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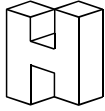
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# **Institut for Nationaløkonomi**

Handelshøjskolen i København

**Working paper 15-98**

## **THE PUBLIC DISCOUNT RATE AND THE UNCERTAIN BUDGETARY FLOWS**

Syed M. Ahsan

Panagiotis Tsigaris

# The Public Discount Rate and the Uncertain Budgetary Flows<sup>1</sup>

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## *Abstract*

Public investment in infrastructure and the like do not usually yield direct pecuniary returns to the public exchequer. Instead public capital leads to increases in factor productivity in the private economy. This paper argues that government typically shares in the latter gains via the tax-expenditure policies. Consequently the discount rate relevant for the investment is ultimately the same rate, as that required for valuing the future gains. However, it is important to note that these productivity gains are subject to aggregate economic shocks. From the perspectives of a counterfactual experiment whereby the risky revenue flows serve as collateral for public borrowing, one may derive the risk discount rate that the capital market would set. This paper applies the above methodology to the United States, uses budget data for the period, 1950-1995, and shows that while the risk discount would typically exceed the risk free rate, it remains well below that facing a private investor. The intuition here is that the portfolio of assets embedded in the state's revenue claims provides additional diversification than is available through financial markets. Consequently even investors holding well-diversified stock portfolios may legitimately view claims on state revenue as vehicles for further risk shifting.

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## The Public Discount Rate and the Uncertain Budgetary Flows

"Mr. Kenneth Clark, the chancellor of the exchequer, conceded yesterday, that the Treasury had made a 'mistake' in its forecast for this year's government revenues...The chancellor said that his scope for cutting had been severely limited by a shortfall in revenues, which meant that borrowing was running well above the forecast.." (*Financial Times*, May 13, 1996)

### 1. Introduction

Government revenue and expenditure are both subject to influences of the aggregate risks embedded in the tax base(s). By taxing uncertain streams of income, profit, and transactions, the government transfers risk onto the budget constraint. Depending on the state of nature, the eventual spending on programs such as social insurance would deviate from the planned level. It is implausible that the risk characteristics of the revenue system would offset those of the expenditure pattern (i.e., both flows covarying together). Question then arises of the *market value* of public revenue, and therefore the true size of expenditure that a government may be able to finance in steady state. To follow through, this paper examines the stochastic nature of budgetary flows, and attempts to measure the systematic component, and quantify the "risk premiums" associated with various tax revenue sources and public outlays.

In parts of the literature there is a presumption that revenue risks were just as costly for the state to bear as for private agents (Bulow and Summers (1984), Gordon (1985) and Hamilton (1987)). Corporate tax revenue is often cited as a prime example. In contrast, the return on human capital risk is believed to be largely idiosyncratic [Eaton and Rosen, (1980a, b) and Varian, 1980], but individuals cannot directly benefit from human capital risks, as they are unable to trade in such risks. Consequently, the government may pool across the latter risk, and improve on the market allocation. Thus a diversified revenue base of a modern economy may indeed present investors with an additional risk-shifting device, and allows them to achieve greater diversification than is otherwise available through the market.<sup>2</sup>

In the financial economics literature, one usually measures the risk premium for a stochastic yield by comparing the latter with an all-encompassing index of the economy wide risk, namely *the market portfolio*. Frequently, the market portfolio is likened to a broad stock market index, which would represent a sizeable part of the corporate economy.<sup>3</sup> However, Shiller (1993) points out that the risks that are traded in today's financial markets are only a small fraction of all the risk society is exposed to.<sup>4</sup> In searching for a broader measure of market risk, it is plausible that one add in the *systematic risk* inherent in the aggregate output shocks. Our hypothesis then is as follows. For a given choice of the market risk index, to the extent government revenue has less than a perfect correlation with the former, it is conceivable that expected utility maximising investors would view claims on public revenue as

vehicles for further risk shifting. Consequently, the market value of risky revenue would be positive.<sup>5</sup> By contrast, was the correlation to be exact [the alternative hypothesis], as has been alleged by Bulow and Summers to be the case with corporate tax revenue, the market value would be negligible: "[Government] can only finance programs whose budgets may have a certainty equivalent of zero" (1984, p21). In this latter event, as they put it, there is no free lunch here.

Given that we analyse the riskiness of a tax revenue flow by treating the latter as an yield on a risky asset, the procedure calls for further motivation. Let us consider the simplest case namely, the corporate income tax (CIT). The total CIT base is clearly the sum of pre-tax net income of all firms. One may therefore interpret the present tax base as a stream of net revenue flows arising from a *portfolio of corporate equities*, where care has been taken to add up each (type of) agent's income by its relative weight in the total. Given a particular tax schedule, in this instance *the rate of tax*, and for simplicity allowing full loss-offsets, the tax revenue would therefore embody the same risk characteristics that afflict equity prices.

Further note that, assuming economic depreciation in the tax schedule, the net income of a firm is the conceptually correct measure of its *return on capital*, and changes in its level essentially constitute the *accrued* capital gains on the firm's equity. Were this source to be the sole revenue claim of a government, and it wished to borrow, lenders could logically demand an expected rate of return that reflects the riskiness of the *de facto* collateral asset, namely the tax base. Claims on future corporate revenue may therefore be seen to be formally similar to yields on risky assets. The case of other tax revenue is similar. One important distinction would be that the underlying portfolio might not be traded in any actual market unlike the CIT base.<sup>6</sup> An obvious example is that of the personal income tax (PIT) revenue. Here the tax base consists of returns to human capital, individual capital and entrepreneurial income. Again the taxable part is in the nature of an income net of direct costs of earning income, and thus has the interpretation of a return on the totality of human, entrepreneurial and physical capital stock attributable to the *personal* sector of the economy. The latter capital is mostly untraded in actual markets, and hence it is conceptually important to develop an appropriate measure of the systematic risk facing such incomes both for descriptive as well as normative purposes. The remaining taxes (notably, payroll, domestic goods and services, foreign trade) may be similarly motivated.

One has to be cautious, however, with treating tax revenue flows similar to the streams of net profit from real or financial assets. The realised values of tax revenue and expenditure flows depend not only on risk factors such as the business cycle, but also on structural changes in the tax and spending codes and the rate schedule, albeit with a lag. These latter elements suggest that the realised revenue yield (or

actual expenditure) in a given period remains, at least nominally, under the direct control of the government, unlike the behaviour of asset returns. A typical example would be an anticipatory counter-cyclical fiscal policy to combat a bad state of nature. However, as Barro (1979) has argued, governments appear not to change the rate schedule and tax code too frequently due to concern over the efficiency costs of such perturbations. Thus even though observed variations in the growth rates of revenues and expenditures may suffer from the noise due to legislative tinkering with the code, we show that one may still obtain an unbiased estimate of the "riskiness" of the tax revenue and expenditure flows.

Our approach to measuring the riskiness of revenue and public expenditure has interesting implications for several related issues in public economics, namely the choice of the social discount rate, size of sustainable debt, and for programs financed by earmarked budgeting. Let us dwell on these briefly, beginning with the discount rate. Note that public investments (e.g., infrastructure) would plausibly lead to enhanced productivity of labour and capital in the private economy. Given a well functioning tax system, such private sector gains would lead to higher government revenue on account of augmented wages, profits, and transactions. The risk characteristics of the augmented revenue flow (or, equivalently, of the debt instrument for which these are collateral claims) may not match that of the market portfolio, or that of other securities actually trading in the market. We show below that the discount rate relevant for evaluating such future gains, that is to say for the public investment decisions in the first place, should reflect the risk inherent in the future returns to physical and human capital.

Turning to the size of debt, were the Bulow-Summers scenario to prevail for overall revenue (admittedly an extreme hypothesis), no positive debt would be sustainable, as the market value of future revenue would be negligible. More generally, however, one can go on to model the limit to debt size under steady state conditions. The implications for ear-marking are rather evident; should the source of finance be very risky, not only would the program output have an unacceptable level of volatility, borrowing against future revenue (from the same source) would appear unwise due to its low market value. Consequently, ear-marking could be practised only in limited instances where, either, the revenue source had little systematic risk, or, by coincidence, both demand for the service and revenue claims were near perfectly correlated.

The plan of the paper is as follows. Our approach calls for a measurement of the degree of riskiness embedded in revenue and government outlays, and hence their market values. We do this for the US economy over the period, 1950-95. In the process we analyse the aggregate revenue and outlay of the *federal* government, as well as their major components. In section 2, we review the revenue and

expenditure profiles, and carry out a preliminary analysis of their mutual correlation as well as against a measure of market risk index (namely, the S&P 500). Based on asset-pricing models, a theory of the evaluation of risky revenue is developed in section 3, which is then applied to the present data set in section 4. Next a discussion and application of the arbitrage pricing theory (APT) is offered in section 5. Implications of the analysis for the public discount rate is reviewed in section 6, while a few concluding remarks are collected in section 7. An appendix deals with the stability of the parameter estimates.

## **2. Revenue and Expenditure Behaviour in the US, 1950-95**<sup>7</sup>

*Revenue and Expenditure Data:* We begin by measuring the variability inherent in the flows of public revenue and expenditure through time. Naturally we deal with real (i.e., net of inflation) values of all variables. Variability may be examined at *level* values of a series or transformations thereof (e.g., first differences). For statistical reasons, stationarity being a principal argument, we have chosen to express the public flows in growth rates per period (Sobel and Holcombe, 1996). Additional refinements are explained as we proceed.

*Revenue Behaviour:* Figure 1 below shows the pattern (i.e., year-on-year real percentage change) of the two largest revenue items of the US federal government over the 1950-1995 period, namely the corporate and the personal income tax revenue. It is obvious that corporate tax revenue has been more volatile than personal income tax revenue. Further note that both the revenue streams tend to move in a pro-cyclical manner over the business cycle. The amplitude of the swings of the corporate tax revenue is however more pronounced.

