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## Tax Rates and Revenue Changes: Behavioural and Structural Factors

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# Tax Rates and Revenue Changes: Behavioural and Structural Factors

John Creedy and Norman Gemmell

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# Abstract

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This paper examines the elasticity of tax revenue with respect to a marginal rate change, at both the individual and aggregate level. The roles of the elasticity of taxable income (the behavioural effect on taxable income of a tax rise) and the revenue elasticity (the structural effect on revenue of a change in taxable income) are highlighted. The revenue elasticity is the central concept in examining fiscal drag, but it has an additional role in the context of the revenue effects of tax changes when incomes respond to rate changes. Illustrations are provided using changes to the New Zealand income tax structure in the 2010 Budget. This reduced all marginal tax rates while leaving income thresholds unchanged.

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**KEYWORDS**            Income tax revenue; elasticity of taxable income; revenue elasticity.

# Preface

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- This paper shows how the change in income tax revenue - at individual and aggregate levels - as a consequence of a marginal tax rate change can be decomposed into several effects. These effects are measured in terms of elasticities, reflecting the proportional change in one variable expressed as a ratio of the proportional change in another variable. The decomposition is useful for revenue forecasting purposes.
- First, there is the obvious positive revenue effect of a rate rise which arises if all incomes remain unchanged: this is the partial elasticity of revenue with respect to the relevant tax rate. Second, there is a negative effect arising from behavioural responses. This latter effect can itself be divided into two multiplicative influences, as follows.
- There is an adjustment to taxable income, measured by the elasticity of taxable income. It captures the net effect of all incentive effects associated with the marginal rate change, and is measured in terms of the net-of-tax rate (rather than the rate itself). In addition, the tax revenue elasticity - the elasticity of tax revenue with respect to a change in income - determines the consequent effect on revenue of the behavioural response.
- The tax revenue elasticity is concerned only with the nature of the income tax structure itself and, when considering aggregation over individuals, the form of the income distribution. Hence the value of this elasticity can be calculated using relatively little data. The elasticity of taxable income is concerned with a wide range of behavioural adjustments associated with tax rate changes, captured in a single measure. Its estimation therefore presents significant difficulties. There is no direct connection between the two elasticity concepts.
- The paper provides a number of technical analyses which derive the conditions (expressed in terms of the elasticities discussed above) under which the tax revenue increases when a particular marginal tax rate increases. These conditions are derived for individuals and for aggregate revenue. It is found that the elasticity of taxable income (of those in the tax bracket affected by the rate change) must be relatively low for a tax rate increase to produce an increase in total revenue.
- The available evidence suggests that for individuals facing the top marginal tax rate in New Zealand, the elasticity of taxable income is around one half, and is possibly higher for the very high income earners. For example, suppose there is an increase in the marginal tax rate from 33 to 36 per cent. This implies that the net-of-tax rate falls by 4.5 per cent. An elasticity of taxable income of one half means that there would be a reduction of at least 2.25 per cent in taxable income.
- The present paper reports an empirical analysis, based on NZ taxable income distribution data, of the elasticity of tax revenue with respect to a marginal rate change. For the pre-2010 tax structure, it was found that, for an elasticity of taxable income in excess of around 0.6, tax revenue from the top tax bracket could fall as a result of a further tax rate increase. As a result of the 2010 change to the income tax structure, the elasticity of taxable income in the top bracket would need to be in excess of around 0.8 before a tax rate increase would be expected to produce a negative revenue change.

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# Tax Rates and Revenue Changes: Behavioural and Structural Factors

## 1 Introduction

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The aim of this paper is to show how the change in income tax revenue - at individual and aggregate levels - as a consequence of a marginal rate change can be decomposed into several effects. The decomposition is useful for revenue forecasting purposes and in contexts where revenue changes form one component of a larger economic model. First, in elasticity terms there is the obvious *ceteris paribus* positive revenue effect of a rate rise which depends (at the individual level) only on the tax structure: this is the partial elasticity of revenue with respect to the relevant tax rate. Second there is a negative effect arising from behavioural responses. This latter effect can itself be divided into two multiplicative influences. The elasticity of taxable income - the response of taxable income to a change in the marginal net-of-tax rate (one minus the marginal rate) - captures the net effect of all incentive effects associated with the marginal rate change.<sup>1</sup> In addition, the tax revenue elasticity - the elasticity of tax revenue with respect to a change in income - determines the consequent effect on revenue of the behavioural response.

The recognition of two basic effects of a rate rise is not of course new. Reference has long been made to 'tax base' and 'tax rate' effects of rate changes and, for example, behavioural and 'mechanical' effects of an increase in the top marginal tax rate in a multi-rate structure were distinguished by Saez (2004).<sup>2</sup> However, when discussing revenue changes resulting from marginal rate changes, the literature on the elasticity of taxable income has not identified the specific role of the revenue elasticity. The latter is the central concept in the literature on 'fiscal drag', which is concerned with the extent to which the non-indexation of tax thresholds leads to increasing average tax rates over time.<sup>3</sup> In this context, the change in gross income is considered to be exogenous and any consequent feedback disincentive effect on income arising from the change in the average tax rate is ignored. Indeed, at the individual level the literature concentrates on changes which do not involve a

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<sup>1</sup> It avoids the considerable complexities of attempting to combine the varied behavioural adjustments into a structural model, as well as providing (under certain assumptions) a convenient method of measuring the marginal excess burden arising from tax changes. However, its use crucially depends on an assumed absence of income effects. The elasticity can be influenced by policy changes concerning, for example, regulations regarding income shifting and the timing of income receipts and tax payments. The seminal paper is Feldstein (1995), with important evidence for the US by Auten and Carroll (1995, 1999) and Auten et al. (2008). Gieritz (2007) and Saez, Slemrod and Gieritz (2009) provide comprehensive reviews of evidence, while Creedy (2010) provides an introduction to the underlying analytics.

