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**Working Paper**

## Control Dilution and Distribution of Ownership

Working paper, No. 16-2000

**Provided in Cooperation with:**

Department of Economics, Copenhagen Business School (CBS)

*Suggested Citation:* Bennedsen, Morten; Fosgerau, Mogens; Wolfenzon, Daniel (2000) : Control Dilution and Distribution of Ownership, Working paper, No. 16-2000, Copenhagen Business School (CBS), Department of Economics, Frederiksberg, <https://hdl.handle.net/10398/7614>

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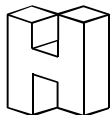
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# **Institut for Nationaløkonomi**

Handelshøjskolen i København

**Working paper 16-2000**

## **CONTROL DILUTION AND DISTRIBUTION OF OWNERSHIP**

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**Mogens Fosgerau**

**Daniel Wolfenzon**

# Control Dilution and Distribution of Ownership.\*

by

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December 2000.

## Abstract:

We investigate the patterns of ultimate *distribution* of ownership in a sample of small closely held corporations. Motivated by recent corporate governance theories we define control dilution as the absence of a single dominating shareholder. Most firms have one or two ultimate owners implying that control is very concentrated. However, we find strong evidence for control dilution in firms with multiple owners and we proceed to analyze three potential explanations: a general strategic motive, a coalition formation argument and the presence of family ownership. We find the strongest evidence for a general strategic motive and we clearly reject that the observed control dilution is due to family ownership. Finally, we show that the probability of control dilution increases with manager ownership and number of owners, but decreases with the age of the firm and with the presence of family ownership.

*Keywords:* Ownership Distribution, Control Allocation, Closely Held Corporations.

*JEL:* G32, G34.

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\*We thank Kasper Nielsen for excellent research assistance.

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# 1 Introduction

Ownership concentration has achieved a lot of attention in recent years both in the theoretical and in the empirical corporate governance literature. In the theory literature, the acceptance of the fact that most firms in most countries around the world have concentrated ownership, has implied a change in focus from the classical conflicts between management and shareholders (Berle and Means 1932, Jensen and Meckling 1976) to conflicts between controlling group of owners and non-controlling group of owners (Shleifer and Vishny 1997).

The empirical literature has taken at least two strands. First, it has accumulated a body of evidence about actual ownership concentration around the world (see e.g. Prose (1992), Berglof and Perotti (1994), Barca (1995), Franks and Mayer (1994), European Corporate Governance Network (1997) and La Porta et.al. (1999) a.o.). Second, starting with Demsetz and Lehn (1985), it has tried to answer two particular questions: What determines ownership concentration in firms? and does ownership concentration have an impact on firms' performance? (see a.o. Morck et.al. (1988), Himmelberg et.al. (1999)).

The present paper is motivated by the observation that where the theoretical literature is focused on the allocation of control between different groups of shareholders, the empirical literature has been almost exclusively focused on the concentration of ownership. There is not a one to one correspondence between control allocation and ownership concentration, rather it is the case that the amount of control that a shareholder with a particular ownership stake achieves over a corporation is determined by the *distribution* of ownership, not only the concentration of ownership. Having an ownership stake of 25 pct. of a big US corporation where the rest of the shares are widely dispersed provides a shareholder with basically ultimate control (La Porta, et.al. (1999)) of all important matters in the corporation. However, having 25 pct. of a closely held corporation, where the ownership is distributed to four shareholders with 55 pct., 25 pct, 15 pct. and 15 pct. respectively, provides a shareholder with almost zero control.

The contribution we add in the following is to investigate empirically patterns of control allocation through analyzing the *distribution* of ownership in small closely held corporations. Corporate governance theories have provided arguments for control concentration in environments with weak investor protection such as protection of investors in closely held corporations. In general, control concentration substitutes legal protection (see survey by La Porta et.al. 2000). However, there might also be reasons for observing control dilution in closely held corporations. In particular, Bennedsen and Wolfenzon (2000) argue that it can be beneficial to allocate control among several large owners, because this forces these controlling owners to cooperate which may induce less expropriation than if one big majority shareholder has the power to unilaterally control the whole corporation. Given these countervailing arguments, we expect to observe both concentrated and dissipated control in our dataset. However, we analyze some systematic patterns in control distribution: In general, the favorite way to keep control concentrated is to keep the firm as a personally owned corporation. On the other hand, there is a strong tendency to diluting control in firms with multiple owners.

To analyze the link between the distribution of ownership and the formation of control requires at least two kinds of information: First, it requires complete ownership information about all owners of a firm. Second, to the extent that some of these owners are other corporations, it requires that we can find the ultimate owners of a firm. These two requirements imply that we cannot study large public traded firms. Our approach is to construct a dataset of 17.705 small (Danish) closely held corporations for which we are able to find the complete ultimate ownership distribution. Closely held corporations are generally characterized by having few and large owners and the absence of a public market for their shares. Hence, in the traditional literature on ownership concentration, they have extremely concentrated ownership. However, even though ownership is extremely concentrated, there are large potentials for conflicts between different groups of owners, as the corporate law literature has frequently documented (see e.g. O’Neals (1987) and

Easterbrook and Fischel (1991)).

Control dilution is a complex phenomenon and we will define it as simply as possible for the main part of the present paper: control dilution is present whenever control is allocated to more than one owner. This implies that control is diluted whenever no single owner has enough shares to control the corporation unilaterally. By this definition, the degree of control dilution becomes a function of the number of owners and the distribution of ownership stakes. In particular, a founder of a firm can choose to keep the firm personally owned or invite one business partner to join the firm in order to avoid control dilution, since this, by our definition, cannot arise in firms with one or two owners.<sup>1</sup> Two thirds of the firms in our sample have a single owner, hence ownership structure is generally chosen to avoid control dilution.

The main insight of the paper is to provide evidence for the fact, that having an ownership structure with multiple owners is highly correlated with the presence of control dilution. We compare the frequency of control dilution in such firms with a random benchmark model, where we simply assume that the ownership distribution for a given number of owners is uniformly distributed on the unit simplex. Our result is that control is significantly more diluted in our sample than in the benchmark model.

After documenting the presence of control dilution, we proceed to characterize these firms, in order to support the hypothesis that founders frequently choose an ownership structure with three or more owners in order to dilute control. Motivated by the theoretical explanations for control dilution we first try to find evidence for strategic control dilution. By strategic control dilution we think of a setting where control has been diluted to avoid a situation where a single shareholder exploits his power in his own interest and against the interest of the remaining shareholders. We find that the tendency to diluting control is higher in manager owner firms.

Second, we analyze a more explicit model of strategic control dilution, namely the

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<sup>1</sup>Control dilution may arise in firms with two owners if ownership is shared equally and neither of the two owners are able to implement their preferred decision without the acceptance from the other owner.

minimum cash flow theory put forward by Bennesen and Wolfenzon (2000). Third, we analyze wheter control dilution is an artifact of family ownership, i.e. that it just reflects that control is allocated to a group of family members in stead of a single large shareholder. We show that this in not the case, since control tends to be more diluted in firms where there is no family ownership.

The final part of the paper replaces the partial analysis with a simultaneous investigation of control dilution. We show that the probability of control dilution increases with manager ownership and number of owners, but decreases with the age of the firm and with the presence of family ownership.

The next section describes our data set. Section 3 provides evidence for control dilution and Section 4 analyzes the three explanations discussed above. In Section 5 we bring all the partial explanations together and develop a logit test for the presence of control dilution. Furthermore, to check the robustness of our results we substitute our simple control dilution definition with a more generalized measure. In section 6 we conclude.

