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Article

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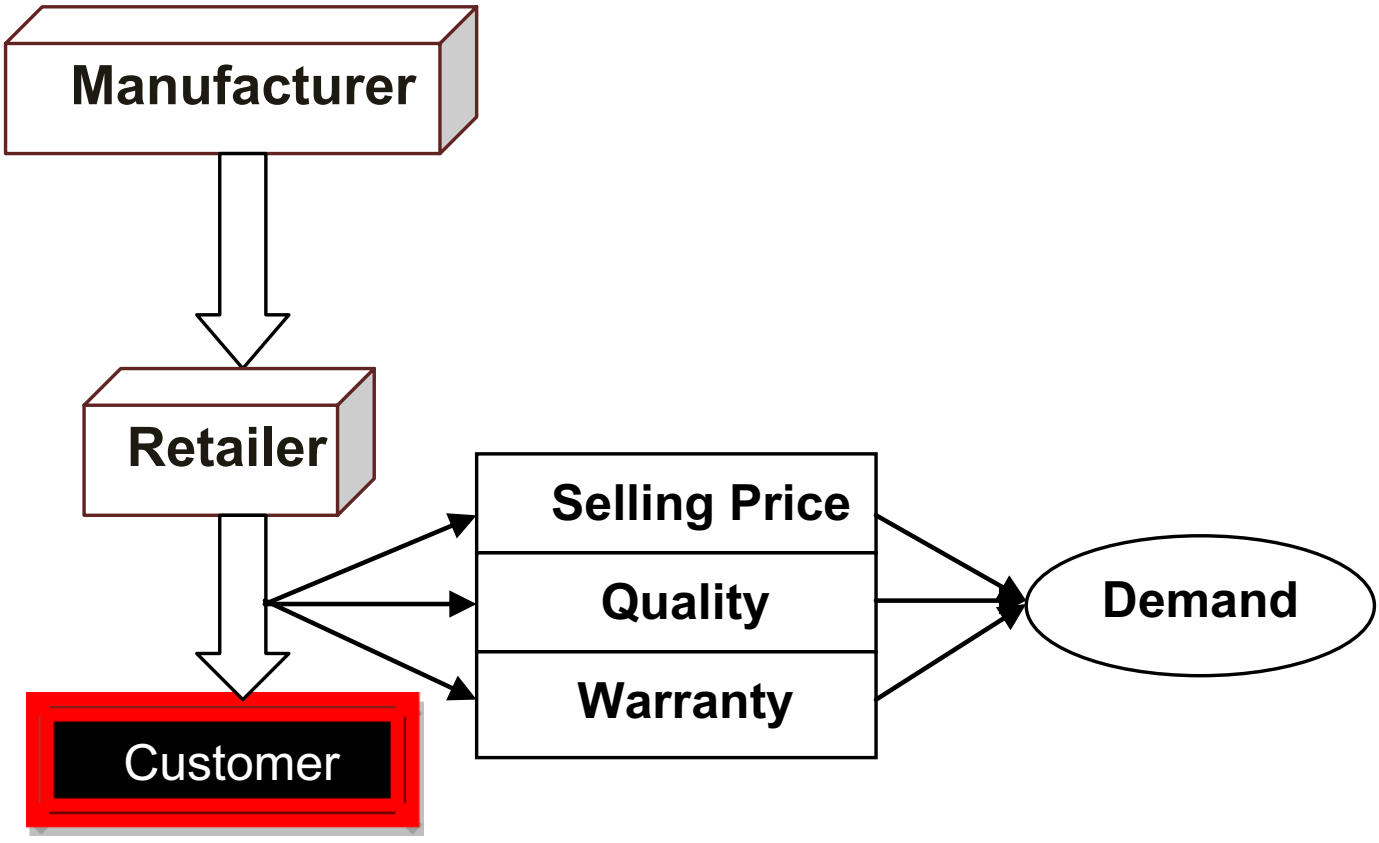

MANAGEMENT | RESEARCH ARTICLE

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Managing a two-echelon supply chain with price, warranty and quality dependent demand

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Abstract: This paper deals with a two-layer supply chain composed of one manufacturer and one retailer for single-type product. The demand function of the end customers depends on quality, warranty, and sales price of the product. The profit functions of the manufacturer and the retailers are maximized under centralized and decentralized approaches. Our study suggests that the joint profit in centralized system is always more than the decentralized system. Finally, the surplus profit in centralized system is shared according to their profits in decentralized system.

Subjects: Behavioral Sciences; Economics; Finance; Business & Industry; Social Sciences

Keywords: pricing; quality; warranty; supply chain

1. Introduction

In competitive business environment, supply chain management has attracted scientists and industrial engineers owing to meet effectively the customers' demand. The customers are more interested for the products which have better quality, and more warranty period in a reasonable price. To improve customer satisfaction, the good management has given emphasize on the aspect of quality in supply chain management. Consequently, quality is a measure of excellence or desirable characteristics of a

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PUBLIC INTEREST STATEMENT

This paper deals with a two-layer supply chain composed of one manufacturer and one retailer for single-type product. It helps to the management of firm/industry how to maintain quality, warranty, and sales price of the product such that total profit of the chain is maximized. Finally, the surplus profit in centralized system is shared according to their profits in decentralized system.

product. Maintenance of good quality level enhances goodwill of the customers that results in higher demand of the customers. On the other hand, below quality product not only harms firm's goodwill but also spreads negative effect on the consumers' demand. However, in maintaining a high-quality level in a global market, complexity in supply chain has become increasingly challenging issue among others. In today's tight production schedules and just-in-time inventory strategies, a quality problem along the supply chain is a very serious issue that significantly impacts on brand integrity and the bottom line. Increasing the number of quality checks along the supply chain may improve products' quality. Although this process slows down the production rate, it may be restored by employing more labour and inventory costs. In order to strike the delicate balance between quality, cost, and managing product complexity, many manufacturers must now consider new ways to generate efficiency. This includes streamlining operations, performing continuous improvement checks, and improving the way they work with suppliers and partners all over the world.

