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Optimal Destabilization of Cartels

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Optimal Destabilization of Cartels

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Abstract: The literature on cartel stability sidelines antitrust policy, whereas the literature on antitrust policy tends to neglect issues of cartel stability. This paper attempts to connect these two interrelated aspects in the context of an augmented quantity leadership model. The cartel is the Stackelberg quantity leader and the fringe firms are in Cournot competition with respect to the residual demand. The antitrust authority decides on its own investigative effort and on the size of the fine that cartel members have to pay when they are detected. For testifying cartel members a leniency program is implemented. Our framework takes into account that these antitrust policy instruments are not costless for society. Our model demonstrates that the optimal antitrust policy exploits the inherent instability of a cartel to reduce its size.

JEL-Classification: L13, L41

Keywords: antitrust, stability, Cournot fringe, oligopoly, leniency.

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1 Introduction

There is a long history of anti-competitive agreements in which firms collude to raise market prices above competitive levels. Such agreements inflict short-term and long-term costs on society. Therefore, the formation of cartels is considered to be illegal in most developed economies. Competition law and antitrust authorities are important elements in combating anti-competitive practices.

Economists have made considerable progress in better understanding the formation of cartels and the effectiveness of antitrust policies. The current literature is mostly concerned with the *sustainability* of a cartel's collusive agreement. A cartel is sustainable, if all firms comply with the collusive agreement. Following Friedman's (1971) seminal paper, this literature usually relies on repeated oligopoly games (supergames) with grim-trigger strategies that form a symmetric subgame perfect Nash equilibrium. Motta and Polo (2003) and Spagnolo (2005) combine this approach with an antitrust authority that has limited resources. The authority *endogenously* determines its antitrust policy. Two basic market outcomes can arise in these supergames. Either the antitrust policy successfully deters the firms from establishing a sustainable cartel, or the antitrust policy fails and an *all-inclusive* cartel arises, that is, all firms become members of the cartel.

Bos and Harrington (2015, p. 133) criticize the existing supergame literature because it focuses on all-inclusive cartels, while in the real world cartels usually compete against some fringe firms.² The existence of fringe firms complicates the analysis considerably, as it raises the issue of cartel *stability*. In the work of d'Aspremont *et al.* (1983, p. 21) a distinction is made between internal and external stability. Internal stability is characterized by the condition that no cartel member wants to become a fringe firm, whereas external stability is conditioned by no fringe firm wanting to become a cartel member. The supergame approach avoids questions of stability by simply assuming that only all-inclusive cartels can arise.³ Furthermore, supergame frameworks usually produce equilibrium conditions rather than explicit equilibrium solutions. This may impede the formulation of policy recommendations.

By contrast, questions of stability have been studied in the context of the so-called leadership approach (e.g., d'Aspremont *et al.*, 1983; Shaffer, 1995), which used to be the standard approach for analysing the formation of cartels. It is embedded in a fully fledged oligopoly framework and produces explicit equilibrium solutions. Leadership models are concerned with stability, but not with antitrust policy and they avoid questions of sustainability by simply assuming that a collusive agreement is enforceable.

The preceding discussion suggests that a comprehensive and tractable model of collus-

²This point has been made also by Bos (2009, pp. 11-12) and by Bos and Harrington (2010, pp. 92-93). Empirical studies such as Harrington (2006), Levenstein and Suslow (2006), and Pham (2019) confirm this position. Hellwig and Hüschelrath (2017) provide a dataset on 114 illegal cartels convicted by the European Commission between 1999 and 2016. The data reveal frequent entries into the cartel and exits from the cartel. Both instances confirm the existence of a fringe.

³Bos and Harrington (2015, pp. 135-136) is the only exception that we are aware of. To address both, sustainability and stability, they sacrifice the endogeneity of the antitrust authority's operations and assume an exogenously given antitrust behavior.

ive behavior ideally would have at least four properties. It addresses: (i) stability and (ii) sustainability in the context of an (iii) endogenous antitrust policy and it is simple enough to produce (iv) explicit equilibrium solutions. Existing models of collusive behavior have one or two of these properties. We attempt to push that limit towards three. Since antitrust policy directly affects the stability of cartels, our framework of collusion addresses *stability* issues in the context of an *endogenous* antitrust policy and it derives an equilibrium solution, the core of which can be explicitly formulated. For this purpose, we revive the leadership approach and enhance it by a welfare maximizing antitrust authority.

Of the four listed desirable properties of a model of collusive behavior, only property (ii), sustainability, is missing. The issue of sustainability features prominently in the supergame approach. In our view, there are at least four good reasons to complement the supergame approach by our leadership approach.

(1) As pointed out before, it is analytically difficult to accommodate both, stability issues and an endogenous antitrust policy in a supergame. By contrast, the simple basic structure of leadership models allows for an extension that captures the endogenous activity of a welfare maximizing antitrust authority. Furthermore, as this paper demonstrates, explicit solutions can be derived.

(2) Leadership models assume perfect enforceability. Supergames replace this assumption with the opposite extreme, no enforceability whatsoever. Studies such as Levenstein and Suslow (2006), Harrington (2006), and Pham (2019) provide elucidating descriptions of actual cartel behavior. They show that perfect enforceability is unrealistic, but that the other extreme, no enforceability, is unrealistic, too. Colluding firms usually develop organizational structures that ensure some degree of enforceability. In other words, whereas the leadership approach is likely to overstate the sustainability of cartels (by taking it for granted), the supergame approach tends to underrate it (by ignoring that colluding firms usually develop some enforcement devices). To obtain a more balanced picture, it is advisable to complement the policy implications derived from the supergame approach by those derived from our leadership approach.

(3) Levenstein and Suslow (2006, p. 78) argue that in the real world the breakup of cartels is pre-dominantly the result of changing economic conditions and not so much the cartel's response to the misbehavior of a cartel member. Misbehavior often results in limited retaliation rather than in the dissolution of the cartel (see also Genesove and Mullin, 2001, pp. 390-394). The grim-trigger strategies underlying most supergames are not fully consistent with this empirical observation.⁴

(4) When firms are sufficiently patient (that is, their discount factor is sufficiently large), sustainability is always guaranteed. Therefore, the leadership model's virtues (addressing stability issues, being expandable by a welfare maximizing antitrust authority, and producing explicit solutions) may outweigh its neglect of sustainability.

These four arguments should not be misinterpreted as an attempt to discredit the supergame approach. They merely suggest that complementary approaches are useful. We think that Shaffer's (1995) quantity leadership model extended by an active antitrust

⁴See also Eaton and Eswaran (1998) and Escrihuela-Villar and Guillén (2011).

authority is such an approach.

In our model, several policy instruments are available to the antitrust authority. It can decide on its investigative effort and on the appropriate size of the fine that detected cartels have to pay. Furthermore, we assume that a leniency program can be implemented that offers testifying firms a discount. The authority's own investigative effort is its most direct option to increase the probability of detection. These efforts are also necessary to turn fines and leniency programs into effective antitrust instruments. Nevertheless, the authority's effort has rarely been addressed in the literature.⁵

In our framework, the probability of detection increases with the number of cartel members, with the authority's effort, and with the size of the fine. Since a leniency program is implemented, the probability of detection also increases with the size of the discount granted to cartel members that cooperate with the antitrust authority.

Antitrust policy, however, is not costless. This cost must be considered in the design of an optimal antitrust policy. We demonstrate how the optimal policy can be derived. This policy creates *internal instability* and *external stability*. Internal instability induces cartel firms to mutate into fringe firms and external stability prevents fringe firms from entering the cartel. Although the optimal antitrust policy reduces the size of the cartel compared to the size without an antitrust policy, it never completely eliminates it.

Though several results of our study are novel, several other results confirm findings that have been derived in the context of supergames. For example, Bos and Harrington (2015, p. 139) point out that with a sufficiently strong antitrust policy, it is always possible to prevent the formation of a cartel. We obtain the same result in the context of our extended quantity leadership model. Also the positive effect of leniency programs on the prevention of large cartels is confirmed.

This paper proceeds as follows. Section 2 surveys the related literature. Section 3 summarizes the original quantity leadership model. In Section 4 we extend this model by including an antitrust authority that combats the formation of cartels. In Section 5 we show how the antitrust authority can use its policy instruments to reduce the size of stable cartels. The derivation of an optimal antitrust policy is outlined in Section 6. Section 7 discusses the underlying economics and the resulting policy implications. Concluding remarks are contained in Section 8. We have placed most of our proofs in the Appendix.

2 Related Literature

This study is primarily concerned with the derivation of an effective antitrust policy. Endowed with vast resources, an antitrust authority might be able to stop any collusionary behavior. At some point along this route, however, the marginal benefit from resources invested in antitrust enforcement falls below the marginal cost of expanding enforcement.

⁵A notable exception is the supergame proposed by Spagnolo (2005). It features an antitrust authority that can decide on its own effort, on the size of the fine, and on the extent of leniency. However, only all-inclusive cartels are considered. Therefore, the link between stability and antitrust policy cannot be analysed.

Therefore, governments endow their authorities with limited resources and expect that these resources are efficiently used to contain collusive behavior.

This institutional framework is reflected in the supergame proposed by Motta and Polo (2003, p. 353). They assume that the authorities have a given budget that they must efficiently allocate for the monitoring and prosecution of cartels. Spagnolo (2005, p. 13) assumes that increasing the conviction probability is costly. These two studies consider only all-inclusive cartels, that is, cartels without a competitive fringe. Therefore, they cannot analyse the link between antitrust policy and cartel stability.⁶

Bos and Harrington (2010, pp. 92-93) list several empirical studies that emphasize the relevance of fringe firms. Therefore, they propose a supergame with heterogeneous capacity-constrained firms, some of which may stay outside the cartel. They show (Bos and Harrington, 2010, p. 101) that a sustainable and stable cartel is made up of the largest firms. The smallest firms prefer the status of a fringe competitor. The fringe firms produce at capacity, whereas the cartel members restrict their output below capacity. In a supergame with quantity setting firms, Escrihuella-Villar (2009, p. 139-140) demonstrates that even homogeneous firms can establish a sustainable and stable cartel. Eaton and Eswaran (1998) and Escrihuella-Villar and Guillén (2011) propose supergames that do not rely on grim-trigger strategies. Instead, the non-cheating members of a cartel continue to operate the cartel without the cheating member.

