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## Labor Market Institutions and the Employment-Productivity Trade-Off : A Wage Posting Approach

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# Labor Market Institutions and the Employment-Productivity Trade-Off: A Wage Posting Approach

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## ABSTRACT

### **Labor Market Institutions and the Employment-Productivity Trade-Off: A Wage Posting Approach\***

This paper analyzes the implications of labor market institutions and policies on the employment-labor productivity trade-off. We consider an equilibrium search model with wage posting and specific human capital investment where unemployment and the distribution of both wages and productivity are endogenous. By means of simulations of this model estimated on French data, we show that the minimum wage allows a high production level to be reached by inducing increased training investment, even if its optimal level is weaker. Considering the payroll tax subsidies implemented to lower the labor cost without removing the minimum wage legislation, we show that this policy has been welfare improving, and has been relatively well managed by spreading subsidies over a large range of wages, and not only at the minimum wage level.

JEL Classification: C51, J24, J31, J38

Keywords: employment, productivity, wage posting, labor costs

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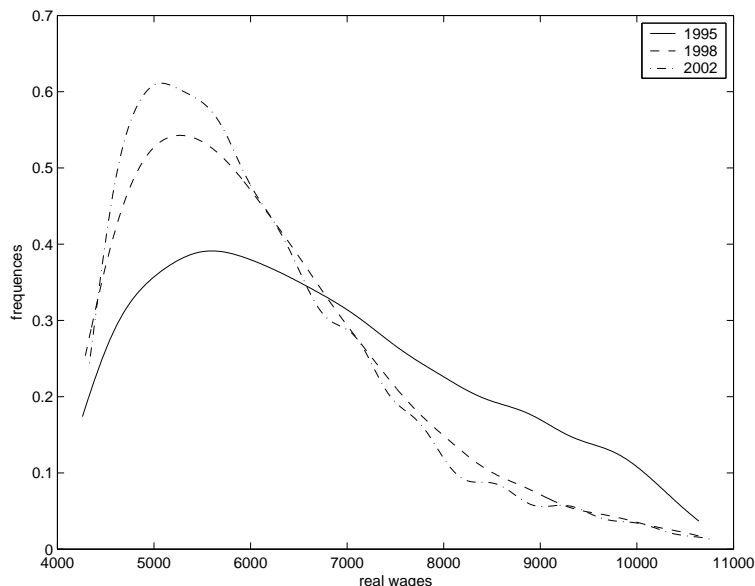
## Introduction

European countries have experienced high rates and high average durations of unemployment for the last two decades. High reservation wages due to generous unemployment benefits, employment protection and a minimum wage are traditionally quoted as responsible for the persistence of a high level of unemployment in continental European countries (Ljunqvist and Sargent [1998], Blanchard and Wolfers [2000], Laroque and Salanié [2000], [2002] and Wasmer [2002]). Simultaneously, European labor productivity (product per hour) appears relatively high (Layard, Nickell, and Jackman [1991]). Beyond a traditional decreasing returns effect, these features could be interpreted as the outcome of a highly frictional labor market where low mobility ensures long enough expected job duration to make human capital investments profitable for firms (see Wasmer [2002] and Acemoglu and Shimer [1999]). This assessment pleads in favor of reexamining the question of the efficiency of labor market institutions and policies by taking into account the productivity channel additionally to the employment effect.

In this paper, we propose to study the potential employment-productivity trade-off in the case of the French labor market where institutions are often quoted as responsible for the unemployment of low-skilled workers. Scholars of the French labor market have already extensively pointed out the negative role played by the minimum wage legislation through increasing labor costs (Laroque and Salanié [2000] and [2002]). We think it is worth quantitatively reassessing the role of the minimum wage legislation by taking the productivity channel into account. But it also seems necessary to analyze in the same framework the payroll tax subsidies for low-wage workers which have been implemented during the nineties in order to compensate for the negative impact on employment of the minimum wage without exacerbating wage inequality. The particular design of this policy tries to avoid, by not concentrating all the subsidies at the minimum wage level, too many reallocations towards badly-paid jobs: it consists of a decreasing reduction of payroll taxes up to 1.33 times the minimum wage, with a maximum reduction of 18.2 points at the minimum wage level. However, as shown in Figure 1, there has been a deformation of the wage distribution of the manual workers towards the bottom. In this paper, we propose the wages and productivity distributions changes. We then evaluate the payroll tax exemptions policy, not only on employment, but also taking into account the productivity channel and ask whether the implemented reform lies on the optimal range of exonerated wages. We compare this implemented policy with a minimum wage decrease, which could have been another option.

In order to quantitatively evaluate the employment plus productivity ef-

Figure 1: Observed wage distributions of manual workers (France)



fects of labor cost policies, we build a structural model. This structural strategy contrasts with recent econometrical exercises (see for instance Kramarz and Philippon [2001] Crépon and Desplatz [2002]) and will allow us to examine some policy experiments. A crucial element in this strategy relies on the way of making productivity endogenous. We propose a model with human capital investment and an identification strategy of the elasticity of the productivity relative to human capital investment based on the observed wage distribution. More precisely, we consider a wage posting model with specific human capital investments, extending the Burdett and Mortensen [1998] wage posting model along the lines Mortensen [2002]. Indeed Bontemps, Robin, and van den Berg [1999], not only provide some empirical evidence in favor of the wage posting approach concerning French wage distribution, but also show that the Burdett and Mortensen [1998] theory of pure dispersion cannot totally explain the skewed wage distribution, unless the assumption of productive heterogeneity across employers is made: this result emphasizes that the productivity distribution plays a central role in the replication of the observed wage distribution, and conversely suggests an identifying strategy to estimate the elasticity of productivity relative to human capital investment. Moreover, Postel-Vinay and Robin [2002] show that the productivity differentials across firms explain about half of the French low-skilled wage variance, while the remaining part is completely due to search friction, without any

room left for individual fixed effects<sup>1</sup>. Robin and Roux [2002] also show that the introduction of general capital in a model *à la* Burdett and Mortensen does not match the observed French wage distribution<sup>2</sup>. Previous empirical evidence, along the lines of Lynch [1992] and Black and Lynch [1996], underlines the significant role of specific firm’s investments in human capital in the distribution of workers’ wages and productivity. On French data, Chéron, Hairault, and Langot [2003] show that the firm-provided training programs, without post-training mobility experience, have an estimated return of 6.1%, after controlling for the firms’ training selectivity<sup>3</sup>, and that low-wage workers gain more from the firm-training programs, even if the “firm-provided training” is the least prevalent among these workers.

We thus build a wage posting model, where both productivity and unemployment are endogenous, along the lines of Mortensen [2000]. In this framework, the expected job duration determines to what extent firms invest in specific human capital. More originally, the wage posting strategies of firms and their training investment are strongly interrelated. The negative association between wage and labor turnover creates incentives for training employees: high productivity allows firms to post high wages and then to expect long duration of the match. This last feature encourages high investment in specific human capital, because the period in which the firm can recoup its investment thus increases. In equilibrium, firms choose different levels of training and wage offers, which results in endogenous within-market productivity differences and consequently a dispersed equilibrium wage offer distribution. Along the lines of Mortensen [2000], the wage posting approach is incorporated into search equilibria, as developed by Pissarides [1990], in order to consistently determine the unemployment and the vacancy rates. It leads to a joint theory of wage (and productivity) and employment where the effect of labor market institutions are not *a priori* determined between the disincentives to create jobs and the reduction of the monopsony power of firms. Furthermore, we include some realistic features in order to deal with the efficiency of labor market policies. We introduce two types of extension. First, we take into account the existence of a minimum wage which influences the labor cost and so the recruiting effort of firms. Secondly, we take into account the transition between short run and long run unemployment. This last extension allows to introduce a time-varying unemployment benefits system, and some heterogeneity of offer arrival rates according to the status of

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<sup>1</sup>The share of the individual effect, interpreted as general human capital, increases with the skill of workers (Postel-Vinay and Robin [2002]).

<sup>2</sup>Mortensen [2002] finds a similar result by using a simple calibrated model.

<sup>3</sup>International evidence shows that the returns of firm-provided training are significant: 2% in Germany (see Pischke [1996]) and 12% in the US (Blanchflower and Lynch [1994]).

individuals (employment, short run or long run unemployment). These features lead to an endogenous distribution of the unemployed workers' reservation wages which allows us to better evaluate the role of the minimum wage legislation.

We estimate crucial parameters of the model on French data using Simulated Method of Moments. Statistical tests do not allow us to reject the hypothesis that the theoretical wage distribution is generated by the same law as the observed one. Simulations show that this type of labor market equilibrium is able to generate a large set of the characteristics consistent with their empirical counterparts, in particular a binding minimum wage.

We thus investigate the different implications of the minimum wage on output<sup>4</sup>. Surprisingly, its optimal level seems to be only slightly weaker than the existing one: a decrease in the minimum wage leads to an employment boost which is not compensated for by the decline of labor productivity. The opposite occurs when considering values below the optimal minimum wage. Removing the productivity channel would have led to a very different conclusion: we show that the short run unemployment allocation would have been binding. Despite the existence of long run unemployed workers, who would be willing to work for a lower wage, it can be shown that no firms would propose a wage below the reservation wage of the short run unemployed workers. In that sense, the minimum wage legislation would be unnecessary. By contrast, taking the productivity channel into account, emphasizes the importance of the minimum wage. Considering the payroll tax subsidies implemented to lower the labor cost without removing the minimum wage legislation, we show that this policy has been welfare-improving, and has been relatively well implemented by allocating subsidies over a large range of wages, and not only at the minimum wage level. The existing exemptions leads to an employment boost which is not completely offset by a strong deterioration of the productivity level. Here again, removing the productivity channel would lead to an opposite recommendation, namely a concentration of the exemptions at the minimum wage level.

