

Fox, Kevin J

Working Paper

Returns to Scale, Technical Progress and Total Factor Productivity Growth in New Zealand Industries

New Zealand Treasury Working Paper, No. 05/04

Provided in Cooperation with:

The Treasury, New Zealand Government

Suggested Citation: Fox, Kevin J (2005) : Returns to Scale, Technical Progress and Total Factor Productivity Growth in New Zealand Industries, New Zealand Treasury Working Paper, No. 05/04, New Zealand Government, The Treasury, Wellington

This Version is available at:

<https://hdl.handle.net/10419/205569>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



<https://creativecommons.org/licenses/by/4.0/>



Returns to Scale, Technical Progress and Total Factor Productivity Growth in New Zealand Industries

Kevin J Fox

NEW ZEALAND TREASURY
WORKING PAPER 05/04

JUNE 2005



MONTH/YEAR

June 2005

AUTHOR

Kevin J Fox
School of Economics, & CAER
University of New South Wales
Sydney 2052
Australia

Email k.fox@unsw.edu.au

Telephone +61 2 9385 3320

Fax +61 2 9313 6337

ACKNOWLEDGEMENTS

The author wishes to thank, without implicating, the staff of the New Zealand Treasury for their suggestions and feedback, particularly Nathan McLellan and Grant Scobie. The paper has also benefited from joint work with W. Erwin Diewert, and valuable comments from Ulrich Kohli.

NZ TREASURY

New Zealand Treasury
PO Box 3724
Wellington 6008
NEW ZEALAND

Email information@treasury.govt.nz

Telephone 64-4-472 2733

Website www.treasury.govt.nz

DISCLAIMER

This document was commissioned by the New Zealand Treasury. However, the views, opinions, findings and conclusions or recommendations expressed in it are strictly those of the author, do not necessarily represent and should not be reported as those of the New Zealand Treasury. The New Zealand Treasury takes no responsibility for any errors, omissions in, or for the correctness of, the information contained in this Paper.

ISSN

1176-9505

Abstract

This paper reviews and applies some recently proposed methods for separating total factor productivity (TFP) growth into contributions from technical progress and returns to scale, allowing for imperfectly competitive markets. The methods are applied to New Zealand data, using a recently available dataset on nine market-sector industries and the aggregate market sector, 1988-2002. The findings suggest that there has been little contribution from technical progress to TFP growth, but increasing returns to scale may have played a substantial role. However, the results are not statistically satisfactory for several industries, and are quite sensitive to the model used. This highlights the need for more work on both data and analysis if a better understanding is to be had of New Zealand's productivity performance.

JEL CLASSIFICATION D24

KEYWORDS Returns to scale; technical progress; monopolistic markups

Table of Contents

Abstract	i
Table of Contents	ii
1 Introduction	1
2 A Simple Linear Regression Approach	3
3 An Alternative Approach	9
4 Results	11
4.1 Total Factor Productivity Growth	11
4.2 A Linear Regression Model for Technical Progress and Returns to Scale	12
4.3 Reverse Regressions	13
4.4 Orthogonal Regression	14
4.5 Application of an Alternative Method	15
5 Conclusion	16
Appendix: A Symmetric Regression Model	17
References	19

Returns to Scale, Technical Progress and Total Factor Productivity Growth in New Zealand Industries

1 Introduction

The concept of productivity growth has been widely used to assess the economic performance of firms, industries and countries. Productivity growth is usually calculated as the growth of outputs relative to the growth of inputs. If outputs grow relatively more quickly then there is a kind of welfare improvement, as relatively more output can be had for relatively less input. However, simply observing productivity growth does not tell us how this is achieved or how it can be replicated.

The standard measure of productivity growth will capture returns to scale as a kind of productivity effect, as a simple ratio of output growth to input growth corresponds to an underlying production function which exhibits constant returns to scale. There is then no explicit theoretical role for returns to scale, although the effects are captured in the empirical measures of productivity growth. There is a large and growing body of theoretical, empirical and historical work, that suggests a large role for increasing returns to growth in explaining economic growth; see for example Morrison (1992), Abramovitz and David (1996), Romer (1996), Devereux, Head and Lapham (1996), and Jones (2004). Hence, this paper aims to empirically disentangle returns to scale and technological change components from estimates of productivity growth for New Zealand manufacturing industries, in order to get a better understanding of the drivers of productivity. Such an understanding can lead to better policy formulation, as discussed below.

Recently the New Zealand Treasury has developed, in collaboration with Statistics New Zealand, an industry-level productivity database of annual data for 1988 to 2002 (Black, Guy and McLellan, 2003). This has opened up the potential for research on the productivity performance of individual industries as opposed to the aggregate level, and in turn, a better understanding of sources of aggregate productivity. It also allows the possibility of developing and applying methods for the understanding of sources of productivity growth in each sector and level of aggregation.

This paper reviews the productivity results that arise from the database, and applies recently introduced methods for separating out the contributions of technical progress and returns to scale to total factor productivity (TFP) growth. These methods are derived

without making the usual assumption of perfectly competitive markets. This more realistic modelling framework allows for the existence of monopolistic markups at the industry level.

The methods presented and applied in this paper are recent developments in a growing area of interest in the economics literature. Arising initially from the persistent finding of procyclical productivity growth, much theoretical and applied work has been done on the possible role of increasing returns to scale in driving TFP growth (e.g. Hall, 1988, 1990; Morrison, 1992; Basu and Fernald, 1997; Burnside, 1996; Nakajima, Nakamura and Yoshika, 1998 and Diewert *et al.*, 2004). Evidence on returns to scale is important in a broader context as well, as it is important for assessing the relevance of theoretical macroeconomic and microeconomic models which assume increasing returns to scale; see for example Romer (1986, 1996), Weil (1989), Baxter and King (1992), Beaudry and Devereux (1995), Benhabib and Farmer (1994), Devereux *et al.* (1996), Hintermaier (2003), Guo (2004) and Jones (2004).

The methods presented here distinguish themselves from other contributions in that they allow for both imperfect competition and multiple outputs. Specifically, some of the theoretical results from Diewert and Fox (2004) and Diewert and Lawrence (2004) are considered and applied to the New Zealand data. The methods are of particular practical use when there are a large number of outputs and/or inputs, implying a typical lack of degrees of freedom using standard econometric methods and aggregate annual data.

One of the theoretical results used is worth giving special emphasis. It has been thought that the assumption of perfect competition underlies the use of index-number techniques. This has led to the qualification of empirical results in various applications. This is particularly true in the context of firms and industries where there may be strong evidence to suggest that a competitive market does not exist. However, Diewert and Fox (2004) show that standard index-number techniques can still be derived in this context. Hence, this result is of considerable relevance in considering the applicability of index-number techniques, and their interpretation, in many contexts.

While there are some data limitations, the empirical results from using alternative methods present a consistent picture; returns to scale are either increasing or constant, and there has been little role played by technical progress in these sectors.

These results imply that a number of policy directions, including the following: 1. access to larger markets through trade and domestic transportation links is desirable in order to be able to take advantage of returns to scale; 2. the empirical observation of procyclical productivity growth can, in some models, be explained by increasing returns to scale; following the logic of these models leads to the somewhat surprising conclusion that productivity can be increased by any type of government spending (Devereux, Head and Lapham, 1996; Guo, 2004); and 3. it is desirable to have an industry policy that not only encourages technological change, but also the exploitation of any returns to scale that exist in a particular industry (such as simple scale of production, or externalities between multiple firms).

The rest of the paper is organized as follows. Section 2 describes the method of Diewert and Fox (2004) that is used in the empirical application of Section 4. This method allows the estimation of returns to scale and technical progress to be reduced to a simple linear regression model where only two parameters are estimated. Section 3 describes the method of Diewert and Lawrence (2004), which is an alternative method for estimating the parameters of interest. This requires a more sophisticated econometric approach;

specifically nonlinear estimation of a system of equations. Empirical results from using the methods are presented in Section 4 using the New Zealand industry productivity database. Section 5 concludes.

2 A Simple Linear Regression Approach

This section introduces one of the results from Diewert and Fox (2004) which is used in the empirical application of Section 4. Consider the case of a single firm or production unit that produces N outputs and uses M inputs for periods $0, 1, \dots, T$.¹ Let $y \equiv [y_1, \dots, y_N]$ denote the vector of positive outputs that is produced by the positive vector of inputs, $x \equiv [x_1, \dots, x_M]$. Assume that in period t , the firm has a feasible set of inputs and outputs, S^t , and that it faces a positive vector of input prices, $w \equiv [w_1, \dots, w_M]$. Assuming that the firm takes these input prices as fixed and beyond its control, the firm's *period t joint cost function*, can be written in general terms as $C(w, y, t) \equiv \min_x \{w \cdot x : (y, x) \text{ belongs to } S^t\}$, where $w \cdot x \equiv \sum_{m=1}^M w_m x_m$ denotes the inner product between the vectors w and x .

We assume that the logarithm of the firm's period t cost function is the following *non-constant returns to scale translog joint cost function*:

$$\begin{aligned} (1) \ln C(w, y, t) &\equiv -rt + \alpha_0 + \sum_{m=1}^M \alpha_m \ln w_m + \sum_{n=1}^N \beta_n \ln y_n \\ &+ (1/2) \sum_{i=1}^N \sum_{j=1}^N \gamma_{ij} \ln y_i \ln y_j + (1/2) \sum_{k=1}^M \sum_{m=1}^M \delta_{km} \ln w_k \ln w_m + \\ &\sum_{m=1}^M \sum_{n=1}^N \phi_{mn} \ln w_m \ln y_n \end{aligned}$$

where the parameters on the right hand side of (1) satisfy the following restrictions:
 $\sum_{n=1}^N \beta_n = k > 0$; $\sum_{j=1}^N \gamma_{ij} = 0$ for $i = 1, \dots, N$; $\gamma_{ij} = \gamma_{ji}$ for all $1 \leq i, j \leq N$; $\sum_{m=1}^M \alpha_m = 1$;
 $\sum_{m=1}^M \delta_{km} = 0$ for $k = 1, \dots, M$; $\delta_{km} = \delta_{mk}$ for all $1 \leq k, m \leq M$; $\sum_{m=1}^M \phi_{mn} = 0$ for $n = 1, \dots, N$;
and $\sum_{n=1}^N \phi_{mn} = 0$ for $m = 1, \dots, M$.

