

Subzar, Mir; Maqbool, Showkat; Raja, Tariq Ahmad; Pal, Surya Kant; Sharma, Prayas

**Article**

## EFFICIENT ESTIMATORS OF POPULATION MEAN USING AUXILIARY INFORMATION UNDER SIMPLE RANDOM SAMPLING

Statistics in Transition New Series

**Provided in Cooperation with:**

Polish Statistical Association

*Suggested Citation:* Subzar, Mir; Maqbool, Showkat; Raja, Tariq Ahmad; Pal, Surya Kant; Sharma, Prayas (2018) : EFFICIENT ESTIMATORS OF POPULATION MEAN USING AUXILIARY INFORMATION UNDER SIMPLE RANDOM SAMPLING, Statistics in Transition New Series, ISSN 2450-0291, Exeley, New York, NY, Vol. 19, Iss. 2, pp. 219-238, <https://doi.org/10.21307/stattrans-2018-013>

This Version is available at:

<https://hdl.handle.net/10419/207896>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*



<https://creativecommons.org/licenses/by-nc-nd/4.0/>

STATISTICS IN TRANSITION *new series, June 2018*  
Vol. 19, No. 2, pp. 219–238, DOI 10.21307/stattrans-2018-013

## EFFICIENT ESTIMATORS OF POPULATION MEAN USING AUXILIARY INFORMATION UNDER SIMPLE RANDOM SAMPLING

Mir Subzar<sup>1</sup>, Showkat Maqbool<sup>1</sup>, Tariq Ahmad Raja<sup>1</sup>, Surya Kant Pal<sup>2</sup>,  
Prayas Sharma<sup>3</sup>

### ABSTRACT

In the present study we have proposed an improved family of estimators for estimation of population mean using the auxiliary information of median, quartile deviation, Gini's mean difference, Downton's Method, Probability Weighted Moments and their linear combinations with correlation coefficient and coefficient of variation. The performance of the proposed family of estimators is analysed by mean square error and bias and compared with the existing estimators in the literature. By this comparison we conclude that our proposed family of estimators is more efficient than the existing estimators. To support the theoretical results, we also provide the empirical study.

**Key words:** Auxiliary information, Bias, Mean Square Error, Simple Random Sampling, Efficiency.

AMS Subject Classification: 62D05

### 1. Introduction

Since last few decades statisticians have proposed estimators for the population parameters. The concept of efficiency is a vital key word for an estimator and it depends on the Mean Square Error (MSE), thus we should know the fundamental of MSE in statistics. Therefore, according to the estimation theory the estimators with the mean square error or variance than lower that of other estimators are said to be efficient estimators and the estimators with the bias lower than that of other estimators are said to be good estimators. Hence, this efficiency in estimation theory is achieved by using the auxiliary information at the design or estimation stage or at both stages. The use of such auxiliary information is made through different methods of estimation such Ratio, Regression and Product methods. So, Cochran (1977) initiated the use of auxiliary information at estimation stage and proposed ratio estimator for population mean. It is a well-established fact that ratio type estimators provide

<sup>1</sup> Division of Agricultural Statistics, SKUAST-K Shalimar, Srinagar-190025, Kashmir, India.

<sup>2</sup> School of Studies in Statistics, Vikram University, Ujjain-456010, Madhya Pradesh P., India.

<sup>3</sup> Department of Decision Sciences, School of Business, UPES, Dehradun, India.  
E-mail: prayassharma02@gmail.com.

better efficiency in comparison to simple mean estimator if the study variable and auxiliary variable are positively correlated and the regression line pass through origin, and if on the other side the correlation between the study variable and auxiliary variable is positive and does not pass through origin but makes an intercept, in that case the regression method provides better efficiency than ratio, simple mean and product type estimator, and if the correlation between the study variable and auxiliary variable is negative, product estimator given by Robson (1957) is more efficient than simple mean estimator.

Further improvements are also achieved on the classical ratio estimator by introducing a large number of modified ratio estimators with the use of known parameters as coefficient of variation, coefficient of kurtosis, coefficient of skewness and population correlation coefficient. For more detailed discussion, one may refer to Cochran (1977), Kadilar and Cingi (2004, 2006), Koyuncu and Kadilar (2009), Murthy (1967), Sharma and Singh (2013), Prasad (1989), Rao (1991), Singh and Tailor (2003, 2005), Singh et al. (2004), Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999) and Yan and Tian (2010).

Further, Subramani and Kumarapandiyam (2012) have taken initiative by proposing modified ratio estimator for estimating the population mean of the study variable by using the population deciles of the auxiliary variable.

Recently, Subzar et al. (2016) have proposed some estimators using population deciles and correlation coefficient of the auxiliary variable, also Subzar et al. (2017) have proposed some modified ratio type estimators using the quartile deviation and population deciles of auxiliary variable, and Subzar et al. (2017) have also proposed an efficient class of estimators by using the auxiliary information of population deciles, median and their linear combination with correlation coefficient and coefficient of variation.

In this paper we have envisaged a new class of improved ratio type estimators for estimation of population mean of the study variable using the information of quartile deviation, median, non-conventional measures of dispersion and their linear combination with correlation coefficient and coefficient of variation. Let  $G = \{G_1, G_2, G_3, \dots, G_N\}$  be a finite population of  $N$  distinct and identifiable units. Let  $y$  and  $x$  denote the study variable and the auxiliary variable taking values  $y_i$  and  $x_i$  respectively on the  $i^{\text{th}}$  unit ( $i = 1, 2, \dots, N$ ). Before discussing the proposed estimators, we will mention the estimators in the literature using the notations given in the next sub-section.

