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# Working Paper Under/Over-Investment and Early Renegotiation in Public-Private Partnerships

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# Under/Over-Investment and Early Renegotiation in Public-Private Partnerships

# Abstract

We consider a public-private partnership in an infrastructure project, which requires specialised expertise during the construction stage for the infrastructure to operationalise. This entails that, after an investment is made to begin building the infrastructure, its construction is completed at a cost, which increases with the investment at an increasing rate, and is higher if the government replaces the firm beforehand. The likelihood of a lower operating cost increases as well with the initial investment. Once the infrastructure is in place, the firm manages it, taking advantage of the (usual) synergy between construction and operation. Given the characteristics of the project, the firm has an incentive to either under-invest or over-invest in early construction, seeking a renegotiation thereafter. We show that, in a renegotiation-proof contract, the marginal cost of the investment facing the government is either above or below the marginal "technological" cost of the investment, at optimum. Accordingly, the resulting investment - although enhanced - is either below or above the efficient level. The contractual payoff of the firm is above its renegotiation payoff in the former case, below in the latter. We further show that when the firm holds private information on the operating conditions, the government may welcome a contractual renegotiation either as a way of containing (avoiding) the distortions due to the informational gap, or as a tool to pass the cost of construction completion onto the firm, or both.

JEL-Codes: D820, H570, H810.

Keywords: public-private partnerships, asset specificity, hold-up, over/under-investment, renegotiation.

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# 1 Introduction

It is now well known that public-private partnerships (henceforth, PPPs) in infrastructure projects are largely subject to renegotiations, and that renegotiations are likely to take place early on, during the construction of the infrastructure, and mostly in favour of the private partner (Guasch [9]). Although this issue is widely acknowledged among the scholars interested in a thorough understanding of PPPs, so far no formal analysis has been conducted to clarify why PPP contracts are subject to firm-led renegotiations during the construction phase, and what consequences this has on both the contractual offer made by the government and the decisions on how much to invest made by the firm. Our paper deals with this issue.

Two essential results are now well known about the renegotiation of contracts in general. First, in agency relationships in which the principal makes a take-it-or-leave-it offer, this offer is designed in such a way that a renegotiation is prevented, if it is undesirable at the time the offer is made (Dewatripoint [6]). In the standard jargon, the contract is *renegotiation-proof.* Second, even if a renegotiation cannot be prevented, as can happen in incomplete contracting environments, it is to the *non*-investing party that the renegotiation is beneficial, provided this party is in a position to hold up the other. Anticipating a hold-up, the investing party is led to economize on the investment, which is thus downsized below the efficient level (Grossman and Hart [8] and Hart and Moore [13]; henceforth, GHM).<sup>1</sup>

The fact that renegotiations do occur in PPPs, and that they usually favour the investing party (the firm), raises questions which, to the best of our knowledge, have not yet received a reply from the literature. One such question is how it can be explained that it is the firm, rather than the (non-investing) government, which seeks a renegotiation in mid-construction when the contract is incomplete. Particularly, the possibility of the firm's incentives to renegotiate in mid-construction resulting from the intrinsic characteristics of the project, has been overlooked so far.<sup>2</sup> Further questions naturally nest on the first one. Are there situations in which the government finds a renegotiation desirable, and is thus uninterested in designing a renegotiation-proof contract? Is the firm moti-

<sup>&</sup>lt;sup>1</sup>Even under incomplete contracting, the principal will usually prefer to offer a renegotiation-proof contract, if the outcome of a renegotiation is perfectly foreseen. See Neeman and Pavlov [21] for a general survey on renegotiation-proof mechanisms. As far as PPPs are concerned, Iossa and Martimort [15] consider an incomplete contracting framework, in which it is nonetheless optimal for the government to offer a renegotiation-proof contract.

<sup>&</sup>lt;sup>2</sup>Previous studies provide answers to the question but in different contexts. One answer is related to the non-benevolence of public officials. Specifically, the firm is encouraged to seek a renegotiation when it expects this to be welcome to public officials who pursue other goals than maximizing the social value of the project (see, for instance, Engel *et al.* [7], Guasch and Straub [11], Iossa and Martimort [16]). Another answer concerns the poorness of enforcing mechanisms. In environments where enforcing institutions are weak, the firm may be able to obtain more than the stipulated compensation by reneging on the contract and bringing the government back the contracting table (Laffont [14], Guash *et al.* [10], Danau and Vinella [4] - [5]). None of these studies deal with the firm's incentives to seek a renegotiation before completing the infrastructure, and with the possibility of those incentives resulting from the characteristics of the project.

vated to under-invest in the project, along the previous findings of the literature, when renegotiation is welcome (if any)? What happens when it is not?

To provide answers to these questions, we focus on a peculiar characteristic of PPP projects, which is potentially to the root of early contractual renegotiations. During the construction stage of a PPP project, the infrastructure is little functional, in general, and may require more specialised expertise to operationalise. This makes the asset more specific to the operator, who thus acquires an advantage over potential replacement operators (Trebilcock and Rosenstock [28]). Particularly, in network industries, specialized assets seem to be the rule rather than an exception (Sidak and Spulber [27]). For instance, such assets include the extensive infrastructure of switching networks in the telecommunications sector (Rothaermel and Hill [23]) as well as the natural monopoly facilities in the water sector (Massarutto [20]). These are both examples of infrastructure projects often developed through PPPs nowadays.

There are two consequences to asset specificity in PPPs. First, the additional investments required to operationalise the asset are intrinsically linked to the way in which the asset is being built, rather than reflecting well-defined investment obligations related, say, to its physical size.<sup>3</sup> Second, the specificity of the asset to its builder involves that the private partner enjoys an advantage over potentially replacing firms. In this respect, in a PPP, it is the investing party who may be able to hold up the non-investing party (the public partner). The most likely juncture for hold-up is mid-construction because the firm has the greatest effective monopoly over asset provision, and project delay imposes pressure on the government. Therefore, the switching costs to governments of replacing private partners are higher during mid-construction than after construction completion, as observed by Vining and Boardman [29]. Switching costs are also especially high as PPP projects are often framed in poorly competitive sectors, in which it is difficult to find alternative operators.<sup>4</sup>

**Overview of the model** To represent these characteristics of PPPs in a formal model, we consider an infrastructure project delegated by a government to a firm through a PPP, which includes three stages, namely, early construction of the infrastructure, completion of the construction, and operation. The project requires the firm making an investment in the first stage. As this investment is non-observable to any party other than the firm itself, it is the source of incentive problems, which also affect the subsequent stages through the externalities inter-linking them. There is a negative externality between the first two stages, in that a higher investment results in a higher cost of construction completion.

<sup>&</sup>lt;sup>3</sup>This characteristics of PPPs is referred to, for instance, in Martimort and Straub [17], who distinguish between well-defined (verifiable) investment obligations, particularly related to the physical characteristics of network projects, and efficiency (non-verifiable) investments, which improve the quality of the projects.

<sup>&</sup>lt;sup>4</sup>Being based on Guasch's empirical finding that renegotiations are less likely to occur in the energy sector, Trebilcock and Rosenstock [28] argue that governments face lower transaction costs in finding alternative suppliers when infrastructure markets are more competitive.

That is, a more sophisticated infrastructure is also more costly to operationalise. Besides, there is a positive externality between the first and the last stage, in that a higher investment results in a higher probability of the cost of operation being low. That is, a higher quality of the infrastructure is more likely to yield savings in operation. Whereas this kind of synergy has been widely captured in PPP models, to the best of our knowledge, the negative externality aforementioned is newly represented in our setting. Given the specificity of the infrastructure to its builder, the replacement of the private partner with another firm during the construction is costly to the government. It both exacerbates the effect of the negative externality raising the cost of construction completion, and weakens the effect of the positive externality making a low cost of operation less likely. The cost of completing the construction is observed by both partners after the investment has been made, but is non-verifiable by a third party (alternatively, it would be infinitely costly to verify it). Hence, the PPP contract is incomplete and susceptible of a renegotiation, unless it is made renegotiation-proof. The peculiarity of the renegotiation game, in which the partners engage, resides in the strategic behaviour of the firm. While choosing the up-front level of investment, the firm takes into account the impact that choice will have on the cost of switching to an alternative operator for the government. The more costly the replacement is, the more prone the government will be to accept a renegotiation and, hence, the more the firm will be motivated to seek it.

**Overview of our findings** We first show that in situations in which a renegotiation takes place, the firm either under-invests or over-invests, relative to a hypothetical complete contracting environment. In an extended acceptation, we shall say that this mirrors the presence of "moral hazard," using this expression to refer to any situation in which the firm has incentives to choose a sub-optimal level of investment, be it below or above the optimal one. Which of the two outcomes arises, depends on how costly it would be for the government to replace the firm while the infrastructure is still being constructed.

Not surprisingly, our analysis tells that, as long as the cost of operation is observed publicly and, hence, no further incentive issue arises after the construction of the infrastructure is completed, the government has an interest in offering the firm a renegotiationproof contract, namely, a contract which the firm will not seek to renegotiate in midconstruction. We highlight that the benefit to this is that, in a contract which is not expected to be renegotiated, the government is able to use the contractual compensation to the firm as an incentive tool to induce some desirable level of investment. By contrast, in a contract which can be subjected to a renegotiation, this possibility is foregone. We further highlight that the way in which moral hazard inter-plays with the renegotiation issue in the optimal contractual design, differs according to whether the firm has an incentive to under- or over-invest. In the former case, the government is interested in inducing more investment, and concedes more to the firm than the payoff the firm would obtain in a renegotiation. The level of investment induced through the contract is, yet, below the efficient level, which would be attained in the absence of incentive problems. Indeed, the marginal cost of the investment facing the government exceeds the marginal "technological" cost of the investment and, hence, the marginal rent to the firm is positive, at optimum. When the firm has an incentive to over-invest instead, the government would like the firm to economize on the investment, and affords to concede less to the firm than it would obtain in a renegotiation. Interestingly, in this case, the government finds it less costly to address moral hazard and prevent a renegotiation jointly, rather than only addressing the renegotiation issue. The level of investment induced through the contract is, yet, above the efficient level, provided the marginal cost of the investment facing the government is now below the marginal technological cost and, hence, the marginal rent is negative, at optimum. In either case, the investment is enhanced up to a point where, at the margin, the efficiency gain is exactly offset by the cost of incentivizing the firm to depart from the investment strategy it would follow in a contract which is not renegotiation-proof.

Private information about the cost of operation on the firm's side affects the strategies and attainments of both partners. On the one hand, the firm can act in a renegotiation process in such a way as to take advantage of the information to be learnt when operation begins. Of course, in a contract which can be subjected to a renegotiation, this in turn influences the firm's investment strategy early on in the relationship. On the other hand, when designing the renegotiation-proof contract the government must account not only for the early incentives to under/over-invest, but also for the later incentives to misrepresent private information. Besides, given that a renegotiation would take place in mid-construction (if any), these latter incentives are relevant not only in the initial contract but also in a renegotiated deal. Interestingly, in this environment, adverse selection may lead the government to turn down the renegotiation-proof contract and welcome a renegotiation. This preference is somewhat unusual, and we identify three possible reasons for it. First, the government prefers a renegotiation when adverse selection is more severe than moral hazard in the initial contract, less severe under firm-led renegotiation. In this case, a renegotiation-proof contract is too costly for the government, in that the firm should be conceded both a rent to be willing to release information, and the equivalent of the renegotiation payoff to be willing to comply with the contract. By contrast, in a renegotiated deal, the payoff of the firm is sufficiently high that the information is elicited without the need to concede a rent for that. Whereas the renegotiation-proof contract triggers a more efficient investment, the renegotiated deal does not require inducing output distortions to contain the information rent. Second, the government also prefers a renegotiation when adverse selection is more severe than the renegotiation problem, and less severe than moral hazard. In this situation, the government must compensate the firm with an information rent, even if a renegotiation is allowed for. However, by doing so, the government can pass the cost of construction completion onto the firm, whereas this is not the case in the renegotiation-proof contract. Third, when adverse selection is more severe than any of the other two problems, the preference of the government for more or less contractual flexibility depends on how costly adverse selection is in the two options.

