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Anatomy of Regional Price Differentials: Evidence From Micro Price Data

Sebastian Weinand
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Anatomy of Regional Price Differentials: Evidence From Micro Price Data*

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Abstract

Our paper uses micro price data collected from Germany’s Consumer Price Index to compile a highly disaggregated regional price index for the 402 counties and cities of Germany. We introduce a multi-stage version of the weighted Country-Product-Dummy method. The unique quality of our price data allows us to depart from previous spatial price comparisons and to compare only exactly identical products. We find that the price levels are spatially autocorrelated and largely driven by the cost of housing. The price level in the most expensive region is about 27 percent higher than in the cheapest region.

Keywords: spatial price comparison, regional price index, PPP, CPD-method, hedonic regression, consumer price data.

JEL Classification: C21, C43, E31, O18, R10.

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1 Introduction

When the International Comparison Program (ICP) was created in 1968, it narrowed a gaping hole in economic statistics. The ICP’s price level estimations facilitated international comparisons of real economic indicators such as the countries’ real GDP, real growth, real per capita income, real investment, real wages, real income distributions, living standards, and poverty rates. The fact remains, however, that the regional differences within countries can be much larger than the differences between countries. Therefore, the ICP’s international comparisons are not sufficient. Comparable price levels and real economic indicators are also needed on the sub-national level. For example, such information is needed for tracking the progress of regional cohesion and for the design of effective social policies. Furthermore, several economic theories can be best put to the test on the basis of regional real economic indicators. Examples are the urban wage premium (e.g. Glaeser and Maré, 2001; Wheeler, 2006; Yankow, 2006), the wage curve theory (e.g. Blanchflower and Oswald, 1995), and the contradictory results of Krugman (1991) and Südekum (2009) concerning the price level differentials between urban and rural regions.

Therefore, the natural extension of the ICP would be National Comparison Programs administered by the national statistical offices cooperating with the ICP. If these offices were completely free to design a data collection process for the purpose of regional price level comparisons, they would subdivide their respective country into many small rural, urban, and metropolitan regions. Then they would draw up a long list of extremely tightly defined representative products (henceforth, we use this term for goods and services) and would record each product’s prices in those regions in which the product is representative. They would complement these prices by data on the regional cost of housing. Based on such an “ideal price data set” the statistical office would be able to regularly compile a regional price index for the complete country.

Even though some attempts in this direction have been undertaken, a sustainable procedure with a thoroughly regionalised data collection process has not yet been established. Official regional price comparisons are currently published by the Office of National Statistics (ONS) of the United Kingdom (e.g. Wingfield et al., 2005; ONS, 2018), by the Bureau of Economic Analysis (BEA) of the United States (e.g. Aten, 2017), and by the Government of Western Australia (GoWA, 2017). The latter index draws on prices from 27 major cities in Western Australia, while the BEA index utilises the prices from 35 metropolitan and 3 urban areas in the United States. The ONS visits 21 locations across the United Kingdom. Considerable thought and resources have been devoted to the compilation of these data sets. Nevertheless, the regions are very large and inhomogeneous (e.g. Scotland is one region) and/or parts of the country are not included in the analysis (e.g. rural U.S. regions). Therefore, none of the data sets can be considered as “ideal”. Notwithstanding
these deficiencies, the official price indices of Western Australia, the United States, and the United Kingdom represent a highly welcome achievement that may encourage other countries to establish similar projects.

Theoretically, compiling an “ideal price data set” appears feasible, because most national statistical offices have decided to collect their Consumer Price Index (CPI) data from different regions. However, the number of sampled regions is usually too small to exploit the price data for a comprehensive interregional price comparison. The Federal Statistical Office of Germany (Statistisches Bundesamt) is a notable exception. It collects its CPI data from about 400 different regions. Though not designed for the purpose of regional price comparisons, it is worldwide probably the best data source for that purpose. It contains not only the prices of all individual products, but also their precise specifications and their outlet types. Furthermore, it includes a large sample of rents along with detailed information about the characteristics of the respective flats and houses.

We utilise this unique data set as our principal data source to compile a spatial price comparison for the 402 regions (295 counties and 107 cities) of Germany. This is the first contribution of our paper. It is the first time that CPI data has been used to create an interregional price comparison that includes the complete household consumption basket for all regions of a complete major industrial country where the average regional size is below 1,000 square kilometre (the size of Scotland is 80,077 square kilometre).

In interregional (and intertemporal) price comparisons it is usual practice to begin the computational procedure by assigning seemingly equivalent products to a group of comparable products (e.g. branded plain yoghurt, 125 grams). The prices of all products assigned to the same group are considered as directly comparable. The initial grouping of products into groups of comparable products, however, may generate tainted price data material giving rise to biased regional price indices (e.g. Silver and Heravi, 2005, p. 463; Silver, 2009, pp. 8-9). This potential contamination is particularly problematic for national statistical offices, because their interregional price indices quite likely find their way into contracts and other legal documents. As a consequence, national statistical offices are extremely reluctant to adopt any methodology that could be challenged in a legal dispute. Working with potentially contaminated price data is such a methodology.

The potential for biased regional price levels depends not only on the degree of contamination in the price material but also on the applied estimation method which, in turn, depends on the completeness of the data. In CPI data sets, very few groups of comparable products are recorded in all regions. A popular method to deal with these data gaps is the Country-Product-Dummy (CPD) approach pioneered by Summers (1973). It regresses the prices of the product groups on two sets of dummy variables. The first set represents the regions (or countries), while the second set represents the various product groups.
If the quality mix of seemingly comparable products assigned to the same product group differs between regions (e.g. higher quality in richer regions), a CPD regression would generate biased estimates of the regional price deviations. To avoid this bias, Kokoski (1991, p. 32), Kokoski et al. (1999, p. 138), and Silver (2009, pp. 13-15) advocate a hedonic CPD regression that expands the set of regressors by variables that capture the qualitative characteristics of the individual products (e.g. taste, design, storage life, outlet type,...). Such an approach relies on the assumption that the impact of the qualitative characteristics on the price is identical for all regions and groups of comparable products. If this assumption is untenable, the regression equation must be further inflated by interaction terms between regional dummies and qualitative characteristics. In our own experimentation with hedonic CPD regressions we also encountered practical problems. Our CPI micro data cover the whole range of consumer products. Even though these data usually contain all the information necessary to unambiguously identify the product, this same information is often insufficient to describe the product’s qualitative characteristics in a satisfactory way. As a consequence, the automation of hedonic CPD regressions turned out to be complex and prone to error.

Therefore, we introduce an alternative approach that rigorously minimises the potential for contaminated price data and, in the context of our own comprehensive CPI data set, is easier to implement into an automated compilation process. Since we know not only the prices of the individual products but also their complementary attributes (precise specification and outlet type), we refrain from any grouping of products into groups of comparable products. Instead, we identify pairs of perfectly matching products. The complementary attributes of such a pair coincide in every respect, except for the region. This Perfect Matches Only (PMO) precept rejects all products that have been observed in only one region, because they are likely to introduce bias in the CPD regression. This bias could be avoided, only if for each basic heading a separate hedonic CPD regression was implemented that includes information on all relevant characteristics. As pointed out before, CPI data usually do not contain this information and, in view of the large number of basic headings, the associated workload would be prohibitive.

The PMO precept defines for each individual product its own vector of regional prices, while the traditional grouping approach defines such a vector for every group of seemingly comparable products. Therefore, with the PMO precept, the number of price vectors is much higher. The gaps within these vectors, however, are larger than in the grouping approach. To deal with these gaps, we embed our PMO precept into the weighted CPD approach advocated by Rao (2001, 2005), Hajargasht and Rao (2010), and Diewert (2005).

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1 This issue is well known from the ICP 2005 where CPD regressions use average prices of product groups. Hill and Syed (2015, p. 524) convincingly demonstrate that this practice is inferior to a CPD regression that is based on individual price quotes. We fully agree with this assessment and add the recommendation that each product dummy must relate to a tightly defined product and not to a group of seemingly very similar products.
We develop a multi-stage variant of this approach. It allows us to analyse our rent data by a separate full-fledged hedonic regression and to merge the resulting regional rent index with the regional price indices derived from the price data. Furthermore, this method solves an analytical problem posed by data confidentiality regulations of the Federal Statistical Office of Germany.\(^2\) We believe that our multi-stage CPD regression based on the PMO precept represents, if not a completely new approach, an important addition to the methodologies available for interregional price comparisons. This is the second contribution of our paper.