Section 1 is devoted to the presentation of the theoretical model, whereas the section 2 presents the calibration and the empirical performances of the model. The quantitative results relative to different policy experiments are discussed in section 3.

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<sup>4</sup>Manning [2004] has recently also proposed analyzing labor policies using Burdett-Mortensen framework.



# 1 The Theoretical Model

Our theoretical framework is based on the Mortensen [2000] style of equilibrium search model with wage posting and firms investment in training. This framework is extended in two ways. First, we take into account the transition between short run and long run unemployment<sup>5</sup>. Secondly, we take minimum wage legislation into account.

The total labor force is composed of employed workers, unemployed workers entitled to unemployment benefits, and unemployed workers excluded from the compensation system, but entitled to minimum income. It is assumed that these three components of the total labor force are not perfectly equivalent in the matching process. An employed worker accepts any offer in excess of that currently earned while unemployed workers of each type accept the first offer which is not lower than the common reservation wage of this type. There will be two different reservation wages in the economy due to differences in unemployment compensation levels and in the intensity of the job search process.

Firms create “job sites” and each job is either vacant or filled. The equilibrium level of vacancies will be endogenously determined by a free entry condition. Each firm also determines the wage<sup>6</sup> and the specific-firm training offered and associated with the job vacancy.

First of all we present the conditions which characterize the flows equilibrium for the two unemployed workers populations and for each job relative to a given wage of the distribution. Then, we determine on the one hand the reservation wages through the derivation of the optimal behavior of workers and on the other hand the vacancy rate, the wage offer distribution and the human capital investment distribution from the derivation of the optimal firm decisions.

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<sup>5</sup>See Albrecht and Vroman [2001] for a Pissarides [1990] style of matching model where the heterogeneity of the reservation wages is due to the exclusion of some of unemployed workers from the unemployment benefit system.

<sup>6</sup>If one focuses on the wage setting rule in this type of economy with search frictions on the labor market (the wage policy), the assumption of firm monopsony power is not rejected on French panel data set (see Cahuc, Postel-Vinay, and Robin [2003]) except for the low skilled worker.

## 1.1 Labor market flows

### 1.1.1 Matching technology

According to Pissarides [1990], the aggregate number of hirings,  $H$ , is determined by a conventional constant returns-to-scale matching technology:

$$H = h(v, h^e e + h^c u^c + h^l u^l)$$

where  $v$  is the number of vacancies,  $h^e \geq 0, h^c \geq 0, h^l \geq 0$  are the exogenous search efficiencies (intensities) for employed, short run and long run unemployed workers which are in number  $e, u^c$  and  $u^l$ , respectively. We normalize  $e + u^c + u^l$  to 1 and we denote  $u \equiv u^c + u^l$  and  $\bar{h} = h^e e + h^c u^c + h^l u^l$ .

Let  $\theta = \frac{v}{\bar{h}}$  be the labor market's tightness, the arrival rates of wage offers for workers are :

- for the employees

$$h^e \lambda(\theta) \equiv \frac{h^e}{\bar{h}} \frac{H}{e + u^c + u^l} = h^e \frac{H}{\bar{h}}$$

- for the short run unemployed

$$h^c \lambda(\theta) \equiv \frac{h^c}{\bar{h}} \frac{H}{e + u^c + u^l} = h^c \frac{H}{\bar{h}}$$

- for the long run unemployed

$$h^l \lambda(\theta) \equiv \frac{h^l}{\bar{h}} \frac{H}{e + u^c + u^l} = h^l \frac{H}{\bar{h}}$$

Accordingly, the average duration spells before a contact are  $1/(h^e \lambda(\theta))$  for the employees,  $1/(h^c \lambda(\theta))$  for the short run and  $1/(h^l \lambda(\theta))$  for the long run unemployed workers.

The transition rate at which vacant jobs are filled is:

$$q(\theta) = \frac{H}{v} = h \left( 1, \frac{\bar{h}}{v} \right)$$

The average vacancy duration is thus  $1/q(\theta)$ .

### 1.1.2 Entries and exits from unemployment

Let  $x_l$  and  $x_c$  denote the endogenous reservation wages of short run and long run unemployed workers, respectively. Steady state level of short run unemployment ( $u^c$ ) is derived from the following equilibrium flows:

$$\underbrace{s(1-u)}_{\text{firings}} = \underbrace{h^c \lambda(\theta) [1 - F(x_c)] u^c}_{\text{hirings}} + \underbrace{\delta u^c}_{\substack{\text{flow into} \\ \text{long run} \\ \text{unemployment}}}$$

where  $s \in [0, 1]$  is the exogenous jobs' destruction rate,  $\delta \in [0, 1]$  is the probability of becoming long run unemployed for short run unemployed workers, and  $F(w)$  denotes the distribution function of wage offers  $w$ . Steady state level of long run unemployment ( $u^l$ ) is given by:

$$\underbrace{\delta u^c}_{\substack{\text{flow out of} \\ \text{short run} \\ \text{unemployment}}} = \underbrace{h^l \lambda(\theta) [1 - F(x_l)] u^l}_{\text{hirings}}$$

These equations show that the fraction of long run unemployed workers in the total unemployed population ( $u^l/u$ ) decreases with the tightness of the labor market ( $\theta$ ): when  $\theta$  increases, the expected duration of unemployment decreases and thus the probability of becoming a long run unemployed worker.

### 1.1.3 Entries and exits from employment at wage $w$ or less

Let  $G(w)$  denote the fraction of employed workers at wage  $w$  or less. This function is derived from the following equilibrium flows:

- If  $x_l \leq w < x_c$ ,

$$\underbrace{(1-u)G(w)h^e \lambda(\theta) [1 - F(w)]}_{\text{voluntary quits}} + \underbrace{s(1-u)G(w)}_{\text{firings}} = \underbrace{h^l \lambda(\theta) F(w) u^l}_{\text{hirings}}$$

- If  $w \geq x_c$ ,

$$\begin{aligned} & \underbrace{h^e \lambda(\theta) [1 - F(w)] (1-u)G(w)}_{\text{voluntary quits}} + \underbrace{s(1-u)G(w)}_{\text{firings}} \\ &= \underbrace{u^c h^c \lambda(\theta) F(w) + u^l h^l \lambda(\theta) F(w)}_{\substack{\text{potential} \\ \text{hirings}}} - \underbrace{u^c h^c \lambda(\theta) F(x_c)}_{\text{rejections}} \end{aligned}$$

## 1.2 Behaviors

### 1.2.1 Workers

Let  $V^n(w)$  denote the value function for an employed worker who earns  $w$ ,  $V^{uc}$  the value function for a short run unemployed worker who is paid the unemployment benefits  $b$ , and  $V^{ul}$  the value function of a long run unemployed worker who is paid a “minimum social income”  $msi$ . It is assumed that the total income of the workers is composed of the labor market earnings and by transfers (government budgetary surplus and firms’ profits<sup>7</sup>) uniformly distributed across households denoted by  $\mathcal{T}$ . Assuming CRRA preferences, these functions solve:

$$\begin{aligned} rV^n(w) &= u((1 - c_n)w + \mathcal{T}) + h^e \lambda(\theta) \int_w [V^n(\tilde{w}) - V^n(w)] dF(\tilde{w}) - s[V^n(w) - V^{uc}] \\ rV^{uc} &= u(b + \mathcal{T}) + h^e \lambda(\theta) \int_{x_c} [V^n(\tilde{w}) - V^{uc}] dF(\tilde{w}) - \delta [V^{uc} - V^{ul}] \\ rV^{ul} &= u(msi + \mathcal{T}) + h^l \lambda(\theta) \int_{x_l} [V^n(\tilde{w}) - V^{ul}] dF(\tilde{w}) \end{aligned}$$

where  $r \geq 0$  and  $c_n \in [0, 1]$  stand for real interest rate and employees’ payroll taxes, respectively. The function  $u(\cdot)$  respects the following restrictions:  $u' > 0$ ,  $u'' < 0$  and  $u(0) = 0$

The reservation wage policies  $x_c$  and  $x_l$  are derived from the two conditions  $V^n(x_c) = V^{uc}$  and  $V^n(x_l) = V^{ul}$ , that lead to:

$$\begin{aligned} u((1 - c_n)x_c + \mathcal{T}) &= u(b + \mathcal{T}) \\ &+ (h^c - h^e) \lambda(\theta) \int_{x_c}^{\bar{w}} [V^n(\tilde{w}) - V^{uc}] dF(\tilde{w}) - \delta [V^{uc} - V^{ul}] \end{aligned} \quad (1)$$

$$\begin{aligned} u((1 - c_n)x_l + \mathcal{T}) &= u(msi + \mathcal{T}) \\ &+ (h^l - h^e) \lambda(\theta) \int_{x_l}^{\bar{w}} [V^n(\tilde{w}) - V^{ul}] dF(\tilde{w}) - s[V^{uc} - V^{ul}] \end{aligned} \quad (2)$$

- It is easy to see that if  $h^c = h^l = h^e$  and  $\delta = 0$  (as in Mortensen [2000]), then  $x_c = b/(1 - c_n)$ .
- The positive probability of losing the unemployment benefit ( $\delta > 0$ ) then accounts for a decrease in the short run unemployed workers’ reservation wage,  $x_c$ <sup>8</sup>. On the contrary, the heterogeneity among search

<sup>7</sup>These two variables are precisely defined in a later section.