<sup>2</sup> Changes in total tax obtained from the top marginal rate are also examined in Saez et al. (2009), in the course of deriving the aggregate excess burden. Following Saez, the two components were also discussed by, for example, Gieritz (2009).

<sup>3</sup> See the survey in Creedy and Gemmell (2002). The revenue elasticity is also used in discussions of local measures of tax progressivity.



movement across tax thresholds, which would otherwise lead to a change in the marginal tax rate.<sup>4</sup> Hence, in addition to the partial elasticity of revenue with respect to the relevant tax rate, one elasticity concerns the way tax revenue changes in response to exogenous income changes while the other elasticity measures the extent to which income declared for tax purposes adjusts when the income tax rate varies.<sup>5</sup>

The revenue elasticity is concerned only with the nature of the income tax structure itself and, when considering aggregation over individuals, the form of the income distribution. The elasticity of taxable income is concerned with a wide range of behavioural adjustments associated with tax rate changes, captured in a single measure. Hence there is no direct connection between the two elasticity concepts. It is shown below how the revenue elasticity has a clear role at the individual level in influencing the change in tax resulting from a rate change. In considering aggregate revenue over all individuals, changes are shown to depend on the revenue elasticity at the arithmetic mean income level within each tax bracket in a multi-rate income tax structure. The restriction of previous analyses to the top marginal rate also involves a substantial simplification. The present paper extends the treatment to deal with changes resulting from any tax rate, and thereby also deals with simultaneous changes in all tax rates.

Section 2 explores the precise relationships among the various elasticities, namely partial elasticity, the individual tax revenue elasticity, the individual elasticity of taxable income and their combination to determine the elasticity of tax revenue with respect to a change in the marginal tax rate. In view of the ubiquitous nature of the multi-step tax function, results are given for this case. Section 3 looks at aggregation over individuals when a single marginal rate changes in a multi-rate tax structure. A diagrammatic method of illustrating the various components is also devised. To illustrate the orders of magnitude involved section 3 applies the aggregate analysis to the New Zealand income tax system. This provides a convenient 'natural experiment' since the New Zealand government's May 2010 Budget involved changes to all marginal income tax rates whilst holding all thresholds constant. The analysis concerns only the effects on income tax revenue, and it is worth recognising that, to the extent that the elasticity of taxable income captures some shifting towards sources which attract lower marginal rates, it does not deal with the potential full consequences for tax revenue. Brief conclusions are in Section 4.

---

<sup>4</sup> In simulations generating aggregate elasticities from individual elasticities, care is also needed to avoid such movements because they can involve huge individual revenue elasticities for tiny changes in gross income. Labour supply incentive effects, in the context of the revenue elasticity with respect to wage rate changes, are examined by Creedy and Gemmell (2005).

<sup>5</sup> The tax rate may vary as a result of a deliberate policy change, or it may change as individuals move across income thresholds, particularly as a result of fiscal drag. As mentioned earlier, such transitions across thresholds are typically not considered in producing revenue elasticities.

## 2 Relationships Among Elasticities

This section demonstrates, at the individual level, how the revenue elasticity and the elasticity of taxable income combine to generate the elasticity of tax with respect to the marginal rate. For convenience, the distinction between gross income and taxable income is ignored, though this distinction is likely to be important for countries with extensive income tax deductions.<sup>6</sup> If there are endogenous, income-related deductions, the following analysis must be in terms of income after deductions have been made.

The literature on the tax revenue elasticity concentrates on the effects of changes in taxation resulting from exogenous changes in taxable income, with tax rates and thresholds held constant.<sup>7</sup> Furthermore, it is usual to assume that the exogenous change in income does not cause the individual to move into a higher tax bracket. Such a movement, where the tax function involves discrete changes in marginal rates, gives rise to a large jump in the elasticity, and this can - when carrying out appropriately tax-share weighted aggregation - distort the aggregate elasticity.

Suppose the multi-step tax function depends on a set of income threshold,  $a_1, \dots, a_K$  and a corresponding set of marginal tax rates  $\tau_1, \dots, \tau_K$ . Let the tax paid by an individual with income of  $y$  be denoted  $T(y) = T(y | \tau_1, \dots, \tau_K, a_1, \dots, a_K)$ . The individual revenue elasticity,  $\eta_{T,y}$ , is thus defined as:

$$\eta_{T,y} = \frac{dT}{dy} \frac{y}{T} \quad (1)$$

and is given by the ratio of the marginal tax rate to the average tax rate faced by the individual. The following uses the general notation,  $\eta_{b,a} = \frac{a}{b} \frac{db}{da}$ , to denote a 'total' elasticity, and  $\eta'_{b,a} = \frac{a}{b} \frac{\partial b}{\partial a}$ , to denote a partial elasticity. The revenue elasticity (where a tax threshold is not crossed) thus has the property that  $\eta_{T,y} = \eta'_{T,y}$ .

