian literature is the Bayes factor, which quantifies the evidence provided by the data in favor of the alternative model  $M_1$  compared to a benchmark model  $M_0$ . The Bayes factor is approximated by the ratio of the marginal likelihoods of the data in each of the two models considered for comparison and is obtained by integrating these densities over the whole parameter space. More precisely, given prior odds  $p(M_0)$  and  $p(M_1)$  for the models and given the observed yield data  $Y$  , the Bayes Theorem

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<sup>9</sup>See Section 2.4.4 for a detailed discussion about the advantages of the  $A_1^{(RS)}(3)$  model and the  $A_2^{(RS)}(3)$  model.

implies:

$$\frac{p(M_1|Y)}{p(M_0|Y)} = \frac{p(Y|M_1)}{p(Y|M_0)} \times \frac{p(M_1)}{p(M_0)}$$

where the ratio of the marginal likelihoods under the two models,  $p(Y|M_1)/p(Y|M_0)$ , denotes the Bayes factor. Assuming un-informative priors  $p(M_0) = p(M_1) = 0.5$ , the Bayes factor is given by the posterior odds.<sup>10</sup> A detailed discussion of Bayes factor can be found in [Kass and Raftery \(1995\)](#).

The larger the Bayes factor, the stronger the evidence in favor of alternative model  $M_1$  compared to the benchmark model  $M_0$ . [Kass and Raftery \(1995\)](#) establish a rule of thumb saying that a Bayes factor exceeding 3 indicates that the data provides 'substantial' evidence in favor of the alternative model versus the benchmark model. Table 2.6 provides results on model comparison with the Bayes factor.

INSERT TABLE 2.6 ABOUT HERE

To begin with, we assess the indication of the Bayes factor regarding model selection between regime-switching models versus the single regime Gaussian model (i.e. the benchmark is the  $A_0^{(SR)}(3)$  model, that is column one of the above table). We notice that the Bayes factor indicates that there is substantial evidence in support of all the other regime-switching models against the single regime Gaussian model. Secondly, we assess that within the regime-switching class of models, the evidence of the Bayes factor seems to be in favor of stochastic volatility models (i.e. the  $A_1^{(RS)}(3)$  and  $A_2^{(RS)}(3)$  model) compared to the Gaussian model. Since the Bayes factor considers the overall relative goodness-of-fit, this might not be surprising. The Gaussian model, precludes by definition time-varying conditional volatility, which in the data has been shown to be counterfactual.

The evidence we found so far shows that the data generating process underlying the U.S. zero coupon yields is seemingly most likely described by a regime-switching model which allows for stochastic volatility in the process of the underlying state variables. More precisely, the  $A_1^{(RS)}(3)$  model and the  $A_2^{(RS)}(3)$  model have shown smaller variances of the measurement errors and smaller average absolute pricing errors. Furthermore model selection analysis by the Bayes factor has shown evidence in favor of these models. Thus, in the next section we investigate the regime probabilities and the ability to match the term structure of unconditional means of the U.S. yields of the  $A_2^{(RS)}(3)$  models.

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<sup>10</sup>In the absence of free parameters and latent variables, where maximum likelihood estimates of the parameters for both models are feasible, the Bayes factor corresponds to a likelihood ratio. In our case, the presence of unknown parameters, latent factors as well as latent regimes, requires that we integrate out the parameters, latent variables and regimes to obtain the marginal likelihood  $p(Y|M_1)$  and  $p(Y|M_0)$ . We refer to Appendix 2.10 for a detailed explanation of the procedure followed.

### 2.4.3 Regimes

Figure 2.1 shows a time series of posterior probabilities of the regime variable, that is, the probability that the economy is either in regime 1 or regime 2 of the  $A_2^{(RS)}(3)$  model. The shaded areas represent periods of recessions identified by the NBER.

INSERT FIGURE 2.1 ABOUT HERE

These plots suggest that regime 2 tends to be associated with recessions, while expansions are related to regime 1. The economy switches for the first time to regime 2 in July 1972 and remains there during the oil crisis in 1973. Also during the recessions in the beginning of the 1980's we are in regime 2, which prevails until the early 1990's (with two short interruptions). The plots show evidence that the first regime is prolonged well beyond the end of the recession in 1982, however, this is a common finding which has previously been documented in e.g. Dai, Singleton, and Yang (2007) and Li, Li, and Yu (2011). In the second half of our sample period the first regime is more pervasive. It is interrupted only three times by the second regime, the last time just before the dot-com crises. Overall, the second regimes prevails more often in the first half of our sample period, where recession appear more often, while the first regime is more persistent in the second half of our sample period.

Figure 2.1 shows that both regimes are rather persistent, that is, the probability for a regime switch is much smaller than the probability of staying in the same regime. This fact is reflected in the transition matrix which shows how likely it is to switch between regimes over the next month. The transition matrix for  $\Delta_t = 1$  month is given as below:

$$\exp(Q\Delta_t) = \begin{bmatrix} 0.739 & 0.261 \\ 0.276 & 0.724 \end{bmatrix}.$$

The transition matrix shows that the probability of switching from regime 1 (2) to regime 2 (1) is 26.1% (27.6%) over the next month, thus, suggesting a strong regime persistence. Additionally, the probability of staying in regime 1 is 73.9% while it is 72.4% for the second regime. The transition matrix shows that both regimes are almost equally persistent. This fact is confirmed in Figure 2.1 where both regimes occur approximately equally often. We relate this finding to the model specification of the RS-ATSM with stochastic volatility, where the volatility is not explicitly regime-dependent and the regimes are thus associated with the level of the yields.

This finding is confirmed when we look at the unconditional means of the yields in both regimes. In general, unconditional means of treasury yields are on average increasing with maturity. In order to see whether our model-implied yields are able

to reproduce these features, we simulate model-implied means and volatilities (along with confidence bands) for each of the regimes and show them against their sample counterparts.

To calculate model implied unconditional means we simulate 100 series of yields, each with the same length as the observed data for every MCMC draw of the estimation period. We condition on the regime variable of the corresponding MCMC draw for each date of our sample period and calculate the latent factors using the parameters from the MCMC draw. We average over the 100 simulated yields and then across the draws to obtain the term structure of unconditional means, as well as the 95% confidence band. Next we compute the unconditional mean of the observed yields for each of the regimes. To do so, we sample the regime for each date of our sample period from the posterior distribution (as explained in Appendix 2.10) and sort out the historical yields according to the regime assigned to each date, then compute sample means for each of the regimes.

Figure 2.2 shows the term structure of unconditional means for each regime for the simulated model-implied yields and their observed sample counterparts.

INSERT FIGURE 2.2 ABOUT HERE

Figure 2.2 confirms our expectation by showing that the unconditional mean of the yields in regime 1 is considerably lower than in the second regime. Additionally, we emphasize that the term structure of unconditional means is upward sloping, replicating the fact that on average investors require higher interest rates for holding longer maturity bonds. The observed yields unconditional mean fall within the 95% confidence bounds of the respective simulated model-implied unconditional first moment.

#### 2.4.4 *Matching the features of bond yields*

In this section we look at the ability of our model implied yields to fit the historical behavior of the U.S. term structure of interest rates. Standard procedure in the literature is to look at four measures, that is, the model's ability to match the stylized facts in terms of the predictability of bond returns as well as the time variability in conditional yield volatilities and their persistence.

The ultimate test of any theoretical model is its ability to match the features of the data it aims to describe and its potential to forecast the dynamic evolution of the variables of interest. In the context of affine term structure models, the overall goodness of fit of the model is measured in terms of its ability to match the cross-section and time-series of observed yields. A tension and trade-off generally arises in fitting both the cross-sectional and time-series properties of yields with affine term structure

models. The first crucially depends on a flexible correlation structure between the state variables determining the short rate, while the second on the persistence and time variation of the conditional volatility of the yields. The Gaussian model (i.e. the  $A_0(3)$  model) performs relatively well in fitting the cross-section of observed yields, while by definition precluding time-varying conditional volatility. On the other hand, the correlated square root diffusion model (i.e. the  $A_3(3)$  model) is able to some extent to replicate the time variability in yield volatilities, but given its restriction in the sign of the correlation structure of risk factors performs worse in terms of the first feature. Following [Dai and Singleton \(2000\)](#), and given the inability of the  $A_3(3)$  model to generate negative correlations between the state variables, as suggested by historical interest rate data, most empirical research concentrates on analyzing the three maximally affine subfamilies consisting of the  $A_0(3)$ ,  $A_1(3)$  and  $A_2(3)$  model. For sufficiently flexible market price of risk specifications the overall fit of the  $A_1(3)$  and  $A_2(3)$  relatively improves, so that combined with the fact that the  $A_0(3)$  precludes time-varying volatility, these models become more appealing.

The regime-switching literature concentrates almost exclusively on the Gaussian model while generally abstaining from analyzing the  $A_1(3)$  and  $A_2(3)$  model, mainly due to the complexity that arises in terms of modelling and most importantly in terms of estimation. In this paper we provide a basis for a general analysis of the whole class of maximally affine term structure models with regime-switches. More precisely, we assess whether there is a benefit in moving firstly from a single-regime Gaussian model to a regime-switching Gaussian model, and secondly within the regime-switching class, moving from a Gaussian specification to stochastic-volatility specifications, that is the  $A_1^{(RS)}(3)$  and  $A_2^{(RS)}(3)$  model. We begin our analysis by looking at the models ability to replicate the Campbell-Shiller regression.

#### *Predictability of excess returns*

An important stylized fact of observed yield data is that expected excess returns are time varying. Starting with [Fama \(1984\)](#), empirical studies on U.S. yield data document that the slope of the yield curve has predictive power for future changes in yields. [Campbell and Shiller \(1991\)](#) show that linear projections of future yield changes on the slope of the yield curve give negative coefficients ( $\beta(\tau) < 0$  in Equation A-2), which are increasing with the time to maturity. [Backus, Foresi, Mozumdar, and Wu \(2001\)](#) and other studies confirm this finding across different sample periods. More precisely, the Campbell-Shiller regression reads as

$$Y_{t+\tau}^{\tau-\tau_1} - Y_t^\tau = \alpha(\tau) + \beta(\tau) \left[ \frac{\tau_1}{\tau - \tau_1} (Y_t^\tau - Y_t^{\tau_1}) \right] + \epsilon_t(\tau) \quad (\text{A-2})$$

where the shortest available maturity is denoted with  $\tau_1$  and  $\tau$  is given in years.  $\alpha(\tau)$  and  $\beta(\tau)$  indicate maturity specific constant and slope coefficients. The results of [Campbell and Shiller \(1991\)](#) imply that an increase in the slope of the yield curve is associated with a decrease in long term yields and vice-versa, hence the current slope of the yield curve is indicative of the direction in which future long rates will most likely move. The expectations hypothesis on the contrary states that risk premia are constant and future bond returns are unpredictable. This empirical failure of the expectations hypothesis is one of the main puzzles in financial economics and being able to reproduce this feature of the yield data is hence important for any term structure model.

Table 2.7 presents the Campbell-Shiller coefficients obtained from the above regression with our sample of historical U.S. yield data, confronted with the coefficients obtained from simulated model-implied yields.<sup>11</sup>

INSERT TABLE 2.7 ABOUT HERE

As we can clearly see from Table 2.7, within the single regime class of models, the models' ability to capture the sign and size of the Campbell-Shiller regression coefficients deteriorates with the number of factors affecting the covariance structure of the latent state variables.<sup>12</sup> A finding which is consistent with the single-regime literature findings of e.g. [Dai and Singleton \(2003\)](#) and [Feldhütter \(2008\)](#). However, moving to the regime-switching class of models, we notice that compared to single regime models, where only the  $A_0^{(SR)}(3)$  model can capture the negative sign of the Campbell-Shiller coefficients (as well as the increase in absolute size of the coefficients as maturity increases), the  $A_1^{(RS)}(3)$  and  $A_2^{(RS)}(3)$  model is able to capture these features if we allow for multiple regimes. These models match the negative sign of the historical Campbell-Shiller coefficients for most maturities and the size of the coefficients decreases with the maturity in a similar fashion to that of the historical data coefficients. The actual magnitude of the model implied and actual regression coefficients are similar, with the models' confidence bands containing the actual data coefficients for most of the maturities (with the 1-year yield as the exception). Turning to models  $A_1^{(RS)}(3)$  and  $A_2^{(RS)}(3)$ , we believe that their improvement in matching the sign and sizes of the Campbell-Shiller coefficients compared to their single-regime counterparts, comes from the flexibility in changing signs for the market price of risk. For regime-switching models in particular the structure of risk

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<sup>11</sup>The ability to replicate the Campbell-Shiller coefficients usually deteriorates with the number of factors entering the volatility matrix of the underlying state variables, i.e. that the Gaussian model outperforms the models with stochastic volatility. In order to see the benefit of the regimes Table 2.7 also includes the  $A_1^{(SR)}(3)$  and the  $A_2^{(SR)}(3)$  model. To obtain model-implied yields as well its observed counterparts we apply the procedure as described in Section 2.4.3.

<sup>12</sup>Since the spacing between maturities in our case is not constant we approximate the unobserved yields, both model-implied and historical ones, following [Campbell and Shiller \(1991\)](#).



premia appears to be one of the fundamental factors affecting the model's ability in matching the Campbell-Shiller regression coefficients. A model specification that allows only the state variables' long run mean to be regime-dependent but not their volatility, requires a regime-dependent market price of factor risk through either the constant of proportionality  $\lambda_0$  or the factor loading  $\lambda_1$ , or both, so that the volatility of the state variable and the risk premia can vary across regimes independently. Our market price of risk specification allows for both  $\lambda_0$  and the factor loading  $\lambda_1$  to be regime dependent, implying that even though the speed of mean reversion are constant under the risk-neutral measure they become regime-dependent under the physical measure, resulting in the observed improvement. It is interesting to confirm through our results in this section, that introducing regimes closes to some extent the wedge between the Gaussian and the correlated square-root diffusion models in terms of fitting the Campbell-Shiller regression coefficients.<sup>13</sup>

#### *Conditional yield volatilities*

Another important feature of the historical U.S. yield data is the time variation and persistence of conditional volatilities of yield changes.

Brandt and Chapman (2002) and Piazzesi (2010) show that conditional yield volatilities are positively varying with interest rates. We are interested in evaluating whether our models are able to reproduce this feature of the data, and hence analyze whether the volatility of our model implied yields is correlated with the level of model-implied yields in a similar fashion. Since regressing yield volatility on the yields themselves would create potential problems of multicollinearity, we regress conditional volatilities on the level, slope and curvature of the yield curve. Litterman and Scheinkman (1991) show that the level, slope and curvature factors explain at least 96% of the variation in excess returns across maturities and are virtually orthogonal and thus, we avoid potential problems of multicollinearity. We then look at the significance, sign and size of the coefficients in order to assess the extent at which the level, slope and curvature factors have explanatory power regarding the time-variation in zero-coupon bond yields.

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<sup>13</sup>Due to the small sample bias it would be interesting to also report model-implied theoretical coefficients, besides the simulated model-implied coefficients and the historical coefficients. Since our model allows for multiple regimes, it is intuitively not so clear how to interpret the comparison of the coefficients on a per-regime basis, hence to be consistent with the existing literature we limit our analysis to simulated model-implied Campbell-Shiller coefficients.

In particular, we run the below regression for our sample of historical yield data and simulated model-implied yields:<sup>14</sup>

$$(Y(t+1, \tau) - Y(t, \tau))^2 = \alpha(\tau) + \beta_1(\tau)Y(t, \tau_1) + \beta_2(\tau) [Y(t, \tau_M) - Y(t, \tau_1)] + \beta_3(\tau) [Y(t, \tau_M) + Y(t, \tau_1) - 2Y(t, \tau_{mid})] + \epsilon_{t, \tau} \text{ for } \tau = 1, \dots, M.$$

The shortest available yield is denoted with  $\tau_1$  while the most long-term yield is indicated with  $\tau_M$ . To calculate the curvature we rely on maturity which lies between  $\tau_1$  and  $\tau_M$  which is given by  $\tau_{mid}$ .

Table 2.8 reports estimates of the regression coefficients for the observed yields and the for the model implied yields of the  $A_1(3)$  and  $A_2(3)$  model. The  $A_0(3)$  model precludes time-varying volatility by definition and is hence omitted from the analysis.

INSERT TABLE 2.8 ABOUT HERE

Table 2.8 shows that volatility is positively correlated with the level of the observed yields. The level coefficient of the actual yield data is positive for all maturities and exhibits a downward trend along the maturity. All models with the stochastic volatility feature capture the positive sign of the level coefficient as well as the decreasing pattern of the slope coefficient. The shorter the time to maturity, the better is the level coefficient of the model implied yields. However, all models fail to replicate the actual magnitude of the level coefficient. A similar reasoning applies to the coefficients of the slope and curvature.

The evidence of the volatility regression is consistent with the results of [Brandt and Chapman \(2002\)](#) who argue that only the class of quadratic term structure models are able to accommodate both the dynamics of conditional expected bond returns and their conditional volatility. The difficulties to match volatility can also be explained by the sample period. [Christiansen and Lund \(2005\)](#) argue that the period of the “monetary experiment”, that is 1979-1982, should be excluded when investigating volatility and the shape of the yield curve. Considering sub-samples may improve the ability of the models to match stylized facts related to volatility.

After having performed the above regression analysis we proceed with a more formal evaluation of the model’s performance with regards to its ability to produce sufficient persistence in the time-variation of yield volatilities so as to be in line with that of the historical data. Following [Dai and Singleton \(2003\)](#), we estimate a GARCH(1,1)

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<sup>14</sup>To obtain model-implied yields and its observed counterparts we apply the same procedure as described in Section 2.4.3.

model<sup>15</sup> for yields with selected maturities using first historical data and then simulated yields for each of the models considered.<sup>16</sup>

In order to examine the benefit of multiple regimes, Table 2.9 reports GARCH estimates for the  $A_1(3)$  and  $A_2(3)$  model for both a single regime and a regime-switching setting.

INSERT TABLE 2.9 ABOUT HERE

The results shown in Table 2.9 indicate that all models capture the persistence in the yield volatility displayed by the historical yield data quite well. This fact holds for all maturities. The  $\beta$ -coefficients for the model-implied GARCH(1,1) coefficients are of similar magnitude to those of the historical data for most maturities, with an average size of circa 0.8, indicating that shocks to conditional variance take quite some time to die out.  $\alpha$ -coefficients for the model-implied GARCH(1,1) regressions are typically lower than those implied by the historical data  $\alpha$ -coefficients, indicating that volatility is slower to react to market movements relative to what the historical results show, i.e. model-implied volatility is less spiky than the historical volatility would imply. For both, the  $A_1(3)$  and the  $A_2(3)$  model, and across all maturities it seems that the regime-switching models estimate the  $\alpha$ - and  $\beta$ -coefficient more accurate than single-regime models.

Overall, we found evidence that introducing regimes in the family of affine term structure models improves the cross-sectional fit, meaning that regime-switching models approximate the yield curve more accurate than single regime models. More importantly we also showed that RS-ATSM with stochastic volatility, and in particular the  $A_1(3)^{(RS)}$  and the  $A_2(3)^{(RS)}$  model, outperform the Gaussian regime-switching, that is the  $A_0(3)^{(RS)}$  model. The superior performance of the stochastic volatility models is reflected in smaller measurement errors, smaller average absolute pricing errors and Bayes factors of beyond three.

We also showed that RS-ATSM with stochastic volatility successfully match some of the stylized facts of the U.S. yield curve such as unconditional first moment and time-varying conditional volatility. Additionally, allowing for multiple regimes improves the ability to replicate the Campbell-Shiller regression coefficients, as shown by the  $A_1^{(RS)}(3)$  model. However, the regime-switching  $A_2^{(RS)}(3)$  and  $A_3^{(RS)}(3)$  model lack the ability to reproduce this stylized fact.

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<sup>15</sup>The GARCH(1,1) model is given as  $\sigma_t = \bar{\sigma} + \alpha\epsilon_t^2 + \beta\sigma_{t-1}^2$ , where  $\epsilon_t$  is the residual of the AR(1) representation of the selected maturity. We use the observed variance of the residuals  $\epsilon_t$ , as a starting estimate for the variance of the first observation.

<sup>16</sup>Instead of simulating 100 series of yields for each MCMC draw of the estimation period we treat the average of the parameters of the estimation period as the true population parameters. Based on this parameters we simulate 1000 series of yield using the usual procedure of Section 2.4.4 and fit a GARCH model to the yields in order to obtain the distribution of GARCH coefficients.

## 2.5 Concluding Remarks

In this paper we embed multiple regimes in an affine term structure model and assess the ability of the RS-ATSM to reproduce historical yields as well as some of the stylized facts of the U.S. yield curve. More precisely, we analyzed the performance of RS-ATSM with a stochastic volatility feature relative to Gaussian models with either a single regime or multiple regimes. We find evidence that RS-ATSM with stochastic volatility successfully describe historical yields while still being able to replicate important features of the U.S. yield curve.

We show that introducing regimes in the family of affine term structure models improves the cross-sectional fit, meaning that regime-switching models approximate the yield curve more accurately than single regime models. Our preferred models, that is the  $A_1^{(RS)}(3)$  model and the  $A_2^{(RS)}(3)$  model, exhibit the smallest measurement error and generate the smallest pricing errors. This finding is supported by the Bayes factor which also shows that these two models are superior.

Additionally, the above mentioned models successfully capture some of the stylized facts of the U.S. yield curve such as unconditional first and second moments and time-varying conditional volatility. We also find that  $A_2^{(RS)}(3)$  model and  $A_3^{(RS)}(3)$  replicate the coefficients of the Campbell-Shiller much closer than the single regime models.