Table 1 provides us with some descriptive statistics on the per capita real growth in various tax revenue sources for the US (federal) government over the same period.<sup>8</sup> These include sample results such as the mean, standard deviation, coefficient of variation and the range. From the table we observe that practically all the mean (per capita) rates of growth have been positive and quite large. The mean growth in (per employee) social insurance contributions (SIC) is the largest with the smallest standard deviation. Highest standard deviation is observed, not unexpectedly, in the case of the corporate income tax (CIT). The *coefficient of variation (CV)*, defined as standard deviation per unit of mean growth, is given in the fourth column. Examined in light of CV, one can observe that the year-on-year fluctuations in corporate tax revenue have been most volatile during the 1950-1995 period.<sup>9</sup> Volatility in this flow is also apparent from the observed large revenue growth due to the expansion of economic activity (1951-52) during the Korean War. Moreover, the recession of the early 80s caused corporate tax revenue to drop by 28 percent in 1983. The table also shows that while both CIT and the personal income tax (PIT) revenue have grown at about the same pace, the latter would appear to be much less

risky. The healthy growth in these revenues can be attributed, mainly, to the long run productivity increase over the period.

Examining the variance of a variable gives us a picture of the total risk of an uncertain stream. We would eventually want to separate total risk into its components, namely *systematic* and *unsystematic*. Only by so partitioning the risk, can we analyse how volatile has the growth in specific categories of tax revenues and government outlays been relative to the return on the market portfolio. Frequently one refers to the *excess* return on the market portfolio, ERM, (i.e., here the mean return on the S&P 500 index *less* the risk free rate), as the *market risk premium*. Figure 2 below compares the market risk premium in period-t with the “excess return” from the corporate tax revenue moved one period forward, i.e., in period (t+1).<sup>10</sup> The graph indicates that the market return does indeed lead the growth in corporate tax revenue by a year. The graph is suggestive of a positive correlation between the two variables. The similarity in the pattern of variability between ERM and CIT revenue is also implied by the summary statistics (Tables 1 and 2).

**Figure 1**

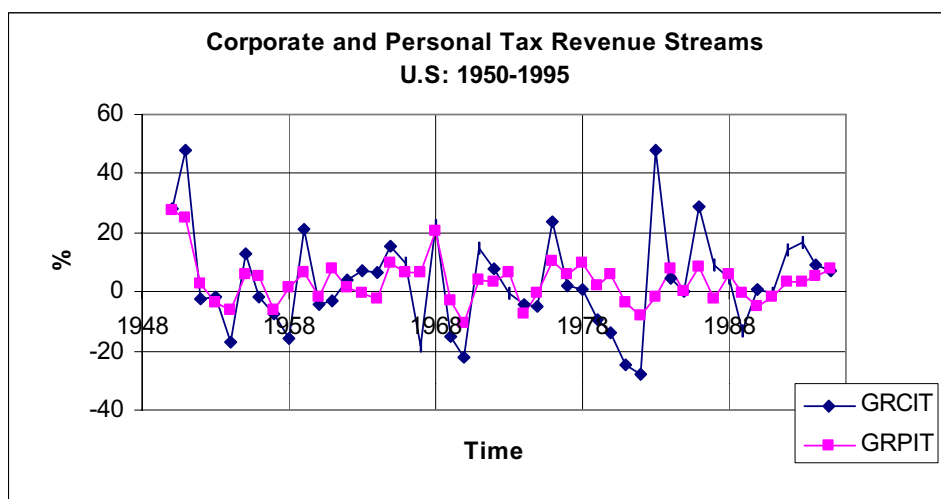


Table 2 shows that over the sample period the mean real return on the stock market index has been 9.17 percent (with a standard deviation of 15.57). The table also shows that the corresponding mean market risk premium (ERM) has been 7.91 percent (with a standard deviation of 15.49). These figures are in accordance with the equity premium literature [e.g., Mehra and Prescott (1985)]. The summary statistics provide an indication that corporate tax revenue flows may have been just as volatile as the stock market yield.<sup>11</sup>



**Table 1: Statistics on Annual Rate of Growth of Tax Revenue, 1950 –1995**

Name	Mean Growth	Standard Deviation	CV	Minimum (Year)	Maximum (Year)
GRPIT	3.10	7.68	2.48	-10.51 (71)	27.83 (51)
GRCIT	3.15	16.64	5.28	-28.00 (83)	47.93 (52)
GRSIC	5.28	6.04	1.14	-3.23 (76)	21.82 (51)
GRET	-0.21	10.46	NA	-17.81 (82)	51.45 (81)
GRT	2.83	6.85	2.42	-9.34 (55)	23.80 (52)

*Notes.* GRPIT: Real rate of growth of the personal income tax revenue per person; GRCIT: Real rate of growth of corporate income tax revenue; GRSIC: Real rate of growth of social insurance tax and contributions per employee; GRET: Real rate of growth of excise tax revenue per person; GRT: Real rate of growth of total tax revenue per person. All data have been adjusted for inflation using the GDP deflator.

**Table 2: S & P 500 Index for the Sample Period: 1950 –1995**

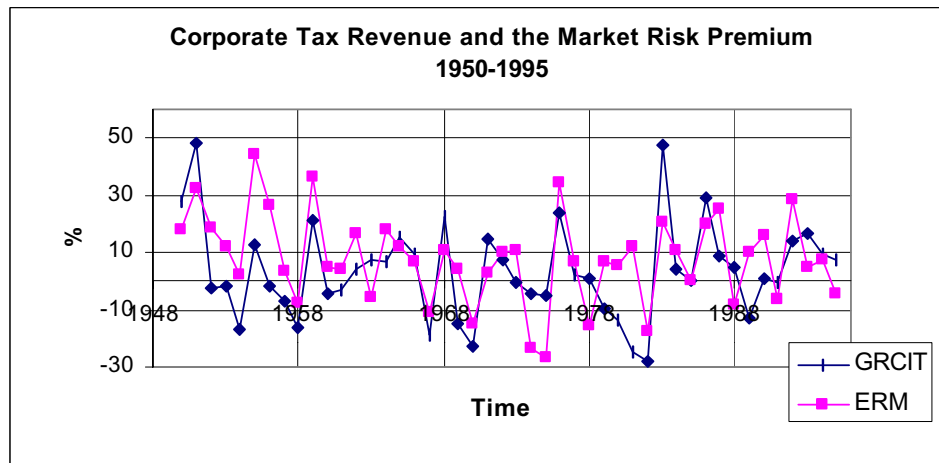
Name	Mean Return	Standard Deviation	CV	Minimum (Year)	Maximum (Year)
RRM	9.17	15.57	1.70	-30.87 (75)	44.40 (55)
ERM	7.91	15.49	1.96	-26.50 (75)	44.52 (55)

Note: RRM = dividend yield plus the capital gains component, and computed as:  $RRM = ((SP500_t + DIV_t) / SP500_{t-1} - 1) \times 100$  less the inflation rate. Inflation was computed using the GDP Deflator. ERM is the total return of the S&P500 index minus the return earned on the safe asset as proxied by the three-month Treasury bill rate.

Below (Table 3) we present the correlation matrix between the market risk premium, (at time-t), and the year-on-year rate of change in tax revenue (at t+1) from different sources. It is seen that all sources of tax revenue have a positive correlation coefficient both with the market excess return, and mutually among the categories. For example, the degree of correlation between the risk premium and the percentage change in corporate tax revenue is 0.567 (with a standard error of 0.124), which happens to be the highest among all tax sources, though statistically less than unity. The correlation of the rate of growth of the personal income tax revenue per person (GRPIT) and the market excess return is 0.380 (with a standard error of 0.139). The latter figure is less than that for the corporate revenue which is rather intuitive, and may indicate support for the theoretical hypothesis expounded by Eaton and Rosen (1980a, b). Table 3 also reveals the pronounced positive correlation between personal tax revenue and other tax revenue sources, but again the coefficients are all less than unity, i.e., a less than perfect relationship. It is therefore plausible that the government may combine different sources of revenue in order to achieve a greater diversification of its revenue stream.

*Expenditure Profile.* We now examine the risk characteristics of government outlays. Figure 3 shows how the risk premium on the stock market varies with the rate of change in the direct payment to individuals, which points to a *negative* relationship, albeit with a time lag. This is clear during the first oil shock in 1974 and during the recession in the early 80s and 90s. Hence it would appear that if the market were to predict a softening of the economy today, transfer payments to households would tend to rise a year hence.

**Figure 2**



**Table 3: Correlation Matrix of Revenue Variables**

	ERM	GRPIT	GRCIT	GRSIC	GRET	GRT
ERM	1.000	0.380 (0.009)	0.567 (0.000)	0.434 (0.003)	0.219 (0.143)	0.530 (0.000)
GRPIT		1.000	0.625 (0.000)	0.417 (0.004)	0.115 (0.447)	0.916 (0.000)
GRCIT			1.000	0.403 (0.005)	0.031 (0.837)	0.819 (0.000)
GRSIC				1.000	0.108 (0.477)	0.579 (0.000)
GRET					1.000	0.181 (0.228)
GRT						1.000

Notes: p-values are shown in parenthesis below the estimated correlation coefficients.<sup>12</sup>

Table 4 below provides a few descriptive statistics on the real growth rate of federal outlays per capita during the sample period, while in Table 5 we present the estimates of the degree of correlation between the risk premium on the stock market and governmental outlays. The mean rate of growth of social insurance payments (GSIP) is 5.05 percent. From the CV figures, it is seen that national defence

spending (GNDF) comes first in terms of volatility. Moreover, the risk premium on the stock market index is found negatively correlated with several categories of the real growth of per capita federal outlays. These are direct payments to individuals (GDPI), total non-defence spending (GTND), and grants to state and local governments (GSLG). The negative correlation between ERM and GDPI signifies a possible insurance role for the state through the budget process. Notice also that most expenditure categories are positively related amongst themselves. A notable exception is that of national defence spending, which appears to grow at the expense of the rest of the federal programs. The observation that national defence grew by 89% in 1952 while direct payments fell by 33% a year earlier further illustrates the zero sum nature of the pattern.

