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Pensions, Savings and Housing: A Life-cycle Framework with Policy Simulations

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Pensions, Savings and Housing: A Life-cycle Framework with Policy Simulations

John Creedy, Norman Gemmell and Grant Scobie

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Pensions, Savings and Housing: A Life-cycle Framework with
Policy Simulations

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Abstract

The objective of the paper is to explore the saving and consumption responses of a representative household to a range of policy interventions such as changes in taxes and pension settings. To achieve this, it develops a two-period life-cycle model. The representative household maximises lifetime utility through its choice of optimal levels of consumption, housing and saving. A key feature of the approach is modelling the consumption of housing services as a separate good in retirement along with the implications for saving. Importantly, the model incorporates a government budget constraint involving a pay-as-you-go universal pension. In addition, the model allows for a compulsory private retirement savings scheme. Particular attention in the simulations is given to the potential impact on household saving rates of a range of policy changes. Typically the effect on saving rates is modest. In most instances, it would take very substantial changes in existing policy settings to induce significant increases in household saving rates.

J.E.L. CLASSIFICATION D12; H24; H31; J26

KEYWORDS Savings; Housing; Retirement; Intertemporal elasticity of substitution; rate of interest; taxation.

Executive Summary

Households make decisions about consumption, saving, housing and retirement income. These are highly interrelated and are influenced by the tax and expenditure policies of the government, while the latter are affected in turn by individuals' behaviour. While abstracting from many complexities, this paper focusses on retirement savings decisions, mortgage borrowing for house purchase and consumption plans in the presence of both income and consumption taxes. Government expenditure is subject to a debt-neutral budget constraint.

A two-period framework is constructed to explore household savings behaviour over the life cycle. The model is based on a representative household that chooses its optimal level of consumption, saving, retirement income and housing subject to its budget constraint. In addition a key feature of the model is the explicit treatment of taxation and expenditure and the constraints imposed by the government's budget balance. The financial sector enters through the role of the interest rate and a mortgage loan-to-value ratio.

The model is calibrated with benchmark values designed to mirror key features of the New Zealand economy. It is then used for an extensive series of policy simulations. These involve comparing the benchmark case with the values of the decision variables after introducing a specific policy change. These include, for example, changes in the rate of interest, the income tax rate, the consumption tax rate, contributions to a compulsory retirement scheme and a change in the rate of taxation on interest income.

Particular attention is given to the response of savings, consumption and housing to changes in various tax rates, pension and savings policies, and demographic changes. In general the responses are typically modest. For example, a rise in the income tax rate of 1.5 percentage points reduces both financial saving and total household saving rates by 0.7 percentage points and implies a 1% fall in house prices.

In view of the fact that New Zealand gives essentially no tax concessions on interest income, this issue was explored in some detail. The model was adjusted to eliminate all tax on interest income. In the first instance financial savings and total savings rise by 17% and 8% respectively. The household saving rate (expressed as a proportion of disposable income) rises by 1.4 percentage points. This is accompanied by a significant shift toward consumption in retirement and lower consumption of housing services, leading to a fall in house prices of some 2.5%.

The loss of tax revenue involves a reduction of 1.7% in the public non-pension expenditure. However, as the extra savings over the working life are drawn down in retirement there is no net effect on aggregate household saving from such a policy.

These results underscore the importance of including a government budget constraint and importantly of the mechanism by which a balanced budget is achieved after a policy intervention that alters the initial level of tax revenue or total expenditure. For example, for some comparisons, we impose a condition that non-pension expenditure per person increases by a set amount while the universal pension is held constant.

Population ageing is expected to create additional fiscal pressures. One approach could be to introduce a compulsory retirement savings contribution and then reduce the universal pension such that total pension income remains constant. With a 10% decline in the ratio of workers to pensioners, this would involve a compulsory contribution rate of 6.5% accompanied by a 22% reduction in the universal pension. Consumption throughout the lifetime would be lower and voluntary savings would decline.

All these results have been obtained as comparative static exercises. They do not allow for the time that adjustments to policy changes would take, or the time path of those adjustments. Furthermore, as the results are for a representative household they are silent on the distributional consequences of policy changes. Despite this, the model provides a rigorous and internally consistent framework for assessing the direction and magnitude of key responses in saving, consumption, housing and pensions to potential changes in a range of tax and retirement income policies.

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Pensions, Savings and Housing: A Life-cycle Framework with Policy Simulations

"You can be young without money, but you can't be old without it."

Tennessee Williams, *Cat on a Hot Tin Roof* (Act 1)

1 Introduction

This paper uses a two-period framework to explore household savings behaviour over the life cycle. A primary objective is to explore the impact on the saving and housing decisions of a representative household, of policy interventions such as tax and retirement income policies. A distinction is drawn between two forms of savings. First, 'financial savings' are defined as interest-bearing savings made in the first (working) period of life in order to augment income in the second (retirement) period. Second, 'housing savings', also made in the first period, are augmented by a mortgage and used to purchase a house. The mortgage is the only form of debt allowed in the model.

It is assumed that households maximise an intertemporal utility function subject to a lifetime budget constraint. Incomes are subject to an income tax. Consumption, other than housing, is subject to a broad-based goods and services tax. At any time the tax revenue from two overlapping generations of workers and pensioners is used to finance an unconditional (non-means-tested) retirement income, in addition to other non-transfer public expenditure per person. Furthermore, the implications of imposing a compulsory private superannuation system, where income obtained by the fund is taxed at a lower rate than other income, are investigated. Comparative static properties of the model are investigated in order to examine the implications for saving and consumption behaviour of a number of policy interventions and other exogenous changes. A key feature of the approach is modelling the consumption of housing services as a separate good in retirement along with the implications for saving.

The present paper concentrates on microeconomic features of saving behaviour, while at the same time ensuring that the government budget constraint remains balanced, implying no change in the level of public debt. The results underscore the critical importance of the assumptions made as to how the government's

budget is balanced following a policy change that affects revenues or expenditures. The analysis contributes to the debate about the role of taxation in saving decisions and the role of housing. Consistent with the microeconomic focus, the rate of interest is assumed to be exogenously given and the banking sector, with possible implications of overseas borrowing, is not modelled explicitly.

The paper ignores business and government saving and, in concentrating on household saving during the working life, does not consider aggregate household saving in a cross-section consisting of both working and retired households. The latter would have to allow for negative saving during the retirement period. Hence the kind of saving rates illustrated here do not correspond to those obtained from national income statistics but refer to savings over the working period only. The model is designed to examine savings and other responses to policy changes, in a setting that captures in a simplified form some of the characteristics of the New Zealand economy, where households are fully informed and rational. The analysis is *not* concerned, for example, with optimal government policy or with the arguments used to justify different forms of superannuation scheme.¹

Section 2 sets out the basic two-period model. In particular, the treatment of housing is explained, since saving for house purchase is a central component of the model. Section 3 examines the use of a Cobb-Douglas type of lifetime utility function, for which the elasticity of substitution between all pairs of goods is equal to one.

Section 4 analyses the government's budget constraint, paying particular attention to the tax-financing of the unconditional pension with overlapping generations. The values of the parameters, variables and policy settings used to calibrate the model are specified in Section 5. The simulation of policy changes commences in Section 6 which sets out the analytical framework for measuring the response to policy interventions. This is followed in Section 7 by an analysis of changes in taxation and expenditure. Other policy changes are examined in Section 8. Section 9 explores non-policy changes, including the impact of demographic change associated with population ageing, together with the preference for housing relative to other goods. The paper concludes in Section 11.

¹ Such arguments include redistribution, risk pooling, administrative efficiency, paternalism and so on. The pension and tax structures examined here are simply taken as given.

2 A Two-period Framework

This section describes the main features of the model, which needs to allow for simultaneously lending for future consumption and borrowing in the form of a mortgage to help finance house purchase. Subsection 2.1 provides an overview, and subsection 2.2 explains the special treatment of housing.

2.1 The Basic Structure of the Model

The approach adopted here is based on a two-period model and focusses on the lifetime plans of a representative household.² The household works and earns income, y_1 , in period 1. Retirement is in period 2, but the model allows for some income, y_2 , during that period. Mortality is known to take place at the end of period 2 and no bequests are planned. While a larger number of periods could be included to capture more details of household life cycle events and housing choices, this level of detail would unnecessarily complicate the analysis without yielding additional insights.³ The household is assumed to maximise utility from consumption of a composite consumption good in each period, denoted c_1 and c_2 . As usual in such models, household structure is ignored, so the household is effectively treated as if it were a single individual.

The representative household maximises a lifetime utility function, expressed in terms of consumption in the two periods and housing, subject to a budget constraint. The household faces an exogenous stream of gross income from employment. However, there is a tax-financed, and non means-tested, pension which is financed on a pay-as-you-go (PAYG) basis. Current taxes from workers, and additional taxable income in retirement, including interest-income, must finance this pension in addition to other non-pension public expenditure per person. The latter does not enter the utility function of the individual. The household solves a utility maximisation problem subject to a budget constraint, while the government must satisfy a government budget constraint involving two generations.

The government budget constraint clearly involves a loss of a degree of freedom in

² On two-period modelling in this context, see Diamond (1977) and the retirement income models of Lindbeck and Persson (2003) and Disney (2005), and the savings portfolio choice model of Aura-Diamond-Geanakoplos (2002). For a review of literature on savings, see Attanasio and Weber (2010).

³ Hayashi, Ito and Slemrod (1988) adopt six periods in their model of saving and housing, which attempts to simulate a variety of life cycle events. Similarly, Coleman (2010) includes a set of four overlapping cohorts of heterogeneous individuals, and three housing tenure choices, in his model of ageing and housing.

setting policy variables: all but one of the policy variables can be set independently. Within the model, non-pension government expenditure is endogenous in that it is solved depending on the consumption and saving choices of the household, income tax and consumption tax settings, and non-pension expenditure. In addition to the government PAYG pension, the model allows for compulsory contributions to a private (defined contribution) pension where the fund's earnings are taxed at a lower rate than the standard income tax rate applied to income from financial savings. This introduces complex inter-relationships among the components, linking the government and individual budget constraints.

2.2 The Treatment of Housing

Period 2 consumption includes consumption of owner-occupied, 'retirement housing', denoted c_H . A house cannot be purchased using retirement income in period 2. Instead, a plan for a positive amount of consumption of owner-occupied housing in period 2 means that the household must acquire a housing asset in period 1, of sufficient value to deliver the desired retirement housing consumption stream in period 2. Retirement housing is a pure investment good which is accumulated in period 1 and consumed in period 2. Consumption of housing in period 1 can be thought of as part of the composite consumption good in period 1. This period 1 housing consumption may or may not be delivered from the same housing stock as the retirement housing asset. That is, the household may be an owner-occupier of its housing asset in period 1 (hence also obtaining a non-retirement housing consumption stream), or it may simply hold this housing asset and simultaneously rent housing. Both are included within c_1 . Given this, the term 'housing consumption' is used below to refer only to housing consumption in period 2, c_H .

