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Turbulence and Unemployment in a Job Matching Model

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ABSTRACT

Turbulence and Unemployment in a Job Matching Model*

According to Ljungqvist and Sargent (1998), high European unemployment since the 1980s can be explained by a rise in economic turbulence, leading to greater numbers of unemployed workers with obsolete skills. These workers refuse new jobs due to high unemployment benefits. In this paper we reassess the turbulence-unemployment relationship using a matching model with endogenous job destruction. In our model, higher turbulence reduces the incentives of employed workers to leave their jobs. If turbulence has only a tiny effect on the skills of workers experiencing endogenous separation, then the results of Ljungqvist and Sargent (1998, 2004) are reversed, and higher turbulence leads to a reduction in unemployment. Thus, changes in turbulence cannot provide an explanation for European unemployment that reconciles the incentives of both unemployed and employed workers.

JEL Classification: E24, J64

Keywords: skill loss, European unemployment puzzle

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1. Introduction

Economists have offered numerous explanations for the emergence of high European unemployment in the 1980s, involving factors such as slow technological growth, weak capital markets, regulatory barriers and overly generous welfare systems.¹ One particularly influential explanation, suggested by Ljungqvist and Sargent (1998) (henceforth LS), is that rapidly evolving industrial structure and technology, combined with high unemployment benefits, have reduced workers’ incentives to exit unemployment. Beginning in the 1980s, the higher “turbulence” of the economic environment drove up the number of unemployed workers with obsolete skills. High unemployment benefits linked to prior high-wage jobs discouraged these workers from accepting new jobs.

While LS provide a compelling rationale for decreased willingness of unemployed workers to accept new jobs, their analysis overlooks an equally important response by employed workers that cuts against their idea. Since displacement induces costly skill obsolescence in a turbulent environment, workers ought to be much more reluctant to part with their existing jobs, instead offering wage concessions to avoid layoffs. Indeed, this intuition became commonly-held among economic observers. Alan Greenspan, for example, offered the following clear statement in 1998:

“...the sense of increasing skill obsolescence has also led to an apparent willingness on the part of employees to forgo wage and benefit increases for increased job security. Thus, despite the incredible tightness of labor markets, increases in compensation per hour have continued to be relatively modest.”

¹See Nickell (2003) for a review of research relating to the European unemployment question.
Based on this argument, an increase in turbulence should have induced a reduction in the rate of job destruction, exerting downward pressure on unemployment and working against the job rejection channel highlighted by LS. But the model used by LS assumes that the job destruction rate is an exogenous constant that cannot vary in response to economic conditions. Thus, the robustness of the LS result to the enhancement of job retention incentives remains an open question.

This paper reassesses the turbulence-unemployment relationship within a more complete job-matching framework of the sort presented in Pissarides (2000). Drawing on the setup of Den Haan, Ramey and Watson (2000), we develop a model in which workers’ skills increase over time from low to high while they are employed at a given job, but may decline from high to low when they are displaced, reflecting economic turbulence. The matching framework offers two important advantages relative to the search-based model considered by LS. First, wage setting is determined by Nash bargaining between the worker and firm, so that the worker is not forced to simply accept or reject the employer’s wage offer. Second, in contrast to LS, workers and firms can alter their wage and job retention decisions in response to higher turbulence.

In response to our earlier paper (Den Haan, Haefke and Ramey (2001)), Ljungqvist and Sargent (2004) (henceforth LS-II) proposed parameter values for our model that generated—in the presence of endogenous job destruction—a positive relationship between turbulence and unemployment. We use these parameter values as the benchmark parameterization to establish an environment most favorable for the LS result. The parameterization specifies a positive exogenous separation rate that does not change as turbulence increases, along with a positive probability that firms and workers choose to separate in response to unfavorable match-specific productivity shocks. Importantly, the benchmark parameterization assumes
that only exogenous separations lead to the possibility of skill loss following displacement.
The probability of skill loss after an exogenous separation serves to measure the degree
of turbulence. In contrast, the probability of skill loss following an endogenously-chosen
separation is zero under the benchmark parameterization.

In this setting, the original LS channel, associated with a decrease in the job accep-
tance rate, continues to operate. Moreover, higher turbulence reduces the overall net value
of employment relationships, so the job destruction rate actually rises. In this case, the
endogenous responses of worker-firm matches serve to reinforce the LS channel under the
benchmark parameterization.

It is clearly unreasonable, however, to assume that workers and firms place no weight
whatsoever on the possibility of skill obsolescence when making wage and job retention deci-
sions. Thus, we consider a perturbation of the benchmark parameterization that introduces
a positive but small probability of skill loss following an endogenous separation. Strikingly,
allowing for a skill loss probability following endogenous separation that is only 3% of the
probability following exogenous separation eliminates the positive turbulence-unemployment
relationship. Increasing this proportion to 5% gives rise to a strong negative relationship
between turbulence and unemployment. We conclude that the job rejection channel empha-
sized by LS is easily outweighed quantitatively by the rise in incentives to preserve jobs, even
when workers face only a tiny probability of skill loss following an endogenous separation.
In other words, the LS turbulence story does not hold up when the plausible responses of

2Thus, in contrast to the view expressed in the Greenspan quote, the increased possibility of skill loss
leads to an endogenous decrease in job security for the benchmark parameterization.

3That is, if the probability of skill loss after an exogenous severance is 10% and the probability of skill loss
after an endogenous severance is 0.3%, then the positive turbulence-unemployment relationship disappears.
employed workers are taken into account.

Central to reversing the LS result is the effect of turbulence on job destruction rates. We show that perturbing the benchmark parameters has little effect on the relationship between turbulence, on one hand, and the number and job rejection rate of low-skilled unemployed workers receiving high unemployment benefits, on the other. Thus, the original LS channel is unaffected by the perturbation. The relationship between turbulence and the job destruction rate, however, is strongly reversed by the perturbation, causing unemployment to fall as turbulence rises.

We consider an alternative explanation for the increase in European unemployment, involving low discount factors. A decrease in the discount factor can represent either an increase in the real interest rate or a decrease in disembodied technological growth, and both have been shown in empirical work to be important in explaining changes in European unemployment rates. We find that the effects are strongly magnified in the presence of turbulence because, as stressed by Ljungqvist and Sargent, low-skilled workers who formerly were high-skilled have high job rejection rates. This exercise shows how the matching model can in principle provide an empirically plausible account of the European experience.

Our model is presented in Section 2, and Section 3 analyzes the turbulence/unemployment relationship for both the benchmark and perturbed parameterization. Section 4 considers the effect of reductions in the discount factor on the unemployment rate in the presence of turbulence. Section 5 concludes. Some details of the equilibrium conditions are given in the Appendix.

