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An Adverse Selection Model of Optimal Unemployment Insurance

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ABSTRACT

An Adverse Selection Model of Optimal Unemployment Insurance*

We derive the shape of optimal unemployment insurance (UI) contracts when agents can exert search effort but face different search costs and have private information about their type. We derive a recursive solution of our dynamic adverse selection problem with repeated moral hazard. Conditions under which the UI agency should always offer separating contracts are identified. We show that the good searcher receives an information rent and that the bad searcher receives the minimal entitlement. From a methodological point of view, we achieve a precise characterization of the sets of jointly feasible entitlements. This allows us to map our analytical results one-to one to a numerical algorithm. According to our results the contract for the good searcher has a decreasing benefit profile, as the one he would be offered in a pure moral hazard environment. In contrast, the contract of the bad searcher is distorted by an adverse selection effect, so that it tends to have an upward-sloping benefit profile. We provide a comparative static analysis of changes in various parameters of our model.

JEL Classification: C61, D82, E61, J64, J65

Keywords: unemployment insurance, adverse selection, moral hazard

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1 Introduction

Governments in all developed countries provide unemployment insurance (UI). There is remarkable empirical evidence that the existence of such a social insurance prolongs unemployment spells (compare the overview articles Atkinson and Micklewright (1991) and Meyer (1995)). This fact can be attributed to the disincentive effects of UI on job search. Designing an optimal UI scheme, the government has to trade off incentives to search against insurance aspects of providing benefits. A contract theoretical framework underlying this interpretation was first introduced by Shavell and Weiss (1979) and later refined and simulated by Hopenhayn and Nicolini (1997). A key result is that UI benefits should decrease with the length of the unemployment spell.

In this paper, we propose to reconsider the optimal design of an UI scheme. The distinguishing feature of our analysis is agents' heterogeneity with respect to their reemployment probability (there are "good" and "bad" searchers). Agents in a heterogeneous population will in general not face the same probability of finding a job (given a certain search effort). And the government will in general lack information about individual reemployment probabilities. Our paper focuses on two questions. First, under which circumstances will it be optimal to offer only one UI contract to all agents - a situation that resembles real world UI schemes in most countries? Second, is a decreasing UI benefit scheme not only optimal in a homogeneous but also in a heterogeneous population? We identify a condition under which it is *never* optimal to offer only one UI contract. Loosely speaking, the condition says that good searchers react more sensitively to search incentives than bad searchers. This condition is met in all numerical specifications of our model. Furthermore, the optimality of a decreasing UI benefit scheme only holds for the good searchers. In contrast, the benefit scheme for the bad searchers is distorted by an adverse selection effect. If this effect is strong, he should face an upward-sloping UI benefit scheme. We investigate these issues theoretically and numerically.

The literature on optimal UI has evolved in several waves and around different questions. Economic aspects considered include job matching, work performance, wage bargaining and human capital acquirement. We will not discuss this extensive literature here in detail and refer the interested reader to the comprehensive overview article by Karni (1999).

The strand of literature on which we build takes a contract theoretical

perspective of the UI design problem. The seminal contributions are Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) (HN hereafter)[1]. In their setup, the probability of receiving a job offer depends on a costly and unobservable search effort. The principal designs a UI benefit scheme in order to guarantee a certain ex-ante-utility for the agent at lowest (expected) cost taking into account the (dynamic) moral hazard of agents. In this framework, the optimal UI benefit scheme should decrease with the length of the unemployment spell. Furthermore, HN show that a government should use the additional instrument of a wage tax after reemployment with a tax rate that should increase with the length of the unemployment spell.

All these contributions have one feature in common: They consider ex-ante identical workers. With search effort hold constant, everybody faces the same probability of receiving a job offer. Our aim is to investigate the shape of optimal contracts in a framework where two types of agents (good and bad), with the type of an agent being private information, face different hazard rates of finding a job so that their resulting search costs are different. From a contract theoretical point of view we consider a dynamic adverse selection problem with repeated moral hazard.

Some steps toward an analysis of this question can be found in the literature already. The issue of adverse selection has been raised first by Mortensen (1983) who applies the seminal Rothschild and Stiglitz (1976) paper to UI. His analysis is, however, static and does not include search incentives. Wang and Williamson (1999) present a numerical welfare analysis of UI in a dynamic economy with moral hazard and heterogeneous agents. Their focus is the effect of (full and partial) experience rating on optimal UI. In contrast in our model agents choose from a set of different UI contracts offered by the principal. Finally, Hopenhayn and Nicolini (2001) address the issue of heterogeneity of agents in a two period model of UI similar to their earlier work HN. They assume, however, that the agents' type is observable and contractable.

In our set-up the principal wants to ensure a certain ex-ante lifetime utility to both agents at lowest cost. In the first period he can offer one or two contracts specifying the benefit levels for each period where the agent remains unemployed. To keep track of the search incentives generated by a contract we introduce the remaining expected life-time utility guaranteed by a contract to an agent as a state variable of the problem (that we call

[1]Recent contributions include Pavoni (2000), Zhao (2001) and Hassler and Mora (2002).

entitlement in the sequel). With this state variable, we are able to give a recursive formulation of two separate cost minimization problems that generate optimal contracts implementing previously specified entitlements. The adverse selection problem can then be stated in terms of these cost minimization problems. Finally we show - in analogy to the standard adverse selection problem (cf. Chap. 2 of Laffont and Martimort (2002)) - that the entitlement constraint of the bad searcher and the incentive constraint of the good searcher must be binding, whereas the entitlement constraint of the good searcher is implied by the other constraints and thus slack. We thus combine a recursive method and an approach to adverse selection problems followed by classical contract theory.

As HN, our paper technically builds on recursive solutions of repeated games and principal agents problems as studied in the papers by Spear and Srivastava (1987), Thomas and Worrall (1990), Abreu, Pearce, and Stacchetti (1990) [APS], Atkeson and Lucas (1992) and Chang (1998). This literature has introduced entitlement utilities as state variables in order to analyze models of repeated moral hazard. The methodological contribution of this paper is a precise characterization of the sets of jointly feasible entitlements which is used one-to-one in our numerical algorithm. This corresponds to the calculation of the set of "sustainable outcomes" in the terminology of infinite-dimensional models, that is characterized theoretically as the fix-point of a set-operator in APS and Chang (1998).

From our solution we can deduce a number of results: : First, the good searcher receives an information rent, whereas the bad searcher receives the minimal entitlement. Second, the contract for the bad searcher is distorted by an adverse selection effect. In the simulation we will see that if this effect is strong, his contract will show an upward-sloping profile. Third, if the incentive constraint of the bad searcher is slack, the contract for the good searcher is identical to the one he would be offered in a pure moral hazard framework, given his entitlement.

We use our recursive solution to derive further results in a a numerical simulation. In particular, we study the effect of various parameters of the model on the two optimal contracts. We are interested in their impact on the adverse selection effect and the information rent for the good searcher. Our findings are as follows:

1. In all our examples, the incentive constraint of the bad searcher is slack. In particular, this means that the good searcher always receives the

same contract as in a pure moral hazard environment with a decreasing benefit profile.

2. The adverse selection effect is strong for low values of the entitlement and for small population shares of the bad searcher. Also, it is reinforced by a fall of the bad searcher's hazard rate.

3. The information rent increases as the entitlement decreases; it is low if the population share of the bad searcher is high or if the agents are highly risk-averse.

We provide an economic intuition for the results of the comparative static exercise and discuss their implications for the application of an optimal UI program with more than one contract.

The paper is organized as follows: In Section 2 the model is introduced and the theoretical solution is presented. In section 3 the model is solved numerically. We also present an extensive comparative statics exercise. Proves are given in the appendix.

2 Model

Our framework for analyzing UI is a dynamic principal-agent model. The principal represents the government (or UI agency) providing social insurance.

The agents' problem The agents are unemployed workers searching for a job. There are two types of agents, differing in their opportunities of finding a new job. There is a continuum of agents, modeled by the unit interval. The fraction of agents of type B ("bad searcher") q and of type G ("good searcher") $1-q$ are common knowledge. Both agents are equally risk-averse, they enjoy the same utility $u(.)$ from consumption. Also, we assume that once an agent has been employed, he will keep his job with a fixed wage rate [2] w until his death in period T. We can thus calculate an agent's total expected lifetime utility in period t as

[2] Technically, the assumption that both agents get the same wage is in no way important for our analysis, it is introduced for simplicity's sake. In the solution to our model we could keep track of the impact of different wages for B and G on the search incentives; qualitatively, the results would not change.

$$W_t = \sum_{l=t}^{T} \beta^l u(w).$$

Subscripts denote the time index, β is the common time discount rate.

The differences between the two types of agents in their search technology is expressed by agent i's probability of remaining unemployed $p_i(a)$, that is a function of the search effort a he exerts. We assume that

$$p_B(a) > p_G(a),$$

i.e. for any given search effort the bad searcher has a higher probability of remaining unemployed than the good searcher (we will sharpen the notion of heterogeneity in condition 2.4 below). Given any benefit scheme $\{b_1, ..., b_T\}$, the agent chooses a in any period when he is unemployed. In his decision to increase his search effort he faces a trade-off between increasing search costs on the one hand and an increasing reemployment probability on the other. The total lifetime utility he expects when remaining unemployed is the key variable determining this decision. We denote by V_{t+1}^i agent i's expected (discounted) lifetime utility from period $t+1$ on. Moreover, in the sequel we will denote by $z_i = u(b_i)$ the utility value of consuming benefit b_i. Thus we can state the agent's problem in period t recursively by

$$V_t^i = \max_a z_t - a + \beta[p_i(a)V_{t+1}^i + (1 - p_i(a))W_{t+1}]. \tag{1}$$

Recall that W_{t+1} is the expected lifetime utility of employment starting in $t+1$. We denote by $\hat{a}_t^i$ the decision of agent i:

$$\hat{a}_t^i = \text{argmax}_a \, z_t - a + \beta[p_i(a)V_{t+1}^i + (1 - p_i(a))W_{t+1}]. \tag{2}$$

Taking the agents' decision problem into consideration, the principal minimizes total expected costs. Hereby the current cost function is the inverse of the agent's utility function $c(z_i) = u^{-1}(z_i) = b_i$. The following property of $c(.)$ is implied by strict concavity of an agent's utility function $u(.)$, i.e. risk aversion.

Condition 2.1 *The current cost function $c(.)$ is increasing and strictly convex.*

The principal's problem The principal's objective is to minimize the cost of providing insurance by the design of optimal contracts for the agents. We will call $\{z_1^i, ..., z_T^i\}$ a contract designed for agent i (with $i = B, G$). In his minimization problem, the principal has to take into consideration the different reemployment probabilities of the agents. Furthermore, he has to take into account the following constraints: First, he has to respect the entitlement $\underline{V}$ (i.e. total expected lifetime utility) the contracts should at least guarantee to agent B and G respectively. Second, he has to guarantee incentive compatibility. We assume that the principal can fully commit to the contract promised in period zero. The principal's problem can thus be stated as

$$\min_{\{z_1^B,...,z_T^B\},\{z_1^G,...,z_T^G\}} q[c(z_1^B) + \beta p_B(\hat{a}_1^B)[c(z_2^B) + \beta p_B(\hat{a}_2^B)[c(z_3^B) + ...]...]] + (1-q)[c(z_1^G) + \beta p_G(\hat{a}_1^G)[c(z_2^G) + \beta p_G(\hat{a}_2^G)[c(z_3^G) + ...]...]]$$

subject to the entitlement constraints (EC)

$$V_1^{b,B} \geq \underline{V}, \tag{3}$$
$$V_1^{g,G} \geq \underline{V}, \tag{4}$$

and the incentive constraints (IC)

$$V_1^{b,B} \geq V_1^{g,B}, \tag{5}$$
$$V_1^{g,G} \geq V_1^{b,G}. \tag{6}$$

Here, $V_t^{j,i}$ denotes total expected lifetime utility in period t by contract j ($j = b, g$) for the unemployed agent i ($i = B, G$).[3] Both the $V_t^{j,i}$s and the $\hat{a}_t^i$s can be calculated from the array of equations 1 and 2.

Remark 2.2 *If agents are homogeneous in their search costs, i.e. they all have the same $p_i(a)$, then the setup is identical to the one considered by Shavell and Weiss (1979) (except for the finite time dimension).*

[3]The superscript j indicates for which agent the contract is designed, i.e., contract b is designed for agent B and contract g for agent G. A priori both agents can of course choose either contract. The IC conditions ensure that they will in fact choose the contract designed for them.