The household saves in period 1 for consumption in period 2. However, the household can borrow an amount, b , in order to purchase a housing asset. This mortgage borrowing incurs an interest cost during period 1 and debt repayment (principal plus interest) is required in period 2. The mortgage is determined by imposing a limit on the amount of borrowing. The constraint takes the form of a loan-to-value ratio (LVR).⁴

These assumptions regarding the housing asset and consumption require some justification. The assumption that consumption of housing in retirement cannot be purchased out of period 2 income is designed to capture two important empirical aspects of housing consumption. First, first-time home purchasing during retirement is in practice unusual. Second, retired households have high home-

⁴ It is shown that this also implies that the mortgage is a proportion of wealth.

ownership rates relative to non-retired households. Where a mortgage is required to enable a house purchase, lenders are generally reluctant to lend to retirees. Furthermore, Appendix A shows – using a more general model which explicitly allows for a wider range of tenancy choice possibilities over the lifetime – that the option of renting in period 1 and purchasing a house in period 2 is not optimal for a very wide range of parameter values.⁵

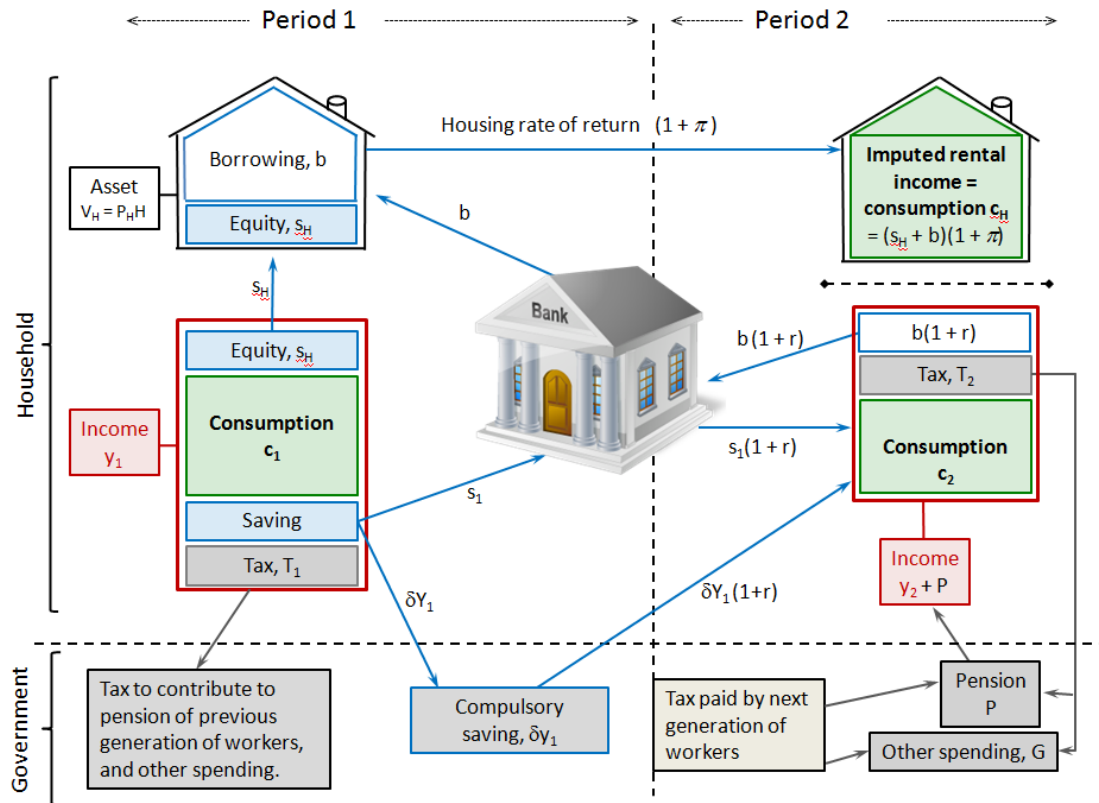
It might be objected that retired households do not fully consume their housing asset during retirement, but consume housing services from an asset that, despite depreciating, has a positive expected value at death. However, even with a bequest motive, from which this model abstracts for simplicity, this cannot be the primary motivation for home-ownership in retirement. However, the risk of living longer than expected, such that the house has fully depreciated before death, is likely to be larger than the risk of dying whilst there is still a residual housing asset value. Hence observing the bequest of a housing asset is compatible with a motivation for home ownership based on a planned housing consumption stream during retirement that does not fully depreciate the asset. This argument is strengthened by the fact that there are few opportunities to enable retirees to realise a fraction of the value of their housing asset towards the end of life. In the present model, households want to be owner-occupiers in period 2 only because of the consumption delivered; the residual house capital stock at death is incidental and delivers no utility.⁶

Figure 1 illustrates, for a single cohort, how the household, banking and government sectors of the model are inter-related. The household saves a fraction of period 1 income, y_1 , in the form of financial saving, s_1 , which earns a return at the rate, r , and contributes $s_1(1 + r)$ to non-housing consumption, c_2 , in period 2. In addition, a fraction of period 1 income is devoted to housing saving, s_H , which earns a return at rate, π , and contributes $s_H(1 + \pi)$ to housing consumption in period 2, c_H . Hence, c_H and c_2 are non-fungible: they cannot be traded in period 2. Both involve the purchase of separate assets in period 1 for consumption streams in period 2. In the present context it is not necessary to impose a condition that the return on both assets should be equal. This is because preferences between c_H and c_2 determined the investment plan in period 1 and their realised returns

⁵ Further support, in the US context, for the assumptions imposed here, is provided by Venti and Wise (2001, p. 129). Henry (2010) and others have argued that ownership of a housing asset in retirement provides insurance against the risk of income shocks during a period of life where negative exogenous shocks to real income are difficult to counteract. In addition, imputed rent is untaxed and immune to the risk of inflation eroding real values.

⁶ Other arguments that might explain why elderly households continue to live in houses with larger asset values than required to deliver an expected consumption stream, but which are difficult to incorporate in simple models, include such things as inertia among the elderly in their consumption habits and familiarity with a particular home configuration, which inhibits an otherwise rational trading-down to smaller houses but greater cash assets.

Figure 1: Household and Government Components of the Model



cannot be exchanged in period 2.

3 The Life-Cycle Model

This section sets out the life-cycle model of consumption and saving and the tax structure. Subsection 3.1 presents optimal solutions for consumption and saving. Subsection 3.2 introduces mortgage borrowing and Subsection 3.3 adds a compulsory pension contribution.

3.1 Optimal Consumption and Saving

It is convenient to write the utility function as:

$$U = c_1^\alpha c_2^\beta c_H^\gamma \quad (1)$$

with α, β and $\gamma > 0$. This form of utility function implies a unit elasticity of substitution between all pairs of goods. This greatly simplifies the analysis. The implications of a non-unit intertemporal elasticity of substitution are further explored in Appendix

C, where it is shown that the main results are not significantly influenced by the assumption of the unit elasticity. Nevertheless, as discussed below, empirical estimates suggest a value of less than one, so that the degree of intertemporal substitution in response to relative ‘price’ changes induced by policy changes is expected to be less than that obtained by the present model.

The representative household chooses values of c_1 , c_2 and c_H to maximise utility, subject to a lifetime budget constraint. This in turn depends on income in each of the two periods, along with income and consumption taxes and a superannuation benefit in the retirement period. Suppose market income in each period, y_1 and y_2 , is taxed at the proportional rate, τ , and in the second period there is a universal untaxed superannuation of P , along with any taxed income of y_2 .⁷ There is an interest-income tax imposed at the same rate, τ .

The housing asset is purchased from savings, s_H , in the first period which, as explained above, appreciates at the rate, π , which in the absence of a capital gains tax, is not subject to taxation. Consumption c_1 and c_2 are assumed to attract indirect taxation in the form of a broad-based goods and services tax, GST, at the fixed tax-exclusive rate, v . The pre-tax price of consumption goods is normalised to 1 in each period. The individual’s budget constraint is thus given by:

$$(c_1 + c_2)(1 + v) + c_H = y_1(1 - \tau) + P + y_2(1 - \tau) + r(1 - \tau)(y_1(1 - \tau) - c_1(1 + v) - s_H) + \pi s_H \quad (2)$$

Consumption and housing expenditure, inclusive of GST, must be equal to net market income, plus the pension, plus the net interest income arising from financial savings in the first period, plus the return from housing savings in the first period. Housing savings can be augmented by obtaining a mortgage: for convenience, discussion of the mortgage is deferred until the next subsection. Using $s_H = c_H / (1 + \pi)$ the budget constraint in (2) can be expressed as:

$$y_1(1 - \tau) + \frac{P + y_2(1 - \tau)}{1 + r(1 - \tau)} = c_1(1 + v) + \frac{c_2(1 + v)}{1 + r(1 - \tau)} + \frac{c_H}{1 + \pi} \quad (3)$$

The net present value of lifetime income, denoted W , is:

$$W = y_1(1 - \tau) + \frac{P + y_2(1 - \tau)}{1 + r(1 - \tau)} \quad (4)$$

Defining $\alpha' = \alpha / (\alpha + \beta + \gamma)$, and so on for β' and γ' , the usual Cobb-Douglas results give optimal values:⁸

$$c_1 = \frac{\alpha' W}{1 + v} \quad (5)$$

⁷ If P is taxable, it is replaced simply by $P(1 - \tau)$.

⁸ For Cobb-Douglas utility, total expenditure on each ‘good’ is a fixed proportion of income (or in this case, net worth), with the constant of proportionality equal to the exponent on the good divided by the sum of exponents.

$$c_2 = \frac{\beta'W \{1 + r(1 - \tau)\}}{1 + v} \quad (6)$$

$$c_H = \gamma'(1 + \pi)W \quad (7)$$

From (7), $s_H = c_H / (1 + \pi) = \gamma'W$ does not depend on s_H .⁹ Financial savings are $s_1 = y_1(1 - \tau) - c_1(1 + v) - s_H$, so that:

$$s_1 = y_1(1 - \tau) - (\alpha' + \gamma')W \quad (8)$$

Hence $\frac{\partial s_1}{\partial r} > 0$ and financial savings unambiguously increase as the rate of interest increases.

3.2 Mortgage Borrowing

The previous results can easily be modified by the addition of a mortgage. If (as discussed in section 2) it is possible to borrow b for a house purchase, then:

$$c_H = (1 + \pi)(s_H + b) \quad (9)$$

and:

$$s_H = \frac{c_H}{1 + \pi} - b \quad (10)$$

Housing savings required for a desired value of c_H are thus reduced by the extent of the mortgage. In this type of model it is necessary to assume that the income tax system treats interest and debt symmetrically: that is, the same net-of-tax interest rate must be applied to interest receipts and payments. Different rates would imply a nonlinear inter-temporal budget constraint, giving rise to corner solutions.¹⁰ Hence, it is required to assume that for tax purposes interest-income is deductible.

On the assumption that the effective mortgage rate is thus $r(1 - \tau)$, the interest paid on the mortgage is $r(1 - \tau)b$. The investment of b yields $b(1 + \pi)$ so that after the principal of b is repaid, and interest income is paid, there remains $\{\pi - r(1 - \tau)\}b$. Hence, the budget constraint is now given by:

$$\begin{aligned} (c_1 + c_2)(1 + v) + c_H &= y_1(1 - \tau) + P + y_2(1 - \tau) \\ &+ r(1 - \tau)s_1 + \pi s_H + \{\pi - r(1 - \tau)\}b \end{aligned} \quad (11)$$

Substitution for s_1 and rearrangement of this constraint produces precisely the same form as in equation (3); all terms in b cancel.¹¹

⁹ Appendix C shows that this result is a special property of the Cobb-Douglas.

¹⁰ Complications arising from nonlinear constraints in two-period models, requiring the application of Kuhn-Tucker conditions, are examined in Creedy (1990).

¹¹ If there is a difference between the mortgage rate and r , then b appears in a revised definition of W . If b were to be in the budget constraint, the only way to solve the model would be to set its value exogenously. Hence it would not be possible to obtain mortgage borrowing as an endogenous variable, arising from optimal lifetime choices.

The value of b is determined by setting a borrowing constraint in the form of a loan-to-value ratio (LRV), $\xi = b / (b + s_H)$, so that:

$$b = \frac{\xi}{1 - \xi} s_H \quad (12)$$

Thus substituting in (9) gives $s_H = c_H (1 - \xi) / (1 + \pi)$ and housing and financial savings are now given by:

$$s_H = \gamma' (1 - \xi) W \quad (13)$$

and:

$$s_1 = y_1 (1 - \tau) - \{\alpha' + \gamma' (1 - \xi)\} W \quad (14)$$

Hence a minor modification is needed to the earlier results. Substitution of (13) into (12) gives $b = \gamma' \xi W$, so that the use of a LVR constraint is equivalent to providing a mortgage that is also proportional to lifetime net worth.

