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**Article**

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# A methodology for stochastic inventory modelling with ARMA triangular distribution for new products

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**Abstarct:** This paper proposes a stochastic inventory policy of continuous review with random demand described with temporal dependence through an autoregressive moving average (ARMA) model with explicative variables, of usefulness in new products without a history of demand data, assuming a triangular distribution. Optimization of the cost function related to the inventory model is obtained considering the expected value and variance marginal stationary of the demand per unit time and stochastic programming. The proposed policy is exemplified with real-world demand data from a Chilean hospital, where the demand of products (drugs) are correlated with other products and autocorrelated. The proposed methodology shows a useful tool for administrators who must decide optimal batch sizes and their reorder points when there is a low availability of demand data and is known to have a temporal structure.

**Subjects: Economics, Finance, Business & Industry; Business, Management and Accounting; Production, Operations & Information Management; Industry & Industrial Studies; Service Industries**

**Keywords: ARMA model; continuous review; triangular distribution**

## 1. Introduction

Supply systems and policy inventories reduce the vulnerability of the supply chain management in enterprises. This reduction is achieved by optimizing inventory levels to meet the demand of products of the companies satisfying customers (Hillier & Lieberman, 2005).

Stochastic inventory models consider the demand for products as a random variable (RV) described by a continuous or discrete distribution. To optimize total costs (TC) of inventory, the cost of placing an order to the supplier has three components: (a) a cost that is independent of the lot size,

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## ABOUT THE AUTHOR

Fernando Rojas is Chemical-pharmaceutical of University of Valparaiso, Chile; Master in Management Science at the University Adolfo Ibañez, Chile, and has a PhD in Management Science candidate at the same university. He currently serves as an adjunct professor at the School of Nutrition and Dietetics, that belongs to the Faculty of Pharmacy, University of Valparaiso, Chile, and is a research associate to the Micro-Bio Innovation Center from the same school. His research interest is in operations research linked to optimal supply decisions, especially in the area of food service.

## PUBLIC INTEREST STATEMENT

The proposed methodology is useful as a tool for administrators who must decide optimal batch sizes and their reorder points when there is a low availability of demand and is known to have a temporal dependence. We propose a methodology based on inventory models of continuous review to be supplied from a single distributor, of utility for new products, assuming a triangular distribution and describing the dependence over time of the demand.

(b) a cost that depends on the quantity of products in the lot, and (c) a cost of shortage due to unsatisfied demand (Gjerdrum, Samsatli, Shah, & Papageorgiou, 2005). Once the inventory model indicators have been defined and distributional assumptions for demand per unit time (DPUT) and for demand during lead-time (LT), in short LTD, have been established, the expected value of the objective function based on the TC of the inventory must be optimized (Namit & Chen, 1999).

The triangular distribution (TRI) is manageable and is known to be useful when a distribution cannot be determined, because the data are difficult to obtain or costly to collect (Glickman & Xu, 2008). This distribution can be used to involve managers in the analytical process by considering their subjective estimates of the minimum, most probable and maximum values. According to Johnson (2002), the TRI distribution has the advantage of being intuitively plausible for practitioners. Assuming that triangularity can help managers deal with new products, which have no historical data and therefore do not offer the possibility of analogies with similar products. Based on this assumption, managers can decide the first lot size to order and reorder point (ROP), which often includes a safety stock (SS) term. As companies initially store the product and learn about their DPUT distribution over time, learning-based approaches can be useful.

The probabilistic treatment of DLT is facilitated when the LT is constant. Nevertheless, often DPUTs are not independent and identically distributed (IID) RVs over time (Kristianto, Gunasekaran, Helo, & Sandhu, 2012). When one wants to optimize inventories composed by multiple products, the possible time dependence of the demand for products that conform the assortment of inventory should be considered in the modelling, which should improve the operational results by using past information (Calfa, 2015). This information can be described by autoregressive moving average models (ARMA) that the interested reader can consult in Box, Jenkins, Reinsel and Ljung (2015). All this statistical structure must be inserted in the mentioned objective function to be optimized by stochastic programming (Rojas & Leiva, 2016).

The motivation of this paper is that the proposed methodology has a useful as a tool for administrators who must decide optimal batch sizes and their reorder points when there is a low availability of demand and is known to have a temporal dependence. Then we propose a methodology based on inventory models of continuous review to be supplied from a single distributor, of utility for new products, assuming a triangular distribution and describing the dependence over time of the DPUT with an ARMA-TRI model. The paper is organized as follows: Section 2 makes a literature review about: (i) modelling of DPUT for TRI distribution and conditional to past information based on ARMA models; (ii) inventory models of continuous review; (iii) stochastic programming; and (iv) financial indicators of the inventory policy. Section 3 exposes the proposed methodology, whereas Section 4 illustrates it with a real-world case study of drugs supply in a Chilean public hospital; and Section 5 provides a discussion and conclusions of the results obtained in this research, as well as their limitations and future research.