## 2 Data

In our analysis we use a data set of Danish closely held corporations from The Ministry of Trade and Industry in Denmark.<sup>2</sup> The complete dataset covers all registered firms in Denmark, i.e. approximately. 140.000 firms, however many of these firms are not active or are subsidiaries of other companies. In Table 2.1 we describe the criteria we have used for selecting our firms. First, we have a grand dataset (KOB) including all active firms which are incorporated or partnerships, but are neither subsidiaries nor public traded. From this data set we construct our ULTIMATE data set by further requiring that we can construct a complete set of ultimate owners for the firms.

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<sup>2</sup>The data is collected from the “Erhvervs- og selskabstyrelsen” in the Danish Ministry of Trade and Industry by “Købmandstandens Oplysningsforbund” (KOB).

Dataset	Criteria
<i>KOB</i> 38025 Firms	<ul style="list-style-type: none"> <li>▷ Active incorporated firms or partnerships with at least one available account.</li> <li>▷ Subsidiaries excluded.</li> <li>▷ Closely Held Corporations.</li> </ul>
<i>ULTIMATE</i> 17.573 Firms	<ul style="list-style-type: none"> <li>▷ Companies in KOB.</li> <li>▷ Perfect ownership information in the 1<sup>st</sup> level.</li> <li>▷ ULTIMATE owners can be found. List of owners end and the sum of shares held by the ultimate owners exceed 95 pct.</li> </ul>

The Danish corporate law requires owners of closely held corporation to report if the ownership stake exceeds 5 pct. of either the voting rights or the income rights of a company. In addition, if there is a change in ownership, owners shall report if the change exceeds or falls short of any 5 pct. interval or limits of 1/3 and 2/3 of the voting or income rights. For partnerships the requirement is that names of all partners are reported. Even though the law is fairly demanding in information requirement, (due to exception rules and missing data) complete ownership information only exists for a small fraction of our firms. Hence, the first criteria we impose is that we have complete information about the distribution of ownership. Table 2.2 describes in the first row, the actual ownership information in the KOB data set. We define complete ownership information as information about more than 90 pct. of the shares.<sup>3</sup>

We are interested in who the ultimate owners of the corporations are. The strength of our data set is that we have the whole universe of corporations in Denmark in our KOB data set. Every time we meet a corporate owner who is not a foreign company, this owner is incorporated in Denmark and therefore included in the dataset. Hence, we can go back and look at who owns this corporation and so forth. Naturally, it may be that there is not complete ownership information about upper level firms and in such cases we exclude the firm from our ULTIMATE set. The second row of Table 2.2 shows the ultimate personal ownership information we have for all the 18.725 firms for which we had more than 90 pct. ownership information in the first level. Our ULTIMATE data set

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<sup>3</sup>Since firms can have up to 10 pct. of own shares, we do not require 100 pct. There are few firms in the interval between 90 and 100 pct. For these firms we normalize share holding so that the total sum is 100.



Table 1: Distribution of ownership information in KOB and distribution of ultimate ownership information, conditioned on at least 90 pct. ownership information at first level.

Interval	0	(0,10]	(10,20]	(20,30]	(30,40]	(40,50]	(50,60]	(60,70]
KOB	698	12387	3286	759	403	391	529	263
ULTIMATE	0	76	47	81	79	310	70	66
Own. Inf. 1 <sup>st</sup> level.								

Interval	(70,80]	(80,90]	(90,95]	(95,100]	>100	SUM
KOB	304	256	66	18659	24	38025
ULTIMATE	66	19	6	<b>17705</b>	0	18525
Own. Inf. 1 <sup>st</sup> level.						

which is used in the following analysis consists of the 17.725 firms for which the ultimate ownership information is between 95 pct and 100 pct.<sup>4</sup>

Ultimative owners can either be personal or non-personal owners. Non-personal owners are typically pension funds, foreign owners or holding companies. The difference is illustrated in Figure 1. Figure 1) provides two examples of firms, illustrating the difference. The traveling company “Nyhavn Rejser A/S” has three corporate owners. Since these owners are in our dataset, we find the owners of these three firms and it is shown that they are all personal owners. The ultimative owners of “Nyhavn Rejser” are thus all persons and the distribution of ownership is Egon K. Nielsen (30 pct), Lillian Fuglsang (25 pct), Per R. Fuglsang (20 pct.), Henrik V. Fuglsang (15 pct.) and Lars K. Mathiasen (10 pct.). Our second firm “Furesøens Mediecenter A/S” is a media company with one personal owner and two corporate owners. One of the corporate owners, “Politikens Lokalaviser A/S” is owned by a major newspaper “Politiken” who is again owned by two foundations. Thus the ultimative ownership distribution of “Furesøens Mediecenter

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<sup>4</sup>We have cleaned for ownership of own shares now, but we only have ownership in single digit percentages. This implies that ownership information sometimes sums to e.g. 97, 98 or 99 pct. depending on the number and distribution of owners. Again we normalize the ownership stakes such that it sums to 100 pct. for all firms in our sample.

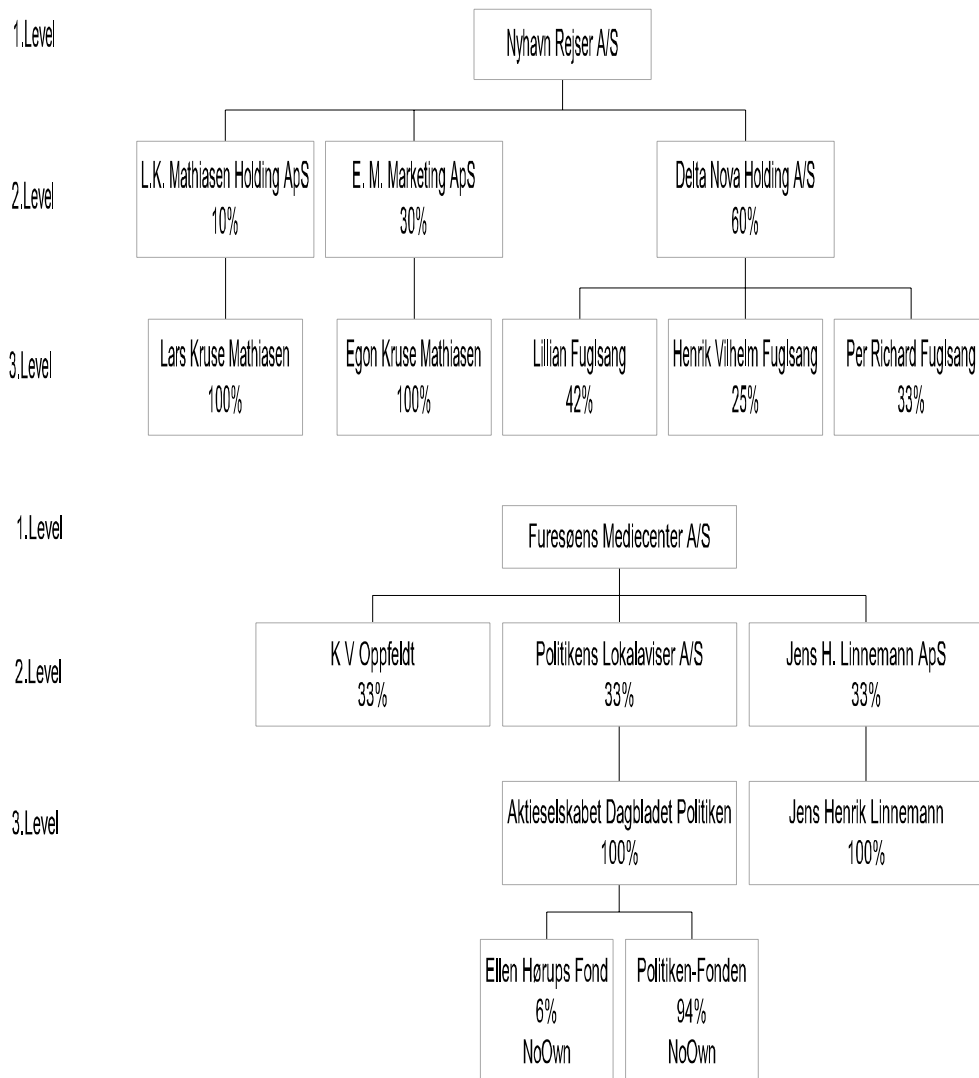


Figure 1: Ultimate ownership. “Nyhavn Rejser” has three corporate owners but only personal ultimate owners. “Furesøens Mediecenter A/S” has both personal and corporate owners and the set of ultimate owners includes two foundations.