\footnote{See Blanchard and Wolfers (2000), Nickell, Nunziata, and Ochel (2005), and Nunziata (2003).}
2. Model

In this paper we analyze the turbulence-unemployment relationship within the job-matching framework of Den Haan, Ramey, and Watson (2000). This framework is similar to the standard job-matching framework of Pissarides (2000), but allows workers’ skill level to vary over time. In particular, we develop a model in which workers’ skills increase over time from low to high while they are employed at a given job, but may decline from high to low when they are displaced, reflecting economic turbulence. The model nests the original setup used in Den Haan, Haefke, and Ramey (2001) to analyze the turbulence-unemployment relationship, as well as the specification used in Ljungqvist and Sargent (2004).

2.1. Employment Relationships. Production takes place within employment relationships consisting of one worker and one firm, who interact through discrete time until the relationship is severed or the worker retires. Workers have either high skills or low skills, indicated by subscripts $h$ and $l$, respectively. The skill level of a given worker may vary over time.

An employment relationship produces output $z$ per period. When a relationship is first formed with a worker having skill level $i$, the initial value of $z$ is drawn from the distribution $\nu_i(z)$. Assume $\nu_h(z) < \nu_l(z)$, i.e., the high-skilled distribution first-order stochastically dominates the low-skilled distribution. For a continuing relationship, the value of $z$ may vary as a consequence of relationship-specific productivity shocks that occur at the start of a period. We consider three kinds of productivity shocks.

- With probability $\gamma^s$ there is a switch of the productivity level $z$. In this case, $z$ is drawn again from the distribution $\nu_i(z)$ for a worker with skill level $i$. If no switch occurs, then the relationship maintains the previous period value of $z$. 
• With probability $\rho^x$ the relationship experiences an *exogenous separation* shock, where severance occurs automatically. Exogenous breakups reflect events that cause the surplus to be negative under all circumstances. Assume that exogenous separations cannot occur in the period that a relationship is newly formed.

• With probability $\gamma^u$, a relationship with a low-skilled worker obtains an *upgrade* of the worker’s skills, wherein he receives a draw from the distribution $\nu_u(z)$ and becomes a high-skilled worker if he does not leave the firm in the current period. As long as he remains high-skilled, subsequent draws in any employment relationship involving this worker are made from the distribution $\nu_h(z)$.

After the current-period productivity parameter is determined, the worker and firm decide whether to continue or sever their relationship, and, if the relationship is continued, they determine the worker’s wage payment. Wages are set according to Nash bargaining, where $\pi$ gives the worker’s bargaining weight, and the disagreement point is severance of the relationship. If the worker and firm agree to sever their relationship following a switch, or if exogenous separation occurs, then they forgo production in the current period and instead enter a matching market in which new employment relationships are formed.

To reflect turbulence in the economy, assume that high-skilled workers may lose their skills when they become unemployed. Following an exogenous breakup shock, a high-skilled worker suffers a *downgrade* of his skills, wherein he becomes low-skilled, with probability $\gamma^{d,x}$. Alternatively, if the worker and firm choose to separate following a switch, then $\gamma^d$

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5 We will specify $\nu_u(z)$ to be a simple transformation of $\nu_h(z)$ in order to limit the number of worker categories that must be considered in the analysis. See equation (6) below and the Appendix for details.
gives the probability that the worker’s skills experience a downgrade.\(^6\) For a high-skilled unemployed worker, assume that a downgrade also occurs with probability \(\gamma^d\) if, following a new match, the worker and firm choose not to begin a relationship after observing their initial draw of \(z\). This simplifies the model by making job acceptance and job continuation decisions for high-skilled workers equivalent.

Finally, a worker retires (at the end of the period) with probability \(\rho^r\), in which case he leaves the labor market and obtains a future value of zero.

2.2. Matching Market. New employment relationships are formed on a matching market. Each period, the number of newly formed relationships is \(m(u, v)\), where \(u\) is the mass of all unemployed workers (both low-skilled and high-skilled) and \(v\) is the mass of firms posting vacancies. We assume that \(m(u, v)\) is homogeneous of degree one and that the total masses of workers and firms are fixed at unity. It follows that the unemployment and vacancy pools will always be of equal size, and the matching probability for unemployed workers and firms posting vacancies will be fixed at the value \(\lambda = m(1, 1)\).

Each period, a proportion \(\rho^r\) of the workers leaves the market through retirement, replaced by an identical number of new entrant workers who flow into the unemployment pool. These new entrant workers are assumed to have low skills. Further, established workers enter the unemployment pool when their employment relationships are severed.

While unemployed, a worker obtains a per period benefit of \(b_j\), where \(j\) is the skill level the worker had in his previous job.\(^7\) Since equilibrium benefit levels will satisfy \(b_{h} > b_{l}\), a

---

\(^6\) Note that Den Haan, Haefke, and Ramey (2001) set \(\gamma^{d,x} = \gamma^d > 0\), while LS-II set \(\gamma^d = 0\).

\(^7\) The unemployment benefit of new entrant workers, \(b_e\), is assumed to be zero, and these workers must remain in an employment relationship for at least one period before they qualify for \(b_l\). Further, low-skilled workers experiencing an upgrade must continue in their relationship for at least one period before they
key feature of the model with turbulence is that—as in LS and LS-II—there are low-skilled unemployed workers with the high level of unemployment benefits.

2.3. Zero Surplus Level. Newly-matched workers and firms choose to accept their match and begin an employment relationship if their initial productivity draw $z$ is sufficiently high. Correspondingly, a worker and firm in an ongoing relationship choose to continue their relationship following a switch if the new draw of $z$ is sufficiently high. We assume that the worker and firm bargain efficiently over the terms of their relationship, and thus they make acceptance and continuation decisions that maximize their joint surplus, defined as follows. Let $z$ denote the current period productivity parameter, and suppose the worker obtains unemployment benefit $b_j$ if the firm and the worker choose not to start or continue a relationship. Then the joint surplus for a relationship with a low-skilled worker is given by

$$s_{lj}(z) = (1 - \tau)z + g_l(z) - b_j - w_{lj}^w - w^f,$$  

where $g_l(z)$ denotes the future joint value from continuing the relationship, $w_{lj}^w$ denotes the worker’s future value from entering the unemployment pool in the current period when he receives an unemployment benefit of $b_j$, $w^f$ indicates the firm’s future value from entering the vacancy pool in the current period, and $\tau$ is the tax rate on income earned in the relationship.\footnote{There is a slight abuse of notation in that the tax rate is actually zero when $z < 0$. In both the benchmark and the perturbed parameterization considered in Section 3, negative values of $z$ do not occur.} The surplus equation for a relationship with a worker who has received an upgrade in the current period is given by

$$s_{hl}(z) = (1 - \tau)z + g_h(z) - b_l - w_{hl}^w - w^f,$$  

where $b_l$ qualify for $b_h$. Further details are given in the Appendix.
and the surplus equation for the remaining new and continuing relationships with high-skilled workers is given by

$$s_{hh}(z) = (1 - \tau)z + g_h(z) - b_h - \gamma^d w^w_{lh} - (1 - \gamma^d) w^w_{hh} - w^f. \tag{3}$$

