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Forthcoming

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Aggregate Output, Capital, and Labor in the Post-War U.S. Economy

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Abstract: New estimates of an aggregate long-term production function for the post-war U.S. economy are reported. The results indicate that this long-term aggregate production function exhibits a slight but statistically significant increasing returns to scale. Since virtually all econometric growth studies assume constant returns to scale, my finding raises serious doubts about the validity of this common practice. I also find that since the war real output has become more sensitive to changes in capital and less sensitive to changes in labor. In particular, I show that the long-run capital and labor elasticities of real output are both in the range of 0.44–0.55. Similar estimates for the capital and labor elasticities of output from earlier studies covering pre-war and the inter-war periods are 0.25 and 0.75, respectively.

Keywords: Aggregate Production Function, Cobb-Douglas Production Function, Returns to Scale, Long-Term, Increasing Returns to Scale, LR Capital Elasticity of Output, LR Labor Elasticity of Output, Estimates of Technical Progress

JEL Codes: E10, E22, E23, E24, E25, O30, O40, O47, O51

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1. Introduction

The literature that covers empirical estimation of aggregate production functions in a partial equilibrium framework is not extensive, although there are many studies on the components of total output. There are several reasons for this. First, from the theoretical point of view, the determination of total output cannot be separated from the determination of other endogenous variables and therefore ideally it should be investigated in a general equilibrium framework. Second, an empirical estimation of an aggregate production function requires simplifying assumptions about the decision making process of the firms about technology, production process and input hiring, and about the similarity of this process across firms and industries. Therefore, assuming the existence of an aggregate production function imposes strict requirements on the underlying sectoral models of production. In particular, it requires that the value added function and the input functions of the capital and labor for each sector must be replicas of the aggregate functions.¹

But this criticism applies to situations in which the data used in the study covers only a short period. Jorgenson (1988, p. 30) touches on this point when he talks about the limitations of the aggregate production function: ‘Over the period 1948–1979 the reallocations are very small relative to the growth of capital and labor inputs and productivity growth. These results show that an aggregate production model may be appropriate for studies of long-term U.S. economic growth. However, an aggregate model can be seriously misleading for relatively short time periods such as the individual business cycles 1948–1953, 1966–1969, and 1973–1979’. He concludes that ‘the aggregate production function model introduced by Cobb and Douglas (1928) and developed by Tinbergen (1942) retains its usefulness in modeling long-term growth trends’ [Jorgenson (1988, p. 31)].

In this study such a long-term aggregate production function is estimated using the quarterly data of the post-war (1948–1983) U.S. economy. With all the drawbacks of this partial equilibrium approach, its use is still worthwhile since the results can be compared with the results of earlier studies that use the same methodology and similar data sets. The next section

¹ Another reason is the belief of many economists that, in order to understand the behavior of the output, one should study the behavior of its components (business cycle approach). Also, from the empirical perspective, attempts to fit a Cobb-Douglas form production function estimated with the time series observations of output, labor and capital often do not produce satisfactory results in the sense that the estimated parameters are not of the size or the sign that the economic theory predicts.

describes the data set used in this study. The estimation results are discussed in section 3. In section 4 the findings are summarized and compared to the results of previous studies. The section ends with some concluding remarks.

2. The data

The quarterly data used in this study consist of real output, real capital stock and employment data for the period 1948-1983 (144 observations). The data for output and capital stock were kindly provided by Robert J. Gordon. While there is nothing unique about the output data, the quarterly stock of capital data was created by using the corresponding annual data published in various issues of *Survey of Current Business*. This was done in two steps:

- (a) The annual series' values were treated as beginning and ending values of the quarterly series;
- (b) Under a fixed exponential rate of depreciation, the quarterly series satisfy $K_t = I_t + (1 - \delta)K_{t-1}$.

The depreciation rates (δ) of each capital stock component were iterated until the fourth quarter's value converged to the end of the year value from the annual series. The resulting estimated annual depreciation rates for the stock of non-residential structures and producers' durable equipment were 6.036% and 14.3%, respectively [see Gordon and Veitch (1984) for more details]. The quarterly employment figures were taken from *Business Conditions Digest*, Bureau of Economic Analysis (February 1984, p. 101).

