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Article

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DEVELOPING SINGLE-ACCEPTANCE SAMPLING PLANS BASED ON A TRUNCATED LIFETIME TEST FOR AN ISHITA DISTRIBUTION

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ABSTRACT

Acceptance sampling plans are statistical procedures that are used for quality control and improvement in cases where it is not possible to test every item in a lot of materials. The outcome of this test determines whether the entire lot is accepted or rejected based on a random sample. In this procedure, an important characteristic of the materials is their lifetime sampling distribution, and this can vary from sample to sample. In this article, a new lifetime distribution, known as an Ishita distribution, is considered for developing a new single-acceptance sampling plan. The new acceptance sampling plan is developed under a condition wherein the mean lifetime test is truncated within a pre-specified time period. Based on this condition, the parameters of the acceptance sampling plan including the operating characteristics function and the minimum sample size are obtained. The producer's risk in relation to the entire lot of materials is derived and illustrated by numerical examples.

Key words: acceptance sampling plans, Ishita distribution, producer's risk, consumer's risk, operating characteristics, truncated lifetime-distribution test.

1. Introduction

Single-acceptance sampling plans (SASP) are usually used to determine whether to accept or reject a lot of materials, while taking into consideration that the important characteristic of the materials is its potential lifetime distribution. Moreover, the most important stage is deciding which unit in a given lot should be selected for testing procedure. From a statistician's point of view, this decision involves two types of errors: a Type I error (α), which reflects the probability of rejecting a good lot of a materials (producer's risk); and a Type II error (β), which occurs when there is a probability of accepting a bad lot (consumer's risk). Several studies consider applying different procedures to SASPs (Al-Omari

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(2015); Al-Omari et al., 2016a, 2016b; Al-Nasser and Al-Omari, 2013 and Al-Omari, 2014). In some plans, a product's quality characteristics are measured on a numerical scale (variable SASP); in other plans, they are measured based on the number of failures (attribute SASP). Kao (1966) provides a detailed comparison between the attribute and variable SASP.

In this article, we aim to develop a new SASP to test a product's attributes under the condition that the experiment is terminated at a pre-assigned time (say t). Here, a binomial experiment is carried out in which a random sample size (say n) is drawn from a large lot and tested for its potential lifetime distribution. In this type of test, if less than a specified accepted number of units (say c) fail, then the consumer accepts this lot; otherwise, they reject the lot. In previous studies the test performed by assuming the lifetime distribution is either gamma, Generalized Exponential, Log Normal, Weibull, or Pareto, etc. (see, for example, Al-Nasser and Gogah, 2017; Gogah and Al-Nasser, 2018; Balakrishnan et al., 2007; and Aslam et al., 2011.) In this article, we propose considering a new lifetime distribution known as the Ishita distribution (Shanker and Shukla, 2017; Shukla and Shanker, 2018).

This paper is organized as follows. Section 2 introduces the Ishita distribution and discusses some of its properties. Section 3 includes the suggested acceptance sampling plans for the truncated Ishita distribution; here, the necessary tables are also provided. The conclusions are presented in section 4.

2. Ishita distribution

In many quality control applications, lifetime distributions, such as those mentioned in the previous section, and many more, play primary roles in analysing and understanding data. These distributions are generally used to obtain the minimum sample size in an acceptance sampling context and other sampling plan parameters, such as the average run length (ARL). Shanker and Shukla (2017) suggest a new lifetime distribution known as the "Ishita distribution", which could also be used as an alternative to the classical lifetime distribution, when obtaining the parameters of any acceptance sampling plan. The one-parameter Ishita probability density function (PDF) is defined as follows:

$$f(x) = \frac{\beta^3}{\beta^3 + 2} (\beta + x^2) e^{-\beta x};, x > 0, \beta > 0, \quad (1)$$

Figure 1 shows the plot of the pdf with the different parameter values.