Formal Assumptions Now we formalize in detail the idea that agents differ in their reemployment probabilities. First, we assume that to achieve a given probability $1 - p_i$ of being reemployed, agent 1 has to exert a higher effort than agent 2. Moreover, to develop the model model formally we make some standard technical assumptions on the $p_i(.)$.

Condition 2.3 *We assume the following for the probability of remaining unemployed $p_i(a)$ of agent i:*

1. *Smoothness*	$p_i(a) \in C^\infty(\mathbf{R})$	
2. *Strict Convexity*	$p_i'(a) < 0$	$p_i''(a) > 0$
3. *Boundary conditions*	$p_i(0) = 1$	$\lim_{a\to\infty} p_i(a) = 0$
4. *Inada conditions*	$\lim_{a\to 0} p_i'(a) = -\infty$	$\lim_{a\to\infty} p_i'(a) = 0$

Condition 2.3 ensures that the agents' problem 1 always has a unique interior solution that can be characterized by a first order condition.

Condition 2.4 *The relation of the two probability functions is assumed to be as follows:*

1. *Given the same effort, type B has a higher probability of remaining unemployed than type G: $p_B(a) > p_G(a)$.*
2. *Given any contract $\{z_1, ..., z_T\}$, in equilibrium $p_B(a_t^B) > p_G(a_t^G)$.*

Condition 2.4 formally characterizes the heterogeneity in our model. Its second part sharpens the basic concept of a bad or good searcher: Given any contract, the bad searcher may exert a higher search effort than the good searcher, but his effort will not be so high that his chance of finding a job exceeds the good searcher's chance. The next lemma shows that the condition is in fact well-known from standard contract theory.

Lemma 2.5 *Property 2 of condition 2.4 is equivalent to the following Spence-Mirrlees property:*

$$\frac{\partial V_t^B}{\partial z_{t+s}} - \frac{\partial V_t^G}{\partial z_{t+s}} > 0 \tag{7}$$

for all t with $1 \le t \le T$ and s with $1 \le s \le T - t$.

Proof. Firstly, we prove that $p_B(a_t^B) > p_G(a_t^G)$ ensues from the Spence-Mirrlees-Property 7. We calculate

$$\frac{\partial V_t^i}{\partial z_{t+1}} = \beta p_i(a_t^i) \frac{\partial V_{t+1}^i}{\partial z_{t+1}} = \beta p_i(a_t^i),$$

where we use the Envelop Theorem. The assertion now follows immediately. Secondly, assume that $p_B(a_t^B) > p_G(a_t^G)$ holds. In the case where $s = 1$, the Spence-Mirrlees property follows from what we have shown above. So let $s > 1$. Then

$$\frac{\partial V_t^B}{\partial z_{t+s}} - \frac{\partial V_t^G}{\partial z_{t+s}} = \beta p_B \frac{\partial V_{t+1}^B}{\partial z_{t+s}} - \beta p_G \frac{\partial V_{t+1}^G}{\partial z_{t+s}},$$

where we have used the Envelop Theorem once more. The assertion follows by induction. ■

Condition 2.3 and 2.4 will be used in all results that follow. For our first result, we need one more assumption on p_i. As we mentioned above, condition 2.3 ensures that the agent's choice of effort 1 can be characterized by the following first order condition:

$$p_i'(a_t^i) = \frac{1}{\beta(V_{t+1}^i - W_{t+1})}. \tag{8}$$

Equation 8 establishes a one-to-one (and smooth) relation between V_t^i and a_t^i. We can therefore define the following function for the next-to-last period:

$$\pi_i(z_T) \equiv p_i(a_{T-1}^i) = p_i\left((p_i')^{-1}\left(\frac{1}{\beta(z_T - u(w))}\right)\right)$$

So $\pi_i(z_T)$ is agent i's probability of remaining unemployed when facing benefit utility z_T in the last period. It is clear that π_i is increasing. We formulate a condition on the elasticity of π_i with respect to z_T, which is -of course- an implicit assumption on p_i.

Condition 2.6 *1. The marginal probability of remaining unemployed $\frac{\partial \pi_i(z)}{\partial z}$ of agent i facing promised utility z is greater for agent G than for agent B:*

$$\frac{\partial \pi_G(z)}{\partial z} > \frac{\partial \pi_B(z)}{\partial z}$$

2. *If the marginal probabilities* $\frac{\partial \pi_i(z)}{\partial z}$ *of agent G and B are equal for two utility values* z^G *and* z^B*, then the utility of G,* z^G*, must be smaller than the utility of B,* z^B*:*

$$\frac{\partial \pi_G(z^G)}{\partial z} = \frac{\partial \pi_B(z^B)}{\partial z} \;\Rightarrow\; z^G < z^B$$

What is the economic content of Condition 2.6? The first part says that agent G reacts more strongly to a change in the promise z than agent B. In other words: The probability of finding a new job depends more critically on the UI benefit promise in the case of agent G than in the case of agent B. Note that here we compare agents G and B that face the same contract. The second part of condition 2.6 says, that whenever the reaction is equal, then agent G must face a *lower* promise than agent B. Summarizing condition 2.6 we can say that the incentive sensitivity of agent G is higher than the one of agent B.

What role do conditions 2.4 and 2.6 play in our analysis? The model presented in this paper incorporates two different paradigms, hidden information and (repeated) hidden action. Condition 2.4 is the typical technical assumption in hidden information models. Condition 2.6 is a condition that ensures in our setup a feature of (pure) repeated hidden action that has been analyzed numerically in Pavoni (2000) and discussed in Hopenhayn and Nicolini (2001): The fact that the (full information) UI contract should fall the more steeply the better a searcher is the agent. Loosely speaking, conditions 2.4 and 2.6 ensure that our model exhibits the standard behavior of a pure hidden information model and a pure hidden action model. We will see in section 3 that both conditions will be met in our functional specification.

Finally, to make our problem interesting, the initial entitlements to total expected lifetime utility have to be below the total lifetime utility from work. We take this as a natural political prerogative. If the entitlements are $V_t^{i,j}$ are higher than W_t for any period posterior to one in that the unemployed agents exert search effort, the efforts would necessarily be zero, and thus the probability of remaining unemployed p_i would be 1.

Condition 2.7 *We assume that the utility entitlement of the unemployed agent is below the one guaranteed by lifetime work:* $\underline{V} < W_1$.

We are now prepared for a first result.

Pooling is Not Optimal The first question we ask is whether and under what circumstances it is actually optimal to offer two contracts in order to screen the agents. The answer gives a first indication that it may indeed be relevant to consider the cost saving potential of a differentiated UI.

Theorem 2.8 *There exists a solution to the Principal's Problem. If Condition 2.6 holds, any solution is separating.*

Proof. see Appendix ■

Economically speaking: It is *not* efficient to offer only one UI contract to two agents that face different search costs in the labor market if the good searcher reduces his effort more than the bad searcher in response to an increase in the entitlement while unemployed (i.e. so that his reemployment chance falls more quickly). In other words: If the good searcher reacts more sensitively to the search incentives set by the principal than the bad searcher, he should *not* offer a single contract to both agents.

Remark 2.9 *In the case of CARA and CRRA utility one can show that to obtain the "no pooling" result of theorem 2.8 it is sufficient to impose assumption 2.6 on the elasticity* $\sigma_\pi^i(z) = \left|\frac{\partial \pi_i(z)}{\partial z}\frac{z}{\pi_i(z)}\right|$.

What makes the theorem very appealing from a more applied perspective is the fact that our numerical implementation with CRRA utility shows that the good searcher is very likely to react more sensitively to the search incentives than the bad searcher. Considering this numerical result as robust we could claim that the UI agency has a definite potential for cost-saving by switching from offering only one to offering two UI contracts.

But what should the optimal contracts look like? The answer to this question is not evident: We can neither apply Shavell and Weiss (1979) solution directly, since we should expect the influence of hidden information on the optimal design of the contracts, nor can we apply standard solutions of adverse selection models that do not incorporate repeated hidden action. Moreover we cannot hope to find a direct recursive formulation of the problem, because both the incentive constraints and the entitlement constraints in the principal's problem have to hold *only* in the first period.

The Solution of the Model In the sequel, we develop a characterization of the solution to the principal's problem in Propositions 2.10, 2.12 and 2.16. The strategy is as follows: We first look at each contract *separately*, giving - in Proposition 2.10 - a recursive formulation of the problem of finding the cost-minimizing contract that provides agent B and agent G with a specified level of ex-ante lifetime utility (*entitlement*) under a given contract. The goal of this proposition is to summarize the cost minimization problem in a compact way, i.e., give two separate recursive formulations, one for each contract. In this formulation the entitlements (as seen by the agent in period t) serve as state variables of the problem (compare HN).

The question left open by proposition 2.10 is which pairs of entitlements are actually jointly feasible under a given contract. The answer to this question has to take into account the laws of motion for the entitlements under a given contract as well as the induced choices of effort by the agents. Proposition 2.12 gives a precise theoretical description of the correspondence mapping pairs of entitlements of the agents today (V_t^B, V_t^G) to jointly feasible policy options $(z_t, V_{t+1}^B, V_{t+1}^G)$, i.e. the benefit today and the entitlements for tomorrow. This proposition is the main theoretical result of the paper and serves as the cornerstone of our recursive numerical implementation. We can now calculate the cost of providing agents with given entitlements in the *first* period.

Finally, we merge the two separately solved cost minimization problems. We can state the original adverse selection problem faced by the principal as a four-dimensional minimization problem in the entitlements of the *first* period. In Proposition 2.16 we further simplify the minimization problem by showing that the entitlement constraint of G must be slack, and that the entitlement constraint of B and the incentive constraint of G must be binding at the solution. This reduces the dimension of the problem from four to two which allows us to solve the problem with high numerical accuracy.

We begin the characterization of the solution by the recursive formulation of contract b, i.e. the contract designed for agent B. As usual in adverse selection problems, we anticipate that only agent B will choose contract b in the end and thus stochastically discount costs at his rate. For the time being, the (total) entitlements of contract b for agent B and agent G, $V^{b,B}$ and $V^{b,G}$, are taken as given. Their optimal values will be calculated in Proposition 2.16 below. The recursive formulation takes the form of a (finite-dimensional) Bellman Equation: The principal minimizes the costs of paying out a benefit

worth z_t (in utility units) today and promising entitlements $V_{t+1}^{b,B}$ and $V_{t+1}^{b,G}$ for tomorrow. In doing so, he has to observe the entitlements of B and G today, $V_t^{b,B}$ and $V_t^{b,G}$, that serve as state variables of the problem. A "law of motion" connects the state and the choice variables. We denote the set of possible choices $(z_t, V_{t+1}^{b,B}, V_{t+1}^{b,G})$ in state $(V_t^{b,B}, V_t^{b,G})$ by $\Gamma_t(V_t^{b,B}, V_t^{b,G})$. This correspondence will be characterized later in Proposition 2.12. Moreover, the choices of effort a_t^i of the agents facing the promised entitlement V_{t+1}^i for tomorrow are taken as given by the principal (thus we incorporate the incentive constraints from the hidden action problem). The recursive formulation is completed by two boundary conditions: In the first period, the entitlements $V_1^{b,B}$ and $V_1^{b,G}$ have to take the values $V^{b,B}$ and $V^{b,G}$ respectively, in the last period, the entitlements $V_T^{b,B}$ and $V_T^{b,G}$ have to take the value of the last benefit z_T. Thus we can state the following proposition without proof:

Proposition 2.10 (Recursive Formulation of Contract b) *The cost functions for contract b, guaranteeing a utility of $V^{b,B}$ to agent B and $V^{b,G}$ to agent G, has the following recursive form:*

$$C_t^B(V_t^B, V_t^G) = \min_{\{z_t, V_{t+1}^B, V_{t+1}^G\} \in \Gamma_t(V_t^B, V_t^G)} c(z_t) + \beta p_B(a^B) C_{t+1}^B(V_{t+1}^B, V_{t+1}^G) \quad (9)$$

subject to

Law of motion for contract b (LOM)

$$\begin{aligned} z_t - a^B + \beta[p_B(a^B)V_{t+1}^B + (1 - p_B(a^B))W_{t+1}] &= V_t^B \\ z_t - a^G + \beta[p_G(a^G)V_{t+1}^G + (1 - p_G(a^G))W_{t+1}] &= V_t^G \end{aligned}$$

Choice of effort in contract b (CE)

$$\begin{aligned} a^B &= \text{argmax}_a\, z_t - a + \beta[p_B(a)V_{t+1}^B + (1 - p_B(a))W_{t+1}] \\ a^G &= \text{argmax}_a\, z_t - a + \beta[p_G(a)V_{t+1}^G + (1 - p_G(a))W_{t+1}] \end{aligned}$$

as well as the boundary conditions

$$V_1^B = V^{b,B} \quad (10)$$
$$V_1^G = V^{b,G} \quad (11)$$
$$V_T^B = z_T \quad (12)$$
$$V_T^G = z_T \quad (13)$$

The correspondence Γ_t maps into the values of jointly feasible z_t, V_{t+1}^B, V_{t+1}^G given the pair of entitlements (V_t^B, V_t^G).