All articles listed in the preceding paragraph neglect the issue of antitrust policy. In a later study, however, Bos and Harrington (2015, p. 135) amend their former framework by an exogenously given antitrust policy and investigate the impact of that policy on the properties of the cartel and the fringe. They assume that the rate of success of the antitrust authority's investigations increases with the number of cartel members. The members of a detected cartel must pay a fine that is proportional to their profits. The fined cartel members can apply for leniency which would reduce the fine. The analysis confirms that antitrust policy affects the stability of cartels. Even though the authors concede that "the relationship between antitrust enforcement and cartel size is too complex for us to provide specific guidance for enforcement policies (p. 148)", their framework suggests that an antitrust policy should be progressively more aggressive for more inclusive cartels.

Bos and Harrington (2015) investigate cartel stability in the context of the supergame approach. Originally, price or quantity leadership models were the backbone of stability analysis. In the price leadership model developed by d'Aspremont *et al.* (1983) the cartel is the Stackelberg *price* leader. The fringe firms take the leader's price as given and set their quantities such that price equals marginal cost. In the quantity leadership model advocated by Shaffer (1995), the cartel is the Stackelberg *quantity* leader and the fringe firms are in Cournot competition with respect to the residual demand. The price

⁶This is true also for studies by Souam (2001) and Mouraviev and Rey (2011). The latter consider a cartel the members of which play sequentially instead of simultaneously. They show how this can facilitate collusion. Souam (2001) proposes a framework in which the antitrust authority takes the market price as a signal for the probability that a cartel exists. Since antitrust enforcement is costly, the antitrust authority should increase its effort with the observed market price. If the market price is sufficiently low, however, collusion should be tolerated. Sustainability is not an issue in his framework.

leadership model with its perfectly competitive fringe might fit industries with a large number of competing firms, whereas the quantity leadership model with its Cournot fringe might be more suitable for markets with a more limited number of firms. The leadership models have inspired further work on the conditions for the successful formation and stability of cartels. For example, Donsimoni (1985), Donsimoni *et al.* (1986), and Prokop (1999) utilize the price leadership model, whereas studies by Konishi and Lin (1999) and Zu *et al.* (2012) are based on the quantity leadership model.

Antitrust policy is not an issue in either type of leadership model. Instead, these studies focus on the formal conditions for the existence and uniqueness of a stable cartel. The present study extends the quantity leadership model by a welfare maximizing antitrust authority. Before we describe this extension in more detail, we provide a summary of the original quantity leadership model advocated by Shaffer (1995).

3 The Original Quantity Leadership Model

Daughety (1990, pp. 1232-33) introduces a model that combines Stackelberg and Cournot competition. The inverse demand function is $P = a - bQ$, where P is the market price, Q is the aggregate quantity produced, and a and b are positive constants. The industry consists of a finite number of $n \geq 2$ identical firms. Some of the firms are independent Stackelberg leaders, while the other firms are independent Stackelberg followers. Only integer numbers of firms are considered. All n firms have a constant marginal cost equal to c . Among each other, the Stackelberg leaders act like Cournot players. The same is true for the Stackelberg followers. Daughety uses this model as a starting point for analyzing the welfare effects of horizontal mergers.

When only one Stackelberg leader exists, Daughety's model simplifies to the quantity leadership model that Shaffer (1995, p. 745) adopts for studying the stability of cartels. In that model, the Stackelberg leader is a cartel comprised of a group of identical firms that coordinate their output decisions in a binding manner. In equilibrium, the cartel always produces more than half of the complete output. Therefore, assigning the role of the Stackelberg leader to the cartel is a reasonable feature of the quantity leadership model.⁷

When $n_C \in (1, \dots, n)$ of the n firms form a cartel, then the number of fringe firms is $n_F = n - n_C$. Since the n_C cartel members act as one firm, we prefer to present the model in terms of n_F instead of n_C . This also simplifies the algebraic derivations without affecting the results.

After the $(n - n_F)$ firms of the cartel have collectively determined their profit maximizing joint output Q_C , each fringe firm determines its profit maximizing output q_F . In other words, the cartel acts as a Stackelberg leader, while the group of fringe firms is the Stackelberg follower. Each fringe firm considers both, the cartel's output, Q_C , and the

⁷Huck *et al.* (2007) provide some experimental evidence that firms that cooperate in a binding manner show leadership behavior, whereas the remaining firms exhibit follower behavior. For additional references that support the leadership role of perfectly colluding firms see Brito and Catalão-Lopes (2011, pp. 3-4).

aggregate output of the other fringe firms, Q_{-F} , as given. Therefore, the output of each fringe firm, q_F , is determined by the Cournot-Nash equilibrium concept.

Shaffer (1995, p. 745) shows that, with $n \in (1, \dots, n)$ and $n_F \in (0, \dots, n - 1)$, the equilibrium is given by the total output

$$Q = Q_C + n_F q_F = \frac{a - c}{b} \frac{2n_F + 1}{2n_F + 2} \quad (1)$$

and the market price

$$P = c + \frac{a - c}{2(n_F + 1)}. \quad (2)$$

The profit of each cartel member is

$$\pi_C(n_F) = \frac{(a - c)^2}{4b(n_F + 1)(n - n_F)} \quad (3)$$

and the profit of each fringe firm is

$$\pi_F(n_F) = \frac{(a - c)^2}{4b(n_F + 1)^2}. \quad (4)$$

Given the choice between being a cartel member or a fringe firm, the firm always chooses the option with the larger profit. If both options generate exactly the same profit, the firm prefers the legal status of a fringe firm to the illegal status of a cartel member. In other words, a firm will stay in a cartel, only if the profit gained by staying a cartel member is *strictly* larger than the profit gained by becoming a fringe firm. This is the condition for *internal stability*.⁸ Formally, a cartel with $(n - n_F)$ members is internally stable, only if

$$\pi_C(n_F) > \pi_F(n_F + 1). \quad (5)$$

External stability requires that no fringe firm has an incentive to become a member of the cartel. This is satisfied, when the profit of the fringe firm is at least as large as the profit the firm would earn after entering the cartel. Formally, the external stability of a cartel with $(n - n_F)$ members requires that

$$\pi_C(n_F - 1) \leq \pi_F(n_F). \quad (6)$$

An all-inclusive cartel ($n_F = 0$) is always externally stable, because no fringe firm exists that could enter the cartel. However, it is easy to show that for $n > 4$, all-inclusive cartels are never internally stable.

A cartel is said to be *stable*, if it satisfies both, internal and external stability (d'Aspremont *et al.*, 1983, p. 21). When n is even (uneven), a stable cartel has exactly two (three) more members than the fringe (Shaffer, 1995, p. 746, Proposition 4) and the profit of each cartel member is smaller than that of each fringe firm (p. 747, Proposition 5). However,

⁸In the original definition of d'Aspremont *et al.* (1983, p. 21) and many subsequent papers a *weakly* larger profit is sufficient for internal stability.

both, the members of the stable cartel and the fringe firms earn a higher profit than in the standard Cournot oligopoly without a cartel (p. 748, Proposition 6). Note that in the standard Cournot oligopoly the market price is

$$P = c + \frac{a - c}{n + 1} \quad (7)$$

and the profit of each firm is

$$\pi(n) = \frac{(a - c)^2}{b(n + 1)^2}. \quad (8)$$

Due to the first-mover advantage, a single firm cartel ($n_F = n - 1$) always prefers the cartel situation to the standard Cournot oligopoly. This can be easily checked by comparing the profits in (3) with those in (8). Also, the welfare arising from a market with a single firm cartel is larger than the welfare arising from a standard Cournot oligopoly. This beneficial “timing effect” (Brito and Catalão-Lopes, 2011, p. 2) has already been discussed in Daughety (1990, p. 1233).

Equation (2) reveals that the market price falls as the number of fringe firms increases. Therefore, welfare increases as cartel members mutate into fringe firms.

The latter two findings have important implications for the introduction of an anti-trust policy. Any such policy should attempt to destabilize the existing stable cartel by providing incentives for the cartel members to become fringe firms. In other words, the policy should change the condition for internal stability such that the size of the stable cartel decreases. However, the policy should stop short of eliminating the cartel, because the ensuing Cournot competition yields a larger market price than the price arising from a single firm cartel. If antitrust policy was costless, the largest welfare would arise from a policy that leads to a stable single firm cartel. In the real world, however, antitrust authorities create social costs. In the following section we complement the quantity leadership model by including an antitrust authority. The policies available to the antitrust authority can increase the number of fringe firms, but also create social costs.

4 Model Extension

Our game has two stages. In the first stage, the antitrust authority decides on its optimal policy, taking into account the reactions of the n firms. A policy specifies the authorities investigative effort, $e \geq 0$, and the size of the fine for detected cartels, $f \geq 0$. Furthermore, a leniency program can be implemented, with $r \geq 0$ denoting the percentage by which the fine of an eligible and cooperating cartel member is reduced, and $\mu \geq 0$ denoting the share of cartel members eligible for that reduction.⁹ Therefore, the variable $d = r\mu$ can

⁹Suppose that the cartel is detected. If $\mu = 0.1$ and the number of cartel members is $n - n_F = 10$, then exactly one member is randomly drawn. This member is regarded as a cooperating firm and receives the reduction. If $\mu = 0.1$ and $n - n_F = 5$, again one member is randomly drawn and that member has a 50% chance of being regarded as a cooperating firm.

be interpreted as the expected discount, that is, as the percentage by which the fine of a member of a detected cartel is reduced.