A warranty is a representation made by a seller or company to a consumer of a product which involves refund, repair, or replacement, if the product faces defective or unsatisfactory performance within a given time period. The majority of customers prefers to purchase a product from a manufacturer with a warranty ensuring the replacement or repairing of the product during the warranty period. A warranty is therefore an effective incentive for customers to purchase a product from a manufacturer among other similar products' quality and reliability of other manufacturers. Price of product has also a direct effect on the consumers' demand. To address these vital issues, we consider the demand of the products which depends not only on price but also on product quality and warranty period. The inclusion of warranty and quality cost have significant impacts on the manufacturer's production policy.

In this paper, we consider a traditional two-level supply chain composed of a manufacturer and a retailer. The manufacturer follows lot for lot production policy. Lead time is assumed as zero. The model investigates profit maximization problem for a demand function which addresses three major issues of a supply chain namely, price, quality level, and warranty period. Maintaining of high quality causes higher cost of technology. The model considers that cost of raw material varies with quality level, i.e. cost of raw material increases when quality level increases and it decreases when level of quality decreases. We restrict the model by introducing optimum level of quality, maximum warranty period, and manufacturer's suggested retail price. Under these difficult scenarios, we develop decentralized and centralized model and find concavity conditions. The model also illustrates through a numerical example. Finally, a profit sharing scheme is discussed for channel coordination.

2. Literature review

Product warranty has become increasingly more and more important in consumer and commercial dealings, and it is widely used to serve many different purposes (Karim & Suzuki, 2005; Wu, 2012; 2013). There are different types of warranties that are used in literature of warranty management in supply chain (Murthy & Djamaludin, 2002), such as (i) free replacement (Rinsaka & Sandoh, 2006; Zhou, Li, & Tang, 2009), (ii) money back in addition to free replacement (Boom, 1998), (iii) outsourcing services (Asgharizadeh & Murthy, 2000; Jackson & Pascual, 2008), etc. Hartman and Laksana (2008) have discussed some warranty contracts including restrictions on repairs and renewals. Sana (2012) has studied an imperfect production system with free minimal repair warranty, allowing shortages due to regular preventive maintenance. Cardenas-Barron and Trevino-Garza (2014) have found out an optimal solution for multiproduct and multiperiod in a three-echelon supply chain network. Cardenas-Barron and Sana have (2014) investigated an issue of channel coordination for a two-echelon supply chain composed of one manufacturer and one retailer. Wu (2014) has developed an optimization model of warranty return policy focusing on no-fault found phenomenon. Esmaeili, Shamsi Gamchi, and Asgharizadeh (2014) has presented a various three-level warranty service contracts in order to obtain a better result using the game theoretical approach. Taleizadeh, Noori-daryan, and Cardenas-Barron (2015) have developed a vendor managed inventory system of deteriorating items for joint optimization of price, replenishment frequency, replenishment cycle, and production rate.

In recent years, quality management has received considerable attention to improve operations performance. Starbird (2001) has developed supply chain contracts considering penalties, rewards, and inspection: provisions for quality. In this model, they have examined the effects of rewards, penalties, and inspection policies on the behavior of an expected cost minimization of the supplier. Kannan and Tan (2005) have empirically examined the extent to which just in time, supply chain management, and quality management are correlated, and how they impact on business performance. They have showed that a commitment to quality and an understanding of supply chain dynamics have the greatest effect on performance.

Pricing is an important decision for the profitability of an enterprise and also plays significant role in demand (Cardenas-Barron, 2012; Kalton & Singh, 1992; Sarkar, Saren, & Wee, 2013). Coordinating pricing decision in supply chain under different channel structure has been extensively studied in the marketing and operation management literature. Variety of coordination contracts (eg. quantity discount (Li & Liu, 2006; Panda, Modak, & Pradhan 2014), two-part tariff (Goering, 2012), revenue sharing (Cachon & Lariviere, 2005), sales rebate (Wong, Qi, & Leung, 2009), buy back (Ding & Chen, 2008), compensation on disposal cost (Panda, Modak, & Basu, 2014), etc. have been used in supply chains as the ways of cutting out channel conflict.

Although pricing decision, warranty period, and level of quality have been widely studied separately in the literature, all these factors altogether have an effect on demand and profitability of an enterprise. This study empirically examines the extent to which pricing decision, warranty period, and level of quality are incorporated in demand, and how they impact on business performance. In this study, we consider a manufacturer–retailer supply chain only. Demand of the product depends on selling price of the product, length of warranty period, and level of quality.

3. Notation

The following notations are used to develop the proposed model.

θ	Quality parameter
$C(\theta) = A_0 + A_1(\theta - \theta_{\min})^2$	Cost of technology
$C_r = \beta_1 + \beta_2(\theta - \theta_{\min})$	Cost of raw material per unit item varies with quality
p_m	Suggested selling price of the retailer by the manufacturer
p_r	Selling price of the retailer
w	Selling price of the manufacturer to the retailer
ρ	Warranty period
$\rho_{\max}$	Maximum warranty period
$\delta(\rho) = \epsilon\left(\frac{\rho}{\rho_{\max}}\right), (\epsilon < 1)$	Percentage of defectives with in ρ
c_0	repairing cost of defective item
$D(p_r, \theta, \rho) = (1 + \frac{\alpha\rho}{\rho_{\max}})(k_0 + k_1(\theta - \theta_{\min}) - k_2(p_r - p_m)), (\alpha < 1)$	demand function

4. The model

4.1. Decentralized decision

In decentralized decision-making, the manufacturer and the retailer are interested in individual profit maximization. Total profit function of the manufacturer is

$$f_1(w, \theta, \rho) = wD(p_r, \theta, \rho) - \epsilon(\frac{\rho}{\rho_{\max}})D(p_r, \theta, \rho)c_0 - [\beta_1 + \beta_2(\theta - \theta_{\min})] D(p_r, \theta, \rho) - A_0 - A_1(\theta - \theta_{\min})^2 \quad (1)$$