Remark 2.11 *Of course the contract g for agent G takes the same form as contract b for agent B in Proposition 2.10.*

In order to make use of Proposition 2.10, we actually have to know the correspondence $\Gamma_t(.,.)$. This is particularly important for any numerical application of the recursive formulation. The following proposition gives a precise characterization of Γ_t. For technical reasons we distinguish between the case where the utility z_t from consuming the UI benefit is bounded from below from the case where it is not. We will discuss this and other issues after stating the proposition.

Proposition 2.12 (Characterization of Γ_t) *The following formulas characterize the correspondence $\Gamma_t(V_t^B, V_t^G)$, that gives all feasible policy options for given entitlements V_t^B and V_t^G in period t.*

1. *For $t \leq T-2$, if no restrictions are imposed on the utility values of benefits z_t, the correspondence of feasible entitlements has the following form*

$$\Gamma_t(V_t^B, V_t^G) = \begin{cases} \{z_t(a), V_{t+1}^B(a), V_{t+1}^G(a)\}_{a\in[0,\infty]} & : \quad V_t^B \leq V_t^G \leq W_t \\ \emptyset & : \quad else \end{cases} \tag{14}$$

   The jointly feasible values $z_t(a)$, $V_{t+1}^B(a)$ and $V_{t+1}^G(a)$ are differentiable functions of a.

2. *Again, be $t \leq T-2$, but let $z_t \geq \underline{z}$. Then there exists a lower bound $\underline{V}^G$ and so that for $\underline{V}^G \leq V_t^G \leq W_t$:*

$$\Gamma_t(V_t^B, V_t^G) = \begin{cases} \{z_t(a), V_{t+1}^B(a), V_{t+1}^G(a)\}_{a\in[\underline{a},\overline{a}](V_t^B,V_t^G)} & : \quad V_t^B \in [\underline{V}_t^B(V_t^G), \overline{V}_t^B(V_t^G)] \\ \emptyset & : \quad else \end{cases} \tag{15}$$

   The jointly feasible values $z_t(a)$, $V_{t+1}^B(a)$ and $V_{t+1}^G(a)$ are differentiable functions, the boundary functions $\underline{a}(V_t^B, V_t^G)$, $\overline{a}(V_t^B, V_t^G)$ and $\underline{V}_t^B(V_t^G)$, $\overline{V}_t^B(V_t^G)$ are continuous. For V_t^G below $\underline{V}^G$, the correspondence $\Gamma_t(V_t^B, V_t^G)$ is the empty set.

3. *In period $T-1$ the correspondence $\Gamma_{T-1}(V_{T-1}^B, V_{T-1}^G)$ is either the empty set or contains only one point of jointly feasible choices $\{z_{T-1}, V_T^B, V_T^G\}$.*

Proof. see Appendix ■

We have two technical remarks on Proposition 2.12.

Remark 2.13 *The upper bound W_t on V_t^B is artificial: Of course the principal can ensure lifetime utilities above the value of secure lifetime income from work. However, this cannot be optimal, since it reduces the search effort to zero, and in view of Condition 2.7 we exclude lifetime utilities above W_t from our considerations.*

Remark 2.14 *The distinction in Proposition 2.12 between the case where the benefit utility z_t is unbounded from the case where it is bounded is made for technical reasons: Some utility functions map onto the real line $\mathbf{R}$, some only onto the half-line $\mathbf{R}_+$ (modulo normalization). An example of the former kind are CARA utility functions, one of the latter CRRA utility functions with risk aversion γ smaller than one.*

With Proposition 2.12 at hand, we can define precisely the notion of feasibility in our model: A pair of entitlements (V_t^B, V_t^G) is called "jointly feasible" if the set $\Gamma_t(V_t^B, V_t^G)$ is non-empty.

Let us now be more specific why this proposition is so important for our purposes. Models including repeated moral hazard, as ours, have been discussed in the framework of a strand of literature building on Spear and Srivastava (1987), Thomas and Worrall (1990), Abreu, Pearce, and Stacchetti (1990) [APS], Atkeson and Lucas (1992) and Chang (1998). We are not bound to provide a detailed discussion of this literature, the following remark, however, relates proposition 2.12 to it.

Remark 2.15 *The sets of jointly feasible entitlements are the finite-dimensional analogon of the set of sequential equilibrium payoffs (of the agents' game) in the infinite-dimensional framework of APS or the set of sustainable outcomes in the (again infinite-dimensional) framework of Chang (1998).*

By introducing entitlements, marginal utilities or sequential equilibrium outcomes as state variables -instead of "intuitive" state variables- we inevitably run into the difficulty of defining precisely the sets of possible values

these state variables can take. APS and Chang (1998) characterize these sets as the largest fix-point of a set operator. Moreover, they show that the fix-point can be obtained by a fix-point iteration of sets. This is theoretically sound. However, it does not provide an entirely satisfactory description of the sets nor *the* definite algorithm to calculate them numerically, in particular if the state space is more than one-dimensional. In fact, the numerical determination of these sets may be a tricky issue in simulations of models building on these methods. In Proposition 2.12 we give -for our model- a "precise" description of the sets of the state variables: The boundaries of the sets are (continuous) functions, in particular, the sets are compact and connected. In the next section we will point out that this is crucial for the numerical implementation of our solution. Moreover, Proposition 2.12 states that the principal's choice problem in a given period is essentially one-dimensional, except for the next-to-last period, where there is only one choice left.

After this methodological digression, we return to the solution of the principal's problem. Given proposition 2.10 it can be stated as follows:

$$\min_{V^{b,B},V^{b,G},V^{g,B},V^{g,G}} qC_1^B(V^{b,B},V^{b,G}) + (1-q)C_1^G(V^{g,B},V^{g,G}) \quad (16)$$

$$s.t. \quad \Gamma_1(V^{b,B},V^{b,G}) \neq \emptyset \quad (17)$$

$$\Gamma_1(V^{g,B},V^{g,G}) \neq \emptyset \quad (18)$$

$$V^{b,B} \geq V^{g,B} \quad (19)$$

$$V^{g,G} \geq V^{b,G} \quad (20)$$

$$V^{b,B} \geq \underline{V} \quad (21)$$

$$V^{g,G} \geq \underline{V}. \quad (22)$$

This is a four dimensional minimization problem, and thus still rather complex. However, as in the case of standard adverse selection problems (compare e.g. the book by Laffont and Martimort (2002), Chap. 2), we are able show that agent B's entitlement constraint and agent G's incentive constraint must be binding, and that agent G's entitlement constraint must be slack at the solution. And so we finally characterize the solution to the principal's problem as follows:

Proposition 2.16 (Solution of the Principal's Problem) *For values of*

$\underline{V}$ that are not too low [4]*, solutions to the principal's problem are solutions to the simplified problem*

$$\min_{V^{g,B},V^{g,G}} qC_1^B(\underline{V}, V^{g,G}) + (1-q)C_1^G(V^{g,B}, V^{g,G})$$
$$s.t. \quad \underline{V} \geq V^{g,B}$$

Proof. see Appendix ■

Two Corollaries Two corollaries ensue from proposition 2.16. The set-up considered by Shavell and Weiss (1979) will be our benchmark, i.e. a set-up where the principal knows the type of the agent and sets the benefits to give optimal search incentives. We will call it the *pure moral hazard environment.* About the contract for type B we learn:

Corollary 2.17 *Type B receives the minimal entitlement utility $\underline{V}$. His contract is distorted with respect to the optimal contract in a pure moral hazard environment such that its value $V^{b,G}$ for type G is reduced.*

Proof. The first assertion is point 2 in the proof of 2.16. To see the second assertion, note that, given that $V^{g,B}$ is chosen optimally for each value of $V^{g,G}$, the cost function of contract g is strictly increasing in $V^{g,G}$. Moreover, in a full information optimum (i.e. the pure moral hazard case for both contracts) the optimal $V^{b,G}$ (optimal with respect to $V^{b,B} = \underline{V}$) can be characterized by a first order condition. We thus obtain a first order reduction of costs for contract g by lowering $V^{g,G} = V^{b,G}$ (constraint 20 is binding!) below the value of $V^{b,G}$ in a full information optimum, whereas there is only a second order increase in costs for contract b. ■

In the case of the contract for type G, we deduce:

Corollary 2.18 *1. Type G receives an information rent, i.e. the utility $V^{g,G}$ that his contract provides him with is greater than $\underline{V}$.*

[4] This condition is needed to avoid corner solutions. In the simulation we found that all reasonable values of $\underline{V}$ were high enough (compare the proof in Appendix C and footnote 5 in Section 3.1).

2. *If the incentive constraint of the bad searcher 19 is slack at the solution, his contract is identical to the optimal contract in the pure moral hazard environment (given the level of entitlement $V^{g,G}$).*

Proof. The fact that $V^{g,G} > \underline{V}$ has been proved in Proposition 2.16; so we look at the second assertion.

In our framework, we can recover the Shavell-Weiss contracts (i.e. the contracts from the pure moral hazard environment) at a given level of entitlement $V^{i,i}$ by solving ($i \neq j$)

$$\begin{aligned} & \min_{V^{i,j}} C_1^i(V^{i,i}, V^{i,j}) \\ s.t. \quad & LOM, CE \end{aligned}$$

and applying forward induction afterwards. This is because by minimizing the costs of contract i with respect to its value for agent j, we just neglect the impact of this value for the optimal contract.

Now, if our objective function is optimized without further restriction, we recover the optimal contract from the pure moral hazard environment, because the value $V^{g,B}$ of contract g for type B does not appear in the cost function of contract b. ■

In our numerical simulations, we found that the incentive constraint of the bad searcher 19 was *always* slack.

We thus recover the rent extraction/efficiency trade-off from a simple adverse selection model without moral hazard (cf. chapter 2 of Laffont and Martimort (2002)), where efficiency here is the search efficiency of agent B.

The corollaries provide first economic insights as to how the unemployment agency should design optimal UI contracts. Further economic aspects will be discussed after a simulation of optimal contracts in the next section.

3 Simulation

3.1 Computational Strategy

The simulation closely follows Proposition 2.10, Proposition 2.12 and Proposition 2.16 in Section 2. The first part of the simulation calculates the cor-

respondence Γ_t by backward induction. As in Proposition 2.12, we use the entitlement of agent G V_t^G as a parameter for the upper and lower bound on the entitlement V_t^B for agent B. We introduce a grid on V_t^G and then calculate the bounds on V_t^B by a bracketing procedure.

More precisely, for a given tuple (V_t^B, V_t^G) of state variables, we check whether the corresponding path of choice variables $(z_t(a), V_{t+1}^B(a), V_{t+1}^G(a))$ (compare the proof of Proposition 2.12, Appendix C) intersects the set of jointly feasible values $(z_t, V_{t+1}^B, V_{t+1}^G)$ as determined in the previous induction step. As defined in the previous section, by "jointly feasible" we refer to tuples (V_{t+1}^B, V_{t+1}^G) such that $\Gamma_{t+1}(V_{t+1}^B, V_{t+1}^G)$ is non-empty; in the case of z_t, we only have to check whether it is above the lower bound $\underline{z}$. By proposition 2.12, we know that for each V_t^G (within the limits of feasible entitlements for G) there exits a $\underline{V}_t^B(V_t^G)$ and $\overline{V}_t^B(V_t^G)$ that limits the range of V_t^B jointly feasible with V_t^G. Since for every V_t^G the set of jointly feasible V_t^B is an interval, we can "encircle" $\underline{V}_t^B$ (and, separately, $\overline{V}_t^B$) by values of V_t^B above and below and then calculate the bound by a (highly precise) bracketing procedure. It is thus proposition 2.12 that guarantees that our algorithm calculates a characterization of the set of jointly feasible entitlements (V_t^B, V_t^G) by stating that the set of jointly feasible entitlements (V_{t+1}^B, V_{t+1}^G) is connected.