In the second stage, each of the n firms decides on its cartel membership and on the optimal output quantities, q_F and Q_C , given the implemented antitrust policy (e, f, d) . We will solve this two stage game by backward induction, starting with the derivation of a subgame perfect equilibrium for the n firms (Section 5), given the authority's antitrust policy. We then derive the welfare maximizing antitrust policy (Section 6).

4.1 Probability of Detection and Antitrust Authority

When the cartel is detected, each member must pay the fine f . Let $p \in [0, 1]$ denote the probability of detection. To capture the interdependencies between the determinants of this probability, we specify it in the following multiplicative way:

$$p(n_F, e, f, d) = g(n - n_F) \cdot h(e, f, d) . \quad (9)$$

The first factor, $g(n - n_F) \in [0, 1]$, takes care of the fact that larger cartels are more likely to be detected than smaller ones.¹⁰ Therefore, $g(n - n_F)$ has a positive first order derivative with respect to $(n - n_F)$ and a negative one with respect to n_F . Furthermore, we assume that $g(n - n_F)$ is concave and, therefore, approaching 1 from below. When no cartel exists ($n_F = n$), the probability of detection, p , should be 0. This requires that $g(0) = 0$.

The second factor of the probability of detection is $h(e, f, d) \in [0, 1]$ and captures the impact of the authority's antitrust policy on the probability of detection. It is through this channel that the antitrust policy can affect the market equilibrium and the associated size of the cartel.

The antitrust authority is part of the government and invests its complete endowment in the detection of cartels. The larger the endowment, the larger the effort, e , and therefore, the probability of detection: $h'_e > 0$. As e rises, the marginal increase in the probability of detection falls: $h''_{ee} < 0$.

The authority can implement a leniency program that offers a reduced fine to some or all cartel firms that inform the authority about the cartel. There is ample evidence that such programs increase the probability of detecting cartels (e.g., Aubert *et al.*, 2006, p. 1242; Brenner, 2009, pp. 642-644). In anticipation of being detected, cartel firms may apply for leniency by providing evidence of a cartel agreement. Furthermore, even if cartel members consider it as unlikely that the antitrust authority will discover anything, they may worry that some fellow member applies for leniency and, because of that worry, applies itself. Harrington (2013, pp. 2-3) denotes these two effects as “prosecution effect” and “preemption” effect, respectively.

The policy variable $d = r\mu$ in $h(e, f, d)$ captures these effects. An expansion of eligibility, μ , or an increase of the percentage r by which the fine of an eligible cartel member is reduced, strengthens the preemptive effect of discounts. However, it lowers the average

¹⁰This is identical to Assumption A4 in Bos and Harrington (2015, p. 135).

fine of the members of a detected cartel, weakening the prosecution effect. We assume that the preemptive effect of d is at least as strong as the prosecution effect. In other words, $h'_d > 0$. Furthermore, we assume that $h''_{dd} < 0$. Our specification allows for discounts $r > 1$, that is, for rewards. For plausibility reasons, however, we restrict the domain of d to the interval $[0, 1)$. Otherwise, the members of a *detected* cartel could expect to receive a reward instead of a fine: $f(1 - d) \leq 0$. This cannot be a sensible antitrust policy.

The third policy instrument is the fine f . A larger fine, f , strengthens both, the prosecution effect and the preemptive effect of the leniency program, but this effect weakens as f rises. Therefore, $h'_f > 0$ and $h''_{ff} < 0$.

A positive effort ($e > 0$) is necessary to turn the leniency program into an effective instrument. Without any effort on the side of the antitrust authority ($e = 0$) the prosecution effect and the preemptive effect of discounts do not exist, regardless of the size of the cartel and the size of the fine. Therefore, the case $e = 0$ must give $p = 0$, which requires that $h(0, f, d) = 0$. When a detected cartel never pays a fine ($f = 0$), the investigative staff is likely to be demoralised and its effort may become completely ineffective, that is, $h(e, 0, d) = 0$. If the fine for detected cartels is positive, the antitrust authority must be able to detect an existing cartel through its own investigative effort e . Therefore, lowering the reduction for testifying cartel members to $r = 0$, or lowering the share of cartel members eligible for the leniency program to $\mu = 0$ (both cases result in $d = 0$) should not imply that the probability of detection falls to 0. Therefore, we need $h(e, f, 0) > 0$.

Summarizing, we assume that $h(e, f, d)$ is a continuous concave function that is strictly increasing and approaching 1 from below. Furthermore, $h(0, f, d) = h(e, 0, d) = 0$ and $h(e, f, 0) > 0$.

4.2 Social Cost

If effort, e , was costless, the authority could choose an infinitely large effort level such that the factor $h(e, f, d)$ approaches 1. Clearly, this is not what we observe in reality. Authorities are limited in their investigative efforts by the size and competence of its staff and the quality of its technical system. If society desires a larger effort by its antitrust authority, it must provide the resources necessary to hire more and better staff and to purchase more effective system. Following the law enforcement literature initiated by Becker (1968), we capture this cost by a continuous social cost function $s(e, f, d)$ with the first order partial derivative $s'_e > 0$.

The antitrust authority can penalize a convicted cartel with a fine f the size of which must have been fixed before the cartel was detected. The size of the fine is determined by the antitrust authority. This is its second antitrust policy instrument. Again following Becker (1968), we assume that the administration of the fine is costless. The payment of the fine represents pure redistribution. No social cost arises from this payment. However, there is a more hidden route by which fines cause a social cost. The antitrust authority has a strong incentive to choose very large fines, because this reduces the expected profits from cartel membership. Excessive fines, however, induce a social cost, because they violate the principle of proportional justice and may increase the risk of convicting innocent firms

(e.g., Allain *et al.*, 2015). Governments have reacted to these concerns by setting upper bounds to legally admissible fines. We formalize this cost in our social cost function $s(e, f, d)$ with $s'_f > 0$.

Leniency programs cause similar social costs. The public may dislike the idea that testifying firms that have broken the law can get away with a discount or, even worse, are rewarded. Lenient treatment of guilty firms may undermine the general respect for the law and may encourage unlawful behavior. The public incorporates into its dislike for leniency programs not only the percentage of the reduction, r , but also the share of cartel members eligible for this reduction, μ . These costs, too, are captured by our social cost function $s(e, f, d)$ with $s'_d > 0$ and $d = r\mu$.

The derivative s'_e can be interpreted as an inefficiency indicator of the antitrust authority, while the derivatives s'_f and s'_d indicate the damage to the rule of law arising from the fine and the expected discount.

In sum, our model extends the quantity leadership model by including an antitrust authority that imposes a rather stylized, but comprehensive antitrust policy. It chooses its effort, e , the fine, f , and the expected discount, d . The probability of detection, $p(n_F, e, f, d)$, is carefully specified, capturing the interdependencies between the relevant forces. The probability of detection increases with the number of cartel members, $(n - n_F)$, with the authority's effort, e , with the expected reduction in the fine for testifying cartel members, d , and with the size of the fine, f . Furthermore, our model introduces a social cost function that is increasing in e , f , and d .

5 Stability

In this section we derive a subgame perfect equilibrium for each given antitrust policy (e, f, d) . The equilibrium is characterized by the number of fringe firms, n_F , their output quantities, q_F , and the output quantity of the cartel, Q_C .

For a given antitrust policy (e, f, d) and given values of n and n_F , the expected value of the fine can be interpreted as a fixed cost of each cartel member. Therefore, the equilibrium quantity and price are still given by expressions (1) and (2). The profit of each fringe firm is still given by (4). Therefore,

$$\pi_F(n_F + 1) = \frac{(a - c)^2}{4b(n_F + 2)^2}. \quad (10)$$

However, a cartel member's profit, defined by expression (3), must be modified. In the presence of the antitrust authority, the expected profit of each cartel member is

$$E[\pi_C(n_F)] = \pi_C(n_F) - p(n_F, e, f, d) f (1 - d) \quad (11)$$

$$= \frac{(a - c)^2}{4b(n_F + 1)(n - n_F)} - g(n - n_F) h(e, f, d) f (1 - d). \quad (12)$$

Accordingly, the formal condition for internal stability becomes

$$E[\pi_C(n_F)] > \pi_F(n_F + 1). \quad (13)$$

The condition's interpretation is unaltered. As long as the expected profit from remaining a cartel member is larger than the profit from becoming a fringe firm, the cartel membership is preferred. However, if the two profits are identical, the firm chooses the riskless and law-abiding option, that is, fringe status.

Inserting the profit definitions (12) and (10), as well as the probability definition (9) in inequality (13) yields

$$A < \bar{T}(n_F) , \quad (14)$$

with

$$\bar{T}(n_F) = \frac{(a-c)^2}{4b} \frac{(n_F+2)^2 - (n_F+1)(n-n_F)}{(n_F+1)(n-n_F)(n_F+2)^2 g(n-n_F)} \quad (15)$$

and

$$A = h(e, f, d) f (1-d) . \quad (16)$$

The variable A characterizes the implemented antitrust policy. The value of A depends on the punishment variables f and d , as well as on the effort e , but not on n and n_F . Increases in the effort, e , and the fine, f , raise the value of A . The impact of the discount d on the value of A is ambiguous, since it increases the probability of detection, but lowers the average fine of the members of a detected cartel, $f(1-d)$. Recall that d was restricted to values smaller than 1. Therefore, A cannot be negative.

$\bar{T}(n_F)$ can be interpreted as a threshold. It is the minimum value of the policy variable A that is required to make a cartel with $(n-n_F)$ members internally instable. The value of $\bar{T}(n_F)$ depends on n and n_F as well as on the market volume $(a-c)/b$. Furthermore, we obtain the following result.

