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Trade Tariff, Wage Gap and Public Spending

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Abstract

This paper studies the interplay between the wage gap and government spending in a small open economy facing a shock in trade policy. We consider a specific factor model with an export sector, which uses skilled labour, and an import-competing sector, which uses unskilled labour. We find the conditions under which there exists an inverse (direct) relation between trade liberalization (protection), which increases (decreases) the skilled-unskilled wage gap, and the level of government expenditure. We also show how either an unbalanced distribution of political bargaining power, or tariff revenue co-financing public spending may break this direct relation.

JEL-Codes: F150, F160, H500.

Keywords: wage gap, trade liberalization, positive political economy.

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1 Introduction

Recent scenarios in international trade policies have been characterized by treaties and actual measures of markets closure through unilateral increases in import tariffs. A central issue for voters and taxpayers, still unsatisfactorily addressed in the economic theory debate, is whether such policies lead to either a higher or a lower taxation and public spending. In order to focus on this issue, one needs to understand first how trade policies influence the redistributive conflicts among taxpayers and second how such changes contribute in determining government equilibrium policies.

Having this in mind, this paper proposes a new theoretical framework that deals with this topic, the interplay between wage gap and government spending in a small open economy facing either a process of trade liberalization or an increase in trade protection. The general aim of this analysis is to explore the link between tariff policies and the size of public spending for public goods. Therefore, we develop a model where public spending is a result of a collective decision-making mechanism where the equilibrium is a compromise among voters’ conflicting interests.

We consider an economy with two sectors: an export sector, which uses capital and skilled labour, and an import-competing sector, which uses capital and unskilled labour. Trade liberalization in the import-competing sector increases the skilled-unskilled wage gap as it increases the skilled and decreases the unskilled wages. The opposite happens following an increase in trade tariffs. We find the conditions under which there exists an inverse (direct) relation between trade liberalization (protection) and the level of government expenditure. Results also show how either an unbalanced distribution of political bargaining power, or tariff revenue co-financing public spending may break this direct relation.

Within the international trade literature various frameworks have been used to analyze the effects of trade liberalization on the wage gap between skilled and unskilled workers. They start from the traditional Heckscher and Ohlin mechanism highlighting that trade liberalization increases the skill premium in countries having a comparative advantage in skill intensive sectors due to a relatively larger endowment of skilled workers. Then, the evidence reported after episodes of trade liberalization for several unskilled-workers abundant countries for which the skill premium has increased, in contrast with the Heckscher and Ohlin prediction,\(^1\) has prompted the development of alternative approaches to explain the impact of trade liberalization on the wage gap. The re-

\(^1\)See Goldberg and Pavcnick (2007) for a survey.
cent developments of the trade literature, including the new trade theory with and without firm heterogeneity explaining the large amount of trade flows generated between relatively similar countries, have allowed to uncover a collection of new channels through which trade liberalization can affect and, generally, increase income inequality.\footnote{Harrison, McLaren and McMillan (2011) draw up a survey on a wide number of such mechanisms complemented in the literature review in Di Comite, Nocco and Oreﬁce (2018).}

In our political economy framework, workers are also voters. Taking this into account, we capture the conﬂicting redistributive interests of skilled and unskilled workers-voters by allowing them to elect their representatives who are responsible of negotiating the size of public spending for public goods of the government. In order to model political negotiation we use a standard Nash bargaining approach,\footnote{See Giuranno (2009, 2010) and Giuranno and Biswas (2019).} whose explanatory power is supported by empirical studies.\footnote{According to Stokman and Thomson (2004), Thomson et al. (2006), Schneider et al. (2006), Hertz and Leuﬀen (2010) and Giuranno and Biswas (2019) cooperative bargaining models are able to predict policy formation in modern democracies better than non-cooperative negotiation approaches. See Muthoo (1999) for a complete treatment of this issue.}

Furthermore, we take into account that in contemporary representative democracies, public policies are not decided solely on the basis of the preferences of the majority group in the society as this would produce abrupt counter-reactions of minorities.\footnote{See, for instance, Antonio de Viti de Marco’s theory of the State (Giuranno and Mosca, 2018). In fact, a non-cooperative approach adopted by the dominant group may lead to counter-reaction of the minority groups that, in turn, may be very costly for the majority.} For this reason, we assume that both groups of voters, the skilled and the unskilled, share government decision-making power inside the legislature. We treat the situations where one group holds full control over government decision making as a special or limit case. In the latter, policy outcome coincides with that of the national median voter when the most numerous group, usually the unskilled one, holds full decision making power. In that case, the representative of the unskilled voters-workers decides policy without taking into account both the preferences of the skilled voters and the tax burden imposed on them.

This paper contributes to a strand of literature that studies the relationship between inequality and public policy.\footnote{See Bourguignon (2018), Arachi, Giuranno and Profeta (2018), Guzi and Kahane (2018) and many others.} A good part of this literature explains the relationship between inequality and government size as dependent on the distance between the income of the national me-
median voter and the average income of the whole economy. Consolidated empirical evidence shows that the national median income is lower than the average income (Meltzer and Richard, 1981, 1983). Thus, a larger distance between the median and the mean income, which is usually associated with higher inequality, leads to a higher demand for public spending for the national median voter because his marginal cost for the provision of public goods is relatively lower the higher such distance. However, the relation between inequality and government spending leads to both different and richer results when conflicting interest groups have to cooperate in order to find mutually beneficial compromises for the provision of public policies. We show that equilibrium policy depends on the distances among the income of the representative of each group.

In this paper, therefore, we propose a new approach to explicitly link two types of literature: one focusing on the relationship between trade policy and wage inequality without considering the implications on public spending and the other one addressing the impact of inequality on public expenditure without taking into account the fact that the economy could be integrated in a global economy.

1.1 Related literature

This paper contributes to the economic literature on economic openness and government size. The existing literature provides ambiguous results about the relation between the level of international economic integration and the scope of government.\(^7\) Cameron (1978) find a positive empirical relation between trade liberalization and government size during the three decades following World War II, and Rodrik (1998) confirms the existence of a positive correlation between an economy’s exposure to international trade and the size of its government. According to Rodrik (1998), such positive relation is due to public spending needed to provide social insurance against the risks of external shocks, which increase with free-trade. Furthermore, Epifani and Gancia (2009) suggest that government size increases because the opening of international trade generates a positive fiscal externality as it lowers the domestic cost of taxation by shifting it abroad.

On the contrary, a number of empirical studies have shown that the relationship between trade liberalization and government size is often negative (Abizadeh, 2005; Benarroch and Pandey, 2008), which makes the stylised facts on this issue quite puzzling. Indeed, also Molana, Montagna and Violato (2011) focus on a narrower aspect of the relationship suggested by Rodrik (1998), that is that of the existence, in an individ-

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\(^7\)See, among others, Liberati (2007, 2013) and Epifani and Gancia (2009).
ual country, of a causal effect from trade-openness to government-size, and their "results question the universality of the compensation hypothesis - or, indeed, that of its rival efficiency hypothesis - and beg for a more careful scrutiny of both the theoretical processes underlying the link between the extent of openness to trade in a country and the relative size of its public sector, as well as the appropriateness of measurements which approximate economic globalisation and government-size."