3.3 A Compulsory SAYG Scheme

In addition to the tax-financed PAYG pension, the model includes a compulsory saving scheme, which requires a proportion, δ , of the first period's income to be placed into an individual retirement fund. The fund's interest earnings are taxed at the lower rate $\tau' < \tau$. The pension from the fund, P' , is not subject to income tax on withdrawal. The scheme corresponds to a system referred to by the letters TtE: contributions are fully taxed initially; earnings are partially taxed; final withdrawals from the fund are exempt from income tax. Of course, expenditures financed from the private pension and the PAYG pension are subject to GST. The public scheme continues to be universal and not subject to means-testing.

The private pension, received in addition to the public pension, is therefore given by:

$$P' = \delta y_1 \{1 + r (1 - \tau')\} \quad (15)$$

The individual's budget constraint needs to be adjusted to allow for the compulsory contribution. In this case it can be shown that net worth, W' , is:

$$W' = y_1 (1 - \tau^*) + \frac{P + y_2 (1 - \tau)}{1 + r (1 - \tau)} \quad (16)$$

where:

$$\tau^* = \tau - \frac{\delta r (\tau - \tau')}{1 + r (1 - \tau)} \quad (17)$$

If $\tau' = \tau$, net worth is not affected, as $\tau^* = \tau$. The tax advantage enjoyed by the compulsory fund therefore implies an effective reduction in the first period's income tax rate.

4 The Government's Budget Constraint

The government faces a budget constraint in financing both the PAYG pension and other expenditure. Holding debt constant, current expenditure must be financed from current tax revenue. The present treatment includes only income and consumption tax, which must finance the pension and additional expenditure of G per person. This non-pension expenditure does not enter the household's utility function so that, for example, any benefits arising from public goods are ignored here.¹² The constraint applies to aggregates, and so the present section deals with distributions of different households from overlapping generations. Hence i subscripts are added to deal with different households and w (for worker) and p (for pensioner) subscripts are added to y and c to distinguish members from the two cohorts.¹³

Let the number of current pensioners and workers be denoted respectively by N_p and N_w . Interest income tax is obtained from both the interest income on financial savings of the currently retired and their interest income on the compulsory fund. The latter is equal to $r\delta\tau' \sum_{i=1}^{N_p} y_{p,1,i}$. Setting total government expenditure equal to total income tax and GST revenue gives the required budget constraint as:

$$\begin{aligned}
 N_p P + (N_p + N_w) G &= \tau \sum_{i=1}^{N_w} y_{w,1,i} + \tau \sum_{i=1}^{N_p} y_{p,2,i} \\
 &+ \tau r \sum_{i=1}^{N_p} (y_{p,1,i} (1 - \tau - \delta) - c_{p,1,i} (1 + v) - s_{p,H,i}) \\
 &+ v \left(\sum_{i=1}^{N_w} c_{w,1,i} + \sum_{i=1}^{N_p} c_{p,2,i} \right) \\
 &+ r\delta\tau' \sum_{i=1}^{N_p} y_{p,1,i} \tag{18}
 \end{aligned}$$

The term on the left-hand side of (18) is total government expenditure, made up of the expenditure on the universal public pension, $N_p P$, and per capita expenditure of all other non-pension payments of G , applied to all individuals, $N_p + N_w$. On the right-hand side, the first line represents income tax from workers and pensioners;

¹² This assumption is common even in standard optimal income tax models. If G enters utility additively, then it will not affect inter-temporal decisions directly. The present analysis is not concerned with optimal policy or with the allocation of expenditure among alternative uses. Models allowing for individuals' preferences over public goods expenditure, along with transfer payments, are discussed by Creedy and Moslehi (2011).

¹³ Writing the constraint in terms of a distribution of y_1 and other values means that it is possible to consider the case where, for example, $y_{w,1,i}$ is not equal to $\bar{y}_{w,1}$. However, in the majority of policy simulations below, the 'representative' individual is considered by setting variables equal to their mean values.

the second line is interest income tax from the savings of pensioners; the third line represents GST revenue from the expenditure of workers and pensioners; the final line is the interest income tax obtained from interest on compulsory contributions to the SAYG.

Growth of real incomes occurs at the rate, g , so that $\bar{y}_{w,1} = \bar{y}_{p,1} (1 + g)$, where the 'bar' indicates arithmetic mean. Hence, each generation of workers receives an income during the working period that is $100g$ per cent higher than that of the previous generation of workers. The appropriate averages, $\bar{c}_{w,1}$, $\bar{c}_{p,2}$, $\bar{c}_{p,1}$ and $\bar{c}_{p,H}$, can be obtained in terms of average net worth using the above results, on the assumption that all individuals have the same tastes. However, net worth includes the value of P , so the above expression does not directly give a reduced-form solution. First, define \bar{W}_p as:

$$\bar{W}_p = \bar{y}_{p,1} (1 - \tau^*) + \frac{P + \bar{y}_{p,2} (1 - \tau)}{1 + r (1 - \tau)} \quad (19)$$

and \bar{W}_w as:

$$\bar{W}_w = \bar{y}_{w,1} (1 - \tau^*) + \frac{P (1 + g') + \bar{y}_{w,2} (1 - \tau)}{1 + r (1 - \tau)} \quad (20)$$

Here $P (1 + g')$ is the pension that current workers can expect to receive when they retire. If pensions are adjusted fully in line with real incomes, then $g' = g$, and if pensions are adjusted in line only with prices, then $g' = 0$ and P is constant in real terms. Substituting gives:

$$\bar{c}_{p,1} (1 + v) + \bar{s}_{p,H} = (\alpha' + \gamma' (1 - \xi)) \bar{W}_p \quad (21)$$

and:

$$\bar{c}_{p,2} v = \frac{v}{1 + v} \beta' \{1 + r (1 - \tau)\} \bar{W}_p \quad (22)$$

Furthermore:

$$v \bar{c}_{w,1} = \frac{v}{1 + v} \alpha' \bar{W}_w \quad (23)$$

Substituting into the government budget constraint and rearranging eventually gives the following form as the solution for G :

$$\begin{aligned} \left(1 + \frac{N_w}{N_p}\right) G &= \left(\frac{N_w}{N_p}\right) \tau \bar{y}_{w,1} \\ &+ \tau r \bar{y}_{p,1} [(1 - \tau) - (\alpha' + \gamma' (1 - \xi)) (1 - \tau^*)] \\ &+ \tau \bar{y}_{p,2} \left[1 - \frac{r (1 - \tau) (\alpha' + \gamma' (1 - \xi))}{1 + r (1 - \tau)}\right] \\ &+ \frac{v \beta'}{1 + v} [\bar{y}_{p,1} (1 - \tau^*) (1 + r (1 - \tau)) + \bar{y}_{p,2} (1 - \tau)] \end{aligned}$$

$$\begin{aligned}
& + \frac{v\alpha'}{1+v} \left[\bar{y}_{w,1} (1 - \tau^*) + \frac{\bar{y}_{w,2} (1 - \tau)}{1 + r (1 - \tau)} \right] \frac{N_w}{N_p} \\
& - r\delta (\tau - \tau') \bar{y}_{p,1} \\
& - P\Omega
\end{aligned} \tag{24}$$

where:

$$\Omega = 1 + \frac{\tau r (\alpha' + \gamma' (1 - \xi))}{1 + r (1 - \tau)} - \frac{v}{1 + v} \left\{ \beta' + \frac{\alpha' (1 + g')}{1 + r (1 - \tau)} \frac{N_w}{N_p} \right\} \tag{25}$$

The approach here has been to solve for G in terms of exogenous variables. It can be seen that the alternative, of solving for the income tax rate, needed to achieve a given G , would require the solution to a quadratic equation. Hence it is more tractable in the present model to consider τ to be exogenous, and allow G to be determined endogenously.

5 Calibrating the Model

Table 1 presents the values of the various parameters chosen to obtain a benchmark solution. Households from different cohorts are characterised by identical representative households, each with the appropriate arithmetic means corresponding to the cohort. In carrying out the calibration exercise, it is important to remember that in the present two-period model the unit of time is not simply a year. Furthermore, the artificial assumption – common to virtually all overlapping generations models – is that the time periods are of equal length, so that one generation of pensioners overlaps with one generation of workers, as in the government budget constraint discussed in the previous section. Thus it cannot be expected that precise calibration of this kind of model to empirical orders of magnitude can be achieved. Given the number of parameters, an extensive calibration exercise involving much trial and error is required. Clearly, absolute values of, for example, y_1 , are largely arbitrary, but considerable effort has been taken to ensure that relative orders of magnitude of major endogenous variables are reasonable. Furthermore, as stressed earlier, the saving rates produced here relate only to savings over the working life.

In considering appropriate values for the rate of interest, the relationship between an annual rate, r_a , and the longer-period rate, given by $1 + r = (1 + r_a)^{30}$, was used. The value chosen for the interest rate of $r = 1.1$ is consistent with an annual rate over thirty years of around 2.5%. In setting values of α and β , the former was normalised to 1, while in thinking about β it is useful to consider that $\beta = 1 / (1 + \rho)$, where ρ is the time preference rate. It is appropriate to impose a value of time preference in excess of the rate of interest: the value of ρ was set at 1.6.

The value of 2.5 for N_w/N_p is based roughly on the midpoint of the 2012 figure of 3.0 and the New Zealand Statistics projection for 2041 of 2.1. The benchmark value of the PAYG pension is set at 255, just under one quarter of the income in the working period. These values produce a value of endogenous non-transfer expenditure per person, G , of approximately 390. As discussed in section 4, at any time the value of G relates to the $N_w + N_p$ people currently alive, while of course the PAYG pension is received only by the N_p non-workers.

Table 1: Benchmark Values

Representative Individual	Symbol	Benchmark
<i>Taste parameters</i>		
Exponent on consumption in first period	α (α')	1.0 (0.612)
Exponent on consumption in second period	β (β')	0.385 (0.235)
Exponent on housing consumption	γ (γ')	0.25 (0.153)
<i>Incomes</i>		
Income in first period of life cycle	y_1	1000
Income in second period of life cycle	y_2	50
Economy characteristics		
Real rate of interest	r	1.1
Real growth rate of incomes	g	0.8
Rate of appreciation of housing	π	1.4
Elasticity of supply of housing	ε_s	0.5
Ratio of number of workers to pensioners	N_w/N_p	2.5
Government policy		
<i>Tax policy</i>		
Income tax rate	τ	0.25
Tax rate applied to SAYG income	τ'	0.20
GST rate	v	0.15
<i>Expenditure policy</i>		
PAYG pension	P	255
Rate of adjustment to PAYG pension	g'	0.8
<i>Other policies</i>		
SAYG Contribution rate	δ	0.035
Mortgage loan to value ratio	ξ	0.5

In setting a suitable loan-to-value ratio, it is assumed that the representative individual is subject to an initial LVR when purchasing a housing asset in period 1. In period 2 the house is owned outright, having repaid the mortgage, such that $LVR = 0$ in period 2. Hence it is required to set ξ to capture the average LVR throughout the working period. For example, an initial LVR of 90%, which falls to 0 over a 30 year mortgage repayment period, could be represented as a 45% LVR on average (that is, half the initial LVR) over the period of the loan. However, mortgage repayment schemes typically involve a fixed repayment per period, which is initially almost all interest on the loan. By the end of the repayment period it is

almost all capital repayment. Hence, the *LVR* falls non-linearly throughout the 30 years and the average annual value is greater than half the initial *LVR*. It can be shown that for each \$1 borrowed over 30 years at 2.5% per annum (approximately the annual equivalent of the benchmark interest rate used here) with monthly repayments, a 90% initial *LVR* is equivalent to an *LVR* of 50% averaged over 360 months. An 80% initial *LVR* yields an equivalent average *LVR* of around 45%.¹⁴ In the benchmark simulations below, $\xi = 0.50$ is therefore adopted, with a reduction simulated by setting it to 0.45. These can be thought of as approximately capturing the impact of setting initial *LVR*s of 90% and 80% respectively. The elasticity of housing supply, ε_s , of 0.5 is in line with the values reported by Sánchez and Johansson (2011).

