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Tax Policy with Uncertain Future Costs: Some Simple Models

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Tax Policy with Uncertain Future Costs: Some Simple Models

Christopher Ball and John Creedy

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Abstract

This paper considers the extent to which the standard argument, that the disproportionate excess burden of taxation suggests the use of tax-smoothing in the face of future cost increases, is modified by uncertainty regarding the future. The role of uncertainty and risk aversion are examined using several highly simplified models involving a possible future contingency requiring an increase in tax-financed expenditure.

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Executive Summary

Governments are often faced with the possibility that higher public expenditure may be needed in the future, although much uncertainty is involved. For example there may be a risk of an earthquake, though the timing and cost implications are subject to considerable uncertainty. A different context is that of population ageing, which is being experienced by most developed economies. While the time profile of the population age distribution in a country can be predicted with a reasonable amount of confidence, it is difficult to know how markets will respond. The question therefore arises of how much, if any, tax smoothing to use, by raising current taxes to build up a fund.

Faced with uncertainty, it is not clear whether governments should take immediate action or wait to collect useful information. Decision makers are more likely to take immediate and larger action, the higher the perceived probability of the contingency arising, the larger the potential cost, and the higher their degree of risk aversion. The aim of this paper is to clarify the precise nature of the relationships, and the orders of magnitude involved, in the context of simple models.

The tax policy choices facing a decision-maker in a multi-period framework, in which a future contingency may or may not arise, are examined. The possible higher future expenditure may be financed using some form of tax smoothing, by increasing the present tax rate.

The incentive for tax smoothing arises from the nature of the excess burden arising from taxation, which depends on the square of the tax rate. It is also thought to depend on the decision-maker's attitude towards risk, as reflected in the degree of risk aversion. If the extra revenue turns out not to be needed, the future tax rate can be reduced accordingly. However, the higher future net income and lower excess burden may not compensate for the initial tax increase.

The models examined in this paper are highly simplified, involving the choice by a single policy-maker or judge faced with information about the probability of a future event and its associated cost. The judge is considered to maximise a welfare or evaluation function that depends on present and future net incomes, associated probabilities and risk aversion. However, the models help to highlight some of the important inter-relationships and trade-offs involved in making policy choices. The value of not acting immediately and waiting until further information becomes available is referred to as the option value.

Numerical illustrations show that the potential future expenditure must be relatively large (as a proportion of income) before it becomes worthwhile to act immediately, that is to sacrifice the option value of waiting to see what the actual outcome will be. In cases where some pre-funding is favoured, the optimal policy does not imply complete smoothing of tax rates.

The degree of risk aversion of the hypothetical judge is found to have little effect on the optimal degree of tax smoothing. In the case of constant relative risk aversion, the effect of varying the degree of risk aversion is found to be negligible; in this case a single parameter affects both risk aversion and the inter-temporal elasticity of substitution of the judge. The alternative case of Epstein-Zin preferences, where the link between risk aversion and inter-temporal substitution is broken, is also considered. With this form of preferences, slightly more sensitivity was found for high potential future (uncertain) costs and low substitution elasticities.

In general, the size of the potential future tax-financed cost and its associated probability were found to be the major determinants of the optimal policy.

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Tax Policy with Uncertain Future Costs: Some Simple Models

1 Introduction

Governments are often faced with the possibility that a future contingency may arise involving higher public expenditure, although much uncertainty is usually involved. For example there may be a risk of an earthquake, though the timing and cost implications are subject to considerable uncertainty. A different context is that of population ageing, which is being experienced by most developed economies. While the time profile of the population age distribution in a country can be predicted with a reasonable amount of confidence,¹ it is very difficult to know how markets will respond since a wide range of behavioural and general-equilibrium adjustments to factor and goods prices is involved. Productivity and labour force participation changes are just two examples of important factors which influence future tax-financed public expenditure. Faced with such uncertainty, despite the more confident expectation of population ageing, it is by no means clear whether governments should take immediate action or wait to see what actually happens, and thereby collect useful information. Intuitively, decision makers are more likely to take immediate and larger action, the higher the perceived probability of the contingency arising, the larger the potential cost, and the higher their degree of risk aversion. But such intuition is rather vague. The aim of this paper is therefore to clarify the nature of the various relationships, and the orders of magnitude involved, in the context of very simple models where uncertainty is involved.

In particular, this paper investigates the tax policy choices facing a government in a multi-period framework in which a future contingency may or may not arise. A single disinterested decision-maker or judge is considered to select an optimal tax policy by maximising a social welfare, or evaluation, function expressed in terms of net incomes in each period. The context is one in which it may be necessary to incur a higher expenditure in future, and this may be financed using some form of tax smoothing, by increasing the present tax rate. The incentive for tax smoothing arises from the fact that the excess burden arising from taxation increases disproportionately with the tax rate, and from the assumption that the judge is averse to risk.²

The question considered here is whether, and to what extent, a decision-maker should act by immediately raising the tax rate above current requirements, even though some or all of the resulting extra revenue that is accumulated may not be needed. If the extra revenue is not in fact needed, the future tax rate can be reduced accordingly. However, the higher net income and lower excess burden may not compensate for the initial tax increase. A further possibility is that it may also be possible to reduce other tax-financed expenditure to help finance the contingency, either in the initial or future periods. The present paper focusses on the essential nature of the trade-offs involved. The approach is to consider an independent

¹ Stochastic population projections suggest quite small confidence intervals; see, for example, Creedy and Scobie (2005).

² An early modern discussion of tax smoothing is Barro (1979). Armstrong et al. (2007) also highlight the concave nature of the government's revenue function, arising from adverse incentive effects. However, these are not modelled here. Davis and Fabling (2002) stress the ability of the government to obtain a rate of return in excess of the cost of borrowing. Again, this feature is not examined here.

judge making the decision on the basis of the expected value of a welfare function, expressed in terms of aggregate amounts of net income and the excess burden of taxation, in a range of situations.³ The welfare function makes explicit the value judgements of the judge. The optimal degree of tax smoothing where the outcome is uncertain can be compared with the optimal policy where future costs are known with certainty.

The analyses are exercises in welfare economics: they consider optimal policies which involve answers to the question of, 'what if a decision maker has a particular type of objective function?'. Thus, it is clear that the results cannot provide direct policy advice. However, it is useful to consider the elements of the decision-making problem, suggesting the type of information needed, and the nature of the possible trade-offs involved. The analysis does indeed highlight the fact that the nature and extent of uncertainty is crucial in considering practical policy issues and therefore cannot be ignored.

The framework may in some ways be compared with the consideration of investment in a multi-period project where the future returns are not known with certainty, there is a non-recoverable sunk cost of investing in the first period and there exists the option of waiting until later periods before making the investment (and thereby waiting for some of the uncertainty to be resolved).⁴ The sunk cost consists of any fixed costs which cannot be recovered (such as the difference between the cost of investing in specific equipment and its resale value) and the foregone value of waiting and obtaining more information, referred to as the option value.⁵ The concept of an option value provides valuable insights: this is discussed further in Appendix A.

In the present context an increase in the tax rate in the first period, in order to accumulate a fund that can be used in the event of a future (possible) expenditure requirement, involves a sunk cost and possibly a positive option value.⁶ This sunk cost arises partly from the nature of the excess burden of taxation which increases more than proportionately with the tax rate. A subsequent reduction in the tax rate, if the extra revenue is not needed, allows the later tax rate to be reduced below the value needed for the other (fully anticipated) tax-financed expenditure. But this cannot fully recover the extra excess burden from the initial tax increase. However, the present context does not require, as in the standard investment framework, a given lumpy amount of investment (such as the construction of a factory) in the period in which it is decided to invest. Here it is possible in the first period to commit to a policy which involves only a small increase in the tax rate, leaving the additional (uncertain) revenue to be obtained by a higher tax rate increase in future, if this turns out to be necessary.

Section 2 begins by examining a two-period model in which there is a probability that an event, which has a given known cost, will occur. In section 3, the model is extended to allow for some uncertainty over the size of the cost, if the event occurs. Section 4 introduces the ability to reduce expenditure on other items in the second period if the need arises.⁷ Despite the fact that the models are highly simplified, closed-form analytical solutions cannot be obtained. Thus, in each case numerical methods are used to provide illustrations. Conclusions are in Section 5.

³ The approach clearly differs from a political economy perspective where different and overlapping generations vote on policies. The outcome may well depend on differences in risk aversion of young and old individuals.

⁴ See, for example, Dixit and Pindyck (1994) and Pindyck (2008).

⁵ In the context of health and long-term care under demographic uncertainty, Lassila and Valkonen (2004, p. 637) find that the longer the time horizon, 'the virtues of using continuously updated demographic information to evaluate future expenditures become evident'.