Our work demonstrates that national statistical offices with a sufficiently regionalised CPI data collection procedure are able to produce, as a byproduct, a reliable regional price index. The actual implementation must respect the specifics of the respective country. Our elaborate multi-stage CPD approach based on the PMO precept offers considerable flexibility and, in our view, ensures the highest possible degree of accuracy. Therefore, we advocate it as a useful reference for future interregional price comparison projects. For such projects it would be interesting to know whether simplified compilation procedures (e.g. grouping of seemingly comparable products) strongly influence the result. The high accuracy of our reference approach allows us to come up with a sound answer. This is our paper’s third contribution.

Its final contribution is an examination of some widely held beliefs that are often based on anecdotal rather than systematic empirical evidence. For example, most economists think that in industrial countries the regional dispersion of housing costs exceeds that of prices of services and even more so that of goods. It is unknown, however, how strong the differences in the dispersion are. Furthermore, it is believed that, with a sufficient level of spatial disaggregation, the regional price levels change only gradually between neighbouring regions.

The remainder of the paper is laid out as follows. Section 2 provides an overview of the available empirical studies on interregional price comparisons. Section 3 describes the data set underlying our own investigation. The applied methodology is explained in Section 4. Section 5 presents the results and Section 6 concludes.

2 Literature Review

Regional price level comparisons differ with respect to their geographical features, their data sources, and their methods for transforming these data sources into regional price levels. The geographical features include not only the country and its coverage (partial or full), but also the size and the number of regions. Table 1 provides an overview of the

\(^2\) The expenditure data necessary for the weighting could be incorporated into the analysis only after one stage of aggregation of the original price data.
various studies and some of their main features.³

**Country:** Currently, official regional price indices exist only for the United Kingdom (ONS, 2018), the United States (Aten, 2017), and Western Australia (Government of Western Australia: GoWA, 2017). For several countries, however, exploratory studies exist: Australia, Brazil, China, the Czech Republic, Germany, India, Italy, Philippines, Poland, and Vietnam (see column “COUNTRY” of Table 1). Janský and Kolvunová (2017) attempt to estimate a regional price index for the complete EU28.

**Coverage:** Regional price level measurement requires regional information. For some regions such information may not be available. Therefore, some studies cover only parts of the country (see column “COV” of Table 1). When the complete country is covered, the regions are usually very large. In most cases, a region’s data are collected from a single metropolitan area within the respective region.⁴

**Size and Number of Regions:** The number of regions ranges from 3 to 440 (see column “#REG.” of Table 1), while the average size of the regions ranges from 761 to 1,098,857 square kilometre (see column “SIZE” of Table 1).

**Primary Data Source:** None of the listed studies is based on an “ideal price data set”. The studies by BBSR (2009), Kawka (2010), ONS (2018), and Ströhl (1994) are special, because they utilise price data that were collected specifically for that purpose. This is a laborious and expensive task. The collection process of the price data for BBSR (2009) and Kawka (2010) took three years. Due to cost considerations, Ströhl (1994) had to confine his analysis to 50 German cities and the ONS (2018) had to content itself with a disaggregation of Britain into 12 large regions. All other studies rely on price data that have been collected for other purposes (see column “DATA” of Table 1). Several of these studies utilise CPI data. Very few studies can draw on micro price data. In many non-OECD countries, sufficiently regionalised CPI data are not available (e.g. China, India, Vietnam), even though in such countries the regional price differences are probably much larger than in OECD countries. Therefore, researchers turned to the data provided by household expenditure surveys.

**Housing:** The studies also differ with respect to the range of products that are included. Most work conducted in developing countries concentrates on food. Less than half of the studies include the cost of housing (see column “HOUS.” of Table 1).

**Methodology:** Depending on the available data set, different computational approaches have been developed to transform the regional data into regional price levels (see column ³Studies that compare the regional price levels of individual products or groups of products without transforming these results into the regions’ overall price levels are not included in this survey. Examples are Hoang (2009) and Majumder et al. (2012) who investigate regional food prices in Vietnam and India, respectively.
⁴For example, Biggeri et al. (2017b) subdivide Italy into 19 regions where each region is represented by its most important city.
“METHODOLOGY” of Table 1). CPI data typically describe the observed market prices of a wide range of products reflecting the consumption patterns of typical households. These data are combined with the households’ average expenditure shares on the various products. Using this information, some studies define a “reference region” and use some standard index formula (e.g. Laspeyres, Fisher, Lowe, Törnqvist) to compute each region’s price level relative to the reference region’s price level. Other studies rely on variants of the GEKS index, following a recommendation by Eurostat-OECD (2012) for the computation of international purchasing power parities. A third group of studies applies some variant of the CPD method. A recent survey of the various methods can be found in Laureti and Rao (2018).

Some authors cannot draw on CPI data, but have to do with household expenditure survey data. In most of these studies a household’s expenditures on some product group are divided by the household’s purchased quantity of that product group to obtain a unit value that can be interpreted as the “implicit price” that this household pays for this product group. One major problem with this approach is the variation in the product group’s quality across households (e.g. Deaton, 1988, p. 420; McKelvey, 2011, p. 157). In response to these concerns, various correction methods have been developed that compute “quality adjusted unit values”. Based on these adjusted unit values and the household expenditures, some studies compute multilateral price indices (e.g. CPD, GEKS). Other studies estimate the parameters of a demand system, and from those a regional cost of living index (COLI) that compares the regional expenditures necessary to achieve a given utility level. A third group of studies exploits Engel’s Law which states that a household’s share of food expenditures falls as its real income increases. If two households located in different regions have identical food expenditure shares, but the nominal income of the first household exceeds that of the second household by 10%, then this implies that the price level in the first household’s region is also 10% higher than in the region of the second household.
<table>
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<th>#REG.</th>
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Table 1: Main features of recent studies on regional price comparisons: country (column heading COUNTRY), coverage of country (COV.), number of regions (#REG.), average size of regions in square kilometre (SIZE), primary data source (DATA), inclusion of housing cost (HOUS.), and applied computational approach (METHODOLOGY).
3 Data

The CPI micro data that we have the privilege of working with were provided to us by
the Research Data Centre (RDC) of the Federal Statistical Office and Statistical Offices
of the Länder. These data are unique in several respects. First, thanks to the federal
structure of Germany, its CPI compilation is based on a profoundly regionalised data
collection process. Second, the price data come with detailed supplementary information
revealing whether two price observations relate to exactly the same product. Third, the
data set includes housing and related costs. Fourth, all prices are collected within one
month. Because of the combination of these four features, the German CPI micro data
come much closer to the rating of an “ideal price data set” than any of the data sets that
were available to the authors of the studies listed in Table 1.

The German territory is subdivided into 402 regions (295 counties and 107 cities). In
each region and each month a large set of consumer price data is collected. In our analysis
we use the data from May 2016. The data includes 381,983 consumer prices for goods,
services, and rents that are classified into 650 categories denoted as basic headings.

The German consumer price data represent a stratified sample where products are se-
lected non-randomly within narrowly defined categories. The hierarchical categorisation
of the products follows the United Nations’ Classification of Individual Consumption by
Purpose (COICOP). At the highest classification level there are 12 divisions (see Table
2). Rents are included in the division: “Housing, water, electricity, gas, and other fuels”.
It turns out that rents are the most relevant category for our interregional price level com-
parisons. Fortunately, the information provided by the rent data exceeds that of goods
and services. This enables us to analyse the rent data using a more sophisticated method
than that applicable to the goods and services. Therefore, we split the data into two
subsets: 366,401 price data assigned to 645 basic headings and 15,582 rent data assigned
to 5 basic headings.

3.1 Price Data

For interregional price level comparisons, the prices for one and the same product must
be available in multiple regions. Whether a pair of products is identical can be examined
by comparing their characteristics documented in the complementary information of our
price data. To each price observation we have not only the price and the region, but also
several other product identifying attributes (amount, unit of measurement, type of outlet
and offer). Depending on the respective basic heading, several additional characteristics
are available (e.g. brand, packaging).