The parameter r which occurs in (1) is a measure of technical progress, which in this case is expressed as exogenous cost reduction.² Usually, $r \geq 0$; if $r < 0$, then the technology exhibits *technological regress*.

A measure of the *local returns to scale* of a multiple output, multiple input firm can be defined as the percentage change in cost due to a one percent increase in all outputs. The technical definition is:³

¹ As is standard, we work in the microeconomic framework of the optimizing unit (e.g. firm), yet the empirical application is to a higher level of aggregation. To the extent that the higher level is a valid aggregate of optimizing units, this is not a problem. Ideally, evidence using firm level data is preferable, but this also provides its own challenges in terms of both data and analysis.

² If we add the quadratic time trend term $-(1/2)st^2$ to the right hand side of (3) where s is a parameter, then our results which follow are still valid with obvious modifications. In particular, $-r$ on the right hand side of (35) should be replaced by $-r - st$. This modification will allow the technical progress term to trend over time. With this more general specification, there will be technological regress in period t if $r + st < 0$.

$$(2) \rho(w,y) \equiv [C(w,y,t)]^{-1} dC(w,\lambda y,t)/d\lambda|_{\lambda=1} = \sum_{n=1}^N \partial \ln C(w,y,t) / \partial \ln y_n = \sum_{n=1}^N \beta_n = k.$$

Thus our measure of (inverse) returns to scale is equal to the sum of the cost elasticities with respect to the N outputs, and the translog cost function defined by (1) has returns to scale equal to the positive parameter k everywhere. In the case of constant returns to scale, the parameter k is equal to 1.⁴

If there are increasing returns to scale or decreasing costs so that the parameter k is less than one, then it is well known that competitive profit maximization breaks down in this case. Hence, since we do not want to restrict k to be equal to or greater than one, it is necessary to allow for a monopolistic profit maximization problem in each period. Thus for period t , we assume that the firm faces the inverse demand function $P_n^t(y_n)$ which gives the market-clearing price for output n as a function of the amount of output y_n that the firm places on the market, for $n = 1, \dots, N$. Assuming that the firm faces the positive input price vector $w^t \equiv [w_1^t, \dots, w_N^t]$, the *firm's period t monopolistic profit maximization problem* is the following unconstrained maximization problem involving the vector of period t output supplies $y \equiv [y_1, \dots, y_N]$:⁵

$$(3) \max_y \left\{ \sum_{n=1}^N P_n^t(y_n) y_n - C(w^t, y, t) \right\}.$$

Assuming that the inverse demand functions and the joint cost function are once differentiable, the observed period t output vector for the firm, y^t , should satisfy the following first-order necessary conditions for (3):

$$(4) y_n^t dP_n^t(y_n^t)/dy_n + P_n^t(y_n^t) = \partial C(w^t, y^t, t) / \partial y_n; \quad n = 1, \dots, N; \quad t = 0, 1, \dots, T.$$

The observed period t price for output n will be $p_n^t \equiv P_n^t(y_n^t)$. Assuming that the demand derivatives $dP_n^t(y_n^t)/dy_n$ are nonpositive, the nonnegative *ad valorem monopolistic markup* m_n^t for the n th output in period t can be defined as follows:⁶

$$(5) m_n^t \equiv - [dP_n^t(y_n^t)/dy_n] y_n^t / p_n^t \geq 0; \quad n = 1, \dots, N; \quad t = 0, 1, \dots, T.$$

The first-order conditions (4) can then be rewritten as follows:

$$(6) p_n^t [1 - m_n^t] = p_n^t M_n^t = \partial C(w^t, y^t, t) / \partial y_n; \quad n = 1, \dots, N; \quad t = 0, 1, \dots, T,$$

where we have defined the *markup factor* for output n in period t , M_n^t , as:⁷

³ This is the reciprocal of the usual returns to scale measure. Hence there are local decreasing costs (and increasing returns to scale) if $\rho(w,y) < 1$ and constant costs (and constant returns to scale) if $\rho(w,y) = 1$.

⁴ In this case, the period t production possibilities set S^t is a cone; i.e., if $(x,y) \in S^t$, then $(\lambda x, \lambda y) \in S^t$ for all $\lambda > 0$. In this constant returns to scale case, the translog cost function defined by (3) with $k = 1$ can provide a second order approximation to an arbitrary joint cost function that is consistent with a constant returns to scale technology. Thus our generalized functional form, where k is no longer restricted to equal 1, is the simplest possible extension of this constant-returns-to-scale type cost function to the case where returns to scale are equal to the arbitrary positive number k .

⁵ In an early related contribution, Appelbaum (1979) specified a Generalized Leontief cost function to represent a monopolist's technology, for the U.S. crude petroleum and natural gas industry, 1947-1971. He found that "the degree of monopoly is statistically significant" (p. 292), and hence "the price taking behavior hypothesis is not appropriate for this industry."

⁶ Later we will assume that the markups m_n^t are constant. This is consistent with the inverse demand functions having the following constant elasticity form: $\ln P_n^t(y_n) \equiv a_n^t - c_n^t \ln y_n$ where a_n^t and c_n^t are positive constants. Basu and Fernald (2002; 976) note that constant markups can be justified from a variety of models.

$$(7) 0 < M_n^t \equiv 1 - m_n^t \leq 1 ;$$

$$n = 1, \dots, N ; t = 0, 1, \dots, T.$$

Assuming differentiability of the period t cost function with respect to the input prices, using Shephard's (1953; 11) Lemma, the cost minimizing vector of input demands for the firm in period t , $x^t \equiv [x_1^t, \dots, x_M^t]$, will be equal to the vector of first order partial derivatives of the cost function with respect to the components of the input price vector:

$$(8) x^t \equiv \nabla_w C(w^t, y^t, t) ; \quad t = 0, 1, \dots, T$$

and the period t observed total cost, $C(w^t, y^t, t)$, will be equal to the inner product of the period t input price and quantity vectors, w^t and x^t respectively:

$$(9) C(w^t, y^t, t) = w^t \cdot x^t ; \quad t = 0, 1, \dots, T.$$

We can rewrite (6) as follows:

$$(10) p_n^t M_n^t = [C(w^t, y^t, t)/y_n^t] \partial \ln C(w^t, y^t, t) / \partial \ln y_n ; \quad n = 1, \dots, N ; t = 0, 1, \dots, T$$

$$= [w^t \cdot x^t / y_n^t] [\beta_n + \sum_{j=1}^N \gamma_{nj} \ln y_j + \sum_{m=1}^M \phi_{mn} \ln w_m]$$

Rearranging (10) leads to the following equations, for $n = 1, \dots, N$ and $t = 0, 1, \dots, T$:

$$(11) [p_n^t y_n^t M_n^t] / w^t \cdot x^t = \beta_n + \sum_{j=1}^N \gamma_{nj} \ln y_j + \sum_{m=1}^M \phi_{mn} \ln w_m .$$

For t fixed, we can sum equations (11) in order to obtain the following equations:

$$(12) \sum_{n=1}^N [p_n^t y_n^t M_n^t] / w^t \cdot x^t = \sum_{n=1}^N [\beta_n + \sum_{j=1}^N \gamma_{nj} \ln y_j + \sum_{m=1}^M \phi_{mn} \ln w_m] ;$$

$$t = 0, 1, \dots, T$$

$$= k \text{ using (2).}$$

Equations (12) can be rearranged to yield the following expressions for period t costs:

$$(13) w^t \cdot x^t = k^{-1} \sum_{n=1}^N p_n^t y_n^t M_n^t ; \quad t = 0, 1, \dots, T.$$

Thus for each period t , an estimate of the firm's (reciprocal) returns to scale k can be obtained as the ratio of period t markup-adjusted revenues, $\sum_{n=1}^N p_n^t y_n^t M_n^t$, divided by period t total cost, $w^t \cdot x^t = \sum_{m=1}^M w_m^t x_m^t$. If there is only one output so that $N=1$, then (13) can be rewritten as $k^{-1} = [M_1^t]^{-1} [w^t \cdot x^t / p_1^t y_1^t]$, which is a standard result in the one output production function literature on this topic: see Basu and Fernald (1997; 253) (2002; 976). The term $w^t \cdot x^t / p_1^t y_1^t$ is observed cost over observed revenue, which in turn is one minus the revenue share of pure profits.

⁷ If there are constant or increasing costs so that the parameter $k \geq 1$, then this situation is consistent with the competitive pricing of outputs. To model this case in what follows, simply set each $M_n^t = 1$ and estimate the parameters k and r . In the production function literature on returns to scale and markups where there is only a single output, the markup factor is defined as price over marginal cost, which is the reciprocal of the markup factor M_n^t ; see Hall (1988) (1990) and Basu and Fernald (1997; 253) (2002; 975) for these single output production function approaches.

Rearranging (10) leads to the following system of equations:

$$(14) \partial \ln C(w^t, y^t, t) / \partial \ln y_n = p_n^t y_n^t M_n^t / w^t \cdot x^t; \quad n = 1, \dots, N; t = 0, 1, \dots, T$$

$$= k p_n^t y_n^t M_n^t / \sum_{j=1}^N p_j^t y_j^t M_j^t \quad \text{using (13).}$$

It turns out that we cannot obtain simple estimating equations in a perfectly general situation where the markup factors M_n^t are allowed to be arbitrary for each output n and for each time period t . However, progress can be made if we assume that within each period all of the markup rates or markup factors are equal to each other, or if we assume that the markup factors are constant across periods for each output. We make the first assumption, as it leads to a model which facilitates estimation of the parameters of interest.⁸

Therefore we have $M_n^t = M^t$ for all $n = 1, \dots, N$ and $t = 0, 1, \dots, T$. Substituting M^t into (14) leads to the following equations for the logarithmic derivatives of the period t cost function with respect to outputs:

$$(15) \partial \ln C(w^t, y^t, t) / \partial \ln y_n = k p_n^t y_n^t / \sum_{j=1}^N p_j^t y_j^t = k s_n^t$$

$$n = 1, \dots, N; t = 0, 1, \dots, T$$

where $s_n^t \equiv p_n^t y_n^t / p^t \cdot y^t$ is the *observed revenue share* of output n in period t . Using (8) and (9), it can be seen that the logarithmic derivatives of the period t cost function with respect to input prices are equal to:

$$(16) \partial \ln C(w^t, y^t, t) / \partial \ln w_m = w_m^t x_m^t / w^t \cdot x^t = S_m^t \quad m = 1, \dots, M; t = 0, 1, \dots, T$$

where $S_m^t \equiv w_m^t x_m^t / w^t \cdot x^t$ is the *observed cost share* of input m in period t .

