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**Working Paper**

## Intergenerational Smoothing of New Zealand's Future Fiscal Costs

New Zealand Treasury Working Paper, No. 13/12

**Provided in Cooperation with:**

The Treasury, New Zealand Government

*Suggested Citation:* Guest, Ross (2013) : Intergenerational Smoothing of New Zealand's Future Fiscal Costs, New Zealand Treasury Working Paper, No. 13/12, ISBN 978-0-478-40351-0, New Zealand Government, The Treasury, Wellington

This Version is available at:

<https://hdl.handle.net/10419/205636>

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# Intergenerational Smoothing of New Zealand's Future Fiscal Costs

Ross Guest

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New Zealand Treasury Working Paper 13/12

July 2013



New Zealand Government

**NZ TREASURY WORKING  
PAPER 13/12**

Intergenerational Smoothing of New Zealand's Future Fiscal Costs

**MONTH/YEAR**

July 2013

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**ISBN (ONLINE)**

978-0-478-40351-0

**URL**

Treasury website at July 2013:  
<http://www.treasury.govt.nz/publications/research-policy/wp/2013/13-12/>  
Persistent URL: <http://purl.oclc.org/nzt/p-1569>

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# Abstract

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This paper applies an overlapping generations model in order to evaluate the implications of intergenerational smoothing of New Zealand's future fiscal costs. The analysis complements the New Zealand fiscal projections of Bell et al. (2010) and the New Zealand tax smoothing analysis in Davis and Fabling (2002). It allows for feedback effects of the tax rate on labour supply through both intratemporal and intertemporal effects which in turn feed back to fiscal projections via taxation revenue. Under Treasury's sustainable debt projections, which implies convergence to a stable 20% net debt to GDP ratio, generations born prior to 1990 are worse off and those born after 2000 are better off (measured by the impact on their remaining lifetime income). However, the magnitudes of the impact on the remaining lifetime income of all generations are small – no greater than 0.7% under the Medium demographic scenario. Those born around 1960 fare the worst, while those born after 2020 fare the best. The losses to current generations are weighed up against the gains to future generations through the social welfare function. The results show that net social gains are possible provided the gains to future generations are given sufficient weight by a low rate of social time preference and a high rate of aversion to variability in aggregate consumption over time. The parameter values required to generate net social gains are close to the bounds of plausible values. The magnitudes of the net social gains/losses range from minus \$90 to plus \$94 per capita per year.

**JEL CLASSIFICATION**

H31, H32, J18, E21

## Executive Summary

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This paper applies an overlapping generations model in order to evaluate the implications of intergenerational smoothing of New Zealand's future fiscal costs. The approach here is to raise the average tax rate at the start of the projection period and keep it constant throughout the projection period in order to reach a target net debt to GDP of 20%. This differs from the approach adopted by New Zealand Treasury (Bell et al, 2010) which projects across the board spending cuts in order to reach a target net debt to GDP of 20%, the figure that Treasury adopts for its sustainable debt scenarios.

The alternative approach here represents tax smoothing in the sense that the tax rate is higher initially but eventually lower than it would be if the tax rate were raised gradually in line with rising government spending in order to balance budgets. The model allows for feedback effects of the tax rate on labour supply through both intratemporal and intertemporal effects which in turn feed back to fiscal projections via taxation revenue.

Tax smoothing implies that current generations will bear a greater tax burden, and future generations a lower burden, than they would under continuously balanced budgets. However the effects are arguably not large. For the baseline demographic projections, no generation is better or worse off by more than 0.7% of remaining lifetime income. This is perhaps not surprising given that the sustainable debt scenario requires only a small increase in the tax to GDP ratio of 0.5% at the most over a decade from 2015.

Those born around 1960 fare the worst, but only suffer a 0.7% drop in their remaining lifetime incomes. This generation is at their peak earning capacity when the tax smoothing policy is introduced, which results in an initial jump in tax rates. They have also retired before the balanced budget scenario yields the payoff of a lower tax rate. Retired workers are also worse off because they pay higher tax rates on their retirement income. Future workers are better off because they escape the higher taxes on earlier generations and reap the gains from lower future taxes.

The losses to current generations can be weighed up against the gains to future generations through the social welfare function. The results show that net social gains are possible provided the gains to future generations are given sufficient weight by appropriate choice of parameters in the social welfare function. The parameter values required to generate net social gains are close to the bounds of plausible values. Depending on the parameter values, the magnitudes of the net social gains/losses range from minus \$90 to plus \$94 per capita per year.

# Table of Contents

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<b>Abstract</b> .....	<b>i</b>
<b>Executive Summary</b> .....	<b>ii</b>
<b>1 Introduction</b> .....	<b>1</b>
<b>2 Intergenerational distribution of the national consumption burden of ageing through fiscal policy</b> .....	<b>2</b>
2.1 Concepts.....	3
2.2 The effects of demographic projections .....	4
<b>3 The simulation model</b> .....	<b>5</b>
3.1 Intergenerational income analysis and social welfare analysis.....	8
<b>4 Data and parameters</b> .....	<b>9</b>
<b>5 Simulation results</b> .....	<b>11</b>
<b>6 Conclusion</b> .....	<b>17</b>
<b>References</b> .....	<b>18</b>
<b>Appendix A – The Simulation Model</b> .....	<b>20</b>

## List of Tables

---

Table 1 – Effect on lifetime income from date of tax smoothing .....	15
Table 2 – Effect on social welfare change of the sustainable debt scenario .....	16

## List of Figures

---

Figure 1 – Intergenerational allocation of the national consumption burden of population ageing through fiscal policy .....	3
Figure 2 – Support ratios. Effect of demographic projections .....	5
Figure 3 – Household labour income (after tax) and consumption .....	7
Figure 4 – Effect of endogenous labour on aggregate participation rate.....	11
Figure 5 – Government spending (ratio to GDP) .....	12
Figure 6 – Tax to GDP ratios – Medium demographic projection .....	12
Figure 7 – Budget balance (ratio to GDP) – Medium demographic projection .....	13
Figure 8 – Net debt to GDP ratios under alternative fiscal regimes – Medium demographic projection .....	13
Figure 9 – Tax to GDP ratios for sustainable debt (=20% GDP) – Various demographic scenarios .....	14
Figure 10 – Intergenerational equity with sustainable debt.....	14



# Intergenerational Smoothing of New Zealand's Future Fiscal Costs

## 1 Introduction

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This paper reports modelling of a response to future fiscal cost pressures in New Zealand. The approach here is to raise the average tax rate at the start of the projection period and keep it constant throughout the projection period in order to reach a target net debt to GDP of 20%. This differs from the approach adopted by New Zealand Treasury (Bell et al, 2010) which projects across the board spending cuts in order to reach a target net debt to GDP of 20%, the figure that Treasury adopts for its sustainable debt scenarios. The alternative approach here represents tax smoothing in the sense that the tax rate is higher initially but eventually lower than it would be if the tax rate were raised gradually in line with rising government spending in order to balance budgets. Tax smoothing implies that current generations will bear a greater tax burden, and future generations a lower burden, than they would under continuously balanced budgets.

The primary aim is to model the implications of this particular type of fiscal adjustment for the lifetime incomes of different generations and for national welfare. Different constant tax rates imply different debt paths and different intergenerational consequences. The methodology is similar to that in Guest (2008a) which was motivated by Australia's Future Fund (FF). The FF is essentially a vehicle for spreading the fiscal costs of population ageing over time, as acknowledged in the Australian Government's 2005-6 Budget Papers, Statement 7: "[the FF] will reduce calls on the budget in the future, at a time when significant intergenerational pressures are expected to emerge." The accumulation of budget surpluses in the FF therefore amounts to tax smoothing.<sup>1</sup> The simulations for New Zealand in this paper imply alternative projections for net Government debt, rather than the accumulation of net assets in a sovereign wealth fund.

Barro (1979) showed that, in a deterministic setting, a constant tax rate over time would minimize the distortions to behaviour arising from taxation. He pointed out that the distortions would increase more than proportionally to increases in the tax rate, drawing on Harberger (1964), cited in Browning (1987). An important distortion, or deadweight loss, arises from the substitution of leisure for work in response to taxation on labour. A policy of tax smoothing would reduce the magnitude of these distortions and therefore lead to a more efficient allocation of resources.

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<sup>1</sup> As at 31 March 2012 the FF held total assets of \$77 billion or 5.3% of GDP.



Empirical studies of tax smoothing have generally found small to modest positive gains in national output. For the U.S. see Cutler et al. (1990), for Europe see Floden (2003) and for Australia see Guest (2008). Cutler et al. (1990) found that the welfare gains from a constant tax rate that returned debt to its 1990 share of GNP after 60 years was 0.017% per year or, in present value terms, 1.1% of 1990 GNP. Floden (2003) found higher gains of up to 0.5% in *annual* consumption (for Italy but lower for most countries). The New Zealand study in Davis and Fabling (2002) found somewhat higher efficiency gains of between 3% and 5% of one year's GDP (2008) in net present value terms. However most of these gains are due to their assumption that the assets accumulated under tax smoothing generate a rate of return above the government's cost of borrowing. This assumption is ruled out in prior studies and here also. Guest (2008a) found even larger gains of around 1% in equivalent *annual* GDP over the projection period. The higher values in Guest (2008a) compared with Davis and Fabling (2002) and Cutler et al. (1990), which both assume a deadweight loss function, may be attributed to differences methodology – in particular a social aversion to variability in aggregate consumption over time, efficiency gains from lower distortions to intertemporal consumption, and a lower time preference rate, among other differences arising from the optimising approach. Floden (2003) uses a Ramsey intertemporal model which is closer to that in this model and, interestingly, produces larger estimates than those in Cutler et al (1990) and, for some countries, larger than Davis and Fabling (2002).