5 One exception are rents. Since 2016 they are collected from a stratified random sample (Goldhammer,2016).
Table 2: The 12 COICOP divisions covering household consumption expenditures and their expenditure weights (WEIGHT, measured in % and compiled in 2010), number of basic headings (#BH) and number of price observations (#PRICES). Source: RDC of the Federal Statistical Office and Statistical Offices of the Länder, Consumer Price Index, May 2016, own computations.

In contrast to the existing studies on interregional price comparisons, we do not treat seemingly equivalent products as if they were directly comparable. Instead, we adhere to our Perfect Matches Only (PMO) precept. Table 3 presents a typical example. It shows the prices, the regions, and complementary information for the basic heading “rice”. As the data are collected independently by the Statistical Offices of the Länder, different spellings occur and the reported values for characteristics such as “amount” and “unit” are often incoherent (e.g. some price collectors write 0.5 kg, others 500 g). These inconsistencies greatly complicate the identification of identical products.

<table>
<thead>
<tr>
<th>REGION</th>
<th>OUTLET</th>
<th>AMOUNT</th>
<th>UNIT</th>
<th>OFFER</th>
<th>CHARACTERISTICS</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>discount store</td>
<td>1 kg</td>
<td>0</td>
<td>0</td>
<td>(Uncle Bens, basmati, bag)</td>
<td>1.69</td>
</tr>
<tr>
<td>D</td>
<td>discount store</td>
<td>0.5 kg</td>
<td>0</td>
<td>0</td>
<td>(Oryza, long grain, bag)</td>
<td>0.99</td>
</tr>
<tr>
<td>A</td>
<td>supermarket</td>
<td>0.5 kg</td>
<td>0</td>
<td>0</td>
<td>(Oryza, short gr., bulk)</td>
<td>0.98</td>
</tr>
<tr>
<td>B</td>
<td>discount store</td>
<td>1000 g</td>
<td>0</td>
<td>0</td>
<td>(Uncle Bens, Basmati, bag)</td>
<td>1.59</td>
</tr>
<tr>
<td>E</td>
<td>discount store</td>
<td>500 g</td>
<td>0</td>
<td>0</td>
<td>(Oryza, long gr., bag)</td>
<td>0.97</td>
</tr>
<tr>
<td>A</td>
<td>supermarket</td>
<td>0.5 kg</td>
<td>1</td>
<td>0</td>
<td>(Oryza, l. grain, bulk)</td>
<td>0.79</td>
</tr>
<tr>
<td>C</td>
<td>supermarket</td>
<td>0.5 kg</td>
<td>0</td>
<td>0</td>
<td>(Oryza, short grain, bulk)</td>
<td>0.96</td>
</tr>
<tr>
<td>E</td>
<td>discount store</td>
<td>0.5 kg</td>
<td>0</td>
<td>0</td>
<td>(reisfit, longgrain, bag)</td>
<td>1.09</td>
</tr>
<tr>
<td>A</td>
<td>discount store</td>
<td>1 kg</td>
<td>0</td>
<td>0</td>
<td>(Reis-fit, med. grain, bag)</td>
<td>1.99</td>
</tr>
<tr>
<td>C</td>
<td>supermarket</td>
<td>500 g</td>
<td>0</td>
<td>0</td>
<td>(Uncle Ben’s, basmati, bulk)</td>
<td>0.79</td>
</tr>
<tr>
<td>B</td>
<td>discount store</td>
<td>1 kg</td>
<td>0</td>
<td>0</td>
<td>(Reisfit, medium gr., bag)</td>
<td>1.89</td>
</tr>
<tr>
<td>C</td>
<td>discount store</td>
<td>1 kg</td>
<td>0</td>
<td>0</td>
<td>(Uncle Bens, Basmati, Bag)</td>
<td>1.89</td>
</tr>
<tr>
<td>B</td>
<td>supermarket</td>
<td>0.5 kg</td>
<td>0</td>
<td>0</td>
<td>(Uncle Ben, basm., bulk)</td>
<td>0.69</td>
</tr>
<tr>
<td>D</td>
<td>discount store</td>
<td>500 g</td>
<td>0</td>
<td>0</td>
<td>(Reisfit, long grain, Bag)</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Table 3: Exemplary consumer price data for rice before data processing (all values fictitious). “OFFER” indicates whether the price is an exceptional offer (= 1) or not (= 0).

In Table 3 none of the fourteen products exactly match. However, a closer look at the data reveals strong similarities between the characteristics as merely some of the spellings and units vary. Correcting and harmonising the spellings and the units of measurement
reduces the number of different products from fourteen to seven. These seven products are listed in the lines of Table 4. The columns of the table indicate the region in which the product has been observed. Since Product 7 has been observed in only one region, it provides no usable information for the interregional price comparison.

<table>
<thead>
<tr>
<th>Product 1 (discount store, 1, kg, 0, Uncle Bens, basmati, bag)</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product 2 (discount store, 1, kg, 0, Reisfit, medium grain, bag)</td>
<td>1.69</td>
<td>1.59</td>
<td>1.89</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Product 3 (discount store, 0.5, kg, 0, Reisfit, long grain, bag)</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>0.99</td>
<td>1.09</td>
</tr>
<tr>
<td>Product 4 (discount store, 0.5, kg, 0, Oryza, long grain, bag)</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>0.99</td>
<td>0.97</td>
</tr>
<tr>
<td>Product 6 (supermarket, 0.5, kg, 0, Uncle Bens, basmati, bulk)</td>
<td>×</td>
<td>0.69</td>
<td>0.79</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Product 7 (supermarket, 0.5, kg, 1, Oryza, long grain, bulk)</td>
<td>0.79</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
</tbody>
</table>

**Table 4**: Price matrix for rice after data processing (lines indicate products, columns indicate regions).

The data processing increases the number of perfectly matching pairs from zero to eight. This is important, because only identical products that have been observed in different regions provide unbiased information for interregional price comparisons. Before the data processing, a comparison between the five regions’ price levels of rice is impossible. After the data processing, regions A, B, and C can be compared to each other, and regions D and E can be compared. However, a direct comparison of regions D or E to regions A, B, or C is still not feasible.

In our original price data set, the problem with inconsistency applies not only to the rice data, but also to the other basic headings. With 366,401 price observations, a manual correction and harmonisation of the different spellings and units is infeasible. Therefore, we apply deterministic string matching algorithms for this purpose. Furthermore, we automatically convert, where possible, the units of measurement to the most frequent units within the basic heading. Our corrections reduce the number of different products by 8.46%, raising the number of estimated price levels by 14.32%.

### 3.2 Rent Data

The German CPI includes both rents and the cost of owner-occupied housing. Roughly 54% of German houses and flats are occupied by renters (Statistisches Bundesamt, 2017, p. 161). This is one reason, why the cost of owner-occupied housing is measured by the rental equivalence approach. This approach assumes that the cost of living in one’s own house or flat is equivalent to the rent that would typically arise for such an accommodation.

The Federal Statistical Office collects rents in existing buildings. It groups the rent data under five basic headings, one covering single-family houses and the other four covering different types of flats. The rent data that are available to us encompass 381 of the 402 regions. Only 315 of the 15,582 rent observations, or less than one observation per
region, refer to single-family houses. In view of this sparse data base and the different characteristics of single-family houses and flats, we exclude the 315 observations on single-family houses from our rent data set.

The literature on the measurement of housing prices (e.g. Wabe, 1971, pp. 249-251) differentiates between house parameters (e.g. living space and quality of the flat’s equipment such as its windows, floors) and locational parameters (e.g. quality of residential area). Both types of information are available in our rent data.6

For 21 regions, the rent data of the Federal Statistical Office do not provide sufficient information to compute a rent level. Furthermore, our rent data cover only a small fraction of tenant changeovers in existing buildings and no flats in newly completed buildings. Therefore, we draw on a second data source. The BBSR (Federal Institute for Research on Building, Urban Affairs and Spatial Development) collects rents for flats without furnishing and with a living space between 40 and 130 square metres. The rents are net of utilities and cover tenant changeovers in existing buildings as well as flats in newly completed buildings. Furthermore, as the data is collected from internet platforms and from newspaper ads, it represents quoted rather than transactional rents. Although the quoted rents are expected to be on average higher than the actual rents that finally are agreed upon, no evidence exists that this difference varies between regions.7 Therefore, the quoted rents serve as an indicator for regional rent level differences and, consequently, become part of the regional rent index numbers. The BBSR has provided us with an average rent per square metre in all 402 regions as of the second quarter 2016.

