

Fux, Sebastian

**Doctoral Thesis**

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**Essays on Return Predictability and Term Structure Modelling**

# Essays on Return Predictability and Term Structure Modelling

**Sebastian Fux**

PhD Series 09.2014

The PhD School of Economics and Management

PhD Series 09.2014

# Essays on Return Predictability and Term Structure Modelling

Sebastian Fux

Supervisor: Jesper Rangvid

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Doctoral Thesis

Department of Finance

Copenhagen Business School

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Sebastian Fux  
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## Preface

This thesis is the result of my Ph.D. studies at the Department of Finance at the Copenhagen Business School. The thesis consists of three essays covering the topics of return predictability and term structure modelling. Each of the three essays is self-contained and can be read independently.

### Structure of the Thesis

The first two essays of the thesis are about return predictability. In the first essay we predict the U.S. equity premia in an out-of-sample fashion. In the return predictability literature it is often argued that the predictability of the U.S. equity premia deteriorates due to model uncertainty, model instability and time-varying coefficients. While accounting for these three sources of deterioration we show evidence that returns are predictable.

The second essay covers the predictability of exchange rates. A firmly held view in international finance is that exchange rates cannot be predicted by macroeconomic or financial variables. In this essay we provide some new evidence on this topic by relying on a large data set consisting of macro-finance variables. The information content of the macro-finance data set is summarized with a few factors extracted by means of Principal Component Analysis. Using this macro-finance factors to predict exchange rates, we find evidence that exchange rates are predictable in-sample as well as out-of-sample (especially over a forecast horizon of twelve months).

The final essay is about term structure models where we develop a regime-switching Affine Term Structure Model with a stochastic volatility feature. We contribute to the literature by analyzing the whole class of maximally-affine regime-switching term structure models. More precisely, we evaluate the performance of the stochastic volatility models relative to the Gaussian model. We find evidence that regime-switching models with stochastic volatility approximate the observed yields more accurately than their Gaussian counterparts. Additionally, we also show that regime-switching Affine Term Structure models with stochastic volatility successfully match some of the most important stylized facts of observed U.S. yield data.

## Acknowledgments

The essays in this thesis have benefited greatly from comments and suggestions from a number of people, however, I would like take the opportunity to thank a number of people for their great support during my Ph.D. studies.

First of all, I would like to thank my supervisor Jesper Rangvid for his invaluable guidance and constant encouragement throughout the last years as a Ph.D. student. I am grateful for all his suggestions and comments which considerably improved the quality of the two first essays in this thesis.

I would also like to thank Desi Volker for the excellent cooperation on the last essay. This essay also also greatly benefited from the comments of Jesper Lund. Furthermore, I thank colleagues and Ph.D. students at the Department of Finance for many rewarding discussion as well as for many hours of fun. In particular, I would like to thank Mads Stenbo Nielsen for always having an open door and for taking his time to discuss my questions. I also wish to thank Carsten Sørensen and Paul Söderling for participating in my pre-defense and for providing constructive comments. Finally, I thank my family for their support throughout the term of my Ph.D studies.

Sebastian Fux  
Zurich, March 2014

## Summary

### English Summary

#### **Chapter I: Stock Return Predictability under Model and Parameter Uncertainty**

The first essay covers the predictability of the U.S. equity premia. Out-of-sample predictability of the U.S. equity premia deteriorates due to structural breaks causing the predictor model and its coefficients to change over time. Additionally, there is only little consensus about the correct specification of the predictor model resulting in considerable model uncertainty. Due to model instability, time-varying parameters and model uncertainty the U.S. equity premia is often neglected. In this essay we rely on a method called Dynamic Model Averaging which accounts for model instability, time-varying coefficients and model uncertainty. We find evidence that Dynamic Model Averaging outperforms several benchmark models statistically and economically. An investor with mean-variance preferences could have increased his utility level by 1.2% by relying on the DMA approach instead of ordinary least squares predictions. Furthermore, we identify interest rate related predictors as the most powerful predictor variables.

#### **Chapter II: Predictability of Foreign Exchange Market Returns in a Data-rich Environment**

In the second essay we predict exchange rates. A firmly held view in international finance is that exchange rates follow a random walk and cannot be predicted by macroeconomic or financial variables over intermediate horizons of one to twelve months. In this essay we provide some new evidence on this topic by using a large number of macro-finance variables to forecast exchange rates. We summarize the information content of macro-finance variables with a few factors (extracted by means of Principal Component Analysis) and we apply this macro-finance factors to predict exchanges rates. We find evidence that this macro-finance factors successfully predict exchanges rates in-sample as well as out-of-sample (especially over a forecast horizon of twelve months).

### **Chapter III: Regime-switching, Affine Term Structure Model**

The final essay is about term structure modeling where we develop a regime-switching Affine Term Structure Model with a stochastic volatility feature. The increased complexity of introducing regime switches in terms of bond pricing and most importantly in terms of estimation has driven most of the literature to focus on Gaussian specifications of the state variable dynamics. Thus, we contribute to the literature by analyzing the whole class of maximally-affine regime-switching term structure models. More precisely, we evaluate the performance of the stochastic volatility models relative to the Gaussian model. We find evidence that regime-switching models with stochastic volatility approximate the observed yields more accurate than their Gaussian counterparts. Additionally, we also show that regime-switching Affine Term Structure models with stochastic volatility successfully match some of the most important stylized facts of observed U.S. yield data.

### **Dansk Resumé**

#### **Kapitel I: Forudsigelse af aktieafkast under model- og parameterusikkerhed**

Første essay omhandler forudsigeligheden af aktieafkast for det amerikanske aktiemarked. Det er velkendt at out-of-sample forudsigelighed af den amerikanske aktieafkast forringes på grund af strukturelle brud, somforårsager prædiktionsmodellen og dens koefficienter til at ændres over tid. Derudover er der kun lidt enighed om den korrekte specifikation af prædiktionsmodellen, hvilket resulterer i betydelig modelusikkerhed. Grundet modelstabilitet, tidsvarierende parametre og modelusikkerhedsår forudsigelsen af aktiekast i det amerikanske aktiemarkeder ofte forsømt i litteraturen. I dette essay bruger vi en metode kaldet Dynamic Model Averaging (DMA) som tager højde for modelstabilitet, tidsvarierende koefficienter og modelusikkerhed. Vi finder beviser for, at Dynamic Model Averaging udkonkurrerer flere benchmarkmodeller både statistisk og økonomisk. En investor med middelværdi-varians præferencer kunne have øget sin nytteværdi niveau med mere end en procent ved at satse påDMA tilgang i stedet for at brugemindste kvadraters metode til at lave forudsigelser. Derudover identificerer vi renterelaterede forklarende variable som den bedste styrke blandt prediktorvariable.



## **Kapitel II: Forudsigelsen af valutaafkast ved hjælp af makro-finansielle variable**

Andet essay forudser vi valutakurser. Et normalt udgangspunkt i international økonomi er, at valutakurserne følger en “random walk” og ikke kan forudsiges ved makroøkonomiske og finansielle variable for perioder af en til tolv måneder. I dette essay giver vi nogle nye beviser vedrørende dette emne ved at gøre brug af en lang række makro-finansielle variable til at forudsige valutakurserne. Indholdet i disse variable sammenfattes med nogle faktorer udvundet ved hjælp af “Principal Component Analysis”, som bruges til at forudsige valutakurser. Vi finder beviser for, at disse makro-finansielle faktorer kan forudsige valutakurser in-sample samt out-of-sample (især over en prognoseperiode på tolv måneder).

## **Kapitel III: Affine rentestruktur model med regime spring**

Det tredje essay omhandler rentestrukturmodeller, hvor vi udvikler en affine rentestrukturmodel med regime spring og stokastisk volatilitetsfunktion. Den øgede kompleksitet med at indføre regime spring i form af obligationsprisfastsættelse og vigtigst i form af estimering har drevet det meste af litteraturen hvor der fokuseres på Gaussian specifikationer for dynamikken for “state” variabelen. Vi bidrager til litteraturen ved at analysere hele klassen af affine rentestrukturmodeller med regime spring. Vi evaluerer resultaterne af de stokastiske volatilitetsmodeller i forhold til den Gaussiske model. Vi finder beviser for, at regime spring modeller med stokastisk volatilitet approksimerer de observerede renter mere præcist end den Gaussiske model. Derudover viser vi også, at regime spring Affine rentestrukturmodeller med stokastisk volatilitet matcher nogle af de vigtigste fakta for observerede amerikanske rentedata

## Introduction

This thesis consists of three essays of which two are about return predictability while the last essay covers term structure models. Return predictability is still a heavily debated issue among financial economists as well as practitioners in the financial industry. The ability to predict stock returns out-of-sample, that is, by relying on information available at time  $t$ , is still controversial. In a recent paper, Goyal and Welch (2008) comprehensively reexamine the performance of 14 predictor variables that have been suggested by the academic literature to be powerful predictors of the U.S. equity premium, that is, the S&P 500 index return minus the short-term interest rate. The authors conclude that none of these predictor variables led to robust predictions across different forecast horizons and sample periods which consistently beat benchmark models such as the historical mean. In a response to Goyal and Welch (2008) Campbell and Thompson (2008) find evidence of out-of-sample predictability by putting some economically meaningful restrictions on the coefficients of the predictive regressions. However, the out-of-sample explanatory power is nil, but nonetheless it is economically significant for investors with mean-variance preferences.

The predictability literature argues that the out-of-sample predictability deteriorates due to structural breaks such as macroeconomic instability, changes in monetary policy, new regulations etc. Thus, not only the predictor model changes over time, but also its coefficients. Goyal and Welch (2008) explain that more sophisticated models accounting for structural breaks might be able to consistently beat historical mean predictions. Additionally, predictability suffers from model uncertainty, meaning that there is only little consensus about the correct predictor variables and hence, the correct specification of the predictor model is unknown. The Bayesian framework accounts for model uncertainty by computing posterior model probabilities for all possible predictor models. Thus, Bayesian forecasts condition on the whole information set as opposed to conditioning on a single predictor variable and lead to more accurate forecasts.

In Essay I of this thesis we resume the issue of structural breaks and model uncertainty and contribute to predictability literature by using an approach that allows the forecasting

model to vary over time while, at the same time, allowing the coefficients in each model to evolve over time. Additionally, a posterior model probability is attached to each of the considered predictor models. We refer this approach as Dynamic Model Averaging (DMA). DMA predictions are given by the weighted average of all considered predictor models, where the posterior model probabilities serve as the weight. Instead of averaging across all possible model combinations, a second approach consists of choosing the best predictor at each point in time. We refer to this approach as Dynamic Model Selection (DMS).

From an econometric perspective, the DMA framework combines a state-space model for the coefficients of each predictor model with a Markov chain model for the correct model specification. The evolution of the predictor model and its coefficients is defined by exponential forgetting. The benefit from the state-space representation is that the coefficients of a particular predictor model and the predictor model itself are allowed to gradually evolve over time and thus, the forecast performance does not deteriorate due to structural breaks.

The forecast evaluation shows that the DMA approach outperforms several benchmark models, such as the recursive ordinary least squares (OLS), historical mean or random walk predictions. More precisely, in terms of Root Mean Squared Forecast Error (RMSFE) and Mean Absolute Forecast Error (MAFE) the DMA and particularly the DMS approach are superior. The DMS approach seems to be more accurate than DMA which shows the importance of choosing the “correct” predictor model over time. Also the evaluation of the predictive density (LOG PL) shows the importance of time-varying coefficients and predictor models since model specifications where the predictor model and its coefficients are allowed to vary more rapidly are favored by this forecast metric. We also find evidence that the DMA and DMS approach economically outperform several benchmark models. A mean-variance investor, who forecasts the market using the DMA (DMS) method, could have gained an annual utility increase of 1.20% (2.91%) at a monthly forecast horizon.

In Essay II we shed some light on exchange rate predictability. Based on the early work of Meese and Rogoff (1983), a firmly held view in international finance is that exchange rates follow a random walk and cannot be predicted by macroeconomic or financial variables.

We challenge this issue by relying on a new approach. Instead of predicting exchange rates by a handful of macro variables, we consider the information content of a large number of macro-finance variables (real business cycle factors, inflation, trade variables, financial market volatility, etc.) in the predictive regressions. Market participants base their investment decision on a large amount of data, which is supposed to be reflected in our data set consisting of more than 100 financial measures and macroeconomic aggregates. To reduce the dimensionality of an investor's information set, we rely on factor analysis to construct macro-finance factors. The benefit of factor analysis is that we are not restricted to a small set of variables that fail to span the information set of financial market participants.

Lustig, Roussanov, and Verdelhan (2010) identify the forward discount as the key predictor for excess returns on a basket of foreign currencies. In this essay we contribute to the existing literature by evaluating if macro-finance factors can enhance the predictability of currency excess returns beyond the information contained in the forward discount in-sample and out-of-sample. The in-sample regression analysis shows that the macro-finance factors are informative about future currency returns both at a monthly and at an annual forecast horizon. The share of explained variation of the currency excess returns rises considerably relative to the forward discount. At a monthly forecast horizon the R-squared is above 4% being around twice that of the forward discount while the R-squared for predictive regression enhanced with macro-finance factors rises to around 20% at an annual forecast horizon. The in-sample regressions also show a strong counter-cyclical behavior of the currency risk premia. More precisely, a factor which captures business cycle information predicts high (low) expected currency returns in economic recessions (expansions). Additionally, we show that factors which capture stock market, interest rate or inflation information also predict exchange rates.

We conclude the forecast exercise by investigating the out-of-sample predictive power of the macro-finance factors relative to predictions based on the forward discount. The continuous evaluation of the forecast performance provides evidence that the macro-finance factors are especially powerful at a longer forecast horizon. At an annual forecast horizon, predictions enhanced with macro-finance factors outperform the forward discount while

this seems not to be the case at a monthly forecast horizon.

Overall, based on our in-sample and out-of-sample analysis we find evidence that macro-finance factors extracted from a large panel of macroeconomic aggregates and financial series contain substantial predictive power when predicting expected currency returns. We find that macro-finance factors contain information about expected currency returns beyond forward discounts, which can be interpreted as interest rate differentials. Macroeconomic fundamentals and financial information contain substantial information about future currency movements that is not contained in interest rates. Thus, the evidence presented in this Essay supports a link between currency returns and the macroeconomy.

In the third Essay we leave the subject of return predictability and turn to term structure models. More precisely, we develop a regime-switching affine term structure model with a stochastic volatility feature. Economic theory suggest that monetary policy does not only affect the short end but the entire yield curve, since movements in the short rate affect longer maturity yields by altering investor expectations of future bond prices. From an economic perspective, it is hence intuitively appealing to allow the yield curve to depend on different macro-economic regimes. In the recent years the literature has further moved on by analyzing regime-switching models in an affine term structure framework, becoming ever more sophisticated. However, the increased complexity of introducing regime switches in terms of bond pricing and most importantly in terms of estimation has driven most of the literature to focus on Gaussian models. With this paper we contribute to the existing literature by analyzing the whole class of maximally-affine regime-switching term structure models. We estimate all models of the affine subfamily, that is, the  $A_0(3)$ ,  $A_1(3)$ ,  $A_2(3)$  and  $A_3(3)$  models (in the sense of Dai and Singleton (2000)) both in a regime-switching and in a single-regime setup and evaluate their relative performance in terms of goodness-of-fit to historical yields as well as in terms of replicating some of the stylized facts of observed U.S. yield data. In particular, we assess whether there is a benefit in moving firstly from a single-regime Gaussian model to a regime-switching Gaussian model, and secondly within the regime-switching class, moving from a Gaussian specification to stochastic-volatility specifications.

We generally expect the models accounting for shifts in the economic regime to outperform

their single-regime counterparts in terms of fitting historical yields. This effect is presumed to be larger for longer maturities, since during the life-span of longer maturity bonds the economy is more likely to be subject to changes in regimes. Our results provide some evidence that regime-switching stochastic volatility models are better equipped for fitting historical yield dynamics compared to the regime-switching Gaussian model as well as to single regime models. This finding is supported by the evidence of the Bayes factor, which shows a substantial improvement of the regime-switching affine term structure models relative to Gaussian models with either a single or multiple regimes.

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## Chapter 1

# Stock Return Predictability under Model and Parameter Uncertainty\*

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## **Abstract**

We consider the problem of out-of-sample predictability of the U.S. equity premia. The lack of ex-ante predictability of the U.S. equity premia is often attributed to structural breaks, that is, model non-stationarity, time-varying coefficients and model uncertainty. Our forecast procedure relies on Dynamic Model Averaging (DMA) which allows to account for structural breaks. From an econometric perspective the DMA approach combines a state-space model for the parameters with a Markov chain for the correct model specification. DMA predictions do not only statistically outperform several benchmark models but also economically. An investor with mean-variance preferences could have increased his utility level by 1.2% by relying on the DMA approach instead of ordinary least squares predictions. The DMA approach identifies interest rate related predictors as the most powerful predictor variables.

## 1.1 Introduction

The question of stock return predictability still bothers both practitioners in the financial industry and financial economists. The characterization of the equity risk premia affects important decisions such as portfolio allocation, savings decisions, pricing of assets and thus remains an important topic. The vast majority of papers about stock return predictability agree that excess returns are predictable in-sample.<sup>1</sup> Nevertheless, the ability to forecast S&P 500 excess returns out-of-sample is still controversial.

Out-of-sample predictability of the U.S. equity premia is often neglected due to structural breaks such as changes in market sentiment, macroeconomic instability, changes in monetary policy, new regulations etc. As a consequence of structural breaks the coefficients of predictor model may change over time and thus, out-of-sample predictability deteriorates. Time-varying coefficients is a widely discussed phenomena in the stock return predictability and we refer to Goyal and Welch (2008), Dangl and Halling (2011) and Pettenuzo and Timmermann (2011) for a recent discussion. However, not only the coefficients of the predictor model maybe time-varying but also the predictor model itself may change over time. Thirdly, out-of-sample predictability suffers from model uncertainty as shown in Cremers (2002) and Avramov (2002). There is only little consensus about the correct specification of the predictor model. Even though the past decades of research have identified a considerable amount of possible predictor variables, it is still unclear what the exact conditioning variables are. For example, the existence of  $K$  different predictor variables results in  $2^K - 1$  possible predictor models. Thus, Bayesian econometricians rely on Bayesian Model Averaging (BMA), meaning that they calculate a posterior model probability for each of the considered predictor models which is used as a weight when averaging across the  $2^K - 1$  point predictions.

In this paper we rely on a method which allows to account for these three sources of uncertainty, namely model non-stationarity, time-varying parameters and model uncertainty.

In particular, we predict the S&P 500 excess returns by relying on a dynamic version of

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<sup>1</sup>The literature about stock return predictability has resulted in a plethora of predictor variables ranging from valuation ratios over nominal interest rates to macro-economic variables etc. We do not intend to summarize stock return predictability literature, instead we refer to Campbell (2000) and Rapach and Zhou (2011) for a more recent survey of the asset pricing literature.

BMA.<sup>2</sup> The benefit of the DMA approach is that the forecasting model varies over time while, at the same time, the coefficients in each predictor model are allowed to gradually evolve. The DMA approach was introduced by Raftery, Karny, and Ettlér (2010) and Koop and Korobilis (2012) forecast inflation by applying the same framework.

From an econometric perspective, the DMA framework combines a state-space model for the coefficients of each predictor model with a Markov chain model for the correct model specification. The evolution of the predictor model and its coefficients is defined by exponential forgetting. The benefit from the state-space representation is that the coefficients of a particular predictor model are allowed to gradually evolve over time and thus, the forecast performance does not deteriorate due to structural breaks. Additionally, the predictor model also varies over time. To allow for a changing model space, we recursively predict S&P 500 excess returns. At each month during our sample period we evaluate the forecast performance of  $2^K - 1$  predictor models and assign posterior predictive model probabilities based on a model's historical forecast performance. Hence, we gauge with  $T \times (2^K - 1)$  predictions. This recursive forecast procedure results in a time-series of posterior predictive model probabilities, which are used when averaging across the  $2^K - 1$  point predictions at each point in time. The gradually evolving time-series of posterior predictive model probabilities justifies the label Dynamic Model Averaging. The parsimony of the DMA approach as well as the efficient estimation method allow us to evaluate this enormous amount of models in real time. DMA predictions are strictly out-of-sample, meaning that they only rely on information available at time  $t$ .

Instead of averaging across all possible model combinations, a second approach to predict S&P 500 excess returns is to choose the predictor variable with the highest posterior model probability at each of the evaluated months. We refer to this approach as Dynamic Model Selection (DMS).

The forecast evaluation shows that DMA, and particularly the DMS approach, outperform several benchmark models. The DMS approach seems to be superior to DMA, showing the importance of choosing the 'correct' predictor model over time. In our main sample

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<sup>2</sup>Classical BMA estimation methods assign a posterior model probability depending on the forecast performance of a predictor model. Each predictor model obtains a single posterior model probability which is used as weight when averaging across the forecasts. We refer to Hoeting, Madigan, Raftery, and Volinsky (1997) for an introduction to BMA.

period we find that in terms of Root Mean Squared Forecast Error (RMSFE) and Mean Absolute Forecast Error (MAFE) the DMS approach is the most accurate. We also show that a mean-variance investor, who forecasts the market using the DMA (DMS) method, achieves considerable utility gains compared to recursive ordinary least squares (OLS), conditional mean and random walk predictions. An investor relying on DMA (DMS) instead of recursive OLS forecasts, could have gained an annual utility increase of 1.20% (2.91%)<sup>3</sup> at a monthly forecast horizon. Finally, the evaluation of the predictive density (LOG PL) also shows the importance of time-varying coefficients and predictor models since model specifications where the predictor model and its coefficients are allowed to vary more rapidly are favored by this forecast metric. Overall, we find evidence that it is important to account for structural breaks, that is changing predictor models, time-varying coefficients and model uncertainty.

The superior performance of the DMA and DMS approach is consistent across different specification of the sample period and priors. As suggested in Goyal and Welch (2008) we consider several sub-samples to account for certain macroeconomic events such as the oil crisis. However, the DMA and DMS are superior for most of the considered sample periods. Additionally, we also conduct a sensitivity analysis regarding the specifications of the priors.<sup>4</sup> The sensitivity analysis reveals an interesting pattern. If we allow the model to vary more rapidly, the forecast performance increases, while it decreases if we allow the coefficients of a predictor model to vary too rapidly. This is intuitively appealing since different predictor variables may predict the U.S. equity premia over the sample period, however, we expect a stable relationship between the predictor variables and the equity premia as suggested by economic theory.

## Related Literature

A large body of the stock return predictability literature neglects out-of-sample predictability. The lack of out-of-sample predictability is often attributed to parameter and model

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<sup>3</sup>Note that these certainty-equivalent gains are annualized percentages.

<sup>4</sup>The posterior model probability is a weighted average of historical posterior model probabilities (age-weighted estimation). By decreasing the forgetting parameter, we shorten the length of the estimation window for the posterior model probabilities and thus, the model changes more frequently. See Section 1.4 for a more detailed discussion.

instability. Time-varying parameters and model non-stationarity have been a long debated issue in the predictability literature (see e.g. Pesaran and Timmermann (1995), Bossaerts and Hillion (1999), Pastor and Stambaugh (2001), Paye and Timmermann (2003), Pesaran and Timmermann (2002), Clements and Hendry (2004), Paye and Timmermann (2006), Rapach and Wohar (2006), Ang and Bekaert (2007), Goyal and Welch (2008)<sup>5</sup>, Lettau and Nieuwerburgh (2008) and Pettenuzo and Timmermann (2011)). All these papers share the conclusion that out-of-sample predictability deteriorates due to either model non-stationarity, meaning that the predictor model between the in-sample selection period and the out-of-sample prediction period model changes, or due to time-varying parameters, that is, the relationship between a predictor variable and the excess returns changes following a structural break.

To resolve model non-stationarity Clements and Hendry (2004) and Rapach, Strauss, and Zhou (2010) suggest to combine individual forecasts by e.g. averaging across forecasts of different predictor models. Forecast combination reduces forecast variance compared to predictions including a single predictor variable, similar to how diversification across individual assets reduces a portfolios' variance. As a consequence, combined forecasts are more stable relative to forecasts based on individual series leading to less volatile and more accurate forecasts.

Rapach, Strauss, and Zhou (2010) implement a recursive OLS-scheme for out-of-sample predictions using the same predictor variables as Goyal and Welch (2008). They combine the individual OLS-predictions by averaging across the predictions, that is, they use constant and equal weights to average across different forecasts. Their paper documents that this combination approach outperforms conditional mean forecasts, a finding which Goyal and Welch (2008) have shown does not hold when using the individual predictor variables. In this article, we relax this assumption of constant and equal weights. Our intention is to assign 'correct' weights to each of the predictor models. The weight assigned to a predictor model is its posterior predictive model probability which depends on the historical forecast performance. The better the recent forecast performance of a predictor model, the higher the posterior predictive model probability. Thus, this particular predictor model is more

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<sup>5</sup>In a response to Goyal and Welch (2008) Campbell and Thompson (2008) show that returns are predictable in an out-of-sample manner by putting restrictions on the predictive regressions.

relevant when averaging across individual forecasts which are weighted by the posterior predictive model probabilities.

To account for time-varying parameters, we rely on a state-space model which is estimated using standard Kalman filter techniques. Johannes, Korteweg, and Polson (2008) and Dangl and Halling (2011) are two recent papers using the state-space framework to predict the S&P 500 returns and thus implicitly account for time-varying parameters. However, our approach distinguishes itself through the econometric framework. Additionally, Johannes, Korteweg, and Polson (2008) focus on stochastic volatility, whereas Dangl and Halling (2011) focus on time-varying coefficients. Both articles share the conclusion that returns are predictable out-of-sample and that predictability is more pronounced during economic downturns, as shown in Dangl and Halling (2011).

Cremers (2002) and Avramov (2002) introduced the Bayesian approach to the stock return predictability literature.<sup>6</sup> Their studies emphasize the effect of model uncertainty, i.e. the effect of uncertainty about the correct specification of the predictor model on stock return predictability and the portfolio selection process. In general, Bayesian methods share the advantage that they condition on the complete information set of a forecasters as opposed to conditioning on a single individual model. The Bayesian framework compares the forecast performance of all possible models simultaneously and assigns a posterior model probability to each model depending on the models' ability to describe the data. Thus, Bayesian forecasts are based on a much richer data set contrary to 'standard' predictions which improves the forecast performance of Bayesian predictions. Both articles find evidence for out-of-sample stock return predictability.<sup>7</sup> In this article we extend their approach by calculating posterior predictive model probabilities for each month of our sample period instead of one posterior model probability which holds for the whole forecast period.

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<sup>6</sup>A third prominent paper in the Bayesian predictability literature is Wright (2008). He uses a Bayesian framework to predict out-of-sample exchange rates. However, also predictions based on Bayesian Model Averaging have difficulties to beat the random walk.

<sup>7</sup>The out-of-sample forecasting scheme in Cremers (2002) is based on a rolling estimation window, each including 20 years of data for the estimation window and 5 years of forecasts. Computational barriers do not allow a recursive estimation which evaluates all possible  $2^{14}$  models each month.

The remainder of the article is organized as follows; Section 1.2 presents the DMA approach. Section 1.3 briefly describes the data. Section 1.4 provides the empirical implementation and the results and Section 1.5 concludes.

## 1.2 Dynamic Model Averaging

The DMA approach is related to conditional dynamic linear models (CDLM), which have recently been discussed in Chen and Liu (2000). Within the class of CDLM models a state-space model is Gaussian and linear conditional on a trajectory of a latent indicator variable. In contrast to CDLM, the composition of the state vector, and not just the specification of the error terms in the measurement and state equation, depend on the unobserved latent variable in the DMA approach.<sup>8</sup> A detailed description of the DMA approach is given in the subsequent section.

### 1.2.1 Econometric Framework

The DMA approach extends the time-varying parameter (TVP) models by allowing the composition of the state vector (regression parameters) in the measurement equation to vary over time. In a TVP model, we denote  $y_t$  as the S&P 500 excess returns,  $z_t = [1, x_{t-1}]$  is a  $1 \times (1 + N)$  predictor vector consisting of a constant and  $N$  predictor variables and  $\theta$  is a  $(1 + N) \times 1$  state vector. Then we assume that the following model holds for the S&P 500 excess returns:

$$y_t = z_t \theta_t + \epsilon_t \tag{1.1}$$

$$\theta_t = \theta_{t-1} + \eta_t. \tag{1.2}$$

The innovations  $\epsilon_t$  and  $\eta_t$  are mutually independent and are distributed as  $\epsilon_t \sim N(0, H_t)$  and  $\eta_t \sim N(0, G_t)$ . Equation 1.1 represents the measurement equation and Equation 1.2 describes the state equation.

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<sup>8</sup>For an excellent text book treatment about state-space models we refer to Harvey (1989) and Frühwirth-Schnatter (2006).



The model in Equation 1.1 and Equation 1.2 allows the parameters  $\theta$  to change over time, while, the set of predictors in  $z_t$  is presumed to be constant. DMA intends to overcome this shortcoming by allowing for a different predictor set  $z_t^{(k)}$ ,  $k = 1, 2, \dots, K$ , to apply at each point in time. The  $K$  different predictor vectors consist of  $z_t = [1, x_{t-1}^{(k)}]$  where  $x_{t-1}^{(k)}$  represents a subset of the predictor variables described in Section 1.3 We introduce the possibility that different models hold at different time points with a time-varying, hidden model indicator  $L_t$ . A model indicator  $L_t \in \{1, 2, \dots, K\}$  determines the composition of  $z_t^{(k)}$  and the corresponding state vector  $\theta_t^{(k)}$ . Thus, we rewrite Equation 1.1 and Equation 1.2 in the sense of a switching linear Gaussian state space model as follows:

$$y_t = z_t^{(k)} \theta_t^{(k)} + \epsilon_t^{(k)} \quad (1.3)$$

$$\theta_t^{(k)} = \theta_{t-1}^{(k)} + \eta_t^{(k)} \quad (1.4)$$

where  $\epsilon_t^{(k)}$  are  $N(0, H_t^{(k)})$  and  $\theta_t^{(k)}$  are  $N(0, G_t^{(k)})$ .

At each month of our sample period, we assess the forecast performance of all  $K$  models, meaning that we calculate a model's posterior predictive model probability. We denote the posterior predictive model probability as  $\pi_{t-1|t,k} = p(L_t = k | Y^{t-1})$  where  $Y^{t-1} = y_1, y_2, \dots, y_{t-1}$ . Thus, at each month during the sample period a predictor model obtains a different posterior predictive model probability. These dynamically evolving predictive model probabilities justify the name Dynamic Model Averaging. Another approach to predict the equity premium consists of only using the best model at each point in time, that is, the model with the highest posterior model probability. We refer to this approach as Dynamic Model Selection (DMS). In contrast to classical, static BMA which addresses the issue where the correct model  $L_t$  and its parameter  $\theta^{(k)}$  are taken to be fixed but unknown, we allow these parameters to vary over time.

We assume that the model indicator  $L_t$  evolves according to a hidden Markov Chain, that is, a latent discrete-valued process. Thus, we need to impose some structure on  $L_t$  which governs its evolution, meaning that we need to specify how predictors enter and leave a model. In case of a hidden Markov chain specification of the model indicator  $L_t$  this is usually done by introducing a transition matrix  $Q$ . The transition matrix has dimension  $K \times K$  and determines the probability of switching from  $L_{t-1}^{(k_{t-1})}$  to  $L_t^{(k_t)}$ . However, if  $K$  is

very large, specifying  $Q$  is challenging, and thus we implicitly estimate  $Q$  using exponential forgetting.

We assume that the prediction of the S&P 500 returns depends on  $\theta_t^{(k)}$  only conditionally on  $L_t = k$ . Thus, we filter and update  $\theta_t^{(k)}$  only conditional on  $L_t = k$ . We circumvent computational difficulties which arise when inference is based on the full sequence of hidden values in the chain by updating  $\theta_t^{(k)}$  only conditionally on  $L_t = k$ .<sup>9</sup> Since  $\theta_t^{(k)}$  is only defined if  $L_t = k$  we can write the probability distribution of  $(\theta_t, L_t)$  as

$$p(\theta_t, L_t) = \sum_{k=1}^K p(\theta_t^{(k)} | L_t = k) \pi_{t,k}. \quad (1.5)$$

This is also the distribution which will be updated if new information becomes available. Estimation of Equation 3 and Equation 4 proceeds recursively, consisting of a prediction step and an updating step where the model indicator  $L_t$  and the state vector  $\theta_t^{(k)}$  (conditional on  $L_t = k$ ) is predicted and updated. Suppose that we know the conditional distribution of the state vector at time  $t - 1$ , then

$$p(\theta_{t-1}, L_{t-1} | Y^{t-1}) = \sum_{k=1}^K p(\theta_{t-1}^{(k)} | L_{t-1} = k, Y^{t-1}) \pi_{t-1|t-1,k} \quad (1.6)$$

where  $p(\theta_{t-1}^{(k)} | L_{t-1} = k, Y^{t-1})$  is given by the following normal distribution:<sup>10</sup>

$$\theta_{t-1} | L_{t-1}^k, Y^{t-1} \sim N\left(\hat{\theta}_{t-1}^{(k)}, \Sigma_{t-1}^{(k)}\right). \quad (1.7)$$

The recursive estimation proceeds with a prediction of the model indicator  $L_t$  and a conditional prediction of the parameter  $\theta_t^{(k)}$  given that  $L_t = k$ . If we were to set up a transition matrix  $Q$  the model prediction step would be

$$\pi_{t|t-1,k} = \sum_{k=1}^K \pi_{t-1|t-1,k} q_{kl}. \quad (1.8)$$

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<sup>9</sup>The approximating assumption that  $\theta_t^{(k)}$  is only conditionally defined on  $L_t = k$  allows us to estimate the model  $K$  times, implying that DMA is still useful for real-time predictions. If we were to run an exact Kalman filter this would imply that we have to estimate the model  $K^T$  times which is computationally feasible only if the total number of observations  $T$  is not too large. For a more detailed discussion about the various approximate filters we refer to Frühwirth-Schnatter (2006).

<sup>10</sup>For details about the priors of  $\theta_{0|0}^{(k)}$  and  $\Sigma_{0|0}^{(k)}$  see Section 1.2.2

$q_{kl}$  is an element of the transition matrix  $Q$  which controls the evolution of the model space. The element  $q_{kl} = Pr [L_t = l | L_{t-1} = k]$  is the probability of switching from model  $k$  at time  $t - 1$  to model  $l$  at time  $t$ . As mentioned previously, in case of a large number of possible models the specification of the transition matrix is cumbersome and real-time prediction becomes infeasible.<sup>11</sup> To circumvent these difficulties we follow the procedure proposed by Raftery, Karny, and Ettler (2010) where a forgetting factor,  $\alpha$ , is introduced. The forgetting factor implicitly defines the transition matrix. Equation 1.8 is thus replaced by

$$\pi_{t|t-1,k} = \frac{\pi_{t-1|t-1,k}^\alpha + c}{\sum_{k=1}^K \pi_{t-1|t-1,k}^\alpha + c} \quad (1.9)$$

where  $\alpha < 1$ . The introduction of  $\alpha$  implies an age-weighted estimation where the model  $j$ -periods in the past gets a weight of  $\alpha^j$ . Thus, the effective size of the estimation window used to calculate  $\pi_{t|t-1,k}$  has length  $h = 1/(1-\alpha)$ . Age-weighted estimation was introduced by Fagin (1964) and Jazwinsky (1970) where they estimated state-space models using exponential forgetting.

The constant  $c$  is set to  $c = 1/(50 \times K)$  and avoids that a posterior model probability is brought to zero. The introduction of the constant  $c$  flattens out the posterior model probabilities and increases the uncertainty about the specification of the correct predictor model which is in accordance with the disagreement about appropriate predictor variables among Bayesian econometricians. However, we note that the constant  $c$  is not crucial and the results do not qualitatively change for different specifications of  $c$ .

Instead of estimating the model by exponential forgetting, one might implement MCMC methods to draw the transition densities between models or an Markov Chain Monte Carlo Model Composition (MC<sup>3</sup>) algorithm to sample over the model space.<sup>12</sup> However, MCMC algorithms are computationally intensive and thus real-time prediction becomes is not possible. Instead, Raftery, Karny, and Ettler (2010) suggest to evaluate the predictive density in the updating step (see Equation 11).

<sup>11</sup>We have  $K = 14$  potential predictors and thus there exist  $2^K - 1 = 2^{14} - 1 = 16383$  different models and hence, the dimension of the transition matrix  $Q$  is  $16383 \times 16383$ . Unless  $k$  is very small,  $Q$  will have so many parameters that inference will be imprecise and the computational burden onerous.

<sup>12</sup>For further details about MC<sup>3</sup> we refer to Madigan and York (1995) and Green (1995).

The second prediction step consists of a parameter prediction and is given as:

$$\theta_t | L_t^k, Y^{t-1} \sim N \left( \hat{\theta}_{t-1}^{(k)}, \Sigma_{t|t-1}^{(k)} \right) \quad (1.10)$$

where  $\Sigma_{t|t-1}^{(k)} = \Sigma_{t-1}^{(k)} + G_t^{(k)}$ . Raftery, Karny, and Ettler (2010) argue that the specification of  $G_t^{(k)}$  is demanding and non-informative. Thus, we rely again on age-weighted estimation and introduce a second forgetting factor,  $\lambda$ , which is slightly below one. Consequently,  $\Sigma_{t|t-1}^{(k)}$  is given by  $\Sigma_{t|t-1}^{(k)} = \lambda^{-1} \Sigma_{t-1}^{(k)}$  and we avoid to specify  $G_t^{(k)}$ .

