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Minna Selene Svane
Growth, Education and the Environment

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Growth, Education and the Environment

by

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Contents

Summary

This dissertation consists of five self-contained chapters on fiscal policy within a two sector endogenously growing economy. The main focus of the dissertation is on educational and environmental issues and in particular on the optimal subsidy to education and the optimal environmental policy. The frameworks, which are used to investigate these issues, are all extensions of the Uzawa–Lucas model of endogenous growth. Chapter 1 and 2 investigate the effects of factor income taxation and subsidization of educational effort, whereas Chapter 3, 4 and 5 investigate the transitional dynamics and the long run effects of environmental policy.

The *first* chapter examines the effects of factor income taxation and subsidization of educational effort in a two sector model of endogenous growth. There is an education externality in the final goods sector and a public input in the education sector. The main result is that labor income taxation as well as subsidization of educational effort have positive effects on the balanced growth rate in the market economy, ceteris paribus. Furthermore, the optimal subsidy rate is unambiguously positive when there are substitution possibilities between public and private input factors in education. Labor income should be taxed at a lower rate than capital income as long as there is a positive education externality in final goods production. The bigger the externality is, the bigger is the difference between the two tax rates. Finally, welfare maximization is not equivalent to growth maximization. In fact, it is only welfare improving to increase the labor income tax and the subsidy to educational effort as long as they are smaller than their optimal levels.

The *second* chapter analyzes the effects of a training leave benefit in a two sector model of endogenous growth, where unemployment is created by the existence of monopoly labor unions. There is a final goods sector and an education sector that provides the facilities to upgrade the skills of the labor force. The main result is that it is optimal to charge a tuition fee instead of giving a benefit in an economy with involuntary unemployment caused by monopoly labor unions. Furthermore, it is only welfare improving to undertake growth enhancing measures as long as the tuition fee is bigger than its optimal level. Moreover, an increase in the training leave benefit leads to a higher rate of balanced growth and at the same time to a lower rate of unemployment. Finally, an increase in the training leave benefit leads to the same transitional

dynamics in the consumption–capital and the capital–labor ratio as an increase in the labor income tax, but the latter has a negative growth rate effect.

The *third* chapter analyses the effects of environmental policy within a two sector endogenously growing economy with pollution. Pollution is either generated by production or by the use of physical capital in production, and can be reduced by public abatement activities. In this generalized Uzawa–Lucas model, the effects of fiscal policy are derived for all core variables and ratios. In addition, the optimal taxation rules are derived. If a pollution tax is not available it turns out that a first best solution may be reached by use of factor income taxation. Additionally, the effects and the possibility of environmental policy are complemented for a small open economy.

The *fourth* chapter simulates the transition path of environmental policy within the two sector endogenous growth model, which was developed and analyzed theoretically in Chapter 3. The policy change is either implemented suddenly, previously announced, or gradually. From a strict welfare point of view, the best policy is the unannounced policy scheme, but in our point of view, the best policy recommendation is a gradual policy scheme. Firstly, it stretches out the adjustment process and secondly the associated welfare loss in comparison with an unannounced policy is negligible. Another main result is that all of the environmental policy schemes only lead to a reduction of the long term growth rate from 2% to 1.98%, when the abatement–output ratio doubles from 1.6% to 3.2%. Qualitative and quantitative results are robust to most parameter changes. However, transitional dynamics are sensitive to changes in the shares of physical capital in production and education.

The *fifth* chapter investigates the effects of an emission standard and taxation within a two sector endogenous growth model with pollution. There are two regimes characterized by a non–binding and a binding emission standard, respectively. The main result is that sustained growth *is* possible, when environmental concerns are taken into account. Furthermore, the outcome of a decentralized economy is inefficient. A capital income tax or a pollution tax is therefore required to reach a first best outcome. If the capital income tax is unavailable as an instrument, then the optimal pollution tax equals the optimal marginal damage of pollution. However, the optimal pollution tax may be below its Pigouvian level, when the optimal capital income tax is high, since a tax on capital income works as an indirect tax on pollution.

Optimal Taxation in a Two Sector Model of Endogenous Growth*

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December 8, 1998

Abstract

This paper examines the effects of factor income taxation and subsidization of educational effort in a two sector model of endogenous growth. There is an education externality in the final goods sector and a public input in the education sector. The main result is that labor income taxation as well as subsidization of educational effort have positive effects on the balanced growth rate in the market economy, ceteris paribus. Furthermore, the optimal subsidy rate is unambiguously positive when there are substitution possibilities between public and private input factors in education. Labor income should be taxed at a lower rate than capital income as long as there is a positive education externality in final goods production. The bigger the externality is, the bigger is the difference between the two tax rates. Finally, welfare maximization is not equivalent to growth maximization.

JEL classification: D62, E62, H21, I28, O41
Keywords: Endogenous growth, education externality, optimal taxation, public spending on education

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1. Introduction

This paper analyzes the effects of factor income taxation and subsidization of educational effort in a two sector model of endogenous growth with public investment in education and a positive education externality in final goods production.

The relevance of such an analysis is supported by three recent empirical papers. Firstly, [4] Mendoza, Milesi–Ferretti & Asea (1995) show that there are significant negative investment effects from factor income taxation and that these are consistent with small negative growth rate effects. Secondly, [2] Hansson & Henrekson (1994) show that educational expenditure by the government has a positive effect on productivity growth, while for instance government transfers and consumption have negative growth rate effects. Thirdly, [1] Barro & Sala–i–Martin (1995) undertake an empirical analysis of a cross section of 87 countries and also find that public spending on education has a positive effect on the growth rate.

The present paper is inspired by four papers, namely [7], [6] Roubini & Milesi–Ferretti (1994a, b), [5] Pecorino (1993) and [9] Sørensen (1993). The first three papers present models that deal with optimal taxation in endogenous growth models, where the tax revenue is redistributed lump sum to consumers. The last paper presents a model, which incorporates government spending on education and training. Thus in contrast to the first three papers, the [9] Sørensen (1993) model uses the tax revenue for a productive purpose.

The first three papers reach different conclusions about the growth maximizing tax structure. The [7], [6] Roubini & Milesi–Ferretti (1994a, b) models have three sectors; a final goods sector that produces both consumption goods and capital goods; an education sector; and a leisure sector. With respect to the latter sector, the authors consider different models of leisure, namely home production, leisure as quality time and no leisure at all. They assume that factor income only arises in the final goods sector, which implies that the optimal taxation rule is to set the capital income tax equal to the labor income tax in a version of their model with no leisure and a balanced government budget. The reason behind this result is that the sectorial allocation of factors is unaffected by taxation as long as both input factors are taxed at a common rate. The only implication of a comprehensive tax is that the interest rate is reduced by one minus the tax rate and that the growth rate is reduced by the fall in the interest rate multiplied by the intertemporal elasticity of substitution. The [5] Pecorino (1993) model also has three sectors; a consumption good sector; a physical capital sector; and an education sector. But in contrast to Roubini & Milesi–Ferretti he assumes that

production in all three sectors give rise to factor income. As a consequence, the factor income from all three sectors is taxed. The implication of this assumption is that the optimal capital income tax is different from the optimal labor income tax except when the factor intensity in the two capital goods sectors as a whole equals the factor intensity in the consumption good sector. Thus, the optimal taxation rule given a balanced budget in the Pecorino (1993) model depends on the relative factor intensity in the three sectors. Thus, a critical assumption is whether factor income arises in all three sectors or just in the final goods sector. Both the [5] Pecorino (1993) model and the [7], [6] Roubini & Milesi-Ferretti (1994a,b) models assume that the government has to raise a revenue in order to finance a lump sum transfer to consumers. As a consequence, factor income taxes neither have to correct a market failure nor an externality in any of these models.

The [9] Sørensen model has two sectors; a final goods sector and an education sector. As in the Roubini & Milesi–Ferretti models, factor income is assumed only to arise in the final goods sector. Compared with the other three models, the Sørensen model assumes that the raised tax revenue is used to finance a public input in education, which is a plausible reason for introducing factor income taxation. Thus, the optimal capital income tax equals the fraction of output which is used on public expenditure on education. Furthermore, the optimal labor income tax depends on the subsidy given to educational effort and is consequently different from the optimal capital income tax. Finally, the optimal subsidy to human capital investment in education can either be positive or negative in the Sørensen model. Unfortunately, these results depend to a large extent on the specification of the production function in the education sector, which is assumed to be of the Leontief form with complementarity between the public and the private input in education.

The model presented in this paper is an extended version of the Uzawa–Lucas model[1] of a closed economy which allows for productive use of the collected tax revenue. Several assumptions are made. First, the representative household is assumed to allocate its entire time between work and education, which means that there is no labor–leisure choice in the model. Second, the government is assumed to be restricted in its ability to borrow and lend, which means that the government budget is balanced every period. Third, factor income is assumed only to arise in the final goods sector, because the human capital input in education usually is left untaxed. Fourth, a

[1] The reproducible input factors – human and physical capital – are used in the final goods sector, whereas the education sector only uses human capital as an input factor, see [3] Lucas (1988).

positive externality due to the level of education is assumed to be present in the final goods sector. Fifth, the tax revenue is used to finance a public input in education and a subsidy to human capital investment in education.

In a large part of the theoretical literature on optimal taxation in endogenous growth models, the tax revenue is redistributed lump sum to consumers, see [7], [6] Roubini & Milesi-Ferretti (1994a, b) and [5] Pecorino (1993). However, this paper presents a model, where the obtained tax revenue is used productively, namely to finance a public input in education and a subsidy to human capital investment in education. Thus, this paper considers the case where government spending affects the productivity of the education sector, whereas previous models typically restrict government spending to be an input in the final goods sector.[2] In addition, the paper allows for a broader set of tax instruments, namely a subsidy to educational effort in addition to factor income taxation. Finally, the paper extends the Sørensen (1993) model by allowing for substitution possibilities between the public and private input in education.

The major conclusions to be drawn are firstly that the growth rate in the market economy is higher when the opportunity cost of education is lower, namely the higher the subsidy rate to educational effort is and the higher the labor income tax rate is. In addition, capital income taxation has no effect on the growth rate in the steady state. Secondly, the optimal subsidy rate is unambiguously positive when the public and private input factors in education are substitutable. Thirdly, human capital grows at a slower pace in the steady state than physical capital as long as there is an education externality in the final goods sector. Finally, labor income should be taxed at a lower rate than capital income when there is a positive spillover from the average skill level to final goods production. Recall that the optimal taxation rule in the Roubini & Milesi–Ferretti models is to set the rate of capital income taxation equal to the rate of labor income taxation.

The basic model is presented in Section 2. Section 3 and 4 derive the balanced growth equilibrium in a centrally planned economy and in a decentralized economy, respectively. Section 5 finds the optimal tax–subsidy structure and Section 6 concludes the paper. The Appendix analyzes an equivalent model, where human capital is assumed to give rise to income in the education sector.

[2] See for instance [1] Barro and Sala-i-Martin (1995), p. 158–159.

2. The Model

This section presents a two sector endogenous growth model of a closed economy. The production side of the economy consists of a large number of identical and perfectly competitive firms. A final goods sector produces consumption goods and physical capital, while an education sector produces human capital. Human capital is assumed to be embodied in people and is consequently a private good, which is both rival and excludable. The final goods sector uses physical capital and human capital as input factors, while the education sector uses human capital and a congested public good. Human capital is assumed to move freely between the two sectors within each period and there is a positive spillover from the average level of human capital to the production of final goods. The government is assumed to tax factor income in order to finance the public input in education and a subsidy to educational effort. Moreover, the government is assumed to balance its budget every period such that total tax revenue equals total government expenditure. The consumption side of the economy consists of a large number of identical infinitely lived households that own the input factors and rent them out to firms. Households are assumed to choose consumption and the allocation of human capital between sectors in order to maximize their life time utility.

2.1. The Final Goods Sector

Firm i produces final goods $Y_{t,i}$ by use of both physical and human capital according to the following Cobb–Douglas production function:

$$Y_{t,i} = AK_{t,i}^{\alpha} \left(u_t H_{t,i}\right)^{1-\alpha} H_t^{\varepsilon} \tag{2.1}$$

where $0 < \alpha < 1$ is the exogenous physical capital share, $\varepsilon \geq 0$ reflects a positive externality in production that arises from the work force's average level of education H_t, which firms do not take into account, A is a productivity parameter, K_t (H_t) is the stock of physical capital (human capital), and $0 < u_t < 1$ is the fraction of human capital that is devoted to the production of final goods. Throughout the paper, depreciation of the physical capital stock is neglected for expositional convenience. Note that due to the presence of the positive education externality, there are constant returns to scale at the firm level, but increasing returns to scale à la Marshall at the aggregate level. Since all firms are identical, the subscripts i may be dropped in the following.

In order to produce final goods, firms rent physical capital from households at the interest rate r_t and hire human capital at the wage rate w_t. Firms are assumed to

maximize their profits:

$$r_t = \alpha A \left(\frac{u_t H_t}{K_t}\right)^{1-\alpha} H_t^{\varepsilon} \tag{2.2}$$

$$w_t = (1-\alpha) A \left(\frac{K_t}{u_t H_t}\right)^{\alpha} H_t^{\varepsilon} \tag{2.3}$$

According to equation (2.2) and (2.3) profits are maximized, when the marginal cost of each factor equals its marginal product. Both factor rewards are seen to increase with the size of the education externality. Note that the private return to human capital is lower than its social return, since firms do not take the positive human capital externality into account. Thus, the smaller the labor share in final goods production and the greater the education externality, the bigger is the difference between the private and the social return to human capital.

2.2. The Education Sector

Production in the education sector is assumed to use two kinds of input factors. These factors are respectively human capital representing students' time[3] and a public good representing buildings and professors.[4] The public good is congested in the sense that it has to increase relative to total output in order to raise the level productivity in education.

In this paper, the production technology in education is assumed to be of the Uzawa–Lucas type with constant returns to human capital. Furthermore, the production function assumes that there is a certain degree of substitutability between the effective labor input $u_t H_t$ and the public input Γ_t.[5] This implies that a small effective labor input, e.g. less qualified students, and a large public input, e.g. a large number of professors lead to the same accumulation of human capital as would a large effective labor input, e.g. well qualified students, and a small public input.

The production technology in the education sector is given by the following function:[6]

$$\dot{H}_t = B\left(1-u_t\right) H_t \Gamma\left(\frac{G_t}{Y_t}\right) \tag{2.4}$$

[3] Recall that human capital is embodied in human beings and therefore is both rival and excludable.

[4] The public sector is assumed to rent the services from school buildings, which implies that the public good in this paper is a flow variable. Some authors have looked at the role of a public good as a stock variable, see e.g. [10] Turnovsky (1996).

[5] Recall that the public and private input factors in education are complements in the [9] Sørensen model.

[6] The [3] Lucas (1988) model is obtained by setting $\Gamma\left(G_t/Y_t\right) = 1$.

where a dot above a variable indicates its derivative with respect to time, B is a productivity parameter, $1-u_t$ is the fraction of human capital that is devoted to education, G_t is the public expenditure on education, $\Gamma(G_t/Y_t)$ is the amount of public input in education, where $\Gamma' > 0$ and $\Gamma'' < 0$. Note that a constant growth rate of labor skills and thereby endogenous growth is obtained, when the time fraction spent in education $(1-u_t)$ and the public input in education $\Gamma\left(G_t/Y_t\right)$ are constant. According to equation (2.4), there is a certain need for infrastructure such as school buildings in the education sector. In the present model, this input is assumed to be publicly financed, but it could just as well have been privately financed. Depreciation of the human capital stock is neglected for expositional convenience.[7]

2.3. The Government

The government taxes households' factor income in order to finance both a subsidy to human capital investment in education, a public input in education, and a lump sum transfer to consumers. The government is assumed to be restricted in its ability to borrow and lend, which implies that it runs a balanced budget in every period. Thus, total tax revenue equals total government expenditure:

$$\tau_k r_t K_t + \tau_h w_t u_t H_t = G_t + s_h w_t \left(1-u_t\right) H_{t,a} + T_t \tag{2.5}$$

where τ_k is the tax rate on capital income, τ_h is the tax rate on labor income, G_t is public expenditure on education, s_h is the subsidy rate on the average level of labor income $w_t\left(1-u_t\right) H_{t,a}$ in the economy which is foregone when households invest in education, and T_t is a lump sum transfer to consumers.[8] The subsidy to educational effort is assumed to depend on the average level of labor income $w_t\left(1-u_t\right) H_{t,a}$, because it probably is the easiest policy to implement in practice. When the subsidy to educational effort depends on the average wage level, the individual household does not take into account that its choice of time spent on education affects the subsidy rate it faces in the future. Alternatively, the subsidy could be assumed to depend on the individual wage level $w_t\left(1-u_t\right) H_t$.

[7] Since older generations are replaced by younger generations and since certain skills deteriorate, if they are not used on a regular basis, it is clearly more realistic to assume that the stock of human capital depreciates.

[8] An implicit assumption in the human capital accumulation function is that taxable income earned by the public input is so small in comparison with the income foregone by students that it is safe to ignore. In Appendix A.1, human capital is treated as a market good that give rise to income in the education sector.

The resource constraint of the entire economy is given by:

$$Y_t = C_t + G_t + \dot{K}_t \tag{2.6}$$

which states that income should equal private and public consumption expenditures and investment in physical capital.

2.4. Households

Households choose consumption C_t and the allocation of human capital between the two sectors u_t in order to maximize their life time utility:

$$U_0 = \int_{t=0}^{\infty} \left(\frac{C_t^{1-\theta}}{1-\theta} - 1 \right) e^{-\rho t} dt \tag{2.7}$$

where θ is the inverse of the intertemporal elasticity of substitution, and ρ is the rate of time preference. Note that the instantaneous utility function is assumed to take the Constant Intertemporal Elasticity of Substitution (CIES) form, and that there is no "leisure activity" in the model.[9] Households maximize their life time utility (2.7) subject to the human capital accumulation function (2.4) and their instantaneous budget constraint:

$$\dot{K}_t = (1-\tau_k) r_t K_t + (1-\tau_h) w_t u_t H_t + s_h w_t (1-u_t) H_{t,a} + T_t - C_t \tag{2.8}$$

which says that consumption and investment in physical capital have to be financed by the net capital and labor income, the subsidy obtained by investing time in education and the lump sum transfer. In the following, time subscripts are left out where unnecessary.

This section has briefly described the model, which is used in the following analyses. Before the growth rate effects of taxation and subsidization are calculated, it proves convenient to solve both the central planner's and the representative agent's problem. This is done in section 3 and 4, respectively.

3. The Planned Economy

This section focuses on the central planner's problem and derives the first order conditions for an optimal growth path given the human capital accumulation function.

[9] In the [7], [6], [8] Roubini & Milesi-Ferretti papers (1994a, b, and 1995) several different specifications of leisure activity are considered.

The central planner maximizes the life time utility of the representative household (2.7) subject to the constraint on human capital accumulation (2.4) and the resource constraint of the economy (2.6). The first order conditions with respect to C, K, H, G, and u are given by:

$$C^{-\theta} e^{-\rho t} = \mu_k \tag{3.1}$$

$$-\frac{\dot{\mu}_k}{\mu_k} = \left(1 - \frac{G}{Y}\right) \alpha A \left(\frac{uH}{K}\right)^{1-\alpha} H^{\varepsilon} \tag{3.2}$$

$$-\frac{\dot{\mu}_h}{\mu_h} = \left(1 + \frac{\varepsilon}{1-\alpha} u\right) B\Gamma \tag{3.3}$$

$$\mu_h B (1-u) H \Gamma' \frac{1}{Y} = \mu_k \tag{3.4}$$

$$\mu_h B \Gamma = \mu_k \left(1 - \frac{G}{Y}\right) (1-\alpha) A \left(\frac{K}{uH}\right)^{\alpha} H^{\varepsilon} \tag{3.5}$$

where μ_k (μ_h) is the shadow price of physical capital (human capital) in the central planner solution. Note that equations (3.1)–(3.5) describe the first best optimal growth path of the economy. Equation (3.1) implies that the marginal utility of consumption in every period should equal the shadow price of consumption (physical capital). Equation (3.2) implies that the rate of change of the shadow price of physical capital should equal the marginal product of capital. Equation (3.3) implies that the rate of change of the shadow price of human capital should equal the marginal product of human capital in the education sector. Equation (3.4) describes the optimal allocation of resources between public expenditure on education and production of physical capital.[10] Thus, if the value of the marginal product of public expenditure on education were higher than the shadow price of physical capital, then it would be optimal to reallocate resources towards the education sector until equality between the two is obtained. Equation (3.5) describes the optimal allocation of human capital between the education sector and the final goods sector. Thus, the value of the marginal product of human capital should be the same in the two sectors. If the marginal product of human capital were higher in the education sector than in the final goods sector, then it would be optimal to reallocate human capital to the education sector until the marginal products in the two sectors were equalized.

The transversality conditions to the maximization problem are:

$$\lim_{t \to \infty} \mu_k K = 0$$

$$\lim_{t \to \infty} \mu_h H = 0$$

[10] The marginal cost of G in terms of foregone output is 1.

These conditions rule out explosive paths by requiring that the present discounted value of each capital good equals zero in the long run. This is a reasonable requirement since optimizing agents do not want valuable assets at the end of their planning horizon. Consequently, the first transversality condition requires that the real interest rate should be positive.

This section solved the central planner's problem. Thus, the following section solves the representative agent's problem.

4. The Market Economy

This section focuses on the representative agent's problem. Firstly, the first order conditions for a balanced growth path in a market economy are derived. Secondly, the balanced growth rate is determined.

The representative household chooses its consumption C and the allocation of human capital u in order to maximize its life time utility (2.7) subject to the human capital accumulation function (2.4) and its instantaneous budget constraint (2.8) taking τ_k, τ_h, s_h and Γ as given. The first order conditions with respect to C, K, H and u are given by:

$$C^{-\theta}e^{-\rho t} = \lambda_k \tag{4.1}$$

$$-\frac{\dot{\lambda}_k}{\lambda_k} = (1-\tau_k)\,r \tag{4.2}$$

$$-\frac{\dot{\lambda}_h}{\lambda_h} = \left(1+\frac{s_h}{1-\tau_h-s_h}u\right)B\Gamma \tag{4.3}$$

$$\lambda_h B\Gamma = \lambda_k(1-\tau_h-s_h)w \tag{4.4}$$

where λ_k (λ_h) is the shadow price of physical capital (human capital) in the representative household problem. Equation (4.1) is identical to equation (3.1). Equation (4.2) implies that the rate of change of the shadow price of physical capital should equal the after–tax marginal product of capital. Equation (4.3) implies that the rate of change of the shadow price of human capital should equal the marginal product of human capital in the education sector *adjusted* for the labor income tax and the education subsidy. Equation (4.4) describes the optimal allocation of human capital between the two sectors. Thus, human capital is optimally allocated between the two sectors when the value of its marginal product equals the opportunity cost of education, which is the

difference between the after–tax wage rate and the education subsidy which is foregone while working.

In the following, the balanced growth rate in the market economy is calculated in three steps. Firstly, two steady state relationships are derived. Then, a semi–reduced expression for the balanced growth rate is determined in which the growth rate is a function of the fraction of human capital allocated to final goods production. And finally, the steady state fraction of human capital in final goods production and thereby the balanced growth rate is obtained.

The first step is to derive two steady state relationships, which are used in the derivation of the balanced growth rate. Note firstly that all endogenous variables should grow at constant rates in the steady state.[11] According to equation (4.1) and (4.2), this implies that a constant steady state growth rate of consumption requires a constant interest rate. Thus, logarithmic differentiation of equation (2.2) yields a relationship between the growth rate of human capital and physical capital, which holds in the steady state:

$$\frac{\dot{H}}{H} = \left(\frac{1-\alpha}{1-\alpha+\varepsilon}\right)\frac{\dot{K}}{K} \tag{4.5}$$

According to equation (4.5), human capital grows at a slower pace than physical capital as long as there is an education externality in the final goods sector. In absence of the externality, the two types of capital grow at the same rate. A further steady state relationship is obtained by logarithmic differentiation of (4.4) and use of (4.5):

$$-\frac{\dot{\lambda}_k}{\lambda_k} = -\frac{\dot{\lambda}_h}{\lambda_h} + \frac{\varepsilon}{1-\alpha}\frac{\dot{H}}{H} \tag{4.6}$$

Equation (4.6) implies that the shadow price of physical capital must decline at the same rate as the shadow price of human capital plus the growth rate of the wage rate. Or equivalently, that the after–tax marginal product of capital should equal the marginal product of human capital in education *adjusted* for the labor income tax and the education subsidy plus the growth rate of wages at each skill level, see equation (4.2) and (4.3).

The second step is to derive a semi–reduced expression for the balanced growth rate in the market economy. This is done by noting that consumption and physical capital must grow at the same rate in the steady state, while human capital grows at a different rate due to the externality. A semi–reduced expression for the balanced growth rate of consumption and physical capital in the market economy is then obtained by

[11] Note that this rate could be zero.

logarithmic differentiation of equation (4.1):

$$g_c \equiv \frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{1}{\theta}\left[-\frac{\dot{\lambda}_k}{\lambda_k} - \rho\right] \tag{4.7}$$

and introduction of equation (4.6), (4.3) and (2.4):

$$g_c = \frac{1}{\theta}\left[B\Gamma\left(1 + \frac{\varepsilon}{1-\alpha} + \left(\frac{s_h}{1-\tau_h-s_h} - \frac{\varepsilon}{1-\alpha}\right)u^*\right) - \rho\right] \tag{4.8}$$

where a variable with a * indicates its steady state value. As can be seen from equation (4.8), the balanced growth rate depends on the fraction of human capital, which is allocated to the final goods sector in the steady state. It is immediately seen that an increase in the fraction of human capital allocated to education has a positive influence on the growth rate as long as $\frac{s_h}{1-\tau_h-s_h} < \frac{\varepsilon}{1-\alpha}$, which happens to be the case whenever the subsidy to educational effort and the tax on labor income are smaller than their optimal levels.[12] In this case, too much human capital is allocated to final goods production, which means that an increase in the time fraction spent in education $(1-u^*)$ is growth enhancing. Equivalently, an increase in $(1-u^*)$ has a negative growth rate effect, whenever the subsidy to educational effort and the tax on labor income are larger than their optimal levels.

The third step is to derive the steady state fraction of time spent at work u^* and thereby the balanced growth rate. In the following, the fraction of human capital allocated to the final goods sector is derived by use of three steady state properties. Firstly, the interest rate must be constant in the steady state, which implies that the growth rate of $\omega \equiv K^{1-\alpha}H^{-(1-\alpha+\varepsilon)}$ must be zero, see equation (2.2). Secondly, consumption and physical capital grow at the same rate in the steady state, which implies that the growth rate of $\chi \equiv C/K$ must be zero. Finally, the fraction of human capital used in the final goods sector must be constant in the steady state, since $0 < u < 1$. These steady state properties yield three equations in ω, χ and u, which determine the steady state fraction of human capital allocated to final goods production as:[13]

$$u^* = \frac{\frac{\theta-1}{\theta} + \frac{\theta-1}{\theta}\frac{\varepsilon}{1-\alpha} + \frac{1}{\theta}\frac{\rho}{B\Gamma}}{1 + \frac{\theta-1}{\theta}\frac{\varepsilon}{1-\alpha} + \frac{1}{\theta}\frac{s_h}{1-\tau_h-s_h}} \tag{4.9}$$

According to (4.9), the time spent at work increases; whenever the marginal product of human capital in education falls due to a decline in the level of productivity in

[12]The optimal factor income taxes and the optimal subsidy to educational effort are obtained in Section 5.

[13]See Appendix A.2 for the derivation of u^* and the steady state values of χ and ω.

	τ_k	τ_h	s_h	A	B	Γ	α	ε	$1/\theta$	ρ
g_c	0	+	+	0	+	+	+	+	+	−
u^*	0	−	−	0	−	−	+	+	−	+

Table 4.1: Effects of changes in tax and subsidy rates, and parameters.

education B or in the public input in education Γ; whenever the opportunity cost of education increases due to a decline in the tax on labor income τ_h or in the subsidy to educational effort s_h; and whenever the difference between the private and the social return to human capital in final goods production increases due to an increase in the education externality ε or a decline in the labor share in final goods production $(1-\alpha)$. In the latter case, more human capital must be allocated to the final goods sector in order to exploit the positive externality stemming from education. In addition, the fraction of time spent at work is higher, the less willing households are to substitute present for future consumption $1/\theta$ and the less patient they are ρ. Note finally that the tax on capital income τ_k and the level of productivity in the final goods sector A have no influence on the time fraction spent at work in the steady state. All of the above-mentioned results are summarized in Table 4.1.

The reduced form of the balanced growth rate in the market economy can now be determined by introduction of (4.9) in (4.8):

$$\begin{aligned} g_c &= \frac{1}{\theta}\left[B\Gamma\left(1 + \frac{\varepsilon}{1-\alpha} + \left(\frac{s_h}{1-\tau_h-s_h} - \frac{\varepsilon}{1-\alpha}\right)\frac{\frac{\theta-1}{\theta} + \frac{\theta-1}{\theta}\frac{\varepsilon}{1-\alpha} + \frac{1}{\theta}\frac{\rho}{B\Gamma}}{1 + \frac{\theta-1}{\theta}\frac{\varepsilon}{1-\alpha} + \frac{1}{\theta}\frac{s_h}{1-\tau_h-s_h}}\right) - \rho\right] \Leftrightarrow \\ g_c &= \frac{1}{\theta}\left[\frac{1}{1 + \frac{1}{\theta}\left(\frac{1+\frac{s_h}{1-\tau_h-s_h}}{1+\frac{\varepsilon}{1-\alpha}} - 1\right)}\left\{B\Gamma\left(1 + \frac{s_h}{1-\tau_h-s_h}\right) - \rho\right\}\right] \end{aligned} \tag{4.10}$$

In the following, the growth rate effects of changes in the taxes, the subsidy and the parameters of the model are determined by use of equation (4.10) and summarized in Table 4.1. Inspection of the semi-reduced expression for the balanced growth rate in the market economy (4.8) reveals that there are two effects at work, namely a direct "growth rate effect" and an indirect "human capital allocation effect". These effects work in the same direction as long as the subsidy to educational effort and the tax on labor income are smaller than their optimal levels, while they work in opposite directions whenever s_h and τ_h are larger than their optimal levels. However, the direct effect always outweighs the indirect effect and the growth rate effects of changes in the tax and subsidy rates and the parameters of the model are determined unambiguously.

According to equation (4.10), the balanced growth rate therefore rises; whenever the marginal product of education increases due to an increase in productivity in education B or in the public input in education Γ; whenever the opportunity cost of education falls due to an increase in the labor income tax τ_h or in the subsidy to human capital investment in education s_h; whenever the difference between the social and the private rate of return to human capital in goods production increases due to an increase in the education externality ε or a decline in the labor share in final goods production $(1-\alpha)$; and whenever households become more patient ρ and more willing to substitute present for future consumption $1/\theta$.[14] Note that an increase in the incentive to accumulate human capital leads to an increase in the balanced growth rate, because education is the engine of growth in this model.

Proposition 4.1. *Both an increase in the labor income tax and an increase in the subsidy to educational effort reduces the opportunity of education, and therefore lead to an increase in the balanced growth rate in a decentralized economy, ceteris paribus.*

Furthermore, it is immediately seen from equation (4.10) that the balanced growth rate effects of the capital income tax τ_k and the exogenous level of productivity in the final goods sector A are zero. The reason behind the zero balanced growth rate effect of capital income taxation is that physical capital not is used as an input factor in education. Thus, an increase in the capital income tax leads to an offsetting increase in the pre–tax interest rate such that the after–tax interest rate and thereby the balanced growth rate are left unaltered. However, the physical capital intensity in final goods production is reduced by an increase in the capital income tax.

In addition, the balanced growth rate effect of labor income taxation becomes zero in the absence of a subsidy to educational effort ($s_h = 0$). The reason behind this result is that the opportunity cost of education becomes equal to foregone wages when $s_h = 0$. Thus, a change in the labor income tax affects the opportunity cost of education and the after–tax wage rate by the same amount. This implies that the allocation of human capital between the final goods sector and education is left unaltered by a change in the labor income tax, which means that the growth rate also is left unaltered. In the presence of a subsidy to educational effort ($s_h > 0$), the growth rate effect of labor income taxation becomes positive, because the opportunity cost of education is reduced less by an increase in the labor income tax, than the after–tax wage rate earned in the final goods sector. This leads to a reallocation of human capital towards education and thereby to an increase in the growth rate.

[14] The partial derivatives of the balanced growth rate are given in Appendix A.3.

Note that changes in the tax and subsidy rates and in public spending on education result in changes in the tax revenue, which are captured by the lump sum transfer to consumers, see equation (2.5). Section 5 therefore derives the optimal tax and subsidy rates.

This section has obtained the balanced growth rate in the market economy and analyzed the balanced growth rate effects of changes in the tax and subsidy rates and the parameters of the model. In the following section, the optimal tax–subsidy structure is derived.

5. Optimal Tax and Subsidy Rates

This section derives the optimal tax and subsidy rates in the model. Throughout the section it is assumed that the government runs a balanced budget every period, where the tax revenue is spend on the subsidy to education and the public input in education, see equation (2.5).

The optimal tax and subsidy rates are derived by comparison of the first order conditions to the central planner's problem (3.1)–(3.5) and the representative household's problem (4.1)–(4.4):

$$\tau_k^{opt} = \left(\frac{G}{Y}\right)^{CPS} \tag{5.1}$$

$$\tau_h^{opt} = \left(\frac{G}{Y}\right)^{CPS} - s_h^{opt} \tag{5.2}$$

$$s_h^{opt} = \left(1-\left(\frac{G}{Y}\right)^{CPS}\right)\frac{\varepsilon}{1-\alpha} \tag{5.3}$$

$$\Gamma'\frac{1-u^{CPS}}{u^{CPS}} = \frac{\Gamma}{(1-\alpha)\left(1-\left(\frac{G}{Y}\right)^{CPS}\right)} \tag{5.4}$$

where the superscript ^{CPS} denotes the central planner solution, and the obtained optimal tax and subsidy rates are welfare maximizing. Note that the optimal tax–subsidy structure (5.1)–(5.4) holds both in the steady state and outside the steady state. Note furthermore that the optimal fraction of output which is spend on education is constant along the balanced growth path.

It is immediately seen that $\tau_k^{opt} = \tau_h^{opt} + s_h^{opt}$. This implies that it is optimal to reduce the social rate of return to physical and human capital by the same amount in the presence of a public input in education, namely by $1 - (G/Y)^{CPS}$. Thus, the optimal rate of capital income taxation τ_k^{opt} equals the fraction of output, which is used as government spending on education. The optimal rate of labor income taxation τ_h^{opt} equals public expenditure on education relative to total output less the optimal subsidy to education. The optimal subsidy rate on education s_h^{opt} is positive and larger, the smaller the optimal public input in education $(G/Y)^{CPS}$, and the bigger the difference between the social and the private return to human capital in production. Finally, the optimal fraction of output, which is used as government spending on education $(G/Y)^{CPS}$ is determined in equation (5.4).[15] It is seen that the more human capital is allocated to the education sector $\left(1 - u^{CPS}\right)$, the higher is the marginal product of public spending in education and thereby the bigger is the optimal fraction of output spent on public input in education.

When the public input in education increases it becomes possible to lower the subsidy to educational effort, because human capital investment and public investment in education are substitutes. The higher the human capital share in final goods production $(1 - \alpha)$ is and the lower the education externality ε is, the smaller is the difference between the private and the social marginal product of human capital in final goods production. As a result, the subsidy to educational effort does not have to be so big in order to exploit the positive externality stemming from the average level of human capital.

According to equation (5.3), the optimal subsidy to educational effort is unambiguously positive in a model with substitution possibilities between input factors in education. In contrast, the optimal subsidy can either be positive or negative in an equivalent model with no substitution possibilities between the public and the private input in education, see [9] Sørensen (1993). Due to the complementarity between the public and the private input in education in the Sørensen model, a high requirement of the public input relative to human capital makes it optimal to charge a tuition fee in order to finance the public expenses associated with the input in education. In

[15] Introduction of the optimal labor income tax and the optimal education subsidy in equation (4.9) yields the optimal fraction of time spend at work:

$$u^{CPS} = 1 - \frac{1}{\theta}\left(1 - \frac{\rho}{B\Gamma\left(1 + \frac{\varepsilon}{1-\alpha}\right)}\right)$$

the presented model, a high input of the public good in education just leads to a low subsidy to educational effort due to the substitutability between the public and the private input in education. As a consequence, it is never optimal to charge a tuition fee when there are substitution possibilities between the input factors in the education sector and a positive education externality is present in final goods production.

Implementation of the optimal policies (5.1)–(5.4) means that the government has to resort to lump sum taxation in order to achieve the first best allocation of resources, since the optimal factor income taxes cannot fully cover the expenses on the optimal subsidy to educational effort and the public input in education. However, both factor income taxation and subsidization of educational effort are needed to achieve the first best optimum even in the presence of a lump sum tax instrument.

If the subsidy alternatively is assumed to depend on the individual wage level $w(1-u)H$, then the optimal subsidy would be given by s_h^{opt} in equation (5.3) multiplied by the optimal fraction of human capital devoted to final goods production u^{CPS}. This means that the optimal subsidy rate is lower, when households take into account that its choice of time spend on education affects the subsidy rate it faces in the future. In Appendix A.1, the optimal tax and subsidy rates are derived in an equivalent model, where human capital is treated as a market good that give rise to income in both sectors. In this case the optimal capital and labor income tax rates are unaltered, but the optimal subsidy to educational effort becomes negative. When agents earn the same wage rate in education as in final goods production, then they tend to allocate too small a fraction of their time to work in the final goods sector. Thus, in order to exploit the positive education externality it becomes optimal to charge a tuition fee, which induces agents to allocate more human capital to final goods production. However, the assumption that human capital is a market good is unrealistic, since the implicit labor income in the production of human capital usually is left untaxed.

Proposition 5.1. *In the presence of a positive education externality in final goods production, labor income should be taxed at a lower rate than capital income. The bigger the externality is and the smaller the labor share in final goods production is, the bigger is the difference between the optimal rate of capital and labor income taxation.*

In Table 5.1, the optimal tax and subsidy rates are given for several special cases of the presented model. Recall that the [3] Lucas (1988) model is obtained, when there is no public input in education $\Gamma = 1$. Firstly, Table 5.1 reveals that it is optimal not to impose any taxes or subsidies in the Lucas model without an education externality in

the final goods sector ($\varepsilon = 0$). Secondly, in the Lucas model with a positive education externality ($\varepsilon > 0$) it is optimal to subsidize education and human capital investment in final goods production at the same rate, because it leads to an increase in the marginal product of human capital *adjusted* for the labor income tax and the subsidy, see (4.3), while leaving the opportunity cost of education unaffected. In this case, the first best optimum can only be achieved, if these subsidies are financed by lump sum taxation $T < 0$. Thirdly, in a version of the presented model with a public input in education ($\Gamma \neq 1$) and no education externality in the final goods sector ($\varepsilon = 0$), the optimal tax structure is to set a comprehensive tax on factor income in order to finance public spending and a zero subsidy to educational effort. In this case, the first best optimum can be obtained without the use of a lump sum tax instrument. Recall fourthly that the first best optimum in the presented model only can be achieved, if a lump sum tax is imposed in addition to the optimal factor income taxes. Hence, the presence of productive public spending ($\Gamma \neq 1$) suggests the efficiency of non–lump sum taxes.

	τ_k^{opt}	τ_h^{opt}	s_h^{opt}	T
$\Gamma = 1, \varepsilon = 0$	0	0	0	0
$\Gamma = 1, \varepsilon > 0$	0	$-s_h^{opt}$	$\frac{\varepsilon}{1-\alpha}$	$-$
$\Gamma \neq 1, \varepsilon = 0$	$\frac{G}{Y}$	$\frac{G}{Y}$	0	0
$\Gamma \neq 1, \varepsilon > 0$	$\frac{G}{Y}$	$\frac{G}{Y} - s_h^{opt}$	$\left(1 - \frac{G}{Y}\right)\frac{\varepsilon}{1-\alpha}$	$-$

Table 5.1: Optimal tax–subsidy structures

Now the steady state growth rate in the centrally planned economy is obtained by introduction of the optimal policies (5.1)–(5.4) in equation (4.10):

$$g^{CPS} = \frac{1}{\theta}\left[B\Gamma\left(1 + \frac{\varepsilon}{1-\alpha}\right) - \rho\right] \tag{5.5}$$

where the optimal growth rate is seen to depend positively on the size of the education externality and the marginal product of human capital in education. Comparison of the balanced growth rate in the centrally planned economy and the market economy reveals that:

$$\begin{aligned} g_c &< g^{CPS}, \quad \text{when } s_h < s_h^{opt} \text{ and } \tau_h < \tau_h^{opt} \\ g_c &> g^{CPS}, \quad \text{when } s_h > s_h^{opt} \text{ and } \tau_h > \tau_h^{opt} \end{aligned} \tag{5.6}$$

Since the central planner growth rate is welfare maximizing, welfare is rising (falling) for increasing growth rates below (above) the optimal growth rate (5.5). The balanced

growth rate in the market economy is lower (higher) than the optimal growth rate, whenever the subsidy to educational effort and the tax on labor income are smaller (bigger) than their optimal levels. Thus, growth maximization is not equivalent to welfare maximization except in cases where the balanced growth rate in the market economy equals the optimal growth rate.

This section derived the optimal tax and subsidy rates that should be imposed in a market economy in order to achieve the first best optimum. The following section summarizes and concludes the paper.

6. Summary and Conclusion

This paper has examined the effects of capital and labor income taxation and subsidization of educational effort in a two sector model of endogenous growth, where the tax revenue is used to finance a congested public input in education.

Households were assumed to allocate their time between work in the final goods sector and education. The analyses revealed that households allocate more time to education, the higher the marginal product of human capital in education is, the lower the opportunity cost of education is, the more willing households are to substitute present for future consumption and the more patient they are.

