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# The impact of reducing the pension generosity on inequality and schooling



by  
**Miguel Sanchez-Romero**  
and **Alexia Prskawetz**

# The impact of reducing the pension generosity on inequality and schooling\*

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## Abstract

In this paper we investigate the impact of a reduction in the pension replacement rate on the schooling choice and on inequality. We develop an overlapping generations model in which individuals differ by their life expectancy and in the cost of attending schooling. Individuals optimally choose their consumption path and their educational attainment. Within our framework we first show how many progressive pension systems are *ex ante* regressive due to the difference in life expectancy across skill groups and, second, we derive the level of progressivity that restores an equal treatment of the pension system across skill groups.

**Keywords:** Human capital, Longevity, Inequality, Life cycle, Social security, Pension, Progressivity

**JEL codes:** E24, J10, J18, H55

## 1 Introduction

As populations age, government programs that redistribute resources from working age population to the dependent elderly, are increasingly getting under fiscal pressure. Policy adjustments have to be implemented that guarantee the fiscal sustainability of these programs. During the last two decades many OECD countries have passed pension reforms that implicitly reduce the generosity of their pension systems (OECD, 2013). However, these reforms are likely to have adverse effects not yet fully investigated. For instance, what will be the effective coverage level for workers that differ by their length of life?

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Recent studies for the US (National Academies of Sciences, Engineering, and Medicine, 2015; Murkin et al., 2017) and Europe (Mackebach, 2019) show that mortality differences by socio-economic groups have widened over the last decades. Since higher socioeconomic groups survive for more years compared to lower socioeconomic groups, they will receive benefits for a longer time period. As Pestieu and Ponthiere (2016, p. 209) nicely summarize these arguments: “Social insurance systems, like pension systems, which were built to reduce well-being inequalities between the surviving old, tend also to exacerbate well-being inequalities between the surviving old and the prematurely dead.” Unless the structure of contributions and benefits is not adjusted to these mortality differentials, welfare programs for the elderly may induce a net transfer from the poor to the rich.

Several papers have recently analyzed the redistributive effects of pension systems when longevity varies across socioeconomic groups both empirically (National Academies of Sciences, Engineering, and Medicine, 2015; Haan et al., 2019; Holzmann et al., 2019) and theoretically (Pestieu and Ponthiere, 2016; Laun et al., 2019; Sanchez-Romero et al., 2019). The main conclusion from the empirical literature is that pension systems are becoming increasingly regressive due to the increase in the longevity gap across socioeconomic groups. However, in the theoretical models the effects are not straightforward. Mortality differences are to some degree known by the individuals and taken into account when forming life cycle decisions such as education, labor force participation, savings, etc. Moreover individuals differ in further dimensions, such as ability and health. As a result, any social program induces incentives and distortions that differ across different population groups.

For instance, based on the case study of Norway, Laun et al. (2019) find that, when individuals differ by education, health and income, proportionally reducing old-age and disability benefits is preferable to other alternative policies such as raising the early retirement age or increasing social contributions; since raising the retirement age will be avoided by claiming disability benefits. However, the former policy increases inequality across socioeconomic groups. Sanchez-Romero et al. (2019) show that the US Social Security reduces regressivity from longevity differences, but would require group-specific life tables to achieve progressivity. In addition, they find that without separate life tables, despite apparent accounting gains, lower income groups would suffer welfare losses and higher income groups would enjoy welfare gains through indirect effects of pension systems on labor supply. Pestieu and Ponthiere (2016) present an excellent discussion on longevity variations and the social protection system including health, education and pension systems. In their study the authors highlight also the importance of heterogenous attitudes towards the risk of longevity. The authors also argue when education is endogenously chosen by individuals, and this decision affects their ex ante life expectancy, new challenges may arise with complex implications for the Welfare State. Therefore, it is important to consider behavioral effects when studying implications of alternative reforms of the welfare state.

In this paper we propose a model that extends the work by Pestieu and Ponthiere

(2016) by introducing heterogeneity in schooling effort, similar to Le Garrec (2015), which endogenously leads to different skill groups in the total population. Higher skill groups are characterized by higher survival and higher levels of income. Following Fehr (2016) we implement a flexible defined benefit pension system in a two-period overlapping generations model that allows to model a flexible level of progressivity of the pension system. Individuals optimally choose the education, consumption and savings. We show that individuals with different educational attainment will face different levels of the implicit tax rate of the pension system. We term the difference between the implicit tax rate of low and high skilled individuals “pension inequality”. Moreover, we analytically derive how the difference in the implicit tax rate between the skill groups is a function of the progressivity of the pension system and the ratio of the relative survival to the relative income advantage of high skilled compared to low skilled individuals. The latter ratio has an intuitive economic meaning. The numerator, i.e. the relative survival advantage of high skilled compared to low skilled individuals, indicates to which extent (how many additional years) the high skilled individuals receive pensions relative to the low skilled. The denominator, i.e. the relative income advantage of high skilled to low skilled individuals, reflects the additional contribution that the higher skilled individuals are paying to the system relative to the low skilled individuals. Within this framework we can show that a mortality differential between low and high skilled individuals will induce regressivity for a pension system of the Bismarckian type; i.e., a flat replacement rate. In case of a progressive pension system of the Beveridge type we find that only if the progressivity of the pension system is above the ratio of the relative survival to the relative income advantage, there will be a redistribution from long lived to short lived individuals. Otherwise even a progressive pension system will become regressive in a situation where mortality differs across the low and high skilled individuals. That is, for a given level of income differential, the higher the mortality differential in a society, the more progressive a pension system needs to be in order to avoid that a progressive pension system becomes regressive.

We next study whether and to which extent a pension reform that reduces the replacement rate (i.e. decreases the generosity of the pension system) may influence the pension inequality. As argued above, in a situation where life expectancy differs by education, a change in the replacement rate will change the implicit tax rate differently for low and high skill individuals. In turn, by changing the implicit tax rate differently for high and low skilled individuals, behavioral effects on individual life cycle decisions, such as educational investment, will result. We show that only if the progressivity of the pension system is equal to the ratio of the relative survival to the relative income differential between high and low skilled individuals, will such a pension reform not introduce any type of pension inequality. I.e. given any mortality and income differential in the society, a pension reform that reduces the pension replacement rate has to adjust the progressivity of the pension system to avoid introducing pension inequality. Whether a decrease in the replacement rate will increase or decrease the

share of the population that is high skilled will depend on the relative risk aversion of individuals. Overall an increase in education due to a lower level of the replacement rate is prevalent in those pensions systems that have a higher rate of progressivity.

We complement our analytical results by studying the progressivity of the pension systems and the ratio of the relative survival to relative income differential by skill group for a set of 21 selected OECD countries. Based on these two values and our analytical results we show for which of the countries a reduction in the generosity of the pension system may induce pension inequality and a change in the share of the population that acquires skills.

## 2 The model

Consider a small open economy populated by overlapping generations. In this economy working-age generations contribute to a pay-as-you-go pension system that pays for the pension benefits of old-age generations. The population is assumed to be stationary (i.e., constant population) and is comprised of a continuum of heterogeneous individuals in each generation.<sup>1</sup>

### 2.1 Individuals

Life is divided into two periods: young and old. In the first period, individuals survive with probability one, they choose their consumption level  $c$  and whether to become skilled workers ( $e_s$ ) or to stay unskilled ( $e_u$ ). Assume the effort of attending school has utility cost  $\phi \in \mathbb{R}$  and differs across individuals (Oreopoulos, 2007; Restuccia and Vandenbroucke, 2013; Le Garrec, 2015; Sanchez-Romero et al., 2016). If  $\phi > 0$  individuals incur a cost by making the effort to continue schooling. A negative value of  $\phi$  simply implies that individuals like going to school.<sup>2</sup> Individuals survive with probability  $\pi(e_i)$  to the second period, which depends on the skill level, and choose their consumption level  $d$ . Throughout, we impose the following assumption

**Assumption 1** *The survival probability increases with the skill level,  $\pi(e_s) > \pi(e_u)$ .*

The effort of attending school  $\phi$  also captures the ignorance of future outcomes derived from decisions made during the schooling period, which is observed during adolescence (Oreopoulos, 2007). Thus, although some individuals are aware that becoming a skilled worker increases their likelihood of survival to old-age, other individuals might not be aware of the positive effects of education, which can be modeled with a large and positive value of  $\phi$ .

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<sup>1</sup>The results presented in this paper hold for any unfunded pension system and constant population growth rate. For expositional simplicity, we opt for modeling a constant population; i.e., zero population growth.

<sup>2</sup>This parameter has been used for analyzing not only the problem of under-education (see, for instance, Oreopoulos, 2007), but also that of over-education (Boll et al., 2016).

The preferences of an individual of type  $\phi$  are described by the following utility function:

$$V(e_i; \phi) = u(c) - \phi \mathbf{1}_{\{e_i=e_s\}} + \beta\pi(e_i)u(d), \quad (1)$$

where  $\mathbf{1}_{\{e_i=e_s\}}$  is an indicator function that takes the value of one if individuals decide to become skilled workers ( $e_s$ ) and takes the value of zero otherwise ( $e_u$ ),  $\beta \in (0, 1]$  is the subjective discount factor, and  $u(\cdot)$  is the period-utility function (with  $u > 0$ ,  $u' > 0$ , and  $u'' < 0$ ).