Our specification of the RS-ATSM allows to analytically solve for bond prices whilst there is still considerable regime-dependence. Introducing priced regime shift risk might be an interesting enhancement of our model specification, however, a market price of regime shift risk proved to be difficult to be estimated using our estimation approach.

## 2.6 Tables

**Table 2.1 Single regime affine term structure models: MCMC parameter estimates**

This table reports parameter estimates and confidence bands for the single regime (denoted with superscript (SR)) extended affine term structure models. The parameter estimate is the average of every 100'th iteration of the estimation period consisting of 300000 iteration (i.e. the variance calibration sample and a burn-in period are excluded). The confidence bounds reported in parenthesis indicate the 95% confidence interval.

|                   | $A_0^{(SR)}(3)$           | $A_1^{(SR)}(3)$             | $A_2^{(SR)}(3)$            | $A_3^{(SR)}(3)$           |
|-------------------|---------------------------|-----------------------------|----------------------------|---------------------------|
| $\kappa_0^Q(1)$   | 0                         | 111.767<br>(99.629;123.144) | 0.525<br>(0.501;0.581)     | 2.140<br>(2.036;2.281)    |
| $\kappa_0^Q(2)$   | 0                         | 0                           | 41.645<br>(37.171;44.798)  | 0.807<br>(0.534;1.133)    |
| $\kappa_0^Q(3)$   | 0                         | 0                           | 0                          | 4.850<br>(4.657;5.024)    |
| $\kappa_0^P(1)$   | 17.848<br>(-1.169;0.311)  | 7.838<br>(10.431;27.682)    | 1.921<br>(0.868;20.042)    | 1.757<br>(0.536;4.566)    |
| $\kappa_0^P(2)$   | 1.017<br>(-2.438;-5.197)  | 4.402<br>(-5.851;13.963)    | 15.743<br>(1.433;36.636)   | 2.781<br>(0.578;7.151)    |
| $\kappa_0^P(3)$   | -7.917<br>(-25.733;7.307) | 45.894<br>(19.914;75.148)   | -8.659<br>(-15.232;-1.929) | 1.836<br>(0.535;4.711)    |
| $\kappa_1^Q(1,1)$ | 0.040<br>(0.038;0.042)    | 2.015<br>(1.996;2.028)      | 0.093<br>(0.090;0.098)     | 0.027<br>(0.025;0.029)    |
| $\kappa_1^Q(1,2)$ | 0                         | 0                           | -0.091<br>(-0.096;-0.086)  | -0.057<br>(-0.060;-0.053) |
| $\kappa_1^Q(1,3)$ | 0                         | 0                           | 0                          | -0.049<br>(-0.056;-0.041) |
| $\kappa_1^Q(2,1)$ | 6.495<br>(6.272;6.616)    | 0.208<br>(0.202;0.212)      | -1.409<br>(-1.415;-1.404)  | -0.013<br>(-0.020;-0.001) |
| $\kappa_1^Q(2,2)$ | 10.017<br>(9.873;10.152)  | 0.314<br>(0.307;0.324)      | 1.503<br>(1.492;1.514)     | 2.473<br>(2.466;2.479)    |
| $\kappa_1^Q(2,3)$ | 0                         | -0.065<br>(-0.069;-0.061)   | 0                          | -0.008<br>(-0.014;-0.002) |
| $\kappa_1^Q(3,1)$ | 0.596<br>(0.436;0.756)    | -0.452<br>(-0.463;-0.441)   | -0.022<br>(-0.028;-0.017)  | -0.019<br>(-0.025;-0.014) |
| $\kappa_1^Q(3,2)$ | 3.220<br>(3.100;3.326)    | -0.531<br>(-0.542;-0.519)   | 0.581<br>(0.573;0.591)     | -0.007<br>(-0.013;0.000)  |
| $\kappa_1^Q(3,3)$ | 0.492                     | 0.153                       | 2.827                      | 2.337                     |

*Continued on next page*

**Table 2.1** – *Continued from previous page*

|                    | $A_0^{(SR)}(3)$                 | $A_1^{(SR)}(3)$                 | $A_2^{(SR)}(3)$                 | $A_3^{(SR)}(3)$                 |
|--------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
|                    | (0.321;0.697)                   | (0.146;0.159)                   | (2.821;2.835)                   | (2.329;2.349)                   |
| $\kappa_1^P(1, 1)$ | -0.111<br>(-0.288;0.051)        | -0.012<br>(-0.039;-0.000)       | -0.137<br>(-0.288;-0.028)       | -0.192<br>(-0.283;-0.110)       |
| $\kappa_1^P(1, 2)$ | -0.787<br>(-1.799;0.219)        | 0                               | 0.036<br>(0.001;0.124)          | 0.614<br>(0.029;1.632)          |
| $\kappa_1^P(1, 3)$ | -0.235<br>(-0.526;0.046)        | 0                               | 0                               | 3.462<br>(1.975;4.817)          |
| $\kappa_1^P(2, 1)$ | 2.370<br>(1.939;2.784)          | 0.014<br>(-0.015;0.046)         | 0.193<br>(0.019;0.405)          | 0.021<br>(0.000;0.066)          |
| $\kappa_1^P(2, 2)$ | 26.359<br>(25.068;27.336)       | -0.253<br>(-0.458;-0.068)       | -0.230<br>(-0.500;-0.112)       | -7.710<br>(-8.994;-6.355)       |
| $\kappa_1^P(2, 3)$ | 7.404<br>(6.921;7.810)          | -0.052<br>(-0.102;-0.000)       | 0                               | 10.907<br>(9.337;12.202)        |
| $\kappa_1^P(3, 1)$ | -11.458<br>(-13.354;-9.935)     | -0.198<br>(-0.277;-0.128)       | -0.400<br>(-0.648;-0.112)       | 0.016<br>(0.000;0.049)          |
| $\kappa_1^P(3, 2)$ | -119.211<br>(-121.397;-116.194) | 1.352<br>(0.941;1.791)          | 0.118<br>(0.021;0.221)          | 3.776<br>(2.681;4.789)          |
| $\kappa_1^P(3, 3)$ | -33.375<br>(-33.903;-32.355)    | -0.163<br>(-0.315;-0.006)       | -1.443<br>(-2.234;-0.503)       | -8.884<br>(-10.363;-7.461)      |
| $\delta_0$         | 0.135<br>(0.132;0.139)          | 0.152<br>(0.136;0.169)          | 0.002<br>(0.000;0.005)          | -0.621<br>(-0.626;-0.609)       |
| $\delta_x(1)$      | 0.025<br>(0.024;0.026)          | 0.000<br>(0.000;0.000)          | 0.006<br>(0.006;0.006)          | 0.000<br>(0.000;0.000)          |
| $\delta_x(2)$      | 0.018<br>(0.017;0.019)          | 0.005<br>(0.005;0.005)          | 0.001<br>(0.000;0.001)          | 0.180<br>(0.180;0.181)          |
| $\delta_x(3)$      | 0.000<br>(0.000;0.000)          | 0.000<br>(0.000;0.000)          | 0.028<br>(0.028;0.029)          | 0.258<br>(0.258;0.259)          |
| $\beta_2(1)$       | 0                               | 0.045<br>(0.036;0.054)          | 0                               | 0                               |
| $\beta_3(1)$       | 0                               | 0.673<br>(0.562;0.793)          | 0.008<br>(0.000;0.029)          | 0                               |
| $\beta_3(2)$       | 0                               | 0                               | 0.026<br>(0.009;0.038)          | 0                               |
| $\sigma^2$         | 7.45e-06<br>(6.70E-06;8.11E-06) | 1.37E-06<br>(1.28E-06;1.44E-06) | 8.51E-07<br>(7.80E-07;9.05E-07) | 4.51E-06<br>(4.27E-06;4.76E-06) |

**Table 2.2 Regime switching affine term structure models: MCMC estimates of regime independent parameters**

This table reports MCMC estimates and confidence bands of the regime independent parameters for all regime switching affine term structure models. The parameter estimate is the average of every 100'th iteration of the estimation sample consisting of 300000 iteration (i.e. the variance calibration sample and a burn-in period are excluded). The confidence bounds reported in parenthesis indicate the 95% confidence interval.

|                    | $A_0^{(RS)}(3)$           | $A_1^{(RS)}(3)$           | $A_2^{(RS)}(3)$           | $A_3^{(RS)}(3)$           |
|--------------------|---------------------------|---------------------------|---------------------------|---------------------------|
| $\kappa_0^Q(1)$    | 0                         | 2.175<br>(2.059;2.279)    | 2.950<br>(2.933;2.970)    | 1.362<br>(1.286;1.406)    |
| $\kappa_0^Q(2)$    | 0                         | 0                         | 0.901<br>(0.880;0.921)    | 7.029<br>(6.811;7.111)    |
| $\kappa_0^Q(3)$    | 0                         | 0                         | 0                         | 9.714<br>(9.453;9.819)    |
| $\kappa_1^Q(1, 1)$ | 0.217<br>(0.207;0.226)    | 0.177<br>(0.168;0.184)    | 1.609<br>(1.604;1.617)    | 0.072<br>(0.069;0.077)    |
| $\kappa_1^Q(1, 2)$ | 0                         | 0                         | -0.299<br>(-0.301;-0.297) | -0.123<br>(-0.129;-0.118) |
| $\kappa_1^Q(1, 3)$ | 0                         | 0                         | 0                         | -0.002<br>(-0.010;0.000)  |
| $\kappa_1^Q(2, 1)$ | 5.003<br>(4.946;5.068)    | -0.058<br>(-0.065;-0.047) | -0.175<br>(-0.188;-0.164) | -0.885<br>(-0.894;-0.880) |
| $\kappa_1^Q(2, 2)$ | 8.746<br>(8.666;8.806)    | 0.990<br>(0.982;0.995)    | 0.033<br>(0.031;0.035)    | 1.676<br>(1.673;1.679)    |
| $\kappa_1^Q(2, 3)$ | 0                         | 1.046<br>(1.039;1.057)    | 0                         | -0.551<br>(-0.562;-0.542) |
| $\kappa_1^Q(3, 1)$ | 1.812<br>(1.797;1.827)    | -0.020<br>(-0.023;-0.016) | 3.364<br>(3.357;3.373)    | -0.173<br>(-0.179;-0.167) |
| $\kappa_1^Q(3, 2)$ | 2.910<br>(2.863;2.966)    | 0.099<br>(0.082;0.108)    | -0.688<br>(-0.690;-0.685) | -0.256<br>(-0.271;-0.243) |
| $\kappa_1^Q(3, 3)$ | -0.008<br>(-0.010;-0.005) | 0.106<br>(0.091;0.116)    | 0.321<br>(0.313;0.329)    | 1.919<br>(1.912;1.924)    |
| $\delta_x(1)$      | 0.067<br>(0.066;0.068)    | -0.004<br>(-0.005;-0.004) | 0.030<br>(0.030;0.030)    | 0.034<br>(0.034;0.034)    |
| $\delta_x(2)$      | 0.080<br>(0.079;0.081)    | 0.003<br>(0.003;0.003)    | -0.005<br>(-0.005;-0.005) | -0.074<br>(-0.074;-0.074) |
| $\delta_x(3)$      | 0.007<br>(0.007;0.008)    | 0.010<br>(0.010;0.011)    | 0.004<br>(0.004;0.004)    | 0.068<br>(0.068;0.069)    |
| $\beta(2, 1)$      | 0                         | 1.864                     | 0                         | 0                         |

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**Table 2.2** – *Continued from previous page*

|              | $A_0^{(RS)}(3)$                 | $A_1^{(RS)}(3)$                   | $A_2^{(RS)}(3)$                 | $A_3^{(RS)}(3)$                 |
|--------------|---------------------------------|-----------------------------------|---------------------------------|---------------------------------|
|              |                                 | (1.453;2.248)                     |                                 |                                 |
| $\beta(3,1)$ | 0                               | 0.124<br>(0.095;0.149)            | 0.715<br>(0.050;1.386)          | 0                               |
| $\beta(3,2)$ | 0                               | 0                                 | 0.143<br>(0.008;0.284)          | 0                               |
| Q(1,1)       | -1.781<br>(-2.275;-1.228)       | -0.489<br>(-1.042;-0.273)         | -0.374<br>(-0.675;-0.187)       | -2.093<br>(-2.739;-1.596)       |
| Q(2,2)       | -0.718<br>(-1.109;-0.438)       | -0.547<br>(-0.897;-0.298)         | -0.396<br>(-0.775;-0.174)       | -1.531<br>(-2.146;-1.002)       |
| $\sigma^2$   | 3.45E-07<br>(3.23E-07;3.68E-07) | 2.75E-07<br>(2.571E-07;2.946E-07) | 2.52E-07<br>(2.37E-07;2.69E-07) | 8.11E-07<br>(7.64E-07;8.62E-07) |



**Table 2.3 Regime switching affine term structure models: MCMC estimates of regime dependent parameters**

This table reports MCMC estimates and confidence bands of the regime dependent parameters for all regime switching affine term structure models. The parameter estimate is the average of every 100'th iteration of the estimation sample consisting of 300000 iteration (i.e. the variance calibration sample and a burn-in period are excluded). The confidence bounds reported in parenthesis indicate the 95% confidence interval.

|                   | $A_0^{(RS)}(3)$          |                           | $A_1^{(RS)}(3)$            |                          | $A_2^{(RS)}(3)$             |                            | $A_3^{(RS)}(3)$           |                           |
|-------------------|--------------------------|---------------------------|----------------------------|--------------------------|-----------------------------|----------------------------|---------------------------|---------------------------|
|                   | Regime 1                 | Regime 2                  | Regime 1                   | Regime 2                 | Regime 1                    | Regime 2                   | Regime 1                  | Regime 2                  |
| $\kappa_0^P(1)$   | -1.083<br>(-2.158;0.040) | -0.787<br>(-2.075;0.376)  | 8.826<br>(2.205;15.781)    | 7.864<br>(3.204;13.172)  | 17.588<br>(3.982;34.303)    | 7.940<br>(1.401;17.046)    | 4.304<br>(0.691;11.728)   | 2.343<br>(0.579;7.518)    |
| $\kappa_0^P(2)$   | 2.327<br>(1.288;3.455)   | 0.601<br>(-0.805;2.215)   | -0.620<br>(-15.319;14.381) | 14.537<br>(2.761;26.127) | 7.164<br>(0.766;18.853)     | 13.977<br>(1.340;34.940)   | 20.089<br>(8.244;32.274)  | 10.525<br>(2.044;21.188)  |
| $\kappa_0^P(3)$   | 1.274<br>(0.182;2.382)   | 2.165<br>(0.789;3.847)    | 3.480<br>(-1.318;8.434)    | 4.672<br>(0.992;8.374)   | -17.291<br>(-29.917;-4.482) | -18.826<br>(-37.449;2.379) | 9.370<br>(3.116;17.493)   | 5.272<br>(0.957;11.314)   |
| $\kappa_1^P(1,1)$ | 0.736<br>(0.221;1.262)   | -1.025<br>(-1.955;-0.056) | 0.643<br>(0.218;1.074)     | 0.331<br>(0.093;0.632)   | 4.298<br>(0.936;7.997)      | 1.738<br>(0.349;3.522)     | 0.262<br>(0.093;0.505)    | 0.203<br>(0.071;0.392)    |
| $\kappa_1^P(1,2)$ | 0.272<br>(-0.131;0.661)  | -1.888<br>(-3.093;-0.636) | 0<br>(-1.250;0.107)        | 0<br>(-0.625;-0.009)     | -0.657<br>(-1.250;0.107)    | -0.262<br>(-0.625;-0.009)  | -0.161<br>(-0.528;-0.004) | -0.100<br>(-0.339;-0.003) |

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Table 2.3 – Continued from previous page

|                    | $A_0^{(RS)}(3)$           |                           | $A_1^{(RS)}(3)$          |                          | $A_2^{(RS)}(3)$           |                             | $A_3^{(RS)}(3)$              |                           |
|--------------------|---------------------------|---------------------------|--------------------------|--------------------------|---------------------------|-----------------------------|------------------------------|---------------------------|
|                    | Regime 1                  | Regime 2                  | Regime 1                 | Regime 2                 | Regime 1                  | Regime 2                    | Regime 1                     | Regime 2                  |
| $\kappa_1^P(1, 3)$ | -0.086<br>(-0.254;0.078)  | -0.549<br>(-0.978;-0.129) | 0                        | 0                        | 0                         | 0                           | -0.156<br>(-0.479;-0.005)    | -0.083<br>(-0.297;-0.002) |
| $\kappa_1^P(2, 1)$ | -0.999<br>(-1.555;-0.487) | 3.173<br>(2.136;4.179)    | 0.032<br>(-0.662;0.745)  | 0.021<br>(-0.408;0.404)  | -1.119<br>(-3.211;-0.044) | -10.692<br>(-12.968;-8.024) | -8.631<br>(-9.757;-7.387)    | -1.978<br>(-3.988;-0.897) |
| $\kappa_1^P(2, 2)$ | 0.433<br>(-0.009;0.869)   | 4.067<br>(2.695;5.424)    | 0.698<br>(0.174;1.336)   | 1.030<br>(0.512;1.541)   | 0.379<br>(0.104;0.791)    | 2.312<br>(1.770;2.848)      | 18.705<br>(15.944;21.035)    | 3.984<br>(1.721;8.440)    |
| $\kappa_1^P(2, 3)$ | 0.351<br>(0.176;0.528)    | 0.929<br>(0.463;1.389)    | 0.889<br>(0.080;1.802)   | 2.283<br>(1.130;3.379)   | 0                         | 0                           | -15.034<br>(-16.920;-13.040) | -2.032<br>(-5.461;-0.214) |
| $\kappa_1^P(3, 1)$ | -0.707<br>(-1.231;-0.183) | 0.397<br>(-0.653;1.377)   | 0.074<br>(-0.161;0.299)  | 0.189<br>(0.067;0.321)   | -3.707<br>(-6.258;-1.205) | -2.088<br>(-6.874;3.985)    | -0.050<br>(-0.148;-0.002)    | -0.081<br>(-0.226;-0.003) |
| $\kappa_1^P(3, 2)$ | -0.461<br>(-0.867;-0.062) | 1.767<br>(0.365;3.026)    | -0.047<br>(-0.236;0.136) | -0.072<br>(-0.227;0.087) | 0.597<br>(0.154;1.033)    | 0.211<br>(-0.943;1.134)     | -0.115<br>(-0.339;-0.004)    | -0.169<br>(-0.425;-0.008) |
| $\kappa_1^P(3, 3)$ | 0.134<br>(-0.034;0.304)   | 1.176<br>(0.717;1.616)    | 0.263<br>(-0.011;0.555)  | 0.036<br>(-0.339;0.418)  | 0.226<br>(0.039;0.493)    | 1.504<br>(0.721;2.243)      | 0.789<br>(0.359;1.448)       | 1.310<br>(0.797;1.866)    |
| $\delta_0$         | 0.053<br>(0.047;0.054)    | 0.065<br>(0.059;0.068)    | 0.063<br>(0.060;0.067)   | 0.073<br>(0.071;0.078)   | -0.008<br>(-0.014;-0.005) | 0.004<br>(-0.002;0.010)     | -0.022<br>(-0.031;-0.018)    | 0.008<br>(0.001;0.015)    |

**Table 2.4**  
**Measurement Errors of the different Affine Term Structure Model Specifications**

This table reports the measurement error of the four different affine term structure models for models with a single regime and models with two regimes. The measurement error is the average of every 100<sup>th</sup> iteration of the estimation sample consisting of 300000 iteration (i.e. the variance calibration sample and a burn-in period are excluded). The confidence bounds reported in parenthesis indicate the 95% confidence interval.

|          | Single regime Models       | Regime-switching Models |
|----------|----------------------------|-------------------------|
| $A_0(3)$ | 27.3477<br>(25.873;28.471) | 5.872<br>(5.687;6.068)  |
| $A_1(3)$ | 11.637<br>(11.306;12.018)  | 5.247<br>(5.070;5.429)  |
| $A_2(3)$ | 9.221<br>(8.943;9.512)     | 5.023<br>(4.866;5.184)  |
| $A_3(3)$ | 21.225<br>(20.657;21.811)  | 9.006<br>(8.742;9.285)  |

**Table 2.5**  
Average absolute pricing errors

This table reports the summary statistics of the four different affine term structure models for models with a single regime and models with two regimes. The absolute pricing errors are calculated over the 495 dates for all seven maturities. The sample period is 11/1971-01/2011.

|                 | Maturity in Years |        |        |        |        |        |        |
|-----------------|-------------------|--------|--------|--------|--------|--------|--------|
|                 | 1                 | 3      | 5      | 7      | 10     | 13     | 15     |
| $A_0(3)^{(SR)}$ |                   |        |        |        |        |        |        |
| Mean            | 30.291            | 27.985 | 18.061 | 12.283 | 14.830 | 20.293 | 24.142 |
| Std             | 3.700             | 4.537  | 2.501  | 3.487  | 3.245  | 2.304  | 3.884  |
| $A_1(3)^{(SR)}$ |                   |        |        |        |        |        |        |
| Mean            | 11.859            | 11.500 | 9.315  | 7.032  | 5.603  | 6.557  | 9.831  |
| Std             | 18.207            | 25.292 | 16.696 | 9.178  | 10.877 | 14.776 | 20.969 |
| $A_2(3)^{(SR)}$ |                   |        |        |        |        |        |        |
| Mean            | 7.213             | 7.366  | 8.754  | 7.372  | 4.360  | 5.411  | 9.074  |
| Std             | 0.789             | 3.647  | 5.638  | 4.448  | 2.493  | 2.922  | 6.489  |
| $A_3(3)^{(SR)}$ |                   |        |        |        |        |        |        |
| Mean            | 18.207            | 25.292 | 16.696 | 9.178  | 10.877 | 14.776 | 20.969 |
| Std             | 2.710             | 15.178 | 8.754  | 3.494  | 5.846  | 7.878  | 11.839 |
| $A_0(3)^{(RS)}$ |                   |        |        |        |        |        |        |
| Mean            | 4.776             | 5.830  | 3.680  | 4.750  | 3.884  | 3.106  | 5.019  |
| Std             | 0.959             | 3.505  | 1.504  | 2.730  | 3.012  | 0.991  | 3.191  |
| $A_1(3)^{(RS)}$ |                   |        |        |        |        |        |        |
| Mean            | 0.959             | 3.505  | 1.504  | 2.730  | 3.012  | 0.991  | 3.191  |
| Std             | 4.014             | 4.643  | 3.060  | 3.755  | 3.083  | 2.624  | 4.698  |
| $A_2(3)^{(RS)}$ |                   |        |        |        |        |        |        |
| Mean            | 4.014             | 4.643  | 3.060  | 3.755  | 3.083  | 2.624  | 4.698  |
| Std             | 7.126             | 7.502  | 7.879  | 7.156  | 4.582  | 5.105  | 9.066  |
| $A_3(3)^{(RS)}$ |                   |        |        |        |        |        |        |
| Mean            | 7.126             | 7.502  | 7.879  | 7.156  | 4.582  | 5.105  | 9.066  |
| Std             | 0.244             | 3.259  | 5.045  | 4.276  | 2.563  | 2.873  | 6.488  |