⁶ The danger that a precautionary fund will be raided by a future government, stressed long ago by Ricardo in the context of the British Sinking Fund, is not considered here. Davis and Fabling (2002) model 'expenditure creep' and report that it can completely erode the efficiency gains from tax smoothing. They conclude that, 'strong fiscal institutions are a prerequisite for achieving the welfare gains from tax smoothing' (2002, p. 16).

⁷ A very different aspect of decision making under uncertainty is considered by Auerbach and Hassett (1998) who discuss the effects of policy variability itself in increasing uncertainty in the economy.

2 A Two-Period Model

Suppose tax revenue is used to finance government expenditure in each of two periods. The term ‘two-period model’ need not be taken literally, since the periods can consist of multiple sub-periods, in which case care is needed in specifying interest and growth rates. The required revenues of b_1 and b_2 per period are obtained from a proportional income tax at the rate, $\tau_1 = b_1/y_1$ and $\tau_2 = b_2/y_2$, where y_i is income in period i . In the present model, labour supply effects of income taxation are ignored, so that the y_i are considered to be exogenous and known with certainty.⁸ In a more complex model, it would be desirable to allow the tax rate to affect the growth rate of income, thereby contributing an additional component of sunk cost. There is exogenous real income growth at the rate g_y , so $\tau_1 = \tau_2$ only if desired expenditure growth, g_b , is the same rate as income growth.

2.1 A Possible Future Cost

There is a probability of p that an event could take place in the second period that involves an additional cost of C . This section assumes that C (along with p) is known, but in the following section there is a distribution of cost values. Two basic options are possible. The first option is simply to wait to see if the uncertain event arises and then, in the second period, take the necessary tax and expenditure decision. If the decision is to wait, the expenditure (if the need arises) must be met from additional taxation in the second period, assuming (in the present section) that reducing b_2 is not viable. The second option is to take action in the first period by contributing to a fund obtained from additional tax in the first period. But of course it would never be desirable to save enough to cover the full contingency, even if there were no uncertainty, since tax smoothing involves tax rates in both periods being higher than in the absence of any chance of the costly ‘event’. Tax smoothing under certainty within this simple framework is examined in Appendix B. Hence, the fund may be designed to meet only a proportion, γ , of the possible cost, the remaining cost being met from additional taxation in the second period if the event actually occurs.⁹

The approach taken here is to suppose that the choice of policy is taken by an independent and thus disinterested judge who is considered to maximise the expected value of a social welfare function expressed in terms of net income and the excess burden of taxation in each period. Of course there is no ‘correct’ form of this function: it represents the value judgements of the decision maker and so a wide range of assumptions could be modelled. The present form has been chosen simply to emphasise the potentially important roles of net income, the excess burden and risk aversion.¹⁰ Disposable income in period i is $y_i(1 - \tau_i)$, where τ_i is the appropriate overall proportional tax rate. The excess burden is assumed to be proportional to the square of the tax rate; hence it can be expressed as $\beta\tau_i^2$. The approximation of the

⁸ Other individual behaviour is ignored, such as saving, which may be affected by uncertainty and views about how the government responds to it.

⁹ Constraints on flexibility of government policy, such as the ability to change tax rates and expenditure, are ignored here. Such constraints are examined by Auerbach and Hassett (2001), who suggested that, faced with uncertainty, the inability to have frequent changes suggests early action but on the other hand inaction may be chosen because of the inability to reverse any adverse effects on particular groups. He concluded that, ‘the optimal policy response over time might best be characterized by great caution in general, but punctuated by occasional periods of apparent irresponsibility’.

¹⁰ As the values of b_i are fixed, it is not necessary here to allow for any benefits arising from this form of tax-financed expenditure. This is discussed further in section 4.

excess burden in terms of τ_i^2 is of course standard.¹¹ These components are assumed to be additive.

Suppose the judge displays constant relative risk aversion of ε . Welfare in each period is therefore:

$$W(y_i(1 - \tau_i) - \beta\tau_i^2) = \frac{1}{1 - \varepsilon} \{y_i(1 - \tau_i) - \beta\tau_i^2\}^{1 - \varepsilon} \quad (1)$$

The social welfare function is the expected value of the present value of welfare in each period, using a discount rate of r .

To consider the effects of varying ε , consider a simple example of a single-period context in which two values of an uncertain outcome, x_1 and x_2 can arise with probabilities p and $1 - p$ respectively, where welfare under each outcome is now $W(x_i)$ for $i = 1, 2$. The expected value of $W(x)$ is thus:

$$E[W(x)] = p \frac{x_1^{1 - \varepsilon}}{1 - \varepsilon} + (1 - p) \frac{x_2^{1 - \varepsilon}}{1 - \varepsilon} \quad (2)$$

The value of x , say x_ε , which – if obtained with certainty – gives rise to the same value as the uncertain prospect is given by:

$$x_\varepsilon = \{px_1^{1 - \varepsilon} + (1 - p)x_2^{1 - \varepsilon}\}^{1/(1 - \varepsilon)} \quad (3)$$

Table 1 illustrates the effects of varying both p and ε for two different combinations of x_1 and x_2 . For example, a person with this kind of welfare function and having $\varepsilon = 0.2$ would regard the certain sum of 468 as equivalent to the uncertain prospect of receiving 200 with probability 0.1, and 500 with probability 0.9. As the probability of the low-value outcome, of 200, increases to 0.8, the certainty equivalent falls to 255; with high risk aversion of $\varepsilon = 2.0$, the certainty equivalent falls to 435 and 227 for $p = 0.1$ and $p = 0.8$ respectively. As ε increases, the certainty equivalent approaches the minimum alternative value of x quite rapidly.

Table 1: Certainty Equivalent for Alternative Prospects and Risk Aversion

	$x_1 = 200; x_2 = 500$			$x_1 = 10; x_2 = 500$		
	$p = 0.1$	$p = 0.5$	$p = 0.8$	$p = 0.1$	$p = 0.5$	$p = 0.8$
$\varepsilon = 0.2$	468	343	255	441	222	82
$\varepsilon = 0.8$	460	322	244	378	103	29
$\varepsilon = 1.5$	447	300	233	194	31	15
$\varepsilon = 2.0$	435	286	227	85	20	12

The form of the welfare function in (1) also has implications for trade-offs between periods in the multi-period framework. For example, in the case where there is no uncertainty, a welfare function of the form $x^{1 - \varepsilon} / (1 - \varepsilon)$ implies a type of aversion to variability in terms of a preference for a steady ‘consumption’ stream. This differs from time preference which of course involves only a preference for present over future consumption.¹²

The following two subsections describe the form of the welfare function under each policy. The third subsection presents numerical examples. In the fourth subsection the role of inequality aversion is examined in more detail.

¹¹ However, the approximation strictly applies to small tax rates, or small increases in rates: see Creedy (2004). The term, β , involves income and the (compensated) labour supply elasticity, but these need not be considered explicitly here as, to keep the model as simple as possible, income is assumed to be exogenous.

¹² The inter-temporal elasticity of substitution is $1/\varepsilon$. Furthermore the consumption discount rate is, from the standard Ramsey equation, $\delta + \varepsilon g$, where g is the growth rate and δ is the pure time preference rate.

2.2 Wait and See

If the event does not occur then, as above, the tax rate in the second period is simply $\tau_2 = b_2/y_2$. When no action is taken in the first period, and the event does occur, the tax rate needed to finance C is denoted C/y_2 so that the tax rate in the second period becomes:

$$\tau_2^H = \tau_2 + \frac{C}{y_2} \quad (4)$$

The expected value of social welfare from waiting, $E(W|\text{wait})$ is expressed as:

$$\begin{aligned} E(W|\text{wait}) = & W(y_1(1 - \tau_1) - \beta\tau_1^2) \\ & + \frac{p}{1+r}W(y_2(1 - \tau_2^H) - \beta(\tau_2^H)^2) \\ & + \frac{1-p}{1+r}W(y_2(1 - \tau_2) - \beta\tau_2^2) \end{aligned} \quad (5)$$

The first line and third lines of (5) simply show the (expected) welfare that arises in each period when the event does not arise. The second line shows the (expected) social welfare in the second period as a result of financing C . Furthermore, $y_2 = (1 + g_y)y_1$ and $b_2 = (1 + g_b)b_1$.