A/S” is K.V.Oppfeldt (33 pct.), Jens H. Linnemann (100 pct.), Politiken-Fonden (31 pct.) and Ellen Hørups Fond (2 pct.). We observe that two of these ultimate owners are foundations which do not have any personal owners themselves.

It is worth remarking, that in general most of our ultimate owners are persons. Non-personal owners are present in only 998 (5.6 pct) of the firms in the ultimate dataset. In the following we focus explicitly on the 1072 firms in the ULTIMATE dataset which have three or more owners. For 773 of these (i.e. 85 pct.) the ultimate owners are all persons.

Table 2 and table 3 provide descriptive statistic for the ULTIMATE dataset and compare this with the descriptive statistic for the whole KOB database. The first table contains all firms. We do not have sales for all firms, thus in addition we have stated equity and assets. From the tables we notice that most of our firms are small, but that there is no particular selection bias with respect to sales. However, it seems that we have selected smaller firms with respect to both equity and assets. Since for most of the paper we focus on firms with three or more owners, we have stated the descriptive statistic for these firms separately. We notice that these firms are in general slightly larger than the whole dataset.

Figure 2 shows the distribution of firms in KOB and ULTIMATE on the year of establishment. In general our firms are young, most of them are established after 1970, but there is no particular selection bias with respect to age in our sample.

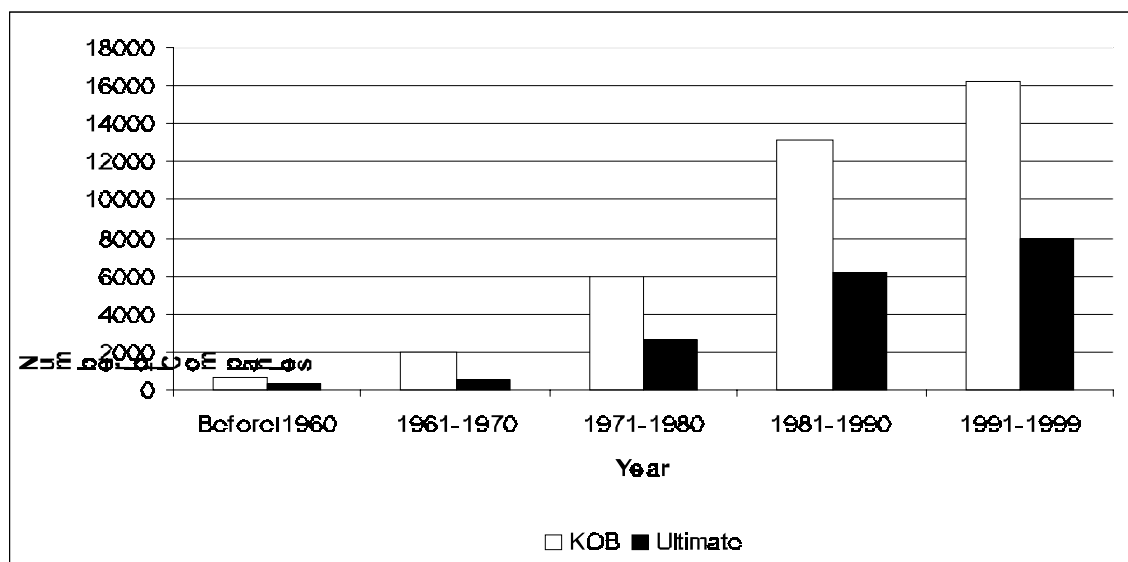
Table 2: Descriptive statistic for KOB and ULTIMATE. Number of firms in parentheses.

	KOB	ULTIMATE	KOB/ULTIMATE
Mean of sales	53481 (11456)	25468(5105)	75998 (6351)
Std. of Mean sales	5855	6569	9137
Median of sales	1665	1474	1835
Mean of Equity	9154 (38025)	5705 (17705)	12158 (20320)
Std. of Mean Equity	1023	1607	1305
Median of Equity	851	654	1090
Mean of assets	36976 (38025)	35280 (17705)	38454 (20320)
Std. of Mean assets	12556	25706	7102
Median of assets	2897	2328	3581

Table 3: Descriptive statistic for KOB and ULTIMATE, firms with three of more owners. Number of firms in parentheses.

	KOB	ULTIMATE	KOB/ULTIMATE
Mean of sales	136182(1955)	68520 (285)	147756 (1670)
Std. of Mean sales	18396	24804	21109
Median of sales	4490	4364	4492
Mean of Equity	19452 (5741)	10138 (998)	21411 (4743)
Std. of Mean Equity	2124	5263	2199
Median of Equity	1733	908	2003
Mean of assets	60373 (5741)	23199 (998)	68195 (4743)
Std. of Mean assets	9734	7037	11686
Median of assets	6338	4328	6845

Figure 2: Distribution of firms according to year of establishment.



### 3 Evidence for Control Dilution in firms with multiple owners

In this section, we provide evidence for control being diluted in firms with more than two owners. In our analysis we focus on possible control structures. To control the firm we assume that an owner or a group of owners must have at least 50 pct. of the shares. For example, in a three owner corporation there are therefore two possible control structures: either there is a single dominating owner with more than 50 pct. of the shares, or it requires two shareholders to control the firm. We denote the former the *one-controller case* and the latter the *multiple controller case*. In general control is diluted in firms with multiple controllers, as any owner must have the support from at least one additional owner to achieve ultimate control of the corporation. Hence, our definition of control dilution is as simple as possible: we define control dilution as the absence of a single dominating owner. In section 5, we provide a more generalized measure of control dilution.

By this definition firms with one or two owners have concentrated control. Table 4 describes the number of firms in our two datasets distributed on the number of owners.

The difference between the second and third column is that the latter provides the number of ultimate owners. The table verifies the non-surprising fact that in general closely held corporations have few owners.<sup>5</sup> In the grand dataset, we have that more than 80 pct. of our firms have one or two owners and this share is even higher in the ULTIMATE dataset. This is consistent with both theory (Bennedsen and Wolfenzon 2000) and other empirical literature (Gomez and Novaez 1999 and, Nagar et.al. 2000).<sup>6</sup>

Table 4: Number of firms in KOB, ULTIMATE 1st level and ULTIMATE by number of owners and share of KOB in ULTIMATE 1st level.