The above multi-step function can be written as:<sup>8</sup>

$$\begin{aligned} T(y) &= \tau_1 (y - a_1) & a_1 < y \leq a_2 \\ &= \tau_1 (a_2 - a_1) + \tau_2 (y - a_2) & a_2 < y \leq a_3 \end{aligned} \quad (2)$$

<sup>6</sup> For discussion of the empirical importance of income-related deductions in personal income tax regimes in OECD countries, see Caminada and Goudswaard (1996) and Wagstaff and van Doorslaer (2001). For the US, Feldstein (1999, p. 675) estimated that total income tax deductions in 1993 amounted to about 60% of estimated taxable income.

<sup>7</sup> The restriction to exogenous income changes is easily controlled in considering individual elasticity values but of course the nature of the overall distribution of income, which is needed to obtain aggregate values, may well be influenced by the incentive effects of the consequent tax changes.

<sup>8</sup> The revenue elasticity properties of this function are examined in more detail in Creedy and Gemmell (2006).

and so on. If  $y$  falls into the  $k$ th tax bracket, so that  $a_k < y < a_{k+1}$ ,  $T(y)$  can be expressed for  $k \geq 2$  as:

$$T(y) = \tau_k (y - a_k) + \sum_{j=1}^{k-1} \tau_j (a_{j+1} - a_j) \quad (3)$$

This can be rewritten as:

$$T(y) = \tau_k (y - a_k^*) \quad (4)$$

where:

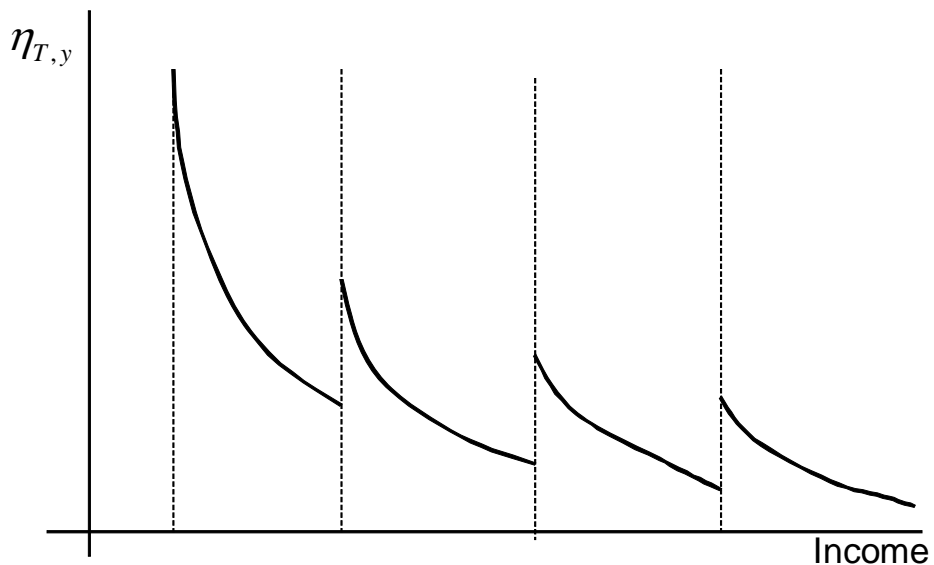
$$a_k^* = \frac{1}{\tau_k} \sum_{j=1}^k a_j (\tau_j - \tau_{j-1}) \quad (5)$$

and  $\tau_0 = 0$ . Thus the tax function facing any individual taxpayer in the  $k$ th bracket is equivalent to a tax function with a single marginal tax rate,  $\tau_k$ , applied to income measured in excess of a single threshold,  $a_k^*$ . Therefore, unlike  $a_k$ ,  $a_k^*$  differs across individuals depending on the marginal income tax bracket into which they fall. For this structure, and supposing that the income thresholds remain fixed, the revenue elasticity is:

$$\eta_{T,y} = \frac{y}{y - a_k^*} \quad (6)$$

and the individual partial elasticity must exceed unity. Within each threshold (for which the marginal rate is fixed) the elasticity declines as income increases. As an individual crosses an income threshold, the revenue elasticity takes a discrete upward jump, before gradually declining again, as shown by the saw-tooth pattern in Figure 1.

**Figure 1:** Variation in the Revenue Elasticity with Income in a Multi-step Tax Function



For the multi-step function the partial individual elasticity,  $\eta'_{T,\tau_j}$ , for  $j < k$  (that is, for changes in marginal tax rates below the tax bracket in which the individual falls) is given by:

$$\eta'_{T,\tau_j} = \frac{\tau_j (a_{j+1} - a_j)}{T(y)} \quad (7)$$

which is simply the tax paid at the rate,  $\tau_j$ , divided by total tax paid by the individual. Furthermore:

$$\eta'_{T,\tau_k} = \frac{\tau_k (y - a_k)}{T(y)} \quad (8)$$

Hence  $\sum_{j=1}^k \eta'_{T,\tau_j}$ .