In addition, the growth rate effects of changes in the tax–subsidy structure were found. On the one hand, it was shown that a change in the capital income tax has no effect on the balanced growth rate in the market economy even though it reduces the physical capital intensity in final goods production. On the other hand, the growth rate effects of a labor income tax and a subsidy to educational effort were found to be unambiguously positive. In fact there were two effects at work behind these results. The first effect is a direct "growth rate" effect, which is positive because both an increase in the labor income tax rate and the subsidy rate tend to increase the tax and subsidy adjusted marginal product of human capital in education. The second effect is a "human capital allocation" effect, which is positive whenever the labor income tax and the subsidy are below their optimal levels, but negative when they are above. It turns out that the "growth rate" effect always dominates the "human capital allocation" effect, which means that both labor income taxation and subsidization of educational effort have positive effects on the balanced growth rate. Furthermore, the balanced growth rate was shown to increase with the growth rate of wages at each skill level, or

put differently with the education externality in the final goods sector and the physical capital share in production.

The paper shows that the optimal subsidy rate on education is unambiguously positive, when there are substitution possibilities between public and private input factors in education. A bigger optimal subsidy to educational effort is required, the smaller the public input in education is, because more human capital is needed as an input in education in order to exploit the positive education externality. In order to give households the right incentives to allocate enough human capital to education, a bigger optimal subsidy is also required, the higher the growth rate of wages at each skill level is. The optimal capital income tax was shown to equal the fraction of output which the government spends on the public input in education, while the optimal labor income tax was shown to be lower than the optimal capital income tax due to the presence of the positive education externality in the final goods sector. If the optimal policies were imposed, the government would have to resort to lump sum taxation in order to achieve the first best allocation of resources, since the optimal factor income taxes cannot fully cover the expenses on the optimal subsidy to educational effort and the public input in education.

Finally, it was found that it is not necessarily welfare improving to achieve a higher balanced growth rate through an increase in the labor income tax and the education subsidy. In fact it is only welfare improving to increase the labor income tax and the subsidy as long as they are smaller than their optimal levels.

There are several ways in which the present paper can be extended. Firstly, the government could be assumed to have no borrowing constraints. This would probably lead to high taxation in the short run in order to build up sufficient assets to finance government spending in the long run. Secondly, the dynamic adjustment to the balanced growth path could be derived in cases, where the initial physical to human capital ratio in the final goods sector either is higher or lower than its steady state value.

A. Appendix

A.1. Human Capital as a Market Good

This appendix considers the case, where human capital is a market good that gives rise to income both in the final goods sector and in the education sector. This assumption changes the household budget constraint (2.8) as follows:

$$\dot{K} = (1-\tau_k)rK + (1-\tau_h)wH + s_h w\,(1-u)\,H + T - C \qquad \text{(A.1)}$$

An implicit assumption in equation (A.1) is that agents take into account that their choice of time spent in education affects the subsidy rate they face in the future.

The representative household maximizes its life time utility (2.7) subject to the constraint on human capital accumulation (2.4) and its budget constraint (A.1). The first order conditions with respect to C, K, H and u are given by:

$$C^{-\theta}e^{-\rho t} = \psi_k \qquad \text{(A.2)}$$

$$-\frac{\dot{\psi}_k}{\psi_k} = (1-\tau_k)\,r \qquad \text{(A.3)}$$

$$-\frac{\dot{\psi}_h}{\psi_h} = \left(-\frac{1-\tau_h}{s_h}\right) B\Gamma \qquad \text{(A.4)}$$

$$\psi_h B\Gamma = \psi_k(-s_h)w \qquad \text{(A.5)}$$

where ψ_k (ψ_h) is the shadow price of physical capital (human capital) in the representative household problem.

Now, the optimal tax and subsidy rates can be derived by comparison of the first order conditions to the central planner's problem (3.1)-(3.5) and the representative household's problem (A.2)-(A.5). The resulting optimal policies are:

$$\tau_k^{opt} = \left(\frac{G}{Y}\right)^{CPS} \qquad \text{(A.6)}$$

$$\tau_h^{opt} = \left(\frac{G}{Y}\right)^{CPS} - \left(1-\left(\frac{G}{Y}\right)^{CPS}\right)\frac{\varepsilon}{1-\alpha}u^{CPS} \qquad \text{(A.7)}$$

$$s_h^{opt} = -\left(1-\left(\frac{G}{Y}\right)^{CPS}\right) \qquad \text{(A.8)}$$

As before, the optimal rate of capital income taxation equals the fraction of output, which is used as government spending on education, and the optimal labor income tax is also unaltered.[16] However, the optimal subsidy to educational effort becomes negative, which means that it is optimal to charge a tuition fee. This result hinges on the fact that households now earn the same wage rate in both sectors. Thus, household maximization leads to a too small fraction of time spent at work in the steady state given the positive education externality in the final goods sector. Thus, in order to induce agents to allocate more human capital to final goods production it becomes optimal to charge a tuition fee.

[16] Compared to the optimal labor income tax in an equivalent model, where education is a non-market activity.

A.2. Derivation of the Steady State Values of u, χ and ω

In the steady state, the values of $\omega \equiv K^{1-\alpha}H^{-(1-\alpha+\varepsilon)}$, $\chi \equiv C/K$ and u are constant. Firstly, the growth rate of ω is derived from the household budget constraint (2.8) and the human capital accumulation function (2.4):

$$\begin{aligned} g_\omega &= (1-\alpha)\left[\left(\alpha(1-\tau_k)+(1-\alpha)(1-\tau_h)+(1-\alpha)s_h\frac{1-u}{u}\right)Au^{1-\alpha}\omega^{-1}-\chi\right] \\ &- (1-\alpha+\varepsilon)B\Gamma(1-u) \end{aligned} \tag{A.9}$$

Secondly, the growth rate of χ is derived from equation (4.7), (4.2), (2.2) and the household budget constraint (2.8):

$$\begin{aligned} g_\chi &= \frac{\alpha(1-\tau_k)-\theta\left(\alpha(1-\tau_k)+(1-\alpha)(1-\tau_h)+(1-\alpha)s_h\frac{1-u}{u}\right)}{\theta}Au^{1-\alpha}\omega^{-1} \\ &+ \chi-\frac{\rho}{\theta} \end{aligned} \tag{A.10}$$

Thirdly, the growth rate of u is derived by logarithmic differentiation of (4.4) and use of equation (4.2), (4.3), (2.8) and (2.4):

$$\begin{aligned} g_u &= \frac{1}{\alpha}B\Gamma\left(1+\frac{s_h}{1-\tau_h-s_h}u\right) \\ &+ (1-\alpha)\left(-(1-\tau_k)+(1-\tau_h)+s_h\frac{1-u}{u}\right)Au^{1-\alpha}\omega^{-1} \\ &- \chi-\frac{\alpha-\varepsilon}{\alpha}B\Gamma(1-u) \end{aligned} \tag{A.11}$$

Now, the steady state values of u, χ and ω are derived by solving the system of three equations (A.9)–(A.11), where $g_\omega = g_\chi = g_u = 0$:

$$u^* = \frac{\frac{\theta-1}{\theta}+\frac{\theta-1}{\theta}\frac{\varepsilon}{1-\alpha}+\frac{1}{\theta}\frac{\rho}{B\Gamma}}{1+\frac{\theta-1}{\theta}\frac{\varepsilon}{1-\alpha}+\frac{1}{\theta}\frac{s_h}{1-\tau_h-s_h}} \tag{A.12}$$

$$\begin{aligned} \chi^* &= \frac{\left(\alpha(1-\tau_k)+(1-\alpha)(1-\tau_h)+(1-\alpha)s_h\frac{1-u^*}{u^*}\right)\rho}{\alpha(1-\tau_k)} \\ &- \frac{\alpha(1-\tau_k)-\theta\left(\alpha(1-\tau_k)+(1-\alpha)(1-\tau_h)+(1-\alpha)s_h\frac{1-u^*}{u^*}\right)}{\alpha(1-\tau_k)} \\ &\frac{1-\alpha+\varepsilon}{1-\alpha}B\Gamma(1-u^*) \end{aligned} \tag{A.13}$$

$$\omega^* = Au^{*(1-\alpha)}\left(\frac{\rho}{\alpha(1-\tau_k)}+\frac{\theta}{\alpha(1-\tau_k)}\frac{1-\alpha+\varepsilon}{1-\alpha}B\Gamma(1-u^*)\right) \tag{A.14}$$

Note that u^* is bounded between 0 and 1, which requires that $\rho < B\Gamma\left(1-\frac{s_h}{1-\tau_h-s_h}\right)$.

A.3. Balanced Growth Rate Effects

The balanced growth rate effects of changes in the tax and subsidy rates and the parameters of the model are:

$$\begin{aligned}
\partial g_c/\partial\tau_k &= 0 \\
\partial g_c/\partial\tau_h &= \frac{1}{\theta}\left[\frac{\frac{s_h}{1-\tau_h}}{\frac{\theta-1}{\theta}\left(1-\frac{s_h}{1-\tau_h}\right)+\frac{1}{\theta}\frac{1}{1+\frac{\varepsilon}{1-\alpha}}}\left\{\rho+\frac{B\Gamma-\left(1-\frac{s_h}{1-\tau_h}\right)\rho}{\left(1-\frac{s_h}{1-\tau_h}\right)+\frac{1}{\theta-1}\frac{1}{1+\frac{\varepsilon}{1-\alpha}}}\frac{1}{1-\tau_h}\right\}\right] > 0 \\
\partial g_c/\partial s_h &= \frac{1}{\theta}\left[\frac{\frac{1}{1-\tau_h}}{\frac{\theta-1}{\theta}\left(1-\frac{s_h}{1-\tau_h}\right)+\frac{1}{\theta}\frac{1}{1+\frac{\varepsilon}{1-\alpha}}}\left\{\rho+\frac{B\Gamma-\left(1-\frac{s_h}{1-\tau_h}\right)\rho}{\left(1-\frac{s_h}{1-\tau_h}\right)+\frac{1}{\theta-1}\frac{1}{1+\frac{\varepsilon}{1-\alpha}}}\right\}\right] > 0 \\
\partial g_c/\partial A &= 0 \\
\partial g_c/\partial B &= \frac{1}{\theta}\left[1+\frac{1}{\theta}\left(\frac{1+\frac{s_h}{1-\tau_h-s_h}}{1+\frac{\varepsilon}{1-\alpha}}-1\right)\right]^{-1}\Gamma\left(1+\frac{s_h}{1-\tau_h-s_h}\right) > 0 \\
\partial g_c/\partial\Gamma &= \frac{1}{\theta}\left[1+\frac{1}{\theta}\left(\frac{1+\frac{s_h}{1-\tau_h-s_h}}{1+\frac{\varepsilon}{1-\alpha}}-1\right)\right]B\left(1+\frac{s_h}{1-\tau_h-s_h}\right) > 0 \\
\partial g_c/\partial\alpha &= \left(\frac{1}{\theta}\frac{1+\frac{s_h}{1-\tau_h-s_h}}{1+\frac{\varepsilon}{1-\alpha}+\frac{1}{\theta}\left(\frac{s_h}{1-\tau_h-s_h}-\frac{\varepsilon}{1-\alpha}\right)}\right)^2\varepsilon\left(\frac{1}{1-\alpha}\right)^2B\Gamma > 0 \\
\partial g_c/\partial\varepsilon &= \left(\frac{1}{\theta}\frac{1+\frac{s_h}{1-\tau_h-s_h}}{1+\frac{\varepsilon}{1-\alpha}+\frac{1}{\theta}\left(\frac{s_h}{1-\tau_h-s_h}-\frac{\varepsilon}{1-\alpha}\right)}\right)^2\frac{1}{1-\alpha}B\Gamma > 0 \\
\partial g_c/\partial\theta &= -\left[\theta+\frac{1+\frac{s_h}{1-\tau_h-s_h}}{1+\frac{\varepsilon}{1-\alpha}}-1\right]^{-2}B\Gamma\left(1+\frac{s_h}{1-\tau_h-s_h}\right) < 0 \\
\partial g_c/\partial\rho &= -\frac{1}{\theta}\left[1+\frac{1}{\theta}\left(\frac{1+\frac{s_h}{1-\tau_h-s_h}}{1+\frac{\varepsilon}{1-\alpha}}-1\right)\right]^{-1} < 0
\end{aligned}$$

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Growth, Training Leave and Unemployment*

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Abstract

This paper analyzes the effects of a training leave benefit in a two sector model of endogenous growth, where unemployment is created by the existence of monopoly labor unions. There is a final goods sector and an education sector that provides the facilities to upgrade the skills of the labor force. The main result is that it is optimal to charge a tuition fee instead of giving a benefit in an economy with involuntary unemployment caused by monopoly labor unions. Furthermore, it is only welfare improving to undertake growth enhancing measures as long as the tuition fee is bigger than its optimal level. Moreover, an increase in the training leave benefit leads to a higher rate of balanced growth and at the same time to a lower rate of unemployment. Finally, an increase in the training leave benefit leads to the same transitional dynamics in the consumption–capital and the capital–labor ratio as an increase in the labor income tax, but the latter has a negative growth rate effect.

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1. Introduction

This paper analyzes the effects of a training leave benefit in a two sector model of endogenous growth, where unemployment is created by the existence of monopoly labor unions.

The main question addressed in this paper is inspired by one of the general policy problems in Western Europe today, namely how to increase economic growth and at the same time reduce unemployment. In order to deal with these two problems, the Danish government introduced a training leave scheme with a leave benefit on the labor market in 1992. The group of persons who can obtain training leave under this scheme includes persons above the age of 25, who are either unemployed, employed or self-employed. In order to obtain training leave, it is required that the person is a member of an unemployment insurance fund and is qualified for unemployment benefit. The leave benefit which is granted during training leave amounts to up to 100 per cent of the maximum rate of unemployment benefit, see [6] the Danish Ministry of Labor. The philosophy behind the implementation of such a training leave scheme is three fold. Firstly, education is an important instrument, when it comes to increasing economic growth. This is supported by [1] Barro & Sala-i-Martin (1995), who in an empirical analysis of a cross section of 87 countries find that male secondary and higher schooling has a highly significant positive effect on the growth rate. Secondly, education is an important instrument when it comes to ensuring the upgrading of the qualifications and skills of the labor force on a current basis. Moreover, the idea of a training leave benefit paid by the government is also supported by the empirical study by Barro & Sala-i-Martin, who find that public spending on education has a positive effect on the growth rate. Thirdly, the existence of unemployment gives an even stronger argument for subsidizing education, simply because the opportunity cost of education for an unemployed person is practically zero when the training leave benefit is close to the unemployment benefit. The analyses undertaken in the present paper suggest that a training leave benefit is both growth enhancing and unemployment reducing, but paradoxically also suggest that the consequence might be a reduction in welfare.

In order to address the question posed above, an endogenous growth model is developed with a separate education sector and an imperfect labor market in which unemployment is created by the existence of monopoly labor unions. As a consequence, the paper deals not only with the particular effects of the Danish training leave scheme, but also with the general question of setting an optimal subsidy to education in an economy with an imperfectly functioning labor market.

Two strands of theoretical literature have been used, namely the literature on factor income taxation in endogenous growth models and the literature on monopoly labor unions. In particular, the endogenous growth models by [3] Lucas (1988), [2] King & Rebelo (1990), [7] Rebelo (1991), and [9] Roubini & Milesi-Ferretti (1995) have been a source of inspiration. These models typically have two sectors – a final goods sector and an education sector – and factor income is assumed only to arise in the final goods sector. The main result in these papers is that factor income taxation is growth reducing, because it leads to a reduction in the after–tax rate of return to capital investment. With respect to the integration of monopoly labor unions in endogenous growth models, only a few attempts have been made. One of these, the [5] Nielsen, Pedersen & Sørensen (1995) paper, has been another source of inspiration. Their model is a one sector model of endogenous growth, where final goods production uses capital, effective labor and effective pollution as input factors. Endogenous growth in their model is driven by private capital accumulation and productive government spending on education and abatement activities. In their model, public spending on e.g. education immediately raises the productivity of labor in final goods production and thereby the growth rate.

The present paper firstly contributes to the existing literature by introducing monopoly labor unions into an endogenous growth model with two sectors. In this model endogenous growth is driven by the accumulation of human capital in a separate education sector and the household choice of allocating time to education is strictly private. Secondly, the paper investigates the positive consequences of introducing a subsidy to education in order to reduce unemployment and at the same time increase economic growth. Finally, the paper investigates the welfare effect of such a training leave benefit scheme.

In order to investigate the above-mentioned issues, the Uzawa–Lucas model[1] is extended to incorporate monopoly labor unions. A final goods sector produces consumption goods and physical capital by use of both physical capital and labor with different skills. Monopoly labor unions determine the wage rate for each skill. In order to maximize the life time utility of its representative member, labor unions set the wage rate as a mark–up over unemployment benefits. Taking this wage rate as given firms then set the fraction of time households spend at work. Households are assumed to be divided into groups with different labor skills and to arbitrage between unemployment

[1] Both reproducible input factors – human and physical capital – are used in the final goods sector, whereas the education sector only uses human capital as an input factor.

and training leave in their remaining time.[2] Households choose to take training leave in the education sector in order to increase their skill level, because they expect to earn higher wages in the future. Endogenous growth is driven by the accumulation of human capital in the education sector. Finally, the government sector is assumed to levy factor income taxes in order to finance both a training leave benefit and an unemployment benefit.

As mentioned earlier one of the hypotheses put forward in this paper is that training leave with a benefit leads to a lower rate of unemployment and at the same time to a higher rate of growth. On the one hand, training leave is expected to have a positive effect on the growth rate, because it raises the level of human capital in society. On the other hand, the training leave benefit is expected to have a negative effect on the growth rate, because it has to be financed through taxation. According to several studies by [8], [9] Roubini & Milesi-Ferretti (1994, 1995) factor income taxation has a negative effect on the growth rate. However, in the present paper the first effect is dominant, which means that the balanced growth rate in the market economy is increasing in the training leave benefit, see Table 3.1. Furthermore, the analyses reveal that a high rate of balanced growth is associated with a low rate of unemployment, see Proposition 3.1.

The main result obtained in this paper is that the presence of labor unions makes it optimal to charge a tuition fee both outside and along the balanced growth path instead of a giving benefit. This is in stark contrast to the philosophy behind the Danish training leave scheme. The reason behind the result is that labor unions set an inoptimally high wage rate which makes the expected future wages too high and thereby training leave too attractive even without a training leave benefit. This results in an excessive growth rate which makes it optimal from a welfare perspective to charge a training leave fee. The imperfect labor market furthermore implies that the opportunity cost of education is the difference between the unemployment benefit rate and the training leave benefit instead of the competitive wage rate as it would be in an economy with a perfect labor market and thereby full employment. In the decentralized economy, the rate of employment is therefore too low due to the inoptimally high wage rate set by labor unions, and the fraction of time spent in education is too high as long as a benefit is given to educational effort. Note that the optimal allocation of time between education and unemployment is obtained when the opportunity cost of education is set equal to the income that a household could obtain, if it does not spend any time in education at all. Another result is that it is welfare improving to undertake growth enhancing measures only as long as the tuition fee is bigger than its optimal level, in which case the balanced growth rate is lower than its optimal level. Finally, the dynamic analysis reveals that the transitional dynamics of the consumption–capital ratio and the capital–labor ratio caused by respectively an increase in the labor income tax and an increase in the training leave benefit are similar, but that the growth rate effects of these two policy changes have opposite signs.

[2] In equilibrium it turns out that households choose the same allocation of time irrespective of their labor skill.

The model is presented in Section 2. Section 3 derives the balanced growth rate in the market economy and determines the effects of a training leave benefit on the rate of unemployment and the rate of balanced growth. Section 4 studies the transitional dynamics of the market economy, while the optimal growth rate and the optimal training leave benefit are determined in Section 5. Section 6 summarizes and concludes the paper.

2. The Model

The model consists of a large number of competitive firms, a large number of infinitely lived households with different labor skills, a monopoly labor union for each of these skills, and a government.

The production side of the economy consists of two sectors. The first sector is a final goods sector that produces consumption goods and physical capital by use of both physical capital and human capital. The stock of human capital is assumed to equal the labor force measured in efficiency units. In the following, the stock of human capital is therefore referred to as the effective labor force. Firms are assumed to have a "right to manage", which means that they choose how much labor to employ at the given wage rate, where the latter is determined by the labor unions. The labor union for each skill is assumed to set the wage rate in order to maximize the life time utility of its representative member. The second sector is an education sector offering education which generates an increase in the skill level of households.

The consumption side of the economy consists of a large number of infinitely lived households with different labor skills i. The total time endowment of each household is set equal to one. Each household works at the wage rate w_i set by their labor union in a fraction of their time u_i set by firms. Furthermore, households determine how much

time to spend on training leave v_i, whereby they arbitrage between being unemployed at the unemployment benefit rate b and taking training leave at the leave benefit rate $(1-p)b$, where the size of the leave benefit $0 < p$ is decided by the government. Thus, the training leave benefit rate is set as a percentage of the unemployment benefit rate. Training leave leads to the accumulation of human capital and takes place in the education sector. Thus, households choose to study, because they expect to earn higher wages in the future due to an increase in their level of human capital.

If on the one hand, the training leave benefit equals the unemployment benefit ($p = 0$), then it would always be beneficial to take training leave instead of being unemployed, since there is no utility from leisure. This means that $p = 0$ is inconsistent with the existence of unemployment. However in the present model, the labor unions generate involuntary unemployment, which means that the model only is consistent, if $p > 0$. If on the other hand, there was no benefit given to training leave in this model ($p = 1$), then households may still choose to take training leave, because they expect to earn higher wages in the future due to an increase in their skill level.

The government is assumed to levy factor income taxes on capital and labor in order to finance the training leave benefit and the unemployment benefit.

2.1. Firms

The representative firm produces final goods Y according to the following constant returns Cobb–Douglas production function:

$$Y = AK^{\alpha}(uH)^{1-\alpha} \tag{2.1}$$

where $0 < \alpha < 1$ is the physical capital share, A is a productivity parameter, K (H) is the stock of physical capital (human capital) and u is the fraction of human capital that is devoted to the production of final goods. Human capital is assumed to be embodied in people and is consequently a private good, which is both rival and excludable. In the following, the stock of human capital is therefore also referred to as the effective labor force.

The competitive representative firm is assumed to maximize its market value at time s:

$$V(s) = \int_s^{\infty} \left\{ Y - wuH - \dot{K} \right\} e^{-\int_s^{\mu} r(\mu)d\mu} ds \tag{2.2}$$

where w is the wage rate for unskilled ("raw") labor, $r(\mu)$ is the interest rate at time μ and a dot above a variable indicates its derivative with respect to time.

The total input of labor in final goods production consists of the following Constant Elasticity of Substitution (C.E.S.) aggregate of different labor skills:

$$uH = n^{\frac{1}{1-E}} \left(\sum_{i=1}^{n} (u_i H_i)^{\frac{E-1}{E}} \right)^{\frac{E}{E-1}} \tag{2.3}$$

where $E > 1$ is the elasticity of substitution between any two different skills of labor and n is the number of different labor skills.

Thus, the representative firm chooses K and uH in order to maximize its market value (2.2) given (2.1) and (2.3). The resulting first order conditions for profit maximum are:

$$r = \alpha A \left(\frac{uH}{K} \right)^{1-\alpha} = \alpha \frac{Y}{K} \tag{2.4}$$

$$w = (1-\alpha) A \left(\frac{K}{uH} \right)^{\alpha} = (1-\alpha) \frac{Y}{uH} \tag{2.5}$$

which imply that the marginal cost of each factor should equal its marginal product. Firms are assumed to have "a right to manage", which means that they choose how much effective labor to employ according to their labor demand function (2.5). The relevant wage index is given by the following cost minimization rule:

$$w \equiv \left(\frac{1}{n} \right)^{\frac{1}{1-E}} \left(\sum_{i=1}^{n} w_i^{1-E} \right)^{\frac{1}{1-E}} \tag{2.6}$$

while the demand for labor given by (2.5) is allocated across different skills i according to:

$$u_i H_i = \left(\frac{w_i}{w} \right)^{-E} \frac{uH}{n} \tag{2.7}$$

Derivation of the latter two equations is given in Appendix A.1.

2.2. Households

The household sector consists of a large number of infinitely lived households with different labor skills i. The representative household i derives utility from consumption of final goods C_i:

$$U_0 = \int_0^{\infty} \left(\frac{C_i^{1-\theta}}{1-\theta} - 1 \right) e^{-\rho t} dt \tag{2.8}$$

where θ is the inverse of the intertemporal elasticity of substitution and ρ is the rate of time preference.

Households are assumed to arbitrage between being unemployed at the unemployment benefit rate b and taking training leave at the leave benefit rate $(1-p)b$, where

$0 < p$ is set by the government. Households choose to study, because they expect to earn higher wages in the future due to the resulting increase in their level of human capital. As a consequence, household i's dynamic budget constraint becomes:

$$\dot{K}_i = (1-\tau_k)rK_i + (1-\tau_h)w_i u_i H_i + (1-p)bv_i H_i + b(1-u_i-v_i)H_i - C_i \quad (2.9)$$

where τ_k is the tax rate on capital income, τ_h is the tax rate on labor income, r is the interest rate, w_i is the wage rate for unskilled ("raw") labor, u_i is the fraction of time spent at work and v_i is the fraction of time spent on training leave. The representative household's time endowment is assumed to be constant and equal to one.

According to (2.9), consumption and investment in physical capital have to be financed by net capital and labor income and the net benefits obtained, while on training leave and during unemployment. Both benefits are assumed to depend on the fraction of time spent in the relevant activity (education or unemployment) times the household's individual skill level H_i. This assumption implies that households are compensated for their individual income loss. Alternatively, it can be assumed that both benefits are related to the average skill level in the economy H_a, which means that households do not take into account that their choice of time spent in education affects the benefits they face in the future. Probably the easiest policy to implement would be to relate the benefits to the average skill level in the economy.

Note that the trade off between training leave and unemployment would cease to exist, if the training leave benefit equals the unemployment benefit ($p = 0$). In this case it is always optimal for households to take training leave instead of being unemployed due to the expected increase in future wages. However, as noted above the case of $p = 0$ is inconsistent with the existence of the involuntary unemployment which is created by the labor unions in the model. In the following it is therefore assumed that $0 < p$.

When a household is on training leave it accumulates human capital in the education sector. The production technology in the education sector is given by the following linear function:[3]

$$\dot{H}_i = Bv_i H_i \quad (2.10)$$

where B is a productivity parameter. The production of human capital is assumed to exhibit constant returns with respect to human capital. Thus, if the time fraction spent in education is constant, then the growth rate of labor skills is constant and endogenous growth is obtained.

[3]This is an Uzawa–Lucas type specification of the education sector, see [3] Lucas (1988).

2.3. Labor Unions

In the labor market, households are organized in labor unions according to their skill i. Thus, the number of labor unions equals the number of labor skills n. Labor unions are assumed to choose the wage rate w_i and the time fraction households spend at work u_i in order to maximize the lifetime utility of its representative member.

The labor union for households with skill i chooses u_i and w_i in order to maximize its representative members utility (2.8) subject to his/her budget constraint (2.9) and the demand for labor with skill i (2.7). Thus, the labor union enforces the division of labor in (2.7) by setting the wage rate for households with skill i:

$$w_i \equiv w = \frac{b}{(1-\tau_h)\frac{E-1}{E}} \quad (2.11)$$

where $0 < (E-1)/E < 1$ indicates the degree of monopoly power of individual labor unions. The smaller the elasticity of substitution between labor skills is and thereby the smaller $(E-1)/E$ is, the greater is the labor unions' monopoly power and thereby the higher a wage rate they can set. According to equation (2.11), the wage rate is set as a mark–up over the unemployment benefit. Note that the wage rate is the same for all workers, since the unemployment benefit rate is assumed to be the same for all households irrespective of their labor skill. As a consequence, the wage rate set by all labor unions is the same and thereby the choice of all households is the same. This implies that all subscripts i can and will be dropped in the following.

Imagine that the monopoly power of labor unions vanished $E \to \infty$, then the wage rate would decline to:

$$w = \frac{b}{1-\tau_h}$$

In which case, the after–tax wage rate would equal the unemployment benefit rate.

2.4. Government

The government is assumed to levy taxes on factor income in order to finance expenditures on training leave benefits and unemployment benefits. Throughout the paper, the government is assumed to run a balanced budget every period:

$$\tau_k rK + \tau_h wuH = (1-p)bvH + b(1-u-v)H \quad (2.12)$$

where the training leave benefit rate is set as a percentage of the unemployment benefit rate. Furthermore, the government is assumed to allow unemployment benefits to rise

with income per unit of effective labor. This indexation rule is justified by the length of the horizon in the following analyses and the fact that social security systems tend to increase compensation as the economy gets richer:[4]

$$b = \bar{b}\frac{Y}{H} \tag{2.13}$$

where $0 < \bar{b} < 1$. The government chooses both the size of the unemployment benefit parameter $\bar{b}$ and the training leave benefit $0 < p$.

The resource constraint of the entire economy is:

$$Y - C - \dot{K} \geq 0 \tag{2.14}$$

which states that income should equal or exceed expenditures on consumption and investment in physical capital.

3. The Market Economy

This section derives the balanced growth rate and the steady state allocation of time between work, education and unemployment in the market economy and investigates how these are influenced by changes in the tax and benefit rates and the parameters of the model. In addition, the relationship between the rate of unemployment and the rate of balanced growth is determined.

The representative household chooses consumption C and the time fraction spent on training leave v in order to maximize its lifetime utility (2.8) subject to the household budget constraint (2.9) and the human capital accumulation function (2.10). The necessary first order conditions with respect to C, v, H and K become:[5]

$$C^{-\theta}e^{-\rho t} = \lambda_k \tag{3.1}$$

$$\lambda_h B = \lambda_k pb \tag{3.2}$$

$$-\frac{\dot{\lambda}_h}{\lambda_h} = B\frac{(1-\tau_h)wu + b(1-u)}{pb} \tag{3.3}$$

$$-\frac{\dot{\lambda}_k}{\lambda_k} = (1-\tau_k)r \tag{3.4}$$

[4] If b alternatively is interpreted to include income earned in the informal sector, then it is furthermore reasonable to assume that this income follows the average income in the formal economy.

[5] The equivalent first order conditions in the case, where both benefit rates are tied to the average level of human capital in the economy are (3.1), (3.2), (3.4) and:

$$-\frac{\dot{\lambda}_h}{\lambda_h} = B\frac{(1-\tau_h)wu + pbv}{pb}$$

where λ_k (λ_h) is the shadow price of physical capital (human capital) and the wage rate w is set by labor unions (2.11). Equation (3.1) implies that the marginal utility of consumption in every period should equal the shadow price of consumption (physical capital). Equation (3.2) describes the optimal allocation of human capital between education and unemployment. Thus, equation (3.2) says that the value of the marginal product of human capital in education should equal value of the opportunity cost of education, which equals the value of the net rate of return to unemployment, namely $b - (1-p)\,b = pb$. Equation (3.3) implies that the rate of change in the shadow price of human capital should equal the marginal product of human capital in the education sector *adjusted* for the labor income tax, the training leave benefit and the unemployment benefit. Finally, equation (3.4) says that the rate of change in the shadow price of physical capital should equal the after–tax marginal product of capital.

In order to derive the balanced growth rate in the market economy it proves necessary to determine the fraction of time households spend at work. Recall therefore that firms have a "right to manage", which means that firms determine the rate of employment by use of their labor demand function (2.5) and the marginal cost of labor w, which is set by the labor unions, see (2.11) and (2.13). Thus, the rate of employment set by firms is:

$$u = (1-\alpha)\,(1-\tau_h)\frac{E-1}{E}\frac{1}{\bar{b}} \tag{3.5}$$

According to equation (3.5), the rate of employment u is lower, the smaller the labor share in final goods production $(1-\alpha)$, and the higher the wage rate set by unions, namely the higher the labor income tax rate τ_h, the higher the unemployment benefit parameter $\bar{b}$, the lower the elasticity of substitution between labor skills E and thereby the larger the monopoly power of labor unions.

In the remainder of this section, it is assumed that the capital income tax τ_k adjusts endogenously in order to balance the government budget (2.12) for a given labor income tax τ_h, a given training leave benefit $(1-p)$ and a given unemployment benefit parameter $\bar{b}$. Note that this assumption implies that the time fraction spent at work is constant over time.

Now, a semi–reduced expression for the balanced growth rate in the market economy with labor unions can be derived by logarithmic differentiation of (3.1) and use of the fact that the shadow price of human capital declines at the same rate as the shadow price of physical capital in the steady state.[6] Thus, the balanced growth rate

[6] This is seen by logarithmic differentiation of (3.2) and use of (2.13).

of consumption in semi-reduced form equals:

$$g_c^* = \frac{1}{\theta}\left[\frac{B}{p}\left(\frac{1}{E-1}u^* + 1\right) - \rho\right] \tag{3.6}$$

where a variable with a * indicates its steady state value. Note the positive relationship between u^* and the balanced growth rate. The higher the steady state fraction of time spent at work is, the higher is the income that a household can obtain, if it does not spend any time in education. Or equivalently, the expected future income is higher, which means that the incentive to invest time in education is greater. As a result, households allocate more time to education, whereby the balanced growth rate increases, since human capital accumulation is the growth engine in this model.

Finally, the balanced growth rate effects of changes in the tax instruments and the parameters of the model are derived by substitution of the steady state fraction of time spent at work u^* (3.5) into equation (3.6):

$$g_c^* = \frac{1}{\theta}\left[\frac{B}{p}\left(\frac{1-\alpha}{E}\frac{1-\tau_h}{\bar{b}} + 1\right) - \rho\right] \tag{3.7}$$

Thus, the balanced growth rate is higher, the more willing agents are to substitute present for future consumption $1/\theta$, the higher the level of productivity in the education sector B, the greater the training leave benefit $(1-p)$, the larger the labor share in final goods production $(1-\alpha)$, the smaller the elasticity of substitution between different labor skills E, and thereby the greater the monopoly power of labor unions, the lower the labor income tax rate τ_h, the smaller the unemployment benefit $\bar{b}$ and the more patient agents are ρ. All of these results are summarized in Table 3.1 at the end of this section. Note furthermore that the capital income tax has no influence on the balanced growth rate. The reason behind this result is that physical capital not is used as an input factor in human capital accumulation, which is the engine behind growth in the present model. Thus, an increase in the capital income tax leads to an offsetting increase in the *pre*–tax interest rate such that the *after*–tax interest rate and thereby the balanced growth rate is left unaltered. However, the physical capital intensity in final goods production is reduced by an increase in the capital income tax.

For the interpretation of the results derived later on it proves convenient to see what happens, if there were no labor unions (NLU) in the model. Thus, the balanced growth rate in a market economy with zero monopoly power of labor unions $(E \to \infty)$ is obtained from (3.7):

$$g_c^{*NLU} = \frac{1}{\theta}\left[\frac{B}{p} - \rho\right] \tag{3.8}$$

In such an economy it turns out that households allocate their time between work, education and unemployment in such a way that the after–tax competitive wage rate $(1-\tau_h)w$ equals the unemployment benefit rate b; and the value of the marginal product of human capital in education B equals the value of the opportunity cost of education pb, which again equals the after-tax return to work minus the training leave benefit. As a result, the balanced growth rate increases, when the opportunity cost of education falls e.g. through an increase in the training leave benefit rate $(1-p)$.

3.1. Growth and Unemployment

So far the balanced growth rates in a market economy with and without labor unions have been determined. In the following, the steady state allocation of time between work, education, and unemployment is derived in order to determine the relationship between growth and unemployment.

Firstly, the growth rates of consumption, physical capital and human capital are determined. The growth rate of consumption is obtained by logarithmic differentiation of (3.1) and use of (3.4) and (2.4):

$$g_c \equiv \frac{\dot{C}}{C} = \frac{1}{\theta}\left[(1-\tau_k)\alpha A\left(\frac{uH}{K}\right)^{1-\alpha} - \rho\right] \tag{3.9}$$

The growth rate of physical capital is derived from the resource constraint of the economy (2.14):

$$g_k \equiv \frac{\dot{K}}{K} = A\left(\frac{uH}{K}\right)^{1-\alpha} - \frac{C}{K} \tag{3.10}$$

while the growth rate of human capital is derived from the human capital accumulation function (2.10):

$$g_h \equiv \frac{\dot{H}}{H} = Bv \tag{3.11}$$

Then, the steady state values of the time fractions spent on training leave and at work, the ratio of consumption to physical capital, and the ratio of physical to human capital can be determined by use of four steady state relationships.

In the steady state, all endogenous variables should grow at constant rates.[7] This implies that a constant growth rate of consumption requires a constant interest rate, which is achieved when human and physical capital grow at the same rate. Furthermore, consumption and physical capital must grow at the same rate in the steady state. Thus, in the derivation of the steady state relationships it proves convenient to define

[7] Note that this rate could be zero.

the ratios $\omega \equiv K/H$ and $\chi \equiv C/K$, since these ratios grow at zero rates in the steady state. The growth rate of the ratio of physical to human capital then becomes:

$$g_\omega = Au^{1-\alpha}\omega^{-(1-\alpha)} - \chi - Bv \tag{3.12}$$

while the growth rate of the ratio of consumption to physical capital becomes:

$$g_\chi = \frac{\alpha(1-\tau_k)-\theta}{\theta} Au^{1-\alpha}\omega^{-(1-\alpha)} - \frac{\rho}{\theta} + \chi \tag{3.13}$$

In addition, the fraction of time spent at work u must grow at a zero rate in the steady state, since $0 < u < 1$. In the previous section it was shown that the time fraction spent at work is constant over time, whenever the capital income tax is assumed to adjust endogenously in order to balance the government budget, see equation (3.5). An expression for the zero growth rate of the time fraction spent at work is derived by logarithmic differentiation of (3.2) and use of (3.3), (3.4) and (3.12):

$$g_u \equiv \frac{\dot{u}}{u} = 0 = \frac{1}{1-\alpha}\left\{-\frac{B}{p}\left(\frac{1}{E-1}u+1\right) - \alpha\tau_k Au^{1-\alpha}\omega^{-(1-\alpha)} + \alpha\left(\chi + Bv\right)\right\} \tag{3.14}$$

The fourth relationship holds both outside and in the steady state and is obtained by rewriting the balanced government budget constraint (2.12):

$$\begin{aligned} v &= \frac{1}{p}Q - \frac{1}{p}u \\ \text{where } Q &= 1 - \frac{\alpha\tau_k + (1-\alpha)\tau_h}{\bar{b}} \end{aligned} \tag{3.15}$$

where it can be seen that a constant u is equivalent to a constant v, whenever Q is constant. Since τ_k changes over time, the time fraction spent on training leave is only constant in the steady state.

Now, the steady state values of u, v, χ and ω are derived by solving the system of four equations (3.12)–(3.15), where $g_\omega = g_\chi = g_u = g_v = 0$, assuming for now that Q is constant. The resulting steady state values of the time fractions spent on training leave v and at work u, the ratio of consumption to physical capital χ, and the ratio of physical to human capital ω are given below:

$$v^* = \frac{\frac{1}{p}\left(1+\frac{1}{E-1}Q^*\right) - \frac{\rho}{B}}{\frac{1}{E-1}+\theta} \tag{3.16}$$

$$u^* = Q^* - \frac{\left(1+\frac{1}{E-1}Q^*\right) - p\frac{\rho}{B}}{\frac{1}{E-1}+\theta} \tag{3.17}$$

$$\chi^* = \frac{\rho}{\alpha(1-\tau_k)} - B\left(\frac{\alpha(1-\tau_k)-\theta}{\alpha(1-\tau_k)}\right)\frac{\frac{1}{p}\left(1+\frac{1}{E-1}Q^*\right) - \frac{\rho}{B}}{\frac{1}{E-1}+\theta} \tag{3.18}$$

$$\omega^* = A^{\frac{1}{1-\alpha}}\left(\frac{Q^*\theta - 1 + p\frac{\rho}{B}}{\frac{1}{E-1}+\theta}\right) \left(\frac{\rho}{\alpha(1-\tau_k)} + B\left(\frac{\theta}{\alpha(1-\tau_k)}\right)\frac{\frac{1}{p}\left(1+\frac{1}{E-1}Q^*\right) - \frac{\rho}{B}}{\frac{1}{E-1}+\theta}\right)^{-\frac{1}{1-\alpha}} \tag{3.19}$$

In the following, the steady state value of τ_k is derived. Recall that the capital income tax rate is assumed to adjust endogenously in order to balance the government budget (2.12) for a given labor income tax τ_h, a given training leave benefit $(1-p)$ and a given unemployment benefit parameter $\bar{b}$. Equation (3.5) shows that the time fraction spent at work in this case is constant over time. Furthermore, the time fraction spent at work in the steady state is given by equation (3.17). Thus, the capital income tax rate can be derived by equating (3.5) and (3.17) using (3.15):

$$\tau_k^* = 1 - \frac{1}{\alpha}\left(1 - \bar{b}\left(\frac{\theta-1}{\theta} + p\frac{1}{\theta}\frac{1}{B}\rho\right) - \frac{\theta-1}{\theta}(1-\alpha)\frac{1}{E}(1-\tau_h)\right) \tag{3.20}$$

According to (3.20), the capital income tax is constant in the steady state for given values of the labor income tax, the benefit rates, and the parameters of the model.[8] The negative effect of an increase in the training leave benefit $(1-p)$ on the steady state capital income tax may immediately seem counter–intuitive. However, an increase in the training leave benefit reduces the time fraction spent in unemployment without changing the time fraction spent at work. Expenses on unemployment benefits therefore fall and since the capital income tax has to balance the government budget is must be reduced in the steady state.

The steady state values of the ratio of consumption to physical capital and the ratio of physical to human capital can be obtained by substitution of the steady state capital income tax (3.20) in (3.18) and (3.19), respectively.