<sup>8</sup>Of course, at equilibrium,  $V^{uc} \geq V^{ul}$ . If this property could be violated, it would be in the short run unemployed workers’ interest to become long run unemployed.

efficiencies, if it is assumed that  $h^c > h^e$ , accounts for a rise of short run unemployed workers' wage claims. Similarly,  $h^l > h^e$  pushes up the reservation wage of long run unemployed workers,  $x_l$ . Indeed, in that case, by accepting a wage offer, an unemployed worker anticipates that his likelihood of earning higher wages in the future is lower.

- Lastly, the perspective, that occurs with probability  $s$ , of becoming a short run unemployed worker through employment destruction implies a decrease in the reservation wage of long run unemployed workers as great as  $V^{uc} - V^{ul}$  is large. This reflects the fact that the position of long run relative to short run unemployed workers is deteriorating.

### 1.2.2 Firms

Let  $k$  be the match specific investment per worker and  $f(k)$  the value of worker's productivity which is an increasing concave function of this investment. It is assumed that whenever an employed worker finds a job paying more than  $w$  (voluntary quit), then the employer seeks for another worker. Whenever an exogenous quit (destruction) occurs, the job gets no value. The expected present value of the employer's future flow of quasi-rent once a worker is hired at wage  $w$ ,  $J(w, k)$ , hence solves:

$$rJ(w, k) = f(k) - (1 + c_p)w - h^e \lambda(\theta)[1 - F(w)][J(w, k) - V] - sJ(w, k)$$

where  $c_p \geq 0$  is the employer's payroll taxes.

In turn, the asset value of a vacant job solves the continuous time Bellman equation:

$$rV = \max_{w \geq x_l, k \geq 0} \{ \eta(w) [J(w, k) - p_k k - V] - \gamma \}$$

where  $\gamma$  is the recruiting cost,  $p_k$  stands for the relative price of one unit of human capital, and  $\eta(w)$  is the probability at which a vacancy with posted wage  $w$  is filled. This probability is defined by:

$$\eta(w) = \frac{\text{Prob}(e|u^l)u^l}{v} + \frac{\text{Prob}(e|u^c)u^c}{v} + \frac{\text{Prob}(e|e)(1-u)}{v}$$

where the first (second, third) term gives the acceptance rate of a job offer for a long term unemployed worker (a short term unemployed worker, an employed worker). These probabilities are:

$$\text{Prob}(e|u^l) = h^l \frac{H}{h} \quad \text{if } w \geq x_l$$

$$\begin{aligned}
\text{Prob}(e|u^c) &= \begin{cases} \frac{h^e H}{h} & \text{if } w \geq x_c \\ 0 & \text{if } w < x_c \end{cases} \\
\text{Prob}(e|e) &= h^e \frac{H}{h} G(w)
\end{aligned}$$

where  $G(w)$  is the fraction of employed workers earning  $w$  or less. These probability functions of the reservation wages are given by:

$$\begin{aligned}
\eta(w) &= \frac{H}{v} \left[ \frac{h^l}{h} u^l + \frac{h^e}{h} (1-u) G(w) \right] \quad \forall w \in [x_l, x_c] \\
\eta(w) &= \frac{H}{v} \left[ \frac{h^l}{h} u^l + \frac{h^c}{h} u^c + \frac{h^e}{h} (1-u) G(w) \right] \quad \forall w \in [x_c, \bar{w}]
\end{aligned}$$

where  $H/v = \lambda(\theta)/\theta$  gives the probability of having a contact for a firm.

Free entry conditions at each wage level imply that  $V = 0$  and expected intertemporal profits are identical whatever  $w \geq \underline{w}$ , where  $\underline{w}$  is the minimum of the wage offered. Actually,  $x_l \leq \underline{w}$ , since it is not in firms interest to offer a wage rejected by all the workers. Hence, labor market tightness  $\theta$ , the wage distribution function  $F(w)$  and the firms' investment in human capital  $k(w)$  can be derived from the system of equations defined by:

$$\gamma = \eta(w) \left[ \max_{k \geq 0} \{ J(w, k) - p_k k \} \right] \quad \forall w \geq \underline{w} \quad (3)$$

with  $F(\underline{w}) = 0$ . Employer have two reasons for offering a wage above the wage  $\underline{w}$ . First, the firm's acceptance rate ( $\eta(w)$ ) increases with the wage offer, since a higher wage is more attractive. Secondly, the firm's retention rate increases with the wage paid by limiting voluntary quits leading to an increase in  $J(w, k)$ . This wage strategy played by firms is strongly interrelated with the human capital investment decisions.

As each employer pre-commits to both the wage offered and the specific capital investment in the match, it is easy to show that the optimal investment solves:

$$f'(k) = p_k(r + s + h^e \lambda(\theta)[1 - F(w)]) \implies k = k(w) \quad \forall w \geq \underline{w} \quad (4)$$

The level of specific human capital hence increases with the level of the wage offer. Indeed, a higher wage reduces the probability that an employee will accept job offers from other firms. The negative association between wage and labor turnover creates incentives for training employees, thus increasing firm-specific productivity, since the expected duration of the match is longer and the period in which the firm can recoup its investment increases.

### 1.3 Labor Market Equilibrium

#### Assumptions on production technology

ASSUMPTION A1 The production function satisfies the following restrictions:  $f(0) > 0$ ,  $f'(0) = \infty$ ,  $f'(k) > 0$  and  $f''(0) < 0$ .

The first assumption is a normalization implying that a worker without training has an inherited strictly positive productivity.

ASSUMPTION A2 The production technology is normalized in order to have  $k(\underline{w}) = 0$ .

This assumption is sufficient to prevent the existence of multiple equilibria for positive vacancy rates.

#### Equilibrium Definition and Properties

A steady state search-matching equilibrium is a reservation wage policy,  $\{x_c, x_l\}$ , a vacancy rate ( $v = \theta \bar{h}$ ), a long run unemployment rate  $u_l$ , a short run unemployment rate  $u_c$ , a wage offer distribution  $F(w)$  and a specific human capital investment function  $k(w)$ . The Appendix A presents the system of equations to solve this equilibrium.

**PROPOSITION 1 There is only one strictly positive level of vacancy rate.**

See appendix B.1 for a proof.

**PROPOSITION 2 There exists a wage interval  $[w_l, x_c[$  over which there is no wage offer**

See appendix B.2 for a proof. This property of the equilibrium suggests that over  $[w_l; x_c[$ , the increase of momentary profits associated with a decrease in wages does not compensate for the loss due to the higher rotation costs in the long run unemployment worker segment.

**COROLLARY If  $w_l > x_l$ , then the support of the wage distribution is formed by two subsets  $[x_l, w_l] \cup [x_c, \bar{w}]$ . Otherwise, all the posted wages are included in the set  $[x_c, \bar{w}]$ .**

This suggests that, if  $w_l < x_l$ , the increase in momentary profits when offering a wage lower than  $x_c$  is never compensated for by the loss due to higher rotation costs.

## The incidence of a minimum wage

The introduction of a minimum wage ( $mw$ ) can affect the properties of the equilibrium in the following ways.

- If the minimum wage is lower than  $x_l$ , it is not a constraint.
- If the minimum wage is greater than  $x_c$ , its value is the lower posted wage:  $\underline{w} = mw$ .
- If the minimum wage is in  $]x_l; w_l[$  then  $\underline{w} = mw$ .
- If the minimum wage is in  $]w_l; x_c[$ , then  $\underline{w} = x_c$ .
- If  $w_l < x_l$  then  $\underline{w} = \max\{x_c, mw\}$ .

## 1.4 Efficiency

In order to evaluate the impact of labor market policies on the equilibrium, we focus on the steady state aggregate output flow net of the recruiting costs defined by:

$$\begin{aligned}
 \mathcal{Y} &= \underbrace{(1-u) \int_{\underline{w}}^{\bar{w}} f(k(w)) dG(w)}_{\text{Output}} && - \underbrace{\gamma v}_{\text{Hiring costs}} \\
 &- \underbrace{p_k h^c \lambda(\theta) u^c \int_{\underline{w}}^{\bar{w}} k(w) dF(w)}_{\substack{\text{training costs} \\ \text{short run unemployed workers}}} && - \underbrace{p_k h^l \lambda(\theta) u^l \int_{\underline{w}}^{\bar{w}} k(w) dF(w)}_{\substack{\text{training costs} \\ \text{long run unemployed workers}}} \\
 &- \underbrace{p_k h^e \lambda(\theta) (1-u) \int_{\underline{w}}^{\bar{w}} \left( \int_{\underline{w}}^w k(w) dF(w) \right) dG(w)}_{\substack{\text{training costs} \\ \text{job-to-job mobility}}}
 \end{aligned}$$

We will also compute the aggregate welfare:

$$\mathcal{W} = (1-u) \left( \int_{\underline{w}}^{\bar{w}} V^n(w) dG(w) \right) + u^c V^{uc} + u^l V^{ul}$$

This implies the need to determine the variations in the government budgetary surplus ( $\mathcal{B}$ ) and firms' profits ( $\Pi$ ) and the way they are distributed



across households. We assume that they are uniformly redistributed *via* lump-sum transfers across all the agents in order not to interfere with the direct distributive effects of policy reforms. As the size of the population is normalized to one, the instantaneous utility functions are  $u((1 - c_n)w + \mathcal{T})$  for the employed workers,  $u(b + \mathcal{T})$  for the short run unemployed workers and  $u(msi + \mathcal{T})$  for the long run unemployed workers, where the total transfers  $\mathcal{T}$  are defined by  $\mathcal{T} = \mathcal{B} + \Pi$ .