Since the right hand side of (1) is a quadratic function in the logarithms of output quantities and the logarithms of input prices, we can apply Diewert's (1976; 118) Quadratic Identity and obtain the following equations, relating the difference in the costs in periods $t-1$ and t , $w^{t-1} \cdot x^{t-1} = C(w^{t-1}, y^{t-1}, t-1)$ and $w^t \cdot x^t = C(w^t, y^t, t)$:

$$(17) \ln C(w^t, y^t, t) - \ln C(w^{t-1}, y^{t-1}, t-1) \quad t = 1, 2, \dots, T$$

$$= (1/2) \{ [\partial \ln C(w^{t-1}, y^{t-1}, t-1) / \partial t] + [\partial \ln C(w^t, y^t, t) / \partial t] \} [(t) - (t-1)]$$

$$+ (1/2) \sum_{n=1}^N \{ [\partial \ln C(w^{t-1}, y^{t-1}, t-1) / \partial \ln y_n] + [\partial \ln C(w^t, y^t, t) / \partial \ln y_n] \} [\ln y_n^t - \ln y_n^{t-1}]$$

$$+ (1/2) \sum_{m=1}^M \{ [\partial \ln C(w^{t-1}, y^{t-1}, t-1) / \partial \ln w_m] + [\partial \ln C(w^t, y^t, t) / \partial \ln w_m] \} [\ln w_m^t - \ln w_m^{t-1}]$$

$$= -r + k \ln Q_T(p^{t-1}, p^t, y^{t-1}, y^t) + \ln P_T(w^{t-1}, w^t, x^{t-1}, x^t)$$

⁸ See Diewert and Fox (2004) for details of method of estimation when it assumed that markups are constant across periods for each output.

where $Q_T(p^{t-1}, p^t, y^{t-1}, y^t)$ is the *Törnqvist* (1936) (1937) *quantity index* for output growth between periods $t-1$ and t and $P_T(w^{t-1}, w^t, x^{t-1}, x^t)$ is the *Törnqvist input price index* for input price growth between periods $t-1$ and t . The logarithms of these two indexes are defined as follows:

$$(18) \ln Q_T(p^{t-1}, p^t, y^{t-1}, y^t) \equiv (1/2) \sum_{n=1}^N [s_n^{t-1} + s_n^t] [\ln y_n^t - \ln y_n^{t-1}] ;$$

$$(19) \ln P_T(w^{t-1}, w^t, x^{t-1}, x^t) \equiv (1/2) \sum_{m=1}^M [S_m^{t-1} + S_m^t] [\ln w_m^t - \ln w_m^{t-1}] .$$

The Törnqvist input price index between periods $t-1$ and t , $P_T(w^{t-1}, w^t, x^{t-1}, x^t)$, can be used in order to define the *implicit Törnqvist input quantity index* between periods $t-1$ and t as follows:

$$(20) Q_T^*(w^{t-1}, w^t, x^{t-1}, x^t) \equiv (w^t \cdot x^t / w^{t-1} \cdot x^{t-1}) / P_T(w^{t-1}, w^t, x^{t-1}, x^t) .$$

Note that the Törnqvist output quantity index and implicit input quantity indexes, $Q_T(p^{t-1}, p^t, y^{t-1}, y^t)$ and $Q_T^*(w^{t-1}, w^t, x^{t-1}, x^t)$ respectively, can be calculated using observable data on output and input prices and quantities for periods $t-1$ and t . Using definitions (9) and (19), we can rewrite (20) as follows:

$$(21) \ln Q_T^*(w^{t-1}, w^t, x^{t-1}, x^t) = -r + k \ln Q_T(p^{t-1}, p^t, y^{t-1}, y^t) ; \quad t = 1, 2, \dots, T.$$

If $T \geq 2$, then the technical change parameter r and the returns to scale parameter k can be estimated by running a linear regression using equations (21) after appending error terms. If there is positive technical progress, then $r > 0$ while if there are increasing returns to scale, then $k < 1$. Hence, a combination of technical progress and increasing returns to scale will cause input growth to be less than output growth. Equation (21) enables us to assess the contribution of each factor in a very simple regression model that has eliminated all of the nuisance parameters that are in the translog cost function that was defined earlier by (1). This is a rather remarkable result which is valid even if M and N are extremely large so that traditional econometric methods for estimating r and k fail.

Equation (21) can be compared with two more familiar methods. A simple rearrangement yields a production-function-type model, with the log of an output aggregate on the left-hand side and the log of an input aggregate on the right-hand side. The difference here is that (21) allows for the existence of multiple outputs, and the inputs are aggregated using a Törnqvist index, rather than the usual Cobb-Douglas index. It is also easy to relate (21) to a simple index-number approach to estimating total-factor-productivity growth. Assuming constant returns to scale ($k=1$), taking exponents and re-arranging, (21) can be expressed as an output index divided by an input index, or total-factor-productivity growth. This result has been derived without assuming perfect competition. Thus, the use of index-number techniques is consistent with the existence of monopolistic behaviour. While we have assumed that monopolistic markups are the same for each output within each period, this is not a restriction if there is only one (aggregate) output.

In addition, as the output and input indexes in (21) are in the form of one-plus-the-growth-rate between periods $t-1$ and t , the logarithm of each approximates the growth rate of output and input, respectively. Hence, (21) can be interpreted as (approximately) an equation relating the growth rate of input to the growth rate of output.

Once an estimate for the returns to scale parameter k has been obtained from the estimation of (21), an estimate for the period t markup factor M^t can be obtained using equations (13):

$$(22) \quad M^t = w^t \cdot x^t k / p^t \cdot y^t ; \quad t = 0, 1, \dots, T.$$

3 An Alternative Approach

As we will see from the empirical application of the above method to the New Zealand data, the results are not entirely satisfactory (see Section 4). Hence, this section discusses the approach of Diewert and Lawrence (2004), which will also be used to see if it can lead to more robust results.

The set up is similar to that of Section 3. However, in order to minimize the number of parameters to be estimated, we assume here that we have only one output for each sector and two aggregate inputs (labour and capital). This is the actual context examined with the New Zealand data in Section 4. We denote the output by y and the inputs by x_1 and x_2 , and assume that there is an aggregate production function for each industry of the form $y=f(x_1, x_2, t)$.

As before, it is assumed that the industry faces an aggregate inverse demand function for its output in period t of the form $p^t = P(y^t, t)$, $\partial P(y^t, t)/\partial y \leq 0$. Consider the following *period t monopolistic profit maximisation problem*:

$$(23) \max_x P[f(x, t)]f(x, t) - w_1^t x_1 - w_2^t x_2$$

where $x \equiv [x_1, x_2]$. The first order necessary conditions for the period t input vector $x^t \equiv [x_1^t, x_2^t]$ to solve (23) are:

$$(24) p^t \nabla_x f(x^t, t) + \{\partial P(y^t, t)/\partial y\} y^t \nabla_x f(x^t, t) = w^t; \quad t = 0, 1, \dots, T$$

where $w^t \equiv [w_1^t, w_2^t]$ is the period t input price vector and $\nabla_x f(x^t, t) \equiv [\partial f(x^t, t)/\partial x_1, \partial f(x^t, t)/\partial x_2]$ is the vector of first order partial derivatives of the period t production function with respect to the components of the input vector.

The period t *ad valorem* monopolist markup factor, m^t , is defined as in (5), and $M^t \equiv 1 - m^t$, as in (7). Equations (24) can then be expressed as follows:

$$(25) w^t = p^t M^t \nabla_x f(x^t, t); \quad t = 0, 1, \dots, T.$$

For simplicity, assume that the elasticity of demand is constant over time, so that $M^t = M$.⁹ There is a possibility that producers may systematically over- or under-value either of the inputs. Hence define, where $\phi_i > 0$, $i = 1, 2$,

$$(26) M_1 \equiv M\phi_1,$$

$$M_2 \equiv M\phi_2,$$

substitute equations (26) into (25), and divide by both sides by p^t to get the estimating equations:

$$(27) w_1^t/p^t = M_1 \partial f(x_1^t, x_2^t, t)/\partial x_1; \quad t = 0, 1, \dots, T;$$

$$w_2^t/p^t = M_2 \partial f(x_1^t, x_2^t, t)/\partial x_2.$$

⁹ This assumption can of course be relaxed. This is considered in Section 4.

If M_1 or M_2 are greater than one, then this implies that the associated input is overvalued relative to its cost.

Follow Diewert and Lawrence (2004) a normalised quadratic (“flexible”) functional form is specified for the production function (McFadden, 1978; Diewert and Wales, 1987):

$$(28) f(x_1, x_2, t) \equiv a + b_1x_1 + b_2x_2 + c_1x_1t + c_2x_2t + dt - (1/2) e^2 [\alpha x_1 - x_2]^2/x_1$$

where a, b_1, b_2, c_1, c_2, d and e are parameters to be estimated. Exogenous technical change is represented by the parameter d , which is positive if technical progress is output augmenting. Following Diewert and Lawrence (2004, p. 5, footnote 6), the parameter α is chosen so that $\alpha x_1 - x_2$ is equal to zero in the first period.¹⁰ A positive value of the parameter c_n implies input n augmenting technical change. The bigger in magnitude is e , the less substitutable are the two inputs. If $e=0$ then the inputs are perfect substitutes. The squaring of e in (28) ensures that f will be concave in x_1 and x_2 . Therefore, as long as the elasticity of demand is constant in each period, the satisfaction of the first order conditions (24) will imply that there is a global maximum for the profit maximisation problems.