In equilibrium, $s_{ij}(z)$ is an increasing function of $z$ and there exists a zero surplus level $\bar{z}_{ij}$ indicating the smallest value of $z$ at which accepting or continuing the relationship yields nonzero surplus to the worker and firm. For values of $z$ below $\bar{z}_{ij}$, the worker and firm will either reject a new match, or they will sever an existing match. The zero surplus level is defined by the following condition:

$$s_{ij}(\bar{z}_{ij}) = 0, \quad (i, j) \in \{(l, e), (l, l), (l, h), (h, l), (h, h)\}. \tag{4}$$

### 2.4. Equilibrium Future Values.

The equilibrium future joint value from continuing a relationship with a low-skilled worker is equal to

$$g_l(z) = \beta(1 - \rho^x)(1 - \gamma^u) \left[ (1 - \gamma^s) \max(s_{ll}(z), 0) + \gamma^s \int_{\bar{z}_{ll}}^{\infty} s_{ll}(y) d\nu_l(y) \right] + \beta(1 - \rho^x) \gamma^u \int_{\bar{z}_{ll}}^{\infty} s_{ll}(y) d\nu_u(y) + \beta(b_l + w^w_{ll} + w^f). \tag{5}$$

---

9 The surplus for a new relationship is equal to that for a continuing relationship because we make the simplifying assumption that unemployed workers with high skills who turn down wage offers are subject to the same probability of skill loss as workers who separate from existing relationships.

10 The discount factor, $\beta$, equals $(1 - \rho^x)/(1 + \rho)$ and includes both the usual discount rate, $\rho$, and the retirement probability, $\rho^r$.

11 The max operator appears because $s_{le}(z) > 0$ but $s_{ll}(z) < 0$ when $\bar{z}_{le} < z < \bar{z}_{ll}$. For these values of $z$, entrants to the job market will accept the job and then return to the unemployment pool (with benefits) after one period, unless a switch or an upgrade makes it more attractive to stay in the relationship.
TURBULENCE AND UNEMPLOYMENT IN A JOB MATCHING MODEL

The distribution \( \nu_u(z) \) is specified as follows:

\[
\nu_u(z) = \begin{cases} 
0, & z < \tilde{z}_{hh} \\
\nu_h(z)/(1 - \nu_h(\tilde{z}_{hh})), & z \geq \tilde{z}_{hh}.
\end{cases}
\]  

Since \( \tilde{z}_{hh} > \tilde{z}_{hl} \), the specification of \( \nu_u(z) \) implies that relationships experiencing an upgrade will choose to continue with probability one. Thus, in equilibrium there will be no unemployed high-skilled workers with low unemployment benefits. Further, the equilibrium future joint value for a relationship with a high-skilled worker is equal to

\[
g_h(z) = \beta(1 - \rho^x)(1 - \gamma^s)\pi_{shh}(y)d\nu_h(y) + \beta(1 - \rho^x)(b_h + \gamma^d w_{lh}^w + (1 - \gamma^d)w_{hh}^w + w^f) \]

\[
+ \beta \rho^x(b_h + \gamma^d, w_{lh}^w + (1 - \gamma^d)w_{hh}^w + w^f).
\]  

The future values from entering the unemployment pool, \( w_{ij}^w \) and \( w_{hh}^w \), are given by

\[
w_{ij}^w = \lambda \beta \int_{\tilde{z}_{ij}}^\infty \pi_{sij}(y)d\nu_i(y) + \beta(b_j + w_{ij}^w), \quad j = e, l, h,
\]

\[
w_{hh}^w = \lambda \beta \int_{\tilde{z}_{hh}}^\infty \pi_{shh}(y)d\nu_h(y) + \lambda \beta(b_h + \gamma^d w_{lh}^w + (1 - \gamma^d)w_{hh}^w) + (1 - \lambda)\beta(b_h + w_{hh}^w),
\]

For firms, the future value from entering the vacancy pool, \( w^f \), depends on the composition of the unemployment pool. Let \( u_{ij} \) denote the mass of unemployed workers who have skill level \( i \) and unemployment benefit \( b_j \). The probability that a firm in the vacancy pool matches with such a worker is given by

\[
\lambda_{ij}^f = \lambda \frac{u_{ij}}{u}, \quad (i, j) \in \{(l, e), (l, l), (l, h), (h, h)\}.
\]
It follows that the future value obtained by a firm in the vacancy pool is
\[ w^f = \sum_{(i,j) \in S} \lambda_{ij}^f \beta \int_{\mathcal{Z}_{ij}} (1 - \pi) s_{ij}(y) d\nu_i(y) + \beta w^f, \quad S = \{(l, e), (l, l), (l, h), (h, h)\}. \] (11)

2.5. Equilibrium Wage Payments and Unemployment Benefits. The wage payment to a worker having skill level \( i \) and unemployment benefit \( b_j \), written \( p_{ij}(z) \), is determined by the Nash bargaining solution. For low-skilled workers the payment must satisfy
\[ p_{lj}(z) + g_{lj}^w(z) = \pi s_{lj}(z) + b_j + w_{lj}, \quad j = e, l, h, \] (12)
where \( g_{lj}^w(z) \) indicates the future value obtained by the worker from continuing the relationship:
\[ g_{lj}^w(z) = \beta(1 - \rho^x)(1 - \gamma^u) \left[ (1 - \gamma^s) \max(\pi s_{ll}(z), 0) + \gamma^s \int_{\mathcal{Z}_{ll}} \pi s_{ll}(y) d\nu_{ll}(y) \right] + \beta(1 - \rho^x) \gamma^u \int_{\mathcal{Z}_{ll}} \pi s_{ll}(y) d\nu_{ll}(y) + \beta(b_l + w_{ll}). \] (13)

For high-skilled workers who qualify for the high unemployment benefit, the wage payment satisfies
\[ p_{hh}(z) + g_{hh}^w(z) = \pi s_{hh}(z) + b_h + \gamma^d w_{lh}^w + (1 - \gamma^d) w_{hh}, \] (14)
where \( g_{hh}^w(z) \) indicates the workers’ future value from continuation:
\[ g_{hh}^w(z) = \beta(1 - \rho^x) \left[ (1 - \gamma^s) \pi s_{hh}(z) + \gamma^s \int_{\mathcal{Z}_{hh}} \pi s_{hh}(y) d\nu_{hh}(y) \right]. \] (15)

---

12 Note that the matching probabilities \( \lambda \) and \( \lambda_{ij}^f \) relate to matches that are made in the current period, and the joint surpluses \( s_{ij}(y) \) in equations (8), (9) and (11) derive from relationships that begin production in the following period.
Finally, the payment to workers who receive an upgrade is determined by

\begin{equation}
\begin{align*}
& \beta (1 - \rho^x) (b_h + \gamma^d w_{lh}^w + (1 - \gamma^d) w_{hh}^w) \\
& + \beta \rho^x (b_h + \gamma^{d,x} w_{lh}^w + (1 - \gamma^{d,x}) w_{hh}^w).
\end{align*}
\end{equation}

Equilibrium wage levels are used to calculate unemployment benefits, \( b_l \) and \( b_h \). \(^{13}\) In particular, we assume that unemployment benefits for workers with skill level \( j \) in their previous jobs are equal to a fraction \( \phi \) of a weighted average of wages earned by all workers with skill level \( j \). Details are given in the Appendix.