As a concluding step, we provide a sketchy extension of the model, which permits to account for the capital structure of the project. With this, we clarify that the government can instruct the firm to run the project with such an amount of debt and equity that breakeven is attained in operation, and the limited liability constraints (if any) are slack in the renegotiation-proof contract. This result points to the conclusion that limited liability on the firm's side is not an additional issue to bind the contractual design, together with those previously discussed.

**Relation with the literature** This paper is first related to the literature on the holdup problem, as pioneered by GHM, from which it emerges that whereas under-investment is the more obvious outcome, over-investment may well occur, in turn, when contractual relationships are repeated over time (Guriev and Kvasov [12]). In our setting, rather than being due to the contractual renewal over time, over-investment (if any) follows from the intrinsic characteristics of the project. As a contribution to the literature, we identify the consequences for the optimal contractual design in both situations with under-investment and with over-investment.

The issue of renegotiation in PPPs and its consequences for the contractual design, have been largely investigated. Iossa and Martimort [15] consider a setting in which contracting is incomplete because the operating conditions are uncertain when the partners sign the contract. Other studies, which we previously mentioned, refer to situations in which the renegotiation comes as a decision of the public authorities, who pursue other purposes than the very efficiency of the project. In such studies as Laffont [14], Guasch *et al.* [10] and Danau and Vinella [4] - [5], renegotiation is an issue because of the weakness of the enforcing institutions and mechanisms in the economy. In this respect, the present work is closer to Iossa and Martimort [15], who consider an environment in which the state of nature is non-verifiable. Unlike those authors, we focus on projects in which the construction of the infrastructure takes two stages, and non-verifiability concerns the cost of completion, *i.e.*, the second stage of construction. Importantly, this entails that non-verifiability comes to matter before any return is derived from the project. Consequently, in our framework, a renegotiation would take place in mid-construction, which is not the case in any of the studies aforementioned.

Our paper is more broadly related to the literature on renegotiation-proof contracts, which was initiated by Dewatripont [6] and spans on a variety of topics, ranging from investment and firm's liquidation (Quadrini [22]) to implementation problems with time dimension (Rubinstein and Wolinsky [24]), to mention only a few.<sup>5</sup> In our framework, a

<sup>&</sup>lt;sup>5</sup>See also Bolton [2] for a discussion on the basic ideas about renegotiation-proof contracts in environments with symmetric but non-verifiable information and in environments with asymmetric information.

renegotiation-proof contract is not necessarily the paramount option for the contractual designer. As a contribution to this strand of literature, we identify situations in which the PPP designer is not best off with a renegotiation-proof contract, and characterize the optimal contract in those situations.

#### 1.1 Outline

The reminder of the paper is organized as follows. Section 2 describes the model. Section 3 presents three benchmarks. First, contracting is complete and the cost of operation is commonly known. Second, the replacement of the firm is costless to the government but contracting is incomplete. Third, contracting is complete but the cost of operation is privately observed by the firm. In the two subsequent sections, the optimal contract is characterized departing from any of these scenarios. In section 4, the focus is on a case where the cost of construction completion is non-verifiable; in section 5, in addition, the cost of operation is taken to be privately observed by the firm. Section 6 provides a sketchy extension of the model to discuss the financial aspect of the project. Section 7 briefly concludes. Mathematical details are relegated to an appendix.

### 2 The model

A government (G) delegates the development of a public project to a private firm (F). The project includes two tasks, namely, the construction of an infrastructure and the supply of a good (or service) to society. F is a Special Purpose Vehicle created by a group of private investors to perform these tasks. The contract between G and F is signed at the beginning of period 0, and the project lasts three periods, namely, 0, 1, 2. There is no discounting between periods.

**Period** 0: *early construction* F begins to build the infrastructure. To perform the basic construction, a physical cost of  $I_0$  is incurred. For simplicity, we set  $I_0 = 0$ . In addition, to make the infrastructure functional in the later operation, an investment (effort) of e is made, which also renders the infrastructure more specific to the builder. Because e is non-observable, it cannot be contracted upon.

**Period 1:** construction completion The construction of the infrastructure is completed. In addition to a physical cost of  $I_1$ , which is again set to 0, for simplicity, this entails an operalisation cost, which depends on the investment made in period 0. Specifically, this cost is given by k(e), where  $k'(\cdot) > 0$  and  $k''(\cdot) > 0$ . That is, the initial investment makes the infrastructure more costly to complete; this effect is more pro-

The author further provides an illustration on the relevance to long-term debt.

nounced the higher the investment is.<sup>6</sup> In the event that the partnership between G and F is terminated at the beginning of period 1, the cost of construction completion amounts to  $(1 + \alpha) k(e)$ , instead of k(e), for some  $\alpha > 0$ . That is, whereas the cost of completion increases with the initial investment regardless of the specific builder, the increase in the cost is nonetheless higher for builders other than F. The cost  $\alpha k(\cdot)$  of switching to an alternative firm should be understood in a broad sense. It may capture, say, a delay in the realization of the project, and/or a limited availability of alternative operators.

**Period 2:** operation and compensation The infrastructure is used to provide the good to society. The provision of a unit of the good has a cost of  $\theta$ . G compensates F for the activity with a transfer of t. Consumption of q units of the good yields a gross surplus of S(q), where  $S'(\cdot) > 0$  and  $S''(\cdot) < 0$ .

It is common knowledge that  $\theta$  will take one of the two values  $(\underline{\theta}, \overline{\theta})$ , such that  $\overline{\theta} > \underline{\theta}$ , with respective probabilities  $\nu(e)$  and  $1 - \nu(e)$ , where  $\nu'(\cdot) > 0$  and  $\nu''(\cdot) < 0.7$  These properties capture the circumstance that more investment in early construction makes a lower cost more likely in operation, although this effect is less pronounced for higher levels of investment. Assume that  $q^* \equiv \underset{q}{argmax} \{S(q) - \theta q\}$  is unique for each value of  $\theta$ . Denote  $q^*(\underline{\theta}) \equiv \underline{q}^*$  and  $q^*(\overline{\theta}) \equiv \overline{q}^*$ , where  $\underline{q}^* > \overline{q}^*$ , of course. Further let  $\underline{w} = S(\underline{q}^*) - \underline{\theta}\underline{q}^*$ ,  $\overline{w} = S(\overline{q}^*) - \overline{\theta}\overline{q}^*$ , and  $\Delta w = \underline{w} - \overline{w} > 0$ , for shortness.

In the event that the firm abandons the project before operation starts, the probability of the cost of operation being low (resp., high) is equal to  $(1 - \beta) \nu(e)$  (resp.,  $1 - (1 - \beta) \nu(e)$ ), for some  $\beta \in (0, 1)$ . That is, due to specialization, the synergy between project stages is weaker if a firm, other than the one which invested up-front, runs the activity thereafter.

In the event that G reneges on the contract, and replaces F with another firm after F has made any investment in the project, G incurs a penalty of P. We take P to be sufficiently high that this situation does not actually occur. That is, G has incentives to renege on the contract neither in mid-construction nor once the infrastructure is in place.

Social values The net social value of the project, when carried on by F, amounts to

$$W(e) = \overline{w} + \nu(e) \Delta w - (e + k(e)).$$
<sup>(1)</sup>

<sup>&</sup>lt;sup>6</sup>Martimort and Straub [17] assume that in the first period of the project (construction) the firm exerts an effort, which affects the production function in the second period (operation). Specifically, once an infrastructure of given characteristics is in place, the firm must ensure a minimum follow-up level of investment/maintenance.

<sup>&</sup>lt;sup>7</sup>The assumption that the values  $\theta$  can take are known *ex ante* is suitable to represent projects in relatively mature sectors and/or delegation phases, for some past experience exists to inform the parties as to the conditions under which the relationship will unfold in the future. See Iossa and Martimort [15], who argue in the same direction to capture situations in which a productivity shock to affect the activity in the future can be foreseen *ex ante*, thus allowing for more complete contracting.

This includes the surplus  $\overline{w} + v(e) \Delta w$ , which G expects to obtain from the management of the infrastructure, net of the total cost of investment e + k(e). The social surplus, which is derived if F itself completes the infrastructure, is given by the economic value of the relationship being continued beyond period 1, net of the cost of investment, namely

$$\beta\nu\left(e\right)\Delta w + \alpha k(e) - e.$$
(2)

The economic value of the continuation of the PPP includes, first, the additional surplus of  $\beta\nu(e) \Delta w$ , which G expects to obtain from operation, if F is still around in that stage of the project. Second, it includes the additional cost of  $\alpha k(e)$ , which G saves as she<sup>8</sup> avoids switching to another operator in construction. As we will see, the sum  $\beta\nu(e) \Delta w + \alpha k(e)$ is, in fact, the gross surplus G and F would share, should the contract be renegotiated. To rule out any unnecessary complications, we take (1) and (2) to be both concave with respect to e and to admit, each, an interior maximum.

**The contract** G addresses a *take-it-or-leave-it* contractual offer to F. The offer consists in a menu of allocations  $\{(\underline{q}, \underline{t}), (\overline{q}, \overline{t})\}$ , each of which is associated with one of the two possible realizations of  $\theta$ . The expected payoff of F from the entire project is given by  $\Pi - (e + k(e))$ , where  $\Pi = \mathbb{E}[t - \theta q]$  is the expected profit from operation, with  $\mathbb{E}$ denoting the expectation operator over the realizations of  $\theta$ . The operational profits of F are respectively given by  $\underline{\pi} = \underline{t} - \underline{\theta}\underline{q}$  and  $\overline{\pi} = \overline{t} - \overline{\theta}\overline{q}$  in the good (low-cost) and bad (high-cost) state of nature. We can thus write  $\underline{t} = \underline{\pi} + \underline{\theta}\underline{q}$  and  $\overline{t} = \overline{\pi} + \overline{\theta}\overline{q}$  so that

$$\Pi = \nu(e) \underline{\pi} + (1 - \nu(e)) \overline{\pi}.$$

**Timing** The relationship between G and F unfolds as follows. In period 0, G offers a take-it-or-leave-it contractual offer to F. If the offer is rejected, then the game is over. If the offer is accepted, then G and F become partners to a PPP. F invests e and begins to construct the infrastructure. In period 1, both partners learn that the cost of construction completion is either k(e) or  $(1 + \alpha) k(e)$ , depending on whether or not F continues the project. In period 2, the infrastructure is in place and  $\theta$  is realized. The operator in place uses the infrastructure to provide the good to society and is compensated by G, depending on the realization of  $\theta$ . In the first part of the analysis, we take this realization to be publicly observable, entailing that the contractual variables can be conditioned on it. In a later stage, we will allow for  $\theta$  to be either observable but non-verifiable or privately observed by the firm.

 $<sup>^{8}</sup>$ All throughout, to avoid confusion, we use the pronoun "she" with reference to the government and the pronoun "it" with reference to the firm.

# 3 Benchmarks

Before studying the optimal contracting in the environment of our interest, we briefly consider three useful benchmarks. In any of them, there is no reason to move away from the pair of efficient quantities  $\{\underline{q}^*, \overline{q}^*\}$ , and one does not need be concerned with the output choice.