During the first period, individuals consume part of their disposable income and save for retirement by purchasing annuities

$$c + s = (1 - \tau)y(e_i), \quad (2)$$

where  $s$  denotes private savings,  $\tau$  is the social contribution rate paid to the PAYG pension system, and  $y(e_i)$  is the gross labor income earned by a worker with skill  $e_i$ , with  $i \in \{s, u\}$ . To simplify the exposition of the model, we impose the following assumption:

**Assumption 2** *The income difference between skilled and non-skilled workers is such that the consumption of skilled workers is always greater than the consumption of non-skilled workers.*

In the second period, individuals consume their wealth, which is equal to the sum of the annuities purchased in the first period and the pension benefits claimed

$$d = \frac{s}{R\pi(e_i)} + \psi [\theta y(e_u) + (1 - \theta)y(e_i)], \quad (3)$$

where  $R \leq 1$  is the market discount factor,  $\psi$  is the maximum pension replacement rate —see Fig. 1, and  $[\theta y(e_u) + (1 - \theta)y(e_i)]$  is the pension base used to calculate the pension benefit.<sup>3</sup> Parameter  $\theta \in [0, 1]$  reflects the extent to which the pension system is more “Beveridgean” (i.e.  $\theta = 1$ ) or “Bismarckian” (i.e.  $\theta = 0$ ).<sup>4</sup> In order to introduce the pension replacement rate —relative to the gross labor income— of an individual with education  $e_i$ , we express the pension benefits claimed as  $y(e_i)f(e_i, \theta)$ . Thus, old-age consumption can be rewritten as

$$d = \frac{s}{R\pi(e_i)} + y(e_i)f(e_i, \theta), \quad (4)$$

where  $f(e_i, \theta)$  is the pension replacement rate of an individual with education  $e_i$

$$f(e_i, \theta) = \begin{cases} \psi & \text{if } e_i = e_u, \\ \psi[1 - \theta\alpha(e_s)] & \text{if } e_i = e_s, \end{cases} \quad (5)$$

<sup>3</sup>In a NDC pension system,  $\psi$  will be a function of the social contribution paid and the average survival probability of the population in the second period.

<sup>4</sup>For a detailed description of Beveridgean and Bismarckian pension schemes in OECD countries and how the economic and demographic composition of the population may affect the design of the social security system see Conde-Ruiz and Profeta (2007).

and  $\alpha(e_s) = \frac{y(e_s) - y(e_u)}{y(e_s)}$  is the relative income advantage of a skilled worker. The term  $\theta\alpha(e_s)$  reflects the degree of progressivity of the replacement rate formula. Hence, for  $\theta = 0$ , Eq. (5) shows that the replacement rate is flat at a value of  $\psi$ , whereas the replacement rate faced by an individual declines with income as  $\theta$  tends to one.

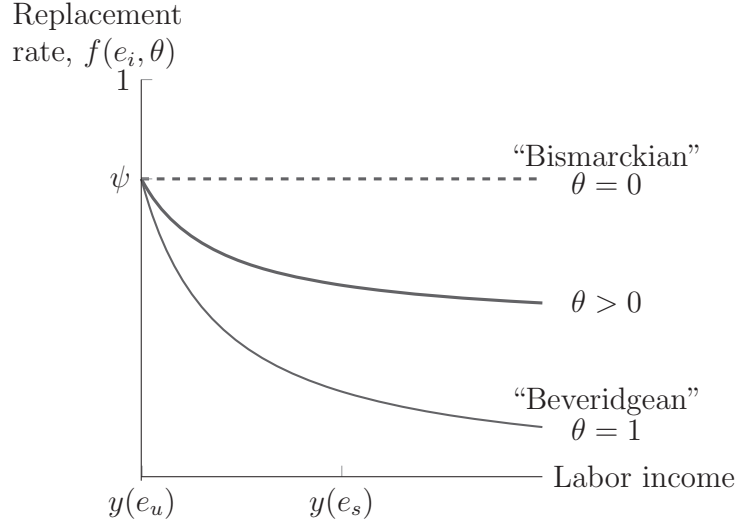


Figure 1: Stylized replacement rate function

Combining (2) and (4), and rearranging terms, we obtain that the lifetime budget constraint of an individual with education  $e_i$  is

$$c + R\pi(e_i)d = (1 - \tau_E(e_i))y(e_i). \quad (6)$$

The left-hand side of (6) is the present value of lifetime consumption. The right-hand side of (6) is the initial wealth of the individual, which includes the gross labor income earned and the social security wealth at the entrance into the labor market,  $-\tau_E(e_i)y(e_i)$ ; where  $\tau_E(e_i)$  is the effective social security tax/subsidy rate:<sup>5</sup>

$$\tau_E(e_i) = \tau - R\pi(e_i)f(e_i, \theta). \quad (7)$$

The effective social security tax/subsidy rate can take positive or negative values. In particular, under an actuarially fair pension system the effective social security tax rate is zero ( $\tau_E = 0$ ), whereas in non-actuarially fair pension systems social contributions paid can generate either implicit taxes ( $\tau_E > 0$ ) or implicit subsidies ( $\tau_E < 0$ ).

<sup>5</sup>Notice that in a two-periods life cycle model the effective social security contribution coincides with the social security wealth.



**Optimal consumption and saving** For the life-cycle model given by (1) and (6) individuals with education  $e_i$  optimally choose in the first period to consume

$$c^*(e_i) = m(e_i)(1 - \tau_E(e_i))y(e_i), \quad (8)$$

where  $(1 - \tau_E(e_i))y(e_i)$  is the individual's human wealth, and  $m(e_i) = (1 + R\pi(e_i)(\beta/R)^{\frac{1}{\gamma}})^{-1}$  is the individual's marginal propensity to consume with respect to human wealth. See proof in A. Moreover, by plugging (8) in (2), and using (7), we have that the optimal saving rate of individuals with education  $e_i$  is

$$\frac{s^*(e_i)}{y(e_i)} = (1 - m(e_i)) - ((1 - m(e_i))\tau + m(e_i)R\pi(e_i)f(e_i, \theta)). \quad (9)$$

The first term on the right-hand side of (9) is the marginal propensity to save, while the negative term in (9) is the reduction in private savings (i.e., crowding-out effect) caused by the pension system. Thus, as it is shown in (9), an increase (resp. reductions) in the social security rate,  $\tau$ , and/or an increase in the pension replacement rate,  $f(e_i, \theta)$ , yield a higher (resp. lower) crowding-out.<sup>6</sup>

**Optimal schooling** Individuals choose whether to become skilled workers ( $e_s$ ) or remain unskilled ( $e_u$ ). This decision depends on the schooling effort  $\phi$ , which differs across individuals. The optimal schooling decision satisfies

$$e_i^* = \begin{cases} e_u & \text{if } \phi \geq \bar{\phi}, \\ e_s & \text{if } \phi < \bar{\phi}. \end{cases} \quad (10)$$

Eq. (10) implies that an individual with utility cost of schooling lower (resp. higher) than  $\bar{\phi}$  will optimally choose to become a skilled (resp. unskilled) worker. The parameter  $\bar{\phi}$  denotes the threshold utility cost of schooling for which an individual is indifferent between staying unskilled and becoming a skilled worker; i.e,  $V(e_u; \bar{\phi}) = V(e_s; \bar{\phi})$ . Equating the expected utility between a skilled worker and an unskilled worker gives

$$\bar{\phi} = u(c^*(e_s)) - u(c^*(e_u)) + \beta[\pi(e_s)u(d^*(e_s)) - \pi(e_u)u(d^*(e_u))]. \quad (11)$$

Eq. (11) is the difference between the utility of consumption of a skilled worker, who also has higher life expectancy, and the utility of consumption of an unskilled worker. From (11) it is straightforward to show that the threshold utility cost of schooling increases the higher is the income of skilled workers; i.e.  $\frac{\partial \bar{\phi}}{\partial y(e_s)} =$

<sup>6</sup>In this model, a reduction in the generosity of the pension system ( $\downarrow \psi$ ), or an increase in the progressivity of the pension system ( $\uparrow \theta$ ), leads to a reduction in the pension replacement rate and, in a mature pension system, also a reduction in the social contribution rate. Therefore, these two policies imply an unambiguous increase in the saving rate for both types workers.

$u'(c^*(e_s)) \left(1 - \tau_E(e_s) - \frac{\partial \tau_E(e_s)}{\partial y(e_s)} y(e_s)\right) > 0$ . Hence, ceteris paribus the income of unskilled workers, more individuals choose to continue schooling when the income of skilled workers rises.

Differentiating (11) with respect to an increase in the life expectancy of skilled workers we can see two opposite effects on  $\bar{\phi}$ .<sup>7</sup> On the one hand, skilled workers enjoy higher utility due to the higher probability of surviving to old-age. On the other, skilled workers loss utility because they have to reduce consumption (i.e. “years-to-consume effect”) in order to finance the additional years lived. To guarantee that the impact of a longer life span on schooling is always positive, also known as the Ben-Porath mechanism (see Ben-Porath, 1967), we impose Assumption 3.