Lemma 1 *The function $\bar{T}(n_F)$ defined by (15) increases in n_F . For $\bar{T}(n_F) \geq 0$, the function $\bar{T}(n_F)$ decreases in n .*

Proof: See Appendix A.

For a cartel with $(n-n_F+1)$ members, the expected profit of each cartel member is

$$E[\pi_C(n_F-1)] = \pi_C(n_F-1) - p(n_F-1, e, f, d) f(1-d) \quad (17)$$

$$= \frac{(a-c)^2}{4b(n_F)(n-n_F+1)} - g(n-n_F+1) h(e, f, d) f(1-d) . \quad (18)$$

External stability requires that

$$E[\pi_C(n_F-1)] \leq \pi_F(n_F) . \quad (19)$$

Inserting the profit definitions (4) and (18), as well as the probability definition (9) in inequality (19) yields

$$T(n_F) \leq A \quad (20)$$

with

$$T(n_F) = \frac{(a-c)^2}{4b} \frac{n_F(2n_F+1-n)+1}{n_F(n-n_F+1)(n_F+1)^2 g(n-n_F+1)} . \quad (21)$$

The weak inequality in (20) reflects the fact that with $E[\pi_C(n_F - 1)] = \pi_F(n_F)$ a firm always prefers the riskless and legal status of the fringe firm.

When conditions (14) and (20) are simultaneously satisfied, the cartel with $(n - n_F)$ members is stable:

$$T(n_F) \leq A < \bar{T}(n_F) . \quad (22)$$

This says that an antitrust authority pursuing a market with n_F fringe firms and a stable cartel with $(n - n_F)$ members, must choose its policy (e, f, d) such that the resulting A -value falls between the two thresholds $T(n_F)$ and $\bar{T}(n_F)$.

Since

$$T(n_F + 1) = \bar{T}(n_F) , \quad (23)$$

the following result is obtained.

Lemma 2 *The function $T(n_F)$ defined by (21) increases in n_F and decreases in n .*

Proof: The proof in Appendix A is valid not only for $\bar{T}(n_F)$, but also for $\bar{T}(n_F - 1)$ and, therefore, for $T(n_F)$. ■

Using Equation (23), the condition for stability, (22), can be written in the form

$$\underbrace{T(n_F)}_{\text{ext. stability}} \leq A < \underbrace{T(n_F + 1)}_{\text{int. stability}} . \quad (24)$$

This has an important implication:

Theorem 1 *If an antitrust policy (e, f, d) generates a stable cartel, then this cartel is unique.*

Proof: By Lemma 2, the sequence of intervals $[T(0), T(1))$, $[T(1), T(2))$, \dots , $[T(n - 1), T(n))$ is connected, but mutually exclusive. Therefore, for each given A -value, only one n_F -value satisfying (24) can exist. ■

By modifying the values of its policy instruments, the antitrust authority can modify the A -value, and therefore, the size of the resulting stable cartels. For example, reducing the effort would lower the A -value and, therefore, the stable n_F -value (enlarge the cartel), whereas increasing the effort would increase the A -value and the stable n_F -value (shrink the cartel).

6 Optimal Antitrust Policy

6.1 Welfare

Welfare does not depend on the budgetary effects of the fines and discounts, because these are of a purely redistributive nature. Therefore, welfare is defined here as the sum of

consumer and producer rent minus the social cost caused by the antitrust policy. Suppose that an antitrust policy leads to a stable cartel with $n_F \in (0, \dots, n - 1)$ members. The associated sum of consumer and producer rent is given by

$$(a - c)Q - 0.5(a - P)Q = \frac{(a - c)^2 (2n_F + 1) (2n_F + 3)}{8b (n_F + 1)^2}, \quad (25)$$

where the values of Q and P are defined by (1) and (2).

When a cartel member becomes a fringe firm, this sum increases by (see Appendix B)

$$\frac{(a - c)^2}{8b} \frac{2n_F + 3}{(n_F^2 + 3n_F + 2)^2} > 0. \quad (26)$$

This would suggest that the antitrust authority should implement a policy that results in a very small stable cartel. However, we know that reducing the size of a stable cartel increases the social cost, $s(e, f, d)$. This cost is the second component of welfare. Combining both components, the welfare function is

$$W(n, e, f, d) = \frac{(a - c)^2 (2n_F + 1) (2n_F + 3)}{8b (n_F + 1)^2} - s(e, f, d), \quad (27)$$

where n_F is determined via definition (16) and condition (24). We want to find the antitrust authority's policy (e, f, d) that generates the maximum welfare. This policy is denoted as the authority's *optimal antitrust policy*, (e^*, f^*, d^*) .

6.2 Cartel Prevention and Passive Antitrust Policy

When $n_F = n$, the cartel is “empty”. If a very aggressive antitrust policy is implemented such that $A > T(n)$, no fringe firm is willing to “enter” this empty cartel. The empty cartel is externally stable. Therefore, a symmetric Cournot oligopoly would arise. However, this is not the outcome the antitrust authority should pursue.

Theorem 2 *The symmetric Cournot oligopoly with $n_F = n$ would generate a smaller welfare than a Stackelberg market with a stable single firm cartel.*

Proof: See Appendix C.

Theorem 2 states that an antitrust policy leading to $n_F = n$ cannot be optimal. Therefore, we can restrict our attention to policies consistent with stable cartels such that $n_F \in (0, \dots, n - 1)$.

Any antitrust policy with $e = 0$ or $f = 0$ leads to $h(e, f, d) = 0$, yielding $p = 0$ for Equation (9). Therefore, such a policy completely eliminates the chance to detect an operating cartel. We denote such policies as *passive* antitrust policies. From Equation

(16) we can see that passive antitrust policies are characterized by $A = 0$. Relationship (24) implies that for such policies the condition for stability becomes

$$T(n_F) \leq 0 < T(n_F + 1) . \quad (28)$$

Only one n_F -value exists that satisfies this condition. We denote this value by n_F^{\min} , because an active antitrust policy would lead to n_F -values that are at least as large as n_F^{\min} , and therefore, to cartels that are never larger than $(n - n_F^{\min})$. Therefore, we can confine our search for the optimal antitrust policy to those policies (e, f, d) that lead to $n_F \in (n_F^{\min}, \dots, n - 1)$.

To find the optimal antitrust policy (e^*, f^*, d^*) , we pursue the following three stage procedure:

1. Find n_F^{\min} .
2. Derive for each given $n_F \in (n_F^{\min}, \dots, n - 1)$ the antitrust policy $(e_{n_F}^*, f_{n_F}^*, d_{n_F}^*)$ that minimizes the social cost, $s(e, f, d)$, of ensuring that at least n_F fringe firms exist.
3. For each of these cost minimizing antitrust policies, compute the resulting welfare. The policy that generates the largest welfare is the optimal antitrust policy (e^*, f^*, d^*) .

In the following, we describe these three stages in more detail. An illustrating example is provided in Section 7.2.

6.3 Stage 1: Finding n_F^{\min}

We know that a passive antitrust policy leads to n_F^{\min} . This is the only n_F -value that satisfies both inequalities in (28). The sign of $T(n_F)$ depends on the sign of the term $n_F(2n_F + 1 - n) + 1$ in expression (21). Therefore, the left inequality of (28) (the external stability condition) gives

$$n - n_F \geq n_F + 1 + \frac{1}{n_F} .$$

This expression implies that in the absence of an active antitrust policy the smallest externally stable cartel and, therefore, the only stable cartel has at least $n_F + 2$ members. Therefore, n_F^{\min} is the largest integer for which the condition $n - n_F^{\min} \geq n_F^{\min} + 2$ is satisfied. Rearranging this condition gives $n_F^{\min} \leq (n - 2) / 2$. Therefore,

$$n_F^{\min} = \begin{cases} (n - 2) / 2 & \text{for even } n \\ (n - 3) / 2 & \text{for uneven } n . \end{cases} \quad (29)$$

This is just a reformulation of Shaffer's (1995, p. 746) Proposition 4. The antitrust authority can restrict its search for the optimal antitrust policy (e^*, f^*, d^*) to policies that lead to $n_F \in (n_F^{\min}, \dots, n - 1)$, where n_F^{\min} is defined by (29).

6.4 Stage 2: Computing the Cost Minimizing Policies

We know that any passive antitrust policy leads to $n_F = n_F^{\min}$, that is, to a stable cartel with $(n - n_F^{\min})$ members.

Theorem 3 *Among all antitrust policies leading to a stable cartel with $(n - n_F^{\min})$ members, the passive policy $(e, f, d) = (0, 0, 0)$ is the cost minimizing policy $(e_{n_F^{\min}}^*, f_{n_F^{\min}}^*, d_{n_F^{\min}}^*)$.*

Proof: The policy $(e, f, d) = (0, 0, 0)$ leads to $A = 0$ and a stable cartel with $(n - n_F^{\min})$ members. Any policy leading to $0 < A < T(n_F^{\min} + 1)$ would generate the same stable cartel and, therefore, the same sum of consumer and producer rent. However, it would cause a larger social cost. ■

Suppose that the antitrust authority wants to shrink the cartel to $(n - n_F)$ members, where $n_F \in (n_F^{\min} + 1, \dots, n - 1)$. For $e = 0$ or $f = 0$ the probability of detection, p , would be 0. Therefore, an active antitrust policy, that is, a policy with $e > 0$, $f > 0$, and $d \geq 0$ is required. From (24) we know that an active antitrust policy (e, f, d) pursuing a cartel with $(n - n_F)$ members must be such that the resulting A -value defined by (16) falls into the interval $[T(n_F), T(n_F + 1))$. An infinite number of active policies exist that satisfy this condition. Since all of these policies lead to the same given n_F -value, the equilibrium quantity, Q , the equilibrium price, P , and therefore, the consumer rent and the producer rent are the same for all of these policies. However, the social cost is not.

