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# Article

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# **OPERATIONS, INFORMATION & TECHNOLOGY | RESEARCH ARTICLE**

# An inventory model for deteriorating items under inflation and permissible delay in payments by genetic algorithm

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Additional information is available at the end of the article

# **OPERATIONS, INFORMATION & TECHNOLOGY | RESEARCH ARTICLE** An inventory model for deteriorating items under inflation and permissible delay in payments by genetic algorithm

Sanjey Kumar<sup>1</sup> and Neeraj Kumar<sup>1\*</sup>

**Abstract:** Inventory models play a leading role in analyzing a lot of realistic situations arising at places like, food and vegetable markets, market yards, oil exploration industries, etc. In the present article, we developed an inventory model for deteriorating items with permissible delay in payment under inflation. In the given model, demand rate is considered as stock-dependent and deterioration rate of each item follows Weibull distribution. The model is developed under two different circumstances depending on whether the credit period is (1) less than the cycle time (2) greater than the cycle time. Also, a new algorithm is developed under these scenarios to obtain the EOQ. Finally results are analyzed and demonstrated with illustrative examples by Genetic Algorithm.

## Subjects: Applied Mathematics; Mathematical Modeling; Non-Linear Systems

Keywords: Weibull distribution; inflation; stock-dependent demand; permissible delay; genetic algorithm

## 1. Introduction

Deterioration of items is a frequent and natural phenomenon which cannot be ignored. In realistic scenario, the life cycle of seasonal product, fruits, electric component, volatile liquid, food, etc. are short and finite usually can undergo deterioration. Thus, the item may not serve the purpose after a period of time and will have to be discarded as it cannot be used to satisfy the future demand of customers. The present article investigates inventory model for deteriorating items with stock-dependent demand rate. The deterioration of inventory in stock during the storage period constitutes

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# PUBLIC INTEREST STATEMENT

The aim of this paper is to widen the concept of demand and supply by model building. This work assumed that demand rate is stock-dependent. Demand is also related to shortage under the conditions of permissible delay in payment. The optimal cycle time is determined to minimize total inventory cost. It will add benefits to the society and individuals who suffers from problems related to partial backlogging. In addition, some managerial insights on the basis of numerical examples are also concluded for maximizing profits and cost.

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an important factor which has attracted the attention of researchers. Dye (2002) developed an inventory model with stock-dependent demand and partial backlogging. Chakrabarty, Giri, and Chaudhuri (1998) extended the Philip's model (1974). Skouri and Papachristos (2003) investigate optimal time of an EOQ model with deteriorating items and time-dependent partial backlogging. Deterioration can't be ignored in business scenarios. Rau, Wu, and Wee (2004) developed an integrated inventory model to determine economic ordering policies of deteriorating items in a supply chain management system. Teng and Chang (2005) developed economic production quantity in an inventory model for deteriorating items. Deterioration refers to decay, damage, or spoilage like foods, drugs, chemicals, electronic components and radio-active substances, deterioration may happen during normal period of storage and the loss is to be taken into account where we analyze inventory systems. Dave and Patel (1983) investigate an inventory model for deteriorating items with time proportional demand. Roychowdhury and Chaudhuri (1983) introduced an order level inventory model considering a finite rate of replenishment and allowing shortages. In this direction their model Misra (1975), Deb and Chaudhuri (1986) considered that deterioration rate is time dependent. Berrotoni (1962) investigates some difficulties of fitting empirical data to mathematical distribution. It can be said that the rate of deterioration increases with age. It may be inferred that the work of Berrotoni (1962) inspired Covert and Philip (1973) to design an inventory model for deteriorating items with Weibull distribution by two parameters. Mandal and Phaujdar (1989) developed a production inventory model for deteriorating items with uniform rate of production and stock-dependent demand. In this direction, some precious works in this area were also done by Padmanabhan and Vrat (1995), Ray and Chaudhuri (1997), Mandal and Maiti (1999).

In all the above models, the time value of money and inflation were not considered because of the belief that the time value of money and inflation would not affect significantly the decisions regarding inventory management. But in real life, the impact of time value of money and inflation cannot be ignored while deciding the optimal inventory policies. Today, inflation has become a permanent feature of the economy. Many researchers have shown the inflationary effect on inventory policy. Bierman and Thomas (1977), Buzacott (1975), Chandra and Bahner (1988), Jesse, Mitra, and Cox (1983), Misra (1979) developed their inventory models assuming a constant inflation rate. An inventory model with deteriorating items under inflation when a delay in payment is permissible is calculated by Liao, Tsai, and Su (2000). Bhahmbhatt (1982) introduced an EOQ model under a variable inflation rate and marked-up price. Ray and Chaudhuri (1997) presented an EOQ model under inflation and time discounting allowing shortages. Both in deterministic and probabilistic inventory models of classical type it is observed that payment is made to the supplier for goods just after getting the consignment. But actually current scenario a supplier grants some credit period to the retailer to increase the demand. In this respect Goyal (1985) just formulated an EOQ model under some conditions of permissible delay in payment. An EOQ model for inventory control in the presence of trade credit is presented by Chung and Huang (2005). Chung, Huang, and Huang (2002) and Chung and Huang (2003) developed an optimal replenishment policy for EOQ models under permissible delay in payments. In recent times to make the real inventory systems more practical and realistic, Aggarwal and Jaggi (1995) extended the model with a constant deterioration rate. Hwang and Shinn (1997) developed lot-sizing policy for exponential demand when delay in payment is permissible. Shah and Shah (1998) presented a probabilistic inventory model with a cost in case delay in payment is permissible. Subsequently Jamal, Sarker, and Wang (1997) developed further following the lines of Aggarwal and Jaggi's (1995) model to take into consideration for shortage and make it more practical and acceptable in real situation. Some of the recent works in this area may relate to Datta, Paul, and Pal (1998), Dye (2002), Chung (2003), Zhou and Yang (2005), Goyal and Chang (2009), and Yang, Teng, and Chern (2010), Kumar, Singh, and Kumari (2013), developed an inventory model with stockdependent demand rate for deterioration items. Kumar, Singh, and Kumari (2014), Tayal, Singh, and Sharma (2015), developed an inventory model for deteriorating items with seasonal products and an option of an alternative market. Kumar and Kumar (2016a), developed an inventory model with stock-dependent demand rate for deterioration items. Recently Kumar and Kumar (2016b) presented an inventory model for deteriorating items stock-dependent demand and partial backlogging.