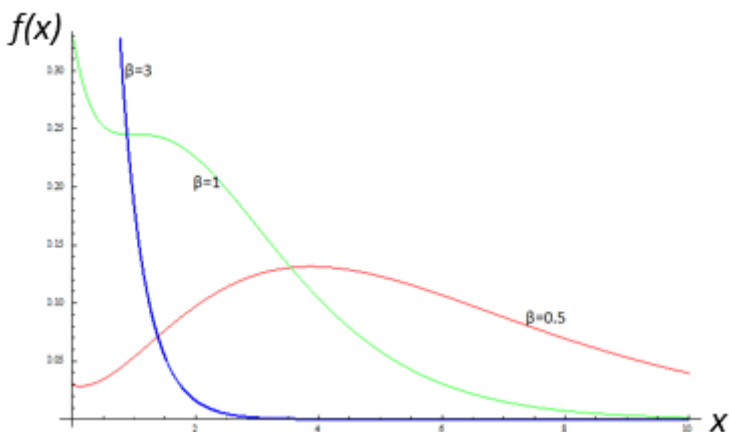


Figure 1. Ishita probability density function with $\beta = 0.5, 1, 3$

The cumulative distribution function (CDF) is defined as follows:

$$F(x) = 1 - \left(1 + \frac{\beta x(\beta x + 2)}{\beta^3 + 2} \right) e^{-\beta x}; \quad x > 0, \beta > 0. \tag{2}$$

Figure 2 provides an illustration of the Ishita CDF.

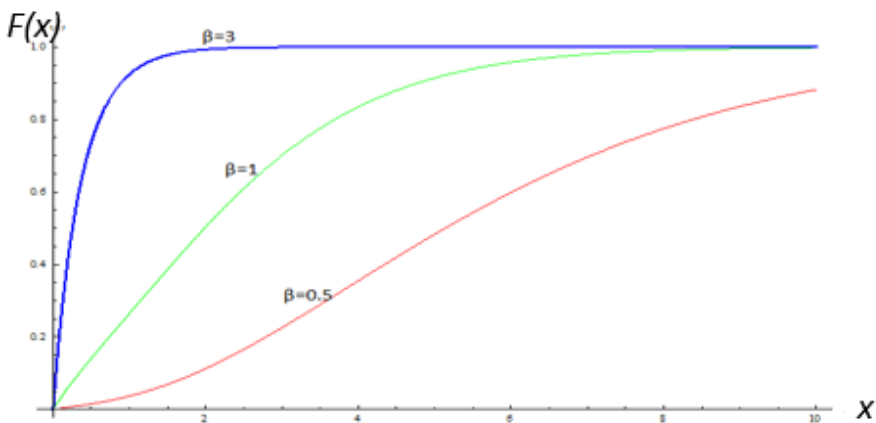


Figure 2. Ishita cumulative distribution function when $\beta = 0.5, 1, 3$

The r th moment of the origin of the Ishita distribution is as follows:

$$E(X^r) = \frac{r! [\beta^3 + (r+1)(r+2)]}{\beta^r (\beta^3 + 2)}; \quad r = 1, 2, 3, \dots$$

For our acceptance sampling plan, based on this distribution, we consider the distribution when parameter $\beta = 3$. Therefore, the moment behaviour that is based on this value can be visualized in Figure 3.

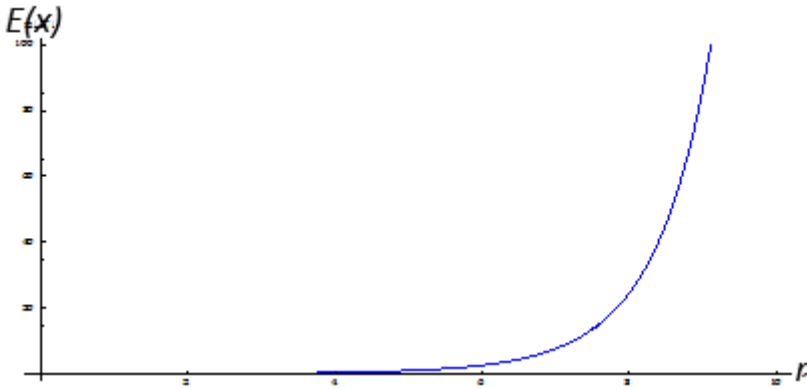


Figure 3. The r th moment value of the Ishita distribution when $\beta = 3$

In general, it can be noted that the distribution mean is

$$E(X) = \frac{\beta^3 + 6}{\beta(\beta^3 + 2)},$$

Then, the maximum likelihood estimator (MLE) $\hat{\beta}$ of β is the solution of the following equation:

$$\frac{6n}{\beta(\beta^3 + 2)} + \sum_{i=1}^n \frac{1+x_i}{\beta + \beta x_i^2} - n\bar{x} = 0,$$

while the moment estimator of the parameter, β , is the solution of the following equation:

$$\bar{x}\beta^4 - \beta^3 + 2\beta\bar{x} = 0,$$

where \bar{x} is the sample mean and n is the sample size. See Shanker and Shukla (2017) for more details on the Ishita distribution and its properties, and Shanker and Shukla (2018) for applications of the Ishita distribution in the context of lifetime-distribution data.