The estimation proceeds with the updating step. As the prediction step, the updating consists of a model and parameter updating. The first step updates the model indicator  $L_t$  and conditional on  $L_t = k$  the state vector,  $\theta_t^{(k)}$  is updated.

The model updating step is given by:

$$\pi_{t|t,k} = \frac{\pi_{t|t-1,k} p_k(y_t | Y^{t-1})}{\sum_{l=1}^K \pi_{t|t-1,l} p_l(y_t | Y^{t-1})} \quad (1.11)$$

where  $p_k(y_t | Y^{t-1})$  is the one-step-ahead predictive density for model  $k$  i.e.

$$y_t | Y^{t-1} \sim N \left( z_t^{(k)} \hat{\theta}_{t-1}^{(k)}, H_t^{(k)} + z_t^{(k)} \Sigma_{t|t-1}^{(k)} z_t^{(k)'} \right). \quad (1.12)$$

The predictive distribution is evaluated at the actual S&P 500 return,  $y_t$ .

The parameter updating equation is:

$$\theta_t^{(k)} | L_t^k, Y^t \sim N \left( \hat{\theta}_t^{(k)}, \Sigma_t^{(k)} \right) \quad (1.13)$$

where

$$\hat{\theta}_t^{(k)} = \hat{\theta}_{t-1}^{(k)} + \Sigma_{t|t-1}^{(k)} z_t^{(k)} \left( H_t^{(k)} + z_t^{(k)} \Sigma_{t|t-1}^{(k)} z_t^{(k)'} \right)^{-1} \left( y_t - z_t^{(k)} \hat{\theta}_{t-1}^{(k)} \right) \quad (1.14)$$

$$\Sigma_t^{(k)} = \Sigma_{t|t-1}^{(k)} - \Sigma_{t|t-1}^{(k)} z_t^{(k)} \left( H_t^{(k)} + z_t^{(k)} \Sigma_{t|t-1}^{(k)} z_t^{(k)'} \right)^{-1} z_t^{(k)} \Sigma_{t|t-1}^{(k)} \quad (1.15)$$

and  $H_t$  is the error variance of the measurement equation.

Finally, the error variance of the measurement equation,  $H_t$ , in Equation 1.15 must be specified. To allow for volatility clusters in the S&P 500 excess return series, we let the

error variance in the measurement equation to change over time. In particular, we use a rolling version of the recursive estimation method of Raftery, Karny, and Ettler (2010). We define

$$\tilde{H}_t^{(k)} = \frac{1}{t^*} \sum_{t-t^*+1}^t \left( \epsilon_t^{(k)} - z_t^{(k)} \Sigma_{t|t-1}^{(k)} z_t^{(k)'} \right) \quad (1.16)$$

where  $\epsilon$  is the innovation in the measurement equation. We use a rolling estimator of the error variance based on 5 years of data. Our estimator  $\hat{H}_t^{(k)}$  of  $H_t^{(k)}$  is given by:

$$\hat{H}_t^{(k)} = \begin{cases} \tilde{H}_t^{(k)} & \text{if } \tilde{H}_t^{(k)} > 0 \\ \hat{H}_{t-1}^{(k)} & \text{otherwise} \end{cases}$$

Thus, in the very rare case that  $\tilde{H}_t^{(k)} < 0$ , we replace it with our previous estimation of  $\tilde{H}_{t-1}^{(k)}$ .

Equation 1.3-1.16 are recursively estimated as new information becomes available. The recursions are initialized by choosing appropriate priors for  $\pi_{0|0,k}$ ,  $\theta_0^{(k)}$  and  $\Sigma_{0|0}^{(k)}$ . Their specification is discussed in Section 1.2.2.

A one-step-ahead recursive forecast is given by the weighted average over all individual model predictions using  $\pi_{t|t-1,k}$  as weights. So, for instance, DMA point predictions are given by:

$$E(y_t | Y^{t-1}) = \sum_{k=1}^K \pi_{t|t-1,k} z_t^{(k)} \hat{\theta}_{t-1}^{(k)} \quad (1.17)$$

where the weights are equal to the posterior predictive model probabilities. In contrast, DMS forecasts are based on the predictor set,  $z_t^{(k)}$ , with the highest posterior predictive model probability,  $\pi_{t|t-1,k}$ .

### 1.2.2 Empirical Implementation

To initialize the recursive estimation, three priors need to be determined: First, the prior probability for each model  $\pi_{0|0,k}$  has to be determined. We use a non-informative prior on the model probability by assigning an equal weight to each model, i.e.  $\pi_{0|0,k} = 1/K$

for  $k = 1, 2, \dots, K$  where  $K$  indicates the total number of estimated predictor models. Additionally, the initial distribution of the state vector  $\theta_{0|0}^{(k)}$  has to be defined. For  $\theta_{0|0}^{(k)}$  and its variance  $\Sigma_{0|0}^{(k)}$  we use a very diffuse prior representing the informativeness about the regression parameters. Specifically, we set  $\theta_{0|0}^{(k)} \sim N(0_k, I_k \times 100)$ .  $I_k$  represents an identity matrix with dimension  $k \times k$  where  $k$  indicates the number of predictor variables in the  $k$ 'th predictor model.

In our base case, the forgetting factors  $\alpha$  and  $\lambda$  are both set to 0.99. As a robustness check, we let  $\alpha$  and  $\lambda$  vary between 0.85 and 0.99. The results are robust with respect to changes in the forgetting parameters (see Section 1.4.3).

### 1.3 Data Overview

We analyze predictability for the excess returns on the S&P 500 index, that is, the total rate of return on the stock market minus the Treasury bill rate. Stock returns are continuously compounded and include dividends.

In a recent study, Goyal and Welch (2008) provide an overview of the out-of-sample performance of several predictors used to forecast the U.S. equity premia. In accordance with their article, we define the following set of predictors:

1. *Dividend-price ratio*, d/p: Difference between the log of dividends paid on the S&P 500 index and the log of stock prices (S&P 500 index), where dividends are measured using a one-year moving sum.
2. *Dividend yield*, d/y : Difference between the log of dividends and the log of lagged stock prices.
3. *Earnings-price ratio*, e/p: Difference between the log of earnings on the S&P 500 index and the log of stock prices, where earnings are measured using a one-year moving sum.
4. *Dividend-payout ratio*, d/e: Difference between the log of dividends and the log of earnings.

5. *Stock variance*, svar: Stock variance is computed as sum of squared daily returns on the S&P 500.
6. *Book-to-market ratio*, b/m: Ratio of book value to market value for the Dow Jones Industrial Average.
7. *Net equity expansion*, ntis: Ratio of twelve-month moving sums of net issues by NYSE-listed stocks to total end-of-year market capitalization of NYSE stocks.
8. *Treasury bill rate*, tbl: Interest rate on a three-month Treasury bill (secondary market).
9. *Long-term yield*, lty: Long-term government bond yield.
10. *Long-term return*, ltr: Return on long-term government bonds.
11. *Term spread*, tms: Difference between the long-term yield and the Treasury bill rate.
12. *Default yield spread*, dfy: Difference between BAA- and AAA-rated corporate bond yields.
13. *Default return spread*, dfr: Difference between long-term corporate bond and long-term government bond returns.
14. *Inflation*, infl: Calculated from the CPI (all urban consumers); since inflation rate data is released in the following month, we use  $x_{i,t-1}$ .

We consider three different out-of-sample evaluation periods. As in Goyal and Welch (2008) we define a long out-of-sample period covering 1965-2008 and a more recent out-of-sample period covering the period between 1976-2008. The latter period accounts for the fact that the out-of-sample predictability of individual economic series decreases significantly after the oil price shock of the mid-1970's. Additionally, Ang and Bekaert (2007) argue that predictability by the dividend yield is not robust to the addition of the 1990's. Thus, we consider a sub-sample covering the years between 1988-2008.

In the DMA framework all predictions are strictly out-of-sample and hence the data snooping criticism does not apply in this study. Data snooping is limited to the choice of the

initial predictor variables. However, the above mentioned predictor variables are often used in the prediction literature and all variables have been identified as having predictive power in earlier studies. Also the automated variable selection process limits the data snooping argument.

## 1.4 Results

Before we describe the results of the DMA and DMS approach we evaluate the predictive power of the individual predictor variables. Let  $y_{t+1}$  denote the S&P 500 excess returns and  $z_t^{(k)}$ , for  $k = 1, 2, \dots, 14$ , indicates a predictor model consisting of a constant and one of the predictor variables described in Section 1.3 We run a standard one-month predictive regression:

$$y_{t+1} = \beta z_t^{(k)} + \nu_{t+1}^{(k)}. \quad (1.18)$$

The results of these regressions are summarized in Table 1.1.

[Insert Table 1.1 about here]

From Table 1.1 we note that only two variables are statistically significant at a 10% significance level: svar and ltr. The adjusted  $R^2$ -statistic for the two predictors is about 1%. Thus, individual predictor variables are not able to explain a vast amount of the variation of the S&P500 excess returns for the sample period we consider.

In the subsequent section we evaluate the predictive power of our predictor variables in greater detail. First, we analyze which predictor variable accurately predicts excess returns over time. We do so by attaching a posterior predictive model probability to every predictor at each point in time. In a second step we conduct a forecast exercise and evaluate the predictive power of the DMA and DMS approach, respectively. We extend our model space and consider all possible model combinations based on our set of predictors.<sup>13</sup> Hence, we assess the ability of the DMA and DMS approach to predict S&P 500 excess

<sup>13</sup>Note that due to computational reasons we restrict the maximum number of predictor variables per model to five.



returns in presence of model instability, time-varying parameters and model uncertainty in Section 1.4.2.

### 1.4.1 What variables are important to predict stock returns?

Figure 1.1 sheds light on which predictors are important over time for our long sample period from 1965-2008 where the forecast horizon is one month. More precisely, Figure 1.1 shows the evolution of the posterior predictive model probabilities, that is, the probability that a predictor variable is useful for forecasting at time  $t$ . The better the historical forecast performance of a predictor variable, the higher the posterior probability and thus, the more useful is the particular variable to predict S&P 500 return at time  $t$ .<sup>14</sup>

[Insert Figure 1.1 about here]

The first fact we note from Figure 1.1 is that the model space changes over time, that is, the set of predictors in the forecasting model varies.<sup>15</sup> The DMA approach identifies interest rate related variables such as *ltr*, *tms*, *dfr* and *dfy* as the most prominent predictor variables. For the first half of our sample period *ltr* is the prevailing predictor variable. After the stock market crash in 1987, there is no single, dominating predictor variable. The best predictor variables are rather equally accurate.

An advantage is that DMA allows for both gradual and abrupt changes in the posterior model probability. In Figure 1.1 the importance of *ltr* changes rapidly whereas *dfy* gradually becomes more important. The rate of change of the posterior model probabilities is to some extent governed by the forgetting parameter  $\alpha$ . In a sensitivity analysis we analyze its impact in more depth.

Subsequently, we identify powerful predictor variables for the US equity premia at a quarterly and an annual forecast horizon. Panel A of Figure 1.2 shows the evolution of the model space for quarterly data. The pattern of the posterior model probabilities for quarterly predictions are different compared to their monthly counterparts. *Ltr* is the only

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<sup>14</sup>For a better readability we only present the posterior model probabilities for the four predictor variables with the highest average posterior model probability.

<sup>15</sup>There is a “convergence” period of 10 years between the initialization of our estimation and the start of our sample period. Thus, the posterior model probabilities already differ in the beginning of our sample period. For a better readability we restrict the analysis to four predictor variables.

predictor variable appearing in both forecast horizons, however, it is by far less important at a quarterly forecast horizon. In addition to ltr, b/m and tbl are the pervasive predictors at a quarterly forecast horizon.

[Insert Figure 1.2 about here]

The posterior model probabilities for an annual forecast horizon are presented in Panel B of Figure 1.2. Two eye-catching facts are presented for annual predictions: First, two predictor variables, namely ltr and e/p outperform the remaining predictor variables, and second, the posterior model probabilities for annual predictions are much smoother compared to their monthly counterparts.

The smoothness of the posterior model probabilities at an annual forecast horizon is due to the age-weighted estimation. The estimation window used in the calculation of the posterior model probabilities includes a period of 100 observations. Thus, the estimation of annual posterior model probabilities is based on a much longer history than for example the monthly posterior model probabilities leading to smoother estimates. We further elaborate on this finding in the Section 1.4.3.

Figure 1.1 and Figure 1.2 show that different explanatory variables are important over time for different forecast horizons. This supports the evidence reported in Pettenuzo and Timmermann (2011) where it is shown that return predictability and thus asset allocation depends crucially on model non-stationarity. We emphasize the benefit of the DMA and the DMS approach that it will pick up appropriate predictors automatically as the forecasting model evolves over time. Thus, the predictive power does neither deteriorate due to model instability nor due to model uncertainty. In the subsequent section we evaluate the forecast performance of DMA and DMS.

### 1.4.2 Forecast Evaluation

We compare the forecast performance of DMA and DMS to several alternative forecast approaches. In particular, Raftery, Karny, and Ettler (2010) connect the DMA framework to usual, static BMA by setting  $\alpha = \lambda = 1$ . The Bayes factor,  $B_{L_m L_n}$ , of two alternative

models  $L^m$  and  $L^n$  is given as the ratio of two marginal likelihoods

$$B_{L_m L_n} = \frac{p(Y^t|L_m)}{p(Y^t|L_n)} \quad (1.19)$$

where  $p(Y^t|L_m) = \prod_t^T p(y_t|Y^{t-1}, L_m)$ . The logarithm of the Bayes factor is

$$\log B_{L_m L_n} = \sum_{t=1}^T \log B_{L_m L_n, t}. \quad (1.20)$$

Conversely, in the DMA framework the Bayes factor is an exponentially age-weighted sum of sample specific Bayes factors which is given as<sup>16</sup>

$$\log \left( \frac{\pi_{T|T,m}}{\pi_{T|T,n}} \right) = \sum_{t=1}^T \alpha^{T-t} \log BB_{L_m L_n} \quad (1.21)$$

where  $BB_{L_m L_n}$  is defined as in Equation 1.20. When  $\alpha = \lambda = 1$ , there is no forgetting and both Bayes factors in Equation 1.20 and Equation 1.21 are equivalent, leading to a recursive but static estimation. Raftery, Karny, and Ettlter (2010) refer to this strategy as recursive model averaging (RMA). RMA is one of the alternative models which we consider.

More precisely, we compare the forecast power of the DMA and DMS approach to the below alternative benchmark models:

- Forecasts based on DMA where  $\lambda = 1$   
This implies that the coefficients of the predictor variables do not vary over time, that is, no forgetting in the coefficients of the predictor variables.
- Forecasts based on RMA where  $\alpha = \lambda = 1$   
This implies that neither the coefficients of the predictor variables nor the predictor models vary over time.
- Forecasts based on DMA where  $\alpha = \lambda = 0.95$   
This implies that the coefficients of the predictor variables and the predictor model are allowed to vary rather rapidly.

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<sup>16</sup>Note that  $c$  in Equation 1.9 is assumed to be zero.

- Forecasts based on DMA where  $\alpha = \lambda = 0.9$   
This implies that the coefficients of the predictor variables and the predictor model are allowed to vary rapidly.
- Forecasts based on DMA where  $\alpha = 0.99$  and  $\lambda = 0.9$   
This implies a stable development of predictor models while coefficients of the predictor variables are allowed to vary rapidly.
- Forecasts based on DMA where  $\alpha = 0.9$  and  $\lambda = 0.99$   
This implies a that the predictor models are allowed to vary rapidly while coefficients of the predictor variables develop stable.
- Forecasts based on a time-varying parameter (TVP) model including all predictors  
This implies that there is only one model with a posterior model probability of 100% which includes time-varying parameters.
- Forecasts based on recursive OLS estimates  
This benchmark was implemented by Rapach, Strauss, and Zhou (2010).
- Conditional mean forecasts
- Random walk forecasts

There exist many metrics for evaluating forecast performance. Two common forecast comparison metrics are the Root Mean Squared Forecast Error (RMSFE) and the Mean Absolute Forecast Error (MAFE). We also calculate the sum of the log predictive likelihoods (LOG PL) as suggested in Björnstad (1990) and Ando and Tsay (2010). The predictive likelihood is the predictive density for  $Y^t$  (given data through time  $t - 1$ ) evaluated at the actual S&P 500 excess returns. Geweke and Amisano (2011) argue that in financial applications, the consideration of the full distribution of asset returns is crucial. Thus, the sum of the log predictive likelihoods is a natural choice when we evaluate the forecasts.

Table 1.2 summarizes the RMSFE, the MAFE and the LOG PL for the considered predictor models.

[Insert Table 1.2 about here]

In terms of the RMSFE and the MAFE, both the model averaging and the model selection forecast method perform very well.<sup>17</sup> Relative to the benchmark models the DMA and DMS approach, where both forgetting parameters are 0.99, are among the models with the smallest forecast error. The DMS is superior across all forecast horizons with regard to the MAFE. Considering the RMSFE, the DMS approach is only outperformed by historical mean forecasts at an annual forecast horizon. We emphasize that also the DMA successfully predicts the US equity premium. A little surprising may be the fact that DMS outperforms DMA in terms of RMSE and MAFE, implying that choosing the ‘correct’ predictor model is more important than averaging across the forecasts of all possible predictor model specifications. This is evidence that the forecast performance deteriorates due to large number of predictor models underlying the DMA approach. It may be interesting to investigate what the optimal amount of data is to predict stock market returns, however, we leave this question for future research.

We emphasize that the DMA and DMS generate smaller forecasts errors than the TVP-model. In contrast to the DMA and DMS approach, the TVP-model does not rely on a model search algorithm and uses all 14 predictor variables to forecast the S&P 500 returns. The finding that DMA and DMS outperform the TVP-model shows the importance of a model search algorithm which identifies the most powerful predictors.

The evaluation of the predictive likelihood reveals an interesting pattern. The sum of the log predictive likelihoods (LOG PL) is the largest, meaning that these forecasts are the most accurate for the forecasts where the two forgetting factors  $\alpha$  and  $\lambda$  are equal to 0.9.<sup>18</sup> Thus, the faster we allow the predictor model and its coefficients to vary over time, the better is the forecast performance. In our base case both forgetting factor are set to 0.99. This leads to an age-weighted estimation where the effective estimation window consists of 100 periods of data. At longer forecast horizons this estimation period seems to be too long and a lower forgetting factor may be appropriate. Allowing for a more rapid change in both the predictor model and its coefficients is crucial when forecasting stock returns.

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<sup>17</sup>Note that we evaluate the forecast performance of all the models after a ‘convergence period’ of 10 years i.e. the recursive estimation of the models starts 10 years prior to the evaluation period.

<sup>18</sup>The sum of the LOG PL is calculated from Equation 1.12. Hence we only report predictive likelihoods for the Bayesian forecast methods.

We further evaluate the impact of different specifications of the forgetting factors on the forecast accuracy in Section 1.4.3.

A limitation of the previously mentioned test statistics is that they do not explicitly account for the risk borne by an investor. To account for this limitation, we calculate the certainty equivalent gains that a mean-variance investor would have obtained if this investor had predicted S&P 500 returns with the DMA or the DMS approach.<sup>19</sup> More precisely, a mean-variance investor maximizes the following utility function:

$$E_t(r_{p,t+1}) - \frac{1}{2}\gamma\text{Var}\{r_{p,t+1}\} \quad (1.22)$$

where  $\gamma$  is the investor's relative risk aversion.  $r_{p,t}$  is the return of a portfolio consisting of a risky asset, that is the S&P 500 index denoted by  $r_{m,t}$ , as well as a risk-free asset denoted by  $r_{f,t}$ . The portfolio is given as  $r_{p,t} = \omega_t r_{m,t} + (1 - \omega_t)r_{f,t}$  where  $\omega_t$  indicates the fraction of wealth invested in the risky asset. The optimal portfolio weight for the risky asset that maximizes the utility of a mean-variance investor is

$$\omega_t = \frac{E_t(r_{m,t+1})}{\gamma\sigma_t^2} \quad (1.23)$$

where  $\sigma_t^2$  is the variance of the risky asset (estimated recursively using all available data) and  $E_t(r_{m,t+1})$  is the expected excess return of the risky asset based on a predictor model. We restrict the portfolio weights to be within  $-50\% \leq \omega_t \leq 150\%$ . This gives two different portfolio weights depending on the forecast method. We denote the portfolio weight  $\omega_{DMA,t}$  ( $\omega_{DMS,t}$ ) when we predict the S&P 500 index using DMA (DMS) and  $\omega_{B,t}$  when predicting with a benchmark model. An investor realizes an average utility level,  $\bar{U}$  of

$$\bar{U} = \frac{1}{T} \sum_{t=1}^{T-1} \left( R_{p,t+1} - \frac{\gamma}{2} \omega_{t+1}^2 \sigma_{t+1}^2 \right) \quad (1.24)$$

during the out-of-sample period. The average utility level, also referred to as the certainty equivalent, denotes a certain return that yields the same utility level as a risky investment

<sup>19</sup>Kandel and Stambaugh (1996), Marquering and Verbeek (2004) and Campbell and Thompson (2008) use this approach to calculate realized utility gains for a mean-variance investor on a real-time basis.

strategy. The calculation of the average utility level enables us to compare different investment strategies. More precisely, the difference between the average utility level achieved by DMA approach, say  $\bar{U}_{DMA}$ , and the average utility level achieved by a benchmark model, say  $\bar{U}_{BM}$ , can be understood as the maximum fee an investor is willing to pay to have access to the additional information available in the DMA approach.

In our calculation we use  $\gamma = 2$ , however, there are no qualitative changes in the results for reasonable values of  $\gamma$ .<sup>20</sup> Table 1.3 relates the economic performance of the DMA and DMS approach to the competing models.

[Insert Table 1.3 about here]

The utility gains or the certainty equivalent,  $\Delta CE$ , associated with the DMA and DMS are noticeable. For example, at a monthly forecast horizon the utility gain of the DMS approach associated with recursive OLS forecasts is 2.91% (annualized percentage return), meaning that an investor would be willing to pay 2.91% of his invested wealth to get access to the information contained in the DMS approach.

The results in Table 1.3 reflect the previous results. The DMA and DMS approach successfully predict S&P 500 returns in the short run which is indicated by the positive certainty equivalents. The DMS generates slightly higher utility gains than the DMA approach, supporting the evidence indicated by the RMSFE and MAFE. At an annual forecast horizon the forecast methods with lower forgetting parameters, which allow for a faster change in the predictor model and the coefficients of the predictor variables outperform the DMA and DMS approach. We attribute this fact to the very long estimation window when forecasting at quarterly and annual horizons and thus, we further investigate this finding in the subsequent sensitivity analysis.

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<sup>20</sup>Mehra and Prescott (1985) propose that the investor's relative risk aversion should vary between 0 and 10. We calculated the certainty equivalent based on  $2 \leq \gamma \leq 5$ , however, there are no qualitative changes in the certainty equivalent. The additional results are available upon request.

### 1.4.3 Sensitivity Analysis

#### 1.4.3.1 Sub-sample Analysis

As part of our sensitivity analysis we consider two sub-samples. Goyal and Welch (2008) and Rapach, Strauss, and Zhou (2010) argue that the out-of-sample predictability deteriorates after the oil price shock in the 1970's. Hence, we analyze a post oil crisis sample ranging from 1976 to 2008. With this in mind, we also evaluate a recent out-of-sample period covering the last 21 years of the full sample covering 1988-2008. The consideration of multiple out-of-sample periods helps to provide us with a good sense of the robustness of the out-of-sample forecasting results, since e.g. Ang and Bekaert (2007) show that predictability is not uniform over time.

To begin with, we consider the posterior predictive model probabilities of the two sub-samples. The interested reader is referred to Figures 1.3 through 1.4 for a visualization of the posterior model probabilities.

[Insert Figures 1.3 and 1.4 about here]

DMA identifies  $l_{tr}$  as the most powerful predictor variable since it exhibits a high posterior predictive model probability in all sub-samples and across different forecast horizons. Additionally, dividend related predictors such as  $d/p$ ,  $d/y$  and  $d/p$  and valuation ratios such as  $b/m$  and  $e/p$  are important in both sub-samples. Overall, there is a large degree of consensus about the posterior model probabilities across the three considered sample periods.

Table 1.4 summarizes the forecast evaluation of the different forecast models for both sub-samples.

[Insert Table 1.4 about here]

In general, the findings from the long sample period are confirmed, meaning that the DMA and DMS approach accurately predict S&P 500 excess returns. Again, the DMS prediction outperform the DMA approach slightly. RMSFE and MAFE show that the DMA and DMS approach are among the best models, especially at shorter forecast horizons. The models with low forgetting parameters exhibit the highest LOG PL indicating that it is important



to allow for rapid changes in both the parameters and the prediction model, thus showing that it is crucial to account for structural breaks.

Table 1.5 shows the economic evaluation of the different forecast models for both subsamples.

[Insert Table 1.5 about here]

Table 1.5 confirms the finding of the economic evaluation of the DMA and DMS approach over the long sample period. The DMS approach is especially successful and almost all differences in the certainty equivalent are positive (again, especially at shorter forecast horizon).

The good performance of the DMA and DMS approach in the short-run relative to the competing models is a general pattern over all three sample periods. By allowing the predictor model and its coefficient to vary more rapidly, we may improve the forecast accuracy of DMA and DMS for longer forecast horizons. We investigate the predictive power of the DMA and DMS procedure for annual predictions in the next section by testing different specifications of the forgetting parameters.

#### 1.4.3.2 Prior Settings

In the previous estimation of the DMA and DMS approach the forgetting parameters were set to  $\alpha = \lambda = 0.99$ . This specification of the forgetting parameter is standard in the state-space literature. However, as already mentioned, especially at an annual forecast horizon lower values of the forgetting parameters may be appropriate. Subsequently we evaluate the effect of different forgetting parameter in the model prediction and parameter prediction step on the forecast accuracy at annual forecast horizons.

To accelerate changes in the model space as well as its coefficients we decrease the value of the forgetting parameter, that is the  $\alpha$  and  $\lambda$ , in the prediction step. The smaller the forgetting parameter, the smaller the size of the estimation window used to calculate the posterior model probabilities and thus, the predictor model and its coefficient vary more rapidly. In particular, we allow the forgetting parameters  $\alpha$  and  $\lambda$  to vary between  $0.85 < \alpha < 0.99$ . Thus, the effective size of the estimation is between 100 and 6.66

years.<sup>21</sup> Figure 1.5 shows the effect of the size of the estimation window on the forecast performance.

[Insert Figure 1.5 about here]

The blue bars in Figure 1.5 show the RMSFE as a function of decreasing  $\alpha$ 's. The forecast errors are lower for model specification with a lower  $\alpha$ , meaning that if we allow the predictor model to vary rapidly the forecast error decreases.

The red bars in Figure 1.5 quantify the effect of changes in  $\lambda$  which governs the updating of the state vector (regression parameters, see Equation 1.10). The RMSFE is rather stable for  $0.96 \leq \lambda < 0.99$ , however for lower values of the forgetting parameter the squared forecast error increases. Thus, there is evidence that forecast performance deteriorates if we allow a predictor model's coefficient to vary to rapidly.

The forecast errors in Figure 1.5 confirm an intuitively appealing finding. It appears that allowing the model to vary over time is more important than time-varying coefficients of the predictor variables. Even in the presence of structural breaks this seems reasonable, since we expect to have a stationary relationship between a predictor variable and the excess stock returns. Thus, we expect to have stable regression parameters over time while the idea that different predictor may hold at different points in time seems intuitively appealing.

Overall, the forecast evaluation shows that DMA and DMS outperform several benchmark models, even by accounting for different sub-samples and various specifications of the forgetting parameters. Thus, the forecast exercise shows the importance to account for model non-stationarity, time-varying parameters and model uncertainty.

## 1.5 Conclusion

In this article we shed some light on ex-ante predictability of S&P 500 excess returns by relying on DMA and DMS. The DMA approach is appealing since it accounts for structural

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<sup>21</sup>A forgetting parameter of 0.99 yields an estimation window of 100 periods. With monthly data this corresponds to an effective window size of 8.3 years. To obtain an estimation window of the same size at an annual forecast horizon, we require a forgetting parameter of approximately 0.87. Hence, we let the forgetting parameter vary between 0.85 and 0.99.

breaks, that is, model non-stationarity, time-varying parameters and model uncertainty. The stock return predictability literature identifies these phenomena as the causes for lack of out-of-sample predictability. Considering these three sources of uncertainty we find that S&P 500 returns are indeed forecastable. The DMA and DMS approach do not only statistically outperform several benchmark models, but also economically as indicated with noticeable utility gains. Additionally, we analyzed what variables are useful for predicting S&P 500 excess returns. DMA identifies interest rate related variables, especially the return on long-term government bonds, as well as valuation ratio such as dividend yields, dividend-payout ratio and book-to-market ratio as the most powerful predictors.

A little surprising may be the fact that the DMA approach is sometimes outperformed by DMS which shows the importance of choosing the appropriate predictor model over time. Each DMA point prediction is based on an enormous amount of information, more precisely, each forecast is a weighted average of 3472 individual predictions. It appears that some of the individual predictions are less accurate and thus, the forecast performance of the DMA approach deteriorates. An interesting question would be to investigate why exactly DMS outperforms DMA and what the most appropriate amount of conditioning information would be, however, we leave that question for future research.

The forecast results of the DMA and DMS strategy are promising compared to our alternative models. However, out-of-sample return predictability remains controversial and will always be a heavily debated issue.

TABLE 1.1: One-month predictive regressions

Variable	Coefficient	t-value	Adj. $R^2$
d/p: dividend-price ratio	0.005	1.125	0.1%
d/y: dividend yield	0.006	1.233	0.1%
e/p: earnings-price ratio	0.007	1.356	0.3%
d/e: dividend-payout ratio	-0.006	-0.664	-0.1%
svar: stock variance	-1.036	-2.495*	1.0%
b/m: book-to-market	0.004	0.507	-0.1%
ntis: net equity expansion	-0.007	-0.058	-0.2%
tbl: T-bill rate	-0.029	-0.387	-0.2%
lty: long-term yield	0.042	0.484	-0.1%
ltr: long-term return	0.153	2.456*	0.9%
tms: term spread	0.197	1.558	0.3%
dfy: default yield spread	0.571	1.140	0.1%
dfr: default return spread	0.311	1.363	0.7%
infl: inflation	-0.247	-0.386	-0.2%

*Notes:* Table 1.1 reports results of in-sample predictive regressions of one-month ahead excess stock returns on the lagged predictive variables. For each regression, the table reports the slope coefficient, the Newey-West corrected t-value, and the adjusted  $R^2$ -statistic. The sample period is 1:1965-12:2008. The '\*' indicates significance at least at a 10% level.

TABLE 1.2: Forecast Evaluation of the DMA and DMS Approach

	h=1			h=3			h=12		
	RMSFE	MAFE	LOG (PL)	RMSFE	MAFE	LOG (PL)	RMSFE	MAFE	LOG (PL)
DMA	4.340	3.278	-1472.525	8.300	6.206	-611.592	18.414	14.589	-206.880
DMS	<i>4.057</i>	<i>3.045</i>	-1439.127	<i>7.588</i>	<i>5.788</i>	-600.782	19.007	<i>14.365</i>	-205.303
DMA, $\lambda = 1$	4.347	3.299	-1440.213	8.328	6.210	-607.265	18.340	14.459	-210.441
DMA, $\alpha = \lambda = 1$	4.349	3.300	-1452.106	8.349	6.229	-611.094	18.386	14.479	-210.057
DMA, $\alpha = \lambda = 0.95$	4.549	3.375	-1391.623	8.079	6.175	-577.159	18.494	14.689	-202.562
DMA, $\alpha = \lambda = 0.9$	4.847	3.660	-1379.736	8.554	6.381	-541.903	18.248	14.502	-178.224
DMA, $\alpha = 0.99, \lambda = 0.9$	4.772	3.687	-1483.865	9.646	6.792	-585.353	18.796	15.135	-187.426
DMA, $\alpha = 0.9, \lambda = 0.99$	4.335	3.264	-1380.069	8.271	6.225	-574.249	18.128	14.307	-198.784
TVP-model (all pred incl)	4.347	3.328	-1517.492	8.235	6.343	-617.675	25.514	20.549	-204.631
Recursive OLS	4.377	3.314	0	8.297	6.284	0	19.773	16.080	0
Historical Mean	4.366	3.300	0	8.235	6.238	0	<i>17.922</i>	14.509	0
Random Walk	6.029	4.663	0	10.972	8.311	0	23.622	18.964	0

Notes: Table 1.2 reports RMSFE, MAFE and LOG (PL) for the different forecast model specifications. The test statistics are calculated for monthly (h=1), quarterly (h=3) and annual (h=12) forecasts. The best model for each test statistic is highlighted in *italic*. The sample period is 1965-2008.

TABLE 1.3: Economic Evaluation of the DMA and DMS Approach

	h=1		h=3		h=12	
	DMA	DMS	DMA	DMS	DMA	DMS
DMA, $\lambda = 1$	1.23	2.95	-0.35	0.21	0.05	-1.54
DMA, $\alpha = \lambda = 1$	1.50	3.21	-0.41	0.16	0.12	-1.47
DMA, $\alpha = \lambda = 0.95$	-0.03	1.69	-2.19	-1.62	-0.03	-1.62
DMA, $\alpha = \lambda = 0.9$	2.17	3.88	0.20	0.77	-0.20	-1.69
DMA, $\alpha = 0.99, \lambda = 0.9$	1.10	2.82	0.16	0.72	-1.34	-2.93
DMA, $\alpha = 0.9, \lambda = 0.99$	0.25	1.97	-0.60	-0.04	-0.55	-2.14
TVP-model (all pred. incl)	0.64	2.36	-1.65	-1.08	5.59	4.00
Recursive OLS	1.20	2.91	-0.72	-0.16	3.19	1.60
Historical Mean	-1.69	0.03	-3.62	-3.06	-2.92	-4.51
Random Walk	-0.29	1.43	-2.02	-1.46	2.04	0.45

*Notes:* Table 1.3 reports certainty-equivalent gains in annualized percentage returns of the DMA (DMS) approach relative to the alternative models. Certainty-equivalent gains are calculated for monthly (h=1), quarterly (h=3) and annual (h=12) forecast horizons. The utility function is  $E(R_p) - \frac{\gamma}{2} \times VAR(R_p)$  with a risk aversion of  $\gamma = 2$ . The optimal portfolio weight of the risky asset is constrained at  $-50\% \leq \omega_t \leq 150\%$ . The sample period is 1965-2008.

TABLE 1.4: Sub-sample Analysis: Forecast Evaluation

	h=1		h=3		h=12	
	RMSFE	MAFE	RMSFE	MAFE	RMSFE	MAFE
DMA	4.344	3.273	7.839	6.049	19.782	16.216
DMS	4.009	3.017	7.250	5.537	21.037	15.856
DMA, $\lambda = 1$	4.400	3.297	7.846	6.035	19.749	16.211
DMA, $\alpha = \lambda = 1$	4.399	3.298	7.864	6.046	19.785	16.273
DMA, $\alpha = \lambda = 0.95$	4.512	3.378	7.912	6.128	20.963	16.703
DMA, $\alpha = \lambda = 0.9$	4.666	3.520	8.280	6.339	20.533	17.228
DMA, $\alpha = 0.99, \lambda = 0.9$	4.525	3.383	7.745	5.996	20.202	16.276
DMA, $\alpha = 0.9, \lambda = 0.99$	4.344	3.270	7.864	6.055	20.044	16.269
TVP-model (all pred incl)	4.322	3.313	8.465	6.850	24.585	20.544
Recursive OLS	4.374	3.321	8.218	6.341	17.995	15.181
Historical Mean	4.356	3.273	7.936	6.020	17.245	14.243
Random Walk	6.036	4.717	10.653	8.214	22.029	17.946

Panel A: 1976-2008

LOG (PL) -153.204  
 LOG (PL) -153.049  
 LOG (PL) -146.851  
 LOG (PL) -154.182  
 LOG (PL) -147.861  
 LOG (PL) -124.398  
 LOG (PL) -149.913  
 LOG (PL) -144.785  
 LOG (PL) -152.230

Panel B: 1988-2008									
	RMSFE	MAFE	LOG (PL)	RMSFE	MAFE	LOG (PL)	RMSFE	MAFE	LOG (PL)
DMA	4.101	3.109	-688.012	7.661	5.683	-288.012	20.011	15.300	-107.027
DMS	3.686	2.775	-663.842	7.444	5.589	-286.734	19.051	14.450	-96.052
DMA, $\lambda = 1$	4.156	3.155	-661.181	7.648	5.645	-283.396	19.943	15.185	-90.803
DMA, $\alpha = \lambda = 1$	4.156	3.154	-665.229	7.624	5.637	-285.341	19.570	14.904	-90.803
DMA, $\alpha = \lambda = 0.95$	4.340	3.252	-645.003	7.797	5.988	-276.144	22.366	17.063	-93.768
DMA, $\alpha = \lambda = 0.9$	4.349	3.409	-619.713	7.879	5.873	-247.964	20.495	15.287	-79.096
DMA, $\alpha = 0.99, \lambda = 0.9$	4.360	3.256	-672.479	7.531	5.667	-283.139	20.089	15.337	-91.357
DMA, $\alpha = 0.9, \lambda = 0.99$	4.101	3.109	-652.460	7.903	5.876	-280.625	22.276	16.841	-93.737
TVP-model (all pred incl)	4.130	3.192	-720.508	8.002	5.978	-295.786	20.071	15.904	-92.784
Recursive OLS	4.111	3.137	0	7.593	5.623	0	20.186	16.831	0
Historical Mean	4.108	3.093	0	7.608	5.517	0	18.737	14.913	0
Random Walk	5.681	4.440	0	10.738	8.070	0	22.472	17.753	0

Notes: Table 1.4 reports RMSFE, MAFE and LOG (PL) for the different forecast model specifications. The test statistics are calculated for monthly ( $h=1$ ), quarterly ( $h=3$ ) and annual ( $h=12$ ) forecasts. The best model for each test statistic is highlighted in *italic*. Panel A shows the results for the sample period 1976-2008 and the sample period in Panel B is 1988:2008.