The model here complements the analyses of both Davis and Fabling (2002) and Bell et al (2010). A key difference is that the present study links feedback effects from the tax rate to labour supply through households' optimal leisure-consumption choice over the lifecycle; this effect on labour supply in turn feeds back to tax revenue. In Davis and Fabling (2002), the feedback effects operate in one direction from the tax rate to labour supply through an assumed constant labour supply elasticity. In a life cycle optimising model the labour supply elasticity with respect to the tax rate is a complex non-linear function of parameters in the model (Ziliak and Kneisner, 2005). Other minor differences include: in Davis and Fabling (2002) labour productivity growth and interest rates are stochastic whereas here labour productivity growth is zero (discussed further below) and the interest rate is constant. The simulations use the fiscal projections from Treasury's Long Term Fiscal Model (LTFM) adjusted for the labour supply response. Given the plans of each generation of households, aggregate consumption and labour supply in a given year are found by summing the consumption and labour supply of generations alive in that year. This overlapping generations framework allows a tracking of the effect of policy changes such as fiscal adjustments on the lifetime incomes of different generations. The model also considers the effect of tax smoothing on national (or social) welfare.

## 2 Intergenerational distribution of the national consumption burden of ageing through fiscal policy

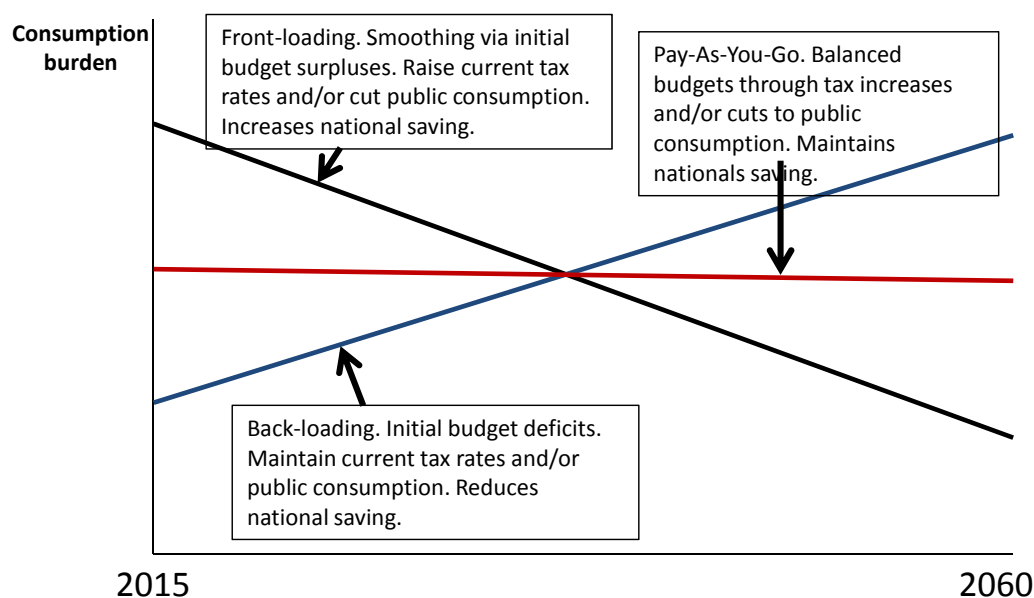
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Much of the projected fiscal cost pressures in New Zealand are attributed to population ageing (see Section 4 for numbers). Popular discussion of the costs of population ageing tends to conflate the national economic burden of ageing with the fiscal costs of ageing. The former refers to the effect of ageing on national consumption per capita over time, which occurs through the effects on the support ratio, labour productivity and the consumption share of GDP.

## 2.1 Concepts

The fiscal costs of ageing refer to the extent to which government revenue falls and/or expenses (including transfer payments) rise under current policies, that is, under existing age-specific public consumption expenditures and tax rates. The way the government responds to these fiscal costs – its fiscal policy – determines the division of the national consumption cost between public sector consumption and private sector consumption, and intergenerationally (between present and future taxpayers). If the government adopts a balanced budget response by progressively raising tax rates or cutting spending, then the fiscal costs of ageing are borne by taxpayers on a Pay-As-You-Go (PAYG) basis. If the budget goes into deficit the consumption burden of ageing is back-loaded onto future taxpayers. If the government prepays the fiscal costs of ageing by running budget surpluses then the fiscal cost is front-loaded onto current generations. The fiscal adjustment in any given year can be met by: (i) cutting public consumption and/or increasing transfers; or (ii) raising taxes and/or reducing transfers. Figure 1 illustrates the three fiscal policy responses just described. Cutting consumption and increasing transfers falls on contemporaneous<sup>2</sup> generations of households.

**Figure 1 – Intergenerational allocation of the national consumption burden of population ageing through fiscal policy**



The intergenerational effect of raising taxes depends on the extent to which the tax increases reduce investment as well as consumption (an issue not pursued further here).

The relationships between current and future consumption, and private and public consumption, can be seen more clearly from national accounting relationships as follows. Let national income be the sum of labour income and capital income:

$$NI = wL + rW \quad (1)$$

and define national consumption as national income minus national saving:

$$\begin{aligned} C &= NI - S \\ &= wL + rW - S \end{aligned} \quad (2)$$

<sup>2</sup> Meaning that a cut in consumption in year  $t$  falls mainly on households in year  $t$ . This is not exactly true, since for example, expenditure on education and defence is treated as consumption but in effect is partly investment to the extent that future generations benefit.

where,  $NI$  is national income,  $C$  is national consumption consisting of private consumption and public consumption,  $S$  is national saving,  $w$  is the average wage rate,  $L$  is labour supply,  $r$  is the interest rate, and  $W$  is national wealth which is defined as capital stock ( $K$ ) minus net foreign liabilities ( $D$ ). Substituting and rearranging, national consumption per capita is

$$\frac{C}{N} = \frac{L}{N} \left( w + r \frac{(K-D)}{L} - \frac{S}{L} \right) \quad (3)$$

The consumption burden of ageing refers to the impact on  $C/N$  through the right hand side variables in (3).<sup>3</sup> The most obvious and probably largest effect is the decline in the support ratio,  $L/N$  (discussed briefly below).

Fiscal policy can shift the consumption burden of ageing to future generations by running budget deficits, which arises from attempting to maintain public consumption and/or existing tax revenue as a share of GDP. This reduces national saving per worker ( $S/L$ ) which, from (3), allows ( $C/N$ ) to be higher. However lower saving reduces national wealth per worker,  $(K-D)/L$ , by either crowding out private investment (which reduces the capital stock,  $K/L$ ) or increasing the current account deficit (which increases foreign liabilities,  $D/L$ ). A lower capital stock lowers output and therefore future consumption possibilities. Higher foreign liabilities require higher debt servicing costs which also lowers the future consumption possibilities. In the opposite case, fiscal policy can shift the consumption burden of ageing to current generations by running initial budget surpluses which raises national saving and hence national wealth, allowing higher future consumption at the expense of current consumption. A middle path is to spread the consumption burden more evenly among generations through a balanced budget response to ageing.

## 2.2 The effects of demographic projections

Figure 2 plots the aggregate support ratios for the Medium Fertility, High Fertility, High Migration, Zero Migration and Low Mortality demographic projections.<sup>4</sup> Cutler et al. (1990) is the seminal study that introduced the notion of a support ratio in population economics. For a New Zealand application see Bryant, Guest and Scobie (2003). Under the Medium projection, the support ratio falls by 11.5% from 2015 to 2060, having already fallen by 3.5% by 2015 from 2005, and by 4.5% from its peak in 1988 (not shown in Figure 2). This implies, other things equal,<sup>5</sup> the 16% fall in the support ratio from its peak in 1988 to its projected level in 2060 would

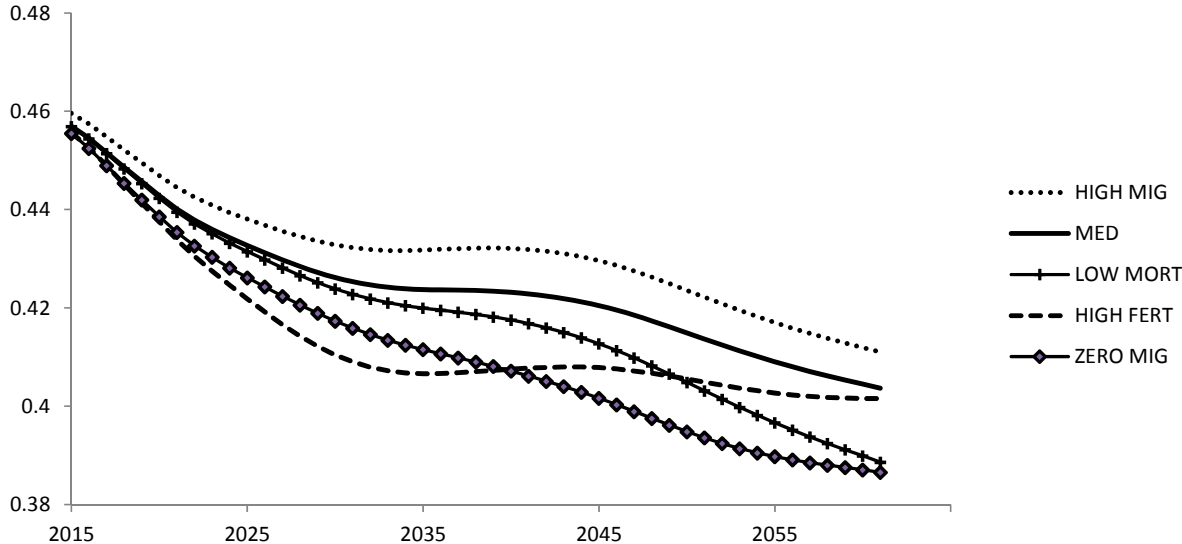
<sup>3</sup> To see the role of labour productivity, express the real wage rate,  $w$ , in (3) as a function of the capital to labour ratio as follows. Assume a constant returns to scale Cobb-Douglas production function and that labour is paid its marginal product. Therefore  $w = \frac{Y}{L} - r \frac{K}{L} = A \left( \frac{K}{L} \right)^\alpha - r \frac{K}{L}$  where  $A$  is a technology parameter. On substituting into (3), we have:  $\frac{C}{N} = \frac{L}{N} \left( \frac{Y}{L} + r \frac{D}{L} - \frac{S}{L} \right)$ .