6 Simulating Policy Changes

The model can be used to examine the potential direction and magnitude of changes in key outcome variables as a result of specific changes in policies or economic conditions. For example, the impact on saving rates, consumption, investment in housing and retirement income of a reduction in the income tax rate can be examined, starting from a benchmark set of parameters and the associated solution. The types of change can be divided into three basic categories. The first category includes ‘tax and expenditure’ policies. These include changes in τ , v and P and, by implication, G . The latter is a policy variable but, as discussed earlier, it is endogenous because of the government budget constraint. For example, it may be desired to examine the effects of a change in the tax mix, from income tax to GST, by reducing τ and increasing v . Similarly, a shift in government expenditure towards non-transfer expenditure involves for example a reduction in the PAYG pension. In each case it is useful to ensure that changes involve similar changes in G . Subsection 6.1 explains how this is achieved, given that G is endogenously determined.

The second category consists of ‘other policy changes’, such as changes in the loan-to-value ratio, ξ , the compulsory contributions rate in the SAYG pension, δ , and the rate of interest, r . The latter is an exogenous variable in the model. Associated with r is the issue of interest-income taxation: hence this second category includes the implications of exempting interest income from taxation. These policy changes would not normally be considered in the context of revenue switching or of revenue raising, although they clearly do have (in some cases small)

¹⁴ These average values rise to 56% and 50% respectively using a 5% annual interest rate. For a standard mortgage calculator, see: <http://www.zyngrule.com/mortgage-calc.php>.

implications for revenue, and hence G . The approach taken when considering this type of change is thus to impose changes which are considered appropriate in the context of realistic policy changes and the calibration of the model. There is no reason here to impose policy changes which imply common changes in G .¹⁵

The third simulation category contains 'economy-wide and demographic' changes. These are changes over which the government would not be expected to have control, and include changes in the demographic ratio, N_w/N_p , the level of income in the first period of life, y_1 , and the preference for the housing good, γ . As with the second category, changes in these variables have varying implications for G , but this is just another endogenous variable that is of interest in comparing changes: there is no reason to impose common changes in G for all of the simulations in this group.

6.1 Changes for which G is Constant

Suppose it is required to compare the effects on savings of alternative tax and expenditure policies. An initial indication is given by partial changes, such as $\partial S/\partial\tau$, $\partial S/\partial P$ and $\partial S/\partial v$. These partial effects can be obtained numerically by imposing small changes in the policy variables and re-solving the model to obtain the corresponding changes in endogenous variables. The government budget constraint means that there is a loss of a degree of freedom in policy choices: the government cannot independently set, for example, the tax rate, τ , and the non-transfer government expenditure per person, G . Hence it is effectively not possible to change just one policy variable at a time, since a change in τ or v or P generates a change in G as well as changes in the endogenous variables that directly or indirectly affect utility. For this reason, partial changes in tax and expenditure variables are not directly comparable. Each partial change involves a different effect on G , and indeed G moves in different directions: it increases when τ and v increase, but falls when P is increased.

It is therefore desirable to adjust the partial changes so that comparisons are made for similar changes in G . Suppose it is required to compare all policy changes such that the associated change in G , denoted ΔG , is the same for all changes. Suppose that, in a reasonable range around the benchmark solution, partial effects are linear, so that the partial changes, $\partial S/\partial\tau$ and so on, are constant. For example, given the partial change, $\partial G/\partial\tau = x$, say, then the change in τ needed to achieve

¹⁵ Indeed, this would give rise to unrealistic changes, especially where the revenue implications are very small.

a change in G of ΔG is given by:

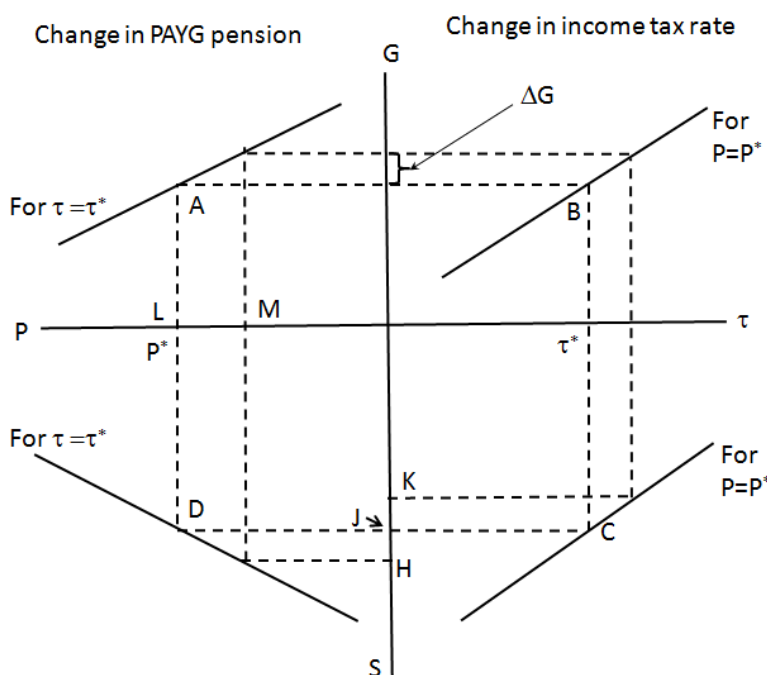
$$\Delta \tau = \Delta G / x \quad (26)$$

Suppose, in addition, that the partial change in savings generated by a change in the tax rate is $\partial S / \partial \tau = y$. Then the change in S resulting from a change in τ of $\Delta \tau$ is given by:

$$\begin{aligned} \Delta S &= y \Delta \tau = \frac{y}{x} \Delta G \\ &= \left(\frac{\partial S / \partial \tau}{\partial G / \partial \tau} \right) \Delta G \end{aligned} \quad (27)$$

Similarly, the effect on savings of a change in v which produces the same effect on G is given simply by replacing τ in (27) with v .

Figure 2: Two Policies Producing A Similar Change in G



Comparisons are illustrated in Figure 2 for two policies. The left hand side of the diagram illustrates the effects on total savings, S , and expenditure, G , of changes in the exogenous PAYG pension, P , for a given tax rate, $\tau = \tau^*$. The right hand side of the diagram shows variations in S and G for variations in the tax rate, τ , for a given pension, $P = P^*$. Hence the points A, B, C and D represent the model's solutions for τ^* and P^* . Each line through the points has a slope given by the respective partial derivative. Hence, from the right hand side of the diagram, a rise in τ which produces a change of ΔG in government expenditure, with P held constant at P^* , is associated with a reduction in savings measured by the length JK. To achieve an equivalent increase in non-transfer expenditure by a policy of

reducing P , with the tax rate held constant at τ^* , it would be necessary to reduce P by LM, which yields an increase in total savings of JH. A similar approach can be extended to allow two policy variables to be combined in a comparable way.

6.2 Changes in The Price of Housing

The model has so far been discussed in terms of the amount spent by the household on housing in period 1. This is the sum of savings s_H and the mortgage, b , and is denoted by $V_{H,1}$. Let $p_{H,1}$ and H_1 denote the price and quantity of housing. Hence, when the house is purchased in period 1:

$$V_{H,1} = p_{H,1}H_1 \quad (28)$$

In examining the comparative statics of the model, the assumption regarding the price elasticity of housing supply, $\varepsilon_s = \frac{dH/H}{dp_H/p_H}$, allows the impact on $p_{H,1}$ to be identified. First, dropping the time subscript, and differentiating (28) gives:

$$\frac{dV_H}{V_H} = \frac{dp_H}{p_H} + \frac{dH}{H} \quad (29)$$

and:

$$\frac{dV_H/V_H}{dp_H/p_H} = 1 + \frac{dH/H}{dp_H/p_H} = 1 + \varepsilon_s \quad (30)$$

Hence:

$$\frac{dp_{H,1}/p_{H,1}}{dV_H/V_H} = \frac{1}{1 + \varepsilon_s} \quad (31)$$

Hence, the proportional response of the house price in period 1 to a change in housing expenditure, V_H , is positive unless supply is infinitely elastic, and is inversely related to the elasticity of housing supply. The effect of, for example, a change in the income tax rate, τ , on the price of housing can thus be obtained as:

$$\eta_{p_H,\tau} = \left(\frac{1}{1 + \varepsilon_s} \right) \eta_{V_H,\tau} \quad (32)$$

7 Tax and Expenditure Policy Changes

This section reports the comparative static results for a number of policy changes to the tax and expenditure structure. These include changes to the income tax rate, the tax rate on consumption, and the level of the public PAYG pension. The impact on different forms of saving and the housing market are reported. In order

to make these three tax and expenditure policies comparable, their effects have been simulated on the basis of a change which, in each case, results in the same change in public non-pension expenditure, as explained in the previous section. In this way the government budget constraint is satisfied and the effect of each of the policy options on any one of the endogenous variables is directly comparable. The following question is therefore relevant. Suppose it is desired to raise G by 5%: what are the implications of financing this extra non-transfer expenditure by raising τ or v , or reducing P ?

7.1 A Change in the Income Tax Rate

Following the method outlined in the previous section, an increase in the income tax rate, τ , from a benchmark value of 25.0% to 26.6% produces a 5% change in non-pension expenditure, G .¹⁶ The reduction in disposable income is reflected in a fall in consumption spending and a decline in financial savings of 6.4%. The demand for housing declines, with a consequent fall in house prices of 1.1%. Retirement consumption falls as a result of lower financial savings. In short, the increase in the tax rate has implications across the life cycle. The full impact obviously depends on how the rise in non-pension expenditure is valued, and this would depend on the exact nature of the additional spending (for example, defence, welfare payments, education or health).

In addition to reducing W , one effect of the increase in τ , because of the interest-income tax, is to change the relative price of consumption in the two periods. However, this effect is extremely small.

7.2 A Change in the Consumption Tax Rate

An increase in the consumption tax rate, v , from the benchmark rate of 15% to 18.2% is needed to obtain a 5% increase in non-pension expenditure, G . The effect of this policy change falls on consumption spending in both periods, while saving and housing and retirement accumulation are unaffected. This is because the relative price of consumption in each period remains unchanged, as does net worth, W . Hence a revenue-neutral change in the tax mix, involving a partial shift from income tax to consumption tax, has a positive effect on savings.¹⁷

¹⁶ A separate document is available providing detailed summary tables of the policy simulations examined here, along with partial effects of changes in policy variables.

¹⁷ This is true even in the absence of interest-income taxation.

7.3 A Reduction in the PAYG Public Pension

In order to achieve a 5% increase in non-pension expenditure, the PAYG pension, P , would need to be reduced by just over 30%. This substantial fall is needed because of the relative sizes of the revenue required for each type of expenditure (and G necessarily applies to all those alive). In order to compensate for the loss of retirement income following the reduction in P , there is an increase in voluntary financial savings of 33.6% accompanied by a reduction in consumption spending of 4.7% in both periods, along with a similar reduced demand for housing services.

The inter-relation between financial savings and a public pension is explored by Hurd *et al.* (2009) using micro-data sets from 12 OECD countries. They find that an extra dollar of pension wealth depresses the accumulation of financial assets on average by 23 to 44 cents, depending on the model used. This is consistent with the present study which finds, as a corollary, that a reduction in the universal pension is accompanied by an increase in financial savings.¹⁸

To explore the further implication of the PAYG pension, it is possible to examine the optimal plans for alternative levels of individual values of income in the first period, y_1 , given the benchmark arithmetic mean value of $\bar{y}_1 = 1000$. Suppose this mean is associated with a lognormal distribution with a variance of logarithms of 0.4.¹⁹ Results are summarised in Table 2 for deciles of the distribution. Unsurprisingly, financial saving rates (as a proportion of gross income) rise with income throughout the distribution. However, the table shows negative saving rates at the lowest two decile values of y_1 , which implies borrowing against the future PAYG pension. This degree of consumption smoothing would in reality not typically be possible, so the saving rates would in fact simply be zero. The finding that the lowest decile income earners have no incentive to save in the face of the public pension is consistent with the findings of Scobie, Gibson and Le (2004) for New Zealand and Moore and Mitchell (1997) and Bernheim (1992) for the United States. Similar results are reported for the UK by Attanasio and Rohwedder (2003), and for Italy by Attanasio and Brugiavini (2003).