2.6. Worker Groups and Tax Rate. In equilibrium, the population of workers is divided into groups according to their employment status, skills and the level of unemployment benefits for which they qualify. The Appendix gives a more detailed breakdown of the worker groups, along with steady state conditions for each group.

Further, we assume that unemployment benefits are the only government expenditures, and that tax revenues are used to finance these benefits. In equilibrium, the tax rate \( \tau \) adjusts to equate total unemployment benefit payments to total tax receipts. The Appendix provides expressions for benefit payments and tax revenues that are used to calculate the equilibrium tax rate.

3. Turbulence and Unemployment

Our principle objective is to analyze the relationship between the degree of turbulence, as measured by \( \gamma^d \) and \( \gamma^{d,x} \), and the equilibrium steady-state unemployment rate, given by the sum of \( u_{le}, u_{ll}, u_{lh}, \) and \( u_{hh} \). In Section 3.1 we use the parameter values of LS-II that

\(^{13}\)Recall that \( b_e \) is assumed to be zero.
produce a positive relationship between turbulence and unemployment under the assumption \( \gamma^d = 0 \). In Section 3.2 we demonstrate the nonrobustness of this positive relationship to a perturbation that raises \( \gamma^d \) slightly. In Section 3.3 we discuss whether interpreting separations as quits versus layoffs imposes plausible restrictions on \( \gamma^d \).

### 3.1. Results for LS-II Parameter Values.

The benchmark parameterization, given in the column I of Table 1, is taken from Ljungqvist and Sargent (2004), who developed it to match evidence on wage-experience profiles.\(^{14}\) The

\[ \text{is one quarter. Since } \gamma^d \text{ is set to zero under the benchmark parameterization, high-skilled workers never face the prospect of skill loss following an endogenous separation.} \]

Table 2 reports unemployment and job destruction results for various levels of \( \gamma^{d,x} \).

Panel A shows that as the skill loss probability increases from zero to 100%, the aggregate unemployment rate rises from 9.9% to 12.7%. The intuition for this result follows LS: when \( \gamma^{d,x} \) has a high value, there are many unemployed low-skilled workers who had high skills in their previous jobs. These workers receive unemployment benefits that are high in comparison to the wage offers that their currently low skills can generate. Consequently, their rejection rates are very high. For example, when the probability of skill loss equals 10%, a low-skilled worker entitled to high unemployment benefits rejects 44.3% of all job offers.

Panel B shows how endogenous job destruction responds as turbulence increases. En-

\(^{14}\)In our earlier paper (den Haan, Haefke and Ramey (2001)), we used a parameterization that implied an unrealistically narrow range of lifetime earnings, with earnings rising too rapidly within this range. Ljungqvist and Sargent (2004) analyzed an alternative parameterization of our model, with a smaller value of \( \gamma^u \) and a larger gap between the mean values of \( z \) for the high- and low-skilled productivity distributions. In the present paper we make use of their parameterization as a benchmark.
dogenous destruction rates, conditional on receiving a switch, for relationships with high- and low-skilled workers are given by \( \nu_h(z_{hh}) \) and \( \nu_l(z_{ll}) \), respectively. Note that these destruction rates both rise with \( \gamma_{d,x} \). Two factors are important in explaining this result. First, higher unemployment raises total unemployment benefit payments, necessitating a higher tax rate. This makes employment relationships relatively less attractive for both skill levels. Second, the prospect of future skill loss reduces the value of high-skilled relationships, and thus raises the zero surplus margin for high-skilled workers.\(^{15}\) Note however that the values of \( \nu_h(z_{hh}) \) seem unreasonably high, while the values of \( \nu_l(z_{ll}) \) seem unreasonably low.

The last column of Panel B shows that the average inflow rate into unemployment, as a percentage of total labor force, remains roughly constant as \( \gamma_{d,x} \) rises. Ljungqvist and Sargent (2004) stress the importance of this finding, as it agrees with empirical evidence showing roughly constant average unemployment inflows over the 1970s and 1980s. In order to reconcile a constant inflow rate with rising destruction rates for each skill level, a sharp shift in skill composition must occur toward the skill level with the lower destruction rate, which for the parameters considered in LS-II is the low level. Higher turbulence does indeed lead to an increase in the fraction of low-skilled employed workers. For example, when \( \gamma_{d,x} \) increases from 0 to 0.5, the fraction of low-skilled workers increases from 20% to 30%. The empirical validity, however, of both the shift in skill composition and the relatively higher job security of low-skilled workers appears questionable.\(^{16}\)

\(^{15}\)Our model is specified to equate the zero surplus margins of high-skilled unemployed workers and high-skilled employed workers. Similarly, the zero surplus margin of low-skilled unemployed workers who receive low unemployment benefits is equal to that of low-skilled employed workers. Thus, the results of Panel B also apply to rejection rates, conditional on obtaining a match, for these two groups of unemployed workers.

\(^{16}\)Indeed, Franz (2000) reviews evidence showing large shifts toward higher skills among German workers.
3.2. Perturbed Parameterization. Now we perturb the parameterization by allowing for a small positive probability of skill loss following endogenous separations. In particular, let the value of $\gamma^d$ be equal to $\varepsilon \gamma^{d,x}$. The benchmark parameterization sets $\varepsilon$ equal to zero, while this section considers values of $\varepsilon$ equal to 0.01, 0.03, and 0.05. If $\varepsilon$ equals 0.05 and $\gamma^{d,x}$ is 10%, for example, then the probability of skill loss following an endogenous separation is 0.5%. Thus, the perturbation allows turbulence to have at least a tiny effect on workers who choose to separate following persistent declines in their match productivity, while maintaining the hypothesis that the effects of turbulence are largely felt after unavoidable layoffs.

Table 3 reports the results. Aggregate unemployment rates are given in Panel A. Note that the column corresponding to the case of $\varepsilon = 0$ coincides with the results from Panel A of Table 2. Strikingly, as $\varepsilon$ rises from zero to 0.03, the positive relationship between turbulence and unemployment disappears. Once $\varepsilon$ reaches 0.05, unemployment actually decreases with turbulence, and the result of LS is reversed. It follows that the turbulence explanation of LS is not robust to adding a tiny probability of skill loss following an endogenous separation.