Consider a change in the individual's tax liability resulting from an exogenous increase in one of the marginal tax rates (with other rates and the thresholds unchanged). This gives rise to a behavioural response, so that:

$$dT = \frac{\partial T}{\partial y} dy + \frac{\partial T}{\partial \tau} d\tau \quad (9)$$

From (9), dividing by  $d\tau$  and writing in elasticity form gives:

$$\eta_{T,\tau} = \eta'_{T,\tau} + \eta_{T,y} \eta_{y,\tau} \quad (10)$$

The first term may be said to reflect a pure 'tax rate' effect of a rate change, with unchanged incomes, while the second term reflects the combined 'tax base' effect, resulting from the incentive effects on taxable income, and the revenue consequences of that income change. When discussing the effect on total revenue of a change in the top income tax rate, Saez et al. (2009) refer to the tax rate effect as 'mechanical' and the second term as the 'behavioural' effect respectively. Thus, their 'behavioural effect' combines the revenue elasticity and elasticity of taxable income effects.<sup>9</sup> Tax paid by the individual increases, when the marginal rate increases, only if:

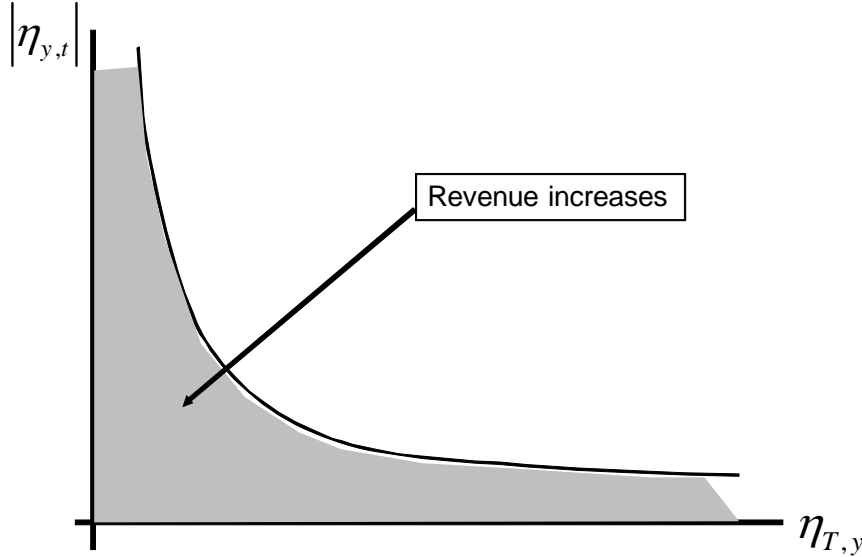
$$(\eta_{T,y}) |\eta_{y,\tau}| < \eta'_{T,\tau} \quad (11)$$

In any progressive income tax structure, the revenue elasticity,  $\eta_{T,y}$ , exceeds 1, unless (as shown in the following section) allowances vary sufficiently with income. Equation (11) shows that, for the individual's tax liability to increase when  $\tau$  increases, the combination of (absolute) elasticities must lie to the south west of a rectangular hyperbola, as shown in Figure 2. The nature of the tax structure determines the position of the hyperbola (via the elasticity,  $\eta'_{T,\tau}$  and the revenue elasticity (which in turn depends on the individual's

<sup>9</sup> Saez et al. (2009, p. 5) do not discuss the separate role of the revenue elasticity in this context. Discussion of the rate and base effects is often discussed in the context of a simple proportional tax structure, with constant average and marginal rate,  $t$ , where the revenue elasticity is everywhere unity.

income).<sup>10</sup> It is of course expected that  $|\eta_{y,\tau}|$  is non-zero only for a change in the marginal rate corresponding to the bracket in which the individual falls, otherwise a rate change affects only the average tax rate faced by the individual.

**Figure 2:** Revenue-Increasing Elasticity Combinations



The individual elasticity of taxable income,  $\eta_{y,1-\tau}$ , measures the behavioural response of taxable income to a change in a marginal net-of-tax rate,  $1 - \tau$ , facing the individual. This can be applied to any particular tax rate (not simply the rate corresponding to the tax bracket in which the individual's income falls), and a subscript is omitted here for convenience. The elasticities  $\eta_{y,1-\tau}$  and  $\eta_{y,\tau}$  are related using  $\eta_{y,1-\tau} = -\left(\frac{1-\tau}{\tau}\right)\eta_{y,\tau}$ .

Hence the elasticity of revenue with respect to the marginal rate faced by an individual in the  $k$ th tax bracket is given by:

$$\eta_{T,\tau_k} = \eta'_{T,\tau_k} - \left(\frac{y}{y - a_k^*}\right) \left(\frac{\tau_k}{1 - \tau_k}\right) \eta_{y,1-\tau_k} \quad (12)$$

The first term,  $\eta'_{T,\tau_k} = \tau_k (y - a_k) / T(y)$ , is the partial or mechanical 'tax rate effect' of the rate change, while the second term combines the behavioural reduction in the tax base (via the individual elasticity of taxable income) with structural effect (via the individual revenue elasticity) to give the consequent effect on revenue.

