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**Doctoral Thesis**

## Economic geography and growth

PhD Series, No. 2.1999

**Provided in Cooperation with:**

Copenhagen Business School (CBS)

*Suggested Citation:* Urban, Dieter (1999) : Economic geography and growth, PhD Series, No. 2.1999, ISBN 8759380985, Copenhagen Business School (CBS), Frederiksberg, <https://hdl.handle.net/10398/7914>

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# **Economic Geography and Growth**

by

Dieter M. Urban

A DISSERTATION

in

Economics

Presented to the Faculty of Economics and Business Administration of the  
Copenhagen Business School in partial fulfillment of the requirements for the degree  
of Doctor of Philosophy.

February 4, 1999

## **Acknowledgements:**

Writing a Ph.D. thesis is a demanding task not only intellectually, but also physically and socially. The more important is the support of supervisors, colleagues, and friends.

First of all, I would like to thank my supervisors. I would like to thank Niels Blomgren-Hansen for offering me a position as Ph.D. student at Copenhagen Business School. This gave me not only the opportunity to get to know the views of Danish economists, but allowed me also to become part of Danish life and culture. Niels Blomgren-Hansen provided me with the most detailed comments, critical questions, and even took paper and pencil, when I was stuck with a calculation. Equally much do I owe to Pascal Raimondos Møller. He taught me the art of writing papers, and was always encouraging, took his time, and introduced me to many economists.

I would also like to thank especially Eric Bond, Kala Krishna, and the Economics Department at PennState University for their hospitality, for the cooperation, and their stimulating comments on my work. In this context, I am also grateful to the external examiners Søren Bo Nielsen, Eric Bond, and Ian Wooten who provided me detailed comments on the final version of my thesis.

I certainly benefitted also from the many comments of the faculty at Copenhagen Business School and the social environment that was provided. In particular, Lisbeth Funding La Cour helped me with the empirical analysis in chapter 5. Preben Alstholt from Denmark's Technical University helped me with a mathematical problem in chapter 3.

Words of thanks are also due to my friends: those who I lived with at Nordisk Kollegiet; those who invited me all over Europe, whenever I needed distraction; those who visited me from all over Europe and shared my passion for Copenhagen. I would like to thank them for understanding that I could not share as much time with them as I wished.

Most of all, I would like to thank my parents. It is fair to say that I would not have finished my Ph.D. thesis without their support and encouragement.

Needless to say, any remaining mistakes or omissions are entirely mine.

Copenhagen, February 1999

Dieter M. Urban

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# 1 A Readers Guide to Geography and Growth\*

## Abstract

This Ph.D. thesis contains 4 chapters on economic geography and economic growth. Chapters 2 and 3 explain the intuition behind the seminal Krugman (1991a) model and develop a technique to analytically solve this model. Chapter 4 embeds this model framework into a neoclassical growth model which explains a poverty trap. Chapter 5 tests this model empirically for Japan and the US using co-integration techniques.

**JEL Classification:** F12, O41

**Keywords:** agglomeration, economic growth

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\*The author thanks Niels Blomgren-Hansen, Søren Bo Nielsen, and Pascalis Raimondos Møller for their comments. The usual caveats apply.

## 1.1 Introduction

This Ph.D. thesis is centered around two major discussions that took place in economics in the 90ies: the convergence debate in growth theory and economic geography. In particular, I explore the relation between income growth of countries/ regions relative to each other and the location of manufacturing industries across countries/ regions<sup>1</sup>.

How do agglomeration processes in the location of manufacturing industries affect the relative growth performance of countries/regions? How does different growth performance feed back on the location of manufacturing industries? Which role do trade openness, integration processes, and globalization play for the relative growth performance and manufacturing agglomeration processes? These questions will be addressed both theoretically and empirically. I hereby follow the statement of Fujita and Thisse (1996), p. 40:

*“The potential connection between growth and agglomeration should be explored more systematically.”*

Before I explain how I address these questions and how I contribute to the existing literature in this thesis, I want to state the most striking stylised facts both covering the convergence debate and the economic landscape, and then give a very crude overview of the existing literature as a point of departure of my own analysis.

## 1.2 Stylised Facts

Since I intend to merge two fields of research, two sets of stylised facts become relevant - the stylised facts of the convergence debate and the stylised facts of the economic landscape<sup>2</sup>. The convergence debate is based on the prediction of neoclassical growth

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<sup>1</sup>Throughout this thesis I define regions as geographic areas such that goods trade is costless within this area, but costly, whenever the border lines are crossed. Countries differ additionally by lack of factor (in particular labour) mobility, non-integrated capital markets, and different jurisdictions.

<sup>2</sup>One may consider agglomeration phenomena on different levels: cities, regions, and countries. One may view regions as a system of cities, and countries as a system of regions in the sense of a development towards ever higher complexity (Krugman, 1996). Since the same principle of agglomeration forces applies to the formation of cities, regions, and countries, I will restrict formal analysis to the simplest form of agglomeration phenomena: the core-periphery pattern relating it to cities,

theory (Solow, 1956). Income of countries with the same structural characteristics (savings rates, technologies, education, etc.), but different initial conditions (initial physical and human capital stocks) converges to a common steady state growth rate, since countries with lower capital stocks have higher marginal products of capital, and higher income growth rates. Also, capital flows from capital-rich to capital-poor countries are re-enforcing convergence of income. Another theoretical channel for income convergence are international technology spillovers.<sup>3</sup> The standard empirical convergence literature tests  $\beta$ - and  $\sigma$ -convergence - the negative relation of average growth rates of a country/region sample on initial income, and the dispersion (variance) of income of a country/region sample over time, respectively.<sup>4</sup>

1.) Barro (1991), Barro and Sala-i-Martin (1992), and Mankiw, Romer, and Weil (1992) find that the average country converges conditional upon structural characteristics towards a common growth path. This supports the neoclassical growth model (Solow, 1956) and endogenous growth models with diminishing returns to capital (Jones and Manuelli, 1990). This view finds support in the First Cohesion Report (1996) of the EC-Commission for the regions of Europe.

Empirically, this view has been challenged by supporters of the club convergence hypothesis such as Quah (1996) and Durlauf and Johnson (1995) who claim that initial conditions matter additionally to structural characteristics. Rich countries converge to a rich country steady state and poor countries to a poor country steady state (poverty trap). The latter observation is explained theoretically in the big push and poverty trap literature.<sup>5</sup>

2.) Lucas (1990) poses the puzzle that capital does not flow in sufficient amounts from rich to poor countries contrary to the predictions of the neoclassical growth and trade theory. A separate literature emerged to explain this puzzle by political risk, and information asymmetries which gave rise to the interpretation of the

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regions and countries in the same way.

<sup>3</sup>See Grossman and Helpman (1991) for the theory and Coe and Helpman (1995) for the empirical evidence.

<sup>4</sup>See Barro and Sala-i-Martin (1995), chapter 11.

<sup>5</sup>A survey is Azariadis (1996).



multinational firm as information intermediary.

3.) Ben-David (1993, 1996) and Sachs and Warner (1995) find that trade liberalization has an impact on the convergence of countries. Countries which reduce trade barriers among each other start converging to each other. This finding is in line with dynamic gains of trade (Baldwin, 1992) in neoclassical trade and growth models. Additionally, Lee (1997) notes that poorer countries start importing more productive capital goods, if trade barriers are reduced. On the contrary, Rauch (1997) reports capital outflows after trade liberalization in Chile from 1974 until 1978 and notes the Italian Mezzogiorno case, where income of North and South Italy diverged after political unification in 1861.

4.) The East -Asian tigers, in particular South-Korea and Taiwan, have been taken as nutshell experiment to test the theories on economic growth and industrialization. Whereas Young's (1995) growth accounting supports the neoclassical model emphasizing the role of factor accumulation, Rodrik (1995) challenges this view on account of a thorough study of the political and economic environment of Taiwan and South Korea. He argues that interventionist policy was useful to overcome a coordination problem of investment to start the catch-up process. This finding supports the big push theory of Murphy, Shleifer, and Vishny (1989) which argues that scale economies require a certain threshold size of the market for an industry, before this industry uses modern mass production technology rather than traditional production methods. Only if all firms can be coordinated to switch technology at the same time, then this switch itself generates so much demand that it becomes profitable for any single firm.

The literature on economic geography - the study of economic activity in space<sup>6</sup> - is centered around four stylised facts which cannot be fully explained by neoclassical location theory<sup>7</sup> which explains location by endowments.<sup>8</sup>

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<sup>6</sup>See Krugman (1991c) for this definition.

<sup>7</sup>The most recent and worked-out formulation of neoclassical location theory is Norman and Venables (1995).

<sup>8</sup>Surveys on empirical tests of implications of economic geography models are Hanson (1998) for America and Amati (1998) for Europe.

1.) The large manufacturing belts in the US<sup>9</sup> - the rust belt at the East coast and the Lakes and the sun belt in California - and the manufacturing belt of Europe - the blue banana - are difficult to explain by endowment differences alone (Krugman, 1991c). It rather seems that these agglomeration phenomena of manufacturing industries as a whole occurred spontaneously by historical accident in a self-enforcing process (cumulative causation)<sup>10</sup>. Accordingly, it remains difficult to explain the emergence of big cities. “With capital and labour mobile in the long run, there would need to be an implausible concentration of immobile resources to produce cities the size of Chicago, Los Angeles, or New York.” (Hanson, 1998, p. 34) Another example is the development of Mexico city. “Mexico city’s share of the national manufacturing labour force rose from 19 per cent in 1930 to 46 per cent in 1960. Over that 30-year period, manufacturing employment in Mexico city grew at an average annual rate of 6.7 per cent, compared to 2.7 per cent in the rest of the country.” (Hanson, 1998, p.33)

2.) Specialization patterns are observed independent of endowment and factor intensity differences across sectors. For example, the US states are known to be highly specialized often without major differences in endowments.<sup>11</sup> Additionally, Amiti (1998) finds that manufacturing in the European regions has become more specialized along with the European integration process. Along with specialization patterns international trade occurs that is not based on endowment differences (intra-industry trade). The puzzle of intra-industry trade is indeed that the bulk of trade is undertaken in goods that have similar factor intensities of production and occurs among countries that have similar endowments.<sup>12</sup>

3.) Economic integration, i.e. the reduction of trade barriers, may lead to divergence of manufacturing distribution. For example, German unification led to

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<sup>9</sup>Kim (1995) and Hanson (1998) give a detailed account on the development of the economic landscape of the US.

<sup>10</sup>The notion of cumulative causation was brought into the discussion by Myrdal (1957) to explain why some countries remain poor in spite of the contrary predictions of neoclassical trade theory.

<sup>11</sup>See Krugman and Venables (1996). Lau (1996) finds some manufacturing industries in Europe that are highly specialized in regions.

<sup>12</sup>See Deardorff (1984). Recently, Davis and Weinstein (1996) and Davis (1997) have challenged this view.

a dry-out of manufacturing in the underdeveloped East-Germany. Lange and Pugh (1998) report a drop in manufacturing employment in East-Germany interpreting it as “deindustrialization” (p. 80). Kim (1995) finds that regional specialization of manufacturing fell after an initial rise in the beginning of the age of industrialisation for the US states. The improvement of transport technology may thus have contributed to less agglomeration.

4.) Puga and Venables (1996) argue that the spread of manufacturing in Asia was not a uniform process. First, Japan developed. When the Asian market grew bigger, industrialization spread over to the four Tiger economies. Recently, China started the industrialization process. This contradicts the prediction of the neoclassical growth theory that the countries with the smallest capital stocks are supposed to grow fastest, if everything else is equal.

The main hypothesis of this thesis is to view the relative income development among countries/ regions and the relative distribution of manufacturing industries as two sides of the same coin: Those countries/ regions that capture the most manufacturing industries also grow fastest. Empirically, this hypothesis has recently gained support by Gallup and Sachs (1998). They show that those countries are richer that have good access to ports. This indicates that transport cost and market access - important features of economic geography<sup>13</sup> - matter for explaining income across countries.

Further, growth and agglomeration are connected by history. “Industrial development in Canada, Mexico, and the United States brought with it the geographic concentration of economic activity.” (Hanson, 1998, p. 31f) Yet, the age of industrialization also sees the introduction of modern mass production technologies and means of transport like the steam engine, the railroads, and the assembly line to mention just a few.

These stylised facts pose a puzzle. If growth and agglomeration are two sides of

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<sup>13</sup>See Krugman (1998).

the same coin, then there cannot occur income convergence according to neoclassical growth theory and manufacturing agglomeration according to theories of self-driven agglomeration processes at the same time (Fingleton and McCombie, 1998). A similar argument can be found in Broadberry (1993). He shows in an empirical analysis of US, UK and German manufacturing labour productivity from 1869 until 1989 persistence of differences and concludes that convergence of income - to the extent that it exists - cannot have occurred through the manufacturing sector.

To understand the contrary predictions of the neoclassical growth theory on one side and theories of agglomeration on the other, the driving assumptions of these separate fields have to be understood. Neoclassical growth theory is based on constant returns to scale technology (CRS), whereas regional economists believe that agglomeration phenomena are caused by increasing returns technology (IRS).<sup>14</sup> For example, Krugman (1998, p.10) states: “Almost all of the interesting ideas in location theory rely implicitly or explicitly on the assumption that there are important economies of scale enforcing the geographic concentration of some activities.”

Because there is no general way of modelling increasing returns (Krugman, 1998) and the predictions may be sensitive to the specific ways, it is helpful to discuss informally in a thought experiment, how one expects the two assumptions to generate different outcomes in growth and location theory, before thoughts are bounded by the mathematical constructs available. This discussion follows next.

### 1.3 Constant Returns versus Increasing Returns to Scale

Increasing returns to scale have been discussed **informally** already by Adam Smith (1776), Allwyn Young (1928), Kaldor (1966), and many others.<sup>15</sup> In particular, it has been noted that IRS is associated with manufacturing production<sup>16</sup>, although scientific

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<sup>14</sup>Crudely defined, CRS implies that larger production plants are just a multiplication of smaller scale production plants, whereas IRS implies that a larger production plant can exploit economies of scale by methods of mass production and reduce average cost, as plant scale rises.

<sup>15</sup>A collection of seminal articles is Buchanan and Yoon (1994).

<sup>16</sup>See Carson (1998) or Fingleton and McCombie (1998).

empirical evidence is mixed on this issue.<sup>17</sup> Additionally, there is a diffuse belief that manufacturing is somehow special.

*“... many people still vaguely believe that manufacturing somehow matters more than any other economic activity; ... Manufacturing, in this way of looking at things, brings more growth, better-paid jobs, fatter export earnings and greater technological progress than any other economic activity.”* (Carson, 1998, p.2)

The difference in the importance of manufacturing may well become understandable, if manufacturing is characterized by mass production and economies of scale, whereas other economic activities (services and agriculture) are not. To understand the importance of this assumption for different predictions in international trade, development economics, and growth theory, it is worthwhile undertaking a simple waiving hands thought experiment.

Suppose production technology has the constant returns to scale (CRS) property, i.e. a complex production plant can be run in a nutshell without efficiency loss. Suppose further that consumers are equally distributed on a real line (in space) with some exogenous income. Finally, there are some costs of transportation for each consumption good depending on the distance of the consumer from the production plant. In such a scenario, one would expect that production is located directly at the place of every single consumer. Every consumer is autark, since transport costs are minimized this way and production is by assumption of CRS not cheaper, if operated on large scale for many consumers. In such a scenario, there is no trade in the same good (intra-industry trade), there is no specialization (other than by endowment differences across space), firms are operated on family size; and economic activity is equally spread in space (no agglomeration). This description fits well to agriculture in the middle ages.

How does this scenario change, if production has the increasing returns to scale property, i.e. production on large scale (industrial mass-production) is cheaper than production on small scale? Then a basic trade-off emerges between firm economies

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<sup>17</sup>See Junius (1997) for a survey. More recent research confirms the existence of increasing returns to scale (Fingleton and McCombie, 1998, and Jun, 1998).

of scale and transport cost. Any production plant will produce for the consumers located nearby the plant. If a firm serves an additional consumer further away as all others, average production cost fall, but transport costs for the new consumer rise. Hence, there are few plants operating in space. Plant size comprises more than just one family. This constitutes a firm, since there is a need for control of the many employees (theory of the firm).

Assume additionally that there are some transportation costs of customer-workers to go to work. Then, there is an incentive for them to move close to the production site. This explains the emergence of cities<sup>18</sup> and the occurrence of migration due to agglomeration forces. In other words, workers move close to firms and firms move close to worker-consumers in a cumulative process.<sup>19</sup>

Suppose, there are different goods in an economy. Once, a plant is located at a site surrounded by many worker-consumers, it pays for other firms producing something else to locate also at this site, because there is already a large market that can be served at low transport cost (agglomeration economies via demand spill-overs). This explains agglomeration clusters of manufacturing as a whole like the rust and sun belt in the US and the blue banana in Europe.

Suppose, there have emerged two cities of equal size. There are no consumers any more located in between the two cities, and there are exactly two goods demanded by consumers produced with identical technology. If transport cost are sufficiently low, it pays for the workers in one city to produce in a single large scale plant good 1, whereas the second city of identical size produces good 2. This explains specialization independent of endowment and technology differences of two sites.

Suppose, there are two sites one of which is bigger than the other in terms of income. Furthermore, there is just one good and there are large transport costs. Production will take place in both cities, since production in the big city at large

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<sup>18</sup>There have always existed cities formed by various types of agglomeration forces, but the emergence of large cities in the 19th century such as Chicago and New York is believed by regional economists to rely on IRS production technologies emerging from the industrial revolution (Hanson, 1998). See also section 1.1.

<sup>19</sup>See Myrdal (1957).

scale and low cost is still more expensive, if transport cost are added, than the more expensive production in the small city without transport cost. If transport cost become negligibly small, it pays in any case to produce at a single location for both cities, i.e. production in one city ceases to exist, because the firms in the larger city can afford to pay higher wages attracting all workers of the smaller city, since large scale operation is cheaper.

Suppose, there is a single owner of a certain product label delivering to both cities which happen to be separated by a border line (e.g. Coca Cola). Then, the question emerges, whether this firm produces in both cities (multi-plant operation of a multi-national firm), or exports from a single production plant in the larger city to the smaller city.

Suppose further, there is only one city. If a rise in production capacity lowers cost, investment may become more profitable the more is invested (investment complementarity). This process may accelerate growth. However, if the size of the market is insufficient, then industrial production may not be profitable at all and a country continues producing in a traditional way (Rosenstein-Rodan, 1943).

Suppose next, there are two cities constituting two countries with complete specialization in two different goods produced with the same capital based technology. Suppose one country invests more increasing its production capacity. With larger production capacity production becomes cheaper and income of this country rises at given international prices. This in turn may induce even larger savings and investments in this country. Consequently, one country is growing faster than the other and the other falls behind (poverty trap).

Suppose finally, there are productivity gains of specialization. A country that has a large final goods market in a largely autarkic world can support more specialized machinery producers, if those require some minimum scale in the presence of IRS to be profitable. The larger variety of specialized tools renders in turn final good production more profitable.<sup>20</sup>

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<sup>20</sup>This story has been especially emphasised by Adam Smith, Allyn Young, and Nicholas Kaldor.

This simple waving hands story suggests that increasing returns technology may potentially explain phenomena as diverse as agglomeration phenomena like the emergence of large cities, or manufacturing belts, the existence of large firms, the existence of the multi-national firm, migration phenomena associated with agglomeration of economic activity, specialization of economic activity, intra-industry trade, or the emergence of underdeveloped regions (poverty traps).

Yet, economic theory was very hesitant to incorporate the assumption of increasing returns to scale into general equilibrium theory. The reason was a logical inconsistency of scale economies in a world with perfect competition. To see this suppose a production technology with constant marginal cost and some fixed cost. Since price equals marginal cost for a price taking firm, the fixed cost will never be covered by the price, firms will always make a loss, and IRS production would cease to exist.<sup>21</sup>

The first to circumvent this problem were Dixit and Stiglitz (1977). They suggest a different market form: monopolistic competition, and firm entry and exit. This allows to model a monopoly premium mark-up of prices on marginal cost such that the mark-up covers the fixed cost. At the same time, entry and exit keep firm profits always at zero and determine the number of firms, since every firm produces a different good in a monopolistically competitive market. In other words product space is endogenous in the Dixit-Stiglitz (1977) set-up, whereas product space is exogenously fixed in standard general equilibrium theory. Unfortunately, this model has been formulated using specific functional forms. It turns out that firm size is always constant. In other words, the effect of decreasing production cost at increasing production volume is not present in this set-up.

Still, this framework proved to be succesful to explain many of the phenomena mentioned above: intra-industry trade (Krugman, 1979, 1980); agglomeration economics (Krugman, 1991a); specialization patterns (Krugman and Venables, 1995,1996); productivity gains of specialization (Ethier, 1982, and Venables, 1996); the theory of

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<sup>21</sup>See Mas-Colell, Whinston, and Green (1995), p. 142ff. Fixed cost can co-exist with perfect competition, if one production factor is fixed (rare management skill) and production occurs beyond the minimum efficiency scale (see Viner, 1932).



the multinational firm (Markusson and Venables, 1995, and Rodriguez-Clare, 1996); big push (Murphy, Shleifer, and Vishny, 1989); accelerating growth (Romer, 1986, 1990); and poverty traps (Martin and Ottaviano, 1996, Baldwin, 1997, Baldwin and Forslid, 1998, and chapter 4 of this thesis).<sup>22</sup>

## 1.4 Agglomeration Economics

In this sub-section, a brief introduction into the major **formal** theoretical contributions explaining agglomeration phenomena shall be made. Fujita and Thisse (1996) distinguish three groups of theories explaining agglomeration of production activities: Marshallian externalities, general equilibrium increasing returns to scale models, and partial equilibrium spatial competition models.

Marshallian externalities build up on information spill-overs, large local specialized labour markets, and specialized non-tradable inputs (Marshall, 1920). Spatial competition models can be distinguished as shipping and shopping models. In Shopping models consumers bear the transport cost (Hotelling, 1929); in shipping models there is complete market segmentation of customers (Hoover, 1937, and Greenhut and Greenhut, 1975). Increasing returns to scale models use two model vehicles of imperfect competition: the first and most popular one is the Chamberlinian model of monopolistic competition - for short the Dixit-Stiglitz-Krugman model. The second is the Cournot oligopoly model (Venables, 1996). In this thesis only theories building upon monopolistic competition are considered. A survey is found in Krugman (1998) or the recent book of Fujita, Krugman and Venables (1998). The most important contributions within this class of models are briefly reviewed next.

Early contributions on this theme include Abdel-Rahman (1988), Fujita (1988), and Rivera-Batiz (1988) who model non-tradable intermediate inputs with increasing returns to scale and consumers enjoy the larger variety of non-tradable goods in big cities compared to small cities. Englmann and Walz (1995) are an endogenous growth version of these ideas.

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<sup>22</sup>A broad survey on increasing returns to scale and monopolistic competition is Matsuyama (1995).

However, the seminal paper is Krugman (1991a). This model uses the observation of the new trade theory explaining intra-industry trade (Krugman, 1980) that the region with the larger home-market has also higher wages, when there are transport cost. If workers migrate to the region with the larger home-market, firms in the smaller region cease to exist and new firms enter in the bigger region to employ the additional workers. Wages become still higher in the larger region attracting even more workers. Therefore, the model provides a microfoundation of Myrdal's (1957) cumulative process. Firms locate close to the consumer market (forward linkage) and worker-consumers move close to firms.<sup>23</sup> Agglomeration of firms is not the only possible outcome of the model. Since there is also an agricultural sector with immobile farmers, there is also some localized domestic demand in the smaller region which may be sufficient to pull back some of the firms. It depends on the relative strength of the home-market effect, the extent of competition effect, and a price index effect<sup>24</sup>, whether firms agglomerate or disperse equally across the two regions. It can be shown that lower trade costs trigger agglomeration by widening the wage differential. If trade costs are lowered further the wage differential narrows again and factor price equalization holds, whenever trade costs are zero.

Since labour is internationally immobile, a different agglomeration process is suggested by Krugman and Venables (1995a) for countries. Starting from two identical countries producing both manufacturing and agricultural goods, one economy may bifurcate to specialize in manufacturing and the other in agriculture, if trade costs are lowered and labour is mobile inbetween the two sectors within a country.

Since one of the most striking empirical observations is that the US regions are highly specialized in different manufacturing industries, whereas Europe is not, Krugman and Venables (1996) present a 2 region model with 2 IRS manufacturing sectors, and labour is mobile across the sectors, but not across the regions. Then the

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<sup>23</sup>In this sense historical accident matters, whether say a region captures a lot of manufacturing. Krugman (1991b) and Matsuyama (1991) note that optimistic or pessimistic expectations about the future may also lead to self-driven agglomeration processes.

<sup>24</sup>These effects will be explained in detail in chapter 3. The seminal paper remains vague about them to the extent that they are not related to the analytics of the model.

two regions specialize, if trade costs are low (like in the US), or remain dispersed, if trade costs are high (like in Europe).

If there are more than two regions, a hierarchy of agglomeration phenomena may occur explaining the hierarchy of cities (Krugman and Fujita, 1995).

A backward linkage is introduced by Venables (1996) as additional agglomeration force into the framework with forward linkages - namely: tradable intermediate inputs produced with increasing returns to scale technology. A large market for final good producers provides also a large market for intermediate goods. Hence, intermediate good producers locate close to final good producers who locate close to the largest final goods market. In this way intermediate inputs re-enforce agglomeration.

Martin and Ottaviano (1996) explain agglomeration by R&D location decisions in an endogenous growth model. Audretsch (1998) shows empirically the importance of R&D location for building agglomeration centers in the high-tech industry.

Krugman and Venables (1995b) provide a continuous space version arguing that countries as natural units of observations vanish and international trade shall be viewed as trade across space. This framework allows to discuss the density of city distribution in space by measuring the wave length of the agglomeration peaks.

Applications of the seminal model discuss policy issues like public infrastructure investment (Martin and Rogers, 1995), or locational implications of customs unions and hub and spoke agreements (Puga and Venables, 1995). Puga and Venables (1996) use this model framework to explain, why industrialization did not spread uniformly to the Asian countries, but started first in Japan, spilled over to the Tiger economies, and finally to China, as the share of manufacturing employment grew in the entire world in the course of history.

Krugman and Livas-Elizondo (1996) explain the emergence of metropolis in the third world such as Mexico city by import substitution policy. If trade barriers are large, manufacturing locates all at the capital, where the largest home-market is. If

trade costs are low, there will be more manufacturing for overseas markets locating closer to ports and border lines. Distribution of manufacturing is more dispersed.

Ricci (1998) argues in a 2-country model with monopolistic competition and stochastic demand and supply shocks that exchange rate flexibility increases specialization, because the country that is specialized in the sector that faces a positive shock appreciates its exchange rate and dampens the volatility of firm profits. This in turn attracts new firms.

Summing up, it is fair to say that all models rely on the same kind of mechanism based on monopolistic competition, increasing returns to scale, and trade cost. Therefore, it is crucial to have a closer examination of the seminal paper to understand the results of the others.

## **1.5 Contributions of This Thesis**

This Ph.D. thesis consists of 4 subsequent chapters which are each self-contained, but successively building upon each other. Each of the chapters addresses a certain scientific problem of the existing literature mentioned above.

Chapter 2 and 3 are complementary chapters. They originate from the fact that the standard economic geography model (Krugman, 1991) is not analyzed in the formal rigorosity that is found in traditional trade theory. Instead of solving the model backward by using expenditure functions and revenue functions like in duality theory to explore terms of trade effects which in turn determine factor prices, the model is solved differently. The question is posed, whether a firm can afford to pay higher real wages in a region that has no manufacturing. If they can, it is concluded that firms do not agglomerate. If they cannot, then it is concluded that complete agglomeration of manufacturing is an equilibrium. This is neither a complete analytical solution to the model, nor does it reveal clearly the mechanics of the model.

### **Chapter 2:**

Chapter 2 discusses the mechanics of the model by looking at the excess demand

system and discussing, how a movement of workers from one region to another affects relative prices, and how this feeds through on relative (nominal and real) wages. In looking at the model in this way, we reveal that the mechanics of the model are not unlike a pure exchange economy which is well understood in economics. This analogy helps forming a simple understanding of the Krugman (1991a) model. The developed simple graphical apparatus allows to present the model and its mechanics in an undergraduate textbook.

### **Chapter 3:**

Chapter 3 provides a complete analytical solution of the Krugman model (1991). The analysis allows to prove the existence of a poverty trap case at an intermediate level of transport cost not mentioned in the seminal paper. Whenever there are sufficiently many firms in one region, firms will tend to locate even in the plane. Whenever there are less firms than a certain threshold level, even the remaining firms will leave and complete agglomeration is the outcome. This phenomenon helps explain, why economic integration of well developed EC-countries triggered income convergence (Ben-David, 1993), and why unification of West- and Eastern Germany ended up with a massive break-down of East-German manufacturing.

Whereas Krugman (1998, p. 11) claims that “... *despite the best efforts of the theorist, all but the simplest models of economic geography usually turn out to be a bit beyond the reach of paper-and-pencil analysis*”, my solution technique can be generalized to many geography and trade models.

### **Chapter 4:**

Chapter 4 integrates economic geography (Krugman, 1991a) into a neoclassical (exogenous) growth framework. I show that such a model captures the ideas of poverty traps and cumulative processes of Myrdal (1957) and Kaldor (1966). I also show that two regimes are possible: a neoclassical growth regime and a poverty trap regime. In the neoclassical growth regime any two countries with identical structural characteristics except a different initial capital stock start converging to the same steady state

income level and an equal distribution of firms. In the poverty trap regime, any two countries start diverging in income level and manufacturing firms agglomerate incompletely. This is my simple solution to the puzzle posed in section 1.1. The question is not, whether there is income convergence or manufacturing agglomeration, but, when there is income convergence (neoclassical regime) and when there is manufacturing agglomeration (poverty trap regime).

Firm agglomeration occurs in this model not due to factor flows but through internal growth in a region. Trade liberalization is shown to eliminate the poverty trap and agglomeration pattern which is just opposite to what Krugman (1991a) suggests. I obtain this result, because I exchange the convergence forces. Instead of an immobile, specific factor, constant returns to scale farming sector, I use capital-labor substitutability as convergence force in this model. This shows similar to Davis (1998) that the role of the farming sector may cause robustness problems, if slight assumption changes compared to the Krugman (1991a) model are undertaken.

## Chapter 5

Chapter 5 tests empirically an endogenous growth version of the model displayed in chapter 4 using time series data for Japan and the US. The testing procedure is distinguished by two properties: 1.) the testing procedure is designed to specifically test the mechanics of the Krugman-type models: Chapter 2 has revealed that the mechanics rely entirely on a terms-of-trade effect. Therefore, one can consider this chapter as an indirect test<sup>25</sup> of the terms-of-trade effect driving Krugman type agglomeration processes. 2.) It is shown that a linearized growth model version of Krugman (1991a) yields a reduced rank hypothesis of a vector error correction model which can be tested using co-integration analysis (Johansen, 1988, 1991). In the Bretton Woods era with fixed exchange rates, I find that income convergence in the sense of cointegration and terms-of-trade effect are not significantly present. The Japanese and the US economy grow independently from each other with Japan being on a faster

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<sup>25</sup>The test is indirect in that we use income data as proxy for “firm agglomeration”, since there is no obvious and unambiguous measure for the latter variable. The theoretical model justifies the use of the income data.

growth path. In the Post-Bretton Woods era with flexible exchange rates, I find that income convergence and terms-of-trade effect cannot be rejected, if one expects oil shocks or other crises to occur in the future.

Possible reasons for the weakness of the evidence may be that 1) my data were not sufficiently disaggregated, 2) the growth link - improved terms of trade - does not feed through on savings and capital accumulation, if capital markets are integrated, 3) Japan and the US are too distant from each other (and trade volume is too small) for agglomeration forces to become measurable, 4) exchange rate interventions and exchange rate bubbles may have prevented the terms of trade measure to follow its “fundamental value”.

## 1.6 Open Questions

Many features of the Krugman-Dixit-Stiglitz model can be criticized. Some features rely on implausible assumptions used for the sake of analytical simplicity: the assumption of iceberg-type transport cost implies that a rise in final goods prices does also rise the transport cost (Fujita and Thisse, 1996). The number of farmers is normalized to be equal to the income share of agriculture. Hence, the isolated impact of these two factors on agglomeration cannot be discussed. Factor specificity of the farming sector and manufacturing do not comply with the mass migration of farmers into cities in the beginning of the age of industrialization. Yet, this assumption may be crucial to constitute a convergence force.

Some features are of a methodological sort: specific functional forms and lack of generality (Fujita and Thisse, 1996); The Marshallian migration process of this model lacks a proper micro-foundation (Ottaviano and Thisse, 1998). There is some unrobustness of slight assumption changes: Davis (1998) shows that transport cost on agricultural goods may revert the home-market effect; Chapter 4 of this thesis argues that the exchange of convergence forces (diminishing returns of capital instead of an immobile farming sector) reverses the role of transport cost. Agglomeration suddenly

occurs, when trade costs are high rather than low.<sup>26</sup> Since these observations are counter-intuitive to what has been argued informally in section 1.2, one can take it as an indication that the Krugman model does not quite capture the mechanics that are described in this section, although it captures most of the predictions.

Some features are of a deeper economic deficiency: The agglomeration force of the Dixit-Stiglitz-Krugman model does not support capital flows to form agglomeration centers, if 1) there is capital-labor substitutability, and 2) capital gains are repatriated. The reason is obvious: if capital flows to a region, more varieties are produced, but the extra income generated flows back to the other region and is largely spent there. Hence, prices and rental rates fall in the region that attains additional capital and capital has an incentive to flow back to its origin. Yet, capital movements are at the heart of every firm relocation. Not labor moves to a region setting up a new business, but a firm invests into a production plant employing local labor.

Along with globalization, (i.e. a reduction of trade cost boosting international trade volume), manufacturing relocates towards the periphery as predicted by chapter 4 of this thesis, not towards the center as predicted by the Krugman-Dixit-Stiglitz model (Carson, 1998). Equivalently, Kim (1995) reports that specialization occurred in the US in the second half of the 19th century, when significant falls of transport cost due to the establishment of a national railway system happened. But specialization decreased thereafter. This may support the view that at low transport cost diffusion of manufacturing distribution occurs contrary to the predictions of Krugman (1991a). Also, relocation is driven by cheap labor which indicates that capital-labor substitutability or low-skill/ high-skill substitutability which is dismissed for the sake of simplicity in most of the economic geography models<sup>27</sup> may play a role. All three stylised facts are captured by chapter 4 of this thesis rather than by the original Krugman (1991a) model.

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<sup>26</sup>This result also shines through in Puga (1998). His model contains the agricultural sector and land-labour substitutability. With intersectoral mobility of labour high and low trade costs induce convergence of manufacturing distribution, whereas agglomeration occurs at an intermediate level of trade cost.

<sup>27</sup>An exemption is Puga (1998).



Some features are simply not fully explored: a complete welfare analysis and an analysis of welfare enhancing measures is still missing. For example the question remains, whether agglomeration is good or bad? If agglomeration is bad, does it happen? Only this constellation would justify policy measures to support the poor region. It may instead be that convergence happens, if agglomeration is good justifying the subsidization of the rich region.<sup>28</sup>

Some features require further exploration by supplementing the existing framework of analysis: How does specialization in a 2 country model with 2 manufacturing sectors affect welfare, if there are different rates of exogenous technological progress of these two sectors. If specialization is spontaneous, and not based on endowments or technologies, does such a model justify a subsidization war for the high-tech industry?

The production technology lacks a micro-foundation: If there are physical IRS in the production technology explaining why manufacturing firms employ many workers, whereas service firms employ substantially less, then social control and free rider problems may offset the advantages of scale economies giving rise to a U-shaped average cost curve. Consequently, agglomeration forces are only present, when firms have not grown to their optimal firm scale, yet. This could explain, why agglomeration phenomena are most striking in young and growing industries such as Silicon Valley or the Bio-tech industry in California. Also, most of the agglomeration phenomena occurred at the beginning of industrialization like the emergence of the large towns New York and Chicago (Kim, 1995).

The Dixit-Stiglitz-Krugman model relies on complementarities generated by the endogeneity of product space. I argued in section 1.2, the most obvious effect of IRS is a cost reduction due to larger scale operation possible in the region with a larger home-market. Kim (1995) shows that firm scale can explain localization and specialization patterns of US manufacturing over the course of history. Also firm scale is an important variable explaining the relative growth performance of European manufacturing (Fingleton and McCombie, 1998). This effect of IRS does not show up

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<sup>28</sup>Preliminary simulations suggest that this case is a possibility in the seminal model.

in the Dixit-Stiglitz-Krugman model, because firm scale is independent of the size of the home market due to the specific functional forms chosen.

Only recently, Ottaviano and Thisse (1998) have developed a framework, where output of a firm depends on market size. They largely confirm the stylized facts produced by the Krugman (1991a) model - but the mechanics of the model is different: Absolute number of firms in both regions, relative distribution of firms, and the degree to which competition of foreign firms is sheltered on a local market (transport cost) determine demand elasticities in the two countries and monopoly price mark-ups. A rise in market size may thus increase competition, rise output of a single firm and thus lower average cost.

To find out which of the two mechanisms - product variety or scale - dominate in practise, one may want to test, whether an increase of wages in an industry goes always along with a rise in prices (Dixit-Stiglitz-Krugman model and neoclassical trade theory) or prices fall, although wages rise (scale effect as described in section 1.2).

Finally, Venables (1996) productivity gains of specialization may be interesting to be modelled in a 2 country growth model. Such a growth model captures the ideas of Kaldor's (1966) 4-stage growth theory. Germany and Japan may have specialized in exports of manufactures, because both countries developed a large market of specialized machinery producers. Because of access to specialized machinery, the manufacturing sectors in both countries gained a comparative advantage. Acceleration processes, as described in section 1.2, may then have contributed to the extraordinary growth performance of the two countries after World War II.<sup>29</sup>

Summing up, the above considerations show that this thesis rather rises new questions, than delivers a final answer to the puzzles of growth and economic geography.<sup>30</sup>

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<sup>29</sup>Barro and Sala-i-Martin (1995) argue that the destruction of physical capital should yield a rapid catch-up in a model with human capital, but this does not explain why 1) Japan and Germany performed better than, say the UK and many other developed countries up to the 70ies and 2) why Japan and Germany ended up with larger shares of manufacturing sectors compared to other developed countries.

<sup>30</sup>Krugman (1998) comes to a similar evaluation on economic geography.

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## 2 Understanding Geography and Trade\*

### Abstract

This paper provides a simple graphical exposition and a rigorous analytical method for monopolistic competition, increasing returns to scale, geography and trade models with transport costs which explain agglomeration or convergence of industries. In the main text, the agglomeration and convergence forces are graphically displayed, whereas the appendix provides the analytical treatment of the model. New light is shed on the Dixit-Stiglitz-Krugman model by an analogy to a heterogeneous agent pure exchange economy.

**JEL Classification:** F12, R60.

**Keywords:** convergence, agglomeration, poverty trap.

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\*The author thanks Kjetil Bjorvatn, Niels Blomgren-Hansen, Maria Lanzeni, Pascal Raimondos-Møller, and participants of the EEA conference in Istanbul 1996, and of seminars at Copenhagen Business School, Copenhagen University, PennState University, and Uppsala University for their comments. Usual caveats apply.



## 2.1 Introduction

The economic geography literature employs several “workhorses”. The most prominent one is perhaps the Dixit-Stiglitz-Krugman monopolistic competition increasing returns to scale model with transport costs explaining industry agglomeration in space. There are two general versions of it: the regional economic model by Krugman (1980,1991) and the international trade model by Krugman and Venables (1995). The regional economic model assumes migration of workers between regions dragging industries with them. Across nations, however, labour is less mobile. Hence, the international trade model assumes that labour is intersectorally mobile and agglomeration of industries occurs, if labour moves from a constant returns to scale agricultural sector to an increasing returns to scale manufacturing sector. However, the models are otherwise identical in structure and the driving forces for the different kind of labour mobilities are very similar, too.

These models have two complications. They are non-linear in structure and may entail multiple equilibria. Hence, most of the analysis is numerical with some suggestive analytical treatments. Rigorous algebra is in particular important for defining conditions for which an even distribution of workers and industries turns into an uneven one (agglomeration condition). Krugman (1991) gives such a condition for total agglomeration, Venables (1995) and Krugman and Venables (1995) give a condition, for which the equal distribution is not a stable equilibrium (“algebra of symmetry-breaking”).

Still, the analysis is far from both the rigour and the economic intuition of the driving effects which is for example provided in the Heckscher and Ohlin type of economic geography models.<sup>31</sup> I provide in chapter 3 of this thesis a complete analytical solution to the Krugman (1991) model and detect a “poverty trap” that was overlooked by the seminal article. Because chapter 3 does not provide any economic

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<sup>31</sup>See, for example, Norman and Venables (1995).

intuition for its results, it is supplemented by this companion paper giving a simple graphical exposition of the model.

The graphical exposition gives a clear-cut intuition for the agglomeration and convergence forces embedded in the model, whereas the analytics of the seminal paper do not reveal this intuition. In particular, the seminal paper argues on the basis of the home market and the extent of competition effect. We confirm the mechanics of the home market effect, but show that the extent of competition effect is better described as a regional composition effect of goods (number of goods effect). Indeed, the model deviates from a standard neoclassical model only because of the endogeneity of product space. The graphical apparatus provides an exposition of the seminal model suitable for undergraduate classes.