### 1.1. Notations

$N$	Population size
$n$	Sample size
$f = n/N$	Sampling fraction
$Y$	Study variable
$X$	Auxiliary variable
$\bar{X}, \bar{Y}$	Population means

$\bar{x}, \bar{y}$	Sample means
$x, y$	Sample totals
$S_x, S_y$	Population standard deviations
$S_{xy}$	Population covariance between variables
$C_x, C_y$	Population coefficient of variation
$\rho$	Population correlation coefficient
$B(\cdot)$	Bias of the estimator
$MSE(\cdot)$	Mean square error of the estimator
$\hat{Y}_i$	Existing modified ratio estimator of $\bar{Y}$
$\hat{Y}_{pj}$	Proposed modified ratio estimator of $\bar{Y}$
$M_d$	Population median of $X$
$\beta_2$	Population kurtosis
$\beta_1$	Population skewness
$QD = \frac{Q_3 - Q_1}{2}$	Population quartile deviation
$HL = median((X_j + X_k) / 2, 1 \leq j \leq k \leq N)$	Hodges-Lehmann estimator
$MR = \frac{X_{(1)} + X_{(N)}}{2}$	Population mid-range
$DM = \frac{D_1 + D_2 + \dots + D_9}{9}$	Decile mean
$G = \frac{4}{N-1} \sum_{i=1}^N \left( \frac{2i - N - 1}{2N} \right) X_{(i)}$	Gini's Mean Difference
$D = \frac{2\sqrt{\pi}}{N(N-1)} \sum_{i=1}^N \left( i - \frac{N+1}{2} \right) X_{(i)}$	Downton's method
$S_{pw} = \frac{\sqrt{\pi}}{N^2} \sum_{i=1}^N (2i - N - 1) X_{(i)}$	Probability weighted moments

### Subscript

- $i$  For existing estimators  
 $j$  For proposed estimators

### 1.2. Estimators in the literature

Kadilar and Cingi (2004) have suggested ratio type estimators for the population mean in the simple random sampling using some known auxiliary information on coefficient of kurtosis and coefficient of variation. They showed that their suggested estimators are more efficient than traditional ratio estimator in the estimation of the population mean.

Kadilar & Cingi (2004) estimators are given by

$$\hat{Y}_1 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x}} \bar{X},$$

$$\hat{Y}_2 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + C_x)} (\bar{X} + C_x),$$

$$\hat{Y}_3 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \beta_2)} (\bar{X} + \beta_2),$$

$$\hat{Y}_4 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_2 + C_x)} (\bar{X}\beta_2 + C_x),$$

$$\hat{Y}_5 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \beta_2)} (\bar{X}C_x + \beta_2),$$

The biases, related constants and the MSE for Kadilar and Cingi (2004) estimators are respectively as follows:

$$B(\hat{Y}_1) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_1^2, \quad R_1 = \frac{\bar{Y}}{\bar{X}} \quad MSE(\hat{Y}_1) = \frac{(1-f)}{n} (R_1^2 S_x^2 + S_y^2 (1-\rho^2)),$$

$$B(\hat{Y}_2) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_2^2, \quad R_2 = \frac{\bar{Y}}{(\bar{X} + C_x)} \quad MSE(\hat{Y}_2) = \frac{(1-f)}{n} (R_2^2 S_x^2 + S_y^2 (1-\rho^2)),$$

$$B(\hat{Y}_3) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_3^2, \quad R_3 = \frac{\bar{Y}}{(\bar{X} + \beta_2)} \quad MSE(\hat{Y}_3) = \frac{(1-f)}{n} (R_3^2 S_x^2 + S_y^2 (1-\rho^2)),$$

$$B(\widehat{Y}_4) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_4^2, \quad R_4 = \frac{\bar{Y}}{(\bar{X}\beta_2 + C_x)} \quad MSE(\widehat{Y}_4) = \frac{(1-f)}{n} (R_4^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\widehat{Y}_5) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_5^2, \quad R_5 = \frac{\bar{Y}}{(\bar{X}C_x + \beta_2)} \quad MSE(\widehat{Y}_5) = \frac{(1-f)}{n} (R_5^2 S_x^2 + S_y^2(1-\rho^2)).$$

Kadilar and Cingi (2006) developed some modified ratio estimators using known value of coefficient of correlation, kurtosis and coefficient of variation as follows:

$$\widehat{Y}_6 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \rho)} (\bar{X} + \rho),$$

$$\widehat{Y}_7 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \rho)} (\bar{X}C_x + \rho),$$

$$\widehat{Y}_8 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + C_x)} (\bar{X}\rho + C_x),$$

$$\widehat{Y}_9 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_2 + \rho)} (\bar{X}\beta_2 + \rho),$$

$$\widehat{Y}_{10} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + \beta_2)} (\bar{X}\rho + \beta_2).$$

The biases, related constants and the MSE for Kadilar and Cingi [6] estimators are respectively given by

$$B(\widehat{Y}_6) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_6^2, \quad R_6 = \frac{\bar{Y}}{\bar{X} + \rho} \quad MSE(\widehat{Y}_6) = \frac{(1-f)}{n} (R_6^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\widehat{Y}_7) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_7^2, \quad R_7 = \frac{\bar{Y}C_x}{\bar{X}C_x + \rho} \quad MSE(\widehat{Y}_7) = \frac{(1-f)}{n} (R_7^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\widehat{Y}_8) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_8^2, \quad R_8 = \frac{\bar{Y}\rho}{\bar{X}\rho + C_x} \quad MSE(\widehat{Y}_8) = \frac{(1-f)}{n} (R_8^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\widehat{Y}_9) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_9^2, \quad R_9 = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + \rho} \quad MSE(\widehat{Y}_9) = \frac{(1-f)}{n} (R_9^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\widehat{Y}_{10}) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{10}^2, \quad R_{10} = \frac{\bar{Y}\rho}{\bar{X}\rho + \beta_2} \quad MSE(\widehat{Y}_{10}) = \frac{(1-f)}{n} (R_{10}^2 S_x^2 + S_y^2(1-\rho^2)).$$

Abid *et al.* (2016) suggested the following ratio estimators for the population mean  $\bar{Y}$  in simple random sampling using non-conventional location parameters as auxiliary information. Estimators suggested by Abid *et al.* (2016) are given as:

$$\hat{Y}_{11} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + TM)} (\bar{X} + TM),$$

$$\hat{Y}_{12} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + TM)} (\bar{X}C_x + TM)$$

$$\hat{Y}_{13} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + TM)} (\bar{X}\rho + TM),$$

$$\hat{Y}_{14} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + MR)} (\bar{X} + MR),$$

$$\hat{Y}_{15} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + MR)} (\bar{X}C_x + MR),$$

$$\hat{Y}_{16} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + MR)} (\bar{X}\rho + MR)$$

$$\hat{Y}_{17} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + HL)} (\bar{X} + HL),$$

$$\hat{Y}_{18} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + HL)} (\bar{X}C_x + HL),$$

$$\hat{Y}_{19} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + HL)} (\bar{X}\rho + HL)$$