This study is developing an inventory model with stock-dependent demand rate with permissible delay in payment in real life situations. It will help the retailers to manage the business. The effect of preservation technique is used to reduce the deterioration and the time value of the money cannot be ignored in determining the optimal inventory decision. The concept of the inflation should be considered especially for long-term investment and forecasting. The model shows the effect due to changes in various parameters by taking suitable numerical examples and sensitivity analysis.

#### 2. Notations and assumptions

#### 2.1. Notations

The following notations are used throughout this paper:

q(t)	Inventory level at time t
S = q(0)	Stock level at the beginning of each cycle after fulfilling backorders
Н	Length of the planning horizon
К	Constant rate of inflation (\$/\$/unit time)
<i>C</i> ( <i>t</i> )	Unit purchase cost for an item bought at time <i>t</i> , i.e. $C(t) = C_0 \cdot e^{\kappa t}$ , where $C_0$ is the unit Purchase cost at time zero
h	Holding cost (\$/unit/year) excluding interest charges
C <sub>o</sub>	Unit purchase cost
C <sub>2</sub>	Shortage cost (\$/unit/time)
C <sub>3</sub>	The ordering cost/cycle
i <sub>e</sub>	Interest earned (\$/time)
i <sub>p</sub>	Interest charged (\$/time)
М	Permissible delay in settling the accounts
Τ <sub>1</sub>	Time at which shortages start ( $0 \le T_1 \le T$ )
Т	Length of a cycle
TCU (T <sub>1</sub> , T)	The Total cost function per unit time
TCU <sub>1</sub> (T <sub>1</sub> , T)	The Total cost function per unit time for $T_1 > M$ (Case I)
TCU <sub>2</sub> (T <sub>1</sub> , T)	The Total cost function per unit time for $T_1 \leq M$ (Case II)

## 2.2. Assumptions

To develop the mathematical model, the following assumptions are being made:

- (i) The inventory system involves only one item.
- (ii) The rate of replenishment is instantaneous.
- (iii) A fraction z(t) of the on hand inventory deteriorates per unit time where  $z(t) = \alpha\beta t^{\beta-1}$ ,  $0 \le \alpha \le 1$ ,  $t \ge 0$ ,  $\beta \ge 1$ .
- (iv) Shortages are allowed and the backlog rate is defined to be  $R(t)/1 + \delta(T-t)$  when inventory is negative. The backlogging parameter  $\delta$  is a positive constant.
- (v) The demand rate R(t) at time t is  $R(t) = \begin{cases} a + b q(t), & 0 \le t \le T_1 \\ a, & T_1 \le t \le T \end{cases}$  where a and b are non-negative constraints.

## 3. Formulation and solution of the model

Based on the above description, during the time interval  $[0, T_1]$ , the inventory level at time t will satisfy the following differential representing the inventory status is given by:

$$\frac{dq(t)}{dt} + \alpha\beta t^{\beta-1}q(t) = -(a+bq(t)) \quad 0 \le t \le T_1$$
(1)

With the boundary condition  $q(T_1) = 0$ , the solution of Equation (1) is:

$$q(t) = a(\alpha \beta T_1^{\beta} + b) \left[ e^{\frac{\alpha \beta}{\beta + 1} \left( T_1^{\beta + 1} - t^{\beta + 1} \right) + b(T_1 - t)} - 1 \right]$$
(2)

Again in the second time interval  $[T_1, T'$  the instantaneous inventory will satisfy. Thus, the differential equation below represents the inventory status:

$$\frac{dq(t)}{dt} = -\frac{a}{1+\delta(T-t)}, \quad T_1 \le t \le T$$
(3)

With the condition  $q(T_1) = 0$ , we get the solution of equation (3) is:

$$q(t) = \frac{a}{\delta} \left[ \log \left\{ 1 + \delta(T - t) \right\} - \log \left\{ 1 + \delta \left( T - T_1 \right) \right\} \right]$$
(4)

Inventory model before and after the deflation is illustrated in Figures 1 and 2.

#### 3.1 Case I ( $M < T_1$ ): Payment before depletion

The holding cost during [0, *T*] is:

$$HC = h \sum_{n=0}^{m-1} C(\eta T) \int_{0}^{t_{1}} q(t) dt = h c_{0} \left( \frac{e^{KH} - 1}{e^{KT} - 1} \right) \left[ \int_{0}^{t_{1}} a \left( \alpha \beta T_{1}^{\beta} + b \right) \left\{ e^{\frac{\alpha \beta}{\beta + 1} (T_{1}^{\beta + 1} - t^{\beta + 1}) + b(T_{1} - t)} - 1 \right\} dt \right]$$
(5)  
$$= h c_{0} \left( \frac{e^{KH} - 1}{e^{KT} - 1} \right) \left[ a \left( \alpha \beta T_{1}^{\beta} + b \right) \left\{ e - \alpha \beta T_{1}^{\beta} - \frac{e^{\frac{\alpha \beta}{\beta + 1}} T_{1}^{\beta + 1} + bT_{1} + 1}{\frac{\alpha \beta}{\beta + 1} T_{1}^{\beta + 1} + bT_{1} + 1} - T_{1} \right\} \right]$$
(6)

(Ignoring the higher order of  $\alpha$ )

The number deteriorating items during  $[0, T_1]$  is:

$$= q(0) - \int_{0}^{T_{1}} (a+bt)dt = S - \left(aT_{1} + \frac{bT_{1}}{2}\right) = \frac{a\alpha T_{1}^{\beta+1}}{\beta+1} + \frac{b\alpha T_{1}^{\beta+2}}{\beta+2}$$
(7)

The deteriorated cost DC is:

$$DC = C_0 \left(\frac{e^{KH} - 1}{e^{KT} - 1}\right) \left\{ \frac{a\alpha T_1^{\beta+1}}{\beta+1} + \frac{b\alpha T_1^{\beta+2}}{\beta+2} \right\}$$
(8)

Figure 1. Inventory model before the deflation.