3. The suggested single-acceptance sampling plan

In this study, we suggest developing an SASP that uses the Ishita distribution to model the lifetime distribution of a product. To begin, suppose a producer claims that the specified mean lifetime distribution of the units is μ_0 . Here, the problem is to find the minimum sample size necessary to ensure a certain average life (μ), when the lifetime-distribution test is terminated at a pre-assigned time (t) and when the number of failures does not exceed a given acceptance number, c . To perform the test according to this plan, a random sample of m units is selected from a lot. If μ_0 can be obtained with a pre-assigned probability, P^* , as specified by the consumer, then the lot is accepted. If not, then it is rejected. Figure 4 provides a visual illustration of the SASP.

Assume that a product's lifetime distribution follows an Ishita distribution. An acceptance sampling plan that is based on truncated lifetime distributions tests consists of the following:

- (1) The number of units, m , in the test.
- (2) The acceptance number, c .
- (3) The maximum test duration time, t .
- (4) The ratio t / μ_0 , where μ_0 is the specified average lifetime distribution.

In this case, the usual practice is to take a random sample from a product lot and then perform a truncated lifetime-distribution test. Accordingly, the lot under investigation will be accepted if there is enough evidence that $\mu \geq \mu_0$, at a certain level of producer risk α and consumer risk β . Now, assume that the lot is large enough to apply the binomial theory.