Arawatari (2015) provides a first attempt to explain such empirical ambiguity by placing the emphasis on the capital-labor ratio changes determined by trade-policy. Arawatari policy predictions are in line with Meltzer and Richard’s reasoning. Government size increases when the capital-labor ratio increases and declines otherwise. In our paper, we highlight a mechanism different from that proposed by Arawatari (2015). We shift our attention to the redistributive conflict that occurs among workers that earn different incomes and which manifests itself politically because workers are at the same time both voters and tax payers who elect their representatives to act in the government. As a result, in our model, where the wage gap is a key determinant of government decision making, we find a number of results often in contrast with the predictions derived by Meltzer and Richard and the subsequent linked literature.

The paper proceeds as follows. Section two develops the economic environment and describes the relationship between trade policy and wages in a small open economy. Section three finds the government equilibrium outcome for a given trade policy. Section four investigates the consequences for public spending of changes in tariff policy when public spending for public goods provision is or is not cofinanced by tariff revenue. We also study the consequences of skilled and unskilled workers having different political influence in the government decision. Section five finds the conditions under which both trade policy and the distribution of political influence leads to efficiency in the provision of public goods. Section six concludes. The Appendixes contain derivations and proofs.  

2 The economy

We consider a small open economy producing two goods, an export good $X$ using skilled labour, $S$, and capital, $K$, and an import-competing good

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8 Arawatari (2015) uses a pre-election probabilistic voting model plugged into a standard Heckscher-Ohlin model.

9 A preliminary version of this paper appeared in POLIS Working Papers 181, Institute of Public Policy and Public Choice - POLIS.
using unskilled labour, L, and capital. Both goods are produced with a constant return to scale technology. The government protects the import-competing sector by imposing an "ad valorem" tariff, denoted by $t \in [0, 1]$. Flexible wages, denoted by $w_S$ and $w_L$ for skilled and unskilled workers respectively, and flexible rate of return to capital, $r$, coupled with perfectly competitive markets ensure full employment of labour and capital.\textsuperscript{10}

The small open economy cannot influence the world prices of these goods, which are respectively denoted by $P^W_X$ and $P^W_Y$. Prices are such that

$$P^W_X = a_{SX}w_S + a_{KX}r \quad (1)$$

$$(1 + t) P^W_Y = a_{LY}w_L + a_{KY}r \quad (2)$$

where, $a_{SX}$ ($a_{LY}$) denotes the requirement of skilled (unskilled) labour to produce one unit of the export (import-competing) good $X$ ($Y$); $a_{KX}$ ($a_{KY}$) denotes the requirement of capital to produce one unit of good $X$ ($Y$). Equations (1) and (2) indicate that the domestic producers earn zero profit due to perfect competition and free entry, which equalise price to unit cost. Notice that while the price of good $X$ is equal to that prevailing at the world level, the price of good $Y$ is $P_Y = (1 + t) P^W_Y$.

The following set of equations gives us the least-cost input choice:

$$a_{SX} = a_{SX} \left( \frac{w_S}{r} \right), a_{LY} = a_{LY} \left( \frac{w_L}{r} \right), a_{KX} = a_{KX} \left( \frac{w_S}{r} \right), a_{KY} = a_{KY} \left( \frac{w_L}{r} \right) \quad (3)$$

The full employment conditions ensure that:

$$S = a_{SX}X \quad (4)$$

and

$$L = a_{LY}Y \quad (5)$$

where, $X$ and $Y$, denote the output levels of good $X$ and $Y$ respectively; and the terms $S$ and $L$ denote fixed endowments of skilled and unskilled labour respectively. For what follows, we also assume $L \geq S$.

Finally, full capital employment is given by

$$K = a_{KX}X + a_{KY}Y \quad (6)$$

where, $K$ denotes fixed endowment of capital.

\textsuperscript{10}As in Topalova (2010) and in Acharyya and Kar (2013), in this specific factor model, capital is mobile between the two sectors and at the equilibrium its rate of return is equal in both sectors.
Given the world commodity prices and tariff rate, the above set of nine equations together determine three factor prices, four input choices, and two output levels. It is evident that the wages consistent with zero-profit and full-employment conditions vary with the trade policy choice as captured here by the tariff rate.\footnote{This is a typical specific factor model \textit{a’ la} Jones (1971), where the intersectoral mobile factor between the import competing sector and the export sector is capital as in Topalova (2010) and in Acharyya and Kar (2013).} Clearly, exogenous changes in the tariff rate would change the equilibrium value of wages. The above set of conditions can be used to derive the precise nature and magnitude of changes in the rate of return on capital and in wages due to changes in tariff rates, as described below (the proof is in Appendix A).

Indeed, the percentage change in the rate of return to capital is given by

\[ \hat{r} = \frac{\alpha_Y}{\alpha} \hat{T} \]  

(7)

In the above equation, and in the rest of the paper, the "hat" over a variable denotes its proportional change (e.g., \( \hat{r} = \frac{dr}{r} \), \( \hat{T} = (1 + t) \)),\footnote{Note that \( \hat{T} = \frac{dT}{1 + t} = \frac{dt}{1 + t} + t \).} and \( \alpha_Y \) and \( \alpha \) will be defined below.

Moreover, the change for the skilled wage is

\[ \hat{w}_S = -\frac{\theta_{KY}}{\theta_{SX}} \frac{\alpha_Y}{\alpha} \hat{T}, \]  

(8)

while that for the unskilled wage is

\[ \hat{w}_L = \frac{\theta_{KY}}{\theta_{LY}} (1 - \frac{\alpha_Y}{\alpha}) \hat{T}, \]  

(9)

where \( \theta_{SX} \equiv \frac{a_{SX}w_X}{P_X} \) is the share of unskilled labour in unit cost of producing good \( X \); cost shares \( \theta_{KY}, \theta_{KX} \) and \( \theta_{SY} \) are similarly defined in Appendix A. Moreover, \( \alpha \equiv \frac{\lambda_{KY}}{\theta_{LY}} \sigma_Y + \frac{\lambda_{KX}}{\theta_{SX}} \sigma_X = \alpha_Y + \alpha_X \) where, \( \alpha_Y \equiv \frac{\lambda_{KY}}{\theta_{LY}} \sigma_Y \) and \( \alpha_X \equiv \frac{\lambda_{KX}}{\theta_{SX}} \sigma_X \), with \( \lambda_{KY} \equiv \frac{a_{KY}Y}{K} \) and \( \lambda_{KX} \equiv \frac{a_{KX}X}{K} \) respectively denoting the share of sector \( Y \) and \( X \) in total employment of capital and \( \sigma_j \) is the elasticity of factor substitution in sector \( j \) (\( j = X, Y \)), as described in Appendix A.