A need for a clarification of the mechanics of the Krugman (1991) model may also be derived from Davis (1998). He shows that a minor change in the model set-up - transport cost for agricultural goods - undermines the home market effect unexpectedly. Unfortunately, his formal proof - though elegant - adds little to the understanding of the mechanics of the model. Instead of his proof by contradiction, we follow the standard approach in general equilibrium theory focusing on the excess demand system and the comparative static effects of worker migration on relative prices and wages. This approach allows us to compare the Krugman increasing returns to scale model directly with a standard neoclassical pure exchange economy.

The analogy to a pure exchange economy sheds new light on the interpretation of trade costs and region size in the model. The analogy may suggest a rethinking of the role of increasing returns in the Dixit-Stiglitz-Krugman model. This is somewhat in contrast to Fujita and Thisse (1996) who attribute a major role in the explanation of agglomeration economics to increasing returns. Whereas we will show that the mechanism of the Dixit-Stiglitz-Krugman model is analogous to a pure exchange economy with two types of heterogeneous agents and three goods with the following properties: 1) each type of consumer is equally endowed with one good, owes the world endowment of the second and nothing of the third. 2) The consumer strictly prefers

the good he is well endowed with to the good he is not endowed with. Furthermore, we obtain the corollary that agglomeration occurs in the Dixit-Stiglitz-Krugman model even *without* transport cost, if a parameter of preference bias towards domestically produced manufacturing goods is introduced into the utility function.

The main text gives an almost entirely graphical treatment to facilitate intuition, whereas the appendix contains the analytical treatment of the model. Section 2 repeats the familiar model set-up for convenience; section 3 gives the equilibrium conditions; section 4 draws an insightful analogy to a pure exchange economy; section 5 explains why there is agglomeration or convergence; and section 6 concludes.

## 2.2 The Model Set-Up

In this section, the basic structure of a typical geography and trade model as developed by Krugman (1991) is presented. The model has two regions with one increasing returns to scale sector (industry) and one constant returns to scale sector (agriculture) in each region. The increasing returns to scale sector is monopolistically competitive. Furthermore, transport costs for industrial goods introduce a geographical dimension into the model.

There are two types of consumers  $j = 1, 2$ , which are only different by their place of residence. Home region's consumers are indexed by 1, foreign region's consumers by 2. Regions are defined as areas for which it is costless to trade industrial goods within them, but costly to trade industrial goods across the border. Furthermore, there is no short run mobility of production factors across borders. However, a long run mobility of labour across borders is considered.

The two types of consumers  $j$  have identical Cobb-Douglas utility functions of the form

$$U_j = C_{Mj}^\mu C_A^{1-\mu}, \quad 0 < \mu \leq 1, \quad (1)$$

where  $C_A$  is consumption of the agricultural good produced with constant returns to scale and  $C_{Mj}$  is an aggregate basket of industrial goods produced in both regions under monopolistic competition and increasing returns to scale. The industrial goods basket  $C_{Mj}$  is further specified by a Dixit-Stiglitz (1977) subutility function:

$$C_{Mj} = \left[ \sum_{i=1}^{n_1+n_2} c_{ij}^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}, \quad \sigma > 1. \quad (2)$$

The demand of consumer  $j$  for a single industrial firm  $i$ 's product is denoted  $c_{ij}$ . There are  $n_1$  firms in the home region and  $n_2$  firms in the foreign. The number of firms is assumed to be sufficiently large. The elasticity of substitution between the industrial goods is denoted  $\sigma$ .

There is factor specificity for industrial production by workers and agricultural production by peasants. Peasants work according to a constant returns to scale technology in perfectly competitive markets. The price for agricultural products serves as numeraire and price equals wage.

Industrial workers have an increasing returns to scale technology of a simple structure: there is a fixed cost  $\alpha$  and a constant marginal cost  $\beta$  for each firm  $i$ . The firm  $i$  uses  $L_{Mi}$  units of labour for producing  $x_i$  goods:

$$L_{Mi} = \alpha + \beta x_i. \quad (3)$$

Every firm produces a different variety in order to exploit potential monopoly profits. In equilibrium, firms will not succeed, however, because free costs of firm entry and exit will assure zero profits. Because firms are assumed to be symmetric, we drop the firm index  $i$  for convenience. However, we will use the index  $i = 1, 2$  in order to distinguish the home and the foreign firms, respectively.

The number of industrial workers in region 1,  $L_1$ , and in region 2,  $L_2$ , are for simplicity assumed to add up to  $\mu$ :<sup>32</sup>

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<sup>32</sup>See Krugman (1991), footnote 1, for a justification of this assumption.

$$L_1 + L_2 = \mu. \quad (4)$$

Without loss of generality, I define the domestic region (region 1) to be the smaller one, i.e. there are less industrial workers than in the foreign region (region 2). The total amount of peasants is  $1 - \mu$ , they are assumed to be equally distributed in both regions, and they are not mobile. Every worker and peasant supplies one unit of work and earns a salary 1, if peasant, and  $w_i$ , if worker in region  $i$ .

Finally, there are trade costs of the Samuelson iceberg-type  $\tau$ , such that only a fraction  $\tau$  of one produced unit of an industrial good arrives at its foreign destination ( $0 < \tau \leq 1$ ). There are neither trade costs for goods delivered to domestic customers, nor trade costs for agricultural goods.

## 2.3 Equilibrium Conditions

In this section, we state some well-known economic relationships that stem from firm optimization, consumer optimization, the zero-profit condition, the labour market equilibrium condition and the goods market equilibrium conditions.

Given the usual assumption that the firm takes into account the impact of its pricing decision on its own demand, but not on other firms' pricing decisions, then the well-known pricing rule for the firm holds:<sup>33</sup>

$$p_i = \gamma \cdot \beta \cdot w_i, \quad (5)$$

where  $\gamma \equiv \sigma / (\sigma - 1)$ , and  $p_i$  denotes the mill price of region  $i$  firms' goods<sup>34</sup>. Prices are constant mark-ups over wages due to the assumptions of constant elasticities of substitution and constant marginal cost. Prices and wages are proportional. An

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<sup>33</sup>This equation is only an approximation which is fairly good for a large number of firms. (It does not imply that  $n_i$  has to be large, because  $n_i$  is normalized to number of firms per country population.) See the discussion in Yang and Heijdra (1993), Dixit and Stiglitz (1993), and d'Aspremont et al. (1996).

<sup>34</sup>The pricing decision for export goods  $p_i^{Export}$  requires the firm to demand the domestic price plus the additional transport cost:  $p_i^{Export} = \gamma \cdot \beta \cdot w_i / \tau$ ,  $i = 1, 2$

increase in wages drives up prices and vice versa. Therefore, prices and wages can be used interchangeably, henceforth.

The optimal output of the firm is known to be determined by the zero profit condition:

$$x_i = \frac{\alpha \cdot (\sigma - 1)}{\beta} \equiv \bar{x}, \quad (6)$$

The equilibrium output of the firm is independent of the number of workers or the number of firms in a region. In fact, it is an exogenously given constant  $\bar{x}$ . All the interesting effects, which drive the agglomeration or convergence process stem from the demand side. This is again a result of the simplifying assumption of constant marginal cost.<sup>35</sup>

The equilibrium number of firms per region follows from the labour market clearing conditions and the output decision:

$$n_i = \frac{L_i}{\alpha \sigma}. \quad (7)$$

This is the third important economic relationship to be kept in mind. If the number of workers increases in a region, workers drag industries with them, and the number of goods increases proportionally.

Since profits are zero, aggregate income in a region is the sum of the income of all workers and peasants in that region:

$$y_i(w_i, L_i) = w_i \cdot L_i + \frac{1 - \mu}{2}. \quad (8)$$

This implies also that the wage bill of all firms in a region equals the sales of all firms in that region.

$$y_i(p_i, n_i) = p_i \cdot n_i \cdot \bar{x} + \frac{1 - \mu}{2} \quad (9)$$

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<sup>35</sup>Hence, one effect, which one might think of, is missing in the model: the larger region does not have larger firm sizes and hence lower production cost under increasing returns to scale. See Krugman (1980), footnote 3, on this issue.

These economic relationships are useful, because every behavioural equation of the model can be interchangeably expressed in terms of the two prices  $p_i$  and the number of goods per region  $n_i$  or equivalently in terms of the two wages  $w_i$  and the labour distribution  $L_i$ <sup>36</sup>. Hence, it will be completely sufficient to describe the model in terms of prices and number of goods which will be determined by the goods market equilibrium conditions. Another example is the CES-price index  $P_j$  that can be written in both ways as function of prices and number of goods or wages and the worker distribution:<sup>37</sup>

$$P_j = P_j(p_1, p_2, n_j) = P_j(w_1, w_2, L_j). \quad (10)$$

An increase in the number of domestic goods will lower the price index even at given prices, because there will be less goods to pay transport costs for than before ( $\partial P_j / \partial n_j < 0$  and  $\partial P_j / \partial L_j < 0$ ). With these ingredients, a short run equilibrium can be defined as an equilibrium of the goods market, the labour market and zero firm profits at a given distribution of labour. Such an equilibrium can be found as the solution of the excess demand curves of the domestic firms  $f(p_1, p_2, n)$ , the foreign firms  $f^*(p_1, p_2, n)$ , and the agricultural sector  $g(p_1, p_2, n)$ <sup>38,39</sup>:

$$\begin{aligned} f(p_1, p_2, n) &= D(p_1, P_1) \cdot y_1 + \frac{E(p_1, P_2) \cdot y_2}{\tau} - \bar{x} = 0 \\ f^*(p_1, p_2, n) &= D(p_2, P_2) \cdot y_2 + \frac{E(p_2, P_1) \cdot y_1}{\tau} - \bar{x} = 0 \\ g(p_1, p_2, n) &= (1 - \mu)(y_1 + y_2) - (1 - \mu) = 0, \end{aligned} \quad (11)$$

where

$$D(p_i, P_i)$$

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<sup>36</sup>From now on  $L$  denotes the amount of domestic workers and  $\mu - L$  the amount of foreign workers. Respectively,  $n$  denotes the number of domestic goods, whereas the number of foreign goods is given by:  $\frac{\mu}{\alpha\sigma} - n$ .

<sup>37</sup>The explicit functional forms are given in appendix A.

<sup>38</sup>The agricultural sectors can be merged into one equilibrium condition, because there are no transport cost for agricultural goods.

<sup>39</sup>The explicit functional forms of the equation system (11) are given in appendix B.

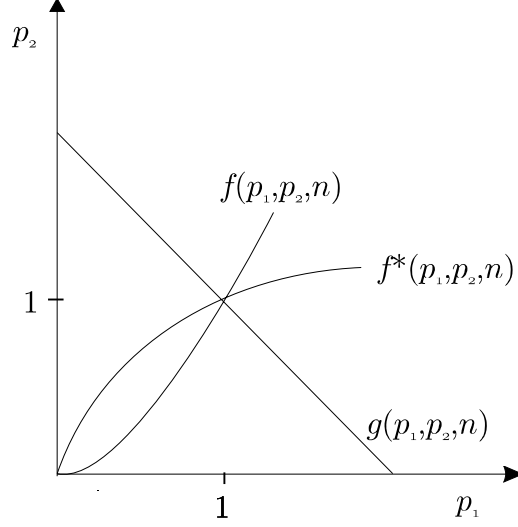


Figure 1: Short Run Equilibrium

describes the fraction of region  $i$ 's income spent on region  $i$ 's firms and

$$E(p_i, P_j), i \neq j$$

describes the fraction of region  $j$ 's income spent on imports from region  $i$ 's firms.<sup>40</sup> The home regions' income  $y_1$  and the foreign regions' income  $y_2$  are defined according to equation (9), and the price indices are defined according to equation (10). These functions describe the excess demand in the three goods markets. The two industrial goods excess demand functions add up domestic demand and exports of a firm and subtract its output. The agricultural goods demand is a constant fraction of world income, whereas supply equals the number of peasants.

Urban (1996) proves the uniqueness of the equilibrium of the system (11). Therefore, we can depict the equilibrium of the goods markets in figure 1. Figure 1 shows the three implicit functions  $f(p_1, p_2, n)$ ,  $f^*(p_1, p_2, n)$  and  $g(p_1, p_2, n)$  in the  $p_2$ - $p_1$ -space for a given  $n$ . The three schedules show the equilibrium in the domestic industrial goods market, the foreign industrial goods market and the market for agricultural products, respectively. The intersection of the three curves is the short run

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<sup>40</sup>The explicit definition for  $D(p_i, P_j)$  is:

$$D(p_i, P_i) = \frac{\alpha \sigma p_i^{-\sigma}}{P_i^{1-\sigma}}$$

and the explicit definition for  $E(p_i, P_j)$  is:

$$E(p_i, P_j) = \frac{\alpha \sigma (p_i/\tau)^{-\sigma}}{P_j^{1-\sigma}}$$



equilibrium. One of the equations is redundant due to Walras law. I drop the excess demand function for the foreign industrial goods market  $f^*(p_1, p_2, n)$ .

The equilibrium condition for domestic firms ( $f$ -schedule) is upward sloping, because any increase in domestic prices for given foreign prices reduces demand for domestic industrial products ( $\partial f / \partial p_1 < 0$ )<sup>41</sup>. In order to restore equilibrium at a given constant supply, foreign prices must also rise (because  $\partial f / \partial p_2 > 0$ ).

The equilibrium condition for agricultural products ( $g$ -schedule) is downward sloping. A constant fraction of world income is spent on agricultural products. Supply is proportional to the number of peasants in the world and thus a constant in this model. If industrial wages in one region rise, world income is rising, thus raising demand for agricultural products at constant supply. In order to restore equilibrium, the industrial wages in the other region must fall, until world income is back at the original level. As mentioned above, prices follow wages.

The way economic geography papers are written does in our opinion not entirely clarify as to how the mechanics of the model operates. Before we analyze this system, we will depart from the previous model by setting up a completely different well-known microeconomic three goods and two heterogeneous agents pure exchange economy. This new set-up will lead to the same demand system as (11). It will be this analogy which sheds new light on the geography and trade models. In particular, it will contribute to the question, how to interpret trade costs and region size in this model.

## 2.4 What are Trade Costs?

As motivation for this section, we pose the question, whether a small open economy like Denmark in the vicinity of both Sweden and Germany is a big or a small country and whether such a country has a relatively high or a low parameter  $\tau$ . The answers to these questions will be crucial for deciding, whether Denmark is subject to an increase or a decline of the manufacturing sector according to the Dixit-Stiglitz-Krugman model

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<sup>41</sup>The rigorous mathematical derivation of the derivatives in this paragraph is part of appendix C.

We set up a well-known pure exchange economy and show that this problem yields the goods market equilibrium conditions (11) under the following two conditions: 1) the consumer  $j$  is equally endowed with one good, owes the world endowment of the second and nothing of the third. 2) The consumer strictly prefers the good he is well endowed with to the good he is not endowed with.

There are two heterogeneous types of agents  $j = 1, 2$  in a pure exchange economy with three goods  $x_1, x_2, X_A$ , where the total endowments of the economy are normalized to  $\bar{x}_1 \equiv \frac{\alpha(\sigma-1)n_1}{\beta}$ ,  $\bar{x}_2 \equiv \frac{\alpha(\sigma-1)n_2}{\beta}$ , and  $\bar{X}_A \equiv 1 - \mu$ . The endowments are distributed in the following way to the types of consumers: Consumer  $j$  is only endowed with good  $x_j$ , but not with good  $x_i$ , whereas  $X_A$ , the numeraire good, is equally distributed among the two types of consumers. This gives rise to the wealth constraint

$$y_j = p_j \bar{x}_j + \frac{1 - \mu}{2}, \quad (12)$$

where  $y_j$  is consumer wealth of all consumers of type  $j$ . Finally, the utility function for consumer  $j$  is given by the following expression:

$$U_j(c_{jj}, c_{ij}, C_{Aj}) = \left( \alpha_j c_{jj}^{\frac{\sigma-1}{\sigma}} + \alpha_i c_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\mu} C_{Aj}^{1-\mu}, j \neq i, \quad (13)$$

where  $c_{ij}$  denotes consumption of consumer  $j$  for good  $x_i$ ,  $C_{Aj}$  is consumption of consumer  $j$  for good  $X_A$  and

$$\alpha_j \equiv \left( \frac{n_j}{n_1 + n_2} \right)^{\frac{1}{\sigma}} \quad (14)$$

$$\alpha_i \equiv \left( \frac{n_i \tau^{\sigma-1}}{n_1 + n_2} \right)^{\frac{1}{\sigma}} \quad (15)$$

are some weighting factors in the utility function such that a consumer  $j$  prefers the good  $x_j$  to the good  $x_i$ . In other words, the consumer prefers the good which she owns to the good that has to be bought from the other type of consumer (bias of preferences towards domestic goods). We can think of the following interpretation:

the two consumers live in different regions and the preferences are biased towards the domestically available goods. The parameter  $\tau$  is then a proxy for the degree to which preferences are biased towards domestic goods. The lower the  $\tau$ , the stronger are domestic goods preferred. Additionally, the weighting factor includes also the size of the country as proxied by the number of goods in the Krugman (1991) model. To yield the same set of equilibrium prices (and wages) country size can be traded off with the degree of preference bias. For instance, China can afford to have more specific tastes than Denmark, and can still achieve higher relative prices (and wages).

The utility function (13) is maximized according to the budget constraint:

$$c_{1j}p_1 + c_{2j}p_2 + C_{Aj} = y_j \quad (16)$$

It is straight forward to show that exactly the demand system (11) emerges.<sup>42</sup> It is this analogy that sheds new light on the Dixit-Stiglitz-Krugman model. The immigration of a worker and the corresponding increase of the number of goods  $n_j$  in the Dixit-Stiglitz-Krugman model acts as if there is an increase in the bias of demand towards domestic goods  $a_j$  (see equations (14) and (15)). If there is a stronger preference for goods of consumer  $j$ , then prices  $p_j$  are rising. In the Dixit-Stiglitz-Krugman model, the rise of prices feeds through on wages. In this sense, the analogy helps to form a simple intuition for the price effects of worker migration.

Further, the analogy suggests that the intuition is somewhat in contrast to Fujita and Thisse (1996) who attribute a major role in the explanation of agglomeration economics to increasing returns, whereas we achieve similar effects in a neoclassical pure exchange economy. The production side of the Dixit-Stiglitz-Krugman model seems not so important for its effect on agglomeration economics than the demand

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<sup>42</sup>The analogy holds only for the short run equilibrium of the model in Krugman (1991). The long run equilibrium could be replicated by our pure-exchange economy, if preferences are endogenous. For example, some type 1 consumers turn into type 2 consumers in the next generation (next time period), if type 2 experiences higher utility in the short run equilibrium today. These consumers will not only change their preferences, but also the endowment. This simple example shows that the application of the Krugman (1991) model is not restricted to economic geography, but also applies to heterogeneous social groups within one economy.

side.<sup>43</sup>

Additionally, we obtain the corollary that the Dixit-Stiglitz-Krugman model will yield the same results, if transport costs are assumed zero ( $\tau = 1$ ) and the utility function (2) is replaced by:

$$C_{Mj} = \left[ \sum_{i \in N_j} c_{ij}^{(\sigma-1/\sigma)} + \sum_{i \in N_k} \alpha c_{ij}^{(\sigma-1/\sigma)} \right]^{\sigma/(\sigma-1)}, \quad \sigma > 1, \quad (17)$$

where  $N_j$  denotes the set of firms in country  $j$ ,  $N_k$  denotes the set of firms in the other country, and  $\alpha \equiv \tau^{(\sigma-1)/\sigma} < 1$  is a parameter of preference bias towards domestically produced manufacturing goods.

Now, we return to the question, whether a small open economy like Denmark is a big or a small region. On the one hand, if trade costs are taken literally to be transport costs as Krugman (1991) seems to suggest, then Denmark is a big region. This is so, because Denmark has a relatively high population density. I.e. a firm finds a relatively large market in its surroundings.<sup>44</sup> Furthermore, Denmark should not be considered a homogeneous region independent of South Sweden and North Germany, because transport costs e.g. from Kolding to Flensburg are not substantially different from transport costs from Kolding to Skagen.

On the other hand, if trade costs are interpreted as a parameter of preference bias towards domestic goods, then the question arises, how different are Danish tastes, relative to European tastes. Clearly, Denmark should then be regarded as a homogeneous region with a unique cultural background and relatively homogeneous tastes inside, but (maybe) somewhat different tastes relative to people in other countries. For example, Danish books are strictly preferred by Danish people relative to, say, Swedish people (who can read Danish books with some inconvenience). Furthermore, Denmark would be a small region, because the absolute number of inhabitants is the relevant measure of region-size in this case.

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<sup>43</sup>What matters in the Krugman-Dixit-Stiglitz model is the endogeneity of product space (see Matsuyama, 1995).

<sup>44</sup>This statement requires just one qualification. Transport costs are higher for the islands. Still, it is not obvious that products are more expensive, for example, in Copenhagen than in Kolding.

After having clarified some interpretations of the model, we will return to the original set-up of sections (2.1) and (2.2), analyze the system (11), and clarify the agglomeration and convergence forces in the model.

## 2.5 Why is there Agglomeration or Convergence?

Suppose for the moment that a worker changes for some arbitrary reason her residence. What is going to happen to the wages and prices? If wages rise in the immigration region and fall in the emmigration region, there will be an incentive for more workers to follow (agglomeration). If wages fall in the immigration region and rise in the emmigration region, there will be an incentive for this worker to go back (convergence)<sup>45</sup>. Because of equation (5), wages are proportional to prices. Hence, we have to examine, how prices change, if a worker moves thereby changing the regional distribution of firms and goods. We will begin with the impact of a movement of a worker on the equilibrium condition for domestic industrial goods.

If an industrial worker moves from the foreign to the domestic region at a given level of prices and wages, then the demand-change for domestically produced industrial goods will be described by the following expression:

$$\begin{aligned} \frac{\partial f}{\partial L} = & w_1 D(p_1, P_1) - w_2 \cdot \frac{E(p_1, P_2)}{\tau} \\ & + y_1 \frac{\partial D(p_1, P_1)}{\partial P_1} \cdot \frac{\partial P_1}{\partial L} + \frac{y_2}{\tau} \cdot \frac{\partial E(p_1, P_2)}{\partial P_2} \cdot \frac{\partial P_2}{\partial L}, \end{aligned} \quad (18)$$

where we made use of (9), (10), and (11). A movement of labour from the bigger to the smaller region has two effects on domestic demand for industrial goods.

*The income effect:* There is one worker more in the domestic region, who spends her income on domestic goods and hence there is one person less in the foreign region,

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<sup>45</sup>In the context of economic geography, convergence means the tendency of increasing returns to scale industries to allocate equally in plane, whereas convergence in the growth literature means the tendency of growth rates of GDP of poorer countries to be bigger than the one in richer countries (This is absolute convergence as defined by Barro and Sala-i-Martin, 1995) The two notions are interrelated, if increasing returns industries have higher technological progress than agriculture. An agglomeration process of industries would then also imply divergence of growth rates and a convergence process of industries would also mean convergence of growth rates.

who could spend some income on exports. Note that the price for the domestic good is reduced for this person due to the absence of trade costs in intra-regional trade. This means that this person now spends relatively more on the domestic good than before. This effect is captured in the first and second term of equation (18).

*The number of goods effect:* If a worker moves from the foreign to the domestic region, firms will relocate, too. Because there are now more firms in the domestic region and less in the foreign, the composition of the price index also changes. At given prices, domestic consumers have fewer import goods to pay transport cost for (and vice versa for the foreign consumer). This lowers the domestic price index for industrial goods and rises the foreign one. If the domestic price index is lowered, the relative price of domestic goods to the price index is increased. This decreases ceteris paribus domestic demand for products of domestic firms. The reverse holds for exports. This effect is captured in the third and fourth term of equation (18). Because these two effects are the key to the understanding of agglomeration and convergence in this model, we will repeat them looking from a different angle that coincides with the graphical exposition that follows in this section.

Suppose that a worker moves from the smaller domestic to the bigger foreign region (rather than vice versa as in the explanation above). On the one hand, the migrated worker increases total income in the foreign region and reduces total income in the domestic region. This typically reduces demand for the domestic firm, because the migrated worker buys less goods from the former home region. This is an agglomeration force, because it raises domestic prices and wages at given foreign prices and wages.

On the other hand, the decrease in the number of workers in the domestic region reduces the number of firms and goods produced. The domestic CES price index accounts for this effect by an increase, because the composition of the index changes towards foreign goods, which are more expensive because of the transport cost and because of the higher labour cost in the bigger foreign region. The increase in the domestic price index reduces the relative price for domestic goods. This increases

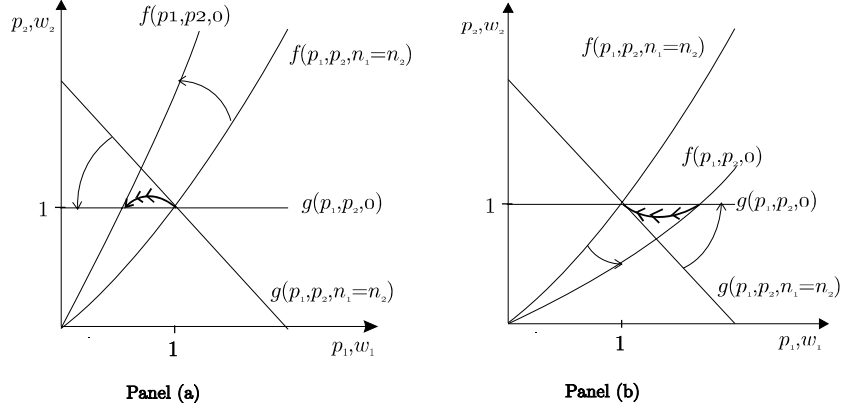


Figure 2: Comparative Statics of Worker Migration

domestic demand for industrial goods and is thus a convergence force in the model, because a decrease in domestic demand lowers domestic prices and wages at given foreign prices and wages. It is ambiguous, i.e. dependent on the parameters of the model, which force dominates ( $\partial f / \partial L \leq 0$ ). This can be demonstrated in figure 2. Figure 2 shows in panel (a) the agglomeration case. Suppose that starting from the equal distribution equilibrium, workers move out continuously from the domestic region making it the smaller region. If the emmigration causes a reduction in demand for domestic goods, this causes prices and wages in the domestic region to fall at given prices and wages in the foreign region. Hence, the  $f$ -schedule shifts leftward.

Additionally, the  $g$ -schedule twists anti-clockwise around the  $(w_1 = 1, w_2 = 1)$ -point. If wages in the domestic region are smaller, than the emmigration will cause a rise in world income which leads to excess demand in the market for agricultural goods. In order to restore equilibrium, domestic wages have to fall at given foreign wages. Both movements of the  $f$ - and the  $g$ -schedule lead to a fall of domestic wages relative to foreign wages. Hence, the emmigration is self-enforcing. The economy ends up at complete agglomeration ( $L=0$ ).

Figure 2 shows in panel (b) the convergence case. Now, emmigration causes a rise in domestic industrial goods demand which increases domestic prices and wages at given foreign prices. Hence, the  $f$ -schedule shifts rightward. If wages in the domestic region are higher than in the foreign region, then the emmigration causes a fall in

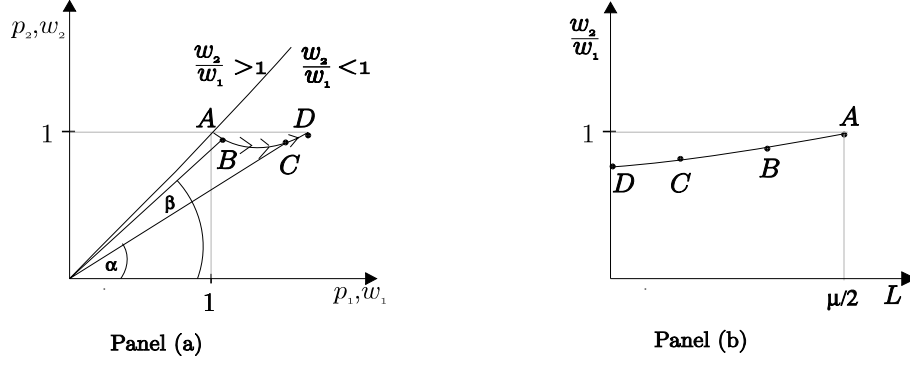


Figure 3: Short Run Equilibrium Condition

world income which leads to excess supply in the market for agricultural products. In order to restore equilibrium, domestic wages have to rise at given foreign wages. The movements of both schedules together induce a rise in domestic wages relative to foreign wages. Hence, the incentive to move out is reversed and the equal distribution equilibrium is stable in the long run.

The last conclusion can graphically be demonstrated more clearly in figure 3. Figure 3, panel (a) repeats the arrowed line from figure 2 (b). Panel 3 (b) is constructed from panel 3 (a) by drawing a ray through the origin to the equilibrium wage combination for a specific worker distribution. Taking the tangens of the angle of this array gives the relative nominal wage  $\left(\frac{w_2(L)}{w_1(L)}\right)$  at this labour distribution. Repeating this procedure for every possible labour distribution gives a schedule depicting the relative nominal wage as a function of the labour distribution. This curve is drawn in panel (b) and describes the **short run equilibrium condition**.<sup>46</sup> The points A,B,C and D correspond in the two panels of figure 3. The schedule is either upward or downward sloping.

Still, this is not the end of the story, because so far only relative nominal wages are considered. A migration decision is rather based on relative real wages. Let's look at the long run steady state condition: equal real wages.

<sup>46</sup>The mathematical derivation of this line is found in appendix D and denoted  $h(W, L)$ .



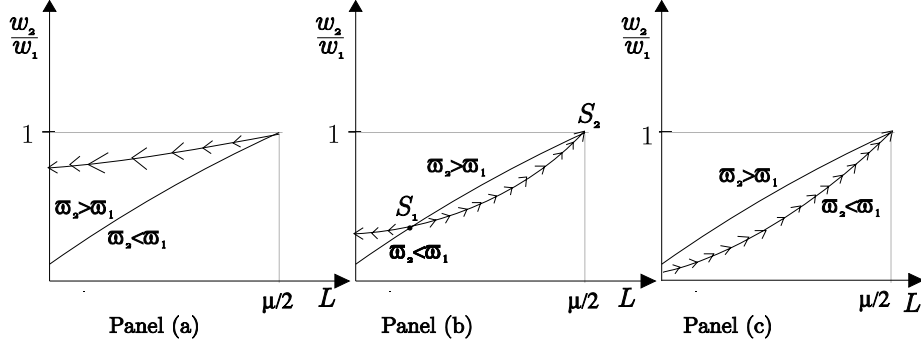


Figure 4: Agglomeration or Convergence

$$\varpi_1 \equiv \frac{w_1}{P_1(p_1, p_2, n)^\mu} = \frac{w_2}{P_2(p_1, p_2, n)^\mu} \equiv \varpi_2 \quad (19)$$

Let's suppose that all nominal wages and prices are equal in both regions. Does this guarantee equal real wages, too? The answer is no. If the domestic region is smaller, i.e. has less workers and less industrial products, more industrial products have to be imported. This raises the domestic CES price index above the foreign one, because the transport cost mark-up has to be paid for more products. But then the domestic real wage is smaller than the foreign. Hence, the domestic nominal wage needs to be bigger than the foreign nominal wage for the real wages to be equal, if the domestic region has less workers. The **equal real wage condition** is downward sloping in the  $\left(\frac{w_2}{w_1}; L\right)$ -space and is depicted in figure 4 as the solid line.<sup>47</sup> Real wages are bigger in the foreign region above this line (agglomeration) and bigger in the domestic region below this line (convergence).

Chapter 3 of this thesis proves that three cases are possible. The short run equilibrium condition is always above the equal real wage condition; then real wages are bigger in the bigger region and workers of the smaller region have an incentive to move to the bigger region thus self-enforcing the agglomeration process (see figure 4, panel a). The short run equilibrium condition is always below the equal real wage condition; then real wages are bigger in the smaller region and the equal distribution equilibrium

<sup>47</sup>The mathematical form of the equal real wage condition is derived in appendix E and is denoted  $k(W, L)$ .

is dynamically stable (see figure 4, panel c).<sup>48</sup> The equal real wage condition cuts the short run equilibrium condition from above (see figure 4, panel b). There will be an unstable intermediate steady state equilibrium  $S_1$  next to the symmetric steady state equilibrium  $S_2$ . For any labour distribution smaller than the one corresponding to  $S_1$ , there is an agglomeration process going on. For any labour distribution greater than the one corresponding to  $S_1$ , the system converges to the equal distribution equilibrium.<sup>49</sup> This case can be considered a “poverty trap”, because it depends on the initial distribution of industries, whether a region becomes industrialized or not.

The precise conditions for each of these cases will be given in chapter 3, proposition 2. If an economy starts out with low scale economies (low  $\sigma$ ), a big agricultural sector (high  $\mu$ ), and high transport costs (low  $\tau$ ), then the economy is likely to be described by the convergence scenario (figure 4 (a)). As transport costs are falling, industries are developing, and economies of scale are rising, the economy will most likely end up in the agglomeration scenario (figure 4 (c)). Whether the “poverty trap” scenario is passed on the way of development (figure 4 (b)), depends on whether the first industries started to be spread even in plane or were already clustered in a few places. Then, the tendency of clustering might have appeared in some regions, whereas it might not have appeared in others. This intermediate stage (poverty trap) might explain different stages in the degree of agglomeration.

## 2.6 Summary

This paper analyses the reasons for agglomeration or convergence in a typical geography and trade model such as Krugman (1991). There are two agglomeration forces (income effect or home market effect and price index effect) and one convergence force (number of goods effect).

**Income effect:** If one worker moves from the smaller to the larger region, then this worker will spend more on goods produced in the larger region than before,

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<sup>48</sup>These two cases are reported in Krugman (1991).

<sup>49</sup>This case is found in Urban (1996).

because there are no transport costs to be paid for these goods anymore. Hence, there is *ceteris paribus* excess demand for goods produced in the larger region. This will raise producer prices and wages and *ceteris paribus* attract even more workers to the larger region.

**Number of goods effect:** If one worker moves from the smaller to the larger region, then output of the existing firms in the larger region rises. Profits increase, since fixed cost are spread over more units of a good, new firms enter, and the number of goods increases in the larger region, whereas the number of goods falls in the smaller region. Next, the price index falls in the larger region, because trade costs have to be paid for less goods than before. Conversely, the price index rises in the smaller region. Hence the ratio of the producer price divided by the consumer price index falls in the larger region and rises in the smaller at given producer prices. This induces excess supply in the goods markets of the larger region and excess demand in the goods markets of the smaller region. Therefore producer prices and wages rise in the smaller region and the worker has an incentive to move back.

**The price index effect:** Since the consumer price index is *ceteris paribus* larger in the smaller region (see above), real wages will be smaller in the smaller region attracting workers to the larger region.

**Agglomeration** occurs, if the income effect and the price index effect overcompensate the number of goods effect; **convergence** of manufacturing distribution occurs, if the number of goods effect overcompensates the income and price index effect.

There exists an interior equilibrium for some parameter range. If a region has a certain critical mass of industries, industries tend to spread equally in space. If a region has not this critical mass of industries (“poverty trap”), this region is going to dry out of industries completely.

Finally, it is shown that the Dixit-Stiglitz-Krugman model behaves like a pure exchange economy with two heterogeneous types of consumers, three goods and the

following properties: 1) Each type of consumer is equally endowed with one good, owes the world endowment of the second, and nothing of the third. 2) The consumer strictly prefers the good he is well endowed with to the good he is not endowed with. A corollary of this analogy is that agglomeration can occur in the Dixit-Stiglitz-Krugman model even at zero transport cost, if consumers prefer domestically produced goods.

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# Appendix

## Appendix 1: Definition of Price Indices

The explicit functional forms for the price indices are:

$$\begin{aligned}
P_1(w_1, w_2, L) &= \gamma\beta \left( \frac{L}{\mu} w_1^{1-\sigma} + \tau^{\sigma-1} \left( \frac{\mu-L}{\mu} \right) w_2^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \\
P_1(p_1, p_2, n) &= \left( np_1^{1-\sigma} + \left( \frac{\mu}{\alpha\sigma} - n \right) (p_2/\tau)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \\
P_2(w_1, w_2, L) &= \gamma\beta \left( \left( \frac{\mu-L}{\mu} \right) w_2^{1-\sigma} + \tau^{\sigma-1} \frac{L}{\mu} w_1^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \\
P_2(p_1, p_2, n) &= \left( \left( \frac{\mu}{\alpha\sigma} - n \right) p_2^{1-\sigma} + n (p_1/\tau)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}.
\end{aligned} \tag{20}$$

## Appendix 2: The Excess Demand Functions

The equation system (11) is written in explicit functional form:

$$\begin{aligned}
\frac{p_1^{-\sigma} (p_1 n \bar{x} + \frac{1-\mu}{2})}{np_1^{1-\sigma} + \tau^{\sigma-1} \left( \frac{\mu}{\alpha\sigma} - n \right) p_2^{1-\sigma}} + \frac{p_1^{-\sigma} (p_2 \left( \frac{\mu}{\alpha\sigma} - n \right) \bar{x} + \frac{1-\mu}{2})}{np_1^{1-\sigma} + \tau^{1-\sigma} \left( \frac{\mu}{\alpha\sigma} - n \right) p_2^{1-\sigma}} &= \frac{\bar{x}}{\mu} \\
\frac{\tau^{\sigma-1} p_2^{-\sigma} (p_1 n \bar{x} + \frac{1-\mu}{2})}{\tau^{\sigma-1} np_1^{1-\sigma} + \left( \frac{\mu}{\alpha\sigma} - n \right) p_2^{1-\sigma}} + \frac{p_2^{-\sigma} (p_2 \left( \frac{\mu}{\alpha\sigma} - n \right) \bar{x} + \frac{1-\mu}{2})}{np_1^{1-\sigma} + \tau^{\sigma-1} \left( \frac{\mu}{\alpha\sigma} - n \right) p_2^{1-\sigma}} &= \frac{\bar{x}}{\mu} \\
p_1 \cdot n + p_2 \cdot \left( \frac{\mu}{\alpha\sigma} - n \right) &= \frac{\mu}{\bar{x}}.
\end{aligned} \tag{21}$$

## Appendix 3: Partial Derivatives of Excess Demand Functions

This part of the appendix calculates the signs of the partial derivatives of the excess demand functions of equation system (11) which is written in explicit functional form in appendix B.

1) The determination of the sign of  $\frac{\partial f}{\partial p_1}$ :

We will first rewrite the first equation in (21) in the following way:

$$f = \frac{f_1}{f_2} + \frac{f_3}{f_4} - \bar{x}, \tag{22}$$

where we used the notation  $n_1$  and  $n_2$  and defined

$$\begin{aligned}
f_1 &\equiv \mu \left( \bar{x} + \frac{1-\mu}{2p_1 n_1} \right) > 0, \\
f_2 &\equiv 1 + \tau^{\sigma-1} \left( \frac{n_2}{n_1} \right) \left( \frac{p_2}{p_1} \right)^{1-\sigma} > 0, \\
f_3 &\equiv \tau^{\sigma-1} \mu y_2 > 0, \\
f_4 &\equiv \tau^{\sigma-1} n_1 p_1 + n_2 p_2^{1-\sigma} p_1^\sigma > 0.
\end{aligned}$$

The partial derivatives are thus:

$$\begin{aligned}
\frac{\partial f_1}{\partial p_1} &= -\frac{1-\mu}{2p_1^2 n_1 \bar{x}} < 0, \\
\frac{\partial f_2}{\partial p_1} &= \tau^{\sigma-1} (\sigma-1) \left( \frac{n_2}{n_1} \right) p_2^{1-\sigma} p_1^{\sigma-2} > 0, \\
\frac{\partial f_3}{\partial p_1} &= 0, \\
\frac{\partial f_4}{\partial p_1} &= \tau^{\sigma-1} n_1 + \sigma n_2 p_2^{1-\sigma} p_1^{\sigma-1} > 0.
\end{aligned} \tag{23}$$

The partial derivative of  $f$  is then given by the following expression:

$$\frac{\partial f}{\partial p_1} = \frac{\frac{\partial f_1}{\partial p_1} f_2 - f_1 \frac{\partial f_2}{\partial p_1}}{f_2^2} - \frac{f_3}{f_4^2 \frac{\partial f_4}{\partial p_1}} < 0, \tag{24}$$

which is readily checked to be negative. **Q.E.D.**

2) The determination of the sign of  $\frac{\partial f}{\partial p_2}$ :

First, we slightly rewrite (22) by defining  $\tilde{f}_3$  and  $\tilde{f}_4$  to replace  $f_3$  and  $f_4$ , respectively:

$$\begin{aligned}
\tilde{f}_3 &\equiv \mu \tau^{\sigma-1} p_1^{-\sigma} \left( p_2 \cdot n_2 \cdot \bar{x} + \frac{1-\mu}{2} \right) > 0, \\
\tilde{f}_4 &\equiv \tau^{\sigma-1} n_1 p_1^{1-\sigma} + n_2 p_2^{1-\sigma} > 0.
\end{aligned} \tag{25}$$

Then, we can formulate the following partial derivatives:

$$\begin{aligned}
\frac{\partial f_1}{\partial p_2} &= 0, \\
\frac{\partial f_2}{\partial p_2} &= \tau^{\sigma-1} (1-\sigma) \left( \frac{n_2}{n_1} \right) p_1^{1-\sigma} p_2^{-\sigma} < 0, \\
\frac{\partial \tilde{f}_3}{\partial p_2} &= \mu \tau^{\sigma-1} p_1^{-\sigma} n_2 \bar{x} > 0, \\
\frac{\partial \tilde{f}_4}{\partial p_2} &= (1-\sigma) n_2 p_2^{-\sigma} < 0.
\end{aligned} \tag{26}$$

The partial derivative of  $f$  is then given by the following expression:

$$\frac{\partial f}{\partial p_2} = -\frac{f_1}{f_2^2 \frac{\partial f_2}{\partial p_2}} + \frac{\frac{\partial \tilde{f}_3}{\partial p_2} \tilde{f}_4 - \tilde{f}_3 \frac{\partial \tilde{f}_4}{\partial p_2}}{\tilde{f}_4^2} > 0, \tag{27}$$

which is readily checked to be positive. **Q.E.D.**

The sign of the partial derivative  $\partial f / \partial L$  is ambiguous. The signs of the partial derivatives of  $g(p_1, p_2, L)$  can be readily seen from the 3rd equation of (21).