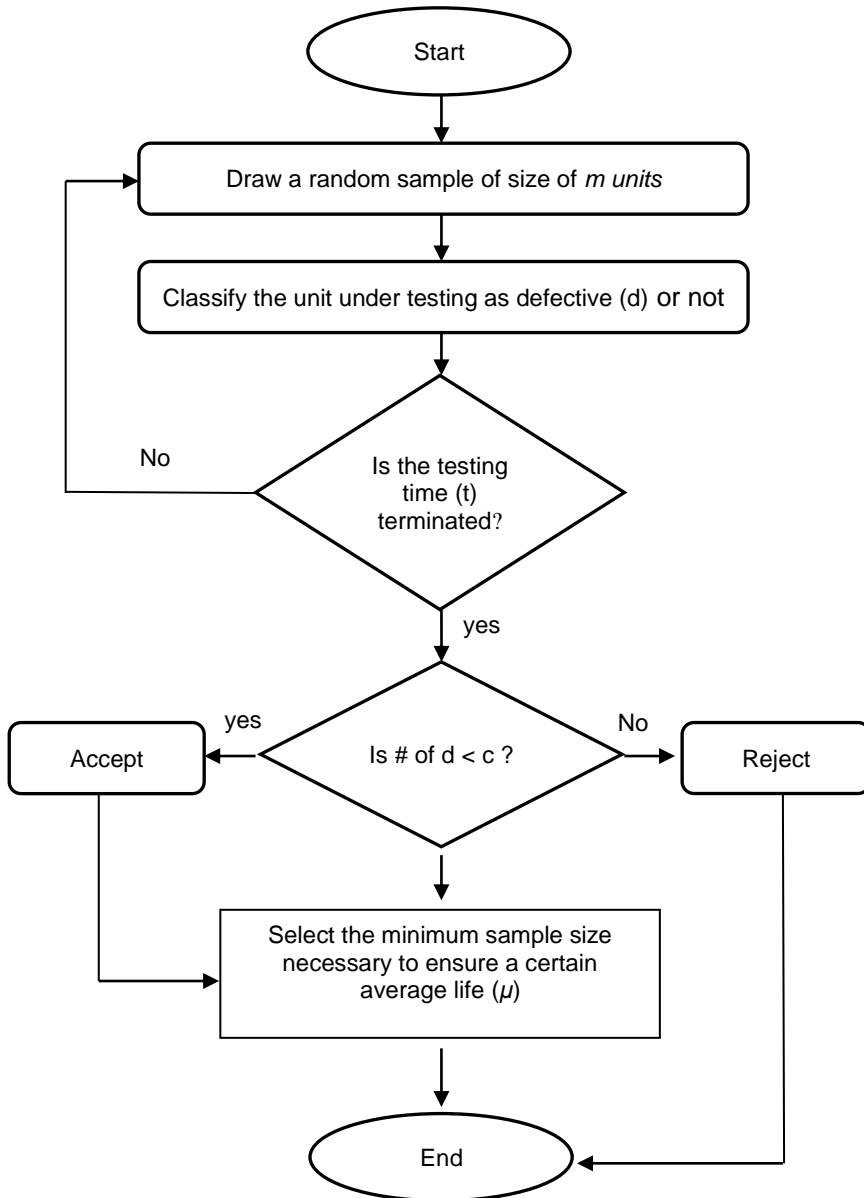


Figure 4. SASP algorithm

3.1. Minimum sample size

Given P^* and assuming that the lot size is large enough to be considered infinite, the probability of accepting a lot can be obtained based on the cumulative binomial distribution function up to the acceptance number, c , and the smallest sample size, m , to ensure that $\mu > \mu_0$ satisfies the following inequality:

$$\sum_{i=0}^c \binom{m}{i} p^i (1-p)^{m-i} \leq 1 - P^*, \tag{3}$$

where $P^* \in (0,1)$, and $p = F(t; \mu_0)$ is a monotonically increasing function of t / μ_0 and is known as the probability of a failure being observed during time t . If the number of observed failures within time t is c , at most, then, from inequality (3), we can confirm, with probability P^* , that $F(t; \mu) \leq F(t; \mu_0)$, which implies $\mu_0 \leq \mu$.

The minimum sample sizes that satisfy the above inequality for $t / \mu_0 = 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712$, with $P^* = 0.75, 0.9, 0.95, 0.99$ and $c = 0, 1, 2, \dots, 10$. The values of t / μ_0 and P^* , presented in this work, are the same as the corresponding values found in Baklizi and El Masri (2004), for the Birnbaum Saunders model; in Kantam et al. (2001), for the log-logistic model; and in Gupta and Groll (1961), for the gamma distribution. Table 1 gives the minimum sample sizes for the new acceptance sampling plan for $\beta = 3$ in the Ishita distribution.

3.2. Operating characteristic function

For the sampling plan $(m, c, t / \mu_0)$, the operating characteristic (OC) function gives the probability of accepting the lot as the following:

$$OC(p) = P_r(\text{Accepting a lot}) = \sum_{i=0}^c \binom{m}{i} p^i (1-p)^{m-i}$$

where $p = F(t; \mu)$ is a function of μ (the parameter of the lot quality). It is of interest to say that, for a fixed t , the average lifetime of the products is increasing $\mu \geq \mu_0$ implying that the failure probability $p = F(t; \mu)$ is a monotonically decreasing function of $\mu \geq \mu_0$, and therefore the operating characteristic function $OC(p)$ is increasing in the average lifetime; which means that as the ratio value $\frac{\mu}{\mu_0}$ is increasing say from "2 to 12", then the operating characteristic function $OC(p)$ is

increasing. Table 2 presents the OC function values as a function of $\mu \geq \mu_0$ for the sampling plan $(m, c = 2, t / \mu_0)$

3.3. Producer's risk

The producer's risk is defined as the probability of the consumer rejecting the lot when $\mu > \mu_0$. This is given by the following:

$$P_r(p) = P_r(\text{Rejecting a lot}) = \sum_{i=c+1}^m \binom{m}{i} p^i (1-p)^{m-i}$$