Since the capital income tax is constant in the steady state and the time fraction spent at work is constant over time, so is the steady state time fraction spent on training leave and thereby the steady state time fraction spent in unemployment. The steady state time fraction spent on training leave can now be determined by substitution of the steady state capital income tax rate (3.20) in equation (3.16):

$$v^* = \frac{\frac{1}{p}\left(1+\frac{1}{E-1}\left\{\frac{1}{\theta} - p\frac{1}{\theta}\frac{1}{B}\rho + \frac{1-\alpha}{b}\left(1+\alpha\frac{\theta-1}{\theta}\frac{1}{E}\right)(1-\tau_h)\right\}\right) - \frac{\rho}{B}}{\frac{1}{E-1}+\theta} \tag{3.21}$$

[8] This implies that the steady state value of Q^* is constant as well.

while the steady state rate of unemployment is derived from equation (3.5) and (3.21):

$$1-u-v^* = 1-\frac{1}{p}\left\{\frac{1}{\theta}-p\frac{\rho}{\theta B}+\frac{1-\alpha}{\bar{b}}\left(1+\alpha\frac{\theta-1}{\theta}\frac{1}{E}+(1+p)\frac{E-1}{E}\right)(1-\tau_h)\right\} \tag{3.22}$$

According to (3.22), the steady state rate of unemployment falls; whenever the opportunity cost of education falls due to an increase in the training leave benefit or a fall in the unemployment benefit; whenever the marginal product of human capital in education increases due to an increase in the level of productivity in education; whenever the expected future after–tax wages increase due to a fall in the labor income tax; and whenever agents become more patient. The results for the time fraction spent on training leave are exactly the opposite. All of these results are summarized in Table 3.1.[9]

	g_c^*	g_c^{*NLU}	τ_k^*	u	v^*	$(1-u-v)^*$
τ_h	$-$	0	$-^\star$	$-$	$-$	$+$
$1-p$	$+$	$+$	$-$	0	$+$	$-$
$\bar{b}$	$-$	0	$+^\star$	$-$	$-$	$+$
$1/\theta$	$+$	$+$	?	0	?	?
ρ	$-$	$-$	$+$	0	$-$	$+$
E	$-$	0	$-^\star$	$+$	?	$-$
A	0	0	0	0	0	0
B	$+$	$+$	$-$	0	$+$	$-$
$1-\alpha$	$+$	0	?	$+$	?	?

Table 3.1: The steady state effects of changes in taxes, benefits and parameters

Recall that the government is assumed not to have a lump sum tax instrument. The implication of this assumption is that changes in either the labor income tax, the benefit rates or the parameters of the model result in an immediate adjustment in the capital income tax in order to balance the government budget at each point in time.

Finally, inspection of Table 3.1 reveals that there is a negative relationship between the balanced growth rate and the rate of unemployment. The intuition behind this results is that all the policy instruments at the government's disposal τ_h, $(1-p)$ and $\bar{b}$ have effects on the time fraction spent in unemployment and on training leave that are

[9]The results indicated with a '$\star$' only holds for an intertemporal elasticity of substitution $1/\theta$ below one ($\theta > 1$), which is normally assumed to be the case.

opposite in signs. Thus, the implementation of a policy that reduces unemployment is growth enhancing, because it increases the time spent in education at the same time.[10] However, an important exception from this result is that both growth and unemployment increase, when the monopoly power of labor unions increases.

Proposition 3.1. *A high unemployment rate is associated with a low balanced growth rate in the market economy for a given elasticity of substitution between labor skills.*

The hypothesis that an increase in the training leave benefit leads to an increase in the balanced growth rate and thereby to a lower unemployment rate is therefore confirmed. However, it is not necessarily welfare improving to reduce unemployment and at the same time achieve a higher growth rate. In fact the analyses in Section 5 reveal that it is only welfare improving to undertake growth enhancing measures as long as the tuition fee is bigger than its optimal level. In which case the market growth rate is below its socially optimal rate, see (5.10).

This section determined the effects of changes in the tax and benefit rates and the parameters of the model on the balanced growth rate and the allocation of time between work, education and unemployment. Furthermore, the relationship between the rate of unemployment and the balanced growth rate was determined. In the following section, the transitional dynamics of the model are characterized.

4. Transitional Dynamics

This section firstly derives the transitional dynamics of the market economy given a balanced government budget. Then, it investigates how the economy adjusts to changes in the training leave benefit, the labor income tax, and the unemployment benefit parameter.

The dynamic evolution of the economy is determined by the system of equations (3.12)–(3.14) which were derived in the previous section. Thus, the capital income tax is assumed to adjust endogenously in order to balance the government budget for given values of the labor income tax, the training leave benefit and the unemployment benefit throughout this section.

[10]Recall that human capital accumulation is the engine of growth in this model.

In the following analyses, it proves convenient to rewrite the dynamic system in terms of the average product of physical capital, which is defined as:[11]

$$z \equiv \frac{Y}{K} = Au^{1-\alpha}\omega^{-(1-\alpha)} \tag{4.1}$$

Insertion of (4.1) in (3.12)–(3.14) and use of (3.15) and the fact that the time fraction spent at work is constant over time yields two equations in z and χ that describe the transitional dynamics of the market economy.[12] The first equation describes combinations of the average product of capital z and the consumption–capital ratio χ, which yield a constant average product of physical capital in the steady state and thereby a constant physical to human capital ratio. This is called the $\dot{z} = 0$–schedule and is given by:

$$\chi = z - \left(\frac{B}{p}\right)^2 \frac{1}{\bar{b}}\left(\frac{1}{E-1}u+1\right)\frac{1}{z} + \frac{B}{p\bar{b}}\left(1 - \frac{(1-\alpha)(1-\tau_h)}{E} - \bar{b}\right) \tag{4.2}$$

see equation (A.16) in Appendix A.2. Several properties of the schedule are revealed by inspection of equation (4.2). Firstly, the $\dot{z} = 0$–schedule approaches a "45–degree"–line from below that intersects the χ–axis at $\frac{B}{p\bar{b}}\left(1 - \frac{(1-\alpha)(1-\tau_h)}{E} - \bar{b}\right)$, when the average product of physical capital approaches infinity ($z \to \infty$). Secondly, the schedule approaches minus infinity, when the average product of physical capital tends to zero ($z \to 0$). And thirdly, the $\dot{z} = 0$–locus is shown in Appendix A.2 to be stable and globally increasing, which means that z is a state–like variable and thereby predetermined in the short run.[13]

The second equation describes combinations of z and χ, which yields a constant ratio of consumption to physical capital in the steady state. This is called the $\dot{\chi} = 0$–schedule and is given by equation (A.17) in Appendix A.2:

$$\chi = \frac{1}{z + \frac{B}{p}\frac{\alpha}{\bar{b}}\frac{\theta}{\theta-\alpha}}\left(z^2 + \left(\frac{B}{p}\frac{\alpha}{\bar{b}} - \frac{\alpha}{\theta-\alpha}D + \frac{1}{\theta-\alpha}\rho\right)z + \frac{1}{\theta-\alpha}\frac{B}{p}\frac{\alpha}{\bar{b}}\rho\right) \tag{4.3}$$

where

$$D = \frac{B}{p}\left(\left(\frac{1}{\alpha}\frac{1}{E-1}+1\right)u + \frac{1-\alpha}{\alpha} + \frac{1-\alpha}{\bar{b}}\tau_h\right) > 0$$

The $\dot{\chi} = 0$–schedule has several properties. Firstly, it is seen to approach a "45–degree"–line from above with an intersect point of 0, when the average product of physical capital approaches infinity ($z \to \infty$). Secondly, the schedule approaches infinity, whenever z approaches $-\frac{B}{p}\frac{\alpha}{\bar{b}}\frac{\theta}{\theta-\alpha}$ from the right. Thirdly, the $\dot{\chi} = 0$–locus intersects the χ–axis at $\frac{\rho}{\theta}$. Fourthly, the schedule is shown in Appendix A.2 to be unstable, which means that χ is a control–like variable that can jump in the short run. Both the $\dot{z} = 0$–schedule and the $\dot{\chi} = 0$–schedule are illustrated in a (z,χ)–diagram, see Figure 4.1.

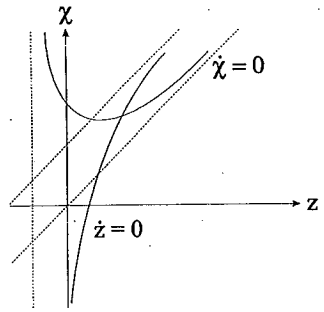

Figure 4.1: The balanced growth equilibrium

In the following analyses it proves convenient to focus on the transitional dynamics in an area around the steady state, see Figure 4.2. In order to do that the dynamic system is linearized around its steady state by means of a first order Taylor expansion, see Appendix A.3. The system turns out to be saddle–path stable for an intertemporal elasticity of substitution below one ($\theta > 1$) and a plausible value of the steady state capital income tax $0 < \tau_k^* < 1$.

The stable saddle path (the dashed line) is upward sloping[14], which implies that both the average product of physical capital and the ratio of consumption to physical capital decline towards their steady state values, if they both start out above their steady state values. From equation (4.1) it is immediately seen that the average product of physical capital is inversely related to the ratio of physical to human capital. Thus, the ratio of physical to human capital increases towards it steady state value, when it starts out below its steady state value.

In the following, it is analyzed how the economy adjusts towards a new steady state in case of a change in the training leave benefit $(1-p)$, the labor income tax τ_h, and the unemployment benefit parameter $\bar{b}$, respectively.

[11] Recall that $\omega \equiv K/H$ and $\chi \equiv C/K$ and note that $g_z = -(1-\alpha)\, g_\omega$.

[12] See Appendix A.2 for the derivation of these two equations.

[13] A state–like variable is defined by [4] Mulligan & Sala–i–Martin (1993) to be a transformation of state variables only, which remains constant in the steady state. They also define a control–like variable as a transformation of control variables that remains constant in the steady state.

[14] The negative eigenvector associated with the stable arm is given in equation (A.24) in Appendix A.3.

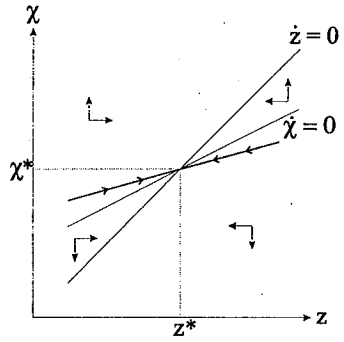

Figure 4.2: Transitional dynamics around the steady state.

In case of an increase in the training leave benefit $(1-p)$; an increase in the labor income tax τ_h; or a fall in the unemployment benefit parameter $\bar{b}$, the $\dot{z}=0$–schedule shifts upwards and to the left, while the $\dot{\chi}=0$–schedule moves downwards to the right, see Figure 4.3. In the resulting new balanced growth equilibrium at point C both the ratio of consumption to physical capital and the average product of physical capital have fallen. In the following, the transitional dynamics of each of the above–mentioned policy changes are treated separately.

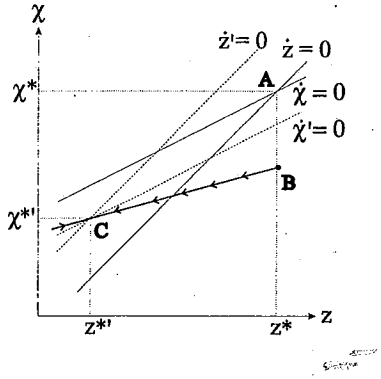

Figure 4.3: Transitional dynamics of an increase in the training leave benefit.

Firstly, an increase in the training leave benefit $(1-p)$ leaves the fraction of time spent at work u unaffected, while the fraction of time spent in education v increases,

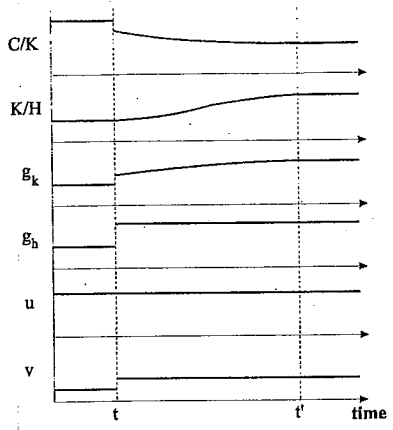

Figure 4.4: Transitional dynamics of core ratios, growth rates and variables.

see Figure 4.4. This results in a higher growth rate of human capital g_h. This leads to an increase in the marginal product of physical capital in final goods production and a resulting increase in physical capital investment. Figure 4.3 illustrates that the immediate reaction to an increase in the training leave benefit is a discrete fall in the ratio of consumption to physical capital from its original steady state value at point A to point B in order to obtain an equivalently higher rate of physical capital accumulation g_k. Subsequently, there is a gradual adjustment from point B towards the new balanced growth path at point C, where the ratio of consumption to physical capital is continuously falling, while the ratio of physical to human capital is continuously increasing. Recall from the previous section that the balanced growth rate in the new steady state C is higher than in the original steady state A.

Secondly, an increase in the labor income tax rate τ_h results in an immediate negative income effect due to a drop in disposable income. The immediate reaction is therefore a discrete fall in the ratio of consumption to physical capital from A to B. Subsequently, there is a gradual adjustment towards the new steady state C, where the ratio of consumption to physical capital is continuously falling, while the ratio of physical to human capital is continuously increasing. The ratio of consumption to physical capital is continuously falling along the transition path, because the fraction of time firms employ households declines and thereby results in a drop in consumption. The ratio of physical to human capital is continuously increasing towards the new

steady state, because households reallocate their time away from education due to the drop in expected future wages. This results in a slow down in the accumulation of human capital. In the long run, these reallocations of effective labor lead to a new balanced growth rate at point C, which is lower than in the original steady state A. It is interesting to note that the transitional dynamics of C/K and K/H caused by respectively an increase in the labor income tax and an increase in the training leave benefit are similar, but that the growth rate effects of these two policy changes have opposite signs. The reason behind this result hinges on the fact that these two policies affect the allocation of time between work, training leave and unemployment differently, see Table 3.1. An increase in the training leave benefit leaves the fraction of time spent at work unaffected and increases the time fraction spent on training leave, whereas an increase in the labor income tax reduces the time fraction spent both at work and on training leave. Since human capital accumulation is the engine of growth in this model, naturally an increase in time spent in education v enhances growth, whereas a decline in v reduces growth.

Thirdly, a decline in the unemployment benefit parameter $\bar{b}$ leads to a reallocation of human capital towards the final goods sector. This implies that physical capital suddenly becomes a scarce input factor in final goods production relative to human capital. The immediate reaction is therefore a discrete fall in the ratio of consumption to physical capital in order to invest more in physical capital accumulation. Subsequently, there is a gradual adjustment towards the new steady state, where the ratio of consumption to physical capital is continuously falling, while the ratio of physical to human capital is continuously increasing. In the new steady state, the balanced growth rate is higher than in the original steady state.

This section described the transitional dynamics of the model and determined how the economy adjusts in case of changes in the training leave benefit, the labor income tax rate, and the unemployment benefit parameter. The following section derives the first order conditions to the central planner's problem and determines the optimal policies in a decentralized economy with and without labor unions.

5. The Planned Economy

This section firstly derives the first order conditions for an optimal growth path and determines the balanced growth rate in a centrally planned economy. Then, the optimal

tax–subsidy structures in a decentralized economy with and without labor unions are determined.

The central planner is assumed to choose consumption C and the fraction of time spent at work u in order to maximize the representative household's lifetime utility (2.8) subject to the resource constraint (2.14) and the human capital accumulation function (2.10). The necessary first order conditions with respect to C, u, H and K become:

$$C_i^{-\theta} e^{-\rho t} = \mu_k \tag{5.1}$$

$$\mu_h B = \mu_k (1-\alpha) \frac{Y}{uH} \tag{5.2}$$

$$-\frac{\dot{\mu}_h}{\mu_h} = B \tag{5.3}$$

$$-\frac{\dot{\mu}_k}{\mu_k} = \alpha \frac{Y}{K} \tag{5.4}$$

where μ_k (μ_h) is the shadow price of physical capital (human capital). Thus, the centrally planned solution is characterized by full employment u^{CPS} in the sense that households spend their entire time either at work u^{CPS} or in education $\left(1-u^{CPS}\right)$. Equation (5.1) is seen to be identical to (3.1). Equation (5.2) describes the optimal allocation of human capital between education and work. Thus, the value of the marginal product of human capital in education should equal the opportunity cost of education, which in the centrally planned economy is the marginal product of human capital. Equation (5.3) implies that the rate of change in the shadow price of human capital should equal the marginal product of human capital in the education sector. Finally, equation (5.4) says that the rate of change in the shadow price of physical capital should equal the marginal product of physical capital.

Once more, the balanced growth rate is derived by logarithmic differentiation of (5.1) and use of the fact that the shadow prices of human and physical capital decline at the same rate in the steady state. The balanced growth rate in the centrally planned economy without labor unions and unemployment therefore becomes:

$$g_c^{*CPS} = \frac{1}{\theta}[B-\rho] \tag{5.5}$$

while the marginal product of human capital in final goods production in the centrally planned economy is obtained by use of (5.4), (5.3) and (2.5):

$$w^{*CPS} = (1-\alpha) A \left(\frac{\alpha A}{B}\right)^{\frac{\alpha}{1-\alpha}} \tag{5.6}$$

Note that w^{*CPS} is equal to the steady state competitive wage rate in a market economy without labor unions and unemployment. According to (5.6), the steady state marginal product of human capital in final goods production rises with the productivity in the final goods sector A and declines with the productivity in education B.

5.1. Optimal Tax–Subsidy Structures

The question addressed in the following is whether a first best outcome characterized by full employment and optimal growth can be implemented in a decentralized economy.

In a market economy with no labor unions (NLU), the optimal value of the training leave benefit $(1-\hat{p})$ is immediately seen by comparison of the market growth rate (3.8) and the optimal growth rate (5.5) to equal zero, see also Figure 5.1. Comparison of

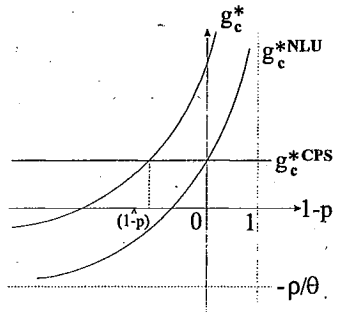

Figure 5.1: The optimal training leave benefit

the first order conditions to the market solution and the planned solution furthermore reveals that the labor income tax τ_h and the capital income tax τ_k should be zero in order to reach the first best solution. This policy $(1-\hat{p})=\hat{\tau}_h=\hat{\tau}_k=0$ results in an optimal allocation of time between work and unemployment, when the unemployment benefit rate b equals the competitive wage rate w^{CPS}. Note that the optimal tax–subsidy structure holds both along and outside the balanced growth path and that a lump sum tax instrument is required to finance unemployment benefits. However, implementation of the above-mentioned optimal policy leads to an indeterminacy in the model, since households become indifferent between working and being unemployed. As a consequence, the following analyses focus on cases that are consistent with a solution to the model, namely a solution where the unemployment benefit rate b is less

than the competitive wage rate w^{CPS}, see Figure 5.2.[15]

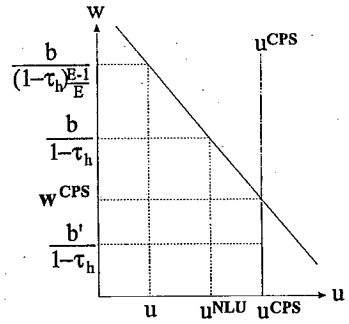

Figure 5.2: The relationship between wages w and employment u.

Figure 5.2 also illustrates that the existence of unemployment in a market economy with labor unions requires that the wage rate (2.11) is larger than or equal to the competitive wage rate. This yields a second constraint on the value of the unemployment benefit, which means that the results of the analyses below hold for the following values of b:

$$(1-\tau_h)\frac{E-1}{E}w^{CPS} \leq b \tag{5.7}$$

Now, the optimal policy in the market economy with an imperfect labor market can be determined. In order to reach a first best solution three conditions should be fulfilled. Firstly, the capital income tax should be zero $\hat{\tau}_k=0$. This is seen by comparison of (3.4) and (5.4). Secondly, full employment should be ensured by giving labor unions the right incentives to set the wage rate (2.11) equal to the marginal product of human capital in production (5.2). Thus, the optimal labor income tax is:

$$\hat{\tau}_h = 1 - \frac{\bar{b}u^{CPS}}{\frac{E-1}{E}(1-\alpha)} \tag{5.8}$$

where $\hat{\tau}_h < 1$, if $b < \frac{E-1}{E}w^{CPS}$. And thirdly, optimal growth should be ensured by giving households the right incentives to allocate time optimally between education and unemployment. This is done by setting the optimal training leave benefit such

[15] This is the case when the labor income tax rate is less than 1, which is a reasonable assumption to make.

that the market growth rate (3.7) equals the optimal growth rate (5.5):

$$(1-\hat{p}) = -\frac{1-\alpha}{E}\frac{1-\hat{\tau}_h}{\bar{b}} < 0 \tag{5.9}$$

where $\hat{\tau}_h$ is given by (5.8). According to equation (5.9), the optimal training leave benefit is negative – even when work effort is subsidized. This implies that it is optimal to charge a tuition fee both outside and along the balanced growth path in a market economy with an imperfect labor market. Setting the tuition fee at the level in (5.9) implies that the opportunity cost of education pb equals the income that a household can obtain, if it does not spend time in education at all, namely $(1-\tau_h)\,wu + b\,(1-u)$, see equation (3.3). According to (5.9), the optimal tuition fee is smaller, the bigger the optimal labor income tax rate τ_h, the higher the unemployment benefit parameter $\bar{b}$, and the smaller the labor share in final goods production $(1-\alpha)$. Moreover, in a situation where labor unions have no monopoly power ($E \to \infty$), the optimal training leave benefit equals zero. Thus, the greater the monopoly power of the labor unions is, the larger is the optimal tuition fee. The reason behind the latter result is that when the unemployment benefit rate is sufficiently high (5.7), then the existence of labor unions leads to a wage rate, which is higher than it would have been in a competitive economy. This gives households an extra incentive to allocate time to education due to the expectation of an even higher future income. In order to counteract the tendency to allocate too much time to education, it becomes optimal for the government from a welfare perspective to charge a tuition fee.

Proposition 5.1. *It is optimal to charge a tuition fee in an economy with involuntary unemployment caused by monopoly labor unions.*

This result solely hinges on the fact that labor unions set excessive wage rates and thereby give households a too large incentive to spend time in education. Even if unemployment benefits were constant over time instead of being indexed to average income, it would still be optimal to charge a tuition fee. However, an important assumption is that the existence of monopoly labor unions leads to a division of labor, where households divide their time between work, education, and involuntary unemployment. In the present model, all households therefore face the same opportunity cost of education, namely the difference between the unemployment benefit and the training leave benefit. However, if households instead were divided into two groups depending on their employment status, then the opportunity cost of education for the

group of employed households would instead be the difference between the wage rate and the training leave benefit plus the possible risk of becoming unemployed at the end of a training leave period. Due to the risk of becoming unemployed, the incentive for employed households to take training leave would probably be too low, which would tend to modify the strong result in Proposition 5.1.

The relationship between the growth rate in the market economy with labor unions and the tuition fee is also illustrated in Figure 5.1. The balanced growth rate in the market economy g_c^* is seen to approach infinity, when the training leave benefit approaches the unemployment benefit ($p \to 0$). The reason is that households allocate more and more time to education as the opportunity cost of education approaches zero. As a result, the growth rate approaches infinity, because human capital accumulation is the engine of growth. Moreover, the balanced growth rate in the market economy approaches $-\frac{\rho}{\theta}$, when the tuition fee approaches infinity. This implies that there is a lower bound for the negative growth rate in the economy, no matter how large an amount is charged in tuition fee. Finally, the optimal tuition fee is seen to approach zero, when the g_c^*–schedule moves towards the g_c^{*NLU}–schedule, which is the case when labor unions lose monopoly power $E \to \infty$.

Comparison of the balanced growth rate in the market economy (3.7) and the centrally planned economy (5.5) furthermore reveals that:

$$\begin{aligned} g_c^* > g_c^{*CPS}, \quad \text{when} \quad -(1-p) < \tfrac{1-\alpha}{E}\tfrac{1-\hat{\tau}_h}{\bar{b}} \\ g_c^* < g_c^{*CPS}, \quad \text{when} \quad -(1-p) > \tfrac{1-\alpha}{E}\tfrac{1-\hat{\tau}_h}{\bar{b}} \end{aligned} \tag{5.10}$$

According to (5.10), the balanced growth rate is higher (lower) in the market economy than in a centrally planned economy, whenever the tuition fee is smaller (greater) than optimal in the steady state. A too small tuition fee therefore leads to an inoptimally high growth rate in the market economy, which implies that the market economy is not welfare optimizing.

Thus, there are two important consequences of introducing a training leave benefit in an economy with involuntary unemployment created by monopoly labor unions. On the one hand, a training leave benefit enhances growth, since households allocate more time to education and thereby accumulate human capital at a faster rate. On the other hand, a training leave benefit reduces welfare, since too much time is allocated to education in the first place due to the inoptimally high wage rate set by labor unions.

This section derived the balanced growth rate in the centrally planned economy, and determined the optimal tax–subsidy structure in an economy with and without

unemployment caused by the existence of monopoly labor unions. The following section summarizes and concludes the paper.

6. Summary and Conclusions

This paper has examined the problem of increasing economic growth and at the same time reduce unemployment by means of a training leave benefit. At the same time it has given an answer to the question of the optimal tuition fee in an economy with an imperfectly functioning labor market.

In order to investigate the effects of a training leave benefit, a two sector endogenous growth model with monopoly labor unions was used. The first sector was assumed to produce both consumption goods and physical capital by use of physical and human capital. Firms in the final goods sector chose how much labor to employ at the wage rate that was determined by monopoly labor unions. A labor union for each household skill maximized the life time utility of its representative member by setting the wage rate as a mark–up over unemployment benefits. Each household was assumed to arbitrage between being unemployed and taking training leave. Households chose to take training leave in order to study in the education sector, because they expected to earn higher wages in the future. Finally, the government levied factor income taxes on both capital and labor in order to finance the training leave benefit and the unemployment benefit. In order to analyze the balanced growth path and transitional dynamics of the market economy, the capital income tax was assumed to adjust in order to balance the government budget.

Several analyses were undertaken in the paper. Firstly, the allocation of the representative household's time between work, unemployment and education was revealed to be affected both by the tax rates and the benefit rates. The time spent at work was shown to fall, whenever the labor income tax or the unemployment benefit increases. In addition, the time spent in unemployment was shown to fall and the time spent on training leave to increase; whenever the marginal product of human capital in education increases due to an increase in the level of productivity in education; whenever the opportunity cost of education falls due to an increase in the training leave benefit or a fall in the unemployment benefit; and whenever the expected future after–tax wages increase due to a fall in the labor income tax.

Secondly, the balanced growth rate in the market economy was shown to increase with the level of productivity in education, and decline with the opportunity cost of

training leave and the labor income tax rate. In addition, the analysis revealed that the balanced growth rate increases with the monopoly power of labor unions, because a low substitutability between labor skills leads to a high wage rate and thereby to a larger allocation of time to education, which is the growth engine in the presented model.

Thirdly, the relationship between the rate of unemployment and the balanced growth rate was shown to be negative such that a high rate of unemployment is associated with a low rate of balanced growth in the market economy.

The dynamic analysis concentrated on the economic adjustments in case of changes in the training leave benefit, the labor income tax rate and the unemployment benefit parameter. The main result of this analysis was that an increase in the training leave benefit left the fraction of time spent at work unaffected, while it increased the fraction of time spent in education. This firstly resulted in a higher growth rate of human capital and thereby in an increase in the marginal product of physical capital, which led to an immediate discrete fall in the ratio of consumption to physical capital in order to obtain a higher rate of physical capital accumulation. Subsequently, there was a gradual adjustment towards the new steady state, where the ratio of consumption to physical capital was continuously falling, while the ratio of physical to human capital was continuously increasing. In the new steady state the balanced growth rate was greater than in the original steady state. It turned out that the transitional dynamics of the consumption–capital ratio and the capital–labor ratio were the same in case of an increase in the labor income tax, but that the latter had a negative effect on the growth rate. The reason being that an increase in the training leave benefit left the fraction of time spent at work unaffected and increased the time fraction spent on training leave, whereas an increase in the labor income tax reduced the time fraction spent both at work and on training leave. Since human capital accumulation is the engine of growth, a decline in the fraction of time spent on training leave naturally reduces growth.

One of the main results of the paper is that it is optimal to charge a tuition fee instead of giving a benefit in an economy with involuntary unemployment caused by monopoly labor unions. The reason behind this result is that labor unions set an inoptimally high wage rate which makes expected future wages too high and thereby training leave too attractive even without the training leave benefit. In order to counteract the resulting excessive growth rate, it becomes optimal from a welfare perspective to charge a tuition fee. The optimal tuition fee was shown to be smaller, the smaller the monopoly power of the labor unions, the higher the unemployment benefit, and

the bigger the labor income tax rate.

Another main result is that it is not necessarily welfare improving to reduce unemployment and at the same time increase economic growth. In fact it is only welfare improving to undertake growth enhancing measures as long as the tuition fee is bigger than its optimal level. Thus, the introduction of a training leave benefit leads to a higher balanced growth rate, but at the same time to a reduction in welfare.

A. Appendix

A.1. The Wage Index and the Labor Demand for Skill i

This appendix derives the relevant wage index given by (2.6) and the demand for labor with skill i given by (2.7).

Define the expenditure function for labor as:

$$\begin{aligned} e(w_1,...,w_n,c) &= \min_{u_iH_i} \sum_{i=1}^{n} w_i u_i H_i \qquad (A.1) \\ \text{s.t. } L(u_1H_1,...,u_nH_n) &= c \end{aligned}$$

where c is a constant. Cost minimization results in the following first order condition:

$$w_i = \lambda n^{\frac{1}{1-E}} \left(\sum_{i=1}^{n} (u_i H_i)^{\frac{E-1}{E}} \right)^{\frac{1}{E-1}} (u_i H_i)^{-\frac{1}{E}} \qquad (A.2)$$

where λ is the shadow price. Multiplication by u_iH_i on both sides of (A.2) and summation over all i yields:

$$e(\cdot) = \lambda n^{\frac{1}{1-E}+\frac{1}{E}} \left(\sum_{i=1}^{n} (u_i H_i)^{\frac{E-1}{E}} \right)^{\frac{1}{E-1}} c^{\frac{E-1}{E}} \qquad (A.3)$$

Divide (A.3) through by $w_nu_nH_n$ and solve for u_nH_n:

$$u_n H_n = e(\cdot)^E w_n^{-E} c^{1-E} n^{-1} \qquad (A.4)$$

Multiplication of (A.4) by w_n and summation over all i and use of (A.1) yields:

$$e(\cdot) = c \left(\frac{1}{n} \sum_{1=1}^{n} w_i^{1-E} \right)^{\frac{1}{1-E}} \qquad (A.5)$$

Thus, the relevant wage index is given by the following cost-minimization rule:

$$w \equiv \left(\frac{1}{n}\right)^{\frac{1}{1-E}} \left(\sum_{i=1}^{n} w_i^{1-E} \right)^{\frac{1}{1-E}} \qquad (A.6)$$

Define the indirect cost function as:

$$\begin{aligned} S(w_1,...,w_n,R) &= \max_{u_iH_i} L(u_1H_1,...,u_nH_n) \qquad (A.7) \\ \text{s.t. } \sum_{i=1}^{n} w_i u_i H_i &= R \end{aligned}$$

Now define $\hat{u}$ as follows:

$$\begin{aligned} e(w_1,...,w_n,\hat{u}) &= R = \hat{u}w \qquad (A.8) \\ S(w_1,...,w_n,R) &= \hat{u} \end{aligned}$$

From (A.8) and (A.6) we know that:

$$S(\cdot) = \frac{R}{w} = R\left(\frac{1}{n}\right)^{\frac{1}{E-1}} \left(\sum_{i=1}^{n} w_i^{E-1} \right)^{\frac{1}{E-1}} \qquad (A.9)$$

Furthermore, we know that:

$$L_i(w_1,...,w_n,R) = -\frac{\partial S(\cdot)/\partial w_i}{\partial S(\cdot)/\partial R} \qquad (A.10)$$

Thus, the demand for labor of skill i is derived from (A.10) and (A.11) as:

$$u_i H_i = \left(\frac{w_i}{w}\right)^{-E} \frac{uH}{n} \qquad (A.11)$$

A.2. The Non–Linear Dynamic System

The dynamic evolution of the economy is determined by equation (3.12)–(3.14), which can be rewritten by use of (3.15) and (4.1) as:

$$-\frac{B}{p}\left(\frac{1}{E-1}+\alpha\right)u - \frac{B}{p} + \alpha\chi = \alpha\tau_k z - \frac{\alpha B}{p}\left(1 - \frac{\alpha\tau_k + (1-\alpha)\tau_h}{\bar{b}}\right) \qquad (A.12)$$

$$\begin{aligned} g_z &= (1-\alpha)\left(\chi - z + \frac{B}{p}\left(1 - u - \frac{(1-\alpha)\tau_h}{\bar{b}}\right) - \frac{B}{p}\frac{\alpha}{\bar{b}}\tau_k\right) \qquad (A.13) \\ g_\chi &= -\frac{\theta - \alpha(1-\tau_k)}{\theta} z - \frac{\rho}{\theta} + \chi \qquad (A.14) \end{aligned}$$

where $g_z = -(1-\alpha)\, g_\omega$. Throughout the analysis $\theta > 1$, which is normally assumed.

Solve (A.12) for τ_k:

$$\tau_k = \frac{1}{z + \frac{B}{p}\frac{\alpha}{\bar{b}}} (\chi - D) \qquad (A.15)$$

where $D = \frac{B}{p}\left(\left(\frac{1}{\alpha}\frac{1}{E-1}+1\right)u + \frac{1-\alpha}{\alpha} + \frac{1-\alpha}{\bar{b}}\tau_h\right) > 0$. It can be seen that an increase in the average product of physical capital leads to a decline in the capital income tax, while an increase in the ratio of consumption to physical capital leads to an increase in the capital income tax.

Introduce τ_k in equation (A.13) and set $g_z = 0$ in order to derive the $\dot{z} = 0$–schedule:

$$\chi = z - \left(\frac{B}{p}\right)^2 \frac{1}{\bar{b}}\left(\frac{1}{E-1}u+1\right)\frac{1}{z} - \frac{B}{p}\left(1-u-\frac{1}{\bar{b}}\left(\alpha+(1-\alpha)\tau_h\right)\right) \qquad \text{(A.16)}$$

The locus is seen to have two asymptotes. One with slope one and intersect point $-\frac{B}{p}\left(1-u-\frac{1}{\bar{b}}\left(\alpha+(1-\alpha)\tau_h\right)\right)$ for z going towards infinity and another, which is the χ–axis for z going towards minus infinity. Furthermore, the locus is stable, since $z > z^*$ leads to a fall in z for a given χ. The slope of the $\dot{z} = 0$–schedule is characterized by the partial derivatives of χ with respect to z:

$$\frac{\partial \chi}{\partial z} = 1 + \left(\frac{B}{p}\right)^2 \frac{1}{\bar{b}}\left(\frac{1}{E-1}u+1\right)\left(\frac{1}{z}\right)^2 > 0$$

which reveals that the $\dot{z} = 0$-schedule is upward sloping globally.

Similarly, the $\dot{\chi} = 0$–schedule is obtained by introduction of τ_k in equation (A.14) setting $g_\chi = 0$:

$$\begin{aligned}\chi &= \frac{z}{\frac{\theta-\alpha}{\theta}+\frac{B}{p}\frac{\alpha}{\bar{b}}\frac{1}{z}}\left(\frac{\theta-\alpha}{\theta}+\left(\frac{\theta-\alpha}{\theta}\frac{B}{p}\frac{\alpha}{\bar{b}}-\frac{\alpha}{\theta}D+\frac{\rho}{\theta}\right)\frac{1}{z}+\frac{B}{p}\frac{\alpha}{\bar{b}}\frac{\rho}{\theta}\left(\frac{1}{z}\right)^2\right) \Leftrightarrow \\ \chi &= \frac{1}{z+\frac{B}{p}\frac{\alpha}{\bar{b}}\frac{\theta}{\theta-\alpha}}\left(z^2+\left(\frac{B}{p}\frac{\alpha}{\bar{b}}-\frac{\alpha}{\theta-\alpha}D+\frac{1}{\theta-\alpha}\rho\right)z+\frac{1}{\theta-\alpha}\frac{B}{p}\frac{\alpha}{\bar{b}}\rho\right) \qquad \text{(A.17)}\end{aligned}$$

This locus also has an asymptote for z going towards infinity, which happens to be the "45–degree"-line. In addition it has a vertical asymptote for z approaching $-\frac{B}{p}\frac{\alpha}{\bar{b}}\frac{\theta}{\theta-\alpha}$. Furthermore, the locus intersects the χ–axis for $\chi = \frac{\rho}{\theta}$. The locus is seen to be unstable, since $\chi > \chi^*$ leads to an increase in χ for a given z.

The slope of the $\dot{\chi} = 0$–schedule is characterized by the partial derivative of χ with respect to z:

$$\frac{\partial \chi}{\partial z} = \frac{1}{\frac{\theta-\alpha}{\theta}z+\frac{B}{p}\frac{\alpha}{\bar{b}}}\left(2\frac{\theta-\alpha}{\theta}z+\frac{\theta-\alpha}{\theta}\frac{B}{p}\frac{\alpha}{\bar{b}}-\frac{\alpha}{\theta}D+\frac{\rho}{\theta}-\frac{\theta-\alpha}{\theta}\chi\right)$$

Thus, the $\dot{\chi} = 0$–schedule can both be upward and downward sloping.

A unique solution requires:

$$-\frac{B}{p}\left(1-u-\frac{1}{\bar{b}}\left(\alpha+(1-\alpha)\tau_h\right)\right) = \frac{B}{p\bar{b}}\left(1-\frac{1}{E}\left(1-\alpha\right)\left(1-\tau_h\right)-\bar{b}\right) > 0 \qquad \text{(A.18)}$$

which means that the positively sloped asymptote of the $\dot{z} = 0$-locus has a positive intersect point with the χ–axis.

Now the steady state value of z, χ and τ_k are derived by use of (A.15)–(A.17). Solve (A.16) and (A.17) in terms of τ_k:

$$\begin{aligned}\chi &= \frac{\theta-\alpha(1-\tau_k)}{\alpha(1-\tau_k)}\left(\frac{B}{p}\left(1-u-\frac{(1-\alpha)\tau_h}{\bar{b}}\right)-\frac{B}{p}\frac{\alpha}{\bar{b}}\tau_k\right)+\frac{\rho}{\alpha(1-\tau_k)} \qquad \text{(A.19)} \\ z &= \frac{\theta}{\alpha(1-\tau_k)}\left(\frac{B}{p}\left(1-u-\frac{(1-\alpha)\tau_h}{\bar{b}}\right)-\frac{B}{p}\frac{\alpha}{\bar{b}}\tau_k+\frac{\rho}{\theta}\right) \qquad \text{(A.20)}\end{aligned}$$

and then substitute these into equation (A.15):

$$\begin{aligned}\tau_k = \frac{1}{\frac{\theta}{\alpha(1-\tau_k)}\left(\frac{B}{p}\left(1-u-\frac{(1-\alpha)\tau_h}{\bar{b}}\right)-\frac{B}{p}\frac{\alpha}{\bar{b}}\tau_k+\frac{\rho}{\theta}\right)+\frac{B}{p}\frac{\alpha}{\bar{b}}} \\ \left[\frac{\theta-\alpha(1-\tau_k)}{\alpha(1-\tau_k)}\left(\frac{B}{p}\left(1-u-\frac{(1-\alpha)\tau_h}{\bar{b}}\right)-\frac{B}{p}\frac{\alpha}{\bar{b}}\tau_k\right)\right. \\ \left.+\frac{1}{\alpha(1-\tau_k)}\rho-\frac{B}{p}\left(\left(\frac{1}{\alpha}\frac{1}{E-1}+1\right)u+\frac{1-\alpha}{\alpha}+\frac{1-\alpha}{\bar{b}}\tau_h\right)\right] \qquad \text{(A.21)}\end{aligned}$$

The solutions to this equation are:

$$\begin{aligned}\tau_k^* &= 1-\frac{1}{\alpha}\left(1-\bar{b}\left(\frac{\theta-1}{\theta}+p\frac{1}{\theta}\frac{1}{B}\rho\right)-(1-\alpha)\left(1-\tau_h\right)\frac{1}{E}\frac{\theta-1}{\theta}\right) \\ \text{or } \tau_k^* &= 1 \qquad \text{(A.22)}\end{aligned}$$

see equation (3.20) for comparison.

The second solution for the capital income tax rate is immediately ruled out, since is does not make sense to tax all capital income away. Furthermore, introduction of $\tau_k = 1$ in the z and the χ expressions implies $\chi \to \infty$ and $z \to \infty$.

A.3. The Linearized Dynamic System

The dynamic system is given by (A.13) and (A.14), where (A.15) have been used:

$$\begin{aligned}g_z &= (1-\alpha)\left(\chi - z + \frac{B}{p}\left(1-u-\frac{(1-\alpha)\tau_h}{\bar{b}}\right)-\frac{B}{p}\frac{\alpha}{\bar{b}}\frac{1}{z+\frac{B}{p}\frac{\alpha}{\bar{b}}}\left(\chi-D\right)\right) \\ g_\chi &= -\frac{\theta-\alpha(1-\frac{1}{z+\frac{B}{p}\frac{\alpha}{\bar{b}}}(\chi-D))}{\theta}z-\frac{\rho}{\theta}+\chi\end{aligned}$$

The system is linearized around the steady state by means of a first order Taylor expansion:

$$\begin{bmatrix} g_z \\ g_\chi \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} z-z^* \\ \chi-\chi^* \end{bmatrix}$$

where:

$$\begin{aligned} a_{11} &= -(1-\alpha)\,\Omega \quad a_{12} = (1-\alpha)\left(1-\frac{\frac{B}{p}\frac{\alpha}{b}}{z^*+\frac{B}{p}\frac{\alpha}{b}}\right) \\ a_{21} &= -\frac{\theta-\alpha\Omega}{\theta} \qquad a_{22} = 1-\frac{\alpha}{\theta}\frac{z^*}{z^*+\frac{B}{p}\frac{\alpha}{b}} \\ \Omega &= 1-\frac{\frac{B}{p}\frac{\alpha}{b}}{z^*+\frac{B}{p}\frac{\alpha}{b}}\tau_k^* \end{aligned} \tag{A.23}$$

and the signs holds for $\theta > 1$ and $0 < \tau_k^* < 1$. The determinant of the Jacobian is given by:

$$-(1-\alpha)\frac{\frac{B}{p}\frac{\alpha}{b}}{z^*+\frac{B}{p}\frac{\alpha}{b}}(1-\tau_k^*) < 0$$

which is negative for $0 < \tau_k^* < 1$. Thus, the system has saddle–path properties, and the eigenvalues are determined by the following characteristic equation:

$$\xi^2 - (a_{11}+a_{22})\,\xi + a_{11}a_{22} - a_{21}a_{12} = 0$$

The two eigenvalues are:

$$\begin{aligned} \xi_1 &= \frac{1}{2}\left[(a_{11}+a_{22}) + \sqrt{(a_{11}+a_{22})^2 + 4a_{21}a_{12}}\right] > 0 \\ \xi_2 &= \frac{1}{2}\left[(a_{11}+a_{22}) - \sqrt{(a_{11}+a_{22})^2 + 4a_{21}a_{12}}\right] < 0 \end{aligned}$$

The eigenvector associated with the negative eigenvalue corresponds to the stable arm and is given by:

$$(\nu_{12}, \nu_{22}) = \left(1, \frac{\frac{\theta-\alpha\Omega}{\theta}}{1-\frac{\alpha}{\theta}\frac{z^*}{z^*+\frac{B}{p}\frac{\alpha}{b}} - \xi_2}\right) \tag{A.24}$$

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Environmental Policy in a Two Sector Endogenous Growth Model *

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Abstract

This paper analyzes policy measures within a two sector endogenously growing economy with pollution. Pollution is either generated by production or by the use of physical capital in production, and can be reduced by public abatement activities. In this generalized Uzawa–Lucas model, the effects of fiscal policy are derived for all core variables and ratios. In addition, the optimal taxation rules are derived. If a pollution tax is not available it turns out that a first best solution may be reached by use of factor income taxation. Additionally, the effects and the possibility of environmental policy are complemented for a small open economy.