The biases, related constants and the mean square error (MSE) for Abid *et al.* (2016) estimators are respectively given by:

$$\begin{aligned} B(\hat{Y}_{11}) &= \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{11}^2, & R_{11} &= \frac{\bar{Y}}{(\bar{X} + TM)} & MSE(\hat{Y}_{11}) &= \frac{(1-f)}{n} (R_{11}^2 S_x^2 + S_y^2 (1 - \rho^2)), \\ B(\hat{Y}_{12}) &= \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{12}^2, & R_{12} &= \frac{\bar{Y}C_x}{(\bar{X}C_x + TM)} & MSE(\hat{Y}_{12}) &= \frac{(1-f)}{n} (R_{12}^2 S_x^2 + S_y^2 (1 - \rho^2)), \\ B(\hat{Y}_{13}) &= \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{13}^2, & R_{13} &= \frac{\bar{Y}\rho}{(\bar{X}\rho + TM)} & MSE(\hat{Y}_{13}) &= \frac{(1-f)}{n} (R_{13}^2 S_x^2 + S_y^2 (1 - \rho^2)), \\ B(\hat{Y}_{14}) &= \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{14}^2, & R_{14} &= \frac{\bar{Y}}{(\bar{X} + MR)} & MSE(\hat{Y}_{14}) &= \frac{(1-f)}{n} (R_{14}^2 S_x^2 + S_y^2 (1 - \rho^2)), \end{aligned}$$

$$\begin{aligned}
 B(\widehat{Y}_{15}) &= \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{15}^2, & R_{15} &= \frac{\bar{Y}C_x}{(\bar{X}C_x + MR)} & MSE(\widehat{Y}_{15}) &= \frac{(1-f)}{n} (R_{15}^2 S_x^2 + S_y^2 (1-\rho^2)), \\
 B(\widehat{Y}_{16}) &= \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{16}^2, & R_{16} &= \frac{\bar{Y}\rho}{(\bar{X}\rho + MR)} & MSE(\widehat{Y}_{16}) &= \frac{(1-f)}{n} (R_{16}^2 S_x^2 + S_y^2 (1-\rho^2)), \\
 B(\widehat{Y}_{17}) &= \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{17}^2, & R_{17} &= \frac{\bar{Y}}{(\bar{X} + HL)} & MSE(\widehat{Y}_{17}) &= \frac{(1-f)}{n} (R_{17}^2 S_x^2 + S_y^2 (1-\rho^2)), \\
 B(\widehat{Y}_{18}) &= \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{18}^2, & R_{18} &= \frac{\bar{Y}C_x}{(\bar{X}C_x + HL)} & MSE(\widehat{Y}_{18}) &= \frac{(1-f)}{n} (R_{18}^2 S_x^2 + S_y^2 (1-\rho^2)), \\
 B(\widehat{Y}_{19}) &= \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{19}^2, & R_{19} &= \frac{\bar{Y}\rho}{(\bar{X}\rho + HL)} & MSE(\widehat{Y}_{19}) &= \frac{(1-f)}{n} (R_{19}^2 S_x^2 + S_y^2 (1-\rho^2)).
 \end{aligned}$$

Abid *et al.* (2016) suggested the following ratio estimators for the population mean  $\bar{Y}$  in simple random sampling using Decile mean, with linear combination of population correlation coefficient and population coefficient of variation as auxiliary information. Estimators suggested by Abid *et al.* (2016) are given as:

$$\begin{aligned}
 \widehat{Y}_{20} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + DM)} (\bar{X} + DM) \\
 \widehat{Y}_{21} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + DM)} (\bar{X}C_x + DM) \\
 \widehat{Y}_{22} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + DM)} (\bar{X}\rho + DM)
 \end{aligned}$$

The biases, related constants and the mean square error (MSE) for Abid *et al.* (2016) estimators are respectively given by:

$$\begin{aligned}
 B(\widehat{Y}_{20}) &= \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{20}^2, & R_{20} &= \frac{\bar{Y}}{(\bar{X} + DM)} & MSE(\widehat{Y}_{20}) &= \frac{(1-f)}{n} (R_{20}^2 S_x^2 + S_y^2 (1-\rho^2)), \\
 B(\widehat{Y}_{21}) &= \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{21}^2, & R_{21} &= \frac{\bar{Y}C_x}{(\bar{X}C_x + DM)} & MSE(\widehat{Y}_{21}) &= \frac{(1-f)}{n} (R_{21}^2 S_x^2 + S_y^2 (1-\rho^2)), \\
 B(\widehat{Y}_{22}) &= \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{22}^2, & R_{22} &= \frac{\bar{Y}\rho}{(\bar{X}\rho + DM)} & MSE(\widehat{Y}_{22}) &= \frac{(1-f)}{n} (R_{22}^2 S_x^2 + S_y^2 (1-\rho^2)),
 \end{aligned}$$

## 2. Improved ratio estimators

Motivated by the estimators mentioned in Section 1.2, in this section we suggest some improved class of estimators by using the auxiliary information median, quartile deviation, Gini's mean difference, Downton's Method, Probability Weighted Moments and their linear combinations with correlation coefficient and



coefficient of variation in survey sampling, and the suggested estimators are given below as:

$$\hat{Y}_{p1} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \Psi_1)} (\bar{X} + \Psi_1)$$

$$\hat{Y}_{p2} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \Psi_2)} (\bar{X} + \Psi_2)$$

$$\hat{Y}_{p3} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \Psi_3)} (\bar{X} + \Psi_3)$$

$$\hat{Y}_{p4} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \Psi_4)} (\bar{X} + \Psi_4)$$

$$\hat{Y}_{p5} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \Psi_5)} (\bar{X} + \Psi_5)$$

$$\hat{Y}_{p6} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \Psi_6)} (\bar{X} + \Psi_6)$$

$$\hat{Y}_{p7} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + \Psi_1)} (\bar{X}\rho + \Psi_1)$$

$$\hat{Y}_{p8} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + \Psi_2)} (\bar{X}\rho + \Psi_2)$$

$$\hat{Y}_{p9} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + \Psi_3)} (\bar{X}\rho + \Psi_3)$$

$$\hat{Y}_{p10} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + \Psi_4)} (\bar{X}\rho + \Psi_4)$$

$$\hat{Y}_{p11} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + \Psi_5)} (\bar{X}\rho + \Psi_5)$$

$$\hat{Y}_{p12} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + \Psi_6)} (\bar{X}\rho + \Psi_6)$$

$$\hat{Y}_{p13} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \Psi_1)} (\bar{X}C_x + \Psi_1)$$

$$\hat{Y}_{p14} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \Psi_2)} (\bar{X}C_x + \Psi_2)$$

$$\hat{Y}_{p15} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \Psi_3)} (\bar{X}C_x + \Psi_3)$$

$$\hat{Y}_{p16} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \Psi_4)} (\bar{X}C_x + \Psi_4)$$

$$\hat{Y}_{p17} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \Psi_5)} (\bar{X}C_x + \Psi_5)$$

$$\hat{Y}_{p18} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \Psi_6)} (\bar{X}C_x + \Psi_6)$$

where  $\Psi_1 = (M_d + G)$ ,  $\Psi_2 = (M_d + D)$ ,  $\Psi_3 = (M_d + S_{pw})$ ,  $\Psi_4 = (QD + G)$ ,  $\Psi_5 = (QD + D)$  and  $\Psi_6 = (QD + S_{pw})$

