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December 2004
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ABSTRACT

Unilingual Versus Bilingual Education System: A Political Economy Analysis*

We consider an economy with two language groups, where only agents who share a language can produce together. Schooling enhances the productivity of students and may modify their language endowment. Under a unilingual system, the language of the politically dominant group is the only language of instruction, and the members of the politically dominated group who attend school shift language. Instead, under a bilingual system, the members of the dominated group who attend school become bilingual. The dominant group chooses the education system, and then individuals decide whether to attend school. While agents do not get utility from speaking their own language, we show that a language conflict of the expected type endogenously arises in the choice between a unilingual and a bilingual system. Democracy (majority rule) always leads to the implementation of the socially optimal education system, while the unilingual system is too often implemented under minority rule. In the presence of productivity spillovers, there may be unanimity for unilingualism, even if this system is assumed to be technologically inferior. The model is consistent with evidence from Finland in 1919 and France in 1863, showing that the choice of bilingualism in education may not be related to the size of language groups.

JEL Classification: I2, J15

Keywords: education, language policies, skills, minorities

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1 Introduction

In 2000, half of the countries in the world had at least one language minority corresponding to more than 10% of their population.\(^1\) This language diversity has recently brought language policies to the forefront of political debate in such countries as Malaysia, Spain, Latvia, the ex-Soviet Muslim States, Belgium or the U.S. As stressed by sociolinguists, one crucial component behind language shift in populations over generations is the choice of the language(s) of instruction in school. For example, Fishman (1977) argues that “for language spread, schools have long been the major formal (organized) mechanisms involved...” (p.116).\(^2\) In other terms, languages which are not given the status of medium of instruction in school tend to be replaced by the languages that are.

The cases of France and Finland provide two illustrations of the importance of language policies for language development. In the late 18th century, around 60% of those living in France did actually not speak French (Grégoire, 1794), but rather other languages.\(^3\) Nowadays, everybody speaks French, and the other languages are spoken by only 5% of the population.\(^4\) Instrumental in this development was the implementation of a unilingual education system from the 1880s which established French as the sole language of instruction in school. At the other end of the spectrum, the bilingual Finnish-Swedish education system implemented upon Finland’s independence from Russia in 1917, has been one of the factors explaining the relative good shape of Swedish in contemporary Finland. The native Swedish-speaking population has remained almost constant in absolute numbers (314,000 native Swedish-speakers in 1920 and 293,000 in 2000) and declined to some extent in relative terms (from 11 percent of the total population in 1920 to 5.9 percent in 2000).\(^5\)

Given the importance of language of instruction choice, we set-up a model for understanding why some multilingual countries choose unilingual education while others maintain language diversity, and for identifying the conditions under which conflict among language groups may arise. In addition, we provide a normative characterisation of the ability of different political systems to implement the socially optimal education system.

We begin by illustrating a historical puzzle that will be elaborated and empirically supported in the first section. One would intuitively guess that the likelihood of observing a unilingual system is directly related to the size of the language majority. As a first step, a comparison of the Finnish and the French cases does not go in that direction. Indeed, while a unilingual system was chosen in 40% French-speaking France, a bilingual system was almost unanimously chosen in

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\(^1\)Data are from the Encyclopaedia Britannica (2003). Worldwide, 48.5% of the countries are multilingual according to a 10% minority definition. Proportions vary across continents: 19.4% of the countries for the Americas, 43.2% for Europe, 44.9% for Asia, 50% for Oceania, and 77.6% for Africa.

\(^2\)See also for example Hagège (1996) for the case of France.

\(^3\)The largest regional language was Occitan, and then came Breton and Alsacian. Other smaller language groups were speaking Basque, Catalan, Flemish, Franco-provençal, Corsican, and Franconian.

\(^4\)Encyclopaedia Britannica (2003)

\(^5\)Data for 1920 are from McRae (1997) and for 2000 from the Encyclopaedia Britannica (2003).
90% Finnish-speaking Finland. One may argue that this contrast is due to national specificities. However, using regional data for 1860s France, we show that the proportion of French-speaking schools is unrelated to the proportion of local French-speakers. Our model explains why majority size may not be the most relevant variable for understanding the choice of education system.

Our economy consists of individuals belonging to one of two language groups, the politically “dominant” and “dominated”, initially unable to communicate. The dominant group decides first on the characteristics of the educational system and then individuals choose whether to attend school or not. Schooling enhances the productivity of the students and can be either unilingual or bilingual. Under a unilingual schooling system, the language of the politically dominant group is the unique language of instruction. As a result, the students belonging to the dominated group shift language when leaving school. In contrast, under a bilingual system, the students from the dominated group become bilingual. Value comes from bilateral production after schooling among agents who speak the same language. Finally, in order to keep the model simple, the individual cost of taking education is assumed to be the same under both language systems. This implies that we give an advantage to the bilingual over the unilingual system, as a student from the dominated group gets a richer language endowment under bilingualism than under unilingualism without bearing an additional cost.

The model has thus two central assumptions. First, individuals take an economic decision on whether to attend school or not. This assumption is in line with the growing literature showing that economic incentives have an impact on school attendance in developing countries (see for example Bourguignon, Ferreira, and Leite, 2003). Second, schooling is a “bundle”, as it simultaneously enhances the productivity (or earnings) of the student and can modify its language endowment. This is because we are interested here in the choice of the language of instruction, and not in language training in general. The positive effect of education on earnings is a well established fact in the literature (see Card, 1999) while the choice of the language of instruction in schooling is an important factor behind language shift according to sociolinguists (e.g. Fishman, 1977 or Hagège, 1996).

Individual education decisions are characterised by communication externalities whose nature varies across educational systems. Under the unilingual system, a member of the dominated group who chooses to take education loses her initial language, and therefore reduces the set of production partners of the other members of the dominated group, unless they choose themselves to attend school and learn the dominant language. For this reason, a positive communication externality links the education decisions across dominated group members, and high educational

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6 Language shift due to schooling needs in general more than one generation to be accomplished, but here we assume it takes places instantaneously in order to keep the model static.

7 Individuals attach no utility to speaking their mothertongue. Adding such a utility term would complicate the analysis, but not qualitatively affect any of the results. We return to this issue at various points in the paper.

8 From a theoretical viewpoint, we are thus assuming away the possibility that agents go to schools that exclusively provide language training.
levels reinforce the individual incentives for taking education (a “bandwagon” effect). Under the bilingual system, in contrast, this effect vanishes, as dominated group members who attend school do not lose their initial language.

We show that the dominant group prefers the education system that maximises the equilibrium education level of the other group, as higher education of the other group translates into a larger number of production partners, since more individuals learn the language of the dominant group. In turn, the dominated group never prefers the unilingual system, as the bandwagon effect arising under unilingual education locks them in equilibria with undereducation (overeducation) when schooling is cheap (expensive).

All political tension arising in equilibrium is of the expected type, i.e., situations in which the dominant group goes for unilingualism while the dominated group prefers a system in which their native language is also a language of instruction. This is an interesting result since it does not rely on any direct utility enjoyed by the agents from speaking their own native language. The dominated group wants its language to be used in schools not because their members “like it” but rather because abandoning it would force them to overinvest in education, due to the bandwagon effect. The dominant group goes for unilingualism in order to free-ride on the educational investment of the other group.

There may be unanimity for the bilingual system as well. This happens for example when education is very cheap. In this case everybody prefers all to take education.

We determine the socially optimal education system. When a benevolent planner can choose the education level of each individual, bilingualism is always the optimal system, as for given education levels more people communicate and hence produce under the bilingual than under the unilingual system. If the central planner can choose the education system but school attendance remains in the hands of the individuals, bilingualism is not necessarily optimal anymore, as the bandwagon effect may induce larger education levels under the unilingual system.

Next, we address the issue of failure in political decision-making, i.e., we analyse the circumstances, if any, under which the political decision process leads to the adoption of the ‘wrong’ type of education system. We show that a democratic rule (the majority decides the education system) always leads to the adoption of the socially optimal decentralised system in our model. In contrast, under minority rule, the unilingual system is implemented too often.

Finally, we extend the model to consider productivity spillovers among production partners, i.e. situations in which the productivity of each agent positively depends on the skill level of the other production partner. Under spillovers, unanimity for a unilingual education system becomes an equilibrium outcome when education is not expensive: in that case, the members of the dominated group are willing to give up their own language in order to benefit from a high education equilibrium. This is an interesting result, as we have attributed an advantage to bilingualism over unilingualism. Why does the existence of spillovers make the dominated willing to support a unilingual system? The reason is that spillovers strengthen the bandwagon effect in
the unilingual system, and do not alter incentives in the bilingual system. More precisely, in the unilingual system, the net gain from schooling for a dominated group member becomes increasing in the number of educated peers through an additional channel: by attending school in a situation in which many peers do so, a dominated group member not only gains many production partners, but also enhances her productivity in the matches with these partners.