#### 3.1 Publicly observed cost of construction completion

First suppose that  $k(\cdot)$  is observed publicly, hence it is contractible. Then, the investment e is contractible as well. G designs the contractual transfers in such a way that F breaks even in expectation, namely  $\Pi = e + k(e)$ . As the objective function of G is given by (1), with those transfers, G attains the payoff  $W(e^*) = \overline{w} + \nu(e^*)\Delta w - (e^* + k(e^*))$ , where  $e^*$  is the level of investment satisfying the following first-order condition with respect to e:

$$\nu'(e^*)\,\Delta w = 1 + k'(e^*).\tag{3}$$

Of course, the efficient level of investment is such that marginal benefit and marginal cost are equal.

#### **3.2** Costless replacement

Next suppose that the project would occasion no additional cost, if a different firm were to complete it in place of F. Formally, this is tantamount to having  $\alpha = \beta = 0$ . In this scenario, if F reneges on the contract and is replaced with a new firm, then G appropriates the investment previously made by F. Given this outcome, F has nothing to gain from reneging. Hence, clearly, F will prefer to abide by the contract. The investment is non-observable and F decides how much to invest in its own interest. The choice of F is the solution to the following problem:

$$\underset{e}{Max}\left\{ \Pi - \left(e + k\left(e\right)\right)\right\}.$$

With  $\nu''(\cdot) < 0$  and  $k''(\cdot) > 0$ , this problem is concave, and the solution is unique and characterized by the following first-order condition:

$$\underline{\pi} - \overline{\pi} \ge \frac{1 + k'(e)}{\nu'(e)}.\tag{4}$$

Henceforth, whenever useful, we refer to (4) as the *moral hazard constraint*, where "moral hazard" must be understood as any situation in which the firm may want to choose a sub-optimal level of investment, be it below or above the optimal one, as we said. In addition to ensuring that (4) is satisfied, G must also warrant that F does not lose money in expectation. It requires permitting F to recover the investments made to undertake

and complete the construction of the infrastructure, namely

$$\Pi \ge e + k\left(e\right). \tag{5}$$

This is the participation constraint of F. With both (4) and (5) being saturated, and taking into account that the level of investment chosen by F is  $e^*$ , transfers are designed to yield the following *ex-post* profits:

$$\underline{\pi}^* = e^* + k \left( e^* \right) + \left( 1 - \nu \left( e^* \right) \right) \frac{1 + k' \left( e^* \right)}{\nu' \left( e^* \right)}$$
$$\overline{\pi}^* = e^* + k \left( e^* \right) - \nu \left( e^* \right) \frac{1 + k' \left( e^* \right)}{\nu' \left( e^* \right)}.$$

In either state of nature, F receives a fixed amount, which permits the repayment of the costs previously incurred. This amount is augmented with a bonus in the low-cost state, and reduced with a penalty in the high-cost state, so that (4) is satisfied. With this contractual allocation, G attains the highest social surplus  $W(e^*)$ .

#### 3.3 Private observation of the cost of operation

As the theory suggests, F is likely to observe the cost of operation privately when it is realized. Let us now consider this case. The following adverse selection constraints must be satisfied in the contractual design:

$$\underline{\pi}\left(e\right) - \bar{\pi}\left(e\right) \ge \Delta \theta \overline{q} \tag{6}$$

$$\bar{\pi}\left(e\right) - \underline{\pi}\left(e\right) \ge \Delta\theta q. \tag{7}$$

The former is the constraint whereby a low-cost firm is not attracted by the allocation targeted to a high-cost firm; the converse is true with the latter constraint. It is straightforward to verify that efficiency is attained even under asymmetric information about  $\theta$ . However, when  $\overline{q}^* > \frac{1+k'(e^*)}{\nu'(e^*)\Delta\theta}$  so that (6) implies (4), which is thus slack, the efficient outcome is reached with a different set of *ex-post* profits, namely

$$\underline{\pi}^* = e^* + k \left( e^* \right) + \left( 1 - \nu \left( e^* \right) \right) \Delta \theta \overline{q}^*$$
$$\overline{\pi}^* = e^* + k \left( e^* \right) - \nu \left( e^* \right) \Delta \theta \overline{q}^*.$$

#### 4 Non-verifiable cost of construction completion

We now turn to consider a (more interesting) case where  $k(\cdot)$  is non-verifiable. This represents real-world situations in which the cost of the additional investment, to be made to complete construction, is intrinsically linked to the way in which the infrastructure is being built, and does not reflect well-defined investment obligations, say, related to the physical characteristics of the infrastructure. In this environment, in addition to the incentive issue concerning the up-front investment to be chosen by the firm, the contract may also be subjected to a renegotiation. We first present the payoffs obtained by the contractual parties in a hypothetical case where the contract is renegotiated, and determine how this affects the level of investment chosen by the firm. We next explore the optimal contractual design, taking into account that G anticipates the firm's strategy.

#### 4.1 Renegotiation payoffs and choice of investment

Suppose one party breaches the contract at the end of period 0, when the investment is sunk. In that case, the partners engage in a renegotiation. If the renegotiation succeeds and a new agreement is reached, then the partnership continues accordingly. If the renegotiation fails, then the partnership is terminated, and F is replaced with another firm (F'), which will complete the project in the place of F.

If the renegotiation succeeds, then the contractual parties agree on F being assigned an expected transfer of  $\mathbb{E}[t^{rn}]$ , where  $t^{rn} \in (\underline{t}^{rn}, \overline{t}^{rn})$ , depending on the unit cost to be realized in period 2. Provided the renegotiation takes place before the cost of construction completion k(e) is incurred (hence, also before the cost of operation  $\theta$  is learnt and incurred), the payoff of F amounts to  $\Pi^{rn} = \mathbb{E}[t^{rn}] - k(e) - \mathbb{E}[\theta q^{rn}]$ , where  $q^{rn} \in (\underline{q}^{rn}, \overline{q}^{rn})$  denotes the levels of output stipulated in the renegotiated contract. The payoff of G is given by  $V^{rn} = \mathbb{E}[S(q^{rn})] - \mathbb{E}[t^{rn}]$ . Therefore, there is a joint surplus of  $\mathbb{E}[S(q^{rn}) - \theta q^{rn}] - k(e)$  to be derived from the continuation of the partnership. Noticeably, because the investment of F is already sunk in the renegotiation stage, it is neglected in the game between the partners. Moreover, because the maximum joint surplus is attained when output is respectively set equal to  $\underline{q}^*$  and  $\overline{q}^*$  in the good and bad state, it is natural that the partners will agree on these levels of output, regardless of those stipulated in the initial contract. With this, the joint surplus amounts to

$$\overline{w} + \nu(e)\,\Delta w - k(e).\tag{8}$$

If the renegotiation fails and the partnership is broken down, then F is not compensated. Hence, the break-up payoff of F is equal to zero. Moreover, F does not recover the investment e. Because of this, a break-up would be undesirable to F. Instead, F might seek a renegotiation and renege on the contract. Suppose that, following a contractual renege, G does replace F with F'. The latter firm receives a *take-it-or-leave-it* contractual offer from G, which includes a menu of allocations  $\left\{ \left(\underline{q}_{F'}, \underline{t}_{F'}\right), \left(\overline{q}_{F'}, \overline{t}_{F'}\right) \right\}$  such that its expected profit is given by

$$\Pi_{F'} = \mathbb{E}_{F'} \left[ t_{F'} - \theta q_{F'} \right] - (1 + \alpha) k \left( e \right),$$

where  $\mathbb{E}_{F'}$  is the expectation operator over the possible realizations of  $\theta$  when F' supplies

the good, and so the set of probabilities is  $\{(1 - \beta) \nu(e), 1 - (1 - \beta) \nu(e)\}$ . The payoff of G amounts to  $W^b = \mathbb{E}_{F'} [S(q_{F'}^b) - t_{F'}^b]$ , where the superscript *b* stands for *b*reak-up. For F' to break even, the transfers must be such that  $\mathbb{E}_{F'} [t_{F'}^b] = (1 + \alpha) k(e) + \mathbb{E}_{F'} [\theta q_{F'}^b]$ . It is straightforward to see that G will recommend the efficient level of output from F' in either state. Thus, G will obtain

$$W^{b}(e) = \overline{w} + \nu(e) \Delta w - k(e) - (\beta \nu(e) \Delta w + \alpha k(e))$$
(9)

$$= W(e) + e - \left(\beta\nu\left(e\right)\Delta w + \alpha k(e)\right).$$
(10)

Actually, this is also the joint surplus in the event of a break-up, since F is left with zero profit in that case. By comparing (9) with (8), the net surplus, which is available when G and F reach a new deal and the relationship continues, is found to be

$$\beta \nu(e) \Delta w + \alpha k(e).$$

In substance, this expresses the economic value of the continuation of the partnership beyond period 1. Assuming that F expects to obtain a share  $\gamma \in (0, 1)$  of the renegotiation surplus, the payoffs that G and F attain in a renegotiation are specified as follows:

$$W^{rn}(e) = W^{b}(e) + (1 - \gamma) \left(\beta \nu \left(e\right) \Delta w + \alpha k(e)\right)$$
$$\Pi^{rn}(e) = \gamma \left(\beta \nu \left(e\right) \Delta w + \alpha k(e)\right).$$

Accounting for the investment made up-front, the net renegotiation payoff of F amounts to  $\Pi^{rn}(e) - e$ . If a renegotiation is foreseen, then F decides to invest  $e^{rn}$ , such that

$$\gamma \left(\beta \nu'\left(e^{rn}\right)\Delta w + \alpha k'(e^{rn})\right) = 1.$$
(11)

The left-hand side of (11) is the surplus F obtains in the renegotiation from the last unit of investment, the right-hand side is the cost incurred thereby. Accordingly, the payoff of G amounts to

$$W^{rn}(e^{rn}) = W^{b}(e^{rn}) + (1 - \gamma) \left(\beta \nu \left(e^{rn}\right) \Delta w + \alpha k(e^{rn})\right)$$
$$= W(e^{rn}) + e^{rn} - \gamma \left(\beta \nu \left(e^{rn}\right) \Delta w + \alpha k(e^{rn})\right).$$

A comparison with the benchmark, in which replacing F is costless to G, highlights the role that the cost of construction completion plays with respect to the choice of the investment made by the firm. Interestingly, it triggers opposite effects in the two scenarios. In the benchmark, the higher the marginal cost  $k'(\cdot)$  is the lower the optimal investment  $e^*$  is. In the renegotiation scenario, the higher the marginal cost  $k'(\cdot)$  is the more surplus is available for the partners to share, and the more F is prone to invest. The main implication is that, at odds with a standard hold-up problem, a renegotiation may not result in under-investment.

**Proposition 1** Suppose F anticipates that the contract will be renegotiated. Then,  $e^{rn} \leq e^*$  if and only if

$$k'(e^*) \le \frac{1 - \gamma\beta}{\gamma(\alpha + \beta)}.$$
(12)

In substance, there are two possible consequences to a renegotiation. On the one hand, as in the familiar GHM setting, G can take advantage of the investment made by F, without sharing the cost of it, since that cost is foregone in the renegotiation stage. Anticipating this, F is induced to economize on the investment up-front. However, whereas in the GHM setting the surplus to be shared in a renegotiation is independent of the foregone investment, here the available surplus is linked to it through the cost of completion  $k(\cdot)$ , which F is still to incur when the contract is renegotiated. More precisely, the surplus *increases* with  $k(\cdot)$ , thus triggering the "opposite" effect previously described. It follows that if the marginal cost of completion is sufficiently low when the investment is  $e^*$ , *i.e.*, one more unit of investment induces only a little increase in surplus, then the former (hold-up) effect prevails, and the usual under-investment problem arises:  $e^{rn} < e^*$ . If the marginal cost of completion is high, instead, when the investment is  $e^*$ , *i.e.*, one more unit of investment induces a significant increase in surplus, then the latter effect prevails, and the firm is led to over-invest:  $e^{rn} > e^*$ . Noticeably, over-investment is more likely to occur the more of the available surplus F obtains in a renegotiation (i.e.,the higher  $\gamma$  is), and the cheaper the construction completion and the operation are as long as F is in charge of them (*i.e.*, the higher  $\alpha$  and  $\beta$  are).