**Assumption 3** *The elasticity of utility with respect to consumption is between zero and one; i.e.  $\eta = du'(d)/u(d) \in (0, 1)$ .*

Assumption 3 is a sufficient, although not necessary, condition that guarantees that a marginal increase in the longevity of skilled individuals leads to a marginal increase in the threshold utility cost of schooling.

## 2.2 The proportion of skilled workers

In this economy there is a continuum of individuals who are heterogeneous by their utility cost of schooling. Let  $g(\phi)$  be the probability density function of the utility cost of schooling within each generation. Let the cumulative distribution function of  $\phi$  be  $G(\phi) = \int_{-\infty}^{\phi} g(x)dx$ . Let us define the proportion of individuals that choose to become skilled workers by  $q$ . Thus, from (11) we have

$$q = G(\bar{\phi}). \quad (13)$$

Fig. 2 shows a stylized density distribution of the utility cost of schooling  $\phi$  in which the gray area represents the value of  $q = G(\bar{\phi})$ . We can see in Fig. 2 how individuals with a  $\phi > \bar{\phi}$  choose to stay unskilled (white area under the curve), while those with a  $\phi < \bar{\phi}$  become skilled workers (gray area under the curve). Thus, we can visually observe that only individuals with a  $\phi$  close to  $\bar{\phi}$  are susceptible to a

<sup>7</sup>Assuming for simplicity no pension benefits, the partial derivative of (11) with respect to  $\pi(e_s)$  gives

$$\frac{\partial \bar{\phi}}{\partial \pi(e_s)} = \beta (u(d(e_s)) - u'(d(e_s))d(e_s)). \quad (12)$$

The first term inside the parenthesis is the additional utility gained by living longer, while the last term inside the parenthesis is the utility cost of living longer. Thus, Assumption 3 guarantees that (12) is positive. Note that the introduction of pension benefits in (12) implies that individuals gain an additional utility from the higher probability of receiving the old-age pension benefits.

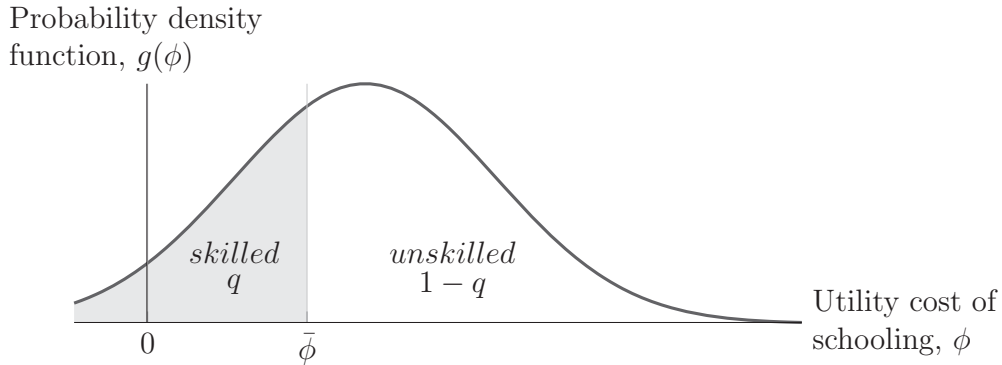


Figure 2: Stylized probability density function of the utility cost of schooling

Note: Function  $q := G(\bar{\phi}) = \int_{-\infty}^{\bar{\phi}} g(\phi)d\phi$  is the cumulative distribution function of the utility cost of schooling at the point  $\bar{\phi}$  where individuals are indifferent between schooling or staying unskilled.

change in  $\bar{\phi}$ .<sup>8</sup> As a consequence, from (13) and Fig. 2 we have that an increase in the threshold value  $\bar{\phi}$  yields a higher proportion of skilled workers because it becomes optimal for some unskilled to continue schooling, i.e.  $G'(\bar{\phi}) > 0$ .

### 2.3 Inequality and pension systems

Pension systems are designed either to treat equally all contributors (Bismarckian) or to distribute from rich workers to poor workers (Beveridgean). However, this distinction is not so clear when life expectancy differs across skill groups. Eq. (7) shows that skilled and unskilled workers do not face the same effective social security tax/subsidy rate since the life expectancy differs across skill groups. As a consequence, even progressive pension systems might induce a regressive distribution of income from low-income workers to high-income workers (Sanchez-Romero and Prskawetz, 2017; Ayuso et al., 2017).

From eqs. (6) and (7) it is clear that a pension system that generates the same effective social security tax/subsidy rate for all contributor types does not cause any redistribution of income across skill groups. Hence, the pension system maintains the relative wealth position of all contributors. Instead, if the effective social security tax rate of unskilled workers is higher, or lower, than that of skilled workers, the pension system will change the wealth position between unskilled and skilled workers. From now on we refer to this inequality as “pension inequality”. Note that pension inequality is defined for any positive or negative difference between the effective taxes

<sup>8</sup>A similar model setting, in which only a set of individuals are affected by a policy change, has been used before for analyzing the implication of compulsory schooling on wealth, health and happiness (Oreopoulos, 2007).

of unskilled and skilled workers.

A simple approach for analyzing whether a pension system induces pension inequality is to calculate the absolute difference between the effective social security tax rate faced by unskilled workers and that of skilled workers, which we denote by  $|\Delta_\tau|$ . From (5) and (7) we have

$$\Delta_\tau = \tau_E(e_u) - \tau_E(e_s) = \psi\pi(e_s) [\varepsilon(e_s) - \theta\alpha(e_s)] R, \quad (14)$$

where  $\varepsilon(e_s) \in [0, 1]$  is the relative survival advantage of a skilled worker with respect to an unskilled worker

$$\varepsilon(e_s) = \frac{\pi(e_s) - \pi(e_u)}{\pi(e_s)}. \quad (15)$$

Propositions 1 and 2 summarize the main results that follow from Eq. (14).

**Proposition 1** *Assuming the same life expectancy across skill groups,  $\pi(e_u) = \pi(e_s)$ , a pension system with*

- (a) *a flat replacement rate ( $\theta = 0$ ) does not redistribute resources across skill groups.*
- (b) *a progressive replacement rate ( $\theta > 0$ ) redistributes income from skilled workers to unskilled workers.*

**Proof.** For  $\pi(e_u) = \pi(e_s)$  and  $\theta = 0$ ,  $\Delta_\tau$  is equal to zero, which proves Proposition 1(a). Similarly, if we assume  $\pi(e_u) = \pi(e_s)$  and a positive value for  $\theta\alpha(e_s)$ , then  $\Delta_\tau = -\psi\pi(e_s)\theta\alpha(e_s)R$ , which is unambiguously negative and proves Proposition 1(b). ■

When  $\pi(e_u) = \pi(e_s)$  Proposition 1 shows that a pension system does not generate pension inequality if the replacement rate is flat, while it reduces the wealth difference across skill groups if the pension system is progressive. Once that the life expectancy differs across skill groups, we show in Proposition 2 that there will be a redistribution from unskilled to skilled workers in pension systems with a flat replacement rate.

**Proposition 2** *Assuming that  $\pi(e_s) > \pi(e_u)$  and defining  $p = \frac{\varepsilon(e_s)}{\alpha(e_s)}$  as the ratio of the relative mortality to the relative income advantage of skilled workers, a pension system with*

- (a) *a flat replacement rate ( $\theta = 0$ ) transfers resources from short-lived and unskilled workers to long-lived and skilled workers.*
- (b) *a progressive replacement rate ( $\theta > 0$ ) (i) implies the same implicit social security tax rate for skilled and unskilled workers when  $\theta = p$ , (ii) redistributes income from skilled workers to unskilled workers when  $\theta > p$ , and (iii) redistributes income from unskilled workers to skilled workers when  $\theta < p$ .*

**Proof.** Given Assumption 1, for a flat replacement rate ( $\theta = 0$ ), we get  $\Delta_\tau = R\psi\pi(e_s)\varepsilon(e_s) > 0$ , which implies that Eq. (14) is unambiguously positive. For  $\pi(e_s) > \pi(e_u)$  and  $p = \frac{\varepsilon(e_s)}{\alpha(e_s)} > 0$ , Eq. (14) shows that the sign of  $\Delta_\tau$  is positive for  $\theta < p$ , negative for  $\theta > p$ , and is equal to zero for  $\theta = p$ . ■

When  $\pi(e_s) > \pi(e_u)$  Proposition 2 shows that a pension system with a flat replacement rate becomes *ex ante* regressive, transferring income from short-lived and unskilled workers to long-lived and skilled workers. In contrast, by allowing a progressive pension system ( $\theta > 0$ ), the government is capable of reducing the difference in the effective social security tax rate paid by the two skill groups.