Total profit function of the retailer is

$$f_2(p_r) = (p_r - w)D(p_r, \theta, \rho) \quad (2)$$

Interaction between the manufacturer and the retailer is considered as a Stackelberg game where the manufacturer is a stackelberg leader. The retailer follows the manufacturer's move and reacts by playing the best move consistent with available information. Objective of leader is to design own move in such a way to maximize own revenue, considering all rational moves follower as a devise. In this game, the retailer maximizes own profit margin depending on manufacturer's wholesale price, level of quality, and warranty of the product. To determine the optimum strategies, we use the backward induction process as follows. For given w, θ, and ρ, the retailer first optimizes its profit function. The necessary condition of optimization of the retailer's profit function, i.e. $\frac{df_2}{dp_r} = 0$ yields

$$p_r = \frac{k_0 + k_2(p_m + w) + k_1(\theta - \theta_{\min})}{2k_2} \tag{3}$$

From Equation 3, we have $dp_r/d\theta = k_1/2k_2 > 0$, i.e. selling price of the retailer increases with increasing quality of the product. One may note that there is no effect of ρ, the decision of warranty of the manufacturer on selling price of the retailer. In response to the retailer's decision, the manufacturer has to maximize its profit function, $f_1(w, \theta, \rho)$, under the condition $0 \le \rho \le \rho_{\max}$, $\theta \ge \theta_{\min}$, and $w < p_m$. Thus, the manufacturer has the following constrained optimization problem

$$\begin{aligned} &\text{Maximize} \quad f_1(w, \theta, \rho) \\ &\text{Subject to} \quad p_r = \frac{k_0 + k_2(p_m + w) + k_1(\theta - \theta_{\min})}{2k_2} \\ &\qquad 0 \le \rho \le \rho_{\max} \\ &\qquad \theta \ge \theta_{\min} \\ &\qquad w < p_m \end{aligned}$$

Solving the above-constrained optimization problem, we have

$$w^* = \frac{(k_1\beta_1 - (k_0 + k_2 p_m)\beta_2)}{(k_1 - k_2\beta_2)} - \frac{c_0\left(k_1^3 - 12A_1k_1k_2 - 2k_2\left(k_1^2 + 3A_1k_2\right)\beta_2 + k_1k_2^2\beta_2^2\right)\epsilon}{\alpha(k_1 - k_2\beta_2)^3} - \frac{2(2k_1 + \beta_2k_2)T}{\alpha^2\rho_{\max}(k_1 - k_2\beta_2)^3} \tag{4}$$

$$\theta^* = \frac{\begin{array}{c}\alpha\rho_{\max}[c_0k_2\left(18A_1k_2 - (k_1 - k_2\beta_2)^2\right)\epsilon - \alpha(k_1 - k_2\beta_2)^2 \\ (k_0 - k_1\theta_{\min} + k_2(p_m - \beta_1 + \beta_2\theta_{\min}))] - 6k_2T\end{array}}{\alpha^2\rho_{\max}(k_1 - k_2\beta_2)^3} \tag{5}$$

$$\rho^* = \frac{\left(6A_1k_2 - (k_1 - k_2\beta_2)^2\right)c_0\alpha\epsilon\rho_{\max} - 2T}{c_0\alpha^2(k_1 - k_2\beta_2)^2\epsilon} \tag{6}$$

where $T = \sqrt{-A_1c_0\alpha^2\epsilon\left(\alpha(k_0 + k_2(p_m - \beta_1))(k_1 - k_2\beta_2)^2 + c_0k_2\left(-9A_1k_2 + (k_1 - k_2\beta_2)^2\right)\epsilon\right)\rho_{\max}^2}$

Now, to verify the concavity of the profit function of the manufacturer in decentralized scenario and whether it has a unique maximum, we compute the Hessian matrix of the manufacturer profit function as follows:

$$H_m = \begin{pmatrix} \frac{\partial^2 f_1}{\partial w^2} & \frac{\partial^2 f_1}{\partial w \partial \rho} & \frac{\partial^2 f_1}{\partial w \partial \theta} \\ \frac{\partial^2 f_1}{\partial \rho \partial w} & \frac{\partial^2 f_1}{\partial \rho^2} & \frac{\partial^2 f_1}{\partial \rho \partial \theta} \\ \frac{\partial^2 f_1}{\partial \theta \partial w} & \frac{\partial^2 f_1}{\partial \theta \partial \rho} & \frac{\partial^2 f_1}{\partial \theta^2} \end{pmatrix}$$

Differentiating f_1 partially, We have

$$\frac{\partial^2 f_1}{\partial w^2} = -k_2\left(1+\frac{\alpha\rho}{\rho_{\max}}\right)$$

$$\frac{\partial^2 f_1}{\partial \rho^2} = -\frac{2c_0\alpha\epsilon\left(k_0-k_2\left(-p_m+\frac{k_0+k_2(p_m+w)+k_1(\theta-\theta_{\min})}{2k_2}\right)+k_1(\theta-\theta_{\min})\right)}{\rho_{\max}^2}$$

$$\frac{\partial^2 f_1}{\partial \theta^2} = -2A_1 - k_1\beta_2\left(1+\frac{\alpha\rho}{\rho_{\max}}\right)$$

$$\frac{\partial^2 f_1}{\partial w\partial \rho} = \frac{c_0k_2\alpha\epsilon\rho}{2\rho_{\max}^2} - \frac{k_2w\alpha}{2\rho_{\max}} + \frac{\alpha\left(k_0-k_2\left(-p_m+\frac{k_0+k_2(p_m+w)+k_1(\theta-\theta_{\min})}{2k_2}\right)+k_1(\theta-\theta_{\min})\right)}{\rho_{\max}} + \frac{k_2\alpha(\beta_1+\beta_2(\theta-\theta_{\min}))}{2\rho_{\max}} + \frac{c_0k_2\epsilon\left(1+\frac{\alpha\rho}{\rho_{\max}}\right)}{2\rho_{\max}}$$

$$\frac{\partial^2 f_1}{\partial w\partial \theta} = \frac{1}{2}k_1\left(1+\frac{\alpha\rho}{\rho_{\max}}\right)+\frac{1}{2}k_2\beta_2\left(1+\frac{\alpha\rho}{\rho_{\max}}\right)$$

$$\frac{\partial^2 f_1}{\partial \rho\partial \theta} = -\frac{c_0k_1\alpha\epsilon\rho}{2\rho_{\max}^2} + \frac{k_1w\alpha}{2\rho_{\max}} - \frac{\alpha\beta_2\left(k_0-k_2\left(-p_m+\frac{k_0+k_2(p_m+w)+k_1(\theta-\theta_{\min})}{2k_2}\right)+k_1(\theta-\theta_{\min})\right)}{\rho_{\max}} - \frac{k_1\alpha(\beta_1+\beta_2(\theta-\theta_{\min}))}{2\rho_{\max}} - \frac{c_0k_1\epsilon\left(1+\frac{\alpha\rho}{\rho_{\max}}\right)}{2\rho_{\max}}$$