Figure 3

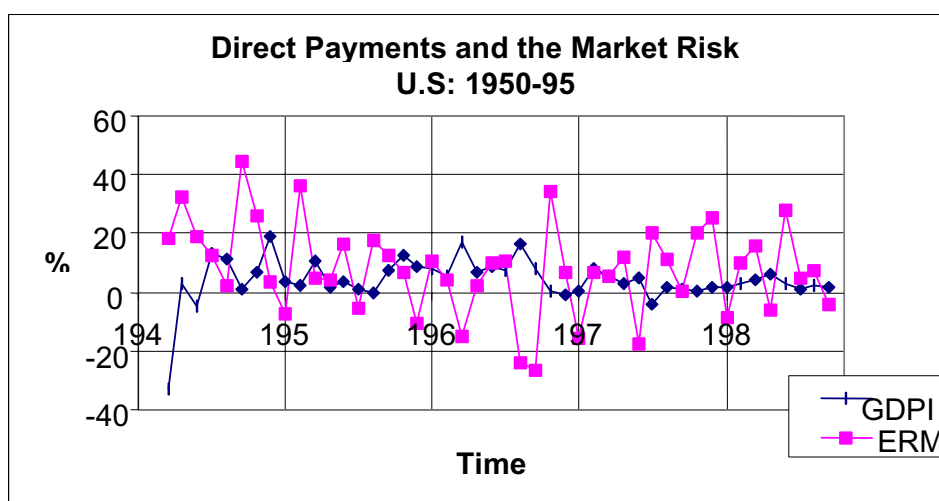


Table 4: Summary Statistics of Annual Federal Outlays, 1950-1995

Name	Mean Growth	Standard Deviation	CV	Minimum (Year)	Maximum (Year)
GSIP	5.05	8.38	1.66	-29.91 (51)	35.06 (50)
GDPI	4.94	8.99	1.82	-32.75 (51)	36.71 (50)
GSLG	6.02	6.79	1.13	-5.60 (73)	24.98 (72)
GATE	3.72	6.71	1.80	-9.68 (51)	18.10 (81)
GNDF	2.54	17.18	6.76	-16.26 (55)	88.87 (52)
GTND	3.65	7.16	1.96	-29.09 (51)	16.25 (75)
GGOV	3.13	7.47	2.39	-9.37 (54)	43.62 (52)

Notation. GGOV: Rate of growth of total governmental outlays per person; GSIP: Rate of growth in payments to individuals per person; GDPI: Percentage change in direct payments to individuals per person; GSLG: Percentage change in grants to state and local governments per person; GSIP = GDP + GSLG, GATE: Percentage change in net interest payments per person; and GTND: Percentage change in non-defence governmental outlays per person. GNDF: Real rate of growth of government defence spending. Again, all figures are net of inflation.

**Table 5: Correlation Matrix of Expenditure Variables**

	<b>ERM</b>	<b>GGOV</b>	<b>GTND</b>	<b>GNDF</b>	<b>GDPI</b>	<b>GSLG</b>	<b>GATE</b>
<b>ERM</b>	1.000	0.017 (0.911)	-0.355 (0.015)	0.198 (0.187)	-0.339 (0.021)	-0.302 (0.041)	0.076 (0.614)
<b>GGOV</b>		1.000	0.1186 (0.432)	0.753 (0.000)	0.007 (0.711)	-0.010 (0.953)	-0.023 (0.881)
<b>GTND</b>			1.000	-0.489 (0.000)	0.695 (0.000)	0.278 (0.061)	0.139 (0.358)
<b>GNDF</b>				1.000	-0.434 (0.003)	-0.204 (0.173)	-0.152 (0.314)
<b>GDPI</b>					1.000	0.303 (0.041)	-0.000 (0.999)
<b>GSLG</b>						1.000	-0.119 (0.430)
<b>GATE</b>							1.000

Notes: The p-values are shown in parenthesis below the corresponding estimated sample correlation figures.

### 3. The Valuation of Risky Flows and Discounting

*The Discount Rate.* The key focus of the present paper is on the appropriate *risk discount rate* for the public sector, which has important bearings on how one appraises the current and future budgetary requirements for public investment. Why do we necessarily relate the discussion to the stochastic properties of public revenue and expenditure? Consider public investment in projects where there exists a certain degree of market failure, such as health, education, and infrastructure. Investment in these sectors would not typically lead to direct revenue gains to the public sector in a *quid pro quo* fashion, thus rendering a direct enumeration of costs and benefits of the project difficult. Instead the investment would give rise to private sector gains via enhanced productivity of human and physical capital.<sup>13</sup> Depending on the design of the tax-transfer mechanism in force, the private gains alluded to above would lead to higher public revenue and/or reduced transfers, in each case reducing the deficit and lessening the need for further borrowing. Viewed in this light, the stochastic behaviour of the future revenue and expenditure implications of current public investment would appear to be the appropriate device for taking riskiness of the project into account. We thus depart from the Arrow-Lind hypothesis that the returns from the public investment were independent of other components of national income. Hence we propose to measure the degree of systematic risk inherent in the stream of future public revenue and spending obligations, and relate these to the determination of the social rate of discount.

We start off with the approach advanced by Hirshleifer (1964, 1966) and Sandmo (1972). The methods used by these authors can be viewed as being consistent with the standard “capital asset pricing model” (CAPM) developed by Lintner (1964), Mossin (1966) and Sharpe (1964). In order to apply this methodology, one needs to conceptualise terms such as the current *market price* of future revenue entitlement. To motivate the discussion, consider a government willing to borrow currently the market value of its future revenue claims (due  $k$ -periods from now). In the process, we assume that the purchasers of government bonds hold a diversified portfolio of assets, and consequently, when valuing the risk embedded in the bond, they would naturally evaluate the riskiness of future tax revenue. In other words, being versed in CAPM, they would “compute” the covariability of the revenue flow with the return on the market portfolio. Once a particular public revenue flow has been assigned a “price”, one is led to the idea of the return that the holder of the entitlement would earn from such a hypothetical debt instrument.

The above reasoning leads to the following equilibrium relationship:

$$(3.1) \quad \begin{aligned} \frac{E(R_{pt} - R_{ft})}{\beta_p} &= E(R_{mt} - R_{ft}), \quad \text{for } \forall \beta_p > 0, \\ E(R_{pt}) &= R_{ft}, \quad \text{for } \forall \beta_p = 0, \end{aligned}$$

where  $E$  denotes the conditional expectations based on all information currently available,  $R_{pt}$  is the “real rate of return” implicit in the (risky) revenue and expenditure flows under consideration,  $R_{mt}$  is the rate of return on the market portfolio in period- $t$ , while  $R_{ft}$  is the risk free rate of a zero-*beta* asset at time- $t$ . The *beta* is the measure of portfolio risk, and is defined as  $\beta = \{ \text{cov}(R_{pt}, R_{mt}) / \sigma_m^2 \}$ , where  $\sigma_m^2$  is the variance of the market portfolio, and the numerator is the covariance between the growth rate of the public flow in question and the return on the market portfolio.

The above relationship states that in equilibrium the expected excess return per unit of risk for any stochastic flow would equal the expected market risk premium.<sup>14</sup> Should a portfolio have an excess return per unit of risk greater (smaller) than that on the market portfolio, and should agents possess homogeneous expectations, the portfolio will be in excess demand (excess supply). Consequently, its “price” would increase (fall), and the expected excess return to fall (rise) until the equality is re-established. Thus the discount rate which explicitly adjusts for the riskiness of the revenue stream, is given by the following security market equilibrium condition:

$$(3.2) \quad \delta = E(R_{pt}) = R_{ft} + \beta [E(R_{mt} - R_{ft})]$$

*Pricing Revenue Entitlement:* Returning to the conceptual experiment of a bond transaction described above, note that rational portfolio investors would be willing to lend to the state at time- $t$  the amount  $P_t$  in exchange of the promise of the stochastic flow  $T_{t+k}$ ,  $k$ -periods later:

$$(3.3) \quad P_t = E_t \sum_{k=0}^{\infty} \frac{T_{t+k}}{(1+\delta)^{k+1}}$$

In the expression above,  $P_t$  may be interpreted as the current “market price” of a claim on the future revenue stream, and  $\delta$  is the risk discount rate as explained above.<sup>15</sup>

*Return to Revenue Entitlement:* Once we have determined that a risky stream  $\{T_{t+k}\}$  has a current market value of  $P_t$ , one may ask what the implicit rate of return is to the investor holding the debt. Clearly the standard answer is the *internal rate of return (IRR)*. To illustrate the use of this concept, it may be instructive to consider some special cases in an explicit stochastic environment. Consider a particular stochastic flow,  $\{T_{t+k}\}$ , which is independently and identically distributed through time. The market value would then be (from 3.3):<sup>16</sup>

$$(3.4a) \quad P_t = \{T_0 / \delta\}, \text{ where } T_0 = E\{T_t\} = E\{T_{t+k}\}, \text{ all } k.$$

In this simple context, the debt yields a return of  $\delta$  per dollar of investment,  $P_t$ . Thus once we use a pre-determined and *constant* magnitude,  $\delta$ , as the discount rate, it also serves as the implicit rate of return, or the IRR.

How does the above valuation change when we allow a secular growth in the stochastic flow,  $T_{t+k}$ , such that  $T_j = (1+g)\{T_{j-1}\}$ , all  $j$ ? In other words, the growth rate is taken to be independent of the state of nature. Again retaining the hypothesis of identically distributed risks, i.e.,  $E\{T_{t+k}\} = \{(1+g)^k E(T_t)\}$ , all  $k$ , we obtain:

$$(3.4b) \quad P_{gt} = \{T_0 / (\delta - g)\},$$

where, as before,  $T_0 = E\{T_t\}$ . Were  $g$  to be *positive*, the investor would value the public flow higher, and thus would pay a larger amount than in (3.4a), i.e.,  $P_{gt} > P_t$ . It would be conceptually helpful to re-write (3.4b) in two equivalent forms:

$$(3.4c) \quad T_0 / P_{gt} = (\delta - g),$$

Or, 
$$(3.4d) \quad \{(T_0 / P_{gt}) + g\} = \delta.$$

In either case, the left hand side is in the nature of an annualised expected return on an investment, while the right hand side is the yield per dollar of investment on an alternative asset that offers a

similar pattern of revenue. It is a matter of indifference whether we describe the valuation as one yielding a return of  $(\delta-g)$  per dollar invested [*and* view the corresponding annual flows to be  $T_0$ , as in (3.4c)], or, one yielding a rate- $\delta$ , with the expected income growing at the non-stochastic rate- $g$ , as in (3.4d). The latter is perhaps more natural.