The bias, related constant and the MSE for the first proposed estimator can be obtained as follows:

MSE of this estimator can be found using Taylor series method defined as

$$h(\bar{x}, \bar{y}) \cong h(\bar{X}, \bar{Y}) + \frac{\partial h(c, d)}{\partial c} \Big|_{\bar{x}, \bar{y}} (\bar{x} - \bar{X}) + \frac{\partial h(c, d)}{\partial d} \Big|_{\bar{x}, \bar{y}} (\bar{y} - \bar{Y}) \quad (2.1)$$

where  $h(\bar{x}, \bar{y}) = \hat{R}_{p1}$  and  $h(\bar{X}, \bar{Y}) = R$ .

As shown in Wolter (1985), (2.1) can be applied to the proposed estimator in order to obtain MSE equation as follows:

$$\begin{aligned} \hat{R}_{pj} - R &\cong \frac{\partial((\bar{y} + b(\bar{X} - \bar{x})) / (\bar{x} + \Psi_k))}{\partial \bar{x}} \Big|_{\bar{x}, \bar{y}} (\bar{x} - \bar{X}) + \frac{\partial((\bar{y} + b(\bar{X} - \bar{x})) / (\bar{x} + \Psi_k))}{\partial \bar{y}} \Big|_{\bar{x}, \bar{y}} (\bar{y} - \bar{Y}) \\ &\cong - \left( \frac{\bar{y}}{(\bar{x} + \Psi_k)^2} + \frac{b(\bar{X} + \Psi_k)}{(\bar{x} + \Psi_k)^2} \right) \Big|_{\bar{x}, \bar{y}} (\bar{x} - \bar{X}) + \frac{1}{(\bar{x} + \Psi_k)} \Big|_{\bar{x}, \bar{y}} (\bar{y} - \bar{Y}) \\ E(\hat{R}_{pj} - R)^2 &\cong \frac{(\bar{Y} + B(\bar{X} + \Psi_k))^2}{(\bar{X} + \Psi_k)^4} V(\bar{x}) - \frac{2(\bar{Y} + B(\bar{X} + \Psi_k))}{(\bar{X} + \Psi_k)^3} Cov(\bar{x}, \bar{y}) + \frac{1}{(\bar{X} + \Psi_k)^2} V(\bar{y}) \\ &\cong \frac{1}{(\bar{X} + \Psi_k)^2} \left\{ \frac{(\bar{Y} + B(\bar{X} + \Psi_k))^2}{(\bar{X} + \Psi_k)^2} V(\bar{x}) - \frac{2(\bar{Y} + B(\bar{X} + \Psi_k))}{(\bar{X} + \Psi_k)} Cov(\bar{x}, \bar{y}) + V(\bar{y}) \right\} \end{aligned}$$

where  $B = \frac{S_{xy}}{S_x^2} = \frac{\rho S_x S_y}{S_x^2} = \frac{\rho S_y}{S_x}$ . Note that we omit the difference of  $(E(b) - B)$ .

$$\begin{aligned} MSE(\bar{y}_{pj}) &= (\bar{X} + \Psi_k)^2 E(\hat{R}_{pj} - R)^2 \cong \frac{(\bar{Y} + B(\bar{X} + \Psi_k))^2}{(\bar{X} + \Psi_k)^2} V(\bar{x}) - \frac{2(\bar{Y} + B(\bar{X} + \Psi_k))}{(\bar{X} + \Psi_k)} Cov(\bar{x}, \bar{y}) + V(\bar{y}) \\ &\cong \frac{\bar{Y}^2 + 2B(\bar{X} + \Psi_k)\bar{Y} + B^2(\bar{X} + \Psi_k)^2}{(\bar{X} + \Psi_k)^2} V(\bar{x}) - \frac{2\bar{Y} + 2B(\bar{X} + \Psi_k)}{(\bar{X} + \Psi_k)} Cov(\bar{x}, \bar{y}) + V(\bar{y}) \\ &\cong \frac{(1-f)}{n} \left\{ \left( \frac{\bar{Y}^2}{(\bar{X} + \Psi_k)^2} + \frac{2B\bar{Y}}{(\bar{X} + \Psi_k)} + B^2 \right) S_x^2 - \left( \frac{2\bar{Y}}{(\bar{X} + \Psi_k)} + 2B \right) S_{xy} + S_y^2 \right\} \\ &\cong \frac{(1-f)}{n} (R^2 S_x^2 + 2BRS_x^2 + B^2 S_x^2 - 2RS_{xy} - 2BS_{xy} + S_y^2) \\ MSE(\bar{y}_{pj}) &\cong \frac{(1-f)}{n} (R^2 S_x^2 + 2R\rho S_x S_y + \rho^2 S_y^2 - 2R\rho S_x S_y - 2\rho^2 S_y^2 + S_y^2) \\ &\cong \frac{(1-f)}{n} (R^2 S_x^2 - \rho^2 S_y^2 + S_y^2) \cong \frac{(1-f)}{n} (R^2 S_x^2 + S_y^2(1 - \rho^2)) \end{aligned}$$

Similarly, the bias is obtained as

$$Bias(\bar{y}_{pj}) \cong \frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_j^2$$

Thus, the bias and MSE of the proposed estimator is given below:

$$B(\hat{Y}_{pj}) = \frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_j^2, \quad R_j = \frac{\bar{Y}}{\bar{X} + \Psi_k} \quad MSE(\hat{Y}_{pj}) = \frac{(1-f)}{n} (R_j^2 S_x^2 + S_y^2(1 - \rho^2)),$$

where  $J = 1, 2, \dots, 6$  and  $k = 1, 2, \dots, 6$

Similarly we can obtain the bias, constant and mean square error for the other proposed estimators as follows:

$$B(\hat{Y}_{pj}) = \frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_j^2, \quad R_j = \frac{\bar{Y}\rho}{\bar{X}\rho + \Psi_k} \quad MSE(\hat{Y}_{pj}) = \frac{(1-f)}{n} (R_j^2 S_x^2 + S_y^2(1 - \rho^2)),$$

where  $J = 7, 8, \dots, 12$  and  $k = 1, 2, \dots, 6$

$$B(\hat{Y}_{pj}) = \frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_j^2, \quad R_j = \frac{\bar{Y}C_x}{\bar{X}C_x + \Psi_k} \quad MSE(\hat{Y}_{pj}) = \frac{(1-f)}{n} (R_j^2 S_x^2 + S_y^2(1 - \rho^2)),$$

where  $J = 13, 14, \dots, 18$  and  $k = 1, 2, \dots, 6$

### 3. Efficiency comparisons

From the expressions of the MSE of the proposed estimators and the existing estimators, we have derived the conditions for which the proposed estimators are more efficient than the usual and existing modified ratio estimators. They are given as follows.