Our model is related to the growing literature on language adoption, and in particular to Lazear (1999), Church and King (1993), and John and Yi (2001). Like in these three papers, agents in our model choose whether they make a costly investment in learning a language that can be used in trade or production with other agents. However, in constrast with these papers, we consider here an investment decision that ties skill acquisition and language acquisition, as languages which are used as media of instruction are learnt while learning other subjects at schools (e.g. while learning mathematics, history...). While in Lazear (1999) agents behave competitively, in our model, just as in Church and King (1993) and John and Yi (2001), the investment decision is strategic and the equilibrium outcome depends on a network externality, i.e. a situation in which the decision of an agent to learn a language increases the number of partners (and thus the utility) of the individuals speaking that language. Our paper differs from the two latter contributions because the type of network externalities under consideration is endogeneised here, as it depends on the choice between a unilingual and a bilingual education system. Another difference is that our explanation of language shift is based on the choice of schooling institutions, while John and Yi (2001) provides an explanation based on geography (more precisely, on the existence of migrations at equilibrium) and on a language transmission rule across generations. Finally, the derivation of language conflict or consensus as an equilibrium outcome are also novel.

2 Motivation and historical evidence

This section studies available evidence on the link between the size of language minorities and the choice of a unilingual education system. One would intuitively guess that a larger minority reduces the likelihood to observe a unilingual system. Instead, here we provide evidence that this

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9 There are other papers studying language. Breton and Mieszkowski (1977) studies in a trade model second language acquisition as a human capital investment. Lang (1986) proposes a language theory of discrimination. Pool (1991) and Laitin (1994) analyse the choice of an official language in multilingual countries. Melitz (2002) shows that having a common language promotes international trade. Saint-Paul (2001) studies mechanisms through which linguistic stratification can arise as an equilibrium outcome. In addition, there is a large literature on language proficiency and earnings (see e.g. Chiswick and Miller, 1995) and a new literature on the possible linguistic organisation of the European Union (see Ginsburgh and Weber, 2004 and van Parijs, 2004).

10 The economics of networks has been extensively studied in the industrial organisation literature, see Farrell and Klemperer (2004) for a recent survey. Research along this line has generally focused on the problem of adaption and coordination from the perspective of profit maximising firms. We take a here a political economy perspective on network adaption.

11 More precisely, John and Yi (2001) assumes that bilingual parents have unilingual children in a certain language if the language distribution in the location is skewed towards that language.
Table 1: Voting on language in Finland (1919-1922)

<table>
<thead>
<tr>
<th>Political Party</th>
<th>Seats</th>
<th>Electoral basis</th>
<th>Choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Democrats</td>
<td>80</td>
<td>FI-SWE</td>
<td>BI</td>
</tr>
<tr>
<td>Agrarians</td>
<td>42</td>
<td>FI</td>
<td>UNI-BI</td>
</tr>
<tr>
<td>National Coalition (conservatives)</td>
<td>28</td>
<td>FI</td>
<td>BI</td>
</tr>
<tr>
<td>National Progressive Party (liberals)</td>
<td>26</td>
<td>FI</td>
<td>mostly BI</td>
</tr>
<tr>
<td>Swedish People Party</td>
<td>22</td>
<td>SWE</td>
<td>BI</td>
</tr>
</tbody>
</table>

Sources: Jackson (1938) and McRae (1997)

may not be the case. We first compare the set-up of the current Finnish and French education systems (see Ortega and Tangerås, 2003, for more details). Next, we provide evidence across French départements (small administrative regions).

2.1 Finland versus France

The current institutional language framework in Finland was set up with the Constitution of 1919 and a series of language laws, the most important being approved in 1922. In 1920, the Swedish-speakers constituted only 11% of the Finnish population (McRae, 1997) and the rest of the population was Finnish-speaking. Nevertheless, a bilingual education system was approved with very large majorities. Table 1 presents the composition of the Finnish Parliament in 1919 and shows that the support came from both language groups. The Constitution recognises Finnish and Swedish as national languages on an equal basis. Concerning the educational system, each municipality has to organise schooling in the minority language (Swedish or Finnish) when a minimum number of parents requires it.

The foundations of the French language policy were decided during the French Revolution (1789-1794). In 1794, the prelate Grégoire presented before the Convention (Parliament) his report on the language situation in France (Grégoire, 1794). Grégoire argued that there were only 15 départements (out of 83) in which French was the only language spoken. Additionally, “it can be stated without exaggeration that six million French, especially in the countryside, do not know the national language; that an equal number is more or less unable to maintain a sustained conversation; and that finally the number of those who speak it does not exceed three million” (Grégoire, 1794). Given that the total French population at that time was around 26 million, this would mean that roughly only 2/5 of the population was native French-speaker (Calvet, 2002). Among the other language groups, the biggest was Occitan (southern-half of the country), and next

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12 Except for a tiny Sami group.
13 The language clauses of the Constitution were approved with a very wide consensus (173 ‘yes’ to 23 ‘no’, 165 ‘yes’ to 22 ‘no’, 183 ‘yes’ to 10 ‘no’ and 183 ‘yes’ to 7 ‘no’, see Eduskunta-Riksdag, 1920, pp. 1028-30).
came Breton and Alsacian. Additionally, small minorities were speaking Franco-provençal, Basque, Catalan, Corsican or Flemish. Each departement (with the exception of the Basses-Pyrénées) had at most two language groups.

As stressed by Hagège (1996), language policy quickly became an important issue in the political choices of the Revolution. After an initial period in which the translation of the decrees into other languages was decided, the newly born Republic controlled by the *montagnards* (radical revolutionaries) chose French-unilingualism in a period of external war and provincial insurrection. In June 1794, Grégoire presented a report which argues that: “Everything we said leads us to the conclusion that, in order to extirpate the prejudices, develop all the truths, the talents, the vertues, merge all the citizens in the national mass, simplify the political mechanisms, we need identity of language” (Grégoire, 1794, p. 341). A series of French-unilingual language decrees were approved, but did not survive the fall of the montagnards in July 1794. As argued by Weber (1976), “The [language] policy foundered. (...) What survived from the shipwreck was the principle” (p. 72).

Comparing Finland and France opens a puzzle: bilingualism arose almost unanimously in a country where the majority represented 90% of the population while unilingualism was the outcome in a country where the biggest language group represented 40% of the population. One may argue that this puzzle can be explained by refering to the particular characteristics of these two countries. For example, French-speakers and Swedish-speakers were over-represented in the élites in France and Finland respectively, so the political power of the language minority in Finland may have been larger than that of the non-French speakers in France. Alternatively, one could argue that the main difference was the existence of more than two language groups in France, and just two in Finland. However, we show now that the same type of puzzle remains when we look at what happened *inside* France.

### 2.2 Looking inside France

In 1863, the Minister of Public Instruction under Napoleon III, Victor Duruy, organised an inquiry into the languages spoken by the population of the by then 89 departments, together with the language(s) of instruction in public schools. The data (see Weber, 1976) show that there was important cross-regional variation in educational systems at that time, before the introduction of the Ferry Laws in 1880-82, which instituted free primary education and legally established French as the only language of instruction in schools (Chervel, 1992).

The data contain information on the linguistic composition of the population in each departement (“French-speakers” versus “non-French speakers”\(^\text{15}\)), together with the number of public schools “using French only”, “using idiom or *patois*\(^\text{16}\) only” or using both.

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14 see Alcouffe and Brummert (1985).

15 Note that the data are quite aggregated, since we get information on the number of inhabitants in “French-speaking” communes and “Non-French-speaking” communes.

16 The term “patois” refers to all the languages and dialects in France, except French.
Figure 1 plots the proportion of public schools using French only versus the proportion of French-speakers in the population for the 34 departments that were not fully French-speaking in 1863. In addition, each of the 55 fully French-speaking departments had a fully French-unilingual system. The scatter plot does not show any pattern of correlation between the two variables.

We next regress the proportion of French-unilingual schools on a number of department-level variables for the 89 departments. The results are reported in Table 2. Column 1 shows that there is a positive relationship between the proportion of French-speakers in the population and the proportion of French-unilingual schools. In addition, the proportion of French-unilingual schools is positively related to the number of teachers in public schools per 10,000 inhabitants. This result relates to the analysis by Grew and Harrigan (1991), which shows that government investment in teachers for public schools started well before the Ferry Laws (1880s). Following the principles set up during the Revolution, these new teachers were trained to teach in French. The first regression also considers the average direct cost of education for parents in each department. Interestingly, a higher cost of education is positively related to the proportion of French-unilingual schools. This may indicate that parents were willing to invest more in education if schools were in French (most likely following a social mobility argument, as French was necessary in skilled occupations).