The actual public flows we work with do not however behave like perpetuities, and thus actual growth,  $(g_t)$ , would be time dependent. (We, of course retain the hypothesis that risks are independently distributed over time.) Moreover, since the measurement of covariance between a risky public flow and the market premium is an issue we pursue below, we find it convenient to write the expected annual return net of the risk free rate. Consequently, the above equilibrium relationship, in view of (3.2), may be restated as:

$$(3.4e) \quad E(T_t / P_{g_t} + g_t - R_{ft}) \equiv E(R_{it} - R_{ft}) = \beta [E(R_{mt} - R_{ft})],$$

where,  $R_{it}$  is interpreted as the return on investment inclusive of the growth element. Thus there is a linear relationship between the expected return on investment, net of the risk free rate, and the market risk premium.

Evidently, if some tax revenue stream were actually independent of any systematic risk, the appropriate discount rate would be the rate on risk free assets, namely  $\delta = R_{ft}$ , as the market beta would be zero. An example may be the wage tax revenue. It is plausible that in a diversified economy such revenue would become riskless in the aggregate.<sup>17</sup> At the other extreme, Hirshleifer and Sandmo highlight a result where public investment leads to a pattern of revenue that perfectly covaries with the market portfolio (and, hence, a beta of unity). The public discount rate, therefore, coincides with the rate applicable to private projects with the identical market risk premium. In such a world, the public sector must emulate private firms in order to achieve allocative efficiency.

There is however a clear parallel between the approaches of Arrow-Lind, Hirshleifer-Sandmo and ours. In all these contexts the rate of discount applicable to a public investment is argued to depend, at least in principle, on the type of investment. While the first two sets of authors' belief in the interdependence of the yield in public and private investments differ, we devise a mechanism that enables us to put the debate to an empirical test.<sup>18</sup> Like Hirshleifer-Sandmo, we agree that the stochastic behaviour of the revenue yielded by the public investment in question (*vis-à-vis* the market premium) is the determining factor. Moreover, we go on to motivate that an evaluation of the particular tax revenue flow being augmented (or, for that matter, of the expenditure category being alleviated) by the said public investment is a plausible way of going about the measurement of the risk of public investment yield.

#### 4. Empirical Results

We have already noted that for econometric purposes, it is instructive to analyse the underlying variability in a flow by studying its growth rate. In the CAPM framework, one usually measures the covariability between the excess return on the market portfolio and that of another asset. Correspondingly, we work with the growth rate in the public sector flows when adjusted for the risk-free rate.<sup>19</sup> It so happens that most of the revenue-expenditure series under review here yield positive net growth in real terms over the whole period. However it is important to stress that these "excess growth" figures in public flows do not arise out of a trade in the corresponding revenue base, and hence are devoid of any useful normative interpretation. However, to the extent the underlying revenue base (or expenditure category) is subject to variability caused by aggregate risks facing the economy, there will be a correspondence between these observed growth figures and the market risk premium. The object of our econometric work is to isolate and interpret these latter effects.

Following equation (3.4e) we posit that the true, but unobservable, theoretical risk premium associated with a public flow is linearly related to the market risk premium over the sample period:

$$(4.1) \quad (R_{i(t+1)}^* - R_{f(t+1)}) = \beta_i (R_{mt} - R_{ft}) + e_{i(t+1)}.$$

In the expression above, the dependent variable, namely,  $(R_{i(t+1)}^* - R_{f(t+1)})$ , is the true unobservable risk premium associated with the source- $i$  at time  $(t+1)$ , and  $e_{i(t+1)}$  is a random element. For reasons already alluded to above, the estimation process is likely to encounter the *measurement error problem* for both the dependent and the independent variables. First, let us take the dependent variable. The true unobservable risk premium associated with the tax revenue or outlays would be measured with an error due to lags in revenue collection and in implementing spending programs, noise arising from tax code and rate schedule changes etc. Focussing on taxes, we may generally write the revenue function as

$$(4.2) \quad T_t = F(\tau_t) B_t(F(\tau), N, Z, P),$$

where  $T_t$  is the real tax revenue the government receives from a given source,  $F(\tau_t)$  is the tax schedule, and  $B_t(F(\tau), N, Z, P)$  denotes the tax base. In addition to the tax schedule itself and the level of population,  $N$ , the tax base is modelled as dependent on the state of nature ( $Z$ ) and, say, productivity shocks ( $P$ ). The realised rate of growth of the actual revenue stream, therefore, may be decomposed into the components as shown in (4.3) below.

The first term,  $\% \Delta F(\tau_t)$ , is the direct impact of the percentage change in the tax schedule on the stochastic rate of growth of the revenue stream. The second term is the growth in the base due to



$$\begin{aligned}
(4.3) \quad R_{i(t+1)} = & \% \Delta F(\tau_t) \text{ due to tax schedule change} \\
& + \% \Delta B_t \text{ due to population growth} \\
& + \% \Delta B_t \text{ due to dis(incentive effects)} \\
& + \% \Delta B_t \text{ due to changes in tax code} \\
& + \% \Delta B_t \text{ due to state of economy and productivity shocks}
\end{aligned}$$

population growth. The third term is the indirect impact of the (dis) incentive effects of changes in the tax schedule on the tax base,  $B_t$ . Although the sign of the first term depends on the direction of the actual tax change, the sign of the third term is generally ambiguous at the analytical level. However, most researchers believe that *increased* taxation does create disincentive effects, especially in the area of labor and capital income taxation, and hence would lead to a long-run reduction in the growth of the revenue streams. The fourth term represents the impact of direct changes in the tax base due to tax code changes.

The final term is the direct impact of the state of the economy, and it is this quantity that may be associated with the "risk premium"; it does measure the degree of variability in the revenue growth that is associated with the business cycle. This however is not observable, and we proceed by expressing the observed revenue growth as follows:

$$(4.4) \quad R_{i(t+1)} = R^*_{i(t+1)} + v_{i(t+1)}$$

where  $R^*_{i(t+1)}$  is the unobservable component due to the business cycle, and  $v_{i(t+1)}$  is a randomly and independently distributed error term. Evidently the latter would depend on changes in the tax schedule, population growth, the incentive effects of the tax rate change on its base, and the impact of the tax code or tax base changes. We further assume that this "error" term has a constant mean,  $\alpha_i$  and variance  $\sigma_v^2$ . We first remove the population growth rate from the observed growth in public flows (except for the corporate tax revenue). [Correspondingly, we net out the rate of change of employment from the growth rate of social insurance contributions.] These steps allow us to remove the secular growth rate, as was discussed earlier.<sup>20</sup>

Even after these adjustments, the mean of the error term could be non-trivial if, for example, a particular tax system causes a reduction (increase) in the average growth rate of the revenue stream due to strong disincentive (incentive) effects. Consequently, if the least square technique were applied when the dependent variable is measured with error it will lead to increased standard errors, though the estimated "riskiness" coefficient would remain unbiased and consistent. In view of (4.1) and (4.4), the estimating regression becomes:

$$(4.5) \quad R_{i(t+1)} - R_{f(t+1)} = \alpha_i + \beta_i (R_{mt} - R_{ft}) + w_{i(t+1)} + e_{i(t+1)}$$

As one can observe the estimation (4.5) could result in a biased constant term. The bias would depend on the expected mean value of the measurement error. In the base case, whereby the mean measurement error is equal to zero, the terms of (4.3) are each zero, except for the last term, and hence the observed growth should be proportional to the market risk premium.<sup>21</sup> In the latter scenario, the model reduces to that given by equation (4.1) since it would have been possible to filter out all the "noise" from the actual public revenue and expenditure growth rates. This is very hard to accomplish in practice. If the first term of (4.3), the rate of change of the tax rate, has an average value that is non-zero and all the other terms were negligible, and we did not filter out this "noise", our estimation would yield a significant intercept term with a zero beta coefficient. The situation gets muddier if we also allow the other components to play a non-trivial role.<sup>22</sup> In principle, one may develop methods that permit a role for additional components affecting the rate of growth of the revenue stream, and indeed an attempt is made to use a more general asset pricing model below. However, in this section of the paper, we stay with the standard CAPM.

The problem of using least squares will be more severe if the independent variable were also measured with error. This would be the case if the return on the market portfolio (i.e., portfolio that includes all assets, tradable and nontradable) were not the same as the excess return on the S&P500 index. In this instance, the estimated coefficient for the riskiness term will be both biased and inconsistent. Since we believe that the true market portfolio consists of assets beyond those traded in the market, we provide a correction of the measurement problem by means of the method of group averages.<sup>23</sup> Accordingly, we estimate the model by first ordering the observed pairs by the magnitude of the independent variable and then allocate these among three groups of approximately equal size. The riskiness estimator is then given by:

$$(4.6) \hat{\beta} = \frac{(\overline{R_{i(t+1)} - R_{f(t+1)}})^3 - (\overline{R_{i(t+1)} - R_{f(t+1)}})^1}{(\overline{R_{mt} - R_{ft}})^3 - (\overline{R_{mt} - R_{ft}})^1}$$

Before we turn to the results, a few more comments on the estimator are in order. The estimated *beta* can often be negative in which case the revenue implications of a public investment project happen to be negatively correlated with the market return. The latter eventuality may arise on the expenditure side of the budget (e.g., in the case of unemployment insurance benefits). This would be a clear example of a systematic risk item that provides additional insurance to society. Hence, an investment that led to alleviation of such spending ought to be discounted at a rate below the risk free rate. Furthermore, we note that one major difference between the beta of a private investment project and

that for the risky national revenue (or, for that matter, uncertain public outlays) is that the latter is unlikely to be greater than one.<sup>24</sup> This is rather intuitive; since the tax revenue derives from a basket of collateral assets (i.e., tax bases), not all being traded, which places an upper limit to the estimated *beta*.