More precisely, the budgetary surplus is defined by:

$$\mathcal{B} = (1 - u) \left( \int_{\underline{w}}^{\bar{w}} [c_p(w) + c_n] w dG(w) \right) - (u^c \times b + u^l \times msi)$$

where the employer's payroll taxes  $c_p(w)$  can be a function of the wage when employment subsidies are introduced. The aggregate firm profits are defined by:

$$\Pi = \mathcal{Y} - (1 - u) \left( \int_{\underline{w}}^{\bar{w}} [1 + c_p(w)] w dG(w) \right).$$

Unlike the net production criterion, the aggregate welfare takes into account the variation in the government surplus and the distributive implications of the different reforms we consider. By distributive implications, we have in mind to capture the variations in the relative situation of workers and their impact on the aggregate welfare for a degree of concavity given by the utility function  $u$ .

## 2 Estimation and Test of the Model

This section describes the implementation of the econometrical method we used to estimate the deep parameters of the model. Given the structure of the model and given that the investments in human capital have no observable counterparts, the structural parameters of the model are estimated using a simulation-based estimation method<sup>9</sup>. This procedure can be easily implemented even when the likelihood function is intractable or when moments cannot be computed using direct integration methods.

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<sup>9</sup>See Gouriéroux, Monfort, and Renault [1993], Smith [1993], Gouriéroux and Monfort [1994] and Gallant and Tauchen [1996] for a general statement of these methods. See Collard, Feve, Langot, and Perraudin [2002] for an applied framework based on labor market dynamics.

## 2.1 Estimation method

We turn to a Simulated Method of Moments (SMM hereafter) which consists of replacing the computation of analytic moments by simulations. The moments underlying the estimation are based on the wages distribution. We focus on the manual workers sub-sample which is particularly concerned by the minimum wage and the probability of being excluded from the unemployment benefit system. This enables us to detect the dimensions along which our simple structural model is capable of mimicking a set of moment restrictions.

The vector  $\Phi$  ( $\dim(\Phi) = 17$ ) contains all the parameters of the model:

$$\Phi = \{h^e, h^c, h^l, \zeta, \gamma, s, \delta, \sigma, b, msi, r, c_n, c_p, p_k, \alpha, A, A_1\}$$

where the parameters  $\{\alpha, A, A_1\}$  come from the specification of the production function:  $f(k) = A_1 + (k + A)^\alpha$ . The value of  $A$  is given by the hypothesis A1. The parameter  $\sigma$  denotes the risk aversion of the workers:  $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ . Finally, the parameter  $\zeta$  denotes the elasticity of the matching function  $H = v^\zeta(h^e e + h^c u^c + h^l u^l)^{1-\zeta}$ .

We restrict the size of the vector of unknown structural parameters:

$$\theta = \{\alpha, p_k, h^e\}$$

This choice is motivated by the absence of empirical evidence for these crucial parameters of the model. The estimation of the vector  $\theta$  is conducted under the following set of restrictions:

- A first vector  $\Phi_1$ , with  $\dim(\Phi_1) = 7$ , defined by

$$\Phi_1 = \{s, \delta, \sigma, msi, r, c_n, c_p\}$$

is fixed on the basis of external information.

1. The destruction rate comes from Cohen, Lefranc, and Saint-Paul [1997]:  $s = 0.0185$ .
2. The parameter  $\delta$  is chosen so that the average duration of short run unemployment corresponds to the allocation spell, *i.e.* 30 months, ( $\delta = 1/30$ ).
3. Microdata suggest that  $\sigma = 2.5$  is an admissible value (see Atanasio, Banks, Meghir, and Weber [1999]).
4. the minimum income  $msi$  is fixed at its 1995 institutional value : 2500Frs.

5. The annual interest rate is fixed at 4%.
6. The payroll taxes on labor  $c_p$  and  $c_n$  equal respectively 40% (firms) and 20% (workers).

- The second vector  $\Phi_2$ , with  $\dim(\Phi_2) = 6$ , defined by

$$\Phi_2 = \{b, h^c, h^l, A_1, \gamma, \zeta\}$$

is calibrated, using the model restrictions, in order to reproduce some stylized facts and assumptions:

1. The unemployment replacement rate ( $b/E(w)$ ) is fixed at 0.6, according to Martin [1996].
2. the unemployment rate  $u$  equals 16.69%.
3. the ratio of long run to short run unemployed worker  $u^l/u$  is 46.84%.
4. the free entry condition to the labor market is respected (no sunk costs linked to the creation of a vacancy).
5. Hiring costs  $\gamma\theta/\lambda(\theta)$  equal to 0.4 as in Mortensen [2002]. These hiring costs correspond approximately to the amount of 2.5% of the wages (Abowd and Kramarz [1998]).
6. The elasticity of employment relative to a minimum wage decrease is bounded by the CSERC [1999] estimations: 20,000 job creations for all the types of workers. Manual workers represent 67.4% of the workers paid between the minimum wage and 1.3 times the minimum wage. So, one can expect that a 1% decrease of the minimum wage will create approximately 14,000 jobs. With  $\zeta = 0.21$  we have matched this estimation of the employment elasticity.

Given a set of moments, the set of calibrated parameters and the set of policy functions, the estimation method is conducted as follows:

**Step 1:** Estimate a  $q$ -dimensional vector of moments,  $\psi \in \Psi \subset \mathbb{R}^q$ , from the data.  $\hat{\psi}_N$  denotes the estimated moments and  $N$  the size of the sample. This set of moments is estimated minimizing the following loss function:

$$Q_N = \left[ \sum_{i=1}^N g(w_i; \psi_N) \right]' \Omega_N \left[ \sum_{i=1}^N g(w_i; \psi_N) \right]$$

where  $\Omega_N$  is a positive definite weighting matrix.  $\{w_i\}'$  represents the  $s$ -dimensional set of wages paid to each manual worker  $i$  in the 1995 set of

observed random variables.  $\psi$  is a  $q$ -dimensional vector of parameters and  $g(w_i; \psi)$  is a  $q$ -dimensional mapping from  $\mathbb{R}^s \times \mathbb{R}^q$  to  $\mathbb{R}^q$ . The choice of moments  $\psi$  is a crucial step for the estimation method. This choice is not driven by the specification of the model, but it should be able to encompass as many features of the data as possible, therefore avoiding any arbitrary choice and reducing biases in estimation. We thus choose a set of moments that describe wage densities as much as possible. In our case,  $g(\cdot)$  takes the form

$$g(w_i; \psi_N) = \begin{bmatrix} w_i - \mu \\ \mathbb{1}_{[w_i < D1]}(w_i - \mu_1) \\ \mathbb{1}_{[Dn \leq w_i < Dn+1]}(w_i - \mu_{n+1}) \\ \mathbb{1}_{[D8 \leq w_i < D9]}(w_i - \mu_9) \end{bmatrix} \quad \text{for } n = 1, \dots, 7$$

where  $Dn$  denotes the wage level for the decile  $n = 1, \dots, 9$ . This minimal set of moments allows us to capture the shape of the observed wage density.

**Step 2:** From the set of equations defining the labor market equilibrium, and given the vector of structural parameters,  $\theta$ , the simulated wage density is computed.

**Step 3:** A SMM estimate  $\tilde{\theta}_N$  for  $\theta$  minimizes the quadratic form:

$$J(\theta) = g'_N W_N g_N$$

where  $g_N = (\hat{\psi}_N - \tilde{\psi}_N(\theta))$ ,  $W_N$  is a symmetric non-negative matrix defining the metric<sup>10</sup> and  $\tilde{\psi}_N(\theta)$  denotes the set of moments implied by the model simulations.

Steps 2 and 3 are conducted until convergence — *i.e.* until a value of  $\theta$  that minimizes the objective function is obtained<sup>11</sup>. Let denote  $\psi_0$  the pseudo-true value of  $\psi$  and  $\theta_0$  the pseudo-true value of  $\theta$ , under standard regularity conditions, as  $N$  goes to infinity,  $\sqrt{N}(\tilde{\theta}_N - \theta_0)$  is asymptotically normally distributed, with a covariance matrix equal to  $(D'_\theta W_N D_\theta)^{-1}$  where  $D_\theta = \partial g_N / \partial \theta$ .

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<sup>10</sup>This matrix is given by the inverse of the covariance matrix of the moments, obtained from actual data.

<sup>11</sup>The minimization of the simulated criterion function is carried out using a Nelder–Meade method for minimization provided in the *Optim* MATLAB numerical optimization toolbox. At convergence of the Nelder–Meade method, a local gradient search method was used to check convergence.

A preliminary consistent estimate of the weighting matrix  $W_N$  is required for the computation of  $\tilde{\theta}_N$ . It may be directly based on actual data, and corresponds to the inverse of the covariance matrix of  $\sqrt{N}(\hat{\psi}_N - \psi_0)$ , which is obtained from step 1.

For identification's sake, we impose the condition that the number of moments exceeds the number of structural parameters. This enables us to conduct a global specification test along the lines of Hansen [1982], denoted  $J - stat = NJ(\theta)$ , which is asymptotically distributed as a chi-square, with a degree of freedom equal to the number of over-identifying restrictions.