It follows that the equilibrium wages change asymmetrically with a reduction of the tariff rate and the functional relationship between wages and tariff can be written as follows

\[ w'_S \equiv \frac{\partial w_S (t)}{\partial t} < 0 \text{ and } w'_L \equiv \frac{\partial w_L (t)}{\partial t} > 0. \]  

(10)

The intuition is simple. Consider an initial situation in which factor and good prices and output levels are at their equilibrium values. A tariff
reduction contracts the domestic production in the import-competing sector through an increased competition from importers as it reduces the price \( P_Y \). Capital employed in the sector producing \( Y \) decreases as the value of its marginal product decreases and, consequently, the wage of unskilled workers employed in the production of \( Y \) declines. Capital released from the import competing sector \( Y \) creates scope for expansion in the export sector \( X \), where production increases. The increase in the use of capital in the export sector increases the demand for skilled workers raising their wage under full employment. Therefore, a reduction in tariff has asymmetric effects on skilled and unskilled wages.

3 Government spending

The government is compounded of the representatives of the two groups of workers.\(^{13}\) The representatives negotiate over the level of public spending for a public good \( g \), to be financed through both a proportional tax \( \tau \) on wages, with \( \tau \in (0, 1) \), and possibly also by tariff revenue denoted by \( V = tI_y(t) \), where \( I_y(t) \) represents the value of imports, and \( V \) follows a Laffer curve shape. The government budget constraint is:\(^{14}\)

\[
g = \tau (w_L L + w_S S) + \beta tI_y(t) \implies \tau = \frac{g - \beta tI_y(t)}{w_L L + w_S S} \tag{11}
\]

where, \( g - \beta tI_y(t) \geq 0 \). The unit cost of the public good is exogenous and equal to one as one unit of the public good is produced making use of one unit of good \( X \) that is chosen as the numeraire. The parameter \( \beta \), without a loss of generality, can take either a value of one or zero and captures the contribution of tariff revenue to the financing of \( g \): when

\(^{13}\)Workers are also voters. It is obvious that each voter votes for a representative of his or her belonging interest group, since there is no incentive for strategic voting. Besides, since voters are all alike within each group, any member can be elected to represent the entire group.

\(^{14}\)The key point addressed by this analysis is the relation between wage gap and public spending for public good \( g \) when trade policy changes. For this reason we consider a tax on labor and the possibility of using tariff revenues. However, this study develops a module that can be extended under different directions in order to adapt it to the context in which the module is employed. A future extension may include, for instance, capital taxation. In this paper we abstract from it in order to disentangle the role of the wage gap and highlight its relevance for public policy formation without polluting the results with other effects. Besides, our model is suitable to understand policy formation for the provision of a national public good under the consideration that in modern democracies labor taxation covers most of the costs for the financing of national public goods. Instead, the taxation of immobile capital is typically a main source of financing of local and regional governments and finances the provision of local public goods and services.
\( \beta = 1 \), tariff revenue cofinances the provision of \( g \); instead, when \( \beta = 0 \), public good provision is only financed through tax revenue. In the trade literature, for instance, Demidova (2017) considers revenue-generating tariffs when she studies the impact of unilateral trade liberalization on a country’s welfare, while Haaland and Kind (2008) point out that they are typically of limited importance in trade between industrialized countries.\(^{15}\)

Thus, tax paid by an unskilled individual is \( \tau w_L = w_L \frac{g - \beta tI_y(t)}{w_L + w_S} \) and tax paid by a skilled individual is \( \tau w_S = w_S \frac{g - \beta tI_y(t)}{w_L + w_S} \).

Each skilled or unskilled individual \( i (i = S, L) \) maximizes his/her utility function
\[
U_i = X_i + aY_i - bY_i^2 + \ln(1 + g)
\]
where \( a, b > 0 \), subject to his/her budget constraint \( P_X X_i + P_Y Y_i = (1 - \tau)w_i + rk_i K \), where \( k_i \) denotes the share of capital owned by individual \( i \).\(^{16}\) Good \( X \) is chosen as the numeraire of the model, with \( P_X^W = 1 \). The indirect utility of individual \( i \) is given by
\[
U_i = (1 - \tau)w_i + rk_i K + f(P) + \ln(1 + g)
\]
where, \( f(P) = (a - P_Y)^2 / 4b \).

The utility of an unskilled individual is therefore
\[
U_L = (1 - \tau)w_L + rk_L K + f(P) + \ln(1 + g) = w_L + rk_L K + f(P) - \frac{w_L}{w_L + w_S} (g - \beta tI_y(t)) + \ln(1 + g) .
\]

Similarly, the utility of a skilled individual is
\[
U_S = w_S + rk_S K + f(P) - \frac{w_S}{w_L + w_S} (g - \beta tI_y(t)) + \ln(1 + g) .
\]

In the legislature, the representatives of both groups, skilled and unskilled, form a government where they negotiate over the amount of public spending for the provision of public good \( g \). In order to approve government budget and implement public policy an agreement is needed. In the case of disagreement, wages cannot be taxed and we obtain \( \tau = g = 0 \). Therefore, even if tariff revenue is positive, it cannot be used for the provision of the public good under government impasse, as

\(^{15}\)Indeed, developed-country models sometimes ignore the effect of tariff revenue on government spending on the grounds that, for these countries, tariff revenue is typically too small, relative to overall tax revenue, to constrain significantly such spending. But this strategy is less appealing for developing countries, which usually rely much more on trade-tax revenue.

\(^{16}\)Note that \( k_L L + k_S S = K \).
the government cannot decide. Hence, the disagreement utility is generated by private consumption only, and for unskilled and skilled it is respectively given by

$$U^d_L = w_L + rk_LK + f(P) \text{ and } U^d_S = w_S + rk_SK + f(P). \quad (12)$$

The net benefit of reaching an agreement on the size of public spending is for unskilled

$$\phi_L = U_L - U^d_L = \ln (g + 1) - \frac{w_L}{Lw_L + Sw_S} (g - \beta I_g (t)) ,$$

while for skilled it reads

$$\phi_S = U_S - U^d_S = \ln (g + 1) - \frac{w_S}{Lw_L + Sw_S} (g - \beta I_g (t)).$$

According to the Nash bargaining axiomatic approach, an agreement will occur if and only if $\phi_L \geq 0$ and $\phi_S \geq 0$. Moreover, note that $\phi_L \geq \phi_S$ as the skilled and unskilled receive the same benefit by public consumption, that is $\ln (g + 1)$, even though the skilled contribute more facing a higher marginal cost ($MC$).

In order to find the equilibrium policy outcome, we maximise the following Nash bargaining product:

$$g = \text{argmax} \left[ \ln (g + 1) - \frac{w_L (g - \beta I_g (t))}{Lw_L + Sw_S} \right] ^\gamma \left[ \ln (g + 1) - \frac{w_S (g - \beta I_g (t))}{Lw_L + Sw_S} \right] ^{1-\gamma}$$

where, $\gamma \in [0, 1]$ is the political bargaining leverage of the unskilled workers. Note that when $\gamma = 1/2$ skilled and unskilled workers have the same bargaining power.