This first regression however does not take into account that in the 55 fully French-speaking departments the possibility of having non-French speaking schools was not even considered. For this reason, we introduce a dummy variable for unilingual French-speaking departments in the
Table 2: The proportion of French-unilingual public schools at the department level in France (1863)

<table>
<thead>
<tr>
<th>Dependent variable: Proportion of French-unilingual public schools</th>
<th>French-speakers in the population</th>
<th>Teachers in public schools (/10,000 inhab.)</th>
<th>Amount paid for education by a family</th>
<th>Log income per head</th>
<th>Unilingual French-speaking department (dummy)</th>
<th>Romance language other than French (dummy)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.108 ** (.046)</td>
<td>.004 ** (.002)</td>
<td>.025 *** (.008)</td>
<td>.044 (.054)</td>
<td>.102 *** (.054)</td>
<td>.303 *** (.044)</td>
</tr>
<tr>
<td></td>
<td>-.015 (.080)</td>
<td>.004 ** (.002)</td>
<td>.024 *** (.008)</td>
<td>.003 (.053)</td>
<td>.256 *** (.057)</td>
<td>.251 *** (.041)</td>
</tr>
<tr>
<td></td>
<td>.094 (.072)</td>
<td>.003 (.002)</td>
<td>.012 *** (.007)</td>
<td>.035 (.044)</td>
<td>.251 *** (.051)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.006 (.066)</td>
<td>.003 (.002)</td>
<td>.013 *** (.007)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The figures reported are the coefficients obtained from OLS estimation. Standard errors in parentheses. *, ** and *** denote significance at 10%, at 5%, and 1% levels, respectively. Data on language are from the Archives Nationales and can be found in Weber (1976). Data on average income levels are from Vapereau (1867). The remaining data are from Ministère de l’Instruction Publique (1878). All data refer to 1863.

regressions of Columns 2 to 4. Column 2 shows that the relationship between the proportion of French-speakers in the population and the proportion of French-unilingual schools becomes non-significant when the dummy for unilingual French-speaking departments is introduced.

The regression in Columns 3 and 4 further includes an indicator for whether the second language spoken in multilingual departments is a Romance language. These two regressions show that the proportion of French-unilingual schools was higher in departments where French was coexisting with another Romance language. This can be easily understood by inspection of Figure 1 if we notice that departments like Ariège, Aude, Corrèze, Haute-Garonne, Ardèche, etc in which Occitan (a Romance language) was spoken in addition to French had a tendency to adopt a fully French unilingual system. Again, in both regressions, the coefficient on the proportion of French-speakers remains statistically non-significant.

It appears thus from this analysis that no clear-cut relation can be established in the case of France at the department level between the size of the language groups and the language(s) of instruction chosen. The model that follows explains why minority size may not be the most relevant variable for understanding the choice of education system.

3 The model

Consider a country inhabited by a continuum of individuals, normalised to unity. There are two language groups in the country, \( m \) and \( n \), of sizes \( M \in (0,1) \) and \( N = 1 - M \), respectively.
Initially, the *ms* speak *mish* and the *ns* speak *nish*. *ms* and *ns* are unable to communicate unless they learn to speak a common language.

Value is created through bilateral production between individuals. Each individual has the opportunity of producing once with every other individual. Bilateral production occurs if and only if the two partners are able to communicate, i.e. if they speak a common language. If they cannot communicate, the value of production is equal to zero.

Individuals choose whether to attend school or not. An individual who takes education becomes skilled and produces $1 + \sigma$ ($\sigma > 0$) when meeting any other agent with whom she is able to communicate. An individual who does not take education produces instead $1$ with any partner speaking the same language.

Schooling also involves language training, the type depending on the educational system. This paper compares a unilingual to a bilingual education system.

Under the unilingual system, *mish* is the unique language of instruction. For this reason, the *m* group is also referred to throughout the paper as the “dominant” group and the *n* group as the “dominated” group. The *ms* who attend school keep their initial language, while the *ns* learn *mish* and lose their initial language. Indeed, as shown by linguists (see e.g. Fishman, 1977, for English, and Hagège, 1996, for the case of France) one crucial factor behind language shift in populations over generations is the choice of the language(s) of instruction in school. In other terms, languages not given the status of medium of instruction in primary school tend to be replaced by the language used in school. Here for simplicity we assume that this language shift takes place in the life span of one generation.

Under the bilingual system, all the members of the dominated group who take education go to schools which have both *nish* and *mish* as languages of instruction. For this reason, they end up speaking both languages after leaving school. The dominant group members go to schools in which *mish* is the unique language of instruction, as in the unilingual system. The bilingual system is thus here a system which protects the language of the dominated group, typically the minority language in democracy.

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17 As e.g. in Diamond (1982) or Lazear (1999).
18 Equivalently, agents consider their expected payoffs when taking decisions. We are assuming away the possibility that agents belonging to a certain language group are concentrated in a particular location. For an analysis including this geographical dimension, see John and Yi (2001).
19 In this model, the value of production for an individual is independent of the skill level of the production partner. In section 7, we consider the case with productivity spillovers.
20 The assignment of the dominant role to the *m* group is arbitrary and without loss of generality. So far we have not specified the relative sizes of the two groups. Nor have we described the political process by which the educational system is chosen. We do not explain here the reasons for which one group becomes the politically dominant group. This is as in Lang (1986), where one group is exogenously assigned the role of the “economically dominant” group because its capital-labour ratio is assumed to be larger than that of the other group.
21 For a dynamic set-up in which the language spoken by the children (exogenously) depends on the language spoken by the parents and the language spoken in the geographical location, see John and Yi (2001).
22 In the case of Finland, for example, there is only one language of instruction (Finnish or Swedish), and then the other language is also taught. One may think that a symmetric bilingual system is a better representation of
The personal cost $c$ of taking education is assumed constant across the population and independent of the educational system. This means that we give an advantage to the bilingual system over the unilingual system, as an $n$ going to school in the bilingual system gets (ceteris paribus) access to more production partners than in the unilingual system after paying exactly the same cost.\footnote{We could have assumed instead different costs for each system. We have not chosen that option for two reasons. The main reason is that the results are rich enough with identical costs across systems. In addition, in a country which initially has two language groups, it is not clear that setting-up a unilingual system is cheaper than implementing a bilingual system. For example, important investments may be needed in order to insure that there is a sufficient number of teachers speaking the unique language of instruction.} Denoting by $\mu_m (\mu_n)$ the fraction of $m$s ($n$s) that take education, we now define the payoffs associated to taking and not taking education in each of the systems.

### 3.1 Payoffs in unilingual education

The expected utility of taking education under a unilingual system, given expected education shares is:\footnote{There is no need to make a distinction between $n$s and $m$s who take education since they speak the same language and have identical skill levels subsequent to becoming educated.}

$$U^{\text{uni}}(\mu_n) = -c + (1 + \sigma)(M + N\mu_n). \quad (1)$$

An individual who attends school pays cost $c$, becomes skilled, and speaks mish when leaving school. She gets $1 + \sigma$ from production with each of the $M$ members of the dominant group and each of the $N\mu_n$ members of the dominated who have shifted language because they have attended school. This means that each skilled individual benefits from more $n$s taking education, due to an expansion of her set of production partners. This positive communication externality (network externality) turns out to be key in educational choice.

An unskilled $m$ has the same production partners as the skilled, as she speaks mish. This agent saves on the cost of education, but gets only a value of 1 when producing:

$$L^{\text{uni}}(\mu_n) = M + N\mu_n. \quad (2)$$

Unskilled $m$s improve their situation when more $n$s take education due to the same positive communication externality playing for the skilled. However, here the size of the externality is smaller since each additional production partner from the $n$ group increases the pay-off of an unskilled agent by 1.

Finally, the expected utility of not attending school for a member of the dominated group is

$$U^{\text{uni}}(\mu_n) = N(1 - \mu_n) \quad (3)$$

the Finnish system. However, the knowledge of Swedish among the Finnish-speaking majority is relatively low. In a previous version of this paper (Ortega and Tangerås, 2003) we considered the case of a symmetric bilingual system, i.e. a system in which both language groups were taught in both languages. We discuss throughout the paper the results under a symmetric bilingual system whenever they are different to the results we get here.

\footnote{We could have assumed instead different costs for each system. We have not chosen that option for two reasons. The main reason is that the results are rich enough with identical costs across systems. In addition, in a country which initially has two language groups, it is not clear that setting-up a unilingual system is cheaper than implementing a bilingual system. For example, important investments may be needed in order to insure that there is a sufficient number of teachers speaking the unique language of instruction.}
under the unilingual system. Indeed, an \( n \) who does not go to school speaks \( n \)ish, and thus can only produce with the \( N(1 - \mu_n) \) members of the dominated group who have not attended school. Clearly, every unskilled \( n \) loses from any of her peers taking education through a contraction of her set of production partners. Hence, education imposes a negative communication externality on those of the \( ns \) who remain unskilled.

### 3.2 Payoffs in bilingual education

Concerning the bilingual system, note first that the expected utilities of the dominant group members are the same as under the unilingual system. This is because the \( ms \) never learn \( n \)ish and thus any positive production with members of the dominated group takes place in \( m \)ish, as in the unilingual system.\(^{25}\) For this reason,

\[
U_m^{\text{bi}}(\mu_n) = U_m^{\text{uni}}(\mu_n)
\]  
\[
U_m^{\text{bi}}(\mu_n) = U_m^{\text{uni}}(\mu_n).
\]

In contrast, bilingual education alters the payoffs of the \( ns \). Indeed, as the bilingual system guarantees that the \( ns \) keep \( n \)ish even if they go to school, neither the value of attending school nor the value of not attending school for an \( n \) depend on the education choices of her peers. More precisely, a skilled \( n \) pays \( c \) for education, produces with everybody, and gets \( 1 + \sigma \) in each case, i.e.,

\[
U_n^{\text{bi}} = -c + 1 + \sigma,
\]

while an \( n \) who chooses to remain unskilled can still produce with all the \( ns \), as the \( ns \) who have become skilled keep \( n \)ish due to bilingual schooling:

\[
U_n^{\text{bi}} = N.
\]

### 3.3 Equilibrium

The timing of the game is as follows. First, anticipating the future levels of education, the educational system is chosen so as to maximise the expected utility of the dominant group. Second, all the individuals independently and simultaneously choose whether to take education. We consider without loss of generality symmetric Nash Equilibria in which all members of each group randomise between education and staying unskilled with the same probability.