## 2. Literature review

### *2.1. DPUT described by TRI distribution conditional to past information*

As stated, usually DPUT are not IID RVs over time. This dependence can be considered by a time dynamic structure and described by ARMA time series models (Kristianto et al., 2012). ARMA model parameters can be estimated via the maximum likelihood (ML) method once the underlying distribution has been defined, where often a normal or Gaussian distribution is assumed (Box et al., 2015), but other distributions might also be considered (Rojas, 2016). ARMA models, or its integrated version to grant stationarity, denominated ARIMA, are widely flexible, easy to estimate and interpret, and besides their prediction is straightforward (Gilbert, 2005). Prediction based on ARMA models may be carried out by the density forecast (DF) technique (Bauwens, Giot, Joachim, & David, 2004; Calfa, 2015; Diebold, Gunther, & Tay, 1998; Diebold, Hahn, & Tay, 1999). When non-normality is detected in the under analysis data, transformations for obtaining normality are usually utilized. However, data transformation brings problems of interpretation in the results. Furthermore, the ARMA model has a linear structure and the distributional assumption is considered for the model error, frequently named white noise.

Therefore, the classic ARMA model is highly restrictive. This restrictiveness was solved in a more general framework of statistical modelling by McCullagh and Nelder (1989) by means of generalized linear models (GLM). The GLM are based on distributions of the exponential family, where the normal model is a particular case. The GLM do not assume a distribution for the model error, but for the response directly, and allow for non-linear structures by a link function that relates the predictor to model mean. Benjamin, Rigby and Stasinopoulos (2003) proposed a GLM version of ARMA models known as generalized ARMA. In this framework it is possible to occupy other probabilistic distributions, such as the TRI distribution, reparametrized with respect to its mean. As mentioned, ARMA models are often used to predict future values, but they may also be employed to estimate mean values. This last aspect is particularly of interest in stochastic inventory models. So, a major difference with previous work developed by Glickman and Xu (2008) or Wanke, Ebwank, Leiva and Rojas (2016) is that in the present proposal we consider the temporal structure of demand that may be present in a forecast for new products, when we know that it may be influenced by a temporal arrangement of the data (modelling), revealing trend, seasonality, cyclicity, among others, which is a very common characteristic in the processes of demand.

### *2.2. Inventory models of continuous review*

A problem of multiple products commonly occurring in inventory policies is to decide what optimal quantities of products must be ordered simultaneously from a same supplier (Hillier & Lieberman, 2005). Continuous review policies provide solutions to problems of inventory management in many real-world situations. Inventory models of continuous review are known as $(Q, r)$ models, which are often used for inventory supply planning. This model is based on the economic order quantity or lot size and reorder point, denoted by $Q$ and $r$, respectively (Rojas, Leiva, Wanke, & Marchant, 2015). Indicators $Q$ and $r$ must be determined to minimize the TC of the inventory management. Such a cost is function of the holding, ordering and shortage costs (Hillier & Lieberman, 2005). When calculating the reorder point for a fixed service level, the LTD distribution and its corresponding probability density function (PDF) must be used. When the LTD distribution is unknown, this PDF can be approximated by any suitable approach. We employ a simultaneous approach to optimize $Q$ and $r$ (Silver, Pyke, & Peterson, 1998; Wanke et al., 2016).

### *2.3. Stochastic programming*

As its name suggests, stochastic programming is a mathematical programming problem (linear, non-linear, integer, etc.) which contains in its formulation some stochastic element that is unknown, but that can be estimated from its probability distribution (Shapiro, Dentcheva, & Ruszczynski, 2014). The expected value of the objective function based on the TC of inventory must be optimized in a continuous review policy of an assortment of products. Stochastic programming can be used to solve this optimization problem by the differential evolution (DE) algorithm, which belongs to the family of genetic algorithms, imitating the natural process of choice in evolutionary fashion (Price, Storn, & Lampinen, 2006; Rojas, & Leiva, 2016; Thangaraj, Pant, Bouvry, & Abraham, 2010; Wanke & Leiva, 2015).

### *2.4. Financial indicators of the inventory policy*

The economical benefits of inventory models are evaluated by efficiency indicators. Some of them are financial, such as TCs from inventories or sales related to rotation of stocks available; whereas others are operational, such as the ability to meet demand with inventory planned by the inventory model (called "fill-rate"), or the expected shortage per cycle (Wanke et al., 2016).

### *2.5. Summary*

In this work, we propose a methodology based on stochastic inventory models of continuous review for multiple products with supply from a single dealer. DPUT of each product is considered as an RV following a continuous probabilistic distribution. DPUTs are assumed to have dependence over time which is described by ARMA models. DPUT of a product may be possibly related to the DPUT of other products of the inventory assortment, which is described by multivariate continuous probabilistic distributions. Optimization of the inventory TC is carried out by stochastic programming using the DE algorithm. Economical evaluation of the results obtained in this study is conducted by financial indicators based on TCs of the inventory assortment of products.