No of Owners	KOB	ULTIMATE 1st level	ULTIMATE 1st level as share of KOB	ULTIMATE
1	22444	13801	0.61	13848
2	9839	2935	0.30	2859
3	3198	653	0.20	664
4	1490	216	0.14	222
5	533	55	0.10	64
+6	521	45	0.09	48
Total	38025	17705	0.47	17705

We next focus on firms with more than two owners for which we have ultimative ownership information. In the appendix we develop a benchmark model which allows us to compare the frequency of control dilution in our sample. It is a one-share-one-vote probability model where we assume that ownership is distributed randomly and for a given set of owners, any possible distribution of ownership is equally likely. In this benchmark model we can solve analytically for the frequency of one and multiple controller firms given the number of owners.

The probability of having a multiple controller firm in the benchmark model is  $1 - \frac{n}{2^n - 1}$ .

Figure 3 illustrates the frequency of one and multiple controller firms in the simplest case

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<sup>5</sup>Notice, that we have not placed any restrictions on firm size, thus, many of our firms are extremely small operations with a single owner.

<sup>6</sup>It is worth emphasizing, that Table 4 reveals a bias in our selection procedure, since we have selected relatively more firms with fewer owners. For instance, our ULTIMATE dataset contains approximately half of the firms from the grand dataset, however it contains only one out of ten firms with six or more owners.

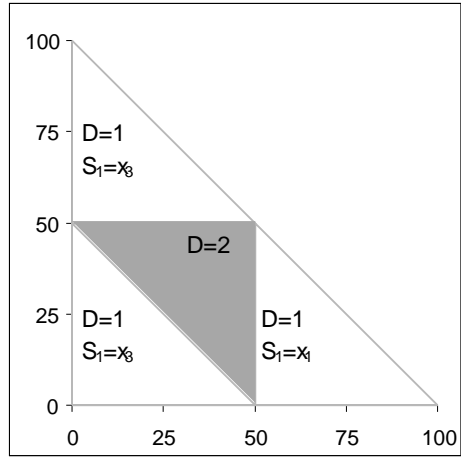


Figure 3: The unit distribution for three owners. The ownership shares  $X_1$  and  $X_2$  are measured on the horizontal and vertical axis and  $X_3 = 1 - X_1 - X_2$ .  $D = 1 \equiv$  one-controller firms.  $D = 2 \equiv$  multiple controller firm.  $S_1$  defines the controlling shareholder in the one-controller case.

of three owners, where  $3/4$  of the firms are one controller firms and  $1/4$  of the firms have multiple controllers.

We are now ready to compare the amount of control dilution in the ULTIMATE sample with the expected amount from the benchmark model in firms with more than two owners. Table 5 states the actual and benchmark (in parentheses) distribution of firms with one and multiple controllers on the number of owners. Relative to our benchmark model, there is a strong tendency to dilution of control in our sample. The last column provides the significance level of difference between observed and predicted values computed with  $\chi^2(1)$ . It shows that for three-, four- and five-owner firms it is strongly significant that there is more control dilution in the sample than predicted from the benchmark model. Only for the 46 firms with six or more owners it is not possible to reject that ownership is uniformly distributed, even though we still have more firms with diluted ownership than predicted.

In a sample of small closely held corporations there are many firms where ownership is distributed equally among the owners. We call such firms for  $1/n$  firms. One comment to the control dilution result above may be that ownership is simply just equal distributed due to reasons which have nothing to do with strategic considerations about control dilution; e.g. that three founders form the corporation together, or a founder gives away the ownership in equal shares to each of his or her children. Even though such comments are not based by any theoretical considerations, we control for this in the following table by removing all  $1/n$  firms and then look at evidence for control distribution again. Remember that we have now stacked the deck against us, since we have only removed firms with multiple controllers. Hence, it might be slightly surprising that Table 6 still shows significant control dilution for three-owner firms at the one percent level. For firms with more than three owners, we cannot in this sample reject the hypothesis of no control dilution. However, more than 50 pct. of this restricted sample are firms with three owners,



Table 5: Number of firms by one or multiple controllers and number of owners. Expected numbers from benchmark model in parentheses. Significance level of difference between observed and expected computed with  $\chi^2(1)$ .

No. of owners	No dominating owner		One dominant owner ( $\geq 50$ pct.)		Sig.
3	442	(166)	222	(498)	0.00
4	170	(111)	52	(111)	0.00
5	55	(44)	9	(20)	0.00
6+	45	(43)	3	(5)	0.30

Table 6: Number of firms by number of owners and presence of dominating owner. Firms with 1/n ownership distribution excluded. Expected number of firms from benchmark model in parentheses. Significance level of difference between observed and expected computed with  $\chi^2(1)$ .

No. of owners	No dominating owner		One dominant owner ( $\geq 50$ pct.)		Sig.
3	120	(86)	222	(257)	0.00
4	61	(52)	57	(57)	0.40
5	26	(24)	9	(11)	0.48
6+	28	(28)	3	(3)	0.79

thus the conclusion for the whole sample still is that we have significant control dilution among small closely held corporations even when all  $1/n$  firms have been removed.

## 4 Partial Explanations for Control Dilution

In this section we will analyze three potential explanations why control is diluted in closely held corporations and we begin by providing evidence for a general strategic motive.

### 4.1 Strategic control dilution.

Corporate control is useful in guiding a firm towards achieving goals that controllers are interested in. As long as this is in the general interest of all owners of the firm, there is no reason to dilute control from a corporate governance perspective. However, when controllers' and non controllers' interests are not perfectly aligned, a recent theme in the corporate governance literature is that diluting control may increase the total value of a corporation, since it may restrict a controller's ability or incentive to take actions that do not benefit the whole group of owners. This general argument in favor of strategic control dilution has been presented in several forms: in a coalition formation context (Bennedsen and Wolfenzon 2000), in a voting context (Bennedsen and Wolfenzon 1999) and in a bargaining context (Gomez and Novaez 1999).

If ownership structure is chosen as an optimal response to potential conflicts between future groups of owners, we should expect to see control dilution more frequently in firms where controllers are more powerful. In particular, controlling owners are more powerful if they in addition to their ownership stakes also participate in the daily management of the firm. There are thus two ways to limit the exploitation of minority shareholders: one is to distribute ownership such that none have more than 50 pct. of the shares, the other is to have a dominating owner but to appoint an outside manager and hereby limit the dominating owner's ability to pursue activities that are not in the interest of all shareholders.

Table 7: Number of firms by presence of dominating owner and manager ownership. Expected numbers under independence of dominating owner and large manager. Significance level of difference between observed and expected computed with  $X^2(1)$ .

No. of owners		No large manager		1+ large manager		Sig
3	No dominant	75	(85)	367	(357)	0.03
	One dominant ( $\geq 50$ pct.)	53	(43)	169	(179)	
4	No dominant	53	(54)	117	(116)	0.64
	One dominant ( $\geq 50$ pct.)	18	(17)	34	(35)	
5	No dominant	13	(15)	42	(40)	0.19
	One dominant ( $\geq 50$ pct.)	4	(2)	5	(7)	
6+	No dominant	17	(18)	29	(28)	0.07
	One dominant ( $\geq 50$ pct.)	2	(1)	0	(1)	
All	No dominant	157	(168)	555	(545)	0.10
	One dominant ( $\geq 50$ pct.)	77	(67)	202	(218)	

We test this hypothesis by analyzing if control dilution happens more frequently in insider owned firms, i.e. where the CEO of the firm is also an owner. However, this raises one potential caveat, namely if a CEO-owner is a significant owner or just a CEO which as part of his compensation package has received a (small) amount of stocks. To distinguish between corporate governance and incentive explanations of control dilution, we define an owner-manager as a CEO who possesses more than 10 pct. of the shares of the corporation according to our ultimate ownership distribution.