\(^{25}\)In the bilingual system, the dominated group members who attend school are bilingual, but this does not affect the payoff of any of the \( ms \), since the \( ms \) never speak \( n \)ish.
Equilibrium education levels

4.1 The unilingual education system

We first derive the equilibrium education levels under the unilingual system. By subtracting (3) from (1), we obtain the net benefit of the representative \(n\) of taking education:

\[
\Delta U^{\text{uni}}_n(\mu_n) = -c + (M + N\mu_n)\sigma + M + N\mu_n - N(1 - \mu_n).
\]

When attending school, this individual pays cost \(c\), becomes skilled, and shifts language from nish to mish. In equation (8), the productivity gain from education is given by \((M + N\mu_n)\sigma\) i.e. the marginal value of education \(\sigma\) times production partners after schooling, namely the \(m\)s and the other skilled \(ns\). In addition, education alters the set of individuals with whom producing is possible. This communication effect is captured by the remaining terms in (8). First, speaking mish after school enables production with the \(M\) native mish speakers and with the \(N\mu_n\) new mish speakers. At the same time, the skilled \(n\) forgets nish and thus can no longer produce with the \(N(1 - \mu_n)\) unskilled \(ns\).

Equation (8) generates an insight crucial to the understanding of the preferences over education systems. For the \(ns\), attending a unilingual school implies both becoming skilled and shifting language. Clearly, both features of unilingual schooling are more attractive the smaller the number of nish speakers and in particular the larger the number of other ns going to the unilingual school. This positive communication externality is thus at the origin of a bandwagon or snowball effect in the decision of taking education of the \(ns\). Indeed, upon inspection of (8), it is easy to check that the net benefit from schooling for an \(n\) is increasing in the number of \(ns\) taking education \((\mu_n)\). This bandwagon effect gives rise to multiple equilibria if sufficiently strong. In addition to the possibility of two extreme equilibria in which either all or none of the \(ns\) take education, a mixed equilibrium may exist.\(^{26}\)

Let us turn now to the choice of a representative \(m\). Her net benefit of attending school is obtained by subtracting (2) from (1):

\[
\Delta U_m(\mu_n) = -c + (M + N\mu_n)\sigma.
\]

Here, only the productivity gain shows up, since schooling does not alter the language endowment of the \(m\) and thus her set of production partners is unchanged by education. At the same time, note that the representative \(m\)'s incentive for taking education positively depends on the educational level of the \(ns\), as the skilled \(ns\) speak mish. Figure 2 characterises the set of equilibrium education levels (consult appendix 9.1 for full analytical details).

Let us first consider the area above the \(M(c - \sigma)\) line i.e. the north-west part of the figure.\(^{27}\)

\(^{26}\)This unstable equilibrium is sometimes referred to as a tipping equilibrium, a term coined by Schelling (1978).

\(^{27}\)\(M(c - \sigma)\) is upward sloping for the following reason: as the cost of education increases, getting educated becomes less attractive, and thus remaining an nish speaker may be a good option even if the size of the \(n\) group shrinks.
In this area, the size $M$ of the dominant group is so large and consequently the productivity gain and communication effect so strong relative to the cost of education, that each individual $n$ prefers to go to school independently of the choice of the other members of the group. In this case, the bandwagon effect does not play a role in educational choices.

Concerning the $ms$, they get educated if schooling is sufficiently cheap (for $c < \sigma$, in which case $(1,1)$ is the unique equilibrium) and abstain from taking education otherwise (in which case $(1,0)$ is the unique equilibrium).

Moving to the south east crossing the $M(c - \sigma)$ line, the bandwagon effect starts playing a role. Indeed, as the $n$ group gets larger and education becomes more expensive, remaining an $n$-ish-speaker and avoiding to pay the cost of education becomes a good choice, but only of course if other members of the dominated group choose the same strategy and do not attend school. For this reason, we get that (in stable configurations) either all the $ns$ take education or none of them does so. Concerning the $ms$, they choose to attend school when education is cheap and the $ns$ are in large numbers (in which case the stable equilibria are $(0,1)$ and $(1,1)$) while they become more and more reluctant to take education as its cost becomes higher (in this other extreme, the stable equilibria are $(0,0)$ and $(1,0)$).

Whenever education becomes sufficiently expensive ($c > 1 + \sigma$), the productivity gain and communication effect are insufficient to cover the cost of education even in case the rest of the population would decide to take education. The bandwagon effect vanishes and nobody takes education independently of the choices of the other agents.

While the analysis gives rise to a series of different equilibrium configurations, the existence of a positive communication externality across groups is the basis for the following general result:

**Proposition 1** Under the unilingual education system, equilibrium education levels are positively
correlated across language groups in the following sense: in the cases with multiple equilibria high (low) education levels among the ns are coupled with high (low) education levels among the ms.

**Proof.** Let \( z^{\text{uni}} = (z^{\text{uni}}_n, z^{\text{uni}}_m) \) and \( y^{\text{uni}} = (y^{\text{uni}}_n, y^{\text{uni}}_m) \) be two distinct equilibria of the game. We need to establish that \( z^{\text{uni}}_n > y^{\text{uni}}_n \) and \( z^{\text{uni}}_m < y^{\text{uni}}_m \) cannot simultaneously hold. Suppose they do. \( y^{\text{uni}}_m > z^{\text{uni}}_m \geq 0 \Rightarrow \Delta U^{\text{uni}}_m(y^{\text{uni}}) \geq 0 \). \( \partial \Delta U^{\text{uni}}_m / \partial \mu_n > 0 \Rightarrow \Delta U^{\text{uni}}_m(z^{\text{uni}}) > \Delta U^{\text{uni}}_m(y^{\text{uni}}) \) for \( z^{\text{uni}}_n > y^{\text{uni}}_n \). Hence, \( z^{\text{uni}}_n > y^{\text{uni}}_n \) and \( y^{\text{uni}}_m > z^{\text{uni}}_m \Rightarrow \Delta U^{\text{uni}}_m(z^{\text{uni}}) > 0 \) and thus \( z^{\text{uni}}_m = 1 \), which contradicts \( y^{\text{uni}}_m > z^{\text{uni}}_m \).

### 4.2 The bilingual education system

We now turn to equilibrium analysis under the bilingual system. Subtract (7) from (6) to get the net benefit of taking education for an \( n \):

\[
\Delta U_n^{\text{bi}} = -c + M + \sigma. \tag{10}
\]

An \( n \) who takes education pays cost \( c \), reaches \( M \) additional partners as she learns mish, and gets an additional amount \( \sigma \) through production, as she is now skilled and she produces with everybody. As we can see, the ns decision to take education is non-strategic under the bilingual system.

The trade-off facing the ms under the bilingual system is identical to the one facing them under the unilingual system as we assume that the ms are never taught in mish:

\[
\Delta U_m(\mu_n) = -c + (M + N\mu_n)\sigma. \tag{11}
\]

![Figure 3: Equilibrium education levels under the bilingual system](image)

The equilibrium levels of education are represented graphically in Figure 3. When education is cheap \( (c < \sigma) \), the productivity effect alone is sufficient to render education profitable for the
members of the dominated group. Anticipating the high educational levels of the other group, the ms find the communication externality sufficiently strong to render education profitable also for them. Hence \((1,1)\) is the unique equilibrium in that case. When education is expensive, \((c > \sigma)\), the productivity gain is too weak relative to the cost of taking education to generate sufficient incentives for the members of the dominant group to take education. The members of the dominated group take education if and only if the communication effect is sufficiently strong, i.e. if and only if the dominant group is sufficiently large relative to the cost of education \((M > c - \sigma)\).\(^{28}\)

5 Welfare

This section considers the welfare properties of the two respective education systems. Define expected welfare under education system \(s \in \{uni, bi\}\) by

\[
W^s(\mu) = N[\mu_n U^s_n(\mu_n) + (1 - \mu_n)U^s_m(\mu_n)] + M[\mu_m U^s_m(\mu_n) + (1 - \mu_m)U^s_m(\mu_n)].
\]

As a first step, we can compare the welfare levels under each system for given education levels:

**Lemma 2** Expected welfare is higher under the bilingual system for given education levels.

**Proof.** Let \(\mu = (\mu_n, \mu_m)\) be an exogenously fixed education level. We just need to show that utility is higher for each of the groups. Subtracting (1) from (6), the net benefit of a skilled \(n\) of having a bilingual education system is \(U^b_n - U^{uni}_n(\mu_n) = N(1 + \sigma)(1 - \mu_n) \geq 0\). Subtracting (3) from (7), the corresponding value for an unskilled \(n\) is \(U^b_n(\mu_n) - U^{uni}_n(\mu_n) = N(1 + \sigma)\mu_n \geq 0\). Finally, for the \(ms\), \(U^b_m(\mu_n) - U^{uni}_m(\mu_n) = U^b_m(\mu_n) - U^{uni}_m(\mu_n) = 0\).

The intuition for this lemma is simple: the bilingual system enables all the \(ns\) to maintain communication and thus production relations with each other, irrespective of their level of education. This is impossible under the unilingual system since the educated \(ns\) under this system lose the ability to speak nish.