What explains this finding? Note first that $\varepsilon$ has little effect on $u_{lh}$, the fraction of unemployed low-skilled workers who are entitled to high unemployment benefits, as may be seen in Panel B of Table 3. Similarly, Panel C indicates that $\nu_l(z_{lh})$, the rejection rate of low-skilled unemployed workers receiving high benefits, is nearly constant across all values of $\varepsilon$. Thus, the effect stressed by LS continues to be present for all of the $\varepsilon$ values. The difference becomes clear when one looks at the breakup probability of high-skilled workers, $\nu_h(z_{hh})$, reported in Panel D. Once $\varepsilon$ rises to 0.03, the relationship reported in Panel B
of Table 2 is reversed, and endogenous job destruction becomes a decreasing function of turbulence. For $\varepsilon$ equal to 0.05, $\nu_h(z_{hh})$ decreases very sharply as turbulence rises.

Of course, the quantitative importance of this result is affected by the specification of the productivity distribution $\nu_h(z)$, which determines the mass of jobs having values of $z$ near the zero surplus margin $z_{hh}$. If the jobs of high-skilled workers are so secure that $z_{hh}$ lies below the support of $\nu_h(z)$, then $\nu_h(z_{hh})$ will equal zero irrespective of any decrease in $z_{hh}$ induced by higher turbulence. On the other hand, $\nu_h(z_{hh})$ will be greatly affected if there exists a large mass of jobs in the neighborhood of $z_{hh}$. Resolving this issue empirically would require data on the number of jobs close to breaking up, which seems difficult to obtain.\(^\text{17}\)

It should be recognized that when the LS channel operates for a particular parameterization, so that turbulence generates a strong increase in job rejection, the reduction in the endogenous destruction rate that we highlight will also tend to be large. The LS channel requires that wage offers given to formerly high-skilled workers be low relative to the unemployment benefits they receive. For this gap to be substantial, there must be a significant difference between the productivity levels of low-skilled and high-skilled jobs. For the benchmark parameterization, productivity realizations under the high-skilled distribution have twice the mean as under the low-skilled distribution. Moreover, the probability that a low-skilled worker upgrades to a high-skilled job is only 2.5% per period. Since the unemployment benefits of formerly high-skilled workers depend on their former productivity, this substantial gap in mean productivity levels implies that formerly high-skilled workers are very likely to reject jobs as low-skilled workers. But it then follows that becoming a low-skilled worker implies a large drop in wages relative to remaining a high-skilled worker.

\(^{17}\)The evidence on wage-experience profiles, however, does provide indirect support for the benchmark parameterization, as Ljungqvist and Sargent (2004) argue.
Thus, high-skilled workers will have very strong incentives to hold on to their jobs when they face a positive probability of skill loss.

To illustrate this point, consider an alternative parameterization in which the productivity distributions are more similar. In particular, let the mean productivity level of low-skilled workers be equal to 1.2, rather than the benchmark value 1.0. We adjust the standard deviation of the low-skilled distribution to maintain the equilibrium values of the endogenous destruction and unemployment rates. Results are reported in Table 4, which recalculates Table 3 using the new parameterization. The effects observed in Table 4 are indeed dampened relative to those of Table 3. In particular, an increase in turbulence does not raise unemployment by as much when \( \varepsilon \) is equal to zero, but it also does not decrease unemployment by as much when \( \varepsilon \) is equal to 0.05.

3.3. Separations as Quits Versus Layoffs. Above we showed that the positive relationship between turbulence and unemployment disappears given only a tiny probability of skill loss following an endogenous separation. In this section we discuss the plausibility of this perturbation of the benchmark parameterization.

LS-II defend their assumption that only exogenous separations can lead to skill loss by arguing that endogenous separations represent worker quits that do not entail a change in skills. As they state: “We see quitters as people who are secure in their skills and inspired to change jobs to take advantage of evident opportunities to make better use of their current skills (p. 462).” They interpret exogenous separations, in contrast, as layoffs that threaten workers with possible loss of skills.

It is clear, however, that the quit/layoff distinction is entirely arbitrary in the context of Nash wage bargaining. Parsons (1986, p. 822) summarizes the key concepts:
“An important implication of this analysis is that no meaningful causal distinction exists between layoffs and quits. The separation rate is a function of the joint distribution of productivity shocks to the firm and to the economy. Job separation conditions are mutually agreed upon, based on complete information on the nature of these shocks. Turnover will occur only when job mobility is efficient because it is in the interest of both parties to agree to such a contract and information and contracting conditions are such that all desirable contracts are attainable.”

Thus, the model as it stands provides no rationale for viewing endogenous separations as quits, contrary to the arguments in LS-II.

Several authors have attempted to impose additional structure in order to obtain a meaningful quit/layoff distinction, while preserving efficient bargaining. The basic idea is expressed by Becker, Landes, and Michael (1977, 1145):

“... a more promising approach relies on the cause of a job or marital separation. A quit could be said to result from an improvement in opportunities elsewhere and a layoff from a (usually unexpected) worsening in opportunities in this job or marriage.”

In following this approach, McLaughlin (1991) adopts a particular bargaining game in which each party can initiate separation. Quits are defined as worker-initiated separations caused by changes in the worker’s productivity outside of the firm, and layoffs are associated with unfavorable changes in productivity within the firm. Mortensen (1994) and Pissarides
extend the matching model to include on-the-job search, with quits tied to the arrival of superior outside offers, and layoffs induced by events within the firm. Each of these models is successful in duplicating the empirical facts that quits are procyclical and layoffs are countercyclical.

LS-II depart from the established literature in associating quits with separations caused by within-firm productivity shifts. Moreover, their assumption leads to the untenable prediction of countercyclical quits.

In the framework of LS-II and this paper, exogenous breakups can be viewed as responses to changes in $z$ such that the surplus becomes permanently negative. Viewed in this way, there is no fundamental distinction, but only a quantitative difference, between exogenous and endogenous separations, as both are optimal responses to a deterioration in $z$. This view accords with our perturbation experiment, where the probability of skill loss after an endogenous severance is a (small) fraction of the probability of skill loss after an exogenous severance.

In practice, most separations result from decisions made by the firm or worker, triggered by changes in the firm’s or worker’s circumstances; that is, they are endogenous. Ideally, empirical data would reveal the particulars of the changes in circumstances and whether they are followed by skill loss. Given the unavailability of this type of data, we think that our

\[18\] An alternative class of models assumes that wage rigidities lead to inefficient separations (e.g., Hashimoto and Yu (1980), Hall and Lazear (1984)). Papers in this vein also adopt the convention that quits result from changes outside the firm, while layoffs are induced by changes within the firm; see Hall (1999, pp. 1154-1156).

\[19\] For example, if the value of $z$ would permanently shift to zero, then this would be sufficient for the firm and the worker to always choose to sever the relationship.

\[20\] An exception may be an industrial accident after which it is physically impossible to continue operations.
reversal of the unemployment-turbulence trade-off for a perturbation of the assumption $\gamma^d = 0$ is problematic for the explanation given by LS and LS-II for the European unemployment puzzle.