As we have pointed out in the preceding section (compare remark 2.15 and the following discussion), the virtue of proposition 2.12 lies in a more "precise" characterization of the sets of jointly feasible entitlements (the set of "sustainable outcomes" in the terminology of Chang (1998)). It is exactly here in the numerical algorithm where this characterization becomes useful: The description of the set of sustainable outcomes as a fixed-point of a set-operator by APS is *mathematically* precise, but poses a serious precision problem in numerical applications with more than one state variable (compare section 8 of Chang).

The second part of our numerical procedure uses the recursive formulation in Proposition 2.10. It calculates a numerical approximation of the cost functions $C_t^i(V_t^B, V_t^G)$ based on a solution of the minimization problems in the backward induction of the principal.

More precisely, we cover the domain of C_t^i (i.e. the set of jointly feasible entitlements (V_t^B, V_t^G) for which $\Gamma_t(V_t^B, V_t^G)$ is non-empty) by a large grid. For each tuple (V_t^B, V_t^G) in the grid ("states of the world"), we solve the minimization problem along the path of choice variables $(z_t(a), V_{t+1}^B(a), V_{t+1}^G(a))$, i.e. we solve it in a. Ignoring the exact value of the limits $\underline{a}$ and $\overline{a}$, we

use a bracketing procedure in which we allocate an extremely high cost to a values delivering choice variables outside the set of feasible values. Note that we make use of the characterization of Γ_t in two ways: First, we rely on the fact that the set of jointly feasible entitlements (V_t^B, V_t^G) is compact and connected. Second, we exploit the reduction of the number of choice variables from three $(z_t, V_{t+1}^B, V_{t+1}^G)$ to one a.

Finally, each cost function is then approximated as a linear combination of complete Chebychev polynomials by regression (for this standard procedure, compare Judd (1998), Chapters 6.4, 6.12 and 12.8).

In the third part, the approximated cost functions C_1^i are combined in the objective function of the principal's problem. According to proposition 2.16, we have to solve a two-dimensional minimization problem [5].

After an initial grid search, the solution is calculated by a Nelder-Mead multidimensional minimization procedure.

3.2 Calibration of the Model

In our calibration, we work with a monthly interval. Therefore we set the discount rate to $\beta = 0.995$ which corresponds to an annual discount rate of 0.95. The overall time-spell is a year, i.e. the number of periods is set to $T = 12$. As for the probability function, we choose $p_i(a) = 1 - \sqrt{1 - \exp(-\theta_i a)}$, where θ_i remains to be determined. We use CRRA utility functions $u(b) = \frac{b^{1-\gamma}}{1-\gamma}$, as common in the UI literature.

Needless to say, the corresponding cost function $c(.) = u^{-1}(.)$ meets the convexity condition 2.1:

$$c(z) = z^{\alpha},$$

where $\alpha = \frac{1}{1-\gamma} \geq 1$. Also, Conditions 2.3, 2.4 and 2.6 are fulfilled by the probability function $p_i(a)$ [6]. So in particular, in our numerical setup the prerequisites of Theorem 2.8 hold.

[5] There is one point to take care of, though: In order to apply proposition 2.16, we have to ensure that the minimal entitlement $\underline{V}$ is so high that $z_1 > \underline{z}$ (compare the proof in Appendix C). We ran alternative minimization routines for low values of $\underline{V}$, showing that the assertion holds.

[6] Our probability function $p_i(a)$ has a slightly more intricate functional form than the one used by HN ($p_{HN}(a) = 1 - \exp(-ra)$). We have chosen it because the latter does not fulfill the Inada condition (cf. Condition 2.3).

In the benchmark case, following HN, we set $\alpha = 2$ (i.e. risk aversion $\gamma = \frac{1}{2}$), which corresponds to intermediate risk aversion on behalf of the agents.

The wage is set to be $w = 100$, so that unemployment benefits become equal to replacement rates. The lower bound on the UI benefits is set to $\underline{z} = 0$, the lowest possible value taken by CRRA utility functions with $0 < \gamma < 1$. As in the proofs to the propositions, in our simulation we have normalized utility from consuming the wage to zero, i.e. all expected lifetime utilities are negative.

We have finally chosen the parameters $\theta_1 = 0.007$ and $\theta_2 = 0.017$ to match reasonable escape rates from unemployment in autarky (compare Meyer (1990)): For type B, the bad searcher, this is then 22.7% per month, for type G, the good searcher, it is 35.8% per month. As a comparison: HN assume a weekly escape rate in autarky of 10% as an average for the US, which corresponds to a monthly escape rate of 34.4%. Other choices of parameters will be discussed in the next section, where we give a detailed comparative statics analysis.

Our first figures, figure 2 to 5, show how the set of feasible entitlements becomes larger and larger along the backward induction. Figures 6 to 14 show optimal UI contracts for different levels of entitlements to the agents and for different values of q, the share of the first agents in the population. The entitlement bounds chosen are $\underline{V} = -20$, -25, -30 and -35 (also denoted in the legend). This corresponds to a certainty equivalent of 68.67%, 61.76%, 55.21% and 49.03% of the wage per period respectively, i.e. the utility of an (unemployed) agent who consumes X% of the wage for all periods and cannot gain employment.[7]

3.3 Results

In this section we will explore the dependence of the optimal contracts on different choices of parameters. Since we know that Theorem 2.8 applies, all solutions are separating. Moreover, for all parameter values we have looked at the incentive constraint of agent B, equation 19 was slack. For contract g designed for the good searcher G this means according to Corollary 2.18 that

[7]In the cases of $\underline{V} = -20, -25$ the UI benefits for agent G exceed a replacement rate of 100% in the first periods. This reflects the fact that our model does not incorporate effects of UI on work effort and its impact on employment.

it is identical to the one considered by Shavell and Weiss (1979). From their paper (compare also HN and Pavoni (2000)) we know that in our setup the optimal contract for the good type G has a falling benefit profile, induced by moral hazard; and this property is reflected by our figures. Changes in the parameter values do not qualitatively change contract g; the falling benefit profile is robust, as our theory predicts (for a detailed numerical discussion of the comparative statics of the contract in a pure moral hazard environment see Pavoni (2000)). However, the level of entitlement for G (and so, in particular, his information rent) varies with the parameters of our model.

We will thus concentrate our discussion on contract b and the information rent for G. The shape of contract b is determined by two effects:

1. a *moral hazard effect* (MH), arising as in the case of type G from the agents' search problem,

2. an *adverse selection effect* (AS), arising from the principal's wish to lower the value of the contract for type G (compare Corollary 2.17) in order to separate the type types.

We know that in the pure MH environment, benefit schemes are falling. What would agent B's contract look like in a *pure AS environment*? As a pure adverse selection environment, we consider a set-up where the type of an agent is still hidden information, but the probabilities of remaining unemployed of type B and G are fixed constants $p_B > p_G$. This is then a typical adverse selection problem as discussed in Chapter 2 of Laffont and Martimort (2002). Due to the assumption of full commitment, the dynamics of the contracts is rather simple. Now, as in the case of the full problem (Cf. Corollary 2.18), agent G receives an information rent, and, given the entitlement $V^{g,G}$, his contract is the first best contract. In the pure AS case, this means his consumption is fully smoothed. Trivially, the following Spence-Mirrlees property holds (as in the full problem, compare Lemma 2.5):

$$\frac{\partial V_t^B}{\partial z_{t+s}} - \frac{\partial V_t^G}{\partial z_{t+s}} > 0 \qquad \forall s \geq 1$$

Therefore, in order to separate the two types, the contract for agent B has to show an *increasing* benefit scheme. [8]

[8] A formal derivation of the solution to the principal's problem in a pure adverse selection environment is available upon request.

Information Rents

$\underline{V}$	Certainty Equivalent	$q = 0.2$	$q = 0.5$	$q = 0.8$
-35	49.03	10.8 %	18.1 %	22.2 %
-30	55.21	7.3 %	13.9 %	17.0 %
-25	61.76	4.8 %	10.5 %	12.9 %
-20	68.67	3.1 %	7.2 %	9.9 %

(rents expressed as percentage increase over minimal entitlement)

Table 1: Information Rents obtained by agent G

Elasticities σ_p of agent B

$\underline{V}$	Certainty Equivalent	$\theta_B = 0.004$	$\theta_B = 0.007$	$\theta_B = 0.010$
-35	49.03	0.0070	0.0116	0.0167
-30	55.21	0.0075	0.0131	0.0188
-25	61.76	0.0079	0.0139	0.0199
-20	68.67	0.0083	0.0146	0.0209

Table 2: Elasticities of unemployment probability w.r.t. benefit level for agent B

So should we expect the benefit scheme for agent B to be rising or falling? Whether the MH or the AS effect dominates depends on the choice of parameters. In the sequel, we discuss the influence of different parameters on the relative weight of the MH and the AS effect. As our *benchmark*, we use the parameterization of the previous subsection [$\alpha = 2$, $\theta_B = 0.007$, $\theta_G = 0.017$, $q = 0.5$].

The entitlement bound $\underline{V}$ For the good searcher, a decrease in the entitlement shifts the contract uniformly downwards (figures 6-9), as can be expected.
We can also see from figures 6 to 9 that for the bad searcher the MH effect prevails for high entitlement bounds $\underline{V}$ and thus his optimal contract falls as

well (figure 6). But as $\underline{V}$ is lowered the AS effect becomes more and more important and the benefit profile becomes hump shaped (figure 8) or even rises (figure 9).
It is interesting to note that at the same time, according to table 1 (compare also table 3 and 4), the information rent for agent G is increasing as $\underline{V}$ falls. We infer that the distortion of contract b away from the contract for B in a pure moral hazard environment (compare Corollary 2.17) increases as $\underline{V}$ is lowered. So broadly speaking we can say that the rent extraction/search efficiency trade-off is more severe for low than for high values of $\underline{V}$. How can this be explained?

Let us first briefly mention a mechanism that reinforces the MH effect as $\underline{V}$ increases but cannot fully account for the observed changes in benefit profiles. The elasticity of the probability of remaining unemployed $\sigma^i_\pi(z)$ with respect to the UI benefits are increasing in the benefit level [9] (compare Remark 2.9 and table 2). This means that at higher levels of utility a reduction of future benefits has a greater effect on the search effort and thus the reemployment chances of the unemployed agent. Hence the MH effect is more likely to matter at higher levels of utility, as we see in the figures. However, the tables show that the elasticities $\sigma^i_\pi(z)$ alter only slightly as the utility $\underline{V}$ is lowered. Thus this "elasticity effect" alone cannot explain the changes we observe in the result of the comparative static exercise.

The main force driving the result is the convexity of the cost function, i.e., the fact that the marginal costs of providing a certain utility increases in the level of utility. Put in terms of the agents' utility function we can say that a reduction of benefits hurts an agent disproportionately badly at a low level of utility. So in particular, a cost neutral shift of a benefit scheme from a flat to a bended profile comes at a greater relative loss of utility at low utility levels than at high utility levels. Loosely speaking, consumption smoothing is more important at low levels of utility than at high levels.

Now we look at contract b. Its utility $V^{b,B}$ for agent B is fixed at $\underline{V}$, and so we concentrate on its value $V^{b,G}$ for G. As we have pointed out above, the rent extraction/search efficiency trade-off is reflected by the AS and the MH effect: The slope of contract b is upward sloping in a pure AS and downward sloping in a pure MH environment. Assume for a moment that the elasticity of the probability of remaining unemployed $\sigma^i_\pi(z)$, that

[9] For reasons of computational ease we have calculated the elasticities w.r.t. to the benefit in the period subsequent to the effort only.

determines the importance of the MH effect, remains constant along shifts of the entitlements (actually we have seen that the increase of $\sigma_\pi^i(z)$ in z even *reinforces* the results). In other words: We assume for a moment that the search efficiency of B is equal across levels of utility.
Now according to what we have said about the preferences for flat versus bended contracts of agents with concave utility functions, we deduce that agent G would prefer to have his consumption smoothed more strongly at low than at high levels of benefits. Since his own contract g is falling, the (flatter) contract b becomes more attractive to him as the utility level is reduced. So in order to ensure that the agents are separated, in equilibrium the principal grants G a higher information rent (in relative terms, i.e. as a fraction of his entitlement) and accepts a stronger distortion of contract b from the pure moral hazard contract for B as $\underline{V}$ is lowered (recall that we hold the MH effect fixed for the moment). The distorted (upward sloping) contract b is then relatively less attractive for G than for B because of the Spence-Mirrlees property, as explained above.