Of course, education levels are not exogenously given, hence we cannot on the background of the above comparisons conclude that the bilingual is superior to the unilingual system from a welfare point of view. We need to adjust for education levels.

\(^{28}\)In a symmetric bilingual system, the \(ms\) get educated in some cases even for \(c - \sigma > 0\), as they can learn nish in school and there is thus also for them a communication effect associated to taking education. More precisely, we get in that case that \((0,1)\) is the unique equilibrium when \(c - \sigma > M\) and \(M < 1 - (c - \sigma)\) simultaneously hold. Multiple equilibria \(\{(1,0), (y^b_n, y^b_m), (0,1)\}\) arise whenever \(M \in (c - \sigma, 1 - (c - \sigma))\). In the rest of the parameter space, the equilibria are as in Fig. 3. We attribute the existence of multiple equilibria in the symmetric bilingual system to a duplication effect: the more people of the other group who learn one’s language, the weaker is the own incentive for learning the other language.
5.1 Centralisation

Suppose there exists a benevolent social planner who is able to enforce the level of education that maximises welfare under each system. Then, the following proposition can be stated:

**Proposition 3** Under centralisation, the bilingual system yields higher expected welfare than the unilingual system. The socially optimal education levels under the bilingual system are \((1, 1) \forall c < \sigma, (1, 0) \forall (c, M) \in (\sigma, 2+\sigma) \times ((c-\sigma)/2, 1)\) and \((0, 0) \forall (c, M) \in (\sigma, \infty) \times (0, \min\{(c-\sigma)/2; 1\})\).

**Proof.** Write \(x^s = (x^s_n, x^s_m)\) welfare maximising education levels under \(s \in \{bi, uni\}\). From Lemma 2, \(W^{bi}(x^{uni}) \geq W^{uni}(x^{uni})\). By optimality of \(x^{bi}\) under the bilingual system, we have \(W^{bi}(x^{bi}) \geq W^{uni}(x^{uni})\). Adding the two inequalities produces \(W^{bi}(x^{bi}) \geq W^{uni}(x^{uni})\). Socially optimal education levels: by differentiating (6)-(5) with respect to \(\mu_m\) and \(\mu_n\) and plugging in (6)-(5), it is easily verified that (subscripts denote partial derivatives):

\[
\begin{align*}
W^{bi}_{\mu_m}(\mu) &= M(\sigma - c - N(1 - \mu_n)\sigma) \\
W^{bi}_{\mu_n}(\mu) &= N(\sigma - c + M(2 + \sigma\mu_m))
\end{align*}
\]

\(W^{bi}_{\mu_n}(\mu) > 0\) for all \(c < \sigma\) implies \(x_n = 1\) for all \(c < \sigma\). \(W^{bi}_{\mu_m}(1, \mu_m) = M(\sigma - c)\) implies \(x_n = 1\) for all \(c < \sigma\), as well. \(W^{bi}_{\mu_n}(\mu) < 0\) for all \(c > \sigma\) implies \(x_m = 0\) for all \(c > \sigma\). \(W^{bi}_{\mu_m}(\mu_m, 0) = N(\sigma - c + 2M),\) hence \(x_n = 1\) for all \((c, M) \in (\sigma, 2+\sigma) \times ((c-\sigma)/2, 1)\) and \(x_n = 0\) for all \((c, M) \in (\sigma, \infty) \times (0, \min\{(c-\sigma)/2; 1\})\).

The central planner would always choose a bilingual system if able to fully control educational levels of the population. It is not hard to understand why. For given education levels, more people communicate, hence produce, under a bilingual than a unilingual system. As welfare is increasing *ceteris paribus* in production, it immediately follows that the bilingual is better than the unilingual system. Unsurprisingly, optimal education levels are decreasing in the cost of taking education. Whenever education costs are relatively high and it is not optimal that all the individuals get educated, the central planner chooses to educate the *ns*, as the education of the *ms* never has an impact on communication. Everything else held equal, optimal education levels are higher the larger \(M\), as the positive impact on communication associated to the *ns* learning mish is in that case larger.

5.2 Decentralised educational choice

In reality, of course, no central planner can perfectly control the amount of effort students put into their studies, even in a system with mandatory education.\(^{29}\) To capture this degree of freedom, consider therefore a situation in which the central planner chooses the education system taking into account the free choices of individuals concerning school attendance. Let \(\gamma^{bi}\) and \(\gamma^{uni}\) be two

\(^{29}\)There is for example a growing empirical literature on the effectiveness of financial incentives for school attendance in developing economies (see e.g. Bourguignon, Ferreira, and Leite (2003) and the references therein).
equilibrium levels of education under each respective education system. While it is still the case that \( W^b(\gamma^{uni}) \geq W^{uni}(\gamma^{uni}) \), it is unclear whether \( W^b(\gamma^b) \geq W^{uni}(\gamma^{uni}) \). In other terms, the central planner may prefer the unilingual system if it leads to a sufficiently higher decentralised education level.

As previously shown, multiple equilibria sometimes arise under the unilingual system. In models with multiple equilibria, predictions generally depend on the equilibria that are under consideration. We thus focus on results that hold for comparisons of all equilibria. Some of our results depend on the exclusion of an interior unstable equilibrium, but most do not. In the subsequent analysis, we discuss the matter of equilibrium selection whenever relevant.

![Figure 4: Optimal decentralised education system](image)

Figure 4 characterises the optimal decentralised education system (appendix 9.3 contains the formal derivations).\(^\text{30}\) Remember that the bandwagon effect is at play only in the unilingual system, since under bilingual education all the ns keep nish. The bandwagon effect works either to the favour or to the disfavour of society, and does so in a non-linear fashion. In the polar case when the dominant group is very large relative to the cost of taking education (region I) the bandwagon effect is bad for society as it may lock the members of the dominated group in a low-education equilibrium. In the other polar case, when the size of the dominant group is very small relative to the cost of education (region IIb), the bandwagon effect is still negative as it may lock agents in a costly high education equilibrium. In the intermediate case (Region IIa), the bilingual system fails to produce sufficient incentives for taking education, while the bandwagon effect can push education levels under the unilingual system to a level which would be impossible to reach under a bilingual system. Thus, despite the fact that the unilingual system is technologically

\(^{30}\text{The figure is drawn assuming } \sigma > 1 \text{. This is irrelevant to the results we have obtained.}\)
inferior to the bilingual system, the unilingual system is nevertheless optimal in this case since it provides stronger incentives for taking education than the bilingual system.\textsuperscript{31}

In the remaining cases, the bandwagon effect does not play any role at equilibrium in the unilingual system, as either education is very cheap relative to the size of the dominant group, or very expensive. As a result, the two systems lead to identical education and welfare levels.

6 The choice of education system

This section analyses how the ms and the ns rank the unilingual education system with respect to the bilingual system, taking into account the equilibrium education levels arising under each system. In particular, we study whether language conflict can endogenously arise in our set-up, and whether this conflict is of the expected type, i.e. a situation in which the ns favour bilingualism while the ms defend unilingualism. In addition, we determine which political rules (if any) enable society to reach the decentralised optimum.

The preferences of the dominant group over education systems can be characterised as follows:

Lemma 4 The dominant group prefers the education system that maximises the dominated group’s equilibrium education level.

Proof. The two systems are identical from the point of view of the ms, save the equilibrium education levels, hence we need only show that the ms prefer more to less education. Write \( u_m(\gamma) \) indirect utility given an equilibrium \( \gamma \):

\[
  u_m(\gamma) = M + N\gamma_n + \gamma_m\Delta U_m(\gamma_n)
  = M + N\gamma_n + \gamma_m(-c + (M + N\gamma_n)\sigma),
\]

and consider two equilibria \( \gamma^s \) and \( \gamma^t \), with \( \gamma^s_n \geq \gamma^t_n \). Subtract \( u_m(\gamma^t) \) from \( u_m(\gamma^s) \) and simplify to obtain

\[
  u_m(\gamma^s) - u_m(\gamma^t) = (\gamma^s_m - \gamma^t_m)\Delta U_m(\gamma^s_n) + \gamma^t_m(\Delta U_m(\gamma^s_n) - \Delta U_m(\gamma^t_n)) + N(\gamma^s_n - \gamma^t_n)
  = (\gamma^s_m - \gamma^t_m)\Delta U_m(\gamma^s_n) + (\gamma^s_n - \gamma^t_n)N(1 + \gamma^t_m\sigma)
\]

The first term is non-negative. This is trivially true for \( \gamma^s_m = \gamma^t_m \). \( \gamma^s_m > \gamma^t_m \geq 0 \) implies \( \Delta U_m(\gamma^s_n) \geq 0 \) and \( \gamma^s_m < \gamma^t_m \leq 1 \) implies \( \Delta U_m(\gamma^s_n) \leq 0 \), hence \( (\gamma^s_m - \gamma^t_m)\Delta U_m(\gamma^s_n) \geq 0 \) even for \( \gamma^s_m \neq \gamma^t_m \). The second term is non-negative [positive] for \( \gamma^s_n \geq [>]\gamma^t_n \), hence \( \gamma^s_n \geq [>]\gamma^t_n \) implies \( u_m(\gamma^s) \geq [>]u_m(\gamma^t) \). Generically \( \gamma^s_n = 1 \) or \( \gamma^t_n = 0 \). In the first case \( s = bi \) and in the second \( s = uni \). \hfill \Box

\textsuperscript{31}In Region IIa, preference for the unilingual system depends on the exclusion of the unstable interior equilibrium under the unilingual system. Intuitively, whenever the size of the dominated group is large and education costly it is better that nobody takes education than only a fraction, since the production surplus then created is insufficient to cover the cost of education.
The intuition for the lemma goes as follows. Note first that the ms attend the same type of school under both educations systems, in the sense that they never learn Mish under any of the two possible institutional arrangements. This means that the only way the ms can expand their set of production partners is by inducing the members of the other group to take education and thus learn Mish. Hence, the best education system is the one that expands to a larger extent the set of production partners.