For a given value of the producer's risk, say ϕ , under a given sampling plan, one may be interested to know the smallest value of μ / μ_0 that will ensure that the producer's risk is ϕ , at most. The value of μ / μ_0 is the minimum positive number for which $p = F\left(\frac{t}{\mu_0} \frac{\mu_0}{\mu}\right)$ satisfies the inequality presented in Inequality 4

$$P_r(p) = \sum_{i=c+1}^m \binom{m}{i} p^i (1-p)^{m-i} \leq \phi \quad (4)$$

Table 3 summarizes the smallest values of the ratio μ / μ_0 that satisfy Inequality (4) for $\beta = 3$ in an Ishita distribution with a given acceptance sampling plan $(m, c, t / \mu_0)$ at the confidence level P^* . As the tables show, we assume that the lifetime distribution follows the Ishita distribution where $\beta = 3$. Table 1 shows the smallest sample sizes necessary to assert that the product's mean lifetime exceeds μ_0 with a probability of at least P^* and an associated acceptance number c . For example, assume that the researcher aims to ensure that the product's mean lifetime distribution is at least 1000 hours, with probability $P^* = 0.90$ with and the acceptance number of $c = 2$, such that the lifetime-distribution test is at least $t = 2356$ hours; that is, $t / \mu_0 = 2.356$. Then, from Table 1, the sample size is $m = 4$ units, which is the number of units that should be tested. Further, if 2 units, at most, out of the 4, fail before the specified time, t , within 1000 hours, then the product lot is accepted with a confidence level of 0.90. Hence, the process is to truncate the time of the test to a time of 2.356 of the specified mean lifetime distribution.

Table 2 summarizes the values of the OC function for the new acceptance sampling plans that were adopted, based on Table 1, for various values of t / μ_0

and P^* with $c=2$. Table 3 shows that the OC values for the sampling plan ($m=4, c=2, t/\mu_0=2.356$) are as follows:

$\frac{\mu}{\mu_0}$	2	4	6	8	10	12
OC	0.352196	0.749243	0.88624	0.940036	0.964799	0.977655

Therefore, if the true mean lifetime distribution is six times the specified mean ($\mu/\mu_0=6$), then the producer's risk is about 0.11376, and the producer's risk is about 0.647804, 0.250757, 0.059964, 0.035201, 0.022345, respectively, for $\mu/\mu_0=2,4,8,10,12$, and approaches zero for large values of μ/μ_0 .

Table 3 provides the minimum ratios of the true mean lifetime distribution to the specified mean lifetime distribution (μ/μ_0) for the acceptance of a product's lot with the producer's risk $\phi=0.05$, with different values of t/μ_0 and the acceptance number c . As an illustration, the smallest value of μ/μ_0 is 8.645 for $P^*=0.90$, $t/\mu_0=2.356$, and $c=2$. That is, the product must have a mean lifetime distribution that is greater than 8645 hours, in order for the lot to be accepted. Hence, Table 3 shows the true mean lifetime distribution that is necessary for accepting 95% of the product's lots.

4. Concluding Remarks

In this paper, we developed a single-acceptance sampling plan for an Ishita distribution (a new lifetime distribution) under the assumption that the mean lifetime is a preassigned quality parameter. The proposed new SASP parameters are obtained to simultaneously measure the consumer and producer's risk in a product as measured by the lifetime distribution of a representative lot of the product. Several computations and numerical examples and their outcomes were presented and discussed.

The operating characteristics of the single-acceptance sampling plan is provided numerically. Based on the results, the consumers and producers are advised to adopt this plan in order to save time and minimize production process cost. Future research could be conducted on single-acceptance sampling plans by assuming the products' has different lifetime distributions.

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APPENDIX

Table 1. Minimum values of m necessary to assert the mean life exceeds μ_0 , with various combinations of P^* , c and t / μ_0 with $\beta = 3$.