3.3 Consumption Expenditure Weights

Our price and rent data are complemented by a two-dimensional system of expenditure weights provided by the Federal Statistical Office. The latest available system of weights is from 2010.

The first dimension of this system are the expenditure shares that a typical German consumer spends on the various basic headings. The expenditure share weights available to us are identical across regions. Moreover, the weights that we use deviate slightly from the original weights published by the Federal Statistical Office, because 16.08% of total expenditures relate to basic headings that are not included in our data set. Therefore, we rescale the weights such that they sum to 100%. The expenditure weights relating to the highest classification level, denoted as divisions, are listed in Table 2. The weights reveal that private households spend most of their expenditures on housing and related components (32.42%). This category includes rents.

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6 Summary statistics for the respective variables can be found in Weinand and Auer (2019, p. 12-13).

7 Faller et al. (2009) find an overall deviation of 8% between quoted and transactional prices for purchases of flats and houses. For rents, they expect that this deviation becomes smaller.
The second dimension of the weighting system are the outlet types. On average, discount stores (36.7%) and specialised shops (26.0%) have the largest market shares in Germany, while the market share of internet and mail-order business (8.7%) is relatively low (see Sandhop, 2012, p. 269). Other outlets are department stores (2.80%), hypermarkets (12.10%), supermarkets (12.40%), other retail (1.00%), and private and public service providers (0.30%). For more than two thirds of the 650 basic headings, we know how expenditures on a particular basic heading are divided between the eight types of outlets. For this subset, the weighting of outlet types varies between different basic headings. For most basic headings, only some of these outlet types are relevant. Milk, for example, has been observed only in hypermarkets, supermarkets, and discount stores. Like the expenditure share weights of basic headings, also the expenditure share weights of outlet types are uniform across regions.

4 Methodology

The compilation of the regional price levels proceeds in four consecutive stages. Here we merely sketch out these stages. More details can be found in the Appendix.

Stage 1: Regional Price Levels Relating to the Same Basic Heading and Outlet Type

Each observation of our price data set comprises the product’s price, the region in which this price was recorded, and some additional characteristics. These additional characteristics allow us to identify those observations that relate to identical products. Identity of products requires not only conformable product characteristics, but also an identical outlet type (e.g. supermarket). This is our PMO precept.

Our price data set exhibits gaps, because none of the products with regionally varying prices is observed in all regions. Therefore, the regional price levels cannot be computed by standard price index formulas. Instead, we apply the CPD regression approach.

Suppose, for example, that we have collected price levels of different “objects” in different regions, but that not all of the objects have been observed in all regions. Let \( r \) (\( r = 1, \ldots, R \)) denote the regions and \( i \) (\( i = 1, \ldots, N \)) the objects. Objects could be products, groups of products, basic headings, or groups of basic headings.

The CPD method introduced by Summers (1973, p. 10-11) assumes that each observed price level, \( p^r_i \), can be obtained by multiplying region \( r \)'s overall price level \( P^r \) by object’s \( i \)'s general value \( \pi_i \) and by a log-normally distributed random variable \( \varepsilon^r_i \): 

\[
p^r_i = P^r \pi_i \varepsilon^r_i.
\]

To transform this relationship into a linear regression model, two sets of dummy variables are introduced. For each region \( s \) (\( s = 1, \ldots, R \)), a dummy variable \( \text{region}^s \) can be defined.
such that \( \text{region}^s = 1 \) when \( r = s \) and \( \text{region}^s = 0 \) otherwise. Similarly, for every object \( j \ (j = 1, \ldots, N) \), a dummy variable \( \text{object}_j \) can be defined such that \( \text{object}_j = 1 \) when \( i = j \) and \( \text{object}_j = 0 \) otherwise. The resulting linear regression model is
\[
\ln p_i^r = \sum_{s=2}^{R} (\ln P^s) \ \text{region}^s + \sum_{j=1}^{N} (\ln \pi_j) \ \text{object}_j + \ln \varepsilon_i^r ,
\]
where the logarithmic price level of the base region is given by \( \ln P^1 = 0 \) and the error term is \( \ln \varepsilon_i^r \sim N(0, \sigma^2) \). Ordinary least squares (OLS) regression of model (1) yields estimates of the object values \( \ln \pi_j \) and the regional logarithmic price levels \( \ln P^s \). For our purposes, only the latter are relevant.

We apply this unweighted CPD regression separately for each combination of basic heading \( b \) and outlet type \( l \). As a result, we obtain for each combination its own vector of estimated regional price levels \( \hat{\ln P}_{bl} = (\hat{\ln P}_{1bl} \ldots \hat{\ln P}_{402bl}) \).

**Stage 2: Regional Price Levels Relating to the Same Basic Heading**

**Rent Levels:** Even though our rent data exhibit some gaps, the information in this data set is richer than in the price data set. Therefore, a hedonic regression technique can be applied that explains a flat’s rent by the region where it is located and by the flat’s other characteristics. The estimation yields “implicit prices” of all of the flats’ characteristics, including its region. Knowing these implicit prices, we can estimate the logarithmic rent levels prevailing in different regions. These are normalised and combined in the vector \( \ln \tilde{P}_{\text{rent}} = (\ln \tilde{P}_{1\text{rent}} \ldots \ln \tilde{P}_{402\text{rent}}) \). In addition, we received from the BBSR regional rent levels related to tenant changeovers in existing buildings and newly completed buildings.

We normalise these logarithmic rent levels and combine them in the vector \( \ln \tilde{P}_{\text{rent}} = (\ln \tilde{P}_{1\text{rent}} \ldots \ln \tilde{P}_{402\text{rent}}) \).

**Price Levels:** A weighted variant of the CPD method was proposed by Rao (2001, p. 15), Rao (2005, p. 575), Hajargasht and Rao (2010, p. S39), and Diewert (2005, pp. 562-563). The weighted version of the CPD regression model (1) is
\[
\sqrt{w_i} \ln p_i^r = \sqrt{w_i} \sum_{s=2}^{R} (\ln P^s) \ \text{region}^s + \sqrt{w_i} \sum_{j=1}^{N} (\ln \pi_j) \ \text{object}_j + \ln \varepsilon_i^r ,
\]
where \( w_i \) is the explicit weight given to object \( i \).

For each of the 645 basic headings we run a separate weighted CPD regression of the type (2). Each of these regressions aggregates the price level vectors \( \hat{\ln P}_{bl} \) compiled in Stage 1 that relate to the same basic heading \( b \), but differ with respect to the outlet type \( l \). The weights reflect the expenditure shares of the respective outlet types. These weighted
CPD regressions yield for each basic heading $b$ the vector of regional price levels estimates $\hat{\ln P}_b = \left( \hat{\ln P}_{b1} \ldots \hat{\ln P}_{b402} \right)$.

**Stage 3: Regional Price Levels Relating to Goods, Services, and Housing**

From Stage 2 we know the two rent vectors $\hat{\ln P}_{rent}$ and $\hat{\ln P}_{rent}$ as well as the 645 price level vectors $\hat{\ln P}_b$. Since most of these 647 vectors are incomplete, their further aggregation into the overall regional price level vector, $\hat{\ln P}$, could be conducted by another weighted CPD regression of the type (2). In this regression, the dummy variables $object_j$ would represent basic headings and the weights $w_i$ the expenditure shares of these basic headings. Regression equation (2) would imply that the values of the regional coefficients $\ln P^r$ are independent from the basic headings. As pointed out before, however, there is a widely held belief that housing costs vary more strongly across regions than the prices of services and that the latter vary more strongly than the prices of goods. In addition, most basic headings are represented by incomplete vectors. As a consequence, a weighted CPD regression including all 647 basic headings is prone to bias.