Moreover, we obtain from (14) that skilled workers face the same effective social security tax as unskilled workers when the degree of progressivity ( $\theta$ ) is equal to the ratio of the relative mortality advantage of skilled workers and the relative income advantage of skilled workers, which we denote by  $p = \frac{\varepsilon(e_s)}{\alpha(e_s)}$ .<sup>9</sup> Thus, any other degree of progressivity ( $\theta \neq p$ ) benefits one skill group at the expense of the other. In

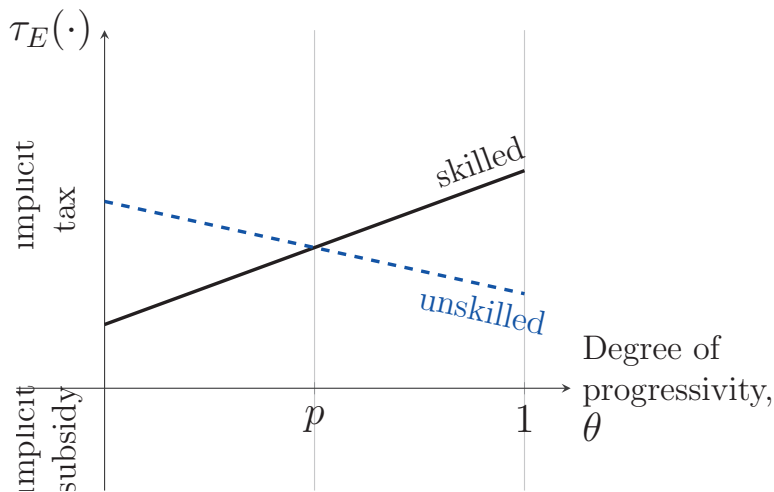


Figure 3: Standardized effective social security tax/subsidy rate ( $\tau_E$ ) for each educational group by degree of progressivity ( $\theta$ )

particular, for a positive gap in life expectancy between skill groups, Figure 3 shows that a pension system whose degree of progressivity is lower than  $p$  (i.e.  $\theta < p$ ) redistributes from short-lived and unskilled workers to long-lived and skilled workers. In contrast, a pension system with a degree of progressivity greater than  $p$  (i.e.  $\theta > p$ ) redistributes from long-lived and skilled workers to short-lived and unskilled workers.

<sup>9</sup>Notice that  $p$  increases (resp. decreases) when the relative mortality advantage of skilled workers increases more (resp. less) than the relative income advantage of skilled workers.

### 3 The impact of reducing the pension replacement rate

In the next decades it is expected that many pension schemes will introduce reforms that reduce the generosity of their systems in order to improve its long-run sustainability. Finland, Germany, Japan, and Spain, for instance, have already introduced automatic adjustment mechanisms, which reduce the replacement rate, to guarantee its sustainability (OECD, 2017b). In this section, we study the impact of this policy on our measure of pension inequality and also on the incentives for becoming a skilled worker.

#### 3.1 Impact on pension inequality

Given that the replacement rate affects pension inequality in a multiplicative way, Eq. (14) implies that a reduction in the replacement rate,  $\psi$ , leads to a less regressive pension system if  $\theta < p$  (lower pension inequality), while this policy diminishes the progressivity of the pension system if  $\theta > p$  (higher pension inequality).<sup>10</sup> In addition, from (14) we have that if a pension system aims at avoiding any pension inequality, while reducing the generosity of the pension system, the progressivity of the pension system should satisfy that  $\theta = p$ .

To see the relevance of this policy we compare the degree of progressivity of the pension system ( $\theta$ ) to the ratio of the relative mortality to the relative income advantage of skilled workers ( $p$ ) for a selection of OECD countries.<sup>11</sup> We derive the value of  $p$  by combining information on relative earnings of men aged 55-65 by educational attainment from OECD (2017a) with male life expectancy at age 65 by educational attainment from Murin et al. (2017). For the case of the US, we instead use our own estimates for the life expectancy at age 65 by educational attainment (see Fig. ??). The degree of progressivity of each pension system ( $\theta$ ) is calculated using the gross pension replacement rate from mandatory pension schemes (public and private) by percentage of individual earnings from OECD (2017b). Therefore, we restrict our analysis to the unfunded component of the pension system in each country. The information is provided for low, median, and high income earners. High income earners are individuals with a wage above 1.5 times the median wage, whereas low income earners are individuals with a wage less than 0.5 times the median wage.<sup>12</sup>

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<sup>10</sup>To study the effect of a decrease in the replacement rate ( $\psi$ ) on pension inequality, we calculate the derivative of (14) with respect to a fall in  $\psi$ .

<sup>11</sup>The sample includes Australia, Austria, Canada, Chile, Czech Republic, Denmark, Finland, France, United Kingdom, Hungary, Italy, Latvia, Mexico, Norway, New Zealand, Poland, Slovak Republic, Slovenia, Sweden, Turkey, United States.

<sup>12</sup>See Conde-Ruiz and Profeta (2007) for an alternative approach to calculate the degree of progressivity of a pension system based on a microeconomic projection of the pension entitlements that correspond to workers aged 55-59 at different levels of earnings. While their approach is more sophisticated, our calculations allow us to include more countries in the analysis.

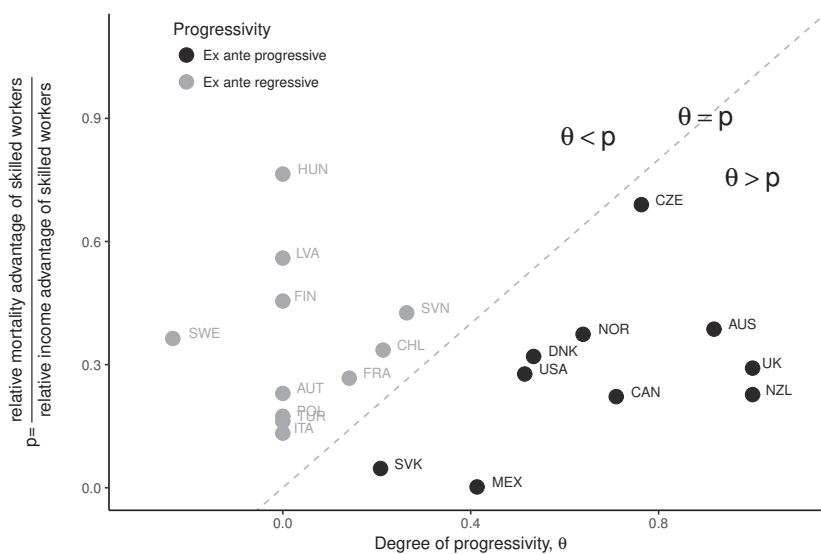


Figure 4: Empirical values of  $p = \varepsilon(e_s)/\alpha(e_s)$  and  $\theta$  for 21 selected OECD countries

*Source:* Values obtained combining information on (male) relative earnings by educational attainment from OECD (2017a) for years 2012–2015, gross pension replacement rates from mandatory pension schemes (public and private) by percentage of individual earnings from OECD (2017b), and on (male) life expectancy at age 65 by educational attainment from Murin et al. (2017) and authors’ calculations for USA combining death records with census data for the year 2015. *Notes:* Calculations done assuming that unskilled workers are comprised of individuals with “below upper secondary education” and skilled workers are formed by individuals with “completed upper secondary or higher education”. All data values are based on period information, which may bias the value of  $p$  downwards.

Fig. 4 shows that despite the fact that many pension schemes include some degree of progressivity in the replacement rate formula (i.e.,  $\theta > 0$ ), the existing longevity gap by socioeconomic status (Murin et al., 2017) leads many pension systems to be *ex ante* regressive (see light grey dots). As a consequence, the fall in the replacement rate will yield a reduction in pension inequality—as measured in (14)—in the *ex ante* regressive pension systems ( $\theta < p$ ), while it will increase pension inequality in the *ex ante* progressive pension systems ( $\theta > p$ ) (see dark gray dots). From Fig. 4 we can also observe that the minimum value of  $p$  is 0,2 percent (Mexico), the maximum is 76 percent (Hungary), and the most frequent value ranges between 20 and 50 percent for the selection of OECD countries, with an average value close to 0.32.<sup>13</sup>

<sup>13</sup>The relative mortality advantage of skilled workers is likely to be underestimated in Mexico and the Slovak Republic, since individuals with middle education have a lower life expectancy than low educated individuals.

## 3.2 Impact on education

Pension systems may also affect the optimal schooling decision of individuals through changes in the effective social security tax/subsidy rate. This is because the effective social security rate has an impact on the expected income earned by workers, on the marginal benefit of education, and ultimately on the educational distribution of the population.