The first order condition with respect to $g$ is

$$\gamma \left[ -\frac{w_L}{Lw_L + Sw_S} + \frac{1}{g+1} \right] + (1 - \gamma) \left[ -\frac{w_S}{Lw_L + Sw_S} + \frac{1}{g+1} \right] = 0,$$

which implies $-\frac{w_L}{Lw_L + Sw_S} + \frac{1}{g+1} \geq 0$ and $-\frac{w_S}{Lw_L + Sw_S} + \frac{1}{g+1} \leq 0$, given that we assume that technology is such that $w_L \leq w_S$.\footnote{Note that in the limit case where $t = 0$, $L = S$ and the technology used in the two sectors is the same, we obtain $w_L = w_S$.} This, in turn, implies that in equilibrium the unskilled voters would like to have higher taxation and public spending than the skilled voters, the reason being that the unskilled workers bear a lower marginal cost of the public good in terms of utility; i.e., $MC_L = \frac{w_L}{Lw_L + Sw_S} \leq \frac{w_S}{Lw_L + Sw_S} = MC_S$.\footnote{Note that in the limit case where $t = 0$, $L = S$ and the technology used in the two sectors is the same, we obtain $w_L = w_S$.}
Following Giuranno (2009), we denote by
\[ L = \frac{w_L}{L} \frac{1}{g+1} + \frac{1}{2g+1} + \ln(g+1) \]
\[ g_L \frac{\partial g}{\partial L} \geq 0 \] and \( \epsilon_S = (1 - \gamma) \frac{w_S}{L} \frac{1}{g+1} + \frac{1}{2g+1} + \ln(g+1) \).

The elasticities of the net gains from implementing policy \( g \) of unskilled and skilled workers respectively. This implies that the equilibrium is a compromise between the first best choice of each group of workers, as the unskilled would always prefer more public good consumption, while the skilled would prefer a reduction in public spending at the equilibrium.

Notice that \( \epsilon_L \geq 0 \) because the net marginal utility of public good consumption of the unskilled \( MU_L \) is weakly positive in equilibrium, while the opposite holds true for the skilled, whose net marginal utility is \( MU_S \), implying \( MU_L \geq 0 \) and \( MU_S \leq 0 \).

It follows that in the Nash bargaining equilibrium (13), skilled and unskilled workers are equally elastic in absolute value; i.e., \( \epsilon_L = -\epsilon_S \), which implies \( |\epsilon_L| = |\epsilon_S| \).

Finally, in the limit case in which \( \gamma = 0 \) (\( \gamma = 1 \)), skilled (unskilled) workers reach their first best which implies that \( \epsilon_S = 0 \) (\( \epsilon_L = 0 \)); that is, \( MU_S = 0 \) (\( MU_L = 0 \)).

4 The relation between tariff and public spending

In this section, we study how exogenous changes in the tariff policy \( t \) influences government spending \( g \). Our analysis is based on the following Lemma.

**Lemma 1** For \( \gamma \in (0, 1) \), tariff policy influences government spending as specified in the following relation:

\[ \frac{dg^*}{dt} \geq 0 \] if

\[ \epsilon_S \left[ (g - \beta V) \left( \frac{S}{\phi_L} + \frac{L}{\phi_S} \right) + \beta V' \left( \frac{w_L}{\phi_L} - \frac{w_S}{\phi_S} \right) \frac{Lw_L + Sw_S}{w_Lw_S' - w_Sw_L'} \right] \leq (1 - \gamma) \frac{L}{\phi_S} \frac{S}{\phi_L}; \]

with: \( V = tI_g(t) \) and \( V' = \partial V/\partial t \).

The proof is in Appendix B.

We use the above Lemma to study how changes in the tariff \( t \) affects the equilibrium level \( g^* \) of public spending. In order to do this, we consider the following conceivable combinations of \( \beta \) and \( \gamma \), as follows:

- case 1) \( \beta = 0 \) and \( \gamma = 1/2 \);

\[ \text{Note that } MU_L = -\frac{w_L}{Lw_L + Sw_S} + \frac{1}{g+1} \text{ and } MU_S = -\frac{w_S}{Lw_L + Sw_S} + \frac{1}{g+1}. \]
case 2) \( \beta = 0 \) and \( \gamma \neq 1/2 \);

case 3) \( \beta = 1 \) and \( \gamma = 1/2 \);

case 4) \( \beta = 1 \) and \( \gamma \neq 1/2 \).

The study of the above four cases leads to the understanding of policy formation when public good provision is (or is not) cofinanced by the tariff revenue and representatives have the same bargaining power and when public good provision is (or is not) cofinanced by the tariff revenue and representatives have different bargaining power.

First, we consider the case where public good provision is not cofinanced by the tariff revenue and representatives have the same bargaining power; that is, \( \beta = 0 \) and \( \gamma = 1/2 \). The results are described in the following Proposition.

**Proposition 1** Assume both the representatives of skilled and unskilled workers have the same political power (\( \gamma = 1/2 \)) and tariff revenue does not cofinance the provision of the public good (\( \beta = 0 \)), then there exists an inverse relation between trade liberalization and the size of government spending for the provision of public good \( g \).

The proof follows from Lemma 1 after considering that for \( \beta = 0 \) and \( \gamma = 1/2 \) it leads to

\[
\frac{dg^*}{dt} \geq 0 \text{ if } \epsilon_S g \left( \frac{S}{\phi_L} + \frac{L}{\phi_S} \right) \leq \frac{1}{2} \left( \frac{L}{\phi_S} - \frac{S}{\phi_L} \right),
\]

which is always satisfied since \( \epsilon_S g \left( \frac{S}{\phi_L} + \frac{L}{\phi_S} \right) \leq 0 \) and \( \frac{1}{2} \left( \frac{L}{\phi_S} - \frac{S}{\phi_L} \right) \geq 0 \).

The other side of the coin of the inverse relation between trade liberalization and the size of government spending is that an increase in trade tariffs implies an increase in \( g \). Indeed, a higher tariff decreases the wage gap between skilled and unskilled workers and this softens the redistributive conflicts between the two working classes. As a result, government provision of public goods increases. The opposite relation is also true: a lower tariff protection following a process of trade liberalization leads to a higher wage gap and to smaller public spending. This, in turn, implies that government spending for public good provision decreases under free-trade \( (t = 0) \) as this leads to a higher domestic wage disparity.

The intuition is that as tariff protection increases, the wage premium decreases. Thus, the marginal cost of public consumption, in terms of additional tax that has to be paid for an increase in public consumption, increases for the unskilled and decreases for the skilled because the unskilled wage is higher, while the skilled wage is lower. Recall that
in the equilibrium before the increase in tariff protection, the unskilled were more willingly to have a larger public consumption than the skilled because their marginal cost was lower than for the skilled. With the increase in tariff protection, the unskilled become able to renegotiate a larger consumption of the public good because the skilled are now more willing to agree on a larger public expenditure as their marginal cost is relatively lower. Furthermore, the unskilled are now willing to increase the public expenditure, even if their marginal cost is increasing, because, in the bargaining equilibrium, their net marginal utility is positive.\textsuperscript{19}

4.1 The role of the political influence of the working classes

We now describe the role of a different political influence in the legislature. In order to do this, we consider the case where $\beta = 0$ and $\gamma \neq 1/2$, with $\gamma \in (0, 1)$. The following Proposition presents the results.