Therefore, we refrain from such an all-encompassing weighted CPD regression and, instead, split the 647 basic headings into three separate segments: housing (2 basic headings, weight 20.99%), services (153 basic headings, weight 25.56%), and goods (492 basic headings, weight 53.45%).

For each segment we conduct a separate weighted CPD regression of the form (2). We obtain the three complete vectors $\hat{\ln P}_{housing}$, $\hat{\ln P}_{services}$, and $\hat{\ln P}_{goods}$, with 402 regional price levels, respectively.

**Stage 4: Overall Regional Price Levels**

We compile the overall regional price level vector $\hat{\ln P}$ from the three vectors $\hat{\ln P}_{housing}$, $\hat{\ln P}_{services}$, and $\hat{\ln P}_{goods}$. Since the latter are complete, we compute for each region, $r$, the weighted arithmetic mean of its three logarithmic index values,

$$\hat{\ln P}^r = w_{housing} \hat{\ln P}^r_{housing} + w_{services} \hat{\ln P}^r_{services} + w_{goods} \hat{\ln P}^r_{goods}, \quad (3)$$

where $w_{housing}$, $w_{services}$, and $w_{goods}$ are the expenditure share weights of the three segments.

We re-normalise the logarithmic price level estimates in (3) so that our final multilateral

---

8 The classification of basic headings into goods (durables, semi-durables, non-durables) and services follows ILO et al. (2004, p. 465-482).

9 Weinand and Auer (2019, p. 35-37) show that exactly the same estimates, $\hat{\ln P}^r$, are obtained when we apply another weighted CPD regression or the GEKS approach where the underlying bilateral price index numbers are computed as weighted Jevons indices.
system of regional index numbers is defined by:

$$\hat{P}_r = \exp (\ln \hat{P}_r - \ln P^\text{Ger})$$

with $\ln P^\text{Ger} = \sum_{r=1}^{402} g_r \cdot \ln \hat{P}_r$, where the weights, $g_r$, are defined as region $r$’s population share.\(^{10}\) This normalisation ensures that the weighted geometric mean of the normalised regional price levels, $\hat{P}_r$, is $P^\text{Ger} = 1$. Therefore, $(\hat{P}_r - 1)$ is the percentage deviation between the price level of region $r$ and the weighted geometric mean of all regional price levels, $P^\text{Ger}$.

## 5 Empirical Results

The regional rent levels (housing) are presented in Section 5.1. Summary statistics and further analysis of the estimated price levels of goods and services are provided in Section 5.2. The overall price levels of the 402 German regions are presented in Section 5.3, along with a comparison of the regional price levels of goods, services, and housing. Furthermore, we examine the spatial correlation of the overall price levels. Finally, Section 5.4 examines whether the overall regional price levels change when simplified data editing procedures are employed.

### 5.1 Housing

As described in Section 4 (Stage 2), we use the CPI rent data of the Federal Statistical Office to compute the logarithmic rent levels of 381 regions, $\ln rent^r$ ($r = 1, \ldots, 381$). 366 of these rent levels were estimated by a hedonic regression. The regression equation and the corresponding regression statistics are documented in the Appendix.

The regression’s adjusted $R^2$ is 0.75. This indicates that our hedonic regression has a high explanatory power.\(^{11}\) The estimated coefficients have the expected signs and are in most cases significant. The estimated rent levels of the seven most populous cities in Germany are above the national average. The rent level in the most expensive region, $P^\text{Ger}$, should be estimated by $\exp (\hat{\ln} P^r - 0.5\hat{\text{var}}(\hat{\ln} P^r))$ and not by $\exp (\hat{\ln} P^r)$. In our regression, however, we cannot estimate the variances, $\hat{\text{var}}(\hat{\ln} P^r)$ in a reliable way. Therefore, we have to do without this adjustment.

\(^{10}\) Referring to the analysis of Goldberger (1968), Kennedy (1981, p. 801) points out that the expected value of the estimator $\exp (\hat{\ln} P^r)$ is not $\exp (\ln P^r)$, but $\exp (\ln P^r + 0.5\hat{\text{var}}(\hat{\ln} P^r))$. This implies that the values of $P^r$ should be estimated by $\exp (\hat{\ln} P^r - 0.5\hat{\text{var}}(\hat{\ln} P^r))$ and not by $\exp (\ln P^r)$. In our regression, however, we cannot estimate the variances, $\hat{\text{var}}(\ln P^r)$ in a reliable way. Therefore, we have to do without this adjustment.

\(^{11}\) Hoffmann and Kurz (2002, p. 18) report values that range from 0.53 to 0.65 for multiple cross-section analysis of West German rent data of the German Socio-Economic Panel. Kholodilin and Mense (2012, p. 17) use rent data of flats located in Berlin, collected from internet ads within the period 2011 to 2012. The goodness of fit of their hedonic regression is 0.65. Behrmann and Goldhammer (2017, p. 22) use the 2017 rents of the German CPI data for twelve of the sixteen Federal States. They report a value of 0.77.
Frankfurt am Main, is 74.23% above the unweighted average rent level of all regions included in the hedonic regression. The cheapest region is 32.34% below that average.

Since the CPI rent data provided by the Federal Statistical Office represent rents that are contractually paid by tenants, we denote them as transactional rents. By contrast, the rents $\ln rent^r (r = 1, \ldots, 402)$ provided by the BBSR stem from internet and newspaper ads and relate to tenant changeovers in existing buildings and newly completed buildings. Therefore, we denote them as quoted rents.

Kendall’s $\tau (= 0.59)$ documents a high similarity between the regional rankings of quoted and transactional rent levels. This similarity is confirmed by Figure 1. The dashed diagonal line indicates equality between quoted and transactional rents. The figure reveals that the regional variation in the (logarithmic) quoted rents exceeds the variation in the (logarithmic) transactional rents. Furthermore, the quoted rents exceed the transactional rents in almost all regions. As shown by the slope of the regression line, this markup increases with the transactional rent level. Transactional rents correspond to existing rental contracts, while the quoted rents correspond to rents that are free to renegotiate. Therefore, the increasing markup may indicate that in large cities (they have the largest transactional rents) the upward trend in rent levels during 2016 is stronger than in more rural regions. In Figure 1, the seven most populous German cities exhibit particularly large markups. This reinforces our decision to also include rents related to tenant changeovers in our regional price comparison.

5.2 Price Levels of Goods and Services

In our price data, we have 6,323 independent data sets, each relating to a different combination of basic heading and outlet type. As outlined in Section 4, in Stage 1 we conduct a separate unweighted CPD regression for each of these data sets and obtain 6,323 price vectors. In Stage 2, these are further aggregated by another weighted CPD regression into the 645 vectors of basic heading price levels, $\hat{\ln P}_b$.

Figure 2 depicts the regional price indices of four basic headings: nuts and raisins (representing COICOP division 01: food and non-alcoholic beverages), women’s shoes (division 03: clothing and footwear), cup of coffee, tea, hot chocolate (division 11: restaurants and hotels), and inpatient care (division 12: miscellaneous goods and services). The regions are ordered by their quoted rent levels. For each of the 402 regions, the solid line shows the level of quoted rents, while the points represent the basic heading’s price level. The figure indicates that the regional price levels of services (bottom panels of Figure 2) fluctuate more than those of goods (top panels of Figure 2). Taking into account all basic headings, this observation remains stable; the coefficient of variation for services is 0.28 and 0.12 for goods.
Figure 1: Transactional rents, $\ln \text{rent}^r$, depicted on horizontal axis and quoted rents, $\ln \tilde{\text{rent}}^r$, on vertical axis (both in €). Population weighted averages as dashed horizontal and vertical lines, weighted least squares regression as solid blue line.

More importantly, Figure 2 reveals that the regional price levels for the basic headings representing goods fluctuate closely around the horizontal axis, implying that they are more or less uncorrelated with the quoted rent levels (nuts and raisins: $\tau = 0.12$, women’s shoes: $\tau = -0.03$). By contrast, the price levels of the basic headings representing services are positively correlated with the quoted rent levels (cup of coffee: $\tau = 0.30$, inpatient care: $\tau = 0.47$). The overall correlation between price levels of those basic headings relating to services and the quoted rent levels is $\tau = 0.13$, while it is $\tau = 0.03$ for basic headings representing goods.