Table 7 shows there are more one-controller firms for any number of owners when the manager is not an owner relative to the frequency of control dilution predicted by our random benchmark model. Hence, this is in accordance with a strategic dilution motive. For the whole sample of firms we notice that the two subsets are significantly different on a 10 pct. level. If we condition on the number of owners, we find that the significance comes from the three owner firms for which we notice a strong tendency of control dilution in manager owned corporations.

This result parallels a result found by Gomez and Novaez. They measure the number of owners with more than 10 pct. ownership stake and find that the frequency of multiple large owners increases if the CEO is also an owner. Notice, there is a no direct connection between the presence of multiple large owners and having one or multiple-controller firms. Since they use a fixed measure of the size of owners they cannot distinguish between one and multiple controllers in firms. In our ULTIMATE sample we have many one-controller firms with two or more owners having an ownership stake larger than 10 pct. However, the two set of results do support each other.

## **4.2 Minimum Cash Flow analysis.**

In this subsection we focus on the concept of minimum cash flow as developed in Bennesen and Wolfenzon 2000, hereafter called BW. The theory focuses on how power struggles in CHCs affect the control formation, and it is shown that given ownership distribution, control is formed such that the amount of cash flow internalized by the controlling group of shareholders is minimized. The intuition is the following: when controlling shareholders pursue projects which are not in the interest of the whole group of owners, this comes with a cost and a benefit. The benefit is distributed among the controlling owners, but the cost is a reduction in the profitability of the corporation and this cost is paid by all owners according to their possession of residual income right in the firm. By forming the controlling coalition in a way that the cash flow internalized by it is less than the cash flow internalized by any other group of owners controlling at least 50 pct. of the corporation, this controlling group maximizes the share of the cost paid by the non-controlling shareholders. Hence, the controlling shareholders receive all benefit from pursuing non-profit maximizing activities and they pay the smallest possible share of the cost.

We define minimum cash flow as the minimum amount of cash flow internalized by any controlling group of owners, i.e. by any group of owners that possesses at least 50 pct. of the shares in the corporation. The minimum cash flow hypothesis has strong implications on how to distribute ownership given the goal is to maximize the value of the firm: it is

Table 8: Number of firms by number of owners and minimum size of minimum cash flow coalition. Expected number according to benchmark model in parentheses.  $X^2$  test of difference.

No. of owners	Minimum size of MCF coalition				Sig.
	1	2	3	4+	
3	222 (498)	442 (166)			0.00
4	52 (111)	163 (89)	7 (22)		0.00
5	9 (20)	13 (25)	42 (16)	0 (3)	0.00
6	2 (4)	2 (5)	13 (7)	2 (3)	0.02

Table 9: Minimum cash flow by number of owners and minimum size of minimum cash flow coalition

No. of owners	Actual					Benchmark model				
	1	2	3	4+	All	1	2	3	4+	All
3	0.62	0.64			0.64	0.67	0.56			0.64
4	0.65	0.52	0.52		0.55	0.63	0.54	0.53		0.58
5	0.65	0.51	0.58		0.58	0.60	0.52	0.52	0.51	0.54
6	0.75	0.53	0.51	0.51	0.54	0.58	0.52	0.51	0.51	0.52

optimal to have few owners; to distribute ownership according to a one share one vote rule; and, to distribute ownership stakes of similar sizes.

In the following we compare the distribution of minimum cash flow in our sample to the distribution predicted by the benchmark model outlined in the appendix. The predicted minimum cash flow size depends on the number of owners and on how many of these that do belong to a minimum cash flow coalition. Table 8 shows the actual and predicted distribution of firms on the number of owners in the minimum cash flow coalition. In cases where there are more than one coalition internalizing the minimum cash flow level, we have stated the minimum number of owners in any such coalition. Relative to Table 5, this table provides additional information on how many owners there are in the controlling (majority) group of owners if control is formed according to the minimum cash flow hypothesis. Not surprisingly given the small number of owners, the controlling coalition in general consists of two owners in firms where control is diluted.

In the appendix we derive the cumulative distribution function for the minimum cash flow defined above in the three owner case. In principle, we can solve the model for any number of owners using the same method, however, the complexity increases significantly when we go to four or five owners. Our alternative approach is to simulate the benchmark model, which we have done for firms with four, five and six owners. The results are reported in the right panel of Table 9.

The left pane of Table 9 shows average minimum cash flow in our sample conditioned on the number of owners and the minimum size of the minimum cash flow coalition. For the one-controller firms we notice in the left panel of Table 9 an increasing relationship between the number of owners and the share of the firm held the dominating owner. The reverse relationship is predicted by the benchmark model. BW is silent about this relationship, they notice that the largest owner should have as much cash flow as possible, but wealth constraints may set this level less than 100.

For the multiple-controller firms there is a decreasing relationship between the cash flow internalized by the minimum cash flow coalition and the number of owners. This is predicted both by the benchmark model and by BW. In their proposition 3, they derive an upper bound for the minimum cash flow internalized by the controlling group of owners which is decreasing in the number of owners. In the three owner case this upper bound is  $2/3$  which is close to the average of 0.64 found in data.

In general, it seems that the minimum cash flow level is higher in the multiple controllers cases relative to the benchmark model for an odd number of owners, but similar for the even number of owners. This happens because we have a large number of firms with equal ownership distribution. For even number of owners this implies that the actual minimum cash flow level reach the lower bound of 50 pct. for firms with no difference in voting and cash flow rights. However, we have assumed that if two groups of owners each have exactly 50 pct. of the shares, one of these groups will have ultimate control. An alternative assumption would be that they have to cooperate which would increase the

minimum cash flow. Unfortunately, we have no way to distinguish between these cases.

The following figure shows the distribution of minimum cash flow in data and in the benchmark model given the number of owners and if the firm is a one or multiple-controller firm. In Panel (a) and (b) we have firms with three owners and a dominating shareholder. The difference is that in Panel (b) we have left out all firms where the dominating shareholder holds exactly 50 pct. of the shares. Notice, in general the benchmark model fits data pretty well in this case. Panel (c) shows multiple controller firms with three owners. Notice that most minimum cash flow coalitions internalize more cash flow than predicted by the benchmark model and many firms attain a minimum cash flow of  $2/3$  which is consistent with the BW theory. Clearly, the result happens because we have many  $1/n$  firms in this case. Panel (d) shows the 4 owner one-controller firms and again there is no significant difference between actual and predicted distribution. The last two figures provide the four owner multiple-controller cases with exactly two owners in the minimum cash flow coalition. Notice from Panel (e) the presence of many firms with a minimum cash flow of 50pct which can be explained by the presence of  $1/n$  firms. Since we have no way to tell if this is the actual cash flow amount internalized by the controlling coalition in these firms, we leave them out in the last panel. It is worth emphasizing that quite a few firms in this group reach the 60 pct. level predicted in BW (Proposition 3).<sup>7</sup>

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<sup>7</sup>There is, however, an important methodological difference in the approach taken in the present paper relative to the BW paper. In the latter the upper limit is calculated given the number of *relevant* owners, i.e. owners that all belong to at least one minimum cash flow coalition. Our approach is to condition on the total number of owners. This explains why we can have a four owner multiple controller firm with a minimum cash flow coalition internalizing more than 60 pct.