<sup>10</sup> Fullerton (1982, p. 9), concentrating on labour supply responses to tax increases, drew a downward sloping convex curve with the labour supply elasticity on the vertical axis and the tax rate on the horizontal axis. For tax revenue to increase when the tax rate increases, the supply elasticity must be sufficiently small; that is, the combination of tax rate and elasticity must lie to the south west of his curve. In simulations, Fullerton (1982, p. 13) actually held the revenue elasticity constant as the tax rate was varied (by increasing average and marginal rates by the same percentage).

### 3 Aggregate Revenue

For tax policy purposes attention is often devoted to aggregate revenue and its variation as component tax rates are changed. This section therefore examines aggregation over individuals. Emphasis is on the effect on total income tax revenue of a change in a single tax rate, and the effect of a simultaneous similar change in all rates. As above, attention is restricted to the case of the multi-rate tax function.<sup>11</sup> First, components of total revenue are examined in subsection 3.1. Aggregate elasticities are derived in subsection 3.2. The potential orders of magnitude involved are then examined in subsection 3.3.

#### 3.1 Components of Total Revenue

When dealing with population aggregates it is necessary to distinguish various tax and revenue terms, for both clarity and succinctness. In the previous section, the tax liability facing an individual with an income of  $y$  was denoted by  $T(y)$ . In the multi-tax form, if income is in the  $k$ th tax bracket a distinction can be made between  $T(y) = \tau_k (y - a_k)$  and the tax paid by the individual at the  $k$ th marginal rate, thereby ignoring tax paid on income falling into lower thresholds.

For aggregate revenue amounts defined over populations, or population sub-groups,  $R$  is used. Thus, in this section  $R$  represents aggregate revenue, while  $R_k$  refers to the aggregate revenue obtained from all individuals whose incomes fall in the  $k$ th tax bracket: that is,  $R_k$  is the aggregate over individuals in the  $k$ th bracket of  $T(y) = \tau_k (y - a_k)$  values. Let  $R_{(k)}$  denote the aggregate amount raised only at the rate  $k$  from individuals who fall into the  $k$ th bracket: that is,  $R_{(k)}$  is the sum over individuals in the  $k$ th bracket of  $\tau_k (y - a_k)$  values. Furthermore,  $R_{(k)}$  refers to the aggregate revenue obtained at the  $k$ th rate from individuals whose incomes fall into higher tax brackets: that is, the number of all individuals in higher tax brackets multiplied by  $\tau_k (a_{k+1} - a_k)$ .

In the multi-step tax function with  $K$  brackets, suppose  $P_k$  people are in the  $k$ th bracket, for, and the arithmetic mean income in each bracket is  $\bar{y}_k$ . Then aggregate revenue is:

$$\begin{aligned} R &= \sum_{k=1}^K R_k \\ &= \tau_1 (\bar{y}_1 - a_1) P_1 \\ &\quad + \{ \tau_2 (\bar{y}_2 - a_2) + \tau_1 (a_2 - a_1) \} P_2 \\ &\quad + \{ \tau_3 (\bar{y}_3 - a_3) + \tau_1 (a_2 - a_1) + \tau_2 (a_3 - a_2) \} P_3 \\ &\quad + \text{etc} \end{aligned} \tag{13}$$

<sup>11</sup> It is assumed that all individuals face the same income thresholds, so that endogenous allowances are not considered here.

Let  $P_k^+ \equiv \sum_{j=k+1}^K P_j$  denote the number of people above the  $k$ th tax bracket. For the top marginal rate, clearly  $P_K^+ = 0$ . Thus aggregate revenue can be written more succinctly as:

$$R = \sum_{k=1}^K \tau_k (\bar{y}_k - a_k) P_k + \sum_{k=1}^{K-1} \tau_k (a_{k+1} - a_k) P_k^+ \quad (14)$$

Thus, using  $R_K^+ = 0$ :

$$R = \sum_{k=1}^K R_{(k)} + \sum_{k=1}^{K-1} R_{(k)}^+ = \sum_{k=1}^K (R_{(k)} + R_{(k)}^+) \quad (15)$$

In this expression,  $R_{(k)} + R_{(k)}^+ \neq R_k$ , although their sums over  $k = 1, \dots, K$  are equal.

### 3.2 Changes in Aggregate Revenue

Consider the response of aggregate revenue to a change in the  $k$ th marginal tax rate. This has two basic components. First, there is the direct effect of the change in the  $k$ th tax rate on tax from that bracket alone. From previous sections, in addition to the partial elasticity this is made up of the behavioural effect of the tax rate change on the incomes of those in the  $k$ th bracket, along with the revenue elasticity effect (which is not a reflection of behaviour but depends on the tax structure). Second, there is an indirect effect on individuals in higher tax brackets, as a result of the term  $\tau_k (a_{k+1} - a_k)$ . Assume first that

there are no behavioural responses. It can be seen that, letting  $\eta'_{R, \tau_k} \equiv \frac{\tau_k}{R} \frac{\partial R}{\partial \tau_k}$ :

$$\eta'_{R, \tau_k} = \frac{\tau_k}{R} \{ (\bar{y}_k - a_k) P_k + (a_{k+1} - a_k) P_k^+ \} \quad (16)$$

For this ‘no behavioural response’ case, these elasticities sum to unity, so that the elasticity of total revenue with respect to an equal proportional change in all rates is unity. Any behavioural response clearly reduces the elasticity below 1, as shown below.