The share of type B agents q Figures 10, 9 and 11 depict the optimal contracts at the entitlement bound $\underline{V} = -35$ (Certainty equivalent: 49.03%) for $q = 0.2$, 0.5 and 0.8 respectively. As the share of the bad searchers q increases, we see a clear shift of contract b dominated by the AS effect to one dominated by the MH effect. Furthermore, the contract for the good searcher is uniformly shifted upwards. Moreover, from table 1 we can see that his information rent rises.

We can explain these observations as follows: With a small proportion of type B agents, the principal concentrates on lowering the costs of contract g. He does so by pushing down the information rent of agent G, thus heavily distorting the value of contract b for agent G (recall that the incentive constraint for the good searcher binds, $V^{b,G} = V^{g,G}$). Since the value of contract b for agent B is fixed at $\underline{V}$, the value $V^{b,G}$ can only be lowered by steeply raising the benefit scheme b. This is true because of the Spence-Mirrlees property.

With an increasing proportion of type B agents, reducing the costs arising from their contract predominates the principal's problem and the issue of paying an information rent to type G agents looses importance. In order to keep the costs of contract b low, the principal prefers to avoid a large distortion of $V^{b,G}$ away from its first best and consequently accepts a higher

Information Rents

$\underline{V}$	Certainty Equivalent	$\theta_B = 0.004$	$\theta_B = 0.007$	$\theta_B = 0.010$
-35	49.03	20.6 %	18.1 %	14.2 %
-30	55.21	14.3 %	13.9 %	10.9 %
-25	61.76	9.7 %	10.5 %	8.2 %
-20	68.67	6.6 %	7.2 %	6.2 %

(rents expressed as percentage increase over minimal entitlement)

Table 3: Information Rents obtained by agent G

information rent paid to type G agents.

Economically speaking, with a high proportion of type G agents, the principal wants to curb the information rent, whereas with a high proportion of type B agents, the principal wants to ensure the search efficiency of type B agents.

Agent B's unemployment probability parameter θ_B The parameter θ_B determines the search capacity of agent B: The higher it is, the lower is his probability of remaining unemployed, given the same search effort a. We have looked at the case where the agents become more similar in their search technology, i.e. θ_B increases while θ_G is held fix. In our analysis, we have therefore kept agent G's search parameter at $\theta_G = 0.017$ and have looked at the cases of $\theta_B = 0.004$, $\theta_B = 0.007$ and $\theta_B = 0.010$. The latter correspond to an escape rate in autarky of 15.6%, 22.7 and 27.7% per month respectively. Figures 12, 8 and 13 show the optimal contracts at a utility level of $\underline{V} = -30$ (certainty equivalent: 55.21%). From these figures we see that the AS effect prevails for low values of θ_B, whereas for high values the MH effect is dominant.

For the good searcher an increase in θ_B shifts his benefit scheme downwards. At the same time an increase in θ_B implies for the bad searcher that the AS aspect looses importance and the MH becomes increasingly important. One reason for this can be seen from table 2. The elasticity of the probability of remaining unemployed is increased considerably as θ_B rises. As explained in the paragraph on $\underline{V}$, this means that the search incentives for agent B are enhanced.

Information Rents

Certainty Equivalent	$\alpha = 1.5$	$\alpha = 2$	$\alpha = 2.5$
49.03	27.3 %	18.1 %	10.4 %
55.21	21.5 %	13.9 %	8.2 %
61.76	17.1 %	10.5 %	5.8 %
68.67	13.7 %	7.2 %	3.7 %

(rents expressed as percentage increase over minimal entitlement)

Table 4: Information Rents obtained by agent G

Risk aversion $\frac{\alpha-1}{\alpha}$ The exponent in the cost function α determines the risk aversion of both agents, i.e., the coefficient of relative risk aversion is $\frac{\alpha-1}{\alpha}$. In this paragraph we look at the impact of a change in risk aversion on the optimal contracts and the information rents for type G.The results for the latter two are shown in table 4. As is apparent from the tables we compare levels of certainty equivalents of the unemployment entitlement bounds rather than levels of utility (which would of course make no sense). All other parameters are identical.

Figures 14, 6 and 15 show the optimal contracts at a replacement rate of 68.67%. In these figures we make two observations. First, the contract for type B, that is dominated by the MH effect at $\alpha = 1.5$, becomes *smoother* as α increases. Furthermore it slightly shifts upwards. Second, we can see that the falling contract for G is shifted upwards as α increases. At the same time his information rent falls (see table 4).

The economic explanation for these two phenomena is straightforward. In our model the coefficient of relative risk aversion is identical to the inverse of the intertemporal elasticity of substitution of consumption, i.e., more risk averse agents have a stronger preference for intertemporal consumption smoothing. Put the other way round: The marginal costs of guaranteeing a certain level of instantaneous utility rise as relative risk aversion rises, so - given that the principal guarantees a specified total expected lifetime utility - falling and rising benefit schemes become relatively more expensive as compared to flat ones. Therefore contracts tend to become flatter as α increases.

For agent G the contract shifts upwards because his contract is falling.

Therefore, as risk aversion increases, he needs to be compensated by higher benefits in order to achieve a given certainty equivalent. This also explains the fall in his information rent. Since his contract becomes disproportionately more expensive in terms of benefits as α increases (recall the convexity of the cost function), the UI agency wants to save information rent. This mechanism resembles the fall of the information rent for G as $\underline{V}$ increases (see above).

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A Proof of Theorem 2.8

Proof. It is clear that there is a solution to the principal's problem. So assume that the relative expected marginal cost is increasing in z_T and that it is greater for agent G than for agent B. By contradiction we prove that there is no pooling solution. So assume that the principal's problem is solved by one contract p, $\{z_1^P, ..., z_{T-1}^P, z_T^P\}$, for both agents, that generates a total expected utility of V_t^B and V_t^G in period t for agent B and G respectively.

We have a look at the "first best" solutions for the last two periods that generate the same utilities V_{T-1}^B and V_{T-1}^G as p. As "first best" solution,

we refer to the solution of the pure moral hazard problem as considered by Shavell and Weiss (1979), i.e. the problem of guaranteeing agent i a utility of V^i_{T-1} at lowest cost. This is stated as follows:

$$\begin{gathered} \min_{z^i_{T-1}, z^i_T} c(z^i_{T-1}) + \beta p_i(a^i) c(z^i_T) \\ s.t. \quad V^i_{T-1} = z^i_{T-1} - a^i + \beta[p(a^i) z^i_T + (1 - p(a^i)) u(w)] \\ 1 = \beta p_i'(a^i)[z^i_T - u(w)] \end{gathered}$$

This is the two period cost minimization problem (the principal's problem in this framework) in the case of agent i, subject to the promise keeping constraint and the first order condition of the agent's problem, determining the choice of effort a^i. Plugging the entitlement constraint into the objective function and making use of the envelop theorem, we calculate the following first order conditions for the principal with respect to z_T (we abbreviate $p_i = p_i(a^i)$):

$$c'(z^B_{T-1}) = -\frac{(p_B')^3}{p_B p_B''} c(z^B_T) + c'(z^B_T) \tag{23}$$

$$c'(z^G_{T-1}) = -\frac{(p_G')^3}{p_G p_G''} c(z^B_T) + c'(z^B_T) \tag{24}$$

The factor of the cost function on the right-hand side is

$$-\frac{(p_i')^3}{p_i p_i''} = \frac{1}{\pi_i(z_T)} \frac{\partial \pi_i(z_T)}{\partial z_T},$$

and so we see that the RHS is identical to the relative expected marginal cost. By condition 2.6 the RHS is greater for agent G than for agent B. [10] We may therefore deduce that the Shavell-Weiss contract of agent B is flatter than its counterpart for agent G, where we define "flatter" in the following sense:

$$\frac{z^G_{T-1}}{z^G_T} > \frac{z^B_{T-1}}{z^B_T}.$$

[10] Note that we could weaken condition 2.6: To ensure that the RHS of G is higher than the RHS of B it is sufficient to assume that the relative marginal probability of remaining unemployed $\frac{1}{\pi_i(z)} \frac{\partial \pi_i(z)}{\partial z}$ is higher for agent G than for agent B.

In the following we will dicuss the last two periods of the pooling contract only and show that it cannot be optimal to offer it to both agents. We will refer to the first best solutions as Shavell-Weiss (SW) contracts.

First, suppose that the pooling contract p is flatter than the SW contract of agent G. Then the principal can offer p and a second contract g' that is identical to contract p except for the last two periods, where z^P_{T-1} and z^P_T are substituted by z^G_{T-1} and z^G_{T-1} from the SW contract. This is incentive compatible: Agent G is indifferent between p and g' by construction. Suppose that agent B (weakly) preferred g' over p. Then for period 1 to $T-2$, he can excert the same effort a^g_1 to a^g_{T-2} (i.e. that he chooses in the case of contract g') when facing contract p, and thus the stochastically discounted utility from the benefits z_1 to z_{T-2} is identical for both contracts. In the last two periods, in contrast, agent B -excerting effort optimally- gains a higher utility from the flatter contract p than from contract g' because of the Spence-Mirrlees property (cf. Lemma 2.5). So agent B cannot prefer g' over p. Offering the two contracts p and g' is also cheaper for the principal, because g' is the (unique) cost-optimizing contract for agent G during the last two periods. Contradiction.

Second, suppose that the pooling contract p is identical to or steeper than the SW contract of agent G. The principal then offers p and a second contract b' that is identical to contract p with z^P_{T-1} and z^P_T substituted by z^B_{T-1} and z^B_T from the SW contract. Since the SW contract of agent B is flatter than the SW contract of agent G, as we have seen, we can infer the contradiction in the same way as in the first case. ■

B Proof of Proposition 2.12

Proof. In order to simplify the proof we introduce a normalization: The utility from consuming the wage w is set to zero. Thus, all W_t become zero, too, and the entitlement utilities of the unemployed agents take non-positive values. Note that the lower bounds for the entitlements, stemming from the lower bound on the benefit utility $\underline{z}$, thus shift downward each period along the backward induction.

First, we look at the agents' problem. Remember it takes the form

$$V^i_t = \max_a z_t - a + \beta[p_i(a)V^i_{t+1} + (1 - p_i(a))W_{t+1}].$$

Given our normalization we obtain the following first order condition at an interior solution:

$$p_i'(a_t^i) = \frac{1}{\beta V_{t+1}^i} \tag{25}$$

By the Inada condition in Condition 2.3 we assure that the interior solution always applies.

The Case of $\mathbf{t = T - 1}$ We start with point 3 of the proposition, i.e. the case of $\Gamma_t(V_t^B, V_t^G)$ with $t = T - 1$. Mathematically speaking, the next-to-last period is different from the previous ones in that there is an additional contraint on the choice variables V_T^i: The boundary conditions 12, 13, namely $V_T^B = V_T^G = z_T$. This is the very reason why, given the pair of state variables (V_{T-1}^B, V_{T-1}^G), there is only one choice left for the principal. First, let's look at the Law of Motion (LOM) for the state variables V_{T-1}^B and V_{T-1}^G:

$$\begin{aligned} z_{T-1} - a_{T-1}^B + \beta p_B(a_{T-1}^B) V_T^B &= V_{T-1}^B, \\ z_{T-1} - a_{T-1}^G + \beta p_G(a_{T-1}^G) V_T^G &= V_{T-1}^G, \end{aligned}$$

where we will drop the time index from the effort variables a_{T-1}^i. In the following, we will denote the difference between the entitlements of the agents by

$$\Delta_t := V_t^G - V_t^B. \tag{26}$$

With this new notation and remembering both our normalization and $V_T^i = z_T$, we solve the LOM for z_{T-1}, equalize both equations and solve for Δ_{T-1}:

$$\Delta_{T-1} = a^B - a^G + \beta p_G(a^G) z_T - \beta p_B(a^B) z_T \tag{27}$$

We want to further simplify equation 27. In the next to last period, the first order condition of the agents' problem 25 takes the following form

$$p_B'(a^B) = p_G'(a^G) = \frac{1}{\beta z_T}. \tag{28}$$

Again by Condition 2.3, the p_i' are strictly increasing functions

$$p_i' : (0, \infty) \longrightarrow (-\infty, 0).$$

Remark B.1 *Given Condition 2.7, the principal will never promise an entitlement above W_t ($= 0$ under our normalization), since at W_t the agents stop searching (i.e. $a_t^i = 0$) and their probability of remaining unemployed becomes $p_i(a_t^i) = 1$.*

Thus in particular $V_T^i = z_T < 0$.