TABLE 1.5: Sub-sample Analysis: Economic Evaluation of the DMA and DMS Approach

Panel A: 1976-2008						
	h=1		h=3		h=12	
	DMA	DMS	DMA	DMS	DMA	DMS
DMA, $\lambda = 1$	2.14	1.54	0.99	1.80	0.14	1.20
DMA, $\alpha = \lambda = 1$	2.08	1.48	1.03	1.84	0.18	1.23
DMA, $\alpha = \lambda = 0.95$	-0.92	-1.52	0.89	1.70	-2.04	-0.99
DMA, $\alpha = \lambda = 0.9$	-1.99	-2.59	4.58	5.39	-0.08	0.97
DMA, $\alpha = 0.99, \lambda = 0.9$	-0.85	-1.45	-0.48	0.33	-1.55	-0.50
DMA, $\alpha = 0.9, \lambda = 0.99$	0.11	-0.49	-0.95	-0.14	-0.01	1.04
TVP-model (all pred incl)	1.76	1.16	2.24	3.05	1.67	2.72
Recursive OLS	-0.06	-0.66	-0.03	0.78	0.47	1.52
Historical Mean	-2.66	-3.26	-0.63	0.18	-2.47	-1.42
Random Walk	1.10	1.70	2.02	2.83	0.30	1.36
	DMA	DMS	DMA	DMS	DMA	DMS
DMA, $\lambda = 1$	-0.72	-1.79	-0.05	-0.08	-0.03	-5.15
DMA, $\alpha = \lambda = 1$	-0.84	-1.91	0.53	0.51	0.08	-5.04
DMA, $\alpha = \lambda = 0.95$	1.44	0.37	-0.23	-0.26	-0.29	-5.41
DMA, $\alpha = \lambda = 0.9$	4.53	3.47	9.31	9.29	7.09	1.97
DMA, $\alpha = 0.99, \lambda = 0.9$	1.25	0.18	0.07	0.05	0.09	-5.03
DMA, $\alpha = 0.9, \lambda = 0.99$	0.06	-1.01	-1.20	-1.23	-0.07	-5.19
TVP-model (all pred incl)	-0.34	-1.41	1.70	1.67	4.27	-0.85
Recursive OLS	4.14	3.07	5.70	5.68	11.41	6.29
Historical Mean	-1.86	-2.92	-0.92	-0.94	2.34	-2.78
Random Walk	0.68	-0.39	4.28	4.26	7.88	2.76

*Notes:* Table 1.5 reports certainty-equivalent gains in annualized percentage returns of the DMA (DMS) approach relative to the alternative models. Certainty-equivalent gains are calculated for monthly (h=1), quarterly (h=3) and annual (h=12) forecast horizons. The utility function is  $E(R_p) - \frac{\gamma}{2} \times VAR(R_p)$  with a risk aversion of  $\gamma = 2$ . The optimal portfolio weight of the risky asset is constrained at  $-50\% \leq \omega_t \leq 150\%$ . Panel A shows the results for the sample period 1976-2008 and in Panel B the sample period is 1988-2008.

FIGURE 1.1: Posterior Probability of Inclusion for Monthly Forecasts

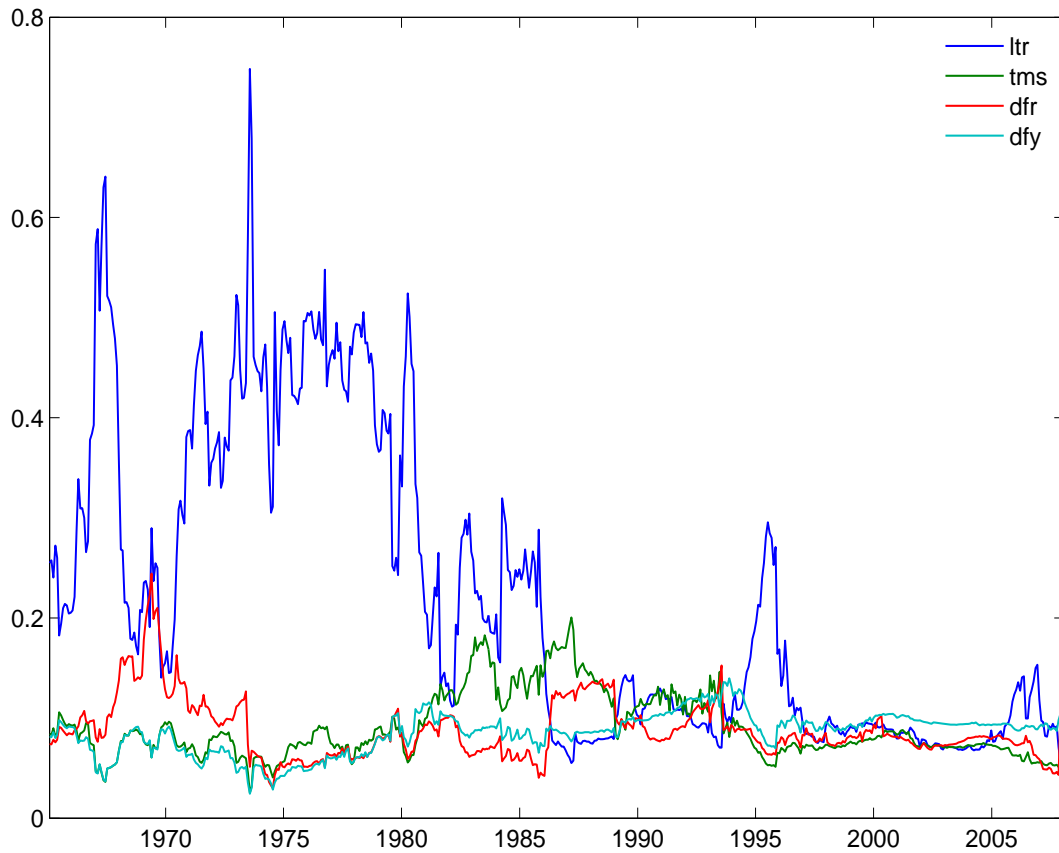


Figure 1.1 shows the most important posterior model probabilities for monthly forecasts. In the above figure  $tr$  denotes the return on a long-term bond,  $tms$  denotes the difference between the long-term yield and the Treasury bill rate,  $dfr$  default return spread and  $dfy$  default yield spread. The sample period is 01/1965-12/2008 and starts after a 'convergence period' of 10 years. Both forgetting factors,  $\alpha$  and  $\lambda$ , are set to 0.99.

FIGURE 1.2: Posterior Probability of Inclusion

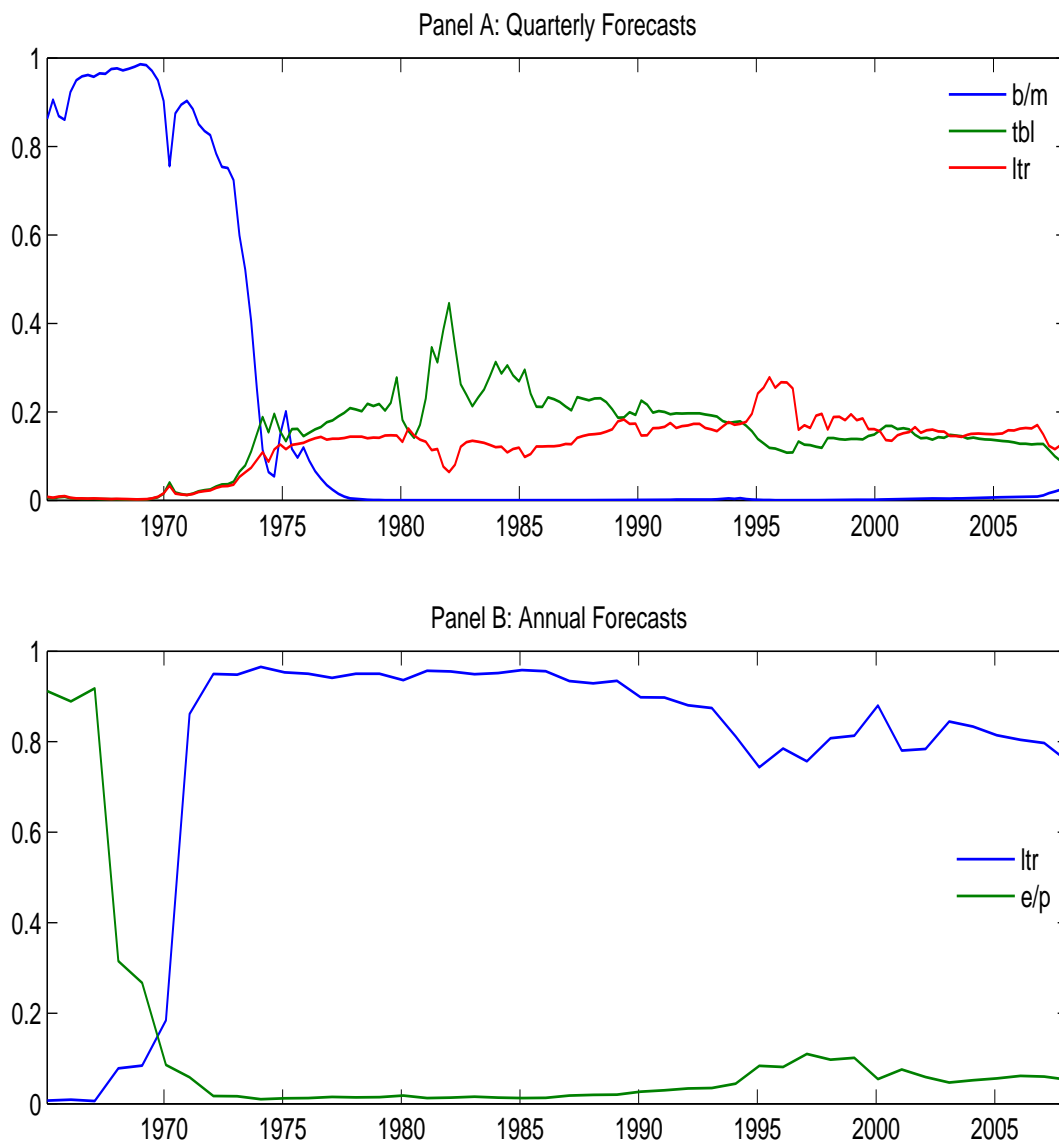


Figure 1.2 shows the most important posterior model probabilities for quarterly (Panel A) and annual forecasts (Panel B). In the above figure  $D/E$  denotes the dividend–payout ratio,  $B/M$  denotes book-to-market ratio,  $TBL$  denotes the Treasury bill rate and  $LTR$  denotes the return on a long-term bond and  $D/Y$  denotes the Dividend-Yield. The sample period is 1965–2008 and starts after a ‘convergence period’ of 10 years. Both forgetting factors,  $\alpha$  and  $\lambda$ , are set to 0.99.

FIGURE 1.3: Posterior Probability of Inclusion

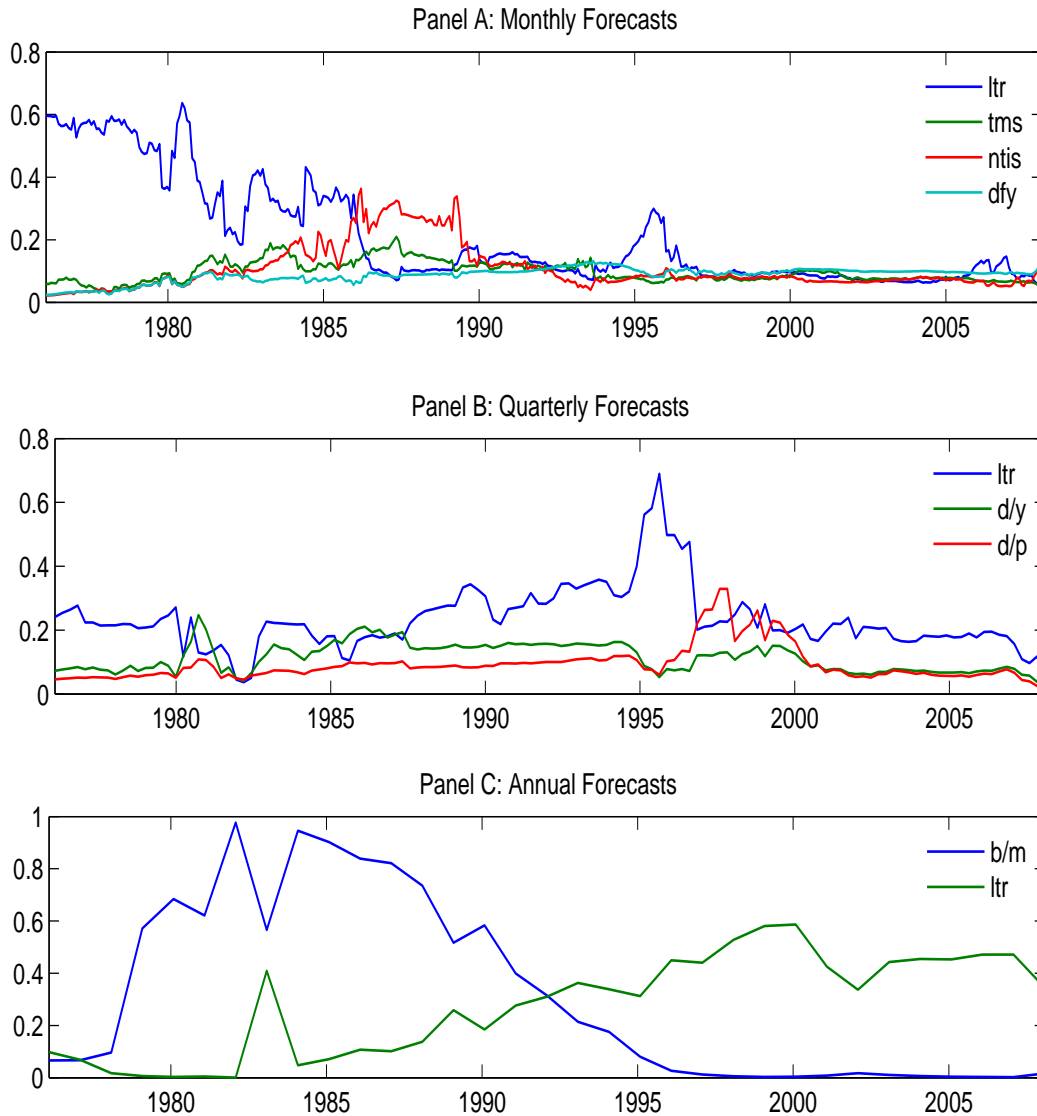


Figure 1.3 shows the most important posterior model probabilities for monthly (Panel A), quarterly (Panel B) and annual forecasts (Panel C). In the above figure  $E/P$  denotes the earnings-price ratio, SVAR denotes the stock variance, NTIS denotes the issuing activity of corporates, LTR denotes the return on a long-term bond,  $D/P$  denotes the dividend-price ratio, DFR denotes the default return spread,  $D/E$  denotes the dividend-payout ratio and  $B/M$  denotes book-to-market ratio. The sample period is 1976-2008 and starts after a 'convergence period' of 10 years. Both forgetting factors,  $\alpha$  and  $\lambda$ , are set to 0.99.

FIGURE 1.4: Posterior Probability of Inclusion

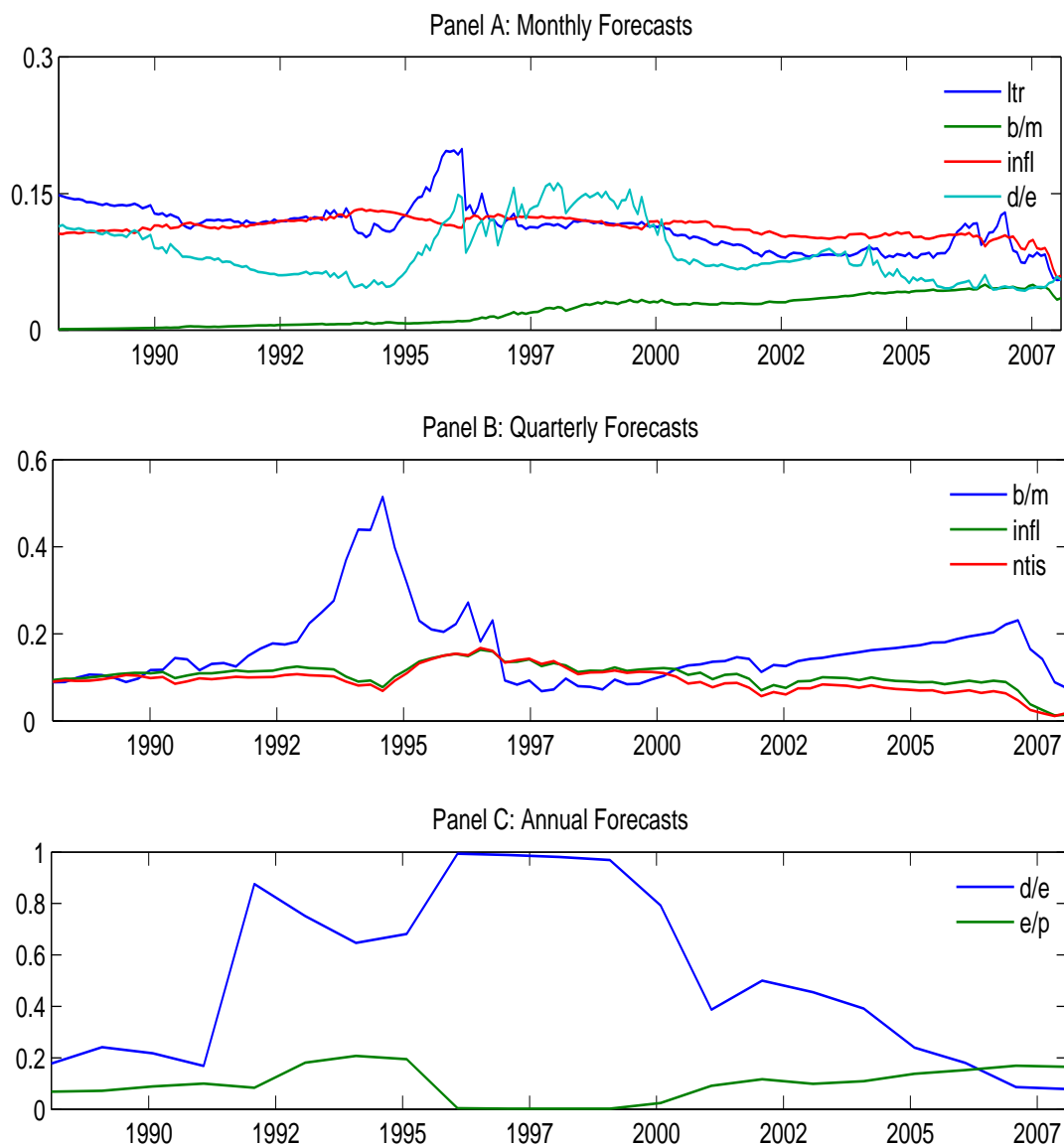


Figure 1.4 shows the most important posterior model probabilities for monthly (Panel A), quarterly (Panel B) and annual forecasts (Panel C). In the above figure  $E/P$  denotes the earnings-price ratio,  $D/E$  denotes the dividend–payout ratio,  $SVAR$  denotes the stock variance,  $LTR$  denotes the return on a long-term bond,  $D/Y$  denotes the dividend yield,  $B/M$  denotes book-to-market ratio and  $NTIS$  denotes the issuing activity of corporates. The sample period is 1988–2008 and starts after a ‘convergence period’ of 10 years. Both forgetting factors,  $\alpha$  and  $\lambda$ , are set to 0.99.

FIGURE 1.5: Sensitivity Analysis: RMSFE as a Forgetting Parameters

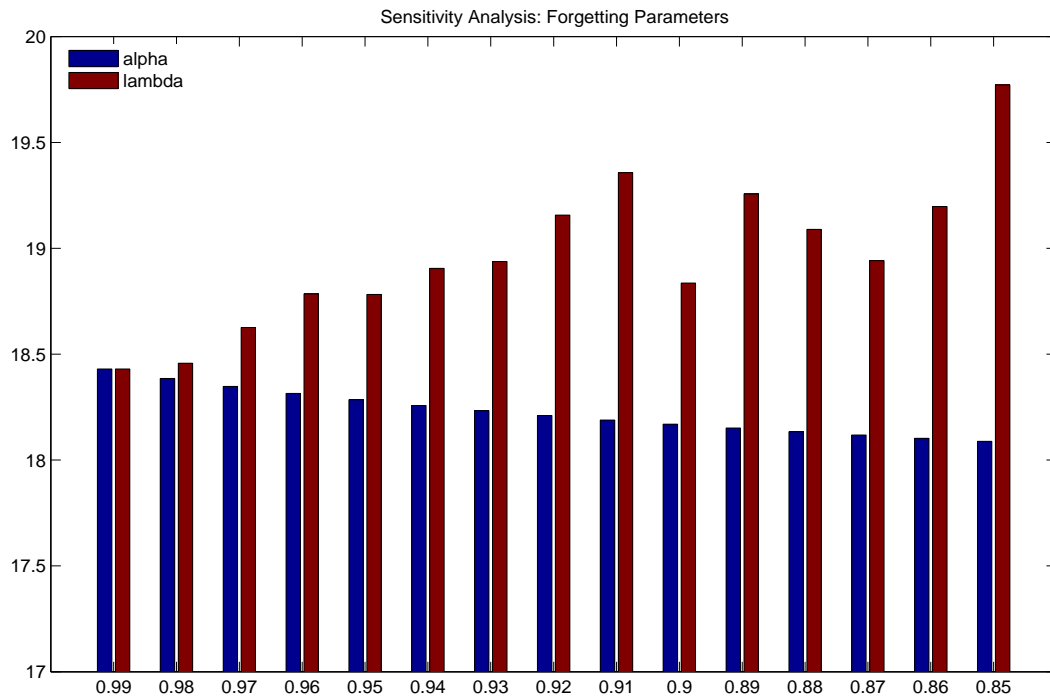


Figure 1.5 shows the RMSFE as a function of the forgetting parameters  $\alpha$  and  $\lambda$ . The forgetting parameters vary in the range of 0.99 and 0.85. The sample period is 1965-2008 and forecast horizon is annual.

## Chapter 2

# Predictability of Foreign Exchange Market Returns in a Data-rich Environment\*

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\*I would like to thank David Scherrer and Desi Volker for useful comments and suggestions. Additionally, I particularly appreciate the guidance from Jesper Rangvid throughout the term of the project.

## **Abstract**

We relate excess returns of a portfolio of currencies to the state of the economy. In particular, we provide fresh evidence on currency return predictability based on macro-finance factors. The macro-finance factors are extracted from an extensive data set covering a broad range of economic and financial activity by means of Principal Component Analysis. We find that “real activity”, “stock market” and “interest rate” factors successfully predict the currency risk premia. Compared to average forward discount predictions, we more than double the share of explained variation over the forecast horizon. In-sample evidence also shows a strong counter-cyclical relation between the macroeconomy and the currency risk premia. Also, the out-of-sample performance of forecasts based on macro-finance factors is striking, especially at longer forecast horizons.



## 2.1 Introduction

Based on the early work of Meese and Rogoff (1983), a firmly held view in international finance is that exchange rates follow a random walk<sup>1</sup> and cannot be predicted by macroeconomic variables over intermediate horizons of one to twelve months. A plethora of papers have investigated the robustness of this result, explanations for this finding, or alternative approaches to forecasting exchange rates but the literature does not seem to have settled on a commonly accepted explanation for this finding yet.<sup>2</sup>

We provide fresh evidence on this topic by examining whether information from the financial markets and macroeconomic fundamentals contain information about future currency movements. Instead of relying on a handful of macro variables suggested by a particular exchange rate model, we consider a large number of macro-finance variables (real business cycle factors, inflation, trade variables, financial market volatility, etc.) for forecasting exchange rates. Recent research argues that market participants act in “data-rich-environment”, that is, investors analyze and monitor hundreds of data series (see Bernanke and Boivin (2003) and Bernanke, Boivin, and Elias (2005) among others). To reduce the dimensionality of an investor’s information set, we rely on factor analysis. The benefit of factor analysis is that we are not restricted to a small set of variables that fail to span the information sets of financial market participants.<sup>3</sup> In particular, we estimate common factors from a monthly panel of 110 measures of financial and economic activity by Principal Component Analysis (PCA). The approach is complemented by relying on model selection techniques to select among competing forecasting models (i.e. models including different sets of factors). Finally, we analyze comprehensively whether currency returns are predictable by the estimated macro-finance factors.

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<sup>1</sup>More precisely, it is said that exchange rates follow a “near random walk”. Due to the convergence of exchange rates to the purchasing power parity levels in the long-run and the fact that currencies accompanied with high interest rates appreciate there is a small degree of predictability.

<sup>2</sup>For example, Mark (1995) early documented exchange rate predictability by monetary fundamentals over long horizons, Engel and West (2005) show that the poor forecasting performance of macro variables can be explained when fundamentals are highly persistent and the discount factor is close to unity, whereas Evans and Lyons (2002) show that order flow is able to forecast exchange rate changes over short horizons. However, the predictive power of certain predictor variables depend crucially on the choice of a particular exchange rate and the sub-sample. As such, they are often subject to criticism and the result of a data-mining exercise.

<sup>3</sup>A second approach which allows to condition on the complete information set of an investor is to implement a Bayesian model selection algorithm. For an example of exchange rate return predictions in a Bayesian framework we refer to Wright (2008).

Lustig, Roussanov, and Verdelhan (2010) identify the average forward discount (henceforth AFD), which is the average interest rate differential across all foreign currencies against the US, as the key predictor for excess returns on a basket of foreign currencies. Our objective is to evaluate if macro-finance factors can enhance the predictability of currency excess returns beyond the information contained in the Dollar forward discount. Our in-sample analysis finds evidence that macro-finance variables are indeed informative about future currency returns and currency excess returns (spot exchange rate changes adjusted for interest rate differentials). For one-month ahead forecasts, we explain up to 4.6% of the variation in the basket of foreign currency excess returns, representing a doubling of the R-squared compared to forecasts based on the AFD. At an annual forecast horizon we obtain a R-squared of about 20%, thus explaining one fifth of the variation in the currency returns over the next year. Additionally, the macro-finance factors reduce the predictive content of the AFD (its coefficient is lower) to some extent, suggesting that the macro-finance factors capture information about the state of the economy not covered by the AFD. Overall, we find evidence that macro-finance factors have predictive power beyond that contained in the AFD.

The factors that are most successful over short horizons are factors related to the stock market and interest rates, whereas a factor capturing business cycle information is the most pervasive for longer forecast horizons. When predicting a carry trade index (CTI) an interest rate related factor, in particular factors capturing the level and slope of the U.S. yield curve appear to have predictive power. However, across specifications, macro factors related to economic aggregates seem to be the most successful and even more successful than pure interest rate factors (interest rates are among the best predictors of foreign exchange returns, see e.g. Lustig, Roussanov, and Verdelhan (2010) and Ang and Chen (2010)), indicating that macro information has a lot to say about currency movements.

The evidence of the in-sample regressions also shows that movements in the currency risk premia is related to cyclical macroeconomic activity. This is in accordance with time-varying risk premia in currency markets developed by Verdelhan (2010). This article shows that in economic downturns risk aversion is high, that is, investors require a compensation for bearing risks related to recessions, meaning that expected excess returns are high

in recessions. A factor which is highly correlated with U.S. industrial production aggregates and employment measures, contains a lot of predictive power at an annual forecast horizon. This real activity factor predicts high expected currency returns in recessions, while predicted expected returns are lower in expansions showing that investors must be compensated for bearing risks related to economic downturns.

We also investigate the out-of-sample predictive power of our macro-finance factors for future returns and excess returns based on adaptive macro-finance indexes as suggested in Bai (2009). The adaptive forecast procedure allows an investor to continuously update his beliefs and dynamically evaluate the predictions of the factor based models against a benchmark. A predictor model is chosen based on its out-of-sample performance. To do so, the out-of-sample performance of a model is evaluated over a training period and at the end of this training period the best model is chosen for the out-of-sample prediction. We compare our out-of-sample forecasts of a basket of currency returns and the CTI with kitchen sink forecasts<sup>4</sup> and forecasts based on the AFD.

The dynamic evaluation of the out-of-sample predictive power of the macro-finance factors shows that they are superior at longer forecast horizons. At an annual forecast horizon, predictions based on macro-finance factors outperform historical mean forecasts as well as forecasts based on the AFD. The superior performance of forecasts based on macro-finance factors is also statistically significant.

From an econometric perspective, we follow Lustig, Roussanov, and Verdelhan (2010) and examine the relationship between macro fundamentals and future returns of a basket of foreign currencies. This is in contrast to much of the earlier literature which has mainly investigated individual exchange rates. However, looking at a basket of foreign currencies (against the U.S. Dollar) has the advantage of averaging out idiosyncratic movements in foreign currencies and allows us to focus on the common component of all our currency pairs, namely the drivers of the U.S. Dollar. In our empirical analysis, we investigate both the predictability of an equally weighted currency return (i.e. the average movement of all exchange rates against the U.S. Dollar) as well as CTI, which weights foreign currency by their interest rate differential against the U.S. short-term interest rate.

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<sup>4</sup>Kitchen sink predictions are based on all eight factors rather than relying on model selection procedure.

### Related Literature

This paper is related to a recent literature in asset pricing that considers the use of large sets of data to extract powerful predictors of financial returns (see e.g. Ludvigson and Ng (2007), Moench (2008), Anderson and Vahid (2007), Ludvigson and Ng (2009), and Cakmakli and Dijk (2012))<sup>5</sup> and a stream of literature that investigates risk premia in FX markets based on currency portfolios (see e.g. Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2009), Ang and Chen (2010), Lustig, Roussanov, and Verdelhan (2010), Lustig, Roussanov, and Verdelhan (2011), Menkhoff, Sarno, Schmeling, and Schrimpf (2012) and Verdelhan (2011)).

Factor models have been shown to successfully predict bond returns (Moench (2008) and Ludvigson and Ng (2009) among others) as well as equity returns (see for example Anderson and Vahid (2007), Ludvigson and Ng (2007), Cakmakli and Dijk (2012)). Ludvigson and Ng (2009) relate excess bond returns to macroeconomic fundamentals and show that macro factors contain substantial information about future bond returns not included in a single forward rate factor, i.e. the Cochrane-Piazzesi factor (see Cochrane and Piazzesi (2005)). Moench (2008) jointly models the term structure and the macroeconomy with a vector-autoregressive model with embedded factors. He finds evidence that the use of macro factors provides better out-of-sample yield forecasts than several benchmark models, especially at a short and medium term forecast horizon. A prominent example of a factor model in the predictability literature is Ludvigson and Ng (2007). Their approach identifies a volatility factor and a risk-premium factor as particularly important to predict the cross-section of expected returns. Furthermore, Cakmakli and Dijk (2012) find evidence that factor models have superior market timing ability compared to widely used predictors such as valuation ratios or interest rate related variables.

For an example of factor models related to currencies we refer to Engel, Mark, and West (2012) who predict bilateral exchange rates using currency factors extracted from a panel of exchange rates. Intuitively, their currency factors contain information about common trends in exchange rates which are difficult to extract from observable fundamentals. In their forecast exercise, they enhance exchange rate predictions models based on observable

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<sup>5</sup>For a survey about factor analysis we refer to Bai and Ng (2008).

variables with exchange rate factors.<sup>6</sup> They conclude that models augmented with factors successfully predict exchange rates in the recent decade for longer forecast horizons, that is, at 8 and 24 quarters, respectively.

Even though factor models based on macroeconomic data seem to accurately predict bond and equity premia, this class of models, to the best of our knowledge, has not been used to predict currency returns yet. Our approach intends to fill this gap in the literature and predicts a portfolio of currencies using factors extracted from a data set covering a broad set of economic and financial activities.

Recent literature suggests to predict portfolios of currencies instead of bivariate currencies. Currency portfolios were introduced by Lustig and Verdelhan (2007) and became popular in recent years. Lustig, Roussanov, and Verdelhan (2010) is closely related to our approach. They employ the AFD (the average interest rate differential across all foreign currencies against the U.S.) and U.S. industrial production to forecast currency returns (a novel carry trade strategy) and show that currency risk premia are counter-cyclical. Our results point into the same direction. For example, we find that one of our factors which captures business cycle information predicts high (low) expected currency returns in economic recessions (expansions), which is similar to what Lustig, Roussanov, and Verdelhan (2010) document in their paper. However, we also show that other factors, such as factors related to the stock market, interest rate variables or inflation aggregates also forecast currency risk premia (and exchange rate changes) and do so in a way consistent with economic intuition. Hence, our results show that exchange rates (and currency risk premia) are predictable with factors extracted from a large set of macro-finance variables. This finding supports the evidence found in Ang and Chen (2010) where it is shown that any factor which potentially affects domestic bond prices has the potential to predict foreign exchange risk premia.

The rest of the paper proceeds as follows. In Section 2.2, we describe our FX and macro data while Section 2.3 details the econometric framework. Section 2.4 presents empirical results and Section 2.5 concludes.

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<sup>6</sup>In particular, they augment a “Taylor rule” model, a monetary model and a model based on deviations of the Purchasing Power Parity with currency factors extracted from the panel exchange rates.

## 2.2 Data

This section describes the data and the way currency excess returns are computed, and provides a description of the macroeconomic data that form the basis for our modelling of currency risk premia.

### 2.2.1 FX data and currency returns

Our FX data covers spot exchange rates and one-month forward exchange rates over the sample period from 12/1983-03/2009. The original source of the data is BBI and WMR/Reuters and we obtain these data via Datastream. The same data have been used in recent work (see e.g. Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011), Lustig, Roussanov, and Verdelhan (2010), Lustig, Roussanov, and Verdelhan (2011) and Menkhoff, Sarno, Schmeling, and Schrimpf (2012)). We denote the spot and forward rates in logs as  $s$  and  $f$ , respectively. Spot and forward rates are end-of-month data (last trading day in a given month).<sup>7</sup>

Excess monthly returns to a U.S. investor for holding foreign currency  $k$  are given by

$$rx_{t+1}^k \equiv i_t^k - i_t - \Delta s_{t+1}^k \approx f_t^k - s_{t+1}^k \quad (2.1)$$

where  $s$  and  $f$  denote the (log) spot and 1-month forward rate (foreign currency unit per U.S. Dollar), respectively and  $\Delta s$  denotes log spot rate changes. FX excess returns are thus composed of the interest rate differential (or carry) minus the depreciation of foreign currency over the maturity of the forward position. The FX excess return for a long position in a foreign currency can be understood as selling the U.S. Dollar in the forward market and buying it back at the future spot rate. Intuitively, this is an *excess* return since this form of currency speculation in the forward market can be equivalently expressed as the return from borrowing funds in U.S. Dollar at the U.S. interest rate, converting them into foreign currency, investing them in the foreign money market and finally converting back to U.S. Dollar at the end of the investment period.

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<sup>7</sup>Our total sample consists of the following 15 countries: Australia, Belgium, Canada, Denmark, Euro area, France, Germany, Italy, Japan, Netherlands, New Zealand, Norway, Sweden, Switzerland, and the United Kingdom.

Our empirical analysis relies on the return of two portfolios of currencies. These are the returns of a portfolio which shorts the Dollar and is long in an equally-weighted basket of foreign currencies. This is labelled the Dollar factor (DOL) in Lustig, Roussanov, and Verdelhan (2010) since it captures the evolution of the value of the U.S. Dollar against a broad set of currencies. The second portfolio is a CTI which takes long and short positions in foreign currency depending on the interest rate differential of a respective currency against the U.S. Dollar. More specifically, an investor goes long the foreign currency (and short the U.S. Dollar) in each currency that has a higher short-term interest rate than the US and short in each foreign currency (and long in the U.S. Dollar) that has a lower short-term interest rate than the US. The CTI averages over the excess returns of all these positions.

It is well-known that uncovered interest parity (UIP) does not hold in the data, which is known as the “forward premium puzzle” introduced by Hansen and Hodrick (1983) and Fama (1984a). By contrast, forward premia or forward discounts are powerful short to medium term predictors of FX excess returns and spot rate changes as shown in Lustig, Roussanov, and Verdelhan (2010). Given that an aggregate measure of forward discounts (average forward discount or Dollar forward discount) has been shown to perform very well in predicting currency returns, we primarily rely on this measure as a benchmark predictor. Our goal is to see if macro-finance factors can enhance the predictability of currency excess returns beyond the information contained in the Dollar forward discount. Descriptive statistics of the FX excess returns are provided in Table 2.1.