An important observation is here in order. Whereas it is clear that F benefits from a higher investment when the marginal cost of construction completion is high, it is less straightforward to draw conclusions as regards G. On the one hand, a higher investment makes the replacement of F more costly to G, who is thus somehow held up in the relationship. On the other hand, a higher investment has a positive impact on the shareable surplus, which makes a renegotiation more convenient to G. All in all, given the link between the investment and the renegotiation surplus, it is unclear whether G prefers to enforce  $e^*$  or, rather, to let over- or under-investment occur. This issue is explored hereafter.

#### 4.2 Renegotiation-proof contract

Suppose G would like to prevent a renegotiation by offering a *renegotiation-proof* contract. To characterize a contract with this feature, it must be taken into account that, as previously seen, F can choose the level of investment strategically in period 0, foreseeing the possibility of reneging on the contract and, hence, causing G to return to the contracting table in period 1.

#### 4.2.1 Moral hazard and renegotiation proofness

Let us begin by considering a case where in period 0 F does not anticipate that it would renege on the contract in the future. Then, F chooses the level of e satisfying (4), which depends on the set of stipulated profits  $\{\underline{\pi}^{rp}, \overline{\pi}^{rp}\}$ , where the index rp stands for renegotiation-proofness. The expected operating profit of F is given by  $\Pi^{rp}(e) =$  $\nu(e) \underline{\pi}^{rp} + (1 - \nu(e)) \overline{\pi}^{rp}$ , if the initial contract is executed. The expected profit net of the cost of construction completion is  $\Pi^{rn}(e)$ , if a renegotiation takes place, instead. Let us next consider a case where in period 0 F anticipates that it would renege on the contract in period 1. Then, the relevant payoffs are  $\Pi^{rp}(e^{rn})$  (instead of  $\Pi^{rp}(e)$ ) and  $\Pi^{rn}(e^{rn})$  (instead of  $\Pi^{rn}(e)$ ). Given the set of strategies available to F, namely  $\{e, e^{rn}\}$ , a renegotiation is not an equilibrium outcome if and only if the following renegotiation-proofness constraint is satisfied:

$$\Pi^{rp}\left(\widetilde{e}\right) - k(\widetilde{e}) \ge \Pi^{rn}\left(\widetilde{e}\right), \,\forall \widetilde{e} \in \left\{e, e^{rn}\right\}.$$
(13)

In this condition, the expected operating profit of F in the renegotiation-proof contract is diminished by the cost of construction completion, whereas the expected renegotiation payoff is not. This is because a renegotiation (if any) would take place before the construction is completed, and the cost of construction completion is implicitly embodied in the payoff of F as determined by the parties in the renegotiation process.

Inspection of (13) further highlights that the incentives of F in the choice of the level of investment inter-play with the temptation of returning to the contracting table with G. In short, this can be labeled as the inter-play between moral hazard and renegotiation in the programme of G. To the root of this is that the expected operating profit  $\Pi^{rp}(\tilde{e})$ depends not only on the investment  $\tilde{e}$  chosen by F, but also on the contractual variables  $\underline{\pi}^{rp}$  and  $\overline{\pi}^{rp}$ . As we said, these are set by G in such a way that the moral hazard constraint is satisfied and, hence, F is motivated to provide a certain level of investment e. In turn, the renegotiation payoff  $\Pi^{rn}(\tilde{e})$  depends on variables other than the contractual ones. In a renegotiation, G cannot use such tools as the *ex-post* profits to induce a certain level of investment. Thus, when a renegotiation is foreseen F chooses some investment  $e^{rn}$  being based on purely exogenous factors. Thus, the firm's investment strategy  $\tilde{e}$  is not neutral with respect to the possibility of a contractual renegotiation.

To offer a formal view on the inter-play between moral hazard and renegotiation, we begin by showing that it is both necessary and sufficient that (13) holds with  $\tilde{e} = e^{rn}$  for the contract to be renegotiation-proof. A clue on necessity has already been provided. Sufficiency follows because e is the optimal choice on the equilibrium path, namely, with (13) being satisfied, whereas  $e^{rn}$  is the optimal choice off the equilibrium path. As a result, (13) is tighter with strategy  $e^{rn}$  than with strategy e.

Lemma 1 The contract is renegotiation-proof if and only if

$$\Pi^{rp}(e^{rn}) \ge \Pi^{rn}(e^{rn}) + k(e^{rn}).$$
(14)

There are two consequences to Lemma 1. First, for the contract to be renegotiationproof, G must concede a rent to F. To see this, consider that (4) is satisfied only if the profits  $\{\underline{\pi}^{rp}, \overline{\pi}^{rp}\}$  are set such that

$$\Pi^{rp}(e) - (e + k(e)) \ge \Pi^{rp}(e^{rn}) - (e^{rn} + k(e^{rn})).$$

This condition holds as an equality if and only if  $e^{rn}$  is the level of investment that G wishes to attain through the contract. This is a very particular case, though. Taken together with (14), the above condition yields

$$\Pi^{rp}(e) \ge e + k(e) + \Pi^{rn}(e^{rn}) - e^{rn}$$

This tells that F cannot be motivated to abide by the contract unless, in the operation stage, it is allowed to recover the investment and the cost of construction completion (namely, e + k(e)) and, in addition, it is also given up the surplus (net of the cost of construction completion) it would obtain in a renegotiation (namely,  $\Pi^{rn}(e^{rn}) - e^{rn}$ ). The second and more interesting consequence to Lemma 1, to be formalized in Lemma 2 below, is that the compensation assigned to F to trigger a desirable choice of the investment, is either above or below the renegotiation payoff. The reason is that by choosing  $e^{rn}$  instead of e, F will either reduce or raise its benefit in the initial contract, relative to the renegotiated one. As long as  $e > e^{rn}$ , the distribution  $\{\nu(e), 1 - \nu(e)\}$  dominates the distribution  $\{\nu(e^{rn}), 1 - \nu(e^{rn})\}$  in the sense of first-order stochastic dominance, and so  $\Pi^{rp}(e) > \Pi^{rp}(e^{rn})$ . This shows that motivating F to make a suitable investment occasions an additional cost to G, on top of that due to the threat of a renegotiation. The converse is true when  $e < e^{rn}$ , in which case G affords to downsize the compensation to F below its renegotiation payoff. That is, it is now cheaper for G to address the moral hazard issue together with the renegotiation issue.

**Lemma 2** The expected profit of F in the renegotiation-proof contract is given by

$$\Pi^{rp}(e) = \gamma \left(\beta \nu \left(e^{rn}\right) \Delta w + \alpha k(e^{rn})\right) + k(e^{rn}) + \left(\nu \left(e\right) - \nu \left(e^{rn}\right)\right) \frac{1 + k'(e)}{\nu'(e)}.$$
 (15)

The value that the expected profit  $\Pi^{rp}(e)$  takes in the optimal contract results from the need to account for the two issues at once. First, to make a renegotiation unattractive to F, G must concede what F would obtain in that event, namely  $\gamma (\beta \nu (e^{rn}) \Delta w + \alpha k(e^{rn}))$ . Second, G must reimburse  $k(e^{rn})$  to F, provided a renegotiation would occur before completing the construction of the infrastructure. The additional term

$$(\nu(e) - \nu(e^{rn})) \frac{1 + k'(e)}{\nu'(e)},$$

which reflects the impact of moral hazard, either augments or diminishes the contractual

payoff of F. Let us interpret this term. Fixed the virtual operating profit  $\Pi^{rp}(e^{rn})$  in such a way that a renegotiation is prevented, G also concedes a reward of  $(1 - \nu (e^{rn})) \frac{1+k'(e)}{\nu'(e)}$ in the good state, and imposes a punishment of  $\nu (e^{rn}) \frac{1+k'(e)}{\nu'(e)}$  in the bad state, in order to motivate F to choose a desirable level of investment. Because the reward and the punishment are drawn based on the distribution  $\{\nu (e), 1 - \nu (e)\}$ , the expected value of the incentive component of the payoff is either positive or negative, depending on whether or not that distribution dominates the (virtual) distribution  $\{\nu (e^{rn}), 1 - \nu (e^{rn})\}$  in a stochastic sense. It is immediate to see that, in a limit case where  $e = e^{rn}$ , the expected value of the incentive compensation would be zero, as in the benchmarks. By contrast, whenever  $e \neq e^{rn}$ , the divergence between real and virtual probability distribution induces an upward or downward shift in the payoff assigned to F for abiding by the contract.

#### 4.2.2 Trade-off between efficiency and cost of over/under-investment

We are now ready to consider the problem of G. Being based on Lemma 2, we see that the payoff of G is given by

$$W^{rp}(e) = \overline{w} + \nu(e) \Delta w - (\nu(e) - \nu(e^{rn})) \frac{1 + k'(e)}{\nu'(e)} - \left[\gamma(\beta\nu(e^{rn}) \Delta w + \alpha k(e^{rn})) + k(e^{rn})\right].$$

The level of investment which maximizes  $W^{rp}(e)$ , denoted  $e^{**}$ , is such that

$$\nu'(e^{**})\Delta w = 1 + k'(e^{**}) + (\nu(e^{**}) - \nu(e^{rn}))\left(\frac{1 + k'(e^{**})}{\nu'(e^{**})}\right)'.$$
(16)

At optimum, the marginal gross surplus drawn by G from the project (the left-hand side of (16)) equals the marginal cost of the investment facing G (the right hand side of (16)). The latter includes the marginal "technological" cost of the investment  $(1+k'(e^{**}))$ and, in addition, the marginal rent to  $F\left(\left(\nu\left(e^{**}\right)-\nu\left(e^{rn}\right)\right)\left(\frac{1+k'(e^{**})}{\nu'(e^{**})}\right)'\right)$ . This is the change in the firm's rent associated with a change in the wedge between the *ex-post* profits, which follows from a marginal raise in the investment, and reflects the need to attenuate moral hazard in an environment where a renegotiation must be prevented as well. When F has an incentive to under-invest, the rent is increased to boost the investment up to a level of  $e^{**}$ , which is, yet, below the efficient level  $e^*$ . Indeed, in this case, the marginal cost of the investment facing G exceeds the marginal technological cost of the investment, at optimum, so that the marginal rent is positive. By contrast, when F has an incentive to over-invest, the rent is deflated to squeeze the investment down to a level of  $e^{**}$ , which is, yet, above the efficient level  $e^*$ . Indeed, now the marginal cost of the investment facing G is below the marginal technological cost of the investment, at optimum, so that the marginal rent is negative instead.

By reformulating (16) as

$$\nu'(e^{**})\,\Delta w - (1 + k'(e^{**})) = (\nu(e^{**}) - \nu(e^{rn})) \left(\frac{1 + k'(e^{**})}{\nu'(e^{**})}\right)',\tag{17}$$

one can interpret the optimal investment  $e^{**}$  as reflecting a trade-off between the net benefit to G of bringing the investment closer  $e^*$  (the left-hand side) and the cost to G of motivating F to depart from  $e^{rn}$  (the right-hand side).