## 3. Methodology

### *3.1. Triangular demand distribution*

Let the continuous RV $Y_t$ be the DPUT for a new product. If $Y_t$ has a TRI distribution around its mode $c$ with parameters $a \in \mathbb{R}$ and $b \in \mathbb{R}$, which is denoted by $Y_t \sim \text{TRI}(a, b, c)$, then its PDF, cumulative distribution function (CDF) (1) and quantile function (QF) (2) are, respectively, given by

$$f_{Y_t}(y_t) = \frac{\mathrm{d}F_{Y_t}(y_t)}{\mathrm{d}y_t} = \begin{cases} \frac{2(y_t-a)}{(b-a)(c-a)}, & \text{if } a \le y_t \le c; \\ \frac{2(b-y_t)}{(b-a)(b-c)}, & \text{if } c \le y_t \le b; \\ 0, & \text{otherwise;} \end{cases}$$

$$F_{Y_t}(y_t) = \mathrm{P}(Y_t \le y_t) = \int_{-\infty}^{y_t} f_{Y_t}(u)\,\mathrm{d}u = \begin{cases} 0, & \text{if } y_t < a; \\ \frac{(c-a)}{(c-b)}\frac{(y_t-a)^2}{(c-a)^2}, & \text{if } a \le y_t \le c; \\ 1 - \frac{(b-c)}{(b-a)}\frac{(b-y_t)^2}{(b-c)^2}, & \text{if } c \le y_t \le b; \\ 1, & \text{if } y_t < b; \end{cases}$$

and

$$F^{-1}_{Y_t}(q) = \begin{cases} a + \sqrt{q(c-a)(b-a)}, & \text{if } 0 \le q \le (c-a)/(b-a); \\ b - \sqrt{(1-q)(b-c)(b-a)}, & \text{if } (c-a)/(b-a) \le q \le 1. \end{cases} \tag{1}$$

In addition, the mean and variance of $X \sim \text{TRI}(a, b, c)$ are, respectively, given by

$$\mu = \mathrm{E}(Y_t) = \frac{a+b+c}{3} \quad \text{and} \quad \sigma^2 = \mathrm{V}(Y_t) = \frac{(b-a)^2}{18}\left(1 - \frac{(c-a)(b-c)}{(b-a)^2}\right). \tag{2}$$

### *3.2. Demand distribution during lead-time*

Furthermore, let the RV $L$ be the LT (lead time) of a item, which is not dependent from the sequence of independent identically distributed RVs $\{Y_t, t \ge 0\}$, where $\mathrm{E}(Y_t) = \mathrm{E}(Y) = \mu$ and $\mathrm{Var}(Y_t) = \mathrm{Var}(Y) = \sigma^2$ are the mean and variance of the DPUT, respectively. Suppose that the orders do not have crossbreeding (Hayya, Bagchi, Kim, & Sun, 2008).

Let $S_L$ be the LTD for the item, a random sum expressed as

$$S_L = \sum_{t=1}^{L} Y_t,$$

with PDF $f_{S_L}(\cdot)$ defined on $[0, \infty)$ (non-negative support), CDF

$$F_{S_L}(s) = \int_0^s f_{S_L}(v)\ \mathrm{d}v$$

and QF $s(q) = F^{-1}_{S_L}(q)$, for $0 < q < 1$. The expectation and variance of $S_L$ are, respectively, expressed as

$$\mu_{S_L} = \mathrm{E}(S_L) = \mathrm{E}(L)\,\mathrm{E}(Y) = \mathrm{E}(L)\mu \tag{3}$$

and

$$\sigma^2_{S_L} = \mathrm{Var}(S_L) = \mathrm{Var}(L)\,\mathrm{E}^2(Y) + \mathrm{E}(L)\,\mathrm{Var}(Y) = \mathrm{Var}(L)\mu^2 + \mathrm{E}(L)\sigma^2. \tag{4}$$

Please note that generally both LT as DPUT can be modelled by any discrete or continuous PDFs.

### 3.3. ARMA model

The assumption of IID RVs for DPUTs $Y_t$ depending over time $t$, for $t = 1, \dots, m$, and for the DLT $S_L$ can be violated. The time series $Y_t$ is described by an ARMA($p$, $q$) model defined by

$$Y_t = \boldsymbol{x}_t^\top \boldsymbol{\beta} + \sum_{j=1}^{p} \phi_j \{y_{t-j} - \boldsymbol{x}_{t-j}^\top \boldsymbol{\beta}\} + \varepsilon_t + \sum_{j=1}^{q} \theta_j \, \varepsilon_{t-j},$$

where $y_t, y_{t-1}, \cdots, y_{t-p}$ are RVs conceived as realizations of stochastic process at different points of time $t, t-1, t-2, \cdots$, which are characterized by a stationary mean $\mathrm{E}(Y_t) = \mathrm{E}(Y_{t-1}) = \mathrm{E}(Y_{t-2}) = \cdots = \mu = \boldsymbol{x}_t^\top \boldsymbol{\beta}$ (Benjamin et al., 2003), $\phi$ y $\theta$ corresponds to the ARMA components of a model of orders $p$ and $q$, respectively. Note that $\boldsymbol{\beta}_t^\top = (\beta_0, \beta_1, \dots, \beta_n)$ is a vector of coefficients associated with $n$ covariates with dependence over time denoted by $\boldsymbol{x}_t^\top = (x_0, x_{1,t}, x_{2,t}, \dots, x_{n,t})$, with $x_0 = 1$. For its part $\varepsilon_{t-j} = y_{t-j} - \mu$ corresponds to residuals o errors uncorrelated in the original scala of measurement of response variable with mean zero and normal distribution. In the ARMA models, it is assumed that the variance of the residuals ($\sigma_\varepsilon^2$) is constant in the time (Gilbert, 2005). All the parameters required are estimated to define the model. Thus, the stationary marginal mean of ARMA model is

$$\mu = \boldsymbol{x}_t^\top \boldsymbol{\beta}, \tag{5}$$

while the stationary marginal standard deviation (SD) of the response variable depend of order of the model. Table 1 presented the stationary marginal SD more usual to response variable according order to ARMA model.