5.3 Overall Price Levels

As described in Section 4 (Stage 3), the regional price indices of the various basic headings are aggregated into the regional price indices of goods, services, and housing. Finally, these three price indices are aggregated to the overall regional price index (Stage 4). The latter are normalised by the population weighted average price level, $\ln P^{\text{Ger}}$. Table 5 contains summary statistics of the estimated price index numbers, $100 \cdot \hat{P}^r$. By definition, the population weighted mean, $100 \cdot P^{\text{Ger}}$, is 100. If we omit the population weights, the (unweighted) mean drops to 98.37. This indicates that regions with larger populations
Figure 2: Estimated price levels $\hat{P}_b^r$ for basic headings $b = \{\text{nuts and raisins, women’s shoes, cup of coffee, and inpatient care}\}$, ordered by quoted rent levels $\hat{P}_{\text{rent}}$ from lowest (region $r = 1$) to highest (region $r = 402$), respectively.

tend to have higher price levels.

<table>
<thead>
<tr>
<th>MIN</th>
<th>Q25</th>
<th>MEDIAN</th>
<th>MEAN</th>
<th>BASE</th>
<th>Q75</th>
<th>MAX</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>90.40</td>
<td>95.33</td>
<td>97.92</td>
<td>98.37</td>
<td>100.00</td>
<td>100.67</td>
<td>114.90</td>
<td>4.09</td>
</tr>
</tbody>
</table>

Table 5: Summary statistics of estimated price index numbers, $100\cdot\hat{P}_r$, with the population weighted average as base ($= 100$).

The seven most populous German cities confirm this observation. The most expensive region is Munich. Its price level is 14.90% above the population weighted average. Frankfurt ($= 11.50\%$), Stuttgart ($= 9.81\%$), Cologne ($= 7.90\%$), Dusseldorf ($= 7.07\%$), Hamburg ($= 6.70\%$), and Berlin ($= 2.56\%$) also exhibit above-average price levels. The distribution is skewed to the right, indicating that strong deviations from the population weighted average more frequently occur in expensive regions than in inexpensive ones.

The overall price index numbers of the 402 German regions are shown in the left hand panel of Figure 3. We also decompose the overall price levels into housing (transactional and quoted rents), goods, and services. These price index numbers are shown in the other three panels of that figure.
Figure 3: Regional price index $100 \cdot \hat{P}_r$ (left panel), housing price index (left centre panel), price index for goods (right centre panel) and price index for services (right panel) normalised by population weighted average price level ($= 100$), respectively.
The index numbers for goods vary only slightly. They range from 92.58 to 103.93. For services, this range expands to 89.07 to 121.35. By contrast, the housing index numbers show strong regional differences. They range from 63.67 to 166.01. Therefore, the overall price level is largely driven by housing.

The left panel of Figure 3 also reveals that the high price levels found in the seven major cities spread out into their neighbouring regions. Moran’s $I = 0.58$ ($p < 0.01$) indicates positive spatial autocorrelation. This positive spatial autocorrelation is mainly driven by housing ($I = 0.65$, $p < 0.01$) rather than by goods ($I = 0.18$, $p < 0.01$) or services ($I = 0.23$, $p < 0.01$).

Figure 4 provides a more comprehensive picture of the spatial autocorrelation structure. It shows the relation between the estimated logarithmic price levels, $\hat{\ln}P_r$, and the (local) Moran’s $I^r$ coefficients of the 402 regions. The u-shaped relation indicates positive spatial autocorrelation especially in those regions with price levels clearly above or clearly below the population weighted average, $\ln P_{Ger} = 0$, while regions with intermediate price levels exhibit very low spatial correlation. This implies that price levels change only gradually as one travels from inexpensive to expensive regions, or vice versa.

Figure 4: Estimated, logarithmic price levels, $\hat{\ln}P_r$, (horizontal axis) and local Moran’s $I^r$ (vertical axis) of our 402 regions. Cubic least squares regression as solid blue line.

5.4 Simplified Compilation Procedures

In Section 3.1 we described the comprehensive editing of the price data. A major part of this editing is necessary to implement the PMO precept in our regional price level

---

\[12\] We compute Moran’s (1950) $I$ based on a row-standardised approach, where each neighbouring region receives a weight according to its population size.
computations. The precept postulates that prices of products are comparable, only if the characteristics of the products coincide in every respect. Without extensive editing of the original price data few products would satisfy this condition (see our illustrative example in Tables 3 and 4).

For the compilation of the regional price levels we use a multi-stage CPD approach to ensure the highest possible accuracy. In Section 4 we described the four stages of this approach in more detail. In Stage 1, CPD regressions aggregate products relating to the same basic heading and outlet type. This yields several vectors of regional price levels for each basic heading, each vector relating to a different outlet type. In Stage 2, the vectors relating to the same basic heading are aggregated. This yields a single vector of regional price levels for each basic heading. In Stages 3 and 4, the rent level vectors as well as the basic heading vectors relating to goods and services, are aggregated into the price levels of housing, goods, and services and these into the overall price levels of the regions. The associated summary statistics depicted in Table 5 are replicated in the bottom line of Table 6.

The editing of the original price data is necessary to conduct Stage 1 of our multi-stage approach which, in turn, is necessary to adhere to the PMO precept. If one ignored the PMO precept, Stages 1 and 2 could be merged. One may ask whether the resulting high degree of accuracy justifies the effort. Would a less rigorous CPD approach generate other regional price levels? Table 6 provides an answer. It presents the summary statistics of two alternative CPD approaches that weaken the PMO precept to different degrees. Both alternatives preserve Stages 3 and 4 of the original multi-stage approach, but merge Stages 1 and 2 into a single stage.

Variant (i) is the more extreme degree of difference. It treats all products within a basic heading as directly comparable, regardless of their qualitative characteristics and their outlet type. Therefore, the time consuming extensive data editing is no longer necessary. Table 6 reveals that in Variant (i) the overall price levels of the regions fluctuate more noticeably around their population weighted average than with the PMO precept. The range of the overall price index is from 77.58 to 129.16, while the PMO precept generates price levels that range from 90.40 to 114.90. This is a considerable deviation. The correlation between the price levels of Variant (i) and the PMO based price levels is $\rho = 0.69$.

Somewhat more encouraging is Variant (ii). It considers only those products as directly comparable that belong to the same basic heading and are sold in the same outlet type. As in Variant (i), this eliminates the need for the extensive data editing. The range of regional price levels narrows to 79.90 and 115.39. The correlation between the price levels obtained in Variant (ii) and those derived from our PMO precept is $\rho = 0.90$.  

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In sum, a CPD approach that drops the product dummies and the outlet dummies and retains only the basic heading dummies and the regional dummies, generates very poor results. A CPD regression that drops only the product dummies but retains all other dummies performs far better, though a loss in accuracy remains.

<table>
<thead>
<tr>
<th></th>
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<th>Q25</th>
<th>MEDIAN</th>
<th>MEAN</th>
<th>BASE</th>
<th>Q75</th>
<th>MAX</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>77.58</td>
<td>92.83</td>
<td>97.04</td>
<td>97.04</td>
<td>100.00</td>
<td>100.47</td>
<td>129.16</td>
<td>7.73</td>
</tr>
<tr>
<td>(ii)</td>
<td>79.90</td>
<td>94.54</td>
<td>98.03</td>
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<td>100.00</td>
<td>101.08</td>
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<td>4.96</td>
</tr>
<tr>
<td>PMO</td>
<td>90.40</td>
<td>95.33</td>
<td>98.37</td>
<td>98.37</td>
<td>100.00</td>
<td>100.67</td>
<td>114.90</td>
<td>4.09</td>
</tr>
</tbody>
</table>

Table 6: Summary statistics of estimated price index numbers, \(100 \cdot \hat{P}_r\), by degree of product definition within a basic heading: (i) none and (ii) outlet type. Population weighted average as base \((= 100)\).