¹⁸ A two-period life cycle model is developed by de Freitas and Oliveira Martins (2013). They use a utility maximisation approach to derive an estimating equation for the household saving rate. This is fitted to data from a sample of 22 OECD countries for 1970 to 2009. They find the saving rate is significantly reduced when the gross replacement rate is raised.

¹⁹ For evidence relating to New Zealand, see Creedy (1997).

Table 2: Saving Rates for Income Deciles

Decile	Income y_1	Saving rate s_1/y_1 (%)
1	365	-10.4
2	480	-3.1
3	590	1.2
4	700	4.0
5	820	6.3
6	960	8.3
7	1140	10.2
8	1400	11.9
9	1840	13.8

8 Impact of Other Policy Changes

This section considers the second group of policy changes discussed in Section 6. The results are based on a 10% change in the policy variable, and unlike the tax and expenditure changes, the level of public non-pension expenditure is not held constant at the same level for all policy simulations.

8.1 A Change in the Loan-to-Value Ratio

In the benchmark case, the mortgage loan-to-value ratio is set at 0.5. Suppose this is reduced to 0.45. The major effect is simply a reallocation of total savings toward housing and a reduction in financial savings of 7.9%, while leaving the overall level of savings, and the housing market, unchanged. This result is consistent with the view that a policy to lower the LVR is aimed at enhancing the stability of the financial system rather than a tool of monetary policy to moderate house price increases. Of course, the comparative static result here has no time dimension to adjustments, whereas in practice there is likely to be a short-term effect on housing demand.

A recent study from by Kuttner and Shim (2013) from the Bank for International Settlements used panel data on 57 countries (including New Zealand) with quarterly time series data to test the effectiveness of non-interest rate policies for stabilising the housing market. The authors concluded that for these policies, including limits on loan-to-value ratios, there was limited evidence that they had any effect on house prices; the results were not particularly robust with respect to a range of statistical methods. They suggest, 'Among the policies considered, a change in housing-related taxes is the only policy tool with a discernable impact on house

price appreciation' (Kuttner and Shim, 2013, p. 1).

8.2 A Change in the Compulsory Saving Rate

In the benchmark case, the compulsory contribution rate is 3.5% of gross income. Suppose this is raised by 10% to 3.85%. The principal impact is a reallocation of saving away from voluntary private savings into the compulsory scheme, such that s_1 falls by 4.1%, with a commensurate fall of 2.3% in total savings (which is used here to refer only to the sum of financial and housing saving) and of course a rise in the private pension.²⁰ Consumption spending rises in both periods by 1%. Taxation receipts from interest income fall as the compulsory savings are taxed at a concessional rate of 20% (in contrast to financial savings which are taxed at the standard tax rate of 25%). Lower tax revenue involves a decline of 1% in G to maintain a balanced budget.

The model can be used to examine the question of whether the existence of a compulsory contribution would result in an increase in the overall level of savings, or would simply lead to a corresponding reduction in private voluntary financial savings. It was found that households fully offset the effect of a compulsory savings scheme by a commensurate reduction in voluntary private savings.²¹

Instead of simply considering the introduction of a SAYG scheme in this way, suppose a scheme is introduced with a compulsory rate of 6% but the universal pension is reduced in a way that preserves the total retirement income from PAYG and SAYG schemes (that is, excluding income from private financial savings), while the tax rate is unchanged. In this case financial savings would fall by 15% and there would be a rise in G of 7%. Alternatively, suppose the SAYG scheme is introduced, with a compulsory rate of 6% and P is reduced to allow G to increase by 5%. In this case financial savings fall by 24% and P needs to be reduced by one third. However total retirement income from the two schemes rises by 10%. Finally suppose the SAYG scheme is introduced, again with a compulsory rate of 6%, and total retirement income is left unchanged at its original level while G is increased by 5%. This could be achieved with a small reduction in τ .

²⁰ A variant of the policy to increase the compulsory rate of saving is to abate the public pension in line with the additional income generated by the compulsory accumulations. This option is explored in section 9.2.

²¹ This contrasts with the finding of Law, Meehan and Scobie (2011) who, in the context of the KiwiSaver programme, found that two thirds of the increase in KiwiSaver accumulations was offset by reductions in other savings. However, the data used in their analysis were collected after the scheme had been in operation for a relatively short time. The present result of full offset is a 'long run' comparison after full adjustment.

8.3 A Change in the Tax Rate on Interest in the Compulsory Pension

The benchmark rate for taxation of the private pension earnings is set at 0.2, in contrast to the rate of taxation on labour and interest income of 0.25. Table 3 presents the results of increasing this by 10% to 0.22. This encourages a compensating rise in financial savings by 0.33%, and reduced consumption by 0.52%, including a reduction in housing consumption of 0.55%. These are small because of the small effect the tax change on W .

8.4 A Change in the Interest Rate

The interest rate is a critical price in determining the intertemporal pattern of consumption. Both income and substitution effects are involved when interest rates change, although in the Cobb-Douglas case the substitution effect always dominates. The net impact of a 10% increase in the interest rate to 1.21 is that consumption falls in period 1 by 0.75% as households are induced to increase savings by 2.7%) in order to shift consumption to period 2 (which rises by 3.7%). Savings for housing rise by 8.2%, in part to offset the higher costs of mortgage repayments, but at the same time the overall demand for housing falls by 0.75% with the higher rate of interest. Interest earnings on the compulsory savings also increase, leading to an increase in the private pension of 4.7%. The higher interest rate implies higher tax revenue which, via the government budget constraint, implies a small increase in G of 0.86%. Appendix C discusses the implications of alternative elasticities of substitution.

8.5 Eliminating Taxation of Interest Income

New Zealand, along with many other countries, has adopted the concept of 'comprehensive income' as the tax base. This is based on the value judgement that people should be taxed each period on their ability to consume, while maintaining their capital intact. This contrasts with the value judgement that people should be taxed on the basis of what they actually spend each period, which leads to the choice of expenditure as tax base. Income derived from capital, such as interest income, simply represents a new source of income and should be taxed along with other sources.²²

²² The New Zealand structure distorts the choice of income sources by exempting most capital gains from income tax.

Nevertheless, an interest income tax implies a different relative price of present versus future consumption, compared with the use of expenditure as the tax base, and this difference increases in high-inflation periods (given that nominal rather than real returns are taxed). For this reason, some people have argued in favour of eliminating or reducing the tax on interest income in order to generate higher levels of savings.

To explore the possible impact of the elimination of interest income tax, a revised version of the current model was constructed in which the tax on interest income is completely removed; Appendix B sets out the modifications required. This removal applies both to the earnings on voluntary financial savings and the income derived from the compulsory savings fund. All other variables were left at their original values.

As a result of the removal of tax on interest, there is a decrease in total tax revenue, so that G is reduced to achieve budget balance. However, the removal of the tax has the effect of raising the effective interest rate, such that the price of consumption, $1/\{1 + r(1 - \tau)\}$ falls in period 2 (the retirement period). This induces substitution toward higher consumption in period 2, which increases by 12%. The consumption of housing and its price fall by 2.4%.

This is accompanied by a rise in the private pension as accumulations of the compulsory saving element are bolstered by the now tax-free income on all savings. There is also a rise in financial saving (17%), reflecting in part a shift out of saving for housing.²³ The financial saving rate increases by 1.5 percentage points, saving for housing falls by 0.2 percentage points. The net result is an increase in the overall household saving rate of 1.4 percentage points, measured as a proportion of gross income. While the underlying relationships are not strictly linear, this result can be used to approximate the impact of a partial reduction in the rate of taxation of interest income. For example, if the tax rate were to be reduced from 0.25 to 0.20 rather than eliminated, the saving rate would rise by approximately 0.28 percentage points.

A question arises as to the extent to which the response of the saving rate is in part a reflection of the Cobb-Douglas utility function, for which the intertemporal elasticity of substitution is one. Suppose this elasticity were actually less than one: empirical estimates suggest that a value below one is more likely in practice. In that case there would be a more muted response to the fall in the relative price of consumption in period 2, and less substitution toward consumption in retirement

²³ With no interest-income taxation, there is also no deductibility of mortgage interest payments, in view of the required symmetry discussed above.

than in the Cobb-Douglas case. This in turn would reduce the need for extra saving to support consumption in period 2. The implication is that the rise in saving rates can be regarded as an upper bound: see also Appendix C for discussion of the CES case.

The increased saving rate following the elimination of the tax on interest income refers to financial saving during the working life (period 1 in the model). As those savings are made to support consumption in retirement. In the long run the net change in aggregate savings (in the cross section of overlapping generations) would be considerably reduced by the decumulation in retirement.

The effect of eliminating the tax on interest income was also examined under the assumption of a 5% increase in non-pension expenditure per person, G . In this case the tax rate on labour income would need to rise from 0.25 to 0.27, while the overall saving rate increases by 1.3 percentage points. If G is held constant at its base level when the tax on interest income is removed then the labour tax rate needs to rise from 0.25 to 0.255, and the overall saving rate rises by 1.8 percentage points.

9 Economy-wide and Demographic Changes

This section considers the third group of comparative static changes examined, which includes the non-policy changes.

9.1 A Rise in Income

Consider the impact of a 10% increase in income during the working years, y_1 . The higher value of W implies greater consumption of both housing and non-housing. At the same time the levels of both financial and housing savings increase by 23% and 8% respectively. Thus, while the financial saving rate rises, the saving *rate* for housing falls despite the absolute increase in the level. This apparent anomaly is simply due to the fact that following the rise in income of 10%, saving for housing rises less than 10%. This serves to underline the point that an increase in household savings during the working life is consistent with an apparent decline in the *rate* of saving. Greater income and the consequent rise in consumption spending means that both income tax and GST revenue rise. A balanced budget

is achieved by increasing G by 11.5%.

9.2 Population Ageing

The future fiscal challenges arising from population ageing and the associated decline in the ratio of workers to pensioners have been well rehearsed; for example, see Treasury (2013). It is important in practice to consider dependency ratios separately from population structure ratios, but in the present model all individuals work in the first period of the life cycle.

A value of N_w/N_p equal to 10% lower than the benchmark of 2.5 implies that, as a result of the decline in tax revenue, government non-pension spending per person falls by nearly 4%. However, reducing spending is only one possible way to achieve a balanced budget in the face of the decline in revenue associated with the falling share of workers in the economy. An alternative approach would be to hold spending constant and raise taxes. In this case, with both P and G held constant, the shortfall in revenue stemming from the ageing population could be met by raising τ from 25.0% to 26.2% or raising the GST rate, v , from 15% to 17.4%.²⁴

A further possible alternative for containing, at least partially, the rising costs of the PAYG pension, P , would be to change the way it is indexed. For example instead of being linked to average wage growth (which preserves its relativity with working-age incomes), it could be linked to a cost of living index (which would preserve its real value over time), or some average of the two.²⁵

The benchmark case assumes that P grows at the same rate as labour incomes (that is, $g' = g$) thus maintaining a constant relation to average wage growth. Suppose instead that indexation of P is adjusted to maintain a balanced budget, with G held constant. The rate of growth of P , set at $g' = 0.8$, the equivalent of 2% per year in the benchmark case, would need to be reduced by one percentage point. In other words the PAYG pension would grow in real terms at half the growth rate of average wages. The overall effect is to reduce lifetime wealth, W . Hence, consumption in both periods falls slightly, the financial and total saving

²⁴ As discussed above, the GST option is expected to have less effect on savings. Experiments with the Treasury's Long Term Fiscal Model (LTFM), for comparable degrees of population ageing, were found to produce very similar tax rate increases in order to maintain NZS and other expenditures at constant real levels. We are grateful to Matthew Bell for obtaining results using the LTFM.