**Proposition 2** Assume both an asymmetric distribution of bargaining power between the representatives of skilled and unskilled workers ($\gamma \neq 1/2$) and tariff revenue does not cofinance the provision of the public good ($\beta = 0$), then the inverse relation between trade liberalization and the size of government spending for the provision of public good $g$ may not hold when $\gamma > \bar{\gamma}$, where $\bar{\gamma} \equiv \frac{L \phi_L}{L \phi_L + S \phi_S} \geq 1/2$. Instead, it holds when $\gamma \leq \bar{\gamma}$.

The proof follows from Lemma 1 after considering that for $\beta = 0$ and $\gamma \neq 1/2$ it leads to

$$\frac{dg}{dt} \geq 0 \text{ if } \epsilon_S g \left( \frac{S}{\phi_L} + \frac{L}{\phi_S} \right) \leq (1 - \gamma) \frac{L}{\phi_S} - \gamma \frac{S}{\phi_L},$$

which is always satisfied when $\gamma \leq \bar{\gamma}$ since $\epsilon_S g \left( \frac{S}{\phi_L} + \frac{L}{\phi_S} \right) \leq 0$ and $(1 - \gamma) \frac{L}{\phi_S} - \gamma \frac{S}{\phi_L} \geq 0$, leading to a direct relation between $t$ and $g$. Instead, when $\gamma > \bar{\gamma}$, we obtain that $(1 - \gamma) \frac{L}{\phi_S} - \gamma \frac{S}{\phi_L} \leq 0$, leading to an ambiguous relation between $t$ and $g$.

Proposition 2 states that, when tariff revenue does not contribute to the financing of $g$ and the bargaining power is asymmetrically distributed in favour of the unskilled workers ($\gamma > \bar{\gamma} \geq 1/2$), government spending for public good provision may either increase or decrease when

\textsuperscript{19}The interested reader may easily verify that the prediction of Proposition 1 reverses in an economy where the employment of the specific factors between the protected sector and the import competing sector turns out to be reversed.
there is a process of trade liberalization. Therefore, when the bargaining power is sufficiently higher for the unskilled, they may use their bargaining leverage to obtain either an increase or a decrease in $g$. In order to understand this, recall that the unskilled have a higher demand than the skilled workers for public good consumption, which is due to their lower marginal contribution to the financing of $g$. In the case under consideration, when the tariff increases, the unskilled face an increase in their marginal cost of $g$, due to both an increase of unskilled wages and tax burden. As a result, the unskilled have to balance their willingness to have a higher public good provision with the higher marginal cost. Therefore, policy outcomes depend on the specific values of the parameters of the model.

Instead, the inverse relation between trade liberalization and the size of government spending holds when the skilled workers have a sufficiently high political influence in the government ($\gamma \leq \tilde{\gamma}$). The reason is that trade liberalization leads to a higher marginal cost of public good provision for the skilled. Given that $\tilde{\gamma} \equiv \frac{L\phi L}{L\phi L + S\phi S}$, the threshold value $\tilde{\gamma}$ depends on the relative net gains of public expenditure of unskilled workers; i.e., $L\phi L$ over the net gains of all workers $L\phi L + S\phi S$. For given $L$ and $S$, the threshold $\tilde{\gamma}$ for the bargaining power of the unskilled increases as the wage gap increases. Thus, as trade liberalization increases the wage gap, the threshold $\tilde{\gamma}$ also increases, meaning that a higher unskilled bargaining power is needed to fall in the situation where the inverse relation between trade liberalization and the size of government spending for the provision of public good may not hold.

### 4.2 The role of tariff revenue cofinancing of public spending

Given the relevance that tariff revenues may have especially for low-income countries (Baunsgaard and Keen, 2010; Cagé and Gadenne, 2018), in this section we study how previous results are affected when tariff revenue cofinances the public good provision, i.e.: $\beta = 1$. Furthermore, we retain for the moment the assumption that the two representatives have the same bargaining power ($\gamma = 1/2$), while it will be dropped in the following subsection.

**Proposition 3** Assume both the representatives of skilled and unskilled workers have the same bargaining power ($\gamma = 1/2$) and tariff revenue cofinances public good provision ($\beta = 1$), then trade liberalization leads to a decrease in the provision of the public good $g$ if the tariff revenue decreases and is ambiguous otherwise.
In order to prove the Proposition, consider that the relation in Lemma 1 becomes
\[
\frac{dg^*}{dt} \geq 0 \text{ if }
\]
\[
\epsilon_S \left[ (g - V) \left( \frac{S}{\phi_L} + \frac{L}{\phi_S} \right) + V' \left( \frac{w_L}{\phi_L} - \frac{w_S}{\phi_S} \right) \frac{Lw_L + Sw_S}{w_L w'_S - w_S w'_L} \right] \leq \frac{1}{2} \left( \frac{L}{\phi_S} - \frac{S}{\phi_L} \right),
\]
which proves the Proposition 3 after considering that the following inequalities hold: \( w_L w'_S - w_S w'_L < 0; \left( \frac{w_L}{\phi_L} - \frac{w_S}{\phi_S} \right) \leq 0; \) given (11) \( g > V; \)
\[
\left[ (g - V) \left( \frac{S}{\phi_L} + \frac{L}{\phi_S} \right) + V' \left( \frac{w_L}{\phi_L} - \frac{w_S}{\phi_S} \right) \frac{Lw_L + Sw_S}{w_L w'_S - w_S w'_L} \right] \geq 0 \text{ when } V' \geq 0
\]
and is ambiguous otherwise; \( \epsilon_S \leq 0 \) and \( \frac{1}{2} \left( \frac{L}{\phi_S} - \frac{S}{\phi_L} \right) \geq 0 \).

The impact on government spending of changes in the tariff rate depends not only on how it influences the wage gap, but also on how it influences tariff revenue for the government. Following the shape of a typical Laffer curve, an increase in tariff revenues due to a change in the tariff rate mitigates the redistributive conflict between skilled and unskilled workers and facilitates government spending.

Thus, according to Proposition (3), when tariff revenue cofinances the provision of public spending, the size of \( g \) increases as tariff increases if this does not lead to a lower government collection of tariff revenue \( \left( V' = \frac{\partial V}{\partial t} \geq 0 \right) \). Clearly, in this case, wages convergence coupled with a weak increase in tariff revenue leads to a less conflicting situation between the two groups in the society and to a larger value of \( g \). Results are ambiguous when \( V' < 0 \).

In the case of a process of trade liberalization, the decrease in tariff leads to a higher wage gap, which in turn reduces public expenditure as in Proposition 1, and such effect is strengthened (weakened) by the decrease in tariff revenue when \( V' > 0 \) (\( V' < 0 \)). This happens for a balanced bargaining power \( (\gamma = 1/2) \).