In the case where there are behavioural effects of marginal rate changes, it is convenient to assume that all those in a given bracket have the same elasticity,  $\eta_{y, \tau_k}$ . In this case, it can be shown that an appropriate adjustment to the average income level within the tax bracket gives:

$$\eta'_{R, \tau_k} = \frac{\tau_k}{R} [ \{ \bar{y}_k (1 + \eta_{y, \tau_k}) - a_k \} P_k + (a_{k+1} - a_k) P_k^+ ] \quad (17)$$

The expression in (17), while quite straightforward, does not bring out the separate elements influencing the elasticity in a transparent way. First, rewrite this as:

$$\eta'_{R,\tau_k} = \frac{\tau_k P_k}{R} \left[ (\bar{y}_k - a_k) - \bar{y}_k \left( \frac{\tau_k}{1 - \tau_k} \right) \eta_{y,1-\tau_k} + (a_{k+1} - a_k) \frac{P_k^+}{P_k} \right] \quad (18)$$

Then multiplying and dividing by  $(\bar{y}_k - a_k^*)$ :<sup>12</sup>

$$\eta'_{R,\tau_k} = \frac{R_k}{R} \left[ \frac{R_{(k)} + R_{(k)}^+}{R_k} - \left( \frac{\bar{y}_k}{\bar{y}_k - a_k^*} \right) \left( \frac{\tau_k}{1 - \tau_k} \right) \eta_{y,1-\tau_k} \right] \quad (19)$$

From equation (6),  $\bar{y}_k / (\bar{y}_k - a_k^*)$  is the revenue elasticity at arithmetic mean income in the  $k$ th bracket. This expression therefore shows how the elasticity, on the left hand side of (19), depends on the elasticity of taxable income of those in the  $k$ th tax bracket, along with the revenue elasticity at  $\bar{y}_k$ , and various tax-share terms. Furthermore, it can be shown that  $\eta'_{R,\tau_k}$  is positive if:

$$\eta_{y,1-\tau_k} < \left( \frac{1 - \tau_k}{\tau_k} \right) \left[ \bar{y}_k - a_k \left( 1 + \frac{P_k^+}{P_k} \right) + a_{k+1} \left( \frac{P_k^+}{P_k} \right) \right] \frac{1}{\bar{y}_k} \quad (20)$$

For the top bracket, the final term within square brackets in equation (20) is zero and the elasticity is positive if:

$$\eta_{y,1-\tau_K} < \left( \frac{1 - \tau_K}{\tau_K} \right) \left( \frac{\bar{y}_K - a_K}{\bar{y}_K} \right) \quad (21)$$

and although the first term in brackets exceeds 1 as long as the top tax rate is less than 0.5, the second term in brackets is likely to be well below 1. Hence the elasticity of taxable income must be relatively low for a tax rate increase to increase aggregate revenue. If all marginal tax rates change, but income thresholds remain fixed, the elasticity of aggregate revenue with respect to a simultaneous equal proportional change in all tax rates is the sum of the separate elasticities over all  $K$ .

### 3.3 Illustrative Examples

In order to provide an illustration of the nature of the relationships involved and the sensitivity to variations in the elasticity of taxable income, it is useful to consider the change to the income tax structure in New Zealand, made in the 2010 Budget. Table <ref>dist</ref> provides summary information regarding the distribution of annual personal taxable incomes in the 2008/09 tax year, the most recent year for which data are available.<sup>13</sup> For comparison purposes the tax rates and thresholds shown in the table relate to the structure in 2009/10. The overall arithmetic mean taxable income is \$35,507.

<sup>12</sup> It can be shown that, for the top marginal rate, (19) reduces to the expression given by Saez et al. (2009, p. 5), although the present form makes the role of the revenue elasticity transparent.

<sup>13</sup> The table is obtained from unpublished Inland Revenue Department data covering 3,304,210 individuals.