**Table 2.6**  
**Model comparison by the Bayes factor**

This table reports the Bayes factor for the ATSM's. The performance of the regime switching models is compared with a single regime Gaussian model denoted with  $A_0(3)^{(SR)}$  as well as among the regime-switching models (denoted with a superscript  $(RS)$ ). A detailed explanation of the calculation of the Bayes factor is in Appendix 2.10.

|                   |                 | Benchmark Model |                 |                 |                 |                 |
|-------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                   |                 | $A_0(3)^{(SR)}$ | $A_0(3)^{(RS)}$ | $A_1(3)^{(RS)}$ | $A_2(3)^{(RS)}$ | $A_3(3)^{(RS)}$ |
| Alternative Model | $A_0(3)^{(SR)}$ | 1               |                 |                 |                 |                 |
|                   | $A_0(3)^{(RS)}$ | 2.047           | 1               |                 |                 |                 |
|                   | $A_1(3)^{(RS)}$ | 5.884           | 2.875           | 1               |                 |                 |
|                   | $A_2(3)^{(RS)}$ | 42.954          | 20.987          | 7.300           | 1               |                 |

**Table 2.7 Campbell-Shiller Regression**

This table reports MCMC estimates and confidence bands of the regime dependent parameters for all regime switching affine term structure models. The parameter estimate is the average of every 100'th iteration of the estimation sample consisting of 300000 iteration (i.e. the variance calibration sample and a burn-in period are excluded). The confidence bounds reported in parenthesis indicate the 95% confidence interval.

| Data             | Maturity in Years        |                           |                           |                           |                          |                          |  |
|------------------|--------------------------|---------------------------|---------------------------|---------------------------|--------------------------|--------------------------|--|
|                  | 3                        | 5                         | 7                         | 10                        | 12                       | 15                       |  |
| $A_0^{(SR)}$ (3) | -0.452<br>(-1.530;0.137) | -1.015<br>(-2.470;-0.255) | -1.491<br>(-3.169;-0.244) | -2.091<br>(-3.911;-0.068) | -2.410<br>(-3.996;0.216) | -2.988<br>(-4.389;0.457) |  |
| $A_1^{(SR)}$ (3) | 1.767<br>(-0.698;3.525)  | 1.998<br>(-0.975;4.008)   | 2.030<br>(-1.329;4.242)   | 1.909<br>(-1.887;4.389)   | 1.562<br>(-2.332;4.104)  | 1.547<br>(-3.026;4.563)  |  |
| $A_2^{(SR)}$ (3) | 1.277<br>(0.824;1.585)   | 1.342<br>(0.850;1.746)    | 1.440<br>(0.882;1.936)    | 1.619<br>(0.955;2.239)    | 1.744<br>(1.014;2.453)   | 1.932<br>(1.099;2.747)   |  |
| $A_0^{(RS)}$ (3) | -0.037<br>(-2.376;1.273) | -0.106<br>(-2.192;1.677)  | -0.315<br>(-2.242;1.892)  | -0.720<br>(-2.736;2.132)  | -1.080<br>(-3.056;2.092) | -1.521<br>(-3.807;2.338) |  |
| $A_1^{(RS)}$ (3) | 0.389<br>(-0.873;1.502)  | 0.062<br>(-1.667;1.542)   | -0.271<br>(-2.426;1.549)  | -0.807<br>(-3.624;1.497)  | -1.262<br>(-4.423;1.193) | -1.764<br>(-5.619;1.194) |  |
| $A_2^{(RS)}$ (3) | 0.221<br>(-0.169;0.833)  | -0.022<br>(-0.657;0.913)  | -0.113<br>(-1.009;1.023)  | -0.191<br>(-1.454;1.127)  | -0.221<br>(-1.693;1.162) | -0.281<br>(-2.006;1.203) |  |

**Table 2.8 Volatility Regression**

This table reports estimated slope coefficients of the volatility regression. The regression is given by  $[Y(t+1, \tau) - Y(t, \tau)]^2 = \alpha(\tau) + \beta_1(\tau)Y(t, 1) + \beta_2(\tau)Y(t, 15) + \beta_3(\tau)[Y(t, 1) + Y(t, 15) - 2 \times Y(t, 7)] + \epsilon(t, \tau)$  where the maturities are denoted in years.  $\beta_1(\tau)$  is the coefficient associated with the level,  $\beta_2(\tau)$  is related with the slope while  $\beta_3(\tau)$  is linked with the curvature. The table compares regression coefficients obtained from actual data with regression coefficients based on simulated yields (in order to account for finite-sample bias). The estimates in the parenthesis indicate the 95% confidence interval. The sample period is from 11/1971-01/2011.

|                 |           | Maturity in Years |                 |                 |                 |                 |                 |                 |
|-----------------|-----------|-------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 |           | 1                 | 3               | 5               | 7               | 10              | 12              | 15              |
| Data            | Level     | 0.214             | 0.117           | 0.075           | 0.061           | 0.052           | 0.048           | 0.047           |
|                 | Slope     | 0.364             | 0.210           | 0.134           | 0.108           | 0.095           | 0.088           | 0.084           |
|                 | Curvature | 0.938             | 0.489           | 0.275           | 0.204           | 0.170           | 0.162           | 0.161           |
| $A_1(3)^{(SR)}$ | Level     | 0.005             | 0.004           | 0.003           | 0.002           | 0.002           | 0.001           | 0.001           |
|                 |           | (-0.002;0.012)    | (-0.002;0.009)  | (-0.001;0.007)  | (-0.001;0.006)  | (-0.001;0.004)  | (-0.001;0.004)  | (-0.001;0.003)  |
|                 | Slope     | -0.130            | -0.109          | -0.086          | -0.068          | -0.049          | -0.039          | -0.034          |
|                 |           | (-0.147;-0.111)   | (-0.123;-0.094) | (-0.097;-0.074) | (-0.077;-0.058) | (-0.057;-0.042) | (-0.045;-0.032) | (-0.039;-0.028) |
| Curvature       |           | -0.183            | -0.154          | -0.121          | -0.096          | -0.070          | -0.055          | -0.048          |
|                 |           | (-0.217;-0.146)   | (-0.180;-0.125) | (-0.143;-0.099) | (-0.114;-0.077) | (-0.084;-0.055) | (-0.067;-0.042) | (-0.059;-0.036) |

*Continued on next page*

Table 2.8 – Continued from previous page

|                 |           | Maturity in Years       |                         |                         |                         |                         |                         |                         |
|-----------------|-----------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
|                 |           | 1                       | 3                       | 5                       | 7                       | 10                      | 12                      | 15                      |
| $A_2(3)^{(SR)}$ | Level     | 0.056<br>(-0.006;0.119) | 0.037<br>(-0.006;0.080) | 0.030<br>(-0.004;0.064) | 0.025<br>(-0.003;0.055) | 0.022<br>(-0.003;0.048) | 0.021<br>(-0.002;0.044) | 0.019<br>(-0.002;0.040) |
|                 | Slope     | 0.209<br>(-0.016;0.439) | 0.139<br>(-0.014;0.293) | 0.111<br>(-0.019;0.235) | 0.096<br>(-0.012;0.205) | 0.083<br>(-0.006;0.176) | 0.077<br>(-0.006;0.164) | 0.070<br>(-0.006;0.148) |
|                 | Curvature | 0.382<br>(-0.031;0.810) | 0.255<br>(-0.025;0.537) | 0.202<br>(-0.037;0.426) | 0.174<br>(-0.025;0.374) | 0.151<br>(-0.014;0.325) | 0.141<br>(-0.011;0.300) | 0.128<br>(-0.013;0.272) |
| $A_1(3)^{(RS)}$ | Level     | 0.000<br>(-0.007;0.005) | 0.000<br>(-0.003;0.002) | 0.000<br>(-0.002;0.002) | 0.000<br>(-0.002;0.001) | 0.000<br>(-0.002;0.001) | 0.000<br>(-0.002;0.001) | 0.000<br>(-0.002;0.001) |
|                 | Slope     | 0.042<br>(0.024;0.057)  | 0.026<br>(0.019;0.031)  | 0.020<br>(0.015;0.025)  | 0.017<br>(0.013;0.021)  | 0.014<br>(0.010;0.018)  | 0.012<br>(0.009;0.016)  | 0.011<br>(0.008;0.015)  |
|                 | Curvature | 0.054<br>(0.021;0.083)  | 0.033<br>(0.024;0.043)  | 0.026<br>(0.019;0.033)  | 0.022<br>(0.016;0.027)  | 0.018<br>(0.013;0.023)  | 0.016<br>(0.011;0.021)  | 0.015<br>(0.010;0.020)  |
| $A_2(3)^{(RS)}$ | Level     | 0.067<br>(0.043;0.091)  | 0.004<br>(-0.007;0.011) | 0.004<br>(-0.004;0.009) | 0.004<br>(-0.002;0.008) | 0.004<br>(0.000;0.007)  | 0.004<br>(0.000;0.007)  | 0.004<br>(0.001;0.006)  |
|                 | Slope     | 0.082<br>(0.034;0.156)  | 0.013<br>(-0.002;0.053) | 0.009<br>(-0.002;0.035) | 0.007<br>(-0.002;0.025) | 0.005<br>(-0.002;0.018) | 0.004<br>(-0.002;0.015) | 0.004<br>(-0.002;0.012) |
|                 | Curvature | 0.115<br>(0.048;0.201)  | 0.016<br>(-0.004;0.064) | 0.012<br>(-0.001;0.043) | 0.010<br>(0.000;0.033)  | 0.008<br>(0.001;0.024)  | 0.007<br>(0.001;0.021)  | 0.006<br>(0.001;0.018)  |



**Table 2.9 GARCH(1,1) model**

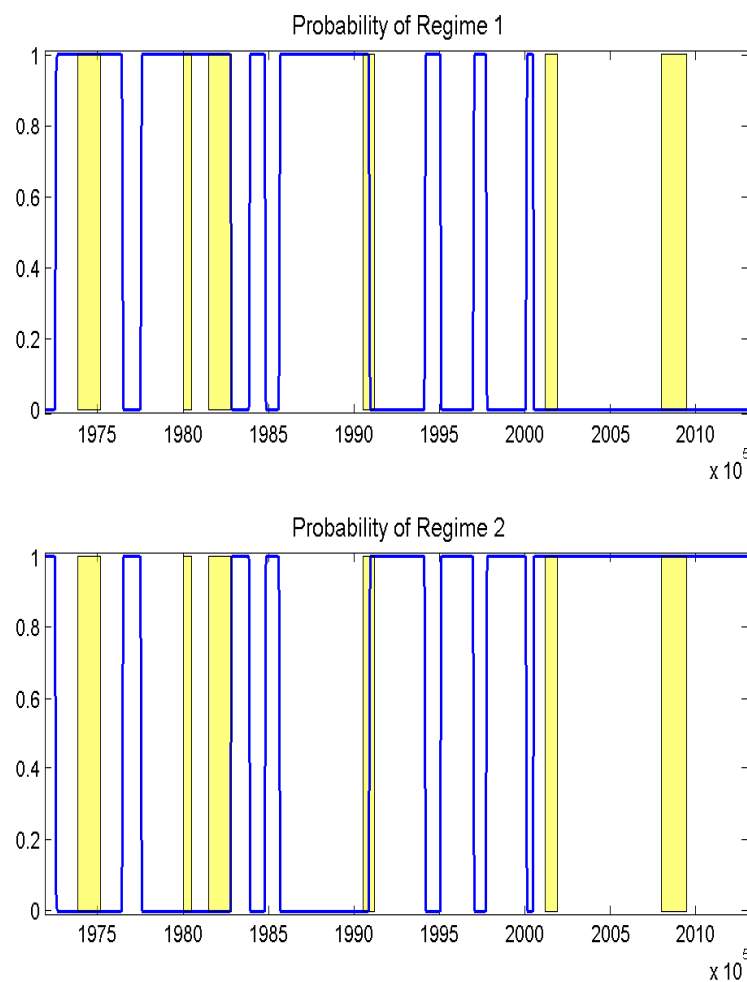
The table presents the Maximum Likelihood estimates of a GARCH(1,1) model:  $\sigma^2 t = c + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2$ , where  $\epsilon_t$  is the innovation from the AR(1) representation of the level of the yields. The estimates in the parenthesis indicate the 95% confidence interval. The sample period is from 11/1971-01/2011.

| Maturity         | 1                      |                        | 3                      |                        | 5                      |                        | 7                      |                        |
|------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
|                  | $\alpha$               | $\beta$                | $\alpha$               | $\beta$                | $\alpha$               | $\beta$                | $\alpha$               | $\beta$                |
| Actual Yields    | 0.236                  | 0.764                  | 0.138                  | 0.840                  | 0.096                  | 0.867                  | 0.085                  | 0.883                  |
| $A_1^{(SR)}$ (3) | 0.028<br>(0.002;0.109) | 0.716<br>(0.099;0.932) | 0.030<br>(0.001;0.116) | 0.709<br>(0.087;0.929) | 0.030<br>(0.001;0.112) | 0.714<br>(0.089;0.933) | 0.030<br>(0.001;0.102) | 0.714<br>(0.112;0.950) |
| $A_2^{(SR)}$ (3) | 0.032<br>(0.002;0.105) | 0.717<br>(0.110;0.932) | 0.033<br>(0.001;0.105) | 0.724<br>(0.122;0.937) | 0.034<br>(0.002;0.102) | 0.727<br>(0.115;0.944) | 0.034<br>(0.001;0.102) | 0.736<br>(0.112;0.950) |
| $A_1^{(RS)}$ (3) | 0.046<br>(0.006;0.123) | 0.820<br>(0.210;0.933) | 0.049<br>(0.007;0.128) | 0.827<br>(0.241;0.934) | 0.052<br>(0.006;0.128) | 0.835<br>(0.237;0.933) | 0.053<br>(0.008;0.130) | 0.840<br>(0.239;0.933) |
| $A_2^{(SR)}$ (3) | 0.038<br>(0.002;0.138) | 0.776<br>(0.100;0.936) | 0.069<br>(0.019;0.166) | 0.905<br>(0.684;0.943) | 0.059<br>(0.020;0.130) | 0.911<br>(0.672;0.950) | 0.051<br>(0.011;0.131) | 0.906<br>(0.568;0.954) |

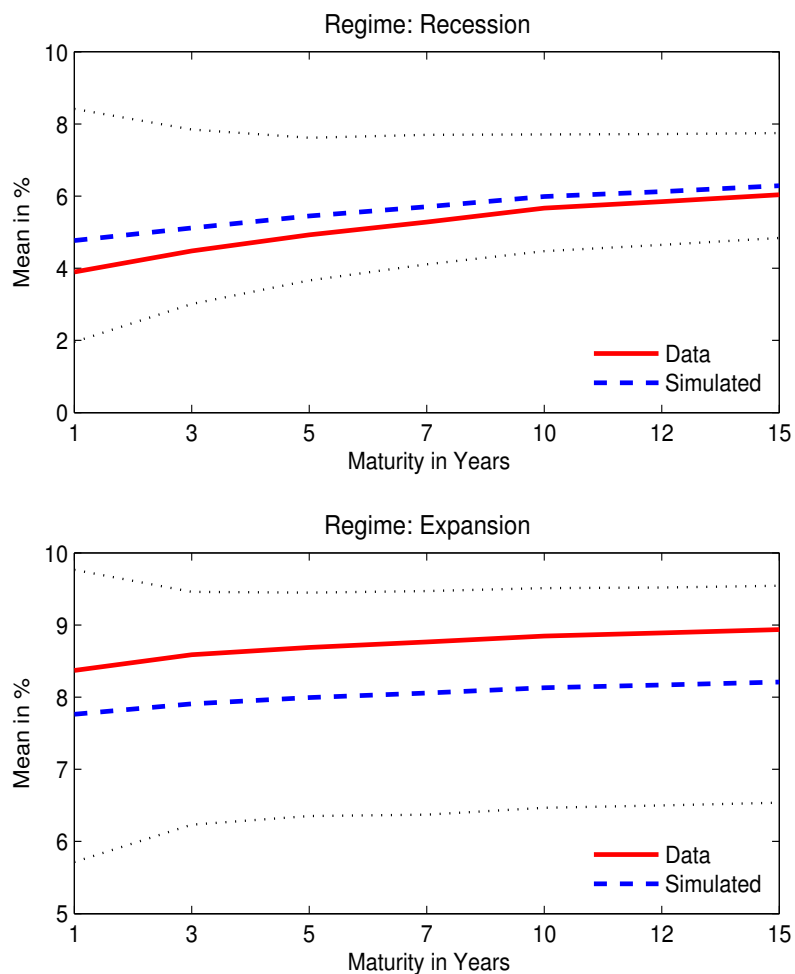
| Maturity         | 10                     |                        | 12                     |                        | 15                     |                        |
|------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
|                  | $\alpha$               | $\beta$                | $\alpha$               | $\beta$                | $\alpha$               | $\beta$                |
| Actual Yields    | 0.102                  | 0.863                  | 0.113                  | 0.845                  | 0.118                  | 0.838                  |
| $A_1^{(SR)}$ (3) | 0.028<br>(0.001;0.123) | 0.715<br>(0.077;0.941) | 0.028<br>(0.001;0.121) | 0.709<br>(0.062;0.944) | 0.028<br>(0.002;0.128) | 0.717<br>(0.084;0.946) |
| $A_2^{(SR)}$ (3) | 0.034<br>(0.002;0.101) | 0.750<br>(0.119;0.954) | 0.033<br>(0.001;0.103) | 0.744<br>(0.121;0.955) | 0.033<br>(0.001;0.102) | 0.743<br>(0.115;0.961) |
| $A_1^{(RS)}$ (3) | 0.056<br>(0.011;0.126) | 0.847<br>(0.298;0.935) | 0.057<br>(0.011;0.126) | 0.853<br>(0.303;0.939) | 0.058<br>(0.011;0.126) | 0.857<br>(0.310;0.943) |
| $A_2^{(SR)}$ (3) | 0.045<br>(0.008;0.116) | 0.902<br>(0.506;0.956) | 0.044<br>(0.008;0.124) | 0.899<br>(0.286;0.956) | 0.043<br>(0.007;0.139) | 0.896<br>(0.241;0.956) |

## 2.7 Figures



**Figure 2.1. Regime Probabilities**

This figure reports a time series of posterior probabilities that the economy is in regime 1 and regime 2, respectively, for the  $A_2^{(RS)}(3)$ .



**Figure 2.2. Actual and Model Implied Unconditional Means**

This figure reports the unconditional means of the yields for all considered maturities for the  $A_2^{(RS)}(3)$  model. Unconditional means are in % and the dotted lines indicate the 95% confidence interval.