#### 3.1 Comparison with the classical ratio estimator

Modified proposed ratio estimators are more efficient than that of the classical ratio estimator if  $MSE(\widehat{Y}_{pj}) \leq MSE(\widehat{Y}_r)$ ,

$$\frac{(1-f)}{n} (R_{pj}^2 S_x^2 + S_y^2 (1-\rho^2)) \leq \frac{(1-f)}{n} (S_y^2 + R^2 S_x^2 - 2R\rho S_x S_y),$$

$$R_{pj}^2 S_x^2 - \rho^2 S_y^2 - R^2 S_x^2 + 2R\rho S_x S_y \leq 0,$$

$$(\rho S_y - RS_x)^2 - R_{pj}^2 S_x^2 \geq 0,$$

$$(\rho S_y - RS_x + R_{pj}^2)(\rho S_y - RS_x - R_{pj} S_x) \geq 0.$$

Condition I:  $(\rho S_y - RS_x + R_{pj} S_x) \leq 0$  and  $(\rho S_y - RS_x - R_{pj} S_x) \leq 0$

After solving the condition I, we get

$$\left( \frac{RS_y - RS_x}{S_x} \right) \leq R_{pj} \leq \left( \frac{RS_x - \rho S_y}{S_x} \right).$$

Hence,

$$MSE(\widehat{Y}_{pj}) \leq MSE(\widehat{Y}_r),$$

$$\left( \frac{\rho S_y - RS_x}{S_x} \right) \leq R_{pj} \leq \left( \frac{RS_x - \rho S_y}{S_x} \right), \text{ or}$$

$$\left( \frac{RS_x - \rho S_y}{S_x} \right) \leq R_{pj} \leq \left( \frac{\rho S_y - RS_x}{S_x} \right). \text{ Where } j = 1, 2, \dots, 18.$$

### 3.2 Comparisons with existing ratio estimators

$$MSE(\widehat{Y}_{pj}) \leq MSE(\widehat{Y}_i),$$

$$\frac{(1-f)}{n} (R_{pj}^2 S_x^2 + S_y^2 (1-\rho^2)) \leq \frac{(1-f)}{n} (R_i^2 S_x^2 + S_y^2 (1-\rho^2)),$$

$$R_{pj}^2 S_x^2 \leq R_i^2 S_x^2,$$

$$R_{pj} \leq R_i,$$

where  $j = 1, 2, \dots, 18$  and  $i = 1, 2, \dots, 22$ .

## 4. Empirical study

The performances of the proposed ratio estimators are evaluated and compared with the mentioned ratio estimators in Section 1.2 by using the data of the three natural populations. For the population I and II we use the data of Singh and Chaudhary (1986) page 177 and for the population III we use the data of Murthy (1967) page 228, in which fixed capital is denoted by X (auxiliary variable) and output of 80 factories is denoted by Y (study variable). We apply the proposed and existing estimators to these data sets and the statistics of these populations are given in Table 1.

From Table 2a and Table 2b, we observe that the proposed estimators are more efficient than all of the estimators in the literature as their Bias, Constant and Mean Square error are much lower than the existing estimators. The percentage relative efficiency (PRE) of the proposed estimators ( $p$ ), with respect to the existing estimators ( $e$ ), is computed by

$$PRE = \frac{MSE \text{ of Existing Estimator}}{MSE \text{ of proposed estimator}} \times 100$$

These PRE values are given in Table 3, for the population I. From this table, it is clearly evident that the proposed estimators are quiet efficient with respect to the estimators in the literature. Similarly, we can calculate the PRE values for population II and population III respectively by using the same formula mentioned above.

**Table 1.** Characteristics of the populations

Parameters	Population 1	Population 2	Population 3
$N$	34	34	80
$n$	20	20	20
$\bar{Y}$	856.4117	856.4117	5182.637
$\bar{X}$	199.4412	208.8823	1126.463

**Table 1.** Characteristics of the populations (cont.)

Parameters	Population 1	Population 2	Population 3
$\rho$	0.4453	0.4491	0.941
$S_y$	733.1407	733.1407	1835.659
$C_y$	0.8561	0.8561	0.354193
$S_x$	150.2150	150.5059	845.610
$C_x$	0.7531	0.7205	0.7506772
$\beta_2$	1.0445	0.0978	-0.063386
$\beta_1$	1.1823	0.9782	1.050002
$M_d$	142.50	150	757.5
$TM$	89.375	162.25	931.562
$MR$	165.562	284.5	1795.5
$HL$	320	190	1040.5
$QD$	184	80.25	588.125
$G$	162.996	155.446	901.081
$D$	144.481	140.891	801.381
$S_{pw}$	206.944	199.961	791.364
$DM$	206.944	234.82	1150.7

**Table 2a.** The Statistical Analysis of the Estimators for the Populations

Estimators	Population I		Population II		Population III	
	Constant	Bias	Constant	Bias	Constant	Bias
$\hat{Y}_r$	4.294	4.940	4.100	4.270	4.601	60.877
$\hat{Y}_1$	4.294	10.0023	4.100	9.1539	4.601	36.5063
$\hat{Y}_2$	4.278	9.9272	4.086	9.0911	4.598	36.4577
$\hat{Y}_3$	4.272	9.8983	4.098	9.1454	4.601	36.5104
$\hat{Y}_4$	4.279	9.9303	3.960	8.5387	4.650	37.2861
$\hat{Y}_5$	4.264	9.8646	4.097	9.142	4.601	36.5117
$\hat{Y}_6$	4.284	9.9578	4.091	9.1147	4.597	36.4453
$\hat{Y}_7$	4.281	9.9432	4.088	9.0995	4.596	36.4251