4. MAGNIFICATION IN THE PRESENCE OF TURBULENCE

In this section we analyze the effects of a reduction in the discount factor on the unemployment rate and document that the effects are stronger in the presence of turbulence. Although we focus on changes in the discount factor, one can expect turbulence to magnify the effects of changes in other parameter values that increase unemployment. We first give a brief overview of some related empirical and theoretical work, and then present the numerical results.

4.1. Changes in the Discount Factor and the Unemployment Rate. A reduction in the discount factor may be interpreted as either an increase in the real interest rate or a reduction in the rate of disembodied technological growth. During the postwar period there have been important changes to both. In particular, real interest rates were higher in the eighties and nineties then in the fifties and sixties. Also, TFP growth has steadily declined during the postwar period. Starting in the late nineties, both developments have been to some extent reversed.\(^{21}\)

Pissarides (2000) discusses the effect of changes in the discount factor on the unemployment rate.\(^{21}\) The importance of movements in the real interest rate and TFP growth for European unemployment rates is documented in Blanchard and Wolfers (2000), Nickell, Nunziata, and Ochel (2005), and Nunziata (2003). Pissarides and Vallanti (2004) use a structural model and also find that changes in TFP growth help to explain changes in the unemployment rate, but they find quantitatively stronger effects for the US than for Europe.
ment rate in job matching models and shows that the effect is ambiguous. The same is true in our model. The intuition underlying this ambiguity can be easily understood by considering the zero surplus condition for a simplified specification with only one skill level ($\gamma^u = 0$), zero taxes, and a constant unemployment benefit. The equation $s(z) = 0$ may be expressed as

$$z = b - [g(z) - w],$$

where $w = w^w + w^f$. It is easy to show that $(1 - \rho^x)\gamma^s = \lambda$ implies $g(z) - w = 0$, and thus $z = b$. Note that if $(1 - \rho^x)\gamma^s = \lambda$, then the chance of getting a new draw of $z$ while in a relationship is equal to the chance of getting matched (and receiving a new draw by the latter route). Thus, for $z = b$ the worker and firm are indifferent between staying in the relationship and breaking up. Since $g(b)$ is equal to $w$, a decrease in the discount factor reduces $g(b)$ and $w$ by equal amounts and leaves $g(b) - w$, and thus $z$, unchanged. Consequently, a change in $\beta$ also does not affect the unemployment rate.

Similarly, the effect of $\beta$ on the unemployment rate in the complete model is influenced by whether the future values $g_i(z_{ij})$ are larger or smaller than the future values from entering the matching pools. Changes in the level of unemployment benefits, taxes, and composition of the unemployment pool introduce additional complications.

For a wide range of parameter values, including those proposed by Ljungqvist and Sargent (2004), a decrease in the discount factor leads to higher unemployment.22 The purpose of this section is to point out that these changes in the discount factor have a bigger effect on unemployment in the presence of turbulence.

22In particular, the last three columns of Table 2 in LS-II document that unemployment increases when $\beta$ falls in most cases considered.
4.2. Changes in the Discount Factor in the Presence of Turbulence.  To illustrate the effects of a change in the discount factor on the unemployment rate, we use the parameter values in columns II and III of Table 1. These are identical to the ones used previously, except that we set \( \gamma^s \) equal to 0.5.\(^{23}\) Further, we specify \( \gamma^{d,x} = \gamma^d = 0.1 \), i.e., both exogenous and endogenous separations lead to a 10% probability of skill loss. The standard deviations of the productivity distributions are increased relative to those of column I in order to obtain an unemployment rate of 5% under the benchmark value of \( \beta = 0.99 \). The values in column II are such that \( \nu_h(z_{hh}) = 0 \) for all values of \( \beta \) considered, so that small changes in the zero surplus margin of high-skilled workers play no role. In column III the standard deviation of the high-skilled productivity distribution is increased, and \( \nu_h(z_{hh}) > 0 \) for all values of \( \beta \). Rejection and destruction rates for high-skilled workers consequently respond to changes in the discount rate.

Results are reported in Table 5. Panel A presents the effects of a decline in \( \beta \) from 0.99 to 0.98, corresponding to a rise in the real (quarterly) interest rate from about 1% to about 2%, under the column II parameterization. This decline in the discount factor raises the unemployment rate from 5% to 8%. Note that the unemployment effect is due solely to responses in the behavior of low-skilled workers, who make up only 18% of the labor force in this example. In particular, \( \nu_l(z_{ll}) \) rises from zero to 8.8% as \( \beta \) falls from 0.99 to 0.98.

Panel B gives the results under the column III parameterization. Since now the high-skilled workers also respond to lower discount factors, the effects are much larger and the

\(^{23}\)Higher \( \gamma^s \) will raise \( g_i(z_{ij}) \) relative to \( w_{ij}^u + w^f \), since the future value of a marginal relationship derives in part from the prospect of switching to a more favorable productivity level \( z > z_{ij} \). The larger gap between \( g_i(z_{ij}) \) and \( w_{ij}^u + w^f \) increases the effect of a change in the discount factor on unemployment.
unemployment rate increases from 5% to 23.7%.\textsuperscript{24} Because of the increase in their rejection and destruction rates, the number of unemployed with high skills increases from 2.2% of the labor force to 10.7% as $\beta$ rises from 0.99 to 0.98. In addition, Ljungqvist and Sargent’s turbulence effect magnifies the increase of the unemployment rate. In particular, the stock of low-skilled unemployed workers who receive high unemployment benefits rises from 0.5% of the total labor force to 6%. Although the probability of skill loss upon displacement is only 10%, it plays a substantial role in the model since the rejection rates of low-skilled workers entitled to high unemployment benefits are so high. The table also shows that a relatively minor decrease in the discount factor from 0.99 to 0.988 leads to an increase in the unemployment rate from 5% to 10%.

In summary, this section shows that the matching model can account for the rise in unemployment through the empirically plausible mechanism of higher discount rates. Thus, the nonrobustness of the Ljungqvist-Sargent channel should not itself be viewed as undermining the salience of the matching model. Further, the quantitative significance of the discount rate channel is greatly increased when skill loss interacts with high unemployment benefits, suggesting that the presence of turbulence may play an important role even if changes in turbulence cannot convincingly account for the increase in European unemployment.