INSERT TABLE 2.1 ABOUT HERE

### 2.2.2 Macro data

Our macro-finance factors are extracted from a data set consisting of 110 monthly variables.<sup>8</sup> The series cover a broad range of measures of economic activity such as industrial production, unemployment, inflation etc. and thus summarize the current state of the U.S. economy. Additionally, we also include financial time-series such as term spreads, defaults

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<sup>8</sup>Note that a few series were only available in quarterly frequency. These series were transformed to monthly data using a cubic spline interpolation method. A detailed description also showing their frequency can be found in the appendix.

spreads, dividend yields and measures of volatility in order to capture the evolution of risk premia in financial markets. Ludvigson and Ng (2009) argue that it is crucial that the data also covers financial information since business cycles are caused by financial shocks as well as macroeconomic shocks. In order to interpret the regression analysis and to identify series that predict currency risk premia, we attach labels to the factors in Section 2.4.1. We find that some factors are highly correlated with macroeconomic fundamentals while other factors summarize financial information.

Similar to Bernanke, Boivin, and Eliasch (2005) and Stock and Watson (2002b), we group these variables in the following 7 categories:<sup>9</sup>

- i) Real Activity (41 series)  
e.g. production data, personal consumption expenditures, housing data, etc.
- ii) Stock Market Valuation (8 series)  
e.g. U.S. stock market indexes, P/E ratios, dividend yields, etc.
- iii) Volatility and Aggregate Uncertainty (26 series)  
e.g. stock market and FX volatility, Debt/GDP ratios, Fama-French Risk Factors, etc.
- iv) Interest Rates and Interest Rates Spreads (20 series)  
e.g. U.S. Treasury rates, corporate rates, U.S. Treasury spreads and corporate spreads, etc.
- v) Price and Wage Variables (21 series)  
e.g. CRB indexes, PPI and CPI data, salary variables, etc.
- vi) Open Economy (5 series)  
e.g. import and export data, current account, etc.
- vii) Monetary Variables (4 series)  
e.g. monetary base, reserves, etc.

Prior to extracting the latent factors all series are transformed to induce stationarity. We compute monthly and annual differences, linearize the level of series and calculate differences of the linearized series to assure stationarity. Additionally, we also standardize

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<sup>9</sup>We note that the macro-finance factors are based on U.S. data only. In an early version we used macro-finance data from the G7 countries, however for a better interpretability of the macro-finance factors restricted our analysis on U.S. data.



the data to have zero mean and unit variance. From this transformed and standardized data set we extract our macro-finance factors using PCA. The entire list of variables, details about their sources and their transformation are given in the appendix.<sup>10</sup>

Descriptive statistics of the macro factors are provided in Table 2.2.

INSERT TABLE 2.2 ABOUT HERE

## 2.3 Econometric Framework

Our approach relies on PCA and therefore allows for a comprehensive and parsimonious empirical modelling of time-varying currency risk premia which is outlined in the following section.

### 2.3.1 A Factor Model and Estimation of its Factors

Following the seminal work of Stock and Watson (2002a) and Stock and Watson (2002b), factor models have become more and more popular for forecasting in recent years since they allow to parsimoniously describe the information contained in a large amount of economic variables.<sup>11</sup> The methodology helps reducing the dimensionality problem that plagues many forecasting and modelling problems in economics and finance. We provide a brief review of the basic methodological framework in the following.

Let  $y_{it}$  ( $i = 1, \dots, N$ ,  $t = 1, \dots, T$ ) denote the panel of macroeconomic and financial data where the cross-section of macro-finance variables available  $N$  is very large, in principle it could be even larger than  $T$ .

We assume that  $y_{it}$  has a factor structure, i.e.

$$y_{it} = \lambda_i' F_t + e_{it} \quad (2.2)$$

where  $F_t$  represents a  $r \times 1$  vector of latent common factors,  $\lambda_i$  is a  $r \times 1$  vector of factor loadings and  $e_{it}$  represents a vector of idiosyncratic disturbances. Note that  $r \ll N$

<sup>10</sup>Most of the data is available on Datastream. In case the data was not available on Datastream the source is indicated in the table in the appendix. Series such as realized volatility or interest rate spreads are calculated by the authors, which is indicated by (ac) in the same table.

<sup>11</sup>See e.g. Breitung and Eickmeier (2005) for a survey.

which implies that information of the comprehensive macroeconomic data set is compactly summarized in a strictly smaller number of factors.

We consider an approximate dynamic factor model as suggested in e.g. Stock and Watson (2002a) and Stock and Watson (2002b) which allows to estimate the macro-finance factors conveniently by asymptotic PCA. Asymptotic PCA for data with a large cross section,  $N$ , and a small number of time series observations,  $T$ , were originally developed by Connor and Korajczyk (1986). Approximate dynamic factor models are appealing due to their simplicity. The estimation of dynamic factor models is more complicated than the estimation its static counterpart.<sup>12</sup> However, Boivin and Ng (2005) compare the forecast performance of the dynamic and static approach and conclude that both methods have similar forecast precision. Thus, we favor the static estimation via PCA in this paper.

We follow the approach suggested by e.g. Stock and Watson (2002a), Stock and Watson (2002b), Moench (2008) and Ludvigson and Ng (2010) to calculate principal components. Accordingly, the factors  $F_t$  are defined by  $\sqrt{T}$  times the  $r$  eigenvectors corresponding to the  $r$  largest eigenvalues of the  $T \times T$  matrix  $y \times y'$ . The factors are normalized such that  $F_t' F_t = I_r$ , where  $I_r$  is the identity matrix of dimension  $r$  and the eigenvalues are sorted in decreasing order. Intuitively, at each point in time  $t$ , the set of factor  $F_t$  is given by a linear combination of each element of the  $N \times 1$  vector  $y_t = (y_{1t}, \dots, y_{Nt})'$ . The factors are chosen such that they minimize the sum of squared residuals of  $(y_{it} - \lambda_i' F_t)^2$  as in a standard linear regression.

We denote the number of factors needed to summarize the information of the data set by  $r$ . In practice, the number of factors is unknown but Bai and Ng (2002) develop model selection criteria which are suited for a panel data setting. In particular, we rely on the below loss function to determine the appropriate number of factors:

$$IC = V(r, F_r) + r\sigma^2 \left( \frac{(N + T - r)\ln(NT)}{NT} \right). \quad (2.3)$$

The fit of a model with  $r + 1$  factors cannot be worse than a model with  $r$  factors; however, efficiency is lost as more factor loadings are estimated. In the above selection

<sup>12</sup>Even though the model specifies a static relationship between  $y_{it}$  and  $F_t$ ,  $F_t$  may still be a dynamic vector process evolving according to  $A(L)F_t = u_t$  where  $A(L)$  is a polynomial in the lag operator. We refer to Forni, Hallin, Lippi, and Reichlin (2005) for a detailed discussion of dynamic factor models.

criteria,  $V(r, F_r)$  is the sum of squared residuals from Equation 2.2 and measures the fit of the model when  $r$  factors are estimated. The latter term of the loss function is a penalty which prevents us from overfitting and determines the number of factors  $r$ . Note that  $\sigma^2$  denotes a consistent estimate of  $(NT)^{-1} \sum_{i=1}^N \sum_{t=1}^T e_{it}^2$  and that the maximum number of estimated factors is set to 20.

The information criterion reported in Table 2.2 achieves its minimum if  $r = 8$ . Thus, the data set is appropriately summarized by 8 factors, i.e.  $r = 1, 2, \dots, 8$ . These 8 factors account for about 54.5% of the variance in the panel data set. The variance explained by each factor decreases in  $r$  since they are sorted in a descending order according to their eigenvalues.

### 2.3.2 Predictive Regressions

We study the predictability of currency portfolio returns and the business cycle dependence of currency risk premia using standard predictive regressions (as in Ludvigson and Ng (2009)). We are particularly interested in whether macro-finance factors provide information beyond forward discounts (or equivalently interest rate differentials vis-a-vis the U.S.), which are shown to be very powerful predictors of currency returns in Lustig, Roussanov, and Verdelhan (2010). This predictability is at the heart of the forward premium puzzle of Fama (1984a) and Hansen and Hodrick (1983). We regress  $h$  period log currency excess returns on lagged predictor variables, which include the AFD, denoted as  $\overline{FD}_t$ , and a set of macro-finance factors. The predictive regression reads as follows

$$rx_{t+h} = \alpha + \beta_{fd} \overline{FD}_t + \beta'_{k,x} F_{k,t} + \varepsilon_{t+h}, \quad (2.4)$$

where  $\beta_{fd}$  denotes the coefficient of the AFD and  $\beta_{r,x}$  indicates the coefficient for the set of macro-finance factors  $F_{k,t}$  included in the regression which represents a subset of  $F_{r,t}$ . The AFD, that is  $\overline{FD}_t = \overline{f_t - s_t}$  is an equally weighted average of the individual forward discounts of the basket of currencies vis-a-vis the USD that we consider. Lustig, Roussanov, and Verdelhan (2010) find this variable to be a more powerful predictor of FX market returns than the portfolio-specific forward discount, echoing previous results in bond markets by Cochrane and Piazzesi (2005).

Even though the extraction of macro-finance factors already represents a substantial reduction of dimensionality and compactly summarizes the information from many economic series in a few  $r$  factors, it is necessary to gauge which of these factors are actually relevant in predicting currency returns. We follow Ludvigson and Ng (2009) in applying a model selection approach guided by the Bayesian information criterion (BIC). With 8 factors there are possibly  $2^8 - 1$  forecast models based on different combinations of factors in the forecast model. Faced by this model uncertainty, we evaluate all possible forecast models and calculate the BIC for each model, which penalizes highly parametrized models. Finally, we present estimation results for the 5 best models with the lowest BIC, which provide a summary of models with significant predictive power and parsimony.

To assess the information content of macro-finance factors for future currency excess returns, we consider both short-term forecast horizons of one month ( $h = 1$ ) and longer-term horizons of one year ( $h = 12$ ). The long-horizon regressions exhibit serial correlation in the error term due to overlapping observations, which is a well-known problem in these types of regressions. To account for this issue, we use two common remedies, HAC robust standard errors by Newey and West (1987) which are based on the optimal number of lags following Andrews (1991) as well as Hansen and Hodrick (1980) standard errors (HH) which are computed with  $h$  lags. Besides autocorrelation, another common econometric pitfall in predictive regression is a potential bias of coefficients in finite samples due to the persistence of the typically used predictors (see Stambaugh (1999)). We account for this well-known problem based on a parametric bootstrap procedure which provides valid inference in small samples. The bootstrap procedure largely follows Mark (1995) and Kilian (1999).<sup>13</sup>

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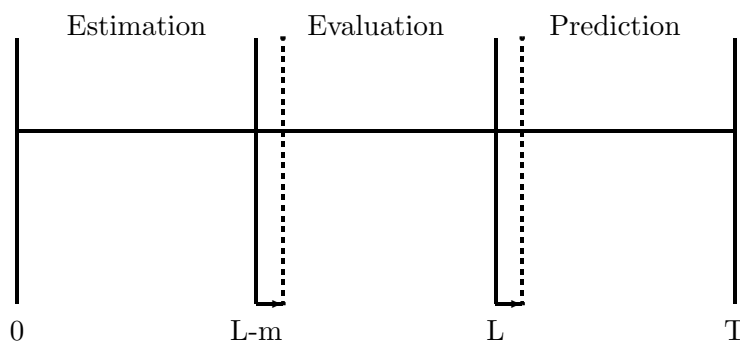
<sup>13</sup>The bootstrap procedure imposes the null hypothesis of non-predictability and assumes an autoregressive structure for the predictive variables  $z_t = (\overline{FD}_t, F_t)'$ . The data generating process is assumed to be given by  $rx_{t+1} = \beta_0 z_t + u_{1t}$  and  $z_{t+1} = \gamma z_t + u_{2t}$ . Based on this model we generate a sequence of pseudo observations  $rx_t^*$  and  $z_t^*$  using the estimated coefficients  $\hat{\beta}_0$  and  $\hat{\gamma}$  and by drawing with replacement from the estimated residuals  $u_{1t}^*$  and  $u_{2t}^*$ . With these pseudo observations at hands we estimate our predictive regression and obtain a distribution of the test statistic of our interest, i.e.  $\theta^* = (\alpha^*, \beta_{fd}^*, \beta_k^*)'$ . This distribution is then used to calculate bootstrapped p-values.

### 2.3.3 Out-of-sample Forecasting Approach

While in-sample analysis is useful for uncovering predictive relations and assessing the evolution of risk premia, an interesting question is whether this information is also useful for predicting out-of-sample. To evaluate the out-of-sample performance of the macroeconomic and financial predictors, we adapt the forecast procedure of Bai (2009). The main idea here is to select continuously good predictors based on their pseudo out-of-sample predictive performance during an evaluation period over which the model performance is evaluated. This adaptive prediction procedure reflects the uncertainty faced by an investor in real-time and allows her the updating of beliefs continuously by re-considering the prediction models.

The investor faces uncertainty about the best model to predict the currency market. Usually, model selection criteria such as AIC, BIC or  $R^2$  are based on in-sample information only, whereas interest typically lies in out-of-sample forecast accuracy. Thus, models chosen based on these criteria may suffer from a lack of predictive power. To overcome this shortcoming we use the predictive least squares principle (PLS) as a model selection criterion.

The picture below briefly summarizes the adaptive forecast procedure.



Graphical Illustration of the Out-of-sample Forecast Procedure

Suppose at time  $t = L$ , an investor faces uncertainty about the appropriate predictor model to support his investment decision. One way to reduce the uncertainty about the correct specification of the predictor model is to evaluate the out-of-sample during a model selection period which we refer to as the “evaluation window”.

At time  $t = L - m$ , an investor predicts currency returns based on the information set available at  $t$  according to the predictive regression given by Equation 2.4. The first forecast at time  $t = L - m + 1$  is denoted with the dashed line. For the first forecast we define the estimation window to have a length of 120 months, which represents our initialization period. After that, the estimation window is expanding. To select the most accurate model we define a model selection period, that is the “evaluation window”. It has length  $m = L - t$ , which we set to 12 months. During that period we calculate the PLS for each of the forecasting models defined by a specific combination of factors  $F_{kt}$ . We sum the PLS during the evaluation window, i.e. from  $L - m + 1$  (first forecast in the evaluation window) to  $L$  (last forecast in the evaluation window). In particular, for the evaluation period the sum of the PLS is given as:

$$PLS(z_i) = \sum_{t=N-m+1}^N (rx_{t+h} - (\alpha + \beta_{fd}\overline{FD}_t + \beta'_{k,x}F_{k,t}))^2.$$

At the end of the evaluation window the investors evaluates the  $PLS(z_i)$  for all possible forecast models and selects the model with the lowest PLS. This model is then used to make a prediction at  $L + 1$ , indicated by the second dashed line in the above figure. Thus, the procedure selects different forecast models based on out-of-sample information, i.e. based on information which is available at time  $t$ .

The length of the evaluation window is somewhat arbitrary. A longer evaluation window has more statistical power while a shorter window is better at depicting the dynamic changes of the economic conditions. The shorter the evaluation window, the better it captures recent developments in the economy. Hence there is a conflict between the length and the quality of the model selection. To analyze the impact of the length of the evaluation window we choose the evaluation window to be 12, 24 and 36 months; however, there are no qualitative differences with respect to the length of the evaluation window. Thus, in Section 2.4.5 we report the out-of-sample forecasting results based on a evaluation window of 12 months.

### 2.3.4 Forecast Evaluation

To evaluate the forecast performance of models augmented by macro-finance factors we compare their forecasts with two benchmarks. First, a forecast where we instead of relying on a model selection procedure use all the eight factors to forecast currency returns. We label this forecast a kitchen sink forecast. Second, we also make use of the AFD to predict currency excess returns. We dynamically compare the predictive power of the macro-finance factor to the benchmark forecasts based on the difference in cumulative prediction errors. This evaluation is beneficial since we are able to evaluate the time series patterns in the forecast performance.

Based on these benchmarks we calculate Theil's U to evaluate the statistical power of the of the factor based model. Theil's U is given by the root mean square error (RMSE) of the forecast based on macro-finance factors relative to the RMSE of the benchmark model such that a value smaller than one indicates that the model beats the benchmark in terms of forecast accuracy. To assess statistical significance we calculate bootstrapped p-values. The bootstrap procedure is a model-based wild bootstrap imposing the null of non-predictability by macro-finance factors.<sup>14</sup>

## 2.4 Results

### 2.4.1 Economic Interpretation of Factors

Before turning to the discussion of predictability results, we briefly discuss what economic information the factors might summarize. The information criterion suggests that our large macro-finance data set is well described by eight common factors, i.e. this is the number of factors for which the information criterion by Bai and Ng (2002) achieves its minimum value. The cumulative variance explained by the macro-finance factors is reported in Table 2.2. In addition, Figure 2.1 serves to interpret the factors from an economic perspective. Following Ludvigson and Ng (2009), each individual series of the data set is regressed on every factor. Figure 2.1 shows the R-squared from this regressions as a bar chart for the eight factors.

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<sup>14</sup>We refer to the appendix for a more detailed explanation of the bootstrap procedure.

Based on the R-squared of these regressions, we attach some economic labels to the series to facilitate the economic interpretation in our predictive regressions. We emphasize that any labelling of the factors is imperfect, because each factor is to some degree influenced by all the variables in our large data set. Nevertheless, it is useful to show that some factors more likely capture relevant macroeconomic information while others are correlated with financial series.

INSERT FIGURE 2.1 ABOUT HERE

Results reported in Figure 2.1 suggest that the first factor, explaining by far the largest share of variance in the data (about 14 %), can be interpreted as a business cycle fluctuation factor. Variables related to industrial production and employment load heavily on the first factor justifying the business cycle labelling. Our regression results indicate that  $f_2$  might be interpreted as a yield curve slope factor as the maximally correlated variables are interest rate spreads.  $f_3$  captures the level of the U.S. yield curve, while  $f_4$  is primarily correlated with inflation variables.  $f_5$  is maximally correlated with equity market valuation ratios and  $f_6$  captures inflation variables and equity index returns. The Sentiment Indexes from the University of Michigan are highly correlated with  $f_7$  and  $f_8$  summarizes the information from consumption expenditures variables as well as information from the indexes published by the Institute of Supply Management. In the subsequent section, we investigate the forecasting properties of these factors for the currency excess returns for a monthly and annual forecast horizon.

### 2.4.2 Results of Monthly Predictive Regressions

Table 2.3 contains the results of predictive regressions for monthly returns of an equally weighted basket of exchange rates against the U.S. Dollar (Panel A) and exchange rate changes of the same basket of currencies (Panel B). Predictive coefficients that are significant based on asymptotically valid standard errors at the 10% level are bold-printed. In addition, we report bootstrap p-values that conduct valid inference in finite samples. Table 2.3 reports the result of the five best model specifications (out of all possible  $2^8 - 1$  combinations) as measured by the BIC. The AFD is controlled for as a predictor in each



regression. As a benchmark, the results using the AFD as a single predictor are reported on the right hand side of the table.

INSERT TABLE 2.3 ABOUT HERE

As evidenced by Panel A of Table 2.3, the short-term predictability of currency excess returns can be raised substantially by augmenting the predictive regressions with macro-finance factors. By including macro-finance factors in the predictive regressions, the R-squared substantially increases, that is, we are able to explain between 2.3% and 4.3% of time-series variation in currency excess returns over the next month (see Panel A). The benchmark regression yields an R-squared of 2.0%, i.e. by adding macro-finance factors to the regression we double the share of explained variation over the forecast horizon.

The benchmark regression shown in Panel B of Table 2.3 shows that the AFD is not statistically different from zero when predicting spot rate changes. Also, the R-squared is very low. Including macro-finance factors in the predictive regression increases the predictive power, however, the explained variation in exchange rate changes remains low, even though the macro-finance factors are statistically significant.

The most prominent macro-finance factors when we predict currency excess returns and exchange rate changes of a basket of currencies are a factor which captures interest rate information ( $f_3$ ) and a factor linked to the stock market ( $f_5$ ). The two factors appear in the three models with lowest BIC criterion. The factors are statistically significant meaning that they contain marginal predictive power when forecasting currency excess returns as well as exchange rate changes. Both factors predict negative currency excess returns.<sup>15</sup>

The evidence shown in Table 2.3 suggests that the estimated macro-finance factors are able to predict aggregate FX market at a one month forecast horizon. In a second step, we examine the predictive power of the macro-finance factors for a carry trade strategy. In particular, we conduct a similar forecast analysis for the CTI, that is an index which

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<sup>15</sup>Note that macro-finance factors are defined up to a constant. To simplify the interpretation of the regression coefficient we transformed the factors such that they are positively correlated with the underlying, economic variables they share the highest correlation. For example, in case of high (low) interest rates in the U.S. the interest rate factor ( $f_3$ ) is mostly positive (negative) and similarly, bull (bear) markets at the U.S. stock exchange are accompanied with positive (negative) values of the stock market factor ( $f_6$ ).

weights the foreign currencies by their interest rate differential against the U.S. short-term interest rate. Table 2.4 summarizes the results of this predictive regression.

INSERT TABLE 2.4 ABOUT HERE

Table 2.4 reports that macro-finance factors successfully predict carry trade returns. Unlike the AFD in the benchmark regression, a factor which is correlated with inflation variables ( $f_4$ ) is statistically significant and considerably rises the explained variation in carry trade returns over the next month. The adjusted R-squared of the predictive regression including the inflation factor is around 3% compared to an adjusted R-squared of -0.3% from the AFD regression. The fact that an inflation factor contains such predictive power might be surprising, however this supports the evidence found by Ang and Chen (2010) where it is shown that any factor which may affect domestic bond prices has the potential to predict foreign exchange risk premia. Thus, inflation could be a possible predictor of FX returns through interest rate variables.

The evidence from monthly predictions support the hypothesis of a link between the state of the economy and exchange rates. Our results are hence more positive and encouraging than most of the “disconnect” literature building on Meese and Rogoff (1983). In a next step, we analyze the predictive power of the macro-finance factors at an annual forecast horizon.

### 2.4.3 Results of Long-Horizon Predictive Regressions

Table 2.5 contains the results of the annual forecast of the aggregate FX market. The AFD factor is a useful predictor as shown by Lustig, Roussanov, and Verdelhan (2010). It is significant based on the bootstrapped p-values and explains about 13% of the time series variation of the aggregate currency return.

INSERT TABLE 2.5 ABOUT HERE

As shown in Table 2.5, including macro-finance factors in the predictive regressions increases the share of explained variation in currency returns to around 20% compared to 13% of the benchmark. More interestingly, if we consider macro-finance factors and in

particular a real activity factor ( $f_1$ ) in the regression analysis, the predictive power of the AFD factor is reduced (its coefficient is lower), suggesting that the AFD factor may to some extent proxy macro economic information. The real activity factor appears in all five top model specifications and is statistically significant across all models. The positive regression coefficient of the real activity factor in Table 2.5 shows that expected currency returns are high (low) in recessions (expansions). This finding suggests a counter-cyclical currency risk premia and we refer to Section 2.4.4 for a more detailed explanation of the counter-cyclical behavior of currency risk premia.<sup>16</sup>

As a final in-sample forecast exercise we predict the CTI and summarize the results in Table 2.6.

INSERT TABLE 2.6 ABOUT HERE

As with the previous regressions, predictions including macro-finance factors substantially increase the share of explained variation in the carry trade returns over the forecast horizon. The top five prediction models (according to the BIC criteria) explain between 10.3% and 13.8% of the time-series variation in carry trade returns which is a substantial increase compared to the adjusted R-squared of 0.6% of the benchmark regression. All macro-finance factors contain marginal predictive power meaning that they are statistically significant (with  $f_8$  as the exception), thus the adjusted R-squared for the regression including macro-finance factors is considerably larger. The most powerful factors are related to interest rates ( $f_2$  and  $f_3$ ), inflation ( $f_4$ ) and equity index returns ( $f_6$ ).

Overall, the evidence from in-sample regression shows that macro-finance factors successfully predict currency excess returns also when we control for the AFD. The in-sample results stress the importance of using information beyond that contained in the AFD when predicting currency returns. Both at annual and monthly forecast horizons, the share of explained variation in currency excess returns over the forecast horizon increases substantially indicating that macroeconomic variables have a lot to say about future currency returns.

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<sup>16</sup>In the recent decade, a large body of the asset pricing literature (see Campbell and Cochrane (1999) and Bansal and Yaron (2004) for two prominent papers) reason that the equity premia shows a counter-cyclical behavior, that is, expected excess returns rise in recessions and fall in expansions. Verdelhan (2010) develops a theoretical model which shows that the currency risk premia also exhibits a counter-cyclical behavior.

The macro-finance factors are particularly powerful when predicting the CTI. We relate this finding to the fact that factors which capture interest rate information are the most prominent in these regressions. More precisely, factors which are correlated with interest rate spreads ( $f_2$ ) and the level of interest rates ( $f_3$ ) are the most powerful factors when predicting the CTI. Also the factor related to inflation aggregates ( $f_4$  and  $f_6$ ) appear in the top models at annual forecast horizon. Thus, in addition to the interest rate information of the AFD it seems that by enhancing predictive regressions with different interest rate information such as interest rate spreads as well as long-term yields (yields of long-term zero-coupon bond load heavily on  $f_3$ ) the predictive power can be raised considerably.

The predictive regressions show evidence that the macro-finance factors contain predictive power, however, we cannot evaluate if the currency risk premia is related to the business cycle. In the following section, we investigate the time varying behavior of the currency risk premia and show that it is strongly counter-cyclical.

#### 2.4.4 Is the currency risk premia counter-cyclical?

The evidence presented so far indicates that excess currency returns are related to macroeconomic variables, but we do not know whether currency risk premia is counter-cyclical, as expected by economic theory. Verdelhan (2010) shows in a habit-based model that the currency risk premia exhibits a counter-cyclical behavior implying that investors require a higher compensation for bearing currency risk in economic downturns. Thus, we expect the currency risk premia to be counter-cyclical with respect to the U.S. business cycle, that is, expected excess returns are high (low) during U.S. recessions (expansions).

Lustig, Roussanov, and Verdelhan (2010) show that the AFD is strongly counter-cyclical meaning that the contemporaneous correlation between AFD and the U.S. industrial production growth is negative. The positive coefficient associated with the AFD in Table 2.5 predicts high (low) expected currency returns in recessions (expansions) showing that investors must be compensated for bearing risks related to recessions.

This finding is confirmed when we investigate the predictive power of the real activity factor which was the most powerful macro factor at an annual forecast horizon. This factor, which captures the business cycle variation, appears in all top models when we predict

currency returns of an aggregate market. To investigate the counter-cyclical behavior of the currency risk premia, Figure 2.2 plots three month moving averages of the real activity factor with three month moving averages of monthly growth rates in U.S. industrial production. The yellow shaded bars display U.S. recession as designated by the National Bureau of Economic Research.

INSERT FIGURE 2.2 ABOUT HERE

Figure 2.2 suggests that the real activity factor is strongly negative correlated with industrial production growth. The correlation between  $f_1$  and industrial production growth rates is about -80.85%. Hence, expansions (recessions) are characterized with high (low) values of industrial production growth and low and mostly negative (high and mostly positive) values of the real activity factor.

By combining this finding with the results of the predictive regression for currency excess returns of Table 2.5, we notice that expected excess returns are high when  $f_1$  is high. Thus, the positive coefficient related to the real activity factor predicts high expected excess returns in recessions and low expected excess returns during expansions.

The counter-cyclical behavior of the dollar risk premia is even more distinct when we add macro factors. The prediction based on macro factors points in the same direction as average forward prediction, hence expected returns further increase (decrease) in recessions (expansion). Overall, the counter-cyclical behavior becomes even more pronounced when we augment the predictive regression with macro factors, showing that that macro factors predict currency excess returns consistent with economic theory.

#### 2.4.5 Out-of-sample analysis

We conclude the forecasting exercise by investigating the out-of-sample predictive power of the macro-finance factors. The performance of the out-of-sample forecast is measured by the difference in the cumulative squared prediction error ( $\Delta SSE$ ) between predictions based on macro-finance factors and a benchmark model, as suggested in e.g. Goyal and Welch (2008) and Bai (2009). As a benchmark we choose a forecast based on a kitchen sink regression, i.e. a regression including all eight factors. In contrast to our preferred forecast

method, we do not rely on a model selection procedure, that is, we do not choose the model with the lowest PLS (see Section 2.3.3). Additionally, we also compare forecasts based on macro-finance factors with a forecast using the AFD. We generate recursive out-of-sample forecasts of the aggregate FX market using an expanding estimation window.

The  $\Delta SSE$  allows a continuous evaluation of the forecast performance over the whole out-of-sample period. These plots avoid a biased judgment based on only single time-point evaluation. The continuous evaluation also allows an analysis of the time-series pattern of forecasts, i.e. one is able to recognize months with a good or bad performance. An increase in a line indicates that the model augmented with the macro-finance factors outperforms the benchmark model whereas a decrease in a line suggests better performance of the benchmark. A good month means that the  $\Delta SSE$  at that month is included in an upward trend along the line.

Figure 2.3 illustrates the out-of-sample forecast performance for the aggregate FX market at a monthly and an annual forecast horizon. The first out-of-sample forecasts for a monthly forecast horizon is at 01/1995, while the out-of-sample forecast evaluation period for annual forecast begins at 12/1995. For the two forecast horizons, the evaluation period ends at 03/2009.

INSERT FIGURE 2.3 ABOUT HERE

The first impression of the left panel in Figure 2.3 indicates the importance of the model selection procedure. The positive  $\Delta SSE$  over the out-of-sample period shows that forecasts relying on the model selection procedure generate smaller cumulative prediction errors than forecasts based on the kitchen sink regression. This holds for the monthly forecast horizon as well as for annual predictions. The steadily increasing  $\Delta SSE$  shows that forecasts based on the model selection procedure outperform the benchmark over almost the entire out-of-sample forecast sample. However, we note that the magnitude of the  $\Delta SSE$  is rather small indicating that both predictor models perform equally well, a fact that we further evaluate in Table 2.7.

On the other hand, the right panel in Figure 2.3 shows that predictions based on macro-finance factors have difficulties to outperform forecasts based on the AFD at a monthly forecast horizon. The  $\Delta SSE$  shows a sharp increase in September 1998 as well as an

abrupt decline in March 2001. A similar pattern is recognized in the left panel, where the benchmark is the kitchen sink regression, although to a lesser extent. At an annual forecast horizon, forecasts enhanced with macro-finance factors outperform AFD predictions. This is shown in the bottom right picture where the  $\Delta SSE$  is positive and increasing suggesting that macro-finance factors successfully predict currency returns in an out-of-sample fashion.

We conduct a similar out-of-sample forecast analysis for the returns of the CTI. Figure 2.4 reports the results from the dynamic forecast evaluation.

INSERT FIGURE 2.4 ABOUT HERE

The dynamic forecast evaluation shows that macro-finance factors accurately predict carry trade returns since the  $\Delta SSE$  is increasing and positive for most of the evaluated out-of-sample forecast period. Macro-finance factors generate smaller cumulative forecast errors compared to kitchen sink regression (left panel) as well as AFD predictions (right panel) at a monthly and an annual forecast horizon. Again, the advantage of macro-finance factors is more distinct at an annual forecast horizon.

The dynamic evaluation of the out-of-sample forecast performance is confirmed in Table 2.7 where we calculate Theil's U and assess its statistical significance as explained in Section 2.3.3. Note that if Theil's U is smaller than one, predictions augmented with macro-finance factors are superior. The bootstrap p-values are computed as the proportion of Theil's U statistics in the bootstrap samples that are smaller than the sample Theil's U. Thus, these p-values are one-sided and test the null of equal predictive performance against the alternative of superior performance of the model including macro-finance factors against the benchmark.

INSERT TABLE 2.7 ABOUT HERE

Panel A of Table 2.7 displays Theil's U for a monthly forecast horizon for out-of-sample forecasts of the aggregate FX market and the CTI. At a monthly forecast horizon, Theil's U suggests that predictions based on the AFD outperform predictions enhanced with macro-finance factors in terms of mean square forecast errors (Theil's U is larger than one for BM 1). However, we note that Theil's U is close to one showing that both models perform

equally well. Additionally, the p-values suggest that we do not reject the null hypothesis of equal predictive performance of the two models. For the case where the kitchen sink regression is the benchmark, Theil's U is smaller than one suggesting that predictions based on our model search algorithm generate smaller forecast errors. The bootstrap p-values show that the improvement by our model search procedure is not statistically significant since we fail to reject the null hypothesis of equal performance of both models. However, the predictive power of forecasts enhanced with macro-finance factors is more pervasive at an annual forecast horizon (see Panel B of Table 2.7). Our results show substantial improvements of predictor models which are augmented with macro-finance factors for predictions of the aggregate FX market as well as the CTI. Macro-finance predictions outperform kitchen sink predictions as well as predictions based on the AFD. This evidence is supported by the bootstrap p-values which show that macro-finance factor prediction statistically outperform kitchen sink predictions as well as average forward predictions.

Overall, the longer the forecast horizon, the better the out-of-sample performance of predictions enhanced with macro-finance factors. Theil's U confirms this finding of the dynamic out-of-sample evaluation and shows that macro-finance factors statistically outperform the benchmark forecasts at an annual forecast horizon.

## 2.5 Conclusion

In this paper, we show that returns of an aggregate currency market and a CTI exhibit strong forecastable patterns. In contrast to most of the "disconnect" literature building on Meese and Rogoff (1983), we find that factors extracted from a large panel of macroeconomic aggregates and financial series contain substantial predictive power when predicting expected currency excess returns.

The in-sample analysis shows that the share of explained variation over the forecast horizon is almost doubled when we forecast the aggregate FX market. We identified a factor capturing business cycles, factors summarizing interest rate information and a factor linked to stock market as the most powerful. Also out-of-sample forecast procedures relying on macro-finance factors outperform the benchmarks, especially at a one year forecast



horizon. Additionally, we find that the currency risk premia exhibits a strong counter-cyclical behavior, that is, expected currency excess returns are low (high) in economic expansion (recessions). Thus, investors have to be compensated for bearing risks associated with economic recessions.

An important finding is that macro-finance factors contain information about expected currency excess returns beyond forward discounts (which are interest rate differentials relative to the U.S.). Thus, macroeconomic fundamentals and financial information contain a lot of information about future currency movements that is not contained in interest rates.

TABLE 2.1: Descriptive Statistics for the FX Data

<b>Panel A: Monthly Data</b>			
Portfolio	DOL	CTI	AFD
mean	1.90	4.62	0.91
median	3.41	6.48	0.96
std	8.68	5.78	0.64
skew	-0.25	-1.41	0.28
kurt	0.51	7.19	-0.57
AC1	0.11	0.14	0.86
SR	0.22	0.80	
<b>Panel B: Annual Data</b>			
Portfolio	DOL	CTI	AFD
mean	2.22	3.50	0.45
median	1.96	4.00	0.40
std	10.28	6.30	1.74
skew	-0.07	-0.94	0.19
kurt	-0.50	2.69	-0.74
AC1	0.94	0.90	0.98
SR	0.22	0.56	

This table reports mean and median returns, standard deviations (both annualized), skewness, and kurtosis of currency portfolios sorted monthly on time  $t-1$  forward discounts. We also report the first order autocorrelation coefficient ( $AC(1)$ ) and annualized Sharpe Ratios (SR). DOL denotes the average return of five currency portfolios and CTI is the carry trade index. All returns are excess returns in USD. The sample period is 09/1983 - 06/2009.

TABLE 2.2: Descriptive Statistics for the Factors

Factor	$AC1(f_i)$	$R_i^2$	$IC_i$
$f_1$	0.933	14.1%	0.885
$f_2$	0.904	8.8%	0.828
$f_3$	0.459	7.3%	0.785
$f_4$	0.767	6.8%	0.747
$f_5$	0.790	5.5%	0.721
$f_6$	0.555	4.7%	0.704
$f_7$	0.192	3.4%	0.700
$f_8$	-0.221	3.2%	0.698

This table reports first order autocorrelation coefficient  $AC1(f_i)$  of the factors extracted from a panel of macroeconomic data. The relative importance of each factor,  $R_i^2$ , is calculated as the fraction of total variance in the data explained by the corresponding factors. We also show an information criterion,  $IC_i$ , which specifies the number of factors needed to capture the common variation in the dataset.  $IC_i$  indicates that 10 factors are sufficient to reflect the information of the dataset, thus  $i = 1, \dots, 10$ . The sample period is 09/1983 - 06/2009.

TABLE 2.3: Predictive regressions for excess returns and spot rate changes of an aggregate FX Market, h=1

<b>Panel A: Currency Excess Returns</b>						
Model	Top 5 Models with Factors					Bench
	(i)	(ii)	(iii)	(iv)	(v)	AFD
$f_3$	<b>0.329</b>					<b>0.329</b>
p(BS)	0.045					0.045
$f_4$			-0.221			
p(BS)			0.216			
$f_5$		<b>-0.318</b>				
p(BS)		0.021				
$f_6$				-0.063	-0.063	
p(BS)				0.870	0.813	
AFD	<b>2.083</b>	<b>2.025</b>	<b>1.946</b>	<b>2.130</b>	<b>2.108</b>	<b>2.084</b>
p(BS)	0.023	0.020	0.044	0.048	0.031	0.027
$R^2$	3.5%	3.4%	2.5%	1.8%	3.2%	2.0%
BIC	1.857	1.858	1.867	1.874	1.879	1.856
<b>Panel B: Spot Rate Changes</b>						
Model	Top 5 Models with Factors					Bench
	(i)	(ii)	(iii)	(iv)	(v)	AFD
$f_3$	<b>0.326</b>					<b>0.326</b>
p(BS)	0.041					0.038
$f_4$			-0.212			
p(BS)			0.231			
$f_5$		<b>-0.313</b>				
p(BS)		0.021				
$f_6$				-0.072	-0.071	
p(BS)				0.848	0.792	
AFD	1.223	1.167	1.093	1.274	1.252	1.225
p(BS)	0.151	0.153	0.223	0.192	0.185	0.179
$R^2$	2.0%	1.8%	0.9%	0.3%	1.7%	0.5%
BIC	1.849	1.850	1.859	1.866	1.870	1.847

This table reports results from in-sample predictive regressions. The dependent variable in Panel A is the excess returns of an aggregate FX market return and in Panel B the spot rate changes of an aggregate FX market. The forecast horizon is one month, h=1. The top 5 model specifications are reported (minimizing the Schwarz criterion) along with results for the benchmark model which contains the average forward discount (AFD). We compute Newey and West (1987) NW standard errors with the optimal number of lags following Andrews (1991) and Hansen and Hodrick (1980) HH standard errors with one lag. Coefficients that are statistically significant (i.e. at the 10% level or below) based on either the NW or HH standard errors are highlighted in bold. p(BS) denotes p-values computed by a parametric bootstrap approach with 1,500 replications. The sample period is 12/1983-03/2009.