Lower A -values allow for lower values of e , f , and d , and therefore, for a lower social cost. Therefore, the antitrust authority should choose a policy that generates the lowest A -value consistent with a stable cartel with $(n - n_F)$ members. In other words, for each given n_F -value the antitrust authority should opt for a policy such that A reaches the lower bound of its admissible interval defined by (24):

$$A = T(n_F) . \quad (30)$$

Choosing a policy with an A -value slightly below $T(n_F)$ would make a cartel with $(n - n_F)$ members externally instable and its size would increase to $(n - n_F + 1)$. Therefore, Equation (30) defines the smallest possible A -value that caps the cartel size at $(n - n_F)$. We denote this condition on the feasible set of policies as the *efficacy condition*. This condition states that the profit from staying a fringe firm, $\pi_F(n_F)$, and the expected profit from entering the cartel, $\pi_C(n_F - 1) - p(n_F, e, f, d) f (1 - d)$, are equal.

An infinite number of policies (e, f, d) satisfy the efficacy condition. Among these policies, the authority should choose the one that causes the lowest social cost, $s(e, f, d)$. For given n_F , this cost minimization problem can be written in the following form:

$$\min_{e, f, d} s(e, f, d) \quad \text{subject to (30)} . \quad (31)$$

The solution to this cost minimization problem is denoted by $(e_{n_F}^*, f_{n_F}^*, d_{n_F}^*)$. We know that this solution is characterized by $e > 0$ and $f > 0$. An interior solution also requires that $d > 0$. In the following it is assumed that the functions $s(e, f, d)$ and

$A = h(e, f, d) f (1 - d)$ are such that a unique solution arises (though not necessarily an interior solution).¹¹

Consider some n_F -value and some corresponding policy, $(e, f, d)'$, that satisfies the efficacy condition specified by Equation (30). This condition implies that a marginal change of one of the three policy variables (e.g., increasing the effort level) must be offset by a change in another policy variable (e.g., reducing the fine) such that the A -value remains on the level that corresponds with the original policy $(e, f, d)'$. At the same time, each policy change generates a change in social cost. If the original policy, $(e, f, d)'$, represented an interior solution to the cost minimization problem (31), then the two changes in social cost exactly offset each other, regardless of the pair of policy variables involved.

It cannot be ruled out that the cost minimization problem does not have an interior solution, but a solution with $d = 0$. Since $A = h(e, f, d) f (1 - d)$, an increase in d raises the value of $h(e, f, d)$, but lowers the value of $(1 - d)$. If the impact on $h(e, f, d)$ is too weak, the value of A may fall and e or f must be raised to compensate for this reduction. Then, all three policy variables would have larger values than in the original policy. Therefore, the new policy would generate a larger social cost. This policy change cannot be optimal. Even if an increase in d increases A such that e or f can be lowered, the increase in the social cost from the larger d -value may not be compensated by the cost reductions that arise from the lower e - and f -value. Again, the policy change would be cost increasing. Since $h(e, f, d)$ is an increasing concave function, cost reducing effects of an increase in d are more likely at lower levels of d than at larger levels. However, if even at $d = 0$ an increase in d and the accompanying change in e or f lead to an increase in the social cost, the cost minimizing policy is a corner solution with $d_{n_F}^* = 0$.

6.5 Stage 3: Selecting the Optimal Antitrust Policy

In the final stage of our three-stage procedure, we could insert the cost minimizing policies $(e_{n_F}^*, f_{n_F}^*, d_{n_F}^*)$ derived in Stage 2 in the welfare function (27) and then maximize welfare with respect to n_F . However, the solution is unlikely to be an integer number. Therefore, the welfare levels associated with the two integer numbers closest to the solution must be calculated. These two welfare levels must be compared to the welfare level arising from $n_F = n_F^{\min}$ (the boundary solution associated with the cost minimizing passive antitrust policy). The largest of these three welfare levels determines the welfare maximizing fringe size, n_F^* . The cost minimizing antitrust policy leading to this fringe size is the optimal antitrust policy (e^*, f^*, d^*) .

We prefer, however, an alternative route to the optimal antitrust policy (e^*, f^*, d^*) . First, we insert $n_F = n_F^{\min}$ and the passive policy $(e, f, d) = (0, 0, 0)$ in welfare function (27) and compute the resulting welfare level. Then we consider the cost minimizing active antitrust policies $(e_{n_F}^*, f_{n_F}^*, d_{n_F}^*)$. The welfare levels corresponding to each integer

¹¹Uniqueness merely requires that in e - f - d -space the plane corresponding to the efficacy condition (30) and a given n_F -value is “more convex” than the isocost-planes defined by the social cost function (e.g., a social cost function that is linear in e , f , and d together with a convex efficacy condition).

$n_F \in (n_F^{\min} + 1, \dots, n - 1)$ are calculated. For this purpose we insert each of these n_F -values together with its corresponding cost minimizing policy $(e_{n_F}^*, f_{n_F}^*, d_{n_F}^*)$ in the welfare function (27). We get a set of welfare levels. From this set we select the maximum value. If this welfare is smaller than the one generated by the passive policy, the optimal fringe size is $n_F^* = n_F^{\min}$. However, if the welfare from the best active antitrust policy is larger than the one from the passive policy, then the optimal fringe size is $n_F^* > n_F^{\min}$. The cost minimizing antitrust policy leading to the stable cartel with $(n - n_F^*)$ members is the optimal antitrust policy (e^*, f^*, d^*) .

7 An Illustrative Application

7.1 A Simple Model Specification

The probability of detection, $p(n_F, e, f, d)$, was defined by Equation (9). This probability is the product of the functions $g(n - n_F)$ and $h(e, f, d)$. In Section 4.1 we described the basic features that these functions should have. In Appendix D we sketch out a rather general class of functions that captures all of these features and has some additional attractive properties. We use this class of functions for all factors determining the probability of detection. More specifically, we assume that

$$g(n - n_F) = \frac{n - n_F}{[(n - n_F)^2 + 1]^{1/2}}$$

and that

$$h(e, f, d) = w(e) \cdot k(d) \cdot m(f) , \quad (32)$$

where

$$w(e) = \frac{e}{(e^2 + 1)^{1/2}} \quad (33)$$

$$k(d) = \frac{d + \rho}{[(d + \rho)^2 + 1]^{1/2}} \quad (34)$$

$$m(f) = \frac{f}{(f^2 + 1)^{1/2}} , \quad (35)$$

with $\rho > 0$.¹²

From (16), (30), and (32) we obtain, for $e > 0$,

$$fm(f) = \frac{T(n_F)}{w(e) k(d) (1 - d)} . \quad (36)$$

This is another formulation of the efficacy condition.

¹²The restriction $\rho > 0$ ensures that for positive effort, e , and fine, f , the probability of detection, p , remains positive, even when $d = 0$.

To keep the model analytically tractable, we also assume that the continuous social cost function is

$$s(e, f, d) = s(z) \quad \text{with} \quad z = \alpha e + \beta f m(f) + \gamma d .$$

The parameter α can be interpreted as the marginal effect of effort on the social cost variable z . Since $\lim_{f \rightarrow \infty} m(f) = 1$, the function $\beta f m(f)$ approximates the simple linear function βf (see graph at the end of Appendix D) and the parameter β approximates the fine's marginal effect on the social cost variable z . The parameter γ is an indicator of the influence of the leniency program's expected discount, d , on the social cost variable z . If the parameter γ is very large relative to the parameters α and β , the social cost of the expected discount dominates the social cost associated with the effort and the fine. As a result, solutions with $d = 0$ can become optimal. A large value of ρ in Equation (34) can have the same effect, because such a ρ -value implies that the positive effect of the expected discount on the probability of detection is very weak; see Equation (52) in Appendix D.

Minimizing the monotonically increasing social cost function, $s(\cdot)$, is equivalent to minimizing the sum

$$\alpha e + \beta f m(f) + \gamma d . \tag{37}$$

Inserting the right hand side of (36) in (37), we can transform the constrained minimization problem (31) into the unconstrained minimization problem

$$\min_e \left(\alpha e + \beta \frac{T(n_F)}{w(e) k(d) (1-d)} + \gamma d \right) . \tag{38}$$

It yields the cost minimizing effort, $e_{n_F, d}^*$, for given n_F and d . The result is stated in Equation (39) of the following theorem:

Theorem 4 *For each $n_F \in (n_F^{\min} + 1, \dots, n - 1)$ and expected discount, $d \in [0, 1)$, the cost minimizing policy $(e_{n_F, d}^*, f_{n_F, d}^*, d)$ that leads to a stable cartel with $(n - n_F)$ members, is defined by*

$$e_{n_F, d}^* = \left[\frac{\beta [(d + \rho)^2 + 1]^{1/2} T(n_F)}{\alpha (d + \rho) (1 - d)} \right]^{1/2} \tag{39}$$

$$f_{n_F, d}^* = \left[\frac{1}{2} F \left(F + (F^2 + 4)^{1/2} \right) \right]^{1/2} \tag{40}$$

with

$$F := \frac{\alpha}{\beta} e_{n_F, d}^* \left((e_{n_F, d}^*)^2 + 1 \right)^{1/2} . \tag{41}$$

Proof: See Appendix E.

For a given cartel size, $(n - n_F)$, and a given expected discount, d , Equations (39) to (41) of Theorem 4 specify the cost minimizing effort, $e_{n_F, d}^*$, and fine, $f_{n_F, d}^*$, such that the efficacy condition (36) is satisfied.