2.3 Act Immediately

An alternative policy is to act immediately, by saving an amount in the first period to finance a proportion, γ , of the anticipated cost. Thus, the tax rate in period 1 is given by:

$$\tau_1^H = \tau_1 + \frac{\gamma C}{y_1(1+r)} \quad (6)$$

Then in the second period, if the event does not happen, this revenue can be used to lower the tax rate below the planned rate needed to finance b_2 . Hence the tax rate in period 2 becomes:

$$\tau_2^L = \tau_2 - \frac{\gamma C}{y_2} \quad (7)$$

If the event does happen, then the rate becomes:

$$\tau_2^H = \tau_2 + \frac{(1-\gamma)C}{y_2} \quad (8)$$

As before, it is assumed that b_1 is not also adjusted. For simplicity, it is further assumed that income in each period is not affected by the tax policy.¹³

Expected social welfare from acting immediately is:

$$\begin{aligned} E(W|\text{act}) = & W(y_1(1 - \tau_1^H) - \beta(\tau_1^H)^2) \\ & + \frac{p}{1+r}W(y_2(1 - \tau_2^H) - \beta(\tau_2^H)^2) \\ & + \frac{1-p}{1+r}W(y_2(1 - \tau_2^L) - \beta(\tau_2^L)^2) \end{aligned} \quad (9)$$

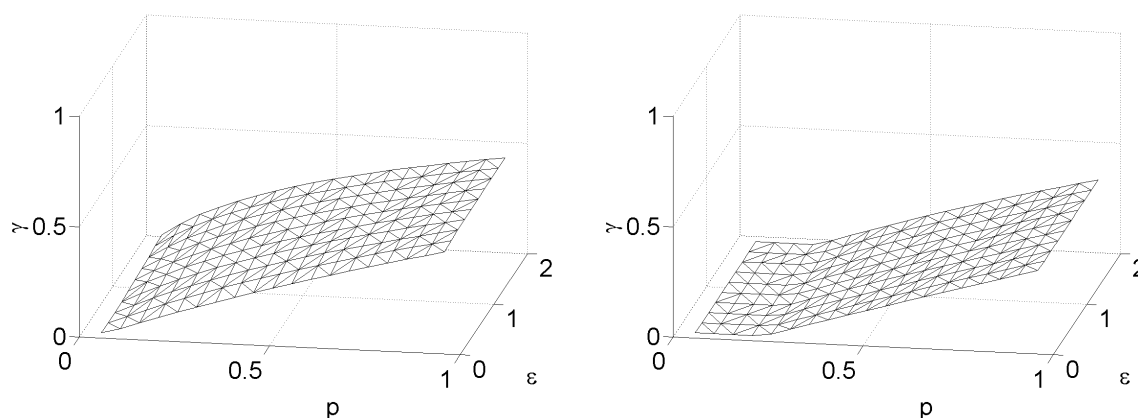
The final line in (9) reflects the fact that, if the event does not take place, the accumulated fund can be used to reduce the second period's tax rate below τ_2 (rather than being used, for example, to increase expenditure on other items). It can easily be seen that when $\gamma = 0$, (9) reduces to (5). Hence waiting is simply an extreme form of acting immediately, and it is necessary only to solve (9) for the value of γ that maximises $E(W|\text{act})$. Furthermore, as p approaches 1, gamma approaches the tax smoothing case discussed in Appendix B.

¹³ In addition to standard incentive effects, there is some debate about whether the use of a fund would increase or reduce income growth.

2.4 Optimal Policy

Suppose that $y_1 = 5000$, $b_1 = 2000$, $\beta = 600$ and $g_y = g_b = r$. These assumptions imply that the tax rate needed to finance the b s is constant at $\tau_1 = \tau_2 = 0.4$. Let $C = 1500$; this is chosen to be a large proportion – just over twenty eight per cent – of the tax base in the second period. Hence if no action is taken in the first period ($\gamma = 0$) a very large increase is required in the second period tax rate if C needs to be financed. The top segment of Figure 1 shows the way in which the optimal value of γ varies as the values of ε and p vary. This clearly shows that the decision depends strongly on the probability of the event taking place, and is not sensitive to the degree of risk aversion. Even for high values of ε , the optima value of γ remains low for lower values of p . Hence in this simple framework it appears that the attitude of the judge to risk is less important than the perceived degree of uncertainty attached to the outcome.

Figure 1: Optimal Gamma for Variations in Probability and Risk Aversion: Cost of 1500 and 500



Reductions in the potential cost, C , have a large effect on the optimal policy. For example, the lower segment of Figure 1 shows the results if C is reduced to 500. It can be seen that the optimal γ remains at zero for p values up to 0.3, and even with $p = 0.75$ gamma is only around 0.23 with tax rates in first and second periods of 0.425 and 0.47. Again there is little variation with respect to risk aversion. Here the certainty case does not imply complete tax smoothing, as the optimal rates in the two periods are 0.435 and 0.46. A higher value of $C = 800$ implies that for $p = 0.75$ the optimal $\gamma = 0.32$, with tax rates of 0.45 and 0.50 in first and second periods respectively.

The effect of a change in the rate of interest has opposing tendencies. Despite the higher discounting of the second period, the higher rate means that, for the same value of γ , a larger fund is accumulated in the second period. This means that tax rates can be lower, hence excess burdens are lower, and the overall effect is to raise the optimal value of γ very slightly.¹⁴

¹⁴ A further characteristic of the models examined here is that, despite the fact that they display unique optimal policies (for given parameters), the concavity of the expected welfare function is quite low. This suggests that the cost of a sub-optimal policy may not be large. Armstrong et al. (2007, pg.31), when considering demographic uncertainty, found that, 'the extent to which the government can mitigate its effects is relatively small'.

2.5 The Role of Risk Aversion

This subsection investigates the role of risk aversion in influencing the optimal degree of tax smoothing. The analysis proceeds by considering two basic relationships. The first is between tax rates in the two periods which arises from the budget constraint. The second relationship is between the two rates, for which the expected social welfare function is constant: it gives what may be referred to as iso-welfare or social indifference curves. The optimal γ is characterised by a point of tangency between these two lines.

For the budget constraint, first rearrange (8):

$$-\gamma C = (\tau_2^H - \tau_2) y_2 - C \quad (10)$$

Substitute this into (6) so that:

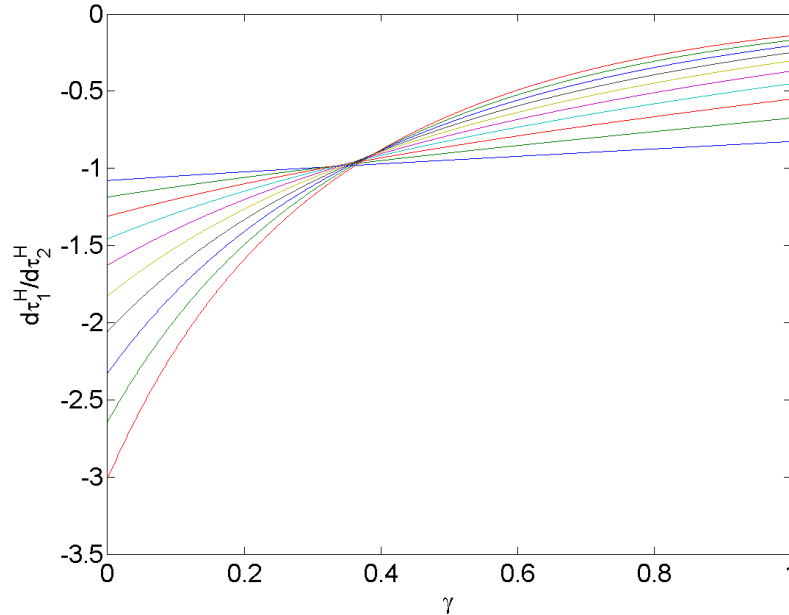
$$\tau_1^H = \tau_1 + \frac{C + \tau_2 y_2}{y_1 (1 + r)} - \frac{\tau_2^H y_2}{y_1 (1 + r)} \quad (11)$$

Consider the case where $g_y = r$, so that, since $y_2 = y_1 (1 + g_y)$:

$$\tau_1^H = \left(\tau_1 + \tau_2 + \frac{C}{y_2} \right) - \tau_2^H \quad (12)$$

The relationship between the two rates is thus a downward sloping 45 degree line.