From this we deduce that the p_i' are one-to-one and onto. Therefore the following function $\gamma(a^G)$ is well defined:

$$\gamma(a^G) := (p_B')^{-1} \circ p_G'(a^G).$$

Now we have everything at hand to define Δ_{T-1} as a function of a^G:

$$\Delta_{T-1}(a^G) = \gamma(a^G) - a^G + \frac{p_G(a^G) - p_B(\gamma(a^G))}{p_G'(a^G)} \tag{29}$$

In order to show point 3 of Proposition 2.12, we have to show that $\Delta_{T-1}(.)$ is invertible.

We do so by proving

$$\Delta_{T-1}'(a^G) > 0. \tag{30}$$

Using the agents' first order condition 25 and

$$\gamma'(a^G) = \frac{p_G''(a^G)}{p_B''(\gamma(a^G))}$$

we calculate

$$\Delta_{T-1}'(a^G) = [p_B(\gamma(a^G)) - p_G(a^G)] \frac{p_G''(a^G)}{(p_G'(a^G))^2}. \tag{31}$$

By Condition 2.3 we know that $p_G''(.) > 0$, and since $p_B(\gamma(a^G)) > p_G(a^G)$ by Condition 2.4 assertion 30 follows.

Finally we observe that -again by Condition 2.3-

$$\lim_{a^G \to 0} \Delta_{T-1}(a^G) = 0. \tag{32}$$

Together with 30 we deduce that as agent B's entitlement V^B_{T-1} approaches agent G's one V^G_{T-1}, the effort of the agent G a^G (as well as the effort of agent B) go to zero. Because of 25 this means that the benefit for the last period z_T has to converge to zero, i.e. the wage consumption utility.

Summarizing our results so far, we can state the following: Given entitlements V^B_{T-1} and V^G_{T-1} such that $\Delta_{T-1} \geq 0$, we can find a unique corresponding choice of effort by agent G a^G (for the time being, we neglect the lower bound $\underline{z}$ on the benefits z_t). From this we can calculate -uniquely- the choice of effort by agent one a^B and the benefit for the last period z_T from equation 25, and the benefit of the next to last period z_{T-1} from LOM. All these functions are differentiable. As Δ_{T-1} goes to zero, the benefit of the last period z_T goes to zero, i.e. the cost of the benefit converges to the wage.

We finally have to look at the set of feasible entitlements V^B_{T-1} and V^G_{T-1}. In the first case of the theorem (without a lower bound $\underline{z}$ on z_t), nothing remains to be shown. In the second case, there is a natural lower bound $\underline{V}^G_{T-1}$, namely the stochastically discounted sum of the bounds on z_{T-1} and z_T. Given $V^G_{T-1} \in [\underline{V}^G_{T-1}, 0]$ we now have to prove that there is a lower and an upper bound $\underline{V}^B_{T-1}(V^G_{T-1})$ and $\overline{V}^B_{T-1}(V^G_{T-1})$ on the corresponding feasible V^B_{T-1}. Because of 25 the lower bound on z_T translates into an upper bound $\overline{a}^G$ on the corresponding choices of effort of agent G. It is attained with equality. By 26 and 30 we find the lower bound

$$\underline{V}^B_{T-1}(V^G_{T-1}) = V^G_{T-1} - \Delta_{T-1}(\overline{a}^G).$$

As for the upper bound $\overline{V}^B_{T-1}(V^G_{T-1})$, one can see intuitively that V^B_{T-1} is bounded by V^G_{T-1} (for a rigorous argument, see point 1 in the proof of 2.16). However, V^B_{T-1} does not necessarily attain this bound, because of an additional constraint: $z_{T-1} \geq \underline{z}$. From the LOM and 25 we know

$$z_{T-1} = V^G_{T-1} + a^G - \frac{p_i(a^G)}{p_i'(a^G)}$$

The right hand side is increasing in a^G, so a lower bound on z_{T-1} implies a lower bound on the effort of the second type, $\underline{a}^G$ (note that because of

our normalization, the reference points for each period have been shifted downwards). Because of 30 a lower bound on Δ_{T-1} ensues. Given V_{T-1}^G, we thus find the upper bound on V_{T-1}^B:

$$\overline{V}_{T-1}^B(V_{T-1}^G) = V_{T-1}^G - \Delta_{T-1}(\underline{a}^G).$$

We see that V_{T-1}^B attains V_{T-1}^G only if the lower bound $\underline{a}^G$ becomes zero (the smallest possible effort). Since $\Delta_{T-1}(.)$ is an increasing function, we see that all values $V_{T-1}^B \in [\underline{V}_{T-1}^B, \overline{V}_{t-1}^B]$ are attainable as long as $\overline{a}^G > \underline{a}^G$. This must be the case for $V_{T-1}^G \geq \underline{V}_{T-1}^G$, since then there are corresponding benefit values z_{T-1}, z_T such that $z_i \geq \underline{z}$. Finally, because of the Theorem of the Maximum both $\underline{a}_{T-1}$ and $\overline{a}_{T-1}$ depend continuously on V_{T-1}^G and since Δ_{T-1} is a smooth function, the lower and the upper bound $\underline{V}_{T-1}^B$ and $\overline{V}_{T-1}^B$ are continuous functions of V_{T-1}^G.

So for period $T-1$, we have shown that the set of feasible values takes the form stated in the theorem. Note in particular that this set is compact and connected.

The Case of $\mathbf{t \leq T-2}$ In this paragraph, we prove the assertion 2 of the proposition by backward induction; point 1 is embedded in what we will show. As we have stated above, the crucial difference between the next to last period and the previous ones is the boundary condition of the last period 12 and 13. Before, for every feasible pair of state variables (V_{T-1}^B, V_{T-1}^G) -or more precisely, for the difference of these state variables Δ_{T-1}- there existed one corresponding choice of effort a_{T-1}^G that determined all choice variables (z_{T-1}, V_T^B, V_T^G). As we will see below, now to each pair of feasible state variables (V_{t-1}^B, V_{t-1}^G) (again, more precisely, to the difference of these state variables Δ_{t-1}) there corresponds a line of possible choices of effort a_{t-1}^G that parameterizes a compact and connected path of choice variables $(z_{t-1}(.), V_t^B(.), V_t^G(.))$.

We have a look at the LOM once more. With the help of the agents' first order condition we transform it into

$$
\begin{aligned}
z_t - a^B + \frac{p_B(a^B)}{p'_B(a^B)} &= V_t^B, \\
z_t - a^G + \frac{p_G(a^G)}{p'_G(a^G)} &= V_t^G,
\end{aligned}
$$

where again we have dropped the time index from a_t^i. This inspires the definition of the following functions ($i = 1, 2$)

$$
f_i(a^i) = a^i - \frac{p_i(a^i)}{p'_i(a^i)}.
$$

From the LOM we can now derive a necessary equation for the choice variables (as represented by the a^is, replacing the V_t^is) to hold:

$$
\Delta_t + f_G(a^G) = f_B(a^B), \tag{33}
$$

where we have used definition 26.

We have a closer look now at f_i. From

$$
f'_i = \frac{p_i p''_i}{(p'_i)^2} > 0 \tag{34}
$$

we can see that it is a strictly increasing function (bearing in mind Condition 2.3). Moreover we calculate

$$
\begin{aligned}
\lim_{a^i \to 0} f_i(a^i) &= 0, &(35)\\
\lim_{a^i \to \infty} f_i(a^i) &= \infty. &(36)
\end{aligned}
$$

Now note that there is a natural lower bound $\underline{V}_t^G$ on each V_t^G, namely the stochastically discounted sum of the $z_{\hat{t}}$s (where $\hat{t} = t, ..., T$). In the case of $T - 1$, we have shown that the set of jointly feasible values V_{T-1}^B, V_{T-1}^G takes the form stated in the theorem. So let $\Gamma_t(V_t^B, V_t^G)$ be non-empty and take the form of a path in the space $(z_t, V_{t+1}^B, V_{t+1}^G)$ for $V_t^B \in [\underline{V}_t^B(V_t^G), \overline{V}_t^B(V_t^G)]$ with $V_t^G \geq \underline{V}_t^G$. We have to show first that then $\Gamma_{t-1}(V_{t-1}^B, V_{t-1}^G)$ is non-empty for $V_{t-1}^B \in [\underline{V}_{t-1}^B(V_t^G), \overline{V}_{t-1}^B(V_t^G)]$ for some continuous functions $\underline{V}_{t-1}^B$, $\overline{V}_{t-1}^B$ when

$V_{t-1}^G \geq \underline{V}_{t-1}^G$ and takes the form of a path in (z_{t-1}, V_t^B, V_t^G).

Put differently, we have to ask for which pairs (V_{t-1}^B, V_{t-1}^G) there are choice variables (z_{t-1}, V_t^B, V_t^G) that are jointly feasible. By the agents' first order condition 25 we can replace V_t^B and V_t^G by the corresponding choices of effort a_{t-1}^B and a_{t-1}^G (we will drop the time index in the sequel). The effort choices a^B and a^G have to satisfy equation 33. Since $\Delta_{t-1} \geq 0$ and by 34, 35 and 36, for all $a^G \geq 0$ we can find a corresponding $a^B \geq 0$. By the LOM, we can furthermore determine z_{t-1} once a^G is given. Thus the number of choice variables $(z_{t-1}(a^G), V_t^B(a^G), V_t^G(a^G))$ is reduced to the "choice" variable a^G. All functions are combinations of differentiable functions and thus differentiable. We will call the projection of the triple of choice variables into the two-dimensional space $(V_t^B(a^G), V_t^G(a^G))$ the curve $\phi_{\Delta_{t-1}}$ parameterized in a^G.

We have reduced the choice problem to one variable, but which a^G correspond to *feasible* triples (z_{t-1}, V_t^B, V_t^G)? First we look at the constraint $z_{t-1} \geq \underline{z}$. As in the preceding paragraph, by the LOM

$$z_{t-1} = V_{t-1}^G + f_G(a^G)$$

it translates into a constraint

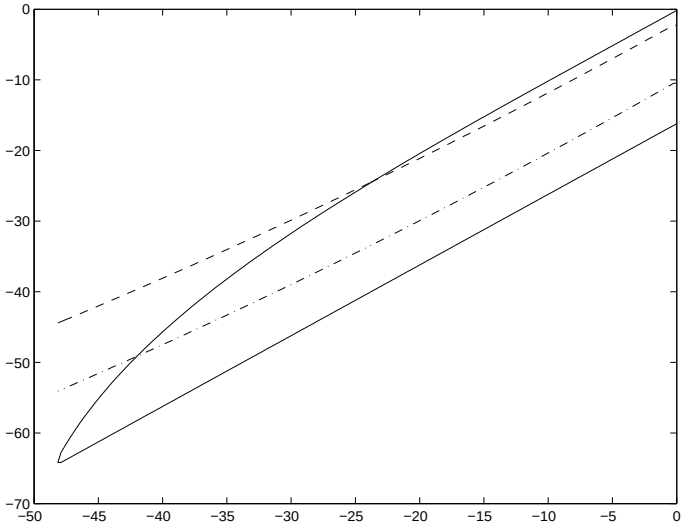
$$a^G \geq \underline{a}^G = \begin{cases} f_G^{-1}(\underline{z} - V_{t-1}^G) & : \quad V_{t-1}^G \leq \underline{z} \\ 0 & : \quad V_{t-1}^G > \underline{z} \end{cases} \tag{37}$$

Second we have to ask: Which of the pairs of entitlements $(V_t^B(a^G), V_t^G(a^G))$ are feasible? Well, those for which $\Gamma_t(V_t^B, V_t^G)$ is non-empty. In other words: Given V_{t-1}^B and V_{t-1}^G, the set of feasible choices is the intersection of the curve $\phi_{\Delta_{t-1}}$ defined by 33, parameterized in a^G with $a^G \geq \underline{a}^G$, and the set of (V_t^B, V_t^G) with $\Gamma_t(V_t^B, V_t^G) \neq \emptyset$. Figure B depicts the intersection for the case of period 4 of 12 in an example from our simulation. The solid lines represent the bounds $\underline{V}_4^B(V_4^G)$ and $\overline{V}_4^B(V_4^G)$, the dotted and the dashed line are curves ϕ_{Δ_3} with two different values for Δ_3.