**Table 2a.** The Statistical Analysis of the Estimators for the Populations (cont.)

Estimators	Population I		Population II		Population III	
	Constant	Bias	Constant	Bias	Constant	Bias
$\widehat{Y}_8$	4.258	9.8348	4.069	9.0149	4.598	36.4546
$\widehat{Y}_9$	4.285	9.9597	4.011	8.763	4.662	37.4882
$\widehat{Y}_{10}$	4.244	9.7711	4.096	9.1349	4.601	36.5106
$\widehat{Y}_{11}$	2.3463	2.986	2.3076	2.9	2.518	32.81
$\widehat{Y}_{12}$	2.0427	2.263	1.973	2.12	2.189	24.79
$\widehat{Y}_{13}$	1.4993	1.219	1.5021	1.229	2.449	31.03
$\widehat{Y}_{14}$	1.6487	1.475	1.7358	1.641	1.774	16.23
$\widehat{Y}_{15}$	1.3718	1.021	1.4185	1.096	1.473	11.23
$\widehat{Y}_{16}$	0.9329	0.472	1.0167	0.563	1.708	15.10
$\widehat{Y}_{17}$	2.233	2.706	2.147	2.51	2.392	29.59
$\widehat{Y}_{18}$	1.930	2.021	1.812	1.788	2.063	22.01
$\widehat{Y}_{19}$	1.398	1.060	1.355	1.980	2.322	27.90
$\widehat{Y}_{20}$	2.107	2.137	1.9301	2.2087	2.276	26.800
$\widehat{Y}_{21}$	1.806	1.483	1.6013	1.3964	1.949	19.650
$\widehat{Y}_{22}$	1.289	0.800	1.1703	0.7459	2.206	25.188
$\widehat{Y}_{p1}$	1.6960	1.5604	1.6651	1.5098	1.9813	20.312
$\widehat{Y}_{p2}$	1.7606	1.6815	1.7136	1.599	2.0598	21.953
$\widehat{Y}_{p3}$	1.5602	1.3205	1.5324	1.2788	2.0681	22.129
$\widehat{Y}_{p4}$	1.8955	1.9490	1.9263	2.0207	1.8608	17.916
$\widehat{Y}_{p5}$	1.9764	2.1191	1.9915	2.1598	1.9299	19.271
$\widehat{Y}_{p6}$	1.7274	1.6187	1.751	1.6696	1.9371	19.416
$\widehat{Y}_{p7}$	0.9671	0.5074	0.9633	0.5053	1.7938	16.650

**Table 2a.** The Statistical Analysis of the Estimators for the Populations (cont.)

Estimators	Population I		Population II		Population III	
	Constant	Bias	Constant	Bias	Constant	Bias
$\widehat{Y}_{p8}$	1.0148	0.5586	0.9997	0.5443	1.8622	17.942
$\widehat{Y}_{p9}$	0.8701	0.4107	0.8666	0.409	1.8693	18.079
$\widehat{Y}_{p10}$	1.1177	0.6777	1.1672	0.7419	1.9130	18.936
$\widehat{Y}_{p11}$	1.1818	0.7577	1.2211	0.8121	1.9909	20.508
$\widehat{Y}_{p12}$	0.9902	0.5318	1.0283	0.5758	1.9991	20.677
$\widehat{Y}_{p13}$	1.4153	1.0866	1.3533	0.9973	1.5540	12.495
$\widehat{Y}_{p14}$	1.4752	1.1806	1.3979	1.0642	1.6184	13.552
$\widehat{Y}_{p15}$	1.2908	0.9038	1.2329	0.8278	1.6252	13.666
$\widehat{Y}_{p16}$	1.6021	1.3923	1.5977	1.3901	1.6667	14.373
$\widehat{Y}_{p17}$	1.6793	1.5298	1.6603	1.5011	1.7410	15.684
$\widehat{Y}_{p18}$	1.4444	1.1317	1.4326	1.1176	1.7489	15.825

**Table 2b.** The Statistical Analysis of the Estimators for the Populations

Estimators	Pop I	Pop II	Pop III	Estimators	Pop I	Pop II	Pop III
	MSE	MSE	MSE		MSE	MSE	MSE
$\widehat{Y}_r$	10960.76	10539.27	189775.1	$\widehat{Y}_{21}$	10386.83	10030.11	116239.3
$\widehat{Y}_1$	17437.7	16673.5	193998.1	$\widehat{Y}_{22}$	9644.04	9472.95	144936.7
$\widehat{Y}_2$	17373.3	16619.6	193746.2	$\widehat{Y}_{p1}$	10208.16	10333.07	119741.5
$\widehat{Y}_3$	17348.6	16666.1	194019.4	$\widehat{Y}_{p2}$	10311.83	10203.59	128249.9
$\widehat{Y}_4$	17376.0	16146.6	198039.9	$\widehat{Y}_{p3}$	10002.72	9929.39	129161.3
$\widehat{Y}_5$	17319.8	16663.3	194026.4	$\widehat{Y}_{p4}$	10540.91	10564.74	107326.5
$\widehat{Y}_6$	17399.5	16639.9	193682.3	$\widehat{Y}_{p5}$	10686.6	10683.87	114349.5



**Table 2b.** The Statistical Analysis of the Estimators for the Populations (cont.)

Estimators	Pop I	Pop II	Pop III	Estimators	Pop I	Pop II	Pop III
	MSE	MSE	MSE		MSE	MSE	MSE
$\widehat{Y}_7$	17387.1	16626.9	193577.6	$\widehat{Y}_{p6}$	10258.09	10264.06	115098.9
$\widehat{Y}_8$	17294.2	16554.4	193730.5	$\widehat{Y}_{p7}$	9306.32	9266.94	100762.1
$\widehat{Y}_9$	17401.1	16338.7	199087.0	$\widehat{Y}_{p8}$	9350.19	9300.31	107457.3
$\widehat{Y}_{10}$	17239.7	16654.2	194020.7	$\widehat{Y}_{p9}$	9223.53	9184.47	108172.8
$\widehat{Y}_{11}$	11429.08	11317.28	184446.20	$\widehat{Y}_{p10}$	9452.18	9469.56	112609.8
$\widehat{Y}_{12}$	10809.99	10649.40	142903.20	$\widehat{Y}_{p11}$	9250.7	9529.64	120761.3
$\widehat{Y}_{13}$	9915.81	9886.21	175238.70	$\widehat{Y}_{p12}$	9327.27	9327.31	121636.1
$\widehat{Y}_{14}$	10134.39	10239.11	98755.61	$\widehat{Y}_{p13}$	9802.37	9688.30	79228.58
$\widehat{Y}_{15}$	9745.79	9772.39	72582.52	$\widehat{Y}_{p14}$	9882.86	9745.56	84709.31
$\widehat{Y}_{16}$	9275.87	9316.02	92644.60	$\widehat{Y}_{p15}$	9645.86	9543.10	85298.12
$\widehat{Y}_{17}$	11189.04	10983.77	167778.60	$\widehat{Y}_{p16}$	10064.19	10024.69	88963.27
$\widehat{Y}_{18}$	10602.02	10365.55	128487.60	$\widehat{Y}_{p17}$	10181.93	10119.77	95756.01
$\widehat{Y}_{19}$	9779.43	9690.50	158990.7	$\widehat{Y}_{p18}$	9841.01	9791.32	96489.39
$\widehat{Y}_{20}$	10934.74	10571.58	153292.6				