²⁵ For an analysis over time of the impact on household saving of changing the method of indexation, see Law (2013).

rates increase slightly, and the housing saving rate falls slightly.²⁶

An alternative policy response to population ageing is to maintain constant total pension income from the PAYG and compulsory SAYG schemes combined, along with G , without raising taxes. This can be done by increasing the compulsory saving rate and simultaneously reducing P . The problem then is to find a compulsory rate of savings and a corresponding reduction in P such that the total retirement income from P combined with the private pension remains constant, as does the level of public expenditure per capita, G . The solution, which can be found by a process of trial-and-error, is to raise the compulsory contribution, δ , from 3.5% to 6.5% of gross income in period 1, and reduce P by 22%. The reduced value of P implies a lower value of W , and hence lower values of consumption in both periods: both fall by 3.3%. Total savings (financial and housing) decline by 7.3%. This decline together with that of P closely matches the rise in private compulsory saving.

9.3 Changes in the Preference for Housing

In the benchmark case, the parameter describing relative housing preferences, γ' , is 0.153. A 10% rise in γ raises γ' to 0.166 and results in a significant shift in the demand for housing at every price level. Consumption is reallocated from non-housing to housing consumption in both periods, with a result that GST revenue falls, leading through fiscal adjustments to a decline on non-pension expenditure in order to achieve a balanced budget. Not surprisingly, there is a marked rise in saving for housing, and the overall saving rate rises. House prices increase by 7.8% and the value of housing rises by 11.6% (on the assumption that the elasticity of supply of housing is 0.5).

With the shift from the consumption of non-housing goods toward housing, the amount of mortgage borrowing increases, and with it financial savings, as these are in part dedicated to the repayment of a larger mortgage. It is commonly argued that the apparent preference New Zealanders have for housing means other forms of saving are reduced. In fact these results demonstrate that for given incomes and a given structure of taxation, a shift in preferences toward housing is associated with a rise in household saving rates, given the LVR constraint.

²⁶ It could be argued that the PAYG pension involves an implicit saving rate, despite being financed by intergenerational transfers. The question arises of whether total savings, allowing for this fall in the implicit component as a result of the lower indexation, remain unchanged. Calculations show that the rise in total savings does not quite match the fall in implicit savings.

10 A Summary of the Policy Simulations

Table 3 provides a summary of the comparative static effects of policy changes examined in Sections 7 to 9. It focuses on saving and the housing market, two of the central themes of this study and, in each case, the value of the PAYG pension, P , is held constant. Alternatively, Figures 3 and 4 provide a graphical summary of the changes. As discussed above, comparisons among a range of policy changes are difficult: there is a danger of comparing policies which have very different scales. For example, in the present context the value of G is adjusted to keep the government budget constraint in balance (debt neutral changes are examined). While, by assumption, this does not affect the behaviour of the representative household, it is obviously an important variable in evaluating policy changes.

Table 3: Summary of Policy Effects

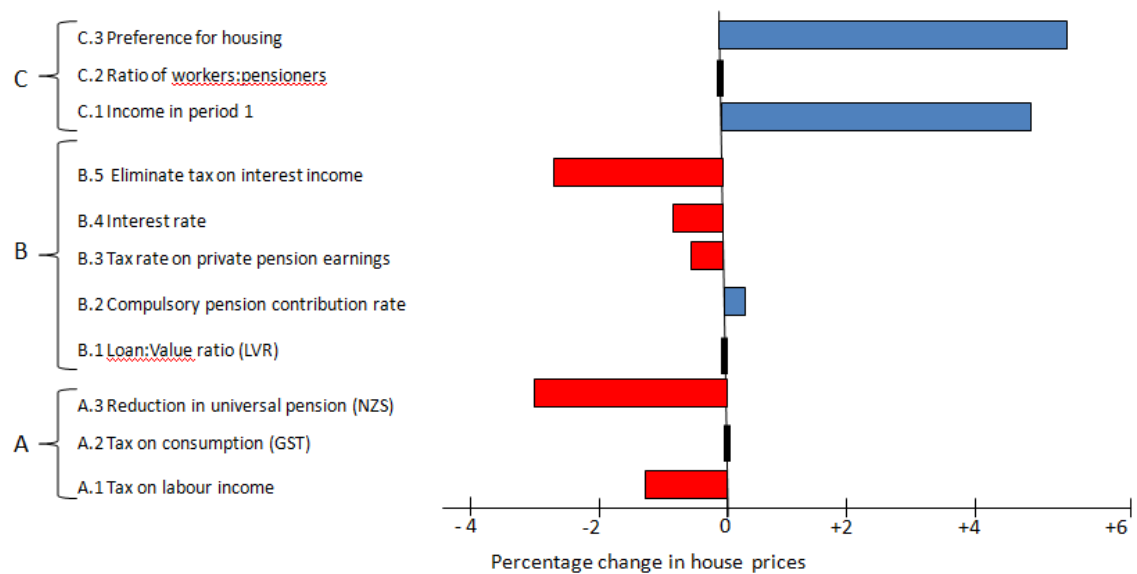
Policy Change	Percentage point change in:			Percentage change in:	
	Financial saving	Housing saving	Total saving rate	Stock of Housing	Price of Housing
<i>A. Tax and Expenditure Policies: producing an increase in G of 5%</i>					
Tax on labour income	-0.57	-0.11	-0.68	-0.54	-1.07
Tax on consumption	0	0	0	0	0
Public PAYG pension	2.95	-0.33	2.62	-1.62	-3.13
<i>B. Other Policy Changes: 10% increases, except for removal of interest income tax</i>					
Loan to value ratio	-0.70	0.70	0	0	0
Contrib rate to private pension	-0.36	0.00	-0.36	0.00	0.01
Tax on private pension earnings	0.03	0.00	0.03	-0.02	-0.03
Interest rate	0.47	-0.05	0.42	-0.25	-0.50
Remove interest income tax	1.52	-0.17	1.35	0	-2.42
<i>C. Economy-wide and demographic changes</i>					
Period 1 income: 10% increase	1.00	-0.11	0.89	2.60	5.49
Ratio N_W/N_P : 10% reduction	0	0	0	-1.13	-2.22
Housing pref: 10% increase in γ	0.26	0.58	0.84	2.63	5.56

Part A of Table 3 refers to the tax and expenditure policies whose impacts were estimated assuming the same change in non-pension expenditure (an increase of 5%); in this way they are directly comparable. Raising the tax rate on labour and interest income reduces the saving rates and lowers demand in the housing market. In contrast, a rise in the rate of GST lowers consumption but, because it does not affect intertemporal price ratios, leaves saving and the housing market unaffected. The increase in G requires a large reduction in the PAYG pension, of over 30%, and not surprisingly this stimulates a relatively large increase in financial savings.

Figure 3: Percentage Point Changes in Saving Rates



Figure 4: Percentage Changes in House Prices



Of the other policy changes shown in Part B, all relevant policy variables are increased by 10% except for the policy of removing the tax on interest income which is clearly a very much larger change (the relevant tax rate falls from 25% to zero). Care must therefore be taken interpreting the larger increase in savings reported by this policy change. Furthermore, the increase is modified when the income tax rate is simultaneously raised in order to keep G constant.²⁷ This is the main change which affects the intertemporal effective price ratio between present and future consumption, so the reduction in the price of consumption in period 2 leads to more saving. The direction of change is unambiguous in the case of Cobb-Douglas utility assumed here, with a high elasticity of substitution.²⁸

The results here represent an upper bound on the increased saving rate for several reasons. First, in reality any policy change would probably be less than the total removal of the tax. Second, higher savings over the working life would be matched by decumulation in retirement, leading to no change in aggregate. Third, the Cobb-Douglas form of the utility function leads to greater substitution towards savings than with an intertemporal elasticity of substitution less than 1.

11 Conclusions

This paper has examined the inter-related choices made by a representative household regarding saving, consumption, housing and retirement income. It has developed a two-period life-cycle model in which the optimal values of these variables are all outcomes of utility maximising behaviour of a representative household, subject to a budget constraint. In addition, an important element of the model is the incorporation of a government budget constraint, in which government pension and all other expenditures are financed on a pay-as-you-go basis. This ensures that any policy changes do not result in budget imbalances and associated changes in public debt levels.

Furthermore, there are critical feedbacks from the government to the household sector via taxes, pensions and non-transfer expenditures. The model incorporates income taxation, including interest-income taxation, as well as a broad-based consumption tax in the form of a GST. It has a universal (non-taxable) public pension and accommodates both private pension savings and a compulsory

²⁷ In the present model this has a relatively small effect because a small amount of revenue is raised by interest-income taxation. In reality, most interest-income tax is raised at higher marginal tax rates, and the tax forms about 6% of total personal tax revenue.

²⁸ Elasticities of substitution below unity imply smaller responses, and for low elasticities the income effect can in fact outweigh the substitution effect of a price change.

saving scheme.

The model is calibrated to a stylised version of the New Zealand economy. It is then used to simulate the responses of the representative household to a change in policies and other exogenous shifts. Particular attention is given to the response of savings, consumption and housing to changes in various tax rates, pension and savings policies, and demographic changes.

In general the responses are typically modest. For example, a 6% rise in the average income tax rate reduces both financial saving and total saving rates by 0.7 percentage points. In view of the fact that New Zealand gives essentially no tax concessions on interest income, this issue was explored in some detail. The model was adjusted to eliminate all tax on interest income. In the first instance financial savings and total saving rates rise by 17% and 8% respectively. Overall household saving rates would rise by 1.4 percentage points. This is accompanied by a significant shift toward consumption in retirement and weaker consumption of housing services leading to a fall in house prices of some 2.5%. The loss of tax revenue is compensated by a reduction of some 1.7% in the public non-pension expenditure, while holding unchanged the real value of the universal pension. In contrast, if the non-pension expenditures were to be also held constant, then tax rates elsewhere would need to be increased.

These results underscore the importance of including a government budget constraint and, in particular, the mechanism by which a balanced budget is achieved after a policy intervention that alters the initial level of tax revenue or total expenditure. Were the public pension to be indexed to a mix of wages and prices such that it grew in real terms at 1.0% rather than 2% (in annual terms), the overall effect would be to reduce lifetime wealth following the fall in the real value of the public pension. Hence, consumption in both periods falls very slightly, the financial and total saving rates increases slightly, and the housing saving rate falls modestly.

Raising the rate of compulsory saving by 10% from its base of 3.5% of gross income leads to offsetting declines in other financial savings; in fact on average households would fully offset the effect of a compulsory savings scheme by a commensurate reduction in their voluntary private savings.

A 10% decrease in the loan-to-value ratio from its benchmark of 0.5 results in a shift of savings from financial savings toward housing, but with little overall impact on total savings. The housing market is unaffected with no long run changes predicted in either prices or the stock of housing.

All these results have been obtained as comparative static exercises. They do

not allow for either the time that adjustments to policy changes would take, or the time path of those adjustments. Despite the strong assumptions of the model, it provides a rigorous and internally consistent framework for assessing the direction and magnitude of key long term responses in saving, consumption, housing and pensions to potential changes in tax and retirement income policies.

Particular attention in the simulations was given to the potential impact on household saving rates of a range of policy changes. Typically the effect on saving rates was found to be modest. In most instances, it would take very substantial changes in existing policy settings to induce significant increases in household saving rates. The main options that would increase household saving rates by more than one percentage point are reductions in the level of the PAYG pension or a substantial cut in the taxation of interest income. In both cases house prices would decline by 2 to 3%. However there are different fiscal implications. While a reduction in the pension would allow for tax cuts or increases in other expenditures, the loss of revenue from reducing taxes on interest income would mean higher taxes or reduced expenditure on non-pension items.