### 4.3 The interplay of political influence and tariff cofinancing

In this subsection we expand further our horizon by analyzing what happens when skilled and unskilled workers have different political influence in the legislature and public spending is cofinanced by tariff revenue. Proposition 4 shows that the mitigating effects of tariff revenue can be either offset or strengthened by an asymmetric distribution of political influence of the two groups of workers, with \( \gamma \in (0, 1) \).

**Proposition 4** Assume \( \beta = 1 \) and \( \gamma \leq \bar{\gamma} \), then trade liberalization leads to a decrease in public spending when tariff revenue decreases and has
ambiguous effects on it otherwise. In addition, when $\gamma > \bar{\gamma}$ the impact on public spending of changes in $t$ is ambiguous.

The proof is a straightforward application of Lemma 1.

Consider a trade liberalization policy where tariff $t$ decreases in the specific case in which it leads to a weak decrease in tariff revenue (i.e., $V' \geq 0$), then government spending decreases if the skilled workers are strong enough ($\gamma \leq \bar{\gamma}$). In order to understand this, one needs to consider that trade liberalization leads to wages divergence. Since the skilled workers are those who are becoming richer and their marginal cost of $g$, in terms of tax burden, increases, they will demand a lower government spending. A distribution of political power sufficiently in favour of the skilled strengthens this effect. However, we find that the policy outcome is ambiguous when political power is distributed in favour of the unskilled ($\gamma > \bar{\gamma}$), since they are in favour of more public good consumption. Hence, co-financing through the tariff revenue may not solve this ambiguity, which depends on the value of the single parameters.

However, in the opposite situation in which a process of trade liberalization leads to a decrease in tariff revenue (i.e., $V' < 0$), this latter effect mitigates the redistributive conflict between skilled and unskilled, even though the final effect on $g$ continues to depend on the parameters values.

4.4 The national median voter

So far we have analyzed the relationship between trade policy changes and the size of public expenditure when a negotiation actually takes place, that is when $\gamma \in (0, 1)$. For completeness, we analyze the limit cases where the representative of either the skilled or unskilled group of voters holds full decision power, that is when either $\gamma = 0$ or $\gamma = 1$.

Corollary 1 Assume that the skilled have full decision making power ($\gamma = 0$), then there exists an inverse relation between trade liberalization and the size of government spending for the provision of public good $g$.

Specifically, when $\gamma = 0$, the first order condition (13) becomes

$$
-\frac{w_S}{Lw_L + Sw_S} + \frac{1}{g+1} = 0,
$$

20Recall that $\bar{\gamma} \equiv \frac{Lw_L}{Lw_L + Sw_S}$, as in Proposition 2.
which implies that the policy outcome corresponds to the skilled first best choice
\[ g_{S}^{fb} = \frac{w_{L}}{w_{S}} L + S - 1. \]

Therefore, given that trade liberalization decreases the wage ratio \( \frac{w_{L}}{w_{S}} \), the size of public spending \( g_{S}^{fb} \) decreases, with \( \frac{dg_{S}^{fb}}{dt} > 0 \). This is independent from the presence of tariff cofinancing.

**Corollary 2** Assume that the unskilled have full decision making power \( (\gamma = 1) \), then there exists a direct relation between trade liberalization and the size of government spending for the provision of public good \( g \).

Indeed, in this case the first order condition (13) becomes
\[
-\frac{w_{L}}{Lw_{L} + Sw_{S}} + \frac{1}{g+1} - \frac{w_{L}(g-\beta I_{t}(t))}{Lw_{L} + Sw_{S}} + \ln (g + 1) = 0,
\]
and it would imply that the policy outcome corresponds to the unskilled first best choice
\[ g_{L}^{fb} = \frac{w_{S}}{w_{L}} S + L - 1. \]

As the ratio \( \frac{w_{S}}{w_{L}} \) increases with trade liberalization, public expenditure \( g_{L}^{fb} \) increases, with \( \frac{dg_{L}^{fb}}{dt} < 0 \). This, again, does not depend on the presence or absence of tariff cofinancing.

Corollary 2 allows us to deal with the case of the national median voter. Indeed, if expenditure would be chosen by majority voting instead of by bargaining between the representative voters in the legislature, the preference of the national median voter would prevail and it would coincide with the outcome obtained in Corollary 2, where \( \gamma = 1 \), as the median voter would belong to the most numerous group in the society.

## 5 Efficiency

In this section we study the efficient policy outcome from the point of view of a benevolent planner and compare it with the Nash bargaining solution. In order to compute the efficient policy outcome, we maximize a weighted social welfare function, where the weights \( \gamma \) and \( 1 - \gamma \) are the political influence of skilled and unskilled voters, as defined in Section 3. Thus, after solving the following maximization problem
\[
g = \arg \max \ [\gamma SU_S + (1 - \gamma) LU_L] = \\
\arg \max \ \gamma S \left\{ w_S + \frac{(a - (1 + t) P_W^S)^2}{4b} \right\} - \frac{w_S (g - \beta t I_g (t))}{Lw_L + Sw_S} + \ln (1 + g) + \\
+ (1 - \gamma) L \left\{ w_L + \frac{(a - (1 + t) P_Y^L)^2}{4b} \right\} - \frac{w_L (g - \beta t I_g (t))}{Lw_L + Sw_S} + \ln (1 + g),
\]

we find that the efficient policy outcome, \( g^e \), is given by

\[
g^e = \frac{\gamma S + (1 - \gamma) L}{\gamma S w_L + (1 - \gamma) Lw_S} (w_L + w_S S) - 1,
\]

which does not depend on the cofinancing of tariff revenue.

Furthermore, we notice that the efficiency equilibrium condition (16) and the Nash bargaining equilibrium (13) equalize under free trade (\( t = 0 \)) when the two groups have the same bargaining power (\( \gamma = 1/2 \)) and the technology is such that wages equalize under free trade with an equal size of the two groups. For the proof, see the Appendix C.

### 6 Concluding remarks

The link between tariff policies and public policies is an open question both in economic theory and in the current economic policy debate characterized by progressive tensions towards the closure of international trade.

This paper intersects two worlds of the economic literature, public economics and international trade, which, although dialoguing between them, are rarely dealt with contextually, proposing a new theoretical module that analyzes the relationship between trade liberalization and the level of government spending.

Trade policy impacts on the internal redistributive conflicts that, in turn, influence government spending. In order to capture those relations, we make use of a specific-factors model, where a change in the relative commodity price increases the real reward of one specific factor and lowers the real reward of the other specific factor.\(^{21}\) Thus, the owners of the specific factors have the strong incentives to try to influence policy. Therefore, in our framework, trade liberalization increases the redistributive conflicts between the two working groups. As a result, government spending decreases, unless there are perturbation due to either an unbalanced distribution of bargaining power or tariff revenue.

\(^{21}\) The reward of the mobile factor, by contrast, is affected ambiguously.
cofinancing public spending. However, bargaining asymmetry or the use of tariff revenue for government spending enhances specific mechanisms that might reverse the direct relation between tariff protection and public expenditure.