**Table 3.** PRE of the Proposed Estimators with the Estimators in the Literature for Population I.

	$\widehat{Y}_{p1}$	$\widehat{Y}_{p2}$	$\widehat{Y}_{p3}$	$\widehat{Y}_{p4}$	$\widehat{Y}_{p5}$	$\widehat{Y}_{p6}$	$\widehat{Y}_{p7}$	$\widehat{Y}_{p8}$	$\widehat{Y}_{p9}$
$\widehat{Y}_r$	107.372	106.293	109.578	103.983	102.565	106.85	117.778	117.225	118.835
$\widehat{Y}_1$	170.821	169.103	174.329	165.428	163.173	169.991	187.381	186.495	189.056
$\widehat{Y}_2$	170.190	168.479	173.685	164.817	162.570	169.363	186.689	185.806	188.358
$\widehat{Y}_3$	169.948	168.239	173.438	164.583	162.339	169.122	186.423	185.542	188.090

**Table 3.** PRE of the Proposed Estimators with the Estimators in the Literature for Population I. (cont.)

	$\widehat{Y}_{p1}$	$\widehat{Y}_{p2}$	$\widehat{Y}_{p3}$	$\widehat{Y}_{p4}$	$\widehat{Y}_{p5}$	$\widehat{Y}_{p6}$	$\widehat{Y}_{p7}$	$\widehat{Y}_{p8}$	$\widehat{Y}_{p9}$
$\widehat{Y}_4$	170.216	168.505	173.712	164.843	162.596	169.389	186.718	185.835	188.387
$\widehat{Y}_5$	169.666	167.960	173.150	164.310	162.070	168.841	186.114	185.234	187.778
$\widehat{Y}_6$	170.447	168.733	173.947	165.066	162.816	169.618	186.970	186.087	188.642
$\widehat{Y}_7$	170.325	168.613	173.823	164.948	162.700	169.498	186.837	185.954	188.508
$\widehat{Y}_8$	169.415	167.712	172.895	164.067	161.830	168.592	185.839	184.960	187.500
$\widehat{Y}_9$	170.462	168.748	173.963	165.081	162.831	169.634	186.988	186.104	188.659
$\widehat{Y}_{10}$	168.881	167.183	172.350	163.550	161.320	168.061	185.253	184.378	186.910
$\widehat{Y}_{11}$	111.960	110.834	114.259	108.425	106.947	111.416	122.814	122.233	123.912
$\widehat{Y}_{12}$	105.895	104.831	108.070	102.552	101.154	105.381	116.161	115.612	117.200
$\widehat{Y}_{13}$	97.1361	96.1595	99.1311	94.0697	92.7873	96.6641	106.552	106.049	107.505
$\widehat{Y}_{14}$	99.2773	98.2792	101.316	96.1434	94.8326	98.7949	108.901	108.387	109.875
$\widehat{Y}_{15}$	95.4705	94.5107	97.4314	92.4568	91.1963	95.0067	104.725	104.230	105.662
$\widehat{Y}_{16}$	90.8672	89.9536	92.7334	87.9987	86.7990	90.4257	99.6762	99.2051	100.567
$\widehat{Y}_{17}$	109.608	108.506	111.860	106.148	104.701	109.076	120.234	119.666	121.309
$\widehat{Y}_{18}$	103.858	102.814	105.991	100.579	99.2085	103.353	113.926	113.388	114.945
$\widehat{Y}_{19}$	95.8001	94.8370	97.7677	92.7759	91.5111	95.3346	105.087	104.590	106.027
$\widehat{Y}_{20}$	107.117	106.040	109.317	103.736	102.322	106.597	117.502	116.946	118.552
$\widehat{Y}_{21}$	101.750	100.727	103.840	98.5382	97.1949	101.255	111.614	111.086	112.612
$\widehat{Y}_{22}$	94.4738	93.5240	96.4141	91.4915	90.2442	94.0148	103.632	103.142	104.559

**Table 3.** PRE of the Proposed Estimators with the Estimators in the Literature for Population I. (cont.)

	$\widehat{Y}_{p10}$	$\widehat{Y}_{p11}$	$\widehat{Y}_{p12}$	$\widehat{Y}_{p13}$	$\widehat{Y}_{p14}$	$\widehat{Y}_{p15}$	$\widehat{Y}_{p16}$	$\widehat{Y}_{p17}$	$\widehat{Y}_{p18}$
$\widehat{Y}_r$	115.96	118.486	117.513	111.817	110.907	113.632	108.909	107.649	111.378
$\widehat{Y}_1$	184.483	188.501	186.954	177.893	176.444	180.779	173.265	171.261	177.194
$\widehat{Y}_2$	183.802	187.805	186.264	177.236	175.792	180.111	172.625	170.629	176.540
$\widehat{Y}_3$	183.541	187.538	185.999	176.984	175.542	179.855	172.379	170.386	176.289
$\widehat{Y}_4$	183.831	187.834	186.292	177.263	175.820	180.139	172.652	170.655	176.567
$\widehat{Y}_5$	183.236	187.227	185.690	176.690	175.251	179.557	172.093	170.103	175.996
$\widehat{Y}_6$	184.079	188.088	186.544	177.503	176.057	180.383	172.885	170.886	176.806
$\widehat{Y}_7$	183.948	187.954	186.411	177.376	175.932	180.255	172.762	170.764	176.680
$\widehat{Y}_8$	182.965	186.950	185.415	176.429	174.992	179.291	171.839	169.852	175.736
$\widehat{Y}_9$	184.096	188.106	186.562	177.519	176.074	180.400	172.901	170.902	176.822
$\widehat{Y}_{10}$	182.389	186.361	184.831	175.873	174.440	178.726	171.297	169.317	175.182
$\widehat{Y}_{11}$	120.915	123.548	122.534	116.595	115.645	118.487	113.562	112.249	116.137
$\widehat{Y}_{12}$	114.365	116.856	115.897	110.279	109.381	112.069	107.410	106.168	109.846
$\widehat{Y}_{13}$	104.905	107.190	106.310	101.157	100.333	102.799	98.5257	97.3864	100.760
$\widehat{Y}_{14}$	107.217	109.553	108.653	103.387	102.545	105.065	100.698	99.5331	102.981
$\widehat{Y}_{15}$	103.106	105.352	104.487	99.4228	98.6131	101.036	96.8363	95.7165	99.0324
$\widehat{Y}_{16}$	98.1347	100.272	99.4489	94.6288	93.8582	96.1643	92.1671	91.1013	94.2573
$\widehat{Y}_{17}$	118.375	120.953	119.961	114.146	113.217	115.998	111.177	109.891	113.698
$\widehat{Y}_{18}$	112.165	114.608	113.667	108.158	107.277	109.913	105.344	104.126	107.733
$\widehat{Y}_{19}$	103.462	105.716	104.848	99.7660	98.9534	101.385	97.1706	96.0469	99.3743
$\widehat{Y}_{20}$	115.685	118.204	117.234	111.552	110.643	113.362	108.650	107.394	111.114
$\widehat{Y}_{21}$	109.888	112.282	111.360	105.962	105.099	107.682	103.206	102.012	105.546
$\widehat{Y}_{22}$	102.030	104.252	103.396	98.3848	97.5835	99.9811	95.8253	94.7172	97.9985