Now, partially differentiating the $f(x_1, x_2, t)$ defined by (28) with respect to x_1 and x_2 and substituting these derivatives into equations (27) yields the following estimating equations:

$$(29) w_1^t/p^t = M_1[b_1 + c_1t + (1/2)e^2(v^t)^2 - e^2\alpha v^t]; \quad t = 0, 1, \dots, T;$$

$$w_2^t/p^t = M_2[b_2 + c_2t + e^2v^t]$$

where v^t is an exogenously determined variable defined as $v^t \equiv [\alpha x_1^t - x_2^t]/x_1^t$. The production function equation itself is added to equations (27) as a third estimating equation as this is necessary to identify the parameters.

In order to minimize potential problems with heteroskedasticity, both sides of the production function are divided through by x_1 for estimation. This leads to the following estimating equation, for $t = 0, 1, \dots, T$:

$$(30) y^t/x_1^t = a(1/x_1^t) + b_1 + b_2(x_2^t/x_1^t) + c_1t + c_2(x_2^t/x_1^t)t + d(1/x_1^t)t - (1/2)e^2[v^t]^2$$

Equations (29) and (30) can then be estimated using nonlinear system of equations estimation. Returns to scale, ρ^t can then be calculated from the fitted production function using the following definition:

$$(31) \rho^t \equiv \partial \ln f(\lambda x_1^t, \lambda x_2^t, t) / \partial \lambda |_{\lambda=1} = [f_1(x_1^t, x_2^t, t)x_1^t + f_2(x_1^t, x_2^t, t)x_2^t] / f(x_1^t, x_2^t, t); \\ t = 0, 1, \dots, T.$$

Technical progress τ^t in each period t is defined as an elasticity type measure, as follows:

$$(32) \tau^t \equiv \partial \ln f(x_1^t, x_2^t, t) / \partial t = [\partial f(x_1^t, x_2^t, t) / \partial t] / f(x_1^t, x_2^t, t); \quad t = 0, 1, \dots, T.$$

¹⁰ In the empirical work that follows, the quantities x_1 and x_2 are indexes set equal to one in the first period, implying that α is equal to one.

4 Results

The methods described in the previous sections are applied to the industry productivity data set provided by the New Zealand Treasury; see Black, Guy and McLellan (2003) (BGM) for details. The data set consists of annual observations, 1988-2002, on capital, labour and output aggregates for nine market-sector industries, plus an aggregate “market sector” and an “Australian Bureau of Statistics Equivalent” (ABSE) market sector. The latter excludes the relatively difficult-to-measure sectors of Business and Property Services (“Business Services”) and Personal and Community Services (“Personal Services”); see Table 1 for a full list of the industries. On the output side, industry values and quantities were taken from the industry income and production GDP accounts. The hours worked series were taken from the Household Labour Force Survey (HLFS), and compensation of employees was taken from the income GDP accounts. An adjustment was made to account of the fact that sole proprietor’s labour income is taken as operating surplus in the System of National Accounts. Industry capital stock was taken as the Statistics New Zealand productive capital stock series. Return to capital was taken as the (adjusted) operating surplus.

This approach assumes that the income side of the national accounts always equals expenditure side, so that by definition the monopolistic markups equal the returns to scale coefficient; see e.g. equation (22). This is standard statistical agency practice, and a common assumption in the productivity literature. As the operating surplus is taken as the residual, ensuring that the national-accounting identity will hold, the construction of alternative capital stock price series using eg, a user-cost approach, would allow the investigation of the existence of monopolistic markups.

Fisher indexes were used in aggregation, except for the ABSE aggregates, where the Törnqvist index was used in order to be consistent with ABS practice. However, this difference turns out to be of little consequence. Also, while the expression in (21) has an implicit Törnqvist index, rather than a direct index, in practice the indexes will often be almost identical. Hence, the direct indexes of BGM are used throughout.

4.1 Total Factor Productivity Growth

Table 1 reports the average industry TFP growth numbers for each of the industries and industry aggregates. These are constructed as output indexes divided by input indexes, so that they measure the output growth that is not explained by input growth.¹¹ The series for the nine industries are plotted in Figure 1.¹² While reporting exactly the same results, compared with the accumulated growth figures of BGM these are perhaps revealing of some facts which were not previously apparent. Figure 1 shows that there was considerable variation in growth from year to year, and that one industry (Mining and Quarrying) had a large positive spike in 1997, representing productivity growth of an impressive 30.5%. As TFP growth captures all changes in output not explained by changes in inputs, which makes no allowance for statistical error, such findings are not

¹¹ For more on this approach to measuring productivity, see e.g. Schreyer (2001), Mawson, Carlaw and McLellan (2003), and McLellan (2004).

¹² The numbers are index numbers, so subtracting one and multiplying by 100 puts them into period-on-period percentage growth terms. The econometrics program SHAZAM was used for all calculations and estimations (White, 1978).

entirely unusual. However, it indicates that there may potentially be data measurement error worth investigating further.

BGM note in their conclusion that “there appears to have been a noticeable improvement in market sector multifactor productivity after 1993” and that “average multifactor productivity growth increased from 0.09% per annum in the period 1988 to 1993 to 1.32% per annum in the period 1993 to 2002.”¹³ However, an investigation of Figure 1 does not seem to suggest a noticeable increase in TFP growth. Figure 2 plots the two aggregates for the Market Sector, and these also do not seem to indicate any long-run change in trend after 1993. The averages reported in Table 1 are split into the periods 1989 to 1994 and 1995 to 2002, whereas BGM took their averages with 1994 included in the latter half of the sample. From Figure 2 we can see that 1994 had substantial positive spike in the plot of TFP growth for the market sector aggregates, so which half of the sample this year is included in will influence the averages and hence the conclusion about relative sub-sample performance. Indeed, from Table 1, we see that for the market sector TFP growth was 0.8% for 1989-1994 and 1.0% for 1995-2000. Hence, only a 0.2% average productivity difference between the two periods. If the astoundingly large increase in TFP growth for Mining and Quarrying in 1997 had not been observed then even this positive advantage of the later period would not have been observed.

In summary, there does not seem to be evidence of a substantial productivity growth improvement from the mid-1990s. Most industries had a poor productivity performance over the whole sample period. The industry which stands out as the star performer is the Transport and Communications Sector (“Transport”). However, this sector experienced a slight productivity slowdown from 6% in the earlier period to 5.2% in the later period.

We turn now to an investigation of possible alternative sources of observed TFP growth.

4.2 A Linear Regression Model for Technical Progress and Returns to Scale

Tables 2 and 3 report the returns to scale and technical progress estimates, respectively, from estimating equation (21) of Section 2. From Table 2 we see from the R^2 values indicate that not much of the variation in the dependent variable (log of the input index) is being explained. The highest R^2 value is 0.672 for the aggregate Market Sector, meaning that approximately 67% of the variation in the dependent variable is explained by the model. The estimates of k are all less than one in magnitude, implying increasing returns to scale. Only for Manufacturing can the null hypothesis of constant returns to scale ($k=1$) be rejected at up to the 10% level of significance. Estimates of r , the technical progress parameter, are reported in Table 2. The estimates are either statistically insignificantly different from zero at the 5% level, or negative. These results are not very satisfactory, as one would not expect technical regress over such a long period, nor would one expect to find such large estimates of returns to scale. The poor fit of the model for almost every sector suggests that not much can be read into these results.

Tables 4 and 5 report results from specifying a quadratic time trend, rather than the simple linear time trend implied by equation (21). This should allow for a more flexible modeling of the underlying relationship with the addition of a single parameter. The R^2 values again

¹³ We use the expression total factor productivity, whereas BGM used the expression multifactor productivity to express the same concept. Also, note that the observation for the first period, 1988, is lost in practice when calculating growth rates between periods. Hence, we refer to productivity growth observations as being from 1989.

indicate that the model does not fit well for most sectors.¹⁴ Again, the estimates of k are much less than one, implying strongly increasing returns to scale. From Table 5, it can be seen that the additional parameter, s , is generally not individually significant, and the p -values for the test of joint significance of the technical progress parameters indicates that they are not statistically different from zero for many sectors, including the aggregate Market Sector. Again there is some evidence of significant technical regress.

As it is unlikely that technical regress took place on average over this period, we impose positive technical progress on the linear-time-trend model by squaring the technical progress parameter and performing a non-linear regression. The results are reported in tables 6 and 7. Again, we find evidence of strong increasing returns to scale (small estimates of k), and little role for technical progress except for the Transport and Communications industry.

In order to investigate further the robustness of these results, we consider running “reverse regressions,” as explained in the following section.

4.3 Reverse Regressions

An interesting feature of the model in equation (21) is that the log of an input index is the dependent variable, whereas usually in production relationships it is an output variable. Thus, our model can be viewed as an inverse production relationship.¹⁵ It could be argued that inputs are more exogenous than outputs, and so the regressions should condition on the input variable. Putting aside issues of possible measurement error and unaccounted for input utilization, it is essentially an issue of which comes first in the production process, inputs or outputs. While it is traditionally taken to be outputs, much attention has been given to the possibility of endogenous inputs. One can think of many examples where economic activity only takes place once output contracts are specified, and producers then determine inputs for a fixed output level.¹⁶ There are of course many occasions where the opposite may be the case, so we also examine “reverse regressions” where the log of the output index is now the dependent variable.

The results are presented in tables 8 and 9, along with the corresponding estimates from the estimates from tables 2 and 3, respectively. The coefficients of returns to scale in these reverse regressions are represented by $1/k$. From Table 8 we see that we get lower estimates of the degree of returns to scale (i.e. the estimates of k are larger). This is a result which is expected from theory if the dependent and independent variables are positively correlated; see e.g., Bartelsman (1995, p. 60). Although we earlier found statistically significant evidence of strong increasing returns to scale (Table 2), we see from Table 8 that the null hypothesis of constant returns to scale ($1/k=1$) cannot be rejected for any industry up to at least the 10% of significance. From Table 9 we see that again the role of technical progress is insignificant.

¹⁴ The R^2 values in tables 2 and 4 cannot of course be directly be compared as there are a different number of parameters in each model.