The shortage cost  $[0, T_1]$  is:

$$SHC = C_2 \cdot C_0 \left(\frac{e^{KH} - 1}{e^{KT} - 1}\right) \int_{T_1}^T q(t) dt$$
(9)

$$= C_2 C_0 \left( \frac{e^{KH} - 1}{e^{KT} - 1} \right) \int_{T_1}^{t} \frac{a}{\delta} \left[ \log \left\{ 1 + \delta(T - t) \right\} - \log \left\{ 1 + \delta(T - T_1) \right\} \right] dt$$
(10)

$$=C_2.C_0\left(\frac{e^{KH}-1}{e^{KT}-1}\right)\frac{a}{\delta}\left[\log\left\{\frac{1}{\delta}+\delta(T-T_1)-(T-T_1)\right\}\right]$$
(11)

The total variable cost is comprised of the sum of the ordering cost, holding cost, backorder cost, deterioration cost, and interest payable minus the interest earned. They are grouped together after evaluating the above cost individually.

The interest earned  $IE_1$  during the time [0, T] is:

$$\begin{split} IE_{1} &= i_{e} \cdot C_{0} \frac{e^{KH} - 1}{e^{KT} - 1} \int_{0}^{T} \left( T_{1} - t \right) \left( a + bq(t) \right) dt \\ &= i_{e} \cdot C_{0} \frac{e^{KH} - 1}{e^{KT} - 1} \left[ \frac{aT_{1}^{2}}{2} + \frac{ab\alpha^{2}\beta^{2}}{\beta + 1} \left( T_{1}^{2(\beta+1)} - T T_{1}^{2(\beta+1)} - \frac{T_{1}^{(\beta+3)}}{2} + \frac{T_{1}^{(2\beta+3)}}{\beta + 1} \right) \\ &- \frac{ab\alpha\beta}{\beta + 1} \left( \frac{T_{1}^{(\beta+2)}}{\beta + 2} - b T_{1}^{(\beta+2)} + \frac{T_{1}^{(\beta+2)}}{\beta + 2} \right) - ab^{2}\alpha\beta \left( \frac{T T_{1}^{(\beta+2)}}{2} + \frac{T_{1}^{(\beta+3)}}{3} \right) \\ &+ abT T_{1}^{2} - \frac{ab^{3} T_{1}^{3}}{3} + ab^{3}T T_{1} \left( T - \frac{T_{1}}{2} \right) \right] \end{split}$$
(12)

The interest payable *IP* per cycle for the inventory not being sold after due date *M*:

$$IP_{1} = i_{p} C_{0} \frac{e^{KH} - 1}{e^{KT} - 1} \int_{M}^{I} q(t) dt$$
(13)

$$= i_{\rho} C_{0} \frac{e^{KH} - 1}{e^{KT} - 1} \int_{M}^{T} a(\alpha \beta T_{1}^{\beta} + b) \left[ e^{\frac{\alpha \beta}{\beta + 1} \left( T_{1}^{\beta + 1} - t^{\beta + 1} \right) + b(T_{1} - t)} - 1 \right] dt$$
(14)

$$= i_{\rho} \cdot C_{0} \left( \frac{e^{KH} - 1}{e^{KT} - 1} \right) \left[ a \left( \alpha \beta T_{1}^{\beta} + b \right) \left\{ \left( \alpha \beta M^{\beta} + b \right) \left( \frac{e^{\frac{\alpha \beta}{\beta + 1} (T_{1}^{\beta + 1} - M^{\beta + 1}) + b(T_{1} - M) + 1}}{\frac{\alpha \beta}{\beta + 1} (T_{1}^{\beta + 1} - M^{\beta + 1}) + b(T_{1} - M) + 1} \right) - e \left( \alpha \beta T_{1}^{\beta} + b \right) \right\} \right]$$
(15)

So, the total variable cost,  $TVC_1$  is defined as:

-

 $TVC_1$  = Ordering cost + holding cost + deterioration cost + shortage cost + interest payable – interest earned.

$$\begin{aligned} \text{TVC}_{1} &= C_{3} + \text{HC} + \text{DC} + \text{SHC} + \text{IP}_{1} - \text{IE}_{1} \end{aligned} \tag{16} \\ \text{TVC}_{1} &= C_{0} \left( \frac{e^{\text{KH}} - 1}{e^{\text{KT}} - 1} \right) \left[ a \left( \alpha \beta T_{1}^{\beta} + b \right) \left\{ h \left( e - \alpha \beta T_{1}^{\beta} - \frac{e^{\frac{\beta \beta}{\beta + 1} T_{1}^{\beta + 1} + b T_{1} + 1}}{\frac{\beta \beta}{\beta + 1} T_{1}^{\beta + 1} + b T_{1}^{\beta + 1} - T_{1}} \right) \right\} \\ &- C_{3} + i_{p} \left\{ \left( \alpha \beta T_{1}^{\beta} + b \right) \left( \frac{e^{\frac{\beta \beta}{\beta + 1} (T_{1}^{\beta + 1} - M^{\beta + 1}) + b (T_{1} - M) + 1}}{\frac{\alpha \beta}{\beta + 1} (T_{1}^{\beta + 1} - M^{\beta + 1}) + b (T_{1} - M) + 1} \right) - e \left( \alpha \beta T_{1}^{\beta} + b \right) \right\} \\ &+ C_{2} \left\{ \frac{\alpha}{\delta} \left[ \log \left\{ \frac{1}{\delta} + \delta \left( T - T_{1} \right) - \left( T - T_{1} \right) \right\} \right] \right\} \\ &+ \left\{ \frac{\alpha \alpha T_{1}^{\beta + 1}}{\beta + 1} + \frac{b \alpha T_{1}^{\beta + 2}}{\beta + 2} \right\} - i_{c} \left\{ \frac{\alpha T_{1}^{2}}{2} + \frac{a b \alpha^{2} \beta^{2}}{\beta + 1} \left( T_{1}^{2(\beta + 1)} - T T_{1}^{2(\beta + 1)} - \frac{T_{1}^{(\beta + 3)}}{2} + \frac{T_{1}^{(2\beta + 3)}}{\beta + 1} \right) \end{aligned} \tag{17} \\ &- \frac{a b \alpha \beta}{\beta + 1} \left( \frac{T_{1}^{(\beta + 2)}}{\beta + 2} - b T_{1}^{(\beta + 2)} + \frac{T_{1}^{(\beta + 2)}}{\beta + 2} \right) - a b^{2} \alpha \beta \left( \frac{T T_{1}^{(\beta + 2)}}{2} + \frac{T_{1}^{(\beta + 3)}}{3} \right) \\ &+ a b T T_{1}^{2} - \frac{a b^{3} T_{1}^{3}}{3} + a b^{3} T T_{1} \left( T - \frac{T_{1}}{2} \right) \right\} \end{aligned}$$