Now, at (w^*, ρ^*, θ^*), we have

$$|H_m| = \frac{2A_1k_2\left(54A_1^2c_0^2k_2^3\alpha\epsilon^2\rho_{\max}+(k_1-k_2\beta_2)^2(\alpha(k_0+k_2(p_m-\beta_1))+c_0k_2\epsilon)T - 6A_1c_0k_2\epsilon\left(\alpha(k_1-k_2\beta_2)^2[\alpha(k_0+k_2(p_m-\beta_1))+c_0k_2\epsilon]\rho_{\max}+3k_2T\right)\right)}{\alpha(k_1-k_2\beta_2)^4\rho_{\max}^3}$$

Second-order principle minors are as follows

$$|H_{12}| = \begin{vmatrix} \frac{\partial^2 f_1}{\partial w^2} & \frac{\partial^2 f_1}{\partial w\partial \rho} \\ \frac{\partial^2 f_1}{\partial \rho\partial w} & \frac{\partial^2 f_1}{\partial \rho^2} \end{vmatrix}$$

$$= \frac{3A_1c_0k_2^2\epsilon\left(-\alpha^2(k_0+k_2(p_m-\beta_1))(k_1-k_2\beta_2)^2\rho_{\max} - c_0k_2\alpha\left(-18A_1k_2+(k_1-k_2\beta_2)^2\right)\epsilon\rho_{\max}-6k_2T\right)}{\alpha(k_1-k_2\beta_2)^4\rho_{\max}^3}$$

$$|H_{23}| = \begin{vmatrix} \frac{\partial^2 f_1}{\partial \rho^2} & \frac{\partial^2 f_1}{\partial \theta\partial \rho} \\ \frac{\partial^2 f_1}{\partial \rho\partial \theta} & \frac{\partial^2 f_1}{\partial \theta^2} \end{vmatrix}$$

$$= \frac{A_1c_0k_2\epsilon\left(c_0k_2\alpha\left(\beta_2(k_1-k_2\beta_2)^2(-4k_1+k_2\beta_2)+6A_1K\right)\epsilon\rho_{\max}\right)}{\alpha(k_1-k_2\beta_2)^4\rho_{\max}^3} - \frac{A_1c_0k_2\epsilon\left(\alpha^2(k_0+k_2(p_m-\beta_1))\beta_2(k_1-k_2\beta_2)^2(4k_1-k_2\beta_2)\rho_{\max}+2TK\right)}{\alpha(k_1-k_2\beta_2)^4\rho_{\max}^3}$$

$$|H_{13}| = \begin{vmatrix} \frac{\partial^2 f_1}{\partial w^2} & \frac{\partial^2 f_1}{\partial w\partial \theta} \\ \frac{\partial^2 f_1}{\partial \theta\partial w} & \frac{\partial^2 f_1}{\partial \theta^2} \end{vmatrix}$$

$$= \frac{\left(3A_1c_0k_2\alpha\epsilon\rho_{\max}-T\right)\left(A_1c_0k_2\alpha\epsilon\rho_{\max}+T\right)}{c_0^2\alpha^2(k_1-k_2\beta_2)^2\epsilon^2\rho_{\max}^2}$$

where $K = \left(2k_1^2 + 8k_1k_2\beta_2 - k_2^2\beta_2^2\right)$

Thus, concavity conditions of the manufacturer's profit function are as follows.

(i) Second-order principle minors are positive, i.e. $|H_{12}| > 0, |H_{23}| > 0, |H_{13}| > 0$. and (ii)the determinant of the third-order Hessian matrix is negative, i.e. $|H_m| < 0$.

Substituting $w = w^*$ and $\theta = \theta^*$ in (3), we get the selling price of the product as

$$p_r^* = \frac{k_1\beta_1 - (k_0 + k_2p_m)\beta_2}{(k_1 - k_2\beta_2)} - \frac{\begin{array}{c} c_0\alpha\left(k_1^3 - 2k_1^2k_2\beta_2 - 3A_1k_2^2\beta_2\right. \\ \left. +k_1k_2\left(-15A_1 + k_2\beta_2^2\right)\right)\epsilon\rho_{\max} + (5k_1 + k_2\beta_2)T \end{array}}{\alpha^2(k_1 - k_2\beta_2)^3\rho_{\max}} \quad (7)$$

Using (4)–(6), demand of the product and profit functions of the manufacturer and the retailer are found as follows

$$D^* = \frac{2k_2\left(3A_1c_0k_2\alpha\epsilon\rho_{\max} - T\right)^2}{c_0\alpha^3(k_1 - k_2\beta_2)^4\epsilon\rho_{\max}^2} \quad (8)$$

$$f_1^* = \frac{\begin{array}{c} 36A_1^2c_0k_2^2\epsilon\left(T_0 + 2k_2T\right) - 216A_1^3c_0^2k_2^4\alpha\epsilon^2\rho_{\max} - A_1(k_1 - k_2\beta_2)^2 \\ (\alpha(k_0 + k_2(p_m - \beta_1)) + c_0k_2\epsilon)\left(T_0 + 8k_2T\right) \end{array}}{\alpha^3(k_1 - k_2\beta_2)^6\rho_{\max}} - A_0 \quad (9)$$

$$f_2^* = \frac{2k_2\left(3A_1c_0k_2\alpha\epsilon\rho_{\max} - T\right)^3}{c_0\alpha^5(k_1 - k_2\beta_2)^6\epsilon\rho_{\max}^3} \quad (10)$$

where, $T_0 = \alpha^2(k_0 + k_2(p_m - \beta_1))(k_1 - k_2\beta_2)^2\rho_{\max} + c_0k_2\alpha(k_1 - k_2\beta_2)^2\epsilon\rho_{\max}$