<sup>4</sup> Projections are produced by Statistics New Zealand. The Medium projection is Statistics New Zealand's Series 5 projection, assuming medium fertility, medium mortality and medium net migration. Labour participation rates for males and females are actual rates for 1987 to 2011 (Statistics New Zealand); the 1987 rates are assumed for years prior to 1987 and the 2011 rates are assumed for years beyond 2011. The High Fertility projection assumes a total fertility rate of 2.5 from 2011-12. The 'very high migration' projection assumes net migration of 25,000 p.a. from 2012, compared with 12,000 in the Medium projection. The Low Mortality projection assumes that life expectancy at birth increases from 81 and 84 years for males and females respectively in 2015, to 95 years for both males and females in 2060.

<sup>5</sup> That is, for given values of labour productivity, saving per worker and foreign liabilities per worker. Population ageing could affect these variables but, on current evidence, the dominant effect of ageing on consumption per capita occurs through the support ratio (see for example Guest, 2007).

imply living standards about 16% lower than they would otherwise have been. Is this large? It implies that average growth in annual consumption per capita would be reduced from 1.5% per annum, for example, to 1.17%. Consumption per capita would therefore be 168 percent higher than today instead of 195% higher in the absence of population ageing (that is, with  $L/N$  remaining constant). Whether these costs are high enough to be a concern for public policy is a value judgement which is not pursued further here.

**Figure 2 – Support ratios. Effect of demographic projections**



Sensitivity to demographic projections is also indicated in Figure 2. The High Fertility projection results in a lower support ratio until 2060 (compared with the Medium projection) when the higher number of workers finally offsets the higher number of young dependants. The drop in the support ratio, relative to Medium, reaches a maximum at 2035 of 4.0%. This implies from (3) that other things constant, national living standards would be 4.0% lower in 2035 than would be the case under the Medium Fertility scenario, but by 2060 the gap would zero. Zero migration lowers the support ratio by 4.5% by 2060, while High Migration raises the support ratio by 1.8% by 2060. Low mortality reduces the support ratio by 3.6% by 2060.

These figures indicate the national consumption cost of ageing from 2015 to 2060. The role of the simulation model is to allow fiscal policy to distribute this consumption cost over time in order to consider the intergenerational equity implications.

### 3 The simulation model

The simulation model is an open economy, overlapping generations model. The essential features of the model are described here and the algebraic detail is given in the Appendix.

Households plan their consumption and labour supply over their entire lifetime, given known values of future income, the tax rate and the interest rate. (The plans of children are effectively made by their parents.) Each generation is characterised by one person household who dies at age 85 with certainty. A period of time is one year duration and a new generation of households is born each period, implying that there are  $h=85$  overlapping generations of households alive at any time. The households supply labour between the age of 15 and 70; hence the retirement decision is exogenous and there are 66 generations of workers.

Households derive utility from consuming a composite index of private goods, leisure and public goods, the latter being exogenous and separable from both private consumption and leisure in generating utility, following the approach in Foertsch (2004). Households plan consumption and leisure over their lifecycle by maximising an intertemporal utility function.<sup>6</sup>

Households have full knowledge of future income, interest rates and tax rates until the policy shock arrives. The policy shock is the unexpected decision by government to adopt a new fiscal regime (tax smoothing in this case) in response to the fiscal pressures from ageing. Until that time households assume that the government will adopt an average tax rate that balances its budget in each period, implying a 'pay-as-you-go' approach to the fiscal costs of ageing and a constant net debt to GDP ratio at its 2015 level. The new fiscal regime is a decision to adopt a constant tax rate from 2015. When this occurs, households re-optimize over the remainder of their lifetimes based on the implied future path of the tax rate; their past decisions are unaffected. For example an individual aged 60 at the time of the shock re-optimises for the remaining 25 years of life. An individual at age 20 re-optimises over the remaining 65 years of life.

The path of the tax rate affects households' optimal plans in two ways. The first is the intratemporal decision to consume goods and services relative to leisure (time spent not working). This is affected by the relative price of leisure which is the after-tax real wage rate. A higher tax rate reduces the price of leisure and therefore discourages labour participation. This is the substitution effect and is the source of the deadweight loss from taxing labour. On the other hand a higher tax rate also reduces disposable income which tends to reduce both consumption and leisure (therefore increasing labour participation). This is the income effect which does not give rise to a deadweight loss because it represents a transfer of income among households, given that taxation ultimately finances spending. The second way in which the path of the tax rate affects household plans is an intertemporal effect. Household saving decisions reflect their allocation of consumption between the present and the future. They balance the cost of saving (their rate of time preference) against the return to saving (the after-tax interest rate). A higher tax rate<sup>7</sup> lowers the return to saving and therefore raises present consumption relative to future consumption. The wedge driven between the return to saving and the cost of saving is the source of the deadweight loss from taxation of capital.

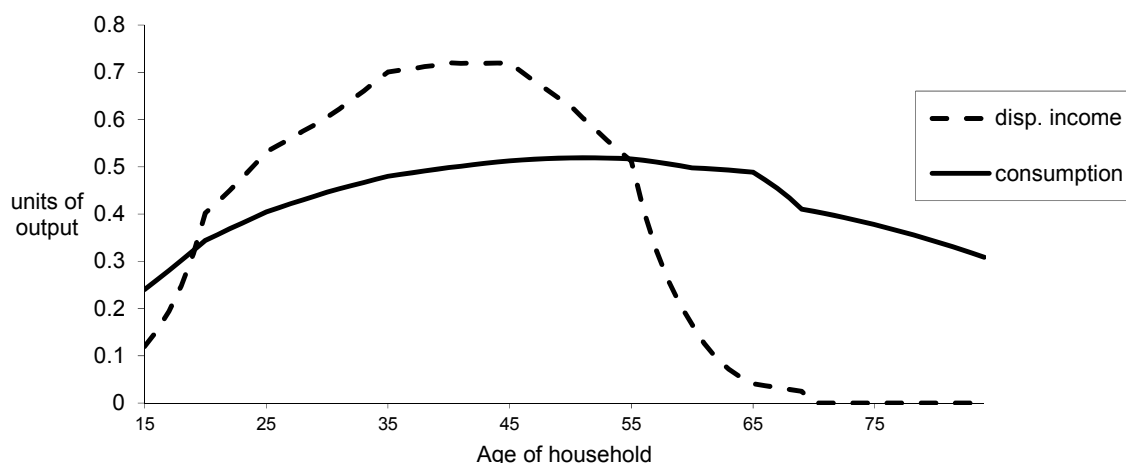
The lifecycle path of disposable labour income and consumption is illustrated in Figure 3 for the generation born in 2015.

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<sup>6</sup> The assumption of fully forward looking consumers could be partially relaxed by partitioning a household's consumption into two parts: the part that is fully forward looking and a part that is determined by a 'rule-of-thumb' such as a constant proportion of current income. Total household consumption would be the sum of the two parts. However this is not done in the current version of the model applied here.

<sup>7</sup> The tax rate is the same on income from both capital and labour. See Guest (2008) for a discussion of this assumption.

**Figure 3 – Household labour income (after tax) and consumption. Balance Budget scenario. Medium fertility**



Households supply labour to a representative firm that combines the aggregate labour with capital according to Cobb-Douglas technology and produces a single good. The firm determines its capital-labour ratio by equating the marginal cost of capital with the cost of capital, which is assumed to be constant. The firm demands labour up to the point where the marginal product of labour is equal to the real wage. The real wage adjusts instantaneously to equate labour demand to labour supply.

Government spending is an exogenously given share of GDP, but since GDP is endogenous through endogenous labour supply the level of government spending is also endogenous.

All government spending other than transfer payments is assumed for simplicity to be government consumption spending. Hence

$$G_j = G_j^C + G_j^T \tag{4}$$

where  $G_j^C$  is government consumption spending and  $G_j^T$  is transfer payments. The government faces the following dynamic budget constraint:

$$D_j^{gov} = D_{j-1}^{gov} (1 + r_j) + G_j - T_j \tag{5}$$

where  $D_j^{gov}$  is government debt (net) and  $T_j$  is total taxes. The balanced budgets simulation implies debt sustainability since the debt to GDP ratio is constant throughout the projection period. For other simulations a sustainable debt path is defined as one that returns to the initial debt to GDP ratio at the end of the projection period.

### 3.1 Intergenerational income analysis and social welfare analysis

We want to compare the effect of the policy shock (the change in tax regime in response to the fiscal costs of ageing) on the remaining lifetime incomes of generations alive at the time of the shock as well as future (unborn) generations.

This gives rise to an ethical dilemma. How should the effect of the shock on an individual aged 60 at the time of the shock be compared with the effect on an individual aged 30? Suppose that the 60 year old has a relatively large change in income over each of the remaining 25 years of life, but when summed over the 25 years amounts to less than the sum of smaller changes in income over each of the remaining 45 years of the 30 year old's life. Who is worse off – the 60 year old who suffers a lot for a short period of time or the 30 year old who suffers less in any year but more in aggregate when summed over their remaining lifetime? This is analagous to the comparison of the social benefits of health interventions on a 60 year old compared with a 25 year old. The approach adopted in this paper is to calculate, over the remaining years of life following the shock, the total change in income as a proportion of the income that would have been received in the absence of the shock. In the above example the 60 year old would have a higher proportional drop in income than the 25 year old.