¹⁵ Bartelsman (1995, p. 60) criticized Hall (1988, 1990) for estimating the inverse of his derived production relationship in his empirical work, because “the estimate of the inverse of a parameter in a linear relationship is not equal to the inverse of the estimate of the parameter”. Our equation (21) is not subject to this criticism as our empirical model is consistent with our theoretical model; output quantities are taken as exogenously given in the cost minimization problem (1) as is the quantity index in the estimating equation (21).

¹⁶ Mines do not typically dig up coal without contracts to sell the output, and clothing manufacturers usually do not start production without orders.

These results can again be seen as somewhat unsatisfactory.¹⁷ The use of system-of-equations estimation (seemingly unrelated regression) did not help in either the direct or reverse regressions. A problem with systems estimation is that one or more poorly specified equations can infect the fit for other equations, and this seems to be the case in this context. In addition, instrumental variables estimation (using lagged values of the variables as instruments) to take into account the possibility of the endogeneity of the right-hand-side variable of the estimating equations was examined, but did not provide any results worth reporting.¹⁸

Hence, we turn to an estimation method which is not commonly used in econometrics.

4.4 Orthogonal Regression

This method is variously known as “orthogonal regression”, “symmetric regression”, or “total least squares”. We try this method, as it will report the same coefficient estimates regardless of which variable is taken to be the dependent variable and which is taken to be the independent variable. The geometry of the problem in the two variable case is straightforward, with the regression errors measured as being orthogonal to the fitted line. Hence, a priori we know that it will yield coefficients which lie between the extremes of those of the direct and reverse regression. As it is not generally well-known to econometricians, the Appendix outlines a derivation of the slope estimator (where the intercept estimator follows residually in the usual way). For more on this approach, see e.g., Adcock (1878), Golub and Van Loan (1980), van Huffel and Vandewalle (1991), del Rio, Riu and Rius (2001), and Madansky (1959).¹⁹

A summary of results are reported in Table 10. While some of the estimates of returns to scale are again too large to be believable (for Primary industry, for example), there are also some more sensible estimates (closer to one). Again there appears to be little role played by technical progress.

Figures 3 and 4 plot the data and regression lines for Personal Services and the aggregate Market Sector, respectively. These provide some insight into the basic problem of estimation with this data set. In Figure 3, The Ordinary Least Squares (OLS) line represents the fit from directly estimating equation (21). The reverse regression line is the corresponding fit from calibrating equation (21) using estimates from the reverse regression approach of Section 4.3. Finally, the orthogonal regression line is from using the symmetric, total least squares approach. The dispersion of the data is such that we can see how different estimators can easily fit different lines of “best fit”. The orthogonal regression line, lying between the two extremes perhaps provides an interesting compromise given the large disparity between the other two regression approaches and the uncertainty about which variable is more exogenous.

While it seems unlikely to find a satisfactory economic model that can explain the data plotted in Figure 3, from Figure 4 we can see why we get more satisfactory results for the aggregate Market Sector. Note, from tables 2 and 8, that the R^2 is the same for both the direct and reverse regressions. Also, the product of the two slope estimates is equal to the

¹⁷ Both the direct and reverse regression approaches were also applied to the aggregate economy sets of Fox, Kohli and Warren (2002, 2003), but also yielded less than satisfactory results.

¹⁸ Results are very sensitive to the instrument set chosen. The problems with finding appropriate instruments for industries in this kind of analysis are highlighted by Burnside (1996), Basu and Fernald (1997), Nordhaus (1990) and Baily (1990).

¹⁹ Madansky (1959, p. 203) notes that the estimator is “well known to econometricians” and references Koopmans (1937) and Tintner (1952). However, more recent econometric texts seem to typically omit this method.

common R^2 ; with a higher R^2 the regression lines will be relatively similar. In Figure 4 there is a clearer relationship between the variables, and so the regression lines are relatively similar. That is, there seems to be relatively less noise in the data at the aggregate level. This suggests that what is needed at the industry level is more work on data quality and quantity if the underlying relations are to be understood.

4.5 Application of an Alternative Method

Finally, we consider the alternative framework of Diewert and Lawrence (2004). With this method we estimate three equations, and have the full fifteen annual observations for each equation, giving a total of 45 observations in the system. There are nine coefficients to be estimated in equations (29) and (30) of Section 3. The coefficient estimates from using nonlinear regression are reported in Table 11 for the Market Sector.²⁰ The results for the other industries were generally very poor, with nonsensical coefficient estimates, low R^2 values for each equation, obviously serious autocorrelation problems from the residual plots, problems in convergence of the nonlinear algorithm, and extreme sensitivity of results to the specification of the coefficient starting values.²¹ The results are also sensitive to the choice of which input is used to normalize the production function, as in equation (30). The best results were obtained when capital was taken as the normalizing input, x_1 . However, the sensitivity of the results suggests that future work on this method should consider a symmetric treatment of inputs, such as the use of a symmetric normalized quadratic form (see e.g. Kohli, 1993).²²

Again, the Market Sector seemed to yield the best results. From Table 11 we see that some of the coefficients are (very) statistically significant. The R^2 values are 0.83 and 0.41 for the capital and labour price equations of (29), respectively, and 0.92 for the production function of (30). However, the markup coefficients M_1 and M_2 are insignificantly different from one, indicating that there are constant returns to scale, given that expenditures equal income in the data set (see equation (22)). A direct test of the null hypothesis of constant returns to scale at the means of the data, using equation (31), also leads to the conclusion that constant returns cannot be rejected. Technical progress estimates calculated from the estimated parameters using equation (32) are small. At the mean of the data, the null hypothesis of zero technical progress can be rejected at the usual significance levels, but it is only 0.1% per annum.

On the whole, this approach does not seem to provide a clearly superior methodology, particularly due to the problems of econometrically estimating nonlinear equations with small, noisy, data sets.

²⁰ The use of alternative optimization algorithms was investigated, but yielded similar results. The reported results are from using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm;

²¹ Various specifications of splined linear time trends were also investigated to allow for a more flexible, representation of the role of the passage of time, but they did not yield statistically satisfactory results. This included making M_1 and M_2 time dependent; see equations (26) of Diewert and Lawrence (2003).

²² The method of imposing symmetry may itself be arbitrary, and different methods may lead to different results. Hence, this is left for future research.

5 Conclusion

This paper has set out the microeconomic theory behind different methods for separating returns to scale and technical progress from total factor productivity. The suggested frameworks allow for the existence of imperfect competition, so that estimates of monopolistic markups may also be found. The methods were applied to industry data for New Zealand, building on the work of Black, Guy and McLellan (2003).

A consistent picture that is observed across the methods and different estimation procedures is that returns to scale are either increasing or constant, and there has been little role played by technical progress in these sectors. These results are not inconsistent with international findings. For example, Denison (1974), Berndt and Khaled (1979) Hall (1990), Morrison (1992), Morrison and Siegel (1997), Basu and Fernald (1997) and Diewert and Fox (2004) have all found evidence of increasing returns to scale, usually using data for the U.S. Our more reasonable estimates are more reasonable than many of the estimates in the cited papers. However, a number of caveats need to be emphasized.

The primary issue here is the data set. This consists of 15 annual observations, 1988-2002, for nine market sectors and two aggregates. A great deal of effort has gone into constructing this data set, and its construction was well overdue given the great interest in New Zealand's recent productivity performance; see e.g. Diewert and Lawrence, (1999). It has provided interesting information on the sources of productivity growth at the industry and aggregate level. However, in order to analyze further the sources of productivity growth it is necessary to impose more structure on the modelling of TFP growth in order to separate out components of interest, such as the role of technical progress, returns to scale and the degree of imperfect competition in the economy. The data set as it stands, at least at the level of the individual industries, does not seem to be able to provide econometrically robust estimates even with very simple models that only require the estimation of two or three parameters.

One of the main conclusions from the Diewert and Lawrence (1999) report on the productivity performance of the New Zealand economy was the necessity for more effort to be put into data construction. The existing data set is an excellent first step in this direction, but this data set needs to be maintained over time to build up a decent time series of data over many years so that more robust results can be obtained and even more rigorous analysis performed. In developing this data, special attention needs to be devoted to the measurement of outputs in service sectors (a priority for statistical agencies around the world), and the investigation of user-cost approaches to the measurement of capital. The latter development will allow the estimation of the degree of imperfect competition that exists in the various sectors.

Finally, while the investigation of productivity at the industry and sector levels are of great interest, the investigation at lower levels of aggregation, particularly the firm level, would shed greater light on the actual sources of measured aggregate productivity growth.

Appendix: A Symmetric Regression Model

A brief derivation of the orthogonal regression approach of Section 4.4 is provided.²³ Consider two N dimensional vectors x and y as being given. These can be interpreted as zero mean vectors. In fitting linear regressions (through the origin) of the type $y = ax + e$ or $x = by + e$, we want a procedure that would have the property that our estimator for a is equal to the reciprocal of the estimator for b . That is, it would not matter which variable we took as being the exogenous variable and which we took as the endogenous variable. Consider the following method for fitting a regression of the type $y = ax + e$:

$$(A1) \min_{x^*, y^*, a} \{ (y - y^*)^T (y - y^*) + (x - x^*)^T (x - x^*) : y^* = ax^* \}$$

$$= \min_{x^*, a} \{ (y - ax^*)^T (y - ax^*) + (x - x^*)^T (x - x^*) \} \equiv f(x^*, a).$$

The solution to (A1) minimizes the sum of the squared distances of each (y_n, x_n) observation from the line through the origin with the equation $y = ax$. To solve this problem, first minimize $f(x^*, a)$ with respect to the components of the x^* vector conditional on a given scalar parameter, a . The first order conditions are as follows:

$$(A2) \nabla_{x^*} f(x^*, a) = 0_N,$$

which yield the following solution for x^* :

$$(A3) x^{**} = [1+a^2]^{-1}[x + ay].$$

Substituting the left-hand side of (A3) into $f(x^*, a)$, as defined by (A1), gives

$$(A4) g(a) \equiv [1+a^2]^{-1}[y - ax]^T[y - ax].$$

Ruling out infinite solutions, the first order necessary condition for minimizing $g(a)$ with respect to a is equivalent to finding a root of the following quadratic equation:

$$(A5) x^T y a^2 + [x^T x - y^T y] a - x^T y = 0.$$

It is assumed that $x^T y > 0$, which implies that the x and y vectors are positively correlated. The largest root of (A5) is given by

$$(A6) a^* = \{ [y^T y - x^T x] + [(y^T y - x^T x)^2 + 4(x^T y)^2]^{1/2} \} / 2x^T y.$$

This root is the desired estimator for the parameter a in the regression line $y = ax$.