To study the impact of reducing the generosity of the pension system on education, we differentiate the proportion of skilled workers,  $q$ , with respect to a fall in  $\psi$ . From (11), (13), and (28), we have

$$\frac{-\partial q}{\partial \psi} = g(\bar{\phi})u'(c^*(e_s))y(e_s) \left[ \frac{-\partial \Delta_\tau}{\partial \psi} + (\Phi - 1) \frac{-\partial \tau_E(e_u)}{\partial \psi} \right], \quad (16)$$

where  $\Phi = \frac{u'(c^*(e_u))y(e_u)}{u'(c^*(e_s))y(e_s)}$  is the ratio of the marginal utility of work between unskilled and skilled workers.<sup>14</sup> See C for a detailed derivation. Eq. (16) is the marginal (utility) gain/loss of reducing the replacement rate to those individuals who are at the margin between staying unskilled or becoming skilled. Note that the right-hand side of (16) is multiplied by  $g(\bar{\phi})$ . Hence, for those individuals whose effort of attending schooling is close to  $\bar{\phi}$ , the fall in the replacement rate leads to a change in the difference between the effective social contribution rate paid by both skill groups (i.e.,  $\frac{-\partial \Delta_\tau}{\partial \psi}$ ) as well as to an income effect and a substitution effect caused by the increase in the disposable income during the working period. This is represented by the second term inside the squared brackets; i.e.,  $(\Phi - 1) \frac{-\partial \tau_E(e_u)}{\partial \psi}$ . On the one hand, individuals use the increase in disposable income to avoid the effort of attending school (*income effect*). On the other hand, since the fall in  $\psi$  reduces the effective tax rate and hence raises the disposable income, this policy makes it more attractive to become a skilled worker (*substitution effect*).

We can distinguish three cases depending on whether the income effect is lower, equal to, or greater than the substitution effect.

For expositional simplicity, we first study the case in which the income effect is equal to the substitution effect ( $\Phi = 1$ ). According to Eq. (13), in this case we just need to differentiate between the case where the progressivity of the pension system  $\theta$  is below and alternatively above  $p$ . We know that for  $\theta < p$  a decrease in the replacement rate makes the pension system less regressive and hence less individuals will invest in education —since the unskilled are now better off—, implying a decrease in the share of skilled workers. On the other hand if  $\theta > p$  a decrease in the replacement rate makes the pension system less progressive, which implies that more individuals will have an incentive to become skilled —since the skilled are now better off—, thereby increasing the share of skilled workers. Note that the extent to which a

<sup>14</sup>If  $\Phi$  is greater than one the marginal utility of work of unskilled exceeds the marginal utility of skilled. If  $\Phi$  is less than one the marginal utility of work of skilled exceeds the marginal utility of unskilled.



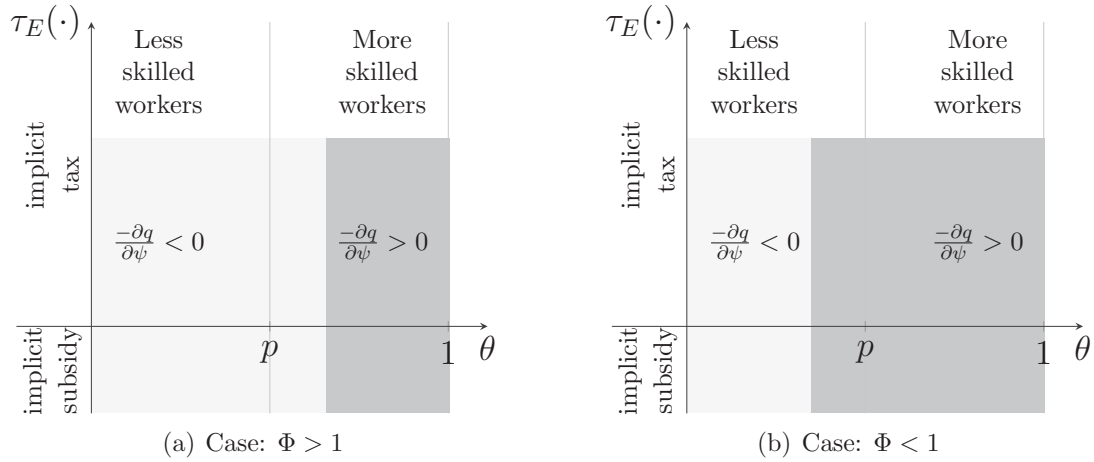


Figure 5: Impact of a reduction in the replacement rate on the proportion of skilled workers by degree of progressivity of the pension system ( $\theta$ )

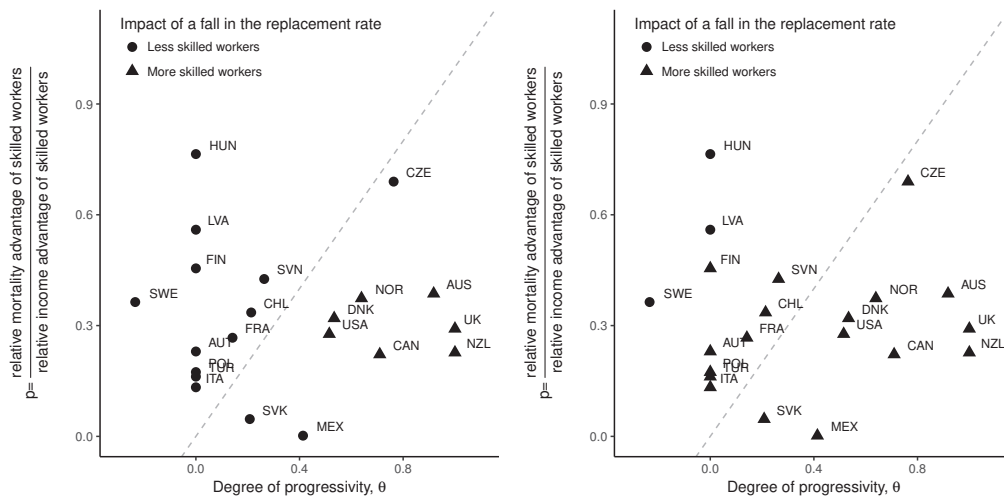
*Notes:* For  $\Phi < 1$  the substitution effect dominates over the income effect, while for  $\Phi > 1$  the income effect dominates over the substitution effect.

decrease in the replacement rate changes the share of skilled workers depends on the absolute difference between  $\theta$  and  $p$ .

Next we relax the assumption of the income effect and substitution effect to be equal.

In case the income effect dominates (see Fig. 5(a)) and  $\theta > p$ , the benefit that skilled workers experience from the reduction in the replacement rate should be large enough to compensate for the effort of attending school. As a consequence, when the income effect dominates, a reduction in the replacement rate will increase the share of skilled people only if  $\theta$  is much larger than  $p$  (i.e.,  $\theta \gg p$ ); otherwise they would opt to stay unskilled. In contrast, in case that the substitution effect dominates (see Fig. 5(b)) and  $\theta < p$ , unskilled workers might find that the additional income they gain from the reduction in the replacement rate is not large enough to compensate for the increase in disposable income they would obtain if they would become skilled. As a consequence, when the substitution effect dominates, a reduction in the replacement rate will increase the share of unskilled people only if  $\theta$  is much smaller than  $p$  (i.e.,  $\theta \ll p$ ); otherwise they would opt to become skilled.

To better understand the impact of a fall in the replacement rate on the distribution of skilled workers for the selection of OECD countries, we assume that the marginal utility of consumption follows a power utility function  $u'(x) = x^{-\gamma}$ , where  $\gamma$  is the relative risk aversion coefficient. We choose two alternative values for the relative risk aversion  $\gamma \in \{0.5, 1.5\}$ , which are within the lower and upper bounds



(a) Relative risk aversion = 1.5  $\Rightarrow \Phi > 1$       (b) Relative risk aversion = 0.5  $\Rightarrow \Phi < 1$

Figure 6: Impact of a reduction in the replacement rate on the proportion of skilled workers by degree of progressivity of the pension system ( $\theta$ ) in 21 selected OECD countries

*Source:* The information collected in Fig. 4 is complemented with the share of total labor income earned by skilled workers. This additional variable is calculated combining information on the share of men aged 55–64 by educational attainment with the relative earnings of men aged 55–64 by educational attainment from OECD (2017a). Calculations done assuming each period lasts forty years, a power marginal utility function  $u'(x) = x^{-\gamma}$ , where  $\gamma$  is the relative risk aversion coefficient, a constant annual real interest rate of 3 percent, a productivity growth rate of 1.5 percent, and a subjective discount factor of 1 percent.

for  $\gamma$  estimated by Chetty (2006).<sup>15</sup> A relative risk aversion of 0.5 implies a  $\Phi$  value that ranges between 0.7 and 0.8 across the countries analyzed, with an average value of 0.75. Hence, the substitution effect dominates over the income effect. A relative risk aversion of 1.5 implies a  $\Phi$  value that ranges between 1.1 and 1.3 across the countries analyzed, with an average value of 1.2. In this last case the income effect dominates over the substitution effect. Moreover, we assume each period lasts forty years, the subjective discount factor is 0.99, and the annual market discount factor is 1.5 percent, which is the result of calculating the difference between an interest rate of 3 percent and a productivity growth rate of 1.5 percent.<sup>16</sup>

<sup>15</sup>An average relative risk aversion of 1 (log utility), as suggested by Chetty (2006), will imply that a reduction in the generosity of the pension system on education depends exclusively on the difference between the effective social contribution rate paid by both skill groups, since  $\Phi$  is close to 1.