The estimation results are summarised in Table 6 below.<sup>25</sup> Since we have chosen to adopt the S&P index as the market portfolio, we also present a measure of the beta between the former and another potential reference portfolio, namely the Dow Jones index. The latter representing a well diversified portfolio of assets ought to have an intercept equal to zero, a beta of unity, and have a very high degree of correlation with the S&P 500 index. The empirical evidence does bear this out; we obtain a beta of unity and the degree of correlation between the two indices is about 96 percent. The above is a useful backdrop against which to interpret our results. Our estimates suggest that there is *systematic* risk associated with the individual and corporate revenues, direct payments to individuals, total tax revenue and the overall budget balance.<sup>26</sup> On the taxation side, both the social security contributions and PIT are among the least risky sources of revenue. Among the expenditure items, total government spending, and interest payments to service debt contain no apparent systematic risk.

Though systematic risk appears to affect mainly the revenue side of the federal ledger, even then the relevant beta coefficients are, as expected, each less than unity. The corporate tax revenue stream, not

Table 6: CAPM Estimates of  $\beta$ 's for Tax Revenues and Outlays using the Method of Group Averages: 1950-95<sup>27</sup>

Name	$\alpha_i = E(v)$	$H_0: \alpha_i = 0$	Beta	$H_0: \beta = 0$	Adj	Var( $e_{it}$ )	$H_0: \beta = 1$	D-W
		$H_a: \alpha_i \neq 0$ t-stats		$H_a: \beta > 0$ t - stats			R-Sq	
ERDJIA	-0.772	-1.233	1.040	24.950 ***	0.932	17.34	0.97	1.93
Taxation Items								
GRPIT	0.721	0.506	0.189	2.071 **	0.068	69.22	-8.76 ***	2.19
GRCIT	-1.695	-0.695	0.647	4.136 ***	0.264	203.05	-2.25 **	2.08
GRSIC	2.818	2.446 **	0.160	2.165 **	0.076	45.32	-11.35 ***	2.41
GRT	-0.098	0.083	0.248	3.258 ***	0.176	48.05	-9.87 ***	2.16
Expenditure Items								
GDPMT	5.050	3.757 ***	-0.236	-2.738 ***	0.126	61.70		1.75
GGLSG	5.759	4.726 ***	-0.148	-1.892 *	0.054	50.72		1.83
GND	-0.889	-0.283	0.272	1.349	0.018	336.14		2.15
GATE	2.419	2.224 **	0.028	0.401	-0.019	40.42		1.70
GRTND	4.379	3.679 ***	-0.251	-3.292 ***	0.179	48.38		2.43
GGOV	1.928	1.361	-0.007	-0.072	-0.023	68.50		2.06

Note: \*\*\* indicates that the beta is statistically significant different from the null hypothesis at the 1 percent level of significance, while \*\* does so at the 5 percent level and \* at the 10 percent level.

surprisingly, has the highest beta, a value of 0.65. Given the standard error, we are able to reject that it is equal to unity at the *five-percent* level.<sup>28</sup> In Figure 4, we provide an illustration of the best-fit line, which clearly does not coincide with the 45-degree line from the origin. The evidence seems to support the proposition that the corporate tax revenue is less than perfectly correlated with the market portfolio. Moreover, the constant term [i.e.,  $\alpha_i$  in eq. (4.5)] is not statistically different from zero. Thus *we find evidence that changes in the tax code/schedule do not affect the mean growth of the CIT revenue*. Given that actual tax rates are not state contingent, these observations may be seen as consistent with Gordon's analytical result (1985) that corporate investment incentives remain largely unaffected by corporate taxation. The relatively low R-squared value, however, is caused by measurement errors in the data leading to a high standard error of the regression.

The total revenue stream is significantly *less* risky than the market portfolio as shown by a beta of 0.248. The rate of growth of payroll (i.e., the social insurance) tax has the least systematic risk (with a beta of 0.16) among the revenue items. However, the constant term is significantly different from zero, which suggests that changes in the structure of the social security tax rate over the period may be the contributing factor.

On the other side of the ledger, direct payments yield a beta of (-) 0.24. It is very intuitive that if the return on the market portfolio were to fall in a given period, future output and employment will be expected to decrease, and as a result payments to individuals (e.g., unemployment insurance and welfare payments) would be predicted to rise. Continuing with the expenditure categories, note that transfers to state and local governments do appear to be less risky than direct payments to households. One should be careful in interpreting the apparent absence any systematic risk in total government spending (GGOV); offsetting influences of defence spending and, say that of transfer payments to households may well neutralise any inherent risk.<sup>29</sup>

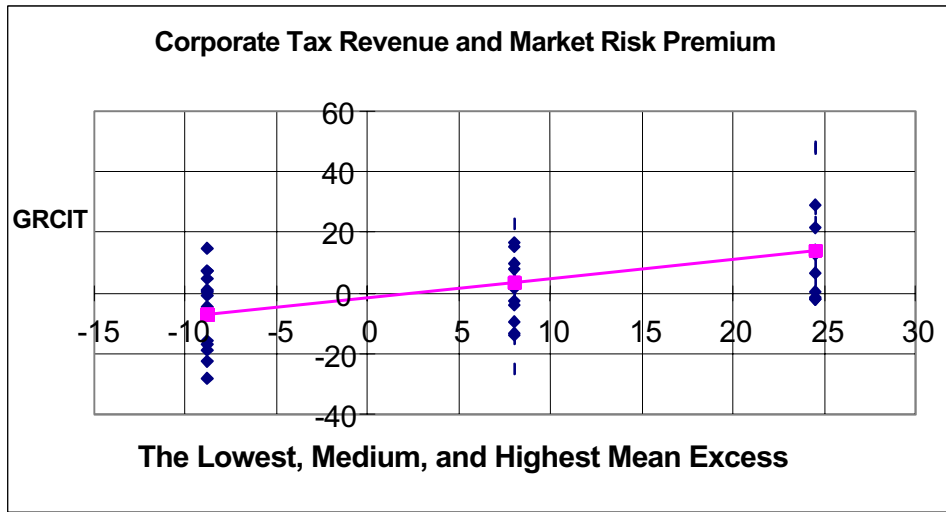
## 5. Macroeconomic Factors and the Volatility of Budgetary Flows

In the preceding section we exclusively focused on the hypothesis that systematic *capital* market risk would explain the riskiness of budgetary flows. Presently we broaden our approach, and explore the extent to which the stock market and the revenue-expenditure flows may each react to movements in macroeconomic variables. This leads us to adapt the *arbitrage pricing theory (APT)* due to Ross (1976) to the task at hand. The APT model measures an asset's riskiness using the "beta" sensitivities to macroeconomic factors. More specifically, the APT model assumes that the rate of return on an asset is a linear function of k factors:

$$(5.1) \quad R_{it}^* = E(R_{it}) + b_{i1}F_{1t} + b_{i2}F_{2t} + \dots + b_{ik}F_{kt} + u_{it}$$

where  $R_{it}^*$  is the  $i$ -th asset's rate of return in period  $t$ , and the constant term is the mean return on the asset, which is conditional on all relevant and available information. The  $F_{kt}$  is a zero mean  $k$ -th factor common to the return on all assets under consideration, while the  $b_{ik}$  is the  $i$ -th asset's "beta" sensitivity to the  $k$ -th systematic factor, and  $u_{it}$  is the idiosyncratic risk of asset  $i$  where  $E(u_{it})=0$ , and  $E(u_{it}^2)=\sigma_i^2$ . Recall that the "asset returns" in the present context are the growth behaviour of tax revenues, public expenditure categories, and the stock market indices.

**Figure 4:** Scatter diagram and best-fit line of the risk premium on the Corporate Tax revenue



The common factors encountered in the finance literature generally include inflation, output, the risk premium and the term structure of interest rates [e.g., see Chen, Roll and Ross (1986) and Burmeister and Wall (1988)]. Proceeding in the latter tradition, our investigation first lead to only one common factor, namely that of short run movements in output growth per person, which appears to be relevant to the growth behaviour considered in the paper.<sup>30</sup>

We use a simple forecasting equation to determine the common factor. We hypothesise that forecasts of expected growth rate of the GDP per person at time- $t$ ,  $E(g_t)$ , is based on the sample mean:<sup>31</sup>

$$(5.2) \quad g_t = a_0 + F_{1t}$$

where  $F_{1t}$  is the factor in question. The estimated constant term will give us an estimate of the long run growth of output per person, the sample mean in our case. The results from this estimation are presented below:

$$(5.3) \quad t = 2.12, \quad (t\text{-stat: } 5.74)$$

The macro factor is therefore proxied by the difference between the actual and the estimated trend GDP growth, i.e.,

$$(5.4) \quad F_{1t} = g_t - \hat{g}_t$$

Thus the one-factor APT model that we estimate next may be restated as:

$$(5.5) \quad R_{i(t+1)} - R_{f(t+1)} = \alpha_0 + \alpha_{1t} F_{1t} + u_{it},$$

where the left-hand side denotes the real net rate of growth in the variable-*i* at time-(*t*+1). For the budgetary flow categories, the net rate should ideally remove all elements other than those subject to influence by the systematic risks considered in the model, i.e., those related to physical and human capital.<sup>32</sup> The implication is that,  $F_{k_s}$  being zero mean variables by construction, the constant term would have the interpretation of risk premium. And, consequently, the coefficients,  $\alpha_k$ , would be interpreted as the relevant "beta". However, in the present case, the constant term reflects both, the risk premium and the mean measurement error:

$$(5.6) \quad R_{i(t+1)} - R_{f(t+1)} = \alpha_0 + \alpha_l + \alpha_{1t} F_{1t} + w_{it} + u_{it}.$$

Table 7 reports the "beta" sensitivities of tax revenue, governmental outlays and the market index to the zero mean GDP factor,  $F_{1t}$ . We again use the method of group averages because we suspect that the estimation of the true macroeconomic factor may involve measurement errors. We obtain new insight from the evidence presented. While the corporate tax revenue and the behaviour of the stock market rate of return was seen earlier to have been one of strong co-variability (i.e., a beta statistically close to unity), they are affected quite differently by the common macroeconomic factor. A one-percent reduction in economic growth in period-*t*, relative to what the investors predicted, will lead to a *two*-percent reduction in the market excess return in period-*t*, (i.e., the relevant *beta* sensitivity being 2.02). And much the same goes for PIT and total revenue growth.<sup>33</sup> However, the same magnitude of unexpected reduction in economic growth leads to a much larger reduction (*five*- percent) in the growth rate of corporate tax revenue. (The *beta* here is 4.90.) Clearly these estimates imply that the consequences of an unanticipated output shock are rather dramatic, especially for corporate revenue.