### 3.4. Parameter estimation

Given $n$ observations of $Y_t$, for $t = 1, \dots, n$, the likelihood function is constructed as the product of conditional PDFs of $Y_t$ given past information $\Omega_{t-1}$. The stationary mean $\mu$ is a function of the ARMA parameters $\boldsymbol{\phi}^\top$, $\boldsymbol{\theta}^\top$ and $\boldsymbol{\beta}^\top$, where each time embodies these conditioning variables. Therefore, the corresponding log-likelihood function is given by

$$l(\boldsymbol{\phi}^\top, \boldsymbol{\theta}^\top, \boldsymbol{\beta}^\top) = \sum_{t=1}^{n} \log(f_{Y_t}(y_t | \mu(\boldsymbol{\phi}^\top, \boldsymbol{\theta}^\top, \boldsymbol{\beta}^\top))). \tag{6}$$

**Table 1. Stationary marginal SD more usual to response variable according order to ARMA model**

| ARMA order | Stationary marginal SD of response variable ($\sigma_y$) |
|---|---|
| 1-0 | $\sigma_y = \sqrt{\frac{\sigma_\varepsilon^2}{1-\phi_1^2}}$ |
| 2-0 | $\sigma_y = \sqrt{\frac{\sigma_\varepsilon^2}{1-\phi_1^2-\phi_2^2}}$ |
| 0-1 | $\sigma_y = \sqrt{\sigma_\varepsilon^2(1+\theta_1^2)}$ |
| 0-2 | $\sigma_y = \sqrt{\sigma_\varepsilon^2(1+\theta_1^2+\theta_2^2)}$ |
| 1-1 | $\sigma_y = \sqrt{\sigma_\varepsilon^2(\frac{1+\theta_1^2-2\phi_1\theta_1}{1-\phi_1^2})}$ |
| 2-1 | $\sigma_y = \sqrt{\sigma_\varepsilon^2(\frac{1+\theta_1^2-2(\phi_1\theta_1+\phi_2\theta_1)}{1-\phi_1^2-\phi_2^2})}$ |
| 1-2 | $\sigma_y = \sqrt{\sigma_\varepsilon^2(\frac{1+\theta_1^2+\theta_2^2-2(\phi_1\theta_1+\phi_1\theta_2)}{1-\phi_1^2})}$ |
| 2-2 | $\sigma_y = \sqrt{\sigma_\varepsilon^2(\frac{1+\theta_1^2+\theta_2^2-2(\phi_1\theta_1+\phi_1\theta_2+\phi_2\theta_1+\phi_2\theta_2)}{1-\phi_1^2-\phi_2^2})}$ |

By differentiating (6) for each of the parameters, it is possible to obtain their maximum likelihood (ML) estimates.

### *3.5. Stochastic programming of inventory model continuous review*

The total expected annual cost assuming inventory shortage is expressed as a sum of (i) the holding cost per unit of product per year, denoted by $C_s$, multiplied by the expected quantity in balance of the items; (ii) the ordering cost, denoted by $C_o$, multiplied by the number of orders per year, and (iii) the penalty cost when have shortage of inventory, denoted by $C_p$, that is, the penalty cost per shortage product unit multiplied by the number of orders per year and by the expected quantity of shortage product units. Thus, for the $(Q, r)$ model, the annual total cost is expressed as

$$C_T = G(Q, r) = \left(\frac{Q}{2} + r - \mu_{S_L}\right)C_s + \frac{12\,\mu}{Q}C_o + W(r)\,\frac{12\,\mu}{Q}C_p, \tag{7}$$

where $\mu$ is calculated from (5) and $\mu_{S_L}$ is defined in (3) (Hadley & Whitin, 1963; Johnson & Montgomery, 1974; Silver et al., 1998). Notice that $\mu$ is multiplied by 12, because the total cost given in (7) is defined on an annual basis and $\mu$ on a monthly basis. In addition, $\mu_{S_L}$ is not altered, because its scope is verified within each safety inventory cycle. For the inventory total cost given in (7), $r - \mu_{S_L} = \mathrm{SS} = k_p\,\sigma_{S_L}$, with the SD of the LTD $\sigma_{S_L}$ being given from (4) which is calculated in stationary marginal form according order to ARMA model of Table 1. $k_p$ is the amount of SDs of the LTD or safety factor (SF) associated with a service level of $p \times 100\%$, for $0 < p < 1$. Note that $k_p$ corresponds to the $p \times 100$th standardized percentile, usually used in the 95th position, ensuring a service level of 95%. In (7), $W(r)$ is the expected shortage per cycle given by

$$W(r) = \int_r^m (s - r) f_{S_L}(s)\ \mathrm{d}s,$$

where $m$ is the maximum value for the LTD, $r$ the already mentioned reorder point and, as also mentioned, $f_{S_L}(\cdot)$ the LTD PDF.

### *3.6. Summary of the methodology*

Algorithm 1 summarizes our methodology in four main steps divided in 10 sub-steps based on the aspects detailed in Sections 3.1–3.6, from the collection of data until the establishment of the TCs to evaluate the optimized system with autocorrelated demand not IID in relation to the IID demand optimized system. We recall this algorithm considers the demand dependent to past information for products, but once all the products are considered, the total cost of the products used in the service are minimized.