## 2.8 Appendix: Derivation of $A(\tau, k)$ and $B(\tau)$

The price  $P(t, \tau, k)$ , of a ZCB at time  $t$ , with maturity  $\tau$  and under regime  $k$  satisfies the following PDDE:

$$\begin{aligned} \frac{1}{2} \text{Tr} \left( \frac{\partial^2 P}{\partial X \partial X'} \Sigma \sigma(x_t) \Sigma' \right) + \frac{\partial P}{\partial X'} \left( \kappa \left( \theta^{(k)} - X_t \right) \right) + \frac{\partial P}{\partial \tau} - \left( \delta_0^{(k)} + \delta_X' X_t \right) P(\tau, X_t, k) \\ + \sum_{j=1, j \neq k}^K Q_{k,j} (P(\tau, X_t, j) - P(\tau, X_t, k)) = 0 \end{aligned}$$

We conjecture that the solution to the above PDDE takes the form:

$$P(t, \tau, k) = e^{A(\tau, k) + B(\tau)' X_t}$$

Computing then the partial derivatives we obtain:

$$\begin{aligned} \frac{\partial P}{\partial X} &= B(\tau)' P(\tau, X_t, k) \\ \frac{\partial^2 P}{\partial X \partial X'} &= B(\tau) B(\tau)' P(\tau, X_t, k) \\ \frac{\partial P}{\partial \tau} &= \left\{ \frac{dA(\tau, k)}{d\tau} + \frac{dB(\tau)'}{d\tau} X_t \right\} P(\tau, X_t, k) \end{aligned}$$

where we used the the fact that  $\frac{\partial A(\tau, k)}{\partial \tau} \frac{\partial \tau}{\partial t} = - \left( \frac{\partial A(\tau, k)}{\partial \tau} \right)$ . Note that the same reasoning applies for  $B(\tau)$ . Substituting the partial derivatives in the PDDE and rearranging the terms (recalling that  $[\sigma(X_t)]_{ii} = \alpha_i + \beta_i' X_t$ ), yields:

$$\begin{aligned} \left\{ \frac{1}{2} \sum_{i=1}^m [\Sigma' B(\tau)]_i^2 \beta_i - \kappa_1' B(\tau) - \delta_X - \frac{dB(\tau)}{d\tau} \right\} X_t P(\tau, X_t, k) + \left\{ \frac{1}{2} \sum_{i=1}^m [\Sigma' B(\tau)]_i^2 \alpha_i \right. \\ \left. + \kappa_0^{(k)'} B(\tau) - \delta_0^{(k)} + \sum_{j=1, j \neq k}^K Q_{k,j} \left( e^{A(\tau, j) - A(\tau, k)} - 1 \right) - \frac{dA(\tau, k)}{d\tau} \right\} P(\tau, X_t, k) = 0 \end{aligned}$$

This must hold  $\forall X$  and  $k$ . Thus,

$$\begin{aligned} \frac{1}{2} \sum_{i=1}^m [\Sigma' B(\tau)]_i^2 \beta_i - \kappa_1' B(\tau) - \delta_X - \frac{dB(\tau)}{d\tau} = 0 \\ \frac{1}{2} \sum_{i=1}^m [\Sigma' B(\tau)]_i^2 \alpha_i + \kappa_0^{(k)'} B(\tau) - \delta_0^{(k)} + \sum_{j=1, j \neq k}^K Q_{k,j} \left( e^{A(\tau, j) - A(\tau, k)} - 1 \right) - \frac{dA(\tau, k)}{d\tau} = 0. \end{aligned}$$

Solving for  $\frac{dB(\tau)}{d\tau}$  and  $\frac{dA(\tau, k)}{d\tau}$  we obtain the following system of ODE's:

$$\begin{aligned} \frac{dB(\tau)}{d\tau} &= \frac{1}{2} \sum_{i=1}^m [\Sigma' B(\tau)]_i^2 \beta_i - \kappa_1' B(\tau) - \delta_X \\ \frac{dA(\tau, k)}{d\tau} &= \frac{1}{2} \sum_{i=1}^m [\Sigma' B(\tau)]_i^2 \alpha_i + \kappa_0^{(k)'} B(\tau) - \delta_0 + \sum_{j=1, j \neq k}^K Q_{k,j} \left( e^{A(\tau, j) - A(\tau, k)} - 1 \right) \end{aligned}$$

## 2.9 Appendix: MCMC Algorithm

In the following section we describe the MCMC algorithm for our particular RS-ATSM where we allow for two regimes. First, we briefly review the conditional distributions which are used in the sampling procedures.

### *The Conditionals*

The conditional density of the latent variables is given as:

$$\begin{aligned} p(X|\theta, K) &= \prod_{n=1}^N \prod_{t=2}^T p(X_{t+1}|X_t, k_t) \\ &= \prod_{n=1}^N \left( \left( \prod_{t=2}^T \frac{1}{\sqrt{\sigma(X_t)}} \right) \exp \left( -\frac{1}{2\Delta_t} \sum_{t=2}^T \frac{\Delta X_{t+1} - \mu_t^{\mathbb{P},(k)} \Delta_t}{\sigma(X_t)} \right) \right) \end{aligned}$$

where we assumed an independent prior for  $X_0$ .

We denote the model implied yields at time  $t$  by

$$\hat{Y}(t, \tau, k) = A^*(\tau, k) + B^*(\tau)X_t.$$

$A^*(\tau, k)$  is regime-dependent scalar and  $B^*(\tau)$  is a  $1 \times N$  vector. Thus, the density  $p(Y|\Theta, X, k)$  can be written as:

$$\begin{aligned} p(Y|\Theta, X, k) &= \prod_{\tau=1}^M \prod_{t=1}^T H_{\tau\tau}^{-\frac{1}{2}} \exp \left( -\frac{(Y(t, \tau) - \hat{Y}(t, \tau, k_t))^2}{2H_{\tau\tau}} \right) \\ &= \frac{1}{\sigma^{MT}} \exp \left( -\frac{1}{2\sigma^2} \sum_{t=1}^T (\epsilon(t, k_t)' \epsilon(t, k_t)) \right) \end{aligned}$$

where  $\epsilon(t, k_t) = Y(t, \tau) - \hat{Y}(t, \tau, k_t)$ .

In addition to these two conditionals, the hybrid MCMC algorithm also depends on the evaluation of the regime variable:

$$p(k|\Theta) = \prod_{t=2}^T (\exp(Q \Delta_t))_{k_{t-1}, k_t}$$

The matrix exponential together with the two conditionals are the main building blocks of the MCMC algorithm.

### *Random-Walk Metropolis-Hastings and Gibbs Sampling Procedures*

#### *Sampling the latent regimes*

The regime variable is sampled using a RW-MH algorithm. For each of the regimes  $k_t = 1, \dots, S$ , at time  $t = 1, \dots, T - 1$  the conditional of  $k_t$  is given as:

$$\begin{aligned} p(k_t|k_{\setminus t}, X, \Theta, Y) &\propto p(Y_t|X_t, k_t, \Theta) \times p(k_t|k_{t-1}, \Theta) \times \\ &\quad p(k_{t+1}|k_t, \Theta) \times p(X_t|X_{t-1}, k_{t-1}, \Theta) \end{aligned}$$

In particular, for  $t = 2, 3, \dots, T - 1$  we calculate:

$$\begin{aligned}
p(k_t = 1 | \cdot) &\propto \exp\left(-\sum_{\tau=1}^M \frac{(Y(t, \tau) - \hat{Y}(t, \tau, 1))}{2H_{\tau\tau}^2}\right) \exp(Q\Delta_t)_{k_{t-1}, 1} \exp(Q\Delta_t)_{1, k_{t+1}} \\
&\quad \frac{1}{\sqrt{\sigma(X_{t-1})}} \exp\left(-\frac{1}{2\Delta t} \varepsilon_t^{(1)} ((\sigma(X_{t-1}))^{-1} \varepsilon_t^{(1)'})\right) \equiv \alpha_1 \\
p(k_t = 2 | \cdot) &\propto \exp\left(-\sum_{\tau=1}^M \frac{(Y(t, \tau) - \hat{Y}(t, \tau, 2))}{2H_{\tau\tau}^2}\right) \exp(Q\Delta_t)_{k_{t-1}, 2} \exp(Q\Delta_t)_{2, k_{t+1}} \\
&\quad \frac{1}{\sqrt{\sigma(X_{t-1})}} \exp\left(-\frac{1}{2\Delta t} \varepsilon_t^{(2)} ((\sigma(X_{t-1}))^{-1} \varepsilon_t^{(2)'})\right) \equiv \alpha_2
\end{aligned}$$

where  $\varepsilon_{t+1}^{(k)} = \Delta X_{t+1} - \mu_t^{\mathbb{P}, (k)}$  for  $k = 1, 2$ . We define  $\tilde{\alpha} = \frac{\alpha_1}{(\alpha_1 + \alpha_2)}$  and draw  $u = \text{unifrnd}(0, 1)$ . We set  $k_t = 1$  if  $u < \tilde{\alpha}_1$  and  $k_t = 2$  otherwise.

For  $t = 1$  the posterior distribution is as

$$p(k_1 | \cdot) \propto \exp\left(-\sum_{\tau=1}^M \frac{(Y(t, \tau) - \hat{Y}(t, \tau, k_1))}{2H_{\tau\tau}^2}\right) \exp(Q\Delta_t)_{k_1, k_2},$$

while for  $t = T$  the posterior is given by

$$\begin{aligned}
p(k_T | \cdot) &\propto \exp\left(-\sum_{\tau=1}^M \frac{(Y(T, \tau) - \hat{Y}(T, \tau, k_T))}{2H_{\tau\tau}^2}\right) \exp(Q\Delta_t)_{k_{T-1}, k_T} \\
&\quad \frac{1}{\sqrt{\sigma(X_{T-1})}} \exp\left(-\frac{1}{2\Delta t} \varepsilon_T^{(k_T)} (\sigma(X_{T-1}))^{-1} \varepsilon_T^{(k_T)'}\right).
\end{aligned}$$

### *Sampling the latent factors*

The latent state variables  $X_t$ , for  $t = 1, 2, \dots, T$  are sampled using a RW-MH algorithm. For  $t = 2, \dots, T - 1$  the conditional of  $X_t$  is given as

$$p(X_t | X_{\setminus t}, k, \Theta, Y) \propto p(Y_t | X_t, k_t, \Theta) \times p(X_t | X_{t-1}, k_{t-1}, \Theta) \times p(X_{t+1} | X_t, k_t, \Theta).$$

For  $t = 1$  the conditional is

$$p(X_1 | X_{\setminus X_1}, k, \Theta, Y) \propto p(Y_1 | X_1, k_1, \Theta) p(X_2 | X_1, k_1, \Theta)$$

while for  $t = T$  the conditional is

$$p(X_T | X_{\setminus X_T}, k_T, \Theta, Y) \propto p(Y_T | X_T, k_T, \Theta) p(X_T | X_{T-1}, k_{T-1}, \Theta)$$

The latent state variables are subject to constraints (e.g. the latent variables entering the volatility are constrained to be positive) hence if a draw violates the constraint it is discarded. The latent factor are sampled using a RW-MH procedure. In particular, we sample new  $X_t^{\text{new}} = X_t^{\text{old}} + \gamma N(0, 1)$

where  $\gamma$  is calibrated and calculate the below posterior distribution:

$$p(X_t|\cdot) \propto \exp\left(-\sum_{\tau=1}^M \frac{(Y(t, \tau) - \hat{Y}(t, \tau, k))}{2H_{\tau\tau}^2}\right) \frac{1}{\sqrt{\sigma(X_t)}} \exp\left(-\frac{1}{2\Delta_t} \varepsilon_{t+1}^{(k)} (\sigma(X_t))^{-1} \varepsilon_{t+1}^{(k)'}\right) \frac{1}{\sqrt{\sigma(X_{t-1})}} \exp\left(-\frac{1}{2\Delta_t} \varepsilon_t^{(k)} (\sigma(X_{t-1}))^{-1} \varepsilon_t^{(k)'}\right).$$

We set  $\alpha = \frac{p(X_t^{\text{new}}|\cdot)}{p(X_t^{\text{old}}|\cdot)}$  and sample  $u = \text{unifrnd}(0, 1)$ . We accept  $X_t^{\text{new}}$  if  $u < \alpha$  and reject otherwise. The parameter  $\gamma$  is calibrated such that the acceptance ratio is between 10% and 30%.

For  $t = 1$  the posterior distribution is as

$$p(X_1|\cdot) \propto \exp\left(-\sum_{\tau=1}^M \frac{(Y(t, \tau) - \hat{Y}(t, \tau, k))}{2H_{\tau\tau}^2}\right) \frac{1}{\sqrt{\sigma(X_1)}} \exp\left(-\frac{1}{2\Delta_t} \varepsilon_2^{(k)} (\sigma(X_1))^{-1} \varepsilon_2^{(k)'}\right),$$

while for  $t = T$  the posterior is given by

$$p(X_T|\cdot) \propto \exp\left(-\sum_{\tau=1}^M \frac{(Y(T, \tau) - \hat{Y}(T, \tau, k))}{2H_{\tau\tau}^2}\right) \frac{1}{\sqrt{\sigma(X_{T-1})}} \exp\left(-\frac{1}{2\Delta_t} \varepsilon_T^{(k)} (\sigma(X_{T-1}))^{-1} \varepsilon_T^{(k)'}\right).$$

### *Sampling the model parameters*

The model parameters are sampled using a RW-MH procedure. In particular, we sample  $\Theta_t^{\text{new}} = \Theta_t^{\text{old}} + \gamma N(0, 1)$  where  $\gamma$  is calibrated. The posterior distribution of the model parameter is given by a subset of the below conditionals:

$$p(\Theta|\cdot) \propto \exp\left(-\sum_{t=1}^T \sum_{\tau=1}^M \frac{(Y(t, \tau) - \hat{Y}(t, \tau, k))}{2H_{\tau\tau}^2}\right) \exp(Q\Delta_t)_{k_{t-1}, k_t} \left(-\frac{1}{\sqrt{\sigma(X_{t-1})}} \exp\left(\frac{1}{2\Delta_t} \varepsilon_t^{(k_t)} (\sigma(X_{t-1}))^{-1} \varepsilon_t^{(k_t)'}\right)\right).$$

We set  $\alpha = \frac{p(\Theta_t^{\text{new}}|\cdot)}{p(\Theta_t^{\text{old}}|\cdot)}$  and sample  $u = \text{unifrnd}(0, 1)$ . We accept  $\Theta_t^{\text{new}}$  if  $u < \alpha$  and reject otherwise. The parameter  $\gamma$  is calibrated such that the acceptance ratio is between 10% and 30%.

### *Sampling the measurement*

The conditional of the variance of the measurement errors is given as:

$$p(D|\Theta_D, X, K, Y) \propto p(Y|\Theta, X)$$

This implies that  $\sigma^2$  can be Gibbs sampled from an inverse Gamma distribution,  $\sigma^2 \sim IG(\sum_{t=1}^T \epsilon(t, k_t)\epsilon(t, k_t)', MT)$ .

## 2.10 Appendix: The Bayes Factor

In this section, we provide details on how to compute the Bayes factor for model comparison. The Bayes Factor summarizes the evidence provided by the data in favor of one of the models considered compared to another, and is given by the ratio of the marginal probabilities of the data under the two models:

$$B = \frac{p(D|M_1)}{p(D|M_2)}$$

When dealing with known single distributions and no free parameters this is just the likelihood ratio. In our case, where we have latent state variables and regimes and unknown parameters, to obtain the marginal probabilities of the data  $p(D)$  we need to integrate out all model parameters, latent factors and regime variables.<sup>17</sup>

### Integrate out the latent state variables and regimes

For each time point  $t = 1, 2, \dots, T$  we compute:

1. For each  $t = 1, 2, \dots, T$  and  $k = 1, 2, \dots, K$  we simulate:

$$s_t^{(k)} \propto \exp \{Q\Delta_t\}_{s_{t-1}, k}$$

2. Having obtained the regime we proceed by simulating the latent state variables given the regime at the particular time step  $X_t$ .
3. We then integrate out the latent regimes and the latent state variables to obtain:

$$\begin{aligned} p(y_t | \Theta) &= \int p(y_t | \Theta_t, X_t, s_t) p(X_t | \cdot) p(s_t | \cdot) dX_t ds_t \\ &= \frac{1}{K} \sum_{k=1}^K \left( \prod_{m=1}^M \exp \left\{ -\frac{1}{2} \frac{(y_t^m - \hat{y}_{s_t}^m)^2}{\sigma_m^2} \right\} \right) \end{aligned}$$

4. Filter the regime for each time point,  $s_t^{(k)}$ , for  $k = 1, 2, \dots, K$ :

$$\begin{aligned} p(s_t^{(1)} | \cdot) &\propto p(y_t | \cdot) p(X_t | \cdot) \times \frac{1}{K} \sum_{k=1}^K \left\{ \exp \{Q\Delta_t\}_{s_{t-1}, 1} \right\} \\ &\equiv \alpha_1 \end{aligned}$$

$$\begin{aligned} p(s_t^{(2)} | \cdot) &\propto p(y_t | \cdot) p(X_t | \cdot) \times \frac{1}{K} \sum_{k=1}^K \left\{ \exp \{Q\Delta_t\}_{s_{t-1}, 2} \right\} \\ &\equiv \alpha_2 \end{aligned}$$

We then draw  $u \sim \text{Bernoulli}\left(\frac{\alpha_1}{\alpha_1 + \alpha_2}\right)$  and if  $u = 1$  we assign  $s_t = 1$ , otherwise if  $u = 0$  we assign  $s_t = 2$ .

5. We simulate new  $X_t$ 's given the regimes filtered above and start over the procedure from step 1 for the next time point.

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<sup>17</sup>This implementation is an adaptation of the procedure described in [Li, Li, and Yu \(2011\)](#) adjusted for the presence of latent state variables.



Once we have carried out this procedure up to time  $t = T$  we obtain:

$$p(D|\Theta^{(g)}) = \prod_{t=1}^T \left( \frac{1}{K} \sum_{k=1}^K \left( \prod_{m=1}^M \exp \left\{ -\frac{1}{2} \frac{(y_t^m - \hat{y}_{s_t}^m)^2}{\sigma_m^2} \right\} \right) \right)$$

**Integrate out the parameters**

Having obtained  $p(D|\Theta^{(g)})$  we integrate out the parameters to obtain the posterior distribution of the data:

$$p(D) = \int p(D|\Theta)\pi(\Theta)d\Theta$$

where  $\pi(\Theta)$  is the prior distribution of the parameters. Since this is not known, we use an importance function  $\pi^*(\Theta)$  to calculate  $p(D)$ , which for a large number of simulations  $g = 1, 2, \dots, G$  approximates the true distribution:

$$p(D) = \frac{\sum_{g=1}^G w_g p(D|\Theta^{(g)})}{\sum_{g=1}^G w_g}, \quad \text{where } w_g = \frac{\pi(\Theta^{(g)})}{\pi^*(\Theta^{(g)})}$$

Choosing  $\pi^*(\Theta) = \frac{p(D|\Theta)\pi(\Theta)}{p(D)}$  we obtain<sup>18</sup>:

$$p(D) = \left( \frac{1}{G} \sum_{g=1}^G p(D|\Theta^{(g)})^{-1} \right)^{-1}$$

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<sup>18</sup>See Kass and Raftery (1995) for a detailed discussion of the choice of the importance function

# Essay 3

## Variance Risk Premia in the Interest Rate Swap market

Desi Volker <sup>1</sup>

### Abstract

In this paper I analyze the time series and cross-sectional properties of variance risk premia in the interest rate swap market. The results presented show that the term structure of variance risk premia displays non-negligible differences in a low interest rate environment, compared to normal times. Variance risk premia have on average been negative and economically significant during the sample. In a low interest rate environment, the variance risk premium tends to display more frequent episodes where it switches sign.

***JEL Classification:*** G12, E43

***Keywords:*** Variance Risk Premia, Swaption Implied-Volatilities

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### 3.1 Introduction

Swaption implied volatilities reflect the market participants expectations of future realized volatility, adjusted by a premium to compensate them for the risk associated with the fact that interest rate volatility is stochastic. The variance risk premium is time-varying and economically significant, and tends to rise in absolute terms, in periods of market turmoil, where uncertainty about the economy and/or investor risk aversion is high. Given the current and protracted low interest rate environment it is important to study whether the properties of variance risk premia embedded in swaptions, differ in a low rate regime compared to normal times. In this paper I analyze the historical behavior of the variance risk premium embedded in interest rate swaptions and assess its characteristics during periods of low and high/normal interest rate levels.

In loose terms the variance risk premium reflects the amount investors are willing to pay during normal times in order to insure against high realized interest rate volatility during periods of market turmoil, while from the option sellers' perspective, it reflects the compensation demanded for taking the risk of incurring significant losses in periods when realized volatility increases significantly and unexpectedly.<sup>2</sup> The variance risk premium will be affected by imbalances in the supply and demand for swaptions, the price associated to event risk, liquidity risk, credit risk, etc. Variance risk premia depend on the whole distribution of the underlying asset returns, that is, not just on the interest rate level and volatility, but also on the higher moments. [Bakshi and Madan \(2006\)](#) for example, show that under a number of assumptions, the variance risk premium is a function of the skewness and kurtosis of the underlying assets' returns. When interest rates are low and close to the zero lower bound, the distribution of interest rates is more closely approximated by a lognormal distribution, with a fat right tail, since the probability associated with an increase in interest rates is higher than it would be under the normal (assuming non-negative rates). Given that short-term nominal interest rates in large part of the developed world are at or near zero and there are no prospects for the situation to change in the near future given inflation expectations are revised downwards, it is relevant to study how this affects variance risk premia in fixed-income markets.

In analogy with the equity literature, I define the variance risk premium as the difference between expected realized future variances and risk neutral variances. Both of these components are not directly observable and a number of approaches are available for measuring them. One approach is to rely on sophisticated dynamic

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<sup>2</sup> However it is important to mention that the results presented in this paper do not correspond to the returns on tradable strategies to exploit the premium, such as at the money straddles or variance swaps.

option-pricing models. Alternatively, one can take a model-free approach and measure realized volatilities from high frequency data as proposed by [Andersen, Bollerslev, Diebold, and Ebens \(2001\)](#); [Barndorff-Nielsen \(2002\)](#) and risk neutral volatilities using a panel of option prices as proposed by [Britten-Jones and Neuberger \(2000\)](#). Lastly, one can use simple parametric models. I use Black-implied ATM swaption volatilities as measures of the unobservable risk neutral volatilities. To measure the unobservable expected realized volatilities, I follow ([Fornari, 2010](#)) and use volatility forecasts based on an asymmetric GARCH model (and conditioning for the information available at each point in time), The variance risk premia obtained span various terms, going from three to twenty-four months and tenors going from two to ten years.