The next step is to determine a social evaluation of the new tax regime. This requires value judgements of an implicit 'social judge' who evaluates only the aggregate consumption (of goods and leisure) of society in the present and the future. This implies that there is no regard for past consumption of generations still alive. The social welfare function applied here is

$$V = \sum_{j=1}^H N_j \left[ \frac{\Pi_j}{1 - \beta_s} \right]^{1 - \beta_s} (1 + \theta_s)^{1 - j} \quad (6)$$

where  $\Pi_j = \sum_{i=0}^h \Pi_{h-i, j-(h-i)+1}$  is the aggregate value of the consumption index of all households alive in period  $j$ ;  $j=1$  in 2015;  $H$  is an arbitrarily long time in the future; and  $V$  is a measure of discounted social welfare, which we will simply call social welfare.

Although the social evaluation in (6) is concerned only with the aggregate consumption index, it accounts for intergenerational equity indirectly through the parameters that weight future consumption. These parameters are  $\beta_s$  and  $\theta_s$ , which are analogous in their role to the parameters  $\beta$  and  $\theta$  in the household's utility function. The parameter  $\theta_s$  is a social rate of pure time preference, which is the rate at which period  $j$  social welfare is discounted in deriving our measure of social welfare. The parameter  $\beta_s$  measures the social degree of aversion to variability in consumption at any given point in time. Both parameters  $\beta_s$  and  $\theta_s$  discount consumption occurring at different time periods.  $\theta_s$  discounts a given level of future consumption according to the distance of that consumption in the future, whereas  $\beta_s$  discounts consumption at a given point in the future according to the size of that consumption.<sup>8</sup>

<sup>8</sup> Although they are analogous, the values of the social and private discount parameters need not be equal. For example, while it may be privately optimal for individuals to adopt a zero rate of pure time preference it may

The higher is  $\theta_s$  the smaller are the future impacts on social welfare from changes in the aggregate consumption index. This will tend to reduce the social weight on the consumption gains relative to the losses because the gains occur in the future. The higher is  $\beta_s$  the smaller the social weight placed on larger consumption gains or losses. This will tend to reduce the social weight on the consumption losses because they are generally larger than the gains even though the gains are spread over a longer period. The simulation outcomes are discussed below in Section 5.

## 4 Data and parameters

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The government spending share of GDP is equal to the values in Treasury's LTFM for 2007 to 2060. Government spending<sup>9</sup> is projected to increase by 6.4% of GDP from 29.6% to 36.0% over the 45 year period from 2014-15 to 2059-60.<sup>10</sup> Health spending grows by 5% of GDP and New Zealand Superannuation (NZS) grows by 3.6% of GDP. However whereas all of the growth in NZS is due to demographic change, almost all of the health spending is due to "non-demographic volume growth" – income growth and input costs. Demographics accounts for roughly 1% of the 5% increase. This is consistent with the pattern in recent decades during which population ageing has accounted for only 10 to 15% of the growth in health spending in New Zealand (Bell et al., 2010). Hence the projected fiscal burden attributable to population ageing is somewhat less than the 6.4% of GDP of projected growth in total spending – approximately 4.5% to 5%. There are small reductions in other spending items as a share of GDP such as education and some welfare expenses.

These figures are based on the "cost pressures" projection. This is a 'no policy change' projection based on bottom-up calculations of growth rates of the main budget expense categories. The growth rates for government consumption expenditure consist of the sum of the growth rates of input prices and output volumes. Input price growth consists of inflation plus (adjusted) real unit input costs which are based on (adjusted) labour productivity growth in the public sector. Output volume growth is the sum of demographically-driven and non-demographically-driven components. The demographically-driven component depends on the growth rates of recipient population age groups. For health spending, this is based on age and gender-specific shares of spending which are assumed constant throughout the projection period. For welfare spending, input price growth is simply equal to the inflation rate and volume growth is determined by demographic growth, in particular the growth rates of recipients for each category of spending. Hence unlike consumption expenditure, welfare spending does not grow with labour productivity growth.

The Government spending share of GDP is assumed to remain constant at its 2055 level thereafter, and constant at its 2007 for all years up to 2007.

The historic and projected age-specific population levels, age specific wage rates and exogenous<sup>11</sup> labour force participation rates were accessed from Statistics New Zealand and

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not be socially optimal, as an implication of the axioms in Koopmans (1960). In particular, if  $\theta_s=0$ , the consumption of generations near to the present would have negligible weightings in social welfare when  $H$  is large. The result would be that the future swamps the present in social importance. It could justify crushing the present generation to yield an infinitely small increase in the utility of each generation in the future.

<sup>9</sup> Defined in the "cost pressures" projections as "core crown expenses excluding financing costs".

<sup>10</sup> Based on spreadsheet projections, derived from the LTFM, provided to the author by NZ Treasury officers.

<sup>11</sup> Exogenous LFPRs are adjusted by households' demand for leisure to generate the endogenous labour supply (see below and the Appendix).



were provided by New Zealand Treasury officers.<sup>12</sup> The population data are the same as those used in Treasury's LTFM. Five demographic projections are compared. (i) A Medium projection which is the base case used for the LTFM and adopts the following long run assumptions: total fertility rate of 1.9, life expectancy at birth increases to 85.6 years for males and 88.7 years for females, and net migration of 10,000 p.a.; (ii) a (very) High Fertility projection which differs from the Medium projection only in that the long run fertility rate is 2.5; (iii) a (very) High Migration projection which differs from the Medium projection only in that long run net migration is 25,000 p.a. (iv) Zero Migration; and (v) Low Mortality in which life expectancy at birth increases to 95 for both males and females.

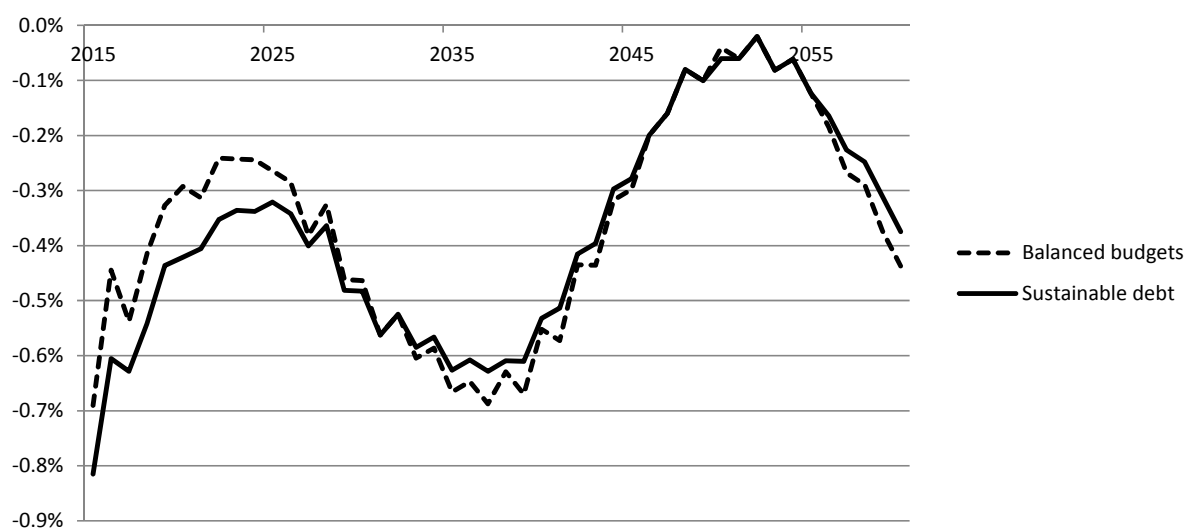
A key parameter in analysing the welfare effects of tax smoothing is  $\psi$ , the intratemporal elasticity of substitution between leisure and consumption. This partly determines the elasticity of labour supply with respect to changes in the after-tax wage. Typical values of this parameter in the literature are in the range 0.5 to 1.0. For example, Foertsch (2004), Auerbach and Kotlikoff (1987) and Altig et al. (2001) all use a value of 0.8 in their dynamic models and this is the value chosen here. The relationship however between  $\psi$  and the labour supply elasticity in a lifecycle optimising model is a complicated non-linear function of the parameters of the model (Ziliak and Kneisner, 2005). Sensitivity tests are reported of labour supply responses over the lifecycle to the range of values of this parameter found in the literature.

Figure 4 plots the effect of endogenous labour on the aggregate labour force participation rate (LFPR). The series plotted is the percentage change in LFPR given by the model compared with that given by the raw demographic data combined with the exogenous LFPR as given by Statistics New Zealand and used in Treasury's LTFM. The endogenous LFPR averages about 0.2% below the exogenous LFPR over the full projection period, but the magnitude is greater (up to 1%) for the first two decades. This represents the response of households to rising tax rates. Note that the response is initially slightly larger under the sustainable debt scenario reflecting the higher initial tax rate.

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<sup>12</sup> Historical population tables from Estimated Resident Population by Age and Sex (1991+). Available from Infoshare <http://www.stats.govt.nz/infoshare>. Population projections from Projected Population of New Zealand by Age and Sex, 2006 (base) – 2061. Available from Table Builder [http://www.stats.govt.nz/tools\\_and\\_services/tools/tablebuilder.aspx](http://www.stats.govt.nz/tools_and_services/tools/tablebuilder.aspx) Age specific participation rates from Labour Force Status by Sex by Age Group (Qrtly-Mar/Jun/Sep/Dec). Available from Infoshare <http://www.stats.govt.nz/infoshare/>. Age specific wage rates from Income by age, sex and labour force status Available from Table Builder: [http://www.stats.govt.nz/tools\\_and\\_services/tools/tablebuilder.aspx](http://www.stats.govt.nz/tools_and_services/tools/tablebuilder.aspx) Age specific earnings for those in paid employment from Earnings by region, sex and age groups. Available from Table Builder [http://www.stats.govt.nz/tools\\_and\\_services/tools/tablebuilder.aspx](http://www.stats.govt.nz/tools_and_services/tools/tablebuilder.aspx).