Now, as the counterpart to (A1), consider the following method for fitting a regression of the type $x = by + e$:

$$(A7) \min_{x^*, y^*, a} \{ (y - y^*)^T (y - y^*) + (x - x^*)^T (x - x^*) : x^* = by^* \}.$$

(A7) is the same as (A1), but with the roles of x and y reversed. The b solution to (A7) is given by (A6) except the roles of x and y must be reversed so that

$$(A8) b^* = \{ [x^T x - y^T y] + [(x^T x - y^T y)^2 + 4(y^T x)^2]^{1/2} \} / 2y^T x.$$

²³ This appendix draws on correspondence and discussions with W. Erwin Diewert.

Assuming that $x^T y > 0$, it is possible to show that

$$(A9) \quad a^* b^* = 1.$$

Hence the regression methods defined by solving (A1) or (A7) are indeed symmetric.

References

- Abramovitz, M. and P. A. David (1996), "Convergence and Deferred Catch-up: Productivity Leadership and the Waning of American Exceptionalism", in R. Landau, T. Taylor and G. Wright (eds.), *The Mosaic of Economic Growth*, Stanford, CA: Stanford University Press, 21-62.
- Adcock, R. J. (1878), "A Problem in Least Squares", *The Analyst* 5, 53-54.
- Appelbaum, E. (1979), "Testing Price Taking Behavior", *Journal of Econometrics* 9, 283-294.
- Bartelsman, E. J. (1995), "Of Empty Boxes: Returns to Scale Revisited," *Economics Letters* 49, 59-67.
- Baily, M.N. (1990), "Comments: Competition, Increasing Returns, and the Solow Productivity Paradox," in *Growth, Productivity, Employment*, P. Diamond (ed.), Cambridge MA: MIT Press.
- Basu, S. and J. G. Fernald (1997), "Returns to Scale in U.S. Production: Estimates and Implications", *Journal of Political Economy* 105, 249-283.
- Basu, S. and J. G. Fernald (2002), "Aggregate Productivity and Aggregate Technology", *European Economic Review* 46, 963-991.
- Baxter, M. and R. G. King (1992), "Productive Externalities and the Business Cycle," Institute for Empirical Macroeconomics, discussion paper no. 53 (Federal Reserve Bank of Minneapolis, Minneapolis, MN).
- Beaudry, P. and M. B. Devereux (1995), "Money and the Real Exchange Rate with Sticky Prices and Increasing Returns to Scale," *Carnegie-Rochester Conference Series on Public Policy* 43, 55-101.
- Benhabib, J. and R. E. A. Farmer (1994), "Indeterminacy and Increasing Returns," *Journal of Economic Theory* 63, 19-41.
- Berndt, E. R. and M. S. Khaled (1979), "Parametric Productivity Measurement and Choice among Flexible Functional Forms", *Journal of Political Economy* 87, 1220-1245.
- Black, M., Melody, G., and N. McLellan (2003), "Productivity in New Zealand 1988 to 2002," New Zealand Treasury Working Paper 03/06.
- Burnside, C. (1996), "Production Function Regressions, Returns to Scale, and Externalities," *Journal of Monetary Economics* 37, 177-201.
- del Rio, F.J., J. Riu and F.X. Rius (2001), "Prediction Intervals in Linear Regression taking into Account Errors on Both Axes," *Journal of Chemometrics* 15, 773-788.
- Denison, E. F. (1974), *Accounting for United States Economic Growth 1929-1969*, Washington, DC: Brookings Institution.

- Devereux, M.B., A.C. Head and B.J. Lapham (1996), Monopolistic Competition, Increasing Returns, and the Effects of Government Spending,” *Journal of Money, Credit, and Banking* 28, 233-254.
- Diamond, P. A. and J. A. Mirrlees (1971), “Optimal Taxation and Public Production II: Tax Rules”, *American Economic Review* 61, 261-278.
- Diewert, W. E. (1974), “Applications of Duality Theory”, pp. 106-171 in *Frontiers of Quantitative Economics, Volume 2*, M. D. Intriligator and D. A. Kendrick (eds.), Amsterdam: North-Holland.
- Diewert, W. E. (1976), “Exact and Superlative Index Numbers”, *Journal of Econometrics* 4, 114-145.
- Diewert, W. E. (1983), “The Measurement of Waste Within the Production Sector of an Open Economy”, *Scandinavian Journal of Economics* 85:2, 159-179.
- Diewert, W.E. and K.J. Fox (2004), “On the Estimation of Returns to Scale, Technical Progress and Monopolistic Markups,” paper presented at the SSHRC International Conference on Index Number Theory and the Measurement of Prices and Productivity, Vancouver, 30 June – 2 July, 2004.
- Diewert, W.E. and D. Lawrence (1999), “Measuring New Zealand’s Productivity,” Treasury Working Paper No. 99/5 March, Wellington, New Zealand.
- Diewert, W.E. and D. Lawrence (2004), “The Role of ICT in Australia’s Economic Performance: Investigation of assumptions influencing the productivity estimates,” Meyrick and Associates, Canberra. Presented at the Asia Pacific Productivity Conference, Brisbane, 14-16 July 2004.
- Diewert, W.E., T. Nakajima, A. Nakamura, E. Nakamura and M. Nakamura (2004), “The Definition and Estimation of Returns to Scale with an Application to Japanese Industries,” unpublished manuscript.
- Fox, K.J., U. Kohli and R.S. Warren, Jr. (2002), “Accounting for Growth and Output Gaps: Evidence from New Zealand,” *Economic Record* 78, 312-326.
- Fox, K.J., U. Kohli and R.S. Warren, Jr. (2003), “Sources of Growth and Output Gaps in New Zealand: New Methods and Evidence,” *New Zealand Economic Papers* 37(1), 67-92.
- Golub, G. H. and C. F. Van Loan (1980), “An Analysis of the Total Least Squares Problem”, *Siam Journal of Numerical Analysis* 17, 883-893.
- Guo, J-T. (2004), “Increasing Returns, Capital Utilization, and the Effects of Government Spending”, *Journal of Economic Dynamics and Control* 28, 1059-1078.
- Hall, R. E. (1988), “The Relationship between Price and Marginal Cost in U. S. Industry”, *Journal of Political Economy* 96, 921-947.

- Hall, R. E. (1990), "Invariance Properties of Solow's Productivity Residual", in *Growth, Productivity, Employment*, P. Diamond (ed.), Cambridge MA: MIT Press.
- Hintermaier, T. (2003), "On the Minimum Degree of Returns to Scale in Sunspot Models of the Business Cycle", *Journal of Economic Theory* 110, 400-409.
- Jones, C.I. (2004), "Growth and Ideas", NBER Working Paper 10767, Cambridge, MA: National Bureau of Economic Research.
- Kohli, U. (1993), "A Symmetric Normalized Quadratic GNP Function and the U.S. Demand for Imports and Supply of Exports," *International Economic Review* 34, 243-255.
- Koopmans, T.C. (1937), *Linear Regression Analysis of Economic Time Series*, Haarlem, Netherlands: DeErven F. Bohn.
- Madansky, A. (1959), "The Fitting of Straight Lines when both Variables are Subject to Error," *Journal of the American Statistical Association* 54, 173-205.
- Mawson, P., K.I. Carlaw and N. McLellan (2003), "Productivity Measurement: Alternative approaches and estimates," New Zealand Treasury Working Paper 03/12, Wellington, New Zealand.
- McLellan, N. (2004), "Measuring Productivity using the Index Number Approach: An Introduction", New Zealand Treasury Working Paper 04/05, Wellington, New Zealand.
- Morrison, C.J. (1992), "Unraveling the Productivity Growth Slowdown in the United States, Canada and Japan: The Effects of Subequilibrium, Scale Economies and Markups," *Review of Economics and Statistics* LXXIV, 381-393.
- Morrison, C.J. and D. Siegel (1997), "External Capital Factors and Increasing Returns in U.S. Manufacturing," *Review of Economics and Statistics* 647-654.
- Nakajima, T., M. Nakamura and K. Yoshioka (1998), "An Index Number Method for Estimating Scale Economies and Technical Progress Using Time-Series of Cross-Section Data: Sources of Total Factor Productivity Growth for Japanese Manufacturing, 1964-1988", *Japanese Economic Review* 49, 310-334.
- Nordhaus, W.D. (1990), "Comments: Investment, Market Power, and Productivity Growth," in *Growth, Productivity, Employment*, P. Diamond (ed.), Cambridge MA: MIT Press.
- Roeger, W. (1995), "Can Imperfect Competition Explain the Difference between Primal and Dual Productivity Measures? Estimates for U.S. Manufacturing," *Journal of Political Economy* 103, 316-330.
- Romer, P.M. (1986), "Increasing Returns and Long-run Growth", *Journal of Political Economy* 94, 1002-1037.
- Romer, P.M. (1996), "Why, Indeed, in America? Theory, History, and the Origins of Modern Economic Growth", *American Economic Review* 86, 202-206.