Figure 2: Slope of Indifference Curves for Alternative Risk Aversion Parameters



To obtain the slope of social indifference curves, first use (10) to substitute in (7) so that:

$$\tau_2^L = \tau_2 + \frac{(\tau_2^H - \tau_2) y_2 - C}{y_2} \quad (13)$$

$$= \tau_2^H - \frac{C}{y_2} \quad (14)$$

Define the terms:

$$Y_1 = y_1 (1 - \tau_1^H) - \beta (\tau_1^H)^2 \quad (15)$$

$$Y_2 = y_2 (1 - \tau_2^H) - \beta (\tau_2^H)^2 \quad (16)$$

$$Y_3 = y_2 \left(1 - \tau_2^H + \frac{C}{y_2} \right) - \beta \left(\tau_2^H - \frac{C}{y_2} \right)^2 \quad (17)$$

Expected social welfare is:

$$E(W) = W\{Y_1\} + \frac{p}{1+r} W\{Y_2\} + \frac{1-p}{1+r} W\{Y_3\} \quad (18)$$

Consider variations in τ_2^H and τ_1^H which leave $E(W)$ unchanged:

$$dE(W) = \frac{\partial W}{\partial Y_1} \frac{\partial Y_1}{\partial \tau_1^H} d\tau_1^H + \left\{ \frac{p}{1+r} \frac{\partial W}{\partial Y_2} \frac{\partial Y_2}{\partial \tau_2^H} + \frac{1-p}{1+r} \frac{\partial W}{\partial Y_3} \frac{\partial Y_3}{\partial \tau_2^H} \right\} d\tau_2^H = 0 \quad (19)$$

Rearranging gives:

$$-\frac{d\tau_1^H}{d\tau_2^H} \Big|_{E(W)} = \left(\frac{1}{1+r} \right) \frac{p(y_2 + 2\beta\tau_2^H) Y_2^{-\varepsilon} + (1-p) \left(y_2 + 2\beta \left(\tau_2^H - \frac{C}{y_2} \right) \right) Y_3^{-\varepsilon}}{(y_1 + 2\beta\tau_1^H) Y_1^{-\varepsilon}} \quad (20)$$

The optimal solution is the value of γ for which:

$$\frac{d\tau_1^H}{d\tau_2^H} \Big|_{E(W)} = 1 \quad (21)$$

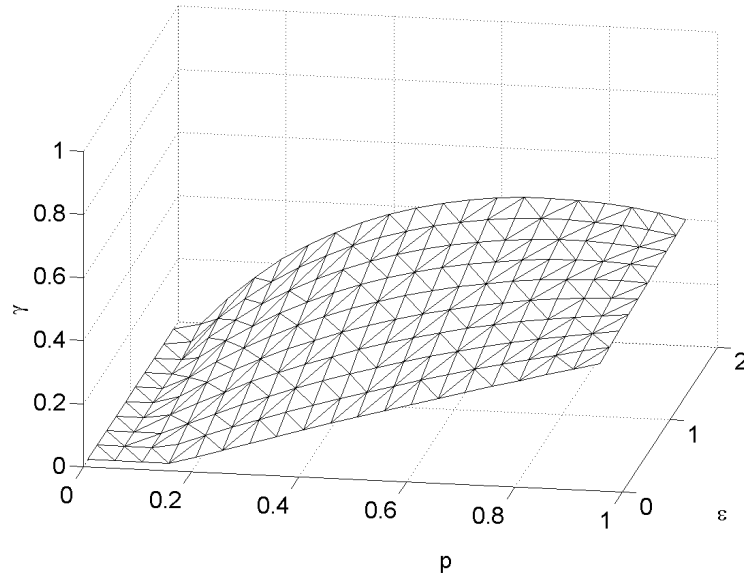
The sensitivity of the optimal value of γ to ε is indicated by the extent to which the right hand side of (20) changes as ε changes, holding the values of τ constant at their optimal values.

Figure 2 shows, for the first case considered above (where $C = 1500$), with $p = 0.6$, the variation in the slope of the indifference curves as risk aversion increases. Each line shows the slope, plotted against gamma, for a given value of ε . The flatter line is for lower risk aversion. The optimal value of γ corresponding to the point where the line intercepts the horizontal line at -1 . It is clear from these schedules that the insensitivity of the degree of tax smoothing with respect to ε arises largely from the nature of the budget constraint: that is, irrespective of the value of C and of the other tax-financed expenditure, the constraint is a downward sloping 45 degree line. The degree of convexity of indifference curves increases as ε increases, corresponding to the greater concavity of the welfare function, but there is little variation where the slope is -1 . As can be seen from (11), the slope of the budget line can differ from -1 where $r \neq g_y$.

2.6 An Alternative Welfare Function

A feature of the welfare function used above is that the measure of relative risk aversion, ε , is directly linked to the intertemporal elasticity of substitution, which is equal to $1/\varepsilon$. An increase in risk aversion (which might, *ceteris paribus*, be thought to increase tax smoothing) is therefore accompanied by a reduction in the intertemporal elasticity of substitution, which in turn makes consumption less sensitive to the interest rate and weakens the motive for consumption (and hence tax smoothing). The fact that these effects oppose each other largely explains the insensitivity observed in the previous subsection. An alternative specification of the welfare function follows what are generally referred to as Epstein and Zin (1989) preferences. In this case preferences, for example over consumption in two periods, are expressed as a constant elasticity of substitution (CES) aggregate of consumption in the first

Figure 3: Choice of Gamma with Epstein-Zin Preferences: $c = 600$ and $\alpha = 0.5$



period and the certainty-equivalent value of consumption in the second period. The certainty equivalent, the value that if known for certain would give the same welfare as the uncertain prospect, is the power mean given in equation (3).¹⁵

Using the notation in the previous subsection, the relevant certainty equivalent, denoted Y_ε , is:

$$Y_\varepsilon = [pY_2^{1-\varepsilon} + (1-p)Y_3^{1-\varepsilon}]^{1/(1-\varepsilon)} \quad (22)$$

Letting α denote the intertemporal elasticity of substitution, the welfare function can be written as:

$$W_{E-Z} = \left[Y_1^{1-\frac{1}{\alpha}} + \left(\frac{1}{1+r} \right) Y_\varepsilon^{1-\frac{1}{\alpha}} \right]^{1/(1-\frac{1}{\alpha})} \quad (23)$$

Consider again the illustrative case examined above where $y_1 = 5000$, $b_1 = 2000$, $\beta = 600$ and $g_y = g_b = r = 0.05$. Letting $C = 600$, Figure 3 shows the resulting choice of γ for variations in ε and p and a value of $\alpha = 0.5$. In this case the optimal choice, and hence degree of tax smoothing, does depend on the degree of risk aversion, particularly for values of p around 0.5.¹⁶

3 Uncertain Costs

The above simple model can be extended to allow for the case where there is some uncertainty about the cost of the event in addition to whether or not it occurs. Suppose that there is a probability p_1 of the event taking place with a relatively low cost of C_L , and a probability of p_2 of the event taking place with a higher cost of C_H . To allow for the chance that the event may

¹⁵ An early proposal to have one concave function to generate the certainty equivalent and another concave function to describe intertemporal substitution between current consumption and the certainty equivalent was made by Selden (1978). For a review of a range of 'exotic' preference functions, see Backus et al. (2004).

¹⁶ This value of p effectively represents the point of maximum uncertainty.

not take place, $p_1 + p_2 < 1$. It was seen in the previous section that, since γ can be varied continuously from $\gamma = 0$, it is only necessary to consider the result of acting immediately. Here, γ is expressed as a proportion of the highest cost. Producing a fund of γC_H in the second period requires saving $\gamma C_H / (1 + r)$ in the first period. It is first necessary to consider the different tax rates. The tax rate in the first period is:

$$\tau_{\gamma,1} = \tau_1 + \frac{\gamma C_H}{y_1 (1 + r)} \quad (24)$$

If the event does happen, the tax rate applicable to the low-cost outcome is:

$$\tau_{L,2} = \tau_2 - \frac{\gamma C_H - C_L}{y_2} \quad (25)$$

and in the case where $\gamma C_H > C_L$, then $\tau_{L,2} < \tau_2$. For the high-cost outcome:

$$\tau_{H,2} = \tau_2 + \frac{C_H - \gamma C_H}{y_2} \quad (26)$$

If the event does not occur at all, the tax rate in the second period is:

$$\tau_{N,2} = \tau_2 - \frac{\gamma C_H}{y_2} \quad (27)$$

The expected value of the welfare function (using the basic iso-elastic function of subsection 2.1) is:

$$\begin{aligned} E(W) = & W \left(y_1 (1 - \tau_{\gamma,1}) - \beta (\tau_{\gamma,1})^2 \right) \\ & + \frac{p_1}{1 + r} W \left(y_2 (1 - \tau_{L,2}) - \beta (\tau_{L,2})^2 \right) \\ & + \frac{p_2}{1 + r} W \left(y_2 (1 - \tau_{H,2}) - \beta (\tau_{H,2})^2 \right) \\ & + \frac{1 - p_1 - p_2}{1 + r} W \left(y_2 (1 - \tau_{N,2}) - \beta (\tau_{N,2})^2 \right) \end{aligned} \quad (28)$$

It is therefore necessary to obtain the value of γ that maximises (28). If this turns out to be zero, then of course the optimal policy is to do nothing in the first period.