\textsuperscript{24}The decrease in the discount factor considered in Table 5 leads to an increase in the destruction rate for each skill level and to an increase in the unemployment inflow rate. It is not difficult, however, to construct alternative parameterizations such that composition effects leads to constant average inflow rates. That is, the decrease in the discount factor would shift the distribution of jobs away from those with high destruction rates, exerting a downward effect on the average destruction rate.
5. Conclusion

Despite a large amount of research, the behavior of European unemployment rates remains difficult to understand fully. In this paper we show that the promising explanation offered by Ljungqvist and Sargent, wherein increased economic turbulence leads to high job rejection rates by workers with obsolete skills, does not take adequate account of the incentives of employed workers. If turbulence has even a tiny effect on the skills of workers who endogenously separate from their jobs, then job destruction rates decline dramatically with an increase in turbulence, and unemployment falls. Thus, changes in turbulence cannot provide an explanation for European unemployment that reconciles the incentives of both unemployed and employed workers.\textsuperscript{25} We show, however, that the level of turbulence may play an important role in determining the quantitative impact of other explanatory factors.

The quantitative magnitude of the examples analyzed in this paper depends crucially on the cross-sectional distribution of the joint surplus, especially on that part of the distribution where relationships are close to breaking up. For example, if high-skilled jobs are extremely secure in the sense that their productivity levels lie far above the zero surplus margin, then a moderate change in the margin will not affect any high-skilled workers. This means that only exogenous breakups matter for these workers, and an increase in turbulence would raise the unemployment rate, just as in LS and LS-II. Similarly, for changes in the discount rate to have a significant effect on the unemployment rate, there must be sufficient mass of either high- or low-skilled workers close to their zero surplus margins. Using survey methods it

\textsuperscript{25}Mortensen and Pissarides (1999) consider an alternative to the turbulence specification of LS that posits an increase in the variability of workers’ productivity levels. They analyze a mean-preserving spread of the cross-sectional distribution of productivity levels across workers of given skills. This type of increased variability obviously leads to an increase of both rejection and destruction rates.
may be possible to discover how close employment relationships are to breaking up. The results in this paper point to the potential importance of this kind of information.

6. Appendix

In this Appendix we present the steady state equations for the different unemployed and employed worker groups, as well as the equations used to calculate the unemployment benefits and the tax rate.

6.1. Worker Groups. The assumption that wage rates affect unemployment benefits makes the model quite complex, since the dependence of wage rates on outside options creates a substantial amount of heterogeneity. To reduce complexity we make the following simplifying assumptions:

1. High-skilled unemployed workers who reject a match are subject to skill loss just like workers who separate from an employer. Therefore we do not need to distinguish between separation and job rejection margins for high-skilled workers.

2. Low-skilled workers who receive a skill upgrade also obtain a productivity draw high enough to ensure that they do not leave their job in the upgrade period.

3. The definition of unemployment benefits, given below, merges the new entrants who receive a draw above $z_{il}$ into the group of low-skilled workers who were either previously employed or had previous switches in their continuing relationships. Thus, in calculating the benefit level $b_l$ we replace the wage payments $p_{le}(z)$ with $p_{ll}(z)$ for new entrants with $z \geq z_{il}$. This simplifies the computations by reducing the number of separate groups of continuing low-skilled workers, with a negligible effect on the results.
New entrants who receive a draw between $z_{lh}$ and $z_{ll}$ do not continue in their relationships unless they receive switches or upgrades in their second period of employment, in which case they no longer receive the payment $p_{le}(z)$.

These assumptions leave us with four distinct unemployment groups and five distinct employment groups. These are $u_{le}, u_{ll}, u_{lh}, u_{hh}, e_{le}, e_{ll}, e_{lh}, e_{hl}$ and $e_{hh}$, where the first subscript indicates the skill level and the second the unemployment benefits. Note that a worker can spend only one period in the groups $e_{lh}$ and $e_{hl}$. In addition, it is necessary to keep track of employed low-skilled workers who were once entitled to high unemployment benefits (that is, those who started employment in $e_{lh}$) for the computation of average productivity levels and unemployment benefits. We refer to this group as $e_{ll}^*$. These workers are identical to those in $e_{ll}$ for given $z$, but have productivity levels satisfying $z > z_{lh}$, whereas the $e_{ll}$ workers have productivity levels that satisfy $z > z_{ll}$.

Let the rejection probabilities be denoted as:

$$
\nu_{le} = \int_{-\infty}^{z_{le}} d\nu_l(y), \quad (18)
$$

$$
\nu_{ll} = \int_{-\infty}^{z_{ll}} d\nu_l(y), \quad (19)
$$

$$
\nu_{lh} = \int_{-\infty}^{z_{lh}} d\nu_l(y), \quad (20)
$$

$$
\nu_{hh} = \int_{-\infty}^{z_{hh}} d\nu_l(y). \quad (21)
$$

Each of the following steady state equations has the same format, with the inflows on the left-hand side and the outflows on the right-hand side.

$u_{le}$ group:

$$
\rho^r = [\rho^r + (1 - \rho^r)\lambda(1 - \nu_{le})] u_{le}; \quad (22)
$$
\[ u_{ll} \text{ group:} \]
\[
(1 - \rho^x) \left[ \rho^x + (1 - \rho^x)(1 - \gamma^u)(1 - \gamma^s(1 - \nu_{ll})) \right] e_{le} \\
+ \left[ \rho^x + (1 - \rho^x)(1 - \gamma^u)\gamma^s\nu_{ll} \right] (e_{ll}^* + e_{ll} + e_{lh}) \\
= \left[ \rho^x + (1 - \rho^x)\lambda(1 - \nu_{ll}) \right] u_{ll};
\]

\[ u_{lh} \text{ group:} \]
\[
(1 - \rho^x) \left[ \rho^x \gamma^{d,x} + (1 - \rho^x)\gamma^s\nu_{hh}\gamma^d \right] (e_{hl} + e_{hh}) + \lambda\nu_{hh}\gamma^du_{hh} \\
= \left[ \rho^x + (1 - \rho^x)\lambda(1 - \nu_{lh}) \right] u_{lh};
\]

\[ u_{hh} \text{ group:} \]
\[
(1 - \rho^x) \left[ \rho^x(1 - \gamma^{d,x}) + (1 - \rho^x)\gamma^s\nu_{hh}(1 - \gamma^d) \right] (e_{hl} + e_{hh}) \\
= \left[ \rho^x + (1 - \rho^x)\lambda(1 - \nu_{hh} + \nu_{hh}\gamma^d) \right] u_{hh};
\]

\[ e_{le} \text{ group:} \]
\[
(1 - \rho^x)\lambda(\nu_{ll} - \nu_{le})u_{le} = e_{le};
\]

\[ e_{ll} \text{ group:} \]
\[
(1 - \rho^x) \left\{ \lambda(1 - \nu_{ll})(u_{le} + u_{ll}) + (1 - \rho^x)(1 - \gamma^u)\gamma^s(1 - \nu_{ll})(e_{le} + e_{ll}^* + e_{lh}) \right\} \\
= [1 - (1 - \rho^x)(1 - \rho^x)(1 - \gamma^u)(1 - \gamma^s\nu_{ll})]e_{ll};
\]

\[ e_{lh} \text{ group:} \]
\[
(1 - \rho^x)\lambda(1 - \nu_{lh})u_{lh} = e_{lh};
\]

\[ e_{ll}^* \text{ group:} \]
\[
(1 - \rho^x)(1 - \rho^x)(1 - \gamma^u)(1 - \gamma^s)e_{lh} = [1 - (1 - \rho^x)(1 - \rho^x)(1 - \gamma^u)(1 - \gamma^s)]e_{ll}^*;
\]
e_{hl} group:

\[ (1 - \rho^r)(1 - \rho^x)s(e_{le} + e_{ll}^* + e_{ll} + e_{lh}) = e_{hl}; \]  