#### Appendix 4: Goods Market Equilibrium Condition

This part of the appendix derives an implicit functional form of the goods market equilibrium condition in  $W$ - $L$ -space which is repeatedly shown in figures 3 and 4.

Reformulating the first two equations from appendix B by using (5) and (6) and (7) yields the following equation system which guarantees goods market equilibrium for domestic industrial products and agricultural products:

$$\begin{aligned}\mu &= w_1 L + (\mu - L) w_2, \\ 1 &= \frac{w_1^{-\sigma} (w_1 L + \frac{1-\mu}{2})}{\frac{L}{\mu} w_1^{1-\sigma} + \tau^{\sigma-1} \left(\frac{\mu-L}{\mu}\right) w_2^{1-\sigma}} + \frac{\tau^{\sigma-1} w_1^{-\sigma} (w_2 (\mu - L) + \frac{1-\mu}{2})}{\tau^{\sigma-1} \frac{L}{\mu} w_1^{1-\sigma} + \left(\frac{\mu-L}{\mu}\right) w_2^{1-\sigma}}.\end{aligned}\quad (28)$$

Expressing this system of equations in relative wages  $W = \frac{w_2}{w_1}$  and rearranging yields:

$$\begin{aligned}\frac{\left(L + \frac{1-\mu}{2w_1}\right)}{\left(\frac{L}{\mu} + \tau^{\sigma-1} \left(\frac{\mu-L}{\mu}\right) W^{1-\sigma}\right)} + \frac{\tau^{\sigma-1} \left(W \cdot (\mu - L) + \frac{1-\mu}{2w_1}\right)}{\left(\tau^{\sigma-1} \frac{L}{\mu} + \left(\frac{\mu-L}{\mu}\right) W^{1-\sigma}\right)} &= 1, \\ \frac{L + W (\mu - L)}{\mu} &= \frac{1}{w_1}.\end{aligned}\quad (29)$$

Plugging the second into the first equation of (29) allows to define an implicit function  $h(W, L)$  in the relative nominal wage  $W$  and the labour distribution  $L$ , which fully characterises goods market equilibrium in both sectors:

$$h(W, L) \equiv \frac{2\mu L + (1 - \mu) (L + W (\mu - L))}{(L + \tau^{\sigma-1} (\mu - L) W^{1-\sigma})} \quad (30)$$

$$+ \frac{\tau^{\sigma-1} (2\mu W (\mu - L) + (1 - \mu) (L + W (\mu - L)))}{(\tau^{\sigma-1} L + (\mu - L) W^{1-\sigma})} - 2 = 0.$$

The implicit functional form  $h(W, L)$  can be solved for  $L$  giving two solutions. Only one of them can be in positive prices because of the uniqueness of the short run equilibrium (Proof see Urban (1996), proposition 1). Hence, the short run equilibrium condition must be either monotonically increasing or monotonically decreasing in  $W - L$ -space. By inspection of the algebraic form of the solution to  $L$  that is a rational function, the schedule of  $h(W, L)$  must also be continuous.

## Appendix 5: Equal Real Wage Condition

This part of the appendix gives a functional form for the equal real wage condition in figures 3, 4 and 5. The definition of the relative real wage can be rewritten in terms of the nominal real wage and the labour distribution by using (20) and (19):

$$\varpi(L, W) = \frac{(\tau^{\sigma-1} L + (\mu - L) W^{1-\sigma})^{\frac{\mu}{1-\sigma}}}{W (L + \tau^{\sigma-1} (\mu - L) W^{1-\sigma})^{\frac{\mu}{1-\sigma}}}.\quad (31)$$

In the steady state, the relative real wage needs to be equal to one ( $\varpi(L, W) = 1$ ). This equation is solved for  $L$  and an implicit function  $k(W, L)$  in  $W$  and  $L$  is defined for which the real wage is one:

$$k(W, L) \equiv -\frac{\mu \left(W^{1-\sigma} - \tau^{\sigma-1} W^{(1-\sigma)(1+\frac{1}{\mu})}\right)}{\tau^{\sigma-1} - W^{1-\sigma} - W^{\frac{1-\sigma}{\mu}} + \tau^{\sigma-1} W^{(1-\sigma)(1+\frac{1}{\mu})}} - L = 0.\quad (32)$$

This is the implicit functional form for the equal real wage condition used in figure 4.



### 3 Increasing Returns and Economic Geography: An Analytical Note\*

#### Abstract

This paper provides an analytical solution to the Krugman (1991a) model explaining industry agglomeration. It is shown there exists a unique short-run equilibrium and multiple long-run equilibria. The latter proves the existence of a “poverty trap” in this model: depending on the initial level of industries we will either see industries spreading evenly in the plane, or moving away from one of the regions. However, it is also shown that this “poverty trap” will not appear if the economy starts developing from an equal distribution of industries.

**JEL Classification:** F12, R60.

**Keywords:** convergence, agglomeration, poverty trap.

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\*The author thanks Niels Blomgren-Hansen, Karsten Junius, Jeffrey Nilsen and Pascal Raimondos-Møller for their comments. Usual caveats apply.

### 3.1 Introduction

The economics of agglomeration have become a growing field of theoretical research.<sup>50</sup> Problems of industry agglomeration are also attracting more and more attention by applied economists. An explanation of agglomeration processes is provided by geography and trade models of the Krugman (1991a) type. These models are appealing, because (i) they endogenously generate a cumulative process, similar to the one informally described in Myrdal (1957), (ii) they do not rely on (unobservable) exogenous externalities, and (iii) they are based on internal increasing returns to scale that is thought to be an important source for agglomeration among regional economists (see Fujita and Thisse, 1996).

Yet, these models lack the analytical rigor that can be found in corresponding models with constant returns to scale production.<sup>51</sup> The reason is that these models are based on a non-linear equation system which does not necessarily obey the standard convexity assumptions and may therefore yield multiple equilibria. A consequence is that the literature is often either based on numerical simulations (e.g. Venables, 1996), or on the analysis of corner solutions (e.g. Krugman, 1991a). If geography and trade models are analytically rigorous, then they usually rely on factor price equalization ruling out some agglomeration and convergence forces (e.g. Helpman and Krugman, 1985, and Martin and Rogers, 1995).

The purpose of this paper is to give a complete analytical treatment of the seminal geography and trade model by Krugman (1991a). In doing so, we uncover some interesting properties of the model that explain the existence of a “poverty trap”. In Krugman’s model, the relocation of industries between two regions is driven by worker migration due to real wage differences. Krugman (1991a) concentrates on the case, in which it pays for a firm to attract workers to a region that had no industries before (total agglomeration). Krugman and Venables (1995) and Venables (1995) analyze the case, in which it pays for a firm to defect from an equal distribution of industries

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<sup>50</sup>A recent survey on the economics of agglomeration is Fujita and Thisse (1996).

<sup>51</sup>See, for example, Norman and Venables (1995).

(symmetry-breaking). The contribution of this paper is to find analytically all steady state industry distributions in between these two extreme cases. Furthermore, we are able to describe the industry reallocation dynamics at *any* initial distribution of industries. In doing so, we identify a particular parameter range for which a “poverty trap” arises: if a region has a certain initial threshold level of industries, they tend to spread evenly in the plane (convergence); On the contrary, if a region does not have this threshold level, all its industries move away (total agglomeration). In this sense, the model shows that initial conditions matter in Krugman’s (1991a) model not only for which region is drying out of industries (determination of the “winner” region), but also whether industries agglomerate or converge (threshold property of the agglomeration process). The integration of “similar” regions (e.g. European Community) causes convergence of industry distribution, the integration of a region with a lot of industries and one with just a few (e.g. German unification) causes agglomeration.

Finally, we show that the “poverty trap” case will not appear if the economy starts developing from an equal distribution of industries. In this particular case, the analysis of symmetry-breaking, as described in Venables (1995) and Krugman and Venables (1995), is sufficient to fully characterize the model.

The mathematical problem solved in this paper is the determination of the exact number of equilibria in a simple fixed point problem with multiple solutions. The well-known fixed point theorems provide little help for this problem, because they prove the uniqueness (non-uniqueness) of an equilibrium. We find a specific solution to this problem for models based on polynomials (i.e. models with CES or Cobb-Douglas functional forms). This solution applies not only to economic geography models, but also to models that exhibit poverty traps and growth (chapter 5 of this thesis).

The rest of this paper is organized as follows. Section 2 describes briefly the Krugman (1991a) model. Subsection 2.1 proves the uniqueness of the short-run equilibrium and subsection 2.2 analyzes the long-run equilibrium and proves the existence of a “poverty trap”. Some conclusions can be found in section 3.

### 3.2 The Krugman (1991a) Model

The model has two sectors, two regions and two consumers. The two sectors – agriculture and industry – differ by their market form: the market for agricultural goods is perfectly competitive; the market for industrial goods is monopolistically competitive. Regions are defined as areas for which it is costless to trade industrial goods within them, but costly to trade industrial goods across them. Consumers  $j = 1, 2$  differ only by their place of residence. It is assumed that there is no short-run mobility of production factors from one region to the other. However, mobility of production factors is allowed in the long-run.

Consumers have identical Cobb-Douglas utility functions containing the agricultural good and the aggregate basket of the industrial goods. The income share attributed to the industrial goods basket is denoted by  $\mu$ . The industrial goods basket is further specified by a Dixit-Stiglitz (1977) subutility function with  $\sigma$  denoting the elasticity of substitution between varieties ( $\sigma > 1$ ). The agricultural good is taken as the numeraire. Based on these standard utility functions, the price index is well known and it is given by the following expression:

$$P_j = \left( \frac{n_i}{n_1 + n_2} (p_i^{ex})^{1-\sigma} + \frac{n_j}{n_1 + n_2} p_j^{1-\sigma} \right)^{\frac{\mu}{1-\sigma}}, \quad i \neq j, \quad (33)$$

where the number of goods in region  $i$  is denoted by  $n_i$ ; the domestic prices of firms in region  $j$  are denoted by  $p_j$ ; and the c.i.f.-prices for export goods are denoted by  $p_i^{ex}$ .

Both goods are produced using only labour. Furthermore, it is assumed that there is factor specificity for industrial production by workers, and for agricultural production by peasants. Peasants work according to a constant returns to scale technology. Workers work according to an increasing returns to scale technology, where marginal cost is constant and where there exists some fixed cost. Firms are assumed to have zero profits.

The sum of workers in both regions ( $L_1 + L_2$ ) is normalised to  $\mu$ . Thus, the total amount of peasants is  $1 - \mu$  and they are assumed to be equally distributed in

both regions and immobile. Every peasant and worker supplies one unit of work and earns a salary of 1, if peasant, and  $w_i$ , if worker in region  $i$ .

Finally, there are transport cost of the Samuelson iceberg-type, such that only a fraction  $\tau$  of one produced unit of an industrial good arrives at its foreign destination ( $0 < \tau < 1$ ). There are no transport cost for goods delivered to domestic customers or for the agricultural goods.

### 3.2.1 The Short-Run Equilibrium

Having described briefly the model, we now present the economic relationships that result from consumer optimization, firm optimization, labour market clearing, product market clearing, and the zero profit condition.

Under the standard assumption that a firm does not take into account the impact of its pricing decision on other firms' pricing decisions and on regional income, the pricing rule for the firm will be: <sup>52</sup>

$$p_i = \frac{\sigma}{\sigma - 1} \cdot \beta \cdot w_i \quad \text{and} \quad p_i^{ex} = \frac{\sigma}{\sigma - 1} \cdot \beta \cdot w_i / \tau, \quad (34)$$

where  $\sigma/(\sigma - 1)$  is the mark-up of prices over marginal cost, and  $\beta$  is the reciprocal of the marginal product of labour. Note, that the pricing decision for export goods  $p_i^{ex}$  requires the firm to take into account the additional transport cost.

The optimal output of the firm is determined by the zero profit condition:

$$x_i = \frac{\alpha \cdot (\sigma - 1)}{\beta} \equiv \bar{x}, \quad (35)$$

where  $\alpha$  is the fixed cost parameter. It is seen that the equilibrium output of the firm is independent of the number of workers or the number of firms in a region.

The equilibrium number of firms per region follows from the labour market clearing

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<sup>52</sup>This equation is an approximation which is only fairly good for a large number of firms (see the discussion in Yang and Heijdra (1993), Dixit and Stiglitz (1993), and d'Aspremont et. al. (1996)). The latter authors show that the mark-up of prices over marginal cost is underestimated by the above approximation. Any correction for this would not change the principle story as long as there is a positive relationship between prices and wages and a positive relationship between number of firms and number of workers.

conditions and the output decision:

$$n_i = \frac{L_i}{\alpha\sigma}. \quad (36)$$

If the number of workers increases in a region, workers drag industries with them and the number of goods increases proportionally. Since profits are zero, aggregate income  $y_i$  in a region  $i$  will be the sum of income of all workers and peasants in that region:

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$$y_i = w_i \cdot L_i + \frac{1 - \mu}{2} = n_i p_i \bar{x} + \frac{1 - \mu}{2}. \quad (37)$$

Finally, the conditions for equilibrium in the goods market are as follows: <sup>54</sup>

$$\frac{\mu p_1^{-\sigma} y_1}{n_1 p_1^{1-\sigma} + t n_2 p_2^{1-\sigma}} + \frac{t \mu p_1^{-\sigma} y_2}{t n_1 p_1^{1-\sigma} + n_2 p_2^{1-\sigma}} = \bar{x}, \quad (38)$$

$$(y_1 + y_2)(1 - \mu) = 1 - \mu, \quad (39)$$

where, for notational simplicity, we write  $t = \tau^{\sigma-1}$ . Equation (38) gives the market clearing condition for the domestic industrial goods, adding up domestic demand and exports of a firm and setting it equal to output. Equation (39) gives the market clearing condition for the agricultural good: the demand for the agricultural good (which is a constant fraction of the world income) equals the supply of the agricultural good (which equals the total number of peasants).

The short-run equilibrium is described by (34), (36), (37), (38) and (39) that determine  $w_i$ ,  $n_i$ ,  $y_i$ , and  $p_i$  for a given labour distribution  $L_i$  and the parameters of the model  $(\tau, \mu, \sigma)$ .<sup>55</sup> However, the above system might yield several solutions at a given distribution of labour.<sup>56</sup> In what follows, it is shown that only one of them is in positive prices and quantities.

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<sup>53</sup>The second equality sign holds, because workers income — or, in other words, labour cost — equals firms sales in a region.

<sup>54</sup>The equilibrium condition for the foreign firm is omitted due to Walras' law. This is the first step, where we differ from Krugman (1991a). He drops the equilibrium condition for the agricultural sector instead. Our proceeding will allow us to summarize the whole model in a single equation.

<sup>55</sup>The parameters  $\alpha$  and  $\beta$  will drop out on the way.

<sup>56</sup>To find an indication for multiple equilibria, insert first equation (37) into equations (38) and (39). Second, suppose  $\sigma = 2$ . One of the two emerging equations entails a polynomial of degree 3 in prices. It is known that an equation system with such a polynomial does not necessarily entail a unique solution. However, some solutions can be complex and thus economically irrelevant.

**Proposition 1:** *The equation system (34), (36), (37), (38) and (39) has a unique solution for  $w_i$ ,  $n_i$ ,  $y_i$ , and  $p_i$  at a given labour distribution  $L_i$ .*

**Proof:** First, we note that (38) and (39) can be rewritten as a system of excess demand functions  $g$  and  $f$ , where  $g \equiv g_1 + g_2 - \bar{x}$  (with  $g_1$  and  $g_2$  being respectively the first and second term at the left hand side of (38)) and  $f \equiv x_1 p_1 n_1 + x_2 p_2 n_2 - \mu$ . It is easily seen that the excess demand functions  $g$  and  $f$  fulfill the gross substitute property. This means that  $g$  depends positively on the industrial price  $p_2$ , and  $f$  depends positively on both industrial prices  $p_1$  and  $p_2$  (at a given number of goods). However, for any excess demand system that fulfills the gross substitute property there exists a unique equilibrium price vector for a given number of goods in both regions (see proposition 17.F.3 in Mas-Colell, et.al. (1995), p. 613). Having shown that, all other endogenous variables are linear transformations of prices and the number of goods and must thus be unique, too. **Q.E.D.**

To facilitate the analysis, we define the relative nominal prices as  $p \equiv \frac{p_2}{p_1}$  and the relative distribution of industries as  $n \equiv \frac{n_2}{n_1}$ . Because the relative distribution of industries equals the relative labour distribution according to equation (36) and relative prices of industrial goods equal relative industrial wages according to equation (34),  $p$  describes also relative wages ( $p = \frac{w_2}{w_1}$ ) and  $n$  describes the relative distribution of labour ( $n = \frac{L_2}{L_1}$ ). The system (38) and (39) can then be rewritten by using (37) as follows:

$$\frac{\mu \left( 1 + \frac{1-\mu}{2} \cdot \frac{1}{n_1 p_1 \bar{x}} \right)}{(1 + t n p^{1-\sigma})} + \frac{t \mu \left( n p + \frac{1-\mu}{2} \cdot \frac{1}{n_1 p_1 \bar{x}} \right)}{(t + n p^{1-\sigma})} = 1 \quad (40)$$

$$\frac{1}{n_1 p_1 \bar{x}} = \frac{1 + n p}{\mu}$$

Substituting the second equation into the first, and solving for  $n$ , we obtain a condition that describes the goods market equilibrium as an implicit relationship of the relative labour distribution  $n$  and the relative nominal wages  $p$ . We call this condition, the

short-run equilibrium condition  $h(p, n)$ :

$$h(p, n) : \quad n = \frac{1 - \mu + (1 + \mu)t^2 - 2tp^\sigma}{p[1 - \mu + (1 + \mu)t^2 - 2tp^{-\sigma}]} \quad (41)$$

The  $h(p, n)$  function can be drawn in the  $p$ - $n$  space.<sup>57</sup> The symmetry point ( $p = 1, n = 1$ ) will be a point of this function. The function can be either upward or downward sloping in the  $p$ - $n$  space.

### 3.2.2 The Long-Run Equilibrium

In the long-run workers are allowed to be mobile, moving to the region which pays the highest real wage.<sup>58</sup> To characterize the migration process at any labour distribution, and thus the firm reallocation incentives, we write the arbitrage condition for the steady state equilibrium:

$$\varpi_1 \equiv \frac{w_1}{P_1} = \frac{w_2}{P_2} \equiv \varpi_2 \quad (42)$$

Using (33), (34), (36) and the definitions for  $p$  and  $n$ , we can rewrite this condition as an implicit function  $k(p, n)$ :

$$k(p, n) : \quad n = \frac{t - p^{\frac{1-\sigma}{\mu}}}{p^{1-\sigma} \left[ tp^{\frac{(1-\sigma)}{\mu}} - 1 \right]} \quad (43)$$

We call the above the *equal real wage condition*. The symmetry point ( $p = 1, n = 1$ ) will be a point of this function. Furthermore, the  $k(p, n)$  function is always downward sloping in the positive orthant of the  $p$ - $n$  space.

Whether a worker will migrate or not depends on whether the *short-run equilibrium condition* lies above or below the *equal real wage condition*. All cutting points of the two curves are interior steady state equilibria. It is obvious that the symmetry point is always a steady state. In what follows we derive the exact conditions under which workers will migrate towards the one region or the other. A diagrammatic illustration is provided later on.

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<sup>57</sup>Without loss of generality, we define region 2 to be the region with the fewest workers (or an equal number of workers). Then, we only need to consider the range  $0 \leq n \leq 1$ .

<sup>58</sup>Real wage rates are identical to the value of the indirect utility function of a worker given a distribution of workers. This follows immediately from the definitions of the CES-price index and the indirect utility function, respectively.



The full characterization of the long-run equilibria is approached in several steps. First, a condition is derived that determines the parameter values for which the symmetry point is a stable steady state.

**Lemma 1:** *The symmetry point  $p = 1, n = 1$  is a stable steady state equilibrium if and only if  $c^{sb}(\mu, \tau, \sigma) < 0$ , where*

$$c^{sb}(\mu, \tau, \sigma) = \tau^{\sigma-1} - \frac{1 - \mu - \sigma(1 - \mu)^2}{1 + \mu - \sigma(1 + \mu)^2}$$

**Proof:** See appendix 1.

If the symmetry point is a stable steady state, then there will occur convergence of industry location in the neighbourhood of this point. If the symmetry point is unstable, i.e.  $c^{sb}(\mu, \tau, \sigma) \geq 0$ , there will occur “symmetry-breaking”<sup>59</sup> and the firms and workers start relocating unevenly in the plane (agglomeration).

Second, we examine the condition for the existence of a corner solution, i.e.  $L_1 = 0$  or  $L_2 = 0$ . A corner solution is a stable equilibrium if and only if real wages are lowest in the region where all industries have disappeared.

**Lemma 2:** *Total agglomeration (either  $L_1 = 0$  or  $L_2 = 0$ ) is a stable steady state equilibrium if and only if  $c^{ta}(\mu, \tau, \sigma) > 0$ , where*

$$c^{ta}(\mu, \tau, \sigma) = 2 - \tau^{\mu\sigma} [(1 + \mu) \tau^{\sigma-1} + (1 - \mu) \tau^{-(\sigma-1)}]. \quad (44)$$

**Proof:** See appendix 2.

The total agglomeration condition describes the set of parameters for which a region dries out of industries completely.<sup>60</sup> If the opposite inequality sign holds, i.e.

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<sup>59</sup>Conditions for symmetry breaking are given in other models of the same type (see Venables, 1995, and Krugman and Venables, 1995).

<sup>60</sup>This condition, although differently derived, is identical to equation (26) in Krugman (1991a). The economic meaning of the parameters  $\mu, \tau$ , and  $\sigma$  in this condition is the same as in Krugman (1991a) and therefore we do not discuss it further.

$c^{ta}(\mu, \tau, \sigma) \leq 0$ , then there will be convergence at  $n = 0$ , i.e. some firms will start relocating from the region with the industries to the region without the industries.

Third, a relationship between the agglomeration/convergence conditions from lemmas 1 and 2 is found.

**Lemma 3:** *The parameter space fulfilling the condition for symmetry-breaking is a subset of the parameter space fulfilling the condition for total agglomeration. Thus, for any  $(\mu, \tau, \sigma)$  with  $0 < \mu \leq 1$ ,  $\sigma > 1$ , and  $0 < \tau < 1$ , the following is true:*

$$\begin{aligned} c^{sb}(\mu, \tau, \sigma) \geq 0 &\implies c^{ta}(\mu, \tau, \sigma) > 0 \text{ and} \\ c^{ta}(\mu, \tau, \sigma) \leq 0 &\implies c^{sb}(\mu, \tau, \sigma) < 0 \end{aligned}$$

**Proof:** See appendix 3.

The importance of this lemma is that there is agglomeration at  $n = 0$  and convergence at  $n = 1$  for some parameters, but there are no allowed parameter values for which there is convergence at  $n = 0$  and agglomeration at  $n = 1$ .

Finally, the maximum number of interior steady states is determined.

**Lemma 4:** *The system (41) and (43) has at most one interior steady state  $n^*$  with  $0 < n^* < 1$ .*

**Proof:** See appendix 4.

Using these four lemmas, the long-run equilibria can be fully characterized. Proposition 2 does exactly that.

**Proposition 2:** *(i) Workers and firms tend to agglomerate completely in one of the two regions independently of the initial labour distribution, if*

$$c^{sb}(\mu, \tau, \sigma) \geq 0.$$

(ii) *Workers and firms tend to spread even in both regions (convergence) independently of the initial labour distribution, if*

$$c^{ta}(\mu, \tau, \sigma) \leq 0.$$

(iii) *There is an unstable intermediate steady state equilibrium at a firm distribution  $n^*$  with  $0 < n^* < 1$  and two stable steady state equilibria at  $n = 0$  and  $n = 1$ , if*

$$c^{sb}(\mu, \tau, \sigma) < 0 < c^{ta}(\mu, \tau, \sigma).$$

**Proof:** (i) If  $c^{sb}(\mu, \tau, \sigma) \geq 0$ , then there is agglomeration at  $n = 1$  (lemma 1). However, if that happens, then  $c^{ta}(\mu, \tau, \sigma) > 0$  (lemma 3) and thus there must also be agglomeration at  $n = 1$  (lemma 2). But then there cannot exist any convergence in between  $0 < n < 1$ , because this would require at least two interior steady states. This, however, contradicts lemma 4. Hence, if  $c^{sb}(\mu, \tau, \sigma) \geq 0$ , there must be agglomeration for any labour distribution  $n$ .

(ii) If  $c^{ta}(\mu, \tau, \sigma) \leq 0$ , then there is convergence at  $n = 0$  (lemma 2). However, if that happens, then  $c^{sb}(\mu, \tau, \sigma) < 0$  (lemma 3) and thus there must also be convergence at  $n = 1$  (lemma 1). But then there cannot exist any agglomeration in between  $0 < n < 1$ , because this would require at least two interior steady states. This, however, contradicts lemma 4. Hence, if  $c^{ta}(\mu, \tau, \sigma) \leq 0$ , there must be convergence for any labour distribution  $n$ .

(iii) If  $c^{sb}(\mu, \tau, \sigma) < 0$ , there is convergence at  $n = 1$  (lemma 1); then the schedule of  $h(p, n)$  lies below the schedule of  $k(p, n)$  in  $p$ - $n$  space for  $n$  slightly below 1. If  $c^{ta}(\mu, \tau, \sigma) > 0$ , there is agglomeration at  $n = 0$  (lemma 2). Then the schedule of  $h(p, n)$  lies above the schedule of  $k(p, n)$  in  $p$ - $n$  space for  $n = 0$ . Hence, there must be at least one cutting point  $n^*$  of the two schedules in between 0 and 1 (intermediate value theorem). Because of lemma 4, there is exactly one.  $n^*$  is unstable, because  $n = 0$  and  $n = 1$  are stable at this parameter constellation. **Q.E.D.**

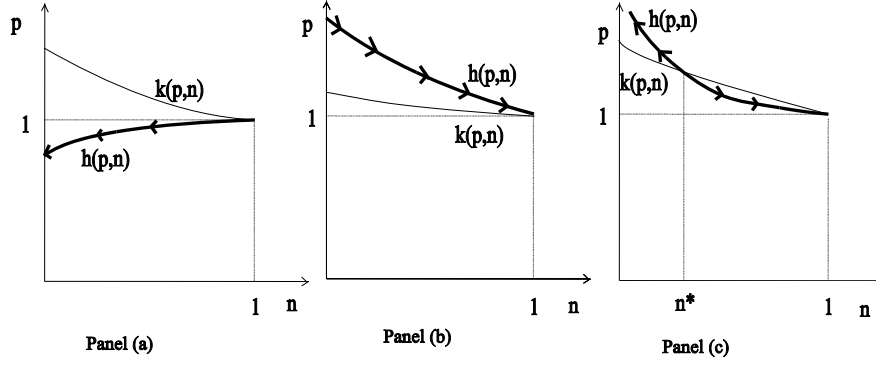


Figure 5: Long Run Equilibria

Proposition 2 can be most easily demonstrated in figure 1. The three cases in proposition 2 correspond to the three panels of figure 1. Panel (a) shows that the short-run equilibrium condition  $h(p,n)$  is below the equal real wage condition  $k(p,n)$  for all labour distributions. Real wages are higher in the larger region and workers and firms of the smaller region have an incentive to move, reinforcing thus the agglomeration process. This corresponds to case (i) in proposition 2.

Panel (b) shows that the short-run equilibrium condition  $h(p,n)$  is above the equal real wage condition  $k(p,n)$  for all labour distributions. Real wages are higher in the smaller region and the workers and firms tend to spread evenly in the plane (convergence). This corresponds to case (ii) in proposition 2.<sup>61</sup>

Panel (c) shows that the short-run equilibrium condition  $h(p,n)$  is *below* the equal real wage condition  $k(p,n)$ , if the initial labour distribution  $n$  is *below* a critical mass  $n^*$ . Then real wages are higher in the bigger region and the few industries in the smaller region relocate to the bigger region (total agglomeration). On the contrary, the short-run equilibrium condition  $h(p,n)$  is *above* the equal real wage condition  $k(p,n)$ , if the initial labour distribution  $n$  is *above* the critical mass  $n^*$ . Then, real wages are higher in the smaller region and industries start to relocate evenly in the plane (convergence). This corresponds to case (iii) in proposition 2.

<sup>61</sup>The cases (i) and (ii) are reported in form of simulations in figure 1 of Krugman (1991a). Note that we derive the schedules of the functions  $h(v,z)$  and  $k(v,z)$  analytically. The precise curvature of the two schedules in figure 1 is suggestive.

The third case will be called a “poverty trap”. In the “poverty trap”, regions that lack the critical mass of industries are stuck in this state because there is no market force that could foster industrialization. If the region had more industries to begin with, market forces would enforce a convergence process.<sup>62</sup>

The “poverty trap” shows that the location of industries might depend on initial conditions. Even if all parameters – like preferences and technology – are the same, convergence or agglomeration may take place depending on the initial distribution of industries. History matters, not only in the determination of the region to be the “winner”, but also in the determination of whether industries tend to agglomerate or spread evenly in the plane.<sup>63</sup> For example the rather underdeveloped Eastern Germany lost a large proportion of its industries and its workers after unification with West Germany.<sup>64</sup> However, the integration of the rather homogeneous European countries seems to enforce a convergence process.<sup>65</sup> This observation may justify that the EU requires new members to have a sufficient stage of development.<sup>66</sup>

Finally, the “poverty trap” case will not appear, if the economy starts developing with a symmetric distribution of industries. It follows from proposition 2 that the condition of “symmetry-breaking” is then sufficient to fully characterize the model.

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<sup>62</sup>The “poverty trap” is usually discussed in the context of growth models (see Barro and Sala-i-Martin, 1995, p. 49ff, for an overview.) There, a poverty trap means that some countries are stuck with a low level of the capital stock. If they had enough capital to begin with, they would converge to the advanced nations. The reason is the switch from a diminishing returns to scale to an increasing returns to scale technology. This is somewhat related to the result in this model. Here the poverty trap arises, if a certain share of the increasing returns to scale and the constant returns to scale sector is prevailed in the economy. A “poverty trap” arises also in Matsuyama (1991). His economic interpretation of the “poverty trap” is similar to the one in this paper. However, the underlying story is very different.

<sup>63</sup>Krugman (1991b) shows that history or expectations might matter for the determination of the “winner” region, if a forward looking migration process is assumed.

<sup>64</sup>See Lange and Pugh (1998).

<sup>65</sup>See Ben-David (1993).

<sup>66</sup>The model predicts also that an integration process which lowers trade cost may cause the economy to move from the convergence case to the “poverty trap” case. This finding follows from proposition 2. To see this, start from the convergence case (ii). Note that a sufficiently small increase in  $\tau$  (decrease of trade cost) induces a rise in  $c^{ta}(\mu, \tau, \sigma)$  and a violation of the condition  $c^{ta}(\mu, \tau, \sigma) < 0$  (which is most easily seen by simulating numerically equation (44)), but not necessarily a violation of  $c^{sb}(\mu, \tau, \sigma) < 0$ , because we proved that the parameter set of case (iii) is non-empty. Therefore, a decrease in trade cost  $\tau$  may lead to a move from case (ii) (convergence case) to case (iii) (poverty trap case) for some parameters  $\mu$  and  $\sigma$  and some initial conditions  $n_0$  with  $0 \leq n_0 \leq 1$ .

### 3.3 Conclusion

This paper gives a complete analytical solution to a monopolistic competition, increasing returns to scale model with transport cost and mobile labour (Krugman, 1991a). It is shown that this model has a unique short-run equilibrium. Furthermore, it is shown that the long run equilibrium is characterized by three groups of parameter ranges: (i) industries agglomerate completely in one of the two regions; (ii) industries tend to spread evenly in the plane; (iii) it depends on the initial distribution of industries whether agglomeration or convergence occurs (“poverty trap”).

The later finding shows that initial conditions matter in Krugman’s (1991a) model not only for the determination of the “winner” region, but also for the determination of agglomeration or convergence. However, the “poverty trap” case will not appear, if the economy starts developing with a symmetric distribution of industries. The condition of “symmetry-breaking” is then sufficient to fully characterize the model.

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# Appendix

## Appendix 1:

Taking the derivative of (41) and evaluating at the symmetry point, yields the following expression:

$$\left. \frac{dn}{dp} \right|_{h(1,1)} = \frac{1 - \mu + (1 + \mu) t^2 - 2t(1 - 2\sigma)}{(1 - t^2)\mu - (1 - t)^2}$$

Taking the derivative of (43) and evaluating at the symmetry point, yields:

$$\left. \frac{dn}{dp} \right|_{k(1,1)} = \frac{(1 - \sigma)[1 - \mu + t(1 + \mu)]}{(1 - t)\mu}$$

The equal distribution is stable (convergence), if

$$\left. \frac{dz}{dv} \right|_{h(1,1)} < \left. \frac{dz}{dv} \right|_{k(1,1)}.$$

Equalizing the two derivatives  $\left. \frac{dn}{dp} \right|_{h(1,1)} = \left. \frac{dn(p)}{dp} \right|_{k(1,1)}$  and solving for  $t$  yields three solutions. One is always negative ( $t = -1$ ) and therefore economically irrelevant. The other is  $t = 1$ , i.e. factor prize equalization holds in the absence of transport cost. And the third is:

$$t = \frac{1 - \mu - \sigma(1 - \mu)^2}{1 + \mu - \sigma(1 + \mu)^2}$$

This expression used in the conditions above gives the lemma 1.

**Q.E.D.**

## Appendix 2:

The equal real wage condition for  $L_1 = 0, L_2 = \mu$  is given in equation (43):

$$\frac{1}{z} = \frac{p^{1-\sigma} \left[ t p^{\frac{1-\sigma}{\mu}} - 1 \right]}{t - p^{\frac{1-\sigma}{\mu}}}$$

or equivalently by using  $(1/n) = L_1/L_2 = 0$  and the definition of  $p = w_2/w_1$  and  $t = \tau^{\sigma-1}$ :

$$\frac{w_2}{w_1} = \tau^\mu$$

Using the definition for real wages of equation (42) and noting that the relative real wage is one gives the condition for the relative price indices:

$$\frac{P_2}{P_1} = \tau^\mu$$

From the short-run equilibrium condition (41), it follows that

$$\frac{1}{n} = \frac{p[1 - \mu + (1 + \mu)t^2 - 2tp^{-\sigma}]}{1 - \mu + (1 + \mu)t^2 - 2tp^\sigma},$$



which yields for  $(1/n) = 0$ :

$$\frac{w_2}{w_1} = p = (0.5(1 - \mu)t^{-1} + 0.5(1 + \mu)t)^{-(1/\sigma)}.$$

The relative real wage at the total agglomeration point can thus be written as:

$$\frac{w_1 P_2}{w_2 P_1} = \tau^\mu (0.5(1 - \mu)\tau^{-(\sigma-1)} + 0.5(1 + \mu)\tau^{\sigma-1})^{1/\sigma}$$

By definition of total agglomeration the real wage in region 1 needs to be smaller than the real wage in region 2, i.e.:

$$\frac{w_1 P_2}{w_2 P_1} < 1$$

Consequently the condition can be stated as in lemma 2.

**Q.E.D.**

### Appendix 3:

First, it is shown that the boundary planes of the two conditions from lemmas 1 and 2 do not intersect for allowed parameter constellations of  $\tau, \sigma, \mu$ :

$$c^{ta}(\mu, \tau, \sigma) = 0$$

$$c^{sb}(\mu, \tau, \sigma) = 0$$

The second equation is solved for  $\tau$  and then plugged into the first expression to give:

$$\left( \frac{(1 - \mu)[1 - \sigma(1 - \mu)]}{(1 + \mu)[1 - \sigma(1 + \mu)]} \right)^{\frac{\mu\sigma}{\sigma-1}} \left( \frac{(1 - \mu)[1 - \sigma(1 - \mu)]}{1 - \sigma(1 + \mu)} + \frac{(1 + \mu)[1 - \sigma(1 + \mu)]}{1 - \sigma(1 - \mu)} \right) = 2$$

This condition is dealt numerically. A grid search procedure in Mathematica shows that  $\mu = 0$  or  $\sigma = 0$  are the only two solutions to this equation. This implies that there is no cutting point of the two equations for  $0 < \mu \leq 1$ , and  $\sigma > 1$ .

Second, it is readily checked that there exist parameter constellations of  $\tau, \sigma$ , and  $\mu$ , such that the three sets defined by

$$c^{ta}(\mu, \tau, \sigma) \leq 0 \text{ and } c^{sb}(\mu, \tau, \sigma) < 0$$

$$c^{ta}(\mu, \tau, \sigma) > 0 \text{ and } c^{sb}(\mu, \tau, \sigma) \geq 0$$

$$c^{ta}(\mu, \tau, \sigma) > 0 \text{ and } c^{sb}(\mu, \tau, \sigma) < 0$$

are non-empty. If the boundary planes do not intersect, there can only be three non-empty sets. See the illustration in figure 2. Figure 2 depicts the boundary planes  $c^{ta}(\mu, \tau, \sigma) = 0$  and  $c^{sb}(\mu, \tau, \sigma) = 0$  for any arbitrary value of  $\tau$  in the  $\mu$ - $\sigma$  space such that they do not intersect. Then there can only be defined three distinct subspaces by these two boundary planes. This implies that the joint conditions  $c^{ta}(\mu, \tau, \sigma) \leq 0$  and  $c^{sb}(\mu, \tau, \sigma) \geq 0$  describe an empty set. Hence, for any  $(\mu, \tau, \sigma)$  in the parameter range holds:

$$c^{sb}(\mu, \tau, \sigma) \geq 0 \implies c^{ta}(\mu, \tau, \sigma) > 0 \text{ and}$$

$$c^{ta}(\mu, \tau, \sigma) \leq 0 \implies c^{sb}(\mu, \tau, \sigma) < 0$$

Or in words: the condition for symmetry-breaking is a subset of the condition for total agglomeration. (If symmetry-breaking occurs, then the system always ends up with total agglomeration.)

**Q.E.D.**

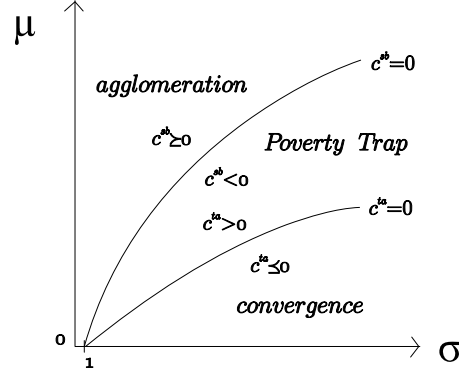


Figure 6: Parameter Range

#### Appendix 4:

If the short-run equilibrium condition (41) and the equal real wage condition (43) are set equal to each other and some terms are rearranged, a function in the variable  $p$  emerges which needs only to be defined for positive wages ( $p > 0$ ).

$$\sum_{i=1}^3 a_i p^{b_i} + a_4 = 0, \quad (45)$$

where

$$\begin{aligned} a_1 &= -a_2 = (1 - \mu) (1 - t^2) > 0 \\ a_3 &= -a_4 = t (1 + \mu) (1 - t^2) > 0 \\ b_1 &= -\sigma < 0 \\ b_2 &= \frac{1 - \sigma}{\mu} < 0 \\ b_3 &= \frac{1 - \sigma (1 + \mu)}{\mu} < 0 \end{aligned}$$

If this equation was a polynomial, Descartes' rule of sign<sup>67</sup> would imply that this gives at most 3 solutions for  $p > 0$ . However, Descartes' rule of sign can still be applied in the following way: Suppose the  $b_i; i = 1, 2, 3$  are rational numbers and  $N \in \mathbb{N}$  is the common denominator of them. Then set  $\hat{b}_i = b_i N$  and define a  $\xi$  such that  $p = \xi^N$ . The equation (45) can thus be rewritten as

$$\sum_{i=1}^3 a_i \xi^{\hat{b}_i} + a_4 = 0 \quad (46)$$

which is a polynomial and Descartes' rule of sign applies. If the polynomial (46) has at most three solutions for  $\xi$ , then it must also have at most three solutions for  $p$  (because there is a one to one mapping between  $p$  and  $\xi$ ). One of them is  $p = 1$ . Suppose that the other two were both interior solutions, i.e.  $p \in (0, 1)$ . If that was true, then there should also exist two solutions for  $p > 1$ , as any interior solution for  $p \in (0, 1)$  must have a corresponding steady state for  $p > 1$  (the result must be independent of the label of the region). However, then there would exist more than 3 solutions. Thus, at most one solution is interior.