In a world with balanced political influence, when the government cofinances the provision of public goods through both a direct wage tax and tariff revenues, a unilateral increase in the tariff will increase public spending when it increases tariff revenues. The reason being that increasing tariff revenue reinforces the effect generated by the reduction in the wage gap. On the contrary, a higher tariff may reduce public spending when the reduction in the wage-gap is associated to a reduction in tariff revenues that more than offset the impact of the smaller wage gap.

However, there may be reasons why the two groups of voters hold different bargaining leverage in the government. In that case, a sufficiently high political influence of skilled workers strengthens the direct relation between tariff protection and public good provision, unless tariff revenue are sufficiently reduced.

Furthermore, we find that free-trade coupled with a balanced political power increases the efficiency of public spending.

To the best of our knowledge, this paper provides the first framework to understand the links among trade policy, wage gap and government spending. It offers a new possible explanation, alternative to that by Arawatari (2015), to the empirical evidence that finds that government spending may either increase (Cameron, 1978; Rodrik, 1998; Epifani and Gancia, 2009) or decrease (Abizadeh, 2005; Benarroch and Pandey, 2008) with trade liberalization and may stimulate further empirical research to clarify the determinants of such results.

The current analysis can be developed in different directions. In the core analysis we consider the case of a small open economy that protects the sector employing skilled workers. However, the framework could be easily adapted to other situations where, for instance, the sector employing unskilled workers is protected.

Finally, following Kovak (2013), Chiquiar (2008), Nicita (2009) and de Viti de Marco (1930), the current analysis can easily be extended in order to study the implications for public finance of the inter-regional inequality effects of trade policies that emerges when the productive factors are unevenly distributed in the space.
References


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bridge University press.


7 Appendix A

From the full employment condition for unskilled labour (5), we obtain
\[ dL = a_{LY}Y + a_{LY}dY \]
that, for a given endowment of unskilled \( L \), leads to the following proportional change
\[ \hat{a}_{LY} + \hat{Y} = 0 \] (17)

Similarly, from the full employment condition for skilled labour (4), we obtain
\[ dS = a_{SX}X + a_{SX}dX, \]
which leads to
\[ \hat{a}_{SX} + \hat{X} = 0 \] (18)

From the full employment condition for capital (6), we obtain
\[ \frac{a_{KX}}{K} \hat{a}_{KX} + \frac{a_{KY}}{K} \hat{X} + \frac{a_{K}Y}{K} \hat{a}_{KY} + \frac{a_{K}Y}{K} \hat{Y} = 0. \]
Then, denoting the share of capital used in sector \( X \) (\( Y \)) as \( \lambda_{KX} \equiv \frac{a_{KX}}{K} (\lambda_{KY} \equiv \frac{a_{KY}}{K}) \) we get
\[ (\hat{a}_{KX} + \hat{X}) \lambda_{KX} + (\hat{a}_{KY} + \hat{Y}) \lambda_{KY} = 0 \] (19)

Substitution of (17) and (18) in (19) yields
\[ (\hat{a}_{KX} - \hat{a}_{SX}) \lambda_{KX} + (\hat{a}_{KY} - \hat{a}_{LY}) \lambda_{KY} = 0 \]

After using the definition of factor substitution elasticity in sector \( X \) and \( Y \), \( \sigma_{X} \equiv \frac{a_{KX} - \hat{a}_{SX}}{\hat{w}_{S} - \hat{r}} \) and \( \sigma_{Y} \equiv \frac{a_{KY} - \hat{a}_{LY}}{\hat{w}_{L} - \hat{r}} \), the above equation boils down to
\[ \sigma_{X} (\hat{w}_{S} - \hat{r}) \lambda_{KX} + \sigma_{Y} (\hat{w}_{L} - \hat{r}) \lambda_{KY} = 0 \] (20)
Now, we use the zero profit conditions \((2)\) and \((1)\) in order to obtain the relationship between changes in wages and in the rate of return to capital, as follows:

\[ P_Y^W d(1 + t) = w_Ld_LY + rda_KY + a_LYdw_L + a_KYdr. \]

Since it must be \(w_Ld_LY + rda_KY = 0\) as the marginal rate of technical substitution is equal to the ratio of input prices,\(^{22}\) we can write \(d(1 + t) = \frac{a_LY}{P_Y^W}dw_L + \frac{a_KY}{P_Y^W}dr; \)

\[
(1 + t)(1 + t) = \frac{a_LY}{P_Y^W}\hat{w}_L + \frac{a_KY}{P_Y^W}\hat{r}; (1 + t) = \frac{a_LY}{P_Y^W}\hat{w}_L + \frac{a_KY}{P_Y^W}\hat{r};
\]

\[
(1 + t) = \theta_{LY}\hat{w}_L + \theta_{KY}\hat{r}
\]

where \(P_Y \equiv (1 + t) P_Y^W, \theta_{LY} \equiv \frac{a_LY}{P_Y^W}\) and \(\theta_{KY} \equiv \frac{a_KY}{P_Y^W}\). Hence, since

\[
d(1 + t) = dt \Rightarrow d\left(\frac{1 + t}{1 + t}\right) = \frac{dt}{1 + t} \Rightarrow (1 + t) = \hat{t} \Rightarrow (1 + t) = \hat{t}_{(1 + t)}^{\hat{t}};
\]

we can rewrite \((21)\) as \(\hat{t}_{(1 + t)}^{\hat{t}} = \theta_{LY}\hat{w}_L + \theta_{KY}\hat{r} = \theta_{LY}\hat{w}_L + \frac{\theta_{KY}}{\theta_{LY}}\hat{r} = \theta_{LY}\hat{w}_L + \theta_{KY}\hat{r} - \frac{\theta_{KY}}{\theta_{LY}}\hat{r} = \frac{\theta_{LY}}{\theta_{LY}}\hat{w}_L - \frac{\theta_{KY}}{\theta_{LY}}\hat{r} = \frac{\theta_{LY}}{\theta_{LY}}\hat{w}_L - \frac{\theta_{KY}}{\theta_{LY}}\hat{r};
\]

\[
(1 + t) = \frac{\theta_{LY}}{\theta_{LY}}\hat{w}_L - \frac{\theta_{KY}}{\theta_{LY}}\hat{r}
\]

where \(\hat{T} = \hat{t}_{(1 + t)}^{\hat{t}}\) given that \(T = 1 + t\).