3. Estimation results

Using the data described in section 2, I first estimate the standard Cobb-Douglas (henceforth CD) production function of the form

$$Y_t = A_0 e^{\omega t} K_t^\alpha L_t^\beta e^{u_t} \quad (1)$$

where Y_t is real GNP, K_t is the real stock of capital; L_t is the number of employees; α and β are capital and labor elasticities of the real output; ω is the exponential rate of technical

progress; A_0 is an efficiency measure of the production process; u_t is the error term with the usual properties; and t is time. The standard prior assumptions are:

$$0 < \alpha < 1, 0 < \beta < 1, \omega > 0, \text{ and } A_0 > 0.$$

It should be emphasized that the residuals of all the estimated eqs. (2), (3) and (4) in this study were analyzed using the Box-Jenkins technique and found to follow Auto-Regressive Moving Average (henceforth ARMA) (2, 0) processes. Consequently, all four equations were estimated using RATS' Filtered Least Squares procedure which is analogous to Cochrane-Orcutt for the ARMA(1, 0) case.² The finding that the residuals in our model follow an ARMA(2, 0) process is not surprising in the light of the fact that many studies based on quarterly time series data conclude that a second-order autoregressive filter is sufficient for the residuals to follow a white noise. [See for example Barro and Rush (1980), Hoffman and Schlagenhauf (1982), or Sims (1972).]

First the CD form production function was estimated without imposing the Constant Returns to Scale (henceforth CRS) condition. The estimate, using quarterly data for the period 1948–1983, is:³

$$\begin{aligned} \log Y_t = & -0.250 + 0.560 \log K_t + 0.540 \log L_t & (2) \\ & (-2.421) \quad (7.887) \quad (3.495) \\ R^2 = & 0.88, \quad DW = 2.32, \quad SEE = 0.0094 \end{aligned}$$

² One may argue that GLS estimates of α and β may be biased as the error term in eq. (2) might be correlated with $\log K$ and $\log L$. Correcting for this potential bias would require a profit maximization approach which would yield a simultaneous equations system that is not easy to estimate. A method of solving such a system involves simplifying assumptions about correlations between managerial and technical efficiencies. Another drawback of that approach is the fact that constant or increasing returns to scale are not compatible with the assumption of both perfect competition and profit maximization and therefore there must be decreasing returns to scale in order for the marginal productivity conditions to have a determinate solution corresponding to maximum profit. Finally, in order to make the results of this study comparable with the results of earlier studies, I decided to follow the single equation methodology.

³ The numbers in parentheses are the t -values.

which implies that the returns to scale parameter, $(\alpha + \beta)$, slightly exceeds one. Note that the significance of all t -values indicates that multicollinearity is not a severe problem. I also computed the determinant of the sample correlation matrix; it is strictly different from zero.

Next, since $(\alpha + \beta)$ is slightly greater than one, the CRS constraint, $\alpha + \beta = 1$, was imposed.⁴ The new equation was estimated twice: once with the technical progress parameter [eq. (3)] and once without it [eq. (4)]:

$$\log(Y/L) = -0.189 + 0.001t + 0.416 \log(K/L) \quad (3)$$

(-3.643) (1.339) (2.681)

$$R^2 = 0.58, \quad DW = 2.35, \quad SEE = 0.0093$$

$$\log(Y/L) = -0.098 + 0.550 \log(K/L) \quad (4)$$

(-7.409) (8.605)

$$R^2 = 0.35, \quad DW = 2.15, \quad SEE = 0.0092$$

The estimated equations' parameters are summarized in table 1.⁵

⁴ The test statistic for testing the hypothesis of CRS — $F(1139)$ — equals 3.90 which is on the border of significance at 5 percent.