(30)

\[ e_{hh} \text{ group:} \]

\[ (1 - \rho^r)\{\lambda(1 - \nu_{hh})u_{hh} + (1 - \rho^x)(1 - \gamma^s\nu_{hh})e_{hl}\} = [1 - (1 - \rho^r)(1 - \rho^x)(1 - \gamma^s\nu_{hh})] e_{hh}. \]  

(31)

\[ 6.2. \text{ Unemployment Benefits.} \quad \text{Unemployment benefits of a worker with skill level } j \text{ in his previous job are equal to a fraction } \phi \text{ of a weighted average of wages earned by workers with skill level } j. \text{ In particular, } b_l \text{ and } b_h \text{ satisfy} \]

\[ e_l b_l = \phi [e_{le} \int_{z_{le}}^{Z_l} p_{le}(z) \frac{dv_{l}(z)}{1 - \nu_l(z_{le})} + e_{ll} \int_{z_{ll}}^{Z_{ll}} p_{ll}(z) \frac{dv_{l}(z)}{1 - \nu_l(z_{ll})} + e_{lh} \int_{z_{lh}}^{Z_{lh}} p_{lh}(z) \frac{dv_{l}(z)}{1 - \nu_l(z_{lh})}], \]  

(32)

\[ e_h b_h = \phi [e_{hh} \int_{z_{hh}}^{Z_{hh}} p_{hh}(z) \frac{dv_{h}(z)}{1 - \nu_h(z_{hh})} + e_{hl} \int_{z_{hl}}^{Z_{hl}} p_{hl}(z)dv_{h}(z)], \]  

(33)

where

\[ e_l = e_{le} + e_{ll} + e_{ll}^* + e_{lh}, \]  

(34)

\[ e_h = e_{hl} + e_{hh}. \]  

(35)

Note that \( b_h \) is simply equal to a fraction \( \phi \) of the average wage of high-skilled workers. Because we merge new entrants with a draw of \( z \) above \( z_{ll} \) with the group of previously-employed low-skilled workers, \( b_l/\phi \) is not exactly equal to the average wage of low-skilled workers but the difference with this average wage is minuscule.

\[ 6.3. \text{ Tax Base and Government Revenue.} \quad \text{Let } \hat{z}_l \text{ denote the average productivity of all low-skilled workers and } \hat{z}_h \text{ the average productivity of all high-skilled workers. These can} \]
be solved from

\[ e_l \hat{z}_l = e_{l0} \int_{\hat{z}_l}^{\hat{z}_h} \frac{dy}{\nu_l(y) - \nu_l(\hat{z}_h)} + e_{l1} \int_{\hat{z}_l}^{\infty} dy \frac{dy}{1 - \nu_l(\hat{z}_l)} \]

\[ + (e_{h0} + e_{hl}) \int_{\hat{z}_h}^{\infty} dy \frac{dy}{1 - \nu_h(\hat{z}_h)} \]

\[ e_h \hat{z}_h = (e_{hh} + e_{hl}) \int_{\hat{z}_h}^{\infty} dy \frac{dy}{1 - \nu_h(\hat{z}_h)} \]

The tax rate then follows from the government budget constraint:

\[ b_{u0} + b_{u1} (u_{lh} + u_{lh}) = \tau (e_l \hat{z}_l + e_h \hat{z}_h). \]

For the parameterizations in columns II and III in Table 1, negative values of \( z \) are possible. For these parameterizations, we adjust the expressions for the tax base to reflect the fact that taxes are zero when profits are negative, that is, there are no subsidies.

**References**


Table 1: Parameter values

I: Parameter values used to study effect of turbulence on unemployment rates; preferred parameter values of Ljungqvist and Sargent (2004)
II & III: Parameter values used to study effects of changes in discount factor on unemployment rates

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<th>Parameter</th>
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<th>II</th>
<th>III</th>
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Table 2: Effects of turbulence

Panel A: Unemployed as fraction of labor force (%)

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<th>$\gamma^{d,x}$</th>
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<th>low skilled &amp; high benefits</th>
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Panel B: Endogenous destruction rates and inflow into unemployment (%)

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<th>average inflow</th>
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Note: Destruction rates are conditional on receiving a switch.
Table 3: Effects of turbulence for perturbed parameterization

Panel A: Unemployment rates (%)

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<tr>
<th>$\gamma^{d,x}$</th>
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<th>0.03</th>
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Panel B: Low-skilled unemployed with high benefits as fraction of labor force (%)

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<th>0.03</th>
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Panel C: Rejection rates of low-skilled unemployed with high benefits (%)  

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<th>0.03</th>
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Note: Rejection rates are conditional on obtaining a match.

Panel D: Endogenous destruction rates for high-skilled employed (%)  

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Note: Destruction rates are conditional on receiving a switch.
Table 4: Effects of turbulence with more similar productivity distributions

Panel A: Unemployment rates (%)

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Panel B: Low-skilled unemployed with high benefits as fraction of labor force (%)

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### Panel C: Rejection rates of low-skilled unemployed with high benefits (%)

<table>
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Note: Rejection rates are conditional on obtaining a match.

### Panel D: Endogenous destruction rates for high-skilled employed (%)

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<th>0.03</th>
<th>0.05</th>
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</table>

Note: Destruction rates are conditional on receiving a switch.
Table 5: Effects of discount factor

Panel A: No endogenous breakups for high-skilled workers

<table>
<thead>
<tr>
<th>β</th>
<th>unemployment rate (%)</th>
<th>average duration</th>
<th>inflow rate (%)</th>
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</thead>
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<tr>
<td>0.99</td>
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Note: The results in this table are based on the parameters under II in Table 1.

Panel B: Endogenous breakups for high-skilled workers

<table>
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<th>β</th>
<th>unemployment rate (%)</th>
<th>average duration</th>
<th>inflow rate (%)</th>
</tr>
</thead>
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Note: The results in this table are based on the parameters under III in Table 1.

Table 6: Different types of workers

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<th>Employment States</th>
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