Suppose that no leniency program exists: $d = 0$. From Equations (39) to (41) of Theorem 4 we can calculate for each n_F the corresponding cost minimizing effort, $e_{n_F,0}^*$, and fine, $f_{n_F,0}^*$. Should the antitrust authority introduce a leniency program, that is, should it increase d ? To answer this question, we use Equations (4) and (17) to express the efficacy condition (30) – the external stability condition (20) with equality sign – in the following form:

$$p(n_F, e, f, d) f (1 - d) = \pi_C(n_F - 1) - \pi_F(n_F) . \quad (42)$$

This reveals that an increase in d must be accompanied by changes in $e_{n_F,d}^*$ and $f_{n_F,d}^*$ such that the expected fine, $p(n_F, e, f, d) f (1 - d)$, remains constant. Increasing d can make sense, only if the accompanying changes of $e_{n_F,d}^*$ and $f_{n_F,d}^*$ are reductions. Otherwise, all three policy instruments would increase, leading to a larger cost and, therefore, to a smaller welfare.

An isolated increase of d reduces the factor $(1 - d)$ and it increases the value of the function $k(d)$ and, therefore, the probability of detection, $p(n_F, e, f, d)$. To examine the overall effect of d on $p(n_F, e, f, d) f (1 - d)$, the following result is helpful.

Corollary 1 *The first order derivative of the expected fine, $p(n_F, e, f, d) f (1 - d)$, with respect to the expected discount, d , is*

$$\frac{\partial [p(n_F, e, f, d) f (1 - d)]}{\partial d} = fp(n_F, e, f, d) \left[\frac{1 - d}{(d + \rho) [(d + \rho)^2 + 1]} - 1 \right] \geq 0 . \quad (43)$$

Furthermore, $\partial^2 [p(n_F, e, f, d) f (1 - d)] / (\partial d)^2 < 0$.

Proof: See Appendix F.

A negative derivative (43) would imply that the expected fine falls as d increases. This would make cartel membership more attractive. Also, social cost increases when d increases. Therefore, an increase in d cannot be welfare increasing. Can this situation arise even for $d = 0$? Recall that the parameter ρ of the function $k(d)$ indicates how insensitively p reacts to changes in d . A large ρ -value says that the positive effect of a d -increase on p is small. If this positive effect is smaller than the reduction in $(1 - d)$, the expected fine would fall. One can compute that for $\rho \geq 0.68233$ and $d = 0$ an increase of d would reduce the expected fine. Therefore, the cost minimizing expected discount would be $d_{n_F}^* = 0$.

For sufficiently small values of ρ and d , however, the derivative (43) is positive. Then an increase in d allows for reductions of $e_{n_F,d}^*$ and $f_{n_F,d}^*$ such that the former value of the expected fine is restored. Note, however, that this policy change is welfare increasing, if and only if it reduces the social cost, $s(z)$ with $z = \alpha e + \beta f^2 / (f^2 + 1)^{1/2} + \gamma d$. The smaller is γ relative to α and β , the greater the likelihood of an overall cost reduction.

To summarize, if for $d = 0$ the derivative (43) is negative, the cost minimizing expected discount ensuring the existence of n_F fringe firms is $d_{n_F}^* = 0$. Even for a positive derivative

(43) we may obtain $d_{n_F}^* = 0$. Only if (43) is positive and at the same time γ is sufficiently small relative to α and β , a positive cost minimizing expected discount arises: $d_{n_F}^* > 0$.

Formally, we can find $d_{n_F}^*$ by the following procedure. In expression (38), we substitute variable e by the right hand side of Equation (39). The new expression is differentiated with respect to d . Setting this derivative equal to 0 and solving for d would give the cost minimizing expected discount $d_{n_F}^*$. Unfortunately, the derivative is complex and we have to utilize a numerical solution process. Usually, this process generates some cost minimizing expected discount $d_{n_F}^* > 0$. However, if ρ is too large and/or γ is very large relative to α and β , the derivative is positive for all $d \in [0, 1)$. Then, the corner solution $d_{n_F}^* = 0$ arises.

Finally, $d_{n_F}^*$ is inserted in (39) to obtain the cost minimizing effort, $e_{n_F}^*$. Inserting this effort in (41) and the resulting F -value in (40) gives the cost minimizing fine, $f_{n_F}^*$. This process of finding the cost minimizing policy $(e_{n_F}^*, f_{n_F}^*, d_{n_F}^*)$ is executed for each given n_F .

To summarize, Theorem 3 defines the cost minimizing policy for $n_F = n_F^{\min}$. This policy is $(e_{n_F^{\min}}^*, f_{n_F^{\min}}^*, d_{n_F^{\min}}^*) = (0, 0, 0)$. For every $n_F \in (n_F^{\min} + 1, \dots, n - 1)$, Theorem 4 and the associated computation of $d_{n_F}^*$ yield the cost minimizing policy $(e_{n_F}^*, f_{n_F}^*, d_{n_F}^*)$. We illustrate the compilations of $(e_{n_F^{\min}}^*, f_{n_F^{\min}}^*, d_{n_F^{\min}}^*)$ and $(e_{n_F}^*, f_{n_F}^*, d_{n_F}^*)$ in the numerical example of Section 7.2.

7.2 Numerical Illustration

Here we present a numerical example that illustrates the three stages of compiling the optimal antitrust policy. Suppose that $n = 10$ and that the social cost function is

$$s(\alpha e + \beta f m(f) + \gamma d) = \left(0.1e + 0.1 \frac{f^2}{(f^2 + 1)^{1/2}} + d \right)^{3/2}. \quad (44)$$

Let the market volume, $(a - c)/b$, be given by $a = 100$, $b = 1$, and $c = 10$. Furthermore, we assume that $\rho = 0.2$.

Stage 1: The lowest relevant number of fringe firms, n_F^{\min} , is obtained from (29): $n_F^{\min} = 4$. From Theorem 2 we know that $n_F = n = 10$ generates a lower welfare than $n_F = 9$. Therefore, we can confine the analysis to $n_F \in (4, \dots, 9)$.

Stage 2: We know that the cost minimizing effort and fine for generating a stable cartel with $n - n_F^{\min} = 10 - 4 = 6$ members, is the passive policy $(e, f, d) = (0, 0, 0)$. This policy is listed in the second, third, and fourth column of Table 1 in the line corresponding to $n_F = 4$.

Raising the number of fringe firms above four requires an active antitrust policy that makes the six firm cartel internally unstable and transforms it into a stable five firm cartel that competes against five fringe firms ($n_F = 5$). Inserting all parameter values in (21) gives $T(5) = 11.405$. Therefore, the antitrust policy must be such that Equation (16) gives

Table 1: Identifying the Optimal Anti-Trust Policy.

n_F	$e_{n_F}^*$	$f_{n_F}^*$	$d_{n_F}^*$	P	<i>rents</i>	$s(z)$	W
4	0	0	0	19	4009.50	0	4009.50
5	3.09	10.11	0.20	17.5	4021.87	1.87	4020.00
6	4.66	22.22	0.25	16.43	4029.33	5.02	4024.31
7	5.81	34.31	0.27	15.62	4034.18	8.85	4025.33
8	7.09	50.71	0.28	15.00	4037.50	14.92	4022.58
9	9.08	82.91	0.30	14.50	4039.87	29.26	4010.61

$A = 11.405$. In view of (33) and (35), this requires a positive effort, e , and a positive fine, f . Whether the antitrust authority should also introduce a positive expected discount, d , depends on the numerical values of the parameters. Inserting all parameter values in (43) and setting the result equal to 0 gives the value $d = 0.32687$. Increasing d up to this level would allow for lower values of e and f without changing the expected fine, $p(n_F, e, f, d) f (1 - d)$. Whether the cost reductions from e and f overcompensate the cost increase from d must be examined.

To compute the cost minimizing expected discount, $d_{n_F}^*$, we follow the procedure described in Section 6.4. Equation (39) of Theorem 4 defines for $n_F = 5$ the cost minimizing effort, $e_{5,d}^*$, as a function of the expected discount, d . Together with $T(5) = 11.405$, we insert this function in expression (38) and differentiate the result with respect to d . Setting this derivative equal to 0, gives the cost minimizing expected discount $d_5^* = 0.2$. Inserting this number back in Equation (39), yields the cost minimizing effort $e_5^* = 3.09$. We insert this number in Equation (41) of Theorem 4 and the resulting F -value in Equation (40) to obtain the cost minimizing fine $f_5^* = 10.11$. Putting everything together, we have derived the cost minimizing policy, $(e_5^*, f_5^*, d_5^*) = (3.09, 10.11, 0.2)$. In Table 1, this policy is listed in the line corresponding to $n_F = 5$.

The cost minimizing policies for $n_F = 6$ to $n_F = 9$ are compiled in an analogous manner. The results are also listed in Table 1.

Stage 3: Finally, we compute for all cost minimizing policies the resulting welfare and select the policy that leads to the largest welfare. Welfare is defined by the welfare function (27), that is, by subtracting the social cost from the sum of consumer and producer rent.

The passive policy $(e, f, d) = (0, 0, 0)$ leads $n_F = 0$. The resulting market price can be computed from Equation (2). The result is listed in the fifth column of Table 1. Equation (25) yields the associated sum of consumer and producer rent (see sixth column). With a passive policy, no social cost arises (see seventh column). Therefore, the associated welfare is identical to the sum of consumer and producer rent. The resulting welfare is $W_4 = 4009.5$ (see last column). This welfare level is the reference for any active antitrust policy. A policy that generates a welfare level below 4009.5 cannot be an optimal antitrust policy.

To increase the number of fringe firms from $n_F = 4$ to $n_F = 5$, the antitrust authority increases its effort, e , its fine, f , and its expected discount, d . These measures increase the expected fine, $p(n_F, e, f, d) f (1 - d)$, such that internal instability is created, inducing one cartel member to mutate into a fringe firm. The additional fringe firm intensifies competition, leading to a lower equilibrium price and to a larger sum of consumer and producer rent. In the numerical example the equilibrium price falls from $P = 19$ to $P = 17.5$, while the sum of the rents increases from 4009.5 to 4021.87, that is by 12.37. We denote this beneficial welfare effect as the positive “competition effect” of the additional fringe firm. Note, however, that the additional fringe firm also causes a negative welfare effect, because the larger values of e , f , and d raise the social cost from 0 to 1.87. We denote this second welfare effect as the negative “cost effect”. In our numerical example, the positive competition effect outweighs the negative cost effect, such that welfare increases from $W_4 = 4009.5$ to $W_5 = 4020$.