I analyze the time-series and cross-sectional properties of variance risk premia, over the full sample, as well as on periods of high/low interest rates and document the following results. Firstly, as expected, variance risk premia have been negative and economically significant during the full sample and on all subsamples. This suggests that volatility risk in the interest rate swap market has been largely priced. There have been however brief periods where variance risk premia have switched sign. These short lived, but consequential episodes reflect periods in which unexpected realized volatility shocks have occurred. Variance risk premia display a high co-movement across terms and tenors and generally tend to spike and fall abruptly in unison, however there are important exceptions. Most of the spikes and abrupt falls coincide with important events and crisis episodes in financial markets and the overall economy. Variance risk premia are increasing in tenor, that is the variance risk premium of shorter tenors is more negative than that of longer tenors. Along the term dimension, the term structure of variance risk premia is increasing with the term. Variance risk premia are quite persistent and the persistence increases with the term. Looking at episodes where the variance risk premium spikes or falls abruptly, one observes that the slope of the term structure in the term dimension switches its sign. Secondly, the main determinants of the time-variation in variance risk premia are, as expected, the interest level and past volatility, which explain most of its variation. In particular an increase in both the short rate and realized volatilities is associated with an increase in the variance risk premium on the full sample. Other measures, such as the interest rate slope, the slope of the volatility curve, swap spreads, credit spreads and the stock market volatility index are significant predictors. An increase in the interest rate slope, the slope of the volatility curve or the swap spreads, is associated with an increase in the variance risk premium. The VIX and corporate credit spread have the opposite effect, reflecting the fact that equity and fixed income volatility risks are distinct and therefore display time-varying correlations. Lastly,

the variance risk premium process displays structural breaks dividing the data into periods belonging to one of two distinctive regimes. The first, with high (negative) level and high dispersion, corresponding to periods where the interest rate level is relatively low, and the second, with a nearly zero level and small dispersion, corresponding to periods where the level of interest rates is high. In the low interest rate subsample, the term structure of variance risk premia across the tenor dimension is upward sloping. While in the high interest rate subsample, it is downward sloping, with the variance risk premium on shorter tenors being less negative than on longer ones. In the low interest rate regime, a change in the short rate will have differential effects on variance risk premia across tenors. In particular an increase in the short rate is associated with an increase in the variance risk premium of the 2 year tenor and a decrease in that of the 10 year tenor, with the overall effect of flattening the term structure of variance risk premia across the tenor dimension in the period when the later has a high slope. In the high interest rate regime, an increase in the short rate increases the variance risk premium across all tenors for a given term. Similar results are found for realized volatility.

The paper is related to a small but increasing literature dealing with the measurement and analysis of variance risk premia in fixed income markets.<sup>3</sup> The most closely related paper is [Fornari \(2010\)](#), which analyzes the compensation for volatility risk across different countries. The sample considered however goes from 1997 to 2006 and therefore does not capture the financial crisis period and the near zero interest rate period that has prevailed since then. Other related papers are [Choi, Mueller, and Vedolin \(2015\)](#) and [Mueller, Vedolin, and Zhou \(2011\)](#) which use options on bond futures to construct model-free expected realized volatility measures and variance risk premia and exploit the information in the latter to forecast real activity and term premia. [Mele and Obayashi \(2013\)](#) use a similar methodology to construct a treasury implied volatility index. [Mele, Obayashi, and Shalen \(2015\)](#) analyze the relation between the VIX and the SRVX, the swap rate volatility index and find significant differences in their behavior, especially during periods of distress in bond markets.

The remainder of the paper is organized as follows. Section [3.2](#) discusses the methodology used for the measurement of the unobservable variance risk premia. Section [3.3](#) analyzes the time series and cross-sectional properties of variance risk premia over the full sample and across subsamples corresponding to periods of high and low interest rates. Section [3.4](#) provides predictive regression results for variance risk premia on a set of likely predictors and Section [3.5](#) concludes.

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<sup>3</sup>More broadly the paper is related to the large literature on equity variance risk premium literature ([Carr and Wu, 2009](#); [Chernov, 2007](#); [Trolle and Schwartz, 2009, 2014](#); [Bollerslev, Gibson, and Zhou, 2011](#))

### 3.2 Constructing Variance Risk Premia

The variance risk premium is defined as the difference between (squared) expected future realized volatilities and (squared) implied volatilities for a given tenor  $\tau$  and term  $h$ :

$$VRP_t^{\tau,h} = E_t^{\mathbb{P}} \left( \int_t^{t+\tau} (\sigma_s^h)^2 ds \right) - E_t^{\mathbb{Q}} \left( \int_t^{t+\tau} (\sigma_s^h)^2 ds \right)$$

For expected volatilities under the risk neutral measure, I use at-the-money swaption implied volatilities with tenors of 2, 5 and 10 years and terms of 3, 6, 12 and 24 months. The data is taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016. To estimate expectations of realized volatility under the physical measure one has to rely in a particular model-free methodology (Bollerslev, Gibson, and Zhou, 2011; Choi, Mueller, and Vedolin, 2015), in an option pricing model (Chernov, 2007; Trolle and Schwartz, 2009, 2014), or in forecasts based on parameter estimates of an assumed model for the historical volatility process (Fornari, 2010). In this paper I follow the latter option and use the filtered historical simulation approach developed by Barone-Adesi, Engle, and Mancini (2008) and used in Fornari (2010). Specifically the methodology works as follows. I model realized volatilities as a GARCH process and produce forecasts of realized volatility based on the model's parameter estimates conditioning on the information available at each point in time. I assume that the historical volatility process for daily log swap returns follows an asymmetric GARCH(1,1) model:

$$\begin{aligned} r_t &= \alpha_0 + \alpha_1 r_{t-1} + \sigma_t \nu_t; & \nu_t &\sim i.i.d. \mathcal{N}(0, 1) \\ \sigma_t^2 &= \beta_0 + \beta_1 \sigma_{t-1}^2 + \beta_2 \epsilon_{t-1}^2 + \beta_3 \max(0, -\epsilon_{t-1})^2 \\ \epsilon_t &= \sigma_t \nu_t | I_{t-1} \sim \mathcal{N}(0, \sigma_t^2) \end{aligned}$$

with  $r_t$  denoting daily logarithmic swap returns with a given maturity  $\tau$  ( $r_t = \log(sr_{t+1}^\tau / sr_t^\tau)$ ),  $\sigma_t$  denotes the conditional volatility for the swap returns,  $I_{t-1}$  denotes the information set available at time  $t$  and  $\epsilon_t$  denote the forecast errors. Since when simulating future paths of conditional volatilities,  $\nu_t$  is bootstrapped from the set of realized forecast errors, the forecasts of future realized volatilities will not suffer from model specification due to the specific choice of the model within the ARCH family. Using daily swap rate data that start a decade earlier to the desired starting date for the expected realized volatilities, I estimate a GARCH(1,1) model on expanding windows. At each day  $t$ , I use the parameter estimates, filtered volatilities and bootstrapped standardized forecast errors from the model estimated on a sample that end at  $t$ , to forecast future realized volatilities. In this way expectations are

formed conditioning on the information available at each point in time. In particular, for each day  $t$  and each swap rate, I simulate 10000 paths of future volatility over different horizons (corresponding to the terms of the ATM swaption implied volatilities employed above) and then take the averages over the options terms and the simulations as the measure of the expected volatility for a particular tenor and term. The series are then annualized to be made comparable to the swaption implied volatilities, and variance risk premia are computed as the difference between the squared expected realized volatility and squared implied volatility for a particular tenor and term. The variance risk premia,  $VRP_t^{\tau,h}$ , obtained from the procedure have a daily frequency spanning the period June 2, 1997 to May 19, 2016 and include tenors of 2, 5, and 10 years and terms of 3, 6, 12 and 24 months.

### 3.3 The Time Series of Swaption Implied Volatilities and Variance Risk Premia

Figure 3.1 displays swaption implied volatilities for tenors of 2, 5 and 10 years and terms of 3, 6, 12 and 24 months along with the one year Treasury rate, while Table 3.1 reports their summary statistics. The figures are in percent, annualized.

[Insert Figure 3.1 and Table 3.1 here.]

The first feature of the data shown in the figure is that there is significant time variation in swaption implied volatilities and a high correlation across the different tenors and terms. Secondly the series appears to display two different regimes, one with a low level and low dispersion, present between 1997 and 2001 and between 2005 and 2008, and one with a high level and dispersion, present between 2001 and 2005 and after 2008. The first, corresponds to a period where the interest rate level is relatively high (given the sample) and one where interest rates are low. During the high interest rate regime, the cross-section across tenors for a given term is almost flat, while in the low interest rate regime it appears to be significantly downward sloping. All series peak during the financial crisis of 2008.

[Insert Figure 3.2 here.]

Figure 3.2 shows the cross section of swaption implied volatilities for the overall sample and extending the tenors to include 1, 2, 3, 4, 5 and 10 years, and the terms to include 6, 12, 60 and 120 months. Again the swaption implied volatilities are downward sloping in the tenor dimension and decreasing with the term. The

steepness of the slope decreases with the term, with longer terms of 60 and 120 months having an almost flat curve.

Figure 3.3 plots swaption implied volatilities  $E^{\mathbb{Q}}[\sigma_t^{\tau,h}]$  and expected realized volatility forecasts  $E^{\mathbb{P}}[\sigma_t^{\tau,h}]$  along with their 95% confidence bounds, for the 2 years tenor, and terms going from 3m to 24 months. The expected realized volatility forecasts and their confidence bounds are computed at each point in time from simulations based on the methodology described above. The figures are in percent and annualized.

[Insert Figure 3.3 here.]

Overall, expected realized volatilities have been lower than risk neutral volatilities, with periods where they overlap corresponding to a high interest rate level, and periods where the gap widens significantly, in 2003 and after 2009. There are however, brief and sudden periods where expected realized volatilities have surpassed risk-neutral volatilities. For longer terms (and tenors, not reported here for brevity) swaption implied volatilities lie well within the 95% confidence bounds. For the shorter tenor and term however, there are brief periods, occurring in the low interest rate regime, where the swaption implied volatility lies beyond the upper bound. This can be explained by the fact that the data displays discernible breaks, with periods of distinct volatility levels and dispersion, while the simulations were based on parameters and forecast errors from a GARCH(1,1) model estimated on the historical data up to that point in time. This suggests that the large spikes in implied volatility were largely unexpected.

Figure 3.4 plots the variance risk premia computed as the difference between squared expected realized volatilities and squared risk neutral volatilities for different tenors and terms.

[Insert Figure 3.4 here.]

As already glimpsed in Figure 3.3, the plots in Figure 3.4 confirm that variance risk premia as defined here have been negative on average, implying a negative premium on average for the investor who hedges against volatility risk and a positive compensation on average for the option seller. There are brief periods, especially after 2008 where the variance risk premia has switched sign, implying that interest rate option sellers, have incurred on average higher and more frequent losses after 2008, and have demanded a higher compensation to account for the increased risk. Option sellers were therefore exposed to sharp losses for brief periods, but these periods were followed with a quick recovery, since risk neutral volatilities reacted more sharply to



the shocks than expected realized volatility and remained higher for next few periods, reflecting heightened risk aversion. Variance risk premia are increasing with the tenor and with term, with shorter tenor and terms displaying also more pronounced and more frequent spikes (both positive and negative). Variance risk premia are quite persistent (see Figure 3.5) and the persistence increases for longer tenor and terms. Figure 3.6 displays minus the variance risk premia series for the 5 year tenor and 3 month term,  $VRP_t^{5y,3m}$ , and the stock market volatility index, VIX.

[Insert Figure 3.5 and Figure 3.6 here.]

The two series have a correlation of -40% and seem to follow similar overall trends, with spikes in crisis periods, such as the Russian debt and currency crisis of 1998 and the global financial crisis of 2008.

A first glance at the time series of variance risk premia suggest there might be changes in the data generating process in periods where interest rates move from a high to a low level and vice-versa. It is important therefore to test more formally for the existence of potential structural breaks. Since there is a suspicion of multiple breaks in the data and I do not want to take a stance on which particular dates the structural breaks occur, I use the methods developed and applied in Bai and Perron (1998, 2003a,b). The authors devise various tests to not only determine the presence of structural change but also the number of breaks and their location along with confidence bounds. Figure 3.7 plots swaption implied volatilities and variance risk premia along with the structural break points determined by the tests.

[Insert Figure 3.7 here.]

Confirming the original suspicion, the tests find three major structural breaks, the first one corresponding to the 9/11 attacks, the second in the end of 2004 and the third to the beginning of the 2008 financial crisis. A fourth break is found for the swaption implied volatilities in the beginning of 2012, however it is not present for the variance risk premia series. Dividing the data according to the structural breaks, one obtains fundamentally two regimes for variance risk premia, one with an almost zero level and very low dispersion, corresponding to a high interest rate environment, and one with a high (negative) level and high dispersion, corresponding to a low interest rate environment. These two regimes present interesting differences in the cross-section of variance risk premia in the tenor dimension, as well as in the term structure of variance risk premia ( see Figure 3.8 and Figure 3.9). Specifically in the high interest rate regime, variance risk premia are slightly decreasing and close to flat

in the tenor dimension with very small differences between the 2 year, 5 year and 10 year tenors for a given term, while in the low interest rate subsample, variance risk premia are strictly and significantly increasing with the tenor. Looking at the term structure of variance risk premia, they are slightly increasing and almost flat in both regimes for longer tenors. For the shortest tenor however, we observe differences in the two regimes, with variance risk premia being linearly increasing with the term (i.e. the variance risk premium with a 3 month term is more negative than that with a 24 month term) for the high interest rate subsample, and it is hump-shaped and increasing for the low interest rate subsample.

[Insert Figure 3.8 and Figure 3.9 here.]

Performing the tests for variance risk premia of various tenors and terms one finds largely similar results, however for longer tenors and terms the level and dispersion in the variance risk premia on the low interest rate regime decreases substantially. In order to see this more clearly, Figure 3.10 depicts principal components of changes in variance risk premia across tenors, for the terms 3, 6, 12 and 24 months.

[Insert Figure 3.10 and Figure 3.11 here.]

It is evident from the figure that the series display very similar patterns, with almost the same spikes and drops. Figure 3.11 relates the most significant and largest changes in variance risk premia to major financial and economic events, such as the Russian debt crisis, the collapse of Bear Stearns and Lehman Brothers, S&P's downgrade of the US credit rating to AA+, etc.

Figure 3.12 displays the term structure of variance risk premia (i.e. in the term dimension) in the dates where the variance risk premium for the 2 year tenor and 3 month term,  $VRP_t^{2y,3m}$ , takes its highest negative values and its highest positive values.

[Insert Figure 3.12 here.]

In days where variance risk premium on the 2 year tenor and 3 month term,  $VRP_t^{2y,3m}$ , the term structure of variance risk premia changes shape and becomes downward sloping (i.e. decreasing with the term), suggesting expectations of a near term event risk. Furthermore during these days, the variance risk premia are decreasing with the tenor.

### 3.4 Predictor Variables of Variance Risk Premia

By construction, the main determinants of variance risk premia are the interest rate level and volatility. The underlying state variables driving the interest rate level and volatility processes will also drive the time-variation in variance risk premia.

Given the tests for structural changes imply the existence of three breaks, I start by running predictive regressions of variance risk premia on the interest rate level, realized volatility, a binary variable indicating the subsamples associated with the breaks (which largely correspond to a high and a low interest rate regime) and interaction terms between the regressors and the binary variable:

$$\begin{aligned} VRP_t^{\tau,h} = & c + \beta_1 r_{t-1} + \beta_2 E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}] + \beta_3 D_{t-1}^i \\ & + \gamma_1 (r_{t-1} \times D_{t-1}^i) + \gamma_2 (E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}] \times D_{t-1}^i) + \epsilon_t^{\tau,h} \end{aligned}$$

Since the breaks are quite distinct, inference on the regression coefficients should not be compromised. Tables 3.2 reports the regression results for variance risk premia with a term of 3 months and 6 months respectively and for tenors of 2, 5 and 10 years.

[Insert Table 3.2 here.]

In the low interest rate regime, a change in the short rate will have differential effects on variance risk premia with short versus long tenors for a given term. In particular an increase in the short rate is associated with an increase in the the variance risk premium for the 2 year tenor, has no significant effect on the 5 year tenor and decreases the variance risk premium on the 10 year tenor, with the overall effect of flattening the term structure of variance risk premia across the tenor dimension. Recall that the cross-section of variance risk premia in the tenors dimension is upward sloping and the slope is higher in the low interest rate regime. Therefore an increase in the interest rate will have the effect of reducing the slope in times where the latter is high.

In the high interest rate regime, an increase in the short rate increases the variance risk premium across different tenors, for a given term. In particular a one standard deviation increase in the short rate will increase variance risk premia by 0.1 to 0.3 standard deviations. The increase in the variance risk premium for the 10 year tenor is larger than the increase in the variance risk premium for the 2 year tenor, with the overall effect of increasing the slope of the cross-section of variance risk premia in the tenor dimension in times when the latter is less steep.

Similar results are found for realized volatility. In the low interest rate regime, an increase in the realized volatility increases the variance risk premium of the 2 year tenor, has no significant effect on the 5 year tenor and decreases the variance risk premium on the 10 year tenor, for a given term. The overall effect is again a flattening of the term structure of variance risk premia across the tenor dimension. In the high interest rate regime, an increase in variance risk premia will increase the variance risk premium of the 2 year tenor, have no significant effect on the 5 year tenor and decrease the variance risk premium on the 10 year tenor, for a given term. The magnitudes of the effects are in the order of 0.15 to 0.3 standard deviations change in variance risk premia for a one standard deviation change in realized volatility.

Next I turn to exploring, what other financial variables are able to explain the time variation in variance risk premia, controlling for the interest rate level and volatility. From an empirical standpoint, even in the absence of causality, understanding which financial and economic variables explain the time variation in variance risk premia is important for forecasting purposes.

Firstly, in order to draw reliable conclusions about the explanatory variables, it is important to examine whether variance risk premia have a unit root. The departure from stationarity due to the presence of either trends or breaks compromises statistical inference and forecasts made based on time series regressions. At first inspection the series exhibits long-run swings consistent with a process with a stochastic trend and that it might suffer from time-instability in the form of structural breaks. The sample autocorrelation function (ACF) suggests that the series is quite persistent (see Figure 3.5). Testing for unit root nonstationarity, the Augmented Dickey-Fuller (ADF) test rejects the unit-root null in favor of the alternative. However, a variance ratio test rejects the hypothesis that the series is a random walk, compromising the results from the Dickey-Fuller test, which applies to homoschedastic series. The Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test rejects the hypothesis that the variance risk premia series are trend stationary. The overall evidence about the presence of a unit root is inconclusive and it remains unclear whether the data has a unit root as it is common with many macro and financial time series.

Secondly, since the main financial variables likely to explain the time-variation in variance risk premia tend to be highly correlated with each other and with the interest level and volatility, it is important to rule out problems arising from multicollinearity. Even imperfect multicollinearity can be problematic, as it inflates the variance of the regression coefficients. This can result in parameter instability and therefore complicates the interpretation of the coefficients.

[Insert Table 3.3 here.]

Table 3.3 shows the correlation matrix for the explanatory variables of interest. These include, the three month interest rate, the variance risk premium for the 2 years tenor, 3 months term, the yield curve slope, the volatility slope, the swap spread, the VIX, Moody's seasoned Baa corporate bond yield relative to the yield on 10-year treasury constant maturity (a measure of credit spread), the Economic Policy Uncertainty Index of (Baker, Bloom, and Davis, 2013) and the Ted Spread. The data has been taken from Bloomberg and FRED (the Federal Reserve Economic Data of the St. Louis Fed), it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016. As we can see, many of the variables of interest are highly correlated with the level of interest rates.

I conduct several multicollinearity tests in order to determine whether this is an issue and single out other problematic regressors. The variance inflation factor (VIF), helps assess whether multicollinearity is problematic for a set of regressors, by evaluating the increase in the variance of an estimated regression coefficient due to the correlation among the regressors. In the case where all explanatory variables are uncorrelated, the variance inflation factor for all coefficients will be one.

[Insert Table 3.4 here.]

Table 3.4 shows the results of the test and confirms that the high correlation of the interest rate level with the other regressors has the potential to inflate the variance of the estimated coefficients. Given that the stationarity tests were inconclusive I consider an Augmented Distributed Lag (ADL) regression model both on the variance risk premium level:

$$VRP_t^{\tau,h} = c + \alpha VRP_{t-1}^{\tau,h} + \beta_1 r_{t-1} + \beta_2 E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}] + \sum_{i=1}^N \gamma_i F_{t-1}^i + \epsilon_t^{\tau,h}$$

and on its difference (assuming the series is difference stationary):

$$\Delta VRP_t^{\tau,h} = c + \alpha VRP_{t-1}^{\tau,h} + \beta_1 r_{t-1} + \beta_2 E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}] + \sum_{i=1}^N \gamma_i F_{t-1}^i + \epsilon_t^{\tau,h}$$

where  $F_{t-1}^i$  is a vector containing the financial variables of interest. Table 3.5 shows results of predictive regressions of the variance risk premium for the 2 year tenor and 3 month term,  $VRP_t^{2y,3m}$ , on these variables.

[Insert Table 3.5 here.]

Since variance risk premia are highly persistent, the lagged variance risk premium explain almost 87% of its variation, with a one standard deviation increase in  $VRP_{t-1}^{2y,3m}$  being associated with a 0.93 standard deviations increase in  $VRP_t^{2y,3m}$ . The short term interest rate is also a significant explanatory variable for the variance risk premium, with a one standard deviation increase in the short rate being associated with a 0.8 standard deviations increase in the variance risk premium. Realized volatility is also highly significant, with a one standard deviation increase in the Overall, an increase in either of the two main determinants of variance risk premia, the interest rate level and realized volatility, is associated with an increase in the variance risk premium, implying a higher compensation demanded by option sellers for taking volatility risk, and symmetrically a higher (negative) premium that investors are willing to pay in order to insure against sudden increases in volatility.

A one standard deviation increase in the slope of the yield curve, implies a 0.47 standard deviation increase in the variance risk premium. The effect is consistent with that of the short rate, since a steeper yield curve implies rising expected future short rates. Similarly, an increase in the slope of the volatility curve, is also associated with an increase in the variance risk premium, since it implies a rise in expected future interest rate volatilities.

The 10-year swap spread is also a significant explanatory variable for variance risk premia. A one standard deviation increase in the swap spread is associated with a 0.14 standard deviation increase in the variance risk premium with a two year tenor and three month term. Since the spread between swap rates and Treasury yields largely reflects a premium demanded for liquidity and default risk, the sign of the coefficient follows economic intuition. The higher the liquidity and default risk in swap markets, the higher will be the premium that investors will have to pay in order to insure themselves against sudden rises in realized volatility.