The total variable cost per unit time, *TCU*, during the cycle period [0, *T*] is given by:

$$\begin{split} TCU_{1} &= \frac{TVC_{1}}{T} = TVC_{1} = C_{0} \bigg( \frac{e^{KH} - 1}{e^{KT} - 1} \bigg) \Bigg[ a \big( \alpha \beta T_{1}^{\beta} + b \big) \Biggl\{ h \bigg\{ e - \alpha \beta T_{1}^{\beta} - \frac{e^{\frac{\alpha \beta}{\beta + 1} T_{1}^{\beta + 1} + b T_{1} + 1}}{\frac{\alpha \beta}{\beta + 1} T_{1}^{\beta + 1} + b T_{1} + 1} - T_{1} \bigg) \Biggr\} \\ &+ C_{3} + i_{\rho} \Bigg\{ \big( \alpha \beta T_{1}^{\beta} + b \big) \bigg\{ \frac{e^{\frac{\alpha \beta}{\beta + 1} (T_{1}^{\beta + 1} - M^{\beta + 1}) + b (T_{1} - M) + 1}}{\frac{\alpha \beta}{\beta + 1} (T_{1}^{\beta + 1} - M^{\beta + 1}) + b (T_{1} - M) + 1} \bigg) - e \big( \alpha \beta T_{1}^{\beta} + b \big) \Biggr\} \\ &+ C_{2} \bigg\{ \frac{a}{\delta} \bigg[ \log \bigg\{ \frac{1}{\delta} + \delta \big( T - T_{1} \big) - \big( T - T_{1} \big) \bigg\} \bigg] \bigg\} \\ &+ \bigg\{ \frac{\alpha \alpha T_{1}^{\beta + 1}}{\beta + 1} + \frac{b \alpha T_{1}^{\beta + 2}}{\beta + 2} \bigg\} - i_{c} \Bigg\{ \frac{a T_{1}^{2}}{2} + \frac{a b \alpha^{2} \beta^{2}}{\beta + 1} \bigg[ T_{1}^{2(\beta + 1)} - T T_{1}^{2(\beta + 1)} - \frac{T_{1}^{(\beta + 3)}}{2} + \frac{T_{1}^{(2\beta + 3)}}{\beta + 1} \bigg) \\ &- \frac{a b \alpha \beta}{\beta + 1} \bigg( \frac{T_{1}^{(\beta + 2)}}{\beta + 2} - b T_{1}^{(\beta + 2)} + \frac{T_{1}^{(\beta + 2)}}{\beta + 2} \bigg) - a b^{2} \alpha \beta \bigg( \frac{T T_{1}^{(\beta + 2)}}{2} + \frac{T_{1}^{(\beta + 3)}}{3} \bigg) \\ &+ a b T T_{1}^{2} - \frac{a b^{3} T_{1}^{3}}{3} + a b^{3} T T_{1} \bigg( T - \frac{T_{1}}{2} \bigg) \bigg\} \bigg] / T \end{split}$$

## 3.2. Case II (T < M): Payment after depletion

The ordering cost  $C_3$ , the holding cost HC, the shortage cost SHC, and the deterioration cost DC during the cycle period (0, *T*) is the same as in case I. The payable per cycle is  $P_T = 0$  when  $T_1 < M < T$  because the supplier can be paid in full (at time *M*) the permissible delay. The interest earned per cycle is:

$$IE_{2} = i_{e}C_{0} \cdot \left(\frac{e^{KH}-1}{e^{KT}-1}\right) \left\{ \int_{0}^{T_{1}} (T_{1}-t)(a+bq(t))dt + (M-T_{1})\int_{0}^{T_{1}} (a+bq(t))dt \right\}$$
(19)

$$= i_{e} \cdot C_{0} \frac{e^{KH} - 1}{e^{KT} - 1} \left[ \left\{ \frac{aT_{1}^{2}}{2} + \frac{ab\alpha^{2}\beta^{2}}{\beta + 1} \left[ T_{1}^{2(\beta+1)} - T T_{1}^{2(\beta+1)} - \frac{T_{1}^{(\beta+3)}}{2} + \frac{T_{1}^{(2\beta+3)}}{\beta + 1} \right] - \frac{ab\alpha\beta}{\beta + 1} \left[ \frac{T_{1}^{(\beta+2)}}{\beta + 2} - b T_{1}^{(\beta+2)} + \frac{T_{1}^{(\beta+2)}}{\beta + 2} - ab^{2}\alpha\beta \left[ \frac{T T_{1}^{(\beta+2)}}{2} + \frac{T_{1}^{(\beta+3)}}{3} \right] + abT T_{1}^{2} - \frac{ab^{3} T_{1}^{3}}{3} + ab^{3}T T_{1} \left( T - \frac{T_{1}}{2} \right) \right\}$$

$$+ a(M - T_{1}) \left\{ T_{1}(1 - bT) - \frac{bT_{1}^{2}}{2} + \frac{\alpha\beta}{\beta + 2} \left( b\alpha\beta T_{1}^{2(\beta+1)} - T_{1}^{\beta+2} \right) \right\} \right]$$
(20)