Finally, in order to determine whether the public sector flows have a high beta vis-à-vis the unexpected movements in the stock market, we add a *second* factor. The latter reflects the unexpected change in the risk premium on the S&P 500 index, i.e., the component that is not explained by the preceding common macroeconomic factor (GDP shock):

$$(5.7) \quad F_{2t} = R_{mt} - R_{ft} - E(R_{mt} - R_{ft} / F_{1t})$$

Consequently, the estimating equation becomes:

$$(5.8) \quad R_{i(t+1)} - R_{f(t+1)} = \beta_0 + \beta_{1t} F_{1t} + \beta_{2t} F_{2t} + w_{it} + u_{it}.$$

The purpose of including this variable in the estimating model is to attribute explicitly the total risk into a part that is due to the macroeconomic factor *and* that due to the stock market. The results are presented in Table 8. As expected the adjusted R-squared has increased since the added variable is significantly different from zero in practically all cases. We again find that the rate of change of budgetary flows is still sensitive to unexpected movements in the stock market, *but the "beta" sensitivities are all significantly lower than encountered earlier in the CAPM framework.* These coefficients are well below unity for *all* variables, except of course, for the Dow Jones index where the estimated beta is precisely equal to unity. A 10 percent unexpected reduction in the stock market return, due to factors other than GDP deviations from trend, is seen to lower the corporate tax revenue by about four percent. This is sharply lower than the figure of 6.5 percent found in the context of the standard CAPM.

**Table 7: The One Factor APT model using the Method of Group Average**

Name	Expected Growth	Sensitivity: Unexpected GDP Shocks	Adj R-Sq	D-W
ERM	7.910	2.200 **	0.070	1.97
H <sub>0</sub> : $\alpha_{11} = 0$ , t-stats	3.591 ***	2.097 **		
GRCIT	3.423	4.903	0.379	2.14
H <sub>0</sub> : $\alpha_{11} = 0$ , t-stats	1.774 *	5.331 ***		
GRPIT	2.217	2.021	0.230	2.04
t-stats	1.989 **	3.804 ***		
GRT	1.863	2.023	0.300	2.13
t-stats	1.978 **	4.507 ***		
GSIC	4.083	0.765	0.032	2.06
t-stats	4.020 ***	1.581		
GDPMT	3.182	-1.569	0.138	1.36
t-stats	2.766 ***	-2.862 ***		
GGLSG	4.589	-0.334	-0.013	2.44
t-stats	4.223 ***	-0.646		
GND	1.259	2.377	0.053	1.84
t-stats	0.474	1.879 *		
GATE	2.641	0.338	-0.009	1.90
t-stats	2.831 ***	0.761		
GRTND	2.391	-0.491	-0.004	1.74
t-stats	2.110 **	-0.909		
GGOV	1.876	0.710	0.012	1.91
t-stats	1.564	1.243		

Notes. The asterisks beside t-stats bear the usual interpretation as described above (e.g., Table 6).

Our analysis of the US federal budgetary flow data therefore reveals that the corporate revenue and total tax revenue are both very sensitive to the systematic capital market risks. Unanticipated GDP fluctuations appear to exert strong influence, quite independent of the stock market deviations, on the market indices, as well as on several major budgetary flows, including the CIT, PIT, total revenue and

transfers to households. We also observe that even though corporate tax revenue appears to be particularly risky, the government by collecting revenue from many different sources presents a more diversified portfolio than the market, thereby incurring a very modest risk premium. Taken in conjunction with the evidence of risk shifting via the social insurance system, our results suggest that the state may indeed be able to absorb the revenue-expenditure risks at a lower cost than private individuals who hold a diversified portfolio of traded assets.

## **6. Discount Rates based on Estimates from Asset Pricing Models**

The estimates based on asset pricing models may be utilised to predict the implied risk premium for public investment financed out of tax revenue, which are shown in Table 9 below. We present results based both on CAPM (Table 6) as well as the APT (Table 8) formulations. Not surprisingly, the corporate tax revenue has the highest estimated risk premium, namely 5.1 and 3.5 percent, respectively, under CAPM and APT. The latter estimates translate to a discount rate of 6.4 and 4.8 percent, respectively. While significantly lower than the private market figure of 9.2 percent, these rates are sharply higher than the real risk free rate of 1.3 percent.

The personal income tax has a much lower risk premium of 1.5 percent (CAPM) or lower (1.0 percent under APT), but these estimates are not particularly robust. Total revenue, which yield strongly significant estimates, point to discount rates in the range of 2.7 to 3.2 percent in the two cases highlighted here. The latter rates are considerably lower than any plausible figures one would have for private risky projects, where many projects would likely be even riskier than the market portfolio. Thus even though the state carries some systematic risk in the budget constraint from the revenue system as a whole, the implied premium is rather small in absolute terms.

Returning to the Arrow-Lind contribution, it is interesting to observe that the difficulties raised by the corporate tax was noted, though not dealt with, by these authors (p.366). Our analysis suggests that, given a well functioning tax system, it would be erroneous to continue to believe in the plausibility of the contemporaneous independence of returns from a public project and GDP. In the context of the hypothesis advanced here, namely that the pecuniary benefits accruing to the public sector from a public investment have the stochastic behaviour of the general (i.e., total) tax revenue, we reach a simple conclusion. Though small in absolute margin, *the risk discount rate for the public sector would exceed the risk free rate, possibly by a multiple in excess of two.*



**Table 8: The General APT Model using the Method of Group Averages: 1950-1995**

Name	Expected Growth	Beta Sensitivity: Unexpected GDP Shocks	Sensitivity: Unexpected “Stock Market” Shocks	Adj. R-sq	D-W
ERM	7.91	2.200		0.070	1.97
$H_0 : \alpha_i = 0$ , t-stats	3.59 ***	2.101 ***			
ERDJIA	N.A	1.442	1.040	0.931	1.93
$H_0 : \alpha_i = 0$ , t-stats	N.A	5.739 ***	23.99 ***		
$H_0 : \alpha_{22}=1$ , t-stats			0.980		
GRCIT	3.423	4.903	0.446	0.529	2.34
$H_0 : \alpha_i = 0$ , t-stats	2.038 **	6.124 ***	3.881 ***		
$H_0 : \alpha_{22} = 1$ , t-stats			-4.814 ***		
GRPIT	2.217	2.021	0.126	0.261	2.12
$H_0 : \alpha_i = 0$ , t-stats	2.030 **	3.883 ***	1.685 *		
GRT	1.863	2.023	0.177	0.407	2.20
$H_0 : \alpha_i = 0$ , t-stats	2.148 **	4.894 ***	2.980 ***		
GSIC	4.083	0.765	0.155	0.368	2.51
$H_0 : \alpha_i = 0$ , t-stats	4.222 ***	1.660 *	2.349 **		
GDPMT	3.182	-1.569	-0.130	0.173	1.26
$H_0 : \alpha_i = 0$ , t-stats	2.823 ***	-2.922 ***	-1.686 *		
GGLSG	4.589	-0.334	-0.136	0.042	2.36
$H_0 : \alpha_i = 0$ , t-stats	4.343 ***	-0.664	-1.880 *		
GND	1.259	2.377	0.144	0.045	1.87
$H_0 : \alpha_i = 0$ , t-stats	0.472	1.871 *	0.787		
GATE	2.641	0.338	0.019	-0.031	1.88
$H_0 : \alpha_i = 0$ , t-stats	2.801 ***	0.753	0.291		
GRTND	2.391	-0.491	-0.171	0.086	1.63
$H_0 : \alpha_i = 0$ , t-stats	2.209 **	-0.953	-2.311 **		
GGOV	1.876	0.710	-0.029	-0.008	1.90
$H_0 : \alpha_i = 0$ , t-stats	1.564	1.230	-0.353		

## 7. Conclusion, Extensions and Further Work

Until now the literature on the proper risk discount rate for public investment has been of an analytical nature. Even then Arrow-Lind wondered if the correlation between returns to a public investment and other components of national income may be so high as to invalidate their analytical result which presumed no correlation (p. 373, 1970). Built on the hypothesis that stochastic behaviour of the benefits of (public) investment accruing to the public sector has the characteristics of budgetary flows, we have provided an analysis of the inter-face between the riskiness inherent in the latter and that exhibited by the stock market returns. We find plausible evidence that the said correlation is highly significant, even though the implied risk premium is fairly small in absolute terms. Whether one views these estimates as having refuted the original Arrow-Lind hypothesis of no correlation, remains an issue for further investigation, presumably utilising more complete constructs of the market risk.

Sandmo had earlier observed that the extent of the approximation of the Hirshleifer-Sandmo model of an ideal stock market economy to the real world was an awkward empirical question (1972, p. 296). We do suggest a procedure and, in our interpretation, the public returns from investment in infrastructure and the like do not match that of any “comparable” private projects, “having the same proportionate time-state distribution of returns” (Hirshleifer, 1966, p272). Government revenue appears to have covariability with market risk in physical capital that is significantly less than *perfect*. Indeed the largest risk premium applying to a public sector flow, i.e., the case of corporate revenue, is seen to be about *half* as large as the market risk premium, the latter presumably being the lower bound to the risk of a private investment project. Public investment projects would thus be candidates for a lower discount rate than the rates prevailing in the private sector.

Our final remark addresses the issue of the appropriate measure of market risk. The stock market indices appear to respond rather significantly to fluctuations in economic aggregates such as GDP deviations from trend. So do most major revenue-expenditure flows. One would thus be tempted to conclude that the stock market risks are an inadequate description of the full gamut of the systematic risk confronting our economic environment. Thus even if it were to access resources at a lower discount rate than the private sector, this need not by any means suggest that the government is enjoying a free lunch; the resulting additional risk shifting through the budget process is the offsetting ledger entry.