Consider next the preferences of the dominated group:

**Lemma 5** The dominated group either prefers the bilingual system or is indifferent between the two systems.

**Proof.** Consider two arbitrary equilibria, $\gamma^b$ and $\gamma^{uni}$ under the bilingual and unilingual systems, respectively. We demonstrate below that $u^b_n \geq u^{uni}_n(\gamma^{uni})$ for all possible equilibria $\gamma^{uni}$. Hence, $u^b_n \geq u^{uni}_n$ and the bilingual system is preferred. Subtract $u^{uni}_n(\gamma^{uni})$ from $u^b_n$:

$$u^b_n - u^{uni}_n(\gamma^{uni}) = (\gamma^b_n - \gamma^{uni}_n) \Delta \gamma^b_n + \gamma^{uni}_n (\Delta U^b_n - \Delta U^{uni}_n(\gamma^{uni})) + U^b_n - U^{uni}_n(\gamma^{uni})$$

The first term is obviously non-negative. So is the second. This is trivially true for $\gamma^b_n = \gamma^{uni}_n$. $\gamma^b_n > \gamma^{uni}_n \geq 0$ implies $\Delta U^b_n \geq 0$ and $\gamma^b_n < \gamma^{uni}_n \leq 1$ implies $\Delta U^b_n \leq 0$, hence $(\gamma^b_n - \gamma^{uni}_n) \Delta U^b_n \geq 0$ even for $\gamma^b_n \neq \gamma^{uni}_n$. Thus $u^b_n \geq u^{uni}_n(\gamma^{uni})$ for all possible equilibria $\gamma^b$ and $\gamma^{uni}$.

The members of the dominated group prefer the bilingual over the unilingual system due to the absence of negative network externalities (the absence of a bandwagon effect) under the bilingual system. In other terms, under the unilingual system the ns are “compelled” in some cases to take education even if it is relatively expensive since this is the only way of learning Mish and avoiding to remain insulated from the rest of society.32

The joint implication of the two lemmas above is that political tension will arise between the two language groups precisely in the circumstances under which the unilingual system provides stronger incentives than the bilingual system for the dominated group to become educated. In this case, the members of the dominant group go for the unilingual system, hoping to lock in the members of the other group in a high-education equilibrium, whereas the members of the dominated group go for a bilingual system precisely in order to avoid the same situation from happening. This situation corresponds to Region II in Figure 5 below, which displays how the preferences of the two groups vary with group size and the relative cost of education. When the

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32 The unilingual system sometimes provides stronger incentives for taking education than the bilingual system. Hence, one might expect circumstances to arise under which the ns nevertheless opt for the unilingual system. This is not the case here, as the unilingual system provides stronger incentives for taking education than bilingualism precisely in those circumstances under which the ns would be better-off if no one took any education at all (i.e. those situations where the dominated group is large and education expensive). In section 7, we show that a preference of the ns for unilingual education is an equilibrium outcome when there are spillovers in production.
cost of taking education is high relative to the size of the dominant group \((c > M + \sigma)\), but still not too high \((c < 1 + \sigma)\), the communication effect is insufficient to render education profitable for any group under the bilingual system, but the bandwagon effect is sufficiently strong to generate positive education levels in the unilingual system. Figure 5 shows that this is the only type of language conflict that we get. This is an interesting result since language conflict is here of the observed type and does not rely on any direct utility enjoyed by the agents from speaking their own native language. In contrast, language conflict is an equilibrium phenomenon. The members of the dominated group want their language to be a language of instruction in school not because they “like it” but rather because abandoning it would force them to overinvest in education. In the same way, the ms may prefer unilingualism not because they “dislike” mish or because they like mish but because a unilingual school guarantees that more people will learn mish and that the value of production is higher for the ms without additional costs. In other terms, the ms free-ride on the costs of speaking a common language, which are entirely paid for by the ns.33

![Fig. 5: Choice of education system](image-url)

Under a wide range of circumstances, there is no political tension over the choice of education system. When education is relatively cheap \((c < M + \sigma)\) both groups either prefer the bilingual system or are indifferent between the two. Education is so cheap that it is important for both groups to generate the strongest possible incentive for taking education, i.e. to eliminate possible negative bandwagon effects. For this reason, a bilingual system is chosen in region I.

When education is expensive \((c > 1 + \sigma)\), the communication effect is too weak relative to the cost of education to generate sufficient incentives for taking education under either system.

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33 Adding an exogenous utility term of speaking one’s mothertongue would only reinforce this result. The members of the dominated group would have an additional reason for selecting a bilingual system. The members of the dominating group would not modify their decision, as they maintain their mothertongue under both systems.
Hence, the choice of system is immaterial.\footnote{If we compare the unilingual system with a symmetric bilingual system, we obtain an additional region of language conflict of the expected type when $c - \sigma > 0$, $M < 1 - (c - \sigma)$, and $M > \overline{M}(c - \sigma)$ are simultaneously satisfied. In addition, for $c - \sigma > 0$, $M < 1 - (c - \sigma)$, and $M < \overline{M}(c - \sigma)$ simultaneously verified, the choices of the $ms$ are indeterminate, as they prefer the unilingual or the bilingual system depending on the equilibrium selection. In the rest of the parameter space, the results are as in Fig. 5.}

Having identified the preferences of the two groups (Figure 5), we are now able to determine the ability of different political rules to reach the decentralised optimum (Figure 4). Alternative political rules can differ at equilibrium in our model only in terms of the solution to the language conflict in region II.

Consider first a democratic system, here represented by majority rule. Under majority rule, political decisions are taken so as to suit the needs of the majority. In the context of our model, we have exogenously assigned political dominance to the $m$ group. Democratic rule thus corresponds to a situation in which the dominant group (the $ms$) constitutes a majority, i.e. $M > 0.5$.\footnote{To understand why $M < 0.5$ does not correspond to a democratic regime with a majority of $ns$, it is sufficient to note that in that case the $ns$ would prefer a unilingual nish system to a unilingual mish system and the majority always prefers a system in which the minority becomes bilingual to one in which the majority becomes bilingual. In other terms, as the $ms$ have been arbitrarily assigned to the role of “dominant group”, a democratic regime with a majority of $ns$ is identical to a democratic regime with a majority of $ms$.} By a comparison of Figure 5 and Figure 4, we find a perfect match between the two figures for $M > 0.5$:

**Proposition 6** Majority rule leads to the adoption of the socially optimal education system.

Thus under a democracy the language conflict arising in region II for $M > 0.5$ is solved through the adoption of a unilingual system, which is socially optimal for these parameter values. In that particular area, a unilingual system is to be preferred from a social viewpoint because it provides (through the bandwagon effect) stronger incentives for the $ns$ to be educated and this is good since education is not very expensive and the $ns$ constitute a relatively small fraction of the population.

Consider next the outcome under minority rule, for example in autocracy, in which case the minority is the dominant group ($M < 0.5$). In that case comparison of Figure 5 and Figure 4 for $M < 0.5$ yields:

**Proposition 7** The unilingual system is implemented too often under minority rule.

Under minority rule, the $ms$ impose a unilingual system in some cases in which society as a whole would have been better-off under a bilingual system. More specifically, according to our model, this problem is likely to arise in situations in which the cost of education is quite high and the dominant group is small. In such a context, education of the $ns$ is not desirable from a social viewpoint since education is expensive, but the bandwagon effect under the unilingual system “forces” the $ns$ to take education and this benefits the $ms$.

We can now come back to the empirical puzzle presented in section 2. Taking into account that a unilingual system will be chosen in the language conflict area II in Figure 5, it is clear from
inspection of that Figure that, in the absence of information concerning the net cost of education $c - \sigma$, the size $M$ of the dominant group does not determine whether a unilingual or a bilingual education system arises in equilibrium.

Finally, Figure 6 compares the equilibrium education levels under the selected decentralised system (from Figures 2, 3, and 5) with the first best education levels (from Proposition 3). The first thing to be noted is that the decentralised systems lead to a full education equilibrium whenever $c < \sigma$, and this corresponds precisely to the case in which the $(1, 1)$ equilibrium is socially optimal. In other terms, the introduction of a compulsory education system is not necessary in our model in order to get full education when this is efficient. A second result is that equilibrium education levels can actually be suboptimally too high, as in area IIb in Figure 6. In this area, the dominant group imposes a unilingual system with education levels $\{(0, 0), (y_n, 0), (1, 0)\}$, while the bilingual system favoured by the dominated group would have lead to the optimal education level $(0, 0)$. In the remaining cases, the political process leads either to the optimum amount of education or to undereducation due to the presence of positive communication externalities from education.