**Figure 4 – Effect of endogenous labour on aggregate participation rate. % change in LFPR relative to exogenous LFPR**



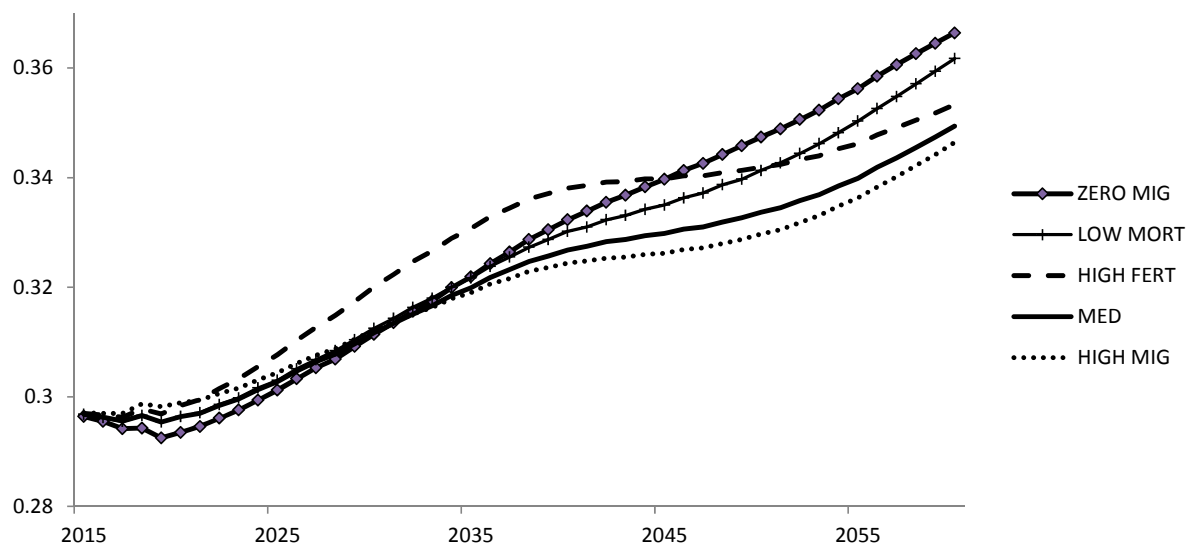
There is zero technical progress in production. This allows for a more transparent analysis of the effects of ageing and accompanying fiscal regimes on labour supply and intergenerational welfare for the following reasons. One reason is that the effect of population ageing on technical progress is, although potentially important, highly uncertain in direction and magnitude according to the theoretical and empirical literature (Guest, 2007). A sensitivity analysis is one way to go but would lengthen the present analysis considerably as there are a number of potential mechanisms. Also, there is the well-known issue of modelling the leisure-consumption choice with technical progress. Technical progress drives up real wages which implies a rising price of leisure and therefore a falling leisure to consumption ratio – it would eventually decline to zero (Kulish et al., 2006; Auerbach and Kotlikoff, 1987). There are more complex utility functions that can deal with technical progress, but this is regarded as beyond the scope here.

Other parameters are the interest rate, rate of time preference, depreciation rate and elasticity of marginal utility with respect to consumption. The values of these along with initial values for government debt, foreign liabilities and the capital stock are given in the Appendix.

## 5 Simulation results

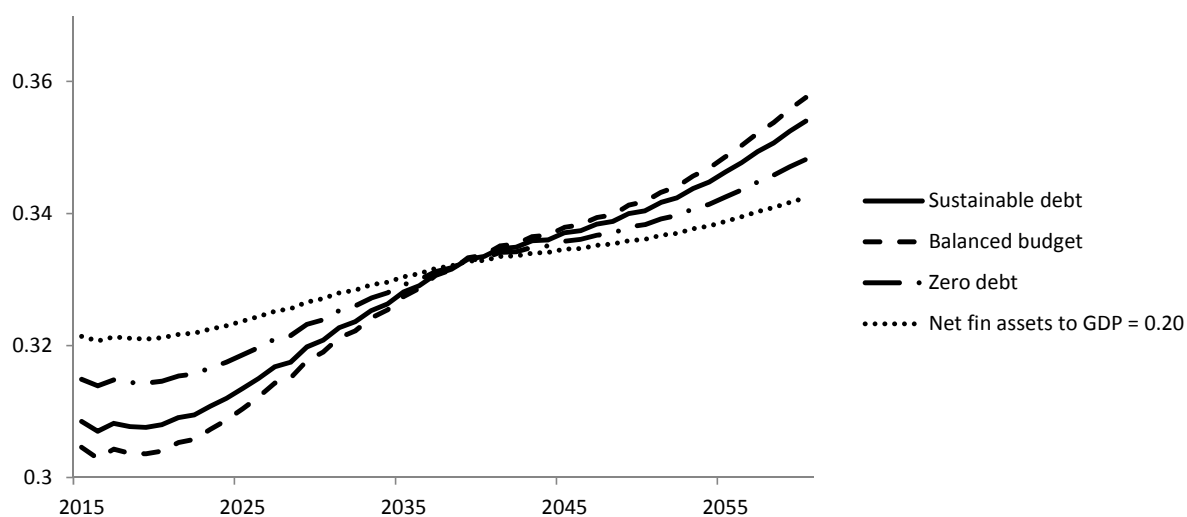
The Government spending share of GDP ( $G/Y$ ) is plotted in Figure 5 for the five demographic projections. The patterns reflect those of the support ratios. Falling support ratios imply rising government spending due mainly to rising spending on NZS and health associated with higher old age dependency. The High Fertility scenario increases government spending to GDP by a maximum of 1.1% in 2035 and by 0.4% in 2060. The other demographic scenarios take longer to impact on GDP; indeed the effect is less than 0.2% of GDP up to 2035. After that the effect is greater. Zero Migration increases spending by the most, 1.7 % of GDP by 2060, compared with 1.2% under Low Mortality. High Migration reduces spending by 0.3% of GDP by 2060.

**Figure 5 – Government spending (ratio to GDP)**



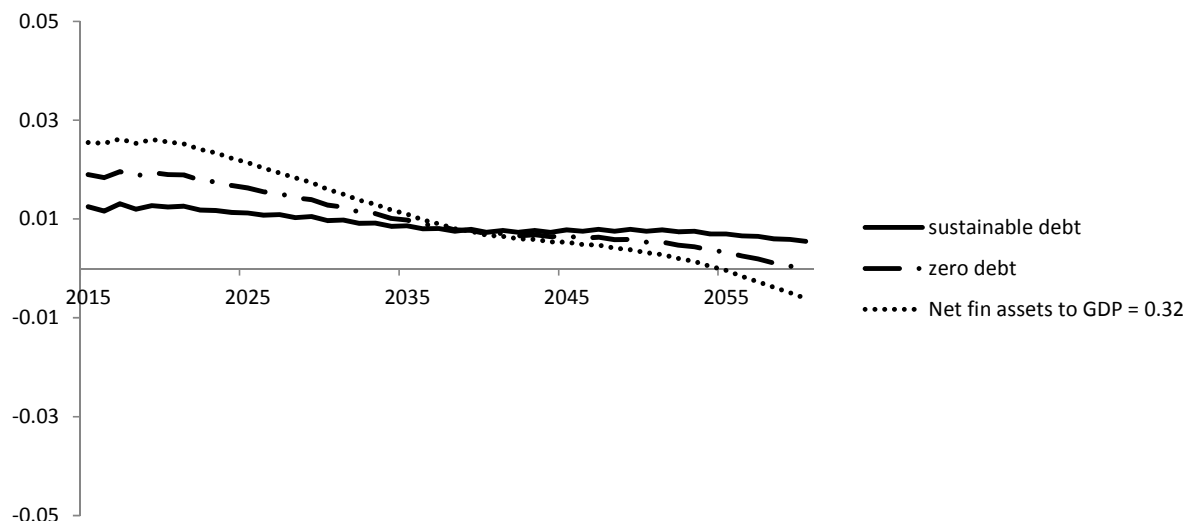
Three tax smoothing regimes are compared with a balanced budget regime. See Figure 6.