Two things remain to be shown:

1. we have to show that the set of (V_{t-1}^B, V_{t-1}^G), for which the intersection is non-empty, takes itself the form of a set bounded by functions $\underline{V}_{t-1}^B$ and $\overline{V}_{t-1}^B$.

Figure 1: Set of jointly feasible entitlements in period 4 of 12



entitlements of type B:type G, $V_4^B : V_4^G$

2. we have to show that if the curve 33 intersects the set of feasible values (V_t^B, V_t^G), it cuts the bounds at most twice, so that the set of feasible choices is connected.

To show the *first assertion*, we look more closely at the family of curves

$$\phi_{\Delta_{t-1}} : a^G \longrightarrow [\phi_B(a^G), \phi_G(a^G)]_{\Delta_{t-1}},$$

where

$$\phi_B(a^G) = \frac{1}{\beta p'_B(f_B^{-1}(\Delta_{t-1} + f_G(a^G)))}, \tag{38}$$

$$\phi_G(a^G) = \frac{1}{\beta p'_G(a^G)}. \tag{39}$$

Since ϕ_G is one-to-one, the curves can also be understood as a function

$$V_t^B = \phi_{\Delta_{t-1}}(V_t^G).$$

We now want to prove the following: The curves are "decreasing" in Δ_{t-1}, i.e.

$$\Delta_{t-1} < \Delta^*_{t-1} \Rightarrow \phi_{\Delta_{t-1}}(V^G_t) > \phi_{\Delta^*_{t-1}}(V^G_t). \tag{40}$$

We do so by calculating the derivative

$$\partial_{\Delta_{t-1}}(\phi_{\Delta_{t-1}})(V^G) = -\frac{1}{(\beta p_1'(a^B))^2} * \beta p_1''(a^B) * \frac{1}{f'_{\theta_1}(\Delta_{t-1} + f_{\theta_2}(a^G))} < 0,$$

which is negative because of 34 and Condition 2.3. The property of $\phi_{\Delta_{t-1}}$ is reflected by its dotted and the dashed representation in Figure B.

By the induction hypothesis, the set of (V^B_t, V^G_t) for which $\Gamma_t(V^B_t, V^G_t) \neq \emptyset$ is compact and connected. Thus we deduce from 40 that there are $\overline{\Delta}_{t-1}$ and $\underline{\Delta}_{t-1}$ so that the curves $\phi_{\Delta_{t-1}}$ intersect the set for $\underline{\Delta}_{t-1} \leq \Delta_{t-1} \leq \overline{\Delta}_{t-1}$ and do not intersect for $\Delta_{t-1} < \underline{\Delta}_{t-1}$ and $\Delta_{t-1} > \overline{\Delta}_{t-1}$ (of course $\underline{\Delta}_{t-1}$ could be smaller than zero, the lower limit for Δ_{t-1}). From this ensues the existence of two functions $\underline{V}^B_{t-1}(V^G_{t-1})$ and $\overline{V}^B_{t-1}(V^G_{t-1})$ limiting the set of feasible pairs (V^B_{t-1}, V^G_{t-1}).

To show the *second assertion*, we have to look more closely at the shape of the curve $\phi_{\Delta_{t-1}}$ as well as the limiting functions $\underline{V}^B_{t-1}(.)$ and $\overline{V}^B_{t-1}(.)$. First, we prove that the derivative of $\phi_{\Delta_{t-1}}$ is *smaller than one*. We do so by showing that

$$D(.) \circ \phi_G^{-1}(V^G_{t-1}) := (\phi_G(.) - \phi_B(.)) \circ \phi_G^{-1}(V^G_{t-1})$$

is increasing in V^G_{t-1}, i.e. the derivative of $\phi_{\Delta_{t-1}}$ is below the one of the diagonal:

$$\partial_{V^G_{t-1}} D(\phi_G^{-1}(V^G_{t-1})) = 1 - \partial_{V^G_{t-1}} \phi_B(\phi_G^{-1}(V^G_{t-1})) > 0$$

Since we know that

$$\partial_{V^G_{t-1}}(\phi_G^{-1})(V^G_{t-1}) < 0$$

by 25, it is sufficient to show that

$$D'(a^G) < 0.$$

Using $a^B := f_B^{-1}(\Delta_{t-1} + f_G(a^G))$ we calculate

$$
\begin{aligned}
D'(a^G) &= -\frac{p''_G(a^G)}{\beta(p'_G(a^G))^2} + \frac{p''_B(a^B)}{\beta(p'_B(a^B))^2} * \frac{f'_G(a^G)}{f'_B(a^B)} \\
&= -\frac{p''_G(a^G)}{\beta(p'_G(a^G))^2} + \frac{p''_B(a^B)}{\beta(p'_B(a^B))^2} * \frac{p_G(a^G)p''_G(a^G)}{(p'_G(a^G))^2} * \frac{(p'_B(a^B))^2}{p_B(a^B)p''_B(a^B)} \\
&= \left(\frac{p_G(a^G)}{p_B(a^B)} - 1\right) * \frac{p''_G(a^G)}{\beta(p'_G(a^G))^2}.
\end{aligned}
$$

The last expression is negative by Condition 2.3 and 2.4.

Now, the second assertion follows if we can show that the derivative of the boundary functions $\underline{V}^B(.)$ and $\overline{V}^B(.)$ is *greater than one*, for then $\phi_{\Delta_{t-1}}$ crosses them at most once. So by induction hypothesis, assume that $\underline{V}^B_t(.)$ and $\overline{V}^B_t(.)$ have a derivative greater or equal than one (note that this is certainly true for the case of $t = T - 1$).

According to what we have shown above, there are $\overline{\Delta}_{t-1}$ and $\underline{\Delta}_{t-1}$ that limit the set of values $\Delta_{t-1} = V^G_{t-1} - V^B_{t-1}$ for which $\phi_{\Delta_{t-1}}$ intersects the set of feasible (V^B_t, V^G_t). From this we might be tempted to deduce immediately both $\underline{V}^B_{t-1}$ and $\overline{V}^B_{t-1}$ must be linear functions with derivative one, for apparently the limits only depend on the difference $\Delta_{t-1} = V^G_{t-1} - V^B_{t-1}$. Note, however, that the starting point $\underline{a}^G$ (see equation 37) for each curve $\phi_{\Delta_{t-1}}$ is shifting upwards as V^G_{t-1} is falling. Thus since by induction hypothesis $\underline{V}^B_t$ and $\overline{V}^B_t$ are more steeply increasing than the $\phi_{\Delta_{t-1}}$, we may deduce that

1. indeed $\underline{V}^B_{t-1}(.)$ is linear with derivative one because the $\phi_{\Delta_{t-1}}$s cross the function $\underline{V}^B_t(.)$ at the lower bound $\underline{V}^G_t$ at a high value for a^G.

2. for lower values of V^G_{t-1} the smallest Δ_{t-1} for which $\phi_{\Delta_{t-1}}$ intersects the set of feasible values (V^B_t, V^G_t) is below the one that would have been obtained with $\underline{a}^G$ fixed. Since the latter one would have corresponded to a linear upper bound $\overline{V}^B_t(.)$ with derivative one, we conclude that $\overline{V}^B_t(.)$ has to rise more steeply than this, i.e. that its derivative is greater than one.

Thus by induction, we have shown that $\phi_{\Delta_{t-1}}$ and $\underline{V}^B_{t-1}(.)$ and $\overline{V}^B_{t-1}(.)$ cross only once and the second assertion on the form of the correspondence

Γ_{t-1} ensues. This concludes the proof of Proposition 2.12. ■

C Proof of Proposition 2.16

Proof. To prove the proposition, we have to show that at the solution

1. the entitlement constraint 22 of type G is slack,
2. the entitlement constraint 21 of type B is binding,
3. the incentive constraint 20 of type G is binding.

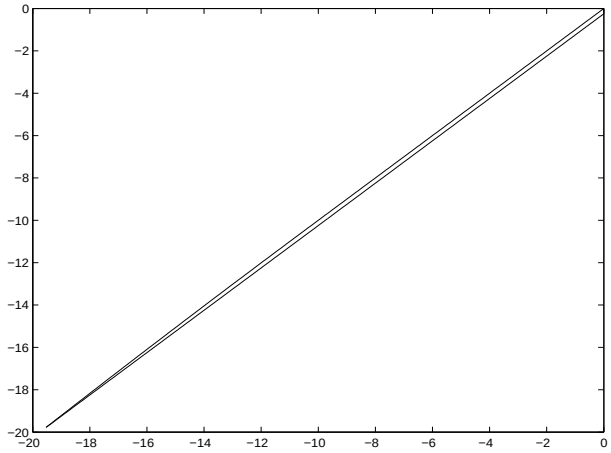
Beginning with 1 we show that for all contracts $V^G > V^B$. The assertion then follows by $V^{b,G} > V^{b,B}$ and agent B's entitlement constraint 21.

So let's consider a feasible UI contract. Given any set of effort choices $(a_1^B, a_2^B, ..., a_{T-1}^B)$ of agent B, the same set of choices would yield a higher value of total expected lifetime utility for agent G than for agent B, $V^G(\vec{a}^B) > V^B(\vec{a}^B)$. This is the case because firstly (total) utility when employed is higher than (total) utility when unemployed (compare the remarks 2.13 and B.1) and secondly by condition 2.4, first part, $p_B(a^B) > p_G(a^B)$ for any $a^B > 0$. Thus, in particular, at the optimum $V^G > V^B$.

We now prove point 2 by contraction. Suppose that for the solution contracts b, $(z_1^b, ..., z_T^b)$, and g, $(z_1^g, ..., z_T^g)$, the constraint 21 did not bind. For sufficiently high $\underline{V}$ we may assume that all $z_t^i > \underline{z}$ for all t, in particular for $t = 1$. But then create new contracts b' and g' by replacing z_1^i by $z_1^i - \epsilon$ $(i = b, g)$ for some $\epsilon > 0$ with $z_1^i - \epsilon > \underline{z}$. These contracts are certainly feasible. They are also incentive compatible, since the entitlements $V_1^{i,j}$ are reduced by the same amount. However, the new contracts b' and g' are less costly for the principal, since the costfunction $c(.)$ is strictly increasing. Contradiction.