**Q.E.D.**

<sup>67</sup>See Itô (1993), p. 36 for a statement of Descartes' rule of sign.

## 4 Neoclassical Growth, Manufacturing Agglomeration, and Terms of Trade\*

### Abstract

This paper presents an integrated view of economic growth, development traps, and economic geography. We explain why there is income convergence among some countries (neoclassical regime) and income divergence among others (poverty trap regime). Income convergence (divergence) and manufacturing industry diffusion (agglomeration) are re-enforcing each other in a cumulative process. Moreover, trade openness may trigger a catch-up process of an economy that is stuck in a “poverty trap”. This catch-up is characterized by an increase in the investment-to-GDP ratio and an improvement of the terms of trade.

**JEL Classification:** F12, O41

**Keywords:** convergence, agglomeration, poverty trap, terms of trade, complementarities

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\*The author thanks Niels Blomgren-Hansen, Markus Bollig, Eric W. Bond, Bjarne Sloth Jensen, Pascal Raimondos-Møller, and seminar participants of Copenhagen Business School, PennState University, the EEA-conference in Toulouse 1997, and the Midwest-International-Economics conference in Bloomington 1997 for their comments. Usual caveats apply.

## 4.1 Introduction

Economic geography is the subfield of economics that explains the location of production factors in space. Growth theory is the subfield of economics that explains national or regional income both across time and across countries or regions. The theory of development traps explains why some countries or regions lack behind. In this paper, we will explore the interrelation of these three subfields.<sup>68,69</sup>

Such an interrelation between growth theory, the location of manufacturing industries, and the backwardness of some regions or countries has been discussed informally among others by Myrdal and Kaldor. In particular, Myrdal (1957) observes and explains disparities both in regional and in national incomes. Additionally, Kaldor (1967) builds his growth theory upon Myrdal's work being more explicit about underlying assumptions and transmission channels. We consider five stylised facts that may capture the main theses of Myrdal (1957) and Kaldor (1967).

1) The same force that explains the agglomeration of economic activity in space also explains national differences in income.

*“The international inequalities are, of course, not dissimilar from the regional inequalities within a country. We will also find that there is a close causal relation between the two.”* (Myrdal, 1957, p. 10)

2) This force is based on the “principle of circular and cumulative causation” (Myrdal, 1957, chapter 2). Myrdal describes this principle as a self-enforcing process that is explicitly thought of as an unstable equilibrium of a dynamical system that drives one country or region into a best position and another country or region

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<sup>68</sup>Lucas (1988) also accounts for all three subfields at once. He starts out with a theory that fits the stylised facts of the US growth experience and explains within this setting 1) why income differs among countries, and 2) why international trade is not insuring convergence of income. Lucas (1988) finally points out that “a national economy is a completely arbitrary unit to consider” (p. 37) and accounts in his setting for the formation of cities by human capital externalities. We will base our model on an endogenous explanation, rather than an externality, to explain many of the same stylised facts.

<sup>69</sup> Thereby we will follow a modeling approach that tries to capture many stylised facts in a model mechanism that is as simple as possible. Of course, none of the stylised facts will be exclusively explained by our model.

into a worst position. Myrdal (1957, p. 27 ff) notes further that migration, capital movements, and trade may keep regional disparities growing.

3) The crucial assumption deviating from neoclassical theory is increasing returns to scale production technology (Kaldor, 1967, lecture 1). Increasing returns to scale applies, according to Kaldor, particularly to a wide range of manufacturing industries and might become effective not so much in terms of firm size, but in terms of process and product differentiation.

*“Economies of scale are derived not only from the expansion from any single industry but from a general industrial expansion ...”* (Kaldor, 1967, p.14)

4) A possible transmission channel may be a terms-of-trade effect.

*“A cumulative process of the same general character, ..., will also be generated by a change in the terms of trade of a community or a region, if the change is large and persistent enough ...”* (Myrdal, 1957, p. 26)

5) Another possible part of a transmission channel may be internal capital accumulation of a region or country that drives up the return on investment in the faster growing regions relative to the slower growing regions. In this sense investment projects may be complements, rather than substitutes.

*“The establishment of a new business or the enlargement of an old one widens the market for others, as does generally the increase of income and demand. Rising profits increase savings, but at the same time investments go up still more, which again pushes up the demand and the level of profits. And the expansion process creates external economies favourable for sustaining its continuation.”* (Myrdal, 1957, p. 25)

We conclude: If the same force explains manufacturing industry agglomeration and income disparities, this calls for a unified approach of growth theory and economic geography. We will set up a model that captures all the above five theses. This raises a question: Why should one try to model income divergence? After all, the neoclassical

view of economic growth, i.e. (conditional) convergence of income at least among some countries, has found large approval among mainstream economists.<sup>70</sup>

The recent empirical convergence literature is inconclusive of the (conditional) convergence hypothesis (Barro, 1991, Barro and Sala-i-Martin, 1992, and Mankiw, Romer, and Weil, 1992 and Cohen, 1996) or the club convergence hypothesis (Baumol, et. al., 1989, Durlauf and Johnson, 1996, and Quah, 1996) for both country and regional data sets. On the one hand, Barro (1991), Barro and Sala-i-Martin (1992), Mankiw, Romer and Weil (1992), and Cohen (1996) find that the average country or region converges conditionally on structural characteristics of the economies. On the other hand, Quah (1997) notes that the population of the converging regions/countries might be double peaked, thus supporting the club convergence hypothesis which says that initial conditions also matter. Additionally, Durlauf and Johnson (1995) reject the conditional convergence hypothesis in favour of multiple regimes or stages of development in a cross section analysis.<sup>71</sup> Quah (1993) notes also that conditional convergence of the average country in a regression analysis is compatible with outlier countries that do not converge.<sup>72</sup>

The theory of development traps explains income of these countries that do not converge to a “rich country” steady state.<sup>73</sup> However, given that there is a theory on economic growth that predicts conditional convergence, and a theory on development traps that predicts some sort of divergence, one may pose the following question: When is a country described by the first theory and when by the second? To answer this question, a unified approach may prove helpful having a **“neoclassical regime”** that has all the properties of a neoclassical growth model, having a **“poverty trap**

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<sup>70</sup>Independently of the empirical convergence literature, there is other empirical evidence supporting the neoclassical growth model. Jones (1995) and Levine and Renelt (1992) show that the time series properties of endogenous growth models are inconsistent with the data and that the growth regressions are unrobust with respect to most independent variables except investment. Furthermore, Young (1995) shows that the East Asian growth miracles can be explained by factor accumulation in the spirit of the neoclassical growth model, rather than by total factor productivity growth. However, Caselli, Esquivel, and Lefort (1996) claim reduced empirical relevance of the Solow model on basis of GMM-estimation.

<sup>71</sup>Jones (1997) adds that the relatively rich countries tend to converge, whereas the relatively poor countries tend to converge from the US per capita income levels.

<sup>72</sup>The classical example is the Italian Mezzogiorno - a region of relative and absolute decline over decades. See Rauch (1997).

<sup>73</sup>Surveys on poverty trap models are Azariadis (1996) and Galor (1996).

**regime**” that explains backwardness, and having a testable condition under which one or the other regime prevails. This paper attempts to provide such an approach.

Our first contribution will be to integrate Myrdal’s and Kaldor’s view on economic growth, development traps and the location of production factors - summarized in the five theses above - into mainstream economics without any sacrifice of neoclassical theory. Additionally, we will state a testable condition under which the one or the other regime applies.

Our second contribution will be to focus on a new agglomeration process of manufacturing industries among countries that is based on a mutual interaction with capital accumulation and growth. Agglomeration of economic activity on different levels like city, region, or nation may be explained by different agglomeration forces.<sup>74</sup> Cities may be formed by localized intermediate inputs (Abdel-Rahman (1988), Fujita (1988), Rivera-Batiz (1988), and in a growth setting Englmann and Walz (1995)). Disparities among regions may be caused by factor movements such as worker migration (Krugman, 1991a), or forward and backward linkages caused by intermediate goods (Venables, 1996). Internationally, frictionless factor movements are less likely to happen than interregionally.<sup>75</sup> But what causes then an unequal distribution of manufacturing industries among countries? One answer is specialization of countries in different sectors (e.g. Krugman and Venables, 1995); another one is information externalities (Grossman and Helpman, 1991); and a third answer is R&D location decisions (Martin and Ottaviano, 1996). The simplest explanation is, however, that there are more manufacturing firms in one country relative to another, because this country has accumulated more capital. This alone does not suffice for an explanation. The missing part is how firm agglomeration feeds back on diverging capital accumulation. How does an increase of agglomeration lead to higher growth of a country relative to another, and higher growth to even higher firm agglomeration? We will

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<sup>74</sup>Fujita and Thisse (1996) survey the literature on agglomeration economics. We consider only endogenous explanations in cumulative processes, such that completely identical countries end up diverging from each other if there is just a small disturbance (idiosyncratic shock).

<sup>75</sup>See Krugman and Venables (1995).

explain this feed-back with a terms-of-trade effect.<sup>76</sup>

Our third contribution will be to explain how trade-liberalization triggers a catch-up process. It is obvious that the agglomeration forces depend crucially on the costs of bridging distances (e.g. transport cost, tariffs, information costs, etc.), because otherwise location does not matter. If agglomeration happens at a high level of trade costs and convergence at a low level, and manufacturing agglomeration or convergence feed through on growth, then we have established a (new) nexus between trade-liberalization and growth.

The rest of the paper is organized as follows: section 2 gives a brief verbal description of the model and its mechanics, and compares related literature; section 3 gives the formal model set-up; section 4 solves the model for the steady states; section 5 provides a stability analysis; section 5.1 discusses the neoclassical growth regime; section 5.2 discusses the “poverty trap” regime; section 5.3 gives the model implications for economic geography; and section 6 concludes.

## 4.2 A Brief Model Description

Our model is a synthesis of an economic geography model (Krugman, 1991a) and a neoclassical growth model (Solow, 1956, and others). There are two countries that have a Dixit-Stiglitz (1977) monopolistic competition production sector with increasing returns on plant level. Labour and capital are immobile. Capital is a durable goods composite of all varieties. Investment is taken literally as foregone consumption. There is intra-industry trade, although trade costs segment the product markets in the two countries and trade is assumed to be balanced. Consequently, the only linkage between the two countries are the terms of trade.

The mechanics of the model are best understood in a thought experiment. Suppose two identical countries grow symmetrically having a capital stock of identical size. For some reason (idiosyncratic shock), country 1’s capital stock grows faster than

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<sup>76</sup>An alternative nexus is given recently in Ben-David and Loewy (1998) based on cross-country technology spillovers embedded in trade flows.



country 2's at one time period. This will increase the number of firms in country 1 relative to country 2 given that output per firm remains constant.<sup>77</sup> Because of trade costs, there is a home market bias in consumption of goods. Additionally, income is higher in country 1, because there is more capital. Hence, there will be stronger demand for any typical variety in country 1 relative to any typical variety in country 2, whereas relative supplies for a typical variety do not change. This will increase country 1's producer price of a typical variety relative to country 2's (terms-of-trade effect).

The savings and investment decision in each country is based on the present and future real interest rate which is equal to the real rental rate of capital. The real rental rate in each country at a given point in time is influenced by three effects: (i) The higher producer prices in country 1 allow *ceteris paribus* for higher rental rates in country 1 (agglomeration force I). (ii) There are less goods to be imported in country 1. Therefore, there are less trade costs to be paid and the consumption price index is thus lower in country 1. This means - everything else equal - that the real interest rate is higher in country 1 (agglomeration force II). (iii) The capital-labor ratio is higher in country 1. By capital-labour substitutability, this implies a higher wage-rental rate in country 1 (convergence force). The net effect of the three forces turns out to be ambiguous and depends on the level of trade costs.

Suppose the real rental rate in country 1 decreases faster than the one in country 2 over the entire transition path towards the steady state (spatial substitutability of investment). Then, investment will be lower in country 1 over the entire transition path and the two capital stocks will eventually converge over time. This implies income convergence and describes thus the neoclassical regime. Suppose, on the contrary, that the real rental rate decreases slower in country 1 than in country 2 over the entire time path (spatial complementarity of investment). Then, future investment will be higher

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<sup>77</sup>This is a standard result in a Dixit-Stiglitz (1977) set-up due to the assumption of CES utility functions and constant variable cost. Suppose there is an expansion in total income. Then demand for each single good is rising. This rises profits for all (symmetric) firms, because the fixed cost can be spread over a larger output. However, the increase in profits causes new firms to enter, such that the original increase in income is now spread over more goods. The amount of income spent on a single good falls back to the original level. Therefore, output of a single firm is a constant in this set-up.

in country 1. This increases further the terms of trade in country 1. Thus, the real rental rate gap might become even bigger self-enforcing the faster capital accumulation in country 1. The cumulative process will eventually stop as the convergence force will begin to dominate at some degree of divergence. This implies income divergence and describes thus the “poverty trap” regime. If in this regime the capital stock is higher in country 1 at any point of time, then a fixed firm size implies an agglomeration of firms in country 1. Hence, a new explanation for manufacturing industry agglomeration is found that is based both on national capital accumulation and a terms-of-trade effect in a cumulative process.

Our model builds upon the literature on big push and poverty traps which was promoted in an influential formal model by Murphy, Shleifer, and Vishny (1989). We share the features of increasing returns technology and demand spillovers to trigger self-enforcing growth processes. However, we pose this idea into an international context allowing us to discuss the importance of trade barriers, and home-market size, and the role of neighbouring countries in boosting or inhibiting growth.

Gali (1995) builds into a model with monopolistic competition an investment complementarity by a competition effect that drives a wedge between the physical marginal product of capital and the marginal revenue product of capital. Instead of the competition effect in a closed economy, we use a terms-of-trade effect in a two country model to generate a relative investment complementarity rather than an absolute one.<sup>78</sup>

Our model is also related to Baldwin, Forslid, and Haaland (1995) which inspired our model set-up and Baldwin and Seghezza (1996). These models have similar production, consumption and market structures as ours. However, their focus is on dynamic gains of trade and on the investment creation of trade liberalization in the symmetric country case. They rule out terms-of-trade effects and exclude the cumu-

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<sup>78</sup>That is a rise in the relative capital stock of two countries rises the ratio of real rental rates, whereas in Gali (1995) an absolute rise in the capital stock rises (locally) the absolute value of the real rental rate.

lative process that we focus on.<sup>79</sup>

### 4.3 The Model Set-up

There are two consumers which differ only by their place of residence in two countries ( $j = 1, 2$ ). A standard logarithmic intertemporal utility function  $U_j$  is assumed<sup>80</sup> that is defined on a consumption basket  $C_j$ :

$$U_j = \int_0^{\infty} e^{-\lambda t} \ln C_j dt, \quad (47)$$

where  $\lambda$  is the time preference rate, and  $t$  is a time index in continuous time.<sup>81</sup> The consumption basket  $C_j$  of a consumer  $j$  is of the Dixit-Stiglitz (1977) type and is defined on all domestic and foreign produced varieties with an elasticity of substitution denoted  $\sigma$  ( $\sigma > 1$ ):

$$C_j = \left( \sum_{i_j=1}^{n_j} \frac{c_{i_j j}^{\frac{\sigma-1}{\sigma}}}{n_1 + n_2} + \sum_{i_k=1}^{n_k} \frac{c_{i_k j}^{\frac{\sigma-1}{\sigma}}}{n_1 + n_2} \right)^{\frac{\sigma}{\sigma-1}}, \quad (48)$$

where the number of goods produced in country  $j$  are indexed  $i_j = 1_j, \dots, n_j$ , and  $c_{i_j j}$  and  $c_{i_k j}$ ,  $j, k = 1, 2$ ,  $k \neq j$ , are consumer  $j$ 's consumption of the varieties  $i_j$  and  $i_k$  produced in country  $j$  and  $k$ , respectively. Additionally, there is no international borrowing and lending and trade will have to be balanced.<sup>82</sup>

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<sup>79</sup>Very recently, Baldwin (1998), Baldwin, Martin, and Ottaviano (1998), and Baldwin and Forslid (1997, 1998) expell the same idea of bifurcation of income convergence/divergence behavior of two economies in dependence of trade cost in a model with monopolistic competition and increasing returns. However, our engine of growth is capital accumulation, and our convergence force Solow's (1956) capital-labor substitutability assumption, whereas the papers above use technological progress as engine of growth and the extent of competition effect of economic geography models (Krugman, 1991a) as convergence force. Consequently, trade openness triggers income *divergence* in Baldwin, Martin, and Ottaviano (1998), whereas in our model trade openness triggers income *convergence*. Also, a larger home market increases firm profits and R&D activity in the papers mentioned above, whereas a larger home market induces a demand bias towards domestic goods and rises the domestic terms of trade in our model.

<sup>80</sup>All results remain valid, if an isoelastic intertemporal utility function is used. However, mathematical proofs would be more complicated.

<sup>81</sup>We suppress the time index whenever obvious.

<sup>82</sup>The assumption of balanced trade has a long tradition in the trade and growth literature: e.g. Stiglitz (1970) and Grossman and Helpman (1991).

With monopolistic competition, each variety  $i_j$  will be produced by a different firm  $i_j$ . Firms differ only by their location. Therefore, firms within a country  $j$  are symmetric and the index  $i_j$  for firm  $i$  in country  $j$  can be collapsed to  $j$  denoting a typical firm in country  $j$ . The production technology is a Cobb-Douglas production function with fixed cost that gives rise to increasing returns to scale at plant level. In particular,  $\alpha$  units of inputs  $v_j$  in form of a basket of labour  $l_j$  and capital  $k_j$  are used to install the production process every day (maintenance work) and  $\beta$  units of the input basket are used to produce each unit of goods for the domestic and the foreign market  $x_j$ :

$$v_j = \alpha + \beta x_j \quad \text{and} \quad v_j = k_j^\delta l_j^{1-\delta}, \quad (49)$$

where  $\delta$  ( $0 < \delta < 1$ ) denotes the income share of capital.<sup>83</sup>

We assume as in Baldwin, Forslid and Haaland (1995) that investment and capital are the same composite of industrial goods as is consumption and goods can be used both for consumption and investment:

$$I_j = \dot{K}_j = \left( \sum_{i_j=1}^{n_j} \frac{\frac{\sigma-1}{\sigma} \iota_{i_j j}}{n_1 + n_2} + \sum_{i_k=1}^{n_k} \frac{\frac{\sigma-1}{\sigma} \iota_{i_k j}}{n_1 + n_2} \right)^{\frac{\sigma}{\sigma-1}}, \quad (50)$$

where  $I_j$  is the investment aggregate used by the firms in country  $j$  to increase the capital stock  $K_j$  of country  $j$ , a dot denotes the time derivative of a variable, and  $\iota_{i_j j}$  and  $\iota_{i_k j}$ ,  $j, k = 1, 2$ ,  $k \neq j$ , are demand of the firms in country  $j$  for investment goods produced by a firm  $i_j$  and  $i_k$  in country  $j$  and  $k$ , respectively. A unit of capital, i.e. a machine, may be assembled at zero cost in different ways from time-varying product spaces, but once it is assembled it performs the same service. A larger product space does not allow for more productive capital (no Smithian growth).<sup>84</sup> Note that we do not allow for the usual depreciation of capital. One can think of capital as a

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<sup>83</sup>It will be this particular type of the production function that guarantees both constancy of factor shares (Kaldor, 1963), and constant returns to scale on industry level (Burnside, 1996).

<sup>84</sup>Smithian growth, i.e. the cost reduction from larger market size and increased specialisation, is discussed in Kelly (1997) in the context of economic geography and growth.

durable composite of intermediate input goods that is permanently maintained. The maintenance cost will show up in the fixed cost parameter  $\alpha$  of the production function.

Additionally, we assume free firm entry and exit which keeps profits at zero. Production factors are immobile.<sup>85</sup> For simplicity, labour supply is inelastic, equally distributed among countries, and normalized to one<sup>86</sup>. Finally, there are trade costs of the Samuelson iceberg-type, such that only a fraction  $\tau$  of one produced unit of a good arrives at its foreign destination ( $0 < \tau < 1$ ).

## 4.4 Equilibrium

The consumption maximization problem of the typical agents in country 1 and 2 may be solved in two stages. First, the demand for any variety is determined for any given time path of expenditure on consumption goods. The corresponding unit expenditure function or ideal CES price index  $P_j$  is found to be:<sup>87</sup>

$$P_j = \left( \frac{n_j p_j^{1-\sigma}}{n_1 + n_2} + \frac{n_k p_k^{ex(1-\sigma)}}{n_1 + n_2} \right)^{\frac{1}{1-\sigma}}, \quad (51)$$

where  $p_j$  and  $p_k^{ex}$  are the domestic producer prices and export prices of firms in country  $j$  and  $k$  charged for consumers in country  $j$ , respectively. Then, the individual budget constraint can be written as follows:<sup>88</sup>

$$\dot{K}_j = I_j = \frac{r_j K_j}{P_j} + \frac{w_j}{P_j} - C_j, \quad (52)$$

where  $r_j$  and  $w_j$  denote nominal rental and wage rates. Investment expenditure equals wage income and rents minus consumption expenditure. Second, the optimal con-

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<sup>85</sup>We make this assumption, because we want to distinguish our agglomeration process from that of Krugman (1991a), Krugman and Venables (1995), Venables (1996), and Martin and Ottaviano (1996). These papers rely on interregional or intersectoral factor (in particular labour) movements and R&D location decisions.

<sup>86</sup>If we did not assume this normalization, then the capital stocks would simply be replaced by the capital-labor ratios. None of the qualitative results obtained in this paper would change, of course.

<sup>87</sup>Note that we take here the symmetry of firms within a country into account.

<sup>88</sup>We use the definition of the expenditure function (and an analogous equation for the investment aggregate  $I_j$ ):

$$P_j C_j \equiv \sum_{i_j=1}^{n_j} p_j c_{i_j j} + \sum_{i_k=1}^{n_k} p_k^{ex} c_{i_k j}$$

sumption expenditure is determined by maximizing utility (47) taking the individual budget constraint (52), a price vector, and the initial condition as given. We assume that private agents do not foresee the impact of their behaviour on decisions of agents in the other country. This assumption excludes strategic interaction and is in line with the monopolistic competition conjecture. The optimization yields the familiar Euler equation:<sup>89</sup>

$$\dot{C}_j = (\rho_j - \lambda) C_j, \quad (53)$$

where  $\rho_j \equiv r_j/P_j$  denotes the real rental rate of capital. Additionally, the familiar transversality condition completes the description of the dynamical system. Note that the steady state condition of the emerging dynamical system will involve equalization of real rental rates of capital across countries.

Firms maximize profits and use a mark-up pricing rule given the imperfect competition conjecture of Dixit and Stiglitz (1977) that firms take the direct impact of their price decision on goods market demand into account, but not the indirect effects on income and the price index:<sup>90</sup>

$$p_j = \frac{\sigma}{\sigma - 1} \beta c(w_j, r_j) \quad \text{and} \quad p_j^{ex} = \frac{\sigma}{\sigma - 1} \beta c(w_j, r_j) / \tau. \quad (54)$$

It is important that prices for foreign consumers contain a transport-cost mark-up on prices for domestic consumers. Furthermore,  $c(w_j, r_j)$  denotes the unit cost function which is given by the following expression:

$$c(w_j, r_j) = (1 - \delta)^{\delta-1} \delta^{-\delta} r_j^\delta w_j^{1-\delta}. \quad (55)$$

Finally, the relative input demand determines after aggregation the wage-rental ratio for a given capital-labour ratio (Recall that labour endowments are normalized to one.):

$$\frac{w_j}{r_j} = \frac{1 - \delta}{\delta} K_j. \quad (56)$$

Capital letters denote aggregates (e.g.  $K_j \equiv n_j k_j$  and  $V_j \equiv n_j v_j$ ). Additionally, the zero profit condition  $n_j p_j x_j = r_j K_j + w_j$  holds due to free firm entry and exit. Hence,

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<sup>89</sup>We follow the standard procedure as in Barro and Sala-i-Martin (1995).

<sup>90</sup>For a discussion of this conjecture see d'Aspremont, et. al. (1996).

we find from the zero profit condition and equation (56) that the rental payments are a constant fraction of income:

$$r_j K_j = \delta n_j p_j x_j. \quad (57)$$

Using the zero profit condition, we derive the following equation for firm output:<sup>91</sup>

$$x_j = \bar{x} = 1, \quad (58)$$

where we normalized without loss of generality  $\beta = 1 - \alpha$  and  $\alpha\sigma = 1$ .<sup>92</sup> Factor market equilibrium requires:

$$n_j = K_j^\delta = V_j. \quad (59)$$

Thus, the number of firms and goods depends on the capital stock of a country. The goods market equilibrium condition for a typical firm in country 1 at any point of time is the last equilibrium condition to be imposed:<sup>93</sup>

$$\frac{p_1^{-\sigma} (r_1 K_1 + w_1)}{n_1 p_1^{1-\sigma} + q n_2 p_2^{1-\sigma}} + \frac{q p_1^{-\sigma} (r_2 K_2 + w_2)}{q n_1 p_1^{1-\sigma} + n_2 p_2^{1-\sigma}} = 1. \quad (60)$$

where  $q \equiv \tau^{\sigma-1}$  proxies the reciprocal of trade costs for notational simplicity. Using the zero profit condition and defining relative producer prices (terms of trade)  $p \equiv p_2/p_1$  and relative firm agglomeration  $n \equiv n_2/n_1$ , equation (60) can be reformulated in the following way:

$$\frac{1}{1 + q n p^{1-\sigma}} + \frac{q n p}{q + n p^{1-\sigma}} = 1, \quad (61)$$

which can be solved for  $n$  to give two solutions  $n = 0$  and

$$n = \frac{q - p^\sigma}{p(q - p^{-\sigma})} \quad \text{with} \quad 0 < n < \infty. \quad (62)$$

This simple equation gives a relationship between the terms of trade and relative firm agglomeration.

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<sup>91</sup>For the derivation, we use the definition of  $V_j$ , equations (49) and (54), and  $c(w_j, r_j) V_j = r_j K_j + w_j$  which is obtained by plugging (57) and its counterpart for labour demand into the definition of  $V_j$ , deviding through by  $c(w_j, r_j)$ , and applying the zero profit condition.

<sup>92</sup>All results of the model are independent of  $\alpha$  and  $\beta$ .

<sup>93</sup>Note that we exploit here the fact that the composition of consumption good and investment good demand is irrelevant for goods market equilibrium, because we assumed investment and the consumption basket to be of the same functional composite of goods.

Defining  $K \equiv K_2/K_1$ , equation (59) may be restated in the following way:

$$n = K^\delta. \quad (63)$$

The degree of firm agglomeration is determined by the relative size of capital stocks. From now on, we can use firm agglomeration  $n$  and relative capital stocks  $K$  interchangeably. Next, the relative consumption price index  $P$  (real exchange rate) of the two countries can be written after some manipulations as:

$$P = p^{\frac{\sigma}{1-\sigma}}, \quad (64)$$

where we used (51) and (62). Define relative (nominal rental rates)  $r \equiv r_2/r_1$ . Then, it follows from (57), (59) and (63) that

$$r = pK^{\delta-1} \quad (65)$$

The relative (nominal) rental rate depends on two factors: the relative capital stocks and the relative producer terms of trade. Now, we can summarize the factor and goods market equilibrium conditions in the following Lemma.

**Lemma 1:** *For  $0 < K \leq 1$  holds: the correspondence  $p = p(K)$  is an upward sloping function below 1;  $P = P(K)$  is a downward sloping function above 1;  $r = r(K)$  is bounded from below by  $p(K)$ ; Finally,  $\lim_{K \rightarrow 0} r(K) = \infty$ .*

**Proof:** See appendix 1.

**Q.E.D.**

Lemma 1 can be shown in figure 1 that depicts the terms of trade  $p(K)$ , relative rental rates  $r(K)$ , and the relative consumption price index  $P(K)$  as dependent on the degree of relative capital stocks  $K$ . Note additionally that relative capital stocks  $K$  and firm agglomeration  $n$  are proportional (equation (63)). If industries are partially agglomerated in country 1 ( $K < 1$ ), then the terms of trade  $p(K)$  are bigger in country 1, whereas the consumption price index  $P(K)$  is smaller. However, the relation of rental rates  $r(K)$  to relative capital stocks  $K$  may be ambiguous.

These results reflect the interplay between terms of trade and agglomeration of industries that is implicit in Krugman (1991a). Suppose the economy starts from an



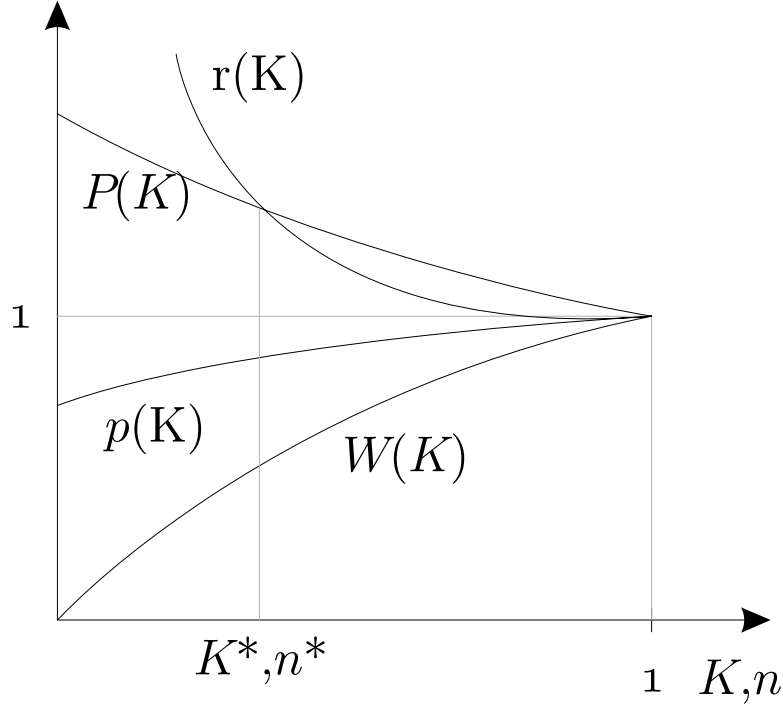


Figure 7: Relative Prices, Wages, and Rental Rates

equal distribution of industries. Then, the relative distribution of production factors changes, because one country is accumulating more capital. Consequently, there will be more purchasing power in the larger country than in the smaller one. Because of trade costs, demand for goods of a typical firm is biased towards domestic firms. This implies that demand for goods of a typical firm in the larger country exceeds the one in the smaller country. However, supply of firms is the same across all firms in the Dixit and Stiglitz (1977) framework (see equation (58)). Thus, goods market clearing requires that relative producer prices fall in the smaller country. The price movement induces the exit of firms in the smaller country and the entry of new firms in the larger (see equation (63)).

The consumption price index of a typical consumer in the large country is below the one in the small country, although (factory gate) producer prices are higher in the large country and a larger share of income is spent on domestic goods (See equation (64)). This is so, because less goods have to be imported in the large country. Hence, there are less goods for which a transport-cost mark-up has to be paid. (See equation

(54)). In this sense, transport costs drive a wedge between relative (factory gate) producer prices and relative consumption price indices.

The ambiguous impact of the distribution of the capital stock on rental rates arises from a convergence force, i.e. capital substitutability, and from an agglomeration force, i.e. the terms-of-trade effect due to the agglomeration of manufacturing industries. The rise in the capital-labour ratio will lower the rental rate relative to the wage rate in the country with more capital; the rise in industrial agglomeration rises the terms of trade in the bigger country and rises the overall factor payments in factor market equilibrium including - in particular - rental rates (see equation (65)).

We close the model by combining the goods and factor market equilibrium conditions and the conditions from firm optimization with the dynamical equations from consumer optimization. Note that the intertemporal budget constraint (52) can be reformulated to yield

$$\dot{K}_j = \frac{n_j p_j}{P_j} - C_j = \frac{r_j K_j}{\delta P_j} - C_j, \quad (66)$$

where equation (58) is used and the second equality sign follows from equation (57). We note from (51), (57), (58), and (59), and Lemma 1 that the real rental rate of capital in a country depends on the level of the two capital stocks in the two countries  $K_1$  and  $K_2$  ( $\rho_j \equiv r_j/P_j = \rho_j(K_1, K_2)$ ). Then the model may be summarized in the following 4-dimensional, non-linear differential equation system with the control variables  $C_1$  and  $C_2$ , the state variables  $K_1$  and  $K_2$ , the national budget constraints (66), and the Euler equations (53):

$$\dot{K}_1 = \frac{\rho_1(K_1, K_2)}{\delta} K_1 - C_1 \quad (67)$$

$$\dot{C}_1 = (\rho_1(K_1, K_2) - \lambda) C_1 \quad (68)$$

$$\dot{K}_2 = \frac{\rho_2(K_1, K_2)}{\delta} K_2 - C_2 \quad (69)$$

$$\dot{C}_2 = (\rho_2(K_1, K_2) - \lambda) C_2, \quad (70)$$

where the transversality conditions are

$$\lim_{t \rightarrow \infty} K_j(t) \mu_j(t) = 0 \quad (71)$$

with the co-state variables  $\mu_j(t)$  for (67) and (69), and the initial conditions are

$$K_j(0) = K_{i0} \quad (72)$$

for  $j = 1, 2$ .

Next, the steady states are calculated. Combining (68) and (70) requires  $\rho \equiv \rho_2/\rho_1 = r(K)/P(K) = 1$  in the steady state. First, we define a benchmark value for the reciprocal transport cost proxy  $q$ , such that

$$q^* \equiv \frac{(2\sigma - 1)(\delta\sigma + 1 - \sigma)}{\delta\sigma - (1 - \sigma)}. \quad (73)$$

Then, we can formulate the following proposition on the equalization of real rental rates of the two countries.

**Proposition 1:** (i) *The steady state condition  $\rho(K) = 1$  has the (trivial) symmetry solution  $\bar{K} = 1$ , if  $q > q^*$ ; moreover, it holds that  $\frac{d\rho(1)}{dK} < 0$  in this case.*

(ii) *The steady state condition  $\rho(K) = 1$  has the solutions  $\bar{K} = \{K^*, 1/K^*, 1\}$ , if  $q < q^*$ , where  $0 < K^* < 1$ ; moreover, it holds that  $\frac{d\rho(1)}{dK} > 0$ ,  $\frac{d\rho(K^*)}{dK} < 0$ , and  $\frac{d\rho(1/K^*)}{dK} < 0$  in this case.*

**Proof:** See appendix 2.

There are two regimes depending on the level of trade costs, and one of the two regimes exhibits multiple equilibria. The first regime will be called **neoclassical regime**; the second regime will be called **poverty trap regime**, henceforth.

Trade costs drive a wedge between relative producer prices and consumption price indices. If this wedge widens sufficiently ( $q < q^*$ ), the intermediate solution  $K^*$  arises (see figure 1). In this case, an increase of the capital stock in the largest country raises the real rental rate above the one in the smallest country in the neighborhood of a symmetric distribution of capital ( $d\rho(1)/dK > 0$ ). In this sense investment projects are local complements in the poverty trap regime (spatial complementarity of investment). If the wedge between producer prices and consumption price indices

is not sufficiently large ( $q > q^*$ ), then an increase of the capital stock in the biggest country leads to a lower real rental rate than in the smallest country ( $d\rho(1)/dK < 0$ ). In this sense investment projects are global substitutes in the neoclassical regime (spatial substitutability of investment).

The steady state variables  $\bar{K}_1, \bar{C}_1, \bar{K}_2, \bar{C}_2$  can be obtained as functions of  $\bar{K}$ .<sup>94</sup> However, we will not focus on their values. For future reference, we will denote the set of steady state vectors  $\bar{x} \equiv (\bar{K}_1, \bar{C}_1, \bar{K}_2, \bar{C}_2)$  and the particular steady state vectors associated with  $\bar{K} = 1$ ,  $\bar{K} = K^*$  and  $\bar{K} = 1/K^*$  by  $x^*$ ,  $x^{**}$ , and  $x^{***}$ , respectively. If an equation holds for any steady state vector, we will also use the notation  $\bar{x}$ .

Finally, we shall point at two interesting properties of the model. First, the model relies on constant factor shares which is one of the stylised facts of growth theory (Kaldor, 1963). Second, the aggregated industry production function  $n_j x_j = K_j^\delta L_j^{1-\delta}$  exhibits constant returns to scale. Hence, the increasing returns to scale assumption on plant level is in line with empirical evidence on the production technology on industry level such as Burnside (1996).

## 4.5 Stability Analysis

We will not follow the standard procedure of a local stability analysis as in Dockner (1985) for 4-dimensional, non-linear differential equation systems, because the Jacobian of the linearized system cannot be signed unambiguously. Instead, we will find a first-order approximation function for the system (67)-(70) that has (i) the same steady state values, (ii) the same Jacobian matrix at the steady state values, and (iii) the Jacobian matrix is unambiguously signed for any single entry. Finally, we use the fact that the qualitative behaviour of the approximation system is equivalent to the original system.

We take the difference in the growth rates of the capital stocks and consumption

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<sup>94</sup>Bars denote steady state values of a variable. Caveat:  $\bar{K}$  denotes the set of all steady state capital stocks (because there are multiple equilibria), whereas  $K^*$  denotes a certain value for one particular steady state capital stock.

using (67)-(70).

$$\frac{\dot{K}_2}{K_2} - \frac{\dot{K}_1}{K_1} = \frac{1}{\delta} (\rho_2(K_1, K_2) - \rho_1(K_1, K_2)) - \frac{C_2}{K_2} + \frac{C_1}{K_1} \quad (74)$$

$$\frac{\dot{C}_2}{C_2} - \frac{\dot{C}_1}{C_1} = \rho_2(K_1, K_2) - \rho_1(K_1, K_2)$$

We would like to express these equations in terms of relative capital and consumption.

For this purpose, we “guess” the following approximation function to the system (74):

$$\frac{\dot{K}}{K} = \frac{a_1}{\delta} \ln \rho(K) - a_2 \ln C + a_2 \ln K \quad (75)$$

$$\frac{\dot{C}}{C} = a_1 \ln \rho(K),$$

where we defined  $C \equiv C_2/C_1$ ,  $a_1 \equiv \bar{\rho}_1$ , and  $a_2 \equiv \bar{C}_2 / \bar{K}_2$ . This approximation is entirely sufficient to describe the behaviour of the terms of trade around the steady state and to pin down the relation of all state variables (capital, income, and firm distribution) between the two countries around the steady state values.<sup>95</sup> However, for the approximation to be valid, we need to show that the approximation (75) is chosen such that this system has the same steady states and the same qualitative dynamic behaviour as the original system (74). The first property is easily confirmed, whereas the second is proven in Lemma 2.

**Lemma 2:** *The Jacobian matrix of the dynamical system (75), (67), and (68) evaluated at any of the steady states has the same eigenvalues as the Jacobian matrix of the dynamical system (67)-(70).*

**Proof:** See appendix 3.

**Q.E.D.**

This lemma will be used for the local stability analysis that is summarized in the next proposition.

**Proposition 2:** *Consider the dynamical system (67)-(72). Assume that the eigenvalues are distinct. Then, this system is locally asymptotically stable if either*

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<sup>95</sup>To recover the absolute values of the state variables, two more equations are necessary: e.g. the dynamical equations governing country 1. We skip them to focus on the idea of the solution method, but use them in the rigorous mathematical derivation in Lemma 2 and appendix 3.

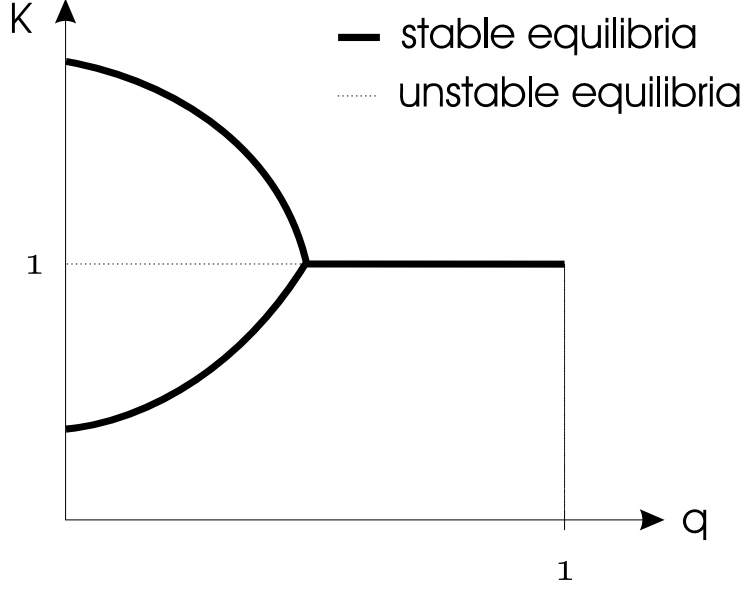


Figure 8: Bifurcation Diagramm

- (i)  $q > q^*$  and  $\bar{K} = 1$  or
- (ii)  $q < q^*$  and  $\bar{K} = K^*$  or
- (iii)  $q < q^*$  and  $\bar{K} = 1/K^*$ .

Furthermore, there exist three corresponding two-dimensional local stable manifolds  $W_{loc}^s(x^*)$ ,  $W_{loc}^s(x^{**})$ , and  $W_{loc}^s(x^{***})$ . On the contrary, the dynamical system (67)-(72) has a one-dimensional local stable manifold  $W_{loc}^s(x^*)$ , if

- (iv)  $q < q^*$  and  $\bar{K} = 1$ .

This local stable manifold is described by  $K_1(t) = K_2(t)$  and  $C_1(t) = C_2(t)$  for  $0 \leq t \leq \infty$ .