TABLE 2.4: Predictive regressions for currency excess returns of Carry Trade Index,  $h=1$ 

<b>Carry Trade Index: Excess Returns</b>						
	Top 5 Models with Factors					Bench
Model	(i)	(ii)	(iii)	(iv)	(v)	AFD
$f_2$		-0.154		-0.120	-0.154	
p(BS)		0.432		0.544	0.445	
$f_4$	<b>-0.325</b>	<b>-0.337</b>			<b>-0.337</b>	
p(BS)	0.016	0.015			0.013	
$f_7$	-0.059	-0.070	-0.053	-0.062	-0.070	
p(BS)	0.987	0.998	0.993	0.993	0.991	
$f_8$					0.025	
p(BS)					0.987	
AFD	-0.431	-0.889	-0.193	-0.544	-0.887	-0.148
p(BS)	0.502	0.407	0.749	0.553	0.426	0.774
$R^2$	2.9%	3.2%	-0.5%	-0.5%	2.9%	-0.3%
BIC	1.069	1.085	1.085	1.104	1.107	1.067

This table reports results from in-sample predictive regressions. The dependent variable are currency excess returns of the carry trade index return (return on the CTI portfolio). The forecast horizon is one month,  $h=1$ . The top 5 model specifications are reported (minimizing the Schwarz criterion) along with results for the benchmark model which contains the average forward discount. We compute Newey and West (1987) NW standard errors with the optimal number of lags following Andrews (1991) and Hansen and Hodrick (1980) HH standard errors with twelve lags. Coefficients that are statistically significant (i.e. at the 10% level or below) based on either the NW or HH standard errors are highlighted in bold. p(BS) denotes p-values computed by a parametric bootstrap approach with 1,500 replications. The sample period is 12/1983-03/2009.

TABLE 2.5: Predictive regressions for aggregate FX Market Returns,  $h=12$ 

<b>Currency Excess Returns</b>						
	Top 5 Models with Factors					Benchmark
Model	(i)	(ii)	(iii)	(iv)	(v)	AFD
$f_1$	<b>3.647</b>	<b>3.575</b>	<b>4.219</b>	<b>4.282</b>	<b>3.815</b>	
p(BS)	0.009	0.002	0.002	0.001	0.002	
$f_2$				-1.029		
p(BS)				0.288		
$f_3$					-0.569	
p(BS)					0.551	
$f_4$	<b>-2.021</b>	<b>-2.013</b>		<b>-2.019</b>	<b>-2.019</b>	
p(BS)	0.009	0.011		0.012	0.023	
$f_8$		1.199				
p(BS)		0.562				
AFD	<b>1.229</b>	<b>1.242</b>	<b>1.230</b>	<b>0.758</b>	<b>1.200</b>	<b>2.142</b>
p(BS)	0.031	0.024	0.031	0.028	0.027	0.004
$R^2$	21.4%	22.5%	18.8%	21.7%	21.4%	12.9%
BIC	4.498	4.503	4.511	4.513	4.517	4.553

This table reports results from in-sample predictive regressions. The dependent variable are currency excess returns of an aggregate FX market return (return on the DOL portfolio). The forecast horizon is one month,  $h = 12$ . The top 5 model specifications are reported (minimizing the Schwarz criterion) along with results for the benchmark model which contains the average forward discount. We compute Newey and West (1987) NW standard errors with the optimal number of lags following Andrews (1991) and Hansen and Hodrick (1980) HH standard errors with twelve lags. Coefficients that are statistically significant (i.e. at the 10% level or below) based on either the NW or HH standard errors are highlighted in bold. p(BS) denotes p-values computed by a parametric bootstrap approach with 1,000 replications. The sample period is 12/1983-03/2009.

TABLE 2.6: Predictive regressions for Carry Trade Index, h=12

<b>Carry Trade Index: Excess Returns</b>						
	Top Models with factors					Benchmark
	(i)	(ii)	(iii)	(iv)	(v)	AFD
$f_2$	<b>-1.457</b>	<b>-1.455</b>	<b>-1.572</b>	<b>-1.456</b>		
p(BS)	0.013	0.034	0.021	0.044		
$f_3$		<b>-0.887</b>	<b>-0.909</b>	<b>-0.883</b>	<b>-0.889</b>	
p(BS)		0.290	0.325	0.305	0.135	
$f_4$	<b>-1.601</b>	<b>-1.631</b>	<b>-1.694</b>	<b>-1.625</b>	<b>-1.513</b>	
p(BS)	0.002	0.002	0.001	0.000	0.000	
$f_6$	<b>-1.140</b>	<b>-1.121</b>		<b>-1.134</b>	<b>-1.246</b>	
p(BS)	0.016	0.009		0.013	0.001	
$f_8$				0.405		
p(BS)				0.472		
AFD	-0.845	-0.834	-0.967	-0.834	-0.352	-0.352
p(BS)	0.047	0.025	0.013	0.0207	0.203	0.151
$R^2$	11.8%	13.4%	10.9%	13.5%	9.8%	0.6%
BIC	3.653	3.654	3.663	3.672	3.675	3.710

This table reports results from in-sample predictive regressions. The dependent variable are spot rate changes of an aggregate FX market return (spot rate changes of the DOL portfolio). The forecast horizon is one year, h=12. The top 5 model specifications are reported (minimizing the Schwarz criterion) along with results for the benchmark model which contains the average forward discount. We compute Newey and West (1987) NW standard errors with the optimal number of lags following Andrews (1991) and Hansen and Hodrick (1980) HH standard errors with twelve lags. Coefficients that are statistically significant (i.e. at the 10% level or below) based on either the NW or HH standard errors are highlighted in bold. p(BS) denotes p-values computed by a parametric bootstrap approach with 1,000 replications. The sample period is 12/1983-03/2009.

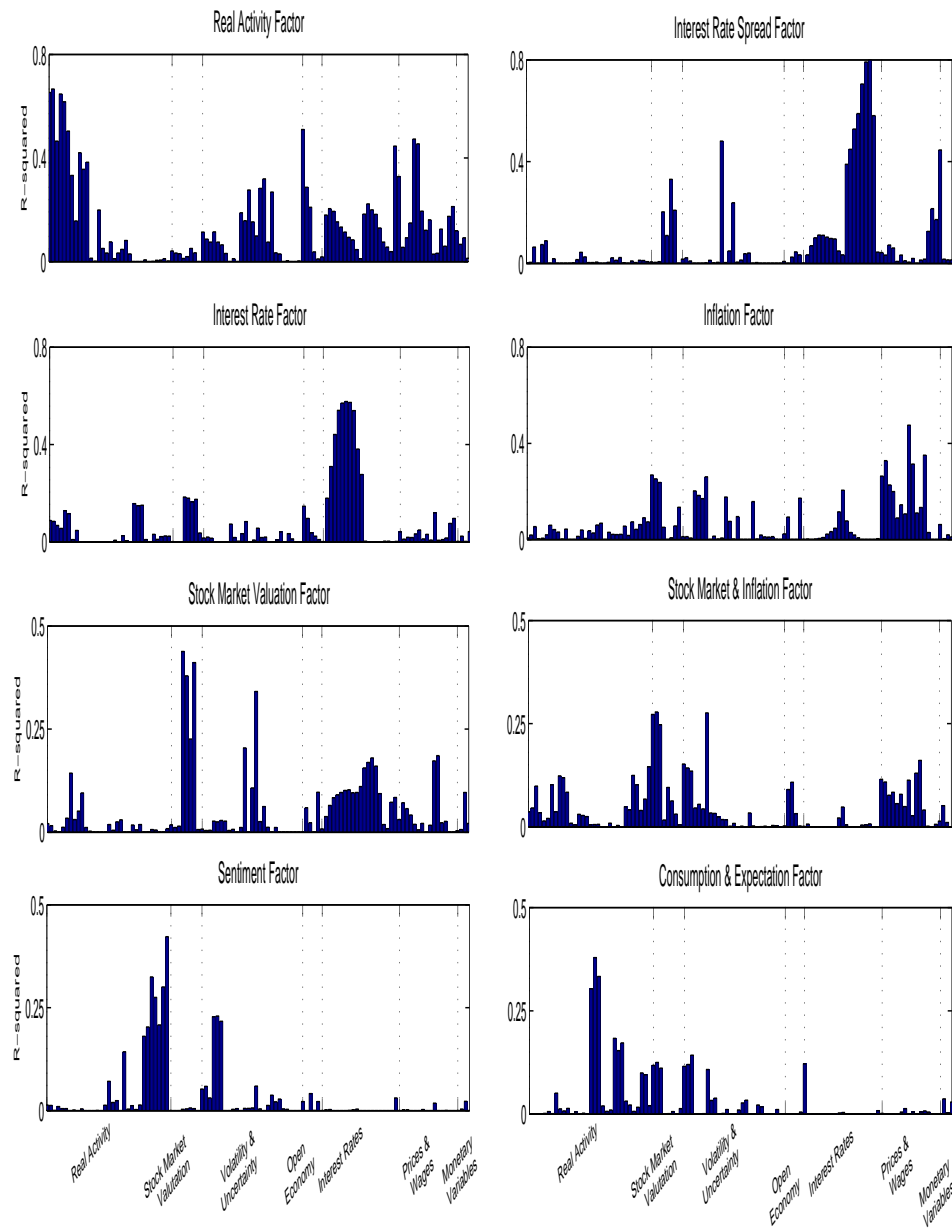
TABLE 2.7: Out-of-sample Forecast Evaluation

Panel A: Out-of-sample Forecast Performance, h=1			
		DOL	CTI
BM 1	Theil's U	1.010	1.009
	$\#TU^{bs} < TU$	0.530	0.513
BM 2	Theil's U	0.981	0.988
	$\#TU^{bs} < TU$	0.357	0.380
Panel B: Out-of-sample Forecast Performance, h=12			
		DOL	CTI
BM 1	Theil's U	0.910	0.885
	$\#TU^{bs} < TU$	0.000	0.000
BM 2	Theil's U	0.894	0.903
	$\#TU^{bs} < TU$	0.000	0.000

This table presents the statistical results of real-time out-of-sample performance for the monthly predictive regressions (Panel A) and annual predictive regressions (Panel B) for the FX aggregate market (DOL) and the carry trade index (CTI). BM 1 denotes that the forward discount forecast is the benchmark model while BM 2 denotes that the kitchen sink regression forecast is the benchmark model. During each out-of-sample month, investors choose a predictor from the base set which generates the smallest cumulative prediction errors in the previous 12 months. The performance is measured by Theil's U. The forecast horizon is one month (Panel A) or one year (Panel B).  $\#TU^{bs} < TU$  denotes bootstrap p-values for testing equal predictive performance of factor enhanced predictions and the respective alternative benchmark models.

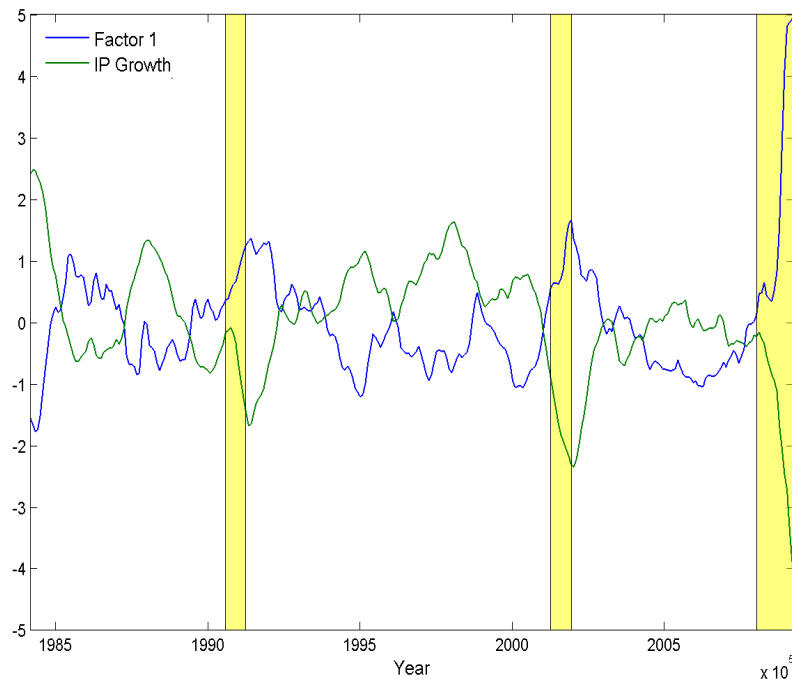


FIGURE 2.1: Factor Interpretation



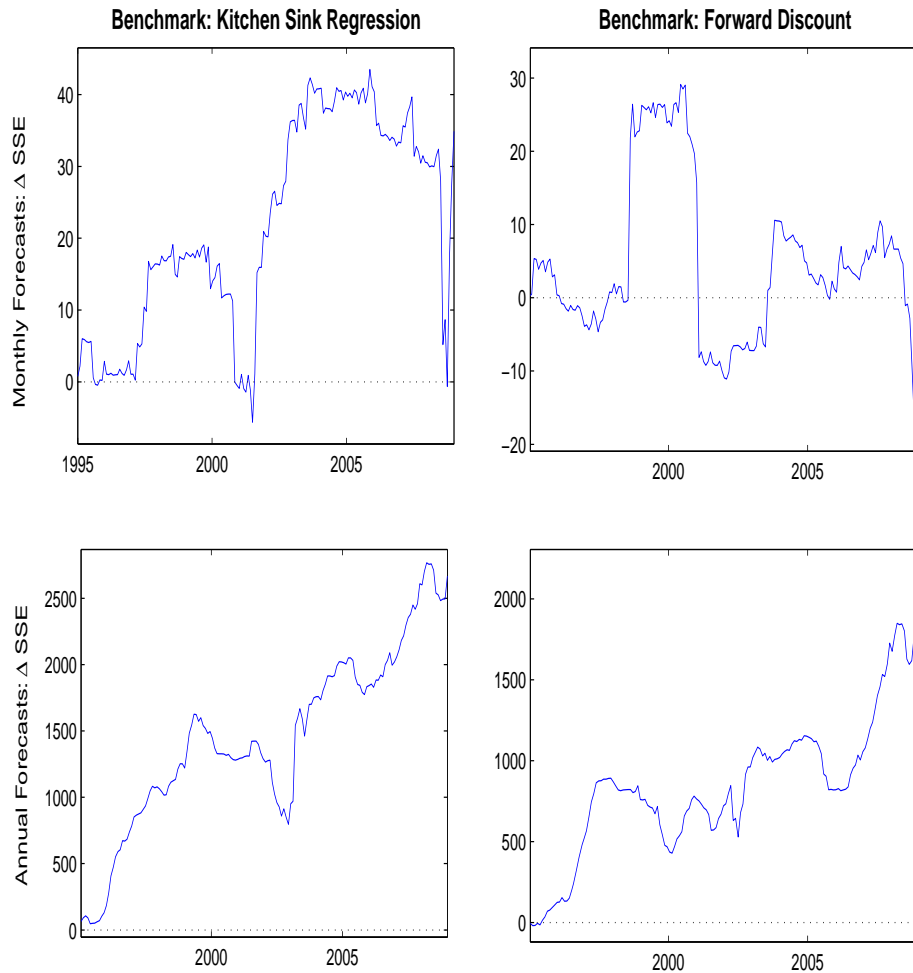
This figure shows the R-squared between the factors and the individual time series which are denoted on the x-axis. The individual time series are grouped into the following seven categories: real activity, stock market valuation, volatility and aggregate uncertainty, interest rates and interest rate spreads, price and wage variables, open economy and monetary variables. The sample period for the regressions is 12/1983-03/2009

FIGURE 2.2: Real Activity Factor and U.S. Industrial Production Growth



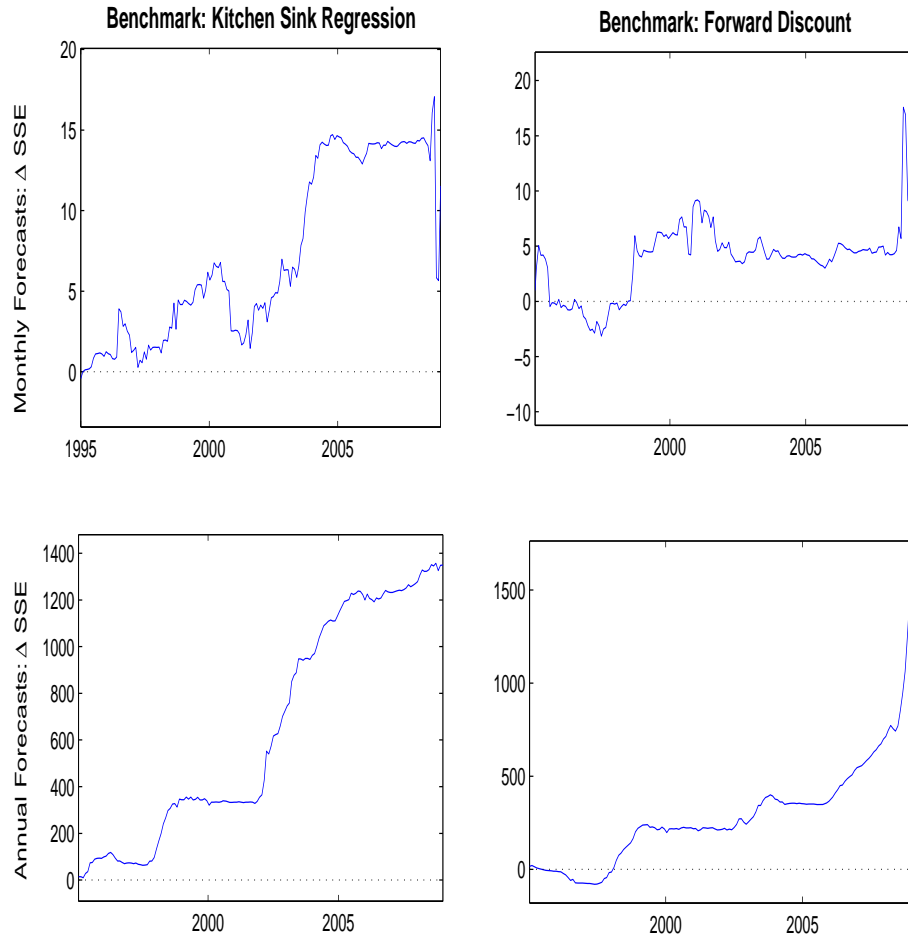
This figure plots three month averages of the real activity factor and U.S. industrial production growth from 02/1984 to 03/2009. The yellow shaded bars display U.S. recession as designated by the National Bureau of Economic Research.

FIGURE 2.3: Dynamic Out-of-Sample Performance for aggregate FX Market based on Adaptive Macro Factors



This figure shows the out-of-sample performance of monthly (Top Panel) and annual (Bottom Panel) prediction models for the FX aggregate market. The performance is measured by the cumulative squared prediction errors of the benchmark ( $u_{B,t}^2$ ) minus those of the alternative ( $u_{A,t}^2$ ),  $\Delta SSE = \sum_t (u_{B,t}^2 - u_{A,t}^2)$ . The benchmark is the historical mean forecast or forecasts based on the average forward discount. The alternative is the conditional forecast using adaptively selected factors constructed from the macro dataset. During each out-of-sample month, investors choose a predictor from the base set which generates the smallest cumulative prediction errors in previous 24 months. The prediction period for panel A is from 12/1995 to 3/2009 and for panel B from 11/1996 to 3/2009.

FIGURE 2.4: Dynamic Out-of-Sample Performance for the Carry Trade Index based on Adaptive Macro Factors



This figure shows the out-of-sample performance of monthly (Top Panel) and annual (Bottom Panel) prediction models for the carry trade index. The performance is measured by the cumulative squared prediction errors of the benchmark ( $u_{B,t}^2$ ) minus those of the alternative ( $u_{A,t}^2$ ),  $\Delta SSE = \sum_t (u_{B,t}^2 - u_{A,t}^2)$ . The benchmark is the historical mean forecast or forecasts based on the average forward discount. The alternative is the conditional forecast using adaptively selected factors constructed from the macro dataset. During each out-of-sample month, investors choose a predictor from the base set which generates the smallest cumulative prediction errors in previous 24 months. The prediction period for panel A is from 12/1995 to 3/2009 and for panel B from 11/1996 to 3/2009.

## 2.A Bootstrap Method

The bootstrap procedure is a model-based wild bootstrap imposing the null of no predictability by macro factors. It is a variant of the approach considered in Clark and West (2006). The wild bootstrap ensures accurate inference in the presence of conditional heteroskedasticity. In each bootstrap iteration the following steps are performed:

- (i) A series of i.i.d. standard normal innovations  $\eta_t$  is drawn.
- (ii) AR(1) models are fitted for both dependent variables, i.e. currency excess returns (DOL) or Carry Trade Index (CTI), as well as each of the macro factors (Fhat). We save the residuals ( $\epsilon_{\text{DOL},t}$ ,  $\epsilon_{\text{CTI},t}$ ,  $\epsilon_{\text{Fhat},t}$ ) from the AR(1) models.
- (iii) Artificial bootstrap series  $\text{DOL}_t^{\text{bs}}$ ,  $\text{CTI}_t^{\text{bs}}$  and  $\text{Fhat}_t^{\text{bs}}$  are constructed based on the estimated AR(1) parameters and the innovations  $\eta_t \in \text{DOL},t$ ,  $\eta_t \in \text{CTI},t$ ,  $\eta_t \in \text{Fhat},t$ . The starting observations of the bootstrap series  $\text{DOL}_t^{\text{bs}}$ ,  $\text{CTI}_t^{\text{bs}}$  and  $\text{Fhat}_t^{\text{bs}}$  are drawn randomly from the actual series.
- (iv) The artificial bootstrap data are then used in the adaptive forecast procedure to generate out-of-sample forecasts for  $\text{DOL}_t^{\text{bs}}$  and  $\text{CTI}_t^{\text{bs}}$  based on models relying on the bootstrapped explanatory macro factors as well as the benchmark models. The corresponding Theil's U statistics ( $TU^{\text{bs}}$ ) are computed.
- (v) We compute bootstrap p-values as the fraction of times that Theil's U in the bootstrap samples is below the one observed in-sample. Hence, these p-values are one-sided and test the null of equal predictive performance against the alternative of superior performance of the model including macro factors vis-a-vis the benchmark. The number of bootstrap iterations is set to 300.

## 2.B Data Description

Category: Real Activity

No	Mnemonic	Trans	Frequ	Description
1	USOPRI35G	annual $\Delta$ ln	M	PRODUCTION OF TOTAL INDUSTRY (EXCLUDING CONSTRUCTION)
2	USOPRI38G	annual $\Delta$ ln	M	PRODUCTION IN TOTAL MANUFACTURING
3	USOPRI49G	annual $\Delta$ ln	M	PRODUCTION OF TOTAL MANUFACTURED CONSUMER GOODS
4	USOPRI53G	annual $\Delta$ ln	M	PRODUCTION OF TOTAL MANUFACTURED DURABLE GOODS
5	USOPRI61G	annual $\Delta$ ln	M	PRODUCTION OF TOTAL MANUFACTURED INTERMEDIATE GOODS
6	USOPRI63G	annual $\Delta$ ln	M	PRODUCTION OF TOTAL MANUFACTURED NON-DURABLE GOODS
7	USHBRM.O	ln	M	HOUSING STARTED - MIDWEST
8	USHBRN.O	ln	M	HOUSING STARTED - NORTHEAST
9	USHBEGUNP	ln	M	HOUSING STARTED
10	USHBRS.O	ln	M	HOUSING STARTED - SOUTH
11	USHBRW.O	ln	M	HOUSING STARTED- WEST
12	USOPL032O	annual $\Delta$ ln	M	CIVILIAN LABOUR FORCE TOTAL
13	USOUN009G	annual $\Delta$ ln	M	UNEMPLOYMENT - SHORT-TERM
14	USOUN015Q	annual $\Delta$ ln	M	UNEMPLOYMENT RATE (% OF CIVILIAN LABOUR FORCE)
15	USOOL012G	annual $\Delta$ ln	M	HELP WANTED ADVERTISING
16	USOOL024Q	annual $\Delta$ ln	M	OVERTIME HOURS - MANUFACTURING, WEEKLY
17	USPERCONB	annual $\Delta$ ln	M	PERSONAL CONSUMPTION EXPENDITURES
18	USCONDURB	annual $\Delta$ ln	M	PERSONAL CONSUMPTION EXPENDITURES - DURABLES
19	USCNXFE.B	annual $\Delta$ ln	M	PERSONAL CONSUMPTION EXPENDITURES - LESS FOOD & ENERGY
20	USCONSRVB	annual $\Delta$ ln	M	PERSONAL CONSUMPTION EXPENDITURES - SERVICES
21	USCONNDRB	annual $\Delta$ ln	M	PERSONAL CONSUMPTION EXPENDITURES - NONDURABLES
22	USCNORCGD	annual $\Delta$ ln	M	NEW ORDERS OF CONSUMER GOODS & MATERIALS
23	USOBS014Q	lv	M	BUSINESS TENDENCY SURVEY: MFG. - CONFIDENCE INDICATOR
24	USNAPMNO	lv	M	ISM MANUFACTURERS SURVEY: NEW ORDERS INDEX
25		lv	M	PURCHASING MANAGER INDEX <sup>i</sup>
26		lv	M	CONSUMER SENTIMENT: PERSONAL FINANCE EXPECTED <sup>ii</sup>
27		lv	M	CONSUMER SENTIMENT: PERSONAL FINANCE CURRENT <sup>ii</sup>
28		lv	M	CONSUMER SENTIMENT: BUSINESS CONDITION 12 MONTHS <sup>ii</sup>
29		lv	M	CONSUMER SENTIMENT: BUSINESS CONDITION 5 YEARS <sup>ii</sup>
30		lv	M	CONSUMER SENTIMENT: BUYING CONDITIONS <sup>ii</sup>
31		lv	M	CONSUMER SENTIMENT: CURRENT INDEX <sup>ii</sup>
32		lv	M	CONSUMER SENTIMENT: EXPECTED INDEX <sup>ii</sup>

Category: Stock Market Valuation

No	Mnemonic	Trans	Frequ	Description
33	S&PCOMP	monthly $\Delta$ ln	M	S&P 500 COMPOSITE - PRICE INDEX
34	S&PINDS	monthly $\Delta$ ln	M	S&P INDUSTRIAL - PRICE INDEX
35	DJINDUS	monthly $\Delta$ ln	M	DOW JONES INDUSTRIALS - PRICE INDEX
36	TOTMKUS(DY)	lv	M	DS MARKET - DIVIDEND YIELD
37	TOTMKUS(PE)	lv	M	DS MARKET - PRICE EARNINGS RATIO
38		lv	M	CYCLICALLY ADJUSTED PRICE EARNINGS RATIO <sup>iii</sup>
39		lv	M	DIVIDEND YIELD <sup>iii</sup>
40		$\Delta$ lv	Q	CAY <sup>iv</sup>

Category: Volatility and Aggregate Uncertainty

No	Mnemonic	Trans	Frequ	Description
41		abs $\Delta$ ln	M	VOLATILITY: S&P 500 COMPOSITE: PRICE INDEX
42		abs $\Delta$ ln	M	VOLATILITY: S&P INDUSTRIAL: PRICE INDEX
43		abs $\Delta$ ln	M	VOLATILITY: DOW JONES INDUSTRIALS: PRICE INDEX
44		lv	M	REALIZED VOLATILITY S&P 500 COMPOSITE (ac)
45		lv	M	REALIZED VOLATILITY S&P INDUSTRIAL (ac)

46		lv	M	REALIZED VOLATILITY DOW JONES INDUSTRIALS (ac)
47		lv	M	FAMA-FRENCH MARKET RISK FACTOR (MKT-RF) (ac)
48		lv	M	FAMA-FRENCH RISK FACTOR (SMB)
49		lv	M	FAMA-FRENCH RISK FACTOR (HML)
50		lv	M	FAMA-FRENCH MOMENTUM FACTOR
51	USEBQDGD%	$\Delta$ 2 lv	Q	GROSS PUBLIC DEBT AS % OF GDP
52	US61PCDLA	$\Delta$ ln	Q	FINANCE COMPANIES - DIRECT COMMERCIAL PAPER
53	US73PCDLA	$\Delta$ ln	Q	DIRECT COMMERCIAL PAPER: BANK HOLDING COS
54	US63CM1AA	$\Delta$ ln	Q	CREDIT MARKET DEBT-MONEY MARKET MUTUAL FUNDS
55		lv	M	TED SPREAD
56		lv	M	REALIZED EXCHANGE RATE VOLATILITY: DEM/USD (ac)
57		lv	M	REALIZED EXCHANGE RATE VOLATILITY: GBP/USD (ac)
58		lv	M	REALIZED EXCHANGE RATE VOLATILITY: JPY/USD (ac)
59		lv	M	REALIZED EXCHANGE RATE VOLATILITY: CAD/USD (ac)
60	USL..NEUE	abs $\Delta$ ln	M	NOMINAL EFFECTIVE TRADE-WEIGHTED EXCHANGE RATE INDEX
61		$\Delta$ 2 lv	Q	UNCERTAINTY - SPF: CPI (CURRENT QUARTER)
62		$\Delta$ 2 lv	Q	UNCERTAINTY - SPF: CPI (4 QUARTERS AHEAD)
63		$\Delta$ 2 lv	Q	UNCERTAINTY - SPF: REAL GDP (CURRENT QUARTER)
64		$\Delta$ 2 lv	Q	UNCERTAINTY - SPF: REAL GDP (4 QUARTERS AHEAD)
65		$\Delta$ 2 lv	Q	UNCERTAINTY - SPF: REAL EXPORTS (CURRENT QUARTER)
66		$\Delta$ 2 lv	Q	UNCERTAINTY - SPF: REAL EXPORTS (4 QUARTERS AHEAD)

## Category: Interest Rates and Interest Rate Spreads

No	Mnemonic	Trans	Frequ	Description
67	FRFEDFD	monthly $\Delta$ lv	M	FEDERAL FUNDS (EFFECTIVE) - MIDDLE RATE
68	FRTBS3M	monthly $\Delta$ lv	M	TREASURY BILL 2ND MARKET 3 MONTH - MIDDLE RATE
69	FRTBS6M	monthly $\Delta$ lv	M	TREASURY BILL 2ND MARKET 6 MONTH - MIDDLE RATE
70	FRTCM1Y	monthly $\Delta$ lv	M	TREASURY CONSTANT MATURITIES 1 YR - MIDDLE RATE
71	FRTCM2Y	monthly $\Delta$ lv	M	TREASURY CONSTANT MATURITIES 2 YR - MIDDLE RATE
72	FRTCM3Y	monthly $\Delta$ lv	M	TREASURY CONSTANT MATURITIES 3 YR - MIDDLE RATE
73	FRTCM5Y	monthly $\Delta$ lv	M	TREASURY CONSTANT MATURITIES 5 YR - MIDDLE RATE
74	FRTCM7Y	monthly $\Delta$ lv	M	TREASURY CONSTANT MATURITIES 7 YR - MIDDLE RATE
75	FRTCM10	monthly $\Delta$ lv	M	TREASURY CONSTANT MATURITIES 10 YR - MIDDLE RATE
76	FRCBAAA	monthly $\Delta$ lv	M	CORPORATE BOND MOODY'S S'ND AAA - MIDDLE RATE
77	FRCBBAA	monthly $\Delta$ lv	M	CORPORATE BOND MOODY'S S'ND BAA - MIDDLE RATE
78		lv	M	SPREAD: 10 YEAR TREASURY - 7 YEAR TREASURY (ac)
79		lv	M	SPREAD: 10 YEAR TREASURY - 5 YEAR TREASURY (ac)
80		lv	M	SPREAD: 10 YEAR TREASURY - 3 YEAR TREASURY (ac)
81		lv	M	SPREAD: 10 YEAR TREASURY - 2 YEAR TREASURY (ac)
82		lv	M	SPREAD: 10 YEAR TREASURY - 1 YEAR TREASURY (ac)
83		lv	M	SPREAD: 10 YEAR TREASURY - 6 MONTH T-BILL RATE (ac)
84		lv	M	SPREAD: 10 YEAR TREASURY - 3 MONTH T-BILL RATE (ac)
85		lv	M	SPREAD: 10 YEAR TREASURY - FEDERAL FUNDS RATE (ac)
86		lv	M	SPREAD: BAA CORPORATE BOND YIELD - AAA CORPORATE BOND YIELD (ac)

## Category: Price and Wage Variables

No	Mnemonic	Trans	Frequ	Description
87	CRBSPOT	annual $\Delta$ ln	M	CRB Spot Index (1967=100) - PRICE INDEX
88	CRBSPFD	annual $\Delta$ ln	M	CRB Spot Index Foodstuffs - PRICE INDEX
89	CRBSPFO	annual $\Delta$ ln	M	CRB Spot Index Fats & Oils - PRICE INDEX
90	CRBSPLV	annual $\Delta$ ln	M	CRB Spot Index Livestock - PRICE INDEX
91	CRBSPMT	annual $\Delta$ ln	M	CRB Spot Index Metals - PRICE INDEX
92	CRBSPRI	annual $\Delta$ ln	M	CRB Spot Index Raw Industrials - PRICE INDEX
93	CRBSPTX	annual $\Delta$ ln	M	CRB Spot Index Textiles - PRICE INDEX
94	USI63...F	annual $\Delta$ ln	M	PRODUVER PRICE INDEX

95	USBCIPPEE	annual $\Delta$ ln	M	PRODUCER PRICE INDEX - PETROLEUM PRODUCTS
96	USOCP009E	annual $\Delta$ ln	M	CONSUMER PRICE INDEX
97	USOCP019F	annual $\Delta$ ln	M	CONSUMER PRICE INDEX FOOD EXCL. RESTAURANTS
98	USOCP041F	annual $\Delta$ ln	M	CONSUMER PRICE INDEX ENERGY
99	USWKNCONB	annual $\Delta$ ln	M	AVG WKLY EARN - NONFARM PAYROLL, CONSTRUCTION
100	USWKNNDURB	annual $\Delta$ ln	M	AVG WKLY EARN - NONFARM PAYROLL, DURABLE GOODS
101	USWKNMANB	annual $\Delta$ ln	M	AVG WKLY EARN - NONFARM PAYROLL, MANUFACTURING

## Category: Open Economy

No	Mnemonic	Trans	Frequ	Description
102	USOXT\$09B	annual $\Delta$ ln	M	IMPORTS
103	USOXT\$03B	annual $\Delta$ ln	M	EXPORTS
104	USOXT\$14B	annual $\Delta$ ln	M	NET TRADE BALANCE
105	USCURBALB	annual $\Delta$ lv	Q	CURRENT ACCOUNT BALANCE
106	USOBP015Q	lv	Q	CURRENT ACCOUNT AS A % OF GDP

## Category: Monetary Variables

No	Mnemonic	Trans	Frequ	Description
107	USOMA027B	annual $\Delta$ ln	M	MONEY SUPPLY - M1
108	USOMA002B	annual $\Delta$ ln	M	MONEY SUPPLY - BROAD MONEY (M2)
109	USI.1..SA	annual $\Delta$ ln	M	INTERNATIONAL RESERVES
110	USI.1B.DA	annual $\Delta$ ln	M	FUND POSITION: SDR'S

## Data Sources:

- i Institute of Supply Management
- ii University of Michigan: Consumer Sentiment Index
- iii Kenneth R. French' Homepage:  
*[http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)*
- iv Sydney C. Ludvigson's Homepage:  
*<http://www.econ.nyu.edu/user/ludvigsons/>*



## Chapter 3

# A Comprehensive Evaluation of Affine Term Structure Models with Regime Shifts\*

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## **Abstract**

We develop and estimate a no-arbitrage multi-factor regime-switching affine term structure model. As a novelty, we assume the vector of latent state variables to follow a mixture of correlated square root diffusion processes instead of Gaussian processes. We assess the models ability to match cross-sectional properties of yields as well as evaluate their ability to capture stylized facts of the U.S yield curve. We find evidence that regime-switching models with state-dependent volatility improve the ability to describe historical yields compared to their Gaussian counterparts as well as single-regime models. Additionally, affine term structure models with multiple regime successfully replicate features of the historical behavior of the U.S. term structure such as yield predictability and time-varying conditional volatility.