## 4. Simulation study and application

Below, we show how inventory management of products can be planned within companies by using the methodology proposed in Section 3 and the computational framework developed for this methodology.

**Algorithm 1** Main methodological steps

1: Collect demand per unit time data for the product $i$ ($i = 1, \ldots, n$) during a period (for example DPUT in each month during 2 years).

2: For the statistical analysis:

2.1 Carry out an autocorrelation study for data collected in Step 1 examining plot of the autocorrelation functions (ACF) and partial autocorrelation (PACF) to detect possible dependence to past information.

2.2 Propose ARMA models considering TRI distribution for the response variable (which is a monthly count), for each product, selecting those through the Akaike criteria was lower.

2.3 Run a check on behavior of residues of the fit of the proposed models, showing the ACF and PACF, density residuals and QQ-plot to show that they follow a TRI distribution.

2.4 To estimate the parameters of the PDF (stationary marginal mean according 5 and SD according Table 1) of the conditional demand past information for each product.

3: For the inventory analysis:

3.1 Select a model of continuous review with shortage, but with an objective function related to minimize CT for each product in a inventory assorment.

3.2 Find the optimal inventory elements $(Q, k)$ based on conditional distributions established and parameters of Step 2.4, asumming normality for DLT.

4: For the financial analysis:

4.1 Compute the TC for the each product of the optimal policy obtained in Step 3.2.

4.2 Determine the corresponding the arguments $(Q, k)$ that maximize inventory TC function, and shortage expected by cycle for each product.

4.3 Repeat steps 4.1 and 4.2 for products considering DPUTs IID, and compare.

### *4.1. Computational framework*

R is a non-commercial and open source software for statistics and graphs, which can be obtained at no cost from http://www.r-project.org. The statistical software R is being currently very popular in the international scientific community. For a use of this software in inventory models (see Rojas et al., 2015). Some R packages related to statistical distributions that may be useful in inventory models are available at http://CRAN.R-project.org (Barros, Paula, & Leiva, 2009; Stasinopoulos & Rigby, 2007). The expected value for demand products is calculated by using ARMA models implemented in the R software by the packages forecast() for time series. The random generator TRI distribution number is made by packaged triangle change distribution of command arima.sim(). The stochastic programming of inventory models is performed by the packaged DEoptim() of the same software.

### *4.2. Simulation study*

#### *4.2.1. Temporal structure of DPUT data*

We use the Monte Carlo method for simulating data ($n$ = 1,000) of DPUT with temporal structure from TRI distribution (parameters $a$ = 1, $b$ = 10, $c$ = 5). The variation of the shape and kurtosis of the distribution is compared when considered TRI distribution with an assumption of IID (without a temporal structure and without co-variate), and then when temporary structures AR and MA are introduced, as well as both (ARMA). Figure 1 shows that the introduction of a positive order autoregressive (AR1P, coefficient $\phi_1$= 0.5) shifts the distribution towards higher values of the variable (right) and it makes slightly more platykurtic. The inverse effects occur with the negative order autoregressive (AR1N, coefficient $\phi_1 = -0.5$). The orders of moving average positive and negative ( MAP, $\theta_1$ =

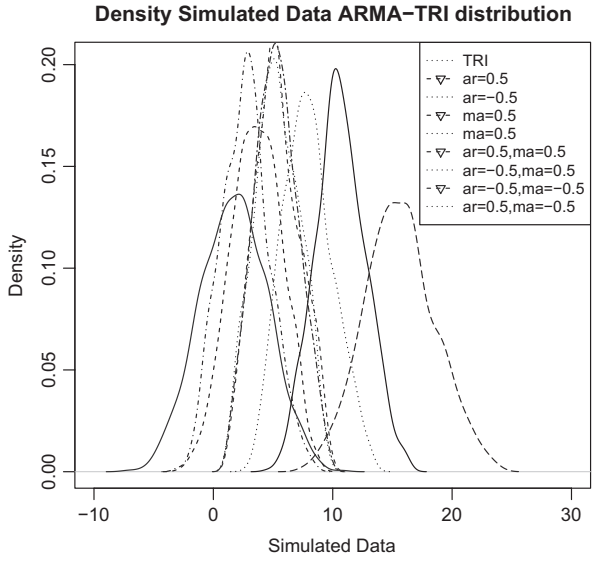



**Figure 1. Simulation study TRI distribution with ARMA structure. Shape and Kurtosis.**

0.5 and MAN, $\theta_1 = -0.5$), operate shifting the values of the variable in the same above address, but with a lower magnitude both in the displacement as the platy-kurtosis. The described effects are accentuated even more in the address described as the AR and MA orders are both positive or both negative, instead are cancelled when they are of different signs, resembling the distribution TRI IID of reference.

*4.2.2. Change in parameter estimation with ARMA-TRI model*

To study the change of the estimation of parameter, we compare the mean and standard deviation (SD) of 1,000 data simulated in time series with ARMA-TRI models respect to IID-TRI distribution (parameters $a = 1$, $b = 10$, $c = 5$). The results are shown in Table 2. Note that in almost all cases the mean and SD of the simulated time series described by ARMA-TRI models are higher to the average and SD of the TRI IID distribution. This change is very important to the modelling in stochastic programming.