**Table 1:** The Distribution of Taxable Income in New Zealand: 2008/09 Tax Year

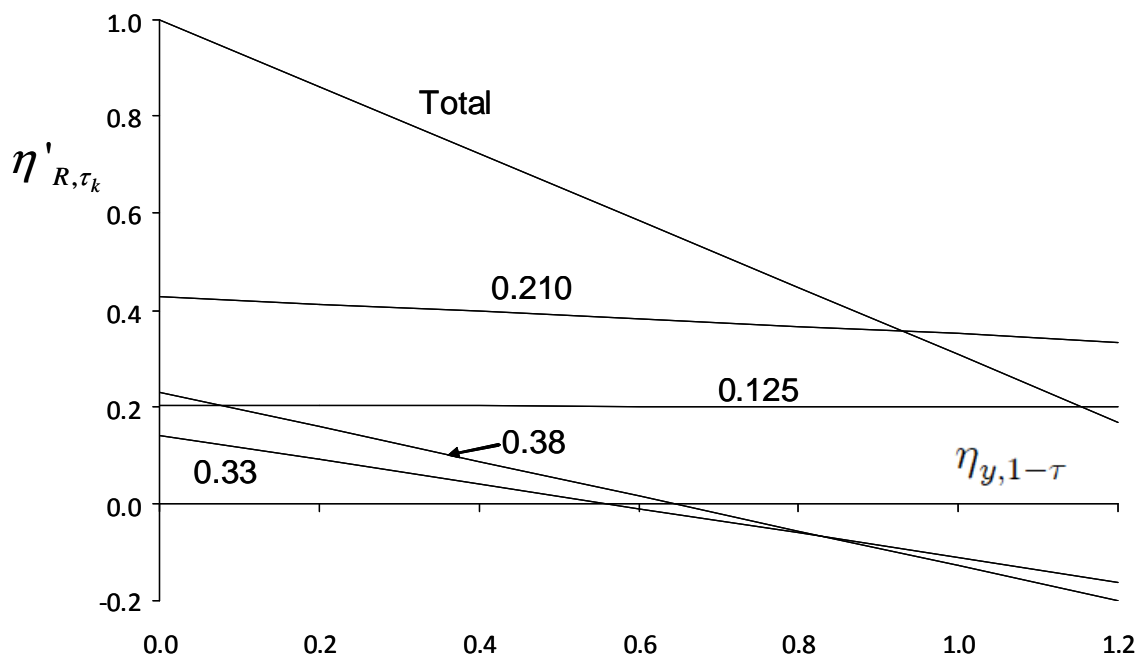
$k$	$a_k$	$\bar{y}_k$	Prop of people	Prop of income
1	1	6748.82	0.241	0.046
2	14000	24080.76	0.434	0.294
3	48000	52414.34	0.224	0.331
4	70000	115480.70	0.101	0.329

Table 2 provides summary information about the pre- and post-2010 Budget tax structures, for the taxable income distribution of Table 1. In the 2010 Budget, all the income thresholds were left unchanged, but the marginal tax rates were reduced, in particular the top marginal rate. Given the relatively low value of the income threshold above which the top rate applies, this tax bracket contributes a higher proportion of total income tax revenue than the other brackets, even though it contains only ten per cent of taxpayers. This compares with the second tax bracket which contains over forty per cent of all taxpayers. The final column of the table reports the revenue elasticity in each tax bracket, evaluated at arithmetic mean income within the bracket. For each tax structure, this elasticity is highest in the third tax bracket because the value of arithmetic mean income in the bracket is relatively closer to the effective income threshold than for the other brackets. For those in the first tax bracket, the average and marginal tax rates are equal and hence the revenue elasticity is unity. The Budget change in the marginal tax rates has little effect on the revenue elasticities.

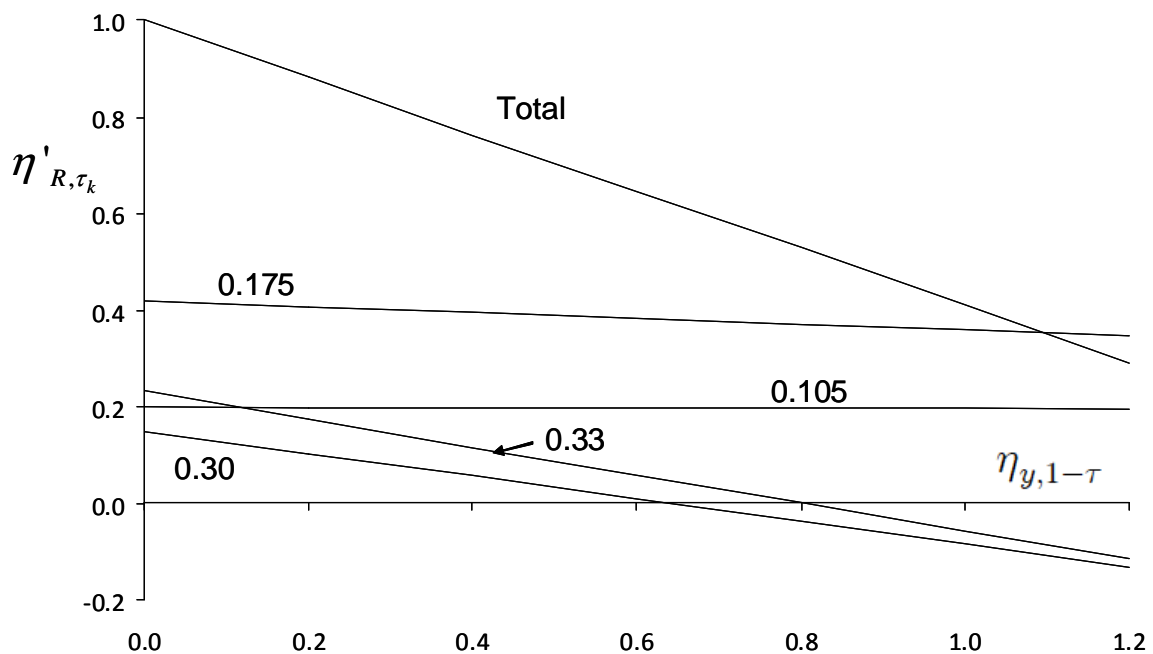
**Table 2:** The New Zealand Income Tax Structure Before and After the 2010