Beyond these traditional statistical tests, we use a simple diagnostic test, following Collard, Feve, Langot, and Perraudin [2002], that locates the potential failures of the structural model. Each element of  $g_N$  measures the discrepancy between the moments computed from the data and those computed from model simulations. A small value for a given element in  $g_N$  indicates that the structural model is able to account for this specific feature of the data, while large values may reveal some failures. The first order condition associated to the minimization of the loss function  $J(\theta)$  leads to:

$$D'_\theta W_N g_N \Big|_{\theta=\tilde{\theta}_N} = 0$$

Using the mean value approximation of  $g_N$ , given that the auxiliary parameters are normally distributed and that the optimal weighting matrix corresponds to the inverse of the covariance matrix  $\Omega_N$ , each element of the following vector of

$$T_N = \left\{ \text{diag} \left[ \Omega_N - D_\theta (D'_\theta W_N D_\theta)^{-1} D'_\theta \right] \right\}^{-1/2} \sqrt{N} g_N$$

is asymptotically distributed as a  $\mathcal{N}(0, 1)$ . The test statistics are computed by replacing  $D_\theta$  and  $\Omega_N$  by consistent estimates (see Collard, Feve, Langot, and Perraudin [2002] for more details).

## 2.2 The empirical performance of the model

**Results of the estimation:** We use data from the French Labor Force Survey (“enquête emploi”) in 1995. We consider this year because afterwards the French labor market experienced large structural reforms. We retain only full-time manual workers. Then, in our particular case,  $\{w_i\}$  consists of the wage set over  $N = 14202$  individuals. Notice that we consider wages, minimum income and unemployment benefits expressed in 1990 French Francs

Table 1 reports estimates for the deep parameters and the global specification test statistic ( $J - stat$ ). First of all, the model is not globally rejected by the data, as the P-value associated to the  $J - stat$  is 97.16%. A second feature that emerges from the table is that all deep parameters are precisely estimated. In the following paragraph, we discuss the model implications for this set of parameter estimates.

Table 1: Parameters Estimates

$\theta$	$\hat{\theta}$	$\hat{\sigma}(\theta)$	$t - stat$
$\alpha$	0.7299	0.0257	28.4222
$p_k$	18.8328	1.0790	17.4536
$h^e$	0.5143	0.0119	43.2492
$J - stat$	1.9425	P-value	97.16%

Beyond the global specification test, one may check the ability of the structural model to reproduce the empirical moments. Table 2 reports observed and simulated values of moments. First of all, all observed moments are significant, making this set of historical moments an exigent table of experience to test our model. Secondly, table 2 shows that the simulated moments are also precisely estimated<sup>12</sup>. Moreover, they match their empirical counterparts relatively well. This is confirmed by the last column of table 2 which reports the diagnostic test. It clearly indicates that, when taken one by one, the model generates moments that are significantly equal to those observed on historical data.

The ability of this kind of model to match the observed wage distribution, here on French data, is in accordance with the Rosholm and Svarer [2000] empirical study on Danish data, based on an alternative empirical methodology developed by Ridder and Van den Berg [1997] and Postel-Vinay and Robin [2002].

**The (hump-shape) distribution of wages.** Does the estimated model generate the hump-shape wages distribution? Figure 2 makes a comparison between the wage distribution generated by the model and the kernel density estimation of the observed real wages<sup>13</sup>. These figures show that our calibration allows the model to reproduce the hump-shape of the wage distribution

<sup>12</sup>Recall that wages are expressed in 1990 Francs, using a deflator equal to 1.116.

<sup>13</sup>Kernel density estimation is a nonparametric technique for density estimation in which a known density function (the kernel) is averaged across the observed data points to create a smooth approximation. We use the SAS procedure KDE.

Table 2: Estimated Moments for both Simulated and Observed Data

Moment	Observed Value			Simulated Value			
	$\hat{\psi}_N$	$\hat{\sigma}(\hat{\psi}_N)$	$t - stat$	$\tilde{\psi}_N(\hat{\theta})$	$\tilde{\sigma}_N(\psi(\hat{\theta}))$	$t - stat$	$T_N$
$\mu$	6304.6799	31.8464	197.9718	6326.7378	18.2335	346.9849	0.8448
$\mu_1$	4471.7044	293.7155	15.2246	4452.0913	13.6150	326.9986	-0.0668
$\mu_2$	4872.3694	353.0800	13.7996	4756.0700	27.2536	174.5113	-0.3304
$\mu_3$	5262.4871	333.6157	15.7741	5040.8162	43.8846	114.8654	-0.6703
$\mu_4$	5587.0088	424.3939	13.1647	5373.9973	63.7304	84.3239	-0.5077
$\mu_5$	5884.5128	405.4031	14.5152	5786.4729	87.9543	65.7895	-0.2477
$\mu_6$	6253.6900	430.6652	14.5210	6279.0869	279.2906	22.4823	0.0775
$\mu_7$	6659.0716	458.8081	14.5138	6849.9954	292.6643	23.4056	0.5403
$\mu_8$	7141.5660	492.0757	14.5131	7506.7023	188.4035	39.8438	0.8032
$\mu_9$	7850.6070	538.7721	14.5713	8248.5618	231.5938	35.9849	0.8181

observed in the data. As suggested by Mortensen [2000] the introduction of an endogenous productivity distribution appears to match the shape of the observed wage distribution, even if the estimation strategy we adopt does not lead to a perfect fit with the wages density.

The model seems to be close to the observed data. Figure 2 shows that the model is able to fit the observed wage density.

**A binding minimum wage.** Moreover, it appears (see Table 3) that the actual French minimum wage ( $mw$ ) is above the highest reservation wage ( $x^c$ ). It is a binding minimum wage, which implies  $F(x^c) = 0$ . Moreover, the model does a good job in replicating employment and unemployment duration, which are endogenous. Notice that these durations are an important determinant of the reservation wage levels.

**Contact probabilities.** The results reported in the Table 3 also show that, beyond the wage distribution, the model allow us to match the estimated probabilities for workers to have a contact on the labor market: the estimated values of these rates on French data are for the employees equal to 0.057 and 0.124 for the unemployed workers (see Postel-Vinay and Robin [2002]).

Figure 2: The observed and predicted wage distributions

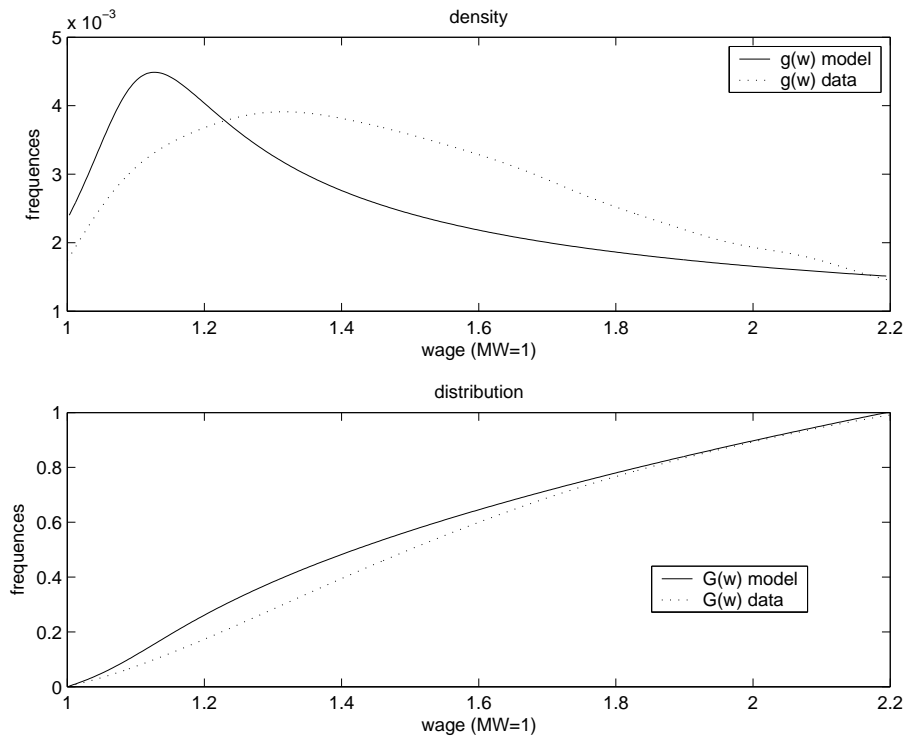




Table 3: Benchmark Equilibrium

$b$	$msi$	$mw$	$E(w)$	$med(w)$	$\underline{w}$	$\bar{w}$
4507	2500	4751	7051	6049	4751	10425
$\delta$	$c_p$	$c_n$	$b/E(w)$	$(\gamma\theta)/\lambda$	$u^l/u$	
0.0333	0.4000	0.2000	0.60	0.3000	0.4575	
$u$	$u^l$	$u^c$	$h^e\lambda$	$h^c\lambda$	$h^l\lambda$	
0.1551	0.071	0.0841	0.0801	0.1520	0.0395	
Reservation Wages						
$x^l$	$w_l$	$x^c$	$F(x^c)$			
601	0	3896	0			
Employment and Unemployment Durations						
model	data	model	data			
32.22	34.00	14.50	17.00			
Human Capital and Welfare per capita						
$E(k)$	$\mathcal{Y}$	$\mathcal{W}$				
323738	11352	-113.308				
Incomes and production are expressed in 1995 French Francs						

### 3 Reassessing French labor cost-reducing policies

In this section our aim is to reexamine the efficiency implications of labor cost-reducing strategies, which are traditionally assessed under their sole employment dimension. In order to show the role of the minimum wage, we firstly determine its optimal level with and without the productivity channel. We pay particular attention to this optimal level relative to the short run unemployment reservation wage. Secondly we investigate the efficiency implications of the recent payroll tax subsidy policy aimed at reducing the damage to employment caused by the minimum wage legislation. This policy is free of the reservation wage limit of a decreasing wage cost as employees do not suffer earnings cuts. Maybe more interestingly, it could induce some very different distributive effects than the minimum wage policy.