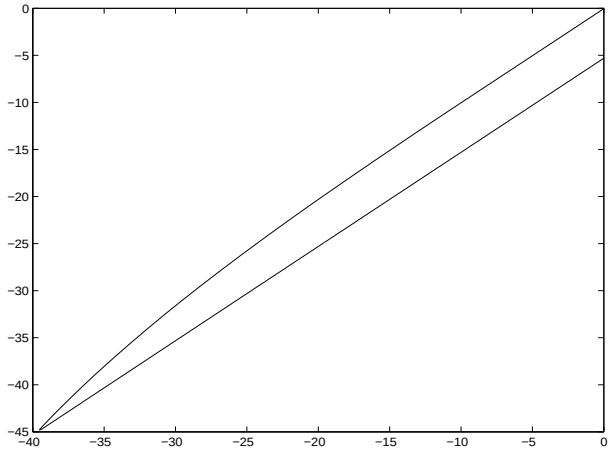
Point 3 is proved by an argument simular to the one in point 2. ■

Figure 2: Set of jointly feasible entitlements in period 11 of 12



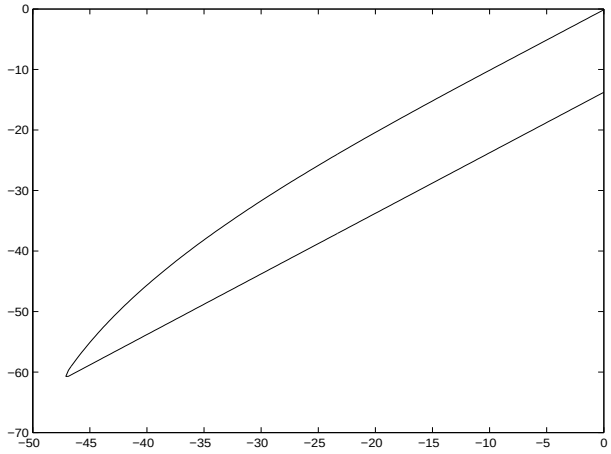
entitlements of type B:type G

Figure 3: Set of jointly feasible entitlements in period 8 of 12



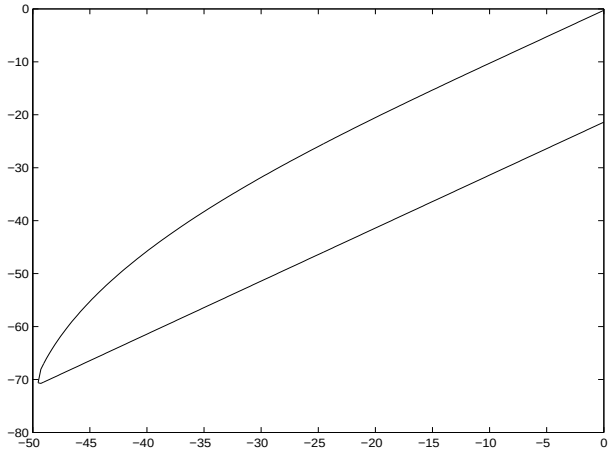
entitlements of type B:type G

Figure 4: Set of jointly feasible entitlements in period 5 of 12



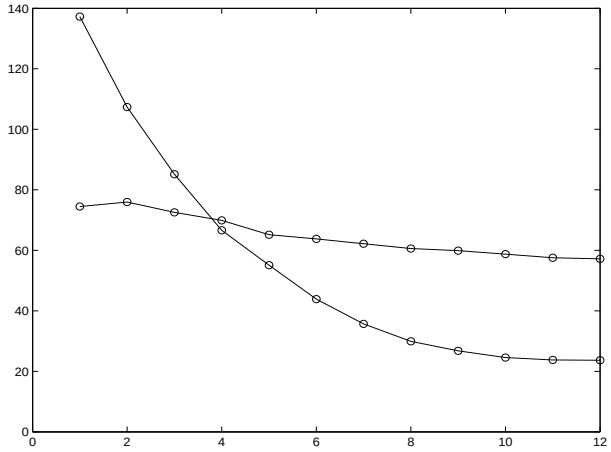
entitlements of type B:type G

Figure 5: Set of jointly feasible entitlements in period 1 of 12



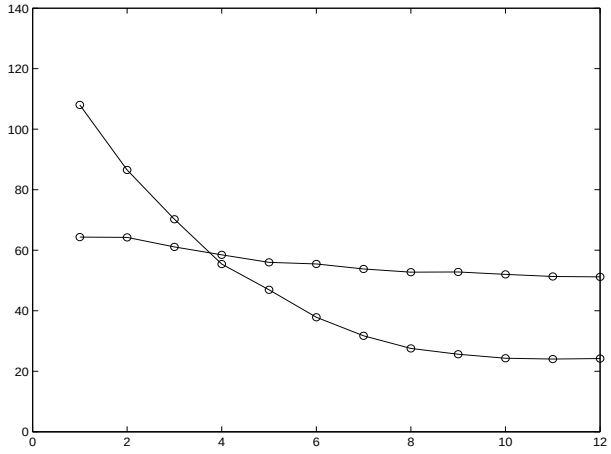
entitlements of type B:type G

Figure 6: UI contracts



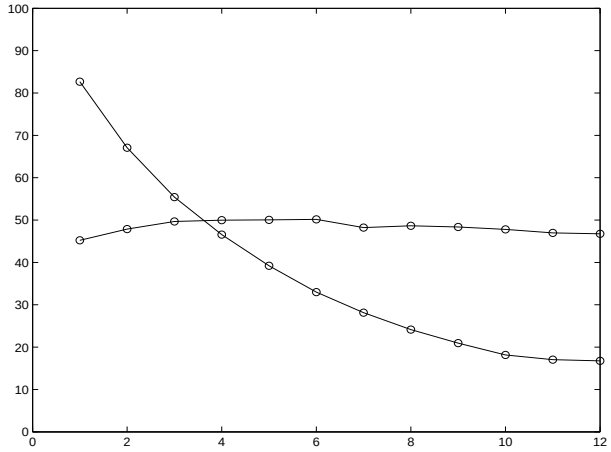
replacement rate/period
Entitlement bound $\underline{V} = -20$
$\cong$ Certainty Equivalent of 68.67% of the wage per period

Figure 7: UI contracts



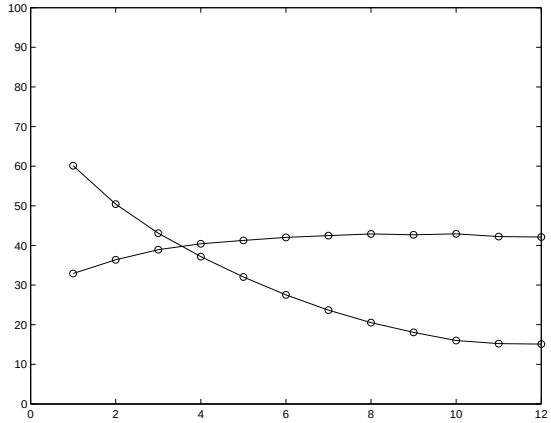
replacement rate/period
Entitlement bound $\underline{V} = -25$
$\cong$ Certainty Equivalent of 61.76% of the wage per period

Figure 8: UI contracts



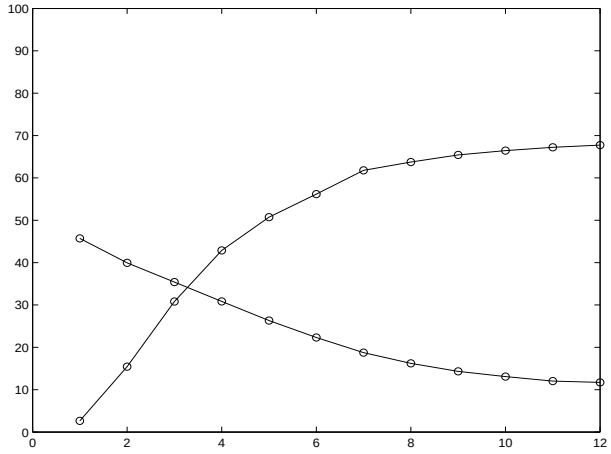
replacement rate/period
Entitlement bound $\underline{V} = -30$
$\cong$ Certainty Equivalent of 55.21% of the wage per period

Figure 9: UI contracts



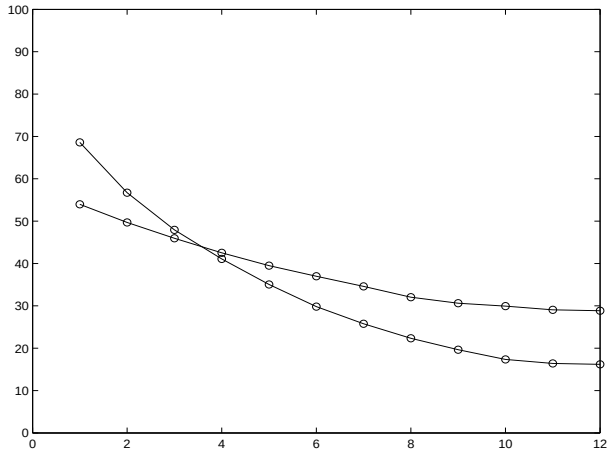
replacement rate/period
Entitlement bound $\underline{V} = -35$
$\cong$ Certainty Equivalent of 49.03% of the wage per period

Figure 10: UI contracts with $q = 0.2$



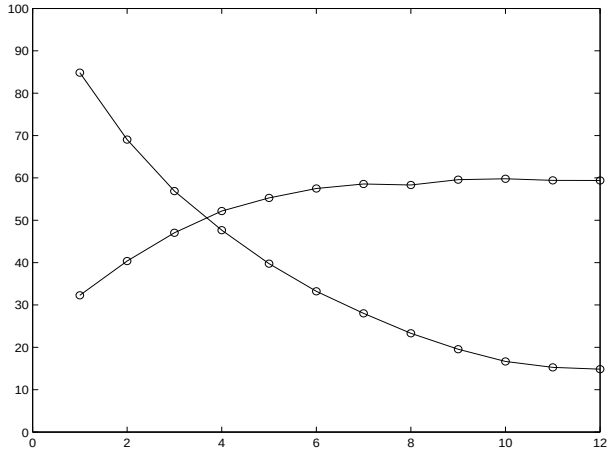
replacement rate/period
Entitlement bound $\underline{V} = -35$
$\cong$ Certainty Equivalent of 49.03% of the wage per period

Figure 11: UI contracts with $q = 0.8$



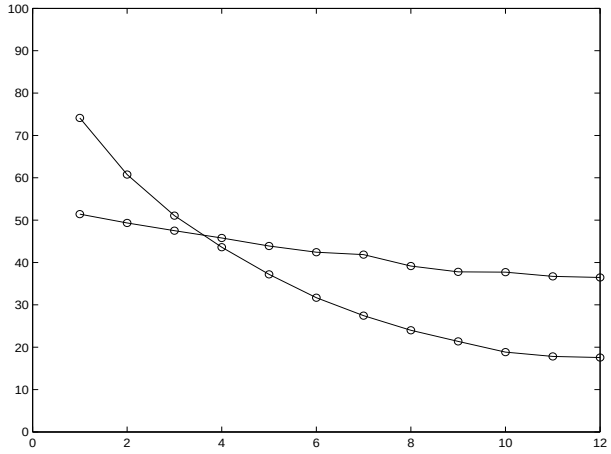
replacement rate/period
Entitlement bound $\underline{V} = -35$
$\cong$ Certainty Equivalent of 49.03% of the wage per period

Figure 12: UI contracts with $\theta_B = 0.004$, $\theta_G = 0.017$



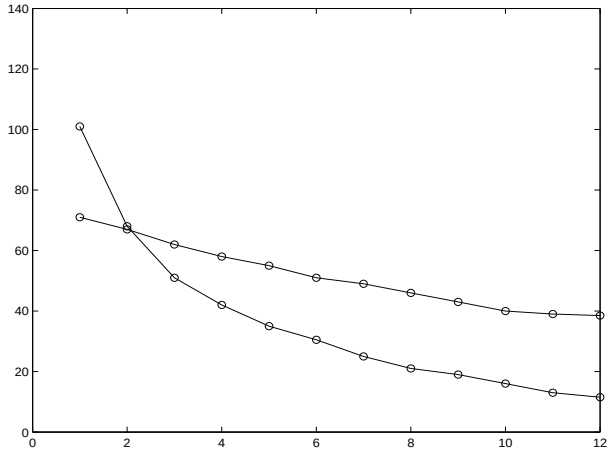
replacement rate/period
Entitlement bound $\underline{V} = -30$
$\cong$ Certainty Equivalent of 55.21% of the wage per period

Figure 13: UI contracts with $\theta_B = 0.010$, $\theta_G = 0.017$



replacement rate/period
Entitlement bound $\underline{V} = -30$
$\cong$ Certainty Equivalent of 55.21% of the wage per period

Figure 14: UI contracts with $\alpha = 1.5$

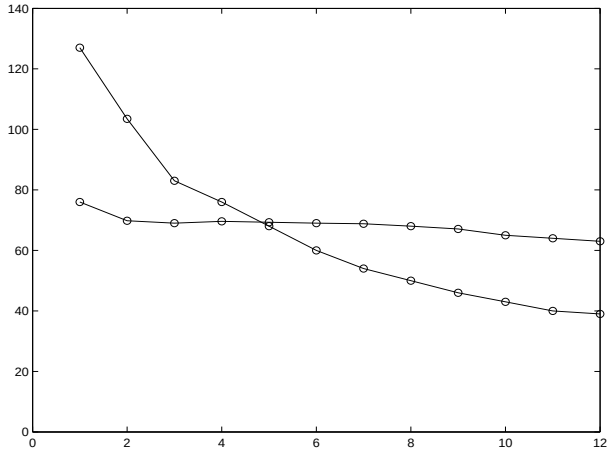


replacement rate/period

Entitlement bound $\underline{V} = -55.7$

$\cong$ Certainty Equivalent of 68.67% of the wage per period

Figure 15: UI contracts with $\alpha = 2.5$



replacement rate/period

Entitlement bound $\underline{V} = -10.3$

$\cong$ Certainty Equivalent of 68.67% of the wage per period

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