$k$	$\tau_k$	$a_k^*$	$R_k/P_k$	$R_k/R$	$\eta_{T(\bar{y}_k),y}$
Tax rates pre-2010 Budget					
1	0.125	1.00	843.48	0.027	1.000
2	0.210	5667.26	3866.83	0.222	1.308
3	0.330	21060.99	10346.61	0.306	1.672
4	0.380	27500.33	33432.53	0.446	1.313
Tax rates post-2010 Budget					
1	0.105	1.00	708.52	0.026	1.000
2	0.175	5600.60	3234.03	0.217	1.303
3	0.300	23267.02	8744.20	0.303	1.798
4	0.330	27515.47	29028.52	0.454	1.313

**Figure 3:** Elasticity of Total Tax Revenue wrt Tax Rates: Pre-2010 Budget



**Figure 4:** Elasticity of Total Tax Revenue wrt Tax Rates: Post-2010 Budget



Figures 3 and 4 show the variations in the elasticity,  $\eta'_{R,\tau_k}$ , for each tax bracket, as the elasticity of taxable income increases. As demonstrated above, the value of each  $\eta'_{R,\tau_k}$  falls linearly with the elasticity of taxable income, but the rate of decrease is less in the post-2010 Budget structure. In each case the elasticity,  $\eta'_{R,\tau_k}$ , for the lowest income tax bracket remains approximately constant. Although the elasticity  $\eta_{T(\bar{y}),y}$  is highest in the third tax bracket, the value of  $\eta'_{R,\tau_k}$  falls slightly faster in the top marginal rate bracket. This is because the value of  $\tau / (1 - \tau)$  is higher for the higher marginal tax rate, along with the fact that the top-rate bracket contributes a higher proportion of aggregate tax revenue.

The reduction in the higher marginal tax rates, resulting from the 2010 Budget, implies that the revenue elasticity,  $\eta'_{R, \tau_k}$ , continues to be positive, for higher values of the elasticity of taxable income.

In these diagrams, the 'total' line is drawn on the assumption that the elasticity of taxable income is the same for all tax brackets. However, this is unrealistic. Some evidence regarding the elasticity for New Zealand taxpayers is reported in Claus et al. (2010). They found that for those in the lower tax brackets, the estimated elasticities were very small, but for the top marginal tax rate the responses were substantial. For top-rate taxpayers, the values were mainly in the range 0.5 to 1.2. These findings have potentially important implications. For the higher marginal tax rates, the diagrams show that if the elasticity is above around 0.6, further increases in the rates could lead to reductions in total income tax revenue.<sup>14</sup> Aggregate revenue is clearly most responsive to changes in the second marginal rate even if, as seems unlikely, the elasticity of taxable income is relatively high.

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<sup>14</sup> As mentioned earlier, the present analysis ignores the revenue obtained when some income is shifted to other lower-taxed sources.

## 4 Conclusions

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This paper has examined the joint role of the elasticity of taxable income (which refers to the behavioural effect on taxable income of a marginal tax rate rise) and the revenue elasticity (which reflects the structural effect on revenue of a change in taxable income) in influencing the revenue effects of tax rate changes. Traditionally, the revenue elasticity has been the central concept in examining fiscal drag, and obtaining local measures of tax progressivity. But it has an additional role in the context of the revenue effects of tax changes when incomes respond to rate changes. This separate effect has not previously been discussed explicitly, even though ‘mechanical’ and behavioural effects have long been distinguished in the literature. The elasticity of tax revenue with respect to a rate change was examined at both the individual and aggregate level.

When a single marginal tax rate in a multi-rate income tax structure is changed, those in the relevant tax bracket adjust their incomes in accordance with the elasticity of taxable income, and this affects the tax paid via the revenue elasticity. There is also a revenue effect on those individuals who are in higher tax brackets, since marginal rate changes in lower tax brackets imply a change in their effective income threshold. But there are no incentive effects on higher-rate taxpayers because only their average tax rate changes. Only if there were no incentive effects would an equal proportional change in all marginal tax rates produce the same proportional increase in total revenue.

Illustrations were provided using the New Zealand income tax structures before and after the 2010 Budget. This reduced all rates while leaving income thresholds unchanged and, in particular, reduced the top marginal rate substantially. The elasticity of total tax revenue with respect to a single tax rate change was found to be particularly sensitive to the elasticity of taxable income in the top two tax brackets. In the pre-Budget structure, an elasticity of taxable income in excess of about 0.6 was found to produce a negative tax response to an increase in the top two marginal rates. When these rates are lower, as in the post-Budget structure, the elasticity of taxable income needs to be over 0.8 before tax revenue in the highest tax bracket is expected to fall in response to an increase in the marginal rate. However, recent estimates of the elasticity of taxable income in the top tax bracket in New Zealand are in the range (with some estimates in excess of 1) where tax revenue may fall. For New Zealand in particular, these results therefore suggest that further detailed empirical investigation of the elasticity of taxable income for taxpayers at different income levels may be warranted to assess whether cuts in some marginal tax rates are likely to be revenue-enhancing.

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