### 3.1 Introduction

Monetary policy affects not only the short end but the entire yield curve, since movements in the short rate affect longer maturity yields by altering investor expectations of future bond prices. From an economic perspective, it is hence intuitively appealing to allow the yield curve to depend on different macroeconomic regimes. It is well documented in the literature that modelling the dynamics of the short rate as a regime-switching process is more appropriate in describing historical short rates (see, for example, Hamilton (1988), Gray (1996), Garcia and Perron (1996), Ang and Bekaert (2002a) and Ang and Bekaert (2002b)). In view of these findings, a number of papers followed by developing and analyzing interest rate models with regime switches, most notably Naik and Lee (1997), Evans (1998), Landén (2000) and Bansal and Zhou (2002), which confirmed that these models are better able in capturing the features of yield curve dynamics compared to their single-regime counterparts. In the recent years the literature has further moved on by analyzing regime-switching models in an affine term structure framework, becoming ever more sophisticated (we refer to e.g., Ang, Bekaert, and Wei (2007) and Dai, Singleton, and Yang (2007)). However, the increased complexity of introducing regime switches in terms of bond pricing and most importantly in terms of estimation has driven most of the literature to focus on Gaussian specifications of the state variable dynamics.

With this paper we contribute to the existing literature by analyzing the whole class of maximally-affine regime-switching term structure models, that is three-factor models with zero, one, two and three factors entering the volatility matrix. In line with the general definition of the single-regime class in Dai and Singleton (2000) the models are referred as  $A_0^{(RS)}(3)$ ,  $A_1^{(RS)}(3)$ ,  $A_2^{(RS)}(3)$ ,  $A_3^{(RS)}(3)$  where the subscript denotes the number of factors entering the volatility matrix and the superscript (RS) indicates regime-switching. We analyze the models performance in terms of overall goodness of fit as well as the ability to match some of the most important stylized facts of observed U.S. yield data. We examine the relative performance of the models along these lines and assess whether there is a benefit in moving firstly from a single-regime Gaussian model to a regime-switching Gaussian model, and secondly within the regime-switching class, moving from a Gaussian specification to stochastic-volatility specifications.

Our specification of the RS-ATSM's allows the intercept of the short rate and the market price of factor risk to be regime-dependent, enabling both the long run mean and the speed of mean reversion of the state variables to be regime-dependent under the physical measure. As indicated by Bansal and Zhou (2002) having a richer and regime-dependent specification of the market prices of factor risks is key for capturing the observed yield curve dynamics. With this specification of the RS-ATSM we are still able to obtain analytical solutions for bond prices whilst allowing for considerable regime-dependence under the physical measure.

We generally would expect the models accounting for shifts in the economic regime to outperform their single-regime counterparts in terms of fitting historical yields. This effect is presumed to be larger for longer maturities, since during the life-span of longer maturity bonds the economy is more likely to be subject to changes in regimes. Our results provide some evidence that regime-switching stochastic volatility models are better equipped for fitting historical yield dynamics, compared to the regime-switching Gaussian model as well as to single-regime models. They display smaller variances of the measurement errors and generally smaller absolute average pricing errors, indicating that the yields implied by the RS-ATSM with stochastic volatility approximate the observed yields more closely. A model selection analysis using the Bayes factors confirms the above, indicating that the evidence provided by the data is in favor of RS-ATSM with stochastic volatility, the data-generating process of which seems more likely to give rise to the observed yields. Summarizing, we show evidence that affine term structure models with stochastic volatility (with one and two factors affecting volatility) display an improved ability to fit historical yields relative to both single-regime models and the regime-switching Gaussian model.

On a second step, we evaluate whether our preferred RS-ATSM models  $A_1^{(RS)}(3)$  and  $A_2^{(RS)}(3)$  are able to successfully match some of the most important stylized facts of U.S. yields. The main features of historical yields that we want our models to replicate are the predictability of bond returns (linear projections of changes in yields on the slope of the yield curve give negative fitted coefficients), the persistence and time-variability in conditional yield volatilities, as well as the term structure of the unconditional means.

The expectations hypothesis implies that excess returns are unpredictable. Conditional on

current information, longer maturity yields are given as expected future short-rates plus a constant risk premium. Several empirical studies have shown that a significant portion of the variability in excess returns is forecastable and that the expectations hypothesis is violated. Fama and Bliss (1987) and Campbell and Shiller (1991) find that the slope of the yield curve has significant predictive power for excess returns, while Cochrane and Piazzesi (2005) find that a single factor, computed as a linear combination of forward rates, predicts an important part of the variation in excess returns, beyond the standard level, slope and curvature factors. In terms of matching these stylized facts of historical yield data, our results show an improvement of our preferred regime-switching stochastic volatility models over single-regime models. More precisely, within the single-regime class of models we find that the ability to capture the Campbell-Shiller regression coefficients decreases with the number of factors that enter the volatility matrix of the latent factors, as documented in the previous literature (see, e.g., Feldhütter (2008)). In particular, within this class of models only the Gaussian model is able to replicate the sign and sizes of the coefficients. For the regime-switching models we find that now the  $A_1^{(RS)}(3)$  and  $A_2^{(RS)}(3)$  models, capture both the negative sign and the decreasing size with maturity of the Campbell-Shiller regression coefficients. Since sufficient variability and persistence in the market prices of risk is key in matching this feature, we conclude that the improvement of these models ability to replicate the failure of the expectations hypothesis is due to our specification of the market price of factor risk. In particular the variability in our extended-affine market price of risk comes both from its dependence in the risk factors (and their conditional volatility) and from the fact that its parameters ( $\lambda_0$  and  $\lambda_x$ ) are regime-dependent.

Another feature of the U.S. bond data is that the conditional volatility of yields displays significant time-variation and persistence (see, e.g., Aït-Sahalia (1996) and Gallant and Tauchen (1997)). Additionally, yield volatility is positively related to interest rates. A regression of squared yield changes on the level, slope and curvature of the U.S yield curve results in a positive coefficients associated with the level factor (see, e.g., Brandt and Chapman (2002) and Piazzesi (2010)). Within the class of RS-ATSM, we expect square root diffusion models to capture the higher moments of historical yield dynamics more closely than the single-regime counterparts. As for the Gaussian models, they preclude

by definition time-varying conditional volatility. We find that RS-ATSM with stochastic volatility successfully capture the  $\beta$ -coefficient of a GARCH(1,1) model. The  $\beta$ -coefficient is around 0.8 and thus implying a rather strong persistence in the volatility of the yields. Furthermore, all specifications of the RS-ATSM with stochastic volatility are able to capture the level effect which showing positive regression coefficients when regressing model implied yield volatilities on the level factor.

Overall, this article shows that introducing regime-shifts in state-dependent volatility models narrows the gap between matching the cross-sectional and time-series properties of bond yields. We find evidence that RS-ATSM with stochastic volatility successfully describe historical yields while still being able to replicate important features of the U.S. yield curve.

The remainder of the paper is organized as follows. In Section 3.2 we present the framework for our regime-switching affine term structure model. Section 3.3 discusses the estimation methodology. Section 3.4 presents the results and Section 3.5 contains concluding remarks. An exposition of the technical details is supplied in the Appendix.

## 3.2 Model Specification

In this section we present the formal set up of the regime-switching affine term structure model. We describe the model in its most general form, however, when estimating the model, we need to impose some restrictions which we explain in greater detail in Section 3.3. We begin by introducing the regime variable, proceed with a parameterization of the state variable dynamics under the risk neutral measure that allows analytical solutions for bond prices and terminate with the specification of the market prices of factor risk.

### 3.2.1 The Regime Variable

We assume a regime variable with discrete support  $k \in 1, 2, \dots, S^1$  and dynamics following a continuous-time Markov chain with infinitesimal matrix under the risk-neutral measure given by  $Q = \{q_{ij}\}_{i,j=1,\dots,S}$ . The intensity matrix is characterized by  $q_{ij} > 0, \forall i \neq$

---

<sup>1</sup>Theoretically there are no restrictions on how many regimes should be included in the analysis, however, for interpretational reasons we restrict our analysis to two regimes, as explained in Section 3.3.

$j$  and  $q_{ii} < 0$ , such that  $q_{ii} = -\sum_{j=1}^S q_{ij}$ ,  $\forall i$ . Hence the full transition rate matrix will be:

$$Q = \begin{bmatrix} -q_{11} & q_{12} & \cdots & q_{1S} \\ q_{21} & -q_{22} & \cdots & q_{2S} \\ \vdots & \vdots & \ddots & \vdots \\ q_{S1} & q_{S2} & \cdots & -q_{SS} \end{bmatrix}$$

Over a small time interval  $\Delta t$  the probability of staying in the same regime will be given by  $1 - q_{ii} \Delta t$ . Thus, letting  $\Delta t$  approach 0, we have that  $\lim_{\Delta t \rightarrow 0^+} 1 - q_{ii} \Delta t = 1$  implying that the probability of staying in the same regime approaches one over an infinite small time period. In a similar vein, the probability that the economy switches from regime  $i$  to regime  $j$  over a small time interval  $\Delta t$  is given by  $q_{ij} \Delta t$ . Thus, if  $\Delta t$  approaches 0, we obtain that  $\lim_{\Delta t \rightarrow 0^+} q_{ij} \Delta t = 0$ , suggesting that the probability of a regime switch approaches zero over an infinite small time period.<sup>2</sup> Due to the Markov property the probability that the economy will be in a given regime in time  $t + 1$  depends only on the current regime and not on the entire history of the regime variable.

### 3.2.2 The Short Rate, the State Variables and Zero-Coupon Bond Pricing

In the absence of arbitrage opportunities the price of a zero-coupon bond at time  $t$  maturing at time  $T$  is given by:

$$P(t, T) = E_t^{\mathbb{Q}} \left[ e^{-\int_t^T r_s ds} \right]$$

where the expectation is taken under the risk-neutral measure.

<sup>2</sup>Over a time interval  $t$  the transition probability matrix is given by the exponential matrix  $\bar{Q} = e^{Q \cdot t}$ , which can be defined by means of a power series  $e^{Q \cdot t} = I + Q t + \frac{(Q t)^2}{2!} + \frac{(Q t)^3}{3!} + \dots$ , where  $I$  is the identity matrix. Over a small time interval we can ignore the quadratic and higher order terms and use the approximation  $\bar{Q} = I + Q \Delta t$ . For an introduction in continuous-time Markov Chains we refer to Karlin and Taylor (1975) and Lando (2004).

We specify the instantaneous short rate  $r_t$  to be an affine function of a vector of unobserved state variables  $X_t = (X_t^1, X_t^2, \dots, X_t^N)$

$$r_t = \delta_0^{(k)} + \sum_{i=1}^N \delta_i X_t = \delta_0^{(k)} + \delta'_X X_t$$

where  $k$  is an indicator for the regime. By allowing the constant term  $\delta_0^{(k)}$  to be regime-dependent we let the short rate's unconditional mean to vary across regimes. We restrict  $\delta_X$  to be regime-independent for analytical tractability.

The dynamics of the latent state variables is given by a mean-reversion square root diffusion process under  $\mathbb{Q}$ :

$$\begin{aligned} dX_t &= \kappa^{\mathbb{Q}} \left( \theta^{\mathbb{Q},(k)} - X_t \right) dt + \Sigma \sqrt{\sigma(X_t)} dW_t^{\mathbb{Q}} \\ &= \left( \kappa_0^{\mathbb{Q},(k)} - \kappa_1^{\mathbb{Q}} X_t \right) dt + \Sigma \sqrt{\sigma(X_t)} dW_t^{\mathbb{Q}} \end{aligned}$$

where  $dW_t^{\mathbb{Q}}$  is an  $N$ -dimensional vector of independent standard Brownian motions under the risk-neutral measure.  $\theta^{\mathbb{Q},(k)}$  is a regime-dependent vector representing the long-run mean of the state variables, while  $\kappa^{\mathbb{Q}}$  is the speed of mean reversion matrix. We keep  $\kappa_1^{\mathbb{Q}}$  constant across regimes in order to obtain closed-form solutions for bond prices.  $\kappa_1^{\mathbb{Q}}$  is a  $(N \times 1)$  vector for each regime while  $\kappa_0^{\mathbb{Q}}$  and  $\Sigma$  are  $(N \times N)$  matrices.

As a novelty for RS-ATSM, we allow the volatility of the latent state variables to be state dependent which introduces conditional heteroskedasticity. In particular, the volatility matrix  $\sigma(X_t)$  is a diagonal matrix, with the  $i^{\text{th}}$  diagonal element given by  $[\sigma(X_t)]_{ii} = \alpha_i + \beta_i X_t$ , where  $\alpha_i \in \{0, 1\}$  and  $\beta_i$  is a  $N \times 1$  vector. Dai and Singleton (2000) classify models according to the number of state variables entering the volatility matrix  $\sqrt{\sigma(X_t)}$ . In their notation, an  $A_m(N)$  denotes a model with a total of  $N$  state variables, of which  $m$  enter the volatility matrix  $\sqrt{\sigma(X_t)}$ . In order for affine specifications to be admissible, restrictions must be imposed on the parameters to ensure positivity of the volatility matrix  $\sqrt{\sigma(X_t)}$ . Dai and Singleton (2000) provide the set of sufficient restrictions on the parameters of an  $A_m(N)$  model to assure admissibility.



The price of a zero-coupon bond,  $P(t, \tau, X, k) = P(t, \tau, k)$ , where  $\tau = T - t$  denotes the time to maturity, satisfies the following partial differential-difference equation (PDDE):

$$\begin{aligned} \frac{1}{2}Tr \left( \frac{\partial^2 P}{\partial X \partial X'} \Sigma \sigma(x_t) \Sigma' \right) + \frac{\partial P}{\partial X'} \left( \kappa \left( \theta^{(k)} - X_t \right) \right) - \frac{\partial P}{\partial \tau} - \\ \left( \delta_0^{(k)} + \delta_{X'} X_t \right) P(\tau, X_t, k) + \sum_{j=1, j \neq k}^K Q_{k,j} \left( P(\tau, X_t, j) - P(\tau, X_t, k) \right) = 0 \end{aligned}$$

subject to the boundary condition  $P(t, 0, k) = 1$ .

Following Duffie and Kan (1996) we conjecture that the solution to the above PDDE is exponentially affine:

$$P(t, \tau, k) = e^{A(\tau, k) + B(\tau)' X_t}.$$

To verify our conjecture we substitute  $\frac{\partial P}{\partial \tau}$ ,  $\frac{\partial P}{\partial X'}$  and  $\frac{\partial^2 P}{\partial X \partial X'}$  in the PDDE and rearrange terms in order to get a system of ordinary differential equations (ODE's). The solution of the ODE's results in a vector  $B(\tau)$  and S scalars  $A(\tau, k)$ . In particular, the set of ODE's that define A and B is given as:<sup>3</sup>

$$\begin{aligned} \frac{dB(\tau)}{d\tau} &= \frac{1}{2} \sum_{i=1}^m [\Sigma' B(\tau)]_i^2 \beta_i - \kappa_1' B(\tau) - \delta_X \\ \frac{dA(\tau, k)}{d\tau} &= \frac{1}{2} \sum_{i=1}^m [\Sigma' B(\tau)]_i^2 \alpha_i + \kappa_0^{(k)'} B(\tau) - \delta_0^{(k)} + \sum_{j=1, j \neq k}^K q_{k,j} \left( e^{A(\tau, j) - A(\tau, k)} - 1 \right). \end{aligned}$$

The above set of ODE's is completely determined by the specification of the short rate and state variable dynamics under the risk neutral measure. We solve these ODE's numerically using the Runge-Kutta method, with initial conditions  $A(0) = 0$  and  $B_{N \times 1}(0) = 0$ .

The continuously compounded yields will then be given by:

$$Y(t, \tau, k) = A^*(\tau, k) + B^*(\tau) X_t$$

where  $A^*(\tau, k) = -\frac{A(\tau, k)}{\tau}$  and  $B^*(\tau) = -\frac{B(\tau)}{\tau}$

<sup>3</sup>For a detailed derivation of the ODE's defining  $A(\tau, k)$  and  $B(\tau)$  we refer to Appendix 3.A.

In order to use the closed-form solution for  $P(t, \tau, k) = \exp(A^*(\tau, k) + B^*(\tau)' X_t)$  in the empirical analysis, we need to know the distribution of  $X_t$  and  $P(t, \tau, k)$  under the historical probability measure  $\mathbb{P}$ . The most general specification of the market price of factor risk that preserves the affine structure of  $X_t$  under  $\mathbb{P}$  is the “extended” specification of Cheridito, Filipovic, and Kimmel (2007). In particular,

$$\Lambda_t^{(k)} = \left( \lambda_0^{(k)} + \lambda_1^{(k)} X_t \right) \left( \Sigma \sqrt{\sigma(X_t)} \right)^{-1}$$

where  $\lambda_0^{(k)}$  is a  $N \times 1$  vector and  $\lambda_1^{(k)}$  is a  $N \times N$  matrix which are both regime-dependent. Using the above market price of factor risk specification, we discretize the process for the latent factors applying the Euler method. For the change of measure we have:

$$dW_t^{\mathbb{Q}} = dW_t^{\mathbb{P}} + \Lambda_t^{(k)} dt$$

Thus, under the historical measure  $\mathbb{P}$  the latent factor process is given as:

$$\begin{aligned} dX_t &= \left( \kappa_0^{\mathbb{Q},(k)} - \kappa^{\mathbb{Q}} X_t \right) dt + \Lambda_t^{(k)} dt + \Sigma \sqrt{\sigma(X_t)} dW_t^{\mathbb{P}} \\ &= \left( \kappa_0^{\mathbb{P},(k)} - \kappa_1^{\mathbb{P},(k)} X_t \right) dt + \Sigma \sqrt{\sigma(X_t)} dW_t^{\mathbb{P}} \end{aligned}$$

where  $\kappa_0^{\mathbb{P},(k)} = \kappa_0^{\mathbb{Q},(k)} + \lambda_0^{(k)}$  and  $\kappa_1^{\mathbb{P}} = \kappa_1^{\mathbb{Q}} - \lambda_1^{(k)}$ . In order to obtain admissibility (in the sense of Dai and Singleton (2000)) we have restricted  $\Sigma$  to be an identity matrix.

### 3.3 Estimation Methodology

In this section, we discuss the MCMC algorithm for estimating the RS-ATSM. MCMC methods have been used in the term structure literature by Eraker (2001), Scott (2002), Sanford and Martin (2005), Ang, Dong, and Piazzesi (2007), Feldhütter (2008), Li, Li, and Yu (2011) among others.<sup>4</sup> MCMC methods are computationally more complex than Maximum Likelihood methods, however, they offer some advantages which we outlay below.

<sup>4</sup>Casella and Robert (2004) provide a thorough introduction in general Monte Carlo Methods while Johannes and Polson (2010) provide a survey of MCMC applications within financial econometrics.

### 3.3.1 Setting up the MCMC Algorithm

An empirical analysis of a regime-switching affine term structure model entails extracting information regarding model parameters, state variables and regimes conditional on observed yields (obtained from zero-coupon bond prices). To do so, we observe  $M$  yields ( $\tau \in 1, \dots, M$ , where  $\tau$  denotes the time to maturity) at time  $t = 1, \dots, T$ , which are stacked in the vector  $Y(t, \tau, k) = Y(t, 1, k, \dots, Y(t, M, k)$ . We assume that all actual yields are observed with an *i.i.d.* measurement error, i.e.

$$Y(t, \tau, k) = A^*(\tau, k) + B^*(\tau)' X_t + \epsilon_t. \quad (3.1)$$

The measurement errors are normally distributed such that  $\epsilon \sim N(0, H)$  where  $H = \sigma^2 I_M$ . Most of the literature in term structure modelling relies on the assumption that at any point in time at least three yields (with three different maturities) are precisely observed. With the  $B^*(\tau)$  matrix being invertible, this allows for a one-to-one mapping from the observed yields to the state variables, which can hence be pinned down exactly. The obtained state variables can then be used to estimate the remaining yields, i.e., those observed with an error, and the dynamics of all yields over time. This assumption leads to tractable estimation of the model, such as with Maximum Likelihood. However, Cochrane and Piazzesi (2005) observe that the fact that we are only able to observe yields imprecisely might hinge on the Markov structure of the term structure and hence partially explain the inability of term-structure models to forecast future excess bond returns. Duffee (2011) notes that the existence of an observation error can potentially create partially hidden factors, where only part of the information regarding the factor can be found in the cross-section, so that models relying strictly on yield data will have difficulties in reliably fitting yield dynamics.

These facts motivated us to use a Bayesian approach which is less vulnerable to these issues than traditional maximum likelihood techniques. More precisely, MCMC methods enable us to relax the restrictive (and unrealistic) assumption of perfectly observed yields, so that we can allow all yields to be observed with an error. We assume that the observation error of the yields for any maturity has the same variance. The intuition behind this choice lies in the fact that the main sources of observation error are market imperfections which

affect bond prices and risk premia and plain measurement error, all of which potentially affect bonds with different maturities in the same way.

The main objective of the estimation analysis is to make inference about the model parameters  $\Theta$ , the latent variables  $\mathbf{X} = \{X_t\}_{t=1}^T$  and the regime variables  $\mathbf{K} = \{k_t\}_{t=1}^T$  based on the observed yields  $\mathbf{Y} = \{Y_t^\tau\}_{t \in 1, \dots, T}^{\tau \in 1, \dots, M}$ .

Characterizing the joint posterior distribution,  $p(\Theta, \mathbf{K}, \mathbf{X} | \mathbf{Y})$ , is difficult due to its high dimension, the fact that the model is specified in continuous time while the yield data is observed discretely and since the state variables transition distributions are non-normal. Furthermore parameters enter the model as solutions to a system of ODE's (the A and B functions derived in the previous section). MCMC allows us to simultaneously estimate parameters, state variables and regimes for non-linear, non-Gaussian state space models as is our RS-ATSM and at the same time accounts for estimation risk and model specification uncertainty.

For interpretational reasons we restrict our analysis to two regimes, thus,  $k = 1, 2$ . Each of the regimes  $k$  is characterized by the following set of parameters:

$$\Theta = \left( \kappa_0^{\mathbb{Q},(k)}, \kappa_1^{\mathbb{Q}}, \delta_0^{(k)}, \delta_X, \lambda_0^{(k)}, \lambda_1^{(k)}, H, \text{ and } Q^{kj} \text{ for } k, j = 1, 2 \right).$$

In addition we also need to filter the regime of the underlying regime process  $\mathbf{K}$ , as well as the latent state variables  $\mathbf{X}$ . The numerical identification of this highly dimensional parameter space proves to be challenging. However, due to the flexibility of the Bayesian techniques we avoid imposing several parameter restrictions as e.g. in Dai, Singleton, and Yang (2007). The only restriction we impose in order to facilitate the estimation is that  $\kappa_0^{\mathbb{Q},(k)}$  is regime-independent, that is  $\kappa_0^{\mathbb{Q},(k)} = \kappa_0^{\mathbb{Q}}$ .

In order to be able to sample from the target distribution  $p(\Theta, \mathbf{K}, \mathbf{X} | \mathbf{Y})$ , we make use of two important results, the Bayes rule and the Hammersley-Clifford theorem.

By Bayes Rule we have:

$$\begin{aligned} p(\Theta, \mathbf{K}, \mathbf{X}|\mathbf{Y}) &\propto p(\mathbf{Y}, \mathbf{X}, \mathbf{K}, \Theta) \\ &= p(\mathbf{Y}|\mathbf{X}, \mathbf{K}, \Theta) p(\mathbf{X}, \mathbf{K}|\Theta) p(\Theta) \end{aligned}$$

where the conditional likelihood function of the yields is given by

$$\begin{aligned} p(\mathbf{Y}|\mathbf{X}, \mathbf{K}, \Theta) &= \prod_{\tau=1}^M \prod_{t=1}^T H_{\tau\tau}^{-\frac{1}{2}} \exp\left(-\frac{\left(Y(t, \tau) - \hat{Y}(t, \tau, k)\right)^2}{2H_{\tau\tau}}\right) \\ &= \frac{1}{\sigma^{MT}} \exp\left(-\frac{1}{2\sigma^2} \sum_{t=1}^T \left(\epsilon_t^{k'} \epsilon_t k\right)\right) \end{aligned}$$

where  $\epsilon_t^k = Y(t, \tau) - \hat{Y}(t, \tau, k)$ .

To derive the joint likelihood  $p(\mathbf{X}, \mathbf{K}|\Theta)$  we rely on a Euler discretization to approximate the continuous-time specification of the latent variable process resulting in the following discrete time process:

$$\Delta X_{t+1} = \mu_t^{\mathbb{P},(k)} \Delta_t + \sqrt{\Delta_t \sigma(X_t)} \varepsilon_{t+1}.$$

The drift under  $\mathbb{P}$  is given by  $\mu_t^{\mathbb{P},(k)} = \left(\kappa_0^{\mathbb{Q}} + \lambda_0^{(k)}\right) - \left(\kappa_1^{\mathbb{Q}} - \lambda_1^{(k)}\right) X_t$ , the measurement error is normally distributed  $\varepsilon_t \sim N(0, I_N)$  and  $\Delta_t$  denotes the discrete time interval between two subsequent observations. Thus, the joint density  $p(\mathbf{X}, \mathbf{K}|\Theta)$  is as

$$\begin{aligned} p(\mathbf{X}, \mathbf{K}|\Theta) &= \prod_{t=1}^{T-1} p(X_{t+1}|X_t, K_t) \exp(Q\Delta_t)_{k_t, k_{t+1}} \\ &= \prod_{n=1}^N \left( \left( \prod_{t=1}^{T-1} \frac{1}{\sqrt{[\sigma(X_t)]_{nn}}} \right) \exp\left(-\frac{1}{2\Delta_t} \sum_{t=1}^{T-1} \frac{[\Delta X_{t+1} - \mu_t^{\mathbb{P},(k)} \Delta_t]_n^2}{[\sigma(X_t)]_{nn}}\right) \right) \\ &\quad \prod_{t=1}^{T-1} \exp(Q\Delta_t)_{k_t, k_{t+1}}. \end{aligned}$$

MCMC is a method to obtain the joint distribution  $p(\Theta, \mathbf{K}, \mathbf{X}|\mathbf{Y})$  which is usually unknown and complex. The Hammersley-Clifford theorem (see Hammersley and Clifford (2012) and

Besag (1974)) states that the joint posterior distribution is characterized by its complete set of conditional distributions:

$$p(\Theta, \mathbf{K}, \mathbf{X} | \mathbf{Y}) \iff p(\Theta | \mathbf{K}, \mathbf{X}, \mathbf{Y}), p(\mathbf{K} | \Theta, \mathbf{X}, \mathbf{Y}), p(\mathbf{X} | \Theta, \mathbf{K}, \mathbf{Y})$$

Given initial draws  $k^{(0)}$ ,  $X^{(0)}$  and  $\Theta^{(0)}$ , we draw  $k^{(n)} \sim p(k | X^{(n-1)}, \Theta^{(n-1)}, Y)$ ,  $X^{(n)} \sim p(X | k^{(n)}, \Theta^{(n-1)}, Y)$  and  $\Theta^{(n)} \sim p(\Theta | k^{(n)}, X^{(n)}, Y)$  and so on until we reach convergence. The sequence  $\{k^{(n)}, X^{(n)}, \Theta^{(n)}\}_{n=1}^N$  is a Markov Chain with distribution converging to the equilibrium distribution  $p(\Theta, \mathbf{K}, \mathbf{X} | \mathbf{Y})$ .

More specifically, at each iteration, we sample from the conditionals:

$$\begin{aligned} & p\left(\kappa_0^{\mathbb{Q},(k)} \mid \kappa_1^{\mathbb{Q}}, \delta_0^{(k)}, \delta_X, \lambda_0^{(k)}, \lambda_1^{(k)}, k, H, Q, X, Y\right) \\ & p\left(\kappa_1^{\mathbb{Q}} \mid \kappa_0^{\mathbb{Q},(k)}, \delta_0^{(k)}, \delta_X, \lambda_0^{(k)}, \lambda_1^{(k)}, k, H, Q, X, Y\right) \\ & \quad \vdots \\ & p\left(k \mid \kappa_0^{\mathbb{Q},(k)}, \kappa_1^{\mathbb{Q}}, \delta_0^{(k)}, \delta_X, \lambda_0^{(k)}, \lambda_1^{(k)}, H, Q, X, Y\right) \\ & p\left(X \mid \kappa_0^{\mathbb{Q},(k)}, \kappa_1^{\mathbb{Q}}, \delta_0^{(k)}, \delta_X, \lambda_0^{(k)}, \lambda_1^{(k)}, k, H, Q, Y\right) \end{aligned}$$

To sample new parameters, we rely on the Random-Walk Metropolis-Hastings (RW-MH) algorithm which is a two-step procedure that first samples a candidate draw from a chosen proposal distribution and then accepts or rejects the draw based on an acceptance criterion specified a priori. For example, we sample a new  $\delta_X$  as  $[\delta_X]^{n+1} = [\delta_X]^n + \gamma N(0, 1)$  where  $\gamma$  is used to calibrate the variance of the proposal distribution. In a second step we calculate the acceptance probability as:

$$\alpha = \min\left(1, \frac{p([\delta_X]^{n+1} | \cdot)}{p([\delta_X]^n | \cdot)}\right).$$

In case that we are able to sample directly from the conditional distribution, we make use of the Gibbs Sampler (GS). The Gibbs Sampling is a special case of the Metropolis-Hastings algorithm in which the proposal distributions exactly match the posterior conditional

distributions and in which proposals are accepted with a probability of one.<sup>5</sup>

After having obtained  $\{K^{(n)}, X^{(n)}, \Theta^{(n)}\}_{n=1}^N$ , the point estimates of the parameters of interest will then be given as the marginal posterior means, that is

$$E(\Theta_i|Y) = \frac{1}{N} \sum_{n=1}^N \Theta_i^{(n)}.$$

Summing up, our hybrid MCMC algorithm looks as below:

$$\begin{aligned} p(k|X, Y, \Theta) &\sim \text{RW-MH} \\ p(X|k, Y, \Theta) &\sim \text{RW-MH} \\ p(\Theta^h|\Theta_{\setminus h}, X, k, Y) &\sim \text{RW-MH} \\ p(\sigma|Y) &\sim \text{GS}. \end{aligned}$$

Both the parameters and the latent factors are subject to constraints and if a draw violates a constraint it can be discarded (see Gelfand, Smith, and Lee (1992)). The efficiency of the RW-MH algorithm depends crucially on the variance of the proposal distribution. Roberts, Gelman, and Gilks (1997) and Roberts and Rosenthal (2001) show that for optimal convergence, we need to calibrate the variance such that roughly 25% of the newly sampled parameters are accepted. To calibrate these variances we run one million iterations where we evaluate the acceptance ratio after 100 iterations. The variance of the normal proposal are adjusted such that they yield acceptance ratios between 10% and 30%. This calibration sample is followed by burn-in period which consist of 700000 iterations. Finally, the estimation period consists of 300000 iterations where we keep every 100th iteration resulting in 3000 draws for inference.<sup>6</sup>

### 3.3.2 Yield Data

The empirical implementation of the MCMC algorithm relies on a set of monthly zero coupon Treasury yields obtained from the Gürkayanak, Sack, and Wright (2007) database,

<sup>5</sup>We refer to Chib and Greenberg (1995) for introductory exposition of the Metropolis-Hastings algorithm and Casella and George (1992) for a detailed explanation of the Gibbs Sampler.

<sup>6</sup>For a complete description of the MCMC algorithm we refer to Appendix 3.B.

with time series November 1971 to January 2011.<sup>7</sup> The maturities included in the estimation are one, three, five, seven, ten, twelve and fifteen years. Given the shorter available sample length for higher maturities, our choice in terms of the data used, is the result of an implicit trade-off between the length of the time series and the highest maturity included, both of relevance in a regime-switching set-up. We emphasize the importance of the sample period, which according to the National Bureau of Economic Research (NBER) is characterized by six recessions and includes the FED's monetary experiment in the 80's, providing a basis for different economic regimes to have potentially occurred. Secondly, relatively longer maturities allow for the possibility of regime changes to have occurred during their life-time, hence including them in the estimation might give rise to more robust results. In the next section we investigate how well regime-switching models fit historical yields and if they are able to match some of the features of observed U.S. yields.

## 3.4 Results

### 3.4.1 MCMC estimates

Table 3.1 presents the parameter estimates from the MCMC estimation for the single regime affine term structure models while regime-independent parameter estimates for the regime-switching model are shown in Table 3.2 and regime-dependent parameters are reported in Table 3.3. Parameter estimates are based on the mean of the MCMC estimation sample. The 2.5% and 97.5% quantile of the MCMC samples are reported in parenthesis.

INSERT TABLE 3.1 TO 3.3 ABOUT HERE

We begin our analysis by evaluating how well the different models are able to describe the conditional distribution of observed U.S. zero coupon bond yields. To assess the cross-sectional fit of the different models we look at several measures, starting with the variance of the measurement error in Equation 3.1, proceeding with the average absolute pricing errors for each of these models and concluding with a model-comparison analysis performed

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<sup>7</sup>The original data set available online at the Board of Governors of the Federal Reserve System, has a daily frequency. We have transformed the data to a monthly frequency by keeping the last day of each month as that months corresponding yield value.



with the Bayes Factor. We then move on to analyzing how well these models manage to match some of the most important features of observed U.S. zero coupon bond yield data, such as the relationship between the slope of the yield curve and expected excess returns, the matching of the unconditional first moment of yields as well as that of the shape and persistence of conditional volatilities of yield changes.

### 3.4.2 Model comparison

The first metric that we examine to compare the different model specifications is the measurement error of Equation 3.1. Mikkelsen (2002) attributes the measurement error to data issues such as rounding errors, observational noise, different data sources, etc. but also to fact that the assumed model is only an approximation to the process that determines interest rates. Hence, the smaller the measurement error, the closer the approximation of observed yields by the model implied yields. In this paper, we focus on fitting a given term structure model to a given set of yields and thus, a small measurement error is taken as an indication of good fit of the term structure model to the actual yield data.

Table 3.4 reports the variance of the measurement error in basis points for all the estimated models.

INSERT TABLE 3.4 ABOUT HERE

The two models with the smallest variance of the measurement error are the  $A_1(3)^{(RS)}$  (where the superscript (RS) denotes regime-switching) and the  $A_2(3)^{(RS)}$  model, showing that RS-ATSM with stochastic volatility match the observed yields most accurately. We also find evidence that the  $A_3(3)$  model is outperformed by the  $A_1(3)$  and the  $A_2(3)$  model. This finding does not only hold for the models with a single regime but also for the regime-switching models and is well documented in e.g. Dai and Singleton (2000) where it is argued that the performance of the  $A_3(3)$  model deteriorates due to the restriction on the conditional correlation among the state variables.

### Pricing errors

We proceed by evaluating the ability to match cross-sectional properties of the yields, that is, the ability of different model specifications to approximate the observed yield curve at

any date during the sample period. For each maturity we calculate the absolute pricing error ( $APE(\tau)$ ), for  $\tau = \{1, 3, 5, 7, 10, 12, 15\}$  years, as below:

$$APE(\tau) = \frac{\sum_{t=1}^T |\hat{Y}(t, \tau) - Y(t, \tau)|}{T}.$$

where  $\hat{Y}(t, \tau)$  denotes simulated model implied yields and  $Y(t, \tau)$  denotes observed yields. To calculate the simulated model-implied yields for each date, we treat the parameter estimates of each MCMC draw after convergence has occurred as the true population parameters and simulate for each maturity a set of yields with the same length as our observed yields sample. The simulated model implied yields for each maturity will then be given as the average over these sets of yields. Table 3.5 provides a summary statistics of the  $APE(\tau)$  for the affine term structure models we have considered.

INSERT TABLE 3.5 ABOUT HERE

Since pricing errors mainly arise due to model misspecification, generally the smaller the pricing error the lower is the likelihood that the model is misspecified. As shown in Table 3.5, pricing errors decrease for models accounting for stochastic volatility as well as multiple regimes. Moving from single regime to multiple regime models seems to generate a significant decrease in average absolute pricing errors across all classes of models regardless of the number of factors affecting the volatility of the risk factors. Furthermore, a passage from the Gaussian regime-switching model to regime-switching models with time-varying conditional volatility decreases the pricing errors further.

In accordance with the evidence from the variance of the measurement error, the pricing errors show that the  $A_1^{(RS)}(3)$  model and the  $A_2^{(RS)}(3)$  model show a better fit to observed yields compared to single regime models as well as to the regime-switching Gaussian model. This subfamily of term structure models lies between the Gaussian model, that is the  $A_0^{(RS)}(3)$  model, and the correlated square-root diffusion, that is the  $A_3^{(RS)}(3)$  model. Dai and Singleton (2000) find that this subfamily of term structure models is superior.<sup>8</sup> Thus

<sup>8</sup>See Section 3.4.4 for a detailed discussion about the advantages of the  $A_1^{(RS)}(3)$  model and the  $A_2^{(RS)}(3)$  model.

in the subsequent sections we follow their approach and analyze the performance of the  $A_1^{(RS)}(3)$  and  $A_2^{(RS)}(3)$  relative to the Gaussian model with either one regime or multiple regimes.

### The Bayes factor

In this section we turn to formally investigate the relative performance of the models to fit historical yields. A widely used means of model selection in the Bayesian literature is the Bayes factor, which quantifies the evidence provided by the data in favor of the alternative model  $M_1$  compared to a benchmark model  $M_0$ . The Bayes factor is approximated by the ratio of the marginal likelihoods of the data in each of the two models considered for comparison and is obtained by integrating these densities over the whole parameter space. More precisely, given prior odds  $p(M_0)$  and  $p(M_1)$  for the models and given the observed yield data  $Y$ , the Bayes Theorem implies:

$$\frac{p(M_1|Y)}{p(M_0|Y)} = \frac{p(Y|M_1)}{p(Y|M_0)} \times \frac{p(M_1)}{p(M_0)}$$

where the ratio of the marginal likelihoods under the two models,  $p(Y|M_1)/p(Y|M_0)$ , denotes the Bayes factor. Assuming un-informative priors  $p(M_0) = p(M_1) = 0.5$ , the Bayes factor is given by the posterior odds.<sup>9</sup> A detailed discussion of Bayes factor can be found in Kass and Raftery (1995).