**Figure 6 – Tax to GDP ratios – Medium demographic projection**



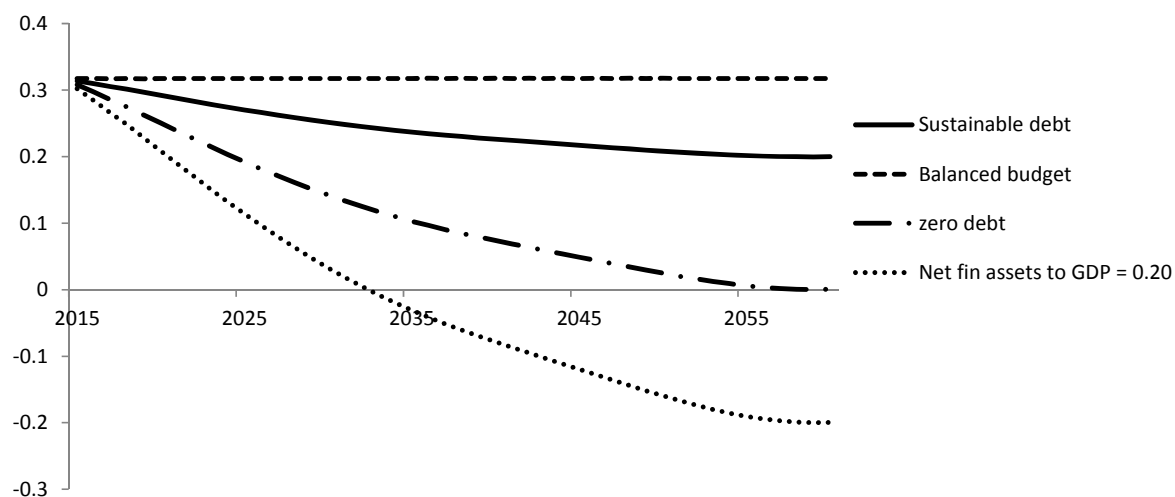
The first is a sustainable debt regime, where this is defined by NZ Treasury as a stable debt to GDP of 20% by 2060. This implies a tax to GDP ratio that starts at 30.9% in 2015 (compared with 30.5% under balanced budgets) and increases to 35.3% in 2060 (compared with 35.6% under balanced budgets). The sustainable debt tax path is very close to the balanced budget path. There is no more than 0.4% of GDP difference at any point over the projection period. Such a small increase in the balanced budget tax rate is all that is needed to reduce debt from the balanced budget level of 31.7% to 20% from 2015 to 2060. This is reflected in small/modest budget surpluses of between 1.3% and 0.6% of GDP over the projection period (see Figure 7 for budget surpluses).

**Figure 7 – Budget balance (ratio to GDP) – Medium demographic projection**



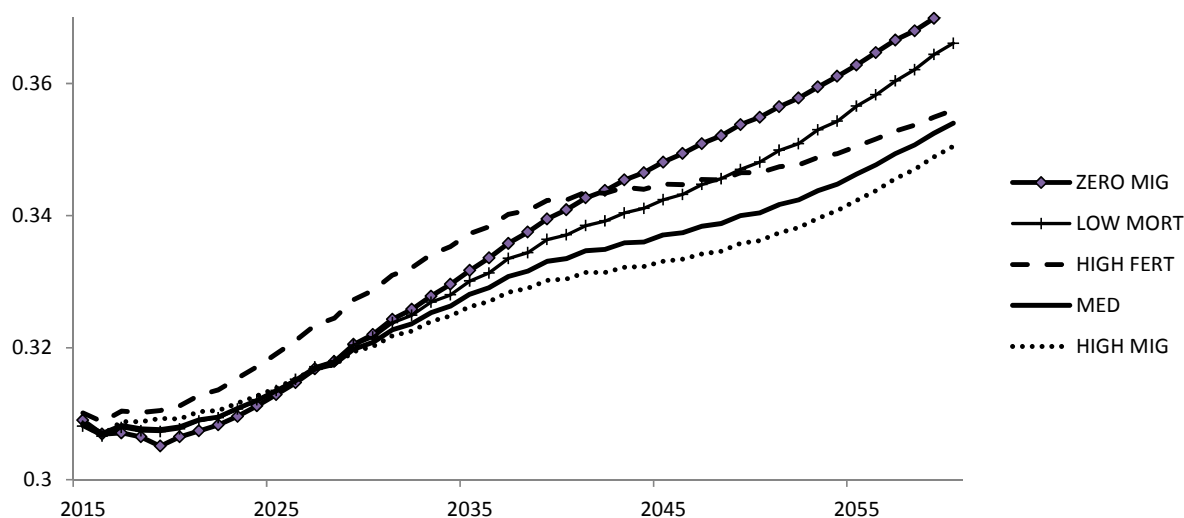
The third budget regime has debt to GDP stabilising at zero rather than 20% (see Figure 8 for the debt ratios in each regime). This requires higher initial budget surpluses of 2% of GDP declining to zero by 2060 (Figure 7). The fourth budget regime is even more extreme tax smoothing where the debt becomes negative and stabilises with net foreign asset of 20% of GDP. The budget surplus starts at 2.6% of GDP in 2015 and steadily declines to a stable budget deficit of 0.5% of GDP. Hence the three regimes represent progressive degrees of smoothing. A higher degree of smoothing implies greater transfer of the consumption cost of ageing to present generations and away from future generations, as discussed earlier.

**Figure 8 – Net debt to GDP ratios under alternative fiscal regimes – Medium demographic projection**



Sensitivity to the demographic projections is illustrated in Figure 9 which shows the tax to GDP ratios under the sustainable debt regime under the five demographic scenarios. It mirrors the pattern of government spending illustrated in Figure 5. The tax ratio is eventually highest under Zero Migration, being 1.6% above that under the Medium projection by 2060, although it takes at least 15 years (until 2030) for the tax ratio to rise above that of the Medium projection. Indeed that is the case for most of the alternative demographic projections – their fiscal implications are slow to take effect, the exception being the High Fertility scenario where the fiscal cost of higher dependents arises sooner.

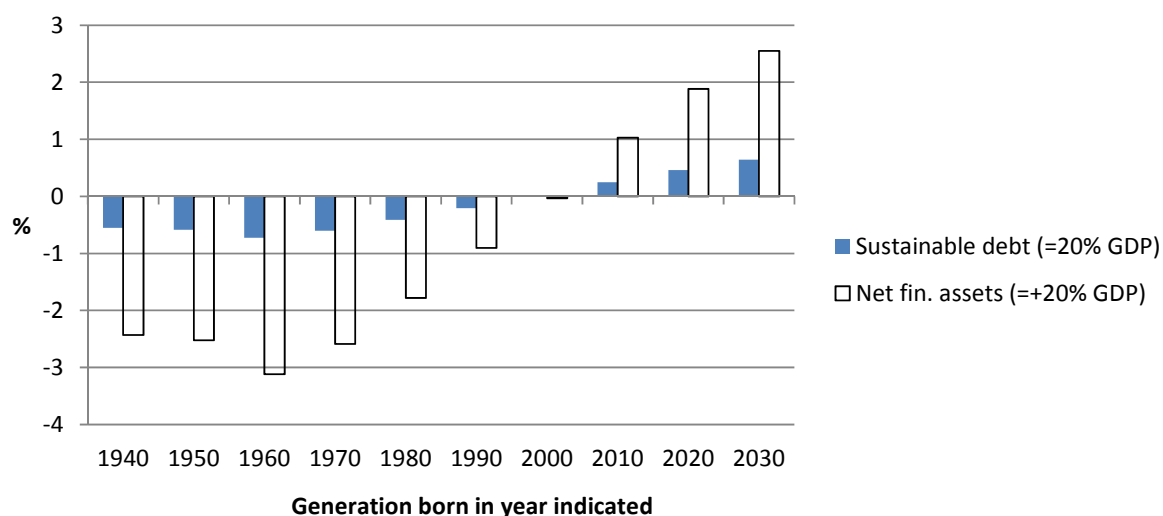
**Figure 9 – Tax to GDP ratios for sustainable debt (=20% GDP) – Various demographic scenarios**



A key aim of the analysis is to determine whether households of different generations are better off or worse off under tax smoothing, and by how much. The method adopted here is to calculate the effect on lifetime income from the year of the policy shift which is 2015.<sup>13</sup> The policy shift is assumed to be unexpected, prior to which balanced budgets are the actual and expected policy.

The results are illustrated in Figure 10 and Table 1. Figure shows percentage effect on remaining lifetime income from 2015 of generations born in the year indicated, due to a change in fiscal regime from a balance. The sustainable debt scenario is compared with the balanced budget scenario for each of the five demographic projections. The balanced budget scenario is chosen as the counterfactual since the principal objective is to examine tax smoothing which implies a comparison with balanced budgets. Note that Table 1 provides more detail by including the extreme form of smoothing (resulting in stable net financial assets of 20% of GDP) for the Medium Fertility scenario, and also reporting results for each of the alternative demographic scenarios in the case of the sustainable debt regime.

**Figure 10 – Intergenerational equity with sustainable debt**



<sup>13</sup> An alternative is to calculate the effect on remaining lifetime utility from 2015, expressed in units of equivalent annual income (see Guest, 2008, for an application of this method).

**Table 1 – Effect on lifetime income from date of tax smoothing**

Figures are percentage changes relative to balanced budget tax regime						
Individual born in year	Median demo. projection		High Fert	High Mig	Zero Mig	Low Mort
	Sustainable debt (=20% GDP)	Net fin. assets (=+20% GDP)	Sustainable debt	Sustainable debt	Sustainable debt	Sustainable debt
1940	-0.55	-2.43	-0.73	-0.57	-0.49	-0.53
1950	-0.59	-2.52	-0.74	-0.60	-0.52	-0.55
1960	-0.73	-3.12	-0.88	-0.76	-0.67	-0.71
1970	-0.60	-2.59	-0.72	-0.63	-0.56	-0.59
1980	-0.41	-1.78	-0.49	-0.44	-0.37	-0.41
1990	-0.20	-0.91	-0.24	-0.24	-0.17	-0.21
2000	0.00	-0.03	0.01	-0.04	0.03	-0.02
2010	0.25	1.03	0.29	0.20	0.29	0.23
2020	0.46	1.88	0.49	0.42	0.51	0.45
2030	0.64	2.55	0.67	0.62	0.70	0.65
<b>Debt ratio in 2060</b>	20%	-20%	20%	20%	20%	20%

The first and important point to note is that the magnitudes of effects on all generations are very small, at most no more than 0.9% on any generation under any demographic scenario. This is perhaps not surprising given the small adjustments to the tax rate required to achieve a sustainable debt (Figure 6). Generations born prior to 1990 are worse off and those born after 2000 are better off. (“Better off” and “worse off” is measured relative to the remaining lifetime income after the tax smoothing shock that occurs in 2015).