To understand the trade-off, first take  $e^{rn} < e^*$ . In this case, G has an interest in incentivizing F to invest more than  $e^{rn}$ , but less than  $e^*$ . Indeed, at  $e^{**}$  G obtains a marginal gain in surplus of

$$\nu'(e^{**})\,\Delta w - (1 + k'(e^{**})) > 0,$$

which is above the marginal gain at  $e^*$  and so  $e^{**} < e^*$ . The reason for this distortion is that inducing F to invest more than  $e^{rn}$  occasions a cost, whose marginal value is

$$(\nu(e^{**}) - \nu(e^{rn})) \left(\frac{1 + k'(e^{**})}{\nu'(e^{**})}\right)' > 0,$$

at optimum. This says that there is an increase in the rent of F. Indeed, to induce F to invest more, G must offer F a higher reward in the good state ( $\pi^{rp}$  is increased) and impose a higher punishment in the bad state ( $\pi^{rp}$  is lowered). Furthermore, with a higher investment, the probability of F being rewarded becomes higher, and the probability of F being punished becomes lower. With a higher reward being given more often and a higher punishment being given less often, there is an increase in the expected value of the lottery of profits facing F. To contain this cost, G induces under-investment ( $e^{**} < e^*$ ).

Next take  $e^{rn} > e^*$ . In this case, somewhat less standard in the literature, F is prone to over-invest, if it does not receive any incentive payment. To increase the (net) surplus, the investment must be downsized below  $e^{rn}$ . That is, the marginal surplus must be higher (less negative) than in  $e^{rn}$ . In the level of investment picked by G, the marginal gain in surplus is given by

$$-\left[\nu'\left(e^{**}\right)\Delta w - \left(1 + k'\left(e^{**}\right)\right)\right] > 0$$

so that  $e^{**}$  is now above the efficient level  $e^*$ . This is because, once again, there is a cost coming along with the surplus gain. The presence of a cost is perhaps less intuitive here. Indeed, as F is induced to economize on the investment, G can afford to shrink the wedge between the *ex-post* profits by reducing both the reward and the punishment ( $\underline{\pi}^{rp}$  is lowered and  $\overline{\pi}^{rp}$  is raised), in compliance with (4). However, with a lower investment, the probability of F being rewarded becomes lower, and the probability of F being punished becomes higher. The fact that a lower reward is given less often and a lower punishment

is given more often, represents a penalty for the government. In marginal terms, the penalty facing G amounts to

$$-\left(\nu\left(e^{**}\right) - \nu\left(e^{rn}\right)\right) \left(\frac{1 + k'\left(e^{**}\right)}{\nu'\left(e^{**}\right)}\right)' > 0,$$

at optimum. Trading off the benefit of bringing the investment closer to its efficient level against the penalty faced to downsize the investment below  $e^{rn}$ , G permits over-investment  $(e^{**} > e^*)$ .

**Proposition 2** The optimal (second-best) investment  $e^{**}$  in the renegotiation-proof contract is such that  $e^{**} \in (e^{rn}, e^*)$  if  $e^{rn} < e^*$ , and such that  $e^{**} \in (e^*, e^{rn})$  if  $e^{rn} > e^*$ .

This result is important, in that it tells that the government herself prefers to compensate the firm so as to trigger under-investment, if the firm would under-invest in the absence of the incentive payment; and to trigger over-investment, if the firm would overinvest in the absence of the incentive payment. Being based on the above explanation, one also gains an intuition about the next result. It says that the ultimate benefit of a renegotiation-proof contract is precisely that G can use the contractual compensation as a tool to incentivize F to choose the desirable level of investment  $e^{**}$ , whereas this would not be possible if a contractual renegotiation were foreseen, in which case F would set the investment to  $e^{rn}$ .

**Corollary 1** With a renegotiation-proof contract, G obtains a net surplus of

$$\frac{\nu(e^{**}) - \nu(e^{rn})}{\nu'(e^{**})} \left[\nu'(e^{**}) \Delta w - (1 + k'(e^{**}))\right].$$
(18)

The conclusion that the government prefers to assign incentive payments to the firm such that the contract is renegotiation-proof, is in line with the previous findings of the literature and, hence, not surprising *per se*. However, this conclusion does not need carry over in all possible situations. We will see that G may display a different preference when F observes the state of nature (the realization of  $\theta$ ) privately.

# 5 Private observation of the cost of operation

In the third benchmark, we saw that, just as moral hazard, also private observation of the cost of operation is costless to G, as long as the contract is complete and no renegotiation is foreseen. We will now consider a case where  $k(\cdot)$  is non-verifiable and, in addition, the firm learns the realization of  $\theta$  privately. This is an interesting case, in that not only moral hazard but also adverse selection inter-plays with the renegotiation issue, with consequences for the strategies and attainments of the partners.

#### 5.1 Private observation and renegotiation

Proceeding as before, we begin by considering a situation in which the contract stipulated by the partners is not executed and a renegotiation game is played. To identify the renegotiation payoffs, it is now necessary to account for the adverse selection constraints. For some given  $\tilde{e}$  previously chosen by F, the partners should agree on *ex-post* profits, denoted  $\underline{\pi}^{rn,1}(\tilde{e})$  and  $\bar{\pi}^{rn,1}(\tilde{e})$ , such that

$$\underline{\pi}^{rn,1}\left(\widetilde{e}\right) - \bar{\pi}^{rn,1}\left(\widetilde{e}\right) \ge \Delta\theta \overline{q}^{rn},\tag{19}$$

$$\underline{\pi}^{rn,1}\left(\widetilde{e}\right) - \bar{\pi}^{rn,1}\left(\widetilde{e}\right) \le \Delta \theta \underline{q}^{rn}.$$
(20)

The pair  $\{\underline{q}^{rn}, \overline{q}^{rn}\}$  is, in turn, the set of quantities agreed upon in the renegotiation process here considered, which are still to be determined. For (19) and (20) to hold, and considering that F would not operate if it were  $\overline{\pi}^{rn,1}(\widetilde{e}) < 0$ , F must be assigned an expected amount of at least  $\nu(\widetilde{e}) \Delta \theta \overline{q}^{rn}$  to be willing to reveal the state of nature. Moreover, in a hypothetical case where the partnership is broken down, G must elicit information from the new firm, which learns the realization of  $\theta$  privately when completing the construction of the infrastructure. Accordingly, G must also satisfy the constraints

$$\underline{\pi}^{bk}\left(\widetilde{e}\right) - \bar{\pi}^{bk}\left(\widetilde{e}\right) \ge \Delta\theta \overline{q}^{bk},\tag{21}$$

$$\underline{\pi}^{bk}\left(\widetilde{e}\right) - \bar{\pi}^{bk}\left(\widetilde{e}\right) \le \Delta \theta \underline{q}^{bk},\tag{22}$$

where  $\{\underline{\pi}^{bk}(\widetilde{e}), \overline{\pi}^{bk}(\widetilde{e})\}$  is the set of operating profits assigned to F' and  $\{\underline{q}^{bk}, \overline{q}^{bk}\}$  is the set of outputs recommended from F'.

The presence of the additional constraints makes the renegotiation game more complex. We first determine the surplus G would obtain if F were replaced with F'. Henceforth, we let  $\underline{w}(q) \equiv S(q) - \underline{\theta}q$  and  $\overline{w}(q) \equiv S(q) - \overline{\theta}q$ , for shortness, in the statement of the results.

Lemma 3 Suppose that the partnership is broken down. The payoff of G is given by

$$W^{b,1}(\widetilde{e}) = W(\widetilde{e}) + \widetilde{e} - B(\widetilde{e}),$$

where

$$B(\tilde{e}) \equiv \beta \nu (\tilde{e}) \Delta w + \alpha k(\tilde{e}) + (1 - \beta) \nu (\tilde{e}) \Delta \theta \overline{q}^{bk}(\tilde{e}) + [1 - (1 - \beta) \nu (\tilde{e})] (\overline{w} - \overline{w}(\overline{q}^{bk}(\tilde{e})))$$

and  $\overline{q}^{bk}(\widetilde{e})$  is such that

$$S'(\overline{q}^{bk}\left(\widetilde{e}\right)) = \overline{\theta} + \frac{(1-\beta)\nu\left(\widetilde{e}\right)}{1-(1-\beta)\nu\left(\widetilde{e}\right)}\Delta\theta.$$

The break-up payoff of G is now below  $W^b(\tilde{e})$ . The reason is twofold. First, output is distorted away from the efficient level in the high-cost state, which yields a reduction in the expected surplus equal to

$$\left[1 - (1 - \beta) \nu\left(\widetilde{e}\right)\right] \left(\overline{w} - \overline{w}(\overline{q}^{bk}\left(\widetilde{e}\right))\right).$$

Second, a rent is conceded to the firm in the low-cost state, which amounts to

$$(1-\beta)\nu(\widetilde{e})\Delta\theta\overline{q}^{bk}(\widetilde{e}).$$

The presence of constraints (19) and (20) also complicates the determination of the renegotiation payoffs. There are two situations to be considered. In one situation, it is not necessary to concede a rent in the renegotiated contract to elicit information. Then, the only change in the renegotiation payoffs is due to the additional cost G incurs in the event of a break-up, as we see from Lemma 3. In the other situation, F must receive an information rent in place of the usual renegotiation payoff, which further calls for inducing output distortions, as in the break-up scenario previously described. Thereby, the renegotiation payoff of G depends on how the cost of asymmetric information compares with the cost of a renegotiation.

Lemma 4 In a renegotiated contract:

(1) If  $\nu(e_1^{rn}) \Delta \theta \overline{q}^* - k(e_1^{rn}) \leq \gamma B(e_1^{rn}) \leq \nu(e_1^{rn}) \Delta \theta \underline{q}^* - k(e_1^{rn})$ , then the payoff of G is given by

$$W_1^{rn} = \overline{w} + \nu \left( e_1^{rn} \right) \Delta w - k(e_1^{rn}) - \gamma B(e_1^{rn}),$$

where  $e_1^{rn}$  is the investment strategy of F, such that  $\gamma B'(e_1^{rn}) = 1$ .

(2) If  $\gamma B(e_1^{rn}) < \nu(e_1^{rn}) \Delta \theta \overline{q}^* - k(e_1^{rn})$ , then the payoff of G is given by

$$W_2^{rn} = \overline{w}\left(\overline{q}_2^{rn}\right) + \nu\left(e_2^{rn}\right)\left(\underline{w} - \overline{w}\left(\overline{q}_2^{rn}\right)\right) - \nu\left(e_2^{rn}\right)\Delta\theta\overline{q}_2^{rn}$$

for some  $\overline{q}_2^{rn} \ge f(e_2^{rn})$ , where

$$S'(f(e_2^{rn})) = \overline{\theta} + \frac{\nu(e_2^{rn})}{1 - \nu(e_2^{rn})} \Delta\theta,$$

and  $e_2^{rn}$  is the investment strategy of F, such that  $q_2^{rn} = \frac{1+k'(e_2^{rn})}{\nu'(e_2^{rn})\Delta\theta}$ .

Not surprisingly, the renegotiation payoff of G in case (1) of Lemma 4 is composed as under symmetric information, except that now the cost of a break-up is higher due to the agency problem which would arise with the new firm. Because of this, F is able to obtain more by renegotiating the contract. In case (2) of the lemma, the renegotiation payoff of G is basically a standard payoff under adverse selection. Indeed, when (19) is sufficiently tight G cannot do better than letting F attain a payoff of  $\nu (e_2^{rn}) \Delta \theta \bar{q}_2^{rn}$ , which is the rent to be conceded for eliciting information. Obviously, F will choose its investment strategy depending on which of the cases (1) and (2) is the relevant one. In case (2), the partners could agree upon distorting output away from the efficient level  $f(e_2^{rn})$ . However, it is clear that none of the partners would have an interest in setting  $q_2^{rn} < f(e_2^{rn})$ . On the one hand, F would then obtain too low an information rent; on the other, G would face too high an efficiency loss. Therefore, it must be the case that  $q_2^{rn} \ge f(e_2^{rn})$ , *i.e.*, only upward distortions may arise.