### *4.3. Application*

The drug supply in pharmacy units of Chilean primary health centres is channelled through their central warehouse, which acts as an intermediary between suppliers and output units (OU). The OUs receive the demand for drugs, including its own pharmacy, which performs dispensing of prescriptions to patients. This warehouse needs the storage, conservation and distribution of such drugs. Supply of warehouse is carried out by a supplier. The warehouse delivers the products on a monthly basis to all OUs by using aggregated demand requirements for each of them in the same period.

**Table 2. Change in parameter estimation ARMA-TRI vs TRI-IID model**

| Model/Parameter | Mean (units) | SD (units) |
|---|---|---|
| TRI-IID | 5.51 | 1.74 |
| AR1($\phi = -0.5$)MA($\theta = 0$)-TRI | 10.5 | 2.42 |
| AR1($\phi = -0.5$)MA($\theta = 0$)-TRI | 3.07 | 2.38 |
| AR($\phi = 0$)MA($\theta = 0.5$)-TRI | 8.18 | 2.18 |
| AR($\phi = 0$)MA($\theta = -0.5$)-TRI | 2.81 | 2.35 |
| AR($\phi = 0.5$)MA($\theta = 0.5$)-TRI | 15.69 | 3.37 |
| AR($\phi = -0.5$)MA($\theta = -0.5$)-TRI | 1.83 | 3.63 |
| AR($\phi = 0.5$)MA($\theta = -0.5$)-TRI | 5.55 | 1.73 |
| AR($\phi = -0.5$)MA($\theta = 0.5$)-TRI | 5.54 | 1.76 |

*4.3.1. The data-set*

To validate the proposed methodology, we use real-world monthly demand data for an assortment of 223 pharmaceutical products, which it has extracted an example of 1 demand of product with time dependence monthly that associated with other demand of product. The associations between demands of pharmaceutical products are common in this area, because usually more than one drug is used to treat the same disease. Then, in ARMA model the DPUT of a product can co-vary as a predictor of another DPUT of product. The products are shipped from the warehouse and delivered to a family health centre, located at the city of Concon, Chile, for a study of supply policy conducted by Fernando Rojas in the University of Valparaiso, Chile, during 24 months of the years 2014–2015 (from January 1 to December 31). In this type of public institutions the availability of data is very limited and so the parameters of the TRI distribution were informed by the pharmacist in charge of the pharmacy.

*4.3.2. Data analysis*

Figure 2 shows a time series monthly of DPUTs, whereas Figure 3 presents (a) ACF, (b) PACF of DPUTs of an illustrative products, respectively. Note that the examination of these figures shows that the probability distributions of these variables are approximated by TRI distribution but not IID. The pharmacist in charge estimated the parameters as $a$ = 50, $b$ = 240 and $c$ = 170 units/month, respectively.

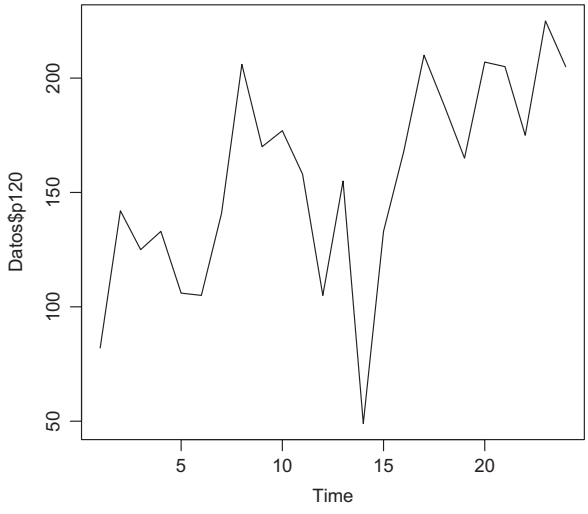
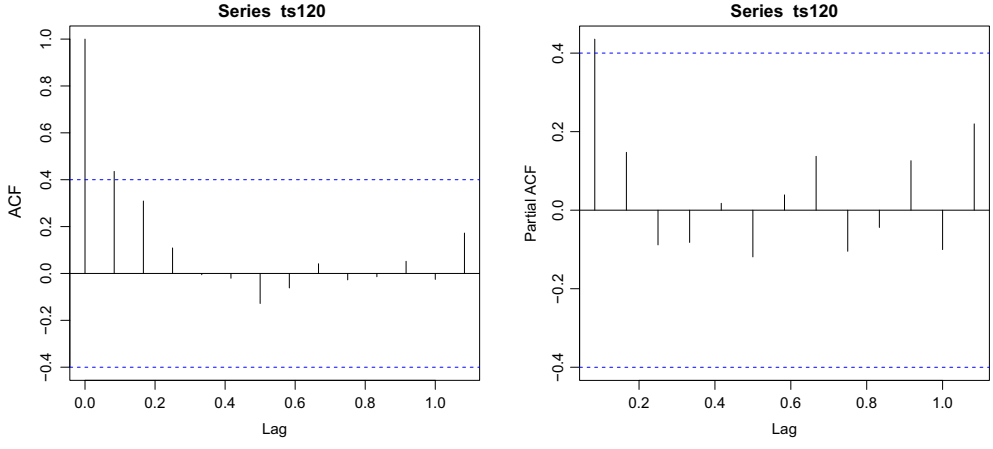