3.1 Optimal Policy

Suppose that, as before, $y_1 = 5000$, $b_1 = 2000$, $\beta = 600$ and $g_y = g_b = r$. Hence $\tau_1 = \tau_2 = 0.4$. There are known probabilities that an event will take place costing either $C_L = 600$ and $C_H = 1000$. These are 11 per cent and nineteen per cent respectively of income, $y_2 = 5250$, in the second period. For $\gamma > 0.6$, the fund available in the second period exceeds the low-cost contingency.

Figure 4 shows the optimal values of γ for variations in the two probabilities, p_1 and p_2 , for a risk aversion parameter of $\varepsilon = 0.1$. Again each three-dimensional diagram shows two perspectives of the same results. The diagram show that, even with very high p_2 , the fund never exceeds the smallest of the two possible costs, C_L . When p_2 is low there is a range of values of p_1 for which $\gamma = 0$ is optimal. As with the simpler model of the previous section, comparisons (not shown here) show that the degree of risk aversion makes very little difference. The main influence on the optimal value of γ is the probability, p_2 . Figure 5 shows, for the corresponding case, the variation in the overall tax rate in the first period. As mentioned above, a value of $\gamma = 0$ corresponds to $\tau_1 = 0.4$. As in the simple model considered earlier, the optimal value of γ shows very little sensitivity with respect to risk aversion.

Figure 4: Optimal Gamma for Variations in Two Probabilities: Risk Aversion of 0.1

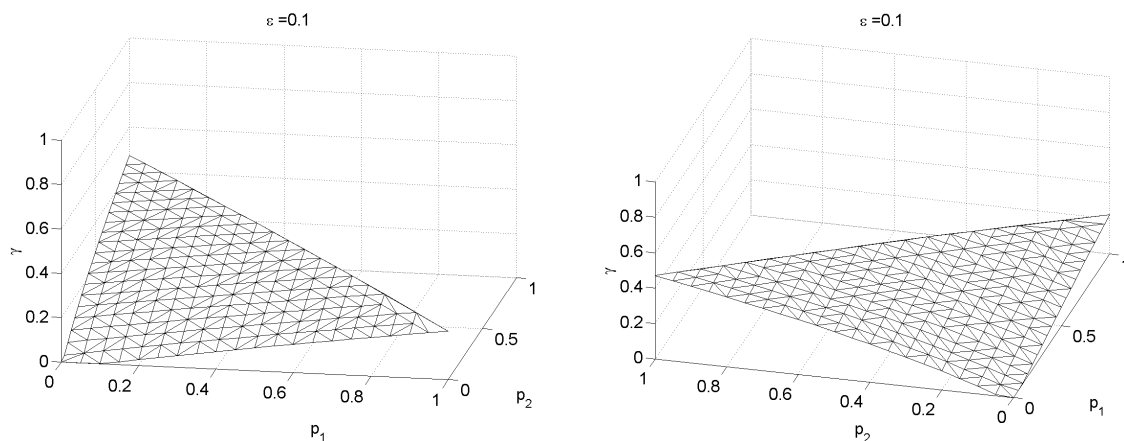


Figure 5: Optimal Tax Rate in Period 1: Risk Aversion of 0.1

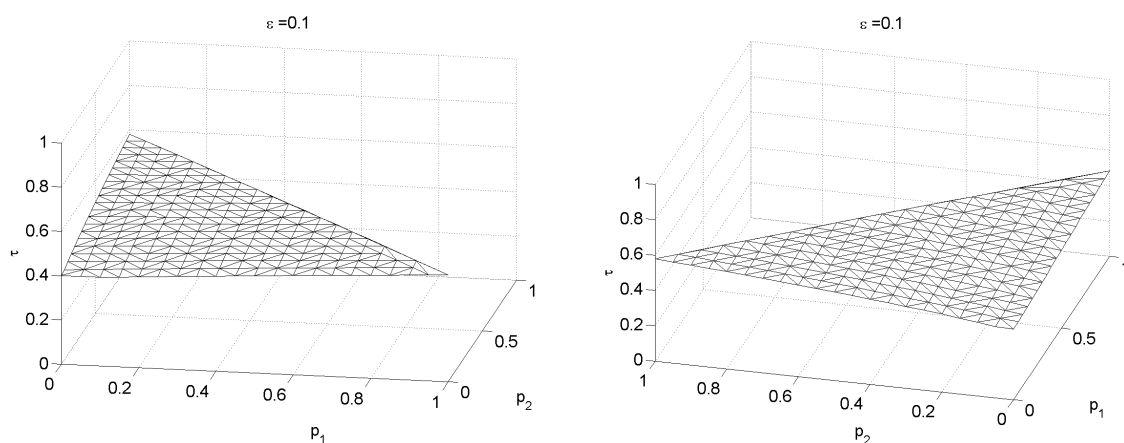
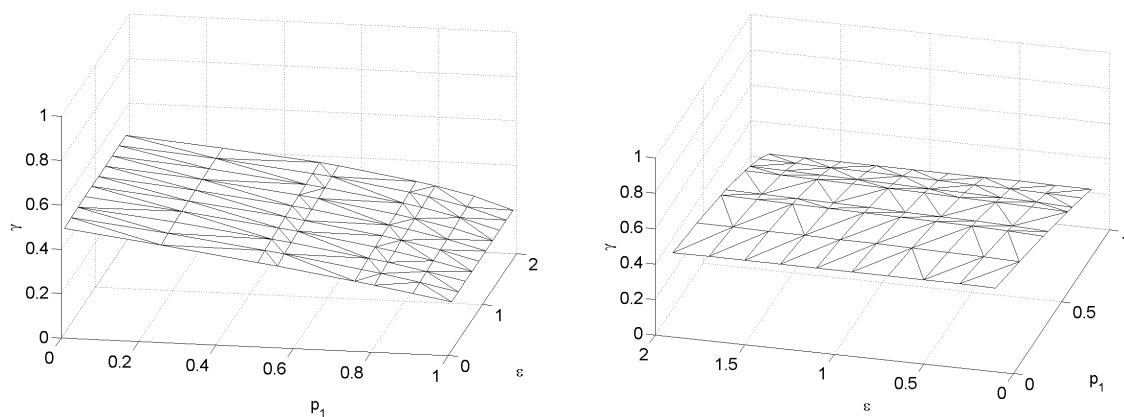


Figure 6: Optimal Gamma with $p_1 + p_2 = 1$



The analysis has not restricted the two probabilities to sum to 1. A modification is to suppose that an event will certainly happen in period that gives rise to an uncertain cost. Figure 6 shows the results from setting $p_2 = 1 - p_1$, so it is possible to show the effect on optimal γ of the joint variation in p_1 and ε . The results are, perhaps not surprisingly, similar to the case of a single cost above, except that $\gamma > 0$ is optimal for all combinations. Again risk aversion has little effect and p_2 dominates.

3.2 An Alternative Welfare Function

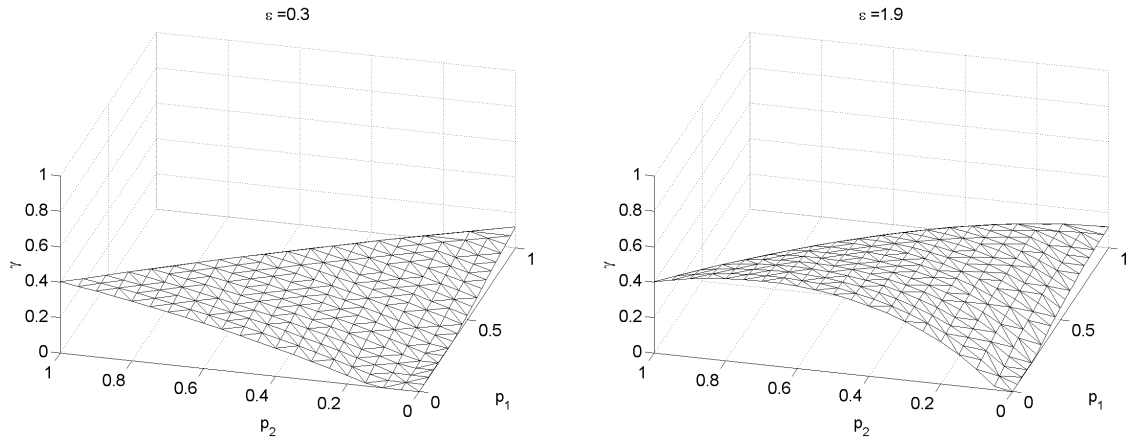
The application of the Epstein-Zin specification for the welfare function in this context involves the certainty equivalent of $Y_\varepsilon = Z^{1/(1-\varepsilon)}$ where:

$$Z = p_1 \left\{ y_2 (1 - \tau_{L,2}) - \beta (\tau_{L,2})^2 \right\}^{1-\varepsilon} + p_2 \left\{ y_2 (1 - \tau_{H,2}) - \beta (\tau_{H,2})^2 \right\}^{1-\varepsilon} + (1 - p_1 - p_2) \left\{ y_2 (1 - \tau_{N,2}) - \beta (\tau_{N,2})^2 \right\}^{1-\varepsilon} \quad (29)$$