4.2. Centralized decision

In centralized decision-making, a single decision-maker takes all decisions to optimize overall channel profit. Total profit function of the centralized channel is

$$\begin{array}{l} f_c(p_r, \theta, \rho) = p_rD(p_r, \theta, \rho) - \epsilon\left(\frac{\rho}{\rho_{\max}}\right)D(p_r, \theta, \rho)c_0 \\ \quad -[\beta_1 + \beta_2(\theta - \theta_{\min})]D(p_r, \theta, \rho) - A_0 - A_1(\theta - \theta_{\min})^2 \end{array} \quad (11)$$

Optimal value of the decisions variables under centralized scenario are found as follows

$$p_r^c = \frac{(k_1\beta_1 - (k_0 + k_2p_m)\beta_2)}{(k_1 - k_2\beta_2)} - \frac{\left(c_0\alpha\left(k_1^3 - 2k_1^2k_2\beta_2 - 3A_1k_2^2\beta_2 + k_1k_2\left(-6A_1 + k_2\beta_2^2\right)\right)\epsilon\rho_{\max} + (2k_1 + k_2\beta_2)T_c\right)}{\alpha^2(k_1 - k_2\beta_2)^3\rho_{\max}}$$

$$\theta^c = \frac{\begin{array}{c} \left(c_0k_2\alpha\left(9A_1k_2 - (k_1 - k_2\beta_2)^2\right)\epsilon\rho_{\max} - \alpha^2(k_1 - k_2\beta_2)^2(k_0 - k_1\theta_{\min}\right. \\ \left. +k_2(p_m - \beta_1 + \beta_2\theta_{\min}))\rho_{\max} - 3k_2T_c\right) \end{array}}{\alpha^2(k_1 - k_2\beta_2)^3\rho_{\max}}$$

$$\rho^c = \frac{c_0\alpha\left(3A_1k_2 - (k_1 - k_2\beta_2)^2\right)\epsilon\rho_{\max} - T_c}{c_0\alpha^2(k_1 - k_2\beta_2)^2\epsilon}$$

where

$$T_c = \sqrt{A_1 c_0 \alpha^2 \epsilon \left(-2\alpha(k_0 + k_2(p_m - \beta_1))(k_1 - k_2\beta_2)^2 + c_0 k_2 \left(9A_1 k_2 - 2(k_1 - k_2\beta_2)^2\right)\epsilon\right)\rho_{\max}^2}$$

Demand of the product under centralized decision is

$$D^c = \frac{k_2\left(-3A_1 c_0 k_2 \alpha\epsilon\rho_{\max} + T_c\right)^2}{c_0\alpha^3(k_1 - k_2\beta_2)^4\epsilon\rho_{\max}^2}$$

Total centralized channel profit is

$$f_c^* = \frac{18A_1^2 c_0 k_2^2\epsilon(\alpha(k_0 + k_2(p_m - \beta_1)) + c_0 k_2\epsilon)}{\alpha^2(k_1 - k_2\beta_2)^4} - \frac{A_1\left(54A_1^2 c_0^2 k_2^4\epsilon^2 + (k_1 - k_2\beta_2)^4(\alpha(k_0 + k_2(p_m - \beta_1)) + c_0 k_2\epsilon)^2\right)}{\alpha^2(k_1 - k_2\beta_2)^6} + \frac{2A_1\left[9A_1 c_0 k_2^3\epsilon - 2k_2(k_0\alpha + k_2 p_m\alpha - k_2\alpha\beta_1 + c_0 k_2\epsilon)(k_1 - k_2\beta_2)^2\right]T_c}{\alpha^3(k_1 - k_2\beta_2)^6\rho_{\max}} - A_0$$

Now, to verify the concavity of the profit function of the centralized channel and its uniqueness, we compute the Hessian matrix as follows:

$$H_c = \begin{pmatrix} \frac{\partial^2 f_c}{\partial p_r^2} & \frac{\partial^2 f_c}{\partial p_r \partial\rho} & \frac{\partial^2 f_c}{\partial p_r \partial\theta} \\ \frac{\partial^2 f_c}{\partial\rho\partial p_r} & \frac{\partial^2 f_c}{\partial\rho^2} & \frac{\partial^2 f_c}{\partial\rho\partial\theta} \\ \frac{\partial^2 f_c}{\partial\theta\partial p_r} & \frac{\partial^2 f_c}{\partial\theta\partial\rho} & \frac{\partial^2 f_c}{\partial\theta^2} \end{pmatrix}$$

We have from the partial derivatives of f_2 as follows:

$$\frac{\partial^2 f_c}{\partial p_r^2} = -\frac{2k_2(\alpha\rho + \rho_{\max})}{\rho_{\max}}$$

$$\frac{\partial^2 f_c}{\partial\rho^2} = -\frac{2c_0\alpha\epsilon(k_0 + k_2(p_m - p_r) + k_1(\theta - \theta_{\min}))}{\rho_{\max}^2}$$

$$\frac{\partial^2 f_c}{\partial\theta^2} = -2A_1 - \frac{2k_1\beta_2(\alpha\rho + \rho_{\max})}{\rho_{\max}}$$

$$\frac{\partial^2 f_c}{\partial p_r\partial\rho} = \frac{\alpha(k_0 + k_1(\theta - \theta_{\min}) + k_2(p_m - 2p_r + \beta_1 + \beta_2\theta - \beta_2\theta_{\min}))\rho_{\max} + c_0 k_2\epsilon(2\alpha\rho + \rho_{\max})}{\rho_{\max}^2}$$

$$\frac{\partial^2 f_c}{\partial p_r\partial\theta} = \frac{(k_1 + k_2\beta_2)(\alpha\rho + \rho_{\max})}{\rho_{\max}}$$

$$\frac{\partial^2 f_c}{\partial\rho\partial\theta} = \frac{\alpha(-(k_0 + k_2 p_m)\beta_2 + k_2 p_r\beta_2 + k_1(p_r - \beta_1 - 2\beta_2\theta + 2\beta_2\theta_{\min}))\rho_{\max} - c_0 k_1\epsilon(2\alpha\rho + \rho_{\max})}{\rho_{\max}^2}$$