**Figure 2. Time series monthly of DPUTs for illustrative product.**




(a) ACF (b) PACF

**Figure 3. (a) ACF, (b) PACF of DPUTs for illustrative product.**

**Table 3. Descriptive statistics of monthly DPUTs of illustrative product**

| Mean | SD | IQR | 0% | 25% | 50% | 75% | 100% | *n* |
|---|---|---|---|---|---|---|---|---|
| 155.625 | 45.15463 | 61.25 | 49 | 131 | 161.5 | 192.25 | 225 | 24 |

**Table 4. ARMA-TRI model coefficients of DPUTs with covariate and temporal structure**

| AR(1) | Intercept | Covariate | $\sigma_\epsilon^2$ | log likelihood | AIC | RMSE | MAPE | MASE |
|---|---|---|---|---|---|---|---|---|
| 0.409 (0.210) | 125.574 (16.021) | 0.189 (0.0717) | 1,178 | −119 | 246 | 34.316 | 24.939 | 0.812 |

Table 3 shows descriptive statistics of monthly DPUTs of illustrative product with mean, SD, interquartile range (IQR), percentile 0, 25, 75, 100 and size sample (*n*). The data-set of this monthly DPUTs is:

{82, 142, 125, 133, 106, 105, 141, 206, 170, 177, 158, 105, 155, 49, 133, 168, 210, 188, 165, 207, 205, 175, 225, 205}.

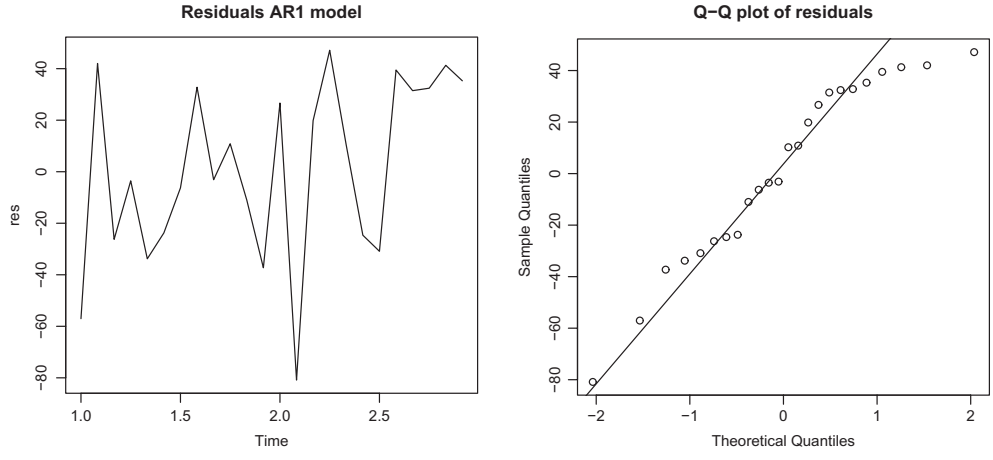
The monthly DPUT of other associated product show a correlation coefficient of 0.5469 with adjusted *p*-values (Holm's method) of 0.0057. The data-set of this monthly DPUTs is:

{100, 0, 100, 100, 100, 100, 200, 300, 200,
200, 200, 100, 100, 0, 100, 0, 300, 400, 400, 300, 200, 0, 200, 100}.

To confirm that original time series is stationary we applied Augmented Dickey–Fuller Test, obtain a statistics Dickey–Fuller = −2.599, for lag order = 2, with *p*-value = 0.3442 for alternative hypothesis of stationarity. This result indicated that it is possible to apply an ARMA model to original data, considering a covariate.

To estimate the parameters and fittings of the ARMA-TRI model according Akaike criteria, it is possible to consult a code in software R, which is available by the author. In this case, in concordance with visual examen to ACF and PACF plots, the best model is AR(1) with covariate. The coefficients necessary to specify the model (in parenthesis shows standard errors) and show a training set error measures of the forecast as root mean squared error (RMSE), mean absolute percentage error (MAPE) and mean absolute scaled error (MASE), obtaining the results shown in Table 4.

To confirm the correct fit of the proposed AR(1)-TRI model, are examined plots of residuals and normal QQ-plots, view Figure 4. Also check Box-Pierce test to corroborate aleatory disposition with statistic $\chi$-squared = 0.063052, degree of freedom (df) = 1, and *p*-value = 0.8017. On the other hand,