Higher average incomes over the working life would result in higher rates of household saving, increased consumption and higher retirement incomes. However some of the increased demand stemming from higher incomes would affect the housing market. In the long run the stock of housing would increase, but in the short run some of the demand would be reflected in higher house prices.

Any potential policy changes which are explicitly designed to raise saving rates should recognise that the long run impact is likely to be modest. An analytical framework such as that developed here can help to understand the complex interactions and provide some guidance on the likely magnitude of policy responses.

Appendix A: Renting versus Ownership

In the two period model of this paper, the assumption was made that the representative individual does not purchase a house in the second (retirement) period of the life cycle. The aim of this appendix is to examine the conditions under which it would be optimal to purchase a house in period 2, while renting in period 1, rather than purchase in period 1. The assumption underlying the above analysis is strengthened if it turns out that very strong conditions are required for purchase in period 2 to be optimal. Given this objective, in what follows there is no need to consider the option of renting in both periods. Furthermore, to simplify the analysis, this appendix abstracts from income and consumption taxation, and transfer payments (such as a tax-financed pension). In addition, no mortgage borrowing is allowed.

The representative consumer must choose optimal values of consumption of non-housing and housing in both periods $[C_1, C_2, C_{H1}, C_{H2}]$. The fundamental choice considered here is to be a renter (type- R) in period 1 and buy a house in which to live in period 2; or to be an owner (type- O) in period 1 and live in the house in both periods. The ‘renter’ must accumulate financial savings in period 1 to fund both house purchase and retirement income in period 2, as well as paying rent in period 1. The ‘owner’ saves in period 1 to fund only retirement income in period 2. Housing consumption, C_{H1} and C_{H2} , can be thought of as being measured in ‘quality units’. For a house owner, consumption is equal to the imputed rent. For a renter, consumption is somewhat below the equivalent imputed rental: there are benefits merely from the fact of ownership which are not appropriated by a renter.

One approach to this problem would be to set up the complete optimisation problem involving the range of discrete choices available. However, progress can be made using a simplified approach to obtain an indication of the condition required for option R to be preferred to option O , as follows. Let superscripts R and O represent consumption in the respective cases. Table 4 gives expressions for C_1, C_2, C_{H1} and C_{H2} , for the R and O cases, in terms of the corresponding savings, and informed by the relevant budget constraint.

Consumption of non-housing by renters in period 1, C_1^R , is equal to exogenous income, y_1 , less financial savings in period 1, S_1^R , less housing rent paid in period 1, R_1 . As mentioned above, the payment of rent gives rise to ‘quality units’ of consumption of $C_{H1}^R = \phi R$, with $\phi < 1$. Consumption of non-housing in period

Table 4: Budget Constraints for Renters and Owners

Renters		Owner-Occupier	
C_1^R	$= y_1 - S_1^R - R_1$	C_1^O	$= y_1 - S_1^O - S_H^O$
C_2^R	$= y_2 + \theta S_1^R(1+r)$	C_2^O	$= y_2 + S_1^O(1+r)$
C_{H1}^R	$= \phi R_1 = \phi \lambda S_H^O$	C_{H1}^O	$= \lambda S_H^O$
C_{H2}^R	$= (1-\theta)S_1^R(1+r)$	C_{H2}^O	$= (1-\lambda+\pi)S_H^O$

Table 5: Consumption Differences

(1)	$C_1^O - C_1^R$	$= S_1^R - S_1^O - (1-\lambda)S_H^O$
(2)	$C_2^O - C_2^R$	$= (1+r)(S_1^O - \theta S_1^R)$
(3)	$C_{H1}^O - C_{H1}^R$	$= (1-\phi)\lambda S_H^O$
(4)	$C_{H2}^O - C_{H2}^R$	$= (1-\lambda+\pi)S_H^O - (1-\theta)S_1^R(1+r)$

2, C_2^R , is equal to $y_2 + \theta S_1^R(1+r)$ where θ is the fraction of the total return to financial saving, $S_1^R(1+r)$, that is allocated to period 2's non-housing consumption. Hence a fraction, $1-\theta$, is allocated to house purchase in period 2. This delivers $C_{H2}^R = (1-\theta)S_1^R(1+r)$ of housing consumption, as shown in the final line of Table 4).

The right-hand column in Table 4 shows the corresponding expressions for consumption in the O case. Here, C_1^O is income, y_1 , less housing equity in period 1, S_H^O , less financial savings, S_1^O . The housing asset, S_H^O , delivers housing consumption in both periods. Let a fraction λ , be delivered in period 1, with $1-\lambda$ in period 2. Housing consumption in the later period also benefits from the appreciation of the asset at rate π . A possible value for λ would be around 0.67 where the working life (period 1) is approximately twice the length of the retirement period 2.

Using the expressions in Table 4, the differences between the four consumption values are given in Table 5. The expressions in the table do not of course represent solutions for the differences between consumption levels: the various values of forms of savings are endogenous. Nevertheless, further insights can be obtained by making the assumption that, for optimal solutions, the period 2 consumption values are the same for R and O -types, so that $C_2^O - C_2^R = 0$ and $C_{H2}^O - C_{H2}^R = 0$. By assumption, R is a home owner in the second period, so that both types enjoy

the benefits of ownership not available to renters.²⁹ From lines (2) and (4) in Table 5, these equalities imply:

$$S_1^R = S_1^O / \theta \quad (\text{A.1})$$

and:

$$S_1^R = \frac{(1 - \lambda + \pi)}{(1 - \theta)(1 + r)} S_H^O \quad (\text{A.2})$$

Substitute for S_1^R in (A.2) using (A.1) to yield:

$$S_1^O = \frac{\theta(1 - \lambda + \pi)}{(1 - \theta)(1 + r)} S_H^O \quad (\text{A.3})$$

Furthermore, using (A.3), line (1) of Table 5 can be written as:

$$\begin{aligned} \Delta C_1 &= C_1^O - C_1^R \\ &= \left[\frac{(1 - \lambda + \pi)}{(1 - \theta)(1 + r)} - \frac{\theta(1 - \lambda + \pi)}{(1 - \theta)(1 + r)} - (1 - \lambda) \right] S_H^O \\ &= \frac{\pi - r(1 - \lambda)}{(1 + r)} S_H^O \end{aligned} \quad (\text{A.4})$$

It is also clear from line (3) of Table 5 that:

$$\begin{aligned} \Delta C_{H1} &= C_{H1}^O - C_{H1}^R \\ &= (1 - \phi)\lambda S_H^O > 0 \end{aligned} \quad (\text{A.5})$$

It is thus possible to make utility comparisons without explicit reference to the nature of the precise utility functions and without solving alternative models. Thus, with $\Delta C_2 = 0$ and $\Delta C_{H2} = 0$ by assumption, and with the above result in (A.5) that $\Delta C_{H1} > 0$, utility in the *O*-type case is unambiguously higher than the *R*-type case if $\Delta C_1 > 0$. This is a sufficient condition, and is thus stronger than a necessary condition. Suppose, in addition, that C_1 and C_{H1} have the same impact on utility, *O*-type utility exceeds that of the *R*-type if $\Delta C_1 + \Delta C_{H1} > 0$.

From equation (A.4), a necessary and sufficient condition for $\Delta C_1 > 0$, is that $\pi > r(1 - \lambda)$: this is a less stringent condition than $\pi > r$ (since $\lambda < 1$). From (A.4) and (A.5), $\Delta C_1 + \Delta C_{H1} > 0$ if:

$$\frac{\pi - r(1 - \lambda)}{(1 + r)} + (1 - \phi)\lambda > 0 \quad (\text{A.6})$$

After rearranging, this becomes:

$$\pi > (1 + r)(\phi - 1)\lambda + r(1 - \lambda) \quad (\text{A.7})$$

²⁹ In the case where the individual does not buy in the second period, then it would not make sense to set the two consumption levels equal. But the emphasis of the analysis is to compare the two special cases, both of which involve home ownership in the second period.

Consider, as suggested above, that $\lambda = 0.67$ and $\phi = 0.8$ (each \$1 of rental housing delivers 80% as much housing consumption as \$1 of imputed rental housing), and $r = 1.1$, which is the two-period analogue of an annual interest rate of 2.5% over 30 years. The condition in (A.7) gives:

$$\begin{aligned}\pi - r(1 - \lambda) &> -0.28 \\ \pi - r &> -1.02 \\ \pi &> 0.08\end{aligned}\tag{A.8}$$

Hence, a value of $\pi - r > -1.02$, or $\pi > 0.08$, implies $\Delta C_1 + \Delta C_{H1} > 0$ and hence the *O*-type case is unambiguously preferred to the *R*-type case, if C_1 and C_{H1} take equal utility weighting. In the model simulations, $\pi > r$, ($\pi = 1.4$), hence $\pi > r(1 - \lambda)$; thus $\Delta C_1 > 0$, and $\Delta C_{H1} > 0$, which is consistent with the *O*-type being unambiguously preferred, regardless of utility weighting.

For the condition $\Delta C_1 + \Delta C_{H1} > 0$ *not* to hold requires $\pi < r(1 - \lambda)$, or $\pi - r$ has to be *more negative* than -1.02 . That is, with $r = 1.1$, the return to housing would have to be less than approximately *minus* the rate of return to financial savings, r , which is an unlikely scenario.

Finally, if there are no additional consumption benefits specific to owner-occupation, that is, $\phi = 1$, then from (A.5) $\Delta C_{H1} = 0$. Hence owner-occupying is unambiguously preferred as long as $\Delta C_1 > 0$. From (A.4) this holds if:

$$\pi > r(1 - \lambda)\tag{A.9}$$

For the previous illustrative values ($\lambda = 0.67$; $r = 1.1$), this simply requires $\pi > 0.363$: the simulations above use $\pi = 1.4$ and $r = 1.1$. If, alternatively, $\pi < r(1 - \lambda)$, then $\Delta C_1 < 0$ and renting is unambiguously preferred.

Appendix B: No Interest-Income Tax

This appendix modifies earlier results by eliminating interest income taxation. The individual's budget constraint is:

$$W \equiv y_1 (1 - \tau) + \frac{P + y_2 (1 - \tau)}{1 + r} = c_1 (1 + v) + \frac{c_2 (1 + v)}{1 + r} + \frac{c_H}{1 + \pi} \quad (\text{B.1})$$

The term in $1 + r (1 - \tau)$ is thus simply replaced by $1 + r$ in many of the previous expressions. As there is no interest income tax, the SAYG scheme contains no particular tax advantage over ordinary financial savings, and the government's budget constraint is modified to:

$$N_p P + (N_p + N_w) G = \tau \sum_{i=1}^{N_w} y_{w,1,i} + \tau \sum_{i=1}^{N_p} y_{p,2,i} + v \left(\sum_{i=1}^{N_w} c_{w,1,i} + \sum_{i=1}^{N_p} c_{p,2,i} \right) \quad (\text{B.2})$$

The solution for G is found to be:

$$\begin{aligned} \left(1 + \frac{N_w}{N_p} \right) G &= \left(\frac{N_w}{N_p} \right) \tau \bar{y}_{w,1} + \tau \bar{y}_{p,2} \\ &+ \frac{v \beta' (1 - \tau)}{1 + v} [\bar{y}_{p,1} (1 + r) + \bar{y}_{p,2}] \\ &+ \frac{v \alpha'}{1 + v} \left[\bar{y}_{w,1} (1 - \tau) + \frac{\bar{y}_{w,2} (1 - \tau)}{1 + r} \right] \frac{N_w}{N_p} \\ &- P \Omega \end{aligned} \quad (\text{B.3})$$

with:

$$\Omega = 1 - \frac{v}{1 + v} \left\{ \beta' + \frac{\alpha' (1 + g')}{1 + r (1 - \tau)} \frac{N_w}{N_p} \right\} \quad (\text{B.4})$$