JEL classification: D62, E62, F43, H21, O41, Q28

Keywords: Endogenous growth, environmental externality, optimal taxation, international capital mobility

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1. Introduction

By analyzing the effects of environmental policy on growth this paper considers an increasingly important topic throughout the industrialized world. It contributes to the literature by analyzing the effects of environmental policy in a fairly general two sector endogenous growth model. This is important, since the results crucially depend on the model specification. Our model is an extension of the generalized Uzawa–Lucas model[1] used by [10] King & Rebelo (1990), which additionally allows for disutility of pollution and public abatement activities that are financed by tax revenues. To keep the analysis general, we investigate two sources of gross pollution, namely pollution generated by final goods production and pollution generated by the use of physical capital in final goods production. The emission of gross pollution can be reduced by public abatement activities. The positive analysis in this paper not only examines the long term growth effects of taxes on consumption, gross pollution, capital and labor income and their channels, but also their influence on all core variables and ratios. The normative analysis addresses the question of the optimal taxation scheme. Furthermore, we investigate the effects and the possibility of environmental policies in a closed economy and a small open economy setting. The analysis of the closed economy is a dynamic general equilibrium analysis, whereas the investigation of the small open economy starts out from conventional assumptions in international trade models, thereby simplifying the analysis by an exogenously given world interest rate, but broadening the analysis by assuming perfect international capital mobility.

In the following, the literature on fiscal policy, environmental policy, and international capital mobility in human capital growth models is reviewed. Several authors, [10] King & Rebelo (1990), [21] Rebelo (1991), [6] Devereux & Love (1994), and [16] Milesi–Ferretti & Roubini (1995), analyze the consequences of tax rate changes for economic growth with production specifications similar to this paper. However, they do not consider the environment and redistribute the tax revenues lump sum to consumers. By incorporating the environment, this paper is able to analyze a broader set of tax instruments, namely a tax on gross pollution in addition to the value added tax and the taxes on labor and capital income. Like in [17] Nielsen et al. (1995), the tax revenues are used to finance public abatement activities.

The effects of environmental policy on economic growth are ambiguous in the literature of endogenous growth with human capital.[2] In the simplest endogenous growth model, the AK model, the growth effect of environmental policy is negative. This is shown both by [8] Gradus & Smulders (1993) for a centrally planned economy by varying the weight on pollution in the utility function and by [11] Ligthart & van der Ploeg (1994) for a decentralized economy. However, [8] Gradus & Smulders (1993) show in a variant of the two sector Uzawa–Lucas model that environmental policy does not affect long term growth. Furthermore, [9] Hettich (1998) shows in a Uzawa–Lucas model with pollution and leisure that the growth effect of a tighter environmental policy depends on the pollution specification. If pollution is caused by the physical capital stock, then higher abatement activities stimulate growth[3], whereas if pollution is complementary to the output level, higher abatement activities does not affect long term growth. This paper shows that a tighter environmental policy has a negative effect on growth in a generalized Uzawa–Lucas model irrespective of the pollution specification.

Finally, [10] King & Rebelo (1990) and [15] Milesi–Ferretti & Roubini (1994) consider the effects of tax rates on growth in small open economies with international capital mobility.[4] In their models, international growth rate differentials can be explained in a residence–based capital tax system, but not in a source–based system.[5] The production processes are similar to those in this paper, but these contributions do not consider an environmental externality.

Irrespective of the chosen pollution specification the major conclusions of our paper are firstly that factor income taxes and the pollution tax are growth reducing, whereas

[1] The reproducible input factors human capital and physical capital are used as input factors both in the education sector and in the final good sector.

[2] In the following, we formally neglect that a better environment in principle could have a positive effect on the productivity of inputs. Environmental quality in such cases not only acts as a public consumption good, but also as a productive public capital good. It has been shown that a better environmental quality may have a stimulating growth effect in the presence of positive environmental spillovers in production, see [3] Bovenberg & Smulders (1995), [7] van Ewijk & van Wijnbergen (1995), and [25] Smulders & Gradus (1996). Such productivity spillovers are certainly conceivable in the field of agriculture and tourism, but it is questionable whether they exist in an aggregate production function.

[3] A higher pollution tax gives rise to more abatement which crowds out consumption. This makes leisure less scarce which stimulates studying and therefore growth.

[4] [22] Rebelo (1992) and [12] Long & Wong (1997) survey the literature on endogenous growth in open economies.

[5] In overlapping generation models of the Uzawa–Lucas type [5] Buiter & Kletzer (1995) explain persistent growth rate differentials independently of the international taxation system. In a variant of the Uzawa–Lucas model augmented with endogenous population [20] Razin & Yuen (1996) find that the growth rate effect of taxes depends on the principle of international taxation and the preference bias between child quantity and child quality.

the consumption tax is a lump sum tax. Secondly, the market allocation without government intervention is inefficient due to the presence of the environmental externality. Thus, there is too much pollution, too little abatement and excessive economic growth in an unregulated market economy. The first best solution can be achieved by setting an optimal pollution tax equal to the marginal damage of pollution. It turns out that an optimal solution instead may be reached by certain combinations of factor income taxes, if the pollution tax is unavailable as an instrument. Thirdly, under a residence–based tax system, a small open economy can choose its own fiscal policy and hence determine its long term growth rate. As long as the environmental problem is a national one, the government can choose the level of pollution and hence the environmental quality. However, under a source–based tax system, a small open economy is partially restricted in its fiscal policy and can as a consequence only lead a second best environmental policy.

The paper is organized as follows. The model is laid out in Section 2 and the first order conditions of the representative agent problem and central planner problem are derived. Section 3 determines the optimal tax rates. Furthermore, the balanced growth rate is derived and the effects of tax and parameter changes on all core variables and ratios are determined. Finally, the small open economy version of the model is analyzed. Section 4 considers an alternative pollution specification and Section 5 concludes the paper.

2. The model

This section presents a two sector endogenous growth model of a closed economy and derives the first order conditions of the representative agent problem and the central planner problem. The first sector produces universal goods which can be used for consumption, abatement activities, and physical capital accumulation, while the second sector is an education sector in which human capital is accumulated.[6] Human capital is assumed to be embodied in people and is consequently a private good, which is both rival and excludable. Both sectors use physical and human capital as input factors and factors move freely between the two sectors. The production of the final good causes a negative environmental externality in form of pollution. The government is assumed

[6]The specification of the two production sectors is similar to [10] King & Rebelo (1990) and [21] Rebelo (1991).

to engage in public abatement activities, which are solely financed by tax revenues. The economy consists of a large number of identical and infinitely lived households that own the factors of production and rent them to firms. Households are assumed to maximize their discounted life time utility and firms are assumed to maximize their market value.

2.1. Technology

In the first sector, final goods Y_t are produced with a constant returns to scale technology using physical capital K_t and human capital H_t as inputs. The technology is assumed to take the Cobb–Douglas form:

$$Y_t = A\left(v_t K_t\right)^{\alpha}\left(u_t H_t\right)^{1-\alpha} \tag{2.1}$$

where A, H_t, K_t, $Y_t > 0$, $0 < \alpha$, u_t, $v_t < 1$. Parameter A reflects the exogenous level of the technology, α is the exogenous physical capital share in final goods production, and v_t and u_t are the endogenous shares of physical and human capital devoted to final goods production, respectively. Human capital is assumed to be a private good, which is embodied in people.[7] In the following, $u_t H_t$ is referred to as the effective labor force.

The flow resource constraint of the closed economy is:[8]

$$Y_t = C_t + \dot{K}_t + Z_t \tag{2.2}$$

where C_t, $Z_t > 0$. Final output Y_t is a universal good since it can be used either for consumption C_t, investment in physical capital $\dot{K}_t$, or for public abatement activities Z_t.[9]

The second sector is an education sector that produces human capital by use of physical capital representing facilities like school buildings and human capital representing education time and knowledge. The human capital accumulation function is assumed to be a constant returns to scale technology:

$$\dot{H}_t = B\left[\left(1-v_t\right)K_t\right]^{\beta}\left[\left(1-u_t\right)H_t\right]^{1-\beta} \tag{2.3}$$

where $B > 0$ reflects the exogenous level of the technology and $0 < \beta < 1$ is the exogenous physical capital share in education. The specification of the final goods

[7]Unlike [13] Lucas (1988), we neglect the possibility of an externality from the average stock of human capital to final goods production.

[8]For simplicity, human and physical capital are assumed to depreciate at a zero rate. However, this assumption does not change the qualitative results.

[9]A dot above a variable indicates its derivative with respect to time.

production function (2.1) and the human capital accumulation function (2.3) ensure that diminishing returns do not arise, when physical and human capital grow at the same rate. Since both inputs H_t and K_t can be accumulated infinitely, the rates of return remain constant along a balanced growth path and unlimited growth is in principle possible.

Final goods production is assumed to cause a negative environmental externality as a side product, which harms utility.[10] The externality is assumed to affect individual utility only and not to harm the production processes, i.e. there is no positive spillover of a better environment to production of goods or human capital accumulation. Of course, it is conceivable that pollution directly affects the productivity in the final goods or the education sector. However, this is an aspect not being analyzed formally in this paper, but there is a discussion of the issue at the end of Section 3.2. Aggregate pollution P_t is a public 'bad', which can be reduced by means of public abatement activities Z_t that consume a part of output, in line with the flow resource constraint (2.2). Public abatement can be interpreted as knowledge about clean production methods. Both P_t and Z_t are modelled as flow quantities, which is justified as long as the balanced growth path is analyzed. If public abatement Z_t increases, the output pollutes less. The net pollution function P_t is assumed to be given by the following functional form:

$$P_t = \left(\frac{Y_t}{Z_t}\right)^{\chi} \tag{2.4}$$

where $\chi > 0$ is the exogenous elasticity of pollution P_t with respect to the output–abatement ratio Y_t/Z_t. According to equation (2.4) pollution is increasing in final goods production and decreasing in public abatement activities. Section 4 analyses an alternative specification of the pollution function, where pollution instead is generated by the use of physical capital in final goods production.

2.2. Firms

The economy consists of a large number of identical and competitive firms. They rent physical capital from households at the interest rate r_t and hire human capital at the wage rate w_t. Firms use these input factors to produce final goods with the technology described by equation (2.1). Firms must pay a pollution tax τ_p according to their gross

[10] Like [19] van der Ploeg & Withagen (1991), [8] Gradus & Smulders (1993) and [11] Ligthart & Ploeg (1994) pollution is modelled as an 'output' factor in this paper. Alternatively, pollution could be modelled as an input factor, see [17] Nielsen et al. (1995) and [2] Bovenberg & de Mooij (1997). Both modelling approaches are equivalent, see [23] Siebert et al. (1980).

pollution. Since pollution is complementary to output, a pollution tax is equivalent to a tax on output. Firms are assumed to maximize their cash flow by choosing $u_t H_t$ and $v_t K_t$ given the pollution tax τ_p levied on total production Y_t:

$$\pi_t = (1-\tau_p)\, Y_t - w_t\,(u_t H_t) - r_t\,(v_t K_t) \tag{2.5}$$

Profits are maximized when the marginal cost of each factor equals its after–tax marginal product:

$$w_t = (1-\tau_p)\,(1-\alpha)\,A\left(\frac{v_t K_t}{u_t H_t}\right)^{\alpha} = (1-\tau_p)\,(1-\alpha)\,\frac{Y_t}{u_t H_t} \tag{2.6}$$

$$r_t = (1-\tau_p)\,\alpha A\left(\frac{v_t K_t}{u_t H_t}\right)^{\alpha-1} = (1-\tau_p)\,\alpha\frac{Y_t}{v_t K_t} \tag{2.7}$$

2.3. Households

Households are assumed to be identical, atomistic agents with perfect foresight over an infinite time horizon. Preferences are restricted to ensure the existence of a sustainable balanced growth path. The necessary conditions imply a specific functional form of the utility function, where consumption is multiplicatively separable from net pollution.[11] For simplicity, we assume an additively separable instantaneous utility function.[12] The corresponding discounted life time utility is given by:

$$U_0 = \int_{t=0}^{\infty} (\ln C_t - \eta \ln P_t)\, e^{-\rho t} dt \tag{2.8}$$

where C_t is consumption, P_t is aggregate net pollution, η is the positive exogenous marginal disutility of pollution, and ρ is the positive exogenous rate of time preference. Utility is seen to be increasing in consumption at a decreasing rate, $U_C > 0$ and $U_{CC} < 0$, while it is decreasing in aggregate pollution at an increasing rate, $U_P < 0$ and $U_{PP} > 0$.[13] Households choose consumption and the allocation of human and physical capital

[11] For the necessary conditions on the utility function, see [25] Smulders & Gradus (1996).

[12] A more general instantaneous utility function is $U_t = \frac{(C_t P_t^{-\eta})^{1-\theta}-1}{1-\theta}$, where θ is the inverse of the intertemporal elasticity of substitution. Since this function does not change the qualitative results concerning the growth rate effects, but complicates the analysis, we use the simplified function (2.8), where the intertemporal elasticity of substitution is equal to one.

[13] A more realistic utility function would allow for increasing marginal disutility of pollution $U_{PP} < 0$, but this only changes the optimal allocation of resources between abatement and final goods production (2.22). All other first order conditions of the central planner are unaffected and so are the obtained results.

between the two sectors in order to maximize their life time utility (2.8) subject to the human capital accumulation constraint (2.3) and the flow budget constraint:

$$\dot{K}_t = (1-\tau_k)\, r_t v_t K_t + (1-\tau_h)\, w_t u_t H_t - (1+\tau_c)\, C_t \tag{2.9}$$

where τ_c is the tax on consumption (value added tax), τ_h is the flat–rate tax on labor income, τ_k is the flat–rate tax on capital income, w_t is the wage rate, and r_t is the rate of return to physical capital. Education is assumed to be a non–market sector which does not give rise to direct income, it can consequently not be taxed by means of income taxes.[14]

2.4. Government

The government levies a pollution tax[15] on final goods production, a capital income tax, a labor income tax and a consumption tax in order to finance public abatement activities. Public spending has to be financed solely by taxes, since the government does not issue bonds. Hence, the government is assumed to run a balanced budget in every period, which is given by:

$$\tau_k r_t v_t K_t + \tau_h w_t u_t H_t + \tau_c C_t + \tau_p Y_t = Z_t \tag{2.10}$$

In the following, time indices of variables are neglected where unnecessary.

2.5. The Market Solution

The representative household chooses its consumption and the allocation of its physical and human capital to final goods production and education, respectively, in order to maximize its life time utility (2.8) subject to the human capital accumulation function (2.3) and the budget constraint (2.9) taking the tax rates τ_c, τ_h, τ_k, and τ_p as given. Since pollution is a public 'bad', economic agents take as given in their maximization problem. The first order conditions with respect to C, K, H, v, and u become:

$$\frac{1}{C} e^{-\rho t} = \lambda_k (1+\tau_c) \quad \Rightarrow \quad -\frac{\dot{\lambda}_k}{\lambda_k} = \frac{\dot{C}}{C} + \rho \tag{2.11}$$

$$-\frac{\dot{\lambda}_k}{\lambda_k} = (1-\tau_k)\, r \tag{2.12}$$

[14] For human capital as a market good, see [16] Milesi–Ferretti & Roubini (1995) in a similar model without environment.

[15] In fact, the pollution tax is a tax on output, since pollution is assumed to be complementary to output.

$$-\frac{\dot{\lambda}_h}{\lambda_h} = (1-\beta)\, B \left[\frac{(1-v)\, K}{(1-u)\, H}\right]^{\beta} \tag{2.13}$$

$$\lambda_k (1-\tau_k)\, r = \lambda_h \beta B \left[\frac{(1-v)\, K}{(1-u)\, H}\right]^{\beta-1} \tag{2.14}$$

$$\lambda_k (1-\tau_h)\, w = \lambda_h (1-\beta)\, B \left[\frac{(1-v)\, K}{(1-u)\, H}\right]^{\beta} \tag{2.15}$$

where λ_k and λ_h are the shadow prices of physical capital and human capital in the market solution, respectively. Equation (2.11) implies that the marginal utility of consumption in every period should equal the after–tax shadow price of physical capital. The first Euler condition (2.12) implies that the rate of change in the shadow price of physical capital should equal the after–tax marginal product of capital in the final goods sector. The second Euler condition (2.13) says that the rate of change in the shadow price of human capital should equal the marginal product of human capital in the education sector. Finally, equations (2.14) and (2.15) describe the optimal allocation of physical and human capital between the two sectors.

The transversality conditions to the maximization problem are:[16]

$$\lim_{t\to\infty} \lambda_k K = 0 \quad \text{and} \quad \lim_{t\to\infty} \lambda_h H = 0 \tag{2.16}$$

Equations (2.14) and (2.15) yield the sectorial allocation of resources as a function of the tax rates and the parameters of the model:

$$\frac{v}{u} = \left(\frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}\right)\left(\frac{1-\tau_k}{1-\tau_h}\right)\frac{1-v}{1-u} \tag{2.17}$$

where equations (2.6) and (2.7) have been used. According to equation (2.17), the after–tax rates of technical substitution between capital and labor must be equalized across sectors in order to achieve an optimal intersectorial allocation of capital and hours. The income taxes on capital and labor affect this allocation in different ways. According to equation (2.17), the capital–labor ratio in the final goods sector v/u increases relative to the capital–labor ratio in the education sector $(1-v)/(1-u)$, whenever the capital income tax declines or the labor income tax increases. In addition, an increase (a decline) in the capital share in final goods production (education) leads to

[16] These conditions rule out explosive paths by requiring that the present discounted value of each capital good equals zero in the long run. This is a reasonable requirement, since optimizing agents do not want valuable assets at the end of their planning horizon. Consequently, the first transversality condition requires that the real interest rate should be positive.

an increase in the capital–labor ratio in final goods production relative to the education sector. Note that the sectorial allocation of resources is unaffected by income taxation, when taxation is comprehensive $\tau_k = \tau_h$. In addition, the sectorial allocation of factors is unaffected by a change in the pollution tax, since it affects the return to the two input factors in the same way.

Finally, the Keynes–Ramsey rule describing the optimal consumption path for the market economy is derived by use of equations (2.11) and (2.12), where r is replaced by equation (2.7):[17]

$$\hat{C} = (1-\tau_k)(1-\tau_p)\,\alpha\frac{Y}{vK} - \rho \quad \equiv \quad R - \rho \tag{2.18}$$

For further analysis it seems to be useful to define R as the return to capital investment net of tax.

It is obvious that the consumption tax τ_c is a lump sum tax, since it is absent in the first order conditions of the market solution (2.11)–(2.15) and hence does not distort the economy. However, all other taxes have effects on the economy. The taxes on capital income and pollution affect the intertemporal incentive to invest in physical capital, described by equation (2.18). In addition, both factor income taxes affect the sectorial allocation of factors, described by equation (2.17).

2.6. The Planned Solution

In contrast to the representative household, a benevolent central planner takes the negative side effects of production into account. Consumption, the amount of public abatement activities, and the allocation of physical and human capital between the two sectors are chosen in order to maximize the discounted life time utility of the representative household (2.8) subject to the human capital accumulation function (2.3) and the resource constraint of the economy (2.2). The first order conditions with respect to C, K, H, Z, v, and u become:

$$\mu_k = \frac{1}{C}e^{-\rho t} \quad \Rightarrow \quad -\frac{\dot{\mu}_k}{\mu_k} = \frac{\dot{C}}{C} + \rho \tag{2.19}$$

$$-\frac{\dot{\mu}_k}{\mu_k} = \left(1-\frac{Z}{Y}\right)\alpha\frac{Y}{vK} \tag{2.20}$$

$$-\frac{\dot{\mu}_h}{\mu_h} = (1-\beta)\,B\left[\frac{(1-v)\,K}{(1-u)\,H}\right]^{\beta} \tag{2.21}$$

[17] A hat above a variable indicates its rate of growth.

$$\mu_k = \chi\eta\frac{1}{Z}e^{-\rho t} \tag{2.22}$$

$$\mu_k\left(1-\frac{Z}{Y}\right)\alpha\frac{Y}{vK} = \mu_h\beta B\left[\frac{(1-v)\,K}{(1-u)\,H}\right]^{\beta-1} \tag{2.23}$$

$$\mu_k\left(1-\frac{Z}{Y}\right)(1-\alpha)\frac{Y}{uH} = \mu_h\,(1-\beta)\,B\left[\frac{(1-v)\,K}{(1-u)\,H}\right]^{\beta} \tag{2.24}$$

where μ_k and μ_h are the shadow prices of physical capital and human capital in the central planner solution, respectively. Note that equations (2.19)–(2.24) describe the first best solution of the economy. Equation (2.19) implies that the marginal utility of consumption in every period should equal the shadow price of physical. The first Euler condition (2.20) implies that the rate of change of the shadow price of physical capital should equal the marginal product of capital in the final goods sector. The second Euler condition (2.21) is identical to equation (2.13). Equation (2.22) describes the optimal allocation of resources between public abatement and production of physical capital. According to equation (2.22), the marginal utility of public abatement activities should equal the shadow price of physical capital. Finally, equations (2.23) and (2.24) describe the optimal allocation of physical and human capital.

After eliminating the shadow prices, equations (2.23) and (2.24) yield the optimal sectorial allocation of resources in the centrally planned economy:

$$\frac{v}{u} = \left(\frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}\right)\frac{1-v}{1-u} \tag{2.25}$$

Finally, the Keynes–Ramsey rule for the centrally planned economy is obtained by use of equations (2.19) and (2.20):

$$\hat{C} = \left(1-\frac{Z}{Y}\right)\alpha\frac{Y}{vK} - \rho \quad \equiv \quad R_S - \rho \tag{2.26}$$

where R_S is defined as the social return to capital investment. As can be seen, the social return equals the private return to capital investment $\alpha\,(Y/vK)$ corrected by the optimal marginal damage of pollution Z/Y, which is external to firms in the market economy.

3. The Balanced Growth Path

In this section, the optimal tax rates are derived. Furthermore, the reduced forms of the balanced growth rate and all core variables and ratios are derived. Finally, the small open economy version of the model is considered.

Along a balanced growth path, the variables C, H, K, Y, and Z grow at the same constant rate, whereas u and v are constant over time. Therefore, the balanced growth rate g can be defined as follows:

$$\begin{aligned} g &\equiv \hat{C} = \hat{H} = \hat{K} = \hat{Y} = \hat{Z} = -\hat{\mu}_i - \rho = -\hat{\lambda}_i - \rho \\ 0 &= \dot{u} = \dot{v} \end{aligned} \tag{3.1}$$

where a hat above a variable indicates its rate of growth and $i = k, h$. According to condition (3.1), the ratios C/K, C/Y, K/H, Z/K, and Z/Y are constant along a balanced growth path. This implies that the level of net pollution P is constant along the balanced growth path. A constant level of P is in accord with sustainable environmental development if the ecosystem is assumed to be a renewable resource and the level of pollution does not exceed the natural regeneration.[18]

3.1. Optimal Tax Rates

This section derives the optimal tax rates in the model. In order to derive the first best tax rates, the first order conditions of the market solution are compared with the corresponding first order conditions of the central planner solution. Comparison of equations (2.15) with (2.24) and (2.12) with (2.20) using equation (2.6) and (2.7) reveals the following two conditions for a first best solution:

$$(1-\tau_h)(1-\tau_p) = (1-\tau_k)(1-\tau_p) = 1-\left(\frac{Z}{Y}\right)^{CPS} \tag{3.2}$$

where the superscript ^{CPS} denotes the central planner solution. Recall that the abatement to output ratio is constant along a balanced growth path. Note that there are more than one possibility to achieve a first best solution in a market economy. Condition (3.2) can be fulfilled in at least three ways, see Table 3.1.

Case 1: A first best solution can be reached by setting tax rates on labor and capital equal to zero and the pollution tax equal to the optimal marginal damage of pollution. In this case, the pollution tax corresponds to a Pigouvian tax. Note that the first best solution can be reached without the use of a lump sum instrument in all three cases. Hence, in Case 1 the pollution tax fulfills two tasks at the same time. It corrects the inefficient input ratio and generates the exact amount of public revenues to provide the optimal level of abatement. But additionally, there are at least two other

[18] See [24] Smulders (1995) for a survey on environmental sustainability in endogenous growth models.

	τ_h	τ_k	τ_p
Case 1	0	0	$\left(\frac{Z}{Y}\right)^{CPS}$
Case 2	$\left(\frac{Z}{Y}\right)^{CPS}$	$\left(\frac{Z}{Y}\right)^{CPS}$	0
Case 3	$1-\sqrt{1-\left(\frac{Z}{Y}\right)^{CPS}}$	$1-\sqrt{1-\left(\frac{Z}{Y}\right)^{CPS}}$	$1-\sqrt{1-\left(\frac{Z}{Y}\right)^{CPS}}$

Table 3.1: Optimal tax rates, when $P = (Y/Z)^{\chi}$

ways to reach a first best solution without setting the pollution tax at its Pigouvian level. *Case 2:* A first best solution can be reached by setting both factor income taxes equal to the optimal marginal damage of pollution and the pollution tax equal to zero. Hence, a comprehensive income tax ($\tau_h = \tau_k$) works in the same way as a pollution tax. Therefore, setting a comprehensive income tax equal to the optimal abatement–output ratio is equivalent to a Pigouvian tax. *Case 3:* A first best solution can be reached in many other ways by setting a comprehensive income tax and using a pollution tax in addition. One of them is represented in Table 3.1 for $\tau_h = \tau_k = \tau_p$. In all three cases, the optimal tax structure holds both along and outside the balanced growth path. Outside the balanced growth path, the tax rates change over time since the optimal abatement–output ratio is not constant. However, along the balanced growth path the tax rates are constant, since the abatement–output ratio is constant. Finally, it can be stated that the non–lump sum income taxes may be efficient, when the tax revenues collected by the government are spent on abatement activities.

3.2. The Market Economy

In the following, we analyze the effects of tax and parameter changes on the reduced forms of all core variables and ratios. At the end of the section, the results are summarized in Table 3.2. Along a balanced growth path, the first order conditions of the market economy (2.13) and (2.18), the human capital accumulation constraint (2.3), and the resource constraint of the economy (2.2) can be rewritten by use of condition (3.1):

$$g = (1-\tau_k)(1-\tau_p)\,\alpha\left(\frac{vK}{uH}\right)^{\alpha-1} - \rho = R - \rho \tag{3.3}$$

$$g = (1-\beta)\,B\left[\frac{(1-v)\,K}{(1-u)\,H}\right]^{\beta} - \rho \tag{3.4}$$

$$g = B\left[\frac{(1-v)\,K}{(1-u)\,H}\right]^{\beta}(1-u) \tag{3.5}$$

$$g = A\left(\frac{vK}{uH}\right)^{\alpha-1} v - \frac{C}{K} - \frac{Z}{K} \tag{3.6}$$

In order to derive the reduced form of the physical to human capital ratio in the final good sector $(vK)/(uH)$, we substitute the term $(1-v)/(1-u)$ in equation (3.4) by (2.17) and set it equal to (3.3):

$$\frac{vK}{uH} = \left\{ \frac{\alpha A}{(1-\beta)B} \left[\frac{\alpha}{1-\alpha} \frac{1-\beta}{\beta} \right]^{\beta} \frac{(1-\tau_p)(1-\tau_k)^{(1+\beta)}}{(1-\tau_h)^{\beta}} \right\}^{\frac{1}{1-\alpha+\beta}} \tag{3.7}$$

According to equation (3.7), the physical to human capital ratio in the final goods sector $(vK)/(uH)$ depends positively on the tax on labor income and negatively on the taxes on pollution and capital income. It is obvious that a higher labor income tax leads to a more capital intensive final goods production, whereas studying becomes more labor intensive. The opposite is true for a higher pollution tax and a higher capital income tax. The effect of a change in the comprehensive income tax on the physical to human capital ratio is identical to that of a change in the pollution tax. This is the reason behind the equivalence of a comprehensive tax and a pollution tax for internalizing the external effect of pollution. Although, the pollution tax is equivalent to a tax on output it does reduce the ratio of physical to human capital in final goods production. The intuition behind this is that an output tax reduces the return to physical capital net of tax rate directly, whereas the rate of return to investment in human capital is left unaffected, because education is a non–market activity. Since the return to capital investment should equal the return to human capital investment, final goods production must become more human capital intensive in order to leave the return to capital investment net of tax unaffected.

The reduced forms of the return to capital investment net of tax and the growth rate, see equation (2.18), can now be obtained by introduction of equation (3.7) in (3.3):

$$R = \left[D(1-\tau_h)^{(1-\alpha)\beta} (1-\tau_p)^{\beta} (1-\tau_k)^{\alpha\beta} \right]^{\frac{1}{1-\alpha+\beta}} \tag{3.8}$$

$$g = \left[D(1-\tau_h)^{(1-\alpha)\beta} (1-\tau_p)^{\beta} (1-\tau_k)^{\alpha\beta} \right]^{\frac{1}{1-\alpha+\beta}} - \rho \tag{3.9}$$

$$\text{where} \quad D \equiv (\alpha A)^{\beta} \left[(1-\beta) B \right]^{(1-\alpha)} \left(\frac{1-\alpha}{\alpha} \frac{\beta}{1-\beta} \right)^{(1-\alpha)\beta} \tag{3.10}$$

According to equation (3.9), the balanced growth rate in the market economy depends positively on the levels of the technology in the final goods sector A and the education

sector B, while it depends negatively on the capital income tax τ_k, the labor income tax τ_h, the comprehensive income tax $\tau_h = \tau_k$, the pollution tax τ_p, and the rate of time preference ρ. It can be seen that a tax on consumption τ_c does not affect the long term growth rate. Furthermore, the value added tax has the characteristics of a lump sum tax, since it does not distort the economy.[19] The result that non–environmental taxes reduce growth has been already shown by [16] Milesi–Ferretti & Roubini (1995) in a similar model without pollution. Furthermore they state that growth and welfare are maximized, when factor income is taxed at a zero rate. Hence, in their model the outcome of the unregulated market economy is a first best solution. However, we consider the environment in addition and show that the long term growth rate and pollution in an unregulated market economy is too high from a welfare perspective, when pollution harms utility.

In the following, the reduced forms of the fractions of human and physical capital that are devoted to final goods production are determined. The fraction of human capital allocated to final goods production is derived by use of equations (3.4) and (3.5) and introduction of (3.9):

$$u = \beta + (1-\beta)\rho \left[D(1-\tau_h)^{(1-\alpha)\beta} (1-\tau_p)^{\beta} (1-\tau_k)^{\alpha\beta} \right]^{-\frac{1}{1-\alpha+\beta}} . \tag{3.11}$$

It can be seen immediately that the fraction of human capital allocated to final goods production depends positively on both factor income taxes, the comprehensive income tax, and the pollution tax. It seems contra–intuitive that an increase in the labor income tax induces agents to spend more time at work. However, the capital–labor ratio v/u and not the absolute value of u is relevant for the allocation of physical and human capital between the two production sectors. As we will see below, a labor income tax also increases the fraction of physical capital allocated to final goods production. However, v rises more than u, since a labor income tax increases the capital–labor ratio in the final goods sector v/u relative to capital-labor ratio in the education sector $(1-v)/(1-u)$, see equation (2.17).

Now, the fraction of physical capital allocated to final goods production is derived

[19] In case the more general instantaneous utility function $U = \frac{(CP^{-\eta})^{1-\theta}-1}{1-\theta}$ is used, the reduced form of the growth rate (3.9) changes to: $g = \frac{1}{\theta}\left\{ \left[D(1-\tau_h)^{(1-\alpha)\beta} (1-\tau_p)^{\beta} (1-\tau_k)^{\alpha\beta} \right]^{\frac{1}{1-\alpha+\beta}} - \rho \right\}$. Obviously, an intertemporal elasticity of substitution $1/\theta$ unequal to unity does not alter the qualitative growth rate effects of parameter and tax rate changes.

by use of equations (2.17) and (3.11) and introduction of (3.9):

$$v = \left[1 + \frac{1-\alpha}{\alpha}\frac{1-\tau_h}{1-\tau_k}\frac{1}{1+\frac{\rho}{\beta\left\{\left[D(1-\tau_h)^{(1-\alpha)\beta}(1-\tau_p)^{\beta}(1-\tau_k)^{\alpha\beta}\right]^{\frac{1}{1-\alpha+\beta}}-\rho\right\}}}\right]^{-1} \tag{3.12}$$

where the fraction of physical capital allocated to final goods production depends positively on the labor income tax, the comprehensive income tax, and the pollution tax, while the effect of the capital income tax is presumed to be negative, but cannot be signed unambiguously.

In the following, the reduced form of the abatement–output ratio is derived. Therefore, the budget constraint of the government (2.10) is rewritten as:

$$(1-\tau_p)\left[\tau_k\alpha + \tau_h(1-\alpha)\right] + \tau_p + \tau_c\frac{C}{Y} = \frac{Z}{Y} \tag{3.13}$$

Introduction of the resource constraint of the economy (3.6) in equation (3.13) yields either:

$$\frac{Z}{Y} = \frac{(1-\tau_p)\left[\tau_k\alpha + \tau_h(1-\alpha)\right] + \tau_p + \tau_c\left[1 - \frac{g}{Av}\left(\frac{vK}{uH}\right)^{1-\alpha}\right]}{1+\tau_c} \tag{3.14}$$

or

$$\frac{C}{Y} = \frac{\left[1 - \frac{g}{Av}\left(\frac{vK}{uH}\right)^{1-\alpha}\right] - (1-\tau_p)\left[\tau_k\alpha + \tau_h(1-\alpha)\right] - \tau_p}{1+\tau_c} \tag{3.15}$$

It can be seen that a consumption tax increases the abatement–output ratio. However, the exclusive use of the consumption tax cannot yield a first best solution, since it does not correct the sectorial allocation of factors. For the derivation of the other effects we have to set the lump sum consumption tax equal to zero, otherwise the effects cannot be signed. Due to tax interaction effects, a higher pollution tax erodes the tax bases of the non–environmental taxes, which finally could lead to a lower abatement–output ratio and thereby to higher pollution. An increase in the pollution tax raises tax revenue directly, but lowers the factor income tax revenue indirectly through a drop in the returns to capital and labor. However, an increase in the pollution tax increases the abatement–output ratio and hence reduces pollution as long as the taxes on capital and labor are between zero and one. The effects of a comprehensive income tax are equivalent to a pollution tax. Isolated increases in either the labor income tax or the capital income tax raise the abatement–output ratio without eroding any other tax bases.

Even though a consumption tax is a lump sum tax, it has a negative effect on the consumption–output ratio, since resources are reallocated towards public abatement

activities, see equation (3.15). However, the effects of all other taxes and parameters on the consumption–output ratio cannot be signed unambiguously. The effects of changes in the tax rates and the parameters on the core variables and ratios are summarized in Table 3.2.[20]

	τ_c	τ_h	τ_k	$\tau_h = \tau_k$	τ_p	A	B	ρ
$\frac{\partial g}{\partial x}$	0	−	−	−	−	+	+	−
$\frac{\partial u}{\partial x}$	0	+	+	+	+	−	−	+
$\frac{\partial v}{\partial x}$	0	+	−*	+	+	+	+	−
$\frac{\partial[(vK)/(uH)]}{\partial x}$	0	+	−	−	−	+	−	0
$\frac{\partial(Z/Y)}{\partial x}$	+	+*	+*	+*	+*	0*	0*	0*

Table 3.2: The effects of changes in taxes and parameters on core variables

By means of the Keynes–Ramsey rule (3.3) and the results in Table 3.2 (second and fifth row), the channels through which taxes affect long term economic growth can be shown. The direct effect of higher taxes on capital and pollution is to reduce the net interest rate R for a given physical to human capital ratio. From equation (3.3), we know that this single effect reduces growth. Due to the indirect effect of these taxes, final good production becomes more labor–intensive, which ceteris paribus stimulates growth due to a higher marginal product of physical capital. Table 3.2 shows that the direct effect dominates the indirect effect. A higher labor tax leads to a more capital–intensive final good production, which lowers growth indirectly, see equation (3.3).

In the preceding analyses, the case of productive environmental spillovers have not been taken into account. However, an improvement in the quality of the environment may increase the productivity in the final goods production or in the education sector.[21] Formally, these cases could have been analyzed in the present setup by adding a multiplicative term $P^{-\epsilon}$ on the left hand side of the production function (2.1) or the human capital accumulation function (2.3). However, it is possible to discuss productive environmental spillovers without a formal analysis. Along a balanced growth path, a better environmental quality is namely equivalent to higher levels of the technology A or B, because the level of pollution is constant along a balanced growth path. From

[20] In table 3.2, the results indicated with a '$*$' are obtained by setting $\tau_c = 0$, whereas '$*$' cannot be signed unambiguously, but is expected to be negative.

[21] Some empirical evidence suggests that pollution may cause productivity loses, see e.g. [14] Margulis (1992), [1] Ballard & Medema (1993), and [4] Brendemoen & Vennemo (1994).

Table 3.2, an increase in productivity is seen to increase economic growth. Hence, environmental improvements stimulate economic growth, ceteris paribus. Whether, this positive growth rate effect dominates the above-mentioned negative growth rate effect depends solely on parameter values.

3.3. The Small Open Economy

So far we have analyzed the effects of environmental policy in a closed economy setting. In this section, we complement the analysis by investigating the possibilities for a small open economy to lead an independent environmental policy. Throughout, the conventional assumptions of international trade models are made: Domestic capital goods and foreign assets F are perfect substitutes, there is international borrowing and lending, and international trade in capital and consumption goods, but international immobility of labor i.e. human capital. The latter assumption ensures that the small open economy will not specialize over time in either final goods production or education. Furthermore, the world interest rate r_f cannot be influenced by a small open economy and is therefore exogenous. This implies that the after–tax return to capital invested domestically and abroad is the same, if a residence–based (world wide) tax system[22] is assumed. In addition, we assume that the environmental externality is national in scope.[23] Given these assumptions, the flow resource constraint of the small open economy becomes:

$$Y + r_f F = C + \dot{K} + \dot{F} + Z \tag{3.16}$$

where $r_f F$ is the interest payment earned on foreign assets and $\dot{F}$ is investment in foreign assets. In the small open market economy, the first order conditions of the representative agent's maximization problem are still given by equations (2.11)–(2.15), but in addition the after–tax return to domestic capital should equal the after–tax return to investment in foreign assets:[24]

$$R \;=\; (1-\tau_k)r \;=\; (1-\tau_f)r_f \tag{3.17}$$

[22]This is the prevailing taxation system, see e.g. [18] OECD (1991). Agents pay taxes in their home country on capital income from foreign investments, but receive a tax credit for any taxes paid abroad on this income.

[23]In case of global environmental problems like the anthropogenic greenhouse effect or the destruction of the ozone layer, a small open economy has no influence on the pollution level at all.

[24]Note that in a pure residence–based taxation system domestic and foreign capital income would be taxed at the same rate.

The small open economy can still determine the domestic interest rate net of tax by use of its tax policy. But at the same time the domestic interest rate net of tax should equal the net of tax world interest rate. As a consequence, the only difference between the closed economy and the small open economy is that the government must accommodate the tax on foreign capital income in order to fulfil equation (3.17). Thus, the tax on foreign capital income τ_f is not a real decision variable of the government. In case the after–tax return is higher on foreign assets, there will be a permanent outflow of capital from the domestic economy. In case the after–tax return is lower on foreign assets, there will be permanent inflow of capital. Obviously, both of these cases are unstable. The endogenous determination of the tax on foreign capital income is derived by introduction of the after–tax interest rate (3.8) in equation (3.17):

$$\tau_f \;=\; 1+\frac{R}{r_f} \;=\; 1-\frac{1}{r_f}\left[D\,(1-\tau_h)^{(1-\alpha)\beta}\,(1-\tau_p)^{\beta}\,(1-\tau_k)^{\alpha\beta}\right]^{\frac{1}{1-\alpha+\beta}} \tag{3.18}$$

It can be seen that for given taxes on labor income, capital income and pollution, there exists a unique feasible value of the tax on foreign assets that equalizes the returns to domestic and foreign investment. Since the equilibrium interest rate net of tax R, see (3.8), is a negative function of the taxes on capital, labor and pollution, the tax on foreign assets is a positive function of these taxes. This result is similar to the result obtained by [15] Milesi–Ferretti & Roubini (1994) except for the effect of the pollution tax.

The key insight from the above analysis is that a small open economy under a residence–based tax system can determine the after–tax return earned by domestic residents as long as the tax rate on foreign capital income is set according to condition (3.18). Thus, the after–tax return on domestic capital equals the world interest rate net of the tax on foreign capital income levied on domestic residents. Hence, a small open economy can decide its own growth rate through taxation of factor income. In fact, the balanced growth rate is the same in the small open economy and in the closed economy (3.9).[25] Furthermore, a small open economy can use efficient instruments to internalize the pollution externality.

As a consequence, the optimal taxation results of Section 3.1 are still valid. However, the government in the small open economy obtains an additional revenue from the tax on foreign assets. This tax revenue must be redistributed in a lump sum manner to guarantee a first best solution. The main difference between the closed and the open

[25]The papers by [10] King & Rebelo (1990) and [22] Rebelo (1992) also show that a residence–based taxation system allows for a wedge between both domestic and world interest rates and growth rates.

economy is that the latter will exhibit no transitional dynamics as long as there are no investment costs present. The domestic capital stock can be changed immediately by borrowing or lending in the international capital markets.