More formally, the positive competition effect of increasing n_F by one is given by Equation (26). For all positive values of n_F , this expression is positive and falling in $n_F \in (0, 1, \dots, N)$. In other words, the positive competition effect falls as n_F rises. Also, the negative cost effect is a function in n_F . However, this function is more complex than the competition effect (26). The negative cost effect usually increases in n_F . Only for very small values of n_F can the cost effect stay roughly constant. Since for small values of n_F the competition effect is quickly decreasing in n_F , also for small n_F -values the welfare change (difference between the competition effect and the cost effect) falls as n_F increases. Imagine that already at n_F^{\min} the negative cost effect outweighs the positive competition effect. Then we get $n_F^* = n_F^{\min}$ and the passive policy $(e, f, d) = (0, 0, 0)$ would be optimal. However, in our numerical example the positive competition effect outweighs the negative cost effect, such that the expansion of the fringe above n_F^{\min} leads to an overall welfare gain.

The maximum welfare is obtained from a policy that leads to $n_F = 7$: $(e^*, f^*, d^*) = (5.81, 34.31, 0.27)$. The corresponding welfare is $W^* = W_7 = 4025.33$. This solution corresponds to a three firm cartel. Reducing the size of the cartel even further would require a larger effort, fine, and discount. However, the associated cost effect outweighs the competition effect.

7.3 Further Discussion

In the numerical example of Section 7.2 the parameter values were given. How does a change in the parameter values affect the optimal antitrust policy (e^*, f^*, d^*) ? We distinguish between the social cost parameters $(\alpha, \beta, \gamma, \rho)$, the market volume parameters (a, b, c) , and the number of firms n .

We know that the optimal policy, (e^*, f^*, d^*) , must be such that the profit from remaining a fringe firm, $\pi_F(n_F)$, and the expected profit from entering the cartel, $\pi_C(n_F - 1) - p(n_F, e, f, d) f (1 - d)$, are identical. The efficacy condition (42) rearranges this relationship. The expected fine, $p(n_F, e, f, d) f (1 - d)$, of a member of a cartel with $(n - n_F + 1)$

members must be equal to the difference $\pi_C(n_F - 1) - \pi_F(n_F)$, where

$$\pi_C(n_F - 1) - \pi_F(n_F) = \frac{(a - c)^2}{4b} \left(\frac{1}{n_F(n - n_F + 1)} - \frac{1}{(n_F + 1)^2} \right). \quad (45)$$

Social cost parameters ($\alpha, \beta, \gamma, \rho$): A small increase of α , β , or γ does not affect the optimal number of fringe firms n_F^* and, therefore, the consumer and producer rent. However, such parameter changes increase the social cost and, therefore, reduce welfare. The changes in e^* , f^* , and d^* must be such that the expected fine, $p(n_F, e, f, d) f(1 - d)$, remains constant. An increase in γ leads to a lower d^* -value, but to larger values of e^* and f^* . For given d , an increase in α leads to a lower effort and a larger fine. The derivative (43) shows that a larger fine increases the effect of an increase of d on the expected fine. This leads to an increase in d , to an additional reduction of effort, and to a secondary negative effect on f that partly offsets the primary increasing effect on f . Overall, we get a smaller effort e^* , a larger fine f^* , and a larger discount d^* . An increase of β results in a larger effort e^* and a smaller fine f^* . The discount d^* is also reduced, because the derivative (43) reveals that a smaller fine reduces the effect of an increase of d on the expected fine. An increase in ρ reduces the positive effect of d on the expected fine. This results in a smaller discount d^* , a larger effort e^* , and a larger fine f^* .

When the parameter changes are sufficiently large, they change the optimal number of fringe firms n_F^* . A sufficiently strong reduction in the social cost parameters α , β , γ , and ρ lowers the social cost of an antitrust policy. This allows the antitrust authority to expand its policy, inducing one cartel member to leave the cartel and to become a fringe firm. The resulting positive competition effect outweighs the negative social cost effect.

Market volume (a, b, c): A small increase in a or a small reduction in c or b increases the market volume without changing the optimal number of fringe firms n_F^* . Equation (25) reveals that the sum of consumer and producer rent increases. This is the welfare increasing effect. We know from Equation (45) that a larger market volume leads to an increased difference $\pi_C(n_F - 1) - \pi_F(n_F)$. To balance this increase, the expected fine must increase, too. This is accomplished by a larger effort e^* and a larger fine f^* . The latter increases the partial derivative $\partial [p(n_F, e, f, d) f(1 - d)] / (\partial d)$. As a consequence, the discount d^* increases, too. In other words, keeping the number of fringe firms constant requires a larger social cost offsetting some of the welfare gain related to the increased producer and consumer rent.

When a sufficiently strong change in market volume occurs, the optimal number of fringe firms, n_F^* , can change. The direction of the change depends on the social cost function. In our numerical example we have chosen the exponential cost function (44). With this cost function, an increase of the market volume leads to a *reduction* of n_F^* . If the exponent of the cost function were not $3/2$, but $1/2$, the optimal number of fringe firms would increase with market volume.

Number of firms n : For given n_F , the increase in n is an enlargement of the cartel. The profits of the fringe firms are not affected by the additional cartel member. This is also true for the aggregate profit of the cartel. However, the profit per cartel member falls and, therefore, the attractiveness of the cartel status. Equation (45) confirms this

consideration. The difference $\pi_C(n_F - 1) - \pi_F(n_F)$ decreases. Furthermore, we know from Equation (9) that larger cartels increase the probability of detection, $p(n_F, e, f, d)$. Therefore, the expected fine, $p(n_F, e, f, d) f (1 - d)$, increases. Both effects lower the values of the optimal policy variables e^* , f^* , and d^* . This saves social cost and increases welfare.

If the reduction in the difference $\pi_C(n_F - 1) - \pi_F(n_F)$ and the increase in the expected fine, $p(n_F, e, f, d) f (1 - d)$, are sufficiently strong, an increase of the number of fringe firms could be welfare increasing. In fact, in our numerical illustration, raising the number of firms from $n = 10$ to $n = 11$ increases the optimal number of fringe firms from $n_F^* = 7$ to $n_F^* = 8$. For $n = 12$, the optimal number of fringe firms stays at $n_F^* = 8$.

8 Concluding Remarks

Studies of antitrust policy are typically embedded in repeated oligopoly games (super-games) with two market outcomes: “no cartel” or “all-inclusive cartel”. However, real world cartels are rarely all-inclusive. They are usually “imperfect”, that is, they compete against some fringe firms. An effective antitrust policy must adjust to this situation.

The existing literature on imperfect cartels is not concerned with the derivation of an optimal antitrust policy. This paper makes a first step in that direction. We demonstrated how a deliberate antitrust policy can exploit the potential instability of cartels to reduce their size. The formal analysis was conducted in the context of a quantity leadership model that was extended by including an antitrust authority. We introduced a carefully specified endogenous probability of detection. To increase this probability, the antitrust authority has three policy instruments at its disposal: its own effort, a fine for detected cartels, and a leniency program for cartel members that cooperate with the authority. Each of these instruments creates a social cost. Taking this cost into consideration, we derived an optimal antitrust policy that makes cartels internally instable (some member wants to leave the cartel) and externally stable (no fringe firm wants to enter the cartel). We showed that a policy that completely blocks the formation of cartels would be too costly. Instead, the antitrust authority should merely reduce the size of the cartel until the resulting gains in the sum of consumer and producer rent (the positive competition effect) no longer overcompensate the resulting increase in social cost (the negative cost effect).

Furthermore, we examined how changes in the market environment affect the optimal antitrust policy. We distinguish between social cost parameters (e.g., the efficiency of the antitrust authority’s operations), market volume parameters, and the total number of firms. Small parameter changes affect the optimal mix of the antitrust policy instruments and the resulting welfare, but not the optimal number of fringe firms. However, larger parameter variations can change the optimal number of fringe firms. For example, a strong reduction in the social cost parameters leads to an increase in the optimal number of fringe firms, that is, to a reduction of the number of cartel members. Also an increase in the total number of firms (fringe firms plus cartel members) is welfare increasing, because

the increase in the social cost of the optimal antitrust policy is overcompensated by the increase in the sum of consumer and producer rent.

Our extended quantity leadership model allows for the derivation of explicit solutions. One disadvantage of the quantity leadership model is its implicit assumption that cartel members cooperate efficiently. Enforceability of the cartel agreement is not an issue. By contrast, in supergames the cartel members do not cooperate at all. Enforceability of any cartel agreement is completely ruled out. The empirical evidence shows that the truth is somewhere in the middle. Cartels are impressively creative in designing cartel agreements that allow for some form of monitoring and dispute settlement. Therefore, a promising area of future research are oligopoly models that analyse the effects of antitrust policy directed at cartels with limited means of cooperation.