Those born around 1960 fare the worst, albeit not by a large amount. They are 0.7% worse off in the Medium demographic scenario. This generation is aged around 45 and therefore at the peak of their earning capacity when the tax smoothing policy is introduced, which results in an initial jump in tax rates. Also this generation has retired before the balanced budget scenario yields a payoff in terms of a lower tax rate which occurs around the late 2030’s. Retired workers are also worse off because they pay higher tax rates on their retirement income. For those born a few years either side of 2000, the losses from higher tax rates for the 15 year period to 2030 are roughly balanced by the gains from lower taxation after 2030. Those who gain the most are future workers, particularly those born after 2020. They escape the higher taxes on earlier generations and reap the gains from lower future taxes. Even so, they are only better off by 0.6% under the sustainable debt scenario. The numbers are of course greater under the more extreme smoothing regime (Figures 10).

Given that existing workers and retirees are worse off while future workers are better off, what is the social gain? This requires value judgements for which we apply the social welfare function (6). The impact on social welfare takes account not only of intergenerational equity but also implicitly the efficiency gains from tax smoothing arising from the reduction in distortions to both the labour-leisure choice arising from taxation of labour income and to the intertemporal consumption allocation arising from the taxation of capital income.

The results are reported in Table 2 by expressing the effect in social welfare in terms of equivalent annual gains in GDP per annum generated by sustainable debt tax smoothing scenario. These were calculated by finding the annual increase in GDP under continual balanced budgets that would generate the same value of social welfare as in the tax smoothing scenario. Results are given for a range of values of the two key parameters in the social welfare function: the social time preference rate,  $\theta_s$ , and the parameter measuring the social aversion to variability in aggregate consumption,  $\beta_s$ . The values chosen for  $\theta_s$  range from zero to 6%; and the values chosen for  $\beta_s$  range from 0.2 to 5. The first point to note is that the effects are mostly small and negative. However gains from smoothing occur for a low

rate of social time preference and a high rate of aversion to consumption variability (see the last row in the Table). The reasons for the gains under those parameter assumptions, as noted above, are that a low value of  $\theta_s$  gives relatively more weight to the future gains from smoothing; and a higher value of  $\beta_s$  gives a relatively low weight to the years of losses which are larger in magnitude but fewer in number. This will tend to reduce the social weight on the consumption losses because they are generally larger than the gains even though the gains are spread over a longer period. Hence net social gains are possible provided the gains to future generations are given sufficient weight. The parameter values required to generate net social gains are close to the bounds of plausible values.

**Table 2 – Effect on social welfare change of the sustainable debt scenario. Base case demographics (Medium fertility)**

Social time pref rate ( $\theta_s$ ) %	Aversion to variability in aggregate consumption ( $\beta_s$ )	% change in equiv. annual GDP	\$ Change in equiv. annual GDP per capita*
0.0	2.0	-0.03%	-\$ 13
3.0	2.0	-0.10%	-\$ 49
6.0	2.0	-0.13%	-\$ 66
3.0	0.2	-0.11%	-\$ 55
3.0	1.0	0.00%	-\$ 1
3.0	5.0	-0.18%	-\$ 90
0.0	5.0	0.19%	\$ 94

\* Calculation is based on a projected 2015 GDP of \$226 billion and a projected population of 4.6 million.

The Table reports magnitudes in equivalent *annual* gains in GDP. This is done by finding the annual increase in GDP under the balanced budget case that would produce the same value of social welfare under the sustainable debt scenario. The magnitudes range from minus \$90 to plus \$94 per capita per year. Although the methodologies are different, these magnitudes are of the same order as those found in Davis and Fabling (2002) who calculated gains for New Zealand of between 3% and 5% of one year’s GDP (not annual GDP), would equate to roughly 0.1% of GDP per year.

## 6 Conclusion

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This paper complements the New Zealand fiscal projections of Bell et al. (2010) and the New Zealand tax smoothing analysis in Davis and Fabling (2002). It allows for feedback effects of the tax rate on labour supply intratemporally and intertemporally which in turn feed back to fiscal projections via taxation revenue. The model allows for efficiency gains from tax smoothing arising from the reduction in distortions to both the labour-leisure choice arising from taxation of labour income and to the intertemporal consumption allocation arising from the taxation of capital income. The overlapping generations framework also allows an analysis of intergenerational income effects; and imposing a social welfare function allows a numerical calibration of the combined equity and efficiency effects of tax smoothing.

For the Medium demographic projections, tax smoothing consistent with sustainable debt implies small gains and losses among generations amounting to no more than 0.7% of remaining lifetime income for any generation. This is not surprising given that the sustainable debt scenario requires only a small increase in the tax to GDP ratio of 0.5% at the most over a decade from 2015. Those born around 1960 fare the worst, but only suffer a 0.7% drop in their remaining lifetime incomes under the sustainable debt scenario for the Medium demographic projection. This generation is at their peak earning capacity when the tax smoothing policy is introduced, which results in an initial jump in tax rates. They have also retired before the balanced budget scenario yields the payoff of a lower tax rate. Retired workers are also worse off because they pay higher tax rates on their retirement income. Future workers are better off because they escape the higher taxes on earlier generations and reap the gains from lower future taxes.

The losses to current generations can be weighed up against the gains to future generations through the social welfare function. The results show that net social gains are possible provided the gains to future generations are given sufficient weight by a low rate of social time preference and a high rate of aversion to variability in aggregate consumption over time. The parameter values required to generate net social gains are close to the bounds of plausible values. The magnitudes of the net social gains/losses for the combinations of parameter values simulated range from minus \$90 to plus \$94 per capita per year.

There are well known limitations of the optimizing framework here, notably the assumption of fully forward looking households who have perfect foresight (except that they do not anticipate the switch in fiscal regime); a high degree of aggregation (the firm produces only one good); a perfect capital market; and a single tax rate applied to both income and capital. There are also a range of practical limitations, including political issues, in a pure form of tax smoothing considered in this paper (see the discussion in Davis and Fabling, 2002). Future work could begin to relax some of these constraints.



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# Appendix A – The Simulation Model

## Firms

A representative firm produces output of a single good according to a Cobb-Douglas production function. Output,  $Y$ , in period  $j$  is given by

$$Y_j = AK_j^\alpha L_j^{1-\alpha} \quad (1)$$

where  $A$  is a constant exogenous technology parameter,<sup>14</sup>  $K_j$  is the capital stock, and  $L_j$  is aggregate labour consisting of the sum of the labour of all generations:  $L_j = \sum_{i=1}^n L_{i,j}$  where  $L_{i,j}$  is the labour of workers of age  $i$  in year  $j$ .

The optimal capital stock,  $K_j$ , is determined by the first order condition that the marginal product of capital (net of depreciation,  $\delta$ ) is equal to the cost of capital,  $r_j$ .

That is,  $\left(\frac{dY_j}{dK_j} - \delta\right) = r_j$ , which gives

$$\left(\frac{K}{L}\right)_j = \left(\frac{A\alpha}{r_j + \delta}\right)^{\frac{1}{1-\alpha}} \quad (2)$$

And investment,  $I_j$ , is given by

$$I_j = K_j - K_{j-1}(1-\delta) \quad (3)$$

Competitive firms equate the price of labour,  $w_j$ , to the marginal product of labour:

$$w_j = (1-\alpha)A\left(\frac{K}{L}\right)_j^\alpha = \left(\frac{Y}{L}\right)_j - (r_j + \delta)\left(\frac{K}{L}\right)_j \quad (4)$$

The wage of each worker is given by

$$w_{i,j} = \alpha_i w_j \quad (5)$$

where  $w_{i,j}$  is the wage of a worker of age  $i$  in year  $j$ ,  $\alpha_i$  is a weight equal to the wage for age  $i$  divided by the average of wages for all age groups which are given by the data.

## Households

Each household consists of one person who dies at age 85 with certainty. A period of time is one duration and a new generation of households is born each period, implying that

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<sup>14</sup> The technology parameter is constant, implying zero technical progress. The reason, as also given in Kulish et al. (2006), is that the leisure to consumption ratio would eventually decline to zero with continual productivity-induced rises in real wages. See Auerbach and Kotlikoff (1987) for a further discussion. It would be possible to specify a non-standard utility function that could deal with this problem in the presence of technical progress, but this is not pursued here.

households live for  $h=85$  periods and that there are  $h$  overlapping generations of households alive at any time.<sup>15</sup> The households supply labour between the age of 15 and 70. Households pay the same single tax rate on income from both capital and labour. Future values of the demographic variables and the parameters are known with certainty, except for the tax smoothing shock which comes as a surprise at which time households must adjust their plans accordingly.

Households derive utility from consuming a composite index of private goods,  $C$ , and leisure,  $S$ . Households also derive utility from consuming public goods,  $G^C$ , which is exogenous and separable from both private consumption and leisure in generating utility, following the approach in Foertsch (2004). Therefore  $G^C$  does not affect the household's choice of private consumption or leisure and can therefore be ignored in solving the household's optimisation problem. The assumption of separability between public and private consumption is quite common, as noted in Foertsch (2004), because of lack of evidence about the substitutability between private and government consumption.

The composite index of consumption and leisure is

$$\Pi_{i,j} = \left[ \mu_i^{1/\psi} C_{i,j}^{\psi-1/\psi} + (1-\mu_i)^{1/\psi} S_{i,j}^{\psi-1/\psi} \right]^{\psi/\psi-1} \quad (6)$$

where  $C_{i,j}$  and  $S_{i,j}$  are the goods consumption and leisure, respectively, of age  $i$  households in period  $j$ . The preference for consumption relative to leisure, captured by the parameter  $\mu_i$ , is assumed to vary over the lifecycle. In particular it is assumed to rise up to middle age and then fall. Hence  $\mu_i$  follows a hump-shape which is given by the quadratic:

$$\mu_i = \xi_1 + \xi_2 i - \xi_3 i^2 \quad (7)$$

The hump-shape pattern on  $\mu_i$  generates a hump-shape path of consumption relative to leisure over the life cycle. This pattern is designed to reflect the observed life cycle pattern of consumption which tends to track the hump-shaped pattern of income to some degree, rising during the household's working life and falling after retirement (see, for example, Deaton, 1999).