# 5.2 Renegotiation-proof contract and the benefit of renegotiation

Besides the two possible renegotiation scenarios, adverse selection brings about further complexity to the renegotiation-proof contract. We will now investigate how the adverse selection constraints affect the contractual design, depending on which of the cases presented in Lemma 4 is relevant.

In case (1), the moral hazard problem can be more or less severe than the adverse selection problem. Formally, this depends on how tight the moral hazard constraint is relative to either of the two adverse selection constraints. When some adverse selection constraint is tighter output is likely to be distorted away from the efficient level so that production is less efficient in the original contract than in a renegotiated deal. Given this outcome, it cannot be taken for granted that G will still opt for a renegotiation-proof contract.

Analogous considerations must be made as regards case (2) of Lemma 4, except that, in that case, output is distorted away from the efficient level not only in the original contract but also in the renegotiated deal. To establish what option makes G better off, it is then necessary to compare the output distortions induced in the two contracts. Furthermore, in case (2), the cost of the construction completion is entirely borne by F, which then obtains an information rent rather than taking advantage of a renegotiated deal. This is an additional reason why G might prefer a renegotiated deal to a renegotiation-proof contract.

**Lemma 5** In the renegotiation-proof contract:

(a) In case (1) of Lemma 4, with  $\overline{q}^* \geq \frac{1+k'(e)}{\nu'(e)\Delta\theta}$ , G attains the payoff

$$W_1^{rp} = \overline{w} \left( \overline{q}_1^{SB} \right) + \nu \left( e_1 \right) \left( \underline{w} - \overline{w} \left( \overline{q}_1^{SB} \right) \right) - \left( \nu \left( e_1 \right) - \nu \left( e_1^{rn} \right) \right) \Delta \theta \overline{q}_1^{SB} - \left( \gamma B(e_1^{rn}) + k(e_1^{rn}) \right)$$

where

$$\overline{q}_{1}^{SB} = \frac{1+k'\left(e_{1}\right)}{\nu'\left(e_{1}\right)\Delta\theta}$$

and  $e_1$  is such that

$$S'\left(\frac{1+k'(e_1)}{\nu'(e_1)\,\Delta\theta}\right) = \overline{\theta} + \frac{\nu(e_1) - \nu(e^{rn,1})}{1-\nu(e_1)}\Delta\theta$$

(b) In case (2) of Lemma 4, with  $\overline{q}^* \leq \frac{1+k'(e_{2,1})}{\nu'(e_{2,1})\Delta\theta} \leq \underline{q}^*$ , G attains the payoff

$$W_{2,1}^{rp} = \overline{w} + \nu (e_{2,1}) \Delta w - (\nu (e_{2,1}) - \nu (e_2^{rn})) \frac{1 + k' (e_{2,1})}{\nu' (e_{2,1})} - (k (e_2^{rn})) + \nu (e_2^{rn})) \Delta \theta \overline{q}_2^{rn}),$$

where  $e_{2,1}$  is such that

$$\nu'(e_{2,1})\Delta w - (1 + k'(e_{2,1})) = (\nu(e_{2,1}) - \nu(e_2^{rn})) \left(\frac{1 + k'(e_{2,1})}{\nu'(e_{2,1})}\right)'$$

(c) In case (2) of Lemma 4, with  $\overline{q}^* \geq \frac{1+k'(e_{2,1})}{\nu'(e_{2,1})\Delta\theta}$ , G attains the payoff

$$W_{2,2}^{rp} = \overline{w} + \nu \left( e_{2,2} \right) \Delta w - \left( 1 - \nu \left( e_{2,2} \right) \right) \left( \overline{w} - \overline{w}(\overline{q}_2^{SB}) \right) - \left( \nu \left( e_{2,2} \right) - \nu \left( e_2^{rn} \right) \right) \Delta \theta \overline{q}_2^{SB} - \left( k \left( e_2^{rn} \right) + \nu \left( e_2^{rn} \right) \Delta \theta \overline{q}_2^{rn} \right),$$

where

$$\overline{q}_{2}^{SB} = \frac{1 + k'\left(e_{2,2}\right)}{\nu'\left(e_{2,2}\right)\Delta\theta}$$

and  $e_{2,2}$  is such that

$$S'\left(\frac{1+k'(e_{2,2})}{\nu'(e_{2,2})\,\Delta\theta}\right) = \overline{\theta} + \frac{\nu(e_{2,2}) - \nu(e_2^{n})}{1-\nu(e_{2,2})}\Delta\theta.$$

In principle, either adverse selection constraint could be binding in the programme of G. However, for simplicity, in the lemma we restricted attention to situations in which only (19) is binding. Specifically, case (a) arises when the constraint is binding in the initial contract and slack in the renegotiated deal; case (b) arises when the constraint is slack in the initial contract and binding in the renegotiated deal; case (c) arises when the constraint is binding in either contract. The case where the constraint is slack in either contract was neglected, as it would boil down to the symmetric information case already seen, with the only difference that here the payoff of G with a break-up would be lower, as we explained.

In case (a), in addition to the renegotiation payoff  $\gamma B(e_1^{rn}) + k(e_1^{rn})$ , G must give up an information rent of  $\Delta \theta \overline{q}_1$  to F. The output of the inefficient type is distorted downwards to contain this rent. In cases (b) and (c), G must concede a profit of  $k(e_2^{rn}) + \nu(e_2^{rn}) \Delta \theta \overline{q}_2^{rn}$ to F, if she wants to prevent a renegotiation. In case (b), not surprisingly, the payoff of F in the renegotiation-proof contract is composed as under symmetric information, provided only the moral hazard constraint is binding, whereas the adverse selection constraints are both slack in the contract. In case (c), the occurrence of an output distortion reflects the need for G of conceding a rent to elicit information from F. In any of these situations, in which the adverse selection constraint whereby the efficient type is unwilling to lie is binding in at least one of the initial and the renegotiated contract, it is not necessarily the case that G is better off with a renegotiation-proof contract.

**Proposition 3** G prefers the renegotiation-proof contract to the renegotiated contract if and only if

$$\left(\nu\left(e_{1}\right)-\nu\left(e_{1}^{rn}\right)\right)\left(\Delta w-\Delta\theta\overline{q}_{1}^{SB}\right)\geq\left(1-\nu\left(e_{1}\right)\right)\left(\overline{w}-\overline{w}\left(\overline{q}_{1}^{SB}\right)\right),$$

in case (a) of Lemma 5; if and only if

$$\left(\nu\left(e_{2,1}\right) - \nu\left(e_{2}^{rn}\right)\right) \left(\Delta w - \frac{1 + k'\left(e_{2,1}\right)}{\nu'\left(e_{2,1}\right)}\right) + \left(1 - \nu\left(e_{2}^{rn}\right)\right)\left(\overline{w} - \overline{w}\left(\overline{q}_{2}^{rn}\right)\right) \ge k\left(e_{2}^{rn}\right),$$

in case (b) of Lemma 5; if and only if

$$(\nu (e_{2,2}) - \nu (e_2^{rn})) \left( \Delta w - \Delta \theta \overline{q}_2^{SB} \right) + (1 - \nu (e_2^{rn})) \left( \overline{w} - \overline{w} (\overline{q}_2^{rn}) \right) \\ \geq k (e_2^{rn}) + (1 - \nu (e_{2,2})) \left( \overline{w} - \overline{w} (\overline{q}_2^{SB}) \right),$$

in case (c) of Lemma 5.

In case (a), with  $e_1 > e_1^{rn}$ , G benefits from a renegotiation-proof contract in that she obtains a bonus of  $\Delta w - \Delta \theta \bar{q}_1^{SB}$  when a good (rather than a bad) state is realized. Relative to the renegotiated contract, G faces an efficiency loss though, since the adverse selection constraint is weak in that contract, and the partners agree on setting output efficiently in either state. The loss to G reflects the fact that F must receive an amount equal to the renegotiation payoff also in the renegotiation-proof contract, and that amount is sufficiently high to costlessly eliminate any incentives of the firm to misrepresent information in the renegotiated contract. Remarkably, if it is  $e_1 < e_1^{rn}$  instead, then there is no point for G in designing a renegotiation-proof contract.

In case (b), making the contract robust to renegotiation is advantageous to G in that, again, a more efficient investment  $(e_{2,1})$  is attained. Furthermore, because adverse selection imposes no restriction in the renegotiation-proof contract, output does not need be distorted away from the efficient level. Nonetheless, there is a cost to G in offering F a renegotiation-proof contract. Indeed, as long as the partners abide by that contract, the cost of construction completion is ultimately borne by the government. By contrast, if the partners return to the contracting table, then that cost is passed onto the firm, which then receives a share of the surplus obtained through the renegotiation instead of

an incentive payment from the government. Because of this, G may find it convenient to allow for a renegotiation.

Case (c) blends together the advantages and disadvantages of a renegotiation-proof contract presented in cases (a) and (b). On the one hand, with  $e_{2,2} > e_2^{rn}$ , G obtains a bonus of  $\Delta w - \Delta \theta \bar{q}_2^{SB}$  through the increase in the likelihood of the good (rather than the bad) state being realized, as in case (a). On the other hand, still as in case (a), output is distorted away from the efficient level in the bad state of nature, whereas no distortion is agreed upon in a renegotiation. Furthermore, by ruling out a return to the contracting table with F, G renounces to the possibility of passing the cost  $k(\cdot)$  onto F, as in case (b). The joint work of these different effects may motivate G to welcome a new negotiation once the initial investment of the firm is sunk.

# 6 Capital structure and limited liability

All throughout not only the expected profit but also the *ex-post* profits of F were taken to be non-negative. This strategy, which is tantamount to introducing limited liability on the firm's side, in addition to the participation constraint, was functional to ruling out a break-up of the partnership. One way of ensuring that the *ex-post* profits are nonnegative, is to instruct the firm to invest enough own funds in the project up-front so that there is a sufficiently high cash flow to accrue to the firm when operating. This suggests that debt finance should not be used massively to launch a PPP project. In fact, as will become apparent in a moment, it is advisable that the early construction of the infrastructure be exclusively financed with equity, and no debt be taken to that end.

For the purpose of this section, we assume that the project can be financed with a mix of equity, debt and public funds, and that the mix of financing sources is contractible. Because the construction of the infrastructure takes two stages and contracting is incomplete, a sensible hypothesis is that a credit, to be denoted C, is awarded to the firm in two tranches. The second tranche is issued conditional on F continuing the activity and the partners committing to repay the debt in the renegotiated contract. As long as the contract becomes complete in the renegotiation stage, the amount of the second tranche of credit is irrelevant. On the other hand, an issue arises with the first tranche of credit, conceded in period 0 to undertake the construction of the infrastructure. Indeed, should F abandon the project after investing e, then there would be no point for G in repaying the debt. One might expect this to enhance the position of G in a renegotiation, as G could save on debt repayment by replacing F with another firm. However, this is not necessarily true since, for the reasons previously explained, G does not welcome a renegotiation and is ready to pay for the investment made early on, regardless of whether it is entirely financed with the firm's own funds or borrowed funds are used as well.