<sup>16</sup>Additional calculations have been performed assuming a market discount factor of 0 percent and

Figure 6 shows that if the relative risk aversion is 0.5 ( $\Phi < 1$ ) a fall in the replacement rate will lead to an increase in the number of skilled workers in all pension systems (see black triangles), except in Hungary, Latvia, and Sweden. If the relative risk aversion is 1.5 ( $\Phi > 1$ ) a fall in the replacement rate will only lead to an increase in the proportion of skilled workers in countries with a sufficiently high degree of progressivity (USA, Denmark, Norway, Canada, Australia, United Kingdom, and New Zealand). This is because the decline in pension inequality is not large enough to compensate for the effort of attending school. If we assume instead a relative risk aversion of 1 (log utility), a fall in the replacement rate will yield an increase the proportion of skilled workers in countries that are ex ante progressive ( $\theta > p$ ) and a decline in the proportion of skilled workers in countries that are ex ante regressive ( $\theta < p$ ).

### 3.3 The combined effect

In the last subsections, we have discussed the impact of a fall in the replacement rate on schooling and inequality on the degree of progressivity of the pensions system. Combining the results from (14) and (16), Proposition 3 summarizes the impact of a fall in the replacement rate on schooling and on pension inequality.

**Proposition 3** *Given (1)–(6), and assumptions 1 and 3, a fall in the replacement rate leads to*

- (a) *less skilled workers and lower pension inequality if* 
$$\begin{cases} \frac{-\partial\Delta_\tau}{\partial\psi} < (1 - \Phi)\frac{-\partial\tau_E(e_u)}{\partial\psi} & \text{for } \Phi < 1, \\ \frac{-\partial\Delta_\tau}{\partial\psi} < 0 & \text{for } \Phi > 1. \end{cases}$$
- (b) *less skilled workers and higher pension inequality if* 
$$0 < \frac{-\partial\Delta_\tau}{\partial\psi} < (1 - \Phi)\frac{-\partial\tau_E(e_u)}{\partial\psi}$$
  
*for*  $\Phi > 1$ .
- (c) *more skilled workers and lower inequality if* 
$$(1 - \Phi)\frac{-\partial\tau_E(e_u)}{\partial\psi} < \frac{-\partial\Delta_\tau}{\partial\psi} < 0$$
  
*for*  $\Phi < 1$ .
- (d) *more skilled workers and higher pension inequality if* 
$$\begin{cases} \frac{-\partial\Delta_\tau}{\partial\psi} > 0 & \text{for } \Phi < 1, \\ \frac{-\partial\Delta_\tau}{\partial\psi} > (1 - \Phi)\frac{-\partial\tau_E(e_u)}{\partial\psi} & \text{for } \Phi > 1. \end{cases}$$

Fig. 7 graphically summarizes Proposition 3.

Fig. 7 shows the sign of the impact of a fall in the replacement rate on the proportion of skilled workers,  $q$ , and on pension inequality induced by the difference in the effective social security rate across skill groups,  $\Delta_\tau$ . Each panel is divided in three shaded areas (light gray, gray, and dark gray), which are the results of combining (14)

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3 percent. The results slightly differ with respect to the benchmark when  $\Phi > 1$ , since a low market discount factor increases the importance of the substitution effect (more skilled workers), while a high discount factor decreases the importance of the substitution effect (less skilled workers).

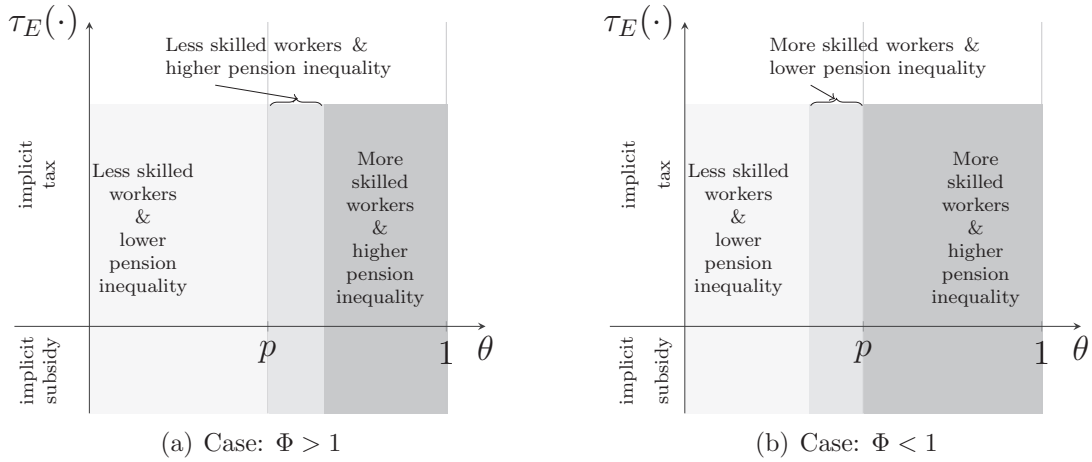
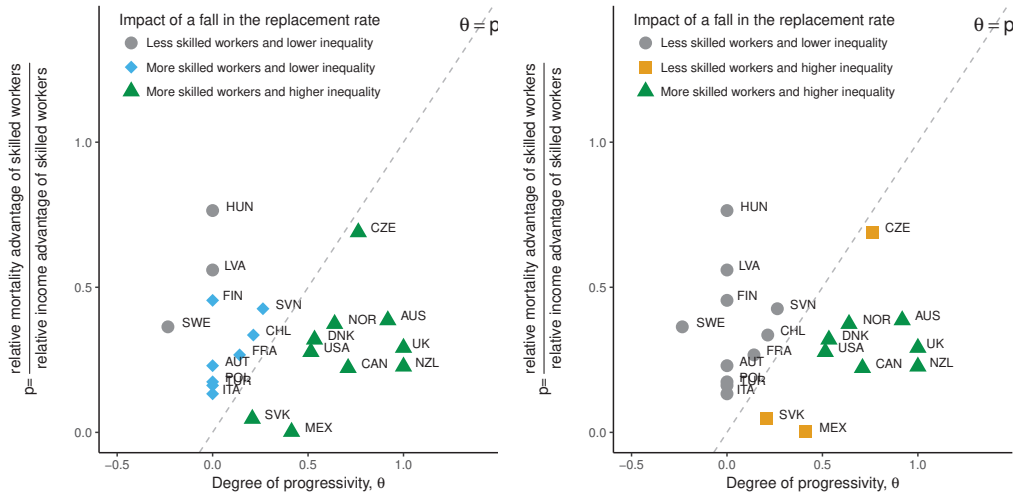


Figure 7: Impact of a reduction in the replacement rate ( $\psi$ ) on the proportion of skilled workers ( $q$ ) and on pension inequality ( $\Delta_\tau$ ) by degree of progressivity of the pension system ( $\theta$ )

and (16). If a pension system lies within the light gray area, a fall in the replacement rate leads not only to a reduction in pension inequality but also to a reduction in the proportion of skilled workers. If a pension system lies within the dark gray area, a fall in the replacement rate leads to an increase the proportion of skilled workers and to an increase in pension inequality. However, if a pension system lies within the gray area, the impact of a fall in the replacement rate on inequality and education depends on the whether the substitution effect dominates over the income effect. In particular, for  $\Phi < 1$ , a lower replacement rate not only reduces pension inequality between education groups, but it also increases the proportion of skilled workers. In contrast, for  $\Phi > 1$ , a fall in the replacement rate leads to a reduction in the number of skilled workers and an increase in pension inequality.

Combining the numerical results shown in figs. 4 and 6 we obtain for a relative risk aversion of 0.5 (see Fig. 8(a)) that a fall in replacement rate: (i) will increase both the proportion of skilled workers and pension inequality (see green triangles) in *ex ante* progressive pension systems; (ii) will increase the proportion of skilled workers and reduce pension inequality (see blue diamonds) in countries with  $\theta \in (0, p)$ ; and (iii) will lead to less skilled workers and lower pension inequality in countries with  $\theta < 0$  (see gray dots).

However, when the income effect dominates over the substitution effect (i.e., relative risk aversion of 1.5), a fall in the replacement rate (i) will lead in countries with a sufficiently high degree of pension progressivity to an increase in the proportion of skilled workers and in pension inequality (see green triangles); (ii) will reduce the proportion of skilled workers and raise pension inequality in *ex ante* progressive countries without a highly progressive system (see yellow squares); and (iii) will reduce



(a) Relative risk aversion = 0.5  $\Rightarrow \Phi < 1$       (b) Relative risk aversion = 1.5  $\Rightarrow \Phi > 1$

Figure 8: Impact of a reduction in the replacement rate ( $\psi$ ) on the proportion of skilled workers ( $q$ ) and on pension inequality ( $\Delta_\tau$ ) by degree of progressivity of the pension system ( $\theta$ ) in 21 selected OECD countries

Source: See figs. 4 and 6.

the proportion of skilled workers and pension inequality in *ex ante* regressive systems (see gray dots). Given that less skilled workers and higher pension inequality is less preferable than having less skilled workers and lower pension inequality on the one side, and more skilled workers and higher pension inequality on the other, the progressivity of the pension system ( $\theta$ ) should be lower but close to  $p$  —as in the case that the relative risk aversion is 0.5— or should be sufficiently progressive so as to be at the frontier between the yellow squares and the green triangles.