**Proof:** See appendix 4.

Proposition 2 resembles a supercritical pitchfork bifurcation with the bifurcation parameter  $q$  and the bifurcation point  $q = q^*$ . We illustrate this in the following bifurcation diagram. The vertical axes shows the position of steady state equilibria in terms of the relative distribution of capital; the horizontal axes shows the level of trade costs. At a high level of trade costs (low  $q$ ), there are three steady states with

the symmetric one ( $\bar{K}=1$ ) being unstable (poverty trap regime). At a low level of trade costs (high  $q$ ), there is only one stable steady state equilibrium at a symmetric distribution of capital (neoclassical regime).

The poverty trap regime emerges if and only if investment projects become locally complementary in the neighborhood of a symmetric distribution of capital and firms.<sup>96</sup> Around a symmetric distribution of capital, an increase of investment in one country relative to the other increases, rather than decreases, the relative real marginal productivity of capital in terms of the consumer price indices inducing more investment to take place in the former than in the latter country. At some degree of divergence in capital stocks and firm distribution the divergence process stops, because investment projects have become local substitutes. A further rise of investment in the booming country lowers the real marginal productivity of capital relative to the declining country. Therefore the divergence process remains incomplete and a certain asymmetric distribution of capital and firms is a stable equilibrium.

The neoclassical regime emerges on the contrary, if investment projects are global substitutes, i.e. a relative rise in investment of one country above investment in the other lowers the real marginal product of capital in the former relative to the latter country. Therefore, only the symmetric distribution of capital can be a stable steady state. Given that there can exist multiple stable local manifolds, it is important to examine one aspect of global stability.

**Proposition 3:** *Consider the dynamical system (67)-(72) and the case  $q < q^*$ . For any given combination of initial conditions  $K_{10}, K_{20} \in \mathbf{R}^+$ , there exists a unique perfect foresight path for the two control variables  $C_1$  and  $C_2$ . Furthermore,  $x^*$  is reached, if  $K_{10} = K_{20}$ ;  $x^{**}$  is reached, if  $K_{10} > K_{20}$ ;  $x^{***}$  is reached, if  $K_{10} < K_{20}$ ;*

**Proof:** See appendix 5.

**Q.E.D.**

This proposition ensures that there exists a unique perfect foresight path. Only

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<sup>96</sup>This follows immediately from the proof of proposition 2 in appendix 4, equations (117)-(120). Note that the investment complementarity is referring to the ratio of capital stocks and the ratio of real rental rates rather than to their absolute values as in the closed economy model of Gali (1995).

one of the three steady states can be reached for any given combination of initial conditions. Therefore, this model does not exhibit expectations driven agglomeration processes as have been found in other dynamic models with increasing returns to scale like Matsuyama (1991), Krugman (1991b), and Kaneda (1995). In particular, we do not need any additional coordination mechanism of expectations as Kaneda's (1995) assumption of "euphoric expectations" to select among multiple perfect foresight path.

#### 4.5.1 The Neoclassical Growth Regime

In this section we discuss in detail the neoclassical regime, i.e. the case where trade costs are relatively low ( $q > q^*$ ). Recall that there is one steady state distribution of capital  $\bar{K} = 1$ . We summarize our results:

**Result 1:** *The neoclassical regime ( $q > q^*$ ) exhibits outphasing growth and convergence of income.*<sup>97</sup>

The dynamic adjustment path is shown in figure 3. The figure presents the unique stable manifold of the 4 dimensional differential equation system (67)-(72). In particular, there is a unique mapping from the state space  $K_2 - K_1$  to the control variable space  $C_2 - C_1$  which follows from the stable manifold theorem (see proposition 2). Even if two structurally identical countries start out with dissimilar capital stocks, i.e. one country is poor and the other is rich, there will be convergence of capital stocks and per capita income. The poorer country will grow faster than the richer country in the transition period to the steady state.

Our neoclassical growth regime differs from, e.g., a Solow or a Ramsey model (without technological progress and population growth) by a different adjustment path. Thus, countries that catch-up do not follow the same path as the leading countries. History does not repeat, as is the case in the Solow and Ramsey model. Once some country is ahead, the catch-up process will change terms of trade and the real marginal product of capital. This will foster income growth of the country lacking

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<sup>97</sup>This follows from proposition 2: the steady state is stable and the relative capital stock approaches one. However, income is a monotone, increasing function of the capital stock.



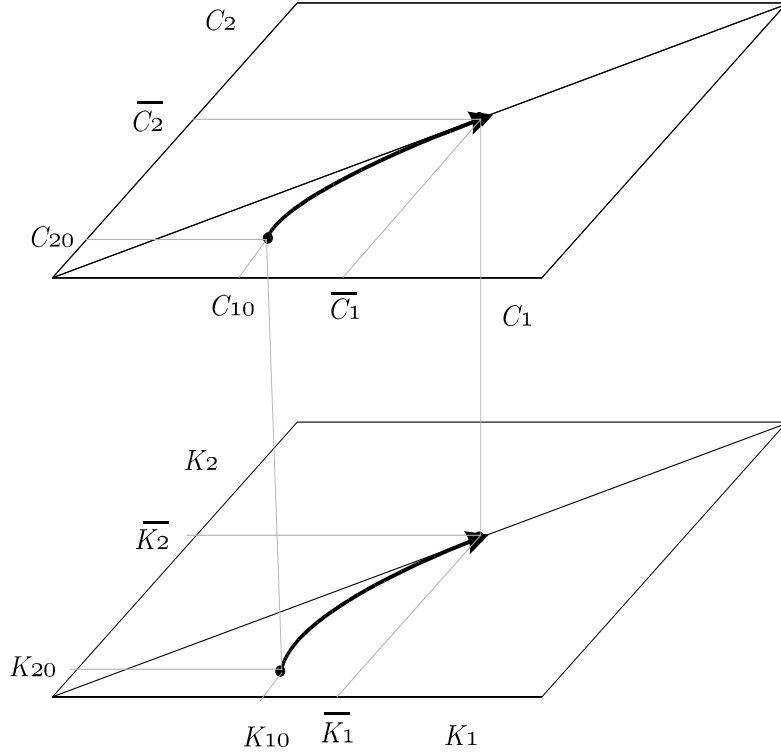


Figure 9: Neoclassical Regime

behind beyond what is predicted by a model with two isolated Ramsey economies. In this sense, the speed of convergence is higher in our neoclassical regime than in the isolated Ramsey economies.

Empirically, it is hard to “detect” the terms-of-trade effect caused by an investment boom, because any terms-of-trade effect due to total factor productivity growth (which is excluded in our model) has to be controlled for. Note that in our model the country that is growing fastest *improves* its terms of trade, because the home market effect together with trade costs causes a demand bias towards domestic goods at a given supply. If total factor productivity growth were the reason for different growth rates, then the faster growing economy is *deteriorating* its terms of trade, because a rise in total factor productivity rises output and decreases its relative price.<sup>98</sup> In a complementary study, the terms-of-trade effect is tested for the US and Japan from 1957 until 1990 in chapter 5 of this thesis. Weak evidence is found in favour of our

<sup>98</sup>This is, for example, the case in Osang and Pereira (1997) which is a two-country, human capital driven endogenous growth model with two sectors, balanced trade, and complete specialization.

model during the flexible exchange rate regime after Bretton-Woods using cointegration techniques.<sup>99</sup>

Furthermore, our model predicts that trade-liberalization triggers a convergence process eliminating poverty traps, if  $q$  passes the threshold  $q^*$ . This adds qualitatively a new dimension to the relation between trade openness and growth as described by dynamic efficiency gains (Baldwin, 1992, and Baldwin and Seghezza, 1996). The bifurcation property of trade openness is in line with the finding of Ben-David (1993) who shows: 1) There is absolute convergence of income in an economy with trade liberalization (EEC6<sup>100</sup> from 1959-1968, EEC3<sup>101</sup> after the mid-sixties, USA and Canada after the Kennedy Round Agreement), or with trade and factor market integration (the convergence of the US states). 2) There is no absolute convergence of economies that are not integrated (e.g. the EEC6 and the EEC3 before trade liberalization, the 25 most developed countries, or the “whole world”).<sup>102</sup> Therefore, this evidence points to a two regime scenario with trade liberalization being the bifurcation parameter as suggested by our model.<sup>103</sup>

Next, our model explains the catch-up process by increased capital accumulation that is triggered by trade liberalization. It has been noted by Young (1995) that factor accumulation rather than total factor productivity growth explains the East

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<sup>99</sup>A positive relation between GDP and international price levels can also be inferred from the cross-country price data of Summers and Heston (1991). Barro and Sala-i-Martin (1995) show that GDP growth and terms of trade are positively correlated. (Note that the original estimates in Barro and Lee, 1994, are revised.) Because Barro and Sala-i-Martin (1995) try to capture all structural characteristics of the economies, we may take this as weak evidence that not differences in structural characteristics that may influence total factor productivity explain the impact of terms of trade on growth. However, the estimates of Barro and Sala-i Martin (1995) may fail the robustness test of Levine and Renelt (1992).

<sup>100</sup>This is the group of countries consisting of France, West Germany, Belgium, the Netherlands, Luxembourg, and Italy.

<sup>101</sup>This is the group of countries consisting of Denmark, Ireland, and UK.

<sup>102</sup>If there is conditional convergence among the EEC6 (Barro and Sala-i-Martin, 1992), but not absolute convergence, then factors other than capital accumulation must drive income convergence. If trade liberalization causes absolute convergence, then trade liberalization must have caused a catch-up in capital stocks. This is the transmission channel in our model.

<sup>103</sup>The role of trade openness as bifurcation parameter may be reversed, if different convergence forces are chosen (see section 5.3). Rauch (1997) gives the examples of Chile 1974-79 and of Italy's political unification 1861, and explains the subsequent economic slumps in an endogenous growth model.

In the relation of trade liberalization and growth, our model deviates in spirit from Myrdal (1957). “The hampering of industrial growth in the poorer southern provinces of Italy, caused by the pulling down of internal tariff walls after Italy's political unification in the last century, is a case in point which has been thoroughly studied ...” (p. 28)

Asian growth miracles. Furthermore, Levine and Renelt (1992) show that the impact of openness on growth stems from investment promotion, not from productivity growth. Finally, Moreno and Trehan (1997) find an empirical link between market size and investment supporting the theoretical link between home-market effect and capital accumulation of our model.

#### 4.5.2 The Poverty Trap Regime

In this section we discuss in detail the poverty-trap regime, i.e. the case where trade costs are relatively high ( $q < q^*$ ). Recall that there are three steady state distributions of capital, one of which is unstable. We summarize our results:

**Result 2:** *In the poverty trap regime ( $q < q^*$ ), income levels tend to diverge monotonically up to some relative ratio  $Y^* = K^{*\delta}$ , if country 2 is taken to be the smaller country.*<sup>104</sup>

The poverty trap case is graphically exposed in figure 4 which is drawn in line with propositions 2 and 3. The figure shows the map of the state space (initial capital distribution) on the control variable space (consumption choices) belonging to the three local stable manifolds  $W_{loc}^s(x^*)$ ,  $W_{loc}^s(x^{**})$ , and  $W_{loc}^s(x^{***})$  which are related to the three steady-state vectors  $x^*$ ,  $x^{**}$ , and  $x^{***}$ , respectively. Proposition 3 ensures that, for  $K_1(0) = K_2(0)$ , consumption is chosen in line with the stable manifold  $W_{loc}^s(x^*)$  that leads to the symmetric steady state  $x^*$ ; if  $K_1(0) > K_2(0)$ , consumption is chosen in line with the stable manifold  $W_{loc}^s(x^{**})$  that leads to the steady state  $x^{**}$  with more capital in country 1; if  $K_1(0) < K_2(0)$ , consumption is chosen in line with the stable manifold  $W_{loc}^s(x^{***})$  that leads to the steady state  $x^{***}$  with more capital in country 2. Because  $W_{loc}^s(x^*)$  is one-dimensional, any slight disturbance of this symmetric growth path, in the sense that one country accumulates more capital at some time period (idiosyncratic shock), will leave the symmetric steady state unachievable. Capital stocks and income will diverge governed by one of the

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<sup>104</sup>The statement follows from proposition 2 that shows the divergence of the capital stocks and from the fact that national income is a monotonic function of capital.

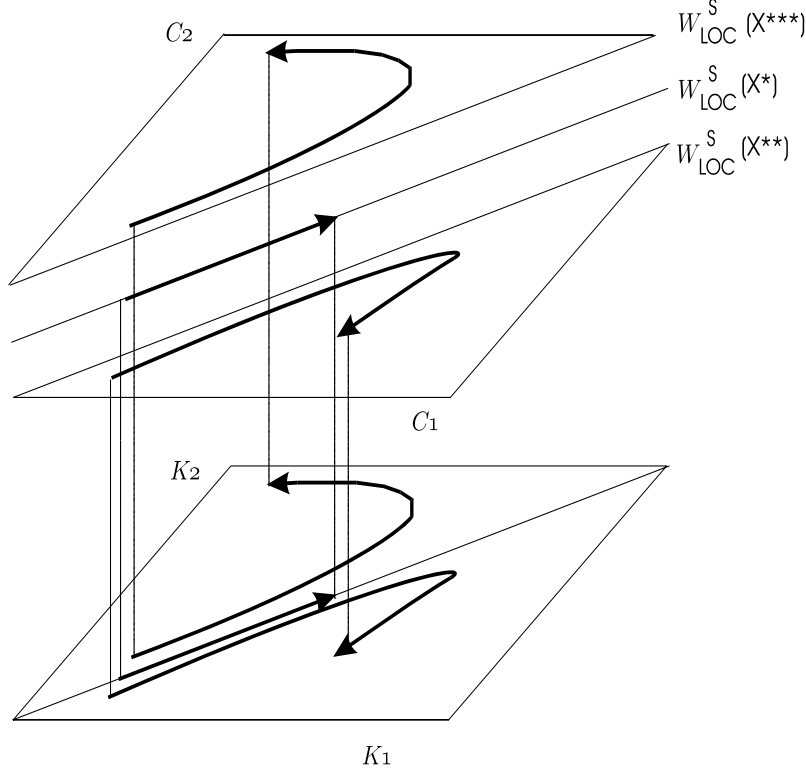


Figure 10: Poverty Trap Regime

other two stable manifolds depending on which country received a positive or negative idiosyncratic shock.<sup>105</sup>

Our model can be distinguished from most of the poverty trap models in a growth setting by explaining income divergence of two countries even though initial conditions are the same except for an idiosyncratic shock. In other words, the ratio of initial conditions matters, not the initial conditions themselves. This has two implications. First, poverty trap models where absolute values of initial conditions matter<sup>106</sup> have difficulties explaining how the rich countries left the poverty trap, whereas the poor countries did not, if all countries started from roughly the same income levels, say in the 17th/18th century.<sup>107</sup> Our model allows some countries to become rich,

<sup>105</sup>We cannot accomplish a global dynamic analysis, but numerical simulations suggest that a typical divergence path would stay close to the symmetric growth path for a long time after an idiosyncratic shock has occurred and will eventually lead to a drastic relative and absolute decline in the country that was originally hit.

<sup>106</sup>These are the poverty trap models corresponding to the club convergence hypothesis. A definition and an overview of convergence hypotheses is given by Galor (1996).

<sup>107</sup>“The very fact that the world at present is so sharply divided between ‘rich’ and ‘poor’ countries is, in the context of the broad sweep of history, something relatively new: it is the cumulative result of the historical experience of two or three hundred years. If we go back a few hundred years for

and others, that are hit by some negative idiosyncratic shock, stay poor. Second, our model is especially suited for explaining the fall-back of highly developed countries like the United Kingdom and Argentina after the turn of the century relative to countries that had initially the same state of development.<sup>108</sup> A wide range of “leapfrogging” models exist that are often based on endogenous growth settings.<sup>109</sup> We show that a neoclassical growth setting can also account for the fall-back of nations, if they are hit by some sufficiently large exogenous shock. There is still one observation to be made concerning the terms of trade.

**Result 3:** *In the poverty trap regime ( $q < q^*$ ), there is a worsening of the terms of trade  $p(t)$  over time in the country that lags behind vis a vis the country that is ahead, where terms of trade are defined in fob-manufacturing-producer prices.*<sup>110</sup>

There has been an extensive discussion in the 50ies, whether developing countries faced a persistent worsening of their terms of trade from 1870 til 1938.<sup>111</sup> Although - strictly speaking - our model is only suitable to developing countries whose export goods are produced with increasing returns to scale and monopolistic competition<sup>112</sup>, our model suggests that a worsening of the terms of trade was in principle explicable, whenever investment projects were locally complements and capital accumulation was poor.<sup>113</sup> Our model suggests that the appropriate policy measure was not to close national markets (import substitution) despite that trade seemed to harm developing countries, but to open national markets in order to eliminate the underlying poverty

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example, to 1700 or 1750, we do not find, as far as we can tell, such large differences in real income per capita between different countries or regions.” Kaldor (1967, p.3)

<sup>108</sup>We may then interpretate the idiosyncratic shock as political turmoil, unfavourable price movements of primary products, and import substitution policy in the case of Argentina and as the loss of colonies in the case of the United Kingdom.

<sup>109</sup>An example is Grossman and Helpman (1991).

<sup>110</sup>Suppose country 2 lags behind. From proposition 2 follows that the relative capital stock  $K(t)$  approaches asymptotically  $K^* < 1$ . From numerical simulations can be inferred that  $K(t)$  changes monotonically. From Lemma 1 follows that  $p(t)$  is monotonically increasing with  $K(t)$ . Therefore, the time path for  $p(t)$  has the same qualitative properties as the time path for  $K(t)$ .

<sup>111</sup>An empirical survey is Spraos (1980).

<sup>112</sup>Spraos (1980) indicates: “Perhaps more important than any of these is the processing of primary products before shipment (for instance, cocoa beans turned into cocoa butter and cocoa paste) which has been increasing all the time, though in developing countries it had gained great momentum only in the last twenty years.” (p. 118) Additionally, mining and agro-business may not a priori be less likely described by increasing returns to scale than manufacturing industries.

<sup>113</sup>Of course, we do not doubt that other explanations can be found. We just want to point out that the terms of trade effect in our poverty trap regime does not run counter to the empirical literature.

trap - a recommendation that finds broad consensus nowadays.

### 4.5.3 Economic Geography

Having shown the interdependence between real marginal product of capital, capital accumulation, and terms of trade, we focus now on the aspect of agglomeration of manufacturing industries. From the analysis so far it follows immediately (by equation (63)) that the faster growth in the country with more capital causes a larger number of firms which we take as a proxy for manufacturing industry agglomeration. A relative increase in domestic capital increases domestic income, which in turn increases demand for any existing domestic variety. The latter increases domestic producer prices relative to foreign (terms-of-trade effect), which leads to positive profits of domestic firms and thus the entry of new domestic firms.

**Result 4:** *At high trade costs ( $q < q^*$ ), there will be partial agglomeration of manufacturing industries in one country.*<sup>114</sup>

Hence, a low growth rate is associated with a decline of manufacturing industries. Indeed, slower growth and a decline of manufacturing industries self-enforce each other in a cumulative process. Conversely, agglomeration of manufacturing industries is explained by faster capital accumulation in one country relative to another. This explanation differs from other explanations in papers on agglomeration and growth - as Bertola (1992), Englmann and Walz (1995), and Martin and Ottaviano (1996). In these papers, agglomeration processes in growth models rely on migration, capital flows with technological spill-overs, and R&D location decisions.

The role of trade costs for triggering agglomeration is reversed compared to Krugman (1991a). This is so, because we exchanged the convergence forces. Krugman's (1991a) convergence force is based on ambiguous terms-of-trade effects caused by an immobile farming sector. As trade costs increase from a very low level, terms of trade increase in the larger country. (We observed the same effect in our model.)

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<sup>114</sup>This follows from result 2 and from equation (63).

However, as trade costs increase further in Krugman (1991a), terms of trade may start to decrease. This effect is not present in our model. We conclude therefore that the role of trade costs is not robust with respect to the specific convergence force used in geography and trade models.

## 4.6 Conclusion

We argued that relative income growth and manufacturing industry distribution among countries are mutually influencing each other. This hypothesis has been formulated by Myrdal (1957) and Kaldor (1967). They argue that a cumulative process may keep some countries poor and others rich, because an expansion process (decline) may rise (lower) the return on investment in the expanding (declining) country thus causing further expansion (contraction) by capital accumulation. The empirical convergence debate suggests, however, that income divergence is a possible, but not a general phenomenon. Therefore, we built a model that explains income divergence in a poverty trap regime, income convergence in a neoclassical regime, and a testable condition under which a country is in one or the other regime. This condition depends on the degree of integration in product markets. If trade barriers are high, income divergence is likely to occur. If trade barriers are low, income convergence is the unique equilibrium. Thus, trade liberalization may trigger a catch-up process of countries that are stuck in a poverty trap.

The interrelation of growth and agglomeration is described by circular causation. Countries grow faster (slower), because they have a lot of (a few) manufacturing industries. Countries have a lot of (a few) manufacturing industries, because they have grown faster (slower) in the past and thereby accumulated more (less) capital. The circular causation relies on a terms-of-trade effect that may or may not feed through on real rental rates (spatial complementarity or substitutability of investment). The countries stuck in the poverty trap experience slower growth, a lower investment-to-GDP ratio, a worsening of their terms of trade, and a decline in manufacturing industries. The countries that catch-up experience a higher growth rate, a higher

investment-to-GDP ratio, an improvement of their terms of trade, and manufacturing industries diffuse to the poor country.

The results in this paper have been derived in a specific model set-up - increasing returns, and monopolistic competition. Following the same sort of argument as Gali (1994) for a closed economy, the pitchfork-bifurcation property may also appear in a set-up with Cournot oligopoly. Our analysis shows that divergence of income and firm agglomeration emerge, whenever investment projects are complementary in the neighborhood of a symmetric distribution of capital and firms.



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## Appendix

### Appendix 1: Proof of Lemma 1.

Taking the derivative of (62) yields:

$$\frac{dn}{dp} = \frac{-\sigma p^\sigma (q - p^{-\sigma}) - (q - p^\sigma) (q - (1 - \sigma) p^{-\sigma})}{p^2 (q - p^{-\sigma})^2} \quad (76)$$

with  $0 < n < \infty$ . (a) Suppose  $p > 1$ , then  $q - p^\sigma < 0$ . Therefore,  $q - p^{-\sigma} < 0$  for  $n$  to be positive by inspection of (62). Then follows by inspection of (76) that  $dn/dp > 0$ , because  $\sigma > 1$  by assumption. (b) Suppose  $p < 1$ , then  $q - p^{-\sigma} < 0$ . Therefore,  $q - p^\sigma < 0$  for  $n$  to be positive by inspection of (62). Then follows by inspection of (76) that  $dn/dp > 0$ , because  $\sigma > 1$  by assumption. This implies that the inverse function  $p \rightarrow n$  exists in the positive range. From (63) follows that  $K$  and  $n$  are proportional. Hence, we have that  $p(K)$  is an invertible function and  $dp/dK > 0$ .  $P(K)$  must then be downward sloping from (64). From (65) follows that  $r(K)$  is bounded from below by  $p(K)$ . Finally, the limit with respect to complete agglomeration ( $K \rightarrow 0$ ) can be taken from (65). **Q.E.D.**

### Appendix 2: Proof of Proposition 1.

We will first show that there are at most 3 solutions to  $\rho(K) \equiv r(K)/P(K) = 1$ . Using equations (64) and (65) yields:

$$r/P = pn^{\frac{\delta-1}{\delta}} p^{\frac{\sigma}{1-\sigma}} = 1. \quad (77)$$

Plugging in the goods market equilibrium condition (62) yields:

$$p^{\frac{1}{1-\sigma}} \left( \frac{q - p^\sigma}{p(q - p^{-\sigma})} \right)^{\frac{\delta-1}{\delta}} = 1. \quad (78)$$

Multiplying out gives a power function of the form:

$$q - p^\sigma - qp^{\frac{1-\sigma+\sigma\delta}{(1-\sigma)(1-\delta)}} + p^{\frac{(1-\sigma)^2(1-\delta)+\delta}{(1-\sigma)(1-\delta)}} = 0. \quad (79)$$

This expression has at most 3 solutions for  $p$  due to Descartes' Rule of Sign. Because there is a one-to-one mapping from  $p$  to  $n$  to  $K$  (Lemma 1), there correspond at most three values for  $n$  and  $K$ . We conclude: one solution is  $K = 1$  (The symmetry solution is always true.); if there exists a second solution  $K^* < 1$ , then the third must be  $(1/K^*) > 1$  because of the symmetry of the model.

Now, we will give a necessary and sufficient condition for the existence of  $K^*$  by restricting our view on  $0 < K \leq 1$ . Recall from Lemma 1:  $\lim_{K \rightarrow 0} r(K) = \infty$ , whereas  $P(0)$  is finite. Hence,

$$\lim_{K \rightarrow 0} \rho(K) \equiv \lim_{K \rightarrow 0} \frac{r(K)}{P(K)} > 1. \quad (80)$$

There will exist the interior solution  $K^*$ , if  $\rho(K) < 1$  for  $K$  slightly below 1 (intermediate value theorem). This is not just a necessary condition for the existence of  $K^*$ , but also a sufficient condition for  $K^*$  to be the only interior solution ( $0 < K^* < 1$ ), because  $\rho(1) = 1$ . (Suppose on the contrary that  $K^*$  exists and  $\rho(K) > 1$ , when  $K$  is slightly below 1, then there will exist at least two interior solutions (or none) for  $0 < K < 1$  which contradicts our findings above.) From (63) follows that there

corresponds a  $n^* = K^{*\delta}$ . We can formulate the necessary and sufficient condition for an interior solution  $n^*$  also in the following way:

$$\frac{dP(1)}{dn} < \frac{dr(1)}{dn}. \quad (81)$$

Evaluating the derivative of the relative price index yields:

$$\frac{dP(n)}{dn} = \frac{\sigma}{1-\sigma} \frac{dp(n)}{dn}. \quad (82)$$

Evaluating this expression at  $n = 1$  and using equation (76) gives us:

$$\frac{dP(1)}{dn} = \frac{\sigma}{1-\sigma} \frac{q-1}{1-2\sigma-q}. \quad (83)$$

Next, the derivative of relative rental rates is found:

$$\frac{dr(n)}{dn} = \frac{dp(n)}{dn} n^{\frac{\delta-1}{\delta}} + p \left( \frac{\delta-1}{\delta} \right) n^{-\frac{1}{\delta}}. \quad (84)$$

We evaluate this expression at  $n = 1$  by using (76):

$$\frac{dr(1)}{dn} = \frac{q-1}{1-2\sigma-q} + \frac{\delta-1}{\delta}. \quad (85)$$

Using (83) and (85) in (81) yields an inequality

$$\frac{q-1}{1-q-2\sigma} \frac{2\sigma-1}{1-\sigma} < \frac{\delta-1}{\delta}, \quad (86)$$

which can be solved for  $q$ :

$$q < q^* \equiv \frac{(2\sigma-1)(\delta\sigma+1-\sigma)}{\delta\sigma-(1-\sigma)} < 1. \quad (87)$$

A similar argumentation holds for  $1/n^*$  and  $1 \leq n < \infty$ , i.e.  $1/K^*$  and  $1 \leq K < \infty$ , by the symmetry property of the model.

Finally, the derivative in (i) follows from  $\rho(1) = 1$ ;  $\rho(K) > 1$ , if  $K < 1$ ; and  $\rho(K) < 1$ , if  $K > 1$ . Correspondingly, the derivatives in (ii) follow from  $\rho(1) = \rho(K^*) = 1$ ; and  $\rho(K) > 1$ , if  $K < K^*$  or  $K > 1/K^*$ ;  $\rho(K) < 1$ , if  $1 > K > K^*$  or  $1/K^* > K > 1$ . (See Lemma 1). **Q.E.D.**

### Appendix 3: Proof of Lemma 2.

Let  $\mathbf{x} \equiv (K_1, C_1, K_2, C_2)$  and the dynamical system (67)-(70) be written in matrix notation as  $\dot{\mathbf{x}} = f(\mathbf{x})$ . Furthermore, let  $\mathbf{y} \equiv (\ln C, \ln K, K_1, C_1)$  and

$$g(\mathbf{y}) \equiv \begin{pmatrix} a_1 \ln \rho(K) \\ \frac{a_1}{\delta} \ln \rho(K) - a_2 \ln C + a_2 \ln K \\ \frac{\tilde{\rho}_1(K_1, K)}{\delta} K_1 - C_1 \\ \left( \tilde{\rho}_1(K_1, K) - \lambda \right) C_1 \end{pmatrix}, \quad (88)$$

where we use

$$\rho_1(K_1, K_2) = \tilde{\rho}_1(K_1, K) \equiv \delta K_1^{\delta-1} \left( \frac{1}{1+K^\delta} + \frac{qp(K)^{1-\sigma}}{1+K^{-\delta}} \right)^{\frac{1}{\sigma-1}}, \quad (89)$$

with  $\partial \tilde{\rho}_1 / \partial K_1 < 0$ , which follows from (51), (57), (58), and (59).<sup>115</sup> Define the invertible matrix  $h$  in the following way:

$$h \equiv \begin{pmatrix} 0 & -\frac{1}{\bar{C}_1} & 0 & \frac{1}{\bar{C}_2} \\ -\frac{1}{\bar{K}_1} & 0 & \frac{1}{\bar{K}_2} & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \quad (90)$$

Then, we find for  $\mathbf{x}$  and  $\mathbf{y}$  in the neighborhood of  $\bar{\mathbf{x}}$  and  $\bar{\mathbf{y}}$  that

$$h\mathbf{x} = \begin{pmatrix} \frac{C_2 - \bar{C}_2}{\bar{C}_2} - \frac{C_1 - \bar{C}_1}{\bar{C}_1} \\ \frac{K_2 - \bar{K}_2}{\bar{K}_2} - \frac{K_1 - \bar{K}_1}{\bar{K}_1} \\ K_1 \\ C_1 \end{pmatrix} \approx \begin{pmatrix} \ln C \\ \ln K \\ K_1 \\ C_1 \end{pmatrix} \equiv \mathbf{y}, \quad (91)$$

where we used the first-order Taylor expansions

$$\begin{aligned} \ln K_2 - \ln K_1 &\approx \frac{1}{\bar{K}_2} (K_2 - \bar{K}_2) - \frac{1}{\bar{K}_1} (K_1 - \bar{K}_1) \\ \ln C_2 - \ln C_1 &\approx \frac{1}{\bar{C}_2} (C_2 - \bar{C}_2) - \frac{1}{\bar{C}_1} (C_1 - \bar{C}_1) \end{aligned} \quad (92)$$

around the steady state vector  $\bar{\mathbf{x}}$ . Furthermore, we calculate the Jacobian Matrix  $B \equiv \frac{dg(\bar{\mathbf{y}})}{d\mathbf{x}}$  evaluated at the steady state vector  $\bar{\mathbf{y}}$ :

$$\frac{dg(\bar{\mathbf{y}})}{d\mathbf{x}} = \begin{pmatrix} \left( \frac{\partial \bar{\rho}_2}{\partial K_1} - \frac{\partial \bar{\rho}_1}{\partial K_1} \right) & 0 & \left( \frac{\partial \bar{\rho}_2}{\partial K_2} - \frac{\partial \bar{\rho}_1}{\partial K_2} \right) & 0 \\ \frac{1}{\delta} \left( \frac{\partial \bar{\rho}_2}{\partial K_1} - \frac{\partial \bar{\rho}_1}{\partial K_1} \right) - \frac{\bar{C}_1}{\bar{K}_1^2} & \frac{1}{\bar{K}_1} & \frac{1}{\delta} \left( \frac{\partial \bar{\rho}_2}{\partial K_2} - \frac{\partial \bar{\rho}_1}{\partial K_2} \right) - \frac{\bar{C}_2}{\bar{K}_2^2} & -\frac{1}{\bar{K}_2} \\ \frac{\partial \bar{\rho}_1}{\partial K_1} \frac{\bar{K}_1}{\delta} + \frac{\bar{\rho}_1}{\delta} & -1 & \frac{\partial \bar{\rho}_1}{\partial K_2} \frac{\bar{K}_1}{\delta} & 0 \\ \bar{C}_1 \frac{\partial \bar{\rho}_1}{\partial K_1} & 0 & \bar{C}_1 \frac{\partial \bar{\rho}_1}{\partial K_2} & 0 \end{pmatrix}, \quad (93)$$

where we used the steady state conditions of (74), i.e.

$$\bar{\rho}_1 = \bar{\rho}_2, \quad (94)$$

$$\frac{\bar{C}_1}{\bar{K}_1} = \frac{\bar{C}_2}{\bar{K}_2}, \quad (95)$$

and by the rules of differentiation and (89):

$$\frac{\partial \ln \bar{\rho}}{\partial K_i} = \frac{1}{\bar{\rho}_2} \frac{\partial \bar{\rho}_2}{\partial K_i} - \frac{1}{\bar{\rho}_1} \frac{\partial \bar{\rho}_1}{\partial K_i}, \quad (96)$$

$$\frac{d \tilde{\rho}_1}{dK_i} = \frac{\partial \bar{\rho}_1}{\partial K_i}, \quad (97)$$

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<sup>115</sup>Recall that bars denote steady state values of any steady state solution.

$$\frac{\partial \ln \bar{K}}{\partial K_1} = -\frac{1}{\bar{K}_1}, \quad (98)$$

$$\frac{\partial \ln \bar{K}}{\partial K_2} = -\frac{1}{\bar{K}_2}, \quad (99)$$

$$\frac{\partial \ln \bar{C}}{\partial C_1} = -\frac{1}{\bar{C}_1}, \quad (100)$$

$$\frac{\partial \ln \bar{C}}{\partial C_2} = -\frac{1}{\bar{C}_2}. \quad (101)$$

Correspondingly, the Jacobian  $A \equiv \frac{df(\bar{\mathbf{x}})}{d\mathbf{x}}$  can be found from the linearization of the dynamical system (67)-(70) around the steady state vectors  $\bar{\mathbf{x}}$ :

$$\frac{df(\bar{\mathbf{x}})}{d\mathbf{x}} = \begin{pmatrix} \frac{\bar{K}_1}{\delta} \frac{\partial \bar{\rho}_1}{\partial K_1} + \frac{\bar{\rho}_1}{\delta} & -1 & \frac{\bar{K}_1}{\delta} \frac{\partial \bar{\rho}_1}{\partial K_2} & 0 \\ \bar{C}_1 \frac{\partial \bar{\rho}_1}{\partial K_1} & 0 & \bar{C}_1 \frac{\partial \bar{\rho}_1}{\partial K_2} & 0 \\ \frac{\partial \bar{\rho}_2}{\partial K_1} \frac{\bar{K}_2}{\delta} & 0 & \frac{\bar{K}_2}{\delta} \frac{\partial \bar{\rho}_2}{\partial K_2} + \frac{\bar{\rho}_2}{\delta} & -1 \\ \bar{C}_2 \frac{\partial \bar{\rho}_2}{\partial K_1} & 0 & \bar{C}_2 \frac{\partial \bar{\rho}_2}{\partial K_2} & 0 \end{pmatrix}. \quad (102)$$

It can be checked that

$$\frac{dg(\bar{\mathbf{y}})}{d\mathbf{x}} = h \frac{df(\bar{\mathbf{x}})}{d\mathbf{x}}, \quad (103)$$

where we used (96) and

$$\frac{\bar{C}_i}{\bar{K}_i} = \frac{\bar{\rho}_i}{\delta} \quad (104)$$

for  $i = 1, 2$ , which follows from the steady state conditions of (67) and (69).

Now, we show that the Jacobian  $A \equiv \frac{df(\bar{\mathbf{x}})}{d\mathbf{x}}$  and the Jacobian  $B \equiv \frac{dg(\bar{\mathbf{y}})}{d\mathbf{y}}$  are similar matrices.<sup>116</sup> Therefore, we need one more preliminary calculation. From the chain rule of matrix differentiation and (91) follows:

$$\frac{dg(\bar{\mathbf{y}})}{d\mathbf{x}} = \frac{dg(h\bar{\mathbf{x}})}{d\mathbf{x}} = \frac{dg(\bar{\mathbf{y}})}{d\mathbf{y}} h. \quad (105)$$

Because  $h$  is invertible, we can write:

$$\frac{dg(\bar{\mathbf{y}})}{d\mathbf{y}} = \frac{dg(\bar{\mathbf{y}})}{d\mathbf{x}} h^{-1}. \quad (106)$$

Then, we may rewrite the Jacobian  $B$  in the following way:

$$B \equiv \frac{dg(\bar{\mathbf{y}})}{d\mathbf{y}} = \frac{dg(\bar{\mathbf{y}})}{d\mathbf{x}} h^{-1} = h \frac{df(\bar{\mathbf{x}})}{d\mathbf{x}} h^{-1} \equiv hAh^{-1}, \quad (107)$$

where the first equality sign follows from (106), the second equality sign follows from (103), and the second identity follows from the definition of  $A$ . Therefore, the matrices  $A$  and  $B$  are similar. However, two similar square matrices  $A$  and  $B$  have the same

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<sup>116</sup>Definition: If  $A$  and  $B$  are square matrices, we say that  $B$  is similar to  $A$ , if there is an invertible matrix  $h$  such that  $B = hAh^{-1}$ . (Brock and Malliaris, 1989, p.349)



characteristic polynomials and eigenvalues (Theorem 4.1 in Brock and Malliaris, 1989, p.349), i.e.

$$|A - \zeta I_4| = |B - \zeta I_4| = 0, \quad (108)$$

where  $\zeta$  denotes the eigenvector and  $I_4$  the  $4 \times 4$  identity matrix. This concludes the proof. **Q.E.D.**

#### Appendix 4: Proof of Proposition 2.

Consider the Taylor linnearization of the system (75), (67), and (68), i.e.

$$\dot{\mathbf{y}} = B (\mathbf{y} - \bar{\mathbf{y}}), \quad (109)$$

where  $\mathbf{y} \equiv (\ln C, \ln K, K_1, C_1)$ ,  $B \equiv \frac{dg(\bar{\mathbf{y}})}{d\mathbf{y}}$  and  $g(\mathbf{y})$  is defined in equation (88) of appendix 3. The characteristic polynomial of the matrix  $B$  is found:

$$|B - \zeta I_4| = \begin{vmatrix} -\zeta & a_1 \frac{d\bar{\rho}}{dK} \bar{K} & 0 & 0 \\ -a_2 & \frac{a_1}{\delta} \frac{d\bar{\rho}}{dK} \bar{K} + a_2 - \zeta & 0 & 0 \\ 0 & \frac{\partial \bar{\rho}_1}{\partial K} \frac{\bar{K}_1}{\delta} & \frac{\partial \bar{\rho}_1}{\partial K} \frac{\bar{K}_1 + \bar{\rho}_1}{\delta} - \zeta & -1 \\ 0 & \bar{C}_1 \frac{\partial \bar{\rho}_1}{\partial K} & \bar{C}_1 \frac{\partial \bar{\rho}_1}{\partial K_1} & -\zeta \end{vmatrix} = 0, \quad (110)$$

where  $\zeta$  denotes the eigenvector and  $I_4$  the  $4 \times 4$  identity matrix. Next, a Gauss-transformation with the Pivotelements (1,1) and (3,4) is undertaken and the second column is changed with the first to form a matrix in Gauss-form:

$$\begin{vmatrix} a_3 & 0 & 0 & 0 \\ \frac{a_1}{\delta} \frac{d\bar{\rho}}{dK} \bar{K} + a_2 - \zeta & -a_2 & 0 & 0 \\ 0 & a_4 & a_5 & 0 \\ 0 & \bar{C}_1 \frac{\partial \bar{\rho}_1}{\partial K} & \bar{C}_1 \frac{\partial \bar{\rho}_1}{\partial K_1} & -\zeta \end{vmatrix} = 0, \quad (111)$$

where we defined

$$\begin{aligned} a_3 &\equiv a_1 \frac{d\bar{\rho}}{dK} \bar{K} - \frac{\zeta}{a_2} \left( \frac{a_1}{\delta} \frac{d\bar{\rho}}{dK} \bar{K} + a_2 - \zeta \right), \\ a_4 &\equiv -\frac{\bar{C}_1}{\zeta} \frac{\partial \bar{\rho}_1}{\partial K} + \frac{\partial \bar{\rho}_1}{\partial K} \frac{\bar{K}_1}{\delta}, \\ a_5 &\equiv \left( \frac{\partial \bar{\rho}_1}{\partial K} \frac{\bar{K}_1 + \bar{\rho}_1}{\delta} - \zeta \right) - \frac{\bar{C}_1}{\zeta} \frac{\partial \bar{\rho}_1}{\partial K_1}. \end{aligned}$$

Because the determinant of a matrix in Gauss form is the product of its diagonal elements, the characteristic polynomial may be written in the following way:

$$\zeta^2 - \zeta \left( \frac{a_1}{\delta} \frac{d\bar{\rho}}{dK} \bar{K} + a_2 \right) + a_1 a_2 \frac{d\bar{\rho}}{dK} \bar{K} = 0 \quad (112)$$

or

$$\zeta^2 - \frac{\zeta}{\delta} \left( \frac{\partial \bar{\rho}_1}{\partial K} \bar{K}_1 + \bar{\rho}_1 \right) - \bar{C}_1 \frac{\partial \bar{\rho}_1}{\partial K_1} = 0. \quad (113)$$

Correspondingly, the 4 eigenvalues are:

$$\begin{aligned}\zeta_{1,2} = & 0.5 \left( \frac{a_1}{\delta} \frac{d\bar{\rho}}{dK} \bar{K} + a_2 \right) \\ & \pm 0.5 \left( \left( \frac{a_1}{\delta} \frac{d\bar{\rho}}{dK} \bar{K} + a_2 \right)^2 - 4a_1a_2 \frac{d\bar{\rho}}{dK} \bar{K} \right)^{1/2}\end{aligned}\quad (114)$$

and

$$\begin{aligned}\zeta_{3,4} = & \frac{1}{2\delta} \left( \frac{\partial \bar{\rho}_1}{\partial K} \bar{K}_1 + \bar{\rho}_1 \right) \\ & \pm \left( \frac{1}{4\delta^2} \left( \frac{\partial \bar{\rho}_1}{\partial K} \bar{K}_1 + \bar{\rho}_1 \right)^2 - 4\bar{C}_1 \frac{\partial \bar{\rho}_1}{\partial K_1} \right)^{1/2}.\end{aligned}\quad (115)$$

Because  $\partial \bar{\rho}_1 / \partial K_1 < 0$ , the last two eigenvalues are real numbers and can be ranked as follows:

$$\zeta_3 < 0 < \zeta_4. \quad (116)$$

The first two eigenvalues are evaluated as follows:

$$\zeta_1 < 0 < \zeta_2, \quad (117)$$

if

$$\frac{d\bar{\rho}}{dK} < 0, \quad (118)$$

and

$$\begin{aligned}\operatorname{Re}(\zeta_1) & > 0, \\ \operatorname{Re}(\zeta_2) & > 0,\end{aligned}\quad (119)$$

if

$$\frac{d\bar{\rho}}{dK} > 0. \quad (120)$$

Note that condition (118) is fulfilled in cases (i), (ii), and (iii) of proposition 2, whereas condition (120) is equivalent to the condition described in case (iv) of proposition 2 which follows from proposition 1.