In this environment, the renegotiation game between G and F is altered as follows. If the partnership is broken down, then F loses the investment e but is left with the credit C, and prefers a break-up to a renegotiation if

$$-e + C \ge \widehat{\Pi}^{rn}(e)$$

where  $\widehat{\Pi}^{rn}(e)$  denotes the renegotiation payoff of F in this context. The fact that F has to repay a debt, if it remains in activity, is relevant in the renegotiation process, provided this process takes place before the payment to the creditor is made. The renegotiation payoff of F is now specified as

$$\Pi^{rn} = \mathbb{E}\left[t^{rn}\right] - \left(k(e) + \mathbb{E}\left[\theta q^{rn}\right] + D\right),$$

where D denotes the debt. Both the joint surplus from the continuation of the partnership after a renegotiation and the surplus from a renegotiation are here reduced by the amount of the debt. They are respectively given by

$$\overline{w} + \nu\left(e\right)\Delta w - \left(k(e) + D\right)$$

and

$$\beta \nu(e) \Delta w + \alpha k(e) - D.$$

Accordingly, the renegotiation payoffs of G and F are formulated as follows:

$$W^{rn}(e) = W^{b}(e) + (1 - \gamma) \left(\beta \nu \left(e\right) \Delta w + \alpha k(e) - D\right)$$
$$\Pi^{rn}(e) = \gamma \left(\beta \nu \left(e\right) \Delta w + \alpha k(e) - D\right).$$

The firm prefers a renegotiation to a break-up if and only if

$$\beta\nu\left(e\right)\Delta w + \alpha k(e) \ge D.$$

When this condition is violated, it is better for F to leave the project and save on debt repayment. First suppose that D is high enough for this to be the case. F prefers the initial contract to a break-up of the partnership, if and only if the following no-break-up constraint holds:

$$\Pi^{rp}\left(e\right) - k(e) \ge D.$$

This constraint is tighter than (14). Next suppose that D is low enough for F to prefer a renegotiation to a break-up. The renegotiation-proofness constraint is written as

$$\Pi^{rp}(e) - (k(e) + D) \ge \gamma \left(\beta \nu(e) \,\Delta w + \alpha k(e) - D\right)$$

or, equivalently, as

$$\Pi^{rp}(e) - k(e) \ge \gamma \left(\beta \nu(e) \,\Delta w + \alpha k(e)\right) + (1 - \gamma) \,D.$$

Clearly, this constraint is tighter than (14). It is also easy to see that, if F uses enough equity in the project, then any issue with limited liability is eliminated. One can thus conclude that the optimal financial structure of the project is such that the early construction rests on equity finance only. There should be enough equity for the initial investment to be recouped and, in addition, for the limited liability constraints to be satisfied, when relevant.

## 7 Concluding remarks

The concerns with under/over-investment and with firm-led renegotiation in midconstruction are not novel to PPP projects, although hitherto the literature has mainly focused on the under-investment problem.<sup>9</sup> Our analysis reveals how the inter-play between the incentive to under/over-invest and that to renegotiate in mid-construction affects the contractual design. In particular, when the firm is keen on over-investing in order to raise its stake in a renegotiation, and it would be too costly for the government to prevent that behaviour through the contract, the best strategy for the government may be to purposely induce an "optimal" amount of over-investment, in equilibrium. This amount was found to reflect the trade-off between the benefit associated with the investment coming closer to the efficient level (the one which would be attained in the absence of incentive problems) and the cost of incentivizing the firm to deviate from its otherwise optimal investment strategy (the one which would yield the highest payoff in a renegotiation).

There is currently a lively debate among scholars and practitioners about the opportunity of introducing flexibility in PPP contracting. The argument, which is mainly put forward (though not yet underpinned by a formal analysis), is that PPPs are long-term and characterized by significant uncertainty in the early stages. In light of this, it looks desirable to preserve the possibility of adjusting contracts when the uncertainty is solved and the environmental conditions become known. Our study suggests that it might be optimal to let PPPs be flexible even within a standard hold-up framework, in which the partners foresee the consequences of their actions (in our setting, the investment in early construction) but the realized state of nature is non-verifiable. Particularly, projects in which the principal might deliberately opt for contractual flexibility were found to be those dogged not only by incentives to invest inefficiently and to return to the contracting table in mid-construction, but also by incentives to misrepresent private information when operation begins. As informational asymmetries seem to be prominent in PPP projects, exactly as they are in most forms of delegated public service provision,<sup>10</sup> our

 $<sup>^{9}</sup>$ An exception is given by Martimort and Straub [17], who identify over-investment in PPP projects with irreversible investments. Also, in Saavedra [25] over-investment is found to arise when a renegotiation is foreseen and the additional benefit the firm expects to obtain, if it invests more, is higher than the additional monetary cost it incurs by doing so.

<sup>&</sup>lt;sup>10</sup>See, for instance, Marty and Voisin [18] for a discussion on the government's informational deficit in

finding offers one theoretical foundation for the pervasiveness of contractual renegotiation in the early stages of PPPs.

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PPPs, and some tools potentially suitable for reducing it. Among theoretical studies, Iossa and Martimort [15] explore the costs and benefits of PPPs in the presence of a variety of problems, including asymmetric information. Iossa and Martimort [16] further model a PPP in which the contractor's revenues are affected by an exhogenous shock, the realization of which is learnt by the private partner after contracting. Private information about the cost of operation, as represented in this study, is also accounted for in Danau and Vinella [4] - [5].

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# Appendix

#### **Proof of Proposition 1**

Denote  $f(e) \equiv \gamma \left(\beta \nu'(e) \Delta w + \alpha k'(e)\right) - 1$ . Under (11),  $f(e^{rn}) = 0$ . Moreover, because the function  $\beta \nu (e) \Delta w + \alpha k(e)$  is concave in e, it is f'(e) < 0,  $\forall e$ . Hence,  $e^{rn} \leq e^*$  if and

only if  $f(e^*) \leq f(e^{rn}) = 0$ . Using (3), we can write

$$\nu'\left(e^*\right) = \frac{1 + k'(e^*)}{\Delta w}.$$

Replacing in the expression of  $f(e^*)$  and rearranging, we obtain

$$f(e^*) = \beta + (\alpha + \beta) k'(e^*) - \frac{1}{\gamma}$$

Therefore, it is  $f(e^*) \leq 0$  (and so  $e^{rn} \leq e^*$ ) if and only if (12) is satisfied.

#### Proof of Lemma 1

*Necessity.* In period 1, F chooses  $max \{\Pi^{rp}(\tilde{e}) - k(\tilde{e}), \Pi^{rn}(\tilde{e})\}$ . In period 0, F decides to invest

$$\widehat{e} = \operatorname{argmax} \left\{ \max \left\{ \operatorname{max} \left\{ \Pi^{rp} \left( \widetilde{e} \right) - k \left( \widetilde{e} \right), \Pi^{rn} \left( \widetilde{e} \right) \right\} - \widetilde{e} \right\} \right\}$$

Suppose that  $\tilde{e}$  is such that  $\Pi^{rp}(\tilde{e}) - k(\tilde{e}) < \Pi^{rn}(\tilde{e})$ . Then, F chooses  $\hat{e} = e^{rn} \equiv argmax \{\Pi^{rn}(\tilde{e}) - \tilde{e}\}$ . Since  $\Pi^{rp}(e^{rn}) - k(e^{rn}) < \Pi^{rn}(e^{rn})$ , F will want to renegotiate. Thus, a renegotiation is not prevented unless (14) is satisfied.

Sufficiency. Provided that (14) holds, the contract is not renegotiated if F chooses  $e^{rn}$  in period 0. Suppose F anticipates that the contract will not be renegotiated, and that it will obtain  $\Pi^{rp}(e) - k(e)$ . Then, given the set  $\{\underline{\pi}^{rp}, \overline{\pi}^{rp}\}$ , F chooses  $\hat{e} = e_1 \equiv argmax \{\Pi^{rp}(e) - (e + k(e))\}$ . F will not want to renegotiate, indeed, if

$$\Pi^{rp}(e_1) - k(e_1) \ge \Pi^{rn}(e_1).$$
(23)

Let us show that (23) is implied by (14). To that end, we respectively rewrite these conditions as

$$\Pi^{rp}(e^{rn}) - (e^{rn} + k(e^{rn})) \ge \Pi^{rn}(e^{rn}) - e^{rn}$$
(24)

and

$$\Pi^{rp}(e_1) - (e_1 + k(e_1)) \ge \Pi^{rn}(e_1) - e_1.$$
(25)

By the definition of  $e^{rn}$  and  $e_1$ , one has  $\Pi^{rn}(e^{rn}) - e^{rn} > \Pi^{rn}(e_1) - e_1$  and  $\Pi^{rp}(e_1) - (e_1 + k(e_1)) > \Pi^{rp}(e^{rn}) - (e^{rn} + k(e^{rn}))$ . Hence, (24) implies (25). Lastly, F prefers  $e_1$  to  $e^{rn}$  in period 0 if

$$\Pi^{rp}(e_1) - (e_1 + k(e_1)) \ge \Pi^{rn}(e^{rn}) - e^{rn}.$$
(26)

Taken together with  $\Pi^{rp}(e_1) - (e_1 + k(e_1)) > \Pi^{rp}(e^{rn}) - (e^{rn} + k(e^{rn})),$  (24) implies (26).

#### Proof of Lemma 2

The value function of G decreases with  $\Pi^{rp}(e)$ . Hence, for any given  $\overline{\pi}^{rp}$  such that (13) holds, G sets  $\underline{\pi}^{rp}$  in such a way that (4) is binding:

$$\underline{\pi}^{rp} = \overline{\pi}^{rp} + \frac{1 + k'(e)}{\nu'(e)}.$$

Accordingly, (13) is reformulated as

$$\overline{\pi}^{rp} \ge \gamma \left(\beta \nu \left(e^{rn}\right) \Delta w + \alpha k(e^{rn})\right) + k(e^{rn}) - \nu \left(e^{rn}\right) \frac{1 + k'\left(e\right)}{\nu'\left(e\right)},$$

and is binding as well. One has

$$\overline{\pi}^{rp} = \gamma \left(\beta \nu \left(e^{rn}\right) \Delta w + \alpha k(e^{rn})\right) + k(e^{rn}) - \nu \left(e^{rn}\right) \frac{1 + k'(e)}{\nu'(e)},\\ \underline{\pi}^{rp} = \gamma \left(\beta \nu \left(e^{rn}\right) \Delta w + \alpha k(e^{rn})\right) + k(e^{rn}) + (1 - \nu \left(e^{rn}\right)) \frac{1 + k'(e)}{\nu'(e)},$$

from which (15) is derived.

#### **Proof of Proposition 2**

If  $e^{**} > e^{rn}$ , then the right-hand side of (17) is positive; hence, the left-hand side must be positive as well, implying that  $e^{**} < e^*$ . Overall, it must be the case that  $e^{rn} < e^{**} < e^*$ . Following a similar reasoning, the converse is found to be true if  $e^{**} < e^{rn}$ , in which case  $e^{rn} > e^{**} > e^*$ .

#### Proof of Corollary 1

Recalling

$$W^{rn}(e^{rn}) = \overline{w} + \nu(e^{rn})\Delta w - k(e^{rn}) - \gamma(\beta\nu(e^{rn})\Delta w + \alpha k(e^{rn}))$$

and

$$W^{rp}(e^{**}) = \overline{w} + \nu (e^{**}) \Delta w - (\nu (e^{**}) - \nu (e^{rn})) \frac{1 + k' (e^{**})}{\nu' (e^{**})} - [\gamma (\beta \nu (e^{rn}) \Delta w + \alpha k(e^{rn})) + k(e^{rn})],$$

we see that  $W^{rp}(e^{**}) \ge W^{rn}(e^{rn})$  if and only if (18) holds.