![Fig. 6: Optimality of the equilibrium education levels under the selected decentralised system](image)

**7 Productivity Spillovers**

We consider now productivity spillovers, i.e. a situation in which the productivity of each agent positively depends on the skill level of the other production partner. Specifically, assume that, with respect to the benchmark, total bilateral production increases by $\sigma$ each time a skilled agent participates to the match. Thus, a match between two unskilled produces still 2, an unskilled-skilled match produces $1 + (1 + \sigma) + \sigma = 2 + 2\sigma$ instead of $1 + (1 + \sigma)$, and a match between two skilled $(1 + \sigma) + (1 + \sigma) + 2\sigma = 2 + 4\sigma$ instead of $(1 + \sigma) + (1 + \sigma)$. Assume also for simplicity that output is equally shared among production partners.
Consider first the unilingual system. With spillovers, the new pay-offs for a skilled, an unskilled \( m \) and an unskilled \( n \) are respectively given by:

\[
\begin{align*}
\mathcal{E}^{uni}(\mu) &= -c + M[(1 + 2\sigma)\mu_m + (1 + \sigma)(1 - \mu_m)] + N(1 + 2\sigma)\mu_n, \\
\mathcal{E}^{uni}_m(\mu) &= M[(1 + \sigma)\mu_m + (1 - \mu_m)] + N(1 + \sigma)\mu_n, \\
\mathcal{E}^{uni}_n(\mu) &= N(1 - \mu_n) = U^{uni}_n(\mu).
\end{align*}
\] (12-14)

Note first that the pays-offs of the skilled and the unskilled \( m \)s (respectively (12) and (13)) are now increasing in \( \mu_m \), as the decision of an additional \( m \) to take education exerts a positive skill externality on all the agents who are able to speak mish. However, as in the benchmark model, the \( m \)s never choose to take education for \( c > \sigma \) (see Ortega and Tangerás, 2003, for analytical details), thus implying that the interaction between the education decisions of the \( m \)s does not play a role in equilibrium. The crucial difference with the benchmark is the existence of a new channel through which the net gain from education for an \( n \) increases with the number of educated \( n \)s. Indeed, while the value of not taking education for an \( n \) remains as in the benchmark (as shown in (14)), spillovers imply that skilled \( n \)s are more productive when they can meet many other skilled \( n \)s. Mathematically, from (1) and (12) \( \mathcal{E}^{uni}_n(\mu_n, 0) - U^{uni}_n(\mu_n, 0) = \sigma \mu_n N \), so with spillovers there is an additional net gain from education equal to \( \sigma \) for each additional \( n \) that goes to school. In other terms, the bandwagon effect in the education decisions among the \( n \)s becomes stronger with spillovers.

Consider next bilingual education, with pay-offs:

\[
\begin{align*}
\mathcal{E}^{bi}_n(\mu) &= -c + M[(1 + 2\sigma)\mu_m + (1 + \sigma)(1 - \mu_m)] + N[(1 + 2\sigma)\mu_n + (1 + \sigma)(1 - \mu_n)], \\
\mathcal{E}^{bi}_n(\mu) &= N[(1 + \sigma)\mu_n + (1 - \mu_n)], \\
\mathcal{E}^{bi}_m(\mu) &= M[(1 + \sigma)\mu_m + (1 - \mu_m)] + N(1 + \sigma)\mu_n = \mathcal{E}^{uni}_m(\mu).
\end{align*}
\] (15-17)

Subtracting (17) from (15), the net pay-off from education for the \( m \)s remains as in the benchmark. The intuition is simple, as in both cases getting educated increases by \( \sigma \) the pay-off obtained in each match with an mish speaker.\(^{36}\) Concerning the \( n \)s, subtracting (16) from (15),

\[
\Delta \mathcal{E}^{bi}_n = -c + M[(1 + \sigma)\mu_m + (1 - \mu_m)] + \sigma.
\] (18)

so the pay-off from education for the \( n \)s is in principle different from that of the benchmark, given by (10). However, as in the benchmark, the \( m \)s do not take education in equilibrium for \( c > \sigma \), thus implying that \( \mu_m = 0 \) and that (18) becomes equal to (10). For these reasons, the equilibrium education levels under bilingualism remain as in the benchmark.

\(^{36}\)In the benchmark, becoming skilled increases from \( 1 \) to \( 1 + \sigma \) the pay-off in a match with an unskilled \( m \), a skilled \( m \) or an \( n \) (necessarily skilled, as she would otherwise not speak mish). With spillovers, the pay-off increases from \( 1 \) to \( 1 + \sigma \) with an unskilled \( m \) and from \( 1 + \sigma \) to \( 1 + 2\sigma \) with a skilled \( m \) or with a skilled \( n \).
Figure 7 displays the preferences of the two groups over education systems for \( c > \sigma \) in the case with spillovers. The main difference with respect to the preferences in the benchmark (see Figure 5) is that there is now sometimes unanimity for the unilingual system i.e. the members of the dominated group are perfectly willing to give up education in their own language and the members of the dominant group perfectly happy to accommodate their wishes.\(^{37}\) This is an interesting result as we have given a technological advantage to the bilingual over the unilingual system.\(^{38}\) Unanimity for the unilingual system arises for \( M < c - \sigma \) but \( M \) sufficiently close to \( c - \sigma \) (region III). In the benchmark, this was formerly a language conflict region (see Figure 5), as the \( ns \) preferred the bilingual system in order to avoid being trapped in a high education equilibrium through the bandwagon effect.\(^{39}\) In contrast, spillovers render beneficial the bandwagon effect in region III and shift the preferences of the \( ns \) towards a unilingual system. In other terms, in area III the \((1, 0)\) equilibrium under unilingualism becomes more profitable for the \( ns \) than the \((0, 0)\) equilibrium under either system because taking education brings about productivity gains that were absent in the benchmark.

\(^{37}\)This result depends on the exclusion of the unstable interior unilingual equilibrium. Otherwise preferences would be indeterminate.

\(^{38}\)Adding an exogenous utility of speaking one’s own language would qualitatively affect the result if this utility was large relative to the spillover effect. In that case, the members of the dominated group would still prefer the bilingual system.

\(^{39}\)In that case, the communication effect is not strong enough to make the \((1, 0)\) equilibrium more attractive than the \((0, 0)\) equilibrium.


8 Conclusion

While many countries are multilingual or have been historically constituted by several language groups, language diversity has not always lead to language conflict between the groups when coming to decide the language of instruction in school, nor has it been always the case that both groups have agreed upon a unilingual or a bilingual system. A possible way of understanding this variety of situations is to assume that agents get some utility from speaking their own language and that compromise over language issues may be reached through political bargaining. However, in that case, we would expect a larger language group to be \textit{ceteris paribus} more likely to obtain an education system in which its language is used as a language of instruction, and this is not what we observe for France in 1863.

Here, we take a different stand on the issue and construct a model around the individual incentives to attend school. We show that these incentives vary across groups depending on the nature of the education system. In particular, under a unilingual system, the language group that does not have its language used in school (the politically “dominated” group) has in general stronger incentives to attend school. For this reason, this group will either bear a large part of the costs necessary to the adoption of a common language, or actively defend a bilingual system in order to avoid paying a large share of these costs. In contrast, the other group will defend more frequently the exclusive use of its language in education in order to free-ride the educational investment of the dominated group. Thus language conflict of the expected type is shown to endogenously arise as the result of an economic conflict.

Should we expect language conflicts to end up in the adoption of the socially optimal language system? According to our model, the answer crucially depends on the nature of political institutions. Specifically, we show that democratic institutions (interpreted as majority rule) choose the right education system, while minority rule chooses unilingual education too frequently.

Finally, when allowing for productivity spillovers among production partners, unanimity for a unilingual education system becomes an additional equilibrium outcome, even if we have attributed an advantage to bilingualism over unilingualism.

References


9 Appendix

Let $\Gamma^s$ be the (finite) set of equilibria under education system $s \in \{bi, uni\}$ and $\gamma^s \in \Gamma^s$ a specific equilibrium under system $s$. Let $u^s_m(\gamma^s)$ be the indirect utility a member of the $m$ group attains under system $s$ given the play of $\gamma^s$ and let $u^s_m$ be the indirect utility when there is a unique equilibrium. As demonstrated below, the bilingual system generically has a unique equilibrium. Write $u^\text{uni}_m = \min\{u^\text{uni}_m(\gamma^\text{uni})|\gamma^\text{uni} \in \Gamma^\text{uni}\}$ ($\nu^\text{uni}_m = \max\{u^\text{uni}_m(\gamma^\text{uni})|\gamma^\text{uni} \in \Gamma^\text{uni}\}$), the minimum (respectively, maximum) indirect utility that is attainable under the unilingual system. We shall say that a member of the $m$ group prefers the bilingual system ($bi \succeq_m uni$) if $u^\text{uni}_m \geq u^\text{bi}_m$, that she prefers the unilingual system ($uni \succeq_m bi$) if $u^\text{uni}_m \geq u^\text{bi}_m$, and that she is indifferent between the two ($bi \sim_m uni$) if the equilibrium is unique also under the unilingual system and $u^\text{uni}_m = u^\text{bi}_m$. These definitions are extended to the $n$ group and to expected welfare in the obvious way.