Because the matrices  $A$  and  $B$  have the same characteristic polynomials (Lemma 2), the qualitative local stability properties are preserved by the transformation from the linearization of system (109) to the linearization of system (67)-(70). In particular, there exist two positive and two negative eigenvalues for system (67)-(70) in the cases (i), (ii), and (iii) of proposition 2 and 3 positive and 1 negative eigenvalue in case (iv). There correspond stable (unstable) eigenvectors to the stable (unstable) eigenvalues. By the stable manifold theorem, the local stable manifolds for the local steady states  $x^*$ ,  $x^{**}$ , and  $x^{***}$  of cases (i), (ii), and (iii) are two-dimensional (i.e. a surface in  $\mathbf{R}^4$ ), whereas the local stable manifold for  $x^*$  in case (iv) is one-dimensional.

Next, it follows from case 1 in Buiter (1984) that a unique solution to the boundary value problem (67)-(72) exists and is stable in cases (i), (ii), and (iii), because the number of positive eigenvalues is equal to the number of control (jump-) variables ( $C_1, C_2$ ).

The boundary value problem (67)-(72) does not have a solution in case (iv), unless we give up one initial condition. Giving up the initial condition for  $K_2(0) = K_{20}$  and

letting  $K_2$  “jump”, yields again a unique and stable solution. If we inspect (114), we see that these are the eigenvalues of the dynamical subsystem (75) which determines convergence/non-convergence of  $K$  towards the steady state  $\bar{K}$ . If these eigenvalues are both positive, there will not be convergence of  $K$ . Therefore we guess that the system (67)-(72) must be restricted in  $K_2(0)$ , such that relative capital ratios are in their steady state right from the beginning. Formally, we guess that

$$K_1(0) = K_2(0) = \kappa \quad (121)$$

will have to hold for any  $\kappa \in \mathbf{R}^+$ . For any time  $\tilde{t}$ , there exists a  $\tilde{\kappa} \in \mathbf{R}^+$  such that  $K_1(\tilde{t}) = \tilde{\kappa}$ . By the property of autonomous differential equation systems,  $\tilde{t}$  can be normalized to zero. Therefore, (121) implies that

$$K_1(t) = K_2(t) \quad (122)$$

for  $t \geq 0$ . From the first equation of (74) follows then that

$$C_1(t) = C_2(t) \quad (123)$$

for  $t \geq 0$ . Furthermore, from (62), (63), and (122) follows that  $p(t) = 1$  for  $t \geq 0$ . The system (67)-(70) collapses to the system of two independent neoclassical growth models. Therefore, the guess in form of condition (121) is valid and yields indeed a stable solution to the boundary value problem (67)-(72) without the initial condition  $K_2(0) = K_{20}$ . **Q.E.D.**

#### Appendix 5: Proof of proposition 3.

We start out with equations (67) and (69). They can be integrated taken  $C_j(0)$ , and  $\rho_j(t)$  to be well-defined (though unknown) functions of time  $t$  as given (Note:  $C_j(0)$  is to be solved for.):

$$C_j(t) = C_j(0) e^{\int_0^t (\rho_j(s) - \lambda) ds} \quad (124)$$

Integrating in the same way (68) and (70) yields

$$K_i(0) = \int_0^\infty C_i(t) e^{-\int_0^t \frac{1}{\delta} \rho_i(s) ds} dt, \quad (125)$$

where we made use of the initial condition (71) and the transversality condition (72). Plugging (124) into (125) yields:

$$C_j(0) = \mu_j(0) K_j(0), \quad (126)$$

where

$$\mu_j(0) = \left( \int_0^\infty e^{-\int_0^t \left(\frac{1-\delta}{\delta}\right) \rho_j(s) + \lambda ds} dt \right)^{-1}.$$

(The three steps are standard in the literature, e.g. Barro and Sala-i-Martin (1995), p. 59ff., in a similar model.) Suppose now:

$$(i) \quad K_{10} = K_{20} = \tilde{\kappa},$$

with any  $\tilde{\kappa} \in \mathbf{R}^+$ . Suppose further that with this initial condition the steady state  $x^{**}$  will be reached, i.e.:

$$(ii) \quad \bar{K}_1 > \bar{K}_2,$$

i.e.  $\bar{K} = K^* < 1$ . Finally, we assume without loss of generality that  $K(0) \leq K(s) \leq K^*$  for  $0 \leq s \leq \infty$ .<sup>117</sup> From this assumption and proposition 1 follows that

$$\rho_1(s) \geq \rho_2(s) \quad (127)$$

for  $0 \leq s \leq \infty$  and a strict inequality for some  $s$ . Consequently,

$$\left(\frac{1-\delta}{\delta}\right) \rho_1(s) + \lambda \geq \left(\frac{1-\delta}{\delta}\right) \rho_2(s) + \lambda \quad (128)$$

for  $0 \leq s \leq \infty$  and a strict inequality for some  $s$ . One may check that this implies

$$\mu_1(0) = \left( \int_0^\infty e^{-\int_0^t \left(\frac{1-\delta}{\delta}\right) \rho_1(s) + \lambda ds} dt \right)^{-1} > \left( \int_0^\infty e^{-\int_0^t \left(\frac{1-\delta}{\delta}\right) \rho_2(s) + \lambda ds} dt \right)^{-1} = \mu_2(0) \quad (129)$$

and therefore by equation (126) and assumption (i)

$$C_1(0) > C_2(0). \quad (130)$$

However, then follows from (74) and assumption (i) that

$$\dot{K}_1(0) < \dot{K}_2(0), \quad (131)$$

i.e.  $\dot{K}(0) > 0$ . Recall that  $K(0) = 1$  (assumption i) and  $K^* < 1$ . Therefore, the direction of movement will always point away from the steady state  $K^*$ , if  $K(0) = 1$ . By the properties of an autonomous differential equation system, the trajectory to the steady state can never pass the threshold  $K = 1$  at any point in time in the direction of the steady state and therefore not reach the steady state. This contradicts assumption (ii). Therefore, there is no perfect foresight path from the initial condition  $K(0) \geq 1$  to the steady state  $K^*$ . By the symmetry property of the model, there is also no perfect foresight path from the initial condition  $K(0) \leq 1$  to the steady state  $(1/K^*)$ . From proposition 2 case (iv) follows that there exists a one-dimensional stable manifold such that  $\bar{K} = 1$  is reached, if  $K(0) = 1$ . This concludes the proof. **Q.E.D.**

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<sup>117</sup>In other words: If  $K_1(s) = K_2(s)$  for  $s \in \{s', s'', \dots\}$ , and  $s' \geq s'' \geq \dots$  on the same trajectory reaching  $x^{**}$  (if it exists), then we normalize by the property of autonomous systems  $s' = 0$ . Then follows that  $K_1(s) > K_2(s)$ , i.e.  $K(s) < 1$ , for  $s > 0$ , because  $s = 0$  is the last point in time, where  $K_1(s) = K_2(s)$  is sustained and before the steady state  $K^* < 1$  is reached. Furthermore,  $x^{**}$  is a stable node which follows from the proof of proposition 2. Therefore, the steady state value is not “overshooting” (as would be the case for a stable focus), i.e.  $K(s) \geq K^*$  for  $s > 0$ .

## 5 Convergence, Terms of Trade, and Cointegration: The Example Japan\*

### Abstract

This paper tests indirectly the mechanics of Krugman's (1991) economic geography model in a growth context with time series methods. A reduced rank hypothesis of cointegration in a vector error correction model is directly derived from a linearization of the theoretical model: a stochastic, dynamic variant of the seminal economic geography model. The model is tested for data on Japan and the US from 1957 until 1990 against its alternative - a stochastic Ak-model. We find that there is weak evidence for the model. In the Bretton Woods era with fixed exchange rates, the model is rejected and the US and Japanese economy grow independently from each other. In the Post-Bretton Woods era, the model is significantly supported, if major shocks such as the oil crises are expected to repeat themselves in the future.

**JEL Classification:** O41

**Keywords:** convergence, terms of trade, cointegration, economic geography, endogenous growth

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\*The author thanks Niels Blomgren-Hansen, Katarina Juselius, Rudolfo Helg, Sheng Cheng Hu, Lisbeth Funding la Cour, Koichi Nakayama, Pascalis Raimondos-Møller, and Katrin Wesche for their comments. Usual caveats apply. This paper was written for the conference "Dynamics, Economic Growth, and International Trade, III" in Taipei, Taiwan, from 24 August 1998 until 26 August 1998. This paper was written, while the author was visiting PennState University and benefited in various ways from the research environment at the Economics Department of PennState University.

## 5.1 Introduction

The purpose of this paper is to provide an indirect test of the mechanics of Krugman's (1991) economic geography model in a growth context using time series methods. The research shall be pursued by deriving the testing hypothesis directly from the theoretical model and applying it to Japanese and US data. With this procedure we account for Granger's criticism:

*“The classical research strategy of the model being provided by the theorists, the parameter estimation by the econometrician, and the interpretation by the applied economist often does not lead to acceptable models, with the suggested specification being rejected by the actual data. It follows that a new strategy is required, and this would be easier to achieve if the gap between theorists and econometricians were reduced, or, even better, removed.”* (Granger, 1992, p. 3)

The mechanics of the seminal economic geography model with monopolistic competition and increasing returns to scale relies on a terms-of-trade effect. Suppose 2 regions are identical to begin with. Then, one worker moves from region 1 to region 2 accompanied by the movement of a firm. Assuming monopolistic competition every firm produces a different good. Hence, there are more goods produced in region 2 than in region 1. Because transport cost have to be paid for foreign goods, demand is biased towards domestic goods. Consequently, there is larger demand for goods produced in region 2, because there are more workers in region 2 and income is higher. Therefore, producer prices must *ceteris paribus* rise in the larger region 2 (income effect). Higher prices induce higher wages and the incentive for workers to move from the smaller region to the bigger becomes stronger (cumulative process).

However, this agglomeration force may be offset by a convergence force. If one good more is produced domestically, then there is one good less for which transport cost have to be paid for and the consumption basket becomes cheaper. Then, the relative price of domestic goods and the ideal consumer price index rises, which reduces demand for the products of the larger region (number of goods effect). This has the

opposite effect on prices and wages. Summing up, the mechanics of the Krugman (1991) model show a relation between firm agglomeration (relative number of firms) and the terms of trade.

The mechanics of the Krugman (1991) model is not directly measurable in a time series analysis. The number of firms is a valid measure only, if firms are symmetric with identical cost functions and identical firm size, as is assumed in the Krugman (1991) model. In reality, firms are heterogeneous and production/income may be the more appropriate measure.<sup>118</sup>

To account for Granger's criticism, we derive the testing procedure directly from a theoretical model. Because non-stationarity is empirically hard to reject for income/production<sup>119</sup>, we have to augment the Krugman (1991) model with an endogenous growth framework to obtain the non-stationarity in the theoretical model. We are careful in preserving the mechanics of the Krugman (1991) model - the terms-of-trade effect -, but we exchange the worker migration process by a capital accumulation process as in Urban (1998a).<sup>120</sup> This exchange of the transmission channel is justified, since worker migration is not a likely channel of agglomeration among countries.<sup>121</sup>

The econometric model derived from the theoretical model is a vector error correction model. In particular, the terms-of-trade effect of the Krugman (1991) model causes cointegration of the income variables of a pair of countries in this vector error correction model. Therefore, the appropriate time series test of the Krugman (1991) model is the cointegration technique as pioneered by Granger (1983), Engle and Granger (1987), Johansen (1988,1991), and Johansen and Juselius (1992).

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<sup>118</sup>The number of employees may be another measure. In this aspect, the Krugman (1991) model simplifies by assuming only one production factor: labour. If there are several production factors and firms are heterogeneous, i.e. some firms work more capital intensive than others, then the indicator number of employees suffers from the same deficiency as the indicator number of firms.

<sup>119</sup>Nelson and Plosser (1982) find that macroeconomic time series are difference stationary rather than trend stationary. We find also non-stationarity in our data set.

<sup>120</sup>Other growth model variants of the Krugman (1991) are Baldwin (1998), Baldwin and Forslid (1998), Baldwin, Martin, and Ottaviano (1998), and Martin and Ottaviano (1996).

<sup>121</sup>The Krugman (1991) model was designed to explain agglomeration among regions. An international trade model (Krugman and Venables, 1995) with intersectoral instead of international mobility of labour is used to explain agglomeration of manufacturing among countries. The mechanics - the terms of trade effect - is exactly the same in both models.

The derivation of the cointegration hypothesis from a stochastic growth model adds first to the economic interpretation of cointegration analysis and second to the income convergence literature in growth theory.

First, the standard economic interpretation of cointegration analysis is an ad hoc disequilibrium adjustment process without micro-foundation towards a long-run equilibrium defined by some static economic model.<sup>122</sup> Lau (1997) shows the link between growth models and integrated, but not cointegrated stochastic processes of the endogenous variables. Daniel (1997) obtains cointegration of the endogenous variables in a growth model by assuming cointegrated stochastic shocks. Pesaran (1997), Ogaki and Park (1997), and Rossana (1998) derive cointegrating relations from intertemporal optimization problems by assuming *non-stationary* stochastic processes.

In contrast, we obtain cointegration in an intertemporal-optimizing growth model with *stationary* stochastic shocks. The non-stationarity of the endogenous variables is caused by an endogenous growth module. Cointegration results directly from the terms-of-trade effect - the mechanics of the Krugman (1991) model. Whenever one country falls behind, its terms of trade strengthen which in turn induces an increase of savings and investment by a wealth effect, until it has caught up to the country that forged ahead. We test this model against a stochastic Ak-model which explains income of a pair of countries as two independent random walks.<sup>123</sup>

Second, the income convergence with time series methods is defined by Leung and Quah (1996) as convergence in distribution of an income variable time series vector in the infinite time limit towards a well defined (random) variable regardless of initial conditions.<sup>124</sup> We operationalize this definition by a testing hypothesis of cointegration, since this definition corresponds to the infinite adjustment process of

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<sup>122</sup>See, e.g. Hansen (1993).

<sup>123</sup>Other than in Kelly (1992) or Leung and Quah (1995), we preserve the spirit of the endogenous growth literature by modelling the Ak-model such that indeed income divergence occurs.

<sup>124</sup>The concept of convergence in form of  $\beta$ - and  $\sigma$ -convergence has been originally developed for a deterministic growth model and for cross country data analysis. See Barro (1991), Barro and Sala-i-Martin (1992), Mankiw, Romer, Weil (1992), Durlauf and Johnson (1995), and Barro and Sala-i-Martin (1995). Leung and Quah (1996) elaborate theoretically on the relation of the time series and cross country analysis of the convergence hypothesis. A theoretical survey on the convergence debate is given by Galor (1996).



a vector error correction model with cointegration towards its cointegration space, if the cointegration space is formed by the income variables. This operationalization is useful for examining convergence forces in the presence of non-stationarity of an income time series vector.

The empirical analysis is finally conducted using data for Japan and the US from 1957-1990. It is found that the theoretical model is rejected during the Bretton Woods era of fixed exchange rates. There is weak evidence in the Post-Bretton Woods era. If one believes that the oil crisis was a unique event of the past, then the theoretical model is rejected, although (almost all) the coefficients of the dynamical system have the correct sign. If one believes that major shocks such as the oil crises are repeating in the future, then the model is accepted. This result does not contradict Daniel (1997) who found that there is only one cointegrating relation between GDP of Japan, UK, and the US. Our result on the mechanics of the Krugman (1991) model complements the research of Davis and Weinstein (1997) who examine the trade flow predictions of the Krugman (1991) model.<sup>125</sup> Our findings that income and terms of trade are (weakly) related to each other for Japanese and US time series data is in line with cross country data analysis such as Spraos (1980) and Barro and Lee (1994).

The rest of the paper is organized as follows: Section 2 lies out the theoretical model. Section 3 derives the cointegration hypothesis from the theoretical model. Section 4 conducts the empirical analysis for Japan and the US. Section 4.1 explores the Bretton Woods era; section 4.2 shows the occurrence of a structural break thereafter; and section 4.3 studies the Post-Bretton Woods era. Section 5 concludes.

## 5.2 The Theoretical Model

We will first set up a stochastic endogenous growth model version of the economic geography model (Krugman, 1991). Then, we will show that this model yields a reduced rank hypothesis in the vector error correction form.

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<sup>125</sup>Other indirect empirical evidence for the economic geography literature is reviewed in Amiti (1998).

There are two types of consumers  $i$  - farmers ( $F$ ) and capitalists ( $C$ ) - in two countries ( $j = 1, 2$ ). There are two sectors - a perfectly competitive agricultural sector ( $A$ ) and a monopolistically competitive manufacturing sector ( $M$ ). The agricultural sector employs  $(1 - \mu)/2$  farmers<sup>126</sup> in each country and the manufacturing sector in each country  $j$  employs  $K_{jt}$  units of capital at a discrete period of time  $t$ . Only capitalists save by maximizing their expected-utility function  $V_j$  subject to a dynamic budget constraint and some initial conditions<sup>127</sup>:

$$V_j = \max_{C_{jt}^C} E_0 \sum_{t=0}^{\infty} \beta^t \ln C_{jt}^C, \quad (132)$$

where  $E_0$  is the expectation operator based on information on all variables until period 0 and  $\beta$  is a discount factor<sup>128</sup>. Both farmers and capitalists of country  $j$  have the same Cobb-Douglas type consumption basket  $C_{jt}^i$  in a period  $t$  defined on the manufacturing sub-basket  $C_{jt}^{Mi}$ , and an agricultural good  $C_{jt}^{Ai}$ :

$$C_{jt}^i = (C_{jt}^{Mi})^{\mu} (C_{jt}^{Ai})^{1-\mu}, \quad (133)$$

where  $\mu$  is the income share spent on the manufacturing basket  $C_{jt}^{Mi}$  and  $i = \{F, C\}$ . The manufacturing basket is defined as a Dixit-Stiglitz (1977) type CES-subutility function on  $n_{jt}$  domestic goods and  $n_{kt}$  foreign goods:

$$C_{jt}^{Mi} = \left( \sum_{j=1}^{n_j} \frac{c_{jjt}^{\frac{\sigma-1}{\sigma}}}{n_{1t} + n_{2t}} + \sum_{k=1}^{n_k} \frac{c_{kjt}^{\frac{\sigma-1}{\sigma}}}{n_{1t} + n_{2t}} \right)^{\frac{\sigma}{\sigma-1}}, \quad (134)$$

where  $c_{jjt}$  and  $c_{kjt}$ ,  $j, k = 1, 2$ ,  $k \neq j$ , are consumer  $j$ 's consumption of the manufacturing goods  $j$  and  $k$  produced in country  $j$  and  $k$  in time period  $t$ , respectively.<sup>129</sup>

Farms employ one unit of labor each and produce one unit of output. Prices for agricultural goods are taken as numeraire. Manufacturing firms differ only by their

<sup>126</sup>We normalize total population in each country to 1. This normalization is chosen such that we remain as close as possible to the seminal Krugman (1991) model.

<sup>127</sup>The consumer optimization problem including the constraints and initial conditions is stated in appendix 2, equations (186)-(190).

<sup>128</sup>We assume  $\beta$  to be identical in both countries, because this implies that both countries have the same long-run savings rates and growth path, after one country has caught up to the other. Yet, this will appear to be relevant for our data set in the empirical analysis, because Japan and the US seem to have roughly the same per capita income in 1990 (which will become obvious in figure 1).

<sup>129</sup>In monopolistically competitive markets every firm produces a different good. Because we will assume firms to be symmetric within a country  $j$ , the index  $j$  denotes a typical firm in country  $j$ .

location. The production technology is of the Ak-type inducing endogenous growth with fixed cost that gives rise to increasing returns to scale on plant level. In particular,  $\alpha$  units of capital  $k_{jt}$  are used to install the production process every day (maintenance work) and  $\beta$  units are used to produce each unit of goods for the domestic and the foreign market  $x_{jt}$ :

$$z_{jt}k_{jt} = \alpha + \beta x_{jt}, \quad (135)$$

where  $z_{jt}$  is a stationary random shock of the production technology. We specify this shock as an AR(1) process<sup>130</sup> of the form:

$$\ln z_{jt} = b_j \ln z_{jt-1} + \varepsilon_{jt}, \quad (136)$$

where  $|b_j| < 1$ , and  $\varepsilon_{jt}$  is i.i.d. normally distributed with zero mean. Present shocks are assumed to be perceived by all agents, but future shocks are not. In other words, everybody knows that there is a recession today, but it is not certain, whether there still will be a recession next year. Additionally, we assume free firm entry and exit which keeps profits at zero. Production factors are immobile.

A unit of capital is assembled by all varieties of manufacturing goods. For simplicity, we assume that capital takes the same CES form as does the consumption basket on manufactured goods:

$$I_{jt} = \left( \sum_{j=1}^{n_j} \frac{\frac{\sigma-1}{\sigma} \iota_{jjt}}{n_{1t} + n_{2t}} + \sum_{k=1}^{n_k} \frac{\frac{\sigma-1}{\sigma} \iota_{kjt}}{n_{1t} + n_{2t}} \right)^{\frac{\sigma}{\sigma-1}}, \quad (137)$$

where  $I_{jt}$  is the investment aggregate used by the manufacturing firms in country  $j$  and  $\iota_{jjt}$  and  $\iota_{kjt}$ ,  $j, k = 1, 2$ ,  $k \neq j$ , are demand of the firms in country  $j$  for investment goods produced by a typical firm  $j$  and  $k$  in country  $j$  and  $k$ , respectively. We also

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<sup>130</sup>The specification that productivity shocks drive a business cycle is taken from the Real Business Cycle literature, e.g. Kydland and Prescott (1982), and Long and Plosser (1983).

This particular autoregressive process of the technology shock is justified by the empirical data employed in later sections. It can be thought as exogenously imposing the autocorrelation structure explained, for example, by “time-to build” specifications (Kydland and Prescott, 1982) without having to overburden the model with further microeconomic specifications.

The lag order of this stochastic process will determine the lag order of the vector error correction model (Rossana, 1998). We choose arbitrarily lag one for illustrative purposes and leave it to the empirical analysis to determine the actual lag length.

assume a 100 per cent depreciation rate such that next period's capital stock is equal to this period's investment ( $K_{jt+1} = I_{jt}$ ).<sup>131</sup> (Note that  $K_{jt} = n_{jt}k_{jt}$ ).

Finally, there are trade costs of the Samuelson iceberg-type for manufacturing goods, such that only a fraction  $\tau$  of one produced unit of a good arrives at its foreign destination ( $0 < \tau < 1$ ). There are no transport cost for agricultural goods. All factors are immobile.

The within-period consumption maximization problem, firms' optimization, and the market clearing conditions are solved straight forwardly following closely chapter 3 of this thesis. The corresponding ideal CES price index  $P_{jt}$  for manufacturing goods is found to be:<sup>132</sup>

$$P_{jt} = \left( \frac{n_{jt}p_{jt}^{1-\sigma}}{n_{1t} + n_{2t}} + \frac{n_{kt}p_{kt}^{ex(1-\sigma)}}{n_{1t} + n_{2t}} \right)^{\frac{1}{1-\sigma}}, \quad (138)$$

where  $p_{jt}$  and  $p_{kt}^{ex}$  are the domestic producer prices and export prices of firms in country  $j$  and  $k$  charged for consumers in country  $j$ , respectively. Firms optimize their profits by the mark-up pricing rule:<sup>133</sup>

$$p_{jt} = \left( \frac{\sigma}{\sigma - 1} \right) \beta r_{jt} \quad \text{and} \quad p_{jt}^{ex} = \tau^{-1} \left( \frac{\sigma}{\sigma - 1} \right) \beta r_{jt}, \quad (139)$$

where  $r_{jt}$  is the rental rate of capital in country  $j$  at time  $t$ . Foreign consumers fully bear the transport cost. Because of free entry and exit of firms, profits are zero. This condition yields an expression for income of capitalists  $y_{jt}^C$  in country  $j$ :

$$n_{jt}p_{jt}x_{jt} = K_{jt}r_{jt} \equiv y_{jt}^C. \quad (140)$$

From the zero profit condition follows that optimal firm output is a constant:

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<sup>131</sup>It is well know that specific stochastic optimization problems with logarithmic functional forms can easily be solved, if this depreciation assumption is employed. See for example Stokey and Lucas (1989).

<sup>132</sup>Note that we take here the symmetry of firms within a country into account.

<sup>133</sup>See d'Aspremont et. al. (1996) for a discussion of this result. Note also that firms optimize under certainty, because contemporary shocks are known and there is no link to the future.

$$x_{jt} = \frac{\alpha(\sigma - 1)}{\beta} \equiv 1, \quad (141)$$

where we normalized without loss of generality  $\alpha\sigma \equiv 1$  and  $\beta \equiv 1 - \alpha$ . From the above equation and the factor market clearing condition we obtain an equation relating the number of firms to the capital stock and the technology shock:

$$n_{jt} = z_{jt} K_{jt}. \quad (142)$$

Note that economy-wide technology shocks are fully absorbed in fluctuations of firm entry and exit. Nominal income of a country  $y_{jt}$  may then be written as follows:

$$y_{jt} = r_{jt} K_{jt} + \frac{1 - \mu}{2} = n_{jt} p_{jt} + \frac{1 - \mu}{2} \quad (143)$$

Real per capita GDP  $y_{jt}^r$  measured by a Laspeyres-index can then be expressed as:

$$y_{jt}^r = n_{jt} \bar{p}_j + \frac{1 - \mu}{2}, \quad (144)$$

where  $\bar{p}_j$  is a base year producer price in country  $j$ .

Finally, the goods market equilibrium condition for a typical manufacturing firm in country 1 and the goods market equilibrium for agricultural goods are obtained:

$$\begin{aligned} \frac{\mu p_{1t}^{-\sigma} y_{1t}}{n_{1t} p_{1t}^{1-\sigma} + q n_{2t} p_{2t}^{1-\sigma}} + \frac{q \mu p_{1t}^{-\sigma} y_{2t}}{q n_{1t} p_{1t}^{1-\sigma} + n_{2t} p_{2t}^{1-\sigma}} &= 1 \\ (y_{1t} + y_{2t})(1 - \mu) &= 1 - \mu, \end{aligned} \quad (145)$$

where  $q \equiv \tau^{\sigma-1}$  for notational simplicity. Following again the steps in chapter 3 of this thesis we summarize the goods market equilibrium conditions in the following excess demand function:

$$h(p_t, n_t) = \frac{1 + \mu + (1 - \mu) n_t p_t}{2(1 + q n_t p_t^{1-\sigma})} + \frac{q[1 - \mu + (1 + \mu) n_t p_t]}{2(q + n_t p_t^{1-\sigma})} - 1 = 0, \quad (146)$$

where we conveniently define the terms of trade  $p_t \equiv \frac{p_{2t}}{p_{1t}}$  and the relative number of firms  $n_t \equiv \frac{n_{2t}}{n_{1t}}$ . This equation can be solved for  $n_t$ :

$$n_t = \frac{1 - \mu + (1 + \mu) q^2 - 2qp_t^\sigma}{p_t [1 - \mu + (1 + \mu) q^2 - 2qp_t^{-\sigma}]} \quad (147)$$

We define for future reference from  $h(p_t, n_t) = 0$  the correspondence  $p_t(n_t)$ . The budget constraint for capitalists in country  $j$  may be written as follows:<sup>134</sup>

$$P_{jt} (I_{jt} + C_{jt}^{MC}) + C_{jt}^{AC} = y_{jt}^C \quad (148)$$

Using (140), (141), (142), the depreciation assumption, and the fact that a constant fraction  $(1 - \mu)$  of income is spent on agricultural goods yields:

$$K_{jt+1} = \mu \pi_{jt} K_{jt} z_{jt} - C_{jt}^{MC}, \quad (149)$$

where we define for convenience  $\pi_{jt} \equiv (p_{jt}/P_{jt})$ . The definition of  $P_{jt}$  in equation (138) is plugged into the definition of  $\pi_{jt}$  to yield:

$$\pi_{1t} = \pi_{1t}(p_t(n_t), n_t) = \left( \frac{1}{1 + n_t} + \frac{qp_t(n_t)^{1-\sigma}}{1 + n_t^{-1}} \right)^{\frac{1}{\sigma-1}} = \pi_{1t}(z_t K_t) \quad (150)$$

and

$$\pi_{2t} = \pi_{2t}(p_t(n_t), n_t) = \left( \frac{1}{1 + n_t^{-1}} + \frac{qp_t(n_t)^{\sigma-1}}{1 + n_t} \right)^{\frac{1}{\sigma-1}} = \pi_{2t}(z_t K_t), \quad (151)$$

where  $z_t \equiv z_{2t}/z_{1t}$ ,  $K_t \equiv K_{2t}/K_{1t}$ , the correspondence  $p_t(n_t)$  from (147), as well as equation (142). Now we make a guess for a consumption function that optimizes expected utility of capitalists around some steady state to be defined later:

$$C_{jt}^M = d_0 \mu \pi_{jt} K_{jt} z_{jt}, \quad (152)$$

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<sup>134</sup>Note that the following relation holds by the definition of the ideal price index:

$$P_{jt} (I_{jt} + C_{jt}^{MC}) = \sum_{j=1}^{n_j} p_{jt} (c_{jzt} + l_{jzt}) + \sum_{k=1}^{n_k} p_{kt}^{ex} (c_{jkt} + l_{jkt}).$$

where  $d_0$  is a parameter yet to be determined. We will later confirm this guess to be valid. Inserting (150), (151), and (152) into (149) yields finally:

$$K_{jt+1} = (1 - d_0) \mu \pi_{jt} (K_t z_t) K_{jt} z_{jt}. \quad (153)$$

This is the stochastic difference equation system summarizing the basic model under the assumption that the guess (152) is valid.

### 5.3 Derivation of the Cointegration Hypothesis

Our final objective is the empirical test of the model (153). Before we can do this, we need to transform the model into the vector error correction form which cointegration tests are based on. We proceed in four steps: first, we show that the deterministic counterpart of (153) yields steady state values for the relative capital stocks  $K_t$  and the terms of trade  $p_t$  given the guess (152) for the consumption function and some further condition. Then, we confirm the guess of the consumption function to be valid around the so-found steady state values. These two results allow us to apply Campbell's (1994) logarithmic approximation method on (147), (150), and (151) around the steady states of  $p_t$  and  $K_t$ . Second, we derive the reduced rank hypothesis of cointegration from the log-linearized system (153) in vector error correction form. Third, we extend the basic model by adding exchange rate shocks. Finally, an alternative hypothesis based on a stochastic Ak-model is formulated.

#### 5.3.1 Basic Version

We proceed by taking the logarithm of the ratio of (153) for country 2 to (153) for country 1:

$$\ln K_{t+1} = \ln \pi_{2t} (z_t K_t) - \ln \pi_{1t} (z_t K_t) + \ln K_t + \ln z_t, \quad (154)$$

where  $\ln z_t \equiv \ln z_{2t} - \ln z_{1t}$  is a stationary stochastic process. This stochastic difference equation is sufficient to describe the behaviour of the relative capital stocks  $K_t$  and terms of trade  $p_t$ . We will next show that this stochastic difference equation is stationary, even if (153) is not. Now, we shut off the stochastic process, i.e. we set

$z_t = 1$ . The remaining deterministic equation can be depicted in figure 1 (see appendix). It depends obviously on the shape of  $\ln \pi_{2t}(K_t) - \ln \pi_{1t}(K_t)$ , whether this difference equation has a unique stable fixed point. In general, this difference equation has multiple fixed points. Under some condition, a unique stable fixed point can be established in proposition 1.

**Proposition 1:** *The deterministic counterpart to the stochastic difference equation given by (154) with (147), (150), and (151) has a unique stable fixed point  $\bar{K}=1$ , and  $\bar{p}=1$ , if*

$$\tau^{\mu\sigma} [(1 + \mu) \tau^{\sigma-1} + (1 - \mu) \tau^{1-\sigma}] > 2 \quad (155)$$

*and the guess for the consumption function (152) is valid.*

**Proof:** See appendix 1.

**Q.E.D.**

The condition in proposition 1 is the same as the one for the “convergence case” in Krugman (1991). There, manufacturing firms tend to spread evenly in space, because producer prices and wages are highest in the smallest region. Here, the country with the most manufacturing has the lowest producer prices, lowest income and savings, grows slowest in number of firms and manufacturing output allowing the other country to catch up. We will assume that condition (155) is true for our data sample<sup>135</sup> and pay no further attention to it, because we want to focus on the empirical investigation of the terms-of-trade effect only.

After we have found that  $\bar{p}=\bar{K}=1$  is a stable fixed point of the deterministic counterpart of the stochastic difference equation (154) conditional on our guess (152) for the consumption function, we can confirm that the initial guess is indeed valid around this steady state. Proposition 2 does exactly this.

**Proposition 2:** *The linear guess for the consumption function (152) is the optimal solution to the maximization problem of capitalists (132) subject to the resource con-*

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<sup>135</sup>If this condition were not true, then the model would predict that income diverges which is certainly not the case for our data sample. Also, the condition implies that transport cost shall be sufficiently large which is likely the case for our data sample US and Japan.



straint (149) and the pricing equation (147) around the steady state  $\bar{K}=1$  and  $\bar{z}_j=1$ ,  $j=1,2$ , if  $d_0=1-\beta$  is chosen.

**Proof:** See appendix 2.

**Q.E.D.**

Since we have established that the steady state solution  $\bar{p}=\bar{K}=1$  is stable (under some condition) and (136) is assumed to be a stationary stochastic process, we can apply as in Campbell (1994) a logarithmic approximation around the steady state values  $\bar{p}=\bar{K}=1$  to the equations (147), (150) and (151). Following this method we obtain for (150) and (151):

$$\begin{aligned}\ln \pi_{1t} &= -\frac{q}{q+1} \ln p_t + \frac{q-1}{2(\sigma-1)(q+1)} \ln n_t, \\ \ln \pi_{2t} &= \frac{q}{q+1} \ln p_t - \frac{q-1}{2(\sigma-1)(q+1)} \ln n_t.\end{aligned}\tag{156}$$

We will exploit the symmetry of these two equations to derive the reduced rank hypothesis of cointegration. Correspondingly, the goods market equilibrium condition (147) is log-linearized as follows:

$$\ln n_t = \gamma_1 \ln p_t,\tag{157}$$

where

$$\gamma_1 \equiv \frac{-4\sigma q - 1 + \mu - (1 + \mu)q^2 + 2q}{1 - \mu + (1 + \mu)q^2 - 2q}.$$

So far, we have a stochastic difference equation system in the endogenous variable capital stocks of firms  $K_{jt}$ . Data on capital stocks are not very reliable, since they depend on depreciation assumptions. Multiplying (153) with  $\ln z_{jt+1}$  and substituting  $n_{jt} = z_{jt}K_{jt}$  for all  $j, t$  transforms the system (153) into one with the endogenous variable number of firms  $n_{jt}$ . This variable is sensitive to the symmetry assumption imposed on firms. If firms were heterogeneous, firm size and number of firms would interact in an unknown way. Therefore, it is not clear, how to measure this variable empirically. To circumvent this problem, one could use another variable: industrial production. However, the model assumes that all industrial production uses

increasing returns technology and all industrial production is exportable. Therefore, it is not clear, whether the data series of industrial production which may contain non-increasing returns production and non-tradable goods is a good measure. Additionally, industrial production data are highly volatile making it difficult to establish economic relations. Hence, we choose to express the theoretical model in terms of real GDP as defined in (144).

Before we can do so, the following two relations have to be noted: First,

$$\Delta \ln y_{jt}^r \approx \Delta \ln n_{jt}, \quad (158)$$

where  $\Delta$  denotes the difference operator; second

$$\ln n_{jt} \approx \ln y_{jt}^r - \ln \bar{p}_j, \quad (159)$$

if the manufacturing sector is sufficiently large, because

$$\lim_{t \rightarrow \infty} y_{jt}^r + \frac{1-\mu}{2} = \lim_{t \rightarrow \infty} y_{jt}^r. \quad (160)$$

In other words, as the economy grows, the share of the agricultural sector in GDP diminishes and GDP is well approximated by the output of all sectors without agriculture.

Now, we are ready to derive the vector error correction form of our theoretical model. Taking the logarithm of (153), plugging in (156) and (157), using (158) and (159), solving for the autoregressive process of the error term (136), and rearranging terms yields:

$$\begin{aligned} \begin{pmatrix} \Delta \ln y_{1t+1}^r \\ \Delta \ln y_{2t+1}^r \end{pmatrix} &= \begin{pmatrix} b_1 - \gamma_2 & \gamma_2 \\ \gamma_2 & b_2 - \gamma_2 \end{pmatrix} \begin{pmatrix} \Delta \ln y_{1t}^r \\ \Delta \ln y_{2t}^r \end{pmatrix} \\ &+ b_0 + \Pi \begin{pmatrix} \ln y_{1t}^r \\ \ln y_{2t}^r \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t+1} \\ \varepsilon_{2t+1} \end{pmatrix}, \end{aligned} \quad (161)$$

where

$$\begin{aligned} \Pi &= \gamma_2 \begin{pmatrix} -(1-b_1) & 1-b_1 \\ 1-b_2 & -(1-b_2) \end{pmatrix}, \\ b_0 &= \begin{pmatrix} (1-b_1) (\ln(\mu\beta) + \gamma_2 \ln \bar{p}_1 - \gamma_2 \ln \bar{p}_2) \\ (1-b_2) (\ln(\mu\beta) - \gamma_2 \ln \bar{p}_1 + \gamma_2 \ln \bar{p}_2) \end{pmatrix}, \end{aligned}$$

$$\gamma_2 = -\frac{(1-q)\gamma_1 + 2q(\sigma-1)}{2(\sigma-1)(1+q)\gamma_1}.$$

The cointegration hypothesis is formulated as a reduced rank  $r < p$  of the  $p \times p$  matrix  $\Pi$  (here  $p = 2$ ) which implies that it can be decomposed into  $\Pi = \alpha\beta'$  with the  $p \times r$  matrices  $\alpha$  and  $\beta$  of full rank (see Johansen 1988, 1995). It is easily seen that  $\Pi$  has rank  $r = 1$  and  $\alpha$  and  $\beta$  are found to be (with an appropriate normalization):

$$\alpha\beta' = \gamma_2 \begin{pmatrix} 1-b_1 \\ -(1-b_2) \end{pmatrix} \begin{pmatrix} -1 & 1 \end{pmatrix}. \quad (162)$$

Additionally, we note that the theoretical model requires that the intercept is not restricted to the cointegrating relation, because there does not exist a  $1 \times r$  vector  $\rho_0$  such that  $\alpha\rho_0' = b_0$ . This can be easily seen by comparing the  $\alpha$  and the  $b_0$  vectors:

$$\begin{pmatrix} 1-b_1 \\ -(1-b_2) \end{pmatrix} \rho_0' \neq \begin{pmatrix} (1-b_1)(\ln(\mu\beta) + \gamma_2 \ln \bar{p}_1 - \gamma_2 \ln \bar{p}_2) \\ (1-b_2)(\ln(\mu\beta) - \gamma_2 \ln \bar{p}_1 + \gamma_2 \ln \bar{p}_2) \end{pmatrix} \quad (163)$$

for every scalar  $\rho_0$ . Therefore the appropriate model specification is the reduced rank hypothesis  $H_1(r)$  in the notation of Johansen (1995, p. 81ff). We are able to obtain this reduced rank hypothesis exactly because the stochastic difference equations (153) are integrated of order 1, but the ratio of the two in equation (154) is stationary.

The empirical implication for economic growth is twofold. First, any two economies grow indefinitely (since we assumed an endogenous growth model). Second, income levels of any two economies fulfilling condition (155) *converge* to each other. The terms of trade tie together the income levels of the two countries. Whenever one country lags behind, its terms of trade strengthen and induce larger investment and economic growth relative to the country that is ahead. In this way we have formulated the convergence hypothesis as a cointegration hypothesis in time series analysis.