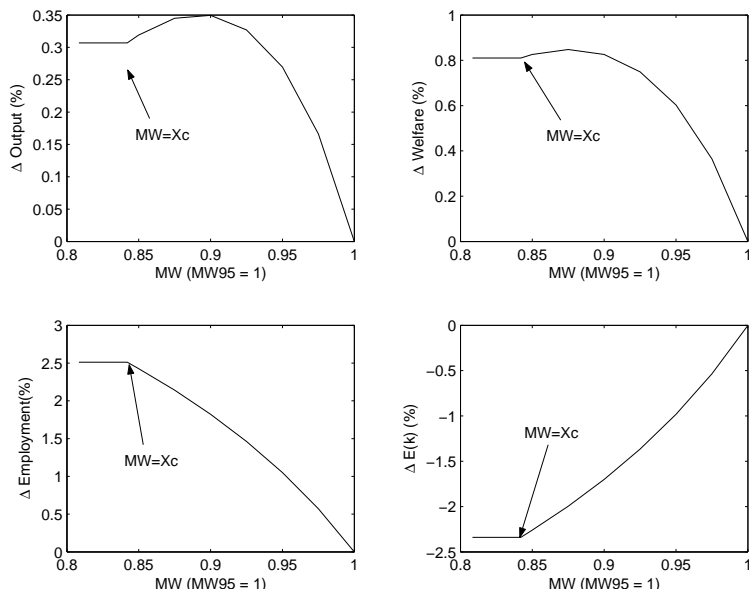
#### 3.1 The optimal minimum wage

It is well known that in the matching model the decrease of the minimum wage leads to a higher vacancy rate and hence to a higher employment level (denoted  $N$  hereafter). Moreover, the magnitude of this effect is dampened by the monopsony power of the firms. But, at the general equilibrium, it can be over-compensated for by the increase of vacancy costs: as usual in matching models, a higher vacancy rate induces a traditional congestion effect and potentially too high hiring costs; secondly, and more specifically to our framework, it can lead to under-investment in human capital due to the reduction of the expected job duration because of the increase in the probability of finding a better job.

By means of simulations, we show that the optimal minimum wage (*i*) is lower than its 1995 value, and (*ii*) is larger than the reservation wage  $x_c$ . Actually, the optimal level for the minimum wage is around 90% of its 1995 value when considering the output criterion (Figure 3,  $\Delta$  Output) or 88% according to the welfare indicator (Figure 3,  $\Delta$  Welfare). The large decrease in unemployment leads to more lump-sum transfers uniformly received by all the agents. Taking into account this effect by considering the aggregate welfare leads to a weaker minimum wage. The difference between the two indicators is however not significant (see Table 4).

The optimal minimum wage is output-increasing (see Table 4), because of the reduction in the rate of unemployment (see Figure 3,  $\Delta$  Employment). This latter effect is partially compensated for by the decrease in human capital investment made by firms (see Figure 3,  $\Delta E(k)$ ). By considering

Figure 3: Optimal minimum wage



the decision rule (eq. (4)), it indeed appears that the decrease in the labor market tightness due to the diminution of wage costs reduces the expected duration of jobs, whatever the level of wages offered: the higher number of vacancies increases the probability for employees to have a contact with another firm. This deters firms from investing much in human capital as they anticipate a shorter job duration. This productivity negative effect on net output is reinforced by the increase of the cost of training due to more job-to-job transitions.

Table 4: The optimal minimum wage level

$mw$	$\mathcal{Y}$	$N$	$E(k)$	$\mathcal{W}$	$\mathcal{B}$
10%	0.3496	1.8218	-1.7001	0.8260	1.6915
12%	0.3479	2.0798	-1.9394	0.8474	1.8704

Variations in % relative to the benchmark calibration

It can be worth comparing these results with the case where the productivity channel is shut off. First we consider that the investment choice of firms for different wage levels is given by the benchmark calibration economy. Surprisingly, a higher vacancy rate now has a positive impact on average productivity (Table 5). Faced with potentially more frequent quits, firms react by offering higher wages. This explains why the average productivity

is pushed up by a composition effect. It can be noticed this latter effect is particularly important: the average human capital stock and the average productivity increase respectively by 7.1446% and by 3.8565% for the optimal 10% decrease in the minimum wage (Table 5). Of course, more vacancies induce some additional costs. Considering production net only of hiring costs is the consistent way to account for all these effects. By using this indicator, we verify that eliminating the human capital investment margin leads to far more gains (Table 5): 5.1980% compared to 0.2554%. Actually maintaining the human capital level on each job unchanged is not sufficient in our theoretical setup to eliminate the productivity channel. It is necessary to eliminate the wage offer game effect on productivity by considering the case where the average productivity is given by its benchmark value ( $E[f(k)]$  constant line in Table 5). The matching effect internal to our model still applies. The net production increase in this latter case (1.2302%) is intermediate between the two previous cases. This analysis reveals that there are two distinct productivity channels in our setup: an investment one, but also a distribution one due to changes in the wages offer distribution.

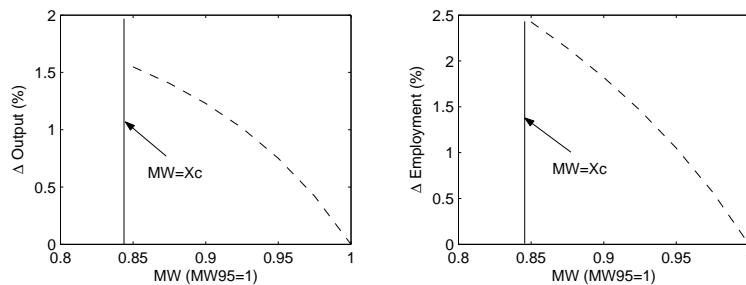
Table 5: Constant or variable productivity –  $\Delta mw = -10\%$

	$(1 - u)E[f(k)] - \gamma v$	$N$	$E[k]$	$E[f(k)]$
$k_i$ variable	0.2554	1.8218	-1.7001	-0.9474
$k_i$ constant	5.1980	1.8218	7.1446	3.8565
$E[f(k)]$ constant	1.2302	1.8218	-	-
Variations in % relative to the benchmark calibration				

Of course, when the productivity channel is totally shut off, it could be optimal to go further in the minimum wage decrease. As can be seen on figure 4,  $\Delta mw = -10\%$  is not the optimal decrease of the minimum wage in this case. It appears, however, that the lowering of the minimum wage is now stopped by the short run reservation wage: when  $x^c = mw$  the net production gain is equal to 1.5485%. Despite the existence of acceptable wage offers between the two reservation wages, firms do not actually make these offers. They are not profitable, given the range of wages included in this interval: the increase of momentary profits for a posted wage included in  $[x_l; x_c[$  is not compensated for by the loss due to the higher rotation costs on

the long run unemployment worker segment. Decreasing the minimum wage below this latter level does not improve production insofar as firms do not propose wages between the two reservation wages: the short run reservation wage becomes binding whereas firms could potentially make offers to long run unemployed workers.

Figure 4: The impact of minimum wage variations when  $E[f(k)]$  is constant



Withdrawing the productivity channel leads to a very different conclusion about the level of the optimal minimum wage and the role of the reservation wage of unemployed workers: in that case, the minimum wage legislation could be removed. By contrast, taking the productivity channel into account emphasizes its important role.

It can be derived from our optimal minimum wage analysis that the labor cost cut must be relatively weak in order to preserve high productivity levels. Moreover, hopefully, decreasing the minimum wage is inherently limited by the high level of the short run reservation wage, despite the existence of acceptable lower wage offers. This is why the payroll tax exemptions policy may have more dramatic consequences since it is free of the reservation wage limit as employees do not suffer earnings cuts.

### 3.2 Reexamining the payroll tax subsidy policy

In order to lower employer labor costs, tax exemptions on the employer-paid payroll taxes were introduced during the nineties. This was seen as a device to counteract the negative impact of the minimum wage on employment without lowering the wages earned by employees. The subsidy increased dramatically in October 1995 and September 1996 (hereafter PTE, for Payroll Tax Exemptions, reform) and finally corresponded to a linear reduction that spanned from 1 to 1.33 times the minimum wage, ranging from 18.6 points at the  $mw$  to roughly 0 points at the extremity of the exoneration interval. It is generally considered to have generated very strong employment effects (Crépon and Desplatz [2002] and Kramarz and Philippon [2001]). However,

these subsidies tend to introduce a bias in favor of the creation of low-paid jobs. To this extent, the danger is a decrease in the aggregate productivity. It is then particularly interesting to draw up a balance-sheet in terms of output, and not uniquely in terms of employment.

There are indeed two fundamental motives for reexamining this subsidy policy through a productivity channel. The first one is similar to the previously studied case of the decrease in the minimum wage and is based on the lowering of the labor cost which leads to more vacancies and more rotations, and hence to less human capital investment (investment channel). The second one is specific to the subsidy policy shape and is related to the bias in favor of the low wages potentially induced by tax exemptions only made from 1 to 1.33 times the minimum wage (distribution channel). For instance Malinvaud [1998] recommends enlarging the range of exonerations at the expense of the lowering of the tax reduction at the minimum wage level.

We first examine the impact on both employment and productivity of the existing subsidy policy. We then determine the optimal range of exonerations among the set of the same linear-decreasing exemptions scheme implying the same ex-ante budgetary cost.