(a) Residuals (b) Normal QQ-plot of residuals

**Figure 4. (a) Plot of AR(1)-TRI model residuals, (b) normal QQ-plot of AR(1)-TRI model residuals.**

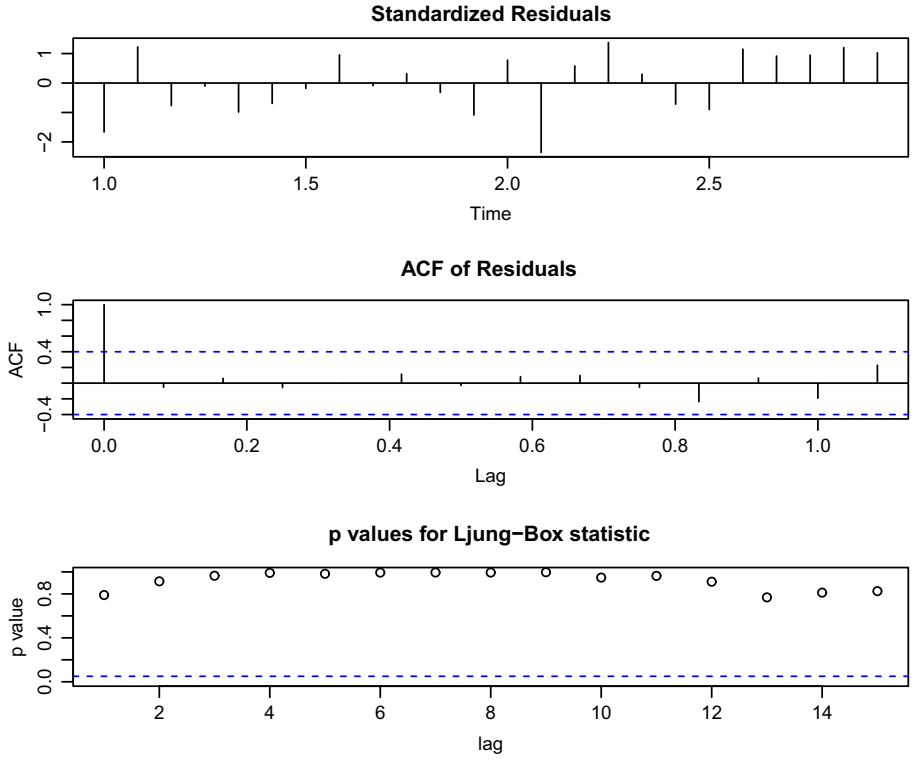



**Figure 5. Check diagnostic AR(1) model for illustrative product.**

Figure 5 also corroborated that standardized residuals of AR(1)-TRI model are random, and haven't autocorrelation and *p*-values for Ljung-Box statistics are not significative for all lags.

Table 5 shows means and SDs of the DPUTs modelled with and without temporal structure, indicators of a model of continuous review inventory and annual financial results compared between the two cases. Lead time is constant of 0.75 month, holding cost ($C_s$) consider is 0.042 USD/unit/year, order cost ($C_o$) is 0.86 USD/order and penalty cost for shortage ($C_p$) is 0.33 USD/unit.

**Table 5. Comparative parameters, indicators and annual financial results of policies continues review according to models of temporal structure**

| | With temporal structure ARMA-TRI model | TRI-IID without temporal structure |
|---|---|---|
| Parameters | Mean (un/month) | Mean (un/month) |
| | 155.5624 | 155.625 |
| | SD (un/month) | SD (un/month) |
| | 37.62098 | 45.15463 |
| Indicators | Lot size (Q) (units) | Lot size (Q) (units) |
| | 291 | 289 |
| | Safety factor ($k_p$) | Safety factor ($k_p$) |
| | 2.06 | 2.06 |
| | Reorder point (r) | Reorder point (r) |
| | 197.27 | 183.78 |
| | Expected Shortage ($W(r)$)(unit/cycle) | Expected Shortage ($W(r)$)(unit/cycle) |
| | 0.28 | 0.23 |
| Annual financial results | Annual total cost inventory (USD/year) | Annual total cost inventory (USD/year) |
| | 15.62 | 14.95 |

## 5. Discussion, conclusions, limitations and future research

### *5.1. Discussion*

In this paper we have shown a useful methodology for managers who make purchasing decisions in situations applicable to new products, or when there is low availability of demand data, which considers the very intuitive TRI distribution with temporal structure, which is very classic to describe the behaviour with tendency, seasonality or cyclicity of many products. The policy inventories raised show an interesting way to achieve significant savings in TC and shortage expected with application to multi-product systems. The proposed supply policy of products is useful in institutions that have a supplier to meet your requirements, and that generally are characterized by high bureaucracy in your administrative systems as public hospitals, where the operation of a supply system as proposed would be facilitated (Bennett & Gilson, 2001; Birkett, Mitchell, & McManus, 2001).

### *5.2. Conclusions*

The parameterization of the mean and standard deviation of DPUT with TRI distribution under a temporal structure of a conditional autoregressive model to information passed as ARMA, achieves an efficient and dynamic characterization of a myopic policy involving the DPUT and DLT. Regarding the key parameters used to model the occupied inventory policy as an example, these are the mean, standard deviation of the DPUT and standardized percentiles of the DLT, we can say that these are broadly sensitive to the underlying temporal structure, and apparently would have Pure and hybrid AR-MA structures where the mean parameter is greatly increased. With respect to the standard deviation, this parameter increases practically in all the temporary structures of modelling, reason why the effect on the percentile of standardized security is but hybrid, since it depends on both described parameters. Althought TRI distribution was occupied in this work the proposal is valid for any PDF that can be parameterized with respect to the conditional mean of a time series model. This condition opens multiple areas of research in the area.

### *5.3. Limitations*

In this paper, we have limited the proposed methodology to the case of a policy of continuous revision considering shortage, with optimum TC as an single objective function. The limitations are that it has been considered a single target on TC, which could be extended to multiple optimization targets, while the shortage has been modelled under a normality assumption, which is also susceptible to improvement.

### *5.4. Future research*

In the future, it is possible to obtain improvements in the modelling raised occupying other than TRI distributions, and even considering that in many cycles demand component is zero, requiring demand model using probability distributions for continuous and/or discrete data zero-inflated. The possibility to perform forecasts based on a conditional autoregressive model whose quality could be evaluated by the probability integral transformed, can update in n-steps ahead of the myopic policy inventories.

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