If in contrast capital income is taxed according to the source–based tax principle, the government becomes partially restricted in its tax policy. Under such a system the net of tax interest rate in the small open economy must equal the world interest rate:

$$R = (1-\tau_k)r = r_f \tag{3.19}$$

Thus, the growth rate of the small open economy is determined by the world interest rate. If on the one hand, the technology of the production processes are identical at home and abroad, the government of the small open economy cannot levy positive taxes on both labor, capital, and pollution, since this would violate condition (3.19) which ensures a stable solution. However, an independent environmental policy can still be lead by the small open economy by setting two of the tax rates and tying the last one according to equation (3.19) given that both taxes and subsidies are available.[26] Thus, the revenues of certain combinations of taxes and subsidies, which does not alter the return to capital net of tax R, and the revenue of the lump sum consumption tax can be used to increase abatement activities and thereby lower pollution. But note that this is a second best environmental policy. Hence, should the EU decide to move towards a source–based income tax system, then a common EU environmental policy might be necessary, since individual member states no longer can lead an effective environmental policy. If on the other hand, the technologies are different then the government must use at least one of the above-mentioned tax instruments to fulfil condition (3.19). As long as the interest rate of the small open economy r is larger than the world interest rate r_f there is a possibility for environmental policy. If the interest rate is smaller than the world interest rate, the small open economy must subsidize the interest rate and finance it by a lump sum tax.

[26] If all foreign tax rates are zero, then $r_f = D^{\frac{1}{1-\alpha+\beta}}$ and the restriction on domestic tax rates becomes:

$$(1-\tau_p) = (1-\tau_h)^{-(1-\alpha)}(1-\tau_k)^{-\alpha}$$

see equation (3.8). Thus, pollution can e.g. be taxed at the rate $\tau_p = s/(1+s)$, if factor income is subsidized at the comprehensive rate $\tau_h = \tau_k = -s$. The revenue from such a policy is negative, which implies that consumption has to be taxed at an appropriate rate in order to balance the government budget and still engage in public abatement activities, see equation (2.10).

4. Capital as the Polluting Factor

After having discussed the possibilities of a small open economy to lead an independent environmental policy, we now turn back to a closed economy setting. In the preceding sections, pollution was assumed to be a function of output and public abatement activities according to equation (2.4). In this section, we assume an another plausible pollution specification. Pollution is now assumed to be a function of physical capital used in the final goods production and public abatement activities. The alternative specification of the net pollution function is:

$$P = \left(\frac{vK}{Z}\right)^{\chi} \tag{4.1}$$

where pollution is seen to be increasing in the use of the dirty factor and decreasing in public abatement. In contrast to the pollution function (2.4), the alternative specification allows for a reduction in pollution without lowering output, through a substitution of physical by human capital in production. This would lead to a cleaner and a more labor–intensive final goods production.

In the following, we derive the growth rate effects of changes in the tax rates and the parameters and compute the optimal tax rates for this pollution specification given that all other things are equal to Section 2.1. In contrast to the setup above, the pollution tax is now levied on the use of physical capital in the final good production. Therefore, the after–tax marginal products of labor and capital change as follows:

$$w = (1-\alpha)\frac{Y}{uH} \tag{4.2}$$

$$r = \alpha\frac{Y}{vK} - \tau_p \tag{4.3}$$

The wage rate is seen to be unaffected by a pollution tax, while the interest rate is a negative function of the pollution tax. The first order conditions of the market solution (2.11)–(2.15) are unchanged. But due to the changes in the wage and interest rate, the sectorial allocation of factors is no longer unaffected by a pollution tax:

$$\frac{v}{u} = \left(\frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}\right)\left(\frac{1-\tau_k}{1-\tau_h}\right)\left(\frac{\alpha\frac{Y}{vK}-\tau_p}{\alpha\frac{Y}{vK}}\right)\frac{1-v}{1-u} \tag{4.4}$$

where equations (2.14), (2.15), (4.2) and (4.3) have been used.

The alternative specification of the pollution function changes the differential equations, which govern the balanced growth path. Thus, along the balanced growth path

equations (3.4)–(3.6) still hold, while the Keynes–Ramsey rule (3.3) of the market economy changes to:

$$g = R - \rho = (1-\tau_k)\left(\alpha\frac{Y}{vK} - \tau_p\right) - \rho \tag{4.5}$$

In order to determine the growth rate effects of changes in the tax rates and the parameters, we derive the reduced form of the growth rate by using equations (3.4), (4.4) and (4.5). The implicit function of the reduced form is called F:

$$F \equiv 0 = (1-\beta)\,B\left(\frac{1-\beta}{\beta}\frac{\alpha}{1-\alpha}\frac{1-\tau_k}{1-\tau_h}\right)^{-\beta} \left\{\frac{1}{\alpha A}\left[\frac{1}{\alpha A}\left(\frac{g+\rho}{1-\tau_k}+\tau_P\right)\right]^{\frac{\alpha}{1-\alpha}}\left(\frac{g+\rho}{1-\tau_k}\right)\right\}^{-\beta} - \rho - g \tag{4.6}$$

The effects of parameter and tax rate changes can be derived by using the implicit function rule.[27] The growth rate is negatively affected by taxes on labor income, capital income[28], and pollution, and by the rate of time preference. Also in this specification, a consumption tax is a lump sum tax and therefore has no effect on economic growth. Growth is stimulated by increases in the levels of technology in both sectors, A and B. These results are similar to the second row of Table 3.2 in section 3.2.

In the following, the optimal tax rates are found by a comparison of the first order conditions of the representative agent problem (2.11)–(2.15) and the central planner problem (A.3)–(A.8).[29] This comparison yields the following two equations that must be fulfilled in order to achieve the first best optimum:

$$(1-\tau_k)\left(\alpha\frac{Y}{vK} - \tau_p\right) = \alpha\left(\frac{Y}{vK}\right)^{CPS} - \left(\frac{Z}{vK}\right)^{CPS} \tag{4.7}$$

$$(1-\tau_h)(1-\alpha)\frac{Y}{uH} = (1-\alpha)\left(\frac{Y}{uH}\right)^{CPS} \tag{4.8}$$

Equation (4.7) can be fulfilled in at least two ways, see Table 4.1. And inspection of equation (4.8) immediately reveals that the optimal labor income tax is zero.

Case 1: A first best solution can be reached by setting taxes on labor and capital income equal to zero and the pollution tax equal to the optimal marginal damage of pollution. Such a pollution tax corresponds to a Pigouvian tax. *Case 2:* Another way

[27] See Appendix A.1 for the partial derivatives of F.

[28] We cannot sign the growth rate effect of the capital income tax, but we expect it to be negative.

[29] See Appendix A.2 for the first order conditions of the central planner solution with this pollution specification.

	τ_h	τ_k	τ_p
Case 1	0	0	$\left(\frac{Z}{vK}\right)^{CPS}$
Case 2	0	$\left(\frac{Z}{vK}\right)^{CPS}\left[\alpha\left(\frac{Y}{vK}\right)^{CPS}\right]^{-1}$	0

Table 4.1: Optimal tax rates, when $P = (vK/Z)^{\chi}$

to achieve a first best solution is to set the labor and pollution tax equal to zero and the capital tax equal to the optimal marginal damage of pollution divided by the optimal marginal product of capital. It can be seen that the effects of a capital income tax are similar to the effects of a tax on pollution. This is not surprising since the use of physical capital in final good production is the dirty input factor. However, there is one difference: the tax base. A tax on capital is levied on the capital income which consists of the marginal product of capital times the use of physical capital in production $[\alpha Y/(vK)]\,vK$, whereas the tax on pollution is levied solely on the use of physical capital in production vK. Therefore, it is necessary to correct the optimal marginal damage of pollution by $[\alpha Y/(vK)]^{-1}$ in order to equate the tax base differences of the two taxes, when a capital income tax is used. Note that both the pollution and the capital income tax lead to a more labor intensive and thereby a cleaner final goods production. Furthermore, there is a rationale for capital income taxation, when capital is the dirty input factor and the pollution tax instrument is not available.

Comparison of Table 3.1 and Table 4.1 reveals that the optimal labor income tax becomes zero, once human capital becomes a clean input factor in production. The discussion of the small open economy version for the alternative pollution specification is similar to the one in Section 3.3 and is therefore neglected.

5. Conclusions

This paper has examined the effects of fiscal policy in a two sector endogenous growth model with pollution, where the tax revenues are used to finance public abatement activities. We investigated two plausible pollution specifications: Pollution is either generated by final goods production or by the use of physical capital in final good production. In both cases, the decentralized outcome is inefficient without government intervention. From a welfare perspective there is too much pollution and economic growth is too high in an unregulated market economy, since economic agents do not take the environmental externality into account. Both factor income taxes and the pol-

lution tax reduce economic growth whereas a consumption tax has the characteristics of a lump sum tax.[30]. The tax on pollution improves welfare as long as the level of pollution is above the optimal level. If productive environmental spillovers are considered additionally there may be a stimulating growth effect of a tighter environmental policy depending on the strength of these spillovers. At least productive environmental spillovers partially offset the negative growth effect of providing a better environmental quality.

When pollution is complementary to final goods production, the government can reach a first best solution either by setting the pollution tax equal to the optimal marginal damage of pollution or by setting a comprehensive income tax at the same level. Furthermore, a combination of the comprehensive income tax and the pollution tax can be used to reach a first best solution. The optimal tax analysis implies that non–lump sum taxes may be efficient, when the tax revenue is spent on public abatement activities. Concerning the growth rate effects of taxes on capital income and pollution, there are two effects at work: Both taxes reduce the growth rate directly through a drop in the net of tax interest rate, but stimulate growth indirectly through an increase in the interest rate, because final good production becomes more labor intensive. However, the direct effect dominates the indirect effect. A higher labor income tax leads to a reduction of the interest rate solely through the indirect effect. Although the pollution tax is equivalent to a tax on output, it does affect the physical to human capital ratio in final goods production, since education is assumed to be a non–market activity. Furthermore, we have shown that a higher pollution tax erodes the tax bases of both factor income taxes. Nevertheless it increases the government revenue and thereby the abatement–output ratio, which is accompanied by a better environmental quality.

When pollution is caused by the use of physical capital in production, the first best solution once more can be reached by setting the pollution tax equal to the optimal marginal damage of pollution. However, the negative externality can also be internalized by a capital income tax that equals the optimal marginal damage of pollution divided by the optimal marginal product of capital. The latter correction is necessary to equate the tax base differences between the capital income tax and the pollution tax. Thus, capital income taxation is well-founded, when capital is a dirty input factor, because it induces firms to use more of the clean input factor in production.

In addition, we investigated a small open economy version of the model with perfect

[30] In a model with identical production processes but augmented by leisure, a consumption tax is growth reducing as well, see [16] Milesi-Ferretti & Roubini (1995).

international capital mobility. We found that under a residence–based income tax system that discriminates between domestic source and foreign source income, a small open economy can lead an independent first best environmental policy, where it chooses its own fiscal policy and thereby determines its own growth rate. However, in order to rule out a permanent inflow or outflow of capital and hence the unstable solution, the tax on foreign capital income is no longer a decision variable of the government. Instead, the government must accommodate any domestic tax rate change by changing the foreign income tax such that the domestic after–tax interest rate equals the after–tax world interest rate.

Under a source–based tax system, the government is partially restricted in its tax policy since the latter must be constrained in such a way that the after–tax interest rate equals the world interest rate. However, in order to lead a first best environmental policy the government must be able to determine its interest rate. Nevertheless, there is still room for an independent second best environmental policy. By means of certain tax–subsidy combinations and the use of the lump sum consumption tax the level of the abatement activities and hence the level of pollution can be varied without changing the interest rate.

A. Appendix

A.1. Partial Derivatives of F

Partial derivatives of the reduced form of the growth rate F:

$$\begin{aligned}
\frac{\partial F}{\partial \tau_h} &= -\beta(1-\beta)B\left(\tfrac{1-\beta}{\beta}\tfrac{\alpha}{(1-\alpha)}(1-\tau_k)\right)^{-\beta}(1-\tau_h)^{\beta-1}\Omega_2^{-\beta} &< 0\\
\frac{\partial F}{\partial \tau_p} &= -\beta\Omega_1\Omega_2^{-\beta-1}\tfrac{\alpha}{1-\alpha}\tfrac{1}{\alpha A}\Omega_3^{\frac{\alpha}{1-\alpha}}\left[1-\tfrac{1}{\alpha A}\tau_p\Omega_3^{-1}\right] &< 0\\
\frac{\partial F}{\partial A} &= \tfrac{1}{A}\beta\Omega_1\Omega_2^{-\beta-1}\tfrac{1}{1-\alpha}\Omega_3^{\frac{1}{1-\alpha}}\left[1-\tfrac{1}{\alpha A}\tau_p\Omega_3^{-1}\right] &> 0\\
\frac{\partial F}{\partial B} &= \tfrac{1}{B}\Omega_1\Omega_2^{-\beta} &> 0\\
\frac{\partial F}{\partial \rho} &= -\beta\Omega_1\Omega_2^{-\beta-1}\tfrac{1}{\alpha A}\tfrac{1}{1-\tau_k}\tfrac{1}{1-\alpha}\Omega_3^{\frac{\alpha}{1-\alpha}}\left\{1-\tfrac{1}{A}\tau_p\Omega_3^{-1}\right\}-1 &< 0\\
\frac{\partial F}{\partial g} &= -\beta\Omega_1\Omega_2^{-\beta-1}\tfrac{1}{\alpha A}\tfrac{1}{1-\tau_k}\tfrac{1}{1-\alpha}\Omega_3^{\frac{\alpha}{1-\alpha}}\left\{1-\tfrac{1}{A}\tau_p\Omega_3^{-1}\right\}-1 &< 0
\end{aligned} \tag{A.1}$$

where the following definitions are used:

$$\begin{aligned}
\Omega_1 &\equiv (1-\beta)\, B \left(\tfrac{1-\beta}{\beta}\tfrac{\alpha}{(1-\alpha)}\tfrac{(1-\tau_k)}{(1-\tau_h)}\right)^{-\beta} &> 0 \\
\Omega_2 &\equiv \tfrac{1}{\alpha A}\left[\tfrac{1}{\alpha A}\left(\tfrac{g+\rho}{1-\tau_k}+\tau_p\right)\right]^{\frac{\alpha}{1-\alpha}}\left(\tfrac{g+\rho}{1-\tau_k}\right) &> 0 \qquad (A.2)\\
\Omega_3 &\equiv \tfrac{1}{\alpha A}\left(\tfrac{g+\rho}{1-\tau_k}+\tau_p\right) &> 0
\end{aligned}$$

To show that the term in the square bracket of the second and third line of equation (A.1) is positive we substitute Ω_3 by $\left(\frac{vK}{uH}\right)^{\alpha-1}$, see equation (4.3). Therefore:

$$1-\frac{1}{\alpha A}\tau_p\Omega_3^{-1} = \frac{1}{\alpha A}\left(\frac{vK}{uH}\right)^{1-\alpha}\left[\alpha A\left(\frac{vK}{uH}\right)^{\alpha-1}-\tau_p\right] = \frac{1}{\alpha A}\left(\frac{vK}{uH}\right)^{1-\alpha} r > 0$$

where it has been used that a positive interest rate is a necessary condition for the existence of a balanced growth path. Then it is straight forward to see that the curly bracket in line fifth and sixth of equation (A.1) is positive as well:

$$1-\frac{1}{A}\tau_p\Omega_3^{-1} = \frac{1}{A}\left(\frac{vK}{uH}\right)^{1-\alpha}\left[A\left(\frac{vK}{uH}\right)^{\alpha-1}-\tau_p\right] > 0$$

A.2. First Order Conditions of the Central Planner Solution

When pollution is caused by the use of physical capital in final goods production, the first order conditions of the central planner problem becomes:

$$\begin{aligned}
\nu_k &= \frac{1}{C}e^{-\rho t} && (A.3)\\
-\frac{\dot{\nu}_k}{\nu_k} &= \left(\alpha\frac{Y}{vK}-\frac{Z}{vK}\right) && (A.4)\\
-\frac{\dot{\nu}_h}{\nu_h} &= (1-\beta)\, B\left[\frac{(1-v)\,K}{(1-u)\,H}\right]^{\beta} && (A.5)\\
\nu_k &= \chi\eta\frac{1}{Z}e^{-\rho t} && (A.6)\\
\nu_h\beta B\left[\frac{(1-v)\,K}{(1-u)\,H}\right]^{\beta-1} &= \nu_k\left(\alpha\frac{Y}{vK}-\frac{Z}{vK}\right) && (A.7)\\
\nu_h\,(1-\beta)\, B\left[\frac{(1-v)\,K}{(1-u)\,H}\right]^{\beta} &= \nu_k\,(1-\alpha)\,\frac{Y}{uH} && (A.8)
\end{aligned}$$

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Transitional Dynamics of Environmental Policy* — A Numerical Simulation of a Two Sector Endogenous Growth Model —

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Abstract

This paper simulates the transition path of environmental policy within a realistically calibrated two sector endogenous growth model. The policy change is either implemented suddenly, previously announced, or gradually. From a strict welfare point of view, the best policy is the unannounced policy scheme, but in our point of view, the best policy recommendation is a gradual policy scheme. Firstly, it stretches out the adjustment process and secondly the associated welfare loss in comparison with an unannounced policy is negligible. Another main result is that all of the environmental policy schemes only lead to a reduction of the long term growth rate from 2% to 1.98%, when the abatement–output ratio doubles from 1.6% to 3.2%. Qualitative and quantitative results are robust to most parameter changes. However, transitional dynamics are sensitive to changes in the shares of physical capital in production and education.

JEL classification: C0, D62, O41, Q28

Keywords: Endogenous growth, environmental externality, transitional dynamics, numerical simulation

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1. Introduction

The main purpose of this paper is to simulate the transitional dynamics of environmental policy within a two sector endogenous growth model, which is developed and analyzed theoretically by [9] Hettich & Svane (1998). We investigate the adjustment processes and the related welfare implications of implementing an increase in green taxes either suddenly, announced or gradually. By calibrating the model realistically we get both a qualitative and a quantitative impression of the consequences of a tighter environmental policy.

The [9] Hettich & Svane (1998) paper determines the long run effects of environmental policy on growth and all core variables within a generalized Uzawa–Lucas model that allows for both disutility of pollution and public abatement activities, which are financed through taxation. However, when it comes to the determination of the welfare implications of environmental policy it proves necessary to have knowledge about the transitional dynamics caused by such a policy. Due to the complexity of the model, welfare effects might be positive in the long run, but negative in the short run such that the overall discounted welfare effect of environmental policy might turn out to be negative. Moreover, policy makers normally want to introduce a policy change gradually or announce it in advance in order to avoid huge costs in the adjustment processes. It is therefore interesting to compare the transitional dynamics of an unanticipated, an anticipated, and a gradual introduction of environmental policy and in particular to determine the welfare effects of these different policy schemes. The present paper simulates numerically the transitional dynamics of the above-mentioned model from an initial balanced growth path to a new balanced growth path for all three policy schemes. In all of these cases, the economy approaches a new balanced growth path asymptotically.[1] Furthermore, we undertake a sensitivity analysis of the exogenous parameters chosen for the simulations.

Whereas almost the entire literature on environmental policy and growth examines the long run effects, little has been done so far to analyze the short and medium term effects. However, there are a few exceptions in the literature. [16] van der Ploeg & Ligthart (1994) derive the transitional dynamics of a linear growth model extended by a renewable environmental resource. By increasing the disutility parameter of pollution of the representative agent they find that the fall in the short run growth rate of

[1] The simulations cover 150 periods. Within this period of time the difference between the simulated growth rates and the balanced growth path values becomes negligible (10^{-10}).

the centrally planned economy is bigger than in the long run growth rate. Note that the evolution of the environmental stock is responsible for and solely determines the transitional dynamics of the economy in their linear model. [15] Perroni (1995) gets a similar result in a two sector growth model, where a composite of two final goods can be used for consumption, and investment in both physical and human capital. In such a model, an environmental tax is growth reducing in the long run. By means of a numerical simulation he finds that short run growth losses are even bigger. Finally, [4] Bovenberg & Smulders (1996) analytically compute the transitional dynamics of a two sector model consisting of a consumption/capital goods sector and a research and development sector that generates knowledge about pollution–augmenting techniques. The renewable environmental resource acts both as public consumption and as a public input into production, where the latter is identical to a productive environmental spillover. They find that if the environment acts mainly as a consumption good, then a tighter environmental policy reduces growth in both the long run and the short run. But if the environment acts mainly as a public investment good, then long run growth rises, while short run growth declines.[2]

There are several conclusions drawn from the analyses undertaken in the present paper. Firstly, from a strict welfare point of view, the best policy is to introduce the optimal pollution tax unannounced. However, in our point of view, the best policy recommendation is a gradual environmental policy scheme. Under such a scheme, the adjustment processes are less severe than under the other schemes simply because a gradual increase in the pollution tax stretches out the adjustment process over several years. Moreover, the welfare loss of a gradual instead of an unanticipated implementation of the policy is negligible. Even though this paper does not take investment costs into account in any of the undertaken analyses, we expect that costs of investment do not have to be very big, before it pays in terms of welfare to implement the policy change gradually. Secondly, the long term growth rate only falls from 2% to 1.98%, when the abatement–output ratio doubles from 1.6% to 3.2%. This result holds for all of the realistically calibrated environmental policy schemes. Thus, a tighter environmental policy leads to a relatively small reduction in the long term growth rate. Thirdly, a previously announced environmental policy just postpones the adjust-

[2] Related papers investigating transitional dynamics within similar models, but without taking the environment into account, are: [5] Caballé & Santos (1993), [6] Chamley (1993), [7] Devereux & Love (1994), [8] Faig (1995), [11] Ladrón-de-Guevara et al. (1997), and [13] Mulligan & Sala-i-Martin (1993).

ment process until after the pollution tax is actually increased instead of leading to a smoothing of consumption over time. Thus, welfare declines when an increase in green taxes is announced in advance instead of being implemented immediately. Note that this result probably hinges on the fact that the intertemporal elasticity of substitution equals is assumed equal to one. Finally, the sensitivity analyses reveal that the shares of physical capital in the two sectors are the key parameters to the understanding of the transitional dynamics in the simulated model. The more similar the two production processes are, the more severe is the adjustment process, since an increase in the pollution tax leads to bigger movements in output and consumption.

The structure of the paper is as follows. The theoretical model is briefly described in Section 2. The model is calibrated to capture the growth rate and abatement expenditures of industrialized economies in Section 3. Section 4 analyses the transitional dynamics of a centrally planned economy, when environmental care suddenly increases. Section 5 simulates the effects of both an unanticipated, an anticipated and a gradual implementation of a tax on the emission of pollution in a decentralized economy. Finally, Section 6 undertakes sensitivity analyses of the chosen parameter, while Section 7 concludes the paper.

2. Model

This section briefly describes the discrete time version of the [9] Hettich & Svane (1998) two sector endogenous growth model.

In the final goods sector a large number of identical and competitive firms produce universal goods Y_t with a constant returns to scale technology using physical capital K_t and human capital H_t as input factors:

$$Y_t = A\left(v_t K_t\right)^{\alpha}\left(u_t H_t\right)^{1-\alpha} = C_t + Z_t + K_{t+1} - K_t + \delta_k K_t \tag{2.1}$$

Parameter A is the level of the technology, α is the physical capital share in final goods production, and $0 < v_t < 1$ ($0 < u_t < 1$) is the fraction of physical (human) capital devoted to final goods production. Universal goods are used for consumption C_t, abatement activities Z_t, and physical capital accumulation. Physical capital is assumed to depreciate at the rate δ_k, while costs of investment are assumed to be zero.

Firms are assumed to rent physical capital from households at the interest rate r_t and hire human capital at the wage rate w_t. The objective of the representative firm

is to maximize its profits by choosing the input of capital and labor:

$$\pi_t = Y_t - w_t\left(u_t H_t\right) - r_t\left(v_t K_t\right) - \tau_p\left(v_t K_t\right) \tag{2.2}$$

where τ_p is a pollution tax levied on the use of physical capital, since pollution is assumed to be caused by the use of physical capital in production. Firm's maximization results in an optimal input mix, where the marginal cost of each factor equals its marginal return:

$$r_t = \alpha\frac{Y_t}{v_t K_t} - \tau_p \tag{2.3}$$

$$w_t = (1-\alpha)\frac{Y_t}{u_t H_t} \tag{2.4}$$

The education sector provides the facilities to accumulate human capital. Education is assumed to be a privately financed activity, where both human capital and physical capital such as school buildings are used as input factors:

$$H_{t+1} - H_t = B\left[\left(1-v_t\right)K_t\right]^{\beta}\left[\left(1-u_t\right)H_t\right]^{1-\beta} - \delta_h H_t \tag{2.5}$$

Parameter B is the level of the technology, β is the physical capital share in education, and δ_h is the rate of depreciation of human capital. Both input factors are assumed to move freely and without costs between the two sectors.

The use of physical capital in final goods production is assumed to cause a negative environmental externality in the form of pollution, which can be reduced by means of public abatement activities Z_t, such as knowledge about clean production methods:

$$P_t = \left(\frac{v_t K_t}{Z_t}\right)^{\chi} \tag{2.6}$$

where χ is the elasticity of pollution with respect to the ratio of physical capital in the final goods sector to abatement. The resulting net emission of pollution P_t is assumed to harm utility. Note that there are no positive spillovers of a better environment to final goods production.

The consumption side of the economy consists of a large number of identical agents. They maximize their life time utility by choosing consumption and the fraction of physical and human capital allocated to final goods production subject to the household budget constraint (2.8) and the human capital accumulation function (2.5):

$$\max_{C_t,\, u_t,\, v_t} U_0 = \sum_{t=0}^{T}\left[\left(\ln C_t - \eta \ln P_t\right)(1+\rho)^{-t}\right] \tag{2.7}$$

$$\text{s.t.} \quad K_{t+1} - K_t = r_t v_t K_t + w_t u_t H_t - C_t - \delta_k K_t \tag{2.8}$$

$$\text{s.t.} \quad H_{t+1} - H_t = B\left[\left(1-v_t\right)K_t\right]^{\beta}\left[\left(1-u_t\right)H_t\right]^{1-\beta} - \delta_h H_t \tag{2.9}$$

where ηP_t^{-1} is the marginal disutility of pollution and ρ is the rate of time preference. Finally, the government is assumed to levy a pollution tax τ_p on the use of physical capital in production in order to finance public abatement activities:

$$\tau_p v_t K_t = Z_t \tag{2.10}$$

The corresponding maximization problem of the central planner and the first order conditions for both maximization problems are given in Appendix A.

The long term effects of environmental policy within this model are described in detail by [9] Hettich & Svane (1998). However, it proves convenient for the interpretation of the policy experiments undertaken here to mention two main results. Firstly, an increase in the pollution tax leads to a reduction in the long term growth rate. There are two effects at work behind this result. On the one hand, a pollution tax leads to a direct reduction in the return to capital investment net of tax, see equation (2.3), which tends to reduce growth. On the other hand, a pollution tax leads to a more labor intensive final goods production and thereby to an increase in the return to capital investment, which indirectly tends to increase growth. However, the direct effect outweighs the indirect effect, which means that a tax on pollution is growth reducing. Secondly, a first best outcome can be obtained by setting the pollution tax rate equal to the ratio of public abatement to physical capital used in final goods production that would have been chosen by a central planner. It can be shown that the optimal pollution tax fulfills two tasks at the same time. It lowers the inefficient high rate of return to capital investment and generates the exact amount of public revenues to provide the optimal level of abatement activities. The centrally planned solution can therefore be obtained by setting the pollution tax at its Pigouvian level without the use of a lump sum tax instrument.

3. Calibration

This section describes the calibration of the model. We calibrate the parameters so as to capture empirical stylized facts of industrialized countries. Additionally, due to the lack of data, we also use parameter values suggested by the literature. In spite of the fact that the model is calibrated, it should be taken into account that the quantitative results of the numerical simulation depend on our choice of exogenous parameter values. As a consequence, a sensitivity analysis is carried out in Section 6.

The calibration is made in order to capture an approximate equilibrium growth rate

in Western Germany of 2%[3] per year ($g = 0.02$), and pollution abatement and control expenditures in the private and public sector as a percentage of GDP of 1.6%[4] in the years 1987–1990 ($Z/Y = 0.016$).

Our choices of exogenous parameters closely follow the literature on simulations of two sector endogenous growth models that are similar to ours. Thus, the share of physical capital in final goods production $\alpha = 0.25$ is taken from [12] Lucas (1988). When it comes to the physical capital share in education β several different levels are used for simulation purposes in the literature. In models with production structures similar to ours, [10] King & Rebelo (1990) set either $\alpha = \beta = 0.33$ or $\beta = 0.05$; whereas [7] Devereux & Love (1994) simulate the transitional dynamics for either $\alpha = \beta = 0.36$, $\beta = 0.26$, or $\beta = 0.05$; and [1] Barro & Sala-i-Martin (1995) vary β between 0.4 and 0.1 in their simulations.[5] As a consequence, we choose something in between, namely $\beta = 0.1$.

According to [1] Barro & Sala-i-Martin (1995, p. 37), the measured depreciation rate for the overall stock of structures and equipment is around 5% per year for the U.S. economy. Taking this as a proxy for the industrialized economies, the rates of depreciation of the physical and human capital stock are assumed to be $\delta_h = \delta_k = 0.05$. In addition, we set the rate of time preference to 2% per year and thereby follow Barro & Sala-i-Martin, who use a base line value of $\rho = 0.02$. Note finally that the intertemporal elasticity of substitution is assumed to equal one, see equation (2.7).[6]

Given the above-mentioned choices of exogenous parameters, the value of the technological level in the two sectors $A = B$, the exogenous disutility of pollution as reflected in η, and the elasticity of pollution χ are calibrated in order to obtain a rate of growth of 2% and an abatement–output ratio of 1.6% in the business as usual scenario

[3] The actual computed value is 1.969%. For the computation we use the annual statistics of the OECD main indicators 13100310 (FDR GDP, current prices, million DM, Western Germany), 13102512 (FDR GDP implicit price level, I/90, Western Germany), and EUROSTAT 123000000 (total working population/male and woman, Western Germany, 1000 Source). Also [10] King & Rebelo (1990) and [7] Devereux and Love (1994) calibrate their models in order to obtain a baseline growth rate of 2%.

[4] [14] OECD (1993), table 14.2A, p. 294.

[5] [7] Devereux & Love (1994) follow [17] Prescott (1988) and set the value of $\alpha = 0.36$. They then follow [10] King and Rebelo (1990) and set $\alpha = \beta$ initially. Furthermore, they calibrate their model such that agents spend about 30% of their available discretionary time working in the final goods sector following both [17] Prescott (1988) and [2] Benhabib et al. (1991).

[6] According to [3] Blanchard & Fischer (1989, p. 44) the empirical estimations of the intertemporal elasticity of substitution vary substantially, but usually lie around or below unity.

(BAU).[7] In the increased environmental care scenario (IEC), η is calibrated such that the abatement–output share is twice as high as in the BAU scenario, namely 3.2%. The values of the parameters are summarized in Table 3.1.

	$A=B$	α	β	$\delta_h=\delta_k$	η	ρ	χ	g	Z/Y
BAU	0.1289	0.25	0.1	0.05	0.3056	0.02	0.08	0.02	0.016
IEC	0.1289	0.25	0.1	0.05	0.6142	0.02	0.08	endogenous	0.032

Table 3.1: Exogenous and calibrated parameter values

In the following sections, three policy experiments are simulated and described, namely an unanticipated, an announced, and a gradual increase in environmental care through an increase in the pollution tax. These experiments are undertaken in order to illustrate the fact that policy makers normally want either to announce a policy change in advance or to implement it gradually in order to avoid harmful adjustment processes that may be associated with huge capital losses. Comparison of the transitional dynamics and the welfare implications of these experiments then provides us with information about the most desirable way to implement an increase in green taxes.

The first experiment illustrates the optimal response of the economy to a sudden increase in environmental care. The increased environmental care (IEC) scenario therefore simulates the transitional dynamics of the centrally planned economy in case of an unanticipated increase in the disutility of pollution as reflected in η. Recall that η was calibrated in order to obtain a growth rate of 2% and an abatement–output ratio of 1.6% in the business as usual scenario (BAU), whereas in the IEC scenario, η is calibrated such that the long run abatement–output ratio is 3.2%. Recall, at this point that the first best solution can be obtained by setting the pollution tax equal to the optimal ratio of abatement to physical capital in production. The transitional dynamics of the centrally planned solution in the IEC scenario therefore corresponds to the adaptation process caused by an unexpected increase in the pollution tax in the decentralized economy, if the pollution tax is set at its optimal level in every period. The latter experiment is called the environmental policy scenario (EP). The second experiment illustrates the transitional dynamics of an increase in the green tax, when the policy action is announced 5 years in advance (EP_5). As a result, this experiment

[7] A is set equal to B, because a difference between the two only affects the units in which the physical to human capital ratio is measured. Furthermore, η and χ are interdependent such that any change in χ implies that η must change in order to maintain an abatement–output share of 1.6% or 3.2%.

investigates to what extent households choose to smooth consumption when they take the knowledge of a future policy change into account. The third policy experiment illustrates the transition path of a gradual increase in environmental policy (EP_grad). In this experiment, the pollution tax is increased linearly within a time span of 10 years and the time path of this tax policy is known by households from the beginning. Since the quantitative results of the three policy experiments may be sensitive to the choice of parameter values, we undertake a sensitivity analysis in Section 6.

4. The Central Planner Solution

The experiment undertaken in this section is called the increased environmental care (IEC) scenario. It illustrates the optimal response of the economy to a sudden increase in environmental care. Thus, this experiment simulates the transition path in a centrally planned economy when the disutility of pollution as reflected in η increases, see Table 3.1. The simulation of the transitional dynamics starts in period 1, where the central planner suddenly faces a higher value of η. The stocks of human H and physical capital K are fixed in period one and correspond to their initial optimal BAU level. In the following period the capital stocks adopt gradually to their new optimal values through changes in investment.[8] The simulated dynamic equations are the first order conditions of the central planner solution, namely the resource constraint (2.1), the human capital accumulation function (2.5), the Keynes–Ramsey rule (A.4), the Euler equation for human capital (A.7), the optimal allocation of factors between sectors (A.8) and between consumption and public abatement (A.9), see Appendix A.

Faced with an unanticipated increase in the disutility of pollution, the central planner chooses consumption C, abatement activities Z, and the fractions of time spent at work u and the fraction of physical capital allocated to final goods production v in order to maximize the life time utility of the representative household.

As mentioned before, increased environmental care leads to a more labor intensive final goods production and to a reduced growth rate in the long run. The immediate response to the increase in the disutility of pollution as reflected in η is therefore a sectorial reallocation of resources. The central planner reduces both the fraction of time spent at work u and the fraction of physical capital allocated to production v in period 1, see Figure 4.1. Note however that v is reduced more than u, which

[8] Recall at this point that the allocation of factors can be altered without any frictions in the simulated model, but that we expect costs of factor mobility between sectors to prevail in practice.

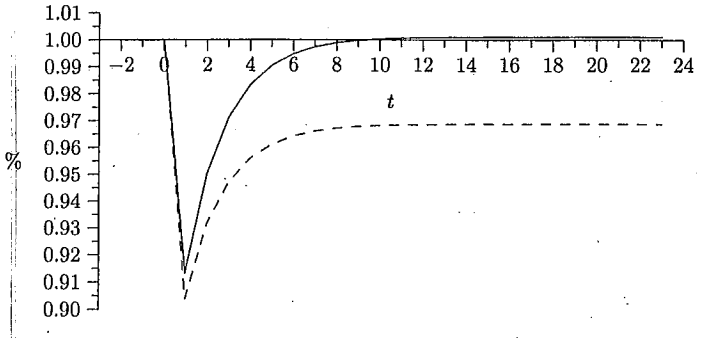

Figure 4.1: Levels of u (solid) and v (dashed)as a % of BAU.

indicates that the capital–labor ratio in final goods production v/u declines relative to the capital–labor ratio $(1-v)/(1-u)$ in education. The combined fall in u and v directly causes a fall in output in period 1, see Figure 4.4. In the following periods, the fractions of human and physical capital allocated to final goods production gradually increase towards their new steady state levels. Along the new balanced growth path, the time fraction spent at work is slightly bigger than before, whereas the fraction of physical capital allocated to production has been reduced, since physical capital is the dirty input factor in production. Comparison of this result and the evolution of the human and physical capital growth rates reveals that increased environmental

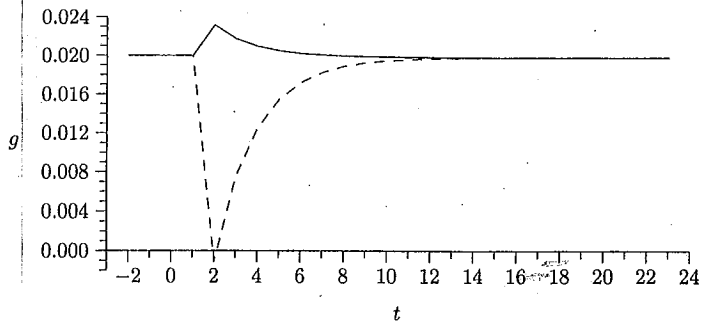

Figure 4.2: Growth rates of human (solid) and physical (dashed) capital.

care leads to a more labor intensive and thereby a cleaner final goods production. As can be seen from Figure 4.2, the growth rate of human capital increases and reaches a maximum of 2.3% in the second period as a response to the shock, while the growth rate of physical capital declines and actually becomes slightly negative −0.1% in the second period.[9] Over time, the return to human capital investment gradually declines, while the return to physical capital investment gradually increases due to the reallocation of resources between the two sectors. Thus, during the transition to the new balanced growth path, the growth rate of human capital slows down to 1.98%, while the growth rate of physical capital rises to 1.98%.

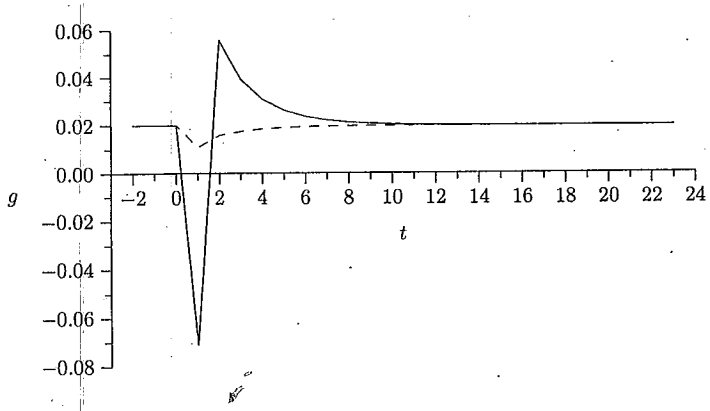

Figure 4.3: Growth rates of output (solid) and consumption (dashed).

Before the unanticipated increase in environmental care takes place in period 1, the balanced growth rates of output and consumption are at their business as usual (BAU) levels of 2% per year, see Figure 4.3. However, like the stocks of human and physical capital, output and consumption also grow at a rate of 1.98% along the new balanced growth path a result of the increase in η. Due to the sudden increase in the disutility of pollution as reflected in η in period 1 it becomes optimal to increase abatement activities Z dramatically in order to equate the marginal utility of abatement and consumption, since the optimal allocation of resources between abatement activities and consumption is determined by $Z = \eta\chi C$, see equation (A.9). Actually, the central planner almost doubles pollution abatement in the first period. The increase

[9] Recall that the stocks of human and physical capital only can be altered gradually by changes in investment and therefore are unaffected in the first period.

in resources used for abatement activities crowds out both consumption and physical capital investment. In addition, final goods production becomes more labor intensive in the long run, since the use of physical capital is responsible for pollution. In order to decrease the physical to human capital ratio in final good production $(vK)/(uH)$, human capital grows above its balanced growth rate and physical capital grows below its balanced growth rate both in the short and medium term. This is a result of the temporary increase in eduction time and the fraction of physical capital allocated to eduction, see Figure 4.1. As a consequence, the growth rate of output declines and actually becomes negative in period 1, namely -7.1%, while the growth rate of consumption drops to 1.1%. In practice, the drop in output growth would be less severe due to costs of reallocating resources from one sector to another. However in the present setup, factors are assumed to move freely and without costs between sectors also in the short run.[10] The evolution of the levels of output Y and consumption C as a percentages of the business as usual scenario (BAU) is illustrated in Figure 4.4.

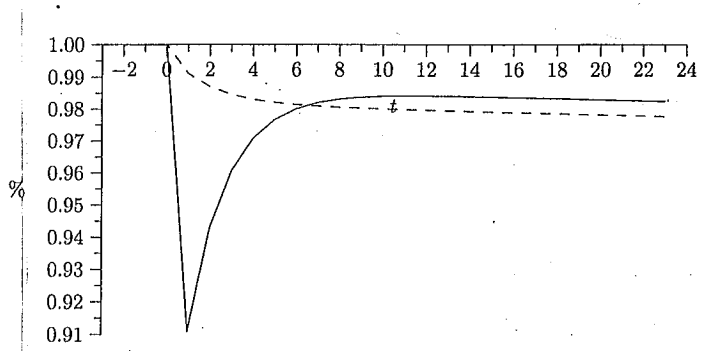

Figure 4.4: Levels of Y (solid) and C (dashed) as a % of BAU.

The level of output is seen to drop dramatically in period 1, whereafter it increases for a period of time. In contrast, the level of consumption as a percentage of BAU scenario gradually declines over time. Although the growth rate of output is above its business as usual (BAU) rate between year 2 and 11, the level of output is always lower than its BAU level. Therefore, the existence of growth rates above 2% is solely

[10] Adjustment costs of capital investment are not taken into account, since the main purpose of the present paper is to simulate the model presented by [9] Hettich & Svane (1998), where costs of adjustment are absent.

a base line effect, which is due to the negative growth rate in year one. The results of the simulation of the IEC scenario are summarized in Table 5.1 at the end of Section 5, where the present discounted welfare is seen to be higher in the IEC scenario than under a business as usual policy (BAU), where abatement activities are unaltered even though the disutility of pollution has increased.

The main conclusions to be drawn is that an unanticipated increase in environmental care only leads to a slight reduction in the growth rate in the long run, but to a negative output growth rate in the period of the shock. Due to the lack of costs related to the reallocation of factors in the short run, the adjustment processes look more severe in the present paper, than they would in practice.