The total variable cost, *TVC*<sub>2</sub> is defined as:

$$TVC_2 = C_3 + HC + DC + SHC - IE_2$$
(21)

h

$$\begin{aligned} TVC_{2} &= C_{0} \left( \frac{e^{KH} - 1}{e^{KT} - 1} \right) \left[ h \left\{ a \left( \alpha \beta T_{1}^{\beta} + b \right) \left( e - \alpha \beta T_{1}^{\beta} - \frac{e^{\frac{a\beta}{\beta+1}} T_{1}^{\beta+1} + b T_{1} + 1}{\frac{a\beta}{\beta+1}} - T_{1} \right) \right\} \\ &+ C_{3} + C_{2} \left\{ \frac{a}{\delta} \left[ \log \left\{ \frac{1}{\delta} + \delta \left( T - T_{1} \right) - \left( T - T_{1} \right) \right\} \right] \right\} \end{aligned}$$
(22)  
 
$$&+ \left\{ \frac{a \alpha T_{1}^{\beta+1}}{\beta+1} + \frac{b \alpha T_{1}^{\beta+2}}{\beta+2} \right\} - i_{c} \left\{ \frac{a T_{1}^{2}}{2} + \frac{a b \alpha^{2} \beta^{2}}{\beta+1} \left( T_{1}^{2(\beta+1)} - T T_{1}^{2(\beta+1)} - \frac{T_{1}^{(\beta+3)}}{2} + \frac{T_{1}^{(2\beta+3)}}{\beta+1} \right) \right. \\ &- \frac{a b \alpha \beta}{\beta+1} \left( \frac{T_{1}^{(\beta+2)}}{\beta+2} - b T_{1}^{(\beta+2)} + \frac{T_{1}^{(\beta+2)}}{\beta+2} \right) - a b^{2} \alpha \beta \left( \frac{T T_{1}^{(\beta+2)}}{2} + \frac{T_{1}^{(\beta+3)}}{3} \right) + a b T T_{1}^{2} \\ &- \frac{a b^{3} T_{1}^{3}}{3} + a b^{3} T T_{1} \left( T - \frac{T_{1}}{2} \right) \right\} \\ &+ a \left( M - T_{1} \right) \left\{ T_{1} \left( 1 - b T \right) - \frac{b T_{1}^{2}}{2} + \frac{\alpha \beta}{\beta+2} \left( b \alpha \beta T_{1}^{2(\beta+1)} - T_{1}^{\beta+2} \right) \right\} \end{aligned}$$

The total variable cost per unit time  $TCU(T_1, T)$  is:

$$\begin{split} TCU_{2} &= \frac{TVC_{2}}{T} = C_{0} \left( \frac{e^{KH} - 1}{e^{KT} - 1} \right) \left[ h \left\{ a \left( \alpha \beta T_{1}^{\beta} + b \right) \left( e - \alpha \beta T_{1}^{\beta} - \frac{e^{\frac{\alpha \beta}{\beta+1}} T_{1}^{\beta+1} + bT_{1} + 1}{\frac{\alpha \beta}{\beta+1}} - T_{1} \right) \right\} \\ &+ C_{3} + C_{2} \left\{ \frac{a}{\delta} \left[ \log \left\{ \frac{1}{\delta} + \delta \left( T - T_{1} \right) - \left( T - T_{1} \right) \right\} \right] \right\} \end{split}$$
(23)  
$$&+ \left\{ \frac{a \alpha T_{1}^{\beta+1}}{\beta+1} + \frac{b \alpha T_{1}^{\beta+2}}{\beta+2} \right\} - i_{c} \left\{ \frac{a T_{1}^{2}}{2} + \frac{a b \alpha^{2} \beta^{2}}{\beta+1} \left( T_{1}^{2(\beta+1)} - T T_{1}^{2(\beta+1)} - \frac{T_{1}^{(\beta+3)}}{2} + \frac{T_{1}^{(2\beta+3)}}{\beta+1} \right) \right. \\ &- \frac{a b \alpha \beta}{\beta+1} \left( \frac{T_{1}^{(\beta+2)}}{\beta+2} - b T_{1}^{(\beta+2)} + \frac{T_{1}^{(\beta+2)}}{\beta+2} \right) - a b^{2} \alpha \beta \left( \frac{T T_{1}^{(\beta+2)}}{2} + \frac{T_{1}^{(\beta+3)}}{3} \right) \\ &+ a b T T_{1}^{2} - \frac{a b^{3} T_{1}^{3}}{3} + a b^{3} T T_{1} \left( T - \frac{T_{1}}{2} \right) \right\} \\ &+ a \left( M - T_{1} \right) \left\{ T_{1} \left( 1 - b T \right) - \frac{b T_{1}^{2}}{2} + \frac{\alpha \beta}{\beta+2} \left( b \alpha \beta T_{1}^{2(\beta+1)} - T_{1}^{\beta+2} \right) \right\} \right] / T \end{split}$$

#### 4. Genetic algorithm

A genetic algorithm (GA) is based on natural selection process to optimized tools that minimizes the total costs in supply chain management. It is an evolutionary computation method to solve inventory problems. This is the more effective methods to find the optimized solution. The genetic algorithm uses three main types of rules at each step to create the next generation from the current population.

#### 4.1. The basic steps to find the optimized solution

Step 1: First one is *Selection rules*, In this we select the individuals, called *parents* that contribute to the population at the next generation.

Step 2: Next one is **Crossover rules**, In this, we perform crossover operation between two parents to form children for the next generation.

Step 3: Last one is *Mutation rules*, In mutation, we apply some random changes to individual parents.

#### 4.2. Parameters

Firstly, we set the different parameters on which the specific GA depends. These are the number of generations (MAXGEN), population size (POPSIZE), the probability of crossover (PCROS), probability of mutation (PMUTE).