An increase in the stock market volatility index VIX and corporate credit spread, as measured by Moody's seasoned Baa 10-year corporate bond yield relative to the yield on 10-year treasury constant maturity, is associated with a decrease in the variance risk premium. The VIX and the swaption variance risk premia have time-varying correlations, displaying both periods of divergence and co-movement (in the full sample analyzed here they are overall negatively correlated for most tenors and terms). The volatility risk and therefore the compensation demanded for this risk is different in equity and government debt markets. [Mele, Obayashi, and Shalen \(2015\)](#) show, for example, that the swap rate volatility index they construct on data going from 2007 to 2013, reacts in an opposite direction to the VIX in periods of distress in

bond markets. This also explains the negative sign of the coefficient for the corporate bond spread.

As expected an increase in the TED spread, is associated with an increase in the variance risk premium, since it reflects the default risk in the interbank loan market. The magnitude of the coefficient is economically more modest, with a one standard deviation increase in the TED spread corresponding to a 0.08 standard deviation in  $VRP_t^{2y,3m}$ .

[Insert Table 3.6 here.]

Table 3.6 reports regression results of variance risk premia for various tenors and terms on the same predictors, where the data has been divided into two subsamples corresponding to the structural break points determined by the tests. The first subsample coincides with a period where interest rate are relatively high and the second with a period where interest rate are low.

The interest rate level has a positive effect on variance risk premia across all tenors and terms in both regimes. The size of the effect however is larger in the regime where interest rates are high. Similarly, an increase in realized volatility is associated with an increase in the variance risk premium in both regimes. For the longest tenors and terms however, the coefficient is insignificant.

Similarly to the level of interest rates, an increase in the slope of the yield curve has a positive effect on the variance risk premium across all tenors and terms, both when interest rates are high and when they are low. The magnitude of the effect is however significantly larger (more than twice as large) for the low interest rate period. The slope of the volatility curve retains the significance and positive effect for most terms and tenors in the high interest rate regime, but it switches the sign of the effect for longer terms and tenors in the low interest rate period.

The swap spread has a significant and positive effect only on the variance risk premium of the longer tenors and terms, with the effect being stronger in the low interest rate period. The VIX and corporate credit spread retain their negative effect on the variance risk premia across all terms and tenors and for both subsamples. The effect of the VIX is considerably stronger in the subsample corresponding to the low interest rate period.

Lastly, since the stationarity tests for variance risk premia are largely inconclusive about the presence of a unit root, I run predictive regressions of first differences of variance risk premia on the set of predictive variables. Table 3.7 presents the regression results for the variance risk premium with a tenor of 2 years and term of 3 months  $VRP_t^{2y,3m}$ , and on the variance risk premium with a tenor of 5 years and

term of 12 months  $VRP_t^{5y,12m}$ .

[Insert Table 3.7 here.]

As expected most coefficients switch their sign given that the variance risk premia are negative on average and we are assessing first differences. An increase in the interest rate level has no significant effect on the  $VRP_t^{2y,3m}$ , and it is associated with a decrease in the change on the variance risk premium with a tenor of 5 years and term of 12 months  $VRP_t^{5y,12m}$ . Changes in the interest rate level have no significant effect. The significance of realized volatility as an explanatory variable for variance risk premia persists across all terms and tenors, with the effect being stronger for shorter terms and tenors. Similarly, the significance of the slope of the yield curve on variance risk premia persists across all tenors and terms. The slope of the volatility curve on the other hand, is only significant for the shorter tenors and terms. VIX, the corporate credit spread and the TED spread become insignificant.

### 3.5 Conclusions

The variance risk premium embedded in swaptions, defined as the difference between expected realized variances and risk neutral variances, reflects the compensation demanded for holding interest rate variance risk. Its time-variation depends on the distribution of interest rates and their volatilities, as well as on other variables that affect the demand and supply for swaptions. Given the current near zero interest rate environment it becomes relevant to study whether any differences in the properties of variance risk premia are observed compared to normal times. The variance risk premium is unobservable, and to measure it I rely on Black-implied swaption volatilities, as proxies of risk neutral volatilities and forecasts of realized volatility based on a GARCH specification for the volatility process, as estimates of expected realized volatilities. To produce the forecasts, I condition on each period's information set.

Analyzing the time-series properties of variance risk premia, I find that the compensation for volatility risk has been economically significant and, as expected, negative on average during the sample, reflecting the fact that investors are willing to pay a premium during normal times in order to insure against high realized volatility during periods of market turmoil (if we think in terms of a variance swap). The series displays however brief periods where it switches sign, implying unexpected shocks in realized variance. Variance risk premia are highly correlated across terms and tenors. The process fluctuates around two distinct regimes, one with high (negative) level and high dispersion, corresponding to periods where the interest rate level is low, and the second with a nearly zero level and no dispersion, corresponding to periods where



the interest rate level is relatively high. During these two regimes, the slope of the term structure of variance risk premia along the tenor dimension displays significant differences. In the low interest rate period, more frequent and severe episodes where the variance risk premium switches sign are observed. Looking at the episodes where the variance risk premium spikes or falls abruptly, we observe that the term structure of variance risk premia in the term dimension, display a switch in the sign of the slope. For the spike episodes, the term structure is significantly upward sloping, while for the abrupt fall episodes it is downward sloping.

Predictive regression results suggest that the main determinants of the variation in variance risk premia are as expected the interest level and past volatility, which explain most of its variation. Other measures, such as credit spreads, swap spreads, the interest rate slope and the stock market volatility index are significant predictors.

### 3.6 Tables

**Table 3.1**  
**Descriptive Statistics of Swaption Implied Volatilities**

The table reports descriptive statistics for Swaption Implied Volatilities with tenors of 2, 5 and 10 years and terms of 3, 6, 12 and 24 months. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

| Swaption Implied Volatility (% p.a) |          |       |       |          |       |       |
|-------------------------------------|----------|-------|-------|----------|-------|-------|
|                                     | Term 3m  |       |       | Term 6m  |       |       |
|                                     | 2 y      | 5 y   | 10 y  | 2 y      | 5 y   | 10 y  |
| <b>Mean</b>                         | 41.36    | 33.05 | 26.45 | 40.21    | 31.97 | 25.99 |
| <b>St. Dev.</b>                     | 22.11    | 15.44 | 11.44 | 20.86    | 14.25 | 10.52 |
| <b>Skewness</b>                     | 00.01    | 00.23 | 00.81 | 00.00    | 00.18 | 00.63 |
| <b>Kurtosis</b>                     | 01.57    | 02.06 | 03.62 | 01.49    | 01.87 | 02.81 |
|                                     | Term 12m |       |       | Term 24m |       |       |
|                                     | 2 y      | 5 y   | 10 y  | 2 y      | 5 y   | 10 y  |
| <b>Mean</b>                         | 37.43    | 29.86 | 24.98 | 32.23    | 26.78 | 23.33 |
| <b>St. Dev.</b>                     | 18.46    | 12.48 | 09.44 | 14.54    | 10.06 | 08.11 |
| <b>Skewness</b>                     | 00.11    | 00.24 | 00.55 | 00.43    | 00.44 | 00.64 |
| <b>Kurtosis</b>                     | 01.52    | 01.88 | 02.40 | 02.00    | 02.19 | 02.52 |
|                                     |          |       |       |          |       |       |
|                                     |          |       |       |          |       |       |

**Table 3.2**  
**Predictive Regressions of Variance Risk Premia**

The table reports predictive regression results of variance risk premia, computed as  $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$ , on their main lagged explanatory variables, i.e. the short term interest rate (the 3 month Treasury rate) and the forecasts of interest rate realized volatility obtained assuming a GARCH(1,1) model for the volatility process. Due to the structural breaks in the data, determined by the methods developed in (Bai and Perron, 1998), a dummy variable  $D_{t-1}^{Highr}$  for the high interest rate regime and interaction terms are included in the regression. The regression results are reported for VRP with tenors of 2, 5 and 10 years and a term of 3 months.  $t$ -statistics are shown in parenthesis, below the reported estimated coefficients. The standard errors are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. All variables are standardized. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

|  | $VRP_t^{\tau=2,5,10 y; h=3m}$ |                  |                  | $VRP_t^{\tau=2,5,10 y; h=6m}$ |                  |                  |
|--|-------------------------------|------------------|------------------|-------------------------------|------------------|------------------|
|  | 2 years                       | 5 years          | 10 years         | 2 years                       | 5 years          | 10 years         |
| $r_{t-1}^{3m}$   | 1.12<br>[7.25]                | 0.11<br>[0.66]   | -0.83<br>[-6.11] | 1.98<br>[13.28]               | 0.35<br>[1.90]   | -0.70<br>[-5.43] |
| $E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}]$                        | 0.39<br>[5.08]                | 0.04<br>[0.47]   | -0.58<br>[-7.30] | 0.61<br>[9.32]                | 0.02<br>[0.22]   | -0.60<br>[-9.95] |
| $D_{t-1}^{Highr}$  | 1.07<br>[8.30]                | 0.60<br>[3.98]   | -0.46<br>[-3.23] | 1.49<br>[12.59]               | 0.56<br>[3.66]   | -0.62<br>[-4.61] |
| $r_{t-1}^{3m} \times D_{t-1}^{Highr}$                          | -1.01<br>[-6.00]              | 0.12<br>[0.63]   | 1.11<br>[7.56]   | -1.90<br>[-11.71]             | -0.06<br>[-0.28] | 1.04<br>[7.50]   |
| $E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}] \times D_{t-1}^{Highr}$ | -0.22<br>[-3.32]              | -0.27<br>[-3.05] | 0.26<br>[2.81]   | -0.36<br>[-5.65]              | -0.17<br>[-1.87] | 0.45<br>[4.95]   |
| <b>Adj <math>R^2</math></b>                                    | 42.96                         | 31.63            | 34.51            | 56.08                         | 44.87            | 47.62            |

**Table 3.3**  
**Correlation Matrix of Variables of Interest**

The table reports the correlation matrix for the explanatory variables of interest. These include in the order reported, the three month interest rate, the variance risk premia computed as  $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$  ( and in this case with tenor 2 years, term 3 months), the yield curve slope, the volatility slope, the swap spread, the VIX, Moody's Seasoned Baa Corporate Bond Yield relative to the yield on 10-Year Treasury Constant Maturity (a measure of credit spread), the Economic Policy Uncertainty Index of (Baker, Bloom, and Davis, 2013) and the TED Spread. The data has been taken from FRED (the Federal Reserve Economic Data of the St. Louis Fed) and Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

| Correlation of Explanatory Variables |          |            |               |               |           |            |            |            |           |
|--------------------------------------|----------|------------|---------------|---------------|-----------|------------|------------|------------|-----------|
|                                      | <b>r</b> | <b>VRP</b> | <b>YSlope</b> | <b>VSlope</b> | <b>SS</b> | <b>VIX</b> | <b>BAA</b> | <b>EPU</b> | <b>TS</b> |
| <b>r</b>                             | 1        | -          | -             | -             | -         | -          | -          | -          | -         |
| <b>VRP</b>                           | -0.61    | 1          | -             | -             | -         | -          | -          | -          | -         |
| <b>YSlope</b>                        | -0.83    | 0.47       | 1             | -             | -         | -          | -          | -          | -         |
| <b>VSlope</b>                        | 0.84     | -0.67      | -0.77         | 1             | -         | -          | -          | -          | -         |
| <b>SS</b>                            | 0.82     | -0.49      | -0.55         | 0.67          | 1         | -          | -          | -          | -         |
| <b>VIX</b>                           | -0.02    | 0.30       | 0.14          | -0.05         | 0.18      | 1          | -          | -          | -         |
| <b>BAA</b>                           | -0.57    | 0.49       | 0.48          | -0.42         | -0.27     | 0.65       | 1          | -          | -         |
| <b>EPU</b>                           | -0.29    | 0.30       | 0.25          | -0.27         | -0.16     | 0.42       | 0.46       | 1          | -         |
| <b>TS</b>                            | 0.33     | -0.15      | -0.29         | 0.32          | 0.40      | 0.50       | 0.21       | 0.10       | 1         |

**Table 3.4**  
**Variance Inflation Factor to assess Multicollinearity**

The table reports results from the variance inflation factor, a measure that assesses the degree of multicollinearity in a given set of regressors, by evaluating the increase in the variance of an estimated regression coefficient that is due to the correlation among the regressors. In the case where all explanatory variables are uncorrelated, the variance inflation factor for all coefficients will be one. A VIF value larger than 10 for a particular variable, suggests that the variable is problematic. The variables of interest include (in the order reported), the variance risk premia computed as  $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$  (and in this case with tenor 2 years, term 3 months) the three month interest rate, realized volatility estimated with a GARCH(1,1) model, the yield curve slope, the volatility slope, the swap spread, the VIX, Moody's Seasoned Baa Corporate Bond Yield relative to the yield on 10-Year Treasury Constant Maturity (a measure of credit spread), the Economic Policy Uncertainty Index of (Baker, Bloom, and Davis, 2013) and the TED Spread. The data has been taken from FRED (the Federal Reserve Economic Data of the St. Louis Fed) and Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

| <b>Variance Inflation Factor</b> |      |       |   |        |        |      |      |      |      |      |
|----------------------------------|------|-------|---|--------|--------|------|------|------|------|------|
|                                  | VRP  | r     | E <sup>ℙ</sup> [σ <sub>t</sub> <sup>τ,h</sup> ] | YSlope | VSlope | SS   | VIX  | BAA  | EPU  | TS   |
| <b>VIF</b>                       | 3.81 | 22.91 | 13.10   | 6.02   | 10.33  | 5.10 | 4.53 | 5.76 | 1.47 | 1.79 |
| <b>VIF</b>                       | -    | 20.24 | 8.23  | 5.00   | 5.66   | 5.01 | 3.92 | 5.59 | 1.47 | 1.77 |
| <b>VIF</b>                       | -    | -     | 7.69  | 2.82   | 5.42   | 2.76 | 3.18 | 3.83 | 1.47 | 1.77 |
| <b>VIF</b>                       | 1.78 | -     | 4.57  | 2.35   | -      | 3.10 | 3.35 | 3.55 | 1.47 | 1.78 |

**Table 3.5**  
**Predictive Regressions of Variance Risk Premia**

The table reports predictive regression results of variance risk premia computed as  $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$  (and in this case with tenor 2 years, term 3 months and tenor 5 years, term 12 months) on a set of lagged explanatory variables. These include the lagged variance risk premia itself, the three month interest rate, realized volatility estimated with a GARCH(1,1) model, the yield curve slope, the volatility slope, the swap spread, the VIX, Moody's Seasoned Baa Corporate Bond Yield relative to the yield on 10-Year Treasury Constant Maturity (a measure of credit spread), and the TED Spread.  $t$ -statistics are shown in parenthesis, below the reported estimated coefficients. The standard errors are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. All variables are standardized. The data has been taken from FRED (the Federal Reserve Economic Data of the St. Louis Fed) and Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

|   | $VRP_t^{2y,3m}$  |                 |                  |                  |                   |
|---|------------------|-----------------|------------------|------------------|-------------------|
|   | (I)              | (II)            | (III)            | (IV)             | (V)               |
| $VRP_{t-1}^{\tau,h}$                    | 0.93<br>[109.70] | -               | -                | 0.87<br>[67.45]  | -                 |
| $r_{t-1}^{3m}$                          | -                | 0.60<br>[35.83] | -                | -                | 0.80<br>[10.30]   |
| $E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}]$ | -                | -               | 0.88<br>[15.71]  | -                | 1.01<br>[18.93]   |
| <b>Yield</b>                            | -                | -               | 0.21             | -0.00            | 0.47              |
| <b>Slope</b> $_{t-1}$                   | -                | -               | [9.70]           | [-0.27]          | [13.20]           |
| <b>Vol</b>                              | -                | -               | 1.08             | 0.03             | 0.99              |
| <b>Slope</b> $_{t-1}$                   | -                | -               | [26.99]          | [2.04]           | [25.46]           |
| <b>Swap</b>                             | -                | -               | 0.41             | 0.03             | 0.14              |
| <b>Spread</b> $_{t-1}$                  | -                | -               | [17.28]          | [4.72]           | [4.40]            |
| <b>VIX</b> $_{t-1}$                     | -                | -               | -0.24<br>[-7.48] | -0.03<br>[-2.82] | -0.39<br>[-10.35] |
| <b>Credit</b>                           | -                | -               | -0.43            | -0.02            | -0.19             |
| <b>Spread</b> $_{t-1}$                  | -                | -               | [-10.72]         | [-1.67]          | [-3.83]           |
| <b>TED</b>                              | -                | -               | 0.08             | 0.02             | 0.08              |
| <b>Spread</b> $_{t-1}$                  | -                | -               | [3.79]           | [1.66]           | [3.78]            |
| <b>Adj <math>R^2</math></b>             | 86.90            | 36.54           | 64.69            | 87.23            | 67.86             |

**Table 3.6**  
**Predictive Regressions of Variance Risk Premia by Interest Rate Regime**

The table reports predictive regression results of variance risk premia computed as  $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$  (and in this case with tenor 2 years, term 3 months and tenor 5 years, term 12 months) on a set of lagged explanatory variables. The data has been divided into two subsets according to the structural break points determined by the methods developed in (Bai and Perron, 1998). Although there are four breaks, the periods correspond to two regimes, one where interest rates are relatively low (based on the given sample), and one where they are relatively high. The regressors include the lagged variance risk premia itself, the three month interest rate, realized volatility estimated with a GARCH(1,1) model, the yield curve slope, the volatility slope, the swap spread, the VIX, Moody's Seasoned Baa Corporate Bond Yield relative to the yield on 10-Year Treasury Constant Maturity (a measure of credit spread), the Economic Policy Uncertainty Index of (Baker, Bloom, and Davis, 2013) and the TED Spread.  $t$ -statistics are shown in parenthesis, below the reported estimated coefficients. The standard errors are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. All variables are standardized. The data has been taken from FRED (the Federal Reserve Economic Data of the St. Louis Fed) and Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

|   | High Interest Rate Regime |                  |                   | Low Interest Rate Regime |                  |                   |
|---|---------------------------|------------------|-------------------|--------------------------|------------------|-------------------|
|   | $VRP_t^{2y,3m}$           | $VRP_t^{5y,12m}$ | $VRP_t^{10y,24m}$ | $VRP_t^{2y,3m}$          | $VRP_t^{5y,12m}$ | $VRP_t^{10y,24m}$ |
| $r_{t-1}^{3m}$                          | 0.57<br>[9.09]            | 0.85<br>[11.65]  | 0.59<br>[6.52]    | 0.45<br>[6.40]           | 0.29<br>[2.78]   | 0.17<br>[2.10]    |
| $E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}]$ | 1.73<br>[13.05]           | 0.49<br>[6.31]   | -0.02<br>[-0.33]  | 0.70<br>[17.17]          | 0.58<br>[10.15]  | -0.01<br>[-0.18]  |
| <b>Yield</b><br><b>Slope</b> $_{t-1}$   | 0.15<br>[4.24]            | 0.17<br>[4.46]   | 0.04<br>[0.79]    | 0.34<br>[11.56]          | 0.61<br>[19.84]  | 0.45<br>[17.62]   |
| <b>Vol</b><br><b>Slope</b> $_{t-1}$     | 1.57<br>[13.04]           | 0.55<br>[8.33]   | -0.08<br>[-1.34]  | 0.72<br>[22.27]          | -0.07<br>[-2.50] | -0.30<br>[-13.20] |
| <b>Swap</b><br><b>Spread</b> $_{t-1}$   | 0.03<br>[0.65]            | 0.00<br>[0.10]   | 0.18<br>[4.75]    | -0.12<br>[-2.09]         | 0.32<br>[4.07]   | 0.25<br>[4.47]    |
| <b>VIX</b> $_{t-1}$                     | -0.17<br>[-5.35]          | -0.04<br>[-1.21] | -0.07<br>[-1.99]  | -0.55<br>[-9.05]         | -0.31<br>[-5.91] | -0.39<br>[-8.54]  |
| <b>Credit</b><br><b>Spread</b> $_{t-1}$ | -0.17<br>[-2.66]          | -0.17<br>[-3.24] | -0.51<br>[-9.49]  | -0.14<br>[-2.01]         | -0.13<br>[-2.00] | -0.02<br>[-0.34]  |
| <b>EPU</b> $_{t-1}$                     | -0.03<br>[-1.40]          | -0.02<br>[-1.25] | -0.01<br>[-0.83]  | 0.02<br>[0.80]           | -0.02<br>[-1.04] | 0.03<br>[2.00]    |
| <b>TED</b><br><b>Spread</b> $_{t-1}$    | -0.02<br>[-0.48]          | 0.04<br>[1.54]   | 0.07<br>[2.52]    | 0.09<br>[1.96]           | 0.01<br>[0.38]   | -0.01<br>[-0.25]  |
| <b>Adj <math>R^2</math></b>             | 56.26                     | 74.75            | 72.56             | 52.06                    | 48.03            | 67.21             |