### 5.3.2 Extension

Our model may be tested as this cointegration hypothesis. However, this test has no power with respect to mechanisms alternative to the terms-of-trade effect of this model that may also generate a cointegrating relation of the same kind, e.g. capital

mobility, capital-labour substitutability, or technology spillovers. In other words we can only find convergence, but not the source of convergence. Therefore, we choose to extend the model by formulating an equation of motion for the terms of trade to allow for testing the significance of the terms-of-trade effect.

The simplest way to introduce an additional equation of motion into the model is a tatonnement process augmented with an  $AR(1)$  stochastic process  $z_{3t}$  as defined by (136):

$$\Delta \ln p_{t+1} = \delta \ln [h(p_t, n_t) + 1] + \ln z_{3t+1}, \quad (164)$$

where  $\delta < 0$  is an adjustment speed parameter and we have assumed that agricultural goods markets clear instantly.<sup>136</sup> Whenever there is excess demand for good 1 the terms of trade  $p_t = p_{2t}/p_{1t}$  will fall. The error term may be justified by “excess volatility” in the nominal exchange rate. We apply the log-linearization to (164) and obtain:

$$\Delta \ln p_{t+1} = \delta \gamma_3 \ln n_{1t} - \delta \gamma_3 \ln n_{2t} + \delta \gamma_4 \ln p_t + \ln z_{3t+1}, \quad (165)$$

where

$$\gamma_3 = \frac{1 - \mu q^2}{(1 + q)^2} > 0,$$

and

$$\gamma_4 = \frac{\mu q^2 (2 - \sigma) - \mu \sigma + \sigma (1 + q)^2 + 2q (\sigma - 1)}{(1 + q)^2} > 0.$$

The vector error correction form of the extended three-dimensional system (153), (164), and (136) can be derived in the same way as the two-dimensional system (161) before:

$$\begin{pmatrix} \Delta \ln y_{1t+1}^r \\ \Delta \ln y_{2t+1}^r \\ \Delta \ln p_{t+1} \end{pmatrix} = \Gamma \begin{pmatrix} \Delta \ln y_{1t}^r \\ \Delta \ln y_{2t}^r \\ \Delta \ln p_t \end{pmatrix} + b_0 + \Pi \begin{pmatrix} \ln y_{1t}^r \\ \ln y_{2t}^r \\ \ln p_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t+1} \\ \varepsilon_{2t+1} \\ \varepsilon_{3t+1} \end{pmatrix}, \quad (166)$$

where

$$\Gamma = \begin{pmatrix} b_1 + \frac{1-q}{2(\sigma-1)(1+q)} & -\frac{1-q}{2(\sigma-1)(1+q)} & -\frac{q}{1+q} \\ -\frac{1-q}{2(\sigma-1)(1+q)} & b_2 + \frac{1-q}{2(\sigma-1)(1+q)} & \frac{q}{1+q} \\ \delta \gamma_3 & -\delta \gamma_3 & \delta \gamma_4 - b_3 \end{pmatrix},$$

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<sup>136</sup>This ad hoc adjustment process is comparable to Dornbusch (1976) in an exchange rate model. He assumes that nominal price levels are sticky for some time, after a monetary shock has occurred. In this paper, we rather assume that the relative prices of foreign to domestic products are sticky, after an exchange rate shock has occurred.

$$\Pi = \begin{pmatrix} \frac{(1-b_1)(1-q)}{2(\sigma-1)(1+q)} & -\frac{(1-b_1)(1-q)}{2(\sigma-1)(1+q)} & -\frac{(1-b_1)q}{1+q} \\ -\frac{(1-b_2)(1-q)}{2(\sigma-1)(1+q)} & \frac{(1-b_2)(1-q)}{2(\sigma-1)(1+q)} & \frac{(1-b_2)q}{1+q} \\ \delta\gamma_3(1-b_3) & -\delta\gamma_3(1-b_3) & \delta\gamma_4(1-b_3) \end{pmatrix},$$

and

$$b_0 = \begin{pmatrix} (1+b_1)\ln(\mu\beta) \\ (1+b_2)\ln(\mu\beta) \\ 0 \end{pmatrix} - \Pi \begin{pmatrix} \ln \bar{p}_1 \\ \ln \bar{p}_2 \\ 0 \end{pmatrix}.$$

This time the matrix  $\Pi$  has rank  $r = 2$ . Therefore, the matrices  $\alpha$  and  $\beta$  are  $3 \times 2$  and can be found to be:

$$\alpha\beta' = \begin{pmatrix} \frac{(1-b_1)(1-q)}{2(\sigma-1)(1+q)} & 0 \\ -\frac{(1-b_2)(1-q)}{2(\sigma-1)(1+q)} & 0 \\ 0 & -\delta\gamma_3(1-b_3) \end{pmatrix} \begin{pmatrix} 1 & -1 & -\frac{2q(\sigma-1)}{1-q} \\ -1 & +1 & -\frac{\gamma_4}{\gamma_3} \end{pmatrix} = \Pi. \quad (167)$$

Note that row 1 of the  $\beta'$ -matrix is the income convergence relation and row 2 the long run goods market equilibrium relation. Row 1 implies that income and the ratio of factory-gate export to import prices shall be positively related for both countries in the long-run. Row 2 mirrors the long run goods market equilibrium. Again, the intercept cannot be restricted to the cointegrating relation. We summarize the results of this section in the following null hypothesis.

**Null Hypothesis:** *The vector error correction model (166) has a  $\Pi$ -matrix (167) of reduced rank  $r = 2$ . The intercept  $b_0$  is not restricted to the cointegration space.*

In the empirical part of this paper, we will test this null hypothesis.<sup>137</sup>

### 5.3.3 Alternative Hypothesis

We now formulate an alternative model against which we test the above model. Naturally, we choose a stochastic Ak-model.<sup>138</sup> In particular, we assume the following

<sup>137</sup>For (166) to comply with the Engle-Granger representation theorems of Johansen (1988), three additional assumptions have to be made (see Johansen, 1995, theorem 4.2). First, the data generating process of  $y_{jt}^r$  needs to be integrated of order 1. This follows from our assumption of an endogenous growth model set-up. (See Lau (1997) for a general proof.) Second, the matrix  $\Gamma$  needs to have full rank which is easily checked to be true. Third, an explosive growth path or a cyclical growth path have to be excluded. The latter can only be checked empirically by calculating the moduli of the eigenvalues of the companion matrix which have to be on or inside the unit circle (see Lütkepohl (1991), p. 11ff). This assumption holds for our data sample.

<sup>138</sup>See Barro and Sala-i-Martin, 1995, for a textbook introduction to the Ak-model. The stochastic counterpart may be found in Lau (1997).

production function:

$$y_{jt} = z_{jt}k_{jt}, \quad (168)$$

where  $y_{jt}$  denotes income of country  $j$  at time  $t$  and  $z_{jt}$  is defined in (136). For simplicity, a constant savings function with savings rate  $s$  similar to Solow (1956) is assumed such that:

$$k_{jt+1} = sy_{jt}. \quad (169)$$

These two equations can be combined with (136) and manipulated to yield:

$$\Delta \ln y_{jt+1} = -b_j \Delta \ln y_{jt} + \ln s + \varepsilon_{jt+1}. \quad (170)$$

This two dimensional difference equation system shows clearly no cointegration between income of the two countries.

**Alternative Hypothesis:** *The vector error correction model (166) has a  $\Pi$ -matrix (167) of reduced rank  $r = 0$ .<sup>139</sup>*

In economic terms the Ak-model implies that there is no convergence in the sense defined above. Two economies grow independently of each other. If the rank of the  $\Pi$ -matrix is zero, then the terms-of-trade effect is not present, either.

## 5.4 Empirical Analysis

We use quarterly data for per capita real GDP of Japan (GDPJP) and the US (GD-PUS) from the first quarter 1957 until the fourth quarter 1990 in 1990 dollar prices. Additionally, we use a real exchange rate variable (TOT) based on factory gate wholesale price indices. All variables are in logs. All data are from the International Financial Statistics of the IMF<sup>140</sup>. For more details see appendix 3. Summers and Heston (1991) have shown that these income statistics lack comparability in absolute value terms, because the law of one price does not hold in practice. Unfortunately, the Summers and Heston (1991) data set is not applicable to a time series analysis, because it contains only annual data. The sample size would not be sufficiently large.

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<sup>139</sup>Of course, a matrix with rank 0 is 0 itself.

<sup>140</sup>However, note footnote 44 in appendix 3.

However, the danger of a data error is limited for our research problem, since we are not interested in absolute values, but in relative values and changes of variables in time.

The three time series are depicted in figure 2 (see appendix). The data on Japanese GDP show a declining positive time trend and a declining variance of GDP over time. The oil crisis in 1974 appears as the only major disturbance of this trend. Japanese GDP starts below the US level and catches-up quickly to US levels. However, the data (until 1990) do not tell, whether US and Japanese GDP converge to some constant ratio or whether Japanese GDP is just described by a faster growth path than US GDP.

The real exchange rate shows on average a sharp real appreciation of the Yen against the dollar, as is predicted by the theoretical model. However, the appreciation trend is not present during the Bretton Woods era until 1970 and is interrupted by larger depreciations accompanying the oil-price shocks. All time series cannot be rejected to be integrated of order one, but can be rejected to be of order two according to augmented Dickey-Fuller (1987) and Philips-Perron (1988) tests.<sup>141</sup>

In a monetary economics history of Japan, Cargill, Hutchison, and Ito (1997) describe four major phases of development: the high-growth period until 1971, the wild-inflation years 1971-1975, 1975-1985, and the bubble economy from 1985 onwards. The contemporary period is described as the “bursting bubble” economy both with a major asset price slump and an ongoing meltdown of the financial system. This period may describe an economy “off” the “efficient frontier” of production which is assumed in our growth models and may even indicate a change in regime from the “neoclassical regime” to the “poverty trap regime” in terms of our growth model in chapter 4 of this thesis. In both cases, the theoretical model would not apply to the contemporary economic situation in Japan. Therefore, we cut off the sample in 1990, fourth quarter.

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<sup>141</sup>We do not report the results, since Hendry and Mizon (1993) argue that these tests can only be rough indications for a vector time series process.

Until 1971 a fixed exchange rate system vis a vis the dollar was kept. Because goods prices appeared “sticky”, the real exchange rate was largely undervalued causing large trade balance surpluses, while the Japanese economy grew faster than the US in this time period. The break-down of the Bretton Woods system was accompanied by four years of trials to “engineer” a nominal devaluation of the Yen which did not succeed, but caused high inflation rates in Japan instead. Since 1975 a steadier monetary policy approach was followed accompanied by a deregulation of the financial system. In general, the Bank of Japan pursued a “leaning against the wind” exchange rate policy with two major intervention periods in the beginning of the 70ies and the end of the 80ies.

We conclude that the terms-of-trade effect, if present, may be disturbed by two major events not regarded in our theoretical model: 1.) considerable real appreciation pressure of the Yen as suggested by the model was suppressed by the policy of fixed exchange rate systems until 1971. 2.) Both oil crises weakened the Yen, although Japan escaped at least the second oil crisis with a smaller real economic back-drop than the US. (One may think of a “safe heaven” argument supporting the dollar in periods of world economic crisis which may strengthen the dollar beyond its “fundamental” value.)

For this reason - and statistical reasons reported later, we decided to use a dummy variable for the first oil price shock. Additionally, we split the sample a priori according to the two different exchange rate regimes from 1957 first quarter until 1971 first quarter and from 1971 second quarter until 1990 fourth quarter. We will first analyse the fixed exchange rate period of the Bretton Woods era, then test in a recursive analysis for a structural break due to the change of the exchange rate regime (which is indeed present), and finally analyse the second period of a flexible exchange rate system in the Post-Bretton Woods era.



### 5.4.1 The Bretton Woods Era

We proceed by testing a generalized version of the vector error correction form (166) for the sample period 1957:1 until 1971:1:

$$\Delta z_t = \Gamma_1 \Delta z_{t-1} + \dots + \Gamma_{k-1} \Delta z_{t-k+1} + \Pi z_{t-1} + D_t + \varepsilon_t, \quad (171)$$

where  $D_t$  is a deterministic term which includes optionally time trends and constants, seasonal dummies, and other dummies. Johansen (1995, p. 81) provides five model variants to be included in the deterministic part. Only, models 3 and 4 - i.e a non-restricted constant, and a non-restricted constant plus a time trend restricted to the cointegration space - imply a linear time trend in the data which comes closest to the decreasing time trend observed in figure 2. Model 3 is required by the theory. Before we can test for the appropriate model, we need to choose the lag-length  $k$  and check the two models for the assumptions on the residuals. Table 1 reports the results (see appendix).

Both models are tested for autocorrelation of residuals, heteroscedasticity of residuals, and normality. Autocorrelation is tested by the Ljung-Box Test with 13 lags, a Lagrange Multiplier Test for first order autocorrelation (LM(1)), and for fourth order autocorrelation of the residuals (LM(4)). Heteroscedasticity is tested by an univariate ARCH(2) test. For testing the assumption of a joint normal distribution of residuals, the Doornik-Hansen test is used. Additionally, univariate normality is tested with the Jarque-Bera test. The test statistics and the limiting distributions are reported in Hansen and Juselius (1995) and the calculations are conducted with the procedure CATS in RATS.

With a lag length  $k=2$ , none of the above statistics indicates any model misspecification.<sup>142,143</sup> Additionally, we report the three autocorrelation functions for the three dimensional system in figure 3 (see appendix). They confirm the absence of autocorrelation. Next,

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<sup>142</sup>We do not apply any of the information criteria to determine the lag length, because our sample period is extremely short. To preserve as many degrees of freedom as possible, we choose the smallest lag length that is still compatible with the assumptions on the residuals.

<sup>143</sup>The test results are dependent on the cointegrating rank  $r$ . To economize on space, we report only the case of  $r=1$ . This will correspond to our choice of the rank hypothesis later.

we select the appropriate model from the options: model 3 and 4. Simultaneously, the cointegrating rank is determined. Johansen (1995) suggests a Trace-Statistic which is formulated for a jointly nested hypothesis of rank and model type to address this model selection problem. The model selection then follows the Pantula (1989) principle. The test results for the two models and the three ranks of at most  $r=0,1,2$  are reported in table 2 (see appendix). Since model  $i=3,4$  on the one hand and model  $i$  with rank  $r=0,1,2$  on the other hand are successively less restrictive, and model 3 with rank 0 cannot be rejected at the 90 per cent significance level, we choose the latter model as an appropriate data generating process for our sample.<sup>144</sup> Model choice 3 is in line with the theoretical model (166). However, rank  $r=0$  contradicts the rank hypothesis  $r=2$ . Instead, this rank hypothesis complies with our alternative theoretical model (170) - the Ak-model. In other words, there is no cointegrating relation between US and Japanese GDP. Income in the US and Japan develop independently of each other following each their own drift. US and Japanese business cycles are independent.

To confirm the hypothesis of rank  $r=0$ , two additional tests - the lambda-max test and the lambda-min test - are applied. See Johansen (1995) for a derivation of the test statistic of the lambda-max test and Bierens (1997) for a derivation of the non-parametric lambda-min cointegration rank test.<sup>145</sup> The results are reported in table 3 (see appendix). Both the lambda-max test and the lambda-min test suggest that contrary to the trace statistic the cointegrating rank is  $r=1$ . We proceed our analysis with this rank assumption and test for stationarity of any of the three variables GDPJP, GDPUS, and TOT. The test is formulated as a restriction  $H\varphi$  on the  $\beta$  matrix (see Johansen, 1995, p. 74). Whereas GDPJP and GDPUS appear non-stationary, the hypothesis of stationarity of the exchange rate variable TOT cannot be rejected. This reflects simply the fact that the Yen exchange rate was successfully kept fixed to the dollar within some bands during the sample period.<sup>146</sup> However, this implies that

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<sup>144</sup>Strictly speaking, model 4 and rank 0 is very weakly rejected again which contradicts the nested hypothesis testing approach. However, we ignore this observation, because this "violation" is only marginal and the hypothesis of model 4, rank 0, is identical to the hypothesis of model 3, rank 0.

<sup>145</sup>Calculations for the lamda-min test are performed with Bierens' (1998) econometrics program EASYREG, version 1.20. I claim sole responsibility for any calculation errors of this program.

<sup>146</sup>Although only the nominal exchange rate was announced fixed, real rigidities proved so strong that also the real exchange rate was fixed in practice.

the increase of the rank from 0 to 1 is entirely caused by the stationarity of TOT (see Johansen, 1995, p. 72ff). GDPUS and GDPJP are still not cointegrated and our conclusion remains the same as above.

Next, we test for weak exogeneity of any of the three variables GDPJP, GDPUS, and TOT. This hypothesis is tested as restriction on the  $\alpha$ -matrix ( $B'\alpha$ ).<sup>147</sup> The results are also reported in table 3. We cannot reject the hypothesis that GDPJP and GDPUS are weakly exogenous. Also the joint hypothesis of GDPJP and GDPUS weakly exogenous and TOT stationary cannot be rejected (table 3). In other words, the  $\Pi$ -matrix consists of 0 entries except for the third element on the third row indicating the stationarity of TOT, which explains that the trace statistics finds rank 0, which is a joint hypothesis for all entries of the  $\Pi$ -matrix to be zero.

We conclude our analysis for the Bretton Woods era by checking the signs of the  $\Pi$ -matrix. The empirical values of this matrix are given by:

$$\Pi = \begin{pmatrix} 0.026 & -0.082 & 0.020 \\ -0.048 & 0.146 & 0.024 \\ 0.004 & 0.005 & -0.243 \end{pmatrix}. \quad (172)$$

The empirical signs are compared to the theoretical signs of matrix (166). In total, 6 out of 9 coefficients have the correct sign. However, the impact of the terms-of-trade effect has not the correct sign for the US economy (element (2,3) of matrix (172)). Also, the assumed goods market equilibrium adjustment process (164) has two wrong signs (elements (3,2) and (3,3) of matrix (172)).

#### 5.4.2 Recursive Analysis

So far, we have assumed that there is a structural break after the Bretton Woods era. Now, we are testing for this hypothesis using the recursive analysis batch procedure in CATS.<sup>148</sup> First, the constancy of the cointegrating rank choice is tested. The trace statistic is calculated for rank at most  $r=0,1,2$  for every sample recursively starting

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<sup>147</sup>See Johansen (1995), p. 77f, and Hansen and Juselius (1995), p. 44ff.

<sup>148</sup>The following analysis is originated by Hansen and Johansen (1992).

from 1957:1-1971:2 until 1957:1-1990:4.<sup>149</sup> Figure 4 reports the results (see appendix). Any test statistic is normalized with its critical value at 90 per cent significance level. Therefore, a value above 1 indicates that a statistics is rejecting the hypothesis of rank at most  $r=0,1,2$ . One can infer from the figure that during the 70ies and 80ies up to three cointegrating relations become significant indicating a structural break after the Bretton Woods era. The statistics shows an upward trend at least for  $r=1$  and  $r=2$ . For the rank hypothesis  $r=2$  to consistently pertain over the entire sample period 1957:1-1990:4, two of the three schedules should be persistently upward sloping (as the schedules are showing the convergence behaviour of the trace statistics with two roots approaching infinity and one root approaching zero under the hypothesis of cointegrating rank  $r=2$ ). This indicates that the data generating process may have changed after the Bretton Woods era.

Second, we test for the constancy of the maximized value of the log-likelihood function used for deriving the ML-estimators of  $\beta$ . The asymptotic distribution is Gaussian and the confidence bands are at 95 per cent significance level. The test results are reported in figure 5 (see appendix). The constancy of the log-likelihood function is rejected in the beginning of the 70ies, but pops into the band again thereafter. This again indicates a structural break after the end of the Bretton Woods era.

Third, we estimate the  $\beta(t)$  matrix that forms the cointegrating relation recursively for each subsample  $t = \{\{1957 : 1 - 1971 : 2\}, \dots, \{1957 : 1 - 1990 : 4\}\}$  and test for constancy relative to the  $\beta$  coefficient<sup>150</sup> obtained from the Bretton Woods era. The test statistic is chi-squared distributed with 2 degrees of freedom. The test statistic is scaled by its 5 per cent critical value. Figure 6 depicts the results (see appendix). The hypothesis of parameter constancy is clearly rejected.

Finally, one-step prediction errors are considered. For any of the sub-samples a prediction one quarter ahead is made and compared to its actual value. This procedure is applied both to the cointegrating relation and the individual time series.

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<sup>149</sup>Unless otherwise stated the recursive analysis is pursued under the hypothesis  $r=1$  which is obtained from the Bretton Woods era.

<sup>150</sup> $\beta$  is calculated as 3 dimensional matrix, as if the rank hypothesis  $r=3$  applied.

The prediction errors are chi-squared distributed with 3 and 1 degree of freedom, respectively.<sup>151</sup> Figure 7 depicts the prediction errors scaled with their critical values (see appendix). There are many prediction errors of the cointegrating relation (panel a) outside the 95 per cent confidence band indicated by shadows which again hints at a structural break. However, only few prediction errors appear for GDPUS and GDPJP which are partially associated with the two oil crises (panel b).<sup>152</sup> The parameter instability is largely caused by and restricted to the real exchange rate TOT. This again is not surprising, since we expect a much larger volatility in a flexible exchange rate system as compared to the fixed exchange rate regime during the Bretton Woods era. We explore the nature of the structural break now in a separate analysis of the Post-Bretton Woods era.

#### 5.4.3 Post-Bretton Woods Era

We pursue a similar analysis for the Post-Bretton Woods era 1971:2 until 1990:4 as for the Bretton Woods era. Table 4 provides the test results for model mis-specification (see appendix). The error terms are less well behaved than in the Bretton Woods era. There is one outlier representing the first oil crisis in 1974. We allow for the option to eliminate the outlier by using an additional dummy variable in  $D_t$  of (171)<sup>153</sup>. With a lag length of  $k=3$  and 1 additional dummy variable, neither autocorrelation, nor heteroscedasticity, nor non-normality can be found.<sup>154</sup> Additionally, we check the autocorrelation functions of the residuals and do not find serious concern of autocorrelation. If the dummy variable is left out, the assumption of normality of residuals is violated.

Next, the trace test statistic is calculated. The results are reported in table 5 (see appendix). Additional to the results for the trace statistic with dummy variables, we report the results without the dummy variables. Two interpretations are possible.

<sup>151</sup>See Lütkepohl (1991, p. 33ff) for a further elaboration on mean square errors.

<sup>152</sup>Lütkepohl (1991, p.147) points out that a few predictions outside the confidence intervals do not necessarily suffice to reject constancy. If a random event is repeated independently many times, it is quite likely that the experiment ends up a few times in the 5 per cent region of rejection.

<sup>153</sup>The dummy variable is described in appendix 3.

<sup>154</sup>The results in the table are calculated for  $r=2$ . This corresponds to the hypothesis chosen later.

If the oil shock outlier is thought to be a unique event that will not repeat in the future, then the violation of the normality condition may be ignored and the usual testing procedure applies without dummy. If the oil shock or similar events are thought to repeat in the future, then the critical values for the trace test are invalid. The correct critical values can be found by simulating them with the software DisCo of Johansen and Nielsen (1993).<sup>155</sup> We define for this program a step dummy which takes the value 1 in the intervall (0.15;0.16) and zero otherwise.

For the scenario *without* dummy, we choose model 3 and rank  $r=0$  according to the Pantula (1989) principle. Additionally, Bierens (1997) non-parametric lambda-min test is performed for the case without dummies.<sup>156</sup> The results are reported in table 6 (see appendix). According to this statistic the correct rank is  $r=1$ . Again, both tests do not support the theoretical model. The signs of the  $\Pi$ -matrix are checked in the case without dummies:

$$\Pi = \begin{pmatrix} 0.072 & -0.154 & -0.001 \\ -0.015 & 0.043 & 0.006 \\ 0.222 & -0.595 & -0.171 \end{pmatrix}. \quad (173)$$

Comparing with (166), we find that 8 out of 9 signs are correct. In particular, the terms-of-trade effect both for the US- and Japanese economy has the correct sign. Only the assumed goods market disequilibrium adjustment process has one wrong sign.

For the scenario *with* dummy, we choose model 3 and rank  $r=2$ . The Pantula principle (1989) would suggest rather model 4 and rank  $r=1$ , but there is heteroscedasticity and non-normality of residuals in this case. This choice supports the theoretical model. Next, the beta matrix is checked for correctness of signs:

$$\beta' = \begin{pmatrix} 1 & -2.263 & -0.152 \\ -0.325 & 1 & 0.439 \end{pmatrix}. \quad (174)$$

In comparison to (167), we find that 5 out of 6 signs are correct. In particular, the terms-of-trade effect in the income convergence relation (row 1) is correct. However,

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<sup>155</sup>See Beyer (1998) for an application of this program to find critical values for different kinds of dummies.

<sup>156</sup>This test does not require the assumption of normality of residuals.

the sign of the terms of trade in the assumed goods market disequilibrium adjustment process is wrong (element (2,3) of matrix (174)). This corresponds to our two observations of the data that the terms of trade have the expected trend, but there are wild fluctuations around this trend.

## 5.5 Conclusion

This paper provides an indirect test of the mechanics of Krugman's (1991) economic geography model in a growth context using time series methods. In particular, it is tested, whether terms-of-trade effects increased the convergence speed of Japan to the US from 1957 until 1990.

To pursue this test, the Krugman (1991) model is augmented with a stochastic, endogenous growth model. It is shown that this model can be written (after log-linearization) as a vector error correction model. Furthermore, the model is appropriately tested as a reduced rank hypothesis of cointegration against its alternative - a stochastic Ak-model.

This hypothesis is tested for data on Japan and the US from 1957-1990. There is weak evidence in favour of the terms-of-trade effect of the Krugman (1991) model. The terms-of-trade effect cannot be found for the period with fixed exchange rates (Bretton Woods era), when exchange rate interventions may have caused the exchange rate to deviate from its "fundamental" value. This implies that Japanese and US income grow independently from each other following different growth path and the terms-of-trade effect driving the Krugman (1991) model is not valid for the US and Japan during this period. In the period with flexible exchange rates, the terms-of-trade effect has the correct sign, but is only significant if one believes that major shocks such as the oil price shocks of the 70ies are repeating events. Still, the theoretical model requires a better module to reflect strong exchange rate fluctuations.

There may not have been stronger evidence for the terms-of-trade effect, because 1) data were insufficiently disaggregated, 2) improved terms of trade do not feed

through on savings and capital accumulation, if capital markets are perfectly integrated, 3) Japan and the US are too distant from each other and trade volume is too small for agglomeration forces to become measurable 4.) nominal exchange rates and real exchange rates do not develop always in line with fundamentals spoiling thereby the statistical significance of the terms-of-trade effect.



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# Appendix

## Appendix 1: Proof of Proposition 1.

In the following we denote fixed points by bars. Since the stochastic process is shut off,  $K_t = n_t$ . We note from (150), (151), and (154) that a fixed point exists, whenever

$$\bar{\pi}_1 = \bar{\pi}_2 \quad (175)$$

or using the definition of  $\bar{\pi}_j$ :

$$\bar{p} / \bar{P} = 1. \quad (176)$$

We follow Urban (1998b) in defining an implicit function  $k(\bar{p}, \bar{n})$  by using (138) that describes all combinations of  $\bar{n}$  and  $\bar{p}$  such that condition (176) is true:

$$k(\bar{p}, \bar{n}) : \quad \bar{n} = \frac{q - \bar{p}^{\frac{1-\sigma}{\mu}}}{\bar{p}^{1-\sigma} \left[ q \bar{p}^{\frac{1-\sigma}{\mu}} - 1 \right]}. \quad (177)$$

A second condition that has to hold is the goods market equilibrium condition (146):  $h(\bar{p}, \bar{n})$ . Whenever the graphs of  $h(\bar{p}, \bar{n}) = 0$  and  $k(\bar{p}, \bar{n})$  cross in  $\bar{p}$ - $\bar{n}$ -space, there is a fixed point. One solution is easily checked to be  $\ln \bar{p} = \ln \bar{n} = \ln \bar{K} = 0$ . There may also exist other fixed points. However, Urban (1998b, proposition 2) proves that this fixed point is unique, if

$$\tau^{\mu\sigma} \left[ (1 + \mu) \tau^{\sigma-1} + (1 - \mu) \tau^{1-\sigma} \right] > 2. \quad (178)$$

To show that this fixed point yields a stable solution to the difference equation (154), it suffices to show that  $\frac{\partial(\ln \pi_2 - \ln \pi_1 + \ln n)}{\partial \ln n} < 1$  at the steady state or

$$\frac{\partial(\ln \pi_2 - \ln \pi_1)}{\partial \ln n} \Big|_{\substack{p=1 \\ n=1}} < 0 \quad (179)$$

which in turn requires

$$\frac{\partial \ln p}{\partial \ln n} \Big|_{\substack{p=1 \\ n=1}} < \frac{\partial \ln P}{\partial \ln n} \Big|_{\substack{p=1 \\ n=1}}. \quad (180)$$

For this to be true

$$\ln p|_{n=1-\varepsilon} > \ln P|_{n=1-\varepsilon} \quad (181)$$

for  $\varepsilon > 0$ , but  $\varepsilon$  close to zero. In terms of  $h(\bar{p}, \bar{n})$  and  $k(\bar{p}, \bar{n})$  this inequality may be rewritten as follows:

$$p \Big|_{h(p, \bar{n}-\varepsilon)} > p \Big|_{k(p, \bar{n}-\varepsilon)} \quad (182)$$

This inequality is proven to be true by Urban (1998b, proposition 2) under assumption (178) and  $\bar{n} = 1$ . **Q.E.D.**

## Appendix 2: Proof of Proposition 2.<sup>157</sup>

First, we make some preliminary considerations. In particular, consumption for agricultural products by capitalists of country  $j$  is well known to be:

$$C_{jt}^A = (1 - \mu) p_{jt} n_{jt} \quad (183)$$

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<sup>157</sup>The proof follows to some extent Chow (1997).

given Cobb-Douglas functional forms for the sub-utility function. By using the second equation of (145), equations (141), (142), (143), and the definitions of  $p_t$ ,  $K_t$ , and  $z_t$  in (183), we obtain:

$$C_{jt}^A = 1 + p_t K_t z_t. \quad (184)$$

Second, we define for convenience:

$$f(K_t z_t) \equiv \ln \pi_2(K_t z_t) - \ln \pi_1(K_t z_t) + \ln K_t + \ln z_t \quad (185)$$

Now, the Consumer optimization problem can be stated as:

$$\max_{C_{jt}^{MC}} E_0 \sum_{t=0}^{\infty} [\mu \ln C_{jt}^{MC} + (1 - \mu) \ln C_{jt}^A] \quad (186)$$

s.t.:

$$K_{jt+1} = \mu \pi_{jt}(K_t z_t) z_{jt} K_{jt} - C_{jt}^{MC} \quad (187)$$

$$\ln K_t = f(K_t z_t) \quad (188)$$

$$\ln z_{jt} = b \ln z_{jt-1} + \varepsilon_{jt} \quad (189)$$

$$\ln z_t = b \ln z_{t-1} + \varepsilon_t, \quad (190)$$

together with (184) and initial conditions for the capital stocks. Note that (188) replaces the goods market equilibrium condition. The first order conditions can be found to be:

$$\frac{\mu}{C_{jt}^{MC}} = \beta E_t \lambda_{1t+1} \quad (191)$$

$$\lambda_{1t} = \beta \mu \pi_{jt}(K_t z_t) z_{jt} E_t \lambda_{1t+1} \quad (192)$$

$$\lambda_{2t} = \beta \frac{\partial f}{\partial K_t} E_t \lambda_{2t+1} + (1 - \mu) \frac{\frac{\partial p_t}{\partial K_t} K_t + p_t}{1 + p_t K_t z_t} + \beta \mu z_{jt} K_{jt} \frac{\partial \pi_{jt}}{\partial K_t} E_t \lambda_{1t+1}, \quad (193)$$

where  $\lambda_{1t}$  is the Lagrange-multiplier associated with constraint (187),  $\lambda_{2t}$  is the Lagrange-multiplier associated with constraint (188), and  $E_t$  is the expectation operator based on information available in period  $t$ . It must be shown that the first order conditions (191)-(193) are fulfilled for the guess (152) at least around a steady state solution.

Combining (191) and (192), taking logarithm, and solving for  $\ln \lambda_{1t}$  yields:

$$\ln \lambda_{1t} = -\ln C_{jt}^{MC} + 2 \ln \mu + \ln \pi_{jt} + \ln z_{jt}. \quad (194)$$

The logarithm is taken from (192) and equation (194) is inserted:

$$\ln \mu - \ln \beta - \ln C_{jt}^{MC} = \ln E_t \left[ \frac{\mu^2 \pi_{jt+1} z_{jt+1}}{C_{jt+1}^{MC}} \right]. \quad (195)$$

The guess (152) for  $C_{jt}^{MC}$  is forwarded one period and plugged into the right hand side of (195) to yield:

$$\begin{aligned} \ln E_t \lambda_{1t+1} &= \ln E_t \left[ \frac{\mu^2 \pi_{jt+1} z_{jt+1}}{C_{jt+1}^{MC}} \right] = \ln E_t \left[ \frac{\mu}{d_0 K_{jt+1}} \right] \\ &= -\ln d_0 - \ln(1 - d_0) - \ln K_{jt} - \ln \pi_{jt} - \ln z_{jt}, \end{aligned} \quad (196)$$

where the second line is obtained by inserting (187). The guess (152) is inserted into the left hand side of (195) and equalized to (196):

$$\ln \beta = \ln(1 - d_0). \quad (197)$$

Since the parameter  $d_0$  is chosen to be  $d_0 = 1 - \beta$ , the guess (152) fulfills the first order conditions (191) and (192). It remains to be shown that the third first order condition holds also at least around the steady state. Equation (196) is exponentiated and inserted into (193):

$$\lambda_{2t} = \beta \frac{\partial f}{\partial K_t} E_t \lambda_{2t+1} + (1 - \mu) \frac{\frac{\partial p_t}{\partial K_t} K_t + p_t}{1 + p_t K_t z_t} + \frac{\mu}{1 - \beta} \frac{\partial \pi_{jt}}{\partial K_t} \frac{1}{\pi_{jt}} \quad (198)$$

Note that the sub-system (188), (198), and the stochastic process (190) are sufficient to describe the dynamic process of  $K_t$ ,  $\lambda_{2t}$ , and  $z_t$ . Therefore, we make a guess for the Lagrange multiplier as a log-linear function of  $K_t$ , and  $z_t$  only:

$$\lambda_{2t} = \phi_0 + \phi_1 \ln K_t + \phi_2 \ln z_t, \quad (199)$$

where  $\phi_i$ ,  $i = 0, \dots, 2$ , are parameters yet to be determined. Next, the conditions (188) and (198) are log-linearized around the candidate steady state  $\bar{K} = 1$ ,  $\bar{z} = 1$ ,  $\bar{\lambda}_2 = \phi_0$  as found in proposition 1:

$$\delta_0 + \delta_1 \ln E_t [\lambda_{2t+1}] + \lambda_{2t} + \delta_3 \ln K_t + \delta_4 \ln z_t = 0, \quad (200)$$

$$\ln K_{t+1} = \Gamma (\ln K_t + \ln z_t), \quad (201)$$

where  $\delta_i$ ,  $i = 0, \dots, 4$ , are functions of all parameters of the model, and  $\Gamma \equiv \frac{\partial f(\bar{K}, \bar{z})}{\partial K_t}$ .<sup>158</sup> Equation (200) can be transformed into:

$$\delta_0 + \delta_1 \phi_0 + (\delta_1 \phi_1 \Gamma + \gamma_1 + \delta_3) \ln K_t + (\delta_1 \phi_1 \Gamma + \delta_4 + (\delta_1 + 1) \phi_2) \ln z_t = 0, \quad (202)$$

where (190), (199), and (201) have been used. If the parameters  $\phi_i$ ,  $i = 0, \dots, 2$ , are chosen such that

$$\begin{aligned} \phi_0 &= \frac{\delta_0}{\delta_1}, \\ \phi_1 &= \frac{\delta_3}{\delta_1 \Gamma + 1}, \\ \phi_2 &= \frac{\delta_1 \phi_1 \Gamma + \delta_4}{1 + \delta_1}, \end{aligned}$$

then the left hand side of (202) vanishes and the third first order condition (193) is also fulfilled approximately around the steady state  $\bar{K} = 1$ ,  $\bar{z} = 1$ ,  $\bar{\lambda}_2 = \delta_0/\delta_1$  given the guess (152). **Q.E.D.**

### Appendix 3: The Data

The following quarterly data are used from the International Financial Statistics of the IMF from 1957:1 until 1990:4.

PPIUS: Wholesale price index of the US (farm gate prices and producer prices on the first production stage) with base year 1990.

PPIJP: Wholesale price index of Japan (farm gate prices and producer prices on the first production stage) with base year 1990.

POPUS: US population (quarterly linear expolation of annual data).

POPJP: Japanese population (quarterly linear expolation of annual data).

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<sup>158</sup>We do not report these functions to economize on space, since they do not yield further insides.

DOLYEN: Nominal dollar/yen exchange rate (quarterly averages).

From these raw data the following derived data are computed:

GDPUS (Real log per capita GDP of US): The logarithm is taken of GDP with base year 1990 divided by population.

GDPJP (Real log per capita GDP of Japan): GDP in 1990 yen prices is divided by population, converted into dollar terms for reasons of comparability using the 1990 first quarter nominal dollar-yen exchange rate, and the logarithm is taken from the result.<sup>159,160</sup>

TOT: The logarithm is taken of the nominal yen-dollar exchange rate, after it is divided by PPIJP and multiplied by PPIUS.

OIL741: Impulse dummy variable with a single entry at 1974:1.

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<sup>159</sup>We do not convert Japanese GDP into dollar terms using the dollar-yen exchange rate time series for two reasons:

1) Since we test the relation between GDP and terms of trade and the latter are highly correlated with the nominal exchange rate, we would construct a correlation into the data.

2.) The idea of a Laspeyres index is to fix prices at a base year level to extract quantity changes. Base year is 1990 for both GDP Japan and US. Consequently, we also use the exchange rate of 1990 to convert Japanese GDP into dollar terms.

<sup>160</sup>The time series 15899B.RZF (Gross Domestic Product of Japan in 1990 prices) contained in the International Financial Statistics-CD of the IMF shows a large break in 1979:1. The otherwise identical OECD Main Indicators data do not show this break. The OECD data appear more credible, but they do not cover our entire sample period. Consequently, we rechain the IFS data such that the growth rate of the OECD data from 1978:4 until 1979:1 is incorporated and base year 1990 keeps its index number (which is identical for both data sources). We are greatly indebted to Koichi Nakajima for pointing this out to us.

Table 1: Mis-Specification Tests 1957:1-1971:1

Equation	Model 3		Model 4	
	ARCH(2) $\chi^2(2)$	Jarque -Bera	ARCH(2) $\chi^2(2)$	Jarque -Bera
$\Delta GDPJP$	0.875	2.973	0.938	2.637
$\Delta GDPUS$	0.493	4.953	2.090	3.941
$\Delta TOT$	0.009	2.262	0.476	0.972
System				
L.-B. (13)	$\chi^2(105)=111.434$	[0.32]	$\chi^2(105)=115.66$	[0.22]
LM (1)	$\chi^2(9)=7.348$	[0.60]	$\chi^2(9)=12.296$	[0.20]
LM (4)	$\chi^2(9)=6.140$	[0.73]	$\chi^2(9)=3.983$	[0.91]
D.-H.	$\chi^2(6)=9.401$	[0.15]	$\chi^2(6)=8.728$	[0.19]

Notes: sample 1957:1-1971:1; k=2; r=1; 3 centered seasonal dummies; Numbers in square brackets are marginal significance levels; The critical value from a chi-square distribution for both the Jarque-Bera and the ARCH(2) Test is 5.99 at 5 per cent significance level.

L.-B.(13): Ljung-Box Test with 13 lags;

LM(i): Lagrange Multiplier Test for i's order serial correlation;

D.-H.: Doornik-Hansen Test for multivariate normality



Table 2: Trace Test of Co-Integrating Rank 1957:1-1971:1

Model 3			Model 4		
r	$-2\ln Q(H_1(r)   H_1(p))$	$C(r)_{0.90}$	$-2\ln Q(H_1(r)   H_1(p))$	$C(r)_{0.90}$	
0	22.399	26.699	39.517	39.077	
1	6.813	13.308	18.053	22.946	
2	0.000	2.706	4.936	10.558	

Note: sample 1957:1-1971:1; k=2; 3 centered seasonal dummies; The critical values  $C(r)_{0.90}$  at 90 per cent significance level are taken from Johansen (1995), p. 214ff.