## 5. Conclusion

From the present study we conclude that our proposed estimators perform better than the existing estimators in the literature as their mean square error and bias are much lower than that of the existing estimators. Hence, we strongly recommend that our proposed estimators are preferred over the existing estimators in practical applications.

## Acknowledgements

The authors are grateful to two anonymous reviewers for giving many insightful and constructive comments and suggestions which led to the improvement of the earlier manuscript.

## REFERENCES

- ABID, M., ABBAS, N., NAZIR, H. Z., LIN, Z., (2016). Enhancing the mean ratio estimators for estimating population mean using non-conventional location parameters, *Revista Colombiana de Estadística*, 39 (1), pp. 63–79.
- ABID, M., SHERWANI, R. A. K., ABBAS, N., NAWAZ, T., (2016). Some improved modified ratio estimators based on decile mean of an auxiliary variable, *Pakistan Journal of Statistic and Operation Research*, 12 (4), pp. 787–797.
- COCHRAN, W. G., (1977). *Sampling Techniques*, Third Edition, Wiley Eastern Limited, New York.
- KADILAR C., CINGI, H., (2006). An improvement in estimating the population mean by using the correlation coefficient, *Hacettepe Journal of Mathematics and Statistics*, 35 (1), pp. 103–109.
- KADILAR, C., CINGI, H., (2004). Ratio estimators in simple random sampling, *Applied Mathematics and Computation*, 151, pp. 893–902.
- KOYUNCU, N., KADILAR, C., (2009). Efficient Estimators for the Population mean, *Hacettepe Journal of Mathematics and Statistics*, 38 (2), pp. 217–225.
- MURTHY, M. N., (1967). *Sampling Theory and Methods*, 1 ed., Statistical Publishing Society, India.
- Prasad, B., (1989). Some improved ratio type estimators of population mean and ratio in finite population sample surveys, *Communications in Statistics-Theory and Methods*, 18, pp. 379–392.
- RAO, T. J., (1991). On certain methods of improving ratio and regression estimators, *Communications in Statistics-Theory and Methods*, 20 (10), pp. 3325–3340.

- ROBSON, D. S., (1957). Application of multivariate Polykays to the theory of unbiased ratio type estimation, *Journal of American Statistical Association*, 52, pp. 411–422.
- SHARMA, P., SINGH, R., (2013). Improved Estimators for Simple random sampling and Stratified random sampling Under Second order of Approximation. *Statistics In Transition- new series*, 14 (3), pp. 379–390.
- SINGH H. P., TAILOR, R., (2003). Use of known correlation coefficient in estimating the finite population means, *Statistics in Transition*, 6 (4), pp. 555–560.
- SINGH, D., CHAUDHARY, F. S., (1986). *Theory and Analysis of Sample Survey Designs*, 1 ed., New Age International Publisher, India.
- SINGH, H. P., TAILOR, R., KAKRAN, M., (2004). Improved estimators of population mean using power transformation, *Journal of the Indian Society of Agricultural Statistics*, 58 (2), pp. 223–230.
- SINGH, H. P., TAILOR, R., (2005). Estimation of finite population mean with known coefficient of variation of an auxiliary, *STATISTICA*, anno LXV, 3, pp. 301–311.
- SISODIA, B. V. S., DWIVEDI, V. K., (1981). A modified ratio estimator using coefficient of variation of auxiliary variable, *Journal of the Indian Society of Agricultural Statistics*, 33 (1), pp. 13–18.
- SUBRAMANI, J., KUMARAPANDIYAN, G., (2012). A class of modified ratio estimators using deciles of an auxiliary variable, *International Journal of Statistical Application*, 2, pp. 101–107.
- SUBZAR, M., ABID, M., MAQBOOL, S., RAJA, T. A., SHABEER, M., LONE, B. A., (2017). A Class of Improved Ratio Estimators for Population Mean using Conventional Location parameters, *International Journal of Modern Mathematical Sciences*, 15 (2), pp. 187–205.
- SUBZAR, M., MAQBOOL, S., RAJA, T. A., SHABEER, M., (2017). A New Ratio Estimators for estimation of Population mean using Conventional Location parameters, *World Applied Sciences Journal*, 35 (3), pp. 377–384.
- SUBZAR, M., RAJA, T. A., MAQBOOL, S., NAZIR, N., (2016). New Alternative to Ratio Estimator of Population Mean, *International Journal of Agricultural Statistical Sciences*, 12 (1), pp. 221–225.
- UPADHYAYA, L. N., SINGH, H., (1999). Use of transformed auxiliary variable in estimating the finite population mean, *Biometrical Journal*, 41 (5), pp. 627–636.
- WOLTER, K. M., (1985). *Introduction to Variance Estimation*, Springer-Verlag.
- YAN, Z., TIAN, B., (2010). Ratio method to the mean estimation using coefficient of skewness of auxiliary variable, *Information Computing and Applications*, 106, pp. 103–110.