## 4 Conclusion

We set up a small-open economy with overlapping generations in which heterogeneous individuals optimally choose their consumption path and their educational attainment. We assume a positive correlation between the length of schooling and the survival probability at old age. To study the impact of a reduction in the generosity of the pension system, we introduce a pay-as-you-go pension system that allows for any combination between a fully Beveridgean pension system and a fully Bismarckian pension system. Within our framework, we show that a pension system with a flat replacement rate redistributes resources from unskilled workers with short lives to skilled workers with long lives. By reducing the generosity of the pension system

with a flat replacement rate, our model shows that the difference between the effective social security tax rate of both skill groups will diminish, but also the proportion of skilled workers. However, if the pension system is sufficiently progressive, a reduction in the pension replacement rate may increase the proportion of skilled workers and reduce wealth inequality.

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## A Solution: Individual problem

Given an optimal schooling choice ( $e_i^*$ ) and an utility cost of continuing schooling ( $\phi$ ), we first maximize the Lagrange function  $\mathfrak{S}$  with respect to the consumption path ( $c, d$ )

$$\begin{aligned} \max_{c,d} \mathfrak{S}(c, d, \lambda; e_i, \phi) &= u(c^*(e_i^*)) - \phi e_i^* + \beta\pi(e_i^*)u(d^*(e_i^*)) \\ &+ \lambda_i [(1 - \tau_E(e_i^*))y(e_i^*) - c^*(e_i^*) - R\pi(e_i^*)d^*(e_i^*)], \end{aligned} \quad (17)$$

where  $\lambda_i > 0$  is the corresponding Lagrange multiplier. The optimal schooling decision is given by

$$e_i^* = \arg \max_{e_i \in \{e_u, e_s\}} V(e_i; \phi). \quad (18)$$

The first-order conditions (FOCs) are:

$$c : \quad u'(c^*(e_i^*)) = \lambda_i, \quad (19a)$$

$$d : \quad \beta\pi(e_i^*)u'(d^*(e_i^*)) = \lambda_i R\pi(e_i^*). \quad (19b)$$

Combining the FOCs we obtain the standard Euler condition

$$u'(d^*(e_i^*)) = u'(c^*(e_i^*))R/\beta. \quad (20)$$

Assuming that the marginal utility of consumption is a standard power function  $u'(c) = c^{-\gamma}$  we have

$$d^*(e_i^*) = c^*(e_i^*)(\beta/R)^{\frac{1}{\gamma}}. \quad (21)$$

Substituting into the budget constraint, we have

$$c^*(e_i^*) = \frac{1 - \tau_E(e_i^*)}{1 + R\pi(e_i^*)(\beta/R)^{\frac{1}{\gamma}}} y(e_i^*). \quad (22)$$

Eq. (22) is the initial consumption of an individual with education  $e_i$ . The first term on the right-hand side is the marginal propensity to consume out of gross labor income of an individual of type  $e_i$ , while the second term is the gross labor income of an individual of type  $e_i$ .

## B The pension system

Consider a stable and mature defined-benefit PAYG pension system with a balanced budget. Given the population and economic characteristics, the budget constraint of the pension system is

$$\tau[y(e_u)(1 - q) + y(e_s)q] = \pi(e_u)\psi y(e_u)(1 - q) + \pi(e_s)\psi[1 - \theta\alpha(e_s)]y(e_s)q, \quad (23)$$



where the left-hand side of Eq. (23) stands for the total social contributions paid by unskilled and skilled workers, respectively, and the right-hand side stands for the total benefits claimed by the surviving retirees of both skill groups. Dividing both sides of Eq. (23) by the total labor income, the social contribution rate,  $\tau$ , is given by

$$\tau = \psi\pi(e_u)(1 - \omega) + \psi[1 - \theta\alpha(e_s)]\pi(e_s)\omega, \quad (24)$$

where  $\omega$  is the share of total labor income earned by skilled workers

$$\omega = \frac{y(e_s)q}{y(e_s)q + y(e_u)(1 - q)}. \quad (25)$$

The first term on the right-hand side of Eq. (24) represents the contribution rate necessary to pay for the pension benefits of unskilled workers, while the second term on the right-hand side accounts for the contribution rate to pay for the pension benefits of skilled workers. Note that by rearranging terms in (24), we can explicitly show how the progressivity of the pension system affects the social contribution rate

$$\tau = \psi [\pi(e_u) + \pi(e_s)\alpha(e_s) (p - \theta) \omega], \quad (26)$$

where  $\varepsilon(e_i) \in [0, 1]$  is the relative survival advantage of an individual with education  $e_i$  with respect to an unskilled worker. Given a replacement rate level  $\psi$ , Eq. (26) shows that the social security contribution rate ( $\tau$ ) declines when the progressivity of the pension system increases ( $\theta$ ). Also, notice that when  $\theta > p$  an increase in the labor income earned by skilled workers, ceteris paribus the income of unskilled workers, yields a reduction in the social security contribution rate. However, a rise in the labor income of skilled workers increases the social security contribution rate when the replacement rate is flat ( $\theta = 0$ ).

In a NDC system, given a social contribution rate  $\tau$ , Eq. (26) shows that the replacement rate is

$$\psi = \frac{\tau}{\pi(e_u) + \pi(e_s)\alpha(e_s) (p - \theta) \omega}. \quad (27)$$

Therefore, an increase in the progressivity of the system ( $\theta$ ) raises the replacement rate or, equivalently, it allows a lower social security contribution for the same level of  $\psi$ . Similar to a DB system, an increase in the labor income of skilled workers leads an increase in the replacement rate  $\psi$  when  $\theta > p$ .

Now, substituting Eq. (26) in the effective social security tax rate  $\tau_E(e_i^*)$  —see Eq. (7)— for  $e_i \in \{e_u, e_s\}$  gives

$$\tau_E(e_i^*) = \begin{cases} \psi [\pi(e_u)(1 - R) + \pi(e_s)\alpha(e_s) (p - \theta) \omega] & \text{if } e_i^* = e_u, \\ \psi [\pi(e_u)(1 - R) + \pi(e_s)\alpha(e_s) (p - \theta) (\omega - R)] & \text{if } e_i^* = e_s. \end{cases} \quad (28)$$

Eq. (28) shows that unskilled and skilled do not face the same effective social security tax/subsidy rate when differences in longevity exists.

## C Impact of $\psi$ on the proportion of skilled workers.

**Proof.** To derive Eq. (16) we differentiate  $q$  w.r.t. a fall in  $\psi$ , which gives

$$\begin{aligned} \frac{-\partial q}{\partial \psi} &= G'(\bar{\phi}) \frac{-\partial \bar{\phi}}{\partial \psi} \\ &= g(\bar{\phi}) \left[ \begin{array}{l} u'(c^*(e_s)) \frac{-\partial c^*(e_s)}{\partial \psi} + \beta \pi(e_s) u'(d^*(e_s)) \frac{-\partial d^*(e_s)}{\partial \psi} \\ -u'(c^*(e_u)) \frac{-\partial c^*(e_u)}{\partial \psi} - \beta \pi(e_u) u'(d^*(e_u)) \frac{-\partial d^*(e_u)}{\partial \psi} \end{array} \right]. \end{aligned} \quad (29a)$$

Substituting the FOCs gives

$$\frac{-\partial q}{\partial \psi} = g(\bar{\phi}) \left[ \begin{array}{l} u'(c^*(e_s)) \left( \frac{-\partial c^*(e_s)}{\partial \psi} + R\pi(e_s) \frac{-\partial d^*(e_s)}{\partial \psi} \right) \\ -u'(c^*(e_u)) \left( \frac{-\partial c^*(e_u)}{\partial \psi} + R\pi(e_u) \frac{-\partial d^*(e_u)}{\partial \psi} \right) \end{array} \right]. \quad (29b)$$

Differentiating the budget constraint (6) w.r.t. the fall in  $\psi$  and plugging the result in (29b) gives

$$\frac{-\partial q}{\partial \psi} = g(\bar{\phi}) \left[ u'(c^*(e_u)) \frac{-\partial \tau_E(e_u)}{\partial \psi} y(e_u) - u'(c^*(e_s)) \frac{-\partial \tau_E(e_s)}{\partial \psi} y(e_s) \right]. \quad (29c)$$

Taking as a common factor  $u'(c^*(e_s))y(e_s)$  in Eq. (29c) we get

$$\frac{-\partial q}{\partial \psi} = g(\bar{\phi}) u'(c^*(e_s)) y(e_s) \left[ \frac{-\partial \tau_E(e_u)}{\partial \psi} \Phi - \frac{-\partial \tau_E(e_s)}{\partial \psi} \right]. \quad (29d)$$

Adding and subtracting  $\frac{-\partial \tau_E(e_u)}{\partial \psi}$  in (29d) gives, after rearranging terms,

$$\frac{-\partial q}{\partial \psi} = g(\bar{\phi}) u'(c^*(e_s)) y(e_s) \left[ \frac{-\partial \Delta_\tau}{\partial \psi} + (\Phi - 1) \frac{-\partial \tau_E(e_u)}{\partial \psi} \right], \quad (29e)$$

which is equivalent to Eq. (16). ■

For convenience we calculate the sign of the impact of a fall in the replacement rate on skill levels as

$$\text{sign} \left[ \frac{-\partial q}{\partial \psi} \right] = \text{sign} \left[ \frac{-\partial \tau_E(e_u)}{\partial \psi} \Phi - \frac{-\partial \tau_E(e_s)}{\partial \psi} \right], \quad (30)$$

which is equivalent to

$$\text{sign} \left[ \frac{-\partial q}{\partial \psi} \right] = \text{sign} [\tau_E(e_s) - \tau_E(e_u) \Phi].$$

Now, assuming  $u'(x) = x^{-\gamma}$ ,  $\Phi$  is given by

$$\Phi = \left( \frac{1 - \tau_E(e_s)}{1 - \tau_E(e_u)} \frac{1 + R^{1-\frac{1}{\gamma}} \beta^{-\frac{1}{\gamma}} \pi(e_u)}{1 + R^{1-\frac{1}{\gamma}} \beta^{-\frac{1}{\gamma}} \pi(e_s)} \right)^\gamma (1 - \alpha(e_s))^{1-\gamma}, \quad (31)$$

where  $\tau_E(e_i)$  is given by (28).