- Schreyer, P. (2001), *OECD Productivity Manual: A Guide to the Measurement of Industry Level and Aggregate Productivity Growth*, Paris: OECD.
- Shephard, R.W. (1953), *Cost and Production Functions*, Princeton: Princeton University Press.
- Tinter, G. (1952), *Econometrics*, New York: John Wiley and Sons.
- Törnqvist, L. (1936), "The Bank of Finland's Consumption Price Index," *Bank of Finland Monthly Bulletin* 10: 1-8.
- Törnqvist, L. and E. Törnqvist (1937), "Vilket är förhållandet mellan finska markens och svenska kronans köpkraft?", *Ekonomiska Samfundets Tidskrift* 39, 1-39 reprinted as pp. 121-160 in *Collected Scientific Papers of Leo Törnqvist*, Helsinki: The Research Institute of the Finnish Economy, 1981.
- van Huffel, S. and J. Vandewalle (1991), *The Total Least Squares Problem: Computational Aspects and Analysis*, Philadelphia: Society for Industrial and Applied Mathematics.
- Weil, P. (1989), "Increasing Returns and Animal Spirits," *American Economic Review* 79, 889-894.
- White, K.J. (1978), "A General Computer Program for Econometric Methods—SHAZAM," *Econometrica*, 239-240.

Table 1: Total Factor Productivity Growth

Sector	1989-2002		1989-1994		1995-2002	
	Mean	s.d.	Mean	s.d.	Mean	s.d.
Primary	1.016	0.076	1.021	0.111	1.013	0.043
Mining and Quarrying	1.003	0.109	0.992	0.093	1.011	0.125
Manufacturing	1.000	0.029	1.003	0.030	0.999	0.029
Utilities	0.999	0.048	1.020	0.036	0.984	0.052
Construction	0.987	0.064	0.969	0.066	1.000	0.063
Trade	1.008	0.029	1.004	0.036	1.011	0.025
Transport	1.060	0.034	1.071	0.033	1.052	0.033
Business Services	0.996	0.026	0.986	0.031	1.004	0.020
Personal Services	1.013	0.025	1.013	0.019	1.013	0.031
Market Sector	1.009	0.016	1.008	0.021	1.010	0.013
ABSE Market Sector	1.011	0.020	1.012	0.022	1.011	0.019

Note: Arithmetic means. A mean value greater than one implies positive TFP growth, while a value less than one implies negative TFP growth. The mean value less one times a hundred gives the average percentage growth in TFP. “s.d.” denotes standard deviation. “ABSE” denotes the ABS Equivalent Market Sector, which excludes the Business Services and Personal Services industries. All productivity indexes were calculated using the Fisher index, except for the ABSE Market Sector, for which the Törnqvist index was used for consistency with ABS methodology.

Table 2: Estimates of Returns to Scale, Linear Time Trend

Sector	\hat{k}	Ho: $k = 1$ t -ratio	R^2	DW
Primary	0.019	-20.488	0.013	1.268
Mining and Quarrying	-0.049	-14.105	0.035	1.764
Manufacturing	0.642	-1.716	0.440	2.071
Utilities	0.189	-4.291	0.077	1.762
Construction	0.433	-6.136	0.647	2.433
Trade	0.242	-6.304	0.252	2.212
Transport	0.409	-2.535	0.204	0.771
Business Services	0.187	-4.226	0.073	2.046
Personal Services	0.326	-2.846	0.136	2.366
Market Sector	0.516	-4.657	0.672	1.783
ABSE Market Sector	0.533	-3.883	0.621	1.952

Note: \hat{k} is the estimate of k from estimating the equation $\ln Q^*(\cdot) = -r + k \ln Q(\cdot)$, where $Q^*(\cdot)$ is an input index and $\ln Q(\cdot)$ is an output index. Ho: $k = 1$ is a test of constant returns to scale, and $k < 1$ implies increasing returns to scale. The critical value for a t -test with 12 degrees of freedom is 1.782 at the 10% level and 2.179 at the 5% level. DW denotes the Durbin-Watson statistic for the null hypothesis of no first-order autocorrelation. For a sample size of 15 observations and one explanatory variable (excluding the intercept), the 5% lower critical value (positive autocorrelation) is 1.08, and the upper critical value (for negative autocorrelation) is $4 - 1.36 = 2.64$.

Table 3: Estimates of r , Linear Time Trend

Sector	\hat{r}	Ho: $r = 0$ t -ratio
Primary	-0.012	-3.264
Mining and Quarrying	-0.019	-2.655
Manufacturing	-0.003	-0.396
Utilities	-0.006	-0.715
Construction	-0.017	-1.931
Trade	-0.010	-2.123
Transport	0.020	1.185
Business Services	-0.023	-3.637
Personal Services	-0.010	-1.036
Market Sector	-0.003	-0.785
ABSE Market Sector	0.001	0.131

Note: \hat{r} is the estimate of r from estimating the equation $\ln Q^*(\cdot) = -r + k \ln Q(\cdot)$, where $Q^*(\cdot)$ is an input index and $\ln Q(\cdot)$ is an output index. $r > 0$ represents positive technical change. Multiplying by 100 gives percentage growth rates. The critical value for a t -test with 12 degrees of freedom is 1.782 at the 10% level and 2.179 at the 5% level.

Table 4: Estimates of Returns to Scale, Quadratic Time Trend

Sector	\hat{k}	Ho: $k = 1$ t -ratio	R^2	DW
Primary	0.011	-26.163	0.439	2.101
Mining and Quarrying	-0.050	-16.160	0.324	2.517
Manufacturing	0.591	-1.822	0.466	2.069
Utilities	0.250	-4.000	0.215	2.010
Construction	0.424	-5.797	0.651	2.407
Trade	0.200	-6.223	0.308	2.385
Transport	0.226	-3.677	0.482	1.076
Business Services	0.308	-2.667	0.114	2.265
Personal Services	0.279	-3.098	0.253	2.670
Market Sector	0.455	-4.984	0.721	1.831
ABSE Market Sector	0.471	-4.330	0.684	2.082

Note: Ho: $k = 1$ is a test of constant returns to scale, and $k < 1$ implies increasing returns to scale. The critical value for a t -test with 11 degrees of freedom is 1.796 at the 10% level and 2.201 at the 5% level. DW denotes the Durbin-Watson statistic for first-order autocorrelation.

Table 5: Estimates of r and s , Quadratic Time Trend

Sector	\hat{r}	Ho: $r = 0$ t -ratio	\hat{s}	Ho: $s = 0$ t -ratio	Ho: $r = s = 0$ p -value
Primary	0.004	0.691	-0.002	-2.890	0.001
Mining and Quarrying	-0.046	-3.272	0.003	2.171	0.011
Manufacturing	0.008	0.481	-0.001	-0.720	0.723
Utilities	0.019	0.958	-0.003	-1.393	0.325
Construction	-0.010	-0.458	-0.001	-0.352	0.212
Trade	-0.003	-0.315	-0.001	-0.950	0.112
Transport	0.041	2.480	-0.004	-2.429	0.051
Business Services	-0.029	-2.641	0.001	0.711	0.013
Personal Services	0.003	0.235	-0.002	-1.312	0.281
Market Sector	0.004	0.644	-0.001	-1.387	0.313
ABSE Market Sector	0.011	1.320	-0.001	-1.477	0.367

Note: $r + st > 0$ represents positive technical change. The critical value for a t -test with 12 degrees of freedom is 1.796 at the 10% level and 2.201 at the 5% level.

Table 6: Estimates of Returns to Scale, Linear Time Trend, Positive Technical Progress Imposed

Sector	\hat{k}	Ho: $k = 1$ t -ratio
Primary	0.073	-16.065
Mining and Quarrying	-0.017	-11.879
Manufacturing	0.661	-1.808
Utilities	0.203	-4.479
Construction	0.440	-5.705
Trade	0.392	-5.793
Transport	0.409	-2.711
Business Services	0.672	-1.757
Personal Services	0.528	-3.644
Market Sector	0.572	-5.931
ABSE Market Sector	0.533	-4.181

Note: Ho: $k = 1$ is a test of constant returns to scale, and $k < 1$ implies increasing returns to scale. The critical value for a t -test with 12 degrees of freedom is 1.782 at the 10% level and 2.179 at the 5% level.

Table 7: Estimates of $r = g^2$, Linear Time Trend

Sector	\hat{g}	Ho: $g = 0$ t -ratio	\hat{r}
Primary	0.000	0.000	0.000
Mining and Quarrying	0.000	0.000	0.000
Manufacturing	0.000	0.000	0.000
Utilities	0.000	0.000	0.000
Construction	0.000	0.000	0.000
Trade	0.000	0.000	0.000
Transport	-0.141	-2.555	0.020
Business Services	0.000	0.000	0.000
Personal Services	0.000	0.000	0.000
Market Sector	0.000	0.000	0.000
ABSE Market Sector	-0.024	-0.284	0.001

Table 8: Estimates of Returns to Scale, Reverse Regression, Linear Time Trend

Sector	$1/\hat{k}$	$\widehat{1/k}$	Ho: $1/k = 1$ t -ratio	R^2	DW
Primary	52.632	0.685	-0.183	0.013	2.808
Mining and Quarrying	-20.408	-0.714	-1.586	0.035	2.395
Manufacturing	1.558	0.686	-1.405	0.440	2.064
Utilities	5.291	0.406	-1.460	0.077	2.213
Construction	2.309	1.494	1.551	0.647	3.100
Trade	4.132	1.041	0.080	0.252	1.680
Transport	2.445	0.498	-1.764	0.204	1.203
Business Services	5.348	0.390	-1.518	0.073	0.835
Personal Services	3.067	0.418	-1.915	0.136	1.341
Market Sector	1.938	1.303	1.154	0.672	2.133
ABSE Market Sector	1.876	1.165	0.628	0.621	2.361

Note: $1/\hat{k}$ is from inverting the estimates of k from Table 2 and is included here for comparison purposes. $\widehat{1/k}$ is the estimate of $1/k$ from estimating the reverse regression $\ln Q(\cdot) = r/k + (1/k) \ln Q^*(\cdot)$ where $Q(\cdot)$ is an output index and $\ln Q^*(\cdot)$ is an input index. Ho: $1/k = 1$ is a test of constant returns to scale, and $1/k > 1$ implies increasing returns to scale. The critical value for a t -test with 12 degrees of freedom is 1.782 at the 10% level and 2.179 at the 5% level. DW denotes the Durbin-Watson statistic for first-order autocorrelation.