The welfare function is:

$$W_{EZ} = \left[\left\{ y_1 (1 - \tau_{\gamma,1}) - \beta (\tau_{\gamma,1})^2 \right\}^{1-\frac{1}{\alpha}} + \left(\frac{1}{1+r} \right) Y_\varepsilon^{1-\frac{1}{\alpha}} \right]^{1/(1-\frac{1}{\alpha})} \quad (30)$$

Figure 7: Optimal Gamma for Epstein-Zin Preferences: Alpha = 0.5



Using the Epstein-Zin welfare function, the optimal choice of γ is shown in Figure 7 for two risk aversion parameters and a value of $\alpha = 0.5$. Other values are, as earlier, $y_1 = 5000$, $b_1 = 2000$, $\beta = 600$ and $g_y = g_b = r = 0.5$. The cost values are assumed to be $C_L = 600$ and $C_H = 1000$. The very high degree of risk aversion does produce a different, and somewhat concave, surface compared with the lower risk aversion. However, the effect of risk aversion becomes much smaller as the two costs are reduced and as the value of α is increased. The concavity in the case of very high risk aversion may be associated with the increase in the certainty equivalent beyond $p_2 = 0.5$ (the point of maximum uncertainty regarding the high cost alternative), allowing for a lower current tax rate.

4 An Option to Reduce other Expenditure

Consider the simple model of section (2) but suppose that the extra cost C , if the need arises, can be met either from additional taxation in the second period or from a combination of reducing b_2 and raising additional tax revenue in period 2. If the choice is made to take precautionary action in the first period by contributing to a fund obtained from additional tax in the first period, the fund may be designed to meet a only proportion, γ , of the possible cost, C , the remaining cost being met from additional taxation in the second period if the event actually occurs, or a combination of reduced b_2 and extra tax.¹⁷ The reduction in b_2 , by a proportion, η , clearly involves a sacrifice, so it is necessary to attach a value to this loss. Suppose the welfare function for each period, i , is similar to that used above, but involves a weighted geometric mean of net income, $y_i (1 - \tau_i)$, and tax-financed expenditure, b_i , with weights ξ and $1 - \xi$ respectively Hence:

$$U_i(y_i(1 - \tau_i), b_i) = \{y_i(1 - \tau_i)\}^\xi b_i^{1-\xi} \quad (31)$$

and, as before, suppose the judge displays constant relative risk aversion of ε . Hence, welfare in each period is now written as:

$$W(y_i(1 - \tau_i), b_i) = \frac{1}{1 - \varepsilon} [U_i(y_i(1 - \tau_i), b_i) - \beta \tau_i^2]^{1-\varepsilon} \quad (32)$$

As γ can be varied continuously from zero, it is only necessary to consider the case where action is taken immediately, by saving enough in the first period to finance a proportion, γ , of the anticipated cost. Thus, in the first period a tax rate of $\tau_{\gamma,1}$ is imposed, given by:

$$\tau_{\gamma,1} = \tau_1 + \frac{\gamma C}{y_1(1 + r)} \quad (33)$$

If the event does happen, and it is possible to reduce expenditure on other items by a proportion, η , then the rate in the second period becomes, $\tau_{\gamma,2}$, given by:

$$\tau_{\gamma,2} = \tau_2 + \frac{(1 - \gamma)C}{y_2} - \frac{\eta b_2}{y_2} \quad (34)$$

Clearly, b -type expenditure cannot be cut beyond the amount needed to finance the additional C -type cost. Hence $\tau_{\gamma,2} \geq \tau_2$ and η must be restricted to the range:

$$\eta \leq \frac{(1 - \gamma)C}{b_2} \quad (35)$$

In the second period, if the event does not happen, the revenue can be used to lower the tax rate below the planned rate needed to finance b_2 . Allowing for the additional possibility that b_2 is reduced below the value it would have without the potential event, then:

$$\tau_{N,2} = \tau_2 - \frac{\gamma C}{y_2} \quad (36)$$

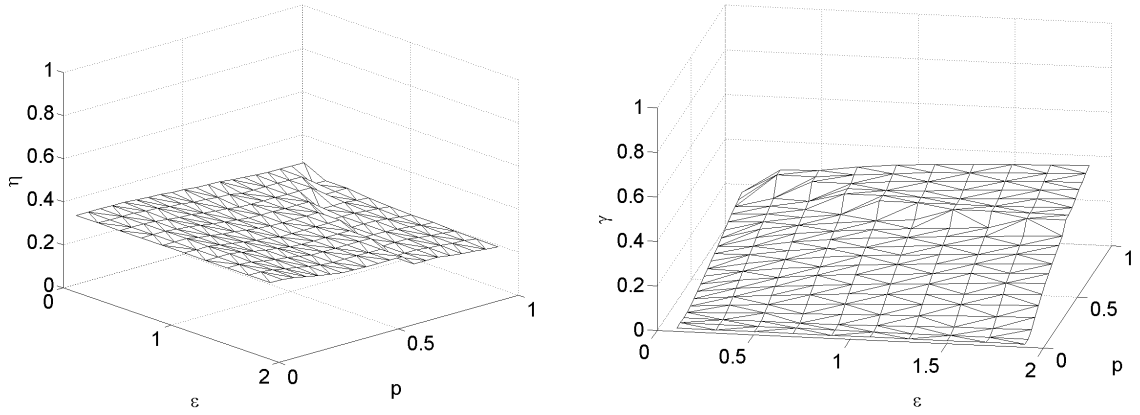
It is assumed that b_1 is not adjusted.

Expected social welfare from acting immediately is:

$$\begin{aligned} E(W|\text{act}) = & W\left(U(y_1(1 - \tau_{\gamma,1}), b_1) - \beta(\tau_{\gamma,1})^2\right) \\ & + \frac{p}{1 + r} \left[W\left(U(y_2(1 - \tau_{\gamma,2}), (1 - \eta)b_2) - \beta(\tau_{\gamma,2})^2\right) \right] \\ & + \frac{1 - p}{1 + r} \left[W\left(U(y_2(1 - \tau_{N,2}), b_2) - \beta(\tau_{N,2})^2\right) \right] \end{aligned} \quad (37)$$

¹⁷ One modification would be to add a limit to the reduction in b_2 allowed.

Figure 8: Optimal Values of Gamma and Eta



The optimal policy therefore involves finding combinations of the two policy variables, η and γ , which maximise (37).

Figure 8 plots combinations of optimal η and γ for alternative values of p and ε , for the following values of other variables: $y_1 = 6000$; $C = 2000$; $b_1 = 2100$; $\beta = 900$; $\alpha = 0.5$; $r = g_y = g_b = 0.05$. As before, the optimal choice is not sensitive to ε but is sensitive to p , with η and γ moving in opposite directions, as expected. In varying these other variables, the results are relatively sensitive to variations in α : raising α leads to higher optimal γ and lower optimal η .