#### 4.3. Chromosome representation

An important issue in applying a GA is to design an appropriate chromosome representation of solutions of the problem together with genetic operators. Traditional binary vectors used to represent the chromosomes are not effective in many non-linear problems. Since the proposed problem is highly non-linear, hence to overcome the difficulty, a real-number representation is used. In this representation, each chromosome V<sub>i</sub> is a string of n numbers of genes  $G_{ii}$  (j = 1, 2, ..., n) where these n numbers of genes, respectively, denote n number of decision variables (X, i = 1, 2, ... n).

#### 4.4. Initial population production

The population generation technique proposed in the present GA is illustrated by the following procedure: For each chromosome  $V_{i}$ , every gene  $G_{i}$  is randomly generated between its boundary (LB,  $UB_i$ ) where  $LB_i$  and  $UB_i$  are the lower and upper bounds of the variables  $X_i$ , i = 1, 2, ..., n, POPSIZE.

#### 4.5. Evaluation

Evaluation function plays the same role in GA as that, which the environment plays in natural evolution. Now, evaluation functions (EVAL) for the chromosome V, is equivalent to the objective function PF(X). These are steps of evaluation:

Step 1: Find EVAL(V<sub>i</sub>) by EVAL(V<sub>i</sub>) =  $f(X_1, X_2, X_3, ..., X_n)$  where the genes  $G_{ii}$  represent the decision variable  $X_{i}$ , j = 1, 2, ..., n, POPSIZE and f is the objective function.

Step 2: Find total fitness of the population:  $F = \sum_{i=1}^{POPSIZE} EVAL(V_i)$ . Step 3: Calculate the probability pi of selection for each chromosome  $V_i$  as  $Y_i = \sum_{j=1}^{i} pj$ 

#### 4.6. Selection

The selection scheme in GA determines which solutions in the current population are to be selected for recombination. Many selection schemes, such as stochastic random sampling, roulette wheel selection have been proposed for various problems. In this paper, we adopt the roulette wheel selection process.

This roulette selection process is based on spinning the roulette wheel POPSIZE times, each time we select a single chromosome from the new population in the following way:

- (a) Generate a random (float) number r between 0 and 1.
- (b) If r < Y, then the first chromosome is V otherwise select the *i*th chromosome V ( $2 \le i \le POPSIZE$ ) such that  $T_i - 1 \le r \le Y_i$ .

#### 4.7. Crossover

Crossover operator is mainly responsible for the search of the new string. The exploration and exploitation of the solution space are made possible by exchanging genetic information of the current chromosomes. Crossover operates on two parent solutions at a time and generates off spring solutions by recombining both parent solution features. After selection of chromosomes for the new population, the crossover operator is applied. Here, the whole arithmetic crossover operation is used. It is defined as a linear combination of two consecutive selected chromosomes  $V_m$  and  $V_a$  and the resulting offspring's  $V'_m$  and are  $V'_n$  calculated as:

$$V'_m = c \cdot V_m + (1 - c) \cdot V_n$$

$$V_n' = c \cdot V_n + (1 - c) \cdot V_m$$

where c is a random number between 0 and 1.

#### 4.8. Mutation

Mutation operator is used to prevent the search process from converging to local optima rapidly. It is applied to a single chromosome  $V_r$ . The selection of a chromosome for mutation is performed in the following way:

Step 1. Set i  $\leftarrow$  1.

Step 2. Generate a random number u from the range [0, 1].

Step 3. If u < PMUTE, then go to Step 2.

Step 4. Set i  $\leftarrow i + 1$ .

Step 5. If  $i \leq$  POPSIZE, then go to Step 2.

Then the particular gene  $G_{ij}$  of the chromosome  $V_p$  selected by the above-mentioned steps is randomly selected. In this problem, the mutation is defined as  $G_{ij}^{mut}$  = random number from the range [0, 1].

#### 4.9. Termination

If the number of iterations is less than or equal to MAXGEN then the process is going on, otherwise it terminates. The GA's procedure is given below:

```
begin
do {
  t ← 0
  while (all constraints are not satisfied)
  {
     initialize Population (t)
  }
     evaluate Population(t)
    while (not terminate)
    {
    t ← t + 1
       select Population(t) from Population(t-1)
       crossover and mutate Population(t)
       evaluate Population(t)
    }
  print Optimum Result
}
end.
```

#### 5. Numerical example and sensitivity analysis

In this paper, the ordering policies have been discussed in two scenarios: payment before total depletion (Case I) and payment after total depletion (Case II). An example is considered to illustrate the effect of the developed model in this paper.

The following inventory parametric values are using  $a = 600, b = 70, \alpha = 0.00010, \beta = 1.0, M = 0.1, \delta = 5.0, C_0 = 0.5, i_e = 0.18, i_n = 0.20, K = 0.1, H = 1 \text{ year}, h = \$ 2.00/\text{unit}, C_3 = 100.0, C_2 = 0.8/\text{unit}.$ 

To solve this problem we used genetic algorithm. In this problem, GA consists of the parameters, POPSIZE = 50, MAXGEN = 50, Cross over probability = 0.75, Mutation probability = 0.005. The solutions of two cases for different parametric values of  $\alpha$ ,  $\beta$ , M and  $\delta$ , are given in Tables 1 and 2.

If we plot the total cost function *TCU*1 and *TCU*2 with some values of  $T_1$  and *T* vs. various parameters then we get strictly convex graph of total cost function given in Figures 3 and 4.

From the above tables and figures, the results can be discussed as follows:

- 1. Table 1 indicates as the values of  $T_1$  and T reduce when the parameter  $\alpha$  increases, but the total cost function (*TCU*) increases in both cases.
- 2. The values of  $T_1$ , T decrease when the parameter  $\beta$  increases, and total cost function (*TCU*) increases in both cases.
- 3. The values of  $T_1$ , T increase when the parameter *M* increases, and the total average inventory cost (*TCU*) decreases both in cases.
- 4. In Table 2 cycle time T decreases as the parameter  $\delta$  increases, but total cost function (TCU) increases in both the cases.