6 Concluding Remarks

The main goal of this paper was to compile sub-national price levels for the 402 counties and cities in Germany. To this end, we introduced a multi-stage CPD approach that is based on the Perfect Matches Only (PMO) precept. This precept bans the assignment of seemingly similar products into groups of directly comparable products. Instead, the computation of regional price levels takes its information only from pairs of identical products. Applied to the German CPI data set, this rigorous approach ensures that the accuracy of the compilations is not impaired by artificially contaminated price information. Our study demonstrates that the regionalised structure of the German CPI data allows for the computation of an accurate regional price index. This index is also unique in its level of spatial disaggregation.

Our results reveal considerable price differentials across the 402 regions. The overall price level in the most expensive region, Munich, is about 27% higher than in the cheapest region. We find that these price differentials are mainly driven by housing. The most expensive region exceeds the cheapest one by 161%. For services the corresponding number is merely 36% and for goods 12%. We also show that the price levels of metropolitan areas tend to be higher than those of more rural areas. The seven most populous cities (Berlin, Hamburg, Munich, Cologne, Frankfurt, Stuttgart and Dusseldorf) exhibit price levels clearly above the German average. Furthermore, our findings reveal regional spillover effects. In the neighbourhood of expensive regions the price levels tend to be higher than in the neighbourhood of inexpensive regions and \textit{vice versa}. This positive spatial autocorrelation can be mainly attributed to housing.

Our regional price index lays the groundwork for any investigation that requires real economic indicators at the sub-national level (e.g. income, wages, productivity, investment, and consumption). Neglecting the issue of regional price disparities produces mis-
leading results. For example, the German Federal Government publishes a yearly report on the current status of cohesion between East and West Germany. In this report it compares the per capita gross domestic products as well as the labour productivities of the five Neue Länder (East Germany without Berlin) to the average of the ten West German States (BMWi, 2018, pp. 88-93). The report completely neglects that, on average, the price levels in the West are considerably higher than those in the East. Therefore, the numbers presented in the report overestimate the gap between the Neue Länder and West Germany. A second example is the measurement of life satisfaction. Deckers et al. (2016, p. 1339) demonstrate the relevance of regional price levels in this important field of research. Drawing on the regional price levels computed by Kawka (2010), they show that, for a given nominal income, life satisfaction falls by 0.1 units (satisfaction is measured on a scale from 0 to 10) as the regional price level increases by 10%. Poverty rates that neglect regional price levels can also be misleading. Accurately reporting regional differences in price levels is indispensable for establishing an index of regional wages, appropriate social security benefits, and other contractual payments.

Our multi-stage CPD approach stands out because of its high degree of accuracy and flexibility. This ensures that it can be easily adopted to other regional price comparison projects based on CPI micro data. The results of our study show that the differentiation between outlets is of utmost importance for the reliability of a regional price index, while the implementation of the PMO precept provides further accuracy. Whether this additional gain in accuracy is worth the effort, depends on how the regional price index will be used. For a regional price index published by a national statistical institute any loss in accuracy is unacceptable, because such an index must be unassailable. For the purpose of economic research, however, a more pragmatic approach that significantly reduces the workload while maintaining a reasonable degree of accuracy might be worth considering.

Finally, it must be pointed out that the computation of regional price levels based on CPI micro data is still in its infancy. Certainly, future studies should examine alternative approaches to the compilation of regional price indices. Some of these alternatives were not realisable with our data set due to data confidentiality restrictions that prevent the linkage of our CPI micro data to “external” data sources, such as expenditure weights and BBSR rents. Notwithstanding these limitations, our study introduces a novel methodology that derives a regional price index from CPI micro data. In our view, this index is unique in terms of accuracy and regional disaggregation and, therefore, represents a useful reference for future projects in the field of regional price comparisons.
Appendix

As outlined in Section 4, the compilation of the regional price levels proceeds in four stages. This Appendix describes Stages 1 and 2 in more detail. Furthermore, it documents the estimation results of the hedonic regression.

Stage 1: Regional Price Levels Relating to the Same Basic Heading and Outlet Type

For individual products neither quantity information nor weights are available. Therefore, we use the unweighted CPD regression (1), where each dummy variable \( object_j \) represents an individual product. Beforehand, however, we split the price data set of each basic heading \( b \) \( (b = 1, \ldots, B) \) into \( L_b \) price data sets each of which relates to a different outlet type \( l \). For example, Table 4 contains a price data set related to the basic heading \( b = \) rice. Since only two different outlet types occur, one may split that price data set into one relating to the outlet type discount stores and a second one relating to the outlet type supermarket, that is, \( L_{\text{rice}} = 2 \).

The price matrix of Table 4 exhibits a peculiarity leading to a modified splitting procedure. In the terminology of the World Bank (2013, p. 98) the price matrix is “not connected”, because regions A, B, and C form one block of regions and regions D and E form a second block of regions and price comparisons between the two blocks are not possible. The standard approach to deal with such price matrices is to exclude the price observations related to one of the two blocks or, even more radical, to exclude the complete basic heading. Clearly, both variants lead to a loss of valuable information. Therefore, we introduce a different approach. Instead of splitting the price matrix into two blocks (one for supermarkets and one for discount stores), we would assign Products 1 and 2 to outlet type “discount store (regions A, B, C)”, Products 3 and 4 to outlet type “discount store (regions D, E)”, and Products 5 and 6 to outlet type “supermarket”. As a consequence, we obtain \( L_{\text{rice}} = 3 \) data sets. The regions within each of these data sets are connected. This splitting approach extracts the maximum information from Table 4. We apply this approach to all basic headings.

The number of resulting outlet types, \( L_b \), differs between the basic headings of our price data set. Within each basic heading, we conduct \( L_b \) separate CPD regressions. Each of them aggregates all price observations relating to the same basic heading, \( b \), and the same outlet type, \( l \), into a vector of \( R = 402 \) estimated regional logarithmic price levels:

\[
\hat{\ln P}_{bl} = (\hat{\ln P}_{bl}^1 \ldots \hat{\ln P}_{bl}^{402})
\]

Due to the gaps in our price data set, some of the \( R = 402 \) regional logarithmic price levels, \( \ln P_{bl}^2 \), cannot be estimated such that the corresponding vector, \( \hat{\ln P}_{bl} \), is incomplete.
Stage 2: Regional Price Levels Relating to the Same Basic Heading

Rent Levels: Our rent data allow us to compute the regional rent levels by the hedonic regression approach. The data base comprises 15,267 flats that are located in 381 of the 402 regions. For 643 observations, the data are incomplete. As a consequence, the number of observations available for the hedonic regression falls to \( N = 14,624 \) and the number of regions to \( R = 366 \).

To indicate the region of a flat, we use dummy variables, \( region^r_i \) (\( r = 1, \ldots, 366 \)), with \( region^r_i = 1 \), if flat \( i \) is located in region \( r \), and \( region^r_i = 0 \) otherwise. Besides its region, each flat is characterised by \( K = 6 \) additional variables: living space in square metres (\( sqm_i \)), length of tenancy in years (\( len_i \)), quality of equipment (\( equ_i \), three levels: low, medium, high), quality of the residential area (\( area_i \), four levels: low, medium, high, very high), private versus social housing (\( priv_i \)), and existence of a built-in kitchen (\( kit_i \)).

To account for regional heterogeneity we incorporate interaction terms for the intercept. A simple Box-Cox test suggests that a logarithmic specification of the regression model is more appropriate than a fully linear or a log-linear specification. Furthermore, a linear specification would most likely suffer from heteroskedasticity. Our hedonic regression model has the following form:

\[
\ln rent_i = \alpha + \sum_{r=1}^{366} \beta_0 r \cdot region^r_i + \beta_1 \ln sqm_i \\
+ \beta_2 priv_i + \beta_3 \ln len_i + \beta_4 priv_i \ln len_i \\
+ \sum_{e=1}^{2} \beta_{5e} equ_{ei} + \sum_{a=1}^{3} \beta_{6a} area_{ai} + \beta_7 kit_i + u_i .
\]  

(5)

The error term \( u_i \) is assumed to be normally distributed with expected value 0 and variance \( \sigma^2 \).\(^{13}\) To avoid perfect multicollinearity, we impose the restriction that \( \sum_{r=1}^{366} \hat{\beta}_0 r = 0 \).

Table 7 contains the summary statistics of the hedonic regression (5)\(^{14}\).