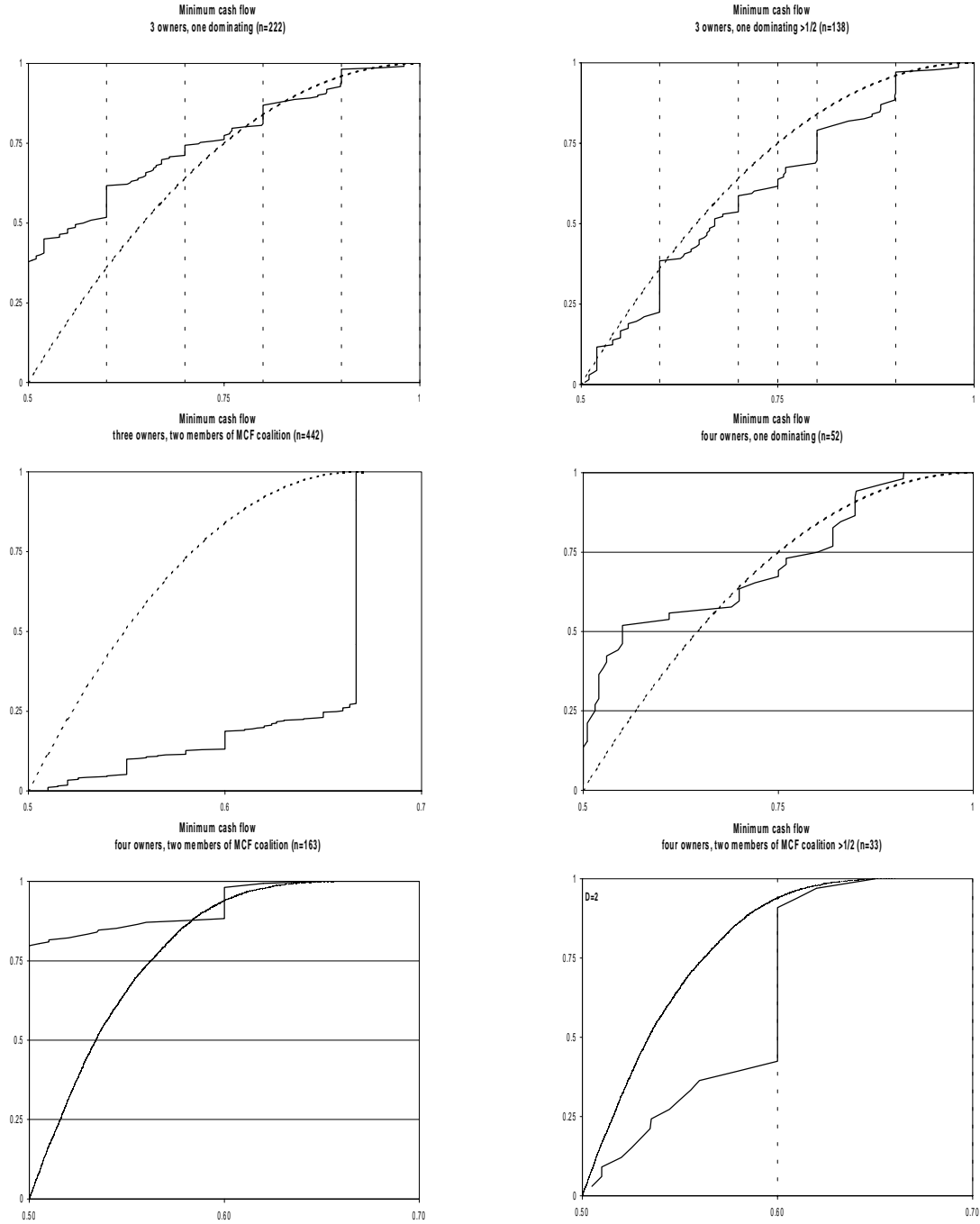


Figure 4: Minimum cash flow in the ULTIMATE dataset condition on three and four owners and on one or two members of the minimum cash flow coalition.



Table 10: Distribution of firms on presence of multiple controllers and family ownership. Expected number under independence in parentheses.

	No dominating owner	One domination owner ( $\geq 50$ pct.)
No family ownership	366 (339)	109 (136)
Family ownership	346 (373)	177 (150)

### 4.3 Family ownership.

In the previous sections we have not controlled for family ownership. The main reason for this is that there is not a clear connection between family ownership and control conflicts in small closely held corporations.

In many firms, family ownership concentrates control in the hand of a unified and single-interested family. What matters in such firms is not the distribution of ownership inside the family, but rather the share of ownership controlled by the family and the share controlled by outsiders. This is typically the case for large public traded corporations, but is obviously also the case for many small closely held corporation. However, families do also conflict, as is easily seen from the many cases of disagreeing family members reaching the court rooms all over the world. Since we have names and complete ownership information, it would be straight forward to aggregate ownership over families. Our main reason not to do so, is that we believe control fights among family members are not unusual in small closely held corporations.

Clearly, the lack of control for family ownership implies the possibility that the observed control dilution is simply an artifact of the presence of family ownership more than it is due to real control dilution. We check for this in Table 10 by splitting the sample on family owned firms and non family owned firms. We use the surname of the owners to define family ownership as firms in which at least two owners have the same surname independently of the family being in the majority or in the minority. As in most other studies of family ownership this introduces a bias in firms where a (female) family member does not carry the family's surname.

Table 10 shows rather surprisingly that there are more one-controller firms among the family owned corporations and less family owned firms among multiple controller corporations. We reject clearly the equal distribution hypothesis with a  $\chi^2$  test for independence using a 1 pct. significant level. Thus, the observed control dilution is not explained by our decision not to control for family ownership, since the presence of family ownership is positively correlated with the *lack* of control dilution in our sample.

Since our data set includes the name of all ultimate owners to our firms, we have also repeated the analysis in the last section for the aggregated data set where all family members are merged together. Obviously this reduces slightly the number of multiple controller firms, but it does not significantly affect the results of our analysis.

To sum up this section we have made three partial analysis to explain why control is diluted in closely held corporations with multiple owners. We found that there are significant evidence for a strategic motive for control dilution. In addition we found that control is diluted in a way that increases the minimum cash flow internalized by any controlling group of owners relative to our random benchmark model. Finally, we strongly rejected the argument, that the observed control dilution is due to the fact that we have not controlled for family ownership.

## 5 Simultaneous Explanations for Control Dilution.

In this section we provide simultaneous tests of a number of explanations for the presence of control dilution in firms with multiple owners. We will first use the presence of control dilution, defined as the absence of a single dominating owner, as the endogenous variable in a logistic regressions. Subsequently, we introduce a more general measure of control dilution to analyze the robustness of our results.

For the time being, let the endogenous variable be the presence of control dilution, defined as the absence of a single dominating shareholder. Our approach is to make a

simple logistic regression assuming that the probability of control dilution is given by:

$$P(\text{no dominating owner}) = \frac{1}{1 + e^{-\beta X}},$$

where  $X$  is a vector of variables and  $\beta$  is a vector of parameters. The explanatory variables are defined in Table 11:

Table 11: Variable definitions

Nodom	No dominating owner.
Islardir	Dummy, a manager owns more than 10 pct. of shares.
IsFam	Dummy, at least two owners with same surname.
JurFor	Dummy, firm is an A/S (larger limited liability company).
Nown4	Firm has 4 owners.
Nown5	Firm has 5 owners.
Nown6+	Firm has 6+ owners.
Assets	Assets from the latest set of accounts.
EstabYear	Year of establishment.

Nodom is the endogenous variable. Islardir captures the general strategic motive discussed in the first part of the previous section. We expect the presence of a large manager among the owners to have a positive effect on control dilution. Isfam captures the presence of family ownership which we, given the analysis above, expect to have a negative effect on control dilution. Jurform is one if the firm is a stock company and zero if it is a partnership. Since many accounting and law firms are organized as partnership with a  $1/n$  ownership structure, we expect control dilution to be negative correlated with this dummy. The next three variables are included because we expect a positive relationship between control dilution and the number of owners in the firm. Since we only have sales measures for a (smaller) subset of our ULTIMATE dataset, we have included the assets of the firm to capture the size of the firm in tradition with the empirical ownership concentration literature.