Table 9: Risk Premiums and Social Discount Rates for the U.S: 1950-1995, Percentages

Items	Risk Premium CAPM Model 1950-1995	Risk Premium CAPM Model 1952-1995	Risk Premium APT Model 1950-1995	Risk Premium APT Model 1952-1995	Discount Rate CAPM Model 1950-1995	Discount Rate APT Model 1950-1995
Market Risk Premium	7.91	7.11	7.91	7.11	9.17	9.17
Corporate Tax Revenue	5.09 ***	3.91 ***	3.53 ***	2.96 ***	6.35	4.79
Personal Income Tax Revenue	1.50 **	0.08	1.00 *	0.56	2.76	2.26
Social Insurance Contributions	1.26 **	0.93	1.23 **	0.98 **	2.52	2.49
Total Revenue	1.96 ***	1.12	1.40 ***	0.99 ***	3.22	2.66
Direct Payments	(1.87) ***	(0.98) **	(1.03) *	(0.89) **		
Grants to State and Local governments	1.17 **	(0.74)	(1.07) *	(0.85)		
Net Interest Payments	(0.22)	0.42	0.25	0.25		
Defense Spending	2.15	(0.26)	1.14	(0.51)		
Total Non Defense Spending	(1.98) ***	(1.15) **	(1.35) **	(0.99) **		
Government Spending	(0.05)	(0.62)	(0.23)	(0.82) **		

## Endnotes

<sup>2</sup> This is especially true for individuals who face high transaction costs, or who are liquidity constrained, or are not aware of the diversification possibilities that the market offers, or when the share prices become too volatile relative to what the fundamentals dictate.

<sup>3</sup> The idea of the portfolio of assets traded in the stock market being used as a proxy for the market portfolio has been challenged by Roll (1977) as the former excludes the non-traded assets (e.g., unlisted assets or claims thereto). However, recently J. Campbell (1996) has re-defined the market portfolio to include the return on human capital.

<sup>4</sup> Shiller observes that corporate dividends account only for three percent of national income. However focusing exclusively on this low value is misleading for a number of reasons. Dividends are not the sole, or even the primary, form of return from share holding.

<sup>5</sup> An alternative approach would be to posit a model of intergenerational risk shifting where revenue risks even if perfectly correlated with the market portfolio would be viewed idiosyncratic by a representative generation, given the belief that the latter's own revenue risks would be borne by all future generations. See Gordon and Varian (1988) and Ahsan and Tsigaris (1998) for further elaboration.

<sup>6</sup> We abstract from the fact that many firms organised as corporate entities for tax purposes are closely held, and hence not traded

<sup>7</sup> The tax revenue and government expenditure data were obtained from the U.S. public accounts.

<sup>8</sup> The real rates of growth of the various tax revenues in this section, except for corporate tax revenue, have been adjusted for population growth. The social insurance taxes have been adjusted for employment growth.

<sup>9</sup> Sobel and Holcombe (1996) also find evidence that, of all revenue categories, corporate tax base has been the most volatile over the business cycle.

<sup>10</sup> The stock market data were obtained from Robert Shiller's home page.

<sup>11</sup> The high variability of the CIT revenue (*vis-à-vis* the market risk premium, ERM) would appear to run counter to the intuition that by allowing *ex-ante* depreciation, CIT revenue may not adequately share in the capital risks (namely, economic depreciation) embodied by capital assets. Below we shall see that the estimating CAPM equation for CIT also yields an insignificant error term, implying a negligible role for changes in the tax code/schedule. See Bulow-Summers (1984) for the distinction between 'income' and 'capital' risks for physical capital.

<sup>12</sup> These results were obtained by running regressions between the variables after they had been standardised (i.e., all variables were expressed as deviations from their respective mean and divided by the standard deviation). The slope coefficients represent the sample coefficient of correlation between the original variables.

<sup>13</sup> It is usual to make a distinction between *public* and *private* benefits and costs arising out of a public investment project. The issues of the social discount rate applies only to the component accruing to the public sector (Arrow-Lind, 1970).

<sup>14</sup> Sandmo's condition for optimal investment required the marginal product of capital to equal the future value of the market risk premium since he effectively let the beta be unity (1972, equations 32 and 33).

<sup>15</sup> In the finance literature this basic valuation formula has been disputed on grounds that asset prices display excessive volatility relative to the anticipated stream of future dividends. (See Shiller, 1981.)

<sup>16</sup> We require that the state balance the budget in the long run.

<sup>17</sup> While such a conclusion would appear to be formally similar to the Arrow-Lind result (1970), the methods are different. The hypothesis here is, much in the spirit of Samuelson (1964) and Vickrey (1964), that the risk is totally idiosyncratic, and by the law of large numbers, government can effectively pool across individual differences. The Arrow-Lind argument, on the other hand, lets the state invest in a variety of projects whose returns are assumed to be uncorrelated with private capital.

<sup>18</sup> So far the discussion in the text has remained within the perfect markets case as required by CAPM. It must be noted that the deeper aspect of the Arrow-Lind contribution deals with incomplete markets. They show that so long as the risks are “publicly borne” by many agents, the total costs of risk bearing are negligible. Even this result, however, requires the assumption of contemporaneous independence of returns cited above.

<sup>19</sup> In empirically determining the systematic risk, we remove the risk free rate from the growth rate of revenue flows, which leaves the residual in a form analogous to the market risk premium. However this step is not necessary for the corporate tax revenue due to the deductibility of interest payments allowed in the tax code.

<sup>20</sup> Appendix 1 presents the results with population growth included in the revenue growth figures.

<sup>21</sup> For example, Summers and Bulow (1984) argued that corporate tax revenue should have an expected return equal to the market risk premium  $E(R_{it+1})=E(R_{mt}-R_{ft})$ , and a beta of unity. Thus the certainty equivalent (or, the market value) of this revenue flow if re-distributed back to the same generation of taxpayers would be zero.

<sup>22</sup> Notice that ordinarily the change in the tax disincentive effects, which alters the effective base, would be indistinguishable from the direct manipulation of the base by the Treasury.

<sup>23</sup> One could have used the instrumental variable technique but there aren't many variables that are highly correlated with the excess return on the market and, at the same time, un-correlated with the error term.

<sup>24</sup> This is true for the country as a whole but it is not true if we had examined each state's corporate tax revenue streams since they may well be industry specific.

<sup>25</sup> The method of group averages can be viewed as a special case of the instrumental variables method. The instrument used is such that it equals -1 if the pair belongs in the 1<sup>st</sup> group, 0 if it belongs to the 2<sup>nd</sup> group and 1 if it belongs to the 3<sup>rd</sup> group. For more details, see Kmenta's text, "Elements of Econometrics".

<sup>26</sup> The net social insurance contribution is the real growth rate of social security tax less the real growth of social insurance payments. Empirical evidence indicates that the social insurance tax and contributions have little systematic risk. Thus the aggregate risk associated with the net social insurance contributions arises from that associated with the social insurance payments. This observation may be verified from the next table of the paper.

<sup>27</sup> Appendix 2 presents the least-squares results.

<sup>28</sup> Regressions were also run with both current and one-period lag market excess returns as explanatory variables. In all cases the lagged variables turned to be insignificant, which is consistent with the efficient markets hypothesis.

<sup>29</sup> Appendix 3 presents the results without the Korean War period.

<sup>30</sup> In our search for the common factors, we also looked at the unexpected inflation. While this yielded a significant negative influence on the market excess return, but had no effect on the behaviour of corporate tax revenue and the other budgetary flows.

<sup>31</sup> We also used a simple AR1 process to determine the factor component of the growth rate of real GDP but the results did not change significantly. The coefficient of the propagation mechanism turned out to be insignificant.

<sup>32</sup> Expected changes in tax rates and policies could affect the estimated coefficients. We discuss stability of the estimated model parameters in the appendix.

<sup>33</sup> On the use of time subscripts, there is going to be a bit of untidiness; note that while the output shock affects the stock market contemporaneously, both these quantities affect the tax variables one period hence. Compare (5.6) and (5.8).

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## Appendix 1: The Robustness of the Riskiness Coefficient: Population and Employment growth

Table 6A: Estimated  $\beta$  for Tax Revenues and Outlays using the Method of Group Averages: 1950-95

Name	$\alpha_i$	$H_0: \alpha_i=0$ $H_a: \alpha_i \neq 0$	Beta	$H_0: \beta = 0$ $H_a: \beta > 0$	Adj R-Sq	Var( $e_{it}$ )	$H_0: \beta = 1$ $H_a: \beta < 1$	D-W
Taxation Items		t-stats		t - stats			t-stats	
GRPIT	1.877	1.309	0.196	2.132 **	0.073	70.16	-8.41 ***	2.18
GRCIT	-1.695	-0.695	0.647	4.136 ***	0.264	203.05	-2.25 **	2.08
GRSIC	4.532	3.901 ***	0.157	2.105 *	0.071	46.10	-11.31 ***	2.31
GRT	1.058	0.886	0.255	3.328 ***	0.183	48.66	-9.73 ***	2.15
Expenditure Items								
GDPMT	6.206	4.587 ***	-0.229	-2.641 ***	0.117	62.50		1.78
GGLSG	6.915	5.653 ***	-0.141	-1.797 *	0.047	51.11		1.78
GND	0.266	0.845	0.279	1.376	0.019	339.86		2.15
GATE	3.575	3.334 ***	0.035	0.507	-0.017	39.27		1.73
GRTND	5.535	4.613 ***	-0.244	-3.176 ***	0.168	49.18		2.44
GGOV	3.084	2.156 **	0.0003	0.003	-0.027	69.91		2.08

Note: \*\*\* indicates that the beta is statistically significant different from the null hypothesis at the 1 percent level of significance, while \*\* does so at the 5 percent level and \* at the 10 percent level.

The inclusion of the population growth rate merely increases the mean measurement error (i.e., the intercept), but leaves the slope coefficient (i.e., the "riskiness" measure) largely unaffected. This is what one would expect *a priori*. Indeed the mean error for all streams increases by the sample average growth rate of the population (i.e., 1.156%, except in the case of SIC, where it rises by the sample average employment growth of 1.714%). This test indicates that our betas' are robust.