#### Proof of Lemma 3

Given the investment  $\tilde{e}$  made by F in period 0, G expects to face a good state with probability  $(1 - \beta) \nu(\tilde{e})$ . Because F' observes the state privately, G must concede a rent of  $(1 - \beta) \nu(\tilde{e}) \Delta \theta \bar{q}^{bk}(\tilde{e})$ . Hence, G obtains a payoff of

$$\overline{w}(\overline{q}^{bk}(\widetilde{e})) + (1-\beta)\nu(\widetilde{e})\left(\underline{w}(\underline{q}^{bk}) - \overline{w}(\overline{q}^{bk})\right) - (1+\alpha)k(\widetilde{e}) - (1-\beta)\nu(\widetilde{e})\Delta\theta\overline{q}^{bk}.$$

Maximization of this function with respect to  $\underline{q}^{bk}$  and  $\overline{q}^{bk}$  yields the set of optimal outputs  $\{q^*, \overline{q}^{bk}(\widetilde{e})\}$ . Thus, the optimized payoff of G is  $W^{b,1}(\widetilde{e})$ .

#### Proof of Lemma 4

For this proof, we let

$$\Gamma\left(\widetilde{e}, \underline{q}^{rn,1}, \overline{q}^{rn,1}\right) \equiv B(\widetilde{e}) - \nu\left(\widetilde{e}\right) \left(\underline{w}(\underline{q}^{*}) - \underline{w}(\underline{q}^{rn,1})\right) - \left(1 - \nu\left(\widetilde{e}\right)\right) \left(\overline{w}(\overline{q}^{*}) - \overline{w}(\overline{q}^{rn,1})\right)$$

be the surplus to be shared in a renegotiation when the set of outputs  $\{\underline{q}^{rn,1}, \overline{q}^{rn,1}\}$  is not necessarily identical to the set  $\{q^*, \overline{q}^*\}$ .

(1) Suppose that (19) and (20) are both slack. The partners agree on the set  $\{\underline{q}^*, \overline{q}^*\}$  so as to share  $\Gamma\left(\tilde{e}, \underline{q}^*, \overline{q}^*\right) = B(\tilde{e})$  in the renegotiation. F obtains an expected profit of  $\gamma B(\tilde{e})$ , net of the cost  $k(\tilde{e})$ , which is still to be incurred after a successful renegotiation. That is, F obtains

$$\mathbb{E}\left[t^{rn,1}\right] - \mathbb{E}\left[\theta q^{rn,1}\right] - k(\tilde{e}) = \gamma B(\tilde{e}).$$

Moreover,  $\mathbb{E}[t^{rn,1}] - \mathbb{E}[\theta q^{rn,1}] = \mathbb{E}[\pi^{rn,1}]$ , which is the expected operating profit. Hence, the expected operating profit in the renegotiation is such that

$$\mathbb{E}\left[\pi^{rn,1}\right] - k(\widetilde{e}) = \gamma B(\widetilde{e}).$$

Regrouping (19) and (20) as  $\nu(e_1^{rn}) \Delta \theta \overline{q}^* \leq \mathbb{E}[\pi^{rn,1}] \leq \nu(e_1^{rn}) \Delta \theta \underline{q}^*$ , we can further write

$$\nu\left(e_{1}^{rn}\right)\Delta\theta\overline{q}^{*}-k(e_{1}^{rn})\leq\mathbb{E}\left[\pi^{rn,1}\right]\leq\nu\left(e_{1}^{rn}\right)\Delta\theta\underline{q}^{*}-k(e_{1}^{rn})$$

(2) Being based on the proof of (1), one deduces that (19) is binding so that  $\mathbb{E}[\pi^{rn,1}] = \nu(\tilde{e}) \Delta \theta \bar{q}^{rn,1}$ , where  $\bar{q}^{rn,1}$  is still to be determined. The payoffs of G and F are respectively given by

$$\overline{w}\left(\overline{q}^{rn,1}\right) + \nu\left(\widetilde{e}\right)\left(\underline{w}(\underline{q}^{rn,1}) - \overline{w}\left(\overline{q}^{rn,1}\right)\right) - \nu\left(\widetilde{e}\right)\Delta\theta\overline{q}^{rn,1}$$
(27)

and

$$\nu\left(\widetilde{e}\right)\Delta\theta\overline{q}^{rn,1} - \left(\widetilde{e} + k(\widetilde{e})\right).$$

We see that  $\underline{q}^{rn,1} = \underline{q}^*$ , since the payoff of F is independent of  $\underline{q}^{rn,1}$ , and the payoff of G attains the maximum in this output level. Furthermore, whereas F would like  $\overline{q}^{rn,1}$  to be as great as possible, in order to obtain a higher rent, G prefers to set it to  $f(\tilde{e}) < \overline{q}^*$ , where

$$S'(f(\widetilde{e})) = \overline{\theta} + \frac{\nu(\widetilde{e})}{1 - \nu(\widetilde{e})} \Delta \theta.$$

Therefore,  $\overline{q}^{rn,1} = q_2^{rn}$ , for some  $q_2^{rn} \ge f(\widetilde{e})$ . F chooses

$$e_{2}^{rn} = argmax \left\{ \nu \left( e \right) \Delta \theta q_{2}^{rn} - \left( e + k(e) \right) \right\}$$

Plugging the optimal values in (27),  $W_2^{rn}$  is derived.

#### Proof of Lemma 5

(a) Because  $\overline{q}^* \geq \frac{1+k'(e)}{\nu'(e)\Delta\theta}$ , (6) is binding and (4) is slack. Proceeding as in the proof of Lemma 4, the payoff of G is written

$$W_1(e, \underline{q}, \overline{q}) \equiv \overline{w}(\overline{q}) + \nu(e) \left(\underline{w}(\underline{q}) - \overline{w}(\overline{q})\right) - \left(\nu(e) - \nu(e_1^{rn})\right) \Delta\theta\overline{q} - \left(\gamma B(e_1^{rn}) + k(e_1^{rn})\right).$$

For any given e, G chooses the output levels  $\underline{q}^*$  and g(e), where

$$S'(g(e)) = \overline{\theta} + \frac{\nu(e) - \nu(e^{rn,1})}{1 - \nu(e)} \Delta\theta.$$

We compute

$$\frac{dW_{1}(e,\underline{q}^{*},g\left(e\right))}{de} = \nu'\left(e\right)\left(\underline{w} - \overline{w}(g\left(e\right)) - \Delta\theta g\left(e\right)\right), \ \forall e,$$

with the terms including g'(e) disappearing under the envelope theorem. Because  $g(e) < \overline{q}^*$  and  $\Delta w > \Delta \theta \overline{q}^*$ , it is  $\frac{dW_1(e, \underline{q}^*, g(e))}{de} > 0$ . Therefore, the optimal investment  $e_1$  is such that (6) and (4) are both binding, namely

$$g(e_1) = \frac{1 + k'(e_1)}{\nu'(e_1) \Delta \theta}.$$

Denoting  $\overline{q}_{1}^{SB} = g(e_{1})$  and  $W_{1}^{rp} = W_{1}(e_{1}, \underline{q}^{*}, g(e_{1}))$ , the proof is completed.

(b) The renegotiation-proofness constraint is written as

$$\Pi^{rp}(e_2^{rn}) - k(e_2^{rn}) \ge \nu(e_2^{rn}) \Delta \theta \overline{q}_2^{rn}$$

Suppose that (6) and (7) are both slack. Then, (4) is binding. Proceeding as before, the payoff of F is found to be

$$\Pi^{rp}(e) = \nu(e_2^{rn})\Delta\theta \overline{q}_2^{rn} + k(e_2^{rn}) + (\nu(e) - \nu(e_2^{rn}))\frac{1 + k'(e)}{\nu'(e)}$$

The payoff of G is given by

$$W_{2,1}(e,\underline{q},\overline{q}) \equiv (1-\nu(e))\,\overline{w}\,(\overline{q}) + \nu(e)\,\underline{w}(\underline{q})$$

$$- \left(\nu(e) - \nu(e_2^{rn})\right)\frac{1+k'(e)}{\nu'(e)} - \left(k\left(e_2^{rn}\right)\right) + \nu\left(e_2^{rn}\right)\right)\Delta\theta\overline{q}_2^{rn} \,.$$

$$(28)$$

Maximizing  $W_{2,1}(e, \underline{q}, \overline{q})$  with respect to  $\underline{q}$  and  $\overline{q}$  yields  $\{\underline{q}^*, \overline{q}^*\}$ , and G obtains

$$W_{2,1}(e,\underline{q}^*,\overline{q}^*) = \overline{w} + \nu(e) \Delta w - (\nu(e) - \nu(e_2^{rn})) \frac{1 + k'(e)}{\nu'(e)} - (k(e_2^{rn})) + \nu(e_2^{rn})) \Delta \theta \overline{q}_2^{rn}).$$

Maximizing  $W_{2,1}(e, \underline{q}, \overline{q})$  with respect to e yields the optimal investment  $e_{2,1}$ , as characterized in the main text. The conditions under which (4) is binding, while (6) and (7) are both slack, are  $\overline{q}^* \leq \frac{1+k'(e_{2,1})}{\nu'(e_{2,1})\Delta\theta} \leq \underline{q}^*$ .  $W_{2,1}^{rp}$  is obtained by plugging the optimal values into  $W_{2,1}(e, q, \overline{q})$ .

(c) From the proof here above one deduces that (6) is binding. The payoff of F is

$$\Pi^{rp}\left(e\right) = k\left(e_{2}^{rn}\right) + \nu\left(e_{2}^{rn}\right)\Delta\theta\overline{q}_{2}^{rn} + \left(\nu\left(e\right) - \nu\left(e_{2}^{rn}\right)\right)\Delta\theta\overline{q},$$

whereas the payoff of G is

$$W_{2,2}(e,\underline{q},\overline{q}) \equiv (1-\nu(e))\,\overline{w}(\overline{q}) + \nu(e)\,\underline{w} - (\nu(e) - \nu(e_2^{rn}))\,\Delta\theta\overline{q} - (k(e_2^{rn}) + \nu(e_2^{rn})\,\Delta\theta\overline{q}_2^{rn})\,.$$

Maximization with respect to the quantities yields  $q^*$  and g(e), which is such that

$$S'(g(e)) = \overline{\theta} + \frac{\nu(e) - \nu(e_2^{rn})}{1 - \nu(e)} \Delta\theta.$$

We compute

$$\frac{dW_{2}(e,\underline{q}^{*},g\left(e\right))}{de} = \nu'\left(e\right)\left(\Delta w + \overline{w} - \overline{w}(g\left(e\right)) - \Delta\theta g\left(e\right)\right), \ \forall e,$$

with the terms including g'(e) disappearing under the envelope theorem. Because  $g(e) < \overline{q}^*$  and  $\Delta w > \Delta \theta \overline{q}^*$ , it is  $\frac{dW_2(e,\underline{q}^*,g(e))}{de} > 0$ . The optimal investment  $e_{2,2}$  is then such that (4) and (6) are both binding, namely

$$g(e_{2,2}) = \frac{1 + k'(e_{2,2})}{\nu'(e_{2,2})\,\Delta\theta},$$

and the optimal output is  $\overline{q}_2^{SB} = g(e_{2,2})$ . The condition  $g(e_{2,2}) \leq \overline{q}^*$  is satisfied because g'(e) < 0 and  $\left(\frac{1+k'(e)}{\nu'(e)}\right)' > 0$  so that

$$g(e_{2,2}) = \frac{1+k'(e_{2,2})}{\nu'(e_{2,2})\,\Delta\theta} < \frac{1+k'(e_{2,1}))}{\nu'(e_{2,1})\,\Delta\theta} < \overline{q}^*.$$

The optimized payoff of G amounts to  $W_{2,2}^{rp} = W_2(e_{2,2}, \underline{q}^*, \overline{q}_2^{SB}).$