P^*	c	t / μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	3	2	2	1	1	1	1	1
	1	5	4	3	3	2	2	2	2
	2	8	6	5	4	4	3	3	3
	3	10	8	6	6	5	4	4	4
	4	12	9	8	7	6	5	5	5
	5	15	11	9	8	7	7	6	6
	6	17	13	11	10	8	8	7	7
	7	19	15	12	11	9	9	8	8
	8	22	16	14	12	11	10	9	9
	9	24	18	15	14	12	11	10	10
0.90	10	26	20	17	15	13	12	11	11
	0	4	3	2	2	1	1	1	1
	1	7	5	4	4	3	2	2	2
	2	10	7	6	5	4	4	3	3
	3	12	9	8	7	5	5	4	4
	4	15	11	9	8	7	6	6	5
	5	18	13	11	9	8	7	7	6
	6	20	15	12	11	9	8	8	7
	7	22	17	14	12	10	9	9	8
	8	25	19	15	14	11	10	10	9
0.95	9	27	20	17	15	13	11	11	11
	10	30	22	19	16	14	13	12	12
	0	5	4	3	2	2	1	1	1
	1	8	6	5	4	3	3	2	2
	2	11	8	7	6	5	4	4	3
	3	14	10	8	7	6	5	5	4
	4	17	12	10	9	7	6	6	6
	5	19	14	12	10	8	7	7	7
	6	22	16	13	12	10	9	8	8
	7	25	18	15	13	11	10	9	9
0.99	8	27	20	17	15	12	11	10	10
	9	30	22	18	16	13	12	11	11
	10	32	24	20	17	14	13	12	12
	0	8	5	4	3	2	2	2	2
	1	11	8	6	5	4	3	3	3
	2	15	10	8	7	5	5	4	4
	3	18	13	10	9	7	6	5	5
	4	21	15	12	10	8	7	6	6
	5	23	17	14	12	9	8	8	7
	6	26	19	15	13	11	9	9	8
7	29	21	17	15	12	10	10	9	
8	32	23	19	16	13	12	11	10	
9	34	25	20	18	14	13	12	11	
10	37	27	22	19	16	14	13	12	

Table 2. OC values for the sampling plan from Table (1) for P^* at $c = 2$ for $\beta = 3$.

P^*	m	t / μ_0	μ / μ_0					
			2	4	6	8	10	12
0.75	8	0.628	0.602233	0.890665	0.957192	0.97918	0.98838	0.992875
	6	0.942	0.570169	0.876254	0.950618	0.975735	0.98637	0.991606
	5	1.257	0.540865	0.861888	0.94385	0.972129	0.984245	0.990255
	4	1.571	0.601762	0.886188	0.954551	0.977643	0.98743	0.992253
	4	2.356	0.352196	0.749243	0.88624	0.940036	0.964799	0.977655
	3	3.141	0.500641	0.829677	0.926184	0.961977	0.977985	0.986153
	3	3.927	0.368648	0.745812	0.881115	0.936076	0.96196	0.975614
	3	4.712	0.266775	0.660235	0.829642	0.904632	0.941746	0.961967
0.90	10	0.628	0.437279	0.813833	0.921665	0.960433	0.977397	0.985921
	7	0.942	0.447328	0.817579	0.923199	0.961184	0.977816	0.986176
	6	1.257	0.385506	0.779748	0.903955	0.950524	0.971384	0.982023
	5	1.571	0.39875	0.785633	0.906547	0.951846	0.97214	0.982493
	4	2.356	0.352196	0.749243	0.88624	0.940036	0.964799	0.977655
	4	3.141	0.19108	0.601946	0.79763	0.886265	0.930449	0.954586
	3	3.927	0.368648	0.745812	0.881115	0.936076	0.96196	0.975614
	3	4.712	0.266775	0.660235	0.829642	0.904632	0.941746	0.961967
0.95	11	0.628	0.365942	0.771385	0.900386	0.948743	0.970382	0.981407
	8	0.942	0.341996	0.753433	0.890665	0.943192	0.966973	0.97918
	7	1.257	0.263901	0.691085	0.855943	0.92306	0.954488	0.970967
	6	1.571	0.247989	0.67441	0.845759	0.916873	0.950542	0.968323
	5	2.356	0.166013	0.580453	0.785718	0.879448	0.926275	0.95187
	4	3.141	0.19108	0.601946	0.79763	0.886265	0.930449	0.954586
	4	3.927	0.099455	0.4663	0.699831	0.820982	0.886219	0.923671
	3	4.712	0.266775	0.660235	0.829642	0.904632	0.941746	0.961967
0.99	15	0.628	0.164133	0.593595	0.797939	0.888306	0.93251	0.956321
	10	0.942	0.188243	0.619592	0.813833	0.897949	0.938652	0.960433
	8	1.257	0.175063	0.601759	0.802013	0.890475	0.93377	0.957109
	7	1.571	0.147227	0.563691	0.776478	0.874204	0.923088	0.949808
	5	2.356	0.166013	0.580453	0.785718	0.879448	0.926275	0.95187
	5	3.141	0.063146	0.398954	0.648376	0.78576	0.862024	0.906613
	4	3.927	0.099455	0.4663	0.699831	0.820982	0.886219	0.923671
	4	4.712	0.050625	0.352196	0.601885	0.749243	0.83473	0.88624

Table 3. Minimum ratio of true mean life over μ_0 at the producer's risk of 0.05 under the $\beta = 3$.