Now, at $(p_r^c, \rho^c, \theta^c)$, we have

$$|H_c| = \frac{4A_1 k_2\left(27A_1^2 c_0^2 k_2^3\alpha\epsilon^2\rho_{\max} + (k_1 - k_2\beta_2)^2(\alpha(k_0 + k_2(p_m - \beta_1)) + c_0 k_2\epsilon)T_c\right)}{\alpha(k_1 - k_2\beta_2)^4\rho_{\max}^3} - \frac{4A_1 k_2\left(3A_1 c_0 k_2\epsilon\left(2\alpha^2(k_0 + k_2(p_m - \beta_1))(k_1 - k_2\beta_2)^2\rho_{\max} + 2c_0 k_2\alpha(k_1 - k_2\beta_2)^2\epsilon\rho_{\max} + 3k_2 T_c\right)\right)}{\alpha(k_1 - k_2\beta_2)^4\rho_{\max}^3}$$

Second-order principle minors are as follows

$$|H_{c12}| = \begin{vmatrix} \frac{\partial^2 f_c}{\partial p_r^2} & \frac{\partial^2 f_c}{\partial p_r \partial \rho} \\ \frac{\partial^2 f_c}{\partial \rho \partial p_r} & \frac{\partial^2 f_c}{\partial \rho^2} \end{vmatrix}$$

$$= \frac{6A_1 c_0 k_2^2 \epsilon \left(-\alpha^2 (k_0 + k_2(p_m - \beta_1))(k_1 - k_2\beta_2)^2 \rho_{\max} - c_0 k_2 \alpha \left(-9A_1 k_2 + (k_1 - k_2\beta_2)^2\right) \epsilon \rho_{\max} - 3k_2 T_c\right)}{\alpha (k_1 - k_2\beta_2)^4 \rho_{\max}^3}$$

$$|H_{c23}| = \begin{vmatrix} \frac{\partial^2 f_c}{\partial \rho^2} & \frac{\partial^2 f_c}{\partial \theta \partial \rho} \\ \frac{\partial^2 f_c}{\partial \rho \partial \theta} & \frac{\partial^2 f_c}{\partial \theta^2} \end{vmatrix}$$

$$= \frac{2A_1 c_0 k_2 \epsilon \left(c_0 k_2 \alpha \left(\beta_2 (k_1 - k_2\beta_2)^2 (-4k_1 + k_2\beta_2) + 3A_1 K\right) \epsilon \rho_{\max} - KT_c\right)}{\alpha (k_1 - k_2\beta_2)^4 \rho_{\max}^3}$$

$$- \frac{2A_1 c_0 k_2 \epsilon \left(\alpha^2 (k_0 + k_2(p_m - \beta_1)) \beta_2 (k_1 - k_2\beta_2)^2 (4k_1 - k_2\beta_2) \rho_{\max}\right)}{\alpha (k_1 - k_2\beta_2)^4 \rho_{\max}^3}$$

$$|H_{c13}| = \begin{vmatrix} \frac{\partial^2 f_c}{\partial p_r^2} & \frac{\partial^2 f_c}{\partial p_r \partial \theta} \\ \frac{\partial^2 f_c}{\partial \theta \partial p_r} & \frac{\partial^2 f_c}{\partial \theta^2} \end{vmatrix}$$

$$= \frac{\left(3A_1 c_0 k_2 \alpha \epsilon \rho_{\max} - T_c\right)\left(A_1 c_0 k_2 \alpha \epsilon \rho_{\max} + T_c\right)}{c_0^2 \alpha^2 (k_1 - k_2\beta_2)^2 \epsilon^2 \rho_{\max}^2}$$

Thus, concavity conditions of the profit function of centralized channel are as follows.

(i) Second-order principle minors are positive, i.e. $|H_{c12}| > 0, |H_{c23}| > 0, |H_{c13}| > 0$. and (ii) the determinant of the third-order Hessian matrix is negative, i.e. $|H_c| < 0$.

5. Numerical illustration

We consider the values of the parameters in appropriate units as follows: $A_0 = \$5000$, $A_1 = \$10000$, $\theta_{\min} = 0.5$, $\beta_1 = 50$, $\beta_2 = 100$, $p_m = \$300$, $\rho_{\max} = 12$ months, $\epsilon = 0.2$, $c_0 = \$200$, $k_0 = 200$, $k_1 = 500, k_2 = 4.6$, and $\alpha = 0.4$. Then, optimal solution in the decentralized system is $w^* = \$230.66$, $\theta^* = 0.6733$, $\rho^* = 9.4986$ months. Test of concavity numerically at the above optimal solution is as follows: $\frac{\partial^2 f_1}{\partial w^2} = -6.05645 < 0$, $\frac{\partial^2 f_1}{\partial \rho^2} = -67.2938 < 0$ and $\frac{\partial^2 f_1}{\partial \theta^2} = -85830.9 < 0$. Second-order principle minors are as follows:

$$|H_{12}| = \begin{vmatrix} \frac{\partial^2 f_1}{\partial w^2} & \frac{\partial^2 f_1}{\partial w \partial \rho} \\ \frac{\partial^2 f_1}{\partial \rho \partial w} & \frac{\partial^2 f_1}{\partial \rho^2} \end{vmatrix} = 305.671 > 0$$

$$|H_{23}| = \begin{vmatrix} \frac{\partial^2 f_1}{\partial \rho^2} & \frac{\partial^2 f_1}{\partial \theta \partial \rho} \\ \frac{\partial^2 f_1}{\partial \rho \partial \theta} & \frac{\partial^2 f_1}{\partial \theta^2} \end{vmatrix} = 4.756990 > 0$$

$$|H_{13}| = \begin{vmatrix} \frac{\partial^2 f_1}{\partial w^2} & \frac{\partial^2 f_1}{\partial w \partial \theta} \\ \frac{\partial^2 f_1}{\partial \theta \partial w} & \frac{\partial^2 f_1}{\partial \theta^2} \end{vmatrix} = 120436 > 0$$

$$|H_m| = \begin{vmatrix} \frac{\partial^2 f_1}{\partial w^2} & \frac{\partial^2 f_1}{\partial w \partial \rho} & \frac{\partial^2 f_1}{\partial w \partial \theta} \\ \frac{\partial^2 f_1}{\partial \rho \partial w} & \frac{\partial^2 f_1}{\partial \rho^2} & \frac{\partial^2 f_1}{\partial \rho \partial \theta} \\ \frac{\partial^2 f_1}{\partial \theta \partial w} & \frac{\partial^2 f_1}{\partial \theta \partial \rho} & \frac{\partial^2 f_1}{\partial \theta^2} \end{vmatrix} = -6066760 < 0$$