5. The Market Solution

In this section, we analyze the effects of two announced tax policy experiments. Before we simulate announcement effects of these policies, we describe the environmental policy scenario (EP) of the market solution, where the pollution tax is increased unexpectedly. The EP scenario corresponds to the IEC scenario provided that the pollution tax is set optimally in every period, and serves as a base line for the two announced tax policy experiments analyzed in this section. The first announced tax policy experiment illustrates the transitional dynamics of the market economy, when an increase in the pollution tax is implemented 5 years after it has been announced (EP_5). This experiment investigates to what extend households smooth consumption when they take the knowledge of a future policy change into account. The second experiment illustrates the transition path of the market economy, when the pollution tax is increased gradually over a period of 10 years (EP_grad). This experiment investigates how a gradual implementation of the pollution tax performs relative to the two other policy alternatives EP and EP_5. In both experiments, the disutility of pollution once more is assumed to increase to the higher level already in the first period, see Table 3.1. However, this does not affect the behavior of the representative agent as long as the government does not change its environmental policy. The results of the different environmental policy scenarios will be summarized at the end of this section in Table 5.1.

The optimal pollution tax is seen from Figure 5.1 to be 0.62% in the business as usual scenario (BAU) and increase immediately in period 1 to 1.36% in the increased environmental care scenario (IEC). The long run level of the optimal pollution tax is

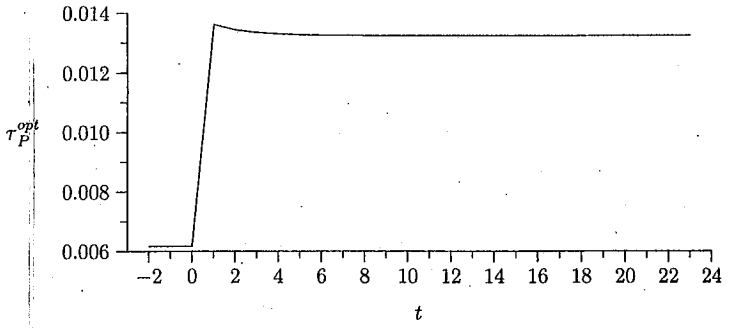

Figure 5.1: The optimal path of the pollution tax.

1.32%, which means that there is a slight overshooting of the optimal pollution tax. If the government chooses the optimal time path of the pollution tax pictured in Figure 5.1, then the market economy would reach a first best outcome. In the following, we choose to let the pollution tax jump to its long run optimal level already in period 1 in the unanticipated environmental policy scenario (EP). This simplifies the simulation of the transition path, because it is much easier to deal with only two values of the pollution tax instead of 150. However, the simplification does not alter the qualitative results and only have a slight effect on the quantitative results, since the variations in the optimal pollution tax are negligible after period 1, see Figure 5.1. Once more, the simulation of the transitional dynamics begins in period 1.

5.1. Anticipated Environmental Policy

This section investigates the transitional dynamics of a decentralized economy, when an increase in the pollution tax is announced 5 years in advance. This anticipated environmental policy scenario (EP_5) illustrates to what extent households choose to smooth consumption when they take the knowledge of a future policy change into account. Policy makers normally want to announce a change in green taxes in advance in order to avoid large swings in output and consumption. Such an announcement gives economic agents the opportunity to reallocate resources prior to the policy change and thereby take some of the adjustment in advance such that the drop in consumption becomes less severe when green taxes actually rise. The latter is more likely to happen when households have a low intertemporal elasticity of substitution and therefore dislike

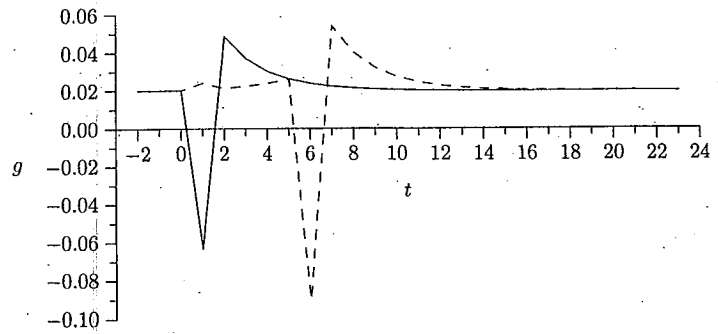

Figure 5.2: Growth of output in the EP (solid), and the EP_5 scenario (dashed).

huge swings in consumption. An announcement of a future increase in the pollution tax (EP_5) may therefore improve welfare compared to an unannounced policy change (EP), if consumption is smoothened sufficiently over time. However, it is definitely not optimal to postpone the policy change, if households do not mind large swings in consumption over time as we will see is the case in the model under consideration. The consequence is a lack of adjustment prior to the tax increase.

In the decentralized economy, households faced with such an anticipated increase in the pollution tax choose consumption C, and the fractions of time u and physical

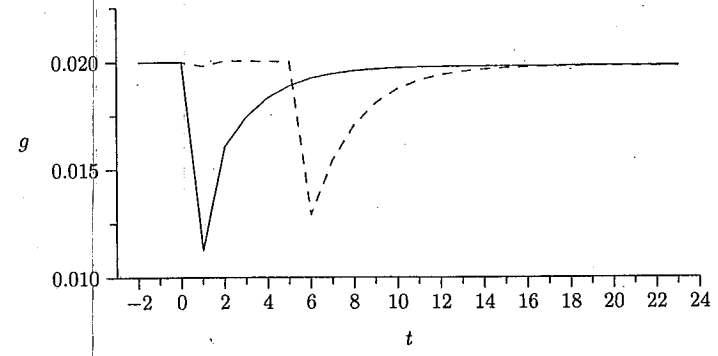

Figure 5.3: Growth of consumption in the EP (solid), and the EP_5 scenario (dashed).

capital v allocated to final goods production in order to maximize their life time utility, see equation (2.7). Note that pollution is a public bad, which households take as given in their maximization problem. Figure 5.2 depicts the growth rate of output in both the anticipated environmental policy scenario (EP_5) and corresponding unanticipated scenario (EP), which is equivalent to the increased environmental care scenario (IEC). Looking at the anticipated scenario EP_5, it can be seen that output growth increases slightly prior to the policy action, see Figure 5.2, whereas consumption growth is practically unaltered, see Figure 5.3. Thus, there is no substantial consumption smoothing effect when the policy change is announced in advance.[11]

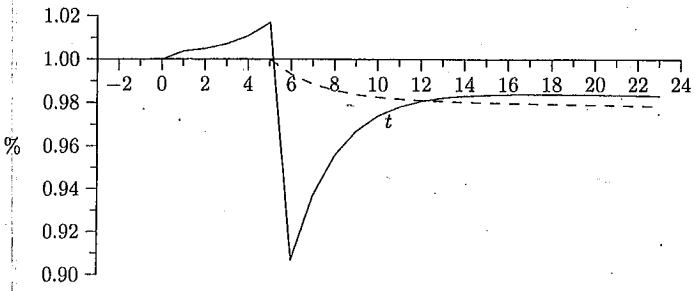

Figure 5.4: Levels of Y (solid) and C (dashed) as a % of BAU.

Figure 5.4 shows that the level of consumption is unaltered prior to the policy action, whereas the level of output is above the business as usual level. After the pollution tax has been increased the levels of output and consumption behave like in the EP scenario, see Figure 4.4. Thus, households allocate greater fractions of human and physical capital to final goods production prior to the policy change. As a result, the decline in output growth once the policy change is implemented has to be bigger in the anticipated case (EP_5), than in the unanticipated case (EP). At the same time, households choose to postpone the drop in consumption until after the implementation of the policy change, see Figure 5.4. Overall, consumption growth exhibits the same pattern as in the environmental policy scenario (EP), but on a slightly smaller scale and

[11] Recall that the intertemporal elasticity of substitution is assumed to be equal to one. A lower intertemporal elasticity of substitution would probably lead to a decline in consumption and output growth already within the announcement period, because consumers are more motivated to smooth consumption.

with the modification that the drop in consumption growth occurs in the period of the policy implementation, see Figure 5.3. In both scenarios EP and EP_5, consumption growth drops once the new policy is implemented, and thereafter converges back to 1.98% per year. Note that the drop in consumption growth is less dramatic in the anticipated case, than in the unanticipated case. In the anticipated case EP_5, public abatement activities increase in the periods prior to the policy change even though the pollution tax rate is unaltered, see Figure 5.5. The reason behind this is that tax

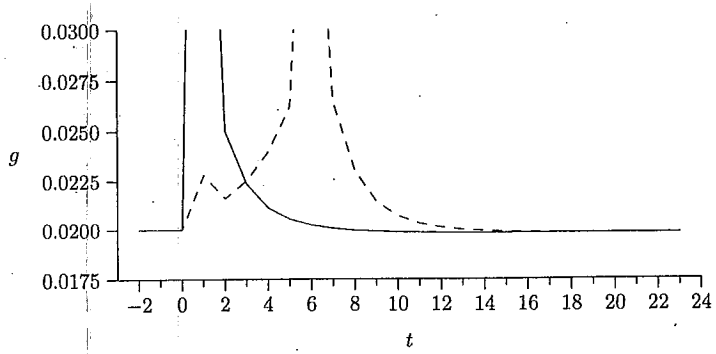

Figure 5.5: Growth of Z in the EP (solid), and the EP_5 scenario (dashed).

revenues and thereby public abatement activities increase due to an increase in the level of the tax base, because vK increases.[12] Once the government levies the new and higher pollution tax, public abatement activities increase dramatically. In fact, the level of abatement activities increases by around 95% in both cases compared to the business as usual scenario, see Table 5.1. This results from an increase in the pollution tax rate and a practically unchanged tax base. During the transition to the new balanced growth path, the growth rate of public abatement activities begins to decline, because the level of the tax base becomes smaller, when labor is substituted for physical capital in final goods production as a reaction to the imposed environmental policy.

Since the need for consumption smoothing is practically absent in our model, the present discounted welfare in the unanticipated scenario (EP) is naturally higher than under the announced policy scenario (EP_5), see Table 5.1. In addition, the pollution

[12] Recall that the government runs a balanced budget every period.

tax rate is unchanged and abatement measures only change through tax base changes during the announced span of time, whereas the abatement measures have been increased already during this period of time in the EP scenario. Thus, pollution is too high in the EP_5 scenario from a welfare point of view during the announced period. For these reasons and since the increase in output growth prior to the policy implementation results in a high level of pollution between the period of announcement and the period of implementation of the policy, the result is a reduction in welfare. Thus, in the announced scenario there are hard times, because welfare as a percentage of BAU decreases during the periods of announcement. Nevertheless there are overall welfare gains compared to the business as usual scenario, where public abatement activities are kept at the low level even though the disutility of pollution has increased. The main conclusion to be drawn from the EP_5 scenario is that the consumption smoothing effect is practically absent and there is consequently is no argument for postponing the increase in the pollution tax rate. Note that this result probably hinges on the assumption of an intertemporal elasticity of substitution of one.

5.2. Gradual Environmental Policy

This section simulates the transition path of the market economy, when the pollution tax is increased gradually over a period of 10 years. Such a gradual implementation of a policy change is commonly proposed by policy makers that aim to avoid severe reductions in output growth like the ones we have seen in the previous scenarios. Even though the simulations do not take investment costs into account, it is interesting to see the magnitude of the welfare loss, if the policy change is phased in over 10 periods instead of being implemented immediately. The EP_grad scenario therefore gives us an idea about how large investment costs should be, before it pays in terms of welfare to implement the policy gradually instead of immediately (EP).

In the gradual environmental policy scenario (EP_grad), the pollution tax is linearly increased by the government every year within the first 10 years. This gradual environmental policy is known by households from period 1. The transitional dynamics of output and consumption growth in this gradual environmental policy scenario is illustrated in Figure 5.6. The paths of the growth rates are quite similar to that of the EP and EP_5 scenarios, however the departures of the growth rates from their steady state values are much smaller and stretched over the first 10 years reflecting the gradual increase in environmental costs during this period. Once again output growth falls and reaches a low of 1.3% in period 2, see Table 5.1. Note that output growth gradually

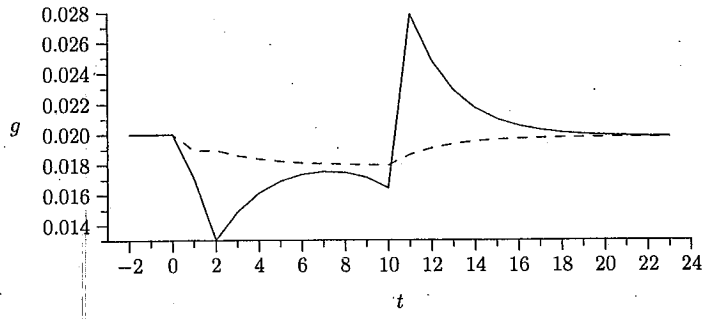

Figure 5.6: Growth of Y (solid) and C (dashed) in the EP_grad scenario.

rises prior to the full implementation of the new and higher pollution tax. Just as in the scenarios described in Section 5.1, output growth rises above its long run level once the increase in the pollution tax is fully implemented in period 11. But in contrast to these scenarios, the minimum output growth in the gradual environmental policy scenario is much bigger and in fact both positive and close to its long run rate. Thus, output growth rates are below their equilibrium values not only in the first period as in the EP scenario, but also during the first 10 years and without being negative, see Table 5.1.[13] Even though the growth rate of output increases a lot between period 10 and 11, the adjustment in the level of output as a percentage of the business as usual scenario (BAU) is smooth, see Figure 5.7.

Table 5.1 also reveals that the level effects as a percentage of the business as usual

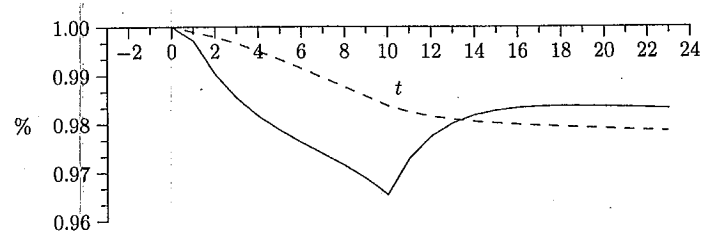

Figure 5.7: Levels of Y (solid) and C (dashed) as a % of BAU.

[13]The minimum output growth rates are negative in the EP and EP_5 scenarios.

Scenario	BAU	IEC	EP	EP_5	EP_grad
announced span of time	–	–	0	5	*
Level effects as a percentage of BAU					
Y in the year of announ.	100	91.1	91.9	100.4	99.7
Y in the year of shock	100	91.1	91.9	90.7	99.7
C in the year of announ.	100	99.1	99.1	99.98	99.9
C in the year of shock	100	99.1	99.1	99.3	99.9
Z in the year of announ.	100	199.2	195.2	100.3	110.9
Z in the year of shock	100	199.2	195.2	194.7	110.9
$(1+\rho)^{-t}\sum_{t=1}^{150} Y_t$	100	97.2	97.2	97.3	97.3
$(1+\rho)^{-t}\sum_{t=1}^{150} C_t$	100	96.9	96.9	97.0	97.0
$(1+\rho)^{-t}\sum_{t=1}^{150} Z_t$	100	194.7	194.6	191.6	192.0
Welfare effects as a percentage of BAU					
year of announ.	100	100.68	100.66	99.995	100.10
year after announ.	100	100.57	100.57	99.997	100.18
year of shock	100	100.68	100.66	100.68	100.10
year after shock	100	100.57	100.57	100.58	100.18
$(1+\rho)^{-t}\sum_{t=1}^{150} U_t$	100	100.23	100.23	100.21	100.22
Macro economic indicators (period)					
long term growth rate	2.0	1.98	1.98	1.98	1.98
max. Y growth	2.0	5.6(2)	4.9(2)	5.4(7)	2.8(11)
max. C growth	2.0	2.0(0)	2.0(0)	2.01(2)	2.0(0)
min. Y growth	2.0	–7.1(1)	–6.3(1)	–9.0(6)	1.3(2)
min. C growth	2.0	1.1(1)	1.1(1)	1.3(6)	1.8(10)

Table 5.1: Results of the numerical simulation.

scenario are much smaller in the gradual environmental policy scenario. Thus, a gradual introduction of an increase in the pollution tax has the nice property of smoothing the adjustment of all core variables. This is seen for the level of output and consumption by comparison of Figure 4.4, 5.4 and 5.7. In addition, discounted welfare under a gradual environmental policy scheme is larger than under an anticipated increase in the pollution tax. The reason behind this result is that the pollution tax is raised from the beginning in the EP_grad scenario. This gives households an incentive to reallocate physical capital towards the education sector already from the beginning, which means that the emission of pollution is reduced already from period 1. Thus, the levels of pollution during the period of gradual implementation of the policy are much lower than under the EP_5 scenario.

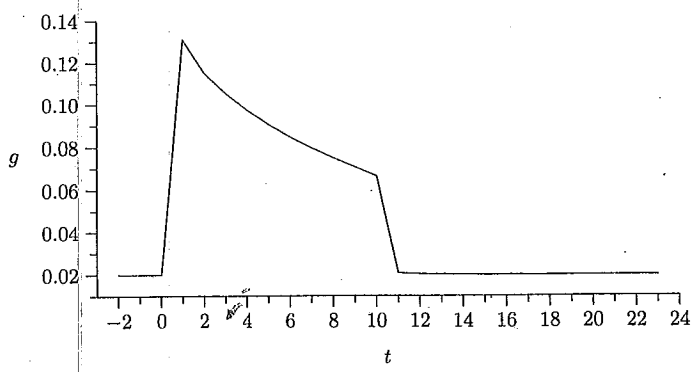

Figure 5.8: Growth of Z in the EP_grad scenario.

According to Figure 5.6, consumption growth slows down a little during the period of the gradual implementation of the pollution tax. But once the new policy is fully implemented consumption growth begins to rise to 1.98%. This pattern was also seen in the anticipated scenario EP_5, but note that the drop in consumption growth in the EP_grad scenario is negligible in comparison. Figure 5.8 reveals that the level of public abatement activities Z increases gradually during the first 10 years in the EP_grad scenario. This is in contrast to the previous scenarios EP and EP_5, where public abatement jumps discretely to a new and higher level, when the pollution tax is increased in period 1 and 5, respectively. In the EP_grad scenario, the level of the tax base gradually decreases during the first 10 years, because human capital is substituted

for physical capital in final goods production. As a result, the growth rate of public abatement activities decline over time and drops to 2.1% already in period 11.

Recall that one of the implications of announcing a future increase in the pollution tax (EP_5) is a welfare loss, because abatement activities are below the optimal level, when the pollution tax is below its Pigouvian level. However, when the pollution tax is increased gradually, the pollution tax rate is closer to its Pigouvian level and the welfare loss much smaller as a consequence. Since we expect costs of factor mobility to prevail in practice, the best policy recommendation in our point of view, but certainly not from a strict welfare point of view, is therefore a gradual environmental policy scheme. Both because its performance with respect to the policy indicators in Table 5.1 are the best of all the considered policy scenarios, but also because the welfare loss in comparison with an unanticipated policy scheme is negligible. The adjustment processes are simply not as tough under a gradual environmental policy scheme.

6. Sensitivity Analysis

This section investigates how sensitive the results obtained in Section 4 and 5 are to changes in the parameters chosen for the simulations, see Table 3.1. One of the main questions posed in this section is by which of the parameters the transitional dynamics of the model are driven.

For the sensitivity analysis we carry out simulations of several different scenarios. Firstly, the capital share in final goods production is set either close to the level in the education sector $\alpha = 0.15$ (S_α1) or at $\alpha = 0.4$ (S_α2). Secondly, the capital share in education is set either at $\beta = 0.05$ (S_β1)[14] or close to the level in the final goods sector $\beta = 0.20$ (S_β2). Thirdly, the rates of depreciation of the physical and human capital stock are set either at $\delta_k = \delta_h = 0.04$ (S_δ1) or at 0.06 (S_δ2). Fourthly, the rate of time preference is set either at $\rho = 0.01$ (S_ρ1) or at 0.03 (S_ρ2). The results from these sensitivity analyses are given in Table 6.1 and 6.2. Just as in the previous sections the model is calibrated to capture a growth rate of 2% and an abatement–output ratio of 1.6% initially and 3.2% along the new balanced growth path. Note however that the changes in the exogenous parameters affect the calibration of the model and thereby the technological levels A and B and the disutility of pollution as reflected in η. Thus, the above-mentioned changes in the exogenous parameters have

[14]This value is also used in sensitivity analyses in [10] King & Rebelo (1990) and [7] Devereux & Love (1994).

Scenario	EP	S_α1	S_α2	S_β1	S_β2
Parameters					
τ_P (%)	1.3	2.5	0.8	1.3	1.3
$A = B$	0.1289	0.1266	0.1318	0.1129	0.1516
α	0.25	0.15	0.4	0.25	0.25
β	0.1	0.1	0.1	0.05	0.20
δ	0.05	0.05	0.05	0.05	0.05
ρ	0.02	0.02	0.02	0.02	0.02
η_{BAU}	0.3056	0.2809	0.3521	0.2786	0.3531
η_{EP}	0.6142	0.5642	0.7081	0.5600	0.7094
Ratios on the new balanced growth path					
C/Y	0.65	0.71	0.56	0.71	0.56
Y/K	0.22	0.27	0.17	0.28	0.17
I/Y	0.09	0.07	0.11	0.07	0.11
K/H	0.12	0.10	0.15	0.07	0.24
v/u	1.76	1.16	2.33	2.51	1.09
$(vK)/(uH)$	0.21	0.12	0.36	0.18	0.26
Level effects as a percentage of BAU					
Y_1	91.9	81.5	96.6	94.5	82.4
C_1	99.1	98.4	99.8	99.7	98.6
Z_1	195.2	172.7	202.1	203.2	171.7
$(1+\rho)^{-t}\sum_{t=1}^{150} Y_t$	97.2	97.2	96.8	97.6	96.5
$(1+\rho)^{-t}\sum_{t=1}^{150} C_t$	96.9	96.9	96.4	97.3	96.2
$(1+\rho)^{-t}\sum_{t=1}^{150} Z_t$	194.6	194.6	194.0	195.5	193.0
Welfare effects as a percentage of BAU					
U_1	100.66	100.43	101.00	100.65	100.74
U_2	100.57	100.40	100.90	100.54	100.71
$(1+\rho)^{-t}\sum_{t=1}^{150} U_t$	100.23	100.21	100.28	100.19	100.31
Political indicators (period)					
long term growth rate	1.98	1.98	1.98	1.99	1.97
max. Y growth	4.9(2)	19.3(2)	2.3(2)	3.2(2)	17.9(2)
max. C growth	2.0(0)	2.0(2)	2.0(0)	2.0(0)	2.0(2)
min. Y growth	−6.3(1)	−16.8(1)	−1.5(1)	−3.6(1)	−15.9(1)
min. C growth	1.1(1)	0.3(1)	1.6(2)	1.5(2)	0.6(1)

Table 6.1: Results of the sensitivity analysis, part I.

Scenario	EP	S_δ1	S_δ2	S_ρ1	S_ρ2
Parameters					
τ_P (%)	1.3	1.2	1.5	1.2	1.5
$A = B$	0.1289	0.1147	0.1432	0.1144	0.1435
α	0.25	0.25	0.25	0.25	0.25
β	0.1	0.1	0.1	0.1	0.1
δ	0.05	0.04	0.06	0.05	0.05
ρ	0.02	0.02	0.02	0.01	0.03
η_{BAU}	0.3056	0.2932	0.3179	0.3896	0.2762
η_{EP}	0.6142	0.5895	0.6386	0.7816	0.5559
Ratios on the new balanced growth path					
C/Y	0.65	0.68	0.63	0.51	0.72
Y/K	0.22	0.21	0.23	0.15	0.28
I/Y	0.09	0.10	0.08	0.13	0.07
K/H	0.12	0.12	0.12	0.11	0.13
v/u	1.76	1.71	1.79	1.94	1.63
$(vK)/(uH)$	0.21	0.21	0.21	0.21	0.21
Level effects as a percentage of BAU					
Y_1	91.9	91.7	92.0	90.3	92.8
C_1	99.1	99.2	99.1	99.0	99.2
Z_1	195.2	195.5	195.0	190.4	198.2
$(1+\rho)^{-t}\sum_{t=1}^{150} Y_t$	97.2	97.4	97.0	96.9	97.5
$(1+\rho)^{-t}\sum_{t=1}^{150} C_t$	96.9	97.0	96.7	96.7	97.1
$(1+\rho)^{-t}\sum_{t=1}^{150} Z_t$	194.6	195.0	194.3	194.0	195.3
Welfare effects as a percentage of BAU					
U_1	100.66	100.64	100.68	101.02	100.55
U_2	100.57	100.55	100.59	100.93	100.45
$(1+\rho)^{-t}\sum_{t=1}^{150} U_t$	100.23	100.22	100.24	100.42	100.18
Political indicators (period)					
long term growth rate	1.98	1.98	1.98	1.98	1.98
max. Y growth	4.9(2)	4.7(2)	4.9(2)	5.2(2)	4.7(2)
max. C growth	2.0(0)	2.0(0)	2.0(0)	2.0(0)	2.0(0)
min. Y growth	-6.3(1)	-6.5(1)	-6.1(1)	-7.8(1)	-5.4(1)
min. C growth	1.1(1)	1.2(1)	1.1(1)	1.0(1)	1.2(1)

Table 6.2: Results of the sensitivity analysis, part II.

implications not only for the transitional dynamics of the model, but also for welfare. As a result, the undertaken sensitivity scenarios cannot be compared with the original BAU scenario, since they have their own BAU scenarios.

There are several implications to be drawn from the sensitivity analysis. Firstly, the effects of an increase in the physical capital share in final goods production α from 0.15 to 0.4 can be seen by comparison of S_α1, EP and S_α2. The first major observation is that an increase in α leads to a decline in the optimal pollution tax rate from 2.5% to 0.8%. The reason behind this is that the return to physical capital is low, when α is low. As a result, less physical capital is allocated to final goods production, which means that the pollution tax base is low. Consequently, the optimal pollution tax has to be higher, the lower the physical capital share in production is, since it has to raise enough revenue to cover expenditures on pollution abatement as a percentage of GDP of 3.2%. Thus, the decline in the optimal pollution tax, when α rises is primarily caused by an increase in the tax base. The second major observation is that the transitional dynamics of an unanticipated environmental policy are dampened by an increase in α.

Secondly, the effects of an increase in the physical capital share in education β from 0.05 to 0.20 can be seen by comparison of S_β1, EP and S_β2. In this case, the optimal pollution tax rate is practically unchanged, since the tax base is not directly affected by changes in β. But in contrast to an increase in α, the transitional dynamics becomes more extreme with a minimum growth rate of -15.9% in period 1 and a maximum growth rate of output of 17.9% in period 2, when β increases to a level close to α. It can therefore be concluded that the closer α and β are to each other, the more severe is the adjustment to the new balanced growth path. Thus, the transitional dynamics of the model are fairly sensitive to the choices of α and β and that the more so the closer they are to each other. The reason behind this result is that reversed factor intensities $\alpha < \beta$ leads to an unstable behavior of the physical to human capital ratio in final goods production, which transmits to the interest rate and thereby to the growth rates. Thus, the closer the model is to a situation, where factor intensities are reversed, the larger are the swings in consumption and output growth.[15]

Thirdly, it turns out that the transitional dynamics of the model are more or less insensitive to changes in the rate of depreciation δ and the rate of time preference ρ. Thus, all ratios along the new balanced growth path are almost unaltered by changes in these two parameters and so are the level effects on output Y, consumption C, and abatement Z. Over all, the effects of changes in ρ are a little bigger than the effects of

[15] See [1] Barro & Sala-i-Martin (1995), p. 197.

changes in δ.

Finally, an unanticipated environmental policy has a greater positive effect on welfare, the larger the shares of physical capital in both sectors are, the higher the rate of depreciation of physical and human capital is, and the more patient agents are.

7. Conclusion

This paper has analyzed the transitional dynamics of environmental policy in a two sector endogenous growth model with pollution. In particular, the undertaken analyses examined the differences between an increase in the pollution tax, which was either unannounced, announced five years in advance, or implemented gradually over a period of ten years.

From a strict welfare point of view it is optimal to introduce the increase in the pollution tax unannounced. However, in our point of view, the best policy recommendation is a gradual environmental policy scheme, since we expect investment costs to prevail in practice. Firstly, the adjustment processes are not as tough as under the two other environmental policy schemes, because a gradual increase in the pollution tax stretches out the adjustment processes over several years. Secondly, consumer welfare is higher under the gradual environmental policy scheme, than under the announced policy scheme, because abatement activities can be increased already within the first five time periods. In addition, the welfare loss of a gradual implementation of the policy is negligible in comparison with the first best policy. Even though, the simulations in this paper do not take investment costs into account, we expect that costs of adjustment do not have to be very big, before it pays in terms of welfare to implement the policy change gradually.

Furthermore, it is interesting to note that the long term growth rate only reduces from 2% to 1.98%, when the abatement–output ratio doubles from 1.6% to 3.2%. This result holds irrespective of the way the increase in the pollution tax is implemented. Thus, the loss in terms of a reduction in the long term growth rate is relatively small, when environmental care increases.

Moreover, it turns out that an announced policy change does not lead to any substantial consumption smoothing over time in the analyzed model. In fact such a policy just postpones the adjustment processes to the actual implementation of the policy. Note however that this result probably would be modified in an equivalent model with an intertemporal elasticity of substitution below one.

Finally, the sensitivity analyses reveal that the shares of physical capital in the two sectors are the key parameters to the understanding of the transitional dynamics in our two sector endogenous growth model. The more similar the production process in the final goods sector is to the production process in the education sector, the more severe are the adjustment processes, since an increase in the pollution tax leads to bigger movements in output and consumption.

A. Appendix

The central planner chooses consumption C, the shares of u and v, and abatement activities Z in order to maximize life time utility subject the resource constraint (2.1) and the human capital accumulation constraint (2.5). The maximization problem of the central planner is:

$$\begin{aligned}
\max_{C_t,\, Z_t,\, u_t,\, v_t} U_0 &= \sum_{t=0}^{T} (\ln C_t - \eta \ln P_t)\,(1+\rho)^t && \text{(A.1)}\\
\text{s.t. } K_{t+1} - K_t &= Y_t - C_t - Z_t - \delta_k K_t && \text{(A.2)}\\
\text{s.t. } H_{t+1} - H_t &= B\left[(1-v_t)\,K_t\right]^{\beta}\left[(1-u_t)\,H_t\right]^{1-\beta} - \delta_h H_t && \text{(A.3)}\\
C_t,\, H_t,\, K_t &\geq 0 \qquad \forall t, \qquad \text{and } H_0,\, K_0 \text{ are given}
\end{aligned}$$

Note that a benevolent planner in contrast to a representative household takes the negative side effects of production into account. Additionally, a central planner uses public abatement activities as a control variable.

The Keynes–Ramsey rule is:

$$\left(\frac{C_{t+1}}{C_t} - 1\right)(1+\rho) = r^i_{t+1} - \delta_k - \rho, \text{ where } i = \textit{market}, \textit{social} \qquad \text{(A.4)}$$

where the interest rate in the market economy and the planned economy are given by:

$$r_t^{market} = \alpha A \left(\frac{u_t H_t}{v_t K_t}\right)^{1-\alpha} - \tau_p \qquad \text{(A.5)}$$

$$r_t^{social} = \alpha A \left(\frac{u_t H_t}{v_t K_t}\right)^{1-\alpha} - \frac{Z_t}{v_t K_t} \qquad \text{(A.6)}$$

The Euler condition for human capital is:

$$(1-\beta)\,B\left[\frac{(1-v_{t+1})\,K_{t+1}}{(1-u_{t+1})\,H_{t+1}}\right]^{\beta} - \delta_h + 1 = (1+\rho)\,\frac{C_{t+1}}{C_t}\,\frac{w_t}{w_{t+1}}\left(\frac{\frac{(1-v_{t+1})K_{t+1}}{(1-u_{t+1})H_{t+1}}}{\frac{(1-v_t)K_t}{(1-u_t)H_t}}\right)^{\beta} \qquad \text{(A.7)}$$

where the wage rate w_t is given in equation (2.4).

The optimal allocation of factors between the two sectors is determined by:

$$\frac{w_t}{r_t^i} = \frac{1-\beta}{\beta}\frac{(1-v_t)\,K_t}{(1-u_t)\,H_t}, \tag{A.8}$$

and the optimal allocation of resources between consumption and public abatement activities in the centrally planned economy is determined by:

$$Z_t = \eta\chi C_t \tag{A.9}$$

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Emission Standards and Growth*

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Abstract

This paper investigates the effects of an emission standard and taxation within a two sector endogenous growth model with pollution. There are two regimes characterized by a non-binding and a binding emission standard, respectively. The main result is that sustained growth *is* possible, when environmental concerns are taken into account. Furthermore, the outcome of a decentralized economy is inefficient. A capital income tax or a pollution tax is therefore required to reach a first best outcome. If the capital income tax is unavailable as an instrument, then the optimal pollution tax equals the optimal marginal damage of pollution. However, the optimal pollution tax may be below its Pigouvian level, when the optimal capital income tax is high, since a tax on capital income works as an indirect tax on pollution.

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1. Introduction

This paper deals with two important topics in the ongoing debate on environmental policy. Firstly with the prospects for sustained economic growth and whether environmental concerns eventually limit growth. Secondly with the implementation of a first best environmental policy in a decentralized economy such that an optimal evolution of output growth and environmental protection is achieved. In order to deal with these topics, this paper extends the Uzawa–Lucas model[1] to allow for disutility of pollution caused by the use of physical capital in production.

The first source of inspiration for the analyses in this paper is the empirical evidence presented by [6] Grossman & Krueger (1995), which shows that there is an inverted U–shape relationship between per capita income and various types of pollution e.g. air pollution such as sulphur dioxide and smoke, and water pollution such as oxygen loss and concentration of several heavy metals.[2] The second source of inspiration is the theoretical work by [16] Stokey (1998), which aims to capture the above-mentioned empirical evidence of an inverted U–shape relationship. Stokey develops both an exogenous and an endogenous growth model. The latter model is an AK–model[3], where growth is assumed to be driven by constant returns to a broad measure of capital that includes both human and physical capital. Stokey then introduces a pollution function in both models that is in line with the empirical evidence[4] and describes pollution as a side product of total production. Thus, pollution is assumed e.g. to be caused by waste products from final consumption.[5] Finally, Stokey introduces a standard of emission into the two models in order to analyze whether there are limits to economic growth. It turns out that sustained growth only is possible in the presence of pollution in the

[1] The reproducible factors human and physical capital are both used as input factors in final goods production. There are constant returns to human capital in the education sector, see [10] Lucas (1998).

[2] [6] Grossman & Krueger examine reduced–form relationships between national GDP and various indicators of local environmental conditions using panel data from the Global Environmental Monitoring System (GEMS), which is a joint project of the World Health Organization and the United Nations Environmental Programme. The measures of pollution pertain to specific cities or sites on rivers, while GDP is measured at the country level.

[3] See [14] Rebelo (1991).

[4] [1] Andreoni & Levinson (1998) show that an inverse U–shaped relationship between pollution and income (the "environmental Kuznets curve") is obtained, when the abatement technology exhibits increasing returns to scale.

[5] Note that [19] the World Bank (1992) finds a positive relationship rather than an inverted U–shape relationship between municipal waste per capita and growth in GDP.

exogenous growth model. Thus, a main result in [16] Stokey (1998) is that environmental concerns in the form of emission standards limit growth within an AK–framework. The reason behind this result is that the return to capital falls as the capital stock grows and the emission standard becomes stricter. Over time the emission standard becomes so strict that there is no further incentive to accumulate capital. This means that growth eventually ceases. Note however that this result hinges on two critical assumptions.

One critical assumption is that pollution is generated by total production, since it is highly questionable, whether waste products from final consumption is a major source of the types of air and water pollution described by [6] Grossman & Krueger. It is much more likely that the above-mentioned types of pollution are caused by the use of physical capital in production provided that this is appropriately interpreted to include the use of consumer durables like cars and houses. Hence, in line with this view the present paper assumes that pollution is generated by the use of physical capital in final goods production, and introduces a pollution function, which is still consistent with the empirical evidence of an inverted U–shape relationship between per capita income and pollution.

Another critical assumption in the Stokey paper is that capital is broadly defined to include both physical and human capital. This is problematic, since one would expect that human capital contributes less to pollution than physical capital. Furthermore, the very idea of a broad definition of capital is that endogenous growth is possible in the long run, if one believes that there are constant returns overall to both kinds of capital. The introduction of an emission standard in an AK–framework therefore indirectly imposes restrictions on human capital. In order to deal with this problem a natural extension of the Stokey model is therefore to develop an endogenous growth model, where human and physical capital are accumulated in separate sectors and the emission standard is imposed specifically on the use of physical capital in production. This paper develops such a model and shows that environmental concerns do not limit growth in any way when the intertemporal elasticity of substitution is smaller than or equal to one, which is normally assumed.

One of the features of the Stokey model and the model presented here is that income eventually reaches a critical level, where pollution is so hampering that it becomes optimal to intensify environmental regulations through a binding emission standard, and thereby moderate economic growth. This results in two regimes characterized by a non–binding and a binding emission standard, respectively. The present paper not

only determines the long term growth rates in these two regimes, but also analyses the transitional dynamics of the model analytically instead of simulating the model like Stokey. Furthermore, it is investigated whether the government should use tax instruments in a decentralized economy in order to implement the first best environmental policy, which is characterized by the optimal path of the emission standard. Finally, the model is extended to include productive public spending.

This paper is closely related to the literature on environmental policy and endogenous growth. Several authors analyze the consequences of environmental policy for economic growth, but the results are ambiguous. [9] Ligthart & van der Ploeg (1994) show that the growth effect of environmental policy is negative in the AK–model, whereas [5] Gradus & Smulders (1993) show that a larger concern for a clean environment has no effect on long term growth in a variant of the two sector Uzawa–Lucas model. Moreover, [4] Bovenberg & de Mooij (1997) present a Barro type model[6] of endogenous growth and explore the link between environmental externalities and distortionary income taxes in a second–best world. They show that an environmental tax reform, which shifts taxes away from output towards pollution, *may* increase growth through two channels. Firstly by improving the efficiency of the tax system as a revenue–raising devise and secondly by increasing the productivity of capital when aggregate pollution declines.[7] This paper does not focus on an environmental tax reform, but shows the growth rate of physical capital is declining, while the growth rate of human capital is increasing during the transition towards the balanced growth path provided that the benevolent central planner imposes an increasingly strict emission standard, when pollution becomes too hampering. Along the balanced growth path, human capital grows at a faster rate than physical capital.

The major conclusions to be drawn from the analyses in this paper are firstly that it *is* possible to allow for environmental concerns and at the same time have sustained growth provided that there are constant returns to educational effort; that human capital is a clean input factor in production; and that physical and human capital are substitutable in final goods production. Secondly, the specification of pollution as generated by the use of physical capital in final goods production agrees with the empirical evidence of an inverted U–shape relationship between income and pollution

[6] See [2] Barro (1990).

[7] If substitution between pollution and other inputs is difficult, then the pollution tax is less powerful in cutting pollution, but more effective in generating revenues to reduce distortionary taxes on output. This tends to increase growth. Moreover, if the positive externality of a better environmental quality on productivity is relatively large, then an increase in the pollution tax also tends to increase growth.

found by [6] Grossman & Krueger. Thirdly, the outcome of an unregulated market economy is inefficient, since pollution is a public "bad" that households and firms take as given in their maximization problems. Final goods production is consequently too capital intensive and the emission of pollution too high from a welfare point of view. In a decentralized economy it therefore proves necessary to tax either physical capital or the emission of pollution in order to enforce the optimal path of the technical environmental standard. Fourthly, the optimal pollution tax equals the optimal marginal damage of pollution when the capital income tax is zero and the government does not have a revenue requirement. Fifthly, the optimal pollution tax might turn out to be below its Pigouvian level, when the optimal capital income tax is high, since a tax on capital income works as an indirect tax on pollution. Finally, the absence of a pollution tax combined with a comprehensive income tax results in a too capital intensive final goods production and an inoptimally high emission of pollution in a second–best world, where the collected tax revenue finances a productive public input in production.

The model is presented in Section 2. The balanced growth equilibrium in a centrally planned economy is derived in Section 3, which also describes the transitional dynamics of the model analytically. Section 4 focuses on the market economy and determines the optimal tax rates. Section 5 extends the model to include productive public spending in order to take into account that the government may have a revenue requirement. Section 6 concludes the paper.

2. Model

The model presented in this section is an extended Uzawa–Lucas model that allows for disutility of pollution caused by the use of physical capital in production. On the production side of the economy, a final goods sector produces consumption goods and physical capital by use of both human and physical capital. Firms rent these factors of production from households and maximize the market value of the firm. In addition, an education sector ensures the upgrading of the skills of the labor force through the accumulation of human capital. Education time and human capital are both input factors in this education process. The use of physical capital in final goods production causes a negative environmental externality in the form of pollution, which harms utility. In a market economy, the representative firm therefore chooses to set a technical environmental standard on its use of physical capital, since a pollution

tax is levied on its emission of pollution. The government is assumed to have several instruments at its disposal, namely a labor income tax, a capital income tax, and a pollution tax. Collected tax revenues are assumed to be redistributed lump sum to consumers. In a centrally planned economy, the government imposes a technical environmental standard on the use of physical capital in production in order to limit the emission of pollution, when pollution becomes too severe from a welfare point of view. On the consumption side of the economy, a large number of identical and infinitely lived households maximize their discounted life time utility by choosing consumption and the allocation of time between the two sectors.

2.1. Technology

In the final goods sector, a large number of perfectly competitive firms produce according to the following Cobb–Douglas production function:

$$Y_t = A\left(K_t z_t\right)^{\alpha}\left(u_t H_t\right)^{1-\alpha} \tag{2.1}$$

where $z_t \in [0;1]$ is a technical environmental standard on the use of physical capital in production. Both physical capital K_t and human capital H_t are used as input factors in production, A is the exogenous level of the technology, α is the exogenous share of physical capital in final goods production, and u_t is the fraction of time spent at work. Human capital is assumed to be embodied in people and is consequently a private good. In the following, $u_t H_t$ is therefore referred to as the effective labor force.