P^*	c	t / μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	38.909	38.897	51.903	32.403	48.594	64.785	80.997	97.188
	1	8.347	9.685	9.112	11.389	9.775	13.031	16.292	19.549
	2	5.627	5.969	6.302	5.765	8.645	7.123	8.906	10.686
	3	4.071	4.622	4.16	5.199	5.862	5.079	6.35	7.619
	4	3.309	3.412	3.854	3.928	4.52	4.058	5.074	6.088
	5	3.119	3.119	3.101	3.196	3.736	4.981	4.31	5.171
	6	2.771	2.915	3.046	3.271	3.224	4.298	3.799	4.558
	7	2.526	2.763	2.639	2.851	2.863	3.817	3.432	4.118
	8	2.489	2.426	2.643	2.542	3.219	3.458	3.155	3.786
	9	2.329	2.361	2.378	2.642	2.937	3.18	2.938	3.525
10	2.202	2.307	2.4	2.418	2.713	2.958	2.762	3.314	
0.9	0	51.887	58.363	51.903	64.869	48.594	64.785	80.997	97.188
	1	12.111	12.52	12.923	16.151	17.079	13.031	16.292	19.549
	2	7.266	7.207	7.965	7.876	8.645	11.525	8.906	10.686
	3	5.054	5.366	6.168	6.461	5.862	7.815	6.35	7.619
	4	4.333	4.449	4.553	4.816	5.891	6.026	7.533	6.088
	5	3.891	3.901	4.162	3.875	4.793	4.981	6.228	5.171
	6	3.386	3.538	3.47	3.807	4.082	4.298	5.374	4.558
	7	3.035	3.278	3.341	3.299	3.585	3.817	4.772	4.118
	8	2.92	3.082	2.941	3.304	3.219	3.458	4.323	3.786
	9	2.702	2.741	2.895	2.972	3.458	3.18	3.976	4.771
10	2.64	2.641	2.854	2.711	3.178	3.617	3.699	4.438	
0.95	0	64.865	77.83	77.88	64.869	97.282	64.785	80.997	97.188
	1	13.99	15.346	16.706	16.151	17.079	22.769	16.292	19.549
	2	8.084	8.44	9.617	9.955	11.811	11.525	14.409	10.686
	3	6.034	6.106	6.168	6.461	7.797	7.815	9.771	7.619
	4	5.014	4.963	5.247	5.691	5.891	6.026	7.533	9.039
	5	4.148	4.29	4.685	4.542	4.793	4.981	6.228	7.472
	6	3.795	3.847	3.889	4.336	4.905	5.442	5.374	6.448
	7	3.541	3.534	3.687	3.739	4.275	4.78	4.772	5.725
	8	3.206	3.3	3.53	3.676	3.812	4.291	4.323	5.187
	9	3.074	3.118	3.151	3.297	3.458	3.915	3.976	4.771
10	2.859	2.973	3.078	2.999	3.178	3.617	3.699	4.438	
0.99	0	103.799	97.297	103.856	97.334	97.282	129.695	162.15	194.564
	1	19.62	20.984	20.477	20.879	24.221	22.769	28.467	34.157
	2	11.349	10.899	11.263	12.02	11.811	15.746	14.409	17.289
	3	7.991	8.316	8.147	8.948	9.689	10.395	9.771	11.724
	4	6.372	6.5	6.623	6.557	7.222	7.854	7.533	9.039
	5	5.173	5.451	5.725	5.856	5.812	6.389	7.988	7.472
	6	4.611	4.772	4.721	4.86	5.709	5.442	6.804	6.448
	7	4.215	4.298	4.374	4.608	4.947	4.78	5.976	5.725
	8	3.92	3.948	4.113	4.045	4.388	5.082	5.365	5.187
	9	3.569	3.68	3.658	3.938	3.962	4.61	4.895	4.771
10	3.403	3.468	3.524	3.567	4.066	4.236	4.522	4.438	