Table 9: Estimates of r , Reverse Regression, Linear Time Trend

Sector	\hat{r}	\tilde{r}	Ho: $r = 0$ t -ratio
Primary	-0.012	0.026	0.256
Mining and Quarrying	-0.019	-0.040	-0.801
Manufacturing	-0.003	0.004	0.330
Utilities	-0.006	0.005	0.158
Construction	-0.017	-0.016	-1.476
Trade	-0.010	0.007	0.489
Transport	0.020	0.123	1.625
Business Services	-0.023	0.031	0.499
Personal Services	-0.010	0.058	0.987
Market Sector	-0.003	0.003	0.644
ABSE Market Sector	0.001	0.008	1.257

Note: \hat{r} is the estimate of r from Table 3 and is included here for comparison purposes. \tilde{r} is the implied r from estimating the coefficient r/k in the reverse regression. $r > 0$ represents positive technical change. Multiplying by 100 gives percentage growth rates.

Table 10: Estimates of k and r , Orthogonal Regression

Sector	\hat{k}	$1/\hat{k}$	\hat{r}
Primary	0.020	50.929	-0.012
Mining and Quarrying	-0.053	-19.003	-0.019
Manufacturing	0.951	1.052	-0.000
Utilities	0.317	3.155	-0.006
Construction	0.474	2.112	-0.017
Trade	0.288	3.467	-0.009
Transport	0.804	1.244	0.045
Business Services	0.322	3.107	-0.020
Personal Services	0.717	1.394	0.003
Market Sector	0.573	1.745	-0.002
ABSE Market Sector	0.613	1.631	0.002

Note: These estimates are from using orthogonal regression (or “total least squares” regression). The procedure is symmetric in the sense the estimate of k from the direct regression equals the inverse of the estimate of $1/k$ from the reverse regression. The estimates of k and the corresponding value of $1/k$ are reported for ease of comparison with the estimates in tables 2 and 8. Estimates of r can be compared with the estimates in Table 9.

Table 11: Coefficient Estimates, Alternative Model, Market Sector

Coefficient	Estimate	Standard Error	<i>t</i> -statistic
M_1	1.019	0.013	79.790
b_1	0.432	0.009	47.496
c_1	0.008	0.000	16.557
e_1	0.359	0.065	5.539
M_2	0.998	0.010	102.285
b_2	0.547	0.004	139.680
c_2	0.001	0.000	1.698
a	0.007	0.005	1.531
d	0.000	0.000	1.444

Note: The *t*-statistic is for testing the null hypothesis that the corresponding coefficient is equal to zero.

Figure 1: TFP Growth for NZ Manufacturing Industries

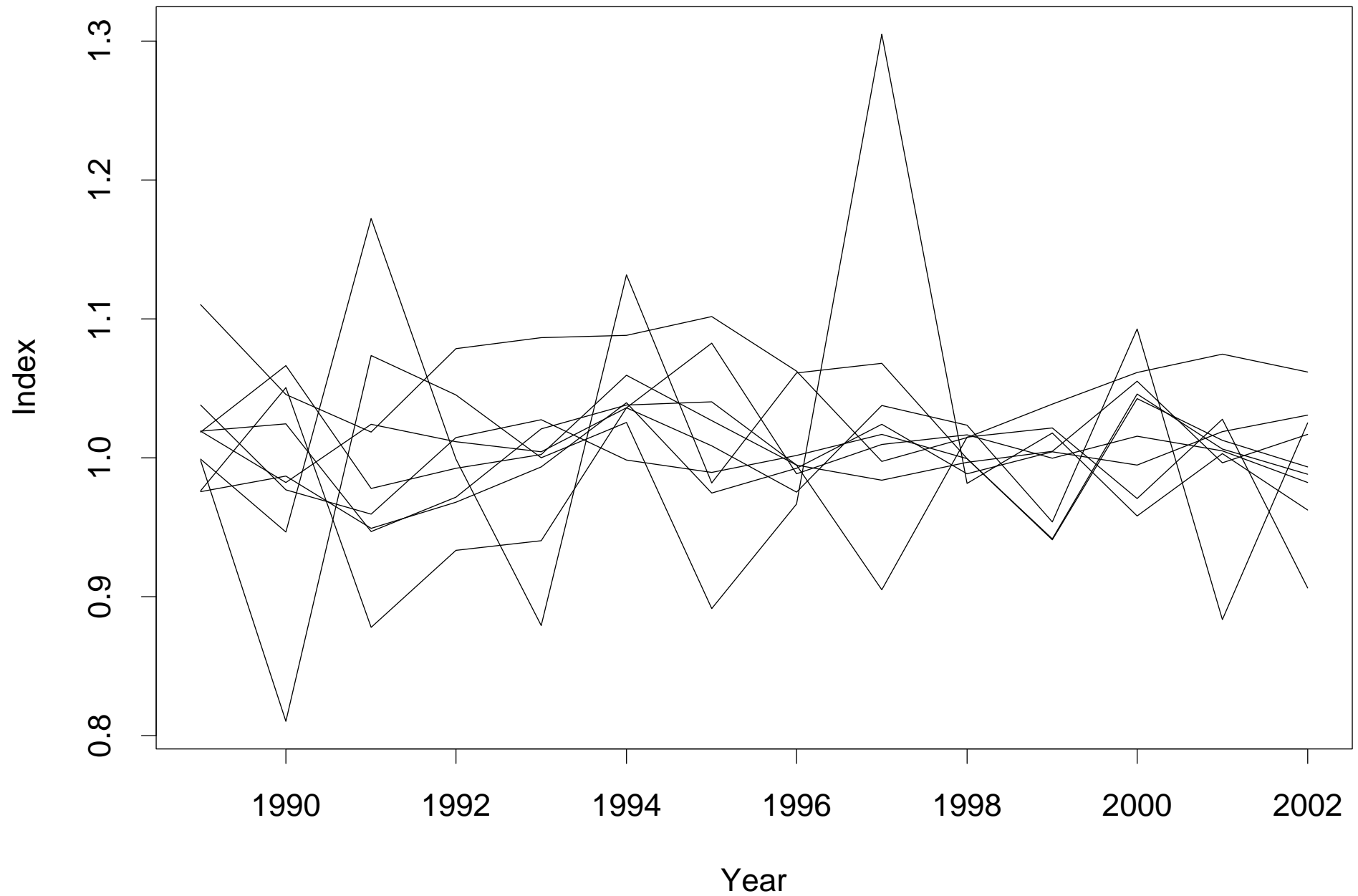


Figure 2: TFP Growth for NZ Market Sector



Figure 3: Regression Lines for Personal Services

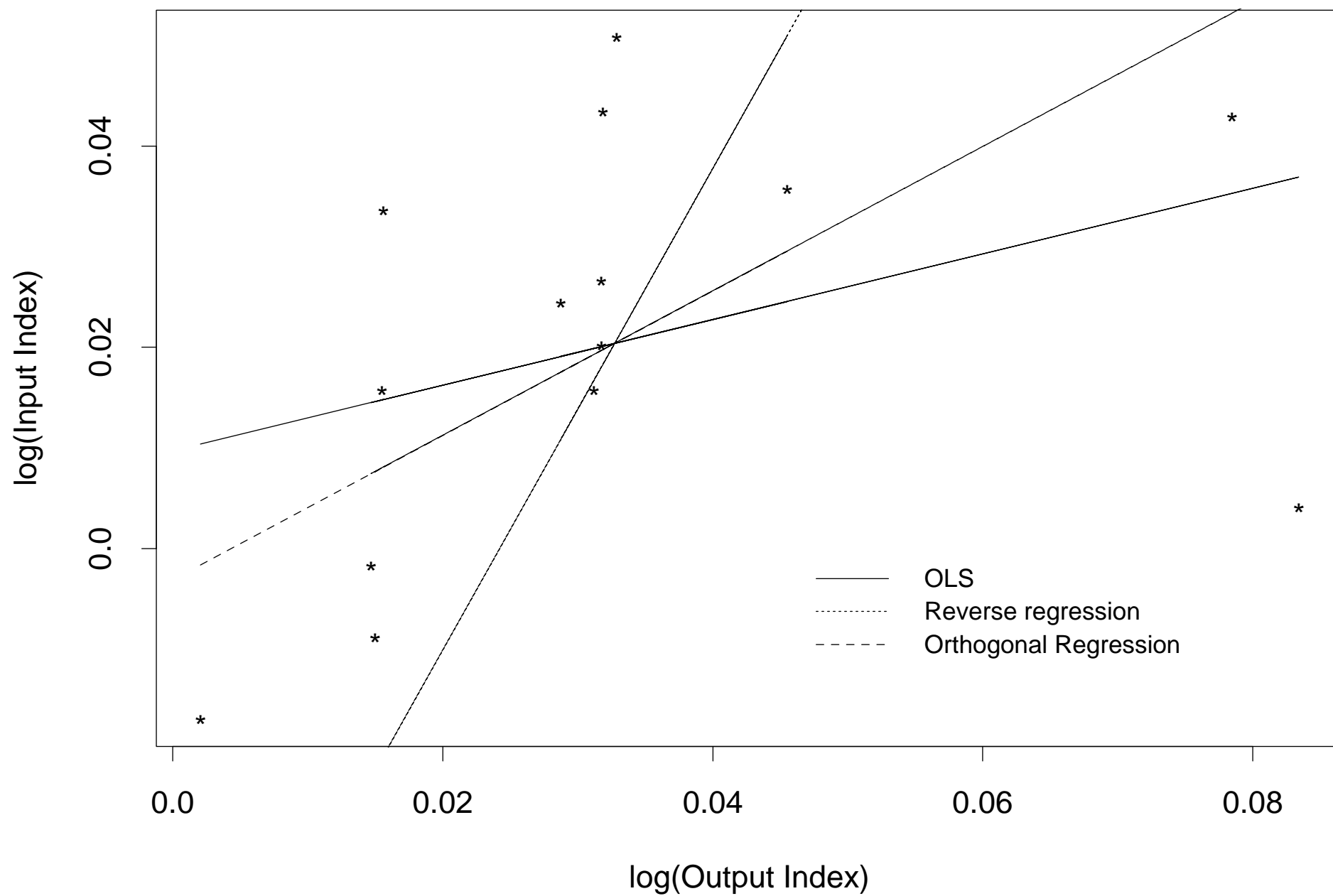


Figure 4: Regression Lines for the Market Sector