Table 1. Case I: Payment before depletion							
Changing parameters	Change in parameter	T <sub>1</sub>	Т				
α	0.00010	0.0822	0.0883	4585.08			
	0.00015	0.0716	0.0853	6375.53			
	0.00020	0.0593	0.0676	7845.87			
в	1.0	0.0822	0.0883	4585.08			
	1.5	0.0721	0.0813	6405.02			
	2.0	0.0543	0.0759	6968.14			
М	0.1	0.0822	0.0883	4585.08			
	0.3	0.0919	0.0953	3695.52			
	0.5	0.0958	0.1053	2655.38			
δ	5.0	0.0822	0.0853	4585.08			
	6.0	0.0873	0.0927	4915.57			
	7.0	0.0892	0.1073	5289.64			

Table 2. Case II: Payment after depletion						
Changing parameters	Change in parameter	<b>T</b> <sub>1</sub>	Т	TCU <sub>2</sub>		
α	0.00010	0.0926	0.1033	3245.50		
	0.00015	0.0816	0.0953	4175.53		
	0.00020	0.0798	0.0886	4855.84		
в	1.0	0.0926	0.1033	3245.50		
	1.5	0.0791	0.0913	3405.02		
	2.0	0.0643	0.0759	4968.14		
Μ	0.10	0.0926	0.1033	3245.50		
	0.12	0.1004	0.1243	2295.56		
	0.13	0.1058	0.1355	2055.30		
δ	5.0	0.0926	0.1033	3245.50		
	6.0	0.973	0.0927	3985.67		
	7.0	0.0976	0.0873	4287.24		

Figure 3. Effect of parameters  $\alpha$ ,  $\beta$ , M, and  $\delta$  on total cost before the deflation.



Figure 4. Effect of parameters  $\alpha$ ,  $\beta$ , M, and  $\delta$  on total cost after the deflation.

In Tables 1 and 2 the parameters  $\alpha$ ,  $\beta$  have a bigger influence than the parameters M and  $\delta$ . Therefore, the parameters  $\alpha$ ,  $\beta$  are necessitate privileged compassion toward the cycle time as well as total average inventory cost.

#### 6. Conclusion

In the present article, we have designed a model has been illustrated for determination of optimal ordering time and total cost with stock dependent demand for deteriorating items following the Weibull distribution. Two cases specifically (i) payment before depletion and (ii) payment after depletion have been taken into account for the consideration of the model which can assist the decision-maker to find the optimal cycle time to minimize the total average inventory cost. From the sensitivity analysis, it is observed that as the rate of deterioration ( $\alpha$  and  $\beta$ ) and backlogging rate  $\delta$  increases, the total average inventory cost increases, which is obvious. Moreover it is also observed that as permissible delay increases, the total average inventory cost decreases i.e. there is an

opposite relation between permissible delay period and the total average inventory cost. The sensitive motive behind this is that the conservatory in permissible delay period offers an opportunity to the consumer to earn more by investing the resource otherwise from the sale proceeds of the inventory which result in lower cost.

A further study would be to extend the purposed model for finite replenishment rate, price dependent demand, fuzzy demand, variable lead time, and many more.

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#### References

Aggarwal, S. P., & Jaggi, C. K. (1995). Ordering policies of deteriorating items under permissible delay in payments. *Journal of the Operational Research Society*, 46, 658–662. http://dx.doi.org/10.1057/jors.1995.90

Berrotoni, J. N. (1962). Practical applications of Weibull distribution. ASQC. Technical Conference Transactions, 303–323.

- Bhahmbhatt, A. C. (1982). Economic order quantity under variable rate of inflation and mark-up prices. *Productivity*, 23, 127–130.
- Bierman, H., & Thomas, J. (1977). Inventory decisions under inflationary conditions. *Decision Sciences*, 8, 151–155. http://dx.doi.org/10.1111/deci.1977.8.issue-1
- Buzacott, J. A. (1975). Economic order quantities with inflation. Operational Research Quaterly, 26, 1188–1191.
- Chakrabarty, T., Giri, B. C., & Chaudhuri, K. S. (1998). An EOQ model for items with Weibull distribution deterioration, shortages and trended demand: An extension of Philip's model. *Computers and Operations Research*, *25*, 649–657. http://dx.doi.org/10.1016/S0305-0548(97)00081-6
- Chandra, J. M., & Bahner, M. L. (1988). The effects of inflation and the value of money. *International Journal of Production Economics*, 23, 723–730.
- Chung, K. J. (2003). An algorithm for an inventory model with inventory-level-dependent demand rate. *Computers & Operations Research*, 30, 1311–1317. http://dx.doi.org/10.1016/S0305-0548(02)00072-2
- Chung, K. J., & Huang, Y. F. (2003). The optimal cycle time for EPQ inventory model under permissible delay in payments. *International Journal of Production Economics*, 84, 307–318.