⁵ I also estimated a more general Constant Elasticity of Substitution production function of the form $Y_t = \gamma [\delta K_t^{-\rho} + (1-\delta)L_t^{-\rho}]^{-\nu/\rho} e^{\varepsilon_t}$ by first transforming it into a log-linear form using Taylor's expansion. The estimated equation is given below:

$$\log Y = -10.9 + 0.890 \log K + 0.940 \log L - 0.318 [\log(K/L)^2], \quad R^2 = 0.88, \quad DW = 1.94, \quad SEE = 0.0087 \quad (1')$$

(-2.69) (1.43) (2.04) (-1.45)

which implies the following values for the original parameters of the production function:

$$\gamma = 0.000018, \quad \delta = 0.486, \quad \nu = 1.83, \quad \rho = 1.39, \quad \sigma = 0.42 \quad (2')$$

(0.001) (2.55) (1.07) (1.78) (3.06)

In (2') the numbers in parentheses are the asymptotic t -values. In order to find these t -values, the variance of the estimates were computed by first taking linear approximations of the coefficients in eq. (1'). The general formula for the approximated variance can be written as

Table 1. Estimated capital and labor elasticities of output, U.S., 1948–1983

Equation Number	Labor elasticity	Capital elasticity	Returns to scale ($\alpha + \beta$)	CRS imposed
(2)	0.54	0.56	1.10	No
(3)	0.45	0.55	1.00	Yes
(4)	0.58	0.42	1.00	Yes

4. Summary and conclusions

The results of this study indicate that the long-term aggregate production function of the post-war U.S. economy exhibits slightly increasing returns to scale. Almost the same result was obtained in a similar study by Douglas (1948) covering the period 1899–1922, Brown (1966) for the period 1890–1960, and Intriligator (1965) for the period 1929–1958. This suggests that the returns to scale has been stable for the U.S. economy for almost a century. The estimated returns to scale parameter equals 1.10 which is exactly what Denison (1974) assumed in his calculations of the sources of growth of the U.S. economy.⁶ My statistical findings support his assumption. As Boskin (1988) points out, virtually all econometric growth studies assume CRS. This paper raises serious doubts about the validity of this practice.

$$\text{Var}(\hat{\phi}) \approx \sum_{i=1}^m \left[\frac{\partial f}{\partial \theta_i} \right]^2 \text{Var}(\hat{\theta}_i) + 2 \sum_{j < i} \left[\frac{\partial f}{\partial \theta_j} \right] \left[\frac{\partial f}{\partial \theta_i} \right] \text{Cov}(\hat{\theta}_j, \hat{\theta}_i)$$

where $\hat{\phi}$ is the estimate with the unknown variance — $\gamma, \delta, \nu, \rho, \sigma$ — and the $\hat{\theta}_i$'s are the estimates from (1'). The resulting estimates are asymptotically efficient. The long-term coefficient of substitution between capital and labor, σ , equals 0.42 and is significant. This finding is consistent with the result obtained by Eisner and Nadiri (1968) but inconsistent with the result of Jorgenson and Stephenson (1969) who claim that the elasticity of substitution between capital and labor is closer to one. It is important to get a reliable estimate of this coefficient since its true value has strong implications for understanding the behavior not only of output, but also of capital and investment. For example, in Jorgenson's formulation of the investment function, which states that investment depends on the gap between the desired and actual capital stock, the assumption of unit elasticity of substitution between capital and labor plays a crucial role. If this elasticity is closer to zero, then the accelerator model of investment becomes relevant. See the survey article of Abel (1988) for further details. Note that, the high value of the R^2 and relatively low values of the t -statistic indicate presence of multicollinearity, and the estimates of γ , ν , and ρ are not statistically significant. It should also be mentioned that among several functional forms tried, the reported estimated production functions gave the most reasonable results.

⁶ Some people have criticized Denison for assuming slightly increasing returns to scale without ever estimating it. But as the results of this study indicate, his assumption ex post seems to have empirical support.

The results indicate that since the war GNP became more sensitive to changes in capital and less sensitive to changes in labor. The long-run capital and labor elasticities of the real output obtained here are different from their estimates for the pre-war or interwar period. While I find both estimated elasticities in the range of 0.45–0.55 for the post-war U.S. economy, for 1899–1922 and 1929–1958 the capital and labor elasticities of output, as reported by Douglas (1948) and Intriligator (1965), are about 0.25 and 0.75, respectively.

The increasing importance of the capital stock in the growth of the U.S. economy has been documented in other studies as well, but the figures are not as high as my estimate. For example, the figures cited in Maddison's survey article (1987) for the weight of the capital stock range from as low as 0.21 to as high as 0.40⁷. It should be noted that these figures were estimated in studies that cover much shorter periods than my study. This suggests that the importance of capital in explaining the long-term growth of the U.S. economy exceeds its importance in explaining short-term business cycle fluctuations.