The larger the Bayes factor, the stronger the evidence in favor of alternative model  $M_1$  compared to the benchmark model  $M_0$ . Kass and Raftery (1995) establish a rule of thumb saying that a Bayes factor exceeding 3 indicates that the data provides 'substantial' evidence in favor of the alternative model versus the benchmark model. Table 3.6 provides results on model comparison with the Bayes factor.

INSERT TABLE 3.6 ABOUT HERE

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<sup>9</sup>In the absence of free parameters and latent variables, where maximum likelihood estimates of the parameters for both models are feasible, the Bayes factor corresponds to a likelihood ratio. In our case, the presence of unknown parameters, latent factors as well as latent regimes, requires that we integrate out the parameters, latent variables and regimes to obtain the marginal likelihood  $p(Y|M_1)$  and  $p(Y|M_0)$ . We refer to Appendix 3.C for a detailed explanation of the procedure followed.

To begin with, we assess the indication of the Bayes factor regarding model selection between regime-switching models versus the single regime Gaussian model (i.e. the benchmark is the  $A_0^{(SR)}(3)$  model, that is column one of the above table). We notice that the Bayes factor indicates that there is substantial evidence in support of all the other regime-switching models against the single regime Gaussian model. Secondly, we assess that within the regime-switching class of models, the evidence of the Bayes factor seems to be in favor of stochastic volatility models (i.e. the  $A_1^{(RS)}(3)$  and  $A_2^{(RS)}(3)$  model) compared to the Gaussian model. Since the Bayes factor considers the overall relative goodness-of-fit, this might not be surprising. The Gaussian model, precludes by definition time-varying conditional volatility, which in the data has been shown to be counterfactual.

The evidence we found so far shows that the data generating process underlying the U.S. zero coupon yields is seemingly most likely described by a regime-switching model which allows for stochastic volatility in the process of the underlying state variables. More precisely, the  $A_1^{(RS)}(3)$  model and the  $A_2^{(RS)}(3)$  model have shown smaller variances of the measurement errors and smaller average absolute pricing errors. Furthermore model selection analysis by the Bayes factor has shown evidence in favor of these models. Thus, in the next section we investigate the regime probabilities and the ability to match the term structure of unconditional means of the U.S. yields of the  $A_2^{(RS)}(3)$  models.

### 3.4.3 Regimes

Figure 3.1 shows a time series of posterior probabilities of the regime variable, that is, the probability that the economy is either in regime 1 or regime 2 of the  $A_2^{(RS)}(3)$  model. The shaded areas represent periods of recessions identified by the NBER.

INSERT FIGURE 3.1 ABOUT HERE

These plots suggest that regime 2 tends to be associated with recessions, while expansions are related to regime 1. The economy switches for the first time to regime 2 in July 1972 and remains there during the oil crisis in 1973. Also during the recessions in the beginning of the 1980's we are in regime 2, which prevails until the early 1990's (with two short interruptions). The plots show evidence that the first regime is prolonged well beyond the end of the recession in 1982, however, this is a common finding which has previously been

documented in e.g. Dai, Singleton, and Yang (2007) and Li, Li, and Yu (2011). In the second half of our sample period the first regime is more pervasive. It is interrupted only three times by the second regime, the last time just before the dot-com crises. Overall, the second regimes prevails more often in the first half of our sample period, where recession appear more often, while the first regime is more persistent in the second half of our sample period.

Figure 3.1 shows that both regimes are rather persistent, that is, the probability for a regime switch is much smaller than the probability of staying in the same regime. This fact is reflected in the transition matrix which shows how likely it is to switch between regimes over the next month. The transition matrix for  $\Delta_t = 1$  month is given as below:

$$\exp(Q\Delta_t) = \begin{bmatrix} 0.739 & 0.261 \\ 0.276 & 0.724 \end{bmatrix}.$$

The transition matrix shows that the probability of switching from regime 1 (2) to regime 2 (1) is 26.1% (27.6%) over the next month, thus, suggesting a strong regime persistence. Additionally, the probability of staying in regime 1 is 73.9% while it is 72.4% for the second regime. The transition matrix shows that both regimes are almost equally persistent. This fact is confirmed in Figure 3.1 where both regimes occur approximately equally often. We relate this finding to the model specification of the RS-ATSM with stochastic volatility, where the volatility is not explicitly regime-dependent and the regimes are thus associated with the level of the yields.

This finding is confirmed when we look at the unconditional means of the yields in both regimes. In general, unconditional means of treasury yields are on average increasing with maturity. In order to see whether our model-implied yields are able to reproduce these features, we simulate model-implied means and volatilities (along with confidence bands) for each of the regimes and show them against their sample counterparts.

To calculate model implied unconditional means we simulate 100 series of yields, each with the same length as the observed data for every MCMC draw of the estimation period. We condition on the regime variable of the corresponding MCMC draw for each date of our sample period and calculate the latent factors using the parameters from the MCMC draw.

We average over the 100 simulated yields and then across the draws to obtain the term structure of unconditional means, as well as the 95% confidence band. Next we compute the unconditional mean of the observed yields for each of the regimes. To do so, we sample the regime for each date of our sample period from the posterior distribution (as explained in Appendix 3.C) and sort out the historical yields according to the regime assigned to each date, then compute sample means for each of the regimes.

Figure 3.2 shows the term structure of unconditional means for each regime for the simulated model-implied yields and their observed sample counterparts.

INSERT FIGURE 3.2 ABOUT HERE

Figure 3.2 confirms our expectation by showing that the unconditional mean of the yields in regime 1 is considerably lower than in the second regime. Additionally, we emphasize that the term structure of unconditional means is upward sloping, replicating the fact that on average investors require higher interest rates for holding longer maturity bonds. The observed yields unconditional mean fall within the 95% confidence bounds of the respective simulated model-implied unconditional first moment.

#### 3.4.4 Matching the features of bond yields

In this section we look at the ability of our model implied yields to fit the historical behavior of the U.S. term structure of interest rates. Standard procedure in the literature is to look at four measures, that is, the model's ability to match the stylized facts in terms of the predictability of bond returns as well as the time variability in conditional yield volatilities and their persistence.

The ultimate test of any theoretical model is its ability to match the features of the data it aims to describe and its potential to forecast the dynamic evolution of the variables of interest. In the context of affine term structure models, the overall goodness of fit of the model is measured in terms of its ability to match the cross-section and time-series of observed yields. A tension and trade-off generally arises in fitting both the cross-sectional and time-series properties of yields with affine term structure models. The first crucially depends on a flexible correlation structure between the state variables determining

the short rate, while the second on the persistence and time variation of the conditional volatility of the yields. The Gaussian model (i.e. the  $A_0(3)$  model) performs relatively well in fitting the cross-section of observed yields, while by definition precluding time-varying conditional volatility. On the other hand, the correlated square root diffusion model (i.e. the  $A_3(3)$  model) is able to some extent to replicate the time variability in yield volatilities, but given its restriction in the sign of the correlation structure of risk factors performs worse in terms of the first feature. Following Dai and Singleton (2000), and given the inability of the  $A_3(3)$  model to generate negative correlations between the state variables, as suggested by historical interest rate data, most empirical research concentrates on analyzing the three maximally affine subfamilies consisting of the  $A_0(3)$ ,  $A_1(3)$  and  $A_2(3)$  model. For sufficiently flexible market price of risk specifications the overall fit of the  $A_1(3)$  and  $A_2(3)$  relatively improves, so that combined with the fact that the  $A_0(3)$  precludes time-varying volatility, these models become more appealing.

The regime-switching literature concentrates almost exclusively on the Gaussian model while generally abstaining from analyzing the  $A_1(3)$  and  $A_2(3)$  model, mainly due to the complexity that arises in terms of modelling and most importantly in terms of estimation. In this paper we provide a basis for a general analysis of the whole class of maximally affine term structure models with regime-switches. More precisely, we assess whether there is a benefit in moving firstly from a single-regime Gaussian model to a regime-switching Gaussian model, and secondly within the regime-switching class, moving from a Gaussian specification to stochastic-volatility specifications, that is the  $A_1^{(RS)}(3)$  and  $A_2^{(RS)}(3)$  model. We begin our analysis by looking at the models ability to replicate the Campbell-Shiller regression.

### **Predictability of excess returns**

An important stylized fact of observed yield data is that expected excess returns are time varying. Starting with Fama (1984b), empirical studies on U.S. yield data document that the slope of the yield curve has predictive power for future changes in yields. Campbell and Shiller (1991) show that linear projections of future yield changes on the slope of the yield curve give negative coefficients ( $\beta(\tau) < 0$  in Equation 3.2), which are increasing

with the time to maturity. Backus, Foresi, Mozumdar, and Wu (2001) and other studies confirm this finding across different sample periods. More precisely, the Campbell-Shiller regression reads as

$$Y_{t+\tau_1}^{\tau-\tau_1} - Y_t^\tau = \alpha(\tau) + \beta(\tau) \left[ \frac{\tau_1}{\tau - \tau_1} (Y_t^\tau - Y_t^{\tau_1}) \right] + \epsilon_t(\tau) \quad (3.2)$$

where the shortest available maturity is denoted with  $\tau_1$  and  $\tau$  is given in years.  $\alpha(\tau)$  and  $\beta(\tau)$  indicate maturity specific constant and slope coefficients. The results of Campbell and Shiller (1991) imply that an increase in the slope of the yield curve is associated with a decrease in long term yields and vice-versa, hence the current slope of the yield curve is indicative of the direction in which future long rates will most likely move. The expectations hypothesis on the contrary states that risk premia are constant and future bond returns are unpredictable. This empirical failure of the expectations hypothesis is one of the main puzzles in financial economics and being able to reproduce this feature of the yield data is hence important for any term structure model.

Table 3.7 presents the Campbell-Shiller coefficients obtained from the above regression with our sample of historical U.S. yield data, confronted with the coefficients obtained from simulated model-implied yields.<sup>10</sup>

INSERT TABLE 3.7 ABOUT HERE

As we can clearly see from Table 3.7, within the single regime class of models, the models' ability to capture the sign and size of the Campbell-Shiller regression coefficients deteriorates with the number of factors affecting the covariance structure of the latent state variables.<sup>11</sup> A finding which is consistent with the single-regime literature findings of e.g. Dai and Singleton (2003) and Feldhütter (2008). However, moving to the regime-switching class of models, we notice that compared to single regime models, where only the  $A_0^{(SR)}(3)$

<sup>10</sup>The ability to replicate the Campbell-Shiller coefficients usually deteriorates with the number of factors entering the volatility matrix of the underlying state variables, i.e. that the Gaussian model outperforms the models with stochastic volatility. In order to see the benefit of the regimes Table 3.7 also includes the  $A_1^{(SR)}(3)$  and the  $A_2^{(SR)}(3)$  model. To obtain model-implied yields as well its observed counterparts we apply the procedure as described in Section 3.4.3.

<sup>11</sup>Since the spacing between maturities in our case is not constant we approximate the unobserved yields, both model-implied and historical ones, following Campbell and Shiller (1991).



model can capture the negative sign of the Campbell-Shiller coefficients (as well as the increase in absolute size of the coefficients as maturity increases), the  $A_1^{(RS)}(3)$  and  $A_2^{(RS)}(3)$  model is able to capture these features if we allow for multiple regimes. These models match the negative sign of the historical Campbell-Shiller coefficients for most maturities and the size of the coefficients decreases with the maturity in a similar fashion to that of the historical data coefficients. The actual magnitude of the model implied and actual regression coefficients are similar, with the models' confidence bands containing the actual data coefficients for most of the maturities (with the 1-year yield as the exception). Turning to models  $A_1^{(RS)}(3)$  and  $A_2^{(RS)}(3)$ , we believe that their improvement in matching the sign and sizes of the Campbell-Shiller coefficients compared to their single-regime counterparts, comes from the flexibility in changing signs for the market price of risk. For regime-switching models in particular the structure of risk premia appears to be one of the fundamental factors affecting the model's ability in matching the Campbell-Shiller regression coefficients. A model specification that allows only the state variables' long run mean to be regime-dependent but not their volatility, requires a regime-dependent market price of factor risk through either the constant of proportionality  $\lambda_0$  or the factor loading  $\lambda_1$ , or both, so that the volatility of the state variable and the risk premia can vary across regimes independently. Our market price of risk specification allows for both  $\lambda_0$  and the factor loading  $\lambda_1$  to be regime dependent, implying that even though the speed of mean reversion are constant under the risk-neutral measure they become regime-dependent under the physical measure, resulting in the observed improvement. It is interesting to confirm through our results in this section, that introducing regimes closes to some extent the wedge between the Gaussian and the correlated square-root diffusion models in terms of fitting the Campbell-Shiller regression coefficients.<sup>12</sup>

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<sup>12</sup>Due to the small sample bias it would be interesting to also report model-implied theoretical coefficients, besides the simulated model-implied coefficients and the historical coefficients. Since our model allows for multiple regimes, it is intuitively not so clear how to interpret the comparison of the coefficients on a per-regime basis, hence to be consistent with the existing literature we limit our analysis to simulated model-implied Campbell-Shiller coefficients.

### Conditional yield volatilities

Another important feature of the historical U.S. yield data is the time variation and persistence of conditional volatilities of yield changes.

Brandt and Chapman (2002) and Piazzesi (2010) show that conditional yield volatilities are positively varying with interest rates. We are interested in evaluating whether our models are able to reproduce this feature of the data, and hence analyze whether the volatility of our model implied yields is correlated with the level of model-implied yields in a similar fashion. Since regressing yield volatility on the yields themselves would create potential problems of multicollinearity, we regress conditional volatilities on the level, slope and curvature of the yield curve. Litterman and Scheinkman (1991) show that the level, slope and curvature factors explain at least 96% of the variation in excess returns across maturities and are virtually orthogonal and thus, we avoid potential problems of multicollinearity. We then look at the significance, sign and size of the coefficients in order to assess the extent at which the level, slope and curvature factors have explanatory power regarding the time-variation in zero-coupon bond yields.

In particular, we run the below regression for our sample of historical yield data and simulated model-implied yields:<sup>13</sup>

$$(Y(t+1, \tau) - Y(t, \tau))^2 = \alpha(\tau) + \beta_1(\tau)Y(t, \tau_1) + \beta_2(\tau) [Y(t, \tau_M) - Y(t, \tau_1)] + \beta_3(\tau) [Y(t, \tau_M) + Y(t, \tau_1) - 2Y(t, \tau_{mid})] + \epsilon_{t,\tau} \text{ for } \tau = 1, \dots, M.$$

The shortest available yield is denoted with  $\tau_1$  while the most long-term yield is indicated with  $\tau_M$ . To calculate the curvature we rely on maturity which lies between  $\tau_1$  and  $\tau_M$  which is given by  $\tau_{mid}$ .

Table 3.8 reports estimates of the regression coefficients for the observed yields and the for the model implied yields of the  $A_1(3)$  and  $A_2(3)$  model. The  $A_0(3)$  model precludes time-varying volatility by definition and is hence omitted from the analysis.

INSERT TABLE 3.8 ABOUT HERE

<sup>13</sup>To obtain model-implied yields and its observed counterparts we apply the same procedure as described in Section 3.4.3.

Table 3.8 shows that volatility is positively correlated with the level of the observed yields. The level coefficient of the actual yield data is positive for all maturities and exhibits a downward trend along the maturity. All models with the stochastic volatility feature capture the positive sign of the level coefficient as well as the decreasing pattern of the slope coefficient. The shorter the time to maturity, the better is the level coefficient of the model implied yields. However, all models fail to replicate the actual magnitude of the level coefficient. A similar reasoning applies to the coefficients of the slope and curvature. The evidence of the volatility regression is consistent with the results of Brandt and Chapman (2002) who argue that only the class of quadratic term structure models are able to accommodate both the dynamics of conditional expected bond returns and their conditional volatility. The difficulties to match volatility can also be explained by the sample period. Christiansen and Lund (2005) argue that the period of the “monetary experiment”, that is 1979-1982, should be excluded when investigating volatility and the shape of the yield curve. Considering sub-samples may improve the ability of the models to match stylized facts related to volatility.

After having performed the above regression analysis we proceed with a more formal evaluation of the model’s performance with regards to its ability to produce sufficient persistence in the time-variation of yield volatilities so as to be in line with that of the historical data. Following Dai and Singleton (2003), we estimate a GARCH(1,1) model<sup>14</sup> for yields with selected maturities using first historical data and then simulated yields for each of the models considered.<sup>15</sup>

In order to examine the benefit of multiple regimes, Table 3.9 reports GARCH estimates for the  $A_1(3)$  and  $A_2(3)$  model for both a single regime and a regime-switching setting.

INSERT TABLE 3.9 ABOUT HERE

The results shown in Table 3.9 indicate that all models capture the persistence in the yield volatility displayed by the historical yield data quite well. This fact holds for all

<sup>14</sup>The GARCH(1,1) model is given as  $\sigma_t = \bar{\sigma} + \alpha\epsilon_t^2 + \beta\sigma_{t-1}^2$ , where  $\epsilon_t$  is the residual of the AR(1) representation of the selected maturity. We use the observed variance of the residuals  $\epsilon_t$ , as a starting estimate for the variance of the first observation.

<sup>15</sup>Instead of simulating 100 series of yields for each MCMC draw of the estimation period we treat the average of the parameters of the estimation period as the true population parameters. Based on this parameters we simulate 1000 series of yield using the usual procedure of Section 3.4.4 and fit a GARCH model to the yields in order to obtain the distribution of GARCH coefficients.

maturities. The  $\beta$ -coefficients for the model-implied GARCH(1,1) coefficients are of similar magnitude to those of the historical data for most maturities, with an average size of circa 0.8, indicating that shocks to conditional variance take quite some time to die out.  $\alpha$ -coefficients for the model-implied GARCH(1,1) regressions are typically lower than those implied by the historical data  $\alpha$ -coefficients, indicating that volatility is slower to react to market movements relative to what the historical results show, i.e. model-implied volatility is less spiky than the historical volatility would imply. For both, the  $A_1(3)$  and the  $A_2(3)$  model, and across all maturities it seems that the regime-switching models estimate the  $\alpha$ - and  $\beta$ -coefficient more accurate than single-regime models.

Overall, we found evidence that introducing regimes in the family of affine term structure models improves the cross-sectional fit, meaning that regime-switching models approximate the yield curve more accurate than single regime models. More importantly we also showed that RS-ATSM with stochastic volatility, and in particular the  $A_1(3)^{(RS)}$  and the  $A_2(3)^{(RS)}$  model, outperform the Gaussian regime-switching, that is the  $A_0(3)^{(RS)}$  model. The superior performance of the stochastic volatility models is reflected in smaller measurement errors, smaller average absolute pricing errors and Bayes factors of beyond three.

We also showed that RS-ATSM with stochastic volatility successfully match some of the stylized facts of the U.S. yield curve such as unconditional first moment and time-varying conditional volatility. Additionally, allowing for multiple regimes improves the ability to replicate the Campbell-Shiller regression coefficients, as shown by the  $A_1^{(RS)}(3)$  model. However, the regime-switching  $A_2^{(RS)}(3)$  and  $A_3^{(RS)}(3)$  model lack the ability to reproduce this stylized fact.

### 3.5 Concluding Remarks

In this paper we embed multiple regimes in an affine term structure model and assess the ability of the RS-ATSM to reproduce historical yields as well as some of the stylized facts of the U.S. yield curve. More precisely, we analyzed the performance of RS-ATSM with a stochastic volatility feature relative to Gaussian models with either a single regime or multiple regimes. We find evidence that RS-ATSM with stochastic volatility successfully

describe historical yields while still being able to replicate important features of the U.S. yield curve.

We show that introducing regimes in the family of affine term structure models improves the cross-sectional fit, meaning that regime-switching models approximate the yield curve more accurately than single regime models. Our preferred models, that is the  $A_1^{(RS)}(3)$  model and the  $A_2^{(RS)}(3)$  model, exhibit the smallest measurement error and generate the smallest pricing errors. This finding is supported by the Bayes factor which also shows that these two models are superior.

Additionally, the above mentioned models successfully capture some of the stylized facts of the U.S. yield curve such as unconditional first and second moments and time-varying conditional volatility. We also find that  $A_2^{(RS)}(3)$  model and  $A_3^{(RS)}(3)$  replicate the coefficients of the Campbell-Shiller much closer than the single regime models.

Our specification of the RS-ATSM allows to analytically solve for bond prices whilst there is still considerable regime-dependence. Introducing priced regime shift risk might be an interesting enhancement of our model specification, however, a market price of regime shift risk proved to be difficult to be estimated using our estimation approach.

## Tables and Figures

TABLE 3.1: Single regime affine term structure models: MCMC parameter estimates

	$A_0^{(SR)}(3)$	$A_1^{(SR)}(3)$	$A_2^{(SR)}(3)$	$A_3^{(SR)}(3)$
$\kappa_0^Q(1)$	0	111.767 (99.629;123.144)	0.525 (0.501;0.581)	2.140 (2.036;2.281)
$\kappa_0^Q(2)$	0	0	41.645 (37.171;44.798)	0.807 (0.534;1.133)
$\kappa_0^Q(3)$	0	0	0	4.850 (4.657;5.024)
$\kappa_0^P(1)$	17.848 (-1.169;0.311)	7.838 (10.431;27.682)	1.921 (0.868;20.042)	1.757 (0.536;4.566)
$\kappa_0^P(2)$	1.017 (-2.438;-5.197)	4.402 (-5.851;13.963)	15.743 (1.433;36.636)	2.781 (0.578;7.151)
$\kappa_0^P(3)$	-7.917 (-25.733;7.307)	45.894 (19.914;75.148)	-8.659 (-15.232;-1.929)	1.836 (0.535;4.711)
$\kappa_1^Q(1,1)$	0.040 (0.038;0.042)	2.015 (1.996;2.028)	0.093 (0.090;0.098)	0.027 (0.025;0.029)
$\kappa_1^Q(1,2)$	0	0	-0.091 (-0.096;-0.086)	-0.057 (-0.060;-0.053)
$\kappa_1^Q(1,3)$	0	0	0	-0.049 (-0.056;-0.041)
$\kappa_1^Q(2,1)$	6.495 (6.272;6.616)	0.208 (0.202;0.212)	-1.409 (-1.415;-1.404)	-0.013 (-0.020;-0.001)
$\kappa_1^Q(2,2)$	10.017 (9.873;10.152)	0.314 (0.307;0.324)	1.503 (1.492;1.514)	2.473 (2.466;2.479)
$\kappa_1^Q(2,3)$	0	-0.065 (-0.069;-0.061)	0	-0.008 (-0.014;-0.002)
$\kappa_1^Q(3,1)$	0.596 (0.436;0.756)	-0.452 (-0.463;-0.441)	-0.022 (-0.028;-0.017)	-0.019 (-0.025;-0.014)
$\kappa_1^Q(3,2)$	3.220 (3.100;3.326)	-0.531 (-0.542;-0.519)	0.581 (0.573;0.591)	-0.007 (-0.013;0.000)
$\kappa_1^Q(3,3)$	0.492 (0.321;0.697)	0.153 (0.146;0.159)	2.827 (2.821;2.835)	2.337 (2.329;2.349)

TABLE 3.1: continued

	$A_0^{(SR)}(3)$	$A_1^{(SR)}(3)$	$A_2^{(SR)}(3)$	$A_3^{(SR)}(3)$
$\kappa_1^P(1, 1)$	-0.111 (-0.288;0.051)	-0.012 (-0.039;-0.000)	-0.137 (-0.288;-0.028)	-0.192 (-0.283;-0.110)
$\kappa_1^P(1, 2)$	-0.787 (-1.799;0.219)	0	0.036 (0.001;0.124)	0.614 (0.029;1.632)
$\kappa_1^P(1, 3)$	-0.235 (-0.526;0.046)	0	0	3.462 (1.975;4.817)
$\kappa_1^P(2, 1)$	2.370 (1.939;2.784)	0.014 (-0.015;0.046)	0.193 (0.019;0.405)	0.021 (0.000;0.066)
$\kappa_1^P(2, 2)$	26.359 (25.068;27.336)	-0.253 (-0.458;-0.068)	-0.230 (-0.500;-0.112)	-7.710 (-8.994;-6.355)
$\kappa_1^P(2, 3)$	7.404 (6.921;7.810)	-0.052 (-0.102;-0.000)	0	10.907 (9.337;12.202)
$\kappa_1^P(3, 1)$	-11.458 (-13.354;-9.935)	-0.198 (-0.277;-0.128)	-0.400 (-0.648;-0.112)	0.016 (0.000;0.049)
$\kappa_1^P(3, 2)$	-119.211 (-121.397;-116.194)	1.352 (0.941;1.791)	0.118 (0.021;0.221)	3.776 (2.681;4.789)
$\kappa_1^P(3, 3)$	-33.375 (-33.903;-32.355)	-0.163 (-0.315;-0.006)	-1.443 (-2.234;-0.503)	-8.884 (-10.363;-7.461)
$\delta_0$	0.135 (0.132;0.139)	0.152 (0.136;0.169)	0.002 (0.000;0.005)	-0.621 (-0.626;-0.609)
$\delta_x(1)$	0.025 (0.024;0.026)	0.000 (0.000;0.000)	0.006 (0.006;0.006)	0.000 (0.000;0.000)
$\delta_x(2)$	0.018 (0.017;0.019)	0.005 (0.005;0.005)	0.001 (0.000;0.001)	0.180 (0.180;0.181)
$\delta_x(3)$	0.000 (0.000;0.000)	0.000 (0.000;0.000)	0.028 (0.028;0.029)	0.258 (0.258;0.259)
$\beta_2(1)$	0	0.045 (0.036;0.054)	0	0
$\beta_3(1)$	0	0.673 (0.562;0.793)	0.008 (0.000;0.029)	0
$\beta_3(2)$	0	0	0.026 (0.009;0.038)	0

TABLE 3.1: continued

	$A_0^{(SR)}(3)$	$A_1^{(SR)}(3)$	$A_2^{(SR)}(3)$	$A_3^{(SR)}(3)$
$\sigma^2$	7.45e-06	1.37E-06	8.51E-07	4.51E-06
	(6.70E-06;8.11E-06)	(1.28E-06;1.44E-06)	(7.80E-07;9.05E-07)	(4.27E-06;4.76E-06)

This table reports parameter estimates and confidence bands for the single regime (denoted with superscript (SR)) extended affine term structure models. The parameter estimate is the average of every 100'th iteration of the estimation period consisting of 300000 iteration (i.e. the variance calibration sample and a burn-in period are excluded). The confidence bounds reported in parenthesis indicate the 95% confidence interval.



TABLE 3.2: Regime switching affine term structure models: MCMC estimates of regime independent parameters

	$A_0^{(RS)}(3)$	$A_1^{(RS)}(3)$	$A_2^{(RS)}(3)$	$A_3^{(RS)}(3)$
$\kappa_0^Q(1)$	0	2.175 (2.059;2.279)	2.950 (2.933;2.970)	1.362 (1.286;1.406)
$\kappa_0^Q(2)$	0	0	0.901 (0.880;0.921)	7.029 (6.811;7.111)
$\kappa_0^Q(3)$	0	0	0	9.714 (9.453;9.819)
$\kappa_1^Q(1,1)$	0.217 (0.207;0.226)	0.177 (0.168;0.184)	1.609 (1.604;1.617)	0.072 (0.069;0.077)
$\kappa_1^Q(1,2)$	0	0	-0.299 (-0.301;-0.297)	-0.123 (-0.129;-0.118)
$\kappa_1^Q(1,3)$	0	0	0	-0.002 (-0.010;0.000)
$\kappa_1^Q(2,1)$	5.003 (4.946;5.068)	-0.058 (-0.065;-0.047)	-0.175 (-0.188;-0.164)	-0.885 (-0.894;-0.880)
$\kappa_1^Q(2,2)$	8.746 (8.666;8.806)	0.990 (0.982;0.995)	0.033 (0.031;0.035)	1.676 (1.673;1.679)
$\kappa_1^Q(2,3)$	0	1.046 (1.039;1.057)	0	-0.551 (-0.562;-0.542)
$\kappa_1^Q(3,1)$	1.812 (1.797;1.827)	-0.020 (-0.023;-0.016)	3.364 (3.357;3.373)	-0.173 (-0.179;-0.167)
$\kappa_1^Q(3,2)$	2.910 (2.863;2.966)	0.099 (0.082;0.108)	-0.688 (-0.690;-0.685)	-0.256 (-0.271;-0.243)
$\kappa_1^Q(3,3)$	-0.008 (-0.010;-0.005)	0.106 (0.091;0.116)	0.321 (0.313;0.329)	1.919 (1.912;1.924)
$\delta_x(1)$	0.067 (0.066;0.068)	-0.004 (-0.005;-0.004)	0.030 (0.030;0.030)	0.034 (0.034;0.034)
$\delta_x(2)$	0.080 (0.079;0.081)	0.003 (0.003;0.003)	-0.005 (-0.005;-0.005)	-0.074 (-0.074;-0.074)
$\delta_x(3)$	0.007 (0.007;0.008)	0.010 (0.010;0.011)	0.004 (0.004;0.004)	0.068 (0.068;0.069)
$\beta(2,1)$	0	1.864 (1.453;2.248)	0	0

TABLE 3.2: continued

	$A_0^{(RS)}(3)$	$A_1^{(RS)}(3)$	$A_2^{(RS)}(3)$	$A_3^{(RS)}(3)$
$\beta(3,1)$	0	0.124 (0.095;0.149)	0.715 (0.050;1.386)	0
$\beta(3,2)$	0	0	0.143 (0.008;0.284)	0
Q(1,1)	-1.781 (-2.275;-1.228)	-0.489 (-1.042;-0.273)	-0.374 (-0.675;-0.187)	-2.093 (-2.739;-1.596)
Q(2,2)	-0.718 (-1.109;-0.438)	-0.547 (-0.897;-0.298)	-0.396 (-0.775;-0.174)	-1.531 (-2.146;-1.002)
$\sigma^2$	3.45E-07 (3.23E-07;3.68E-07)	2.75E-07 (2.571E-07;2.946E-07)	2.52E-07 (2.37E-07;2.69E-07)	8.11E-07 (7.64E-07;8.62E-07)

This table reports MCMC estimates and confidence bands of the regime independent parameters for all regime switching affine term structure models. The parameter estimate is the average of every 100'th iteration of the estimation sample consisting of 300000 iteration (i.e. the variance calibration sample and a burn-in period are excluded). The confidence bounds reported in parenthesis indicate the 95% confidence interval.

TABLE 3.3: Regime switching affine term structure models: MCMC estimates of regime dependent parameters

	$A_0^{(RS)}(3)$		$A_1^{(RS)}(3)$		$A_2^{(RS)}(3)$		$A_3^{(RS)}(3)$	
	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
$\kappa_0^P(1)$	-1.083 (-2.158;0.040)	-0.787 (-2.075;0.376)	8.826 (2.205;15.781)	7.864 (3.204;13.172)	17.588 (3.982;34.303)	7.940 (1.401;17.046)	4.304 (0.691;11.728)	2.343 (0.579;7.518)
$\kappa_0^P(2)$	2.327 (1.288;3.455)	0.601 (-0.805;2.215)	-0.620 (-15.319;14.381)	14.537 (2.761;26.127)	7.164 (0.766;18.853)	13.977 (1.340;34.940)	20.089 (8.244;32.274)	10.525 (2.044;21.188)
$\kappa_0^P(3)$	1.274 (0.182;2.382)	2.165 (0.789;3.847)	3.480 (-1.318;8.434)	4.672 (0.992;8.374)	-17.291 (-29.917;-4.482)	-18.826 (-37.449;2.379)	9.370 (3.116;17.493)	5.272 (0.957;11.314)
$\kappa_1^P(1,1)$	0.736 (0.221;1.262)	-1.025 (-1.955;-0.056)	0.643 (0.218;1.074)	0.331 (0.093;0.632)	4.298 (0.936;7.997)	1.738 (0.349;3.522)	0.262 (0.093;0.505)	0.203 (0.071;0.392)
$\kappa_1^P(1,2)$	0.272 (-0.131;0.661)	-1.888 (-3.093;-0.636)	0 (-3.093;-0.636)	0 (-1.250;-0.107)	-0.657 (-1.250;-0.107)	-0.262 (-0.625;-0.009)	-0.161 (-0.528;-0.004)	-0.100 (-0.339;-0.003)
$\kappa_1^P(1,3)$	-0.086 (-0.254;0.078)	-0.549 (-0.978;-0.129)	0 (0.174;1.336)	0 (0.512;1.541)	0 (0.104;0.791)	0 (1.770;2.848)	-0.156 (-0.479;-0.005)	-0.083 (-0.297;-0.002)
$\kappa_1^P(2,1)$	-0.999 (-1.555;-0.487)	3.173 (2.136;4.179)	0.032 (-0.662;0.745)	0.021 (-0.408;0.404)	-1.119 (-3.211;-0.044)	-10.692 (-12.968;-8.024)	-8.631 (-9.757;-7.387)	-1.978 (-3.988;-0.897)
$\kappa_1^P(2,2)$	0.433 (-0.009;0.869)	4.067 (2.695;5.424)	0.698 (0.174;1.336)	1.030 (0.512;1.541)	0.379 (0.104;0.791)	2.312 (1.770;2.848)	18.705 (15.944;21.035)	3.984 (1.721;8.440)

TABLE 3.3: continued

	$A_0^{(RS)}(3)$		$A_1^{(RS)}(3)$		$A_2^{(RS)}(3)$		$A_3^{(RS)}(3)$	
	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
$\kappa_1^P(2,3)$	0.351 (0.176;0.528)	0.929 (0.463;1.389)	0.889 (0.080;1.802)	2.283 (1.130;3.379)	0 (-16.920;-13.040)	0 (-5.461;-0.214)	-15.034 (-16.920;-13.040)	-2.032 (-5.461;-0.214)
$\kappa_1^P(3,1)$	-0.707 (-1.231;-0.183)	0.397 (-0.653;1.377)	0.074 (-0.161;0.299)	0.189 (0.067;0.321)	-3.707 (-6.258;-1.205)	-2.088 (-6.874;3.985)	-0.050 (-0.148;-0.002)	-0.081 (-0.226;-0.003)
$\kappa_1^P(3,2)$	-0.461 (-0.867;-0.062)	1.767 (0.365;3.026)	-0.047 (-0.236;0.136)	-0.072 (-0.227;0.087)	0.597 (0.154;1.033)	0.211 (-0.943;1.134)	-0.115 (-0.339;-0.004)	-0.169 (-0.425;-0.008)
$\kappa_1^P(3,3)$	0.134 (-0.034;0.304)	1.176 (0.717;1.616)	0.263 (-0.011;0.555)	0.036 (-0.339;0.418)	0.226 (0.039;0.493)	1.504 (0.721;2.243)	0.789 (0.359;1.448)	1.310 (0.797;1.866)
$\delta_0$	0.053 (0.047;0.054)	0.065 (0.059;0.068)	0.063 (0.060;0.067)	0.073 (0.071;0.078)	-0.008 (-0.014;-0.005)	0.004 (-0.002;0.010)	-0.022 (-0.031;-0.018)	0.008 (0.001;0.015)

This table reports MCMC estimates and confidence bands of the regime dependent parameters for all regime switching affine term structure models. The parameter estimate is the average of every 100'th iteration of the estimation sample consisting of 300000 iteration (i.e. the variance calibration sample and a burn-in period are excluded). The confidence bounds reported in parenthesis indicate the 95% confidence interval.

TABLE 3.4: Measurement Errors of the different Affine Term Structure Model Specifications

	Single regime Models	Regime-switching Models
$A_0(3)$	27.3477 (25.873;28.471)	5.872 (5.687;6.068)
$A_1(3)$	11.637 (11.306;12.018)	5.247 (5.070;5.429)
$A_2(3)$	9.221 (8.943;9.512)	5.023 (4.866;5.184)
$A_3(3)$	21.225 (20.657;21.811)	9.006 (8.742;9.285)

This table reports the measurement error of the four different affine term structure models for models with a single regime and models with two regimes. The measurement error is the average of every 100'th iteration of the estimation sample consisting of 300000 iteration (i.e. the variance calibration sample and a burn-in period are excluded). The confidence bounds reported in parenthesis indicate the 95% confidence interval.

TABLE 3.5: Average absolute pricing errors

	Maturity in Years						
	1	3	5	7	10	13	15
$A_0(3)^{(SR)}$							
Mean	30.291	27.985	18.061	12.283	14.830	20.293	24.142
Std	3.700	4.537	2.501	3.487	3.245	2.304	3.884
$A_1(3)^{(SR)}$							
Mean	11.859	11.500	9.315	7.032	5.603	6.557	9.831
Std	18.207	25.292	16.696	9.178	10.877	14.776	20.969
$A_2(3)^{(SR)}$							
Mean	7.213	7.366	8.754	7.372	4.360	5.411	9.074
Std	0.789	3.647	5.638	4.448	2.493	2.922	6.489
$A_3(3)^{(SR)}$							
Mean	18.207	25.292	16.696	9.178	10.877	14.776	20.969
Std	2.710	15.178	8.754	3.494	5.846	7.878	11.839
$A_0(3)^{(RS)}$							
Mean	4.776	5.830	3.680	4.750	3.884	3.106	5.019
Std	0.959	3.505	1.504	2.730	3.012	0.991	3.191
$A_1(3)^{(RS)}$							
Mean	0.959	3.505	1.504	2.730	3.012	0.991	3.191
Std	4.014	4.643	3.060	3.755	3.083	2.624	4.698
$A_2(3)^{(RS)}$							
Mean	4.014	4.643	3.060	3.755	3.083	2.624	4.698
Std	7.126	7.502	7.879	7.156	4.582	5.105	9.066
$A_3(3)^{(RS)}$							
Mean	7.126	7.502	7.879	7.156	4.582	5.105	9.066
Std	0.244	3.259	5.045	4.276	2.563	2.873	6.488

This table reports the summary statistics of the four different affine term structure models for models with a single regime and models with two regimes. The absolute pricing errors are calculated over the 495 dates for all seven maturities. The sample period is 11/1971-01/2011.