Finally, most corporate governance theories have taken a static approach to the analysis of conflicts between different group of owners. However, even in closely held corporations ownership changes significantly over time. In our grand dataset we have ownership information over a five year period for 27.125 firms and more than 1/3 of these did have at least one change in ownership during the period from 1994 to 1998.

There exist little theoretical analysis of the dynamic aspect of control dilution in closely held corporations. In the absence of formal theory, we have no clear expectation of the connection between age and control dilution. For example one scenario could be that initially the founder chooses a control diluted  $1/n$  ownership structure because this maximizes the expected value of the firm. However, at that point he introduces a conflict between controlling and non-controlling owners. Later on it may be in the interest of one or more controlling owners to achieve more control through private negotiations, even if it is not beneficial to the whole set of owners. If this is the case, the ownership structure may tend to include less control dilution over time. These argument is consistent with Bebchuck (1999) who analyze dynamic control concentration in public traded corporations. On the other hand, if firm value is higher the more diluted control there is in a firm, there could be other factors implementing this over time. For example, it may be in the interest of a large dominating owner to reduce her ownership stake as a commitment not to pursue future actions that are not in the interest of all owners. To distinguish between such informal stories of dynamic control dilution, we have included the year of establishment in the regression.

The model was first estimated including all second order interactions, then parameters were eliminated hierarchically such that main effects were included as long they had not been eliminated from corresponding interactions. The Hosmer-Lemeshow test did not reveal problems with goodness of fit. Two models have been selected for presentation. Model 1 includes main effects only whereas model 2 includes the interaction between the presence of a large CEO among the owners and family ownership. Model 2 is param-

Table 12: Estimation results from Logit regression. Standard errors in parentheses.

	Model 1		Model 2	
Islardir	0.3778	(0.1709)		
Isfam	-0.4644	(0.1524)		
IsFam=1*Largedir=0			0.1064	(0.2971)
IsFam=0*Largedir=1			0.8057	(0.2539)
IsFam=1*Largedir=1			0.1555	(0.2451)
Jurfor	-0.3396	(0.1487)	-0.3203	(0.1493)
EstabYear	0.0183	(0.0056)	0.0186	(0.0056)
assets	-3.15E-7	(4.87E-7)	-3.48E-7	(4.82E-7)
Nown4	0.6584	(0.1857)	0.6505	(0.1855)
Nown5	1.2906	(0.3775)	1.2787	(0.3769)
Nown6+	2.0091	(0.6088)	2.053	(0.6112)
Constant	-35.5268	(11.1965)	-36.4575	(11.1478)
-2 log likelihood		1121.0		1116.1
-2 log likelihood, constant only		1195.7		1195.7
Observations		998		998

terized such that the t-test of each level of the interaction variables can be read directly from the parameter estimates.

Model 1 confirms the insight derived in the previous section: The presence of a manager owning more than 10 pct. increases the probability of control dilution and this effect is significant on a 5 pct. level. On the other hand, if there is a family among the owners the probability of control dilution decreases. This effect is also significant on a 5 pct. level. As expected, the dummy for stock company is significant and negative. In addition, we notice that control dilution increases with the number of owners in the firm, but that the coefficient for assets, our proxy for size, is insignificant.

It is interesting to notice that the year of establishment is positively correlated with control dilution and this effect is highly significant. This implies that younger firms have more diluted ownership. One possible interpretation of this is that for small closely held corporations the evolution of ownership among surviving firms tends to dissolve any

initial control dilution, which support the first scenario discussed above. Thus, we find supporting evidence for the Bebchuck 1999 prediction that control has tendency of being concentrated over time.

In model 2 an interaction between Isfam and Islardir is included and the model is re-parameterized. This reveals that the effects of these two variables are driven by the interaction, which is to say that the probability of control dilution increases, in non-family firms, when a manager owns more than 10 pct. of the shares. This effect is significant at the 1 pct. level. The additional effect of family ownership is not significant. The sign and significance of the rest of the variables are similar to Model 1.

### **A generalized control dilution measure.**

In the previous analysis we have chosen a very simple measure of control dilution, namely the absence of a single dominating shareholder. However, this measure captures only one aspect of control dilution, namely that at least two shareholders must collaborate to achieve control, and there may be additional interesting features of control dilution. As an example, consider two three owner firms. In the first a dominating owner has 52 pct. of the shares, one minority share holder possesses 43 pct. and the last minority owner has only 5 pct. of the shares. In the second, a dominating owner has 90 pct. and the two minority shareholder have 5 pct. each. With our simple measure of control dilution these two firms are equal from a control perspective. However, it may be reasonable to argue that control is more diluted in the former where the dominating owner is not much larger than the second largest owner relative to the latter firm, where the dominating owner owns almost the whole corporation.

To check the robustness of our previous analysis, we now define a more general measure of control dilution. We define  $1/n$  firms as the perfect diluted ownership structure and measure control dilution as the (negative) distance from the  $1/n$  distribution:

$$\text{Dist} \equiv \sqrt{\sum_{i \in N} (X_i - \frac{1}{n})^2}.$$

In our example above, this measure of control dilution implies that the first of our two firms will have a more diluted control structure than the second firm. We know from data that there is a significant point mass at  $dist = 0$ , i.e. that we have a large portions of firms with ownership equally distributed among owners. Thus our estimation approach is first to produce a Probit regression for Dist being greater than zero, and second, to make a truncated TOBIT regression on the firms with Dist being greater than zero. The results are presented in Table 13 and Table 14.

The test for  $1/n$  distribution is rather consistent with the test for control dilution above. We notice that family ownership is significant and make it less likely that ownership is distributed equally among the owners. Having a manager with a large ownership share is significant and make it more likely that ownership is equally distributed. A stock company has less equal distribution than a partner ship and the more owners the more likely it is that ownership is not equally distributed. The establishment year and the size of the firm is insignificant here.

Given that ownership is not equally distributed, Table 14 does not provide us with a clear answer to what determines the degree of control concentration. The most significant result is that younger firms in this subsample have more concentrated control, but the coefficient is very close to zero. The only other significant variable is that having four owners imply more uneven distribution of ownership.

Table 13: Probit regression for 1/n ownership:

Dependent Variable: Dist&gt;0

Method: ML - Binary Probit

Variable	Coefficient	Std.Error	z-Statistic	Prob.
ISFAM>0	0.461506	0.083052	5.556813	0.0000
LARGEDIR>0	-0.25269	0.097075	-2.60307	0.0092
JURFOR	0.320494	0.092639	3.459597	0.0005
EstabYear	-7.27E-05	0.000123	-0.58869	0.5561
LOG(BALAN)	0.000936	0.029109	0.033069	0.9736
NOWN=4	-0.12151	0.1001	-1.21392	0.2248
NOWN=5	0.005161	0.167713	0.030775	0.9754
NOWN>5	0.362095	0.196398	1.843681	0.0652

Error Distribution

Mean dependent VAR	0.520121	S.D. dependent VAR	0.499846
S.E. of regression	0.48796	Akaike info criterion	1.346014
Sum squared resid	234.7719	Schwarz criterion	1.385465
Log likelihood	-660.969	Hannan-Quinn criter.	1.361012
Avg. log likelihood	-0.66496	Total obs	994

Table 14: Truncated TOBIT regression given  $DIST > 0$ .