The above results show that principle minors of the Hessian matrix are alternate in sign, i.e. manufacturer's profit function in decentralized scenario is concave for above numerical setting of the parameters. Optimal selling price, demand, profit of the manufacturer, and the retailer in decentralized scenario are respectively found as follows: $p_r^* = \$296.49$, $D^* = 398.70$ units, $f_1^* = \$47193.3$ and $f_2^* = \$26246.9$. Again, an optimal solution of the centralized decision is $p_r^c = \$249.28$, $\rho^c = 9.6516$ months, $\theta^c = 0.894$. To test the concavity of the profit function numerically for centralized scenario at the above values, we have $\frac{\partial^2 f_c}{\partial p_r^{\,2}} = -12.1598 < 0$, $\frac{\partial^2 f_c}{\partial \rho^2} = -135.109 < 0$ and $\frac{\partial^2 f_c}{\partial \theta^2} = -152172 < 0$. Now, the second-order principle minors are as follows:

$$|H_{c12}| = \begin{vmatrix} \frac{\partial^2 f_c}{\partial p_r^{\,2}} & \frac{\partial^2 f_c}{\partial p_r \partial \rho} \\ \frac{\partial^2 f_c}{\partial \rho \partial p_r} & \frac{\partial^2 f_c}{\partial \rho^2} \end{vmatrix} = 1232.18 < 0$$

$$|H_{c23}| = \begin{vmatrix} \frac{\partial^2 f_c}{\partial \rho^2} & \frac{\partial^2 f_c}{\partial \theta \partial \rho} \\ \frac{\partial^2 f_c}{\partial \rho \partial \theta} & \frac{\partial^2 f_c}{\partial \theta^2} \end{vmatrix} = 1.64526 \times 10^7 < 0$$

$$|H_{c13}| = \begin{vmatrix} \frac{\partial^2 f_c}{\partial p_r^{\,2}} & \frac{\partial^2 f_c}{\partial p_r \partial \theta} \\ \frac{\partial^2 f_c}{\partial \theta \partial p_r} & \frac{\partial^2 f_c}{\partial \theta^2} \end{vmatrix} = 240402 < 0$$

$$|H_c| = \begin{vmatrix} \frac{\partial^2 f_c}{\partial p_r^{\,2}} & \frac{\partial^2 f_c}{\partial p_r \partial \rho} & \frac{\partial^2 f_c}{\partial p_r \partial \theta} \\ \frac{\partial^2 f_c}{\partial \rho \partial p_r} & \frac{\partial^2 f_c}{\partial \rho^2} & \frac{\partial^2 f_c}{\partial \rho \partial \theta} \\ \frac{\partial^2 f_c}{\partial \theta \partial p_r} & \frac{\partial^2 f_c}{\partial \theta \partial \rho} & \frac{\partial^2 f_c}{\partial \theta^2} \end{vmatrix} = -2.4266 \times 10^7 < 0$$

By the above values of the second-order derivatives, we see that the principle minors of the Hessian matrix are alternate in sign, i.e. channel's profit function in centralized scenario is concave for above numerical setting of the parameters. The optimal demand and channel profit under centralized scenario are $D^c = 803.6$ units and $f_c^* = \$99992.3$, respectively.

Comparing demand of the product and channel profit between decentralized and centralized scenarios, one may easily observe that $D^c > D^*$ and $f_c^* > f_1^* + f_2^*$. Also, we have $p_r^c > p_r^*$, $\rho^c > \rho^*$, and $\theta^c > \theta^*$. This is quite obvious as indicated in supply chain literature that cooperative integrated decision is always more profitable than decentralized decision. In the next subsection, we demonstrate a profit sharing mechanism assuming that the manufacturer and the retailer jointly take the centralized decision which is the channel best decision. Then, the members share the total channel profit in a portion that ensures win-win profit. The sensitivity analysis of the key parameters has been done in Table 1.

From the above table, one may easily note that wholesale price (w^*) and selling prices (p_r^* & p_r^c) are highly sensitive on β_2, k_1, and k_2; moderately sensitive on ϵ and α and less sensitive on other parameters. Level of quality (θ^* & θ^c) in decentralized and centralized decision is highly sensitive on β_2, k_1, and k_2; moderately sensitive on ϵ and A_1 and less sensitive on other parameters. Warranty period (ρ^* & ρ^c) in decentralized and centralized decision is highly sensitive on β_1, β_2, ϵ, α, and k_2; moderately sensitive on k_0, k_1 and lower sensitive on A_1. Demands (D^* & D^c) and profits (f_1^*, f_2^*, f_c^*) are highly sensitive on β_2 and ϵ; moderately sensitive on α, β_1, k_0, and k_2 while less sensitive on k_1 and A_1. k_1 and k_2 are two key parameters of demand function and high degree of fluctuation of these parameters may cause no real solution of the system. For example, 40% increase of the parameter k_1 fails to provide any real solution of the model and 40% decrease of the parameter k_2 provides any complex solution of the model.