Households maximise the following intertemporal utility function:

$$U = \sum_{i=1}^h \frac{\Pi_{i,j}^{1-\beta}}{1-\beta} (1+\theta)^{1-i} + v(G^C_j) \quad (8)$$

with respect to  $C_{i,j}$  and  $S_{i,j}$  after substituting for  $\Pi_{i,j}$ , and subject to a lifetime budget constraint:

$$\begin{aligned} & \sum_{i=1}^h C_{i,j} \left(1+r_j(1-t_j)\right)^{1-i} \\ &= \sum_{i=1}^h \left(p_{i,j} L_{i,j} + G^T_{i,j}\right) \left(1+r_j(1-t_j)\right)^{1-i} + Q \left(1+r_j(1-t_j)\right)^{1-(h-6)} - \bar{A}^{1-h} \end{aligned} \quad (9)$$

<sup>15</sup> The population projections include migrants and people over age 85. The model implicitly assumes that migrants are indistinguishable from the incumbent population in their lifecycle consumption patterns; and also that people aged 85 and over have the same consumption as 85 year olds.

where the right hand side is the present value of lifetime income. The latter includes transfer payments<sup>16</sup>,  $G_{i,j}^T$ , and inheritance,  $Q$ , which is received when the household is aged 60 less a target bequest,  $\bar{A}$ ; <sup>17</sup>  $t_j$  is the tax rate in year  $j$  applying to income from both labour and capital; and  $p_{i,j} = w_{i,j}(1-t_j)$  is the after-tax wage, and therefore relative price of leisure, facing a household of age  $i$  in year  $j$ .

The solution to the household's intratemporal optimisation problem yields the following relation between consumption of goods and leisure as a function of the relative price of leisure:

$$\frac{\mu_i S_{i,j}}{(1-\mu_i)C_{i,j}} = p_{i,j}^{-\psi} \quad (10)$$

Define total expenditure at each age as

$$Z_{i,j} = C_{i,j} + p_{i,j}S_{i,j} \quad (11)$$

Rearranging this and substituting into (10) yields

$$C_{i,j} = \frac{\mu_i Z_{i,j}}{\mu_i + (1-\mu_i)p_{i,j}^{1-\psi}} \quad (12)$$

and

$$S_{i,j} = \frac{p_{i,j}^{-\psi} (1-\mu_i) Z_{i,j}}{\mu_i + (1-\mu_i)p_{i,j}^{1-\psi}} \quad (13)$$

Define  $P_{i,j}$  as the price of the consumption index,  $\Pi_{i,j}$ , which implies that it is the minimum  $Z_{i,j}$  such that  $\Pi_{i,j}=1$ . Hence  $Z_{i,j}=P_{i,j}\Pi_{i,j}$ . Using this definition of  $P_{i,j}$  and substituting equations (14) and (15) into the expression for  $\Pi_{i,j}$  yields

$$P_{i,j} = \left[ \mu_i + (1-\mu_i)p_{i,j}^{1-\psi} \right]^{\frac{1}{1-\psi}} \quad (14)$$

Substituting (14) into  $Z_{i,j}=P_{i,j}\Pi_{i,j}$  and then substituting the resulting expression for  $Z_{i,j}$  into (12) and (13) yields

$$C_{i,j} = \mu_i \left( \frac{1}{P_{i,j}} \right)^{-\psi} \Pi_{i,j} \quad (15)$$

$$S_{i,j} = (1-\mu_i) \left( \frac{p}{P} \right)_{i,j}^{-\psi} \Pi_{i,j} \quad (16)$$

<sup>16</sup> For simplicity, total transfer payments paid by the government in a given year are allocated evenly across all households alive in that year, rather than being allocated to certain generations. Hence  $f_{i,j} = \frac{f_j}{N_j}$

<sup>17</sup> Households leave a bequest equal to 10 percent of their total lifetime pre-tax income. The bequest is received by the generation 30 years younger, which is a simplification for the purpose of generating lifetime budgets because the demographic data used for the simulations reflects the actual patterns of age-specific fertility.

The first order condition for the solution of the household's intertemporal optimisation problem yields the Euler equation for the evolution of the consumption index over the lifecycle:

$$\frac{\Pi_{i,j} - \Pi_{i-1,j-1}}{\Pi_{i-1,j-1}} = \frac{1}{\beta} \left( r_j(1-t_j) - \left( \frac{P_{i,j} - P_{i-1,j-1}}{P_{i-1,j-1}} \right) - \theta \right) \quad (17)$$

The balance of financial assets at age  $i$  in year  $j$  is given by

$$A_{i,j} = \begin{cases} A_{i-1,j-1} \left( 1 + r_j(1-t_j) \right) + \left( w \frac{L}{N} \right)_{i,j} (1-t_j) - C_{i,j} + G^T_{i,j} & i = 1, \dots, 11, 13, \dots, 18 \\ A_{i-1,j-1} \left( 1 + r_j(1-t_j) \right) + \left( w \frac{L}{N} \right)_{i,j} (1-t_j) - C_{i,j} + G^T_{i,j} + Q & i = 12 \end{cases} \quad (18)$$

Note that the wage of a worker,  $w_{i,j}$ , is multiplied by  $(L/N)_{i,j}$  to reflect the fact that there are  $L_{i,j}$  workers but  $N_{i,j}$  households of age  $i$  in year  $j$ .

The solution to the household's optimisation problem is obtained numerically as follows. Specify a trial value of  $\Pi_{i,j}$  for  $i=1$ , then solve forward for  $\Pi_{i,j}$  for  $i=1, \dots, h$  according to the Euler equation (17). For  $i=1, \dots, h$  calculate  $C_{i,j}$  and  $S_{i,j}$  according to (15) and (16). Then calculate  $A_{h,j}$ ; if it does not equal the target bequest<sup>18</sup>, then adjust  $\Pi_{i,j}$  for  $i=1$  and repeat the algorithm iteratively until the target bequest is met within a degree of tolerance.

The labour supply of households aged  $i$  in year  $j$ ,  $L_{i,j}$ , is given by  $L_{i,j} = e_{i,j} \bar{L}_{i,j}$  where  $\bar{L}_{i,j}$  is the exogenously given size of the labour force of age  $i$  in year  $j$  and  $e_{i,j}$  is work intensity defined as  $e_{i,j} = \frac{1}{S_{i,j}}$ . The notion of work intensity here follows that in Barro and Sala-i-Martin (1995, p.322) where no distinction is drawn between an increase in  $e_{i,j}$  that reflects a rise in effort from one that reflects a rise in hours worked. Both amount to an increase in labour supply. Our model implies, for example, that a 1 percent increase in demand for leisure gives rise to a 1 percent decline in labour supply in terms of either effort or hours worked. The total resources available to the household from which to provide work effort are therefore normalised to  $e_{i,j} S_{i,j} = 1$ .

The labour market clearing condition is

$$L_j = \sum_{i=1}^h L_{i,j} \quad (19)$$

where  $L_j$  is labour demand and the right hand side is the labour supply of households. Competitive firms demand labour up to the point where the marginal product of labour is equal to the real wage, according to (4). Labour supply depends on the real wage via the demand for leisure of each household. The real wage adjusts instantaneously to equate labour demand to labour supply. Firms then adjust their demand for capital in response to the

<sup>18</sup> The target bequest is set exogenously as the actual value of national wealth per person in 2015.

level of employment in order to maintain the desired capital-labour ratio, which is determined by the interest rate according to (2).

The standard national accounting identity gives the evolution of foreign liabilities:

$$D_j = D_{j-1}(1+r_j) + \sum_{i=1}^h C_{i,j} + G_j^C + I_j - Y_j \quad (20)$$

The parameter values are given in Table A1. The household's rate of time preference,  $\theta$ , is equal to  $r\beta\gamma$  which is the rate that would, if both the tax rate and the parameter  $\mu_t$  were constant, ensure that consumption grows at the long run rate of growth of output.<sup>19</sup> The capital elasticity of output,  $\alpha$ , is calibrated such that the initial capital to output ratio is equal to 3.0, an approximate historical value for New Zealand. The initial tax to GDP ratio is set equal to 0.3, the approximate estimated value for New Zealand in 2015 according to the LTFM. The values of the elasticities,  $\beta$  and  $\psi$ , are set equal to 2 and 0.8, respectively, which are common values used in related studies in the literature, see for example Foertsch (2004).

There is no assumption that the economy is in a steady state prior to the tax smoothing policy shock, nor that the economy converges to a steady state. Nevertheless, the properties of the overlapping generations model lead to fairly well-behaved state variables. In particular, debt and the capital stock do not take extreme values at any point over the projection period.

**Table A1. Base case parameter values and initial values.**

Constant interest rate	0.03
Household's rate of time preference, $\theta$	0.01
Depreciation	0.06
Capital elasticity of output: $\alpha = \left(\frac{\bar{K}}{\bar{Y}}\right)_0 (r_0 + \delta)$	0.3
Initial capital to output ratio $\left(\frac{\bar{K}}{\bar{Y}}\right)$	3.0
Initial tax rate on all income, $t$	0.3
Foreign liabilities to GDP ratio, $D/Y$ , in 2015	1.0
Elasticity of marginal utility w.r.t. consumption index, $\beta$	2.0
Elasticity of substitution between consumption and leisure, $\psi$	0.8

<sup>19</sup> This equation for  $\theta$  is not, however, a condition for a stable equilibrium in OLG models.