### 3.2.1 The Payroll Tax Exemption reform

The PTE reform has changed the wage distribution as already depicted in the introduction (Figure 1). It is possible to quantitatively capture this effect by computing the fraction of the jobs paid under 1.33 times the minimum wage before and after 1995. From the *Enquête Emploi* data base, it appears that this fraction has changed from 37.83% in 1995 to 45.33% in 1998. It is particularly important to be able to generate such changes with our estimated model. We conclude that this fraction was equal to 41% for our benchmark pre-1996 estimated model and is now at 45% when the exemptions policy is introduced. Faced with a new environment, the wage offer strategies lead to similar wage distribution changes as those observed. This result gives strong support to our investigation: what are the consequences of the 1996 reform on the net production level?

The results relative to the PTE reform are reported in Table 6. This policy would have increased the net production in the economy. This result is due to the large employment boost reflected by the 2 points decrease in unemployment. This policy succeeds in supporting more vacancies and job creation in the economy. The employment scale effect we get is consistent with econometric studies on this topic (Kramarz and Philippon [2001], Crépon and Desplatz [2002], Laroque and Salanié [2000] and [2002]). However, the

human capital investment shrinks greatly, decreasing the capital stock by 2.03%.

Table 6: The PTE Reform

$b$	$msi$	$mw$	$E(w)$	$med(w)$	$\underline{w}$	$\bar{w}$
4507	2500	4751	70486	5955	4751	10410
$\delta$	$c_p$	$c_n$	$b/E(w)$	$(\gamma\theta)/\lambda$	$u^l/u$	
0.0333	0.4000	0.2000	0.60	0.4651	0.428	
$u$	$u^l$	$u^c$	$h^e\lambda$	$h^c\lambda$	$h^l\lambda$	
0.1366	0.0586	0.0780	0.090	0.1708	0.0444	
Reservation Wages						
$x^l$	$w_l$	$x^c$	$F(x_c)$			
605	0	4002	0			
Employment and Unemployment Durations						
model	Bench.	model	Bench.			
31.76	32.22	12.45	14.50			
Variations (in %)						
$\mathcal{Y}$	$N$	$E(k)$	$\mathcal{W}$	$\mathcal{B}$		
0.3399	2.1834	-2.0399	1.0150	-1.0555		

Even if output is increased relative to the benchmark case, its level remains inferior to the value obtained with the decrease in the minimum wage. The endogenous variation in productivity is responsible for this result and especially a strong negative composition effect via the distribution channel. It is possible to evaluate the magnitude of this effect by comparing the variable productivity case with the constant investment one. By a composition effect, the average human capital  $E[k]$  and the average productivity  $E[f(k)]$  are decreased in the  $k_i$  constant case (Table 7). This average productivity deterioration due to the biased scheme of exemptions is particularly significant insofar as we know that higher rotations per se lead to a strong positive effect on productivity (Table 5) when human capital investments are considered as given. This decrease is exacerbated in the variable case because of the decrease in human capital investments following higher rotations in the economy. It is worth emphasizing that the last effect dominates the

composition effect. It is only when the average productivity is maintained artificially unchanged that the PTE reform is as efficient to increase the net (only of hiring costs) production (1.4275% gain) as the optimal decrease of the minimum wage (1.2302% gain). However, recall that the global balance on the net production criterion basis is positive.

Table 7: Constant or variable productivity

	$(1 - u)E[f(k)]$ $-\gamma v$	$N$	$E[k]$	$E[f(k)]$
$k_i$ variable	0.2625	2.1834	-2.0399	-1.1283
$k_i$ constant	1.0962	2.1834	-1.3802	-0.3209
$E[f(k)]$ constant	1.4275	2.1834	-	-

Variations in % relative to the benchmark calibration

As the payroll tax exemptions reform implies some direct budgetary cost, the welfare criterion can lead to a less optimistic evaluation. However, the exemptions do not constitute the total budgetary cost: it is necessary to take into account the lowering in the unemployment allocations and the increase in payroll tax collection induced by the employment boom. Globally, it appears that the PTE reform is not self-financed: the average cost of a job creation ( $\Delta\mathcal{B}/\Delta n$ ) is equal to 24330 French Francs whereas the return ( $\Delta\mathcal{Y}/\Delta n$ ) is of 22492 French Francs. Despite this budgetary cost, the PTE reform implies an increase in Welfare ( $\Delta\mathcal{W}=1.0150\%$ , Table 6) relative to the benchmark economy, but also, and more unexpectedly, compared to the optimal minimum wage level ( $\Delta\mathcal{W}=0.8474\%$ , Table 4). As long as the minimum wage is lowered until its optimal value, the employee value goes down because of the decrease in the average employee wage. This is not the case when payroll tax exemptions are introduced. Of course, we have to take into account the decrease in dividends and in government lump-sum transfers in this latter case. But this decrease is spanned over all agents, while the decrease of the minimum wage only concerns the employees at the bottom of the distribution. The reduction of the employment costs for the lower-paid workers are supported by all the agents if the instrument is the payroll taxes subsidies, whereas the incidence only applies to the lower-paid workers if the instrument is the minimum wage. Given the concavity of the utility function, these changes in the distribution of total earnings are not neutral and lead



to a superiority of the exemptions tax policy over a decrease in the minimum wage.

### 3.2.2 Targeting subsidies around the minimum wage or spreading over a larger range?

Do the PTE reforms lie on the optimal range of exonerated wages? We take as given the shape of the policy and its direct cost. Our model is particularly well suited for studying the consequences of this kind of policy. There is clearly a trade-off for a given budgetary cost: either the subsidies lie on a narrow range in order to greatly reduce the labor cost or they are spread over a larger range in order to avoid a deformation of the wage distribution towards the bottom.

Let us first notice that the PTE reform is an intermediate case between a policy concentrating all the exemptions at the minimum wage level and another one spreading the payroll tax exemptions over the whole wage distribution. The first case magnifies the positive employment effect and the negative productivity impact. Table 8 shows that the balance is clearly in disfavor of this policy. The second case tries to attenuate the job allocation distortions, but at the expense of the magnitude of the decrease in the labor cost: only 2.05 points of payroll tax exemptions are possible in order to respect the same budgetary cost as the PTE reform. This is why this policy is dominated by the PTE reform, even if the human capital stock is higher (Table 8).

Table 8: Comparison with two polar cases

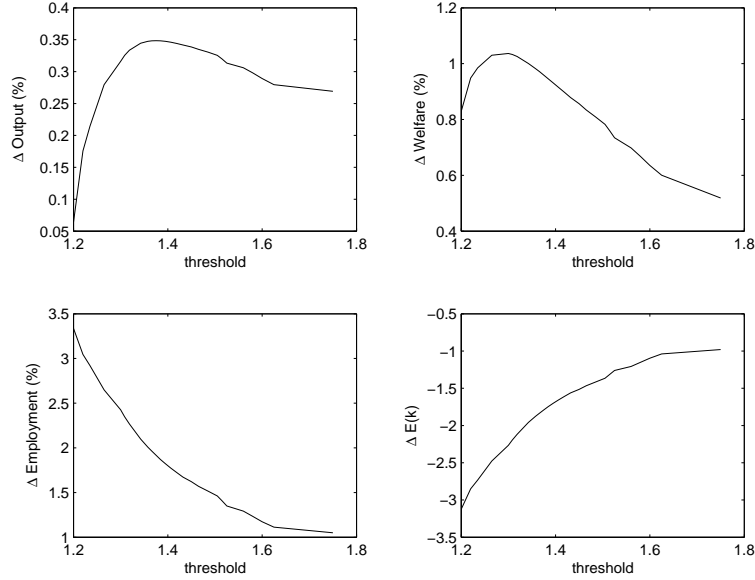
	$\mathcal{Y}$	$\mathcal{W}$	$N$	$E[k]$
PTE Reform	0.3184	1.0560	2.2084	-2.0818
Uniform exonerations	0.1063	0.1705	0.3469	-0.3250
Exonerations at the $mw$	-0.4215	-0.0692	3.5951	-3.1394

Variations in % relative to the benchmark calibration

Given the same ex-ante (direct) budgetary cost, should we go further towards higher subsidy at the minimum wage or, on the contrary, spread out the subsidies over a larger range? It appears that the first strategy is output-degrading whereas the second is output-improving (see Figure 5).

More precisely, with regard to the production criterion, the optimal subsidy scheme continuously increases from 0% for jobs paid above 1.40 times the minimum wage to 13.5% for jobs paid at the minimum wage (see Figure 5). With respect to the PTE reform, this particular shape optimally

Figure 5: The optimal subsidy scheme



solves the trade-off between employment and productivity: the net output increases by 0.3461%. The employment increase is lower (1.7731% in this case, 2.1834% in the case of the PTE reform), but the capital decrease is lessened (-1.6559% in this case, -2.0399% in the case of the PTE reform). Whereas only taking into account the employment side would have led to concentrate the exemptions around the minimum wage level, we show that the conclusion dramatically changes when the productivity incidence is considered. In this context, spreading out the exemptions over a larger range of the distribution appears as more efficient.

Considering the welfare criterion leads to putting more weight on the unemployment decrease by taking into account the concavity of the utility function. This is why the optimal scheme ranges up to 1.3 times the minimum wage, allowing more exonerations at the minimum wage level.

Whatever the criterion considered, the optimal profiles are so close to the PTE one that one may conclude that the balance between the lowering of labor costs and the wages range covered by exemptions is almost perfect. Our analysis gives strong support for the PTE reform implemented in France in the nineties.

## 4 Concluding remarks

The analysis of the French labor market illustrates the importance of taking into account both employment and productivity effects in order to arrive at a proper evaluation of labor market institutions and policies.