As before, this can be extended to allow for uncertainty about the cost of the event in addition to whether or not it occurs. Suppose that there is a probability p_1 of the event taking place with a relatively low cost of C_L , and a probability of p_2 of the event taking place with a higher cost of C_H . Suppose it is planned to accumulate a proportion, γ , of the anticipated higher cost in the first period. Thus, the tax rate in the first period is:

$$\tau_{\gamma,1} = \tau_1 + \frac{\gamma C_H}{y_1(1+r)} \quad (38)$$

If funds are needed, the alternative tax rates for low and high cost outcomes respectively are:

$$\tau_{L,2} = \tau_2 - \frac{\gamma C_H - C_L}{y_2} - \frac{\eta b_2}{y_2} \quad (39)$$

If more than C_L is accumulated, then of course $\tau_{L,2}$ is accordingly lower than otherwise.

$$\tau_{H,2} = \tau_2 + \frac{C_H - \gamma C_H}{y_2} - \frac{\eta b_2}{y_2} \quad (40)$$

These expressions reflect the reduction in planned expenditure in period 2 to help finance the contingency (again assuming that b_1 is not also adjusted). As in the simple case of one known cost level, the value of η must be restricted to ensure that $\tau_{L,2} > \tau_2$ and $\tau_{H,2} > \tau_2$. If the funds are not needed, then:

$$\tau_{N,2} = \tau_2 - \frac{\gamma C_H}{y_2} \quad (41)$$

Expected utility is:

$$\begin{aligned} E(W|\text{act}) = & W\{U(y_1(1-\tau_{\gamma,1}), b_1) - \beta\tau_{\gamma,1}^2\} \\ & + \frac{p_1}{1+r} [W\{U(y_2(1-\tau_{L,2}), (1-\eta)b_2) - \beta\tau_{L,2}^2\}] \\ & + \frac{p_2}{1+r} [W\{U(y_2(1-\tau_{H,2}), (1-\eta)b_2) - \beta\tau_{H,2}^2\}] \\ & + \frac{1-p_1-p_2}{1+r} [W\{U(y_2(1-\tau_{N,2}), b_2) - \beta\tau_{N,2}^2\}] \end{aligned} \quad (42)$$

Figure 9: Optimal Eta for alternative Risk Aversion

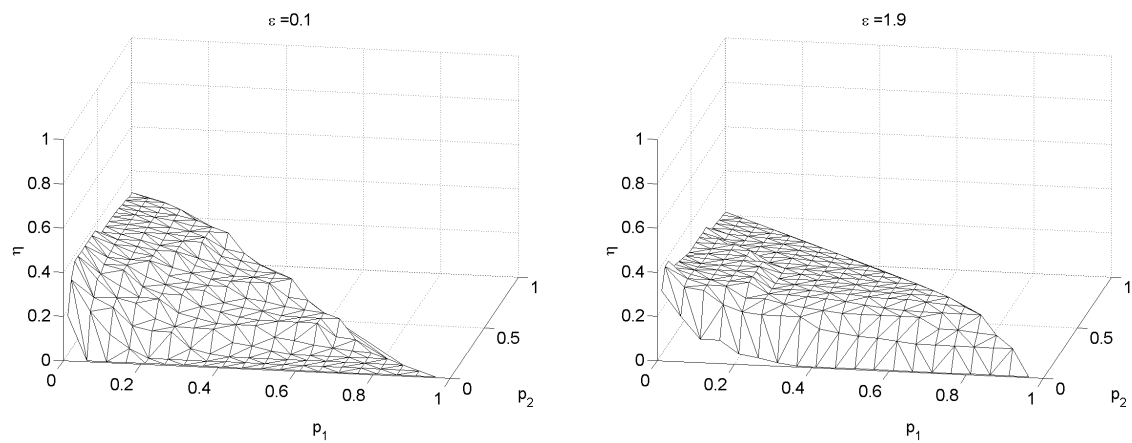
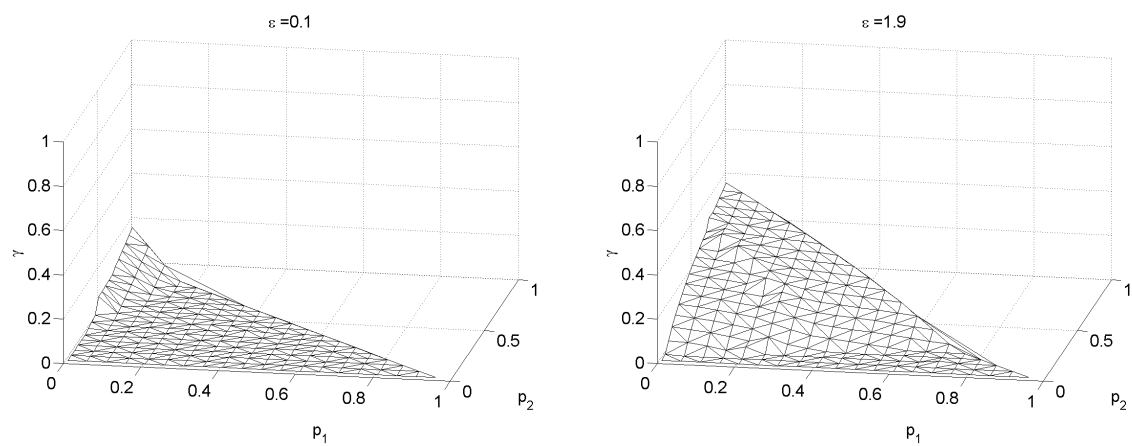


Figure 10: Optimal Gamma for Alternative Risk Aversion



Figures 9 and 10 show optimal values of η and γ for variations in the probabilities p_1 and p_2 , in each case for two different values of risk aversion, ε , for the following values: $y_1 = 6000$; $C_H = 2500$; $C_L = 1500$; $b_1 = 3000$; $\beta = 900$; $\alpha = 0.4$; $r = g_y = g_b = 0.05$. In this case higher risk aversion does lead to higher amounts of prefunding with consequently lower reductions in the second period other expenditure. Again higher values of γ are found to be associated with higher ε and lower η .

5 Conclusions

This paper has examined the extent to which the standard tax smoothing argument, arising from the disproportional excess burden of taxation, is modified by the existence of uncertainty both about whether the need for extra expenditure will arise, and about the level of expenditure if the need arises. This is important in view of the fact that considerable uncertainty regarding future expenditure requirements exists in practice. A range of simple two-period models has been examined in which a single decision maker decides on the level of tax smoothing, and associated precautionary fund, which maximises an objective function, or ‘social welfare function’, expressed in terms of net income and the excess burden of taxation in each period.

Although these are highly simplified models, they do help to highlight some of the important inter-relationships and trade-offs involved. Numerical illustrations showed that the potential future expenditure must be relatively large (as a proportion of income) before the ‘option value’, the cost of waiting to see what the actual outcome will be, is sacrificed. In cases where some pre-funding is favoured, the optimal policy does not imply complete smoothing of tax rates. Importantly, the degree of risk aversion of the judge was found to have little effect on the optimal degree of smoothing. In the case of the standard iso-elastic welfare function (where the same parameter affects risk aversion and the inter-temporal elasticity of substitution), the effect of varying risk aversion was negligible. In the case of Epstein-Zin preferences, where the link between risk aversion and inter-temporal substitution is broken, slightly more sensitivity was found for high potential future (uncertain) costs and low substitution elasticities. In general, the size of the potential future tax-financed cost and its associated probability were found to be the major determinants of the optimal policy.

The simple models examined here are concerned only with aggregate values, using proportional income taxation, and therefore no consideration has been given to distributional issues. It would be of interest to consider the possible effects of using a progressive income tax structure where, in addition, the decision maker has some concern for inequality. Allowing for income inequality would also allow consideration of ‘political economy’ approaches to decision making, such as the use of majority choice by members of the population, rather than decision making by a disinterested judge.¹⁸ Further complications could be introduced by allowing explicitly for incentive effects of income taxation.

¹⁸ Such an approach would also need to allow for the fact that, at any date, the population consists of overlapping generations of voters.

A Sunk Costs and the Value of Waiting

Suppose a project yields a profit of π_1 in year 1. In the second period it yields either no profit, with a probability of $1 - p$, or a profit of π_H with probability p .¹⁹ The rate of interest is r . Suppose there is a fixed cost of investing, in the first period of operation, of S . This cannot be recovered if, depending on the outcome in period 2, it is decided not to continue in production. But importantly the sunk cost may actually be higher than S because investing in the first period involves giving up the 'option value' of waiting to obtain more information. This can be seen as follows. The expected net present value (NPV) of the project is:

$$E(NPV) = \pi_1 + \frac{p\pi_H}{1+r} - S \quad (\text{A.1})$$

Hence the conventional NPV criterion suggests that investment is worthwhile if:

$$S < \pi_1 + \frac{p\pi_H}{1+r} \quad (\text{A.2})$$

First, compare the fixed cost with an annual fee. The fee must be compared, not with the fixed cost, but with the equivalent annual fee, C , defined by $S = C + \frac{C}{1+r}$. Hence:

$$C = \frac{S}{\left(1 + \frac{1}{1+r}\right)} \quad (\text{A.3})$$

Suppose there is an annual fee of F . The expected NPV in this case, in view of the fact that if the lower (zero) profit outcome arises, no production takes place and hence no fee is paid, is:

$$E(NPV)_F = (\pi_1 - F) + \frac{p(\pi_H - F)}{1+r} \quad (\text{A.4})$$

Starting the project in the first period is worthwhile so long as:

$$F < \left(1 + \frac{p}{1+r}\right)^{-1} \left\{ \pi_1 + \frac{p\pi_H}{1+r} \right\} \quad (\text{A.5})$$

The maximum fixed cost, S_{\max} , and its associated equivalent annual amount, C_{\max} , under which investment in the project in the first period is not worthwhile, is obtained by replacing the inequality in (A.2) with an equality. This may be compared with the maximum fee, F_{\max} , given by replacing the inequality in (A.5) with an equality, such that the project with a fee becomes not worthwhile. It can be seen that $F_{\max} > C_{\max}$ if:

$$1 > \frac{1 + \frac{p}{1+r}}{1 + \frac{1}{1+r}} \quad (\text{A.6})$$

This clearly holds, since $p < 1$.²⁰ Thus the standard NPV expression in (A.1) does not provide an appropriate evaluation criterion. It is not only the fixed cost, S , that cannot be recovered after being incurred in period 1. Starting the project in the first period foregoes the option value of waiting. Hence it can be said that the sunk cost – the amount which cannot be recovered at any stage after starting the project – is greater than the fixed cost by an amount that reflects the option value.