Furthermore, from the zero profit condition \((1)\) and the normalization \(P_X^W = 1\), we obtain \(0 = (a_{SX}w_s + a_{KX}r)\frac{d(a_{SX}w_s + a_{KX}r)}{P_x^W}\), which becomes

\[
\frac{d(a_{SX}w_s + a_{KX}r)}{P_X} = \frac{d(a_{SX}w_s + a_{KX}r)}{P_X} = \frac{d(a_{SX}w_s + a_{KX}r)}{P_X} = \frac{ds}{P_X}(wa_{SX} + a_{SX}dw_s + rda_{KX} + a_{KX}dr) = 0.
\]

As it must be \(w_sda_{SX} + rda_{KX} = 0\), we obtain \(\frac{a_{SX}w_s}{P_X} + \frac{a_{KX}r}{P_X} = 0\) that, making use of \(\theta_{SX} \equiv \frac{a_{SX}w_s}{P_X}\) and \(\theta_{KX} \equiv \frac{a_{KX}r}{P_X}\), can be rewritten as

\[
\hat{w}_s = -\frac{\theta_{KX}}{\theta_{SX}}\hat{r}
\]

Substitution of \((22)\) and \((23)\) in \((20)\) yields the change in the rate of return to capital \((7)\), as shown below:

\[
\sigma_X(\hat{w}_s - \hat{r})\lambda_{KX} + \sigma_Y(\hat{w}_L - \hat{r})\lambda_{KY} = 0;
\]

\[
\sigma_X\left(-\frac{a_{KX}}{\theta_{SX}}\hat{r} - \hat{r}\right)\lambda_{KX} + \sigma_Y\left(\hat{r} - \frac{\theta_{KX}}{\theta_{LY}}\right)\lambda_{KY} = 0;
\]

\[
-\hat{r}\sigma_X\lambda_{KX}\left(\frac{\theta_{KX}}{\theta_{SX}} + 1\right) + \frac{\sigma_Y\lambda_{KX}}{\theta_{LY}} - \hat{r}\sigma_Y\lambda_{KY}\left(\frac{\theta_{KY}}{\theta_{LY}} + 1\right) = 0;
\]

\[
\frac{\sigma_Y\lambda_{KX}}{\theta_{LY}} = \hat{r}\left[\frac{\sigma_Y\lambda_{KX}}{\theta_{LY}}\right] = \frac{\sigma_Y\lambda_{KX}}{\theta_{LY}}\left(\theta_{KY} + \theta_{LY}\right) + \frac{\sigma_X\lambda_{KX}}{\theta_{SX}}\left(\theta_{KX} + \theta_{SX}\right) = 0;
\]

Then, since \(\left(\theta_{KY} + \theta_{LY}\right) = 1\) and \(\left(\theta_{KX} + \theta_{SX}\right) = 1\), we can write \(\frac{\sigma_Y\lambda_{KX}}{\theta_{LY}} = \hat{r}\left(\frac{\sigma_Y\lambda_{KX}}{\theta_{LY}} + \frac{\sigma_X\lambda_{KX}}{\theta_{SX}}\right)\). Defining \(\alpha \equiv \frac{\lambda_{KX}}{\theta_{LY}}\sigma_Y + \frac{\lambda_{KX}}{\theta_{SX}}\sigma_X = \alpha_Y + \alpha_X\), with \(\alpha_Y \equiv \frac{\lambda_{KX}}{\theta_{LY}}\sigma_Y\) and \(\alpha_X \equiv \frac{\lambda_{KX}}{\theta_{SX}}\sigma_X\), we obtain \(\alpha_Y\hat{T} = \hat{r}\alpha\), which leads to equation \(7)\).

\(^{22}\)See Krugman, Obstfeld and Melitz, 2015, pp. 449-451.
Finally, substitution of (7) in (22) and (23) yields the changes in the domestic wages (8) and (9).

8 Appendix B

Denote by \( Z \) the left hand side of the first order condition (13):

\[
Z \equiv \gamma \frac{w_L}{L_L + S w_S} + \frac{1}{g+1} + (1 - \gamma) \frac{w_S}{L_L + S w_S} + \frac{1}{g+1} = 0
\]

Clearly, the second order condition is negative; that is,

\[
Z_g < 0.
\]

Therefore, the sign of \( \frac{dg}{dt} \equiv -\frac{Z_t}{Z_g} \) depends on the sign of \( Z_t \equiv \frac{dZ}{dt} \).

Denoting \( w'_S \equiv \frac{\partial w_s}{\partial t} \) and \( w'_L \equiv \frac{\partial w_L}{\partial t} \), we compute

\[
Z_t = \gamma \frac{w_L}{(L_L + S w_S)^2} \phi_L - \left[ S \frac{(w_L w'_S - w_S w'_L)}{(L_L + S w_S)^2} (g - \beta t I_y (t)) + \frac{w_L}{L_L + S w_S} \beta (I_y (t) + t I'_y (t)) \right] \frac{\partial \phi_L}{\partial g} + \phi_L^2 \frac{(1 - \gamma) w_L w'_S - w_S w'_L}{L_L + S w_S} \phi_s + \left[ L \frac{w_L w'_S - w_S w'_L}{(L_L + S w_S)^2} (g - \beta t I_y (t)) - \frac{w_S}{L_L + S w_S} \beta (I_y (t) + t I'_y (t)) \right] \frac{\partial \phi_s}{\partial g}.
\]

Therefore, the sign of \( Z_t \) is non negative when

\[
(\frac{w_L w'_S - w_S w'_L}{(L_L + S w_S)^2}) \frac{S \phi_L}{\phi_L^2} - \left[ S (g - \beta t I_y (t)) + \frac{w_L (L_L + S w_S)}{w_L w'_S - w_S w'_L} \beta (I_y (t) + t I'_y (t)) \right] \frac{\partial \phi_L}{\partial g} + \frac{w_L w'_S - w_S w'_L}{(L_L + S w_S)^2} (1 - \gamma) \phi_s + \left[ L (g - \beta t I_y (t)) - \frac{w_S (L_L + S w_S)}{w_L w'_S - w_S w'_L} \beta (I_y (t) + t I'_y (t)) \right] \frac{\partial \phi_s}{\partial g} \geq 0.
\]

After substituting \( t I_y (t) = V \geq 0 \), considering that \( \tau \geq 0 \), \( (I_y (t) + t I'_y (t)) = V' = \frac{\partial V}{\partial t}, w_L w'_S - w_S w'_L < 0 \) and making use of the equilibrium relationship \( \epsilon_L = -\epsilon_s \), expression (24) can be rewritten as

\[
\epsilon_s \left[ \frac{S (g - \beta V)}{\phi_L} + \frac{\frac{L L_L + S w_S}{w_L w'_S - w_S w'_L} \beta V'}{\phi_s} \right] + \frac{L (g - \beta V)}{\phi_s} - \frac{w_S (L_L + S w_S)}{w_L w'_S - w_S w'_L} \beta V' \leq (1 - \gamma) \frac{L}{\phi_s} - \gamma \frac{S}{\phi_L},
\]

which leads to (14).
9 Appendix C

In order to calculate equation (16), we maximise the social welfare function (15) with respect to \( g \). The first order condition is

\[
\gamma S \left( -\frac{w_S}{Lw_L + Sw_S} + \frac{1}{1 + g} \right) + (1 - \gamma) L \left( -\frac{w_L}{Lw_L + Sw_S} + \frac{1}{1 + g} \right) = 0,
\]

which leads to (16).

Let us compare the efficiency condition (16) with the Nash bargaining equilibrium condition (13). The two conditions equalize in the limit case when with \( t = 0, L = S \) and \( w_L = w_S \). \(^{23}\) Moreover, if in addition \( \gamma = 1/2 \), both equations (16) and (13) become

\[
g = L + S - 1;
\]

that is, the normative and the positive solutions become equal.

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\(^{23}\)Note that, in order to simplify the discussion we are assuming that the technology is such that wages equalise under free-trade.