TABLE 3.6: Model comparison by the Bayes factor

		Benchmark Model			
		$A_0(3)^{(SR)}$	$A_0(3)^{(RS)}$	$A_1(3)^{(RS)}$	$A_2(3)^{(RS)}$
Alternative Model	$A_0(3)^{(SR)}$	1			
	$A_0(3)^{(RS)}$	2.047	1		
	$A_1(3)^{(RS)}$	5.884	2.875	1	
	$A_2(3)^{(RS)}$	42.954	20.987	7.300	1

This table reports the Bayes factor for the ATSM's. The performance of the regime switching models is compared with a single regime Gaussian model denoted with  $A_0(3)^{(SR)}$  as well as among the regime-switching models (denoted with a superscript  $(RS)$ ). A detailed explanation of the calculation of the Bayes factor is in Appendix 3.C.

TABLE 3.7: Campbell-Shiller Regression

	Maturity in Years						
	3	5	7	10	12	15	
Data	-0.452	-1.015	-1.491	-2.091	-2.410	-2.988	
$A_0^{(SR)}$ (3)	-0.695	-1.137	-1.423	-1.579	-1.390	-1.410	
	(-1.530;0.137)	(-2.470;-0.255)	(-3.169;-0.244)	(-3.911;-0.068)	(-3.996;0.216)	(-4.389;0.457)	
$A_1^{(SR)}$ (3)	1.767	1.998	2.030	1.909	1.562	1.547	
	(-0.698;3.525)	(-0.975;4.008)	(-1.329;4.242)	(-1.887;4.389)	(-2.332;4.104)	(-3.026;4.563)	
$A_2^{(SR)}$ (3)	1.277	1.342	1.440	1.619	1.744	1.932	
	(0.824;1.585)	(0.850;1.746)	(0.882;1.936)	(0.955;2.239)	(1.014;2.453)	(1.099;2.747)	
$A_0^{(RS)}$ (3)	-0.037	-0.106	-0.315	-0.720	-1.080	-1.521	
	(-2.376;1.273)	(-2.192;1.677)	(-2.242;1.892)	(-2.736;2.132)	(-3.056;2.092)	(-3.807;2.338)	
$A_1^{(RS)}$ (3)	0.389	0.062	-0.271	-0.807	-1.262	-1.764	
	(-0.873;1.502)	(-1.667;1.542)	(-2.426;1.549)	(-3.624;1.497)	(-4.423;1.193)	(-5.619;1.194)	
$A_2^{(RS)}$ (3)	0.221	-0.022	-0.113	-0.191	-0.221	-0.281	
	(-0.169;0.833)	(-0.657;0.913)	(-1.009;1.023)	(-1.454;1.127)	(-1.693;1.162)	(-2.006;1.203)	

This table reports MCMC estimates and confidence bands of the regime dependent parameters for all regime switching affine term structure models. The parameter estimate is the average of every 100'th iteration of the estimation sample consisting of 300000 iteration (i.e. the variance calibration sample and a burn-in period are excluded). The confidence bounds reported in parenthesis indicate the 95% confidence interval.



TABLE 3.8: Volatility Regression

		Maturity in Years									
		1	3	5	7	10	12	15			
Data	Level	0.214	0.117	0.075	0.061	0.052	0.048	0.047			
	Slope	0.364	0.210	0.134	0.108	0.095	0.088	0.084			
	Curvature	0.938	0.489	0.275	0.204	0.170	0.162	0.161			
$A_1(3)^{(SR)}$	Level	0.005	0.004	0.003	0.002	0.002	0.001	0.001	0.001		
		(-0.002;0.012)	(-0.002;0.009)	(-0.001;0.007)	(-0.001;0.006)	(-0.001;0.004)	(-0.001;0.004)	(-0.001;0.004)	(-0.001;0.003)		
	Slope	-0.130	-0.109	-0.086	-0.068	-0.049	-0.039	-0.034	-0.034		
		(-0.147;-0.111)	(-0.123;-0.094)	(-0.097;-0.074)	(-0.077;-0.058)	(-0.057;-0.042)	(-0.045;-0.032)	(-0.039;-0.028)			
$A_2(3)^{(SR)}$	Curvature	-0.183	-0.154	-0.121	-0.096	-0.070	-0.055	-0.048			
		(-0.217;-0.146)	(-0.180;-0.125)	(-0.143;-0.099)	(-0.114;-0.077)	(-0.084;-0.055)	(-0.067;-0.042)	(-0.059;-0.036)			
	Level	0.056	0.037	0.030	0.025	0.022	0.021	0.019			
		(-0.006;0.119)	(-0.006;0.080)	(-0.004;0.064)	(-0.003;0.055)	(-0.003;0.048)	(-0.002;0.044)	(-0.002;0.040)			
$A_1(3)^{(RS)}$	Slope	0.209	0.139	0.111	0.096	0.083	0.077	0.070			
		(-0.016;0.439)	(-0.014;0.293)	(-0.019;0.235)	(-0.012;0.205)	(-0.006;0.176)	(-0.006;0.164)	(-0.006;0.148)			
	Curvature	0.382	0.255	0.202	0.174	0.151	0.141	0.128			
		(-0.031;0.810)	(-0.025;0.537)	(-0.037;0.426)	(-0.025;0.374)	(-0.014;0.325)	(-0.011;0.300)	(-0.013;0.272)			
$A_1(3)^{(RS)}$	Level	0.000	0.000	0.000	0.000	0.000	0.000	0.000			
		(-0.007;0.005)	(-0.003;0.002)	(-0.002;0.002)	(-0.002;0.001)	(-0.002;0.001)	(-0.002;0.001)	(-0.002;0.001)			
	Slope	0.042	0.026	0.020	0.017	0.014	0.012	0.011			
		(0.024;0.057)	(0.019;0.031)	(0.015;0.025)	(0.013;0.021)	(0.010;0.018)	(0.009;0.016)	(0.008;0.015)			
$A_1(3)^{(RS)}$	Curvature	0.054	0.033	0.026	0.022	0.018	0.016	0.015			
		(0.021;0.083)	(0.024;0.043)	(0.019;0.033)	(0.016;0.027)	(0.013;0.023)	(0.011;0.021)	(0.010;0.020)			

TABLE 3.8: Volatility Regression

		Maturity in Years										
		1	3	5	7	10	12	15				
$A_2(\beta)^{(RS)}$	Level	0.067 (0.043;0.091)	0.004 (-0.007;0.011)	0.004 (-0.004;0.009)	0.004 (-0.002;0.008)	0.004 (0.000;0.007)	0.004 (0.000;0.007)	0.004 (0.000;0.007)	0.004 (0.001;0.006)			
	Slope	0.082 (0.034;0.156)	0.013 (-0.002;0.053)	0.009 (-0.002;0.035)	0.007 (-0.002;0.025)	0.005 (-0.002;0.018)	0.004 (-0.002;0.015)	0.004 (-0.002;0.012)	0.004 (-0.002;0.012)			
Curvature		0.115 (0.048;0.201)	0.016 (-0.004;0.064)	0.012 (-0.001;0.043)	0.010 (0.000;0.033)	0.008 (0.001;0.024)	0.007 (0.001;0.021)	0.007 (0.001;0.021)	0.006 (0.001;0.018)			

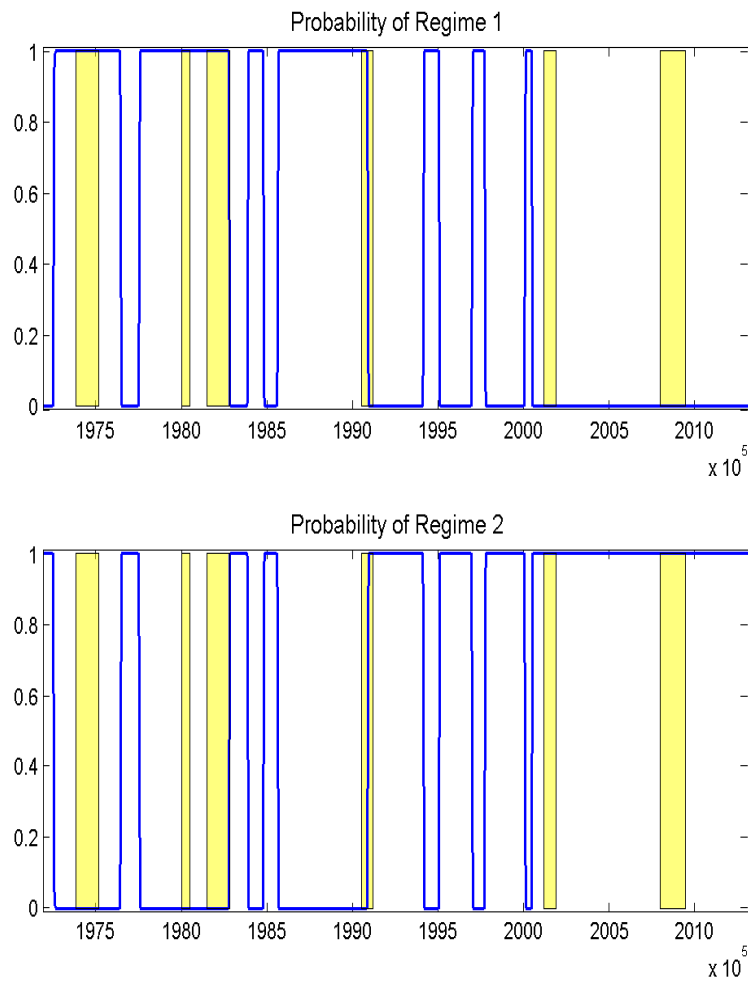
This table reports estimated slope coefficients of the volatility regression. The regression is given by  $[Y(t+1, \tau) - Y(t, \tau)]^2 = \alpha(\tau) + \beta_1(\tau)Y(t, 1) + \beta_2(\tau)Y(t, 15) + \beta_3(\tau)[Y(t, 1) + Y(t, 15) - 2 \times Y(t, 7)] + \epsilon(t, \tau)$  where the maturities are denoted in years.  $\beta_1(\tau)$  is the coefficient associated with the level,  $\beta_2(\tau)$  is related with the slope while  $\beta_3(\tau)$  is linked with the curvature. The table compares regression coefficients obtained from actual data with regression coefficients based on simulated yields (in order to account for finite-sample bias). The estimates in the parenthesis indicate the 95% confidence interval. The sample period is from 11/1971-01/2011.

TABLE 3.9: GARCH(1,1) model

Maturity	1		3		5		7	
	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$
Actual Yields	0.236	0.764	0.138	0.840	0.096	0.867	0.085	0.883
$A_1^{(SR)}(3)$	0.028 (0.002;0.109)	0.716 (0.099;0.932)	0.030 (0.001;0.116)	0.709 (0.087;0.929)	0.030 (0.001;0.112)	0.714 (0.089;0.933)	0.030 (0.001;0.102)	0.714 (0.112;0.950)
$A_2^{(SR)}(3)$	0.032 (0.002;0.105)	0.717 (0.110;0.932)	0.033 (0.001;0.105)	0.724 (0.122;0.937)	0.034 (0.002;0.102)	0.727 (0.115;0.944)	0.034 (0.001;0.102)	0.736 (0.112;0.950)
$A_1^{(RS)}(3)$	0.046 (0.006;0.123)	0.820 (0.210;0.933)	0.049 (0.007;0.128)	0.827 (0.241;0.934)	0.052 (0.006;0.128)	0.835 (0.237;0.933)	0.053 (0.008;0.130)	0.840 (0.239;0.933)
$A_2^{(SR)}(3)$	0.038 (0.002;0.138)	0.776 (0.100;0.936)	0.069 (0.019;0.166)	0.905 (0.684;0.943)	0.059 (0.020;0.130)	0.911 (0.672;0.950)	0.051 (0.011;0.131)	0.906 (0.568;0.954)
Maturity	10		12		15			
	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$		
Actual Yields	0.102	0.863	0.113	0.845	0.118	0.838		
$A_1^{(SR)}(3)$	0.028 (0.001;0.123)	0.715 (0.077;0.941)	0.028 (0.001;0.121)	0.709 (0.062;0.944)	0.028 (0.002;0.128)	0.717 (0.084;0.946)		
$A_2^{(SR)}(3)$	0.034 (0.002;0.101)	0.750 (0.119;0.954)	0.033 (0.001;0.103)	0.744 (0.121;0.955)	0.033 (0.001;0.102)	0.743 (0.115;0.961)		
$A_1^{(RS)}(3)$	0.056 (0.011;0.126)	0.847 (0.298;0.935)	0.057 (0.011;0.126)	0.853 (0.303;0.939)	0.058 (0.011;0.126)	0.857 (0.310;0.943)		
$A_2^{(SR)}(3)$	0.045 (0.008;0.116)	0.902 (0.506;0.956)	0.044 (0.008;0.124)	0.899 (0.286;0.956)	0.043 (0.007;0.139)	0.896 (0.241;0.956)		

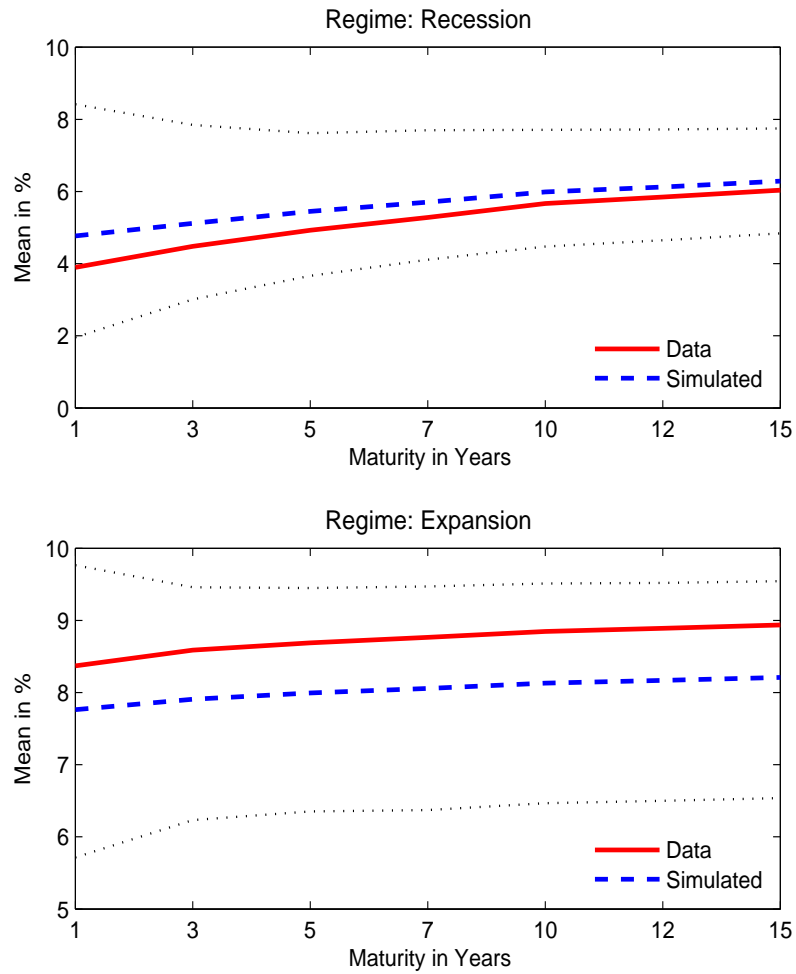
The table presents the Maximum Likelihood estimates of a GARCH(1,1) model:  $\sigma^2 t = c + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2$ , where  $\epsilon_t$  is the innovation from the AR(1) representation of the level of the yields. The estimates in the parenthesis indicate the 95% confidence interval. The sample period is from 11/1971-01/2011.

FIGURE 3.1: Regime Probabilities



This figure reports a time series of posterior probabilities that the economy is in regime 1 and regime 2, respectively, for the  $A_2^{(RS)}(3)$ .

FIGURE 3.2: Actual and Model Implied Unconditional Means



This figure reports the unconditional means of the yields for all considered maturities for the  $A_2^{(RS)}(3)$  model. Unconditional means are in % and the dotted lines indicate the 95% confidence interval.

### 3.A Derivation of $A(\tau, k)$ and $B(\tau)$

The price  $P(t, \tau, k)$ , of a ZCB at time  $t$ , with maturity  $\tau$  and under regime  $k$  satisfies the following PDDE:

$$\begin{aligned} \frac{1}{2} \text{Tr} \left( \frac{\partial^2 P}{\partial X \partial X'} \Sigma \sigma(x_t) \Sigma' \right) + \frac{\partial P}{\partial X'} \left( \kappa \left( \theta^{(k)} - X_t \right) \right) + \frac{\partial P}{\partial \tau} - \left( \delta_0^{(k)} + \delta_X' X_t \right) P(\tau, X_t, k) \\ + \sum_{j=1, j \neq k}^K Q_{k,j} (P(\tau, X_t, j) - P(\tau, X_t, k)) = 0 \end{aligned}$$

We conjecture that the solution to the above PDDE takes the form:

$$P(t, \tau, k) = e^{A(\tau, k) + B(\tau)' X_t}$$

Computing then the partial derivatives we obtain:

$$\begin{aligned} \frac{\partial P}{\partial X} &= B(\tau)' P(\tau, X_t, k) \\ \frac{\partial^2 P}{\partial X \partial X'} &= B(\tau) B(\tau)' P(\tau, X_t, k) \\ \frac{\partial P}{\partial \tau} &= \left\{ \frac{dA(\tau, k)}{d\tau} + \frac{dB(\tau)'}{d\tau} X_t \right\} P(\tau, X_t, k) \end{aligned}$$

where we used the the fact that  $\frac{\partial A(\tau, k)}{\partial \tau} \frac{\partial \tau}{\partial t} = - \left( \frac{\partial A(\tau, k)}{\partial \tau} \right)$ . Note that the same reasoning applies for  $B(\tau)$ . Substituting the partial derivatives in the PDDE and rearranging the terms (recalling that  $[\sigma(X_t)]_{ii} = \alpha_i + \beta_i' X_t$ ), yields:

$$\begin{aligned} \left\{ \frac{1}{2} \sum_{i=1}^m [\Sigma' B(\tau)]_i^2 \beta_i - \kappa_1' B(\tau) - \delta_X - \frac{dB(\tau)'}{d\tau} \right\} X_t P(\tau, X_t, k) + \left\{ \frac{1}{2} \sum_{i=1}^m [\Sigma' B(\tau)]_i^2 \alpha_i \right. \\ \left. + \kappa_0^{(k)'} B(\tau) - \delta_0^{(k)} + \sum_{j=1, j \neq k}^K Q_{k,j} \left( e^{A(\tau, j) - A(\tau, k)} - 1 \right) - \frac{dA(\tau, k)}{d\tau} \right\} P(\tau, X_t, k) = 0 \end{aligned}$$

This must hold  $\forall X$  and  $k$ . Thus,

$$\begin{aligned} \frac{1}{2} \sum_{i=1}^m [\Sigma' B(\tau)]_i^2 \beta_i - \kappa'_1 B(\tau) - \delta_X - \frac{dB(\tau)}{d\tau} &= 0 \\ \frac{1}{2} \sum_{i=1}^m [\Sigma' B(\tau)]_i^2 \alpha_i + \kappa_0^{(k)'} B(\tau) - \delta_0^{(k)} + \sum_{j=1, j \neq k}^K Q_{k,j} \left( e^{A(\tau,j) - A(\tau,k)} - 1 \right) - \frac{dA(\tau, k)}{d\tau} &= 0. \end{aligned}$$

Solving for  $\frac{dB(\tau)}{d\tau}$  and  $\frac{dA(\tau, k)}{d\tau}$  we obtain the following system of ODE's:

$$\begin{aligned} \frac{dB(\tau)}{d\tau} &= \frac{1}{2} \sum_{i=1}^m [\Sigma' B(\tau)]_i^2 \beta_i - \kappa'_1 B(\tau) - \delta_X \\ \frac{dA(\tau, k)}{d\tau} &= \frac{1}{2} \sum_{i=1}^m [\Sigma' B(\tau)]_i^2 \alpha_i + \kappa_0^{(k)'} B(\tau) - \delta_0^{(k)} + \sum_{j=1, j \neq k}^K Q_{k,j} \left( e^{A(\tau,j) - A(\tau,k)} - 1 \right) \end{aligned}$$

### 3.B MCMC Algorithm

In the following section we describe the MCMC algorithm for our particular RS-ATSM where we allow for two regimes. First, we briefly review the conditional distributions which are used in the sampling procedures.

#### The Conditionals

The conditional density of the latent variables is given as:

$$\begin{aligned} p(\mathbf{X}|\mathbf{K}, \Theta) &= \prod_{t=1}^{T-1} p(X_{t+1}|X_t, K_t) \\ &= \prod_{n=1}^N \left( \left( \prod_{t=1}^{T-1} \frac{1}{\sqrt{[\sigma(X_t)]_{nn}}} \right) \exp \left( -\frac{1}{2\Delta_t} \sum_{t=1}^{T-1} \frac{[\Delta X_{t+1} - \mu_t^{\mathbb{P},(k)} \Delta_t]_n^2}{[\sigma(X_t)]_{nn}} \right) \right) \end{aligned}$$

where we assumed an independent prior for  $X_0$ .

We denote the model implied yields at time  $t$  by

$$\hat{Y}(t, \tau, k) = A^*(\tau, k) + B^*(\tau)X_t.$$

$A^*(\tau, k)$  is regime-dependent scalar and  $B^*(\tau)$  is a  $1 \times N$  vector. Thus, the density  $p(Y|\Theta, X, k)$  can be written as:

$$\begin{aligned} p(\mathbf{Y}|\Theta, \mathbf{X}, \mathbf{K}) &= \prod_{\tau=1}^M \prod_{t=1}^T H_{\tau\tau}^{-\frac{1}{2}} \exp \left( -\frac{(Y(t, \tau) - \hat{Y}(t, \tau, k_t))^2}{2H_{\tau\tau}} \right) \\ &= \frac{1}{\sigma^{MT}} \exp \left( -\frac{1}{2\sigma^2} \sum_{t=1}^T (\epsilon(t, k_t)' \epsilon(t, k_t)) \right) \end{aligned}$$

where  $\epsilon(t, k_t) = Y(t, \tau) - \hat{Y}(t, \tau, k_t)$ .



In addition to these two conditionals, the hybrid MCMC algorithm also depends on the evaluation of the regime variable:

$$p(\mathbf{K}|\Theta) = \prod_{t=1}^{T-1} (\exp(Q\Delta_t))_{k_t, k_{t+1}}$$

The matrix exponential together with the two conditionals are the main building blocks of the MCMC algorithm.

## Random-Walk Metropolis-Hastings and Gibbs Sampling Procedures

### Sampling the latent regimes

The regime variable is sampled using a RW-MH algorithm. For each of the regimes  $k_t = 1, \dots, S$ , at time  $t = 1, \dots, T - 1$  the conditional of  $k_t$  is given as:

$$p(k_t|k_{\setminus t}, X, \Theta, Y) \propto p(Y_t|X_t, k_t, \Theta) \times p(k_t|k_{t-1}, \Theta) \times \\ p(k_{t+1}|k_t, \Theta) \times p(X_t|X_{t-1}, k_{t-1}, \Theta)$$

In particular, for  $t = 2, 3, \dots, T - 1$  we calculate:

$$p(k_t = 1|.) \propto \exp\left(-\sum_{\tau=1}^M \frac{(Y(t, \tau) - \hat{Y}(t, \tau, 1))^2}{2H_{\tau\tau}^2}\right) \exp(Q\Delta_t)_{k_{t-1}, 1} \exp(Q\Delta_t)_{1, k_{t+1}} \\ \frac{1}{\sqrt{\sigma(X_{t-1})}} \exp\left(-\frac{1}{2\Delta_t} \varepsilon_t^{(1)} ((\sigma(X_{t-1}))^{-1} \varepsilon_t^{(1)})'\right) \equiv \alpha_1$$

$$p(k_t = 2|.) \propto \exp\left(-\sum_{\tau=1}^M \frac{(Y(t, \tau) - \hat{Y}(t, \tau, 2))^2}{2H_{\tau\tau}^2}\right) \exp(Q\Delta_t)_{k_{t-1}, 2} \exp(Q\Delta_t)_{2, k_{t+1}} \\ \frac{1}{\sqrt{\sigma(X_{t-1})}} \exp\left(-\frac{1}{2\Delta_t} \varepsilon_t^{(2)} ((\sigma(X_{t-1}))^{-1} \varepsilon_t^{(2)})'\right) \equiv \alpha_2$$

where  $\varepsilon_{t+1}^{(k)} = \Delta X_{t+1} - \mu_t^{\mathbb{P}, (k)}$  for  $k = 1, 2$ . We define  $\tilde{\alpha} = \frac{\alpha_1}{(\alpha_1 + \alpha_2)}$  and draw  $u = \text{unifrnd}(0, 1)$ . We set  $k_t = 1$  if  $u < \tilde{\alpha}_1$  and  $k_t = 2$  otherwise.

For  $t = 1$  the posterior distribution is as

$$p(k_1|\cdot) \propto \exp\left(-\sum_{\tau=1}^M \frac{(Y(t, \tau) - \hat{Y}(t, \tau, k_1))^2}{2H_{\tau\tau}^2}\right) \exp(Q\Delta t)_{k_1, k_2},$$

while for  $t = T$  the posterior is given by

$$p(k_T|\cdot) \propto \exp\left(-\sum_{\tau=1}^M \frac{(Y(T, \tau) - \hat{Y}(T, \tau, k_T))^2}{2H_{\tau\tau}^2}\right) \exp(Q\Delta t)_{k_{T-1}, k_T} \\ \frac{1}{\sqrt{\sigma(X_{T-1})}} \exp\left(-\frac{1}{2\Delta t} \varepsilon_T^{(k_T)} (\sigma(X_{T-1}))^{-1} \varepsilon_T^{(k_T)'}\right).$$

### Sampling the latent factors

The latent state variables  $X_t$ , for  $t = 1, 2, \dots, T$  are sampled using a RW-MH algorithm.

For  $t = 2, \dots, T - 1$  the conditional of  $X_t$  is given as

$$p(X_t|X_{\setminus t}, k, \Theta, Y) \propto p(Y_t|X_t, k_t, \Theta) \times p(X_t|X_{t-1}, k_{t-1}, \Theta) \times p(X_{t+1}|X_t, k_t, \Theta).$$

For  $t = 1$  the conditional is

$$p(X_1|X_{\setminus X_1}, k, \Theta, Y) \propto p(Y_1|X_1, k_1, \Theta)p(X_2|X_1, k_1, \Theta)$$

while for  $t = T$  the conditional is

$$p(X_T|X_{\setminus X_T}, k_T, \Theta, Y) \propto p(Y_T|X_T, k_T, \Theta)p(X_T|X_{T-1}, k_{T-1}, \Theta)$$

The latent state variables are subject to constraints (e.g. the latent variables entering the volatility are constrained to be positive) hence if a draw violates the constraint it is discarded. The latent factor are sampled using a RW-MH procedure. In particular, we sample new  $X_t^{\text{new}} = X_t^{\text{old}} + \gamma N(0, 1)$  where  $\gamma$  is calibrated and calculate the below posterior

distribution:

$$p(X_t|\cdot) \propto \exp\left(-\sum_{\tau=1}^M \frac{(Y(t, \tau) - \hat{Y}(t, \tau, k))^2}{2H_{\tau\tau}^2}\right) \frac{1}{\sqrt{\sigma(X_t)}} \exp\left(-\frac{1}{2\Delta_t} \varepsilon_{t+1}^{(k)} (\sigma(X_t))^{-1} \varepsilon_{t+1}^{(k)'}\right) \frac{1}{\sqrt{\sigma(X_{t-1})}} \exp\left(-\frac{1}{2\Delta_t} \varepsilon_t^{(k)} (\sigma(X_{t-1}))^{-1} \varepsilon_t^{(k)'}\right).$$

We set  $\alpha = \frac{p(X_t^{\text{new}}|\cdot)}{p(X_t^{\text{old}}|\cdot)}$  and sample  $u = \text{unifrnd}(0, 1)$ . We accept  $X_t^{\text{new}}$  if  $u < \alpha$  and reject otherwise. The parameter  $\gamma$  is calibrated such that the acceptance ratio is between 10% and 30%.

For  $t = 1$  the posterior distribution is as

$$p(X_1|\cdot) \propto \exp\left(-\sum_{\tau=1}^M \frac{(Y(t, \tau) - \hat{Y}(t, \tau, k))^2}{2H_{\tau\tau}^2}\right) \frac{1}{\sqrt{\sigma(X_1)}} \exp\left(-\frac{1}{2\Delta_t} \varepsilon_2^{(k)} (\sigma(X_1))^{-1} \varepsilon_2^{(k)'}\right),$$

while for  $t = T$  the posterior is given by

$$p(X_T|\cdot) \propto \exp\left(-\sum_{\tau=1}^M \frac{(Y(T, \tau) - \hat{Y}(T, \tau, k))^2}{2H_{\tau\tau}^2}\right) \frac{1}{\sqrt{\sigma(X_{T-1})}} \exp\left(-\frac{1}{2\Delta_t} \varepsilon_T^{(k)} (\sigma(X_{T-1}))^{-1} \varepsilon_T^{(k)'}\right).$$

### Sampling the model parameters

The model parameters are sampled using a RW-MH procedure. In particular, we sample  $\Theta_t^{\text{new}} = \Theta_t^{\text{old}} + \gamma N(0, 1)$  where  $\gamma$  is calibrated. The posterior distribution of the model

parameter is given by a subset of the below conditionals:

$$p(\Theta|\cdot) \propto \exp\left(-\sum_{t=1}^T \sum_{\tau=1}^M \frac{(Y_{t,\tau} - \hat{Y}_{t,\tau, k})^2}{2H_{\tau\tau}^2}\right) \exp(Q\Delta_t)_{k_{t-1}, k_t} \\ \left(-\frac{1}{\sqrt{\sigma(X_{t-1})}} \exp\left(\frac{1}{2\Delta_t} \varepsilon_t^{(k_t)} (\sigma(X_{t-1}))^{-1} \varepsilon_t^{(k_t)'}\right)\right).$$

We set  $\alpha = \frac{p(\Theta^{\text{new}}|\cdot)}{p(\Theta^{\text{old}}|\cdot)}$  and sample  $u = \text{unifrnd}(0, 1)$ . We accept  $\Theta_t^{\text{new}}$  if  $u < \alpha$  and reject otherwise. The parameter  $\gamma$  is calibrated such that the acceptance ratio is between 10% and 30%.

### Sampling the measurement

The conditional of the variance of the measurement errors is given as:

$$p(D|\Theta_D, X, K, Y) \propto p(Y|\Theta, X)$$

This implies that  $\sigma^2$  can be Gibbs sampled from an inverse Gamma distribution,  $\sigma^2 \sim IG(\sum_{t=1}^T \varepsilon(t, k_t) \varepsilon(t, k_t)', MT)$ .

### 3.C The Bayes Factor

In this section, we provide details on how to compute the Bayes factor for model comparison. The Bayes Factor summarizes the evidence provided by the data in favor of one of the models considered compared to another, and is given by the ratio of the marginal probabilities of the data under the two models:

$$B = \frac{p(D|M_1)}{p(D|M_2)}$$

When dealing with known single distributions and no free parameters this is just the likelihood ratio. In our case, where we have latent state variables and regimes and unknown parameters, to obtain the marginal probabilities of the data  $p(D)$  we need to integrate out all model parameters, latent factors and regime variables.<sup>16</sup>

#### Integrate out the latent state variables and regimes

For each time point  $t = 1, 2, \dots, T$  we compute:

1. For each  $t = 1, 2, \dots, T$  and  $k = 1, 2, \dots, K$  we simulate:

$$s_t^{(k)} \propto \exp \{Q \Delta_t\}_{s_{t-1}, k}$$

2. Having obtained the regime we proceed by simulating the latent state variables given the regime at the particular time step  $X_t$ .
3. We then integrate out the latent regimes and the latent state variables to obtain:

$$\begin{aligned} p(y_t|\Theta) &= \int p(y_t | \Theta_t, X_t, s_t) p(X_t | \cdot) p(s_t | \cdot) dX_t ds_t \\ &= \frac{1}{K} \sum_{k=1}^K \left( \prod_{m=1}^M \exp \left\{ -\frac{1}{2} \frac{(y_t^m - \hat{y}_{s_t}^m)^2}{\sigma_m^2} \right\} \right) \end{aligned}$$

4. Filter the regime for each time point,  $s_t^{(k)}$ , for  $k = 1, 2, \dots, K$ :

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<sup>16</sup>This implementation is an adaptation of the procedure described in Li, Li, and Yu (2011) adjusted for the presence of latent state variables.

$$\begin{aligned} p(s_t^{(1)}|\cdot) &\propto p(y_t|\cdot) p(X_t|\cdot) \times \frac{1}{K} \sum_{k=1}^K \left\{ \exp \{Q\Delta_t\}_{s_{t-1},1} \right\} \\ &\equiv \alpha_1 \end{aligned}$$

$$\begin{aligned} p(s_t^{(2)}|\cdot) &\propto p(y_t|\cdot) p(X_t|\cdot) \times \frac{1}{K} \sum_{k=1}^K \left\{ \exp \{Q\Delta_t\}_{s_{t-1},2} \right\} \\ &\equiv \alpha_2 \end{aligned}$$

We then draw  $u \sim \text{Bernoulli}\left(\frac{\alpha_1}{\alpha_1 + \alpha_2}\right)$  and if  $u = 1$  we assign  $s_t = 1$ , otherwise if  $u = 0$  we assign  $s_t = 2$ .

5. We simulate new  $X_t$ 's given the regimes filtered above and start over the procedure from step 1 for the next time point.

Once we have carried out this procedure up to time  $t = T$  we obtain:

$$p(D|\Theta^{(g)}) = \prod_{t=1}^T \left( \frac{1}{K} \sum_{k=1}^K \left( \prod_{m=1}^M \exp \left\{ -\frac{1}{2} \frac{(y_t^m - \hat{y}_{s_t}^m)^2}{\sigma_m^2} \right\} \right) \right)$$

### Integrate out the parameters

Having obtained  $p(D|\Theta^{(g)})$  we integrate out the parameters to obtain the posterior distribution of the data:

$$p(D) = \int p(D|\Theta)\pi(\Theta)d\Theta$$

where  $\pi(\Theta)$  is the prior distribution of the parameters. Since this is not known, we use an importance function  $\pi^*(\Theta)$  to calculate  $p(D)$ , which for a large number of simulations  $g = 1, 2, \dots, G$  approximates the true distribution:

$$p(D) = \frac{\sum_{g=1}^G w_g p(D|\Theta^{(g)})}{\sum_{g=1}^G w_g}, \quad \text{where } w_g = \frac{\pi(\Theta^{(g)})}{\pi^*(\Theta^{(g)})}$$

Choosing  $\pi^*(\Theta) = \frac{p(D|\Theta)\pi(\Theta)}{p(D)}$  we obtain<sup>17</sup>:

$$p(D) = \left( \frac{1}{G} \sum_{g=1}^G p(D|\Theta^{(g)})^{-1} \right)^{-1}$$

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<sup>17</sup>See Kass and Raftery (1995) for a detailed discussion of the choice of the importance function

## Conclusion

This thesis contains two essays about return predictability and an essay about term structure models. The first essay sheds some light on the predictability of the U.S. equity premia while the second essay predicts exchange rates. Finally, in the last essay we develop a regime-switching Affine Term Structure model with a stochastic volatility feature and compare its performance with several benchmark models.

More precisely, the first essay covers the predictability of the U.S. equity premia in the presence of structural breaks such as changes in monetary policy, macroeconomic instability, new regulations etc. As a consequence of such structural breaks the out-of-sample predictability of the U.S. equity premia diminishes. By using an approach which accounts for structural breaks we do not only statistically outperform several benchmark models but also economically. In the second essay we predict a basket of exchange rates. As a novelty we base our predictions on a large macro-finance data set which mirrors the current state of the economy rather than a few predictor variables. Our in-sample analysis finds evidence that macro-finance variables are indeed informative about future exchange rate movements and that the currency risk premia exhibit a strong counter-cyclical behavior. We also find some nil evidence of out-of-sample predictability, however, we do not always outperform the benchmark models. In the last essay we develop a regime-switching Affine Term Structure model with stochastic volatility. We find evidence that this model outperforms single-regime models as well as regime-switching Gaussian models in terms of goodness of fit. Additionally, we also show that this model successfully replicates features of the U.S. yield curve such as predictability of bond returns, the persistence and time-variability in conditional yield volatilities, as well as the term structure of the unconditional means.

The insights of the first two essays should be combined to get a better understanding of the predictability literature. We show that by using a method which considers structural breaks and by conditioning the predictions on large amount of macro-finance data forecast performance improves. However, out-of-sample predictability of the equity as well as currency returns is still controversial and additional work is needed to understand the characterization of the equity risk premia and the currency risk premia.



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