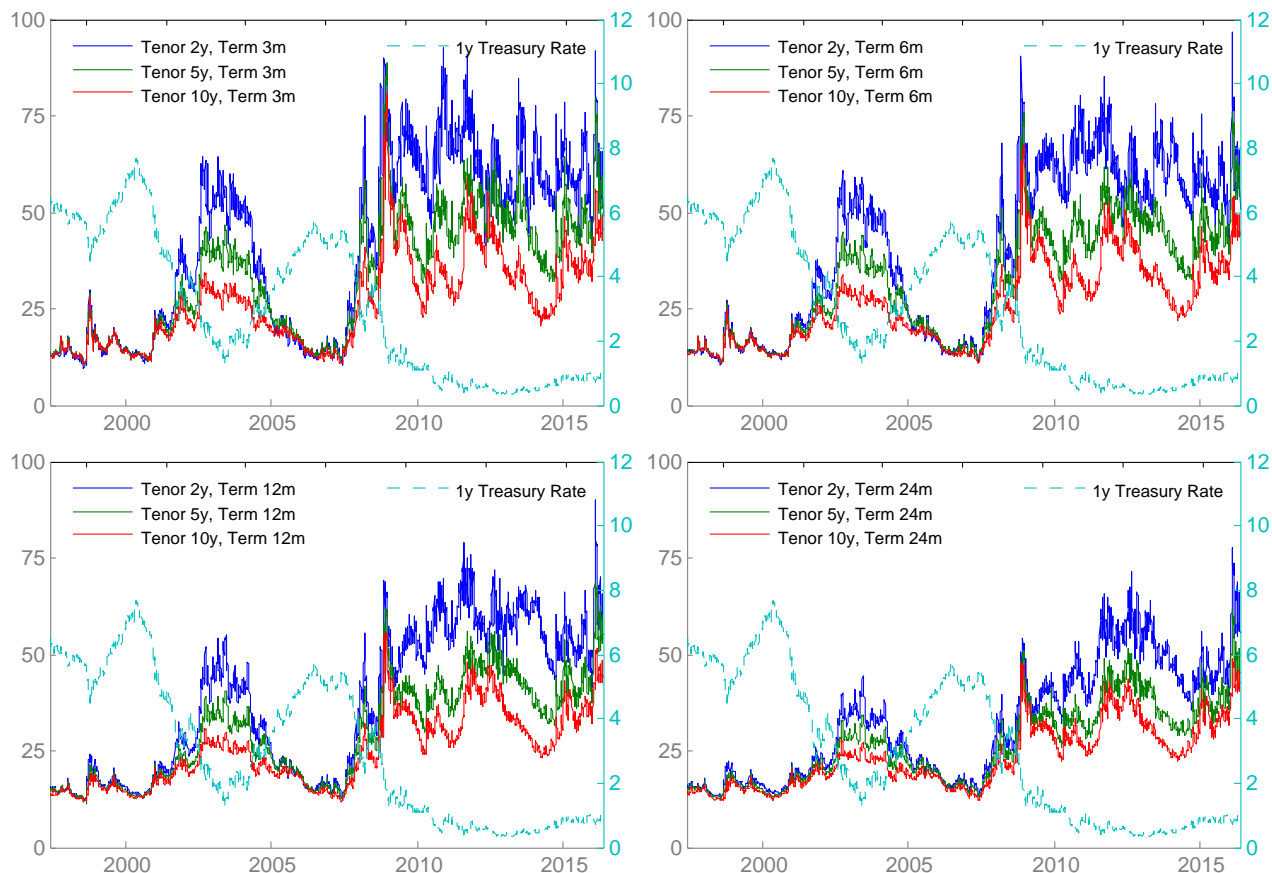
**Table 3.7**  
**Predictive Regressions of Changes in Variance Risk Premia**

The table reports predictive regression results of first differences in variance risk premia on a set of lagged explanatory variables. The variance risk premia are computed as  $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$  (and in this case the results for the tenor 2 years, term 3 months are presented). These include the lagged variance risk premia itself, the three month interest rate, realized volatility estimated with a GARCH(1,1) model, the yield curve slope, the volatility slope, the swap spread, the VIX, Moody's Seasoned Baa Corporate Bond Yield relative to the yield on 10-Year Treasury Constant Maturity (a measure of credit spread), the Economic Policy Uncertainty Index of (Baker, Bloom, and Davis, 2013) and the TED Spread.  $t$ -statistics are shown in parenthesis, below the reported estimated coefficients. The standard errors are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. All variables are standardized. The data has been taken from FRED (the Federal Reserve Economic Data of the St. Louis Fed) and Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

|  | $\Delta VRP_t^{2y,3m}$ |                  | $\Delta VRP_t^{5y,12m}$ |                  |
|--|------------------------|------------------|-------------------------|------------------|
| $r_{t-1}^{3m}$                                 | -0.10<br>[-1.52]       | -0.09<br>[-1.32] | -0.18<br>[-2.71]        | -0.18<br>[-2.64] |
| $\Delta r_{t-1}^{3m}$                          | -                      | -0.01<br>[-0.77] | -                       | 0.00<br>[-0.25]  |
| $E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}]$        | -0.33<br>[-6.04]       | -0.31<br>[-5.62] | -0.11<br>[-2.22]        | -0.10<br>[-2.09] |
| $\Delta E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}]$ | -                      | -0.07<br>[-2.17] | -                       | -0.03<br>[-0.85] |
| <b>Yield Slope</b> $_{t-1}$                    | -0.12<br>[-3.98]       | -0.12<br>[-3.81] | -0.10<br>[-3.32]        | -0.10<br>[-3.31] |
| <b>Vol Slope</b> $_{t-1}$                      | -0.31<br>[-6.20]       | -0.30<br>[-5.97] | -0.06<br>[-1.34]        | -0.06<br>[-1.33] |
| <b>Swap Spread</b> $_{t-1}$                    | -0.02<br>[-0.93]       | -0.02<br>[-0.79] | 0.03<br>[1.02]          | 0.03<br>[1.01]   |
| <b>VIX</b> $_{t-1}$                            | 0.02<br>[0.55]         | 0.02<br>[0.58]   | 0.05<br>[1.34]          | 0.05<br>[1.33]   |
| <b>Credit Spread</b> $_{t-1}$                  | 0.06<br>[1.45]         | 0.06<br>[1.25]   | -0.04<br>[-0.88]        | -0.04<br>[-0.87] |
| <b>EPU</b> $_{t-1}$                            | -0.02<br>[-0.99]       | -0.02<br>[-0.86] | -0.01<br>[-0.37]        | -0.01<br>[-0.34] |
| <b>TED Spread</b> $_{t-1}$                     | 0.02<br>[0.73]         | 0.02<br>[0.67]   | -0.01<br>[-0.32]        | -0.01<br>[-0.32] |
| <b>Adj <math>R^2</math></b>                    | 2.23                   | 2.66             | 0.44                    | 0.53             |

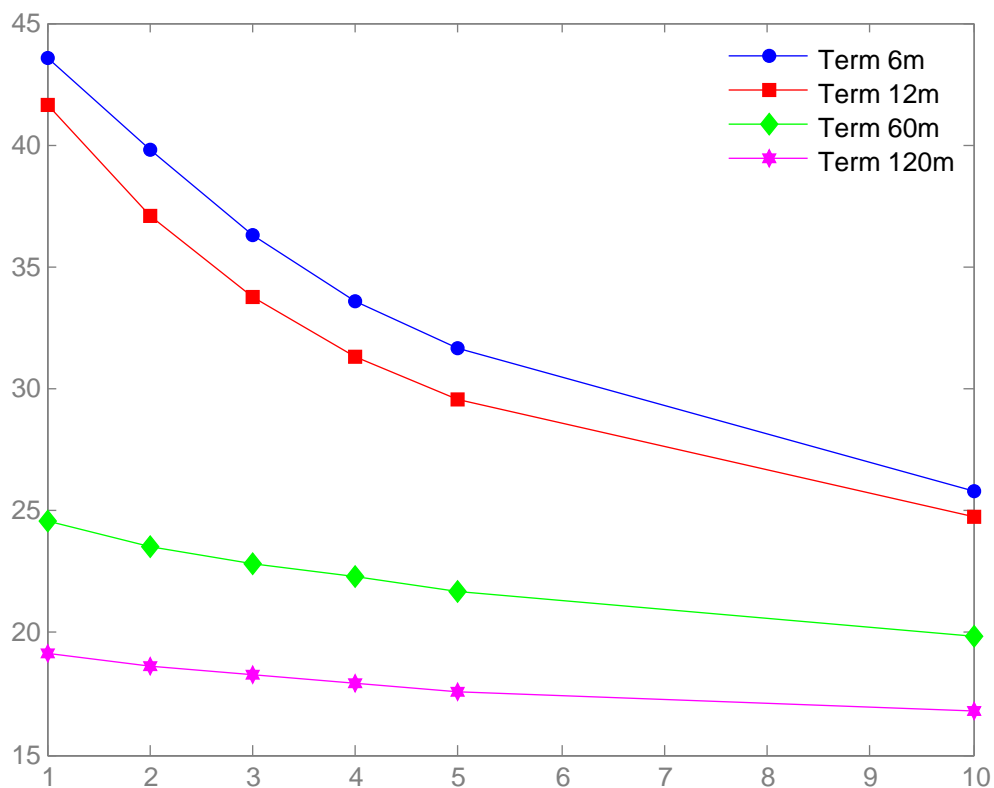


### 3.7 Figures



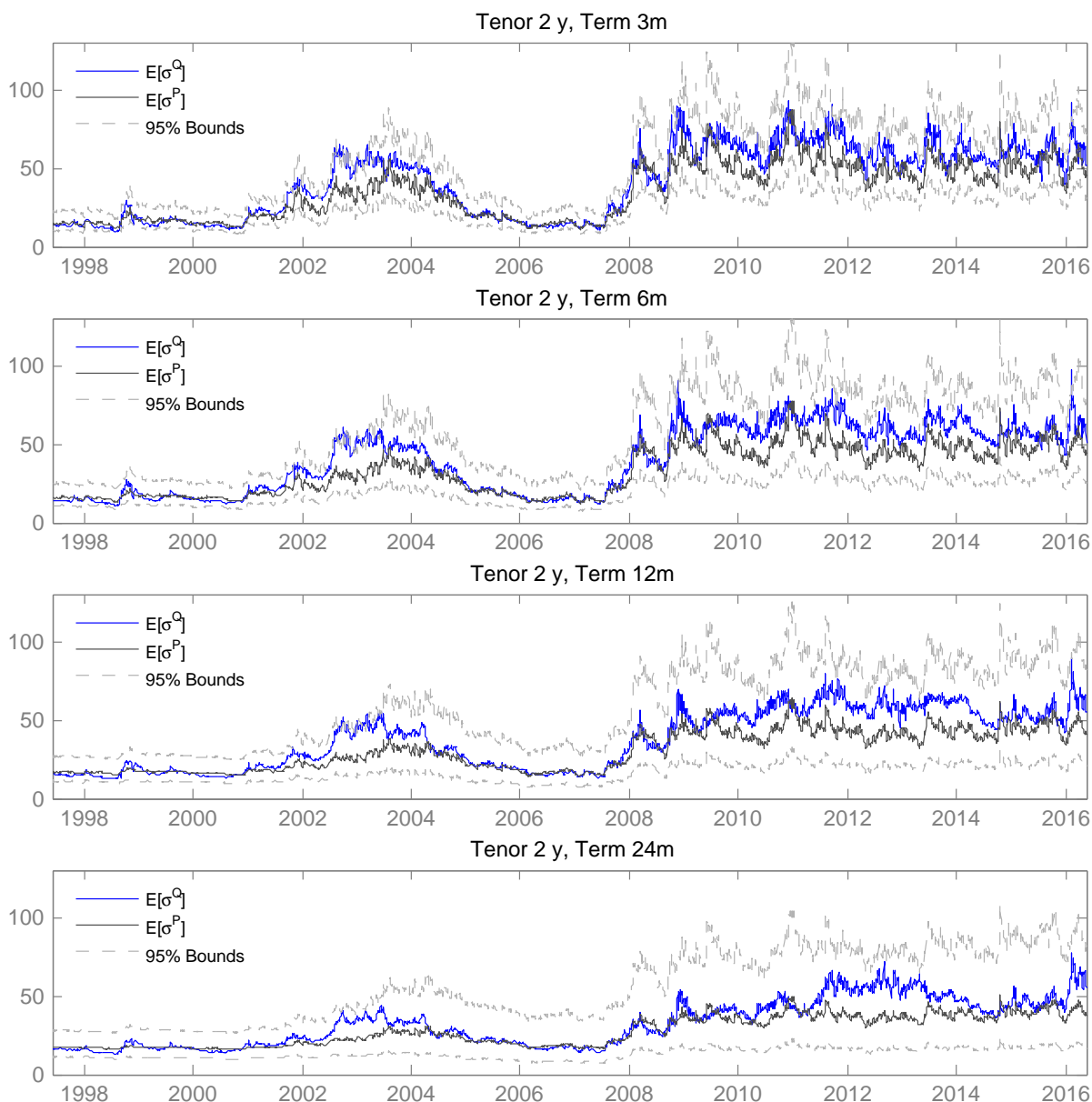
**Figure 3.1. Swaption Implied Volatilities**

This figure plots Swaption implied volatilities for tenors of 2, 5 and 10 years and terms of 3, 6, 12 and 24 months along with the one year Treasury rate. The figures are in percent, annualized. The data is from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.



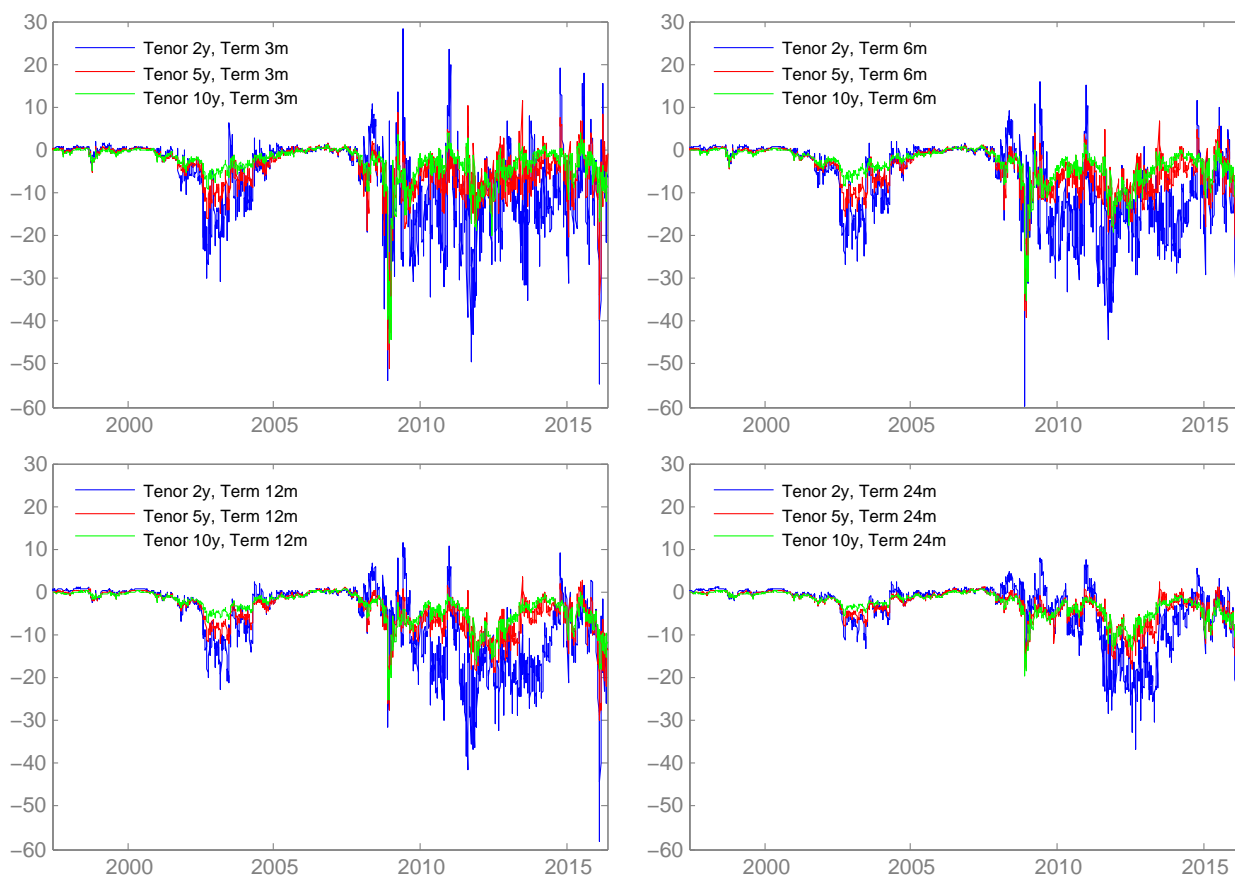
**Figure 3.2. Cross Section of Swaption Implied Volatilities**

This figure plots the cross-section of Swaption implied volatilities for tenors of 1, 2, 3, 4, 5 and 10 years and terms of 6, 12, 60 and 120 months. The figures are in percent, annualized. The data is from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.



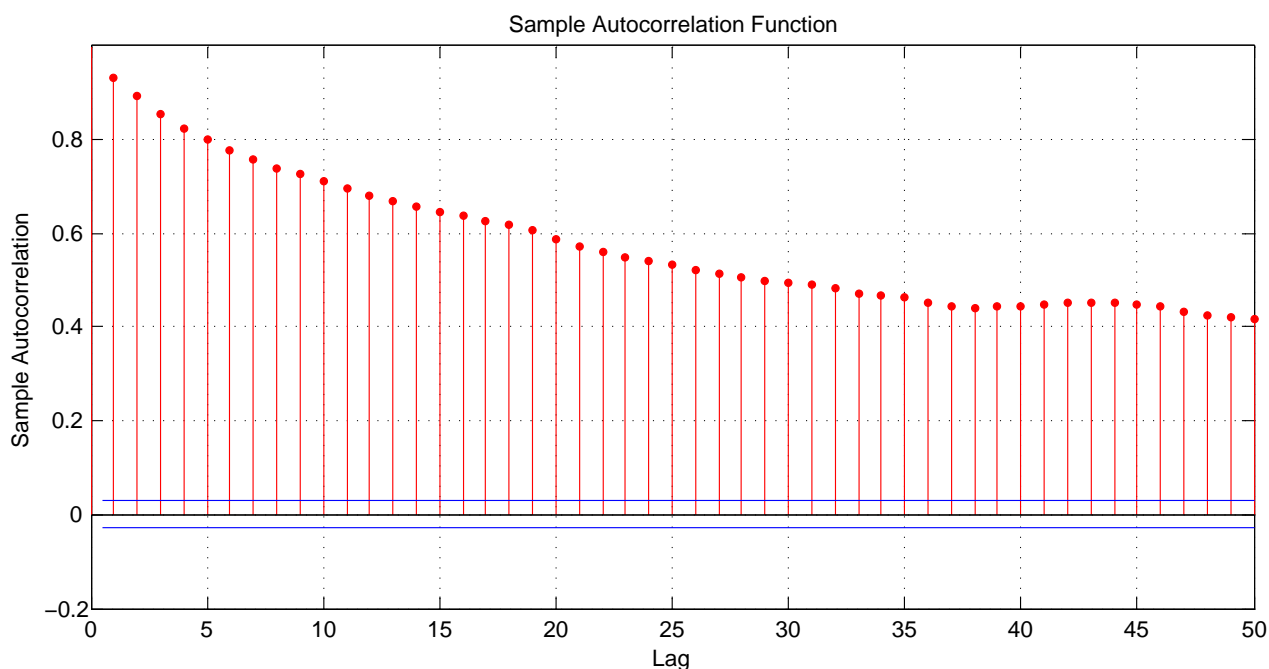
**Figure 3.3. Swaption Implied Volatilities and Forecasts of Realized Volatility**

This figure plots Swaption implied volatilities and expected realized volatility forecasts  $E^{\mathbb{P}}[\sigma_t^{\tau,h}]$  along with their 95% confidence bounds, for the 2 years tenor, and terms going from 3m to 24 months. The expected realized volatility forecasts and their confidence bounds are computed at each point in time from simulations based on parameter estimates from a GARCH(1,1) model and conditioning at the information available at each point in time. The figures are in percent, annualized. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.



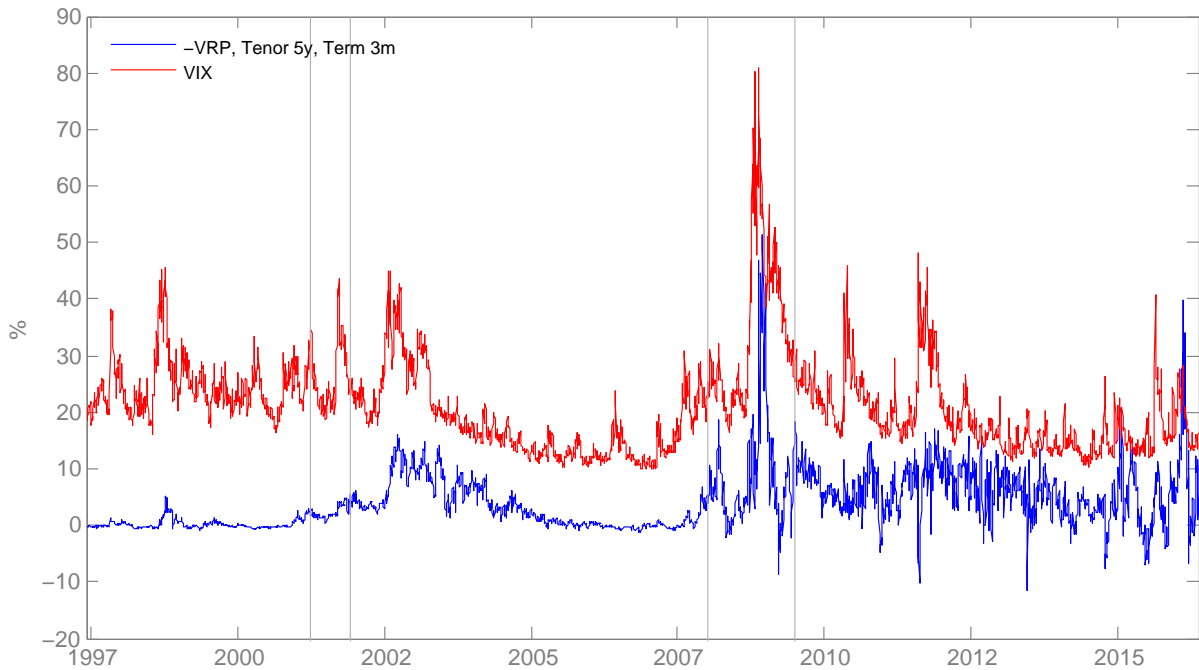
**Figure 3.4. Variance Risk Premia**

The figure plots variance risk premia, computed as  $VRP_t^{\tau,h} = E^{\mathbb{P}}[\sigma_t^{\tau,h}]^2 - E^{\mathbb{Q}}[\sigma_t^{\tau,h}]^2$ , with tenor of 2 years and terms of 3 months. The expected realized volatility forecasts  $E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2]$  are computed at each point in time from simulations based on parameter estimates from a GARCH(1,1) model and conditioning at the information available at each point in time. The figures are in percent, annualized. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.