Appendix A

Proof of Lemma 1: (15) can be written as

$$\bar{T}(n_F) = \frac{(a-c)^2}{4b} \frac{I(n_F)}{g(n-n_F)}$$

with

$$I(n_F) = \frac{2n_F^2 + (5-n)n_F + 4 - n}{(n_F + 1)(n - n_F)(n_F + 2)^2}. \quad (46)$$

Using the quotient rule for the differentiation of (46) with respect to n_F , gives the positive denominator

$$(n_F + 1)^2(n - n_F)^2(n_F + 2)^4 > 0$$

and the numerator

$$(4n_F + 5 - n)(n_F + 1)(n - n_F)(n_F + 2)^2 \quad (47)$$

$$- [2n_F^2 + (5 - n)n_F + 4 - n](n_F + 2)^2(n - n_F) \quad (48)$$

$$- [2n_F^2 + (5 - n)n_F + 4 - n]2(n_F + 2)(n_F + 1)(n - n_F) \quad (49)$$

$$+ [2n_F^2 + (5 - n)n_F + 4 - n](n_F + 1)(n_F + 2)^2. \quad (50)$$

The expression in line (47) is equal to

$$(2n_F + 5 - n)(n_F + 1)(n - n_F)(n_F + 2)^2 + 2n_F(n_F + 1)(n - n_F)(n_F + 2)^2. \quad (51)$$

We add to the first summand of (51) the expression in line (48) and obtain

$$(2n_F + 1)(n - n_F)(n_F + 2)^2 > 0.$$

Next we add to the second summand of (51) the expressions in lines (49) and (50), factor out $(n_F + 2)(n_F + 1)$, simplify the remaining term to get

$$\begin{aligned} & n_F [4n_F^2 + 5(3-n)n_F + 2n^2 - 11n + 22] + 2n^2 - 10n + 8 \\ &= n_F \left[\left(2n_F + \frac{5}{4}(3-n) \right)^2 + \frac{7}{16}(3-n)^2 + n + 4 \right] + 2n^2 - 10n + 8. \end{aligned}$$

For $n_F \geq 1$, this expression and, therefore, the expression in lines (47) to (50) are positive and so is the derivative of $I(n_F)$ with respect to n_F :

$$I(n_F + 1) > I(n_F) .$$

In addition, $g(n - n_F)$ is increasing in $(n - n_F)$, and therefore, decreasing in n_F :

$$g(n - (n_F + 1)) < g(n - n_F) .$$

Therefore, we get

$$\frac{I(n_F + 1)}{g(n - (n_F + 1))} > \frac{I(n_F)}{g(n - n_F)}$$

which is identical to $\bar{T}(n_F + 1) > \bar{T}(n_F)$.

Let N denote the numerator and D the denominator of (15). We know that $\partial g(n - n_F)/\partial n > 0$ and $D > 0$. The derivative of (15) with respect to n is

$$\begin{aligned} & \frac{(-n_F - 1)D - (n_F + 1)(n_F + 2)^2 [[\partial g(n - n_F)/\partial n] (n - n_F) + g(n - n_F)] N}{D^2} \\ & = \frac{-(n_F + 1) - (n_F + 1)(n_F + 2)^2 [[\partial g(n - n_F)/\partial n] (n - n_F) + g(n - n_F)] \bar{T}(n_F)}{D} . \end{aligned}$$

For $\bar{T}(n_F) \geq 0$, this derivative is negative. ■

Appendix B

Equation (25) shows the sum of consumer and producer rent. When the number of fringe firms increases from n_F to $n_F + 1$, this sum changes by

$$\begin{aligned} & \frac{(a - c)^2}{8b} \left(\frac{(2(n_F + 1) + 1)(2(n_F + 1) + 3)}{((n_F + 1) + 1)^2} - \frac{(2n_F + 1)(2n_F + 3)}{(n_F + 1)^2} \right) \\ & = \frac{(a - c)^2}{8b} \left(\frac{(2n_F + 3)(2n_F + 5)(n_F + 1)^2}{(n_F + 2)^2(n_F + 1)^2} - \frac{(2n_F + 1)(2n_F + 3)(n_F + 2)^2}{(n_F + 1)^2(n_F + 2)^2} \right) \\ & = \frac{(a - c)^2}{8b} \left(\frac{(2n_F + 3)(2n_F + 5)(n_F + 1)^2 - (2n_F + 1)(2n_F + 3)(n_F + 2)^2}{(n_F^2 + 3n_F + 2)^2} \right) \\ & = \frac{(a - c)^2}{8b} \left(\frac{(2n_F + 3)((2n_F + 5)(n_F + 1)^2 - (2n_F + 1)(n_F + 2)^2)}{(n_F^2 + 3n_F + 2)^2} \right) \\ & = \frac{(a - c)^2}{8b} \left(\frac{2n_F + 3}{(n_F^2 + 3n_F + 2)^2} \right) . \end{aligned}$$

Appendix C

Proof of Theorem 2: We know that the price (2) associated with $n_F = n - 1$ is smaller than the price (7) associated with $n_F = n$. As a consequence, the sum of consumer and producer rent is larger for $n_F = n - 1$ than for $n_F = n$. From Lemma 1 we know that $\bar{T}(n_F)$ is increasing in n_F . Therefore, a policy that eliminates the single firm cartel must increase the value of A from $T(n - 1) \leq A < \bar{T}(n - 1)$ to $A \geq \bar{T}(n - 1)$. From (16) and the definition of $h(e)$ we know that $\partial A / \partial e > 0$ and $\partial A / \partial f > 0$. Therefore, the increase in A required to eliminate the single firm cartel necessitates larger values of e and f . These larger values translate into a higher social cost. The increase in social cost and the fall in the sum of consumer and producer rent result in a lower welfare. ■

Appendix D

The probability of detection specified by (9) is the product of three functions that all belong to the same general class of functions. This class is defined by

$$y(x) = \frac{x + \rho}{\left((x + \rho)^\theta + 1\right)^{1/\theta}},$$

with $\theta > 0$ and $\rho \geq 0$. The first order derivative is

$$y'(x) = y(x) \frac{1}{(x + \rho) \left[(x + \rho)^\theta + 1\right]} = \frac{1}{\left[(x + \rho)^\theta + 1\right]^{(1+\theta)/\theta}}. \quad (52)$$

Note that $0 < y'(x) < 1$ for all $x \geq 0$. The second order derivative is

$$y''(x) = -y(x) \left(\frac{(x + \rho)^\theta + \theta}{(x + \rho)^2 \left[(x + \rho)^\theta + 1\right]^2} \right) < 0.$$

Furthermore

$$\lim_{x \rightarrow \infty} y(x) = 1$$

and

$$y(0) = \frac{\rho}{(\rho^\theta + 1)^{1/\theta}}.$$

Therefore, the parameter ρ determines the minimum value that the function $y(x)$ can take. For $\rho = 0$ we get $y(0) = 0$.

Equation (52) implies that

$$\frac{y'(x)}{[y(x)]^2} = \frac{\left[(x + \rho)^\theta + 1\right]^{1/\theta}}{(x + \rho)^2 \left[(x + \rho)^\theta + 1\right]}.$$

For $\theta = 1$ and $\rho = 0$, this expression simplifies to

$$\frac{y'(x)}{[y(x)]^2} = \frac{1}{x^2}. \quad (53)$$

As depicted in Figure 1, the function $g(x) = x \cdot y(x)$ with $y(x) = (x + 1) / [(x + 1)^2 + 1]^{0.5}$ approximates the function $v(x) = x$.

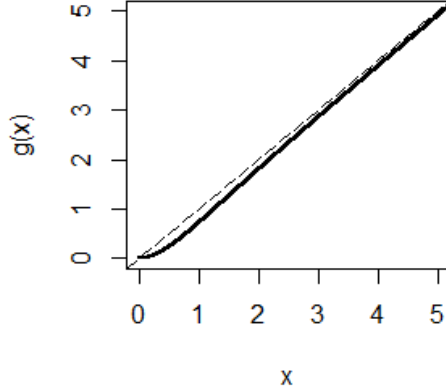


Figure 1: Graph of the Function $g(x) = x \cdot y(x)$.

Appendix E

Proof of Theorem 4: Differentiating the expression in (38) with respect to e , and setting the result equal to 0 yields

$$\frac{[w(e)]^2}{w'(e)} = \frac{\beta}{\alpha} \frac{T(n_F)}{k(d)(1-d)}. \quad (54)$$

Exploiting relationship (53) for the left hand side of (54), inserting expression (34) in the right hand side of (54), and taking the square root, gives the optimal effort (39).

Expression (33) gives

$$w(e_{n_F}^*) = \frac{e_{n_F}^*}{[(e_{n_F}^*)^2 + 1]^{1/2}}. \quad (55)$$

We know from Equation (39) that

$$\frac{T(n_F)}{k(d)(1-d)} = (e_{n_F}^*)^2 \frac{\alpha}{\beta}. \quad (56)$$

Inserting (55) and (56) in (36) gives

$$f_{n_F}^* m(f_{n_F}^*) = \frac{\alpha}{\beta} e_{n_F}^* \left((e_{n_F}^*)^2 + 1 \right)^{1/2} .$$

Substituting $m(f_{n_F}^*)$ by the function defined in (35) and solving for $f_{n_F}^*$ yields the optimal fine defined by (40). ■

Appendix F

Proof of Corollary 1: The first order derivative is

$$\begin{aligned} \frac{\partial [pf(1-d)]}{\partial d} &= f \left[\frac{\partial p}{\partial d} (1-d) - p \right] \\ &= f \left[\frac{\partial p}{\partial k(d)} \frac{\partial k(d)}{\partial d} (1-d) - p \right] \\ &= pf \left[\frac{1}{k(d)} \frac{\partial k(d)}{\partial d} (1-d) - 1 \right] \\ &= pf \left[\frac{1}{k(d)} \frac{k(d)}{(d+\rho) [(d+\rho)^2 + 1]} (1-d) - 1 \right] \\ &= pf \left[\frac{1-d}{(d+\rho) [(d+\rho)^2 + 1]} - 1 \right] \end{aligned}$$

which is (43). The second order derivative is

$$\frac{\partial^2 [pf(1-d)]}{(\partial d)^2} = pf \left[\frac{-d(d+\rho) [(d+\rho)^2 + 1] - (1-d) [3(d+\rho)^2 + 1]}{(d+\rho)^2 [(d+\rho)^2 + 1]^2} \right] < 0 .$$

■

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