Table 3: Lamda-Max/-Min, Stationarity, and Weak Exogeneity Tests 1957:1-1971:1

Test		$-2\ln Q(H_1(r)   H_1(r+1))$	$C(r)_{0.90}$
$\lambda_{\max}$	r=0	15.59	13.39
	r=1	6.81	10.60
	r=2	0.0	2.71
		$\hat{g}_m(r)$	$C(r)_{0.90}$
	r=0	0.0105 (m=4)	(0;0.017)
$\lambda_{\min}^*$	r=1	0.0588 (m=3)	(0;0.034)
	r=2	126.7 (m=3)	(0;0.111)
H $\phi$	GDPJP	$\chi^2(2)=15.36$ [0.00]	
	GDPUS	$\chi^2(2)=15.56$ [0.00]	
	TOT	$\chi^2(2)=2.80$ [0.25]	
B' $\alpha$	GDPJP	$\chi^2(2)=0.12$ [0.73]	
	GDPUS	$\chi^2(2)=0.61$ [0.43]	
	TOT	$\chi^2(2)=8.46$ [0.00]	

Note: sample 1957:1-1971:1; k=2; 3 centered seasonal dummies; model 3; Numbers in square brackets are marginal significance levels; H, B are unit vectors with entry at position i=1,2,3 for variable i=GDPJP, GDPUS, TOT;

The joint test on GDPUS and GDPJP exogenous and TOT stationary is  $\chi^2(4)=3.35$  [0.50].

\* The test statistic is derived in Bierens (1997). The rejection interval is from Bierens (1997), table 2.

The discretization parameter m of the continuous weighting function is optimally chosen according to Bierens (1997), section 4.2 and table 1. The calculation is performed by EASYREG, version 1.20.

Table 4: Mis-Specification Tests 1971:2-1990:4

Equation	Model 3		Model 4	
	ARCH(3) $\chi^2(3)$	Jarque -Bera	ARCH(3) $\chi^2(3)$	Jarque -Bera
$\Delta$ GDPJP	0782	4.863	0.570	4.141
$\Delta$ GDPUS	1.301	2.464	2.258	2.189
$\Delta$ TOT	0.524	0.236	0.434	0.273
System				
L.-B. (19)	$\chi^2(147)=159.61$	[0.23]	$\chi^2(147)=166.47$	[0.13]
LM (1)	$\chi^2(9)=10.411$	[0.32]	$\chi^2(9)=10.36$	[0.32]
LM (4)	$\chi^2(9)=3.364$	[0.95]	$\chi^2(9)=3.245$	[0.95]
D.-H.	$\chi^2(6)=6.036$	[0.42]	$\chi^2(6)=4.979$	[0.55]

Notes: sample 1971:2-1990:4; k=3; r=2; 3 centered seasonal dummies; impulse dummy OIL741;  
Numbers in square brackets are marginal significance levels; The critical values from a chi-square distribution for the Jarque-Bera Test is 5.99 and for the ARCH(3) Test is 7.81 at 5 per cent significance level.

L.-B.(19): Ljung-Box Test with 19 lags;

LM(i): Lagrange Multiplier Test for i's order serial correlation;

D.-H.: Doornik-Hansen Test for multivariate normality;

Table 5: Trace Test of Co-Integrating Rank 1971:2-1990:4

r	Model 3				Model 4			
	L.R. w/o dum.	C(r) <sub>0.90</sub> w/o du.	L.R. w. dum.	C(r) <sub>0.90</sub> w. du.	L.R. w/o dum.	C(r) <sub>0.90</sub> w. dum.	L.R. w. dum.	C(r) <sub>0.90</sub> w. du.
0	22.15	26.699	22.34	21.255	31.04	39.077	27.63	33.155
1	9.05	13.308	8.80	7.783	13.29	22.946	9.53	17.485
2	0.18	2.706	0.02		4.40	10.558	0.74	

Note: sample 1971:2-1990:4; k=3; 3 centered seasonal dummies; The critical values  $C(r)_{0.90}$  w/o dum. at 90 per cent significance level are taken from Johansen (1995), p. 214ff.; The critical values  $C(r)_{0.90}$  w. du. are obtained from 50 000 repetitions of the simulation program DisCo of Johansen and Nielsen (1994) by using an impulse dummy which is 1 for the sample period interval (0.15;0.16) and zero otherwise, where the sample length is normalized to 1; There are no critical values obtainable for r=2.  
L.R. w. dum.: Likelihood Ratio Test Statistic for the sample with oil shock dummy;  
L.R. w/o dum.: Likelihood Ratio Test Statistic for the sample without oil shock dummy;

Tabel 6: Bierens' (1997) Non-parametric  $\lambda_{\min}$ -Test, 1971:2-1990:4

	$H_0$ :	$\hat{g}_m(r)$	$C(r)_{0.90}$
	$r=0$	0.0105 (m=4)	(0;0.017)
$\lambda_{\min}$	$r=1$	0.0588 (m=3)	(0;0.034)
	$r=2$	126.7 (m=3)	(0;0.111)

The test statistic is derived in Bierens (1997). The rejection interval is from Bierens (1997), table 2. The discretization parameter  $m$  of the continuous weighting function is optimally chosen according to Bierens (1997), section 4.2 and table 1. The calculation is performed by EASYREG, version 1.20.

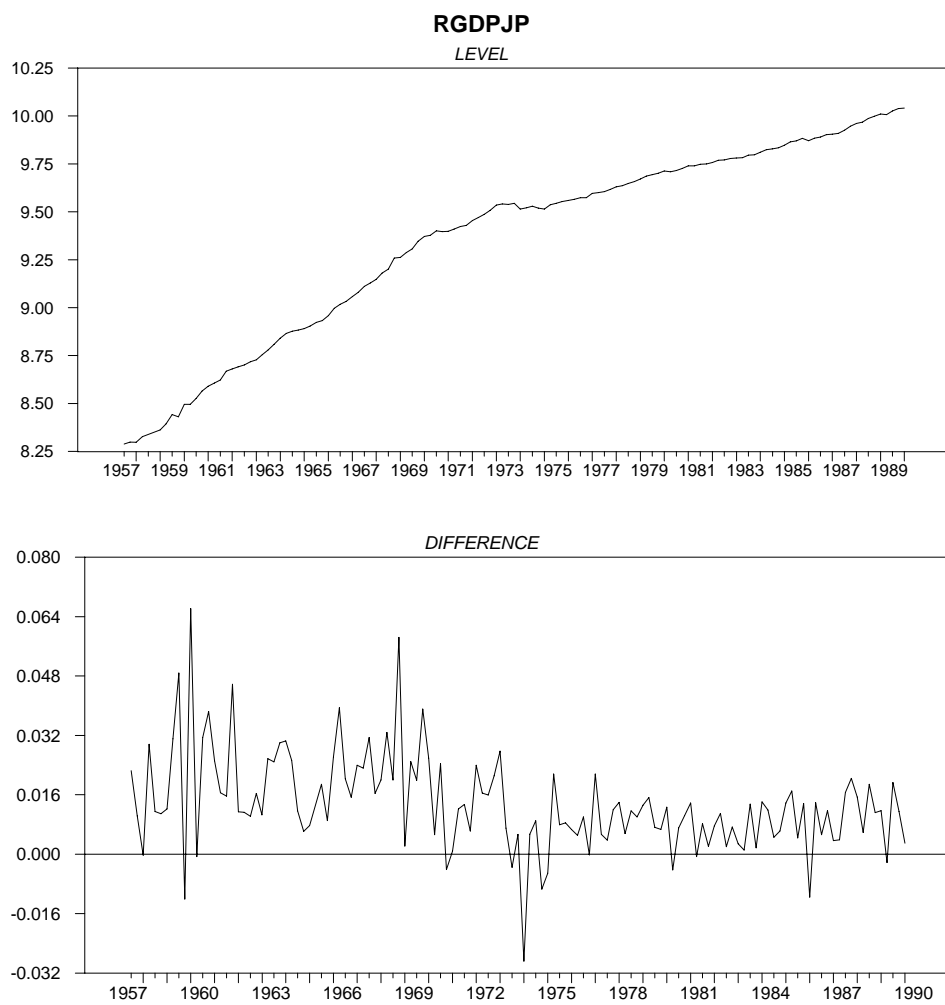


Figure 2a: Japanese Real GDP

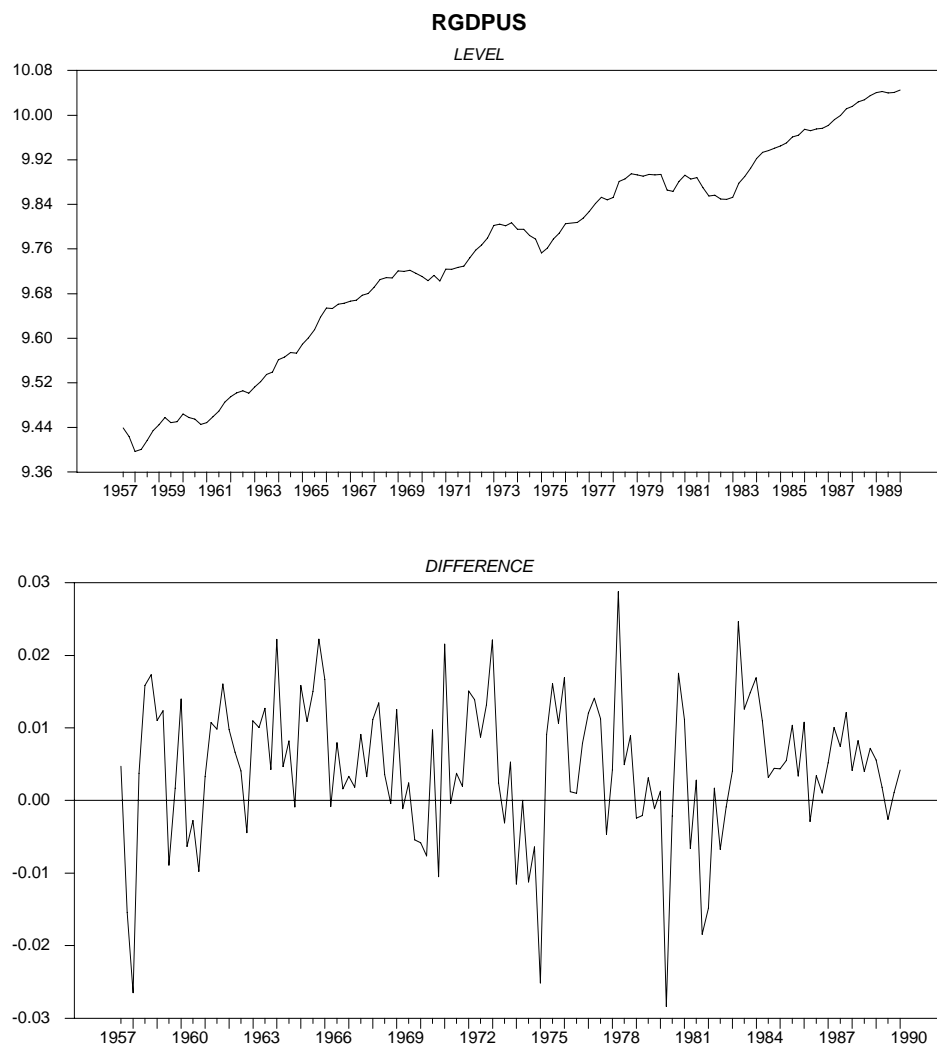


Figure 2b: US Real GDP

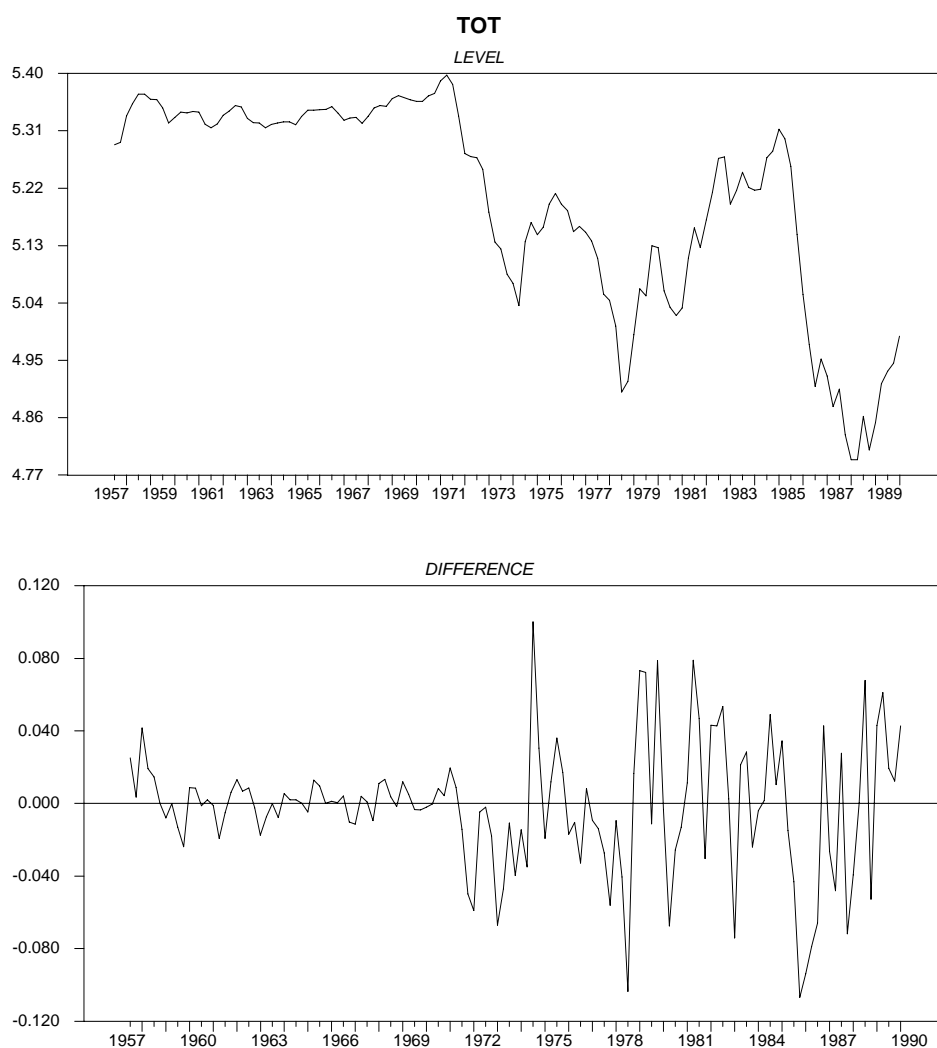


Figure 2c: Terms of Trade



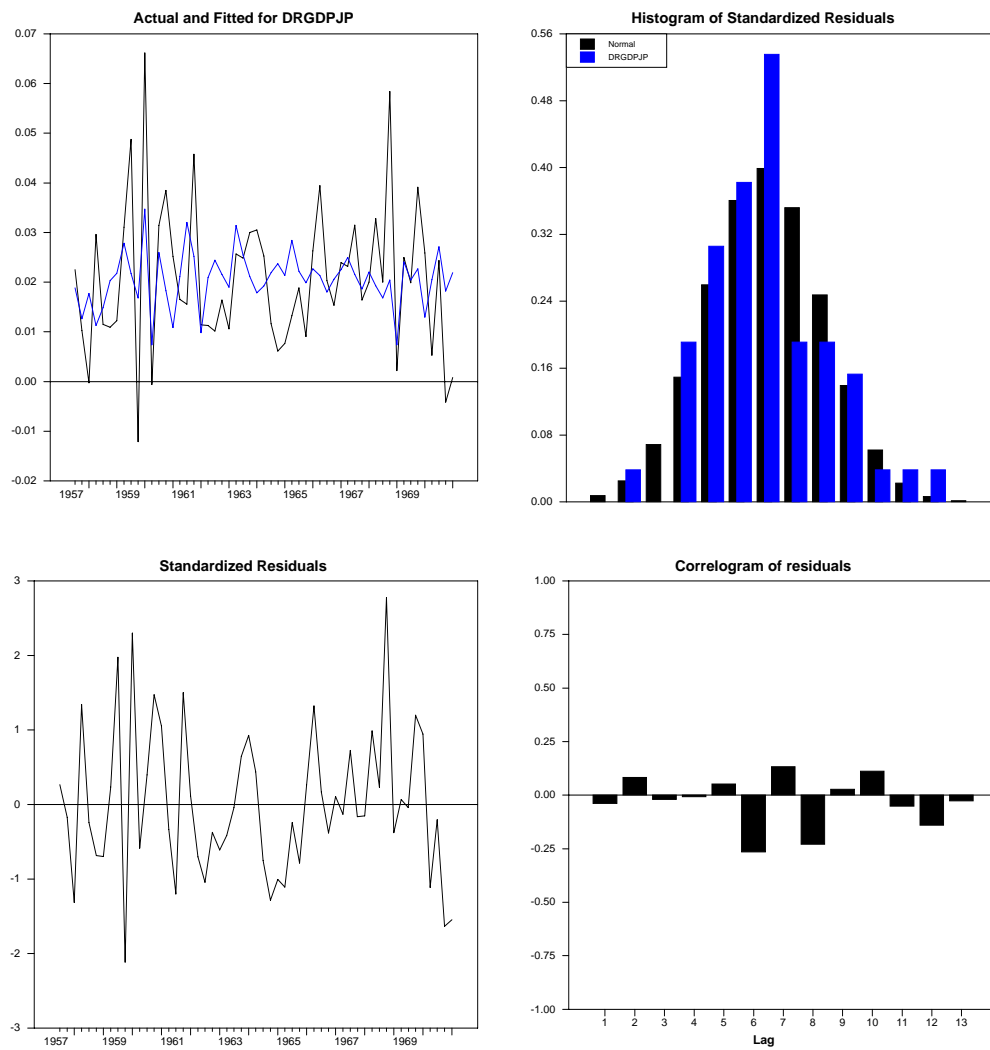


Figure 3a: Autocorrelogram for Residuals of Japanese GDP

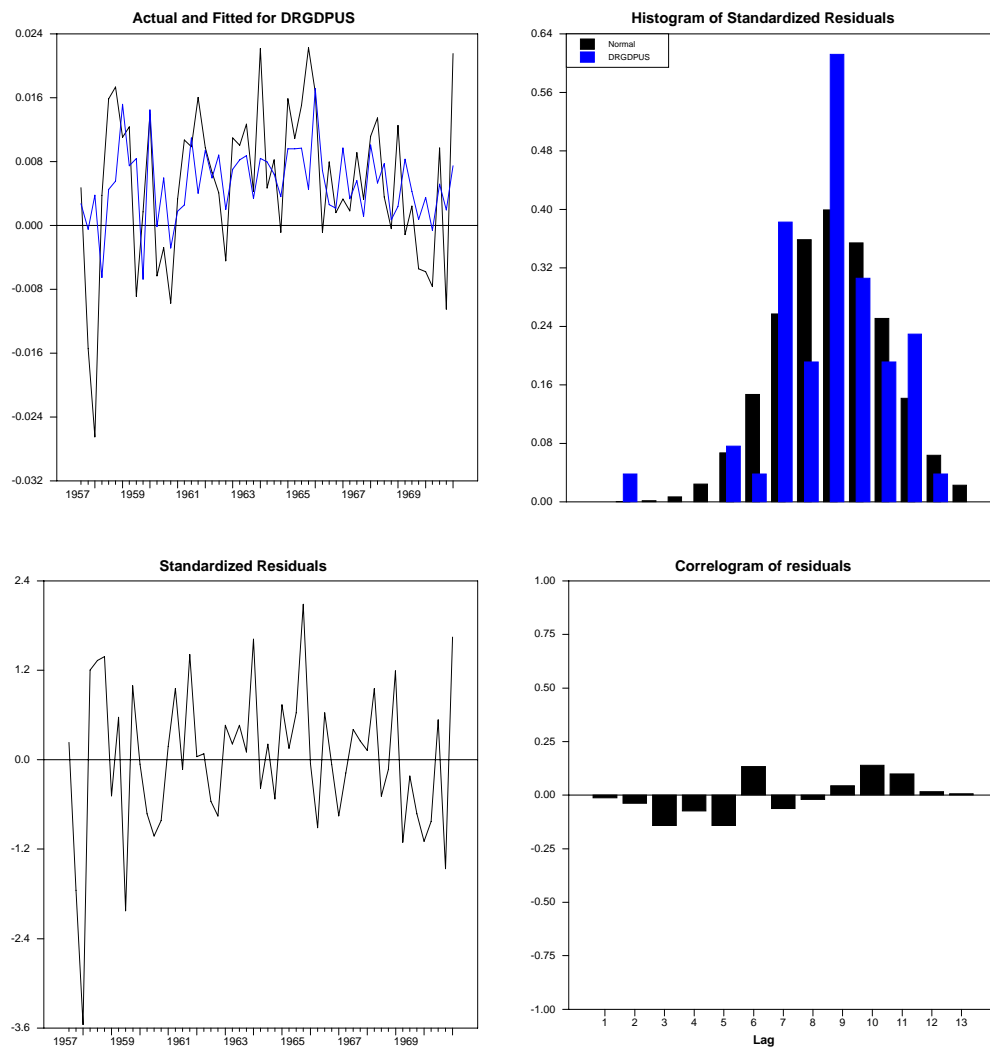


Figure 3b: Autocorrelogram of the Residuals of US GDP

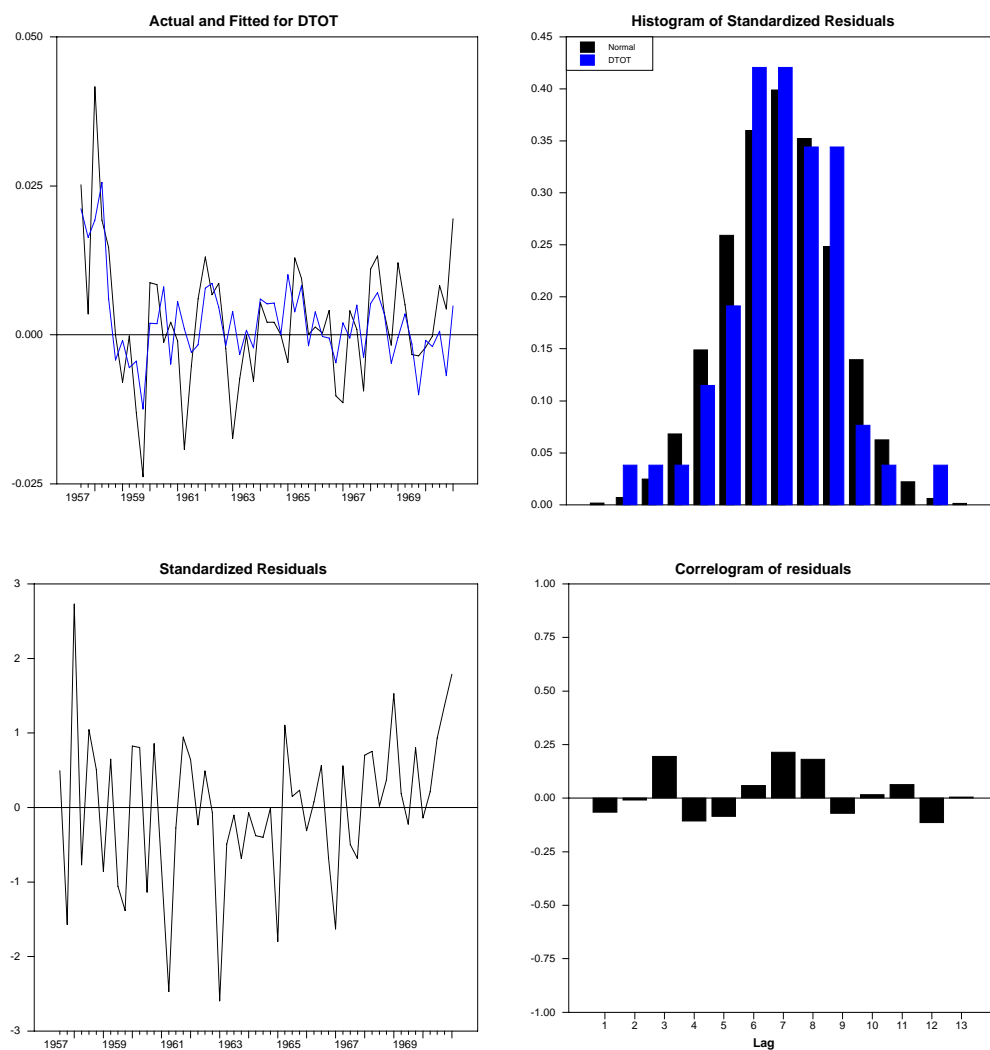


Figure 3c: Autocorrelogram of Residuals of Terms of Trade

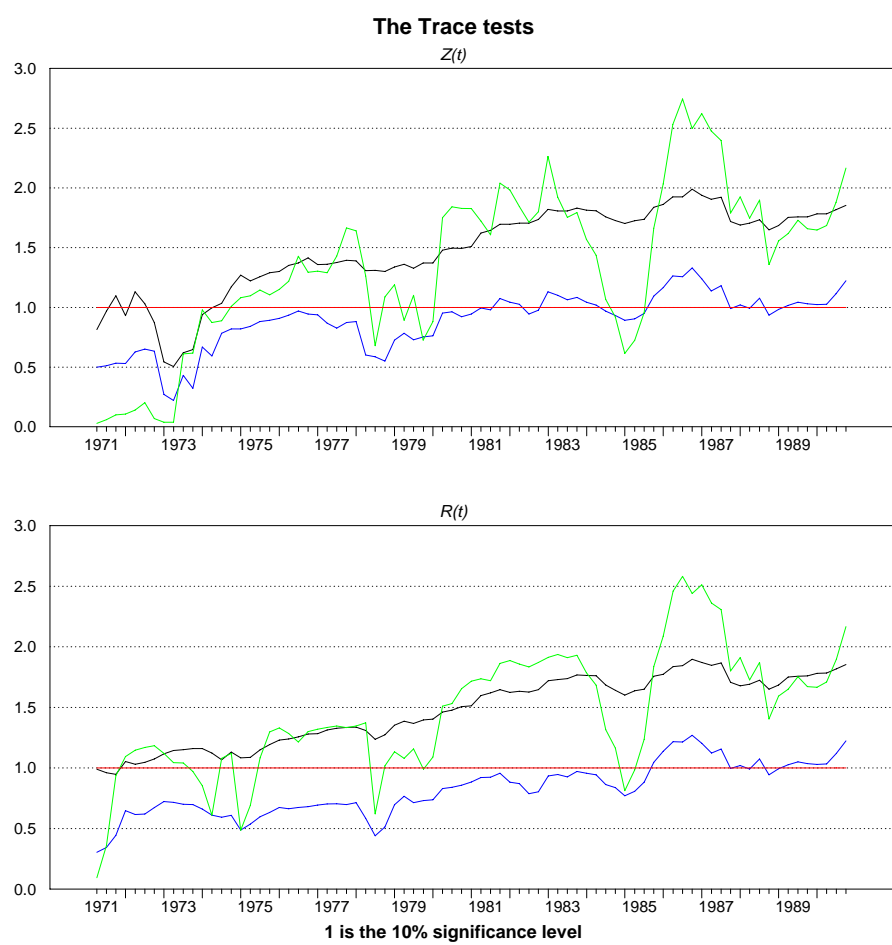


Figure 4: Recursive Trace Test

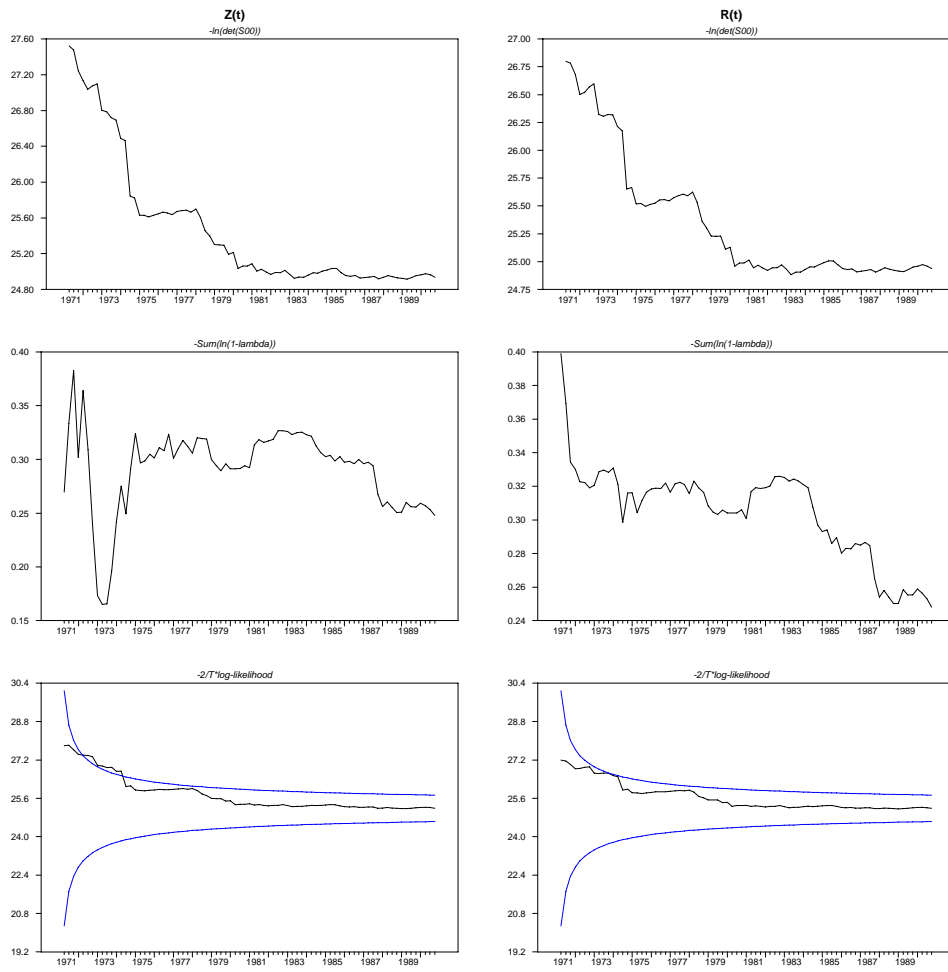


Figure 5: Stability of Maximized Log-Likelihood Function

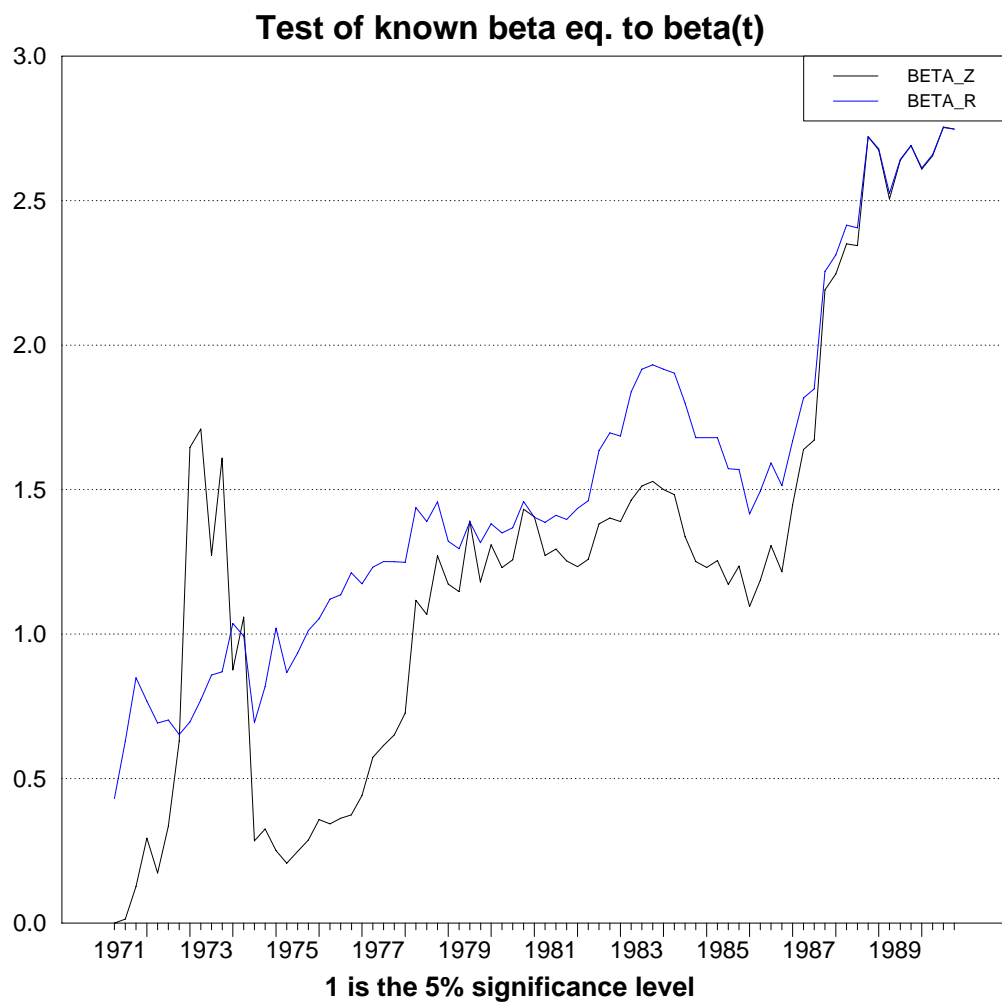


Figure 6: Stability of Beta-Coefficients

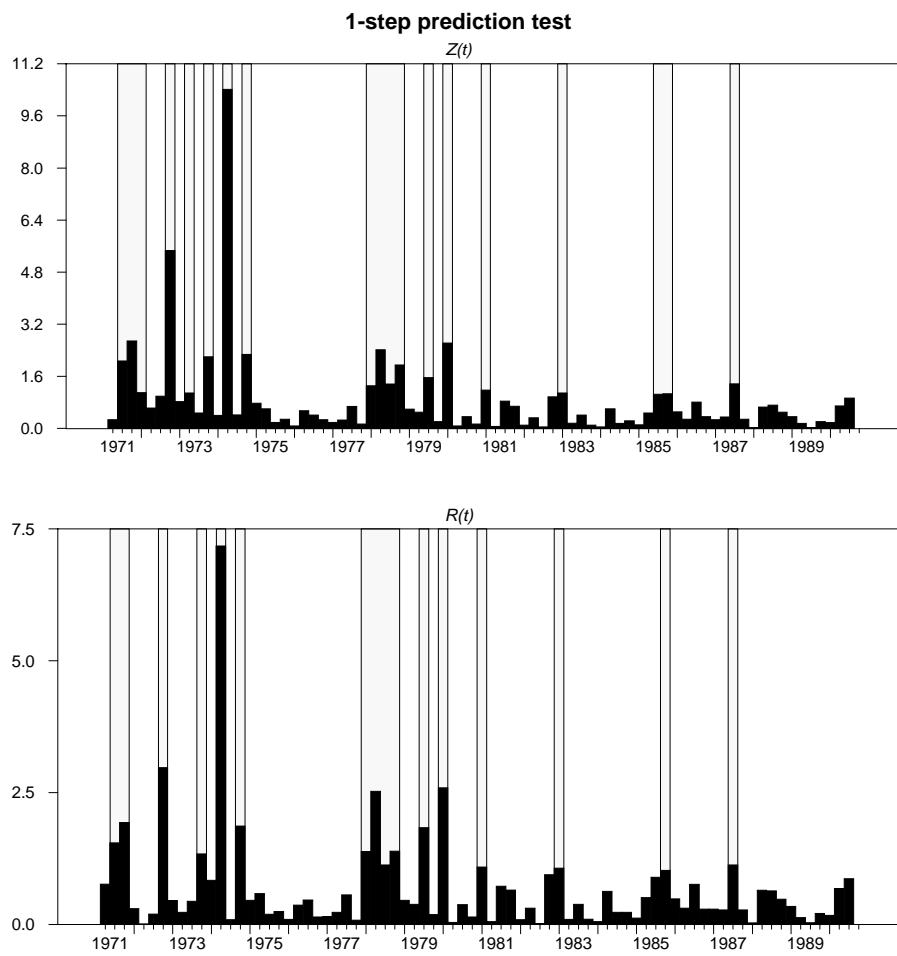


Figure 7a: 1-Step Prediction Error of System

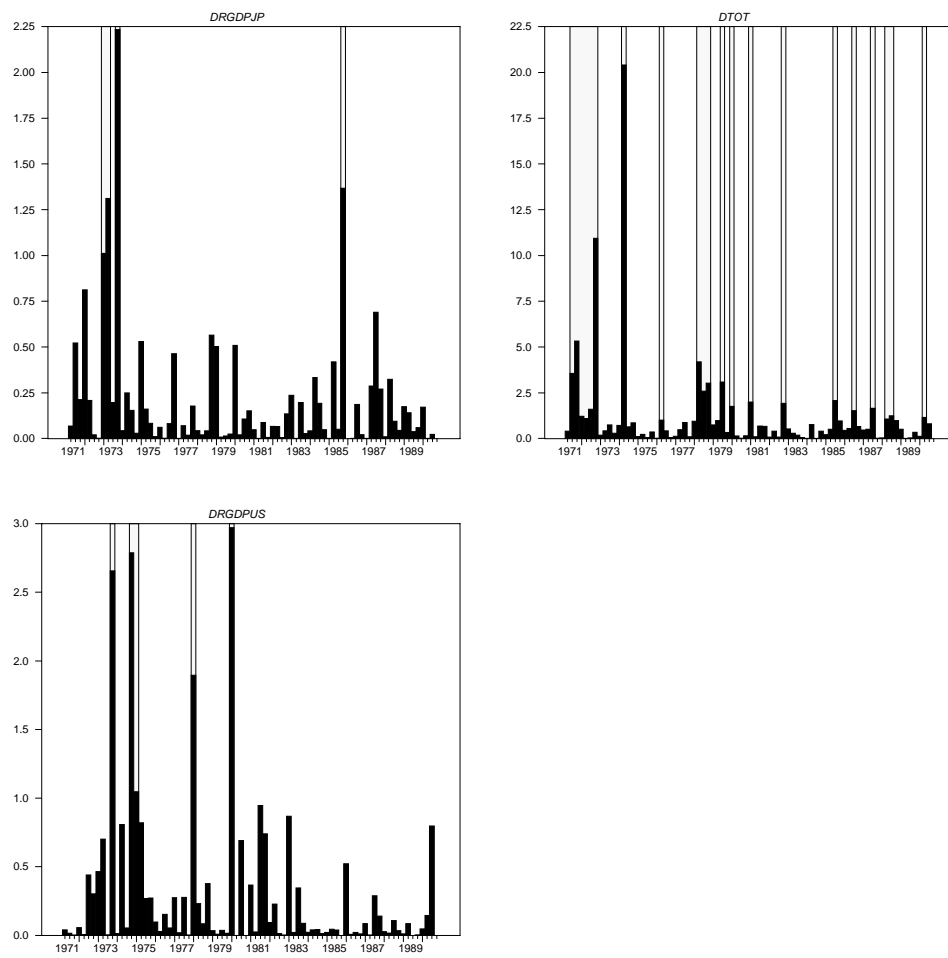


Figure 7b: 1-Step prediction Error of Individual Time Series



## Denish Summary/ Resume på Dansk

Denne Ph.D. afhandling søger af sammenknytte to nationaløkonomiske problemkredse: Økonomisk vækst og økonomisk geografi. Afhandlingens centrale spørgsmål er: Vil økonomisk vækst føre til industriel koncentration eller industriel spredning? Hvordan påvirker en ændring i industriens lokalisering regional og national vækst? Hvilken indflydelse har den stadig tættere integration på økonomisk vækst og industrikoncentration?

Afhandlingen består af fem kapitler. Kapitlerne er nært forbundne, men kan læses uafhængigt af hinanden.

Kapitel 1 er et introduktionskapitel. Det indeholder en oversigt over litteraturen på området og et resume af de i denne afhandling fremlagte forskningsbidrag.

Udgangspunktet for min forskning har været Krugman's banebrydende artikel "Economic Geography and Increasing Returns" fra 1991. Heri udvikler han en model, der kan forklare, under hvilke omstændigheder, industrier tenderer at koncentrere sig i en land eller en region. Artiklen mangler imidlertid en fulstændig analytisk løsning af modellen, og intuitionen bag dens centrale sammenhænge er uklar.

Kapitel 2 tilstræber at råde bod på disse svagheder. Kapitellet indeholder dels en rigoristisk analytisk løsning, dels en simpel grafisk illustration af Krugman's model. Det påpeges, at der er en nær analogi mellem Krugman's model og neoklassisk udenrigshandelsteori baseret på antagelsen om heterogene agenter.

I kapitel 3 vises, at Krugman's model har en unik kortsigtet ligevægt, men multiple langsigtede ligevægte. Afhængig af industriens initiale fordeling vil en relativt mindre industrialiseret økonomi enten af-industrialiseres eller konvergere mod samme udviklingstrin som den mere industrialiserede økonomi. Den mulige eksistens af en "fattigdomsfælde", hvoraf en økonomi ikke kan udvikle sig ved egen kraft, giver en teoretisk begrundelse for, at aftaler om fri bevægelse for varer og kapital (eksempelvis inden for EU) kædes sammen med aftaler om strukturstøtte til de mindre industrialiserede regioner.

I kapitel 4 integreres økonomisk geografi og neoklassisk vækstteori i en model, der simultant forklarer økonomisk vækst og industriens fordeling. Det påvises, at der er to regimer: Et neoklassisk "catching-up" regime, hvor økonomierne gradvis konvergerer, og et fattigdomsregime", hvor forskellen i økonomisk udvikling udvides. Det påvises, at afvikling af handelsbarrierer *kan* eliminere fattigdomsfælden, således at mere tilbagestående lande med tiden vil tilnærme sig de mere udviklede økonomier.

Kapitel 5 er en empirisk test af den model, der udvikles i kapitel 4, mod data for USA og Japan. Testen viser, at modellen ikke kan afvises for så vidt angår perioden efter Bretton Woods fast-kurs systemets sammenbrud i 1972. Under Bretton Woods systemet synes de to økonomer at have udviklet sig uafhængigt af hinanden, hvorfor modellen må afvises for denne periode.