9.1 Equilibrium education levels under the unilingual system

This section derives generic equilibrium education levels under the unilingual system. We begin by deriving the equilibrium education levels of the $n$ group. Remember that

$$\Delta U^\text{uni}_n(\mu_n) = -c + (M + N\mu_n)\sigma + M + N\mu_n - N(1 - \mu_n).$$

Note first that $\partial \Delta U^\text{uni}_n(\mu_n) / \partial \mu_n = N(1 + 2\sigma) > 0$. Monotonicity of $U^\text{uni}_n(\cdot)$ in $\mu_n$ and $\Delta U^\text{uni}_n(1) = 1 + \sigma - c$ imply that $\gamma^\text{uni}_n = 0$ is the unique equilibrium for $c > 1 + \sigma$. Next, $\Delta U^\text{uni}_n(0) = M(1 + \sigma) - N - c$ and monotonicity of $U^\text{uni}_n(\cdot)$ in $\mu_n$ imply that $\gamma^\text{uni}_n = 1$ is the unique equilibrium for $M > \overline{M}$ with

$$\overline{M} = \frac{1 + c}{2 + \sigma}.$$

Finally, with $c - \sigma < 1$ and $M < \overline{M}$, the bandwagon effect generates multiple equilibria. $\gamma^\text{uni}_n = 0$ is an equilibrium since $\Delta U^\text{uni}_n(0) < 0$, $\gamma^\text{uni}_n = 1$ is an equilibrium since $\Delta U^\text{uni}_n(1) > 0$, and $\gamma^\text{uni}_n = (\overline{M} - M)/N$ is an equilibrium since $\Delta U^\text{uni}_n((\overline{M} - M)/N) = 0$.

Turn next to the education levels of the $m$ group. Recall that

$$\Delta U^\text{uni}_m(\mu_m) = -c + (M + N\mu_m)\sigma.$$

Note first that $\partial \Delta U^\text{uni}_m(\mu_m) / \partial \mu_m = N\sigma > 0$. Monotonicity of $U^\text{uni}_m(\cdot)$ in $\mu_m$ and $\Delta U^\text{uni}_m(1) = \sigma - c$ imply that $\gamma^\text{uni}_m = 0$ is the unique equilibrium for $c > \sigma$. Next, $\Delta U^\text{uni}_m(0) = M\sigma - c$ and monotonicity of $U^\text{uni}_m(\cdot)$ in $\mu_m$ imply $\gamma^\text{uni}_m = 1$ for all $M > \underline{M}$ with

$$\underline{M} = \frac{c}{\sigma}.$$

For $c < \sigma$ and $M < \underline{M}$, the equilibrium education level $\gamma^\text{uni}_m$ depends on $\gamma^\text{uni}_n$. First, $\gamma^\text{uni}_n = 1$ implies $\gamma^\text{uni}_m = 1$ since $\Delta U^\text{uni}_m(1) = \sigma - c > 0$. Second, $\gamma^\text{uni}_n = 0$ implies $\gamma^\text{uni}_m = 0$ since $\Delta U^\text{uni}_m(0) =$
By plugging in the relevant expressions, we obtain in Figure 3.

\[ \gamma_n^{uni} = (\overline{M} - M)/N \] implies \( \gamma_n^{uni} = 1 \) for \( 2c < \sigma \) and \( \gamma_n^{uni} = 0 \) for \( 2c > \sigma \). The equilibrium configurations are depicted in Figure 2.

### 9.2 Equilibrium education levels under the bilingual system

As opposed to the unilingual case, the bilingual case generates a unique equilibrium for almost every parameter configuration. Let \( \gamma^b = (\gamma^b_n, \gamma^b_m) \) denote this equilibrium. An inspection of eq. (10) reveals \( \Delta U^b_m(\mu) > 0 \) for all \( c < \sigma \) hence \( \gamma^b_n = 1 \) for all \( c < \sigma \). \( \Delta U^b_m(1, \mu_m) = \sigma - c > 0 \) within the same parameter range, hence \( \gamma^b_n = (1,1) \) for all \( c < \sigma \). Consider next \( c > \sigma \). In this case \( \Delta U^b_m(\mu) < 0 \) follows from inspection of eq. (11), hence \( \gamma^b_n = 0 \) for all \( c > \sigma \). \( \Delta U^b_n(\mu_n,0) = M + \sigma - c \). Thus \( \gamma^b = (1,0) \) for all \( c \in (\sigma, 1 + \sigma) \) and \( M \in (c - \sigma, 1) \), whereas \( \gamma^b = (0,0) \) for all \( c > \sigma \) and \( M \in (0, \min\{c - \sigma; 1\}) \). The equilibrium configurations are depicted in Figure 3.

### 9.3 Welfare

This appendix derives the socially optimal welfare system under decentralised education choice. Expected welfare \( W^s(\gamma^s) \) in equilibrium \( \gamma^s = (\gamma^s_n, \gamma^s_m) \) under system \( s \in \{bi, uni\} \) is defined by

\[ W^s(\gamma^s) = Nu^s_n(\gamma^s) + Mu^s_m(\gamma^s), \]

with

\[ W^b(\gamma^b) - W^uni(\gamma^uni) = N(u^b_n(\gamma^b) - u^uni_n(\gamma^uni)) + M(u^b_m(\gamma^b) - u^uni_m(\gamma^uni)) \]

where

\[ u^s_k(\gamma^s) = \gamma^s_kU^s_k(\gamma^s) + (1 - \gamma^s_k)\overline{U}_k(\gamma^s), \ k \in \{n, m\}. \]

By plugging in the relevant expressions, we obtain

\[ u^b_n(\gamma^b) - u^uni_n(\gamma^uni) = (\gamma^b_n - \gamma^uni_n)(M + \sigma - c) + N\gamma^uni_n(1 - \gamma^uni_n)(2 + \sigma) \]

\[ u^b_m(\gamma^b) - u^uni_m(\gamma^uni) = N(\gamma^b_n - \gamma^uni_n) + (\gamma^b_m - \gamma^uni_m)(M\sigma - c) + N(\gamma^uni_n - \gamma^uni_m)(\gamma^uni_m\sigma). \]

Consider first the case with \( c < \sigma \) and \( M \in (0, \overline{M}) \). In this case the unilingual system gives rise to multiple equilibria, and the bilingual equilibrium implements the full education equilibrium.

\[ W^b(1,1) - W^uni(\gamma^uni) = N(1 - \gamma^uni_n)(M2 + \sigma\gamma^uni_n) + N\gamma^uni_n(2 + \sigma) \]

\[ \geq 0 \]
implies a welfare preference for the bilingual system.

For $c \in (0, \sigma)$ and $M \in (\overline{M}, 1)$, both systems implement the full education equilibrium. Since $W^{\text{bi}}(1, 1) = W^{\text{uni}}(1, 1)$, the choice of education system does not affect welfare in this case.

For $c \in (\sigma, 1 + \sigma)$ and $M \in (0, (c - \sigma)/2)$, $\Gamma^{\text{uni}} = \{(0, 0), ((\overline{M} - M)/N, 0), (1, 0)\}$ and $\Gamma^{\text{bi}} = \{(0, 0)\}$. By plugging in the correct expressions and simplifying, we obtain:

$$W^{\text{bi}}(0, 0) - W^{\text{uni}}(\gamma^{\text{uni}}_n, 0) = N\gamma^{\text{uni}}_n(2 + \sigma + c - \sigma - 2M) \geq 0.$$ 

In this case the bilingual system is optimal from a welfare point of view.

For $c \in (\sigma, 1 + \sigma)$ and $M \in ((c - \sigma)/2, c - \sigma)$, education levels are still the same as above. Now, $W^{\text{uni}}(1, 0) > W^{\text{bi}}(0, 0) = W^{\text{uni}}(0, 0)$. If we restrict attention to the set of stable equilibria, the unilingual system becomes optimal. However, $W^{\text{uni}}((\overline{M} - M)/N, 0) > W^{\text{uni}}(0, 0)$ if and only if $M > 0.5$, hence preferences are indeterminate if $M < 0.5$ and all equilibria are included in the comparison.

For $c \in (\sigma, 1 + \sigma)$ and $M \in (c - \sigma, \overline{M})$, the unilingual equilibria are the same as above, but now $\Gamma^{\text{bi}} = \{(1, 0)\}$. By straightforward computations, we obtain

$$W^{\text{bi}}(1, 0) - W^{\text{uni}}(\gamma^{\text{uni}}_n, 0) = N(1 - \gamma^{\text{uni}}_n)(2M - (c - \sigma) + N\gamma^{\text{uni}}_n(2 + \sigma)) \geq 0,$$

hence the bilingual system is optimal.

For $c \in (\sigma, 1 + \sigma)$ and $M \in (\overline{M}, 1)$ and for $c > 1 + \sigma$ both systems give rise to identical equilibria, $(1, 0)$ in the first case and $(0,0)$ in the second. The choice of education system has no effect on welfare in this final case because $W^{\text{bi}}(1, 0) = W^{\text{uni}}(1, 0)$ and $W^{\text{bi}}(0, 0) = W^{\text{uni}}(0, 0)$.

The optimal decentralised education system is depicted in Figure 4.