Appendix C: A CES Utility Function

This appendix considers the more general case of a utility function with a constant, but non-unit, elasticity of substitution between pairs of goods.³⁰ Suppose the utility function takes the more general form, where $\eta \neq 1$:

$$U = \alpha c_1^{1-\frac{1}{\eta}} + \beta c_2^{1-\frac{1}{\eta}} + \gamma c_H^{1-\frac{1}{\eta}} \quad (\text{C.1})$$

The parameter η is the elasticity of substitution between each pair of goods. The lifetime budget constraint is not affected by the mortgage, and is therefore given by:

$$W \equiv y_1 (1 - \tau) + \frac{P + y_2 (1 - \tau)}{1 + r (1 - \tau)} = c_1 (1 + v) + \frac{c_2 (1 + v)}{1 + r (1 - \tau)} + \frac{c_H}{1 + \pi} \quad (\text{C.2})$$

where τ is the proportional income tax rate, v is the tax-exclusive GST rate and P is the universal pension financed on a pay-as-you-go basis. It can be shown that the solution for the maximisation of (C.1) subject to (C.2) is as follows. First, define the term, K , where:

$$K^{-1} = 1 + \frac{\beta}{\alpha} \left\{ \frac{\alpha}{\beta (1 + r (1 - \tau))} \right\}^{1-\eta} + \frac{\gamma}{\alpha} \left\{ \frac{\alpha}{\gamma (1 + \pi) (1 + v)} \right\}^{1-\eta} \quad (\text{C.3})$$

The optimal value of consumption in each period is:

$$c_1 = \frac{1}{1 + v} KW \quad (\text{C.4})$$

$$c_2 = \frac{1}{1 + v} \left\{ \frac{\alpha}{\beta (1 + r (1 - \tau))} \right\}^{-\eta} KW \quad (\text{C.5})$$

and housing consumption is:

$$c_H = \left\{ \frac{\alpha}{\gamma (1 + \pi) (1 + v)} \right\}^{-\eta} KW \quad (\text{C.6})$$

In this case, $s_H = (1 - \xi) c_H / (1 + \pi)$ and, unlike the Cobb-Douglas case, it depends on π . Furthermore, financial saving in the first period, s_1 , is given by:

$$\begin{aligned} s_1 &= y_1 (1 - \tau - \delta) - c_1 (1 + v) - s_H \\ &= y_1 (1 - \tau - \delta) - \left[1 + \frac{1 - \xi}{1 + \pi} \left\{ \frac{\alpha}{\gamma (1 + \pi) (1 + v)} \right\}^{-\eta} \right] KW \end{aligned} \quad (\text{C.7})$$

³⁰ In considering a suitable value of this elasticity, a wide range of estimates is reported in the literature. Gunning *et al.* (2008) provide a review of estimates. Based on values reported from 15 empirical studies, mainly from the US, the median value was 0.5 and the mean was 0.66. A study by Diamond and Zodrow for the Treasury used 0.8 for New Zealand. See also Havránek (2013).

Consider the elasticity of c_1 with respect to a change in the rate of interest. Define $X = 1/K$.

$$E_{c_1,r} = \frac{r}{c_1} \frac{dc_1}{dr} = \frac{r}{K} \frac{dK}{dr} + \frac{r}{W} \frac{dW}{dr} \quad (\text{C.8})$$

and

$$\frac{dK}{dr} = -\frac{1}{X^2} \frac{dX}{dr} \quad (\text{C.9})$$

so that:

$$E_{c_1,r} = -\frac{r}{X} \frac{dX}{dr} + \frac{r}{W} \frac{dW}{dr} \quad (\text{C.10})$$

With:

$$\frac{dX}{dr} = -(1-\eta) \left\{ \frac{\alpha}{\beta(1+r(1-\tau))} \right\}^{-\eta} \left\{ \frac{1-\tau}{1+r(1-\tau)} \right\} \quad (\text{C.11})$$

Hence:

$$E_{c_1,r} = rK(1-\eta) \left\{ \frac{\alpha}{\beta(1+r(1-\tau))} \right\}^{-\eta} \left\{ \frac{1-\tau}{1+r(1-\tau)} \right\} + \frac{r}{W} \frac{dW}{dr} \quad (\text{C.12})$$

Furthermore, rewriting $c_2 = \frac{1}{1+v} \left\{ \frac{\alpha}{\beta(1+r(1-\tau))} \right\}^{-\eta} KW$ as:

$$c_2 = c_1 \left\{ \frac{\alpha}{\beta(1+r(1-\tau))} \right\}^{-\eta} \quad (\text{C.13})$$

so that writing:

$$Y = \left\{ \frac{\alpha}{\beta(1+r(1-\tau))} \right\}^{-\eta} \quad (\text{C.14})$$

the elasticity of c_2 with respect to r is:

$$E_{c_2,r} = E_{c_1,r} + \frac{r}{Y} \frac{dY}{dr} \quad (\text{C.15})$$

Using

$$\frac{r}{Y} \frac{dY}{dr} = \eta r (1-\tau) \quad (\text{C.16})$$

$$E_{c_2,r} = E_{c_1,r} + \eta r (1-\tau) \quad (\text{C.17})$$

Also c_H is proportional to c_1 , since:

$$c_H = \left\{ \frac{\alpha}{\gamma(1+\pi)(1+v)} \right\}^{-\eta} (1+v) c_1 \quad (\text{C.18})$$

The constant of proportionality does not depend on the interest rate. Hence:

$$E_{c_H,r} = E_{c_1,r} \quad (\text{C.19})$$

Housing savings, s_H , are given by:

$$\begin{aligned} s_H &= \left(\frac{1-\xi}{1+\pi} \right) c_H \\ &= \left(\frac{1-\xi}{1+\pi} \right) \left\{ \frac{\alpha}{\gamma(1+\pi)(1+v)} \right\}^{-\eta} (1+v) c_1 \end{aligned} \quad (\text{C.20})$$

Financial savings, s_1 , can be expressed as:

$$s_1 = y_1 (1 - \tau - \delta) - (1 + v) c_1 \left[1 + \left(\frac{1 - \xi}{1 + \pi} \right) \left\{ \frac{\alpha}{\gamma (1 + \pi) (1 + v)} \right\}^{-\eta} \right] \quad (\text{C.21})$$

and letting the term in square brackets, which does not depend on r , be denoted by Φ , the elasticity of s_1 with respect to r is given by:

$$E_{s_1, r} = - (1 + v) \Phi \frac{c_1}{s_1} E_{c_1, r} \quad (\text{C.22})$$

It is seen above that the elasticity, $E_{c_1, r}$, contains the term, $\frac{r}{W} \frac{dW}{dr}$, reflecting the elasticity of net worth with respect to the interest rate. As shown earlier:

$$W = y_1 (1 - \tau^*) + \frac{P + y_2 (1 - \tau)}{1 + r (1 - \tau)} \quad (\text{C.23})$$

Hence:

$$E_{W, r} = - \frac{\{P + y_2 (1 - \tau)\} (1 - \tau) r}{\{1 + r (1 - \tau)\}^2 W} \quad (\text{C.24})$$

It was found above that changes in the rate of interest had a negligible effect on housing consumption. This is caused by two factors. First, the government budget constraint led to an increase in the PAYG pension, P , so that W was almost constant. Secondly, for the Cobb-Douglas utility function, the only influence of r on c_H is via its effect on W . Figure 5 shows the variation in housing consumption with the rate of interest, for different values of η , for a fixed value of W , and holding other parameters at their benchmark values. It is clear that the relative lack of sensitivity is shared by other values of η . For $\eta < 1$, c_H increases slightly as r increases, while for $\eta > 1$, c_H falls slightly. Hence the result in the paper arises not from the choice of η but from the operation of the government budget constraint. The absolute values of c_H and other variables clearly do depend on η as, unlike the case of $\eta = 1$, the taste coefficients do not determine expenditure shares in such a simple manner.

It was also found that in the Cobb-Douglas case of $\eta = 1$, the value of s_H does not depend on house price appreciation, π . The results presented here for $\eta \neq 1$ show that s_H does depend on π . Figure 6 shows, for other parameters set at their benchmark values, the variation in s_H with π for different values of π . Compared with the constant Cobb-Douglas case, s_H falls slightly as π increases for $\eta < 1$, and rises slightly as π increases for $\eta > 1$. However, again the variations in slopes around any given value of π are small.

As mentioned above, the absolute values vary if the preference parameters are held constant. For this reason, elasticity values calculated on the assumption that the values of α , β and γ are fixed would show larger differences than the slopes,

Figure 5: Variations in Housing Consumption with Rate of Interest

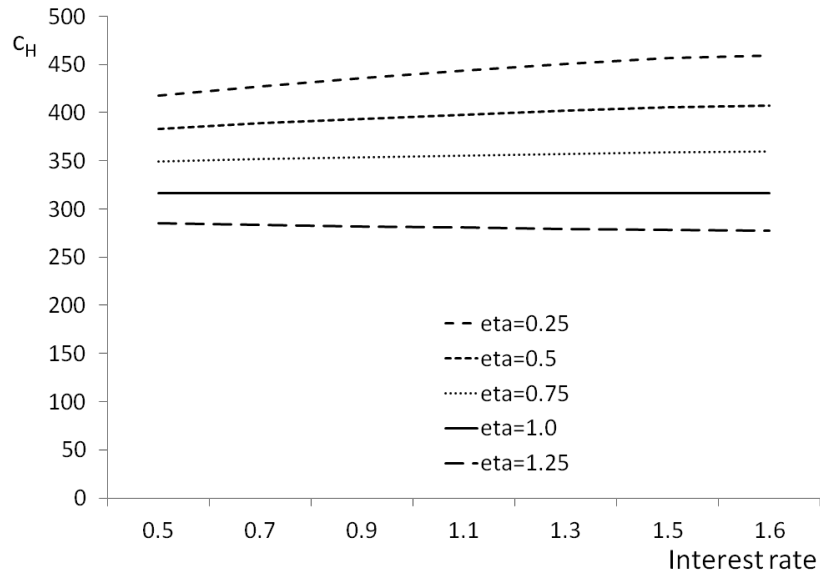
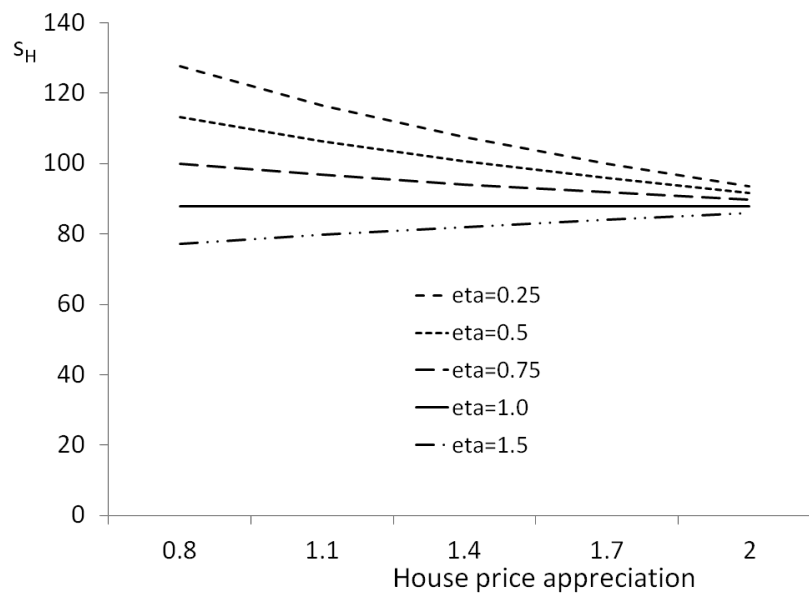


Figure 6: Variations in Housing Savings with House Price Appreciation



while the latter are more relevant for many of the comparative static comparisons. More appropriate comparisons of elasticities would be for values of α , β and γ which give similar 'benchmark' values of the major endogenous variables of interest. However, calculations show that the term, $E_{W,r}$, is a relatively large component of the various elasticities derived in this appendix. Yet it has been seen that the role of the budget constraint, involving an endogenous change in P , is to leave W virtually unchanged when the rate of interest changes.

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