An example of a technical environmental standard z_t could be a certain filter that limits the emission of polluted smoke from production enterprises. In practice it often turns out that such a technical environmental standard is easier to implement for the government than a conventional emission standard, where actual emissions have to be measured on a current basis in order to ensure enforcement. Thus, z_t is not an emission standard in the conventional sense of a maximum limit on the actual emission of pollution. Nevertheless, z_t is referred to as an emission standard in the following, because it indirectly limits the emission of pollution via a technical standard on the use of physical capital in production.

When $z_t = 1$, then the dirtiest production method is used. Section 3 on the centrally planned economy shows that the government begins to introduce a technical environmental standard on the use of physical capital $z_t < 1$ in order to limit the emission of pollution, when income reaches a certain level. This standard applies to the use of physical capital, because pollution is assumed to be generated by the use

of physical capital in production. As argued earlier, the major contribution to air and water pollution is probably the use of physical capital in production and not total output as assumed in [16] Stokey (1998). As a consequence, labor is assumed to be a clean input factor in the following.

The flow resource constraint of the economy is:

$$\dot{K}_t = Y_t - C_t \tag{2.2}$$

which states that physical capital is accumulated as long as income Y_t is greater than consumption C_t.[8]

In the education sector, human capital is produced according to the following Uzawa–Lucas function:[9]

$$\dot{H}_t = B\left(1 - u_t\right) H_t \tag{2.4}$$

where B is the exogenous level of the technology and $(1 - u_t)$ is the fraction of time devoted to education. Human capital is assumed to move freely and costlessly between final goods production and education. Note that constant returns to human capital ensure endogenous growth, when the fraction of time spent in education is constant. Human capital accumulation is therefore the engine of growth in this model..

The use of physical capital in final goods production is assumed to cause a negative environmental externality, which harms utility. This implies that the marginal cost of pollution begins to outweigh the marginal benefit of a higher output, when production reaches a certain level. As a result, it becomes optimal for the government in a centrally planned economy to set standards of the use of physical capital in production in order to limit the emission of pollution. In addition, it proves necessary in a market economy to tax either capital income or the emission of pollution in order to induce firms to choose the optimal path of the emission standard, see Section 4. Thus, aggregate pollution is a public "bad", which can be reduced either by a technical environmental standard or through taxation.

Pollution is assumed to be increasing in the use of physical capital in final goods production, and declining as emission standards becomes stricter:

$$P_t = K_t z_t^{\delta}, \qquad \delta > 1 \tag{2.5}$$

[8] In Section 5, the model is extended to include productive public spending G_t. This changes the flow resource constraint to:

$$\dot{K}_t = Y_t - C_t - G_t \tag{2.3}$$

[9] See [10] Lucas (1988).

where $z_t \in [0;1]$ is taken as an index of the rate of emission of pollution, and δ is the elasticity of pollution with respect to the emission standard. As emission standards become stricter, $\delta > 1$ ensures that pollution is decreasing for a given capital stock. As a consequence, pollution is an increasing and convex function of the actual use of physical capital given a fixed potential use of physical capital K_t. According to (2.5), the abatement technology exhibits increasing returns to scale, since $2K$ and $\frac{1}{2}z$ creates less pollution than K and z.[10]

As mentioned earlier, the pollution function in [16] Stokey (1998) depends on total output and is consistent with the empirical evidence of an inverted U–shape relationship between per capita income and certain types of pollution. It turns out that the pollution function in (2.5) also is consistent with this evidence, see Appendix A.1.

2.2. Firms

The production side of the economy consists of a large number of identical and perfectly competitive firms. Firms are assumed to maximize profits by choosing the input of physical and human capital, and its technical environmental standard given the pollution tax τ_{pt} levied on its emission of pollution.[11] Thus, in a decentralized economy the government can regulate the emission of pollution by setting the pollution tax τ_{pt}. Capital is rented from households at the interest rate r_t, while "raw" labor is hired at the unskilled wage rate w_t. The representative firm's maximization problem is:[12]

$$\begin{aligned} \max_{K_t,\, H_t,\, z_t} \Pi_t &= A\left(K_t z_t\right)^{\alpha}\left(u_t H_t\right)^{1-\alpha} - r_t K_t - w_t u_t H_t - \tau_{pt} K_t z_t^{\delta} \\ \text{s.t.} \quad K_t &> 0,\ H_t > 0, \text{ and } z_t \leq 1 \end{aligned} \tag{2.6}$$

[10]Note that the relationship between pollution and income is inverse U–shaped if and only if the abatement technology exhibits increasing returns to scale, see [1] Andreoni & Levinson (1998). An example of such a technology is the technolgy of sweeping a floor. The inputs to abatement are a floor with a layer of dust one centimeter thick and a person providing one hour of sweeping. Now consider a doubling of the two inputs to abatement, namely a dust layer of two centimeters and a person providing two hours of sweeping. Provided that the person can sweep just as fast in both cases, four times the dust is cleaned up in the latter case, which implies increasing returns to scale.

[11]Like [11] Nielsen et al. (1995) and [4] Bovenberg & de Mooij (1997) pollution is modelled as an 'input' factor in this paper. Alternatively, pollution could be modelled as an 'output' factor, see [13] van der Ploeg & Withagen (1991), [5] Gradus & Smulders (1993), [9] Ligthart & Ploeg (1994), and [8] Hettich & Svane (1998). Both modelling approaches are equivalent, see [15] Siebert et al. (1980).

[12]This is a constrained maximization problem with an inequality constraint. See Appendix A.2 for the derivation of the first order conditions.

The first order conditions for profit maximization are:

$$r_t = \alpha \frac{Y}{K} - \tau_{pt} \frac{P}{K} \tag{2.7}$$

$$w_t = (1-\alpha)\frac{Y}{uH} \tag{2.8}$$

$$\tau_{pt} \leq \frac{1}{\delta}\alpha \frac{Y}{P} \tag{2.9}$$

where equation (2.9) holds with inequality, if the technical environmental standard is non–binding $z_t = 1$. If the technical environmental standard turns out to be binding $z_t < 1$, then the marginal cost of pollution τ_{pt} should equal its marginal return. Note that an emission standard of $z_t = 0$ implies that the marginal returns to capital, labor, and pollution are zero.

2.3. Households

A large number of identical, atomistic households with perfect foresight over an infinite horizon choose consumption C_t and the allocation of time between work u_t and education $(1-u_t)$ in order to maximize their discounted life time utility:

$$U_0 = \int_{t=0}^{\infty} \left(\frac{C_t^{1-\theta}}{1-\theta} - \frac{\eta}{\gamma} P_t^{\gamma} \right) e^{-\rho t} dt, \qquad \gamma > 1 \tag{2.10}$$

where θ is the inverse of the intertemporal elasticity of substitution, ρ is the rate of time preference, $\eta P_t^{\gamma-1}$ is the marginal disutility of pollution, and $\gamma > 1$ ensures that the marginal disutility of pollution is increasing in pollution, which is realistic to assume. Note that the disutility of pollution is related to the flow of new pollutants and not to the stock.[13] As a consequence, the following analyses are concentrated on forms of pollution that can be reduced quickly in the absence of new inflows. The instantaneous budget constraint of the representative household is:

$$\dot{K}_t = (1-\tau_k)\, r_t K_t + (1-\tau_h)\, w_t u_t H_t + T_t - C_t \tag{2.11}$$

where τ_k is the capital income tax, τ_h is the labor income tax, and the revenue from taxation is redistributed to consumers in a lump sum manner T_t. Section 5 alternatively assumes that the collected tax revenue is spent on a congested public input in production.

[13]In order to capture types of pollution such as deforestation and depletion of the ozone layer [16] Stokey analyzes a model with exogenous technological change, where pollution accumulates as a stock that affects utility.

Thus, the representative household chooses C_t and u_t in order to maximize life time utility (2.10) subject to its budget constraint (2.11) and the human capital accumulation function (2.4) taking the time paths of taxes τ_h, τ_k, τ_{pt} and the technical environmental standard z_t as given. Since pollution is a public "bad", households take it as given in their maximization problem.[14] The first order conditions with respect to C_t, u_t, H_t, and K_t are:

$$C_t^{-\theta} e^{-\rho t} = \lambda_{kt} \tag{2.12}$$

$$-\frac{\dot{\lambda}_{kt}}{\lambda_{kt}} = (1-\tau_k)\, r_t \tag{2.13}$$

$$-\frac{\dot{\lambda}_{ht}}{\lambda_{ht}} = B \tag{2.14}$$

$$\lambda_{ht} B = \lambda_{kt} (1-\tau_h)\, w_t \tag{2.15}$$

where λ_{kt} and λ_{ht} are the shadow prices of physical and human capital in the market economy, respectively. Equation (2.12) implies that the marginal utility of consumption in every period equals the shadow price of physical capital. The first Euler equation (2.13) implies that the rate of change in the shadow price of physical capital should equal the after–tax marginal product of capital in the final goods sector. The second Euler equation (2.14) says that the rate of change in the shadow price of human capital should equal the constant marginal product of human capital in education. Finally, equation (2.15) determines the optimal allocation of human capital between the two sectors. Note that the labor income tax has lump sum characteristics if it is constant over time, since it affects the opportunity cost of education in the same way as it affects the net of tax wage rate.[15] This means that taxation of labor income has no effect on the sectorial allocation of human capital and thereby no influence on the dynamic evolution of the economy.

2.4. Government

The government levies taxes on both the emission of pollution and factor income accruing from capital and labor, and the collected tax revenue is redistributed lump sum to consumers. The government is assumed to be restricted in its ability to borrow and lend, which means that it runs a balanced budget every period:[16]

$$\tau_k r_t K_t + \tau_h w_t u_t H_t + \tau_{pt} P_t = T_t \tag{2.17}$$

In the following, time subscripts are left out where unnecessary.

This section has briefly described the model. The following two sections describe the outcome of a centrally planned economy and a market economy, respectively.

3. The Planned Economy

This section derives the outcome of a centrally planned economy. The following section, then investigates whether it is possible to implement this first best outcome in a decentralized economy. In the following, the first order conditions to the central planner problem and the reduced forms of the balanced growth rates of all endogenous variables are derived. In addition, the transitional dynamics of the centrally planned economy are investigated.

The central planner chooses consumption C, the allocation of human capital between the two sectors u, and the emission standard z in order to maximize the life time utility of the representative consumer subject to the resource constraint of the entire economy (2.2), the human capital accumulation function (2.4), and the constraint on the emission standard $z \leq 1$. Hence, the central planner faces a constrained maximization problem with an inequality constraint. The first order conditions with respect to C, z, K, H, and u are:

$$C^{-\theta} e^{-\rho t} = \mu_k \tag{3.1}$$

$$\eta P^{\gamma-1} e^{-\rho t} \leq \mu_k \frac{1}{\delta} \alpha \frac{Y}{P} \tag{3.2}$$

$$-\frac{\dot{\mu}_k}{\mu_k} = \left(1-\frac{1}{\delta}\right) \alpha \frac{Y}{K} \tag{3.3}$$

$$-\frac{\dot{\mu}_h}{\mu_h} = B \tag{3.4}$$

$$\mu_h B = \mu_k (1-\alpha) \frac{Y}{uH} \tag{3.5}$$

[14] As a result, the first order conditions are independent of the level of pollution, since pollution is assumed to enter the utility function additively.

[15] If the labor income tax e.g. increases over time, then it works as an indirect tax on education and therefore no longer has lump sum characteristics.

[16] The government budget constraint changes to:

$$\tau_k r_t K_t + \tau_h w_t u_t H_t + \tau_{pt} P_t = G_t + T_t \tag{2.16}$$

when the model is extended to include productive public spending G_t in Section 5.

where μ_k and μ_h are the shadow prices of physical and human capital in the central planner solution, respectively, and equation (3.2) holds with inequality (equality) for $z = 1$ ($z < 1$). Equation (3.1) is equivalent to (2.12). Equation (3.2) determines the optimal value of the emission standard:

$$z = 1, \qquad \text{if } \mu_k > \frac{\eta K^{\gamma-1} e^{-\rho t}}{\frac{1}{\delta}\alpha A \left(\frac{uH}{K}\right)^{1-\alpha}} \tag{3.6}$$

$$z = \left(\frac{\mu_k \frac{1}{\delta} \alpha A \left(\frac{uH}{K}\right)^{1-\alpha}}{\eta K^{\gamma-1} e^{-\rho t}} \right)^{\frac{1}{\delta\gamma-\alpha}}, \qquad \text{if } \mu_k \leq \frac{\eta K^{\gamma-1} e^{-\rho t}}{\frac{1}{\delta}\alpha A \left(\frac{uH}{K}\right)^{1-\alpha}} \tag{3.7}$$

According to (3.6) and (3.7) there are two regimes. Before the critical date t^*, the marginal utility of consumption μ_k still outweighs the disutility of pollution caused by an extra unit of production $(\eta K^{\gamma-1} e^{-\rho t})/(\frac{1}{\delta}\alpha A \left(\frac{uH}{K}\right)^{1-\alpha})$, which means that the emission standard equals one (regime I) and is non–binding. After the critical date t^*, it becomes necessary to impose an increasingly strict emission standard (regime II) in order to equalize the marginal disutility of pollution and the marginal return to pollution, see equation (3.2). The first Euler equation (3.3) implies that the rate of change in the shadow price of physical capital should equal the marginal product of capital in the final goods sector. Using (3.6) and (3.7) this implies that the shadow price of physical capital evolves over time according to:

$$-\frac{\dot{\mu}_k}{\mu_k} = \left(1 - \frac{1}{\delta}\right) \alpha A \left(\frac{uH}{K}\right)^{1-\alpha}, \qquad \text{if } z = 1 \tag{3.8}$$

$$-\frac{\dot{\mu}_k}{\mu_k} = \left(1 - \frac{1}{\delta}\right) \left(\frac{\mu_k \frac{1}{\delta} \left(\alpha A \left(\frac{uH}{K}\right)^{1-\alpha}\right)^{\frac{\delta\gamma}{\alpha}}}{\eta K^{\gamma-1} e^{-\rho t}} \right)^{\frac{\alpha}{\delta\gamma-\alpha}}, \qquad \text{if } z < 1 \tag{3.9}$$

which means that the social return to capital investment equals the private marginal product of physical capital corrected by the optimal marginal damage of pollution. The second Euler equation (3.4) is equivalent to (2.14) and equation (3.5) describes the optimal allocation of human capital between the two sectors.

The remainder of this section firstly derives the long run growth rates of consumption, the two capital stocks, the emission standard, and the level of pollution in regime II. Then, the balanced growth rate effects of changes in the parameters of the model are determined. And finally, the transitional dynamics of the model from the first to the second regime are described.

Several conditions must hold along a balanced growth path, namely the first order conditions of the planned economy (3.1)–(3.5), the human capital accumulation constraint (2.4), and the resource constraint of the economy (2.2):[17]

$$g_c \equiv \frac{\dot{C}}{C} = \frac{1}{\theta}\left[\left(1 - \frac{1}{\delta}\right)\alpha \frac{Y}{K} - \rho\right] \tag{3.10}$$

$$g_c = \frac{1}{\theta}\left[B + \alpha\left(g_k + g_z - g_h\right) - \rho\right] \tag{3.11}$$

$$g_h \equiv \frac{\dot{H}}{H} = B\left(1 - u\right) \tag{3.12}$$

$$g_k \equiv \frac{\dot{K}}{K} = \frac{Y}{K} - \frac{C}{K} \tag{3.13}$$

Along the balanced growth path all variables should grow at constant possibly zero rates. In order to obtain a constant growth rate of physical capital both the output to capital ratio and the consumption to capital ratio must be constant, see equation (3.13). In addition, a constant growth rate of human capital requires that the fraction of time allocated to production u grows at a zero rate, see equation (3.12). In the second regime, the first condition for balanced growth is obtained by use of equation (2.1) and (3.7):

$$-\frac{\dot{\mu}_k}{\mu_k} - \rho = \left(1 - \gamma - \frac{1-\alpha}{\alpha}\delta\gamma\right) g_k + \frac{\delta\gamma}{\alpha}\left(1 - \alpha\right) g_h \tag{3.14}$$

which ensures that physical capital and output grow at the same rate. The second condition for balanced growth is obtained by use of equation (3.1):

$$-\frac{\dot{\mu}_k}{\mu_k} - \rho = \theta g_k \tag{3.15}$$

which ensures that consumption and physical capital grow at the same rate. The relationship between the balanced growth rates of physical and human capital can now be derived by use of these two conditions:

$$g_h = \left(1 + \frac{\alpha}{1-\alpha}\frac{\theta + \gamma - 1}{\delta\gamma}\right) g_k \tag{3.16}$$

According to (3.16) human capital grows at a faster pace than physical capital in the second regime ($z < 1$).[18]

[17] Equation (3.11) is obtained by logarithmic differentiation of (3.5) and use of (3.1).

[18] Obviously, the emission standard grows at a zero rate in regime 1. Thus, a constant output to capital ratio is obtained, when human and physical capital grow at the same rate.

Now, the production function (2.1) is rewritten in order to obtain the emission standard:

$$z = \left(\frac{Y}{AK^{\alpha}(uH)^{1-\alpha}} \right)^{\frac{1}{\alpha}} \tag{3.17}$$

The rate at which the emission standard improves along the balanced growth path can now be obtained by logarithmic differentiation of (3.17) and use of the fact that output and physical capital grow at the same steady state rate:[19]

$$g_z \equiv \frac{\dot{z}}{z} = -\frac{1-\alpha}{\alpha}(g_h - g_k) \tag{3.18}$$

According to (3.18), the emission standard declines along the balanced growth path in regime II, since human capital grows at a faster rate than physical capital.

Finally, the level of total pollution is derived from (2.5) and use of (3.17):

$$P = K, \qquad \text{if } z = 1 \tag{3.19}$$

$$P = K\left(\frac{Y}{AK^{\alpha}(uH)^{1-\alpha}} \right)^{\frac{\xi}{\alpha}}, \qquad \text{if } z < 1 \tag{3.20}$$

Logarithmic differentiation of (3.20) and use of (3.16) implies that the growth rate of total pollution along the balanced growth path is given by:

$$g_p \equiv \frac{\dot{P}}{P} = -\frac{\theta - 1}{\gamma} g_k \tag{3.21}$$

According to (3.19)–(3.21), total pollution increases at the same rate as the physical capital stock before the critical date t^*, and declines afterwards, if the intertemporal elasticity of substitution $1/\theta$ is below one ($\theta > 1$), which is normally assumed.[20] Another realistic situation arises, if $\theta = 1$, then total pollution is constant along the balanced growth path, while the emission standard is still declining. The final possibility $\theta < 1$ is ruled out in the following analyses, since an ever increasing level of total pollution would be problematic in practice.[21]

In the second regime, the balanced growth rate of consumption, output and physical capital in the centrally planned economy is determined by use of (3.11), (3.16) and

[19] The emission standard improves at a rate that offsets the difference between the rate of human capital accumulation and the rate of physical capital accumulation such that output and physical capital grow at the same rate in the steady state.

[20] According to [3] Blanchard & Fischer (1989, p. 44) the empirical estimations of the intertemporal elasticity of substitution vary substantially, but usually lie around or below unity.

[21] An intertemporal substitution elasticity $1/\theta$ below one ensures that the marginal return to pollution is decreasing in Y for incomes above the critical level Y_{t^*}, see also Appendix A.1.

(3.18):

$$g_c = g_y = g_k = \left(\frac{1}{1 + \frac{1}{\theta}\frac{\alpha}{1-\alpha}\frac{\theta+\gamma-1}{\delta\gamma}} \right) \frac{1}{\theta}[B - \rho] \tag{3.22}$$

and the balanced growth rate of human capital is derived by introduction of (3.22) in (3.16):

$$g_h = \left(\frac{1 + \frac{\alpha}{1-\alpha}\frac{\theta+\gamma-1}{\delta\gamma}}{1 + \frac{1}{\theta}\frac{\alpha}{1-\alpha}\frac{\theta+\gamma-1}{\delta\gamma}} \right) \frac{1}{\theta}[B - \rho] \tag{3.23}$$

Furthermore, the balanced growth rate of the emission standard is derived by use of (3.22), (3.16) and (3.18):

$$g_z = -\left(\frac{\frac{\theta+\gamma-1}{\delta\gamma}}{1 + \frac{1}{\theta}\frac{\alpha}{1-\alpha}\frac{\theta+\gamma-1}{\delta\gamma}} \right) \frac{1}{\theta}[B - \rho], \tag{3.24}$$

and the balanced growth rate of pollution is derived from (3.21):

$$g_p = -\left(\frac{\frac{\theta-1}{\gamma}}{1 + \frac{1}{\theta}\frac{\alpha}{1-\alpha}\frac{\theta+\gamma-1}{\delta\gamma}} \right) \frac{1}{\theta}[B - \rho] < 0, \qquad \text{if } \theta > 1 \tag{3.25}$$

There are four main conclusions to be drawn at this point. Firstly, there is a critical date t^*, before which the emission standard is non–binding $z = 1$ and after which the emission standard becomes increasingly strict $z < 1$. Secondly, total pollution increases before the critical date t^* and declines thereafter, if the intertemporal elasticity of substitution $1/\theta$ is below one. Thirdly, human capital grows at a faster rate than physical capital along the balanced growth path in regime II. The intuition behind this is that the level of total pollution becomes so hampering at time t^* that it is optimal to intensify environmental regulations and thereby substitute human capital for physical capital in production. Thus, it *is* possible to find ways to take environmental concerns into account in an endogenous growth framework and at the same time have sustainable growth provided that there are constant returns to human capital accumulation; that pollution is caused by the use of physical capital in final goods production instead of total output; and that physical and human capital are sufficiently substitutable in production. Fourthly, the rate of return to capital is constant in regime II, while the rate of return to human capital is declining. The reason behind this result is that even though skill levels increase over time, the stock of human capital becomes less and less productive, because ever stricter restrictions are imposed on the use of physical capital.[22] However, real wages are still increasing over time, because the rate of increase

[22] Recall that human and physical capital are complementary as input factors in final goods production.

regime II	B	α	γ	δ	θ	ρ
$g_c = g_y = g_k$	+	−	+	+	−	−
g_h	+	+	−	−	−	−
g_z	−	+	+	+	+	+
g_p	−	+	+	−	−	+

Table 3.1: Balanced growth rate effects

in the skill level of households outweighs the rate of decline in the rate of return to human capital.

The growth rate effects of changes in the parameters of the model are summarized in Table 3.1 for $\theta > 1$. Note that the parameters α, γ, and δ have effects on the balanced growth rates in regime II. Thus, an increase in the share of physical capital α in production leads to a reallocation of resources from production towards education and thereby to a higher growth rate of human capital and a lower growth rate of physical capital. This results in a decline in the marginal disutility of pollution caused by an extra unit of production, which tends to reduce the rate at which pollution declines. At the same time, the rate of decline in the emission of pollution is also reduced, because the emission standard becomes stricter at a slower pace, see equation (3.18).

Table 3.1 also reveals that the growth rate effects of δ are similar to γ except for the effect on the rate of decline in pollution. On the one hand, an increase in the elasticity of pollution with respect to the emission standard δ leads to a *direct* increase in the rate of decline in pollution $g_p = g_k + \delta g_z$, see equation (2.5). On the other hand, an increase in δ results both in an decline in the marginal return to pollution and in an increase in the marginal return to capital, see equation (3.2) and (3.3). The consequence is a higher growth rate of physical capital g_k, and thereby a smaller rate of decline in the optimal emission standard g_z, because the growth rates of the two capital stocks become closer to each other, see equation (3.18). This leads to an *indirect* reduction in the rate of decline in pollution. However, the *direct* effect outweighs the *indirect* effect implying that an increase in the elasticity of pollution with respect to the emission standard δ leads to a faster rate of decline in pollution.

In contrast, inspection of Table 3.1 reveals that an increase in the marginal disutility of pollution as reflected in γ leads to a slower rate of reduction in pollution solely through the *indirect* effect. Hence, an increase in the marginal disutility of pollution γ results in an equivalent increase in the marginal return to pollution through an increase

in the accumulation of physical capital. This reduces the rate of decline in the optimal emission standard g_z and thereby the rate of decline in pollution.

3.1. Transitional Dynamics

In the following, the transitional dynamics of the centrally planned economy from regime I to the balanced growth path in regime II are investigated.[23] In order to do that the dynamic systems in regime I and II are rewritten in terms of the consumption–capital ratio C/K, the average product of physical capital Y/K and the time fraction spent at work u, which is convenient since all these variables have to be constant along a balanced growth path, see Appendix A.3.

The two dynamic systems are illustrated in Figure 3.1,[24] where point C is the long term equilibrium in regime II.[25] Firstly, the $g_{C/K} = 0$–schedule was derived by use of the Keynes–Ramsey rule and the resource constraint of the economy. Note that the $g_{C/K} = 0$–schedule, which describes combinations of C/K and Y/K that yield a constant consumption–capital ratio, is unaltered by a shift from the first to the second regime, see equation (A.3). Secondly, the $g_{Y/K} = 0$–schedules were derived by use of the resource constraint of the economy and the human capital accumulation function. The $g_{Y/K} = 0$–schedule, which describes combinations of C/K and Y/K that yield a constant average product of capital, is vertical in regime I, but becomes positively sloped in regime II, see equation (A.6) and (A.12) respectively. Finally, the $g_u = 0$–schedule was obtained by use of the condition for an optimal sectorial allocation of human capital. The $g_u = 0$–schedule, which describes combinations of C/K and u that yield a constant time fraction spent at work, is less steep and has a higher intersection point in regime I than in regime II, see equation (3.26) and (A.10).[26]

Figure 3.1 illustrates the transitional dynamics of an economy with initial endow-

[23] [16] Stokey (1998) simulates the transitional dynamics of both the endogenous and the exogenous model.

[24] Both systems are shown in Appendix A.3 to be saddle path stable. Superscripts I and II on the schedules in Figure 3.1 refer to regime I and II, respectively.

[25] The long term equilibrium in regime I is given by the intersection of the $g_{C/K} = 0$–schedule and the $g^I_{Y/K} = 0$–schedule, but is never reached by the economy.

[26] The $g^{I*}_u = 0$–schedule holds in regime I, when the average rate of return to capital investment is at its long term regime I value (A.9):

$$g_u^{1*} = -\chi + Bu + \left[\frac{1-\alpha}{\alpha} + \frac{1}{\alpha(\delta-1)}\right] B \tag{3.26}$$

see equation (A.4).

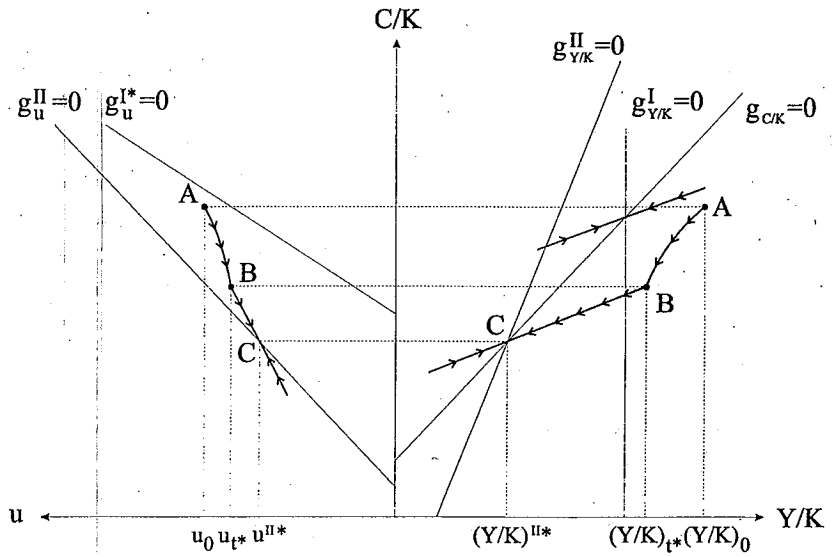


Figure 3.1: Transitional dynamics from regime I to regime II.

ments of physical and human capital of K_0 and H_0, respectively. The dynamic system in regime I (0, t^*) describes the time paths of the consumption–capital ratio C/K, the average product of capital Y/K and the time fraction spent at work u until the critical point in time t^* is reached, where the emission standard imposed by the government becomes binding. The dynamic system in regime II (t^*, ∞) then describes the evolution of the economy when the emission standard becomes increasingly strict over time. At time 0 it is already known that an increasingly strict emission standard will be imposed – in order to equalize the marginal disutility of pollution and the marginal return to pollution – once pollution becomes too hampering. This means that the endogenous variables must change continuously at time t^*, since no new information is revealed at this time. As a consequence, the economy follows an unstable path in regime I from point A to B, which exactly reaches the stable saddle path in regime II at time t^*. From point B the economy then follows a stable saddle path in regime II towards its long term equilibrium at point C. The expressions for the endogenous variables at each point in time along the transition path are derived in three steps, see Appendix A.4. Firstly, the stable saddle path in regime II is derived. Secondly, the

values of the endogenous variables at time t^* are determined. And finally, the unstable path in regime I is derived by determining the initial value of the consumption–capital ratio, which will ensure that the economy arrives at the stable saddle path for regime II exactly at time t^*.[27]

At time 0, the economy is at point A on the unstable path in regime I, where the initial average product of physical capital is assumed to be higher than its long run regime I value indicated by the $g_{Y/K}$–schedule. This could for instance be the case in a pre–industrialized economy, where the effective labor force probably is abundant relative to the physical capital stock.[28] At time 0, it is already known that it becomes optimal to intensify environmental regulations at time t^*, and thereby moderate economic growth and consumption. In order to take this information into account, the time spent at work u in regime I begins to decline from point A to B in order to reallocate resources towards education.[29] This speeds up human capital accumulation. Moreover, the consumption–capital ratio and the average product of physical capital are continuously declining in regime I between time 0 and time t^* towards point B due to the accumulation of physical capital, but also in order to avoid a sudden decline in consumption at time t^*. As a consequence, a part of the substitution of human capital for physical capital in final goods production has already taken place when the emission standard becomes binding. Environmental concerns are taken into account through a stricter emission standard when the marginal disutility of pollution has become so high that it is necessary to slow down physical capital accumulation, because physical capital is a dirty input factor in production. In the second regime (t^*, ∞), the adjustment which was already begun in regime I is continued and C/K, Y/K, and u are declining further from point B towards their new and lower balanced growth path levels at point C. Along the transition path, the growth rate of human capital is increasing while the growth rate of physical capital is declining, since Y/K is declining at a faster rate than C/K, see equation (3.13). Recall that physical capital accumulation is high initially, because the initial marginal product of physical capital is high. Along the balanced

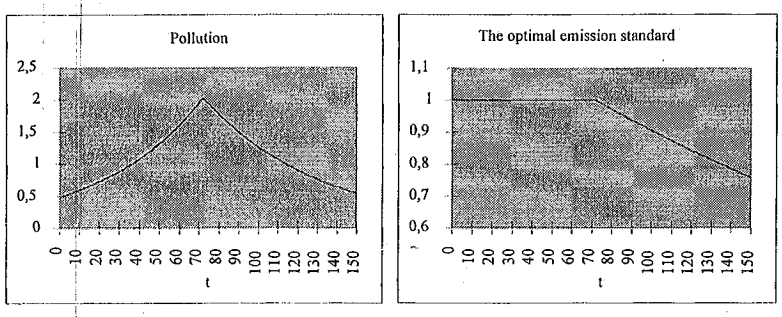
[27] The consumption–capital ratio is a control–like variable, while the average product of capital is a state–like variable, which is predetermined in the short run.

[28] Another situation arises, if the effective labor force is low initially relative to the physical capital stock. In this case, the initial average product of capital might be lower than its long run regime I value, which means that the average product of physical capital would be increasing until the regime shift takes place at time t^*, where the emission standard becomes binding.

[29] The unstable path from A to B in the (u, C/K)–diagram is drawn given the time path of Y/K in regime I (A.17).

growth path in regime II, the growth rate of human capital is higher and the growth rate of physical capital lower than their initial growth rates, and human capital grows at a faster rate than physical capital.

The time paths of pollution P_t and the emission standard z_t can also be illustrated graphically using a calibrated discrete time version of the model.[30]



It can be seen that total pollution increases until the critical time t^* is reached. After time t^*, total pollution begins to decrease as the emission standard becomes stricter. Thus, the evolution of total pollution in the centrally planned economy is in line with the empirical evidence of an inverted U–shape relationship between income and pollution. This indicates that it might be possible to implement the first best outcome in a decentralized economy.

This section derived the long run growth rates of all endogenous variables and analyzed the transitional dynamics of the centrally planned economy. The following section concentrates on the market economy and investigates whether it is possible to implement the first best outcome through taxation.

4. The Market Economy

This section focuses on the market economy and determines the optimal tax rates.

Recall at this point that the representative household chooses consumption and the fraction of time spent at work in order to maximize its life time utility subject to its budget constraint and the human capital accumulation function taking the time paths of the tax rates and the technical environmental standard as given. The resulting first order conditions (2.12) and (2.13), where r is replaced by (2.7), can then be used to derive the Keynes–Ramsey rule that describes the optimal consumption path in the decentralized economy:

$$g_c = \frac{1}{\theta}\left[(1-\tau_k)\left\{\alpha\frac{Y}{K} - \tau_{pt}\frac{P}{K}\right\} - \rho\right] \tag{4.1}$$

In the decentralized economy, the transitional dynamics towards the steady state may differ from the optimal path, if taxes are not set at their optimal rates, because neither firms nor households take the negative environmental externality into account. Hence, the question addressed in the following is whether the optimal environmental policy, which is determined by the time path of z set by the central planner, can be enforced through taxation in a decentralized economy.

The first best tax rates are derived by comparison of the first order conditions to the central planner problem (3.1)–(3.5) and the representative agent problem (2.12)–(2.15). In order to reach a first best solution, tax rates must be set in such a way that the evolution of the shadow price of physical capital is the same in the market economy and in the centrally planned economy. Thus, comparison of (2.13), (3.8) and (3.9) reveals that the following conditions must be fulfilled:

$$\begin{aligned}(1-\tau_k)\left(\alpha\frac{Y}{K} - \tau_p\right) &= \left(1-\frac{1}{\delta}\right)\alpha\frac{Y}{K}, \qquad \text{if } z = 1\\ (1-\tau_k)\left(\alpha\frac{Y}{K} - \tau_p\frac{P}{K}\right) &= \left(1-\frac{1}{\delta}\right)\alpha\frac{Y}{K}, \qquad \text{if } z < 1\end{aligned}$$

where the representative firm chooses the time path of the technical environmental standard given the tax on its emission of pollution levied by the government. Thus, the tax instruments τ_k, τ_{pt} and the emission standard z are alternative instruments in the present model.[31] Naturally, it is preferable to use an emission standard as long as the government does not have a revenue requirement. However, direct regulation of emissions implies a private marginal return to capital of $\alpha Y/K$, which is higher than the social marginal return to capital, and therefore gives households wrong investment incentives. In order to attain a first best outcome it is therefore necessary to use the available tax instruments instead. According to the above conditions, a first best solution in a decentralized economy can therefore either be reached by setting an optimal

[30] The discrete time version of the model was simulated for $A = B = 0.111208$, $\alpha = 0.4$, $\eta = 0.05$, $\delta = 10$, $\delta_h = \delta_k = 0.05$, $\gamma = 1.1$, $\theta = 2$, and $\rho = 0.02$, where δ_h (δ_k) is the rate of depreciation of human (physical) capital.

[31] Another possibility is to allow for a market in pollution vouchers. [16] Stokey (1998) analyzes such a voucher system and finds that it works essentially like a pollution tax.

pollution tax; or an optimal capital income tax; or a combination of the latter two in order to give firms the right incentives to choose the optimal technical environmental standard, see Table 4.1.[32] The labor income tax has lump sum characteristics and is set equal to zero in the following.

	τ_k	τ_{pt}	T
Case 1			
$z=1$	0	$\frac{1}{\delta}\alpha\left(\frac{Y}{K}\right)^{CPS}$	$\frac{1}{\delta}\alpha Y$
$z<1$	0	$\frac{1}{\delta}\alpha\left(\frac{Y}{Kz^{\delta}}\right)^{CPS}$	$\frac{1}{\delta}\alpha Y$
Case 2			
$z=1$	$\frac{1}{\delta}$	0	$\frac{1}{\delta}\alpha Y$
$z<1$	$\frac{1}{\delta}$	0	$\frac{1}{\delta}\alpha Y$

Table 4.1: Optimal tax rates

Case 1: If on the one hand the capital income tax is unavailable as an instrument, then a first best solution can be reached by setting the pollution tax equal to the optimal marginal damage of pollution. In this case, the pollution tax corresponds to a Pigouvian tax.[33] Note that the optimal pollution tax is increasing over time along the balanced growth path in the second regime ($z < 1$) as the optimal emission standard becomes stricter. Note furthermore that environmental taxation is necessary in the first regime even though the emission standard is not binding. The reason behind this is that irrespective of the regime, the use of physical capital causes a negative environmental externality, which harms utility.

Case 2: If on the other hand the pollution tax is unavailable as an instrument, then a first best solution can be reached by setting the capital income tax equal to the inverse of the elasticity of pollution with respect to the emission standard. Hence, a capital income tax works as an indirect tax on pollution. Both in Case 1 and 2, the collected tax revenue is redistributed lump sum to consumers.

According to the above, the effects of a capital income tax and a pollution tax are similar, which is not surprising, since pollution is assumed to be generated by the use of physical capital in production. However, the tax bases of the two tax instruments are

[32]Note that the government has two effective tax instruments at is disposal and only one externality to correct.

[33]The first best tax on pollution equals the optimal marginal environmental damage, when the government has no revenue requirement or can use lump sum taxes and transfers to meet its fiscal needs, see [12] Pigou (1947).

different, because the tax on capital is levied on capital income, which consists of the marginal product of capital times the use of physical capital in production $(\alpha Y/K)\,K$, while the pollution tax is levied on the emission of pollution Kz^{δ}. As long as capital is homogeneous both tax instruments can be used to obtain the first best solution. However, in a world with heterogeneous capital it is no longer irrelevant whether the capital income tax or the pollution tax is at the government's disposal. Imagine for instance that some types of physical capital pollute more than others and thereby that the marginal damage of pollution is sector specific. In such a case, a differentiated capital income tax would be necessary to internalize the negative externalities from pollution. This contrasts with the conventional arguments in economic theory for a uniform capital income tax in the presence of heterogenous capital, which say that the return to capital investment should be taxed at the same rate in all sectors such that a reallocation of capital from one sector to another does not lead to an increase in output.

This section showed that an optimal environmental policy could be enforced in a market economy through taxation. In the following section, the model is altered to include productive public spending.

5. Productive Public Spending

So far the analyses have not taken into account that the government may have a need for a tax revenue. The model is therefore extended to include productive public spending in order to investigate whether it is still possible to implement a first best allocation of resources between the two sectors.

In this section, the tax revenue obtained by the government is therefore alternatively assumed to finance a congested public input in final goods production. Such a public good could for instance be government services such as highways, water systems, police and fire services, and courts, which are all subject to some degree of congestion. For a given quantity of the public good, the quantity available to each user therefore declines as other users start to congest the facilities. In the following, it is assumed that the quantity of the public good has to rise at the same rate as output in order to keep the quantity available to each user constant.

The alternative production function is given by:

$$Y = A\left(Kz\right)^{\alpha}\left(uH\right)^{1-\alpha}\Gamma\left(\frac{G}{Y}\right) \tag{5.1}$$

where G are expenditures on government services, $\Gamma' > 0$ and $\Gamma'' < 0$. According to (5.1) there is a certain need for infrastructure such as highways in the final goods sector. Once more, all variables should grow at constant possibly zero rates along the balanced growth path. Thus, the consumption–capital ratio, the output–capital ratio, and the fraction of human capital allocated to production u should be constant. In the long run, $\Gamma(G/Y)$ is also constant implying that the optimal growth rates of all variables are the same as without productive use of public spending, see equation (3.22)–(3.25). Furthermore, the first order conditions to the market solution are still given by (2.12)–(2.15) and the marginal products of capital and labor net of tax are given by (2.7)–(2.9), when (5.1) is used.

The optimal tax rates in this alternative setup can now be obtained by comparison of the first order conditions to the central planner problem (A.34)–(A.39) and the representative agent problem (2.12)–(2.15). In the previous section, the labor income tax had lump sum characteristics, which meant that it could take any constant value without altering the outcome. However, comparison of (A.39) and (2.15), and use of (2.8) and (5.1) now suggests that labor income taxation should be used to finance part of the productive public input in production:

$$\tau_h = \left(\frac{G}{Y}\right)^{CPS} \qquad \forall\, z \tag{5.2}$$

The labor income tax instrument comes into action, because the government now has an extra externality to correct due to increased congestion for a given level of expenditures on government services G when human and physical capital accumulates. This means that the labor income tax no longer has lump sum characteristics, since it can change over time outside the steady state. An increase in the labor income tax over time would for instance work like an indirect tax on educational effort, and therefore alter the allocation of resources between sectors. Hence, it is necessary to use the labor income tax instrument to attain the first best outcome, when the tax revenue finances a public input in production. Furthermore, the optimal tax rates must ensure that the evolution of the shadow price of physical capital is the same in the market economy (2.13) as in the centrally planned economy (A.40), which means that:[34]

$$\begin{aligned}
(1-\tau_k)\left(\alpha\frac{Y}{K}-\tau_p\right) &= \left(1-\frac{1}{\delta}\right)\alpha\frac{Y}{K}\left(1-\frac{G}{Y}\right), && \text{if } z=1\\
(1-\tau_k)\left(\alpha\frac{Y}{K}-\tau_p\frac{P}{K}\right) &= \left(1-\frac{1}{\delta}\right)\alpha\frac{Y}{K}\left(1-\frac{G}{Y}\right), && \text{if } z<1
\end{aligned}$$

[34]The first order conditions to the central planner problem, the evolution of the shadow price of physical capital, and the time path of the optimal emission standard are given in Appendix A.5.

Once more there are several ways that the above conditions can be fulfilled depending on which tax instruments are available to the government. Table 5.1 summarizes three different cases, where the optimal labor income tax in all cases is given by (5.2). A lump sum transfer instrument T, which makes it possible to reallocate the tax revenue to consumers in a non–distorting way, is assumed to be available in all three cases.