We show in this paper that a wage posting model is able to replicate the heterogeneity of the observed wage distribution for low skilled workers in France during the nineties. This gives some empirical relevance to the endogenous underlying distribution of productivity generated by training investment at the firm level. Then we analyze the role of the minimum wage legislation on this equilibrium outcome. It appears that the existence of a minimum wage creates more unemployment, but stimulates specific human capital by increasing the expected duration of jobs. This already known qualitative effect here receives a quantitative validation on French data. It explains why the optimal level of the minimum wage is only slightly inferior to its current level and remains superior to the (highest) reservation wage. Employer payroll tax exonerations are then not necessary to evade a potential limit to the lowering of the labor cost due to a too high reservation wage. However, we show that the payroll tax subsidy experiment aimed at preventing a rise in wage inequality has led to an output rise despite the productivity decline. This reform appears particularly well balanced between the lowering of labor costs and the wages range covered by exemptions. It is an exemplary case for any labor market reforms which should take into account the negative impact on human capital investment.

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## A Extensive definition of the equilibrium

The equilibrium, is defined by the following set of equations:

$$\begin{aligned}
u &= \frac{s(h^l \lambda(\theta) + \delta)}{h^l \lambda(\theta) \{h^c \lambda(\theta) [1 - F(x_c)] + \delta\} + s(h^l \lambda(\theta) + \delta)} \\
u^c &= u \frac{h^l \lambda(\theta)}{h^l \lambda(\theta) + \delta} \\
u^l &= u \frac{\delta}{h^l \lambda(\theta) + \delta} \\
&= u((1 - c_n)x_c + \mathcal{T}) \\
&= u(b + \mathcal{T}) + (h^c - h^e)\lambda(\theta) \int_{x_c}^{\bar{w}} [V^n(\tilde{w}) - V^{uc}] dF(\tilde{w}) - \delta [V^{uc} - V^{ul}] \\
&= u((1 - c_n)x_l + \mathcal{T}) \\
&= u(msi + \mathcal{T}) + (h^l - h^e)\lambda(\theta) \int_{x_l}^{\bar{w}} [V^n(\tilde{w}) - V^{ul}] dF(\tilde{w}) - s[V^{uc} - V^{ul}] \\
\frac{\gamma\theta}{\lambda(\theta)} &= \frac{h^l}{\bar{h}} u^l \left( \frac{s + h^e \lambda(\theta)}{s + h^e \lambda(\theta) [1 - F(w)]} \right) \times \\
&\quad \left( \frac{\max_{w \geq x_l, k \geq 0} \{f(k) - (1 + c_p)w - p_k k(r + s + h^e \lambda(\theta) [1 - F(w)])\}}{r + s + h^e \lambda(\theta) [1 - F(w)]} \right) \quad \forall w \in [x_l, x_c] \\
\frac{\gamma\theta}{\lambda(\theta)} &= \frac{s}{s + h^e \lambda(\theta) [1 - F(w)]} \times \\
&\quad \left( \frac{\max_{w \geq x_c, k \geq 0} \{f(k) - (1 + c_p)w - p_k k(r + s + h^e \lambda(\theta) [1 - F(w)])\}}{r + s + h^e \lambda(\theta) [1 - F(w)]} \right) \quad \forall w \in [x_c, \bar{w}] \\
f'(k) &= p_k(r + s + h^e \lambda(\theta) [1 - F(w)]) \quad \forall w \in [x_l, \bar{w}] \\
\mathcal{T} &= \mathcal{B} + \Pi \\
\mathcal{B} &= (1 - u) \left( \int_{\underline{w}}^{\bar{w}} [c_p(w) + c_n] w dG(w) \right) - (u^c \times b + u^l \times msi) \\
\Pi &= \mathcal{Y} - (1 - u) \left( \int_{\underline{w}}^{\bar{w}} [1 + c_p(w)] w dG(w) \right)
\end{aligned}$$

This system allows us to determine the equilibrium unemployment rate  $u^l$ ,  $u^c$  and  $u \equiv u^l + u^c$ , the vacancy rate given that  $v \equiv \theta \bar{h}$ , the reservation wages  $x_l$  and  $x_c$ , the distribution of the wage offer  $F(w)$  and the associated investment in human capital for each wage  $k = k(w)$



## B Proofs of the propositions

### B.1 Proof of proposition 1

- If  $\underline{w} = x_c$  and  $F(x_c) = 0$ ,  $\theta$  is given by the equation (3) and is such that

$$\frac{\gamma\theta}{\lambda(\theta)} = \left( \frac{s}{s + h^e \lambda(\theta)} \right) \left( \frac{f(k(\underline{w})) - (1 + c_p)\underline{w} - p_k k(\underline{w})(r + s + h^e \lambda(\theta))}{r + s + h^e \lambda(\theta)} \right)$$

Evaluated for  $w = \underline{w}$ , we find that  $\theta$  solves, stemming from the fact that  $f(0) > 0$  and  $k(\underline{w}) = 0$ :

$$\frac{\gamma\theta}{\lambda(\theta)} = \left( \frac{s}{s + h^e \lambda(\theta)} \right) \left( \frac{f(0) - (1 + c_p)\underline{w}}{r + s + h^e \lambda(\theta)} \right)$$

Let us denote  $\Phi(\theta) = \frac{\gamma\theta}{\lambda(\theta)}$  and  $\Psi(\theta) = \left( \frac{s}{s + h^e \lambda(\theta)} \right) \left( \frac{f(0) - (1 + c_p)\underline{w}}{r + s + h^e \lambda(\theta)} \right)$ , then  $\Phi(0) = 0$ ,  $\Phi'(\theta) > 0$ ,  $\Psi(0) = \left( \frac{f(0) - (1 + c_p)\underline{w}}{r + s} \right) > 0$  and  $\Psi'(\theta) < 0$ . This implies that there exists only one positive equilibrium value of  $\theta$ , hence of  $v$ .

- If  $\underline{w} = x_l$ ,  $\theta$  is given by the equation (3) but is now such that

$$\frac{\gamma\theta}{\lambda(\theta)} = \frac{h^l}{\bar{h}} u_l \left( \frac{f(k(\underline{w})) - (1 + c_p)\underline{w} - p_k k(\underline{w})(r + s + h^e \lambda(\theta))}{r + s + h^e \lambda(\theta)} \right)$$

Evaluated for  $w = \underline{w}$ , we find that  $\theta$  solves, stemming from the fact that  $f(0) > 0$  and  $k(\underline{w}) = 0$ :

$$\frac{\gamma\theta}{\lambda(\theta)} = \frac{h^l}{\bar{h}} u_l \left( \frac{f(0) - (1 + c_p)\underline{w}}{r + s + h^e \lambda(\theta)} \right)$$

Since  $u_l$  is decreasing in  $\theta$ , then  $\tilde{\Psi}(\theta) = \frac{h^l}{\bar{h}} u_l \left( \frac{f(0) - (1 + c_p)\underline{w}}{r + s + h^e \lambda(\theta)} \right)$  is strictly decreasing in  $\theta$ , implying that there exists only one positive level of vacancy rate.

### B.2 Proof of proposition 2

The proof of the discontinuity of the wage distribution support follows the one proposed in the seminal paper of Mortensen [1990]. Let us denote by  $\pi(w)$  the following profit flow:

$$\pi(w) = f(k(w)) - p_k k(w)(r + s + h^e \lambda(\theta)[1 - F(w)])$$

- For any  $w \in [x_l; x_c[$ , then the intertemporal expected profit associated with a filled job is given by:

$$\frac{h^l}{\bar{h}} u^l \left( \frac{s + h^e \lambda(\theta)}{s + h^e \lambda(\theta)[1 - F(w)]} \right) \left( \frac{\pi(w) - (1 + c_p)w}{r + s + h^e \lambda(\theta)[1 - F(w)]} \right) \quad (5)$$

Evaluating this expression for  $w = x_c^-$ , we have:

$$\frac{h^l}{\bar{h}} u^l \left( \frac{s + h^e \lambda(\theta)}{s + h^e \lambda(\theta)[1 - F(x_c^-)]} \right) \left( \frac{\pi(x_c^-) - (1 + c_p)x_c^-}{r + s + h^e \lambda(\theta)[1 - F(x_c^-)]} \right) \quad (6)$$

- Now, consider that  $w = x_c$ , from the definition of  $G(w)$  over  $w \in [x_c, \bar{w}]$ , this intertemporal expected profit turns out to be:

$$\frac{s}{s + h^e \lambda(\theta)[1 - F(x_c)]} \left( \frac{\pi(x_c) - (1 + c_p)x_c}{r + s + h^e \lambda(\theta)[1 - F(x_c)]} \right) \quad (7)$$

Comparing equations (6) and (7) for  $x_c^- \rightarrow x_c$ , we find that (6) < (7) as long as  $h^e \lambda(\theta)[1 - F(x_c)] + s > 0$  which is guaranteed until  $s > 0$ . This shows that there is no wage offer over the interval  $[x_c^-; x_c[$ .

- There must exist a critical wage offer  $w_l$  such that there is no wage offer over the interval  $[w_l; x_c[$ . This critical point of the wage distribution can be derived by equalizing the condition (5) evaluated for  $w = w_l$  with the condition (7), and by taking into account the restriction  $F(x_c) = F(w_l)$ :

$$\frac{\gamma\theta}{\lambda(\theta)} = \frac{h^l}{\bar{h}} u^l \left( \frac{s + h^e \lambda(\theta)}{s + h^e \lambda(\theta)[1 - F(x_c)]} \right) \times \left( \frac{pf(k(w_l)) - (1 + c_p)w_l - p_k k(w_l)(r + s + h^e \lambda(\theta)[1 - F(x_c)])}{r + s + h^e \lambda(\theta)[1 - F(x_c)]} \right)$$