Dependent Variable: Dist

Method: ML - Censored Normal (TOBIT)

Truncated sample

Variable	Coefficient	Std.Error	z-Statistic	Prob.
ISFAM>0	0.003374	0.025148	0.134156	0.8933
LARGEDIR>0	-0.04454	0.026195	-1.70011	0.0891
JURFOR	0.025075	0.027091	0.925552	0.3547
EstabYear	6.97E-05	3.46E-05	2.013882	0.044
LOG(BALANC)	0.013976	0.00795	1.757889	0.0788
NOWN=4	0.050406	0.028629	1.760685	0.0783
NOWN=5	-0.04273	0.049785	-0.85832	0.3907
NOWN>5	-0.07959	0.055137	-1.44346	0.1489

Error Distribution

Mean dependent VAR	0.298	S.D. dependent VAR	0.180234
S.E. of regression	0.128422	Akaike info criterion	-0.38678
Sum squared resid	16.24478	Schwarz criterion	-0.34239
Log likelihood	201.2272	Hannan-Quinn criter.	-0.3699
Avg. log likelihood	0.202442	Total obs	517



## 6 Conclusion

The aim of the present paper is to analyze ownership distribution and control dilution in small closely held corporations. We hope this may be a first step into extending the literature on determinants of ownership concentration to explaining ownership *distribution* and *control* concentration.

The paper derives a number of insights. First, it confirms that most small closely held corporations have concentrated control and that this control concentration is created through having at most two owners of the corporation. Second, we show that firms with multiple owners in general have an ownership structure that dilutes control over the corporation. Third, we have provided evidence for control being more diluted when managers have significant ownership shares, supporting a corporate governance view control dilution: it is beneficial to dilute control in manager owned firms, because it forces the powerful owner-manager to cooperate with a group of owners and through this internalize a larger set of interests. Fourth, the observed pattern of control dilution forces any group of majority owners to internalize a significant amount of cash flow rights, an observation which is consistent with recent theories on control formation in closely held corporations. Fifth, we argued that control dilution is not an artifact of family ownership, since control is less diluted in family owned firms. Finally, we have shown that ownership dilution is affected by the age of the firm. In particular, this highlight the need for dynamic theories of optimal ownership structure, a highly underdeveloped field in corporate governance.

In the present paper we have focused on explaining control dilution. An obvious next step is to investigate whether the distribution of ownership and the dilution of control matters? One promising way to approach this question could be to analyze if our generalized dilution measure affects firm performance. We have left this question for future research.

## Appendix: The Benchmark Model

Define the set of owners as  $N$  and let  $|N| = n \geq 1$ . Assume  $\{Y_i\}_1^{n-1}$  is uniformly distributed on the unit interval and let  $\sigma$  be an ordering such that  $0 = Y_0 \leq Y_{\sigma(1)} \leq \dots \leq Y_{\sigma(n-1)} \leq Y_{\sigma(n)} = 1$ . implying that the ownership share of each owner is  $X_i = Y_i - Y_{i-1}$ .

Denote the event of a one controller as  $D = 1$ , from which it follows that  $D = 1 \Leftrightarrow S_1 \geq 1/2$ , where  $S_1$  is the share holding of the largest shareholder. The probability of having a one controller firm is,

$$\begin{aligned} P(D = 1) &= P(\max X_i \geq 1/2) \\ &= nP(X_1 \geq 1/2, X_1 = S_1) \\ &= nP(Y_i \geq 1/2, \forall i) = \frac{n}{2^{n-1}} \end{aligned} \tag{1}$$

We solve for the distribution the amount of cash flow internalized by the minimum cash flow coalition under the benchmark model in the case of three owners and conditioned on the size of the minimum cash flow coalition.

### One-controller firms.

From above, this case is defined by  $d = 1$  and  $S_1$  is the minimum cash flow size, which is equal to the ownership share of the dominating owner. We next derive the cumulative distribution function for the minimum cash flow share conditioned on the firm has one controller using equation (1):

$$\begin{aligned} P(S_1 \leq t \mid D = 1) &= 1 - P(S_1 > t \mid D = 1) \\ &= 1 - \frac{P(S_1 > t, D = 1)}{P(D = 1)} \\ &= 1 - \frac{2^{n-1}}{n} P(S_1 > t, D = 1) \\ &= 1 - \frac{2^{n-1}}{n} nP(S_1 > t, D = 1, S_1 = X_1) \\ &= 1 - 2^{n-1}(1 - t)^{n-1}. \end{aligned}$$

From this we find the distribution function for the minimum cash flow given a one controller firm,

$$f_{S_1|D=1}(t) = (n-1)2^{n-1}(1-t)^{n-2}.$$

The expected value of the minimum cash flow in the one controller firm becomes:

$$\begin{aligned} E(S_1 \mid D=1) &= (n-1)2^{n-1} \int_{1/2}^1 t(1-t)^{n-2} dt \\ &= (n-1)2^{n-1} \int_0^{1/2} (1-x)x^{n-2} dx \\ &= (n-1)2^{n-1} \left[ \frac{x^{n-1}}{n-1} - \frac{x^n}{n} \right]_0^{1/2} \\ &= (n-1) \left[ \frac{2n-n+1}{2n(n-1)} \right] \\ &= \frac{1}{2} + \frac{1}{2n}. \end{aligned}$$

### Multiple controller firms.

We cannot generally solve the two controller case, except in the most relevant case for three owners. If we notice that, in the three owner case,  $x_3 = 1 - x_1 - x_2$ , we can plot the set of feasible ownership distributions in a two dimensional triangle, as was done in Figure 3. The distribution function on this triangle is  $f(x_1, x_2) = 2$ ,  $x_1, x_2 \geq 0, x_1 + x_2 \leq 1$ .

From equation (1) (or directly from Figure 3), we notice that in this case  $P(D=1) = \frac{3}{4}$  and  $P(D=2) = 1 - P(D=1) = \frac{1}{4}$ . The cumulative distribution function of  $S_1$  given  $D=1$  is

$$\begin{aligned} P(S_1 \leq t \mid D=1) &= -4t^2 + 8t - 3, \quad \frac{1}{2} \leq t \leq 1 \\ \text{and } E(S_1 \mid D=1) &= \frac{2}{3}. \end{aligned}$$

Let  $S_2 = \min_i (1 - x_i) = 1 - S_1$  be the minimum cash flow size in the case where two owners control the firm. Then  $S_2 \leq \frac{2}{3}$  and when  $D=2$  we have  $S_2 \geq \frac{1}{2}$ . The cumulative distribution function for the minimum cash flow stake for any controlling groups of owners

is,

$$\begin{aligned}
P(S_2 \leq t \mid D = 2) &= \frac{P(S_2 \leq t, D = 2)}{P(D = 2)} \\
&= 4P(S_2 \leq t, D = 2) \\
&= 12P(S_2 \leq t, D = 2, X_1 = S_1) \\
&= 12P(X_1 \geq 1 - t, D = 2, X_1 > X_2, X_2 > 1 - 2X_1) \\
&= 12 \int_{1-t}^{1/2} \int_{1-2x_1}^{x_1} 2dx_2 dx_1 \\
&= 24 \int_{1-t}^{1/2} 3x_1 - 1 dx_1 \\
&= 24 \left[ \frac{3}{2}x_1^2 - x_1 \right]_{1-t}^{1/2} \\
&= -15 + 48t - 36t^2,
\end{aligned}$$

and the distribution function is,

$$f_{S_2|D=2}(t) = 48 - 72t.$$

The expected value of the minimum cash flow conditioned on having a two controller case is,

$$E(S_2 \mid D = 2) = \int_{1/2}^{2/3} 48t - 72t^2 dt = \frac{5}{9}.$$

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