**Figure 3.5. Autocorrelation Function for Variance Risk Premia**

The figure plots the autocorrelation function of variance risk premia with tenor of 2 years and terms of 3 months, computed as  $VRP_t^{\tau,h} = E^{\mathbb{P}}[\sigma_t^{\tau,h}]^2 - E^{\mathbb{Q}}[\sigma_t^{\tau,h}]^2$ . The expected realized volatility forecasts  $E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2]$  are computed at each point in time from simulations based on parameter estimates from a GARCH(1,1) model and conditioning at the information available at each point in time. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.



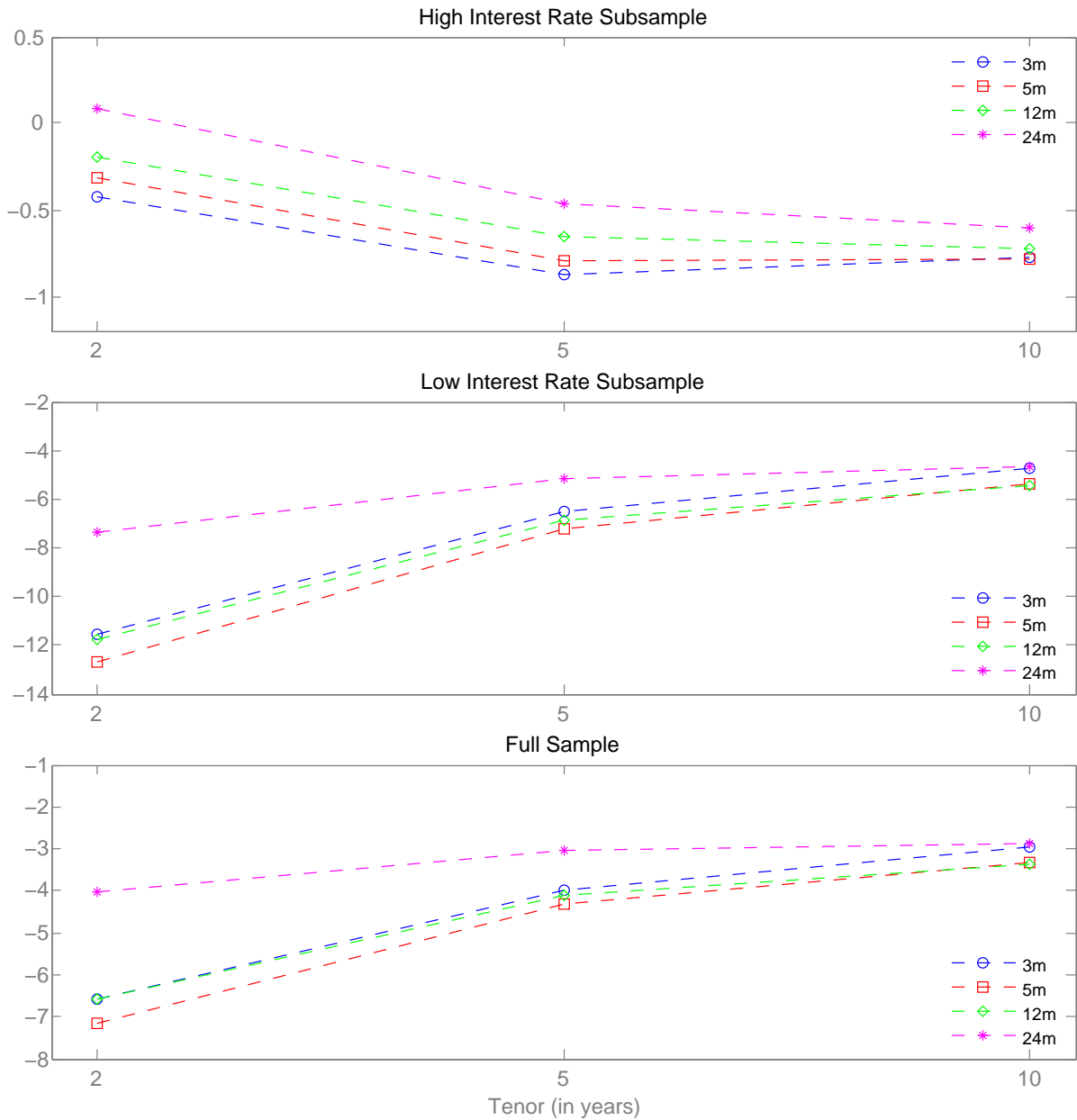
**Figure 3.6. Swaption Variance Risk Premia and the VIX**

The figure plots minus the variance risk premia, computed as  $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$ , with a tenor of 5 years and terms of 3 months along with the VIX. The figures are in percent, annualized. The data has been taken from FRED (the Federal Reserve Economic Data of the St. Louis Fed) and Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.



**Figure 3.7. Structural Breaks in the Data**

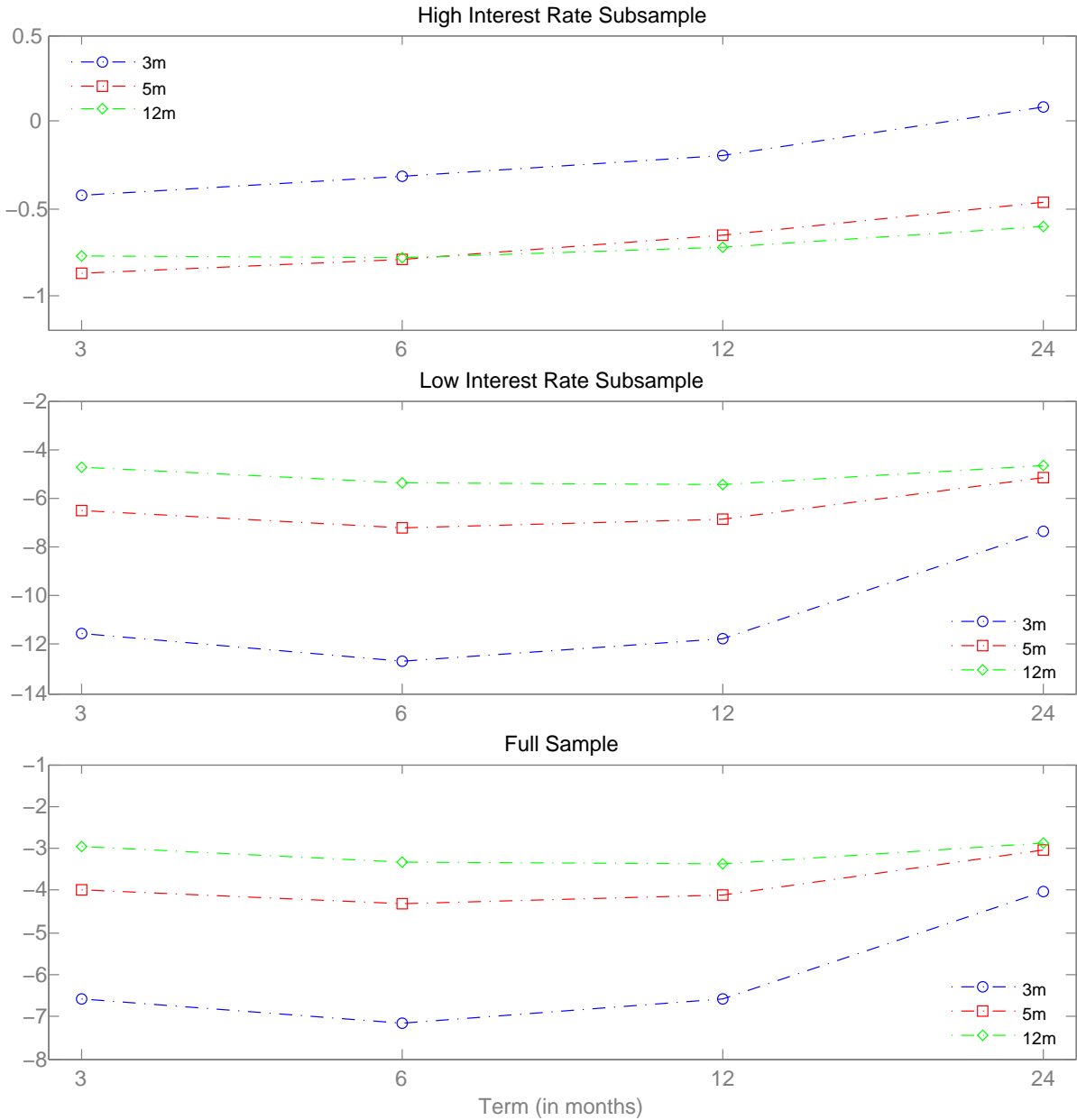
The figure plots the Swaption implied volatility and variance risk premia, with tenor 1 year and term 6 months, along with structural break points (in grey dashed vertical lines), determined by the methods developed in (Bai and Perron, 1998). In red dashed lines, are reported the means of the data by the regimes corresponding to the break points. The green vertical lines denote two events associated with the structural breaks. The variance risk premia are computed as  $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$ . The expected realized volatility forecasts  $E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2]$  are computed at each point in time from simulations based on parameter estimates from a GARCH(1,1) model and conditioning at the information available at each point in time. The figures are in percent, annualized. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.



**Figure 3.8. Cross-Section of Variance Risk Premia by Subsamples**

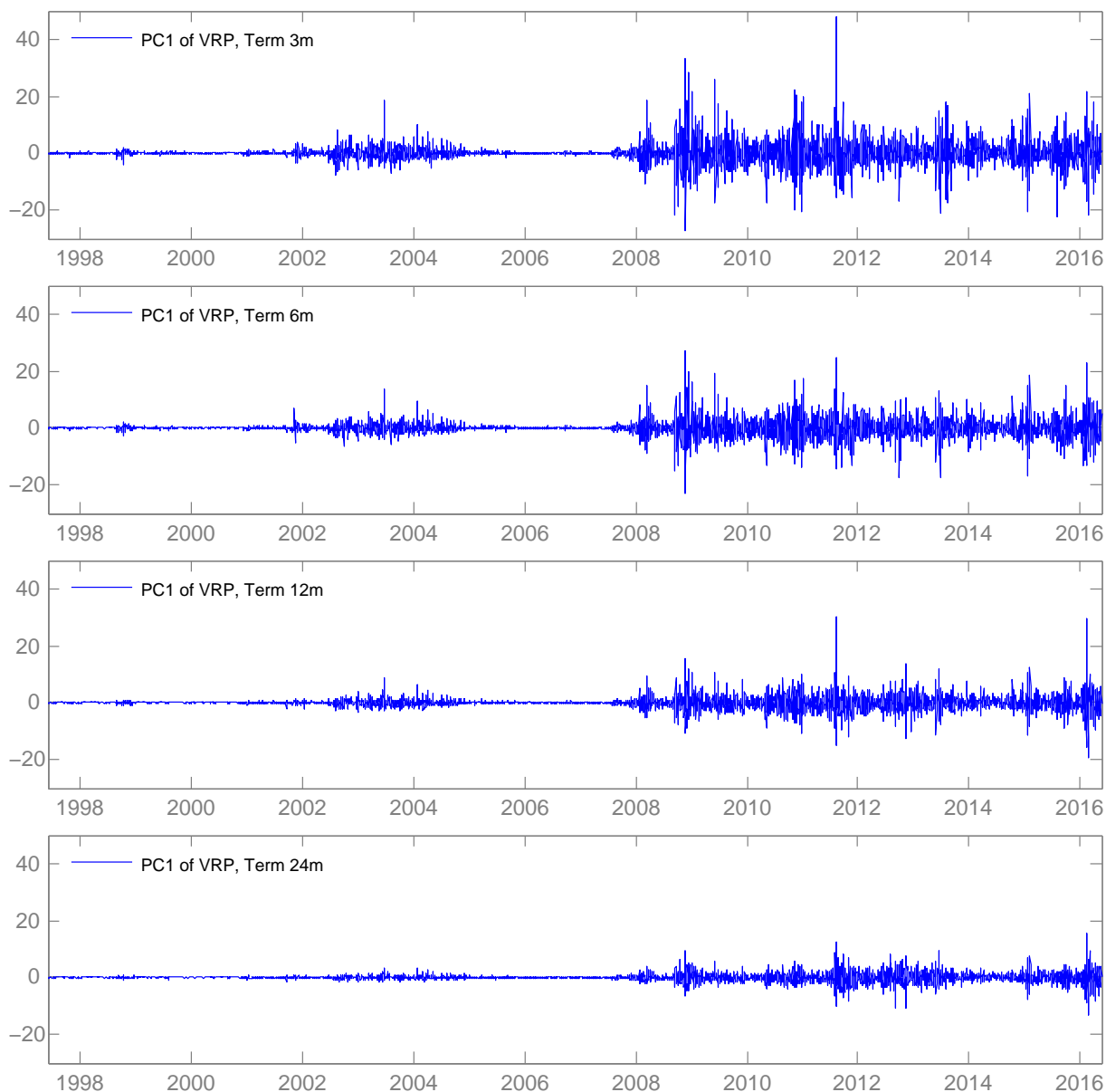
The figure plots the cross-section of variance risk premia in the tenor dimension by the subsamples corresponding to the break points. The variance risk premia are computed as  $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$ . The expected realized volatility forecasts  $E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2]$  are computed at each point in time from simulations based on parameter estimates from a GARCH(1,1) model and conditioning at the information available at each point in time. The figures are in percent, annualized. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.





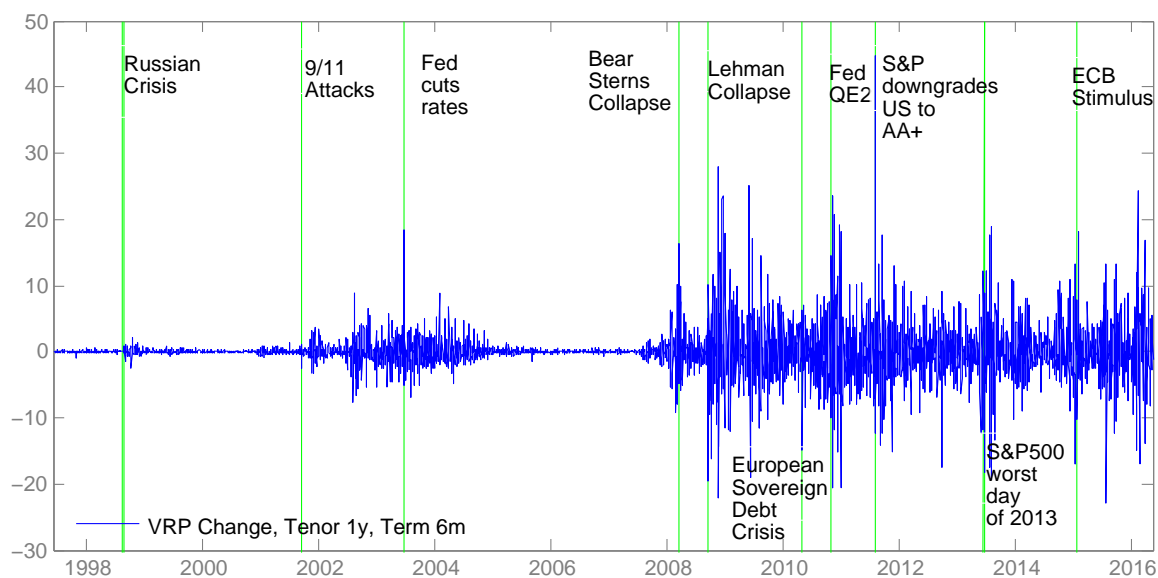
**Figure 3.9. Term Structure of Variance Risk Premia by Subsamples**

The figure plots the term structure of variance risk premia (i.e. in the term dimension) by the subsamples corresponding to the break points. The variance risk premia are computed as  $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$ . The expected realized volatility forecasts  $E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2]$  are computed at each point in time from simulations based on parameter estimates from a GARCH(1,1) model and conditioning at the information available at each point in time. The figures are in percent, annualized. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.



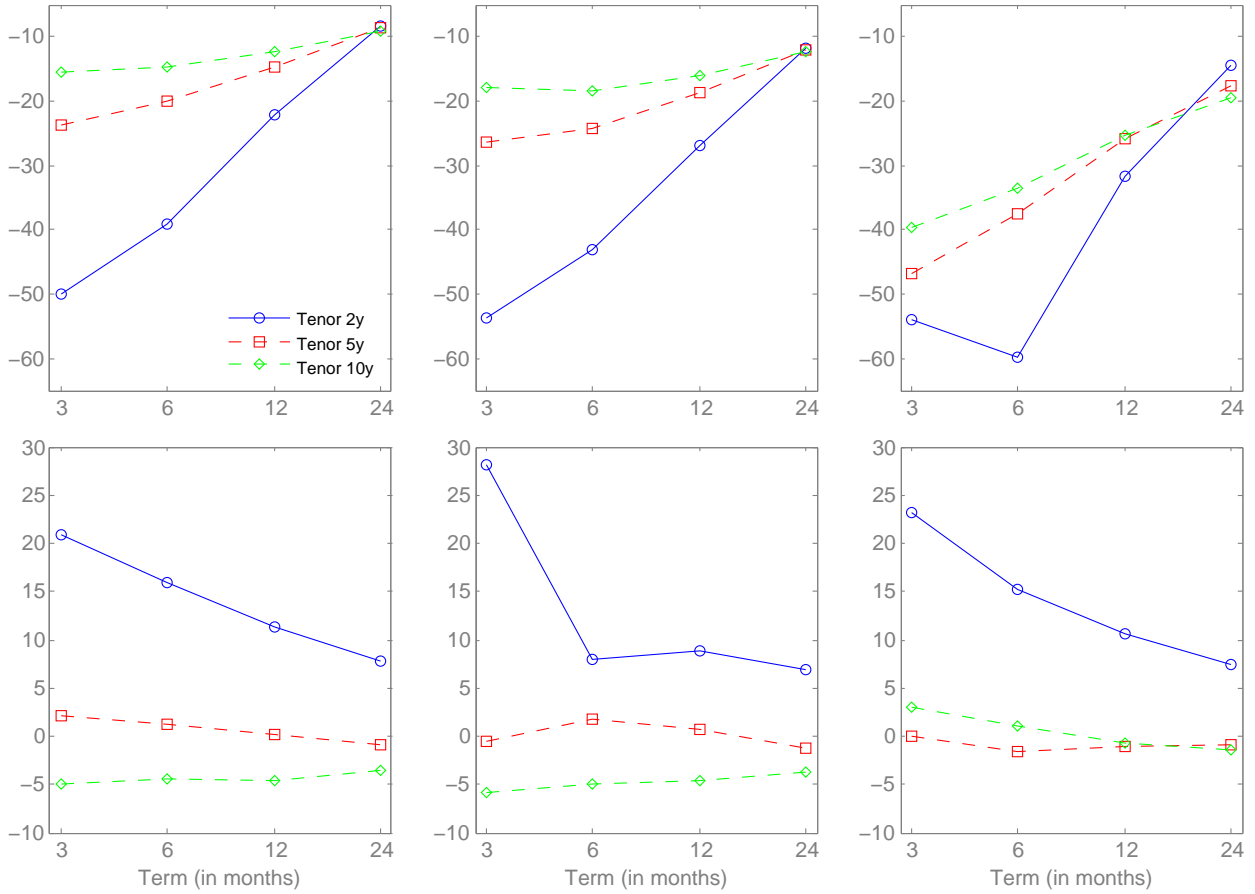
**Figure 3.10. Principal Components of Changes in Variance Risk Premia**

The figure plots the principal components of changes in variance risk premia, for terms of 3, 6, 12 and 24 months. The variance risk premia are computed as  $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$ . The expected realized volatility forecasts  $E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2]$  are computed at each point in time from simulations based on parameter estimates from a GARCH(1,1) model and conditioning at the information available at each point in time. The figures are in percent, annualized. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.



**Figure 3.11. Changes in Variance Risk Premia and Major Financial/Economic Events**

The figure plots changes in variance risk premia (for the 1 year tenor, 6 months term), along with the major financial and economic events (green vertical lines) associated with the the largest changes. The variance risk premia are computed as  $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$ . The expected realized volatility forecasts  $E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2]$  are computed at each point in time from simulations based on parameter estimates from a GARCH(1,1) model and conditioning at the information available at each point in time. The figures are in percent, annualized. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.



**Figure 3.12. Term Structure of Variance Risk Premia on Particular Dates**

The figure plots the term structure of variance risk premia (i.e. in the term dimension) in the dates where the variance risk premium for the 2 year tenor and 3 month term,  $VRP_t^{2y,3m}$ , takes its highest negative values (the three plots on the top) and its highest positive values (the three plots on the bottom). The figures are in percent, annualized. The variance risk premia are computed as  $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$ . The expected realized volatility forecasts  $E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2]$  are computed at each point in time from simulations based on parameter estimates from a GARCH(1,1) model and conditioning at the information available at each point in time. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

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