## D Sensitivity analysis

### D.1 Different relative risk aversion coefficients

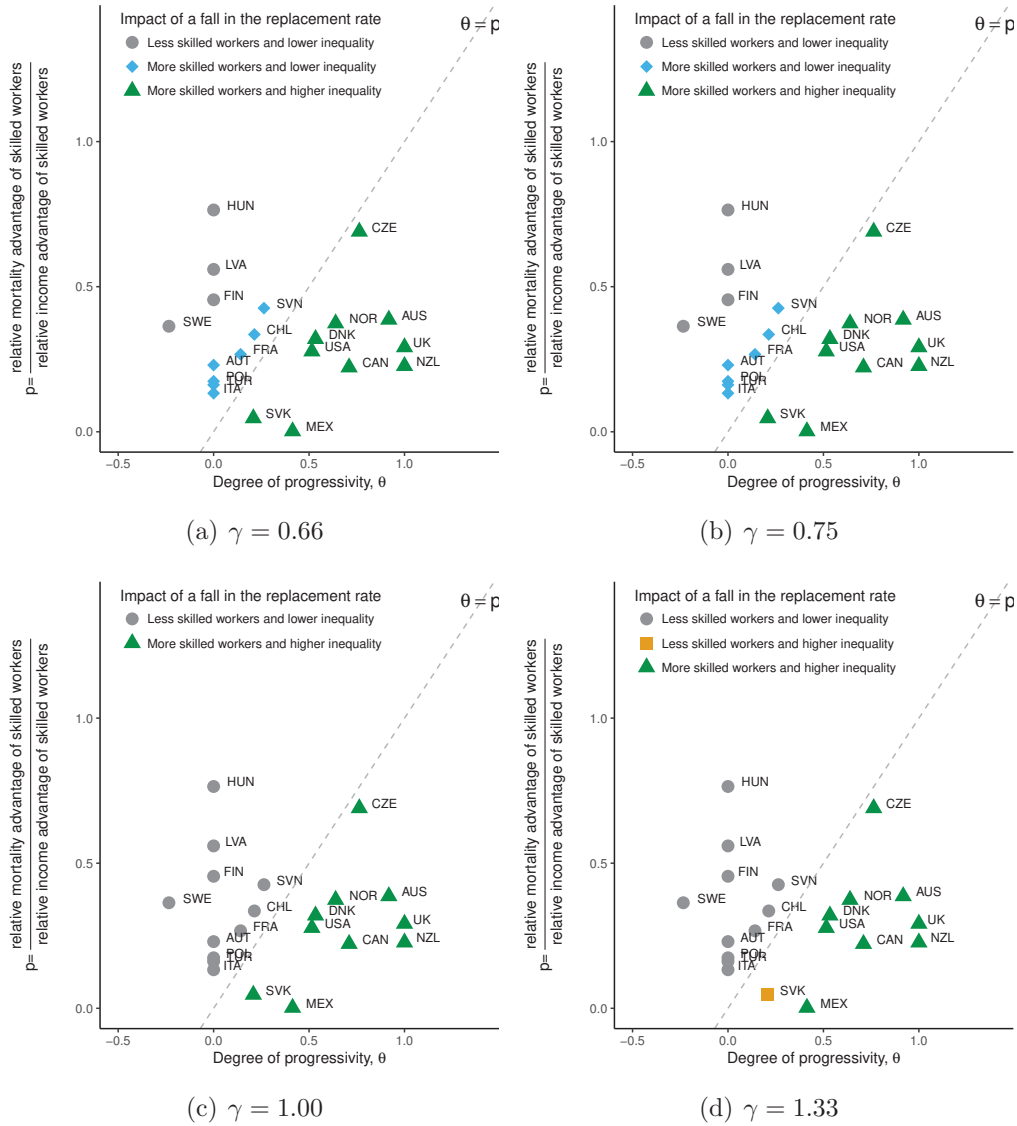


Figure 9: Impact of a reduction in the replacement rate ( $\psi$ ) on the proportion of skilled workers ( $q$ ) and on pension inequality ( $\Delta_\tau$ ) by degree of progressivity of the pension system ( $\theta$ ) in 21 selected OECD countries

## D.2 Different market discount factors

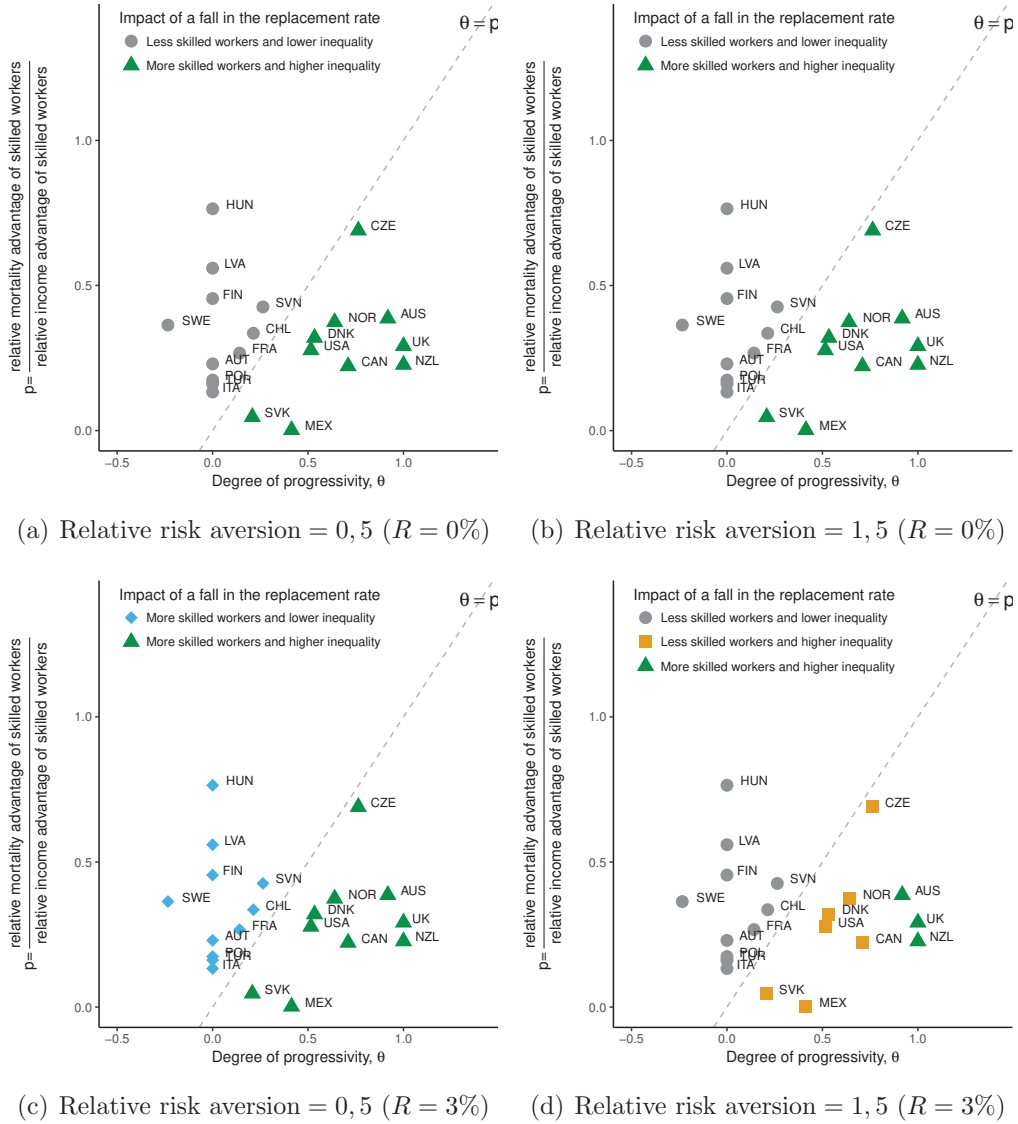


Figure 10: Impact of a reduction in the replacement rate ( $\psi$ ) on the proportion of skilled workers ( $q$ ) and on pension inequality ( $\Delta_\tau$ ) by degree of progressivity of the pension system ( $\theta$ ) in 21 selected OECD countries

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