Table 1. Sensitivity analysis

	Changes (%)	w^*	θ^*	ρ^*	p_r^*	D^*	f_1^*	f_2^*	p_r^c	θ^c	ρ^c	D^c	f_c^*
	−40	5.3	17.3	1.1	4.2	0.5	0.4	0.8	10.1	28.1	2.1	1.1	0.8
	−20	2.0	6.5	0.4	1.6	0.2	0.16	0.3	3.8	10.5	0.8	0.4	0.3
A_1	+20	−1.3	−4.3	−0.3	−1.1	−0.1	−0.1	−0.2	−2.5	−6.9	−0.5	−0.2	−0.2
	+40	−2.3	−7.4	−0.45	−0.18	−0.2	−0.2	−0.3	−4.2	−11.8	−0.9	−0.4	−0.3
	−40	−2.1	2.7	21.2	−0.5	10.5	17.8	16.1	−1.1	4.3	21.0	10.5	16.9
	−20	−1.0	1.3	10.6	−0.2	5.2	8.6	7.8	−0.5	2.1	10.5	5.2	8.2
β_1	20	1.0	−1.3	−10.6	0.2	−5.0	−8.2	−7.4	0.6	−2.1	−10.5	−5.1	−7.8
	40	2.1	−2.6	−21.2	0.5	−9.9	−16.0	−14.5	1.2	−4.1	−21.0	−9.9	−15.2
	−40	44.0	66.9	64.9	37.7	33.7	25.6	54.5	78.8	98.4	88.3	72.6	54.5
	−20	18.7	28.3	17.1	15.5	8.4	6.8	12.9	39.8	58.6	37.7	19.2	14.3
β_2	+20	−18.9	−26.4	1.1	−14.7	0.5	0.4	0.8	−35.5	−45.2	2.2	1.1	0.8
	+40	−43.5	−58.1	21.12	−32.7	10.4	8.4	16.0	−69.7	−75.6	47.5	24.5	18.1
	−40	8.5	9.0	87.9	4.4	35.0	23.4	21.5	10.6	14.6	96.6	35.5	22.6
	−20	3.9	3.2	57.9	1.9	12.7	7.7	7.1	4.6	5.3	7.7	12.9	7.5
ϵ	+20	−3.5	−2.1	−51.8	−1.6	−8.1	−3.7	−3.4	−3.8	−3.3	−51.4	−8.2	−3.6
	+40	−6.8	−3.5	−78.8	−3.0	−13.4	−5.1	−4.7	−7.3	−5.6	−88.1	−13.6	−4.9
	−40	−11.1	−4.6	−74.6	−4.8	−18.2	−4.6	−4.4	−11.6	−7.5	−78.7	−18.3	−4.5
	−20	−4.3	−2.5	−32.8	−2.0	−9.6	−4.2	−3.9	−4.7	−3.9	−42.1	−9.7	−4.1
α	+20	3.2	2.6	25.2	1.6	10.2	6.0	5.6	3.7	4.2	34.8	10.3	5.8
	+40	5.7	5.3	60.5	2.9	20.7	13.1	12.0	6.9	8.6	59.8	20.9	12.6
	−40	−5.7	−2.2	−18.4	−5.4	−8.6	−14.0	−12.7	−5.9	−3.5	−18.3	−8.7	−13.3
	−20	−2.8	−1.1	−9.2	−2.7	−4.3	−7.1	−6.5	−2.9	−1.8	−9.1	−4.4	−6.8
k_0	+20	2.8	1.1	9.2	2.7	4.5	7.5	6.8	2.9	1.8	9.1	4.5	7.1
	+40	5.7	2.3	18.4	5.4	9.0	15.4	13.9	6.0	3.7	18.3	9.1	14.6
	−20	−18.3	−34.3	2.0	−14.2	0.9	0.8	1.4	−34.4	−94.0	4.1	2.0	1.5
	−10	−9.7	−32.1	−1.5	−7.6	−0.7	−0.5	−1.0	−18.2	−51.2	−2.9	−1.4	−1.1
k_1	+10	11.8	34.1	6.7	9.5	3.2	2.6	4.9	23.3	57.7	13.8	6.8	5.3
	+20	26.7	36.1	19.7	21.9	9.7	7.8	14.9	57.5	135.4	44.1	22.6	16.7
	−20	31.3	74.2	35.3	26.3	−5.8	−6.6	−2.1	65.2	141.5	65.7	7.6	3.6
	−10	12.9	32.8	12.1	10.7	−4.6	−4.7	−1.9	24.1	55.9	19.6	−1.2	−1.8
k_2	+10	−10.1	−29.5	−5.7	−8.2	6.9	6.7	5.5	−17.9	−47.1	−7.1	6.2	5.8
	+20	−18.9	−58.1	−7.0	−15.1	15.9	15.2	14.0	−33.3	−92.9	−6.3	16.3	14.7

6. Profit sharing mechanism through adjustment of wholesale price for channels best outcome

In this mechanism, the manufacturer offers an incentive to the retailer by sharing the surplus profit if both of them jointly adopt centralized decisions. Under profit sharing mechanisms, the system performance is first optimized and the resultant benefit is then shared between the manufacturer and the retailer. This solution can be considered as a cooperative solution. Its implementation, however, depends on the development of a profit sharing scheme that is acceptable to both parties. The channel members can make an agreement that they will divide the surplus profit proportionally according to their decentralized profit. Surplus profit for accepting centralized policy is $f_{sp} = f_c^* - (f_1^* + f_2^*)$. The manufacturer and the retailer will get additional profits $[f_1^*/(f_1^* + f_2^*)]f_{sp}$ and $[f_2^*/(f_1^* + f_2^*)]f_{sp}$, respectively. Thus, under profit sharing mechanism, the profits of the manufacturer and the retailer are

$$f_1^{ps} = f_1^* + \frac{f_1^*}{(f_1^* + f_2^*)} f_{sp}$$

$$f_2^{ps} = f_2^* + \frac{f_2^*}{(f_1^* + f_2^*)} f_{sp}$$

For implementation of profit sharing policy, we propose that the surplus profit can be shared between them by just adjusting wholesale price properly. Suppose, w^{ps} be the wholesale price under profit sharing mechanism, then

$$w^{ps} = p_r^c - \frac{f_2^c}{D^c}$$

Thus, through proper choice of wholesale price, profit sharing mechanism can be implemented and the decentralized channel can achieve profit equal to centralized profit. This also assures win–win outcomes for all the channel members.

7. Conclusions

In this research work, a mathematical model has been developed for determining optimal selling price of the product, level of quality, and length of warranty period in a two-echelon supply chain. These three factors are studied separately in literature of profit maximization. In this study, we develop decentralized and centralized model considering price-, quality-, and warranty-dependent demand. Optimal solutions are found in closed form and the model is also illustrated through a numerical example. As far as the authors' knowledge goes, there exists no study which analyzes these three factors simultaneously.

The present model can be extended further considering quality , cost for warranty, and demand of the customers as stochastic variables which are the major limitations of the model.

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