http://dx.doi.org/10.1016/S0925-5273(02)00465-6

- Chung, K. J., & Huang, T.-S. (2005). The algorithm to the EOQ model for Inventory control and trade credit. *Journal of the Operational Research Society*, 42, 16–27.
- Chung, K. J., Huang, Y. F., & Huang, C. K. (2002). The replenishment decision for EOQ inventory model under permissible delay in payments. *Opsearch*, 39, 327–340.
- Covert, R. P., & Philip, G. C. (1973). An EOQ model for items with weibull distribution deterioration. AIIE Transactions, 5, 323–326. http://dx.doi.org/10.1080/05695557308974918
- Datta, T. K., Paul, K., & Pal, A. K. (1998). Demand promotion by upgradation under stock-dependent demand situation—A model. International Journal of Production Economics, 55, 31–38. http://dx.doi.org/10.1016/S0925-5273(98)00033-4
- Dave, U., & Patel, L. K. (1983). (T.So) policy inventory model for deteriorating items with time proportional demand. *Journal of the Operational Research Society*, 20, 99–106.
- Deb, M., & Chaudhuri, K. S. (1986). An EOQ model for items with finite rate of production and variable rate of deterioration. Opsearch, 23, 175–181.
- Dye, C. Y. (2002). A deteriorating inventory model with stockdependent demand and partial backlogging under conditions of permissible delay in payments. *Opsearch*, 39, 189–201.
- Goyal, S. K. (1985). Economic order quantity under conditions of permissible delay in payments. *Journal of the Operational Research Society*, 36, 335–338. http://dx.doi.org/10.1057/jors.1985.56
- Goyal, S. K., & Chang, C. T. (2009). Optimal ordering and transfer policy for an inventory with stock-dependent demand. European Journal of Operational Research, 196, 177–185. http://dx.doi.org/10.1016/j.ejor.2008.02.029
- Hwang, H., & Shinn, S. W. (1997). Retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments. *Computers and Operations Research*, 24, 539–547. http://dx.doi.org/10.1016/S0305-0548(96)00069-X
- Jamal, A. M., Sarker, B. R., & Wang, S. (1997). An ordering policy for deteriorating items with allowable shortage and permissible delay in payment. *Journal of the Operational Research Society*, 48, 826–833. http://dx.doi.org/10.1057/palarave.jors.2600428
- Jesse, R. R., Mitra, A., & Cox, J. F. (1983). EOQ formula: Is it valid under inflationary conditions? *Decision Sciences*, 14,
- 370–374. http://dx.doi.org/10.1111/deci.1983.14.issue-3 Kumar, N., & Kumar, S. (2016a). Effect of learning and salvage worth on an inventory model for deteriorating items with inventory-dependent demand rate and partial backlogging with capability constraints. Uncertain Supply Chain Management, 4, 123–136.
- http://dx.doi.org/10.5267/j.uscm.2015.11.002 Kumar, N., & Kumar, S. (2016b). Inventory model for noninstantaneous deteriorating items, stock dependent demand, partial backlogging, and inflation over a finite time horizon. *International journal of Supply and operations Management*, 3, 1168–1191.
- Kumar, N., Singh, S. R., & Kumari, R. (2013). Learning effect on an inventory model with two-level storage and partial backlogging under inflation. *International Journal of Services and Operations Management*, 16, 105–122. http://dx.doi.org/10.1504/IJSOM.2013.055575

Kumar, N., Singh, S. R., & Kumari, R. (2014). Effect of salvage value on a two-warehouse inventory model for deteriorating items with stock-dependent demand rate and partial backlogging. *International Journal of Operational Research*, 19, 479–496. http://dx.doi.org/10.050/1160000116

#### http://dx.doi.org/10.1504/IJOR.2014.060415

Liao, H. C., Tsai, C. H., & Su, C. T. (2000). An inventory model with deteriorating items under inflation when a delay in payment is permissible. *International Journal of Production Economics*, 63, 207–214. http://dx.doi.org/10.1016/S0925-5273(99)00015-8

Mandal, M., & Maiti, M. (1999). Inventory of damagable items with variable replenishment rate, stock-dependent demand and some units in hand. Applied Mathematical Modelling, 23, 799–807.

http://dx.doi.org/10.1016/S0307-904X(99)00018-9

Mandal, B. N., & Phaujdar, S. (1989). An inventory model for deteriorating items and stock-dependent consumption rate. *Journal of the Operational Research Society*, 40, 483–488. http://dx.doi.org/10.1057/jors.1989.75

Misra, R. B. (1975). Optimum production lot-size model for a system with deteriorating inventory. *International Journal* of Production Research, 13, 495–505. http://dx.doi.org/10.1080/00207547508943019

- Misra, R. B. (1979). A note on optimal inventory management under inflationfl. Naval Research Logistics Quarterly, 26, 161–165. http://dx.doi.org/10.1002/(ISSN)1931-9193
- Padmanabhan, G., & Vrat, P. (1995). EOQ models for perishable items under stock dependent selling rate. European Journal of Operational Research, 86, 281–292.
- Philip, G. C. (1974). A generalized EOQ model for items with weibull distribution deterioration. *AIIE Transactions*, 6, 159–162.

http://dx.doi.org/10.1080/05695557408974948

Rau, H. L., Wu, M. Y., & Wee, H. M. (2004). Deteriorating item inventory model with shortage due to supplier in an integrated supply chain. *International Journal of Systems Science*, 35, 293–303.

http://dx.doi.org/10.1080/00207720410001714833

Ray, J., & Chaudhuri, K. S. (1997). An EOQ model with stock-dependent demand, shortage, inflation and time discounting. *International Journal of Production Economics*, 53, 171–180.

#### http://dx.doi.org/10.1016/S0925-5273(97)00112-6

- Roychowdhury, M., & Chaudhuri, K. S. (1983). An order-level inventory model for deteriorating items with finite rate of replenishment. *Opsearch*, 20, 99–106.
- Shah, N. H., & Shah, Y. K. (1998). A discrete-in-time probabilistic inventory model for deteriorating items under conditions of permissible delay in payments. *International Journal of Systems Science*, 29, 121–125.

#### http://dx.doi.org/10.1080/00207729808929504

Skouri, K., & Papachristos, S. (2003). Optimal stopping and restarting production times for an EOQ model with deteriorating items and time-dependent partial backlogging. International Journal of Production Economics, 81–82, 525–531.

#### http://dx.doi.org/10.1016/S0925-5273(02)00356-0

Tayal, S., Singh, S. R., & Sharma, R. (2015). An inventory model for deteriorating items with seasonal products and an option of an alternative market. Uncertain Supply Chain Management, 3, 69–86.

#### http://dx.doi.org/10.5267/j.uscm.2014.8.003

Teng, J. T., & Chang, C. T. (2005). Economic production quantity models for deteriorating items with price and stock dependent demand. *Computers & Operations Research*, 32, 297–308.

#### http://dx.doi.org/10.1016/S0305-0548(03)00237-5

Yang, H. L., Teng, J. T., & Chern, M. S. (2010). An inventory model under inflation for deteriorating items with stockdependent consumption rate and partial backlogging shortages. *International Journal of Production Economics*, 123, 8–19. http://dx.doi.org/10.1016/j.ijpe.2009.06.041

Zhou, Y. W., & Yang, S. L. (2005). A two-warehouse inventory model for items with stock-level-dependent demand rate. International Journal of Production Economics, 95, 215–228. http://dx.doi.org/10.1016/j.ijpe.2003.12.007



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