	τ_k	τ_{pt}	T_t
Case 1			
$z=1$	0	$\frac{1}{\delta}+\left(1-\frac{1}{\delta}\right)\left(\frac{G}{Y}\right)^{CPS}$ $\quad \alpha\left(\frac{Y}{K}\right)^{CPS}$	$1-\left(\frac{G}{Y}\right)^{CPS}$ $\quad \frac{1}{\delta}\alpha Y$
$z<1$	0	$\frac{1}{\delta}+\left(1-\frac{1}{\delta}\right)\left(\frac{G}{Y}\right)^{CPS}$ $\quad \alpha\left(\frac{Y}{Kz^{\delta}}\right)^{CPS}$	$1-\left(\frac{G}{Y}\right)^{CPS}$ $\quad \frac{1}{\delta}\alpha Y$
Case 2			
$z=1$	$\frac{1}{\delta}+\left(1-\frac{1}{\delta}\right)\left(\frac{G}{Y}\right)^{CPS}$	0	$1-\left(\frac{G}{Y}\right)^{CPS}$ $\quad \frac{1}{\delta}\alpha Y$
$z<1$	$\frac{1}{\delta}+\left(1-\frac{1}{\delta}\right)\left(\frac{G}{Y}\right)^{CPS}$	0	$1-\left(\frac{G}{Y}\right)^{CPS}$ $\quad \frac{1}{\delta}\alpha Y$
Case 3			
$z=1$	$\left(\frac{G}{Y}\right)^{CPS}$	$\frac{1}{\delta}\alpha\left(\frac{Y}{K}\right)^{CPS}$	$1-\left(\frac{G}{Y}\right)^{CPS}$ $\quad \frac{1}{\delta}\alpha Y$
$z<1$	$\left(\frac{G}{Y}\right)^{CPS}$	$\frac{1}{\delta}\alpha\left(\frac{Y}{Kz^{\delta}}\right)^{CPS}$	$1-\left(\frac{G}{Y}\right)^{CPS}$ $\quad \frac{1}{\delta}\alpha Y$

Table 5.1: Optimal tax rates

Case 1: Imagine first that the capital income tax is unavailable as an instrument. In this situation, the first best outcome can be reached by setting the pollution tax both to target the pollution objective $\frac{1}{\delta}\alpha\left(Y/P\right)^{CPS}$ and to finance part of the optimal expenditures on the public input $\left(1-1/\delta\right)\left(G/Y\right)^{CPS}\alpha\left(Y/P\right)^{CPS}$.[35] Thus, the optimal pollution tax is above its Pigouvian level, when the government has a revenue requirement and the capital income tax is zero.

Case 2: Imagine secondly that the pollution tax is unavailable as an instrument. In this situation, the first best can be reached by setting the capital income tax such that it both targets the pollution objective $\frac{1}{\delta}$ and finances part of the optimal expenditures on the public input in production $\left(1-1/\delta\right)\left(G/Y\right)^{CPS}$. Once more the effects of the pollution tax and the capital income tax are similar and the only difference between the two are their tax bases.

Case 3: Imagine thirdly that capital and labor income are taxed at the same comprehensive rate. This means that factor income taxation fully finances expenditures

[35]Recall that the remaining part is financed by the optimal labor income tax.

on the productive public input and internalizes the congestion externality. Hence, the pollution tax solely has to target the pollution objective, which means that the optimal pollution tax should be set at its Pigouvian level, see Case 1 in Table 4.1.

In fact a first best solution can be reached in many other ways by setting taxes on capital and labor income and using a pollution tax in addition. The higher the optimal capital income tax is, the more likely it is that the optimal pollution tax should be below its Pigouvian level. Summing up on Case 1–3, it can be concluded that any of the two tax rates can be set equal to zero, but that the practical implementation of one of the taxes might be easier that the other.

Imagine now a second–best world, where the government levies a comprehensive income tax $\tau = \tau_h = \tau_k > 0$ and for some reason is unable to use the pollution tax instrument $\tau_p = 0$ and the lump sum transfer instrument $T = 0$. In practice it might be reasonable to levy a comprehensive income tax, since it eliminates potential tax evasion problems that arise when one type of income can be redefined as another type of income for tax purposes. Hence, in order to balance the government budget (2.16) every period in this second–best situation, capital and labor income must be taxed at the following comprehensive rate:

$$\tau = \frac{G}{Y} \tag{5.3}$$

which exactly covers the expenses on the productive public input in production. Since pollution is caused by the use of physical capital in production and human capital is a clean input factor, it is obvious that capital income is taxed at a too low rate from a welfare point of view, see also Case 2 in Table 5.1. Hence, final goods production becomes too physical capital intensive in this second best situation and results in a too high level of pollution. On the one hand, the absence of a pollution tax combined with a comprehensive income tax leads to a *direct* increase in the after–tax private return to capital investment, because pollution is taxed at a zero rate. This tends to increase growth. On the other hand, a too capital intensive final goods production leads to an *indirect* reduction in the pre–tax return to capital investment which tends to decrease growth. If the *direct* effect outweighs the *indirect* effect, then the growth rate in the market economy is too high from a welfare point of view. Moreover, the adjustment process from the first to the second regime may be affected, since taxes are not set at their optimal rates.

The analyses in this section revealed that it is still possible to implement a first best allocation of resources between the two sectors when the government has a revenue

requirement as long as a lump sum transfer instrument is available.

6. Conclusions

This paper has examined the effects of a technical environmental standard and taxation in a two sector endogenous growth model, where pollution is caused by the use of physical capital in production. The first question was whether sustained economic growth is possible within an endogenous growth framework when environmental concerns are taken into account. The second question was whether enforcement of a first best environmental policy is possible through taxation in a market economy.

The main property of the model, which was used to answer these questions, is that it has two regimes. One which is characterized by a non–binding technical environmental standard and another in which the emission standard becomes increasingly strict. In the first regime, the economy evolves along an unstable path, where total pollution is increasing at the same rate as physical capital, because pollution is assumed to be generated by the use of capital in production. In the second regime, the economy evolves along a stable saddle path towards its balanced growth equilibrium, where total pollution is declining over time. The unstable path in regime I exactly reaches the stable saddle path in regime II when total pollution becomes so hampering that it is optimal to intensify environmental regulations either through an increasingly strict emission standard or an increasing pollution tax. Imagine an economy with a scarce initial physical capital stock relative to its initial effective labor force and thereby a high initial marginal product of physical capital. The growth rate of physical capital in this economy will be high initially and falling along the transition path in regime I as physical capital is accumulated. Already from the beginning it is known that the emission standard will become binding in the future. In order to take this information into account, the time fraction spent at work begins to decline already in regime I in order to reallocate resources towards education and to smooth consumption over time. As a consequence, the growth rate of human capital is increasing along the unstable path in regime I. At the time of the regime shift, a part of the substitution of human capital for physical capital in production has therefore taken place already. In the transition towards the new balanced growth path in regime II, the adjustment which was begun in regime I is continued and the consumption to capital ratio, the average product of capital, and the fraction of time spent at work are continuously declining towards their new and lower steady state levels. Moreover, the emission of pollution

will be declining in the second regime due to the increasingly strict emission standard. The pollution function specified in this paper is therefore in line with the empirical evidence of an inverted U-shape relationship between income and pollution, see [6] Grossman & Krueger.

Hence in contrast to results obtained by [16] Stokey (1998), this paper shows that it is indeed possible to find ways to take environmental concerns into account and still have sustainable growth in an endogenous growth model provided that there are constant returns to educational effort; that human capital is a clean input factor in production; and that physical and human capital are sufficiently substitutable in production.

In the second regime both an increase in the elasticity of pollution with respect to the emission standard δ and an increase in the marginal disutility of pollution as reflected in γ lead to a higher growth rate of physical capital, and a lower growth rate of human capital in the long run, which results in a smaller optimal rate of decline in the emission standard. These *indirect* effects tend to reduce the rate of decline in pollution and is the reason why an increase in γ leads to a slower rate of reduction in pollution. However, there is an additional *direct* effect of an increase in the elasticity pollution with respect to the emission standard δ, which tends to increase the rate of decline in pollution. It turns out that the direct effect outweighs the indirect effect, which means that a higher elasticity of pollution with respect to the emission standard results in a faster rate of reduction in pollution.

In the model, pollution is seen by firms and households as a public "bad", which they take as given in their maximization problems. Hence, the outcome of an unregulated market economy is inefficient, because the private marginal return to capital investment is too high when the pollution tax and the capital income tax are zero. Therefore, final goods production becomes too capital intensive and the resulting emission of pollution is too high from a welfare point of view. The question is therefore whether a first best environmental policy can be implemented through taxation in a market economy.

It turns out that the government can induce firms to choose the optimal path of the technical environmental standard in several ways. If on the one hand, the pollution tax instrument is used, then the pollution tax rate should be set at its Pigouvian level, namely equal to the socially optimal marginal damage of pollution, which is constant in the first regime ($z = 1$) and increasing along the balanced growth path in the second regime ($z < 1$) as the emission standard becomes stricter. Note that environmental taxation is necessary in both regimes due to the presence of the negative environmental

externality even though the emission standard is non-binding in regime I. If on the other hand, the capital income tax instrument is used, then a first best solution can be reached by setting the capital income tax rate equal to the inverse of the elasticity of pollution with respect to the emission standard. The difference between the two taxes is that the capital income tax is levied on the marginal product of capital times the use of physical capital in production, while the pollution tax is levied on the emission of pollution. Furthermore, the implementation of one of the taxes might be easier than the other as a practical matter. However, a first best outcome can be reached in the decentralized economy through taxation.

Finally, the model was extended to include a productive public input. Two conclusions can be drawn from this alternative setup. Firstly, the optimal pollution tax might be below its Pigouvian level, when the government requires a certain revenue to finance a public input in production and the optimal capital income tax is high. Secondly, the absence of a pollution tax combined with a comprehensive income tax results in a too capital intensive final goods production and thereby in a too high level of pollution in a second-best world, where a lump sum transfer instrument is unavailable. Note that the long term growth rate and the adjustment process from the first to the second regime may be affected when taxes are not set at their optimal rates.

A. Appendix

A.1. The Relationship between Y and P

In a static version of the model there is neither accumulation of physical capital nor of human capital. This means that consumption equals production:

$$C = Y = A(Kz)^{\alpha} H^{1-\alpha}$$

The central planner maximizes the representative agent's utility by choosing the emission standard z:

$$\max_{z} U = \frac{C^{1-\theta}}{1-\theta} - \frac{\eta}{\gamma} P^{\gamma} \qquad \text{s.t.} \qquad z \leq 1$$

where P is given by equation (2.5). The first order condition for optimum is:

$$\begin{aligned} \alpha Y^{1-\theta} &> \eta\delta P^{\gamma}, \quad \text{if } P = K \\ \alpha Y^{1-\theta} &= \eta\delta P^{\gamma}, \quad \text{if } P < K \end{aligned}$$

and the resulting relationship between per capita income Y and pollution P is:

$$P = K = \left(\frac{1}{AH^{1-\alpha}}Y\right)^{\frac{1}{\alpha}} \quad \text{for } Y \leq \hat{Y}$$
$$P = \left(\frac{\alpha}{\eta\delta}Y^{1-\theta}\right)^{\frac{1}{\gamma}} \quad \text{for } Y > \hat{Y}$$

where the critical level of income above which emission standards come into play is given by:

$$\hat{Y} = \left[\left(\frac{\alpha}{\eta\delta}\right)^{\frac{1}{\gamma}} A^{\frac{1}{\alpha}} H^{\frac{1-\alpha}{\alpha}}\right]^{\frac{1}{\frac{1}{\alpha}-\frac{1-\theta}{\gamma}}}$$

At income levels below $\hat{Y}$, pollution increases with income and no emission standards are in place. At income levels above $\hat{Y}$, pollution is declining in Y if the intertemporal elasticity of substitution is below one ($\theta > 1$), increasing in Y if $\theta < 1$ and is constant when the utility function is logarithmic $\theta = 1$. Thus, the relationship has an inverted U-shape if $\theta > 1$. The graph in Figure A.1 is drawn for $A = 0.111208$, $\alpha = 0.4$, $\eta = 0.05$, $\delta = 10$, $\gamma = 1.1$, $\theta = 2$, and $H = 1$.

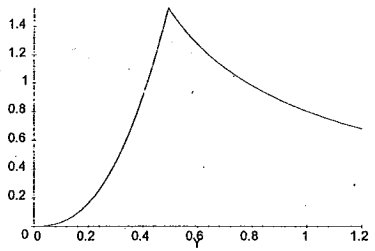


Figure A.1: Relationship between per capita income Y and pollution P

A.2. Firm Maximization

The representative firm is assumed to maximize profits by choosing the input of physical capital K and human capital H, and the technical environmental standard z:

$$\begin{aligned} \max_{K_t,\, H_t,\, z_t} \Pi_t &= A\left(K_t z_t\right)^{\alpha}\left(u_t H_t\right)^{1-\alpha} - r_t K_t - w_t u_t H_t - \tau_{pt} K_t z_t^{\delta} \\ \text{s.t.} \quad z_t &\leq 1 \end{aligned}$$

The Lagrangian for this problem becomes:

$$\mathcal{L} = A\left(K_t z_t\right)^{\alpha}\left(u_t H_t\right)^{1-\alpha} - r_t K_t - w_t u_t H_t - \tau_{pt} K_t z_t^{\delta} - \lambda_z\left(z - 1\right)$$

where $\lambda_z \geq 0$ is the Kuhn–Tucker multiplier. The first order conditions for profit maximization are:

$$r_t = \alpha\frac{Y}{K} - \tau_{pt}\frac{P}{K}$$
$$w_t = (1-\alpha)\frac{Y}{uH}$$
$$\tau_{pt} = \frac{1}{\delta}\alpha\frac{Y}{P} - \lambda_z\frac{z}{\delta P}$$

where the complementary slackness conditions associated with the technical environmental standard are:

$$\lambda_z = 0, \qquad \text{if } z < 1$$
$$\lambda_z > 0, \qquad \text{if } z = 1$$

see [18] Varian, p. 502–505.

A.3. Stability of the Dynamic Systems in Regime I and II

Along the balanced growth path, the average product of physical capital $\omega \equiv Y/K$, the consumption–capital ratio $\chi \equiv C/K$, and the fraction of time spent at work u have to grow at zero rates. The dynamic system in both regimes is therefore rewritten in terms of these three variables and the stability of the two systems is checked.

Firstly, the dynamic equations (2.2) and (3.3) can be rewritten in terms of the average product of physical capital and the consumption–capital ratio:[36]

$$g_k = \omega - \chi \tag{A.1}$$
$$g_{\mu_k} \equiv \frac{\dot{\mu}_k}{\mu_k} = -\left(1 - \frac{1}{\delta}\right)\alpha\omega \; . \tag{A.2}$$

and the growth rate of the consumption–capital ratio in both regimes can be derived by logarithmic differentiation of (3.1) and use of (A.1) and (A.2):

$$g_\chi = -\left[1 - \frac{1}{\theta}\left(1 - \frac{1}{\delta}\right)\alpha\right]\omega + \chi - \frac{\rho}{\theta} \tag{A.3}$$

Regime I $(0, t^*)$ is characterized by a non–binding emission standard $z = 1$. In this regime, the growth rate of the fraction of time spent at work is derived by logarithmic differentiation of (3.5) and use of (3.4), (3.6), (A.2), (A.1) and (2.4):

$$g_u^I = \frac{1}{\delta}\omega^I - \chi^I + Bu^I + \frac{1-\alpha}{\alpha}B \tag{A.4}$$

The average product of physical capital is given by:

$$\omega^I \equiv \frac{Y}{K} = A\left(\frac{uH}{K}\right)^{1-\alpha} \tag{A.5}$$

[36] Note that these relationships hold in both regimes.

Logarithmic differentiation of (A.5) and use of (2.4), (A.1), (A.2) and (A.4) yields the following growth rate of the average product of physical capital:

$$g_\omega^I = -(1-\alpha)\left(1-\frac{1}{\delta}\right)\omega^I + \frac{1-\alpha}{\alpha}B \tag{A.6}$$

The dynamic system in regime I can now be written in matrix form as:

$$\begin{bmatrix} g_\omega^I \\ g_\chi^I \\ g_u^I \end{bmatrix} = \begin{bmatrix} a_{11}^I & 0 & 0 \\ a_{21}^I & a_{22}^I & 0 \\ a_{31}^I & a_{32}^I & a_{33}^I \end{bmatrix} \begin{bmatrix} \omega^I \\ \chi^I \\ u^I \end{bmatrix} + \begin{bmatrix} b_1^I \\ b_2^I \\ b_3^I \end{bmatrix} \tag{A.7}$$

where

$$\begin{array}{lll} a_{11}^I = -(1-\alpha)\left(1-\frac{1}{\delta}\right), & & b_1^I = \frac{1-\alpha}{\alpha}B \\ a_{21}^I = -\left[1-\frac{1}{\theta}\left(1-\frac{1}{\delta}\right)\alpha\right], & a_{22}^I = 1, & b_2^I = -\frac{\rho}{\theta} \\ a_{31}^I = \frac{1}{\delta}, & a_{32}^I = -1, \quad a_{33}^I = B, & b_3^I = \frac{1-\alpha}{\alpha}B \end{array}$$

A first order Taylor expansion linearizes the system around the steady state:

$$\begin{bmatrix} g_\omega^I \\ g_\chi^I \\ g_u^I \end{bmatrix} = \begin{bmatrix} a_{11}^I & 0 & 0 \\ a_{21}^I & a_{22}^I & 0 \\ a_{31}^I & a_{32}^I & a_{33}^I \end{bmatrix} \begin{bmatrix} \omega^I - \omega^{I*} \\ \chi^I - \chi^{I*} \\ u^I - u^{I*} \end{bmatrix} \tag{A.8}$$

where the steady state equilibrium solutions in regime I ω^{I*}, χ^{I*} and u^{I*} are given by:

$$\begin{array}{l} \omega^{I*} = \frac{B}{\alpha\left(1-\frac{1}{\delta}\right)} \\ \chi^{I*} = \frac{B}{\alpha\left(1-\frac{1}{\delta}\right)} - \frac{1}{\theta}\left[B-\rho\right] \\ u^{I*} = 1 - \frac{1}{B}\frac{1}{\theta}\left[B-\rho\right] \end{array} \tag{A.9}$$

The determinant of the Jacobian in regime I is:

$$\det \begin{vmatrix} -(1-\alpha)\left(1-\frac{1}{\delta}\right) & 0 & 0 \\ -\left[1-\frac{1}{\theta}\left(1-\frac{1}{\delta}\right)\alpha\right] & 1 & 0 \\ \frac{1}{\delta} & -1 & B \end{vmatrix} = -(1-\alpha)\left(1-\frac{1}{\delta}\right)B < 0$$

where the eigenvalues are given by the diagonal elements, since the Jacobian is recursive. Thus, there is one negative and two positive eigenvalues, which means that the dynamic system in regime I is saddle path stable.

Regime II (t^*, ∞) is characterized by a binding emission standard $z < 1$. In this regime, the growth rate of the fraction of time spent at work is derived by logarithmic differentiation of (3.5) and use of (3.4), (3.7), (A.2), (A.1) and (2.4):

$$g_u^{II} = -\frac{(\delta-1)\gamma}{\delta\gamma-1}\chi^{II} + Bu^{II} + \frac{1-\alpha}{\alpha}\frac{\delta\gamma}{\delta\gamma-1}B + \frac{\rho}{\delta\gamma-1} \tag{A.10}$$

The average product of physical capital is given by:

$$\omega^{II} \equiv \frac{Y}{K} = A\left(\frac{uH}{K}\right)^{1-\alpha}\left(\frac{\mu_k\frac{1}{\delta}\alpha A\left(\frac{uH}{K}\right)^{1-\alpha}}{\eta K^{\gamma-1}e^{-\rho t}}\right)^{\frac{\alpha}{\delta\gamma-\alpha}} \tag{A.11}$$

where equation (3.7) has been used. Logarithmic differentiation of (A.11) and use of (2.4), (A.1), (A.2) and (A.3) yields the following growth rate of the average product of physical capital:

$$\begin{aligned} g_\omega^{II} = & -\frac{(1-\alpha)\delta\gamma + (\gamma-1)\alpha + \alpha^2\left(1-\frac{1}{\delta}\right)}{\delta\gamma-\alpha}\omega^{II} + \frac{\gamma-1}{\delta\gamma-1}\chi^{II} \\ & + \frac{1-\alpha}{\alpha}\frac{\delta\gamma}{\delta\gamma-1}B + \frac{\rho}{\delta\gamma-1} \end{aligned} \tag{A.12}$$

The dynamic system in regime II can now be written in matrix form as:

$$\begin{bmatrix} g_\omega^{II} \\ g_\chi^{II} \\ g_u^{II} \end{bmatrix} = \begin{bmatrix} a_{11}^{II} & a_{12}^{II} & 0 \\ a_{21}^{II} & a_{22}^{II} & 0 \\ 0 & a_{32}^{II} & a_{33}^{II} \end{bmatrix} \begin{bmatrix} \omega^{II} \\ \chi^{II} \\ u^{II} \end{bmatrix} + \begin{bmatrix} b_1^{II} \\ b_2^{II} \\ b_3^{II} \end{bmatrix} \tag{A.13}$$

where

$$\begin{array}{lll} a_{11}^{II} = -\frac{(1-\alpha)\delta\gamma+(\gamma-1)\alpha+\alpha\left(1-\frac{1}{\delta}\right)\alpha}{\delta\gamma-\alpha}, & a_{12}^{II} = \frac{\gamma-1}{\delta\gamma-1}, & b_1^{II} = \frac{1-\alpha}{\alpha}\frac{\delta\gamma}{\delta\gamma-1}B + \frac{\rho}{\delta\gamma-1} \\ a_{21}^{II} = -\left[1-\frac{1}{\theta}\left(1-\frac{1}{\delta}\right)\alpha\right], & a_{22}^{II} = 1, & b_2^{II} = -\frac{\rho}{\theta} \\ & a_{32}^{II} = -\frac{(\delta-1)\gamma}{\delta\gamma-1}, \quad a_{33}^{II} = B, & b_3^{II} = \frac{1-\alpha}{\alpha}\frac{\delta\gamma}{\delta\gamma-1}B + \frac{\rho}{\delta\gamma-1} \end{array}$$

A first order Taylor expansion linearizes the system around the steady state:

$$\begin{bmatrix} g_\omega^{II} \\ g_\chi^{II} \\ g_u^{II} \end{bmatrix} = \begin{bmatrix} a_{11}^{II} & a_{12}^{II} & 0 \\ a_{21}^{II} & a_{22}^{II} & 0 \\ 0 & a_{32}^{II} & a_{33}^{II} \end{bmatrix} \begin{bmatrix} \omega^{II} - \omega^{II*} \\ \chi^{II} - \chi^{II*} \\ u^{II} - u^{II*} \end{bmatrix} \tag{A.14}$$

where the steady state equilibrium solutions in regime II ω^{II*}, χ^{II*} and u^{II*} are given by:

$$\begin{array}{l} \omega^{*II} = \frac{\frac{1-\alpha}{\alpha}\delta\gamma B + \frac{\theta+\gamma-1}{\theta}\rho}{a_{11}^{II}a_{22}^{II} - a_{21}^{II}a_{12}^{II}} \\ \chi^{II*} = \left[1-\frac{1}{\theta}\left(1-\frac{1}{\delta}\right)\alpha\right]\omega^{II*} + \frac{\rho}{\theta} \\ u^{II*} = \frac{1}{B}\frac{(\delta-1)\gamma}{\delta\gamma-1}\chi^{II*} - \frac{1-\alpha}{\alpha}\frac{\delta\gamma}{\delta\gamma-1} - \frac{1}{B}\frac{\rho}{\delta\gamma-1} \end{array} \tag{A.15}$$

The Jacobian in regime II is seen to be block recursive, which means that one of the eigenvalues is given by the last diagonal element $\xi_3^{II} = B$, and the remaining two eigenvalues are determined by the following characteristic equation:

$$\xi^{II2} - \left(a_{11}^{II} + a_{22}^{II}\right)\xi^{II} + a_{11}^{II}a_{22}^{II} - a_{21}^{II}a_{12}^{II} = 0$$

The determinant of the upper block in the Jacobian matrix is negative, which means that there is one negative and one positive eigenvalue:

$$\xi_1^{II} = \frac{1}{2}\left[a_{11}^{II} + a_{22}^{II} - \sqrt{a_{11}^{II2} + a_{22}^{II2} - 2\left(a_{11}^{II}a_{22}^{II} - 2a_{21}^{II}a_{12}^{II}\right)}\right] < 0$$
$$\xi_2^{II} = \frac{1}{2}\left[a_{11}^{II} + a_{22}^{II} + \sqrt{a_{11}^{II2} + a_{22}^{II2} - 2\left(a_{11}^{II}a_{22}^{II} - 2a_{21}^{II}a_{12}^{II}\right)}\right] > 0$$

Hence, the dynamic system in regime II is saddle path stable. Note that the speed of adjustment along the stable saddle path in regime II is faster, the larger $\left|\xi_1^{II}\right|$ is.

A.4. Transitional Dynamics

The method used in the following is briefly described on page 137–141 in [17] Turnovsky and an example of its use is given in [7] Haufler & Nielsen.

Regime I $(0, t^*)$, where $z = 1$: The economy evolves along an unstable path, which is derived by use of both the negative eigenvalue a_{11}^I and the positive eigenvalue a_{22}^I:

$$\chi_t^I = A_1^I e^{a_{11}^I t} + A_2^I e^{a_{22}^I t} + \chi^{*I} \tag{A.16}$$
$$\omega_t^I = \frac{a_{11}^I - a_{22}^I}{a_{21}^I} A_1^I e^{a_{11}^I t} + \frac{a_{22}^I - a_{22}^I}{a_{21}^I} A_2^I e^{a_{22}^I t} + \omega^{*I} \quad \Leftrightarrow$$
$$\omega_t^I = \frac{a_{11}^I - a_{22}^I}{a_{21}^I} A_1^I e^{a_{11}^I t} + \omega^{*I} \tag{A.17}$$

where A_1^I and A_2^I are constants to be determined. Eliminating $A_1^I e^{a_{11}^I t}$ from these equations yields the unstable path:

$$\chi_t^I - \chi^{*I} = A_2^I e^{a_{22}^I t} + \frac{a_{21}^I}{a_{11}^I - a_{22}^I}\left(\omega_t^I - \omega^{*I}\right) \tag{A.18}$$

Regime II (t^*, ∞), where $z < 1$: The economy evolves along a stable path, which is derived by use of only the negative eigenvalue ξ_1^{II}:

$$\chi_t^{II} = A_1^{II} e^{\xi_1^{II} t} + \chi^{*II} \tag{A.19}$$
$$\omega_t^{II} = \frac{\xi_1^{II} - a_{22}^{II}}{a_{21}^{II}} A_1^{II} e^{\xi_1^{II} t} + \omega^{*II} \tag{A.20}$$

where A_1^{II} is a constants to be determined. Eliminating $A_1^{II} e^{\xi_1^{II} t}$ from these equations yields the stable arm of the saddlepoint in regime II:

$$\chi_t^{II} - \chi^{*II} = \frac{a_{21}^{II}}{\xi_1^{II} - a_{22}^{II}}\left(\omega_t^{II} - \omega^{*II}\right) = \frac{\xi_1^{II} - a_{11}^{II}}{a_{12}^{II}}\left(\omega_t^{II} - \omega^{*II}\right) \tag{A.21}$$

Finally, the constants A_1^I, A_2^I and A_1^{II} are determined by use of the fact that the solutions for χ_t and ω_t must be continuous at time t^*. Hence, equating $\chi_{t^*}^I = \chi_{t^*}^{II}$

and $\omega_{t^*}^I = \omega_{t^*}^{II}$ in (A.16)–(A.17) and (A.19)–(A.20) yields two equations in the three unknown constants A_1^I, A_2^I and A_1^{II} :

$$A_1^{II} e^{\xi_1^{II} t^*} - A_1^I e^{a_{11}^I t^*} - A_2^I e^{a_{22}^I t^*} = \chi^{*I} - \chi^{*II} \tag{A.22}$$
$$\frac{\xi_1^{II} - a_{22}^{II}}{a_{21}^{II}} A_1^{II} e^{\xi_1^{II} t^*} - \frac{a_{11}^I - a_{22}^I}{a_{21}^I} A_1^I e^{a_{11}^I t^*} = \omega^{*I} - \omega^{*II} \tag{A.23}$$

A third equation must therefore be obtained from the first order conditions to the central planners problem in order to determine A_1^I, A_2^I and A_1^{II}. Hence, use of equation (3.1) and (3.2) reveals that the following relationship between ω and χ must hold at time t^*.[37]

$$\omega_{t^*} = \frac{\eta\delta}{\alpha} K_{t^*}^{\theta+\gamma-1} \chi_{t^*}^{\theta} \tag{A.24}$$

The physical capital stock K at the critical time t^* can be obtained by use of the resource constraint of the economy (2.2):

$$K_{t^*} = K_0 e^{\int_0^{t^*} (\omega_s - \chi_s) ds}$$

where K_0 is the initial stock of physical capital. This means that the constants A_1^I, A_2^I and A_1^{II} now can be determined by use of equation (A.16), (A.17), (A.22), (A.23) and (A.24). Thus, at time t^*:

$$\chi_{t^*} = A_1^I e^{a_{11}^I t^*} + A_2^I e^{a_{22}^I t^*} + \chi^{*I} \tag{A.25}$$
$$\omega_{t^*} = \frac{a_{11}^I - a_{22}^I}{a_{21}^I} A_1^I e^{a_{11}^I t^*} + \omega^{*I} \tag{A.26}$$

which yields the first constant:

$$A_1^I = \frac{a_{21}^I}{a_{11}^I - a_{22}^I} e^{-a_{11}^I t^*}\left(\omega_{t^*} - \omega^{*I}\right) \tag{A.27}$$

Substitution of this in (A.22) and (A.23) yields the remaining two constants:

$$A_2^I = \frac{a_{21}^{II}}{\xi_1^{II} - a_{22}^{II}} e^{-a_{22}^I t^*}\left(\omega_{t^*} - \omega^{*II}\right) - \frac{a_{21}^I}{a_{11}^I - a_{22}^I} e^{-a_{22}^I t^*}\left(\omega_{t^*} - \omega^{*I}\right) - e^{-a_{22}^I t^*}\left(\chi^{*I} - \chi^{*II}\right) \tag{A.28}$$
$$A_1^{II} = \frac{a_{21}^{II}}{\xi_1^{II} - a_{22}^{II}} e^{-\xi_1^{II} t^*}\left(\omega_{t^*} - \omega^{*II}\right) \tag{A.29}$$

The initial value of the control–like variable χ is now determined in order to ensure that the unstable path (regime I) (A.18) just reaches the stable path (regime II) (A.21)

[37] Recall that $\omega \equiv Y/K$ and $\chi \equiv C/K$.

at the critical time t^*. Thus, the initial value of the consumption–capital ratio χ_0 is derived by evaluating equation (A.16) at time 0:

$$\begin{aligned} \chi_t^I &= A_1^I e^{a_{11}^I t} + A_2^I e^{a_{22}^I t} + \chi^{*I} \Rightarrow \\ \chi_0 &= A_1^I + A_2^I + \chi^{*I} \end{aligned} \tag{A.30}$$

where A_1^I, A_2^I, and χ^{*I} are given by equation (A.27), (A.28) and (A.9), respectively.

The initial value of the time fraction spent at work is determined by the definition of ω:

$$u_0 = \frac{(\omega_0)^{\frac{1}{1-\alpha}}}{A\left(\frac{H_0}{K_0}\right)} = \frac{\left(\frac{a_{11}^I - a_{22}^I}{a_{21}^I} A_1^I + \omega^{*I}\right)^{\frac{1}{1-\alpha}}}{A\left(\frac{H_0}{K_0}\right)} \tag{A.31}$$

where ω_0 is given by equation (A.17). The transition path of u can now be determined by use of the two dynamic systems (A.7) and (A.13) and the time paths of ω and χ. In regime I $(0,\, t^*)$, where $z = 1$:

$$g_u^I = a_{31}^I \omega^I + a_{32}^I \chi^I + a_{33}^I u^I + b_3^I$$

which by use of equation (A.16) and (A.17) yields:

$$\begin{aligned} g_u^I &= a_{33}^I u_t^I + \left[a_{31}^I \omega^{*I} + a_{32}^I \chi^{*I} + b_3^I\right] \\ &\quad + \left[\left(a_{31}^I \frac{a_{11}^I - a_{22}^I}{a_{21}^I} + a_{32}^I\right) A_1^I e^{a_{11}^I t} + a_{32}^I A_2^I e^{a_{22}^I t}\right] \end{aligned}$$

This is a first order differential equation with a constant coefficient and a variable term and the solution is:

$$u_t^I = \bar{C}^I e^{a_{33}^I t} + e^{a_{33}^I t} \int_0^t \left[\left(a_{31}^I \frac{a_{11}^I - a_{22}^I}{a_{21}^I} + a_{32}^I\right) A_1^I e^{a_{11}^I t} + a_{32}^I A_2^I e^{a_{22}^I t}\right] e^{-a_{33}^I t} dt \tag{A.32}$$

where $\bar{C}^I$ is a constant to be determined.

In regime II $(t^*,\, \infty)$, where $z < 1$:

$$g_u^{II} = a_{32}^{II} \chi^{II} + a_{33}^{II} u^{II} + b_3^{II}$$

which by use of equation (A.19) yields:

$$g_u^{II} = a_{33}^{II} u^{II} + \left(a_{32}^{II} \chi^{*II} + b_3^{II}\right) + a_{32}^{II} A_1^{II} e^{\xi_1^{II} t}$$

The associated solution is:

$$u_t^{II} = \bar{C}^{II} e^{a_{33}^{II} t} + e^{a_{33}^{II} t} a_{32}^{II} A_1^{II} \int_0^t e^{\left(\xi_1^{II} - a_{33}^{II}\right) t} dt \tag{A.33}$$

The constants $\bar{C}^I$ and $\bar{C}^{II}$ can be determined by use of two facts. Firstly, the initial fraction of time spent at work u_0 is given both by equation (A.31) and (A.32). Secondly, the time path of u should be continuous at time t^*, which means that $u_{t^*}^I = u_{t^*}^{II}$.

A.5. First Order Conditions of the Central Planner Solution

When tax revenues finance a productive public input in production, the first order conditions to the central planner problem become:

$$C^{-\theta} e^{-\rho t} = \mu_k \tag{A.34}$$

$$\eta P^{\gamma-1} e^{-\rho t} = \mu_k \frac{1}{\delta} \alpha \frac{Y}{P}\left(1 - \frac{G}{Y}\right) \tag{A.35}$$

$$\frac{\Gamma'}{\Gamma} = 1 \tag{A.36}$$

$$-\frac{\dot{\mu}_k}{\mu_k} = \left(1 - \frac{1}{\delta}\right) \alpha \frac{Y}{K}\left(1 - \frac{G}{Y}\right) \tag{A.37}$$

$$-\frac{\dot{\mu}_h}{\mu_h} = B \tag{A.38}$$

$$\mu_h B = \mu_k (1 - \alpha) \frac{Y}{uH}\left(1 - \frac{G}{Y}\right) \tag{A.39}$$

which naturally are quite similar to the first order conditions obtained in Section 3 except for the term $(1 - G/Y)$ and equation (A.36) determining the optimal allocation of resources between productive public spending and physical capital accumulation. The evolution of the shadow price of physical capital in the centrally planned economy becomes:

$$\begin{aligned} -\frac{\dot{\mu}_k}{\mu_k} &= \left(1 - \tfrac{1}{\delta}\right) \alpha A \left(\tfrac{uH}{K}\right)^{1-\alpha} \left(1 - \tfrac{G}{Y}\right) \Gamma, && \text{if } z = 1 \\ -\frac{\dot{\mu}_k}{\mu_k} &= \left(1 - \tfrac{1}{\delta}\right) \left(\frac{\mu_k \frac{1}{\delta} \alpha A \left(\frac{uH}{K}\right)^{1-\alpha} \left(1 - \frac{G}{Y}\right) \Gamma}{\eta K^{\gamma-1} e^{-\rho t}}\right)^{\frac{\alpha}{\delta\gamma - \alpha}}, && \text{if } z < 1 \end{aligned} \tag{A.40}$$

where equation (A.37) and (A.41) have been used. The optimal value of the emission standard is given by (A.36):

$$\begin{aligned} z &= 1, && \text{if } \mu_k > \frac{\eta K^{\gamma-1} e^{-\rho t}}{\frac{1}{\delta} \alpha A \left(\frac{uH}{K}\right)^{1-\alpha} \left(1 - \frac{G}{Y}\right) \Gamma} \\ z &= \left(\frac{\mu_k \frac{1}{\delta} \alpha A \left(\frac{uH}{K}\right)^{1-\alpha} \left(1 - \frac{G}{Y}\right) \Gamma}{\eta K^{\gamma-1} e^{-\rho t}}\right)^{\frac{1}{\delta\gamma - \alpha}}, && \text{if } \mu_k \leq \frac{\eta K^{\gamma-1} e^{-\rho t}}{\frac{1}{\delta} \alpha A \left(\frac{uH}{K}\right)^{1-\alpha} \left(1 - \frac{G}{Y}\right) \Gamma} \end{aligned} \tag{A.41}$$

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Resumé

Denne afhandling består af fem selvstændige kapitler, der alle beskæftiger sig med skattepolitik i endogene vækstmodeller med to sektorer. Afhandlingen beskæftiger sig med uddannelses- og miljøpolitik og i særdeleshed med at bestemme den optimale uddannelsesstøtte og den optimale miljøpolitik. De økonomiske modeller, der anvendes i samtlige kapitler, er alle udvidelser af Uzawa–Lucas modellen. Kapitel 1 og 2 undersøger effekterne af faktorindkomstbeskatning og subsidiering af uddannelse, mens kapitel 3, 4 og 5 undersøger den transitoriske dynamik og langsigtseffekterne af miljøpolitik.

Det *første* kapitel undersøger effekterne af faktorindkomstbeskatning og uddannelsesstøtte i en endogen vækstmodel med to sektorer. Der er en uddannelseseksternalitet i færdigvaresektoren og et offentligt input i uddannelsessektoren. Hovedresultatet er, at arbejdsindkomstbeskatning og uddannelsesstøtte har positive effekter på den langsigtede vækstrate i en markedsøkonomi, ceteris paribus. Derudover er den optimale uddannelsesstøtte altid positiv, når der er substitutionsmuligheder mellem det offentlige og det private input i uddannelsessektoren. Desuden er den optimale arbejdsindkomstskat lavere end den optimale kapitalindkomstskat, sålænge der er en positiv uddannelseseksternalitet i færdigvaresektoren. Jo større eksternaliteten er, jo større er forskellen mellem de to skattesatser. Endeligt er det ikke nødvendigvis velfærdsmaksimerende at maksimere vækstraten. Det er faktisk kun velfærdsforbedrende at øge arbejdsindkomstskatten og uddannelsesstøtten, sålænge de er lavere end deres optimale niveauer.

Det *andet* kapitel analyserer effekterne af en uddannelsesorlovsydelse i en to sektor endogen vækstmodel med ufrivillig arbejdsløshed, der er skabt af monopolistiske fagforeninger. Der er en færdigvaresektor og en uddannelsessektor, som sørger for at forbedre arbejdsstyrkens færdigheder. Hovedresultatet er, at det er optimalt at opkræve et undervisningsgebyr i stedet for at yde uddannelsesstøtte i en økonomi med ufrivillig arbejdsløshed, der er skabt af monopolistiske fagforeninger. Derudover er det kun velfærdsforbedrende at fremme væksten, sålænge undervisningsgebyret er større end dets optimale niveau. En stigning i uddannelsesorlovsydelsen medfører en højere vækstrate på langt sigt samtidig med, at arbejdsløsheden reduceres. Endelig vises det, at en stigning i uddannelsesorlovsydelsen medfører den samme transitoriske dynamik som en stigning i arbejdsindkomstskatten, men at sidstnævnte har en negativ effekt på

vækstraten.

Det *tredie* kapitel analyserer effekterne af miljøpolitik i en to sektor endogen vækst model med forurening. Forureningen er enten et bi–produkt af færdigvareproduktionen eller af brugen af fysisk kapital i produktionen og kan reduceres ved hjælp af offentlig forureningsbekæmpelse. I denne generaliserede Uzawa–Lucas model bestemmes effekten af forskellige skattepolitiske tiltag på de vigtigste variabler og forhold.. Desuden findes de optimale skatteregler. Hvis udledningen af forurening ikke kan beskattes, så er det stadigt muligt at opnå en optimal løsning ved hjælp af faktorindkomstbeskatning. Derudover analyseres effekterne af og mulighederne for at føre miljøpolitik i en lille åben økonomi.

Det *fjerde* kapitel simulerer den transitoriske dynamik i den to sektor endogene vækstmodel, der blev opstillet og analyseret teoretisk i kapitel 3. Miljøpolitikken indføres enten pludseligt, forudannonceret eller gradvist. Udfra et velfærdssynspunkt er det bedst at indføre politikken pludseligt, men efter vores mening er det bedst at indføre en ændring i miljøpolitikken gradvist. For det første bliver tilpasningsprocessen udjævnet over tid, og for det andet er velfærdstabet i sammenligning med en pludselig ændring i forureningsskatten forsvindende lille. Et andet resultat er at uafhængigt af hvorledes miljøpolitikken indføres, så medfører den kun et fald i den langsigtede vækstrate fra 2% til 1.98%, når forholdet mellem forureningsbekæmpelsen og færdigvareproduktionen stiger fra 1.6% til 3.2%. De kvalitative og kvantitative resultater er robuste overfor ændringer i de fleste parametre, men den transitoriske dynamik er dog følsom overfor ændringer i andelen af fysisk kapital i produktion og uddannelse.

Det *femte* kapitel undersøger effekterne af en emissionsstandard og beskatning i en to sektor endogen vækstmodel med forurening. Der er to regimer, der er karakteriseret af henholdsvis en ikke–bindende og en bindende emissionsstandard. Hovedresultatet er, at vedvarende vækst er mulig, selv når der tages hensyn til miljøet. Markedsøkonomien er inefficient, og det er derfor nødvendigt enten at beskatte kapitalindkomst eller at pålægge virksomhederne en grøn afgift for at opnå en optimal løsning. Hvis det ikke er muligt at beskatte kapitalindkomst, så vil den optimale grønne afgift være lig med den optimale marginale skade af forureningen. Imidlertid kan den optimale grønne afgift være mindre end det Pigouvianske niveau, når den optimal kapitalindkomstskat er høj, fordi beskatning af kapitalindkomst virker som en indirekte skat på forurening.

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