To compute each region’s rent level, we define a reference flat and compile for each region the logarithm of the rent that, according to our hedonic regression, must be paid for this reference flat.\(^{15}\) Our reference flat is privately financed and it has a built-in

\(^{13}\) Weinand and Auer (2019, p. 34-35) show that the predicted rent, \( \hat{\ln rent}^r \), is not affected when in (5) instead of \( \ln (rent_i) \) the endogenous variable \( \ln (rent_i/sqm_i) \) is used.

\(^{14}\) For the interpretation of coefficients relating to dummy variables some care is warranted, because the endogenous variable is logarithmic. Elaborating a comment by Halvorsen and Palmquist (1980, p. 474), Kennedy (1981, p. 801) recommends to compute the adjusted coefficient

\[ \hat{\beta}^* = e^{\hat{\beta} - 0.5 \text{var}(\hat{\beta})} - 1 . \]

This adjusted coefficient indicates the percentage change in the rent caused by a change of the dummy variable from the value 0 to the value 1.

\(^{15}\) Clearly, as no interaction terms between the regional dummy variables and the other variables are...
Dependent variable: ln (rent\(_i\))

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate (SE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>2.621 (0.031)***</td>
</tr>
<tr>
<td>region(_i) = Frankfurt</td>
<td>0.556 (0.043)***</td>
</tr>
<tr>
<td>region(_i) = Munich</td>
<td>0.511 (0.012)***</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>region(_i) = Stuttgart</td>
<td>0.414 (0.013)***</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
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<td>region(_i) = Hamburg</td>
<td>0.301 (0.041)***</td>
</tr>
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<td>...</td>
</tr>
<tr>
<td>region(_i) = Cologne</td>
<td>0.282 (0.015)***</td>
</tr>
<tr>
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<tr>
<td>region(_i) = Dusseldorf</td>
<td>0.257 (0.017)***</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>region(_i) = Berlin</td>
<td>0.185 (0.008)***</td>
</tr>
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<td>...</td>
<td>...</td>
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<tr>
<td>region(_i) = Wunsiedel</td>
<td>−0.389 (0.055)***</td>
</tr>
<tr>
<td>ln (sqm(_i))</td>
<td>0.846 (0.007)***</td>
</tr>
<tr>
<td>priv(_i) = social housing</td>
<td>−0.254 (0.040)***</td>
</tr>
<tr>
<td>ln (len(_i))</td>
<td>−0.046 (0.001)***</td>
</tr>
<tr>
<td>ln (len(_i)) priv(_i) = social housing</td>
<td>0.019 (0.005)***</td>
</tr>
<tr>
<td>equ(_i_1) = low</td>
<td>−0.039 (0.004)***</td>
</tr>
<tr>
<td>equ(_i_2) = high</td>
<td>0.107 (0.006)***</td>
</tr>
<tr>
<td>area(_i_1) = low</td>
<td>−0.042 (0.006)***</td>
</tr>
<tr>
<td>area(_i_2) = high</td>
<td>0.052 (0.004)***</td>
</tr>
<tr>
<td>area(_i_3) = very high</td>
<td>0.141 (0.008)***</td>
</tr>
<tr>
<td>kit(_i) = no</td>
<td>−0.044 (0.004)***</td>
</tr>
</tbody>
</table>

Number of observations = 14624
Residual standard error = 0.183
Adjusted \(R^2 = 0.748\)
\(F\)-statistic: 116.9 on 375 and 14248 degrees of freedom, \(p\)-value: < 0%
Significance level: *** < 0.1%, ** < 1%, * < 5%

Table 7: Estimated coefficients of hedonic regression model (5) with White’s (1980) heteroskedasticity-robust standard errors in brackets. Regional fixed effects for variable \(region\(_i\)\) in descending order from highest \((r = Frankfurt)\) to lowest \((r = Wunsiedel)\).

The quality of the equipment and the residential area are classified as medium. Additionally, we assume for the reference flat a living space of 65 square metres and a length of tenancy of 7 years. Both values nearly coincide with the respective median of all flats in the rent data. For each of the 366 regions included in the hedonic regression, we are able to compute the predicted logarithmic rent that must be paid for the reference flat.

As pointed out before, the rent data of 15 other regions were incomplete. For these regions, we merely know the rent and the size of the flats. Therefore, we do not include
these regions in the hedonic regression. Instead, we calculate the region’s average rent per square metre as a simple geometric mean and we multiply this number by 65, the size of the reference flat. We combine the predicted logarithmic rents (from the hedonic regression) and the logarithms of the 15 average rents to the vector \( \ln rent^r \). Therefore, the normalised logarithmic rent levels are

\[
\ln \hat{P}_{rent}^r = \ln rent^r - \ln rent^1, \quad \text{for } r = 1, \ldots, 381.
\]  

(6)

All regional rent levels are combined in the vector \( \ln \hat{P}_{rent} = (\ln \hat{P}_{rent}^1 \ldots \ln \hat{P}_{rent}^{402}) \), with 21 values missing. This vector represents the five basic headings covered by the rent data set of the Federal Statistical Office.

As pointed out in Section 3.2, we received from the BBSR a complementary data set. It shows the regional logarithmic rent levels, \( \ln \tilde{rent}^r \), related to tenant changeovers in existing buildings and newly completed buildings. The normalised logarithmic rent levels

\[
\ln \tilde{P}_{rent}^r = \ln \tilde{rent}^r - \ln \tilde{rent}^1, \quad \text{for } r = 1, \ldots, 402,
\]  

(7)

are combined in the vector \( \ln \tilde{P}_{rent} = (\ln \tilde{P}_{rent}^1 \ldots \ln \tilde{P}_{rent}^{402}) \).

We split the total expenditure weight of rents (20.99%) into the weight of transactional rents (19.10%) and the weight of quoted rents (1.89%).\(^{16}\) The rent level vectors \( \ln P_{rent} \) and \( \ln \tilde{P}_{rent} \) complement the 645 vectors \( \ln P_b \) relating to goods and services that are estimated from the price data set. Consequently, their total weight is 79.01%.\(^{17}\)

**Price Levels:** We aggregate the \( L_b \) estimated vectors \( \ln \tilde{P}_bl = (\ln \tilde{P}_bl^1 \ldots \ln \tilde{P}_bl^{402}) \) relating to basic heading \( b \), into the basic heading’s vector of estimated regional logarithmic price levels, \( \ln \tilde{P}_b = (\ln \tilde{P}_b^1 \ldots \ln \tilde{P}_b^{402}) \). For example, according to our price data, milk is sold in three different outlet types, namely in hypermarkets, supermarkets, and discount stores. Accordingly, in Stage 1 we computed three vectors \( \ln \tilde{P}_bl \), with \( b = \text{milk} \) and \( l = (\text{hypermarket}, \text{supermarket}, \text{discount store}) \). For these three outlet types, expenditure weights are available. Therefore, we apply the weighted CPD regression model (2), where the price levels \( \ln \tilde{P}_bl \) replace the price levels \( p_i^r \) and the dummy variables \( object_j \) represent outlet types, with \( object_j = 1 \) when \( l = j \) and \( object_j = 0 \) otherwise. The weights \( w_l \) replace the weights \( w_i \) and reflect the expenditure share of outlet type \( l \) within basic heading \( b \).

\(^{16}\) This decomposition reflects the average tenant changeover rate in Germany in 2016. This rate was nearly 9% (Techem, 2017): \( (9/100) \cdot 20.99\% = 1.89\% \).

\(^{17}\) 47 of these basic headings exhibit a uniform price in all regions (e.g. books and cigarettes). Their combined expenditure weight is 12.25%.

\(^{18}\) A justification for the use of expenditure shares can be found in Rao (2005, footnote 4, p. 575).
Adding to each vector the logarithmic price level of the reference region \( r = 1 \), we end up with \( B = 645 \) different vectors \( \hat{\ln P}_b = \left( \hat{\ln P}_b^1 \ldots \hat{\ln P}_b^{402} \right) \). Again, some of the logarithmic regional price levels, \( \ln P_r^b \), cannot be estimated such that the corresponding vector, \( \hat{\ln P}_b \), is incomplete.

**References**