The increasing importance of the capital input in the growth of the U.S. economy since 1948 is emphasized also in studies that analyze the source of the U.S. economic growth [Boskin (1988) Christensen and Jorgenson (1973), Jorgenson (1986, 1988)]. For example, Jorgenson recently argued that 'comparing the contribution of capital input with other sources of output growth for the period 1948–1979 as a whole makes clear that capital input is the most significant source of growth' (1988, p. 25). This increasing importance of the capital input in the long-term growth of the U.S. economy can be explained by the fact that the production over the period became more capital intensive. In addition, some part of the existing capital stock reflects the substantial technological improvements that have occurred since the war, the capital stock time series captures a large fraction of this technical progress. The idea is that \$1 worth of 1950's capital investment is not the same as \$1 worth of 1980's capital investment due to the technological progress witnessed since the war in the fields of computers, computerized automation, robotics, etc. This explains why my estimate for technical progress is relatively low. This may also explain Jorgenson's (1988) finding that most of the aggregate growth of U.S. economy can be attributed to the combined contribution of capital and labor to the total output.

⁷ See Maddison (1987, p. 660, table 8). A possible reason for the differences between my estimates and the previous estimates might be the difference between the data sets. My data is quarterly while the previous studies use annual data.

References

- Abel, A.B., 1988, Consumption and investment, NBER working paper no. 2580 (National Bureau of Economic Research, Cambridge, MA).
- Barre, R.J. and M. Rush, 1980, Unanticipated money and economic activity, in: S. Fischer, ed., *Rational expectations and economic policy* (University of Chicago Press, Chicago, IL) 23–48.
- Boskin, M.J., 1988, Tax policy and economic growth: Lesson's from the 1980s, *Journal of Economic Perspectives* 2, 71–97.
- Brown, M., 1966, *On the theory and measurement of technological change* (Cambridge University Press, New York).
- Christensen, L.R. and D.W. Jorgenson, 1973, Measuring the performance of the private sector in the U.S. economy, 1929–1969, in: M. Milton, ed., *Measuring economic and social performance* (National Bureau of Economic Research, New York) 233–238.
- Cobb, C.W. and P.H. Douglas, 1928, A theory of production, *American Economic Review* 18, 139–165.
- Denison, E.F.. 1974, *Accounting for United States economic growth 1929–1969* (Brookings Institution, Washington, DC).
- Douglas, P.H., 1948, Are there laws of production? *American Economic Review* 38, 1–49.
- Eisner, R. and M.I. Nadiri, 1968, Investment behavior and neo-classical theory, *Review of Economics and Statistics* 50, 369–382.
- Gordon, R.J. and J. Veitch, 1984, Fixed investment in the American business cycle, 1919–1983, NBER working paper no. 1426 (National Bureau of Economic Research, Cambridge, MA).
- Hoffman, D.L. and D.E. Schlagenhauf, 1982, An econometric investigation of the monetary neutrality and rationality propositions from an international perspective, *Review of Economics and Statistics* 64, 562–571.
- Intriligator, M.D., 1965, Embodied technical change and productivity in the U.S., 1929–1958, *Review of Economics and Statistics* 47, 65–70.
- Jorgenson, D.W., 1986, Productivity and economic growth, Discussion paper (Harvard Institute of Economic Research, Cambridge, MA).
- Jorgenson, D.W., 1988, Productivity and postwar U.S. economic growth, *Journal of Economic Perspectives* 2, 23–41.

- Jorgenson, D.W. and J.A. Stephenson, 1969, Issue in the development of the neoclassical theory of investment behavior, *Review of Economics and Statistics* 51, 346–353.
- Maddison, A., 1987, Growth and slowdown in advanced capitalist economies: Techniques of quantitative assessment, *Journal of Economic Literature* 25, 649–698.
- Sims, C.A., 1972, Money, income and causality, *American Economic Review* 62, 540–552.
- Tinbergen, J., 1942, Zur Theorie der langfristigen Wirtschaftsentwicklung, *Weltwirtschaftliches Archiv* 55. 511–549.