## Appendix 2: Least Square Estimation of the CAPM

Table 6B: Estimated  $\beta$  for Tax Revenues using the Least Squares Technique: 1950-95

Name	Alpha	$H_0: \alpha = 0$ $H_a: \alpha \neq 0$	Beta	$H_0: \beta = 0$ $H_a: \beta > 0$	Adj R-Sq	Var( $e_{it}$ )	$H_0: \beta = 1$ $H_a: \beta < 1$	D-W
Taxation Items		t-stats		t - stats			t-stats	
GRPIT	0.651	0.482	0.198	2.526 **	0.107	66.35	-10.22 ***	1.06
GRCIT	-1.387	-0.604	0.608	4.570 ***	0.306	191.21	-2.94 ***	1.72
GRSIC	2.715	2.501 **	0.173	2.747 ***	0.127	42.80	-13.13 ***	1.41
GRT	-0.070	0.076	0.244	3.786 ***	0.228	44.99	-10.32 ***	1.12

Table 6C: Estimated  $\beta$  for Public Outlays, 1950-95

Name	Alpha	$H_0: \alpha = 0$ t - stats	Beta	$H_0: \beta = 0$ t - stats	Adj. R-Sq	D-W	Var( $e_{it}$ )
GDPMT	4.630	3.49 ***	-0.183	-2.38 **	0.094	1.24	63.99
GGSL	5.676	4.83 ***	-0.137	-2.02 **	0.064	1.35	50.21
GND	-0.549	-0.18	0.228	1.29	0.015	0.74	337.22
GATE	2.395	2.27 **	0.031	0.51	-0.017	1.56	40.33
GTND	3.780	3.14 ***	-0.176	-2.52 **	0.106	1.26	52.70
GGOV	1.850	1.35 **	0.003	0.03	-0.023	1.57	68.51



### Appendix 3: Riskiness of Flows without the Korean War Period

*CAPM*: Next, we re-run all the regressions *excluding* the 1951-1952 Korean War years. First, we look at the CAPM estimates which, given in Table 6D, change quite sharply. Essentially all the beta coefficients decline. In particular, we find that there is no evidence of systematic risk in PIT. These results suggest that the threat of a war may well contribute to the volatility of the revenue flows, and thus result in a higher public discount rate. Comparing the risk discount rates based on this set of beta's and those discussed in the text, (i.e., Table 9, inclusive of the War period), the new values all decline by about a full percentage point for projects financed with CIT, PIT and total revenue. Specifically, the declines are 1.18, 0.95, 0.84 percent, respectively, in the three cases. Thus, the "peace dividend" comes in the form of a one-percentage point drop in the borrowing cost for public projects, which is significant.

*APT*: We now turn to the results from the arbitrage model, which are given in Table 8A below. Once again, a similar pattern as the one described in the preceding paragraph emerges.

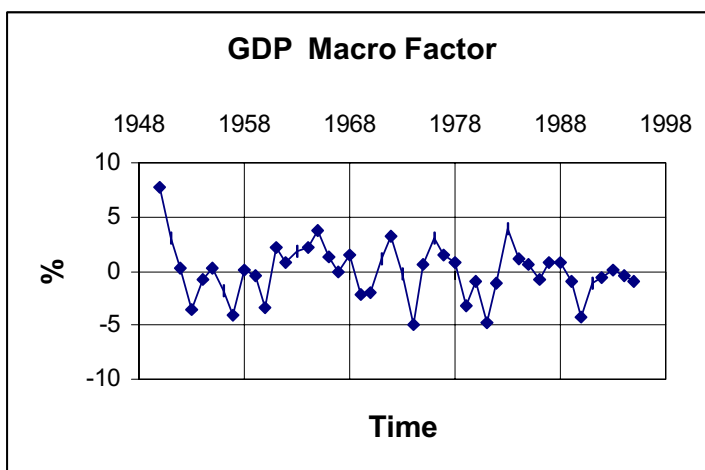
### Appendix 4: Predicting the long run economic growth rate.

The expected long run growth rate of GDP *per person* is found to be:

$$\hat{g}_t = 2.12$$

*t* - *stats* 5.74

Thus the resulting GDP growth figure of 2.12 percent is consistent with the findings of Maddison (1991), which clearly reflects the secular growth in total factor productivity. The residuals from this regression form a proxy for the GDP macroeconomic factor and are shown in the figure below.



From the above figure we observe that the macroeconomic growth factor behaves like a white noise error. The mean of the factor is zero with a variance of 6.26 which appears relatively large. The auto-correlation function shows the extent of correlation between neighbouring data points in the series (i.e., between  $F_{1t}$  and that of  $F_{1t-1}, F_{1t-2}, F_{1t-3}, F_{1t-4}$  etc). As seen from the next table, the auto-correlations provide an indication that the macroeconomic factor is indeed a white noise term. Furthermore, the partial correlation measures the correlation between  $F_{1t}$  and  $F_{1t-p}$ , after the effects of  $F_{1t-1}, F_{1t-2}, F_{1t-3}, F_{1t-4} \dots$  have been taken into account. The diagram does indicate that the partial autocorrelations are all relatively small just as the autocorrelation. All the sample autocorrelations and partial autocorrelations are statistically insignificant as is indicated by the high probability of a type I error and the low t-statistics. The Langrange multiplier and the Ljung-Box-Pierce statistics indicate that the factor behaves randomly (see Table on next page).

Lag	Rho	t-stats	LM-test	Q Test	p-value	Partial	t-stats
1	0.26	1.73	1.79	3.36	0.07	0.26	1.73
2	-0.15	-1.00	1.01	4.51	0.11	-0.23	-1.53
3	-0.14	-0.93	0.97	5.57	0.14	-0.04	-0.26
4	0.00	0.00	0.00	5.57	0.23	0.03	0.20
5	-0.03	-0.20	0.20	5.62	0.34	-0.09	-0.60
6	-0.20	-1.33	1.39	7.75	0.26	-0.19	-1.26
7	-0.08	-0.53	0.55	8.09	0.32	0.03	0.20
8	-0.12	-0.80	0.82	8.88	0.35	-0.21	-1.40
9	-0.17	-1.13	1.20	10.58	0.30	-0.17	-1.13
10	-0.05	-0.33	0.37	10.75	0.38	-0.02	-0.13
11	0.16	1.07	1.12	12.31	0.34	0.08	0.53
12	0.09	0.16	0.62	12.80	0.38	-0.11	-0.73

Table 6D: Estimated  $\beta$  for Tax Revenues and Outlays using the Method of Group Averages: 1952-95

Name	$\alpha_i + \alpha_{11}$	$H_0: \alpha_{11}=0$ $H_a: \alpha_{11} \neq 0$ t-stats	Beta	$H_0: \beta = 0$ $H_a: \beta > 0$ t - stats	Adj R-Sq	Var( $e_{it}$ )	$H_0: \beta = 1$ $H_a: \beta < 1$ t-stats	D-W
Taxation Items								
GRPIT	0.407	0.376	0.078	1.089	0.004	39.86	-12.80 ***	2.16
GRCIT	-2.057	-0.906	0.550	3.633 ***	0.221	176.00	-2.97 ***	1.99
GRSIC	2.458	2.438 **	0.131	1.947 *	0.061	34.68	-12.94 ***	2.04
GRT	-0.365	-0.399	0.158	2.598 ***	0.118	28.57	-13.80 ***	2.01
Expenditure Items								
GDPMT	4.951	4.389 ***	-0.138	-1.836 **	0.052	43.42		1.84
GGLSG	5.573	4.442 ***	-0.104	-1.249	0.013	53.70		1.75
GND	-2.074	-1.722 *	-0.036	-0.447	-0.019	49.51		2.15
GATE	2.548	2.380 **	0.059	0.830	-0.007	39.10		1.78
GRTND	4.390	4.313 ***	-0.162	-2.395 **	0.099	35.35		2.41
GGOV	1.470	1.732 *	-0.087	-1.535	0.031	24.56		2.36

Note: \*\*\* indicates that the beta is statistically significant different from the null hypothesis at the 1 percent level of significance, while \*\* does so at the 5 percent level and \* at the 10 percent level.

Table 8A: The General APT model using the Method of Group Averages: 1952-1995

Name	Expected Growth	Beta Sensitivity: Unexpected GDP Shocks	Sensitivity: Unexpected "Stock Market" Shocks	Adj. R-sq	D-W
ERM	7.110	1.658		0.020	2.06
$H_0 : \alpha_i = 0$ , t-stats	3.120 ***	1.388			
GRCIT	1.853	4.379	0.416	0.463	2.19
$H_0 : \alpha_i = 0$ , t-stats	1.116	5.039 ***	3.703 ***		
$H_0 : \alpha_{22} = 1$ , t-stats			-5.200 ***		
GRPIT	0.964	1.629	0.079	0.247	2.08
$H_0 : \alpha_i = 0$ , t-stats	1.165	3.760 ***	1.405		
GSIC	3.388	0.349	0.138	0.086	2.28
$H_0 : \alpha_i = 0$ , t-stats	3.868 ***	0.762	2.336 **		
GRT	0.762	1.656	0.139	0.422	2.42
$H_0 : \alpha_i = 0$ , t-stats	1.168	4.852 ***	3.145 ***		
GDPMT	3.970	-1.392	-0.126	0.199	1.75
$H_0 : \alpha_i = 0$ , t-stats	4.346 ***	-2.911 ***	-2.045 **		
GGLSG	4.831	-0.355	-0.120	0.023	2.32
$H_0 : \alpha_i = 0$ , t-stats	4.395 ***	-0.617	-1.618		
GND	-2.330	0.410	-0.072	-0.010	1.22
$H_0 : \alpha_i = 0$ , t-stats	-2.205 **	0.741	-1.001		
GATE	2.969	0.900	0.035	0.040	1.78
$H_0 : \alpha_i = 0$ , t-stats	3.226 ***	1.870 *	0.556		
GRTND	3.235	-0.132	-0.140	0.070	2.11
$H_0 : \alpha_i = 0$ , t-stats	3.551 ***	-0.276	-2.268 **		
GGOV	0.853	0.074	-0.115	0.075	1.89
$H_0 : \alpha_i = 0$ , t-stats	1.168	0.194	-2.331 **		