¹⁹ This example is based largely on the numerical example given by Pindyck (2008). A similar example is given by Auerbach and Hassett (2001) to illustrate the potential benefits from waiting.

²⁰ Clearly, it may not hold if, instead, the lowest profit in the first period is positive, though less than F_{\max} .

Instead of having the choice of starting the project in the first period, or not at all, suppose instead that the investor waits until the second period before making a decision. If circumstances which give rise to zero profit arise, no investment takes place and no fixed cost needs to be lost. The expected NPV (evaluated in period 1) in this ‘waiting case’ is thus simply:

$$E(NPV)_W = \frac{p(\pi_H - S)}{1 + r} \quad (\text{A.7})$$

Suppose that, from (A.2), the fixed cost is $\pi_1 + \frac{p\pi_H}{1+r}$, so that it is not worth investing in the first period. Substituting this value in (A.7) gives:

$$E(NPV)_W = \frac{p}{1+r} \left\{ \pi_H \left(1 - \frac{p}{1+r} \right) - \pi_1 \right\} \quad (\text{A.8})$$

Hence if $\pi_H > \pi_1 / \left(1 - \frac{p}{1+r} \right)$, it is worth waiting and then investing in the project in the second period. In this case the value of waiting is $E(NPV)_W$ since, by assumption, the value of acting in the first period, $E(NPV)$, is zero. But in general the value of the option to wait, or the option value, V , is:

$$V = E(NPV)|_W - E(NPV) \quad (\text{A.9})$$

This option value need not, of course, always be positive. Similarly, the true sunk cost, S^* , exceeds the fixed cost, S , since it must allow for the foregone option value. Hence it is:

$$S^* = S + V \quad (\text{A.10})$$

B Tax Smoothing under Certainty

Consider the simple model of Section 2 where, instead of uncertainty, it is known for certain that the additional cost of C will arise in the second period. In this case the tax rate in the first period is:

$$\tau_1^T = \tau_1 + \frac{\gamma C}{(1+r)y_1} \quad (\text{B.1})$$

and the rate in the second period is:

$$\tau_2^T = \tau_2 + \frac{(1-\gamma)C}{y_1} \quad (\text{B.2})$$

The evaluation function is thus:

$$\begin{aligned} E(W) = & W \left(y_1 (1 - \tau_1^T) - \beta (\tau_1^T)^2 \right) \\ & + \frac{1}{1+r} W \left(y_2 (1 - \tau_2^T) - \beta (\tau_2^T)^2 \right) \end{aligned} \quad (\text{B.3})$$

writing $Y_i = y_i (1 - \tau_i^T) - \beta (\tau_i^T)^2$ and $W_i = W \{Y_i\}$, this is maximised when:

$$\frac{dE(W)}{d\gamma} = W_1^{-\varepsilon} \frac{dY_1}{d\gamma} + \frac{1}{1+r} W_2^{-\varepsilon} \frac{dY_2}{d\gamma} = 0 \quad (\text{B.4})$$

so that:

$$(1+r) \left(\frac{W_2}{W_1} \right)^\varepsilon = - \frac{dY_2/d\gamma}{dY_1/d\gamma} \quad (\text{B.5})$$

Furthermore:

$$- \frac{dY_2/d\gamma}{dY_1/d\gamma} = \left(\frac{y_2}{y_1} \right) \frac{y_2 + 2\beta\tau_2^T}{y_1 + 2\beta\tau_1^T} \quad (\text{B.6})$$

The left hand side of (B.5) involves a ratio of marginal valuations for the two periods and the right hand side involves a ratio of ‘marginal costs’ of changing γ . In a two-dimensional diagram the optimum is represented by a tangency between an iso-welfare line and a nonlinear ‘relative price’ line. The value of γ that achieves this tangency solution gives the optimal degree of tax smoothing. This nonlinear equation can be solved numerically.

C Indifference Curves with E-Z Preferences

For the simple model examined in subsection 2.6 of Section 2, the certainty equivalent is:

$$Y_\varepsilon = [pY_2^{1-\varepsilon} + (1-p)Y_3^{1-\varepsilon}]^{1/(1-\varepsilon)} \quad (C.1)$$

and the welfare function is:

$$W_{EZ} = \left[Y_1^{1-\frac{1}{\alpha}} + \left(\frac{1}{1+r} \right) Y_\varepsilon^{1-\frac{1}{\alpha}} \right]^{1/(1-\frac{1}{\alpha})} \quad (C.2)$$

with:

$$Y_1 = y_1 (1 - \tau_1^H) - \beta (\tau_1^H)^2 \quad (C.3)$$

$$Y_2 = y_2 (1 - \tau_2^H) - \beta (\tau_2^H)^2 \quad (C.4)$$

$$Y_3 = y_2 \left(1 - \tau_2^H + \frac{C}{y_2} \right) - \beta \left(\tau_2^H - \frac{C}{y_2} \right)^2 \quad (C.5)$$

Consider variations in τ_2^H and τ_1^H which leave W_{E-Z} unchanged:

$$dW_{EZ} = \frac{\partial W}{\partial Y_1} \frac{\partial Y_1}{\partial \tau_1^H} d\tau_1^H + \frac{\partial W}{\partial Y_\varepsilon} \frac{\partial Y_\varepsilon}{\partial \tau_2^H} d\tau_2^H = 0 \quad (C.6)$$

$$dW_{EZ} = - \left(1 - \frac{1}{\alpha} \right) (y_1 + 2\beta\tau_1^H) Y_1^{-\frac{1}{\alpha}} d\tau_1^H + \left(\frac{1}{1+r} \right) \left(1 - \frac{1}{\alpha} \right) Y_\varepsilon^{-\frac{1}{\alpha}} \frac{\partial Y_\varepsilon}{\partial \tau_2^H} d\tau_2^H = 0 \quad (C.7)$$

$$\left. \frac{d\tau_1^H}{d\tau_2^H} \right|_{W_{EZ}} = \left(\frac{1}{1+r} \right) \frac{\frac{\partial Y_\varepsilon}{\partial \tau_2^H} Y_\varepsilon^{-\frac{1}{\alpha}}}{(y_1 + 2\beta\tau_1^H) Y_1^{-\frac{1}{\alpha}}} \quad (C.8)$$

with:

$$\frac{\partial Y_\varepsilon}{\partial \tau_2^H} = \left(\frac{1}{1-\varepsilon} \right) (pY_2^{1-\varepsilon} + (1-p)Y_3^{1-\varepsilon})^{\frac{\varepsilon}{1-\varepsilon}} \left\{ p \frac{\partial Y_2^{1-\varepsilon}}{\partial \tau_2^H} + (1-p) \frac{\partial Y_3^{1-\varepsilon}}{\partial \tau_2^H} \right\} \quad (C.9)$$

where:

$$\frac{\partial Y_2^{1-\varepsilon}}{\partial \tau_2^H} = -(1-\varepsilon) (y_2 + 2\beta\tau_2^H) Y_2^{-\varepsilon} \quad (C.10)$$

$$\frac{\partial Y_3^{1-\varepsilon}}{\partial \tau_2^H} = -(1-\varepsilon) \left(y_2 + 2\beta \left(\tau_2^H - \frac{C}{y_2} \right) \right) Y_3^{-\varepsilon} \quad (C.11)$$

Then:

$$\left. \frac{d\tau_1^H}{d\tau_2^H} \right|_{W_{EZ}} = - \left(\frac{1}{1+r} \right) \frac{Y_\varepsilon^{-\frac{1}{\alpha}} \left\{ p (y_2 + 2\beta\tau_2^H) Y_2^{-\varepsilon} + (1-p) \left(y_2 + 2\beta \left(\tau_2^H - \frac{C}{y_2} \right) \right) Y_3^{-\varepsilon} \right\}}{(y_1 + 2\beta\tau_1^H) Y_1^{-\frac{1}{\alpha}}} \quad (C.12)$$

This may be compared with the result using the basic iso-elastic welfare function, given in subsection 2.5 above.

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