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Adult Equivalence Scales, Inequality and Poverty in New Zealand

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Adult Equivalence Scales, Inequality and Poverty in New Zealand

John Creedy and Catherine Sleeman

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D I S C L A I M E R

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Abstract

This paper examines the sensitivity of inequality and poverty measures to the choice of adult equivalence scales and the type of income unit examined. Comparisons are made using parametric equivalence scales, and income units include individuals, equivalent adults and households. The results are based on HES data for total expenditure. A variety of equivalence scales, for New Zealand, Australia, the UK and the OECD are examined. The implications of varying the poverty line are also considered.

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Adult Equivalence Scales, Inequality and Poverty in New Zealand

1 Introduction

The three fundamental features of any empirical study of inequality and poverty relate to the income concept, the income unit and the time period of analysis. This paper is concerned with the first two features and examines the sensitivity of several inequality and poverty measures to the choices of the adult equivalence scale, used to adjust total household income, and the unit of analysis. Standard measures of inequality and poverty were designed for homogenous populations. Achieving the required homogeneity involves creating, as Ebert (1997, p.235) aptly put it, ‘an (artificial) income distribution for a fictitious population’.¹ In creating an income distribution, household income is adjusted using an adult equivalence scale so as to provide a more accurate reflection of ‘living standard’. A unit of analysis or income recipient is then defined to form the fictitious population.

Section 2 considers the alternative concepts and measures used. It introduces the two-parameter functional form of the adult equivalence scale which is used in the empirical analyses, and discusses the use of three units of analysis (or weights attached to households in computing inequality measures), namely the household, the equivalent adult and the individual. The corresponding inequality and poverty measures are also briefly described. Section 3 presents the main empirical comparisons for New Zealand. These demonstrate the sensitivity of New Zealand’s inequality and poverty measures to the parameters of the equivalence scales, and in addition to the chosen unit of analysis. This section also considers the effects on inequality of varying the inequality aversion coefficient, along with the effects on poverty measures of varying the poverty line. Section 4 identifies the roles of both the equivalence scale and unit of analysis in the context of the redistributive effect of direct taxation. Emphasis is placed on the degree of reranking involved when using alternative scales. Section 5 examines a range of equivalence scales designed for New Zealand, the United Kingdom, Australia and the OECD. To provide comparable estimates of parameters, the two-parameter equivalence scale function is fitted to the various scales used. The resulting scales are applied to the New Zealand data and the resulting inequality and poverty measures are contrasted. Brief conclusions are provided in section 6.

¹ Cowell (1984) discussed nine alternatives, arising from a distinction between three types of income recipient and three income measures.

2 Alternative Concepts and Measures

This section describes the equivalence scales, along with the inequality and poverty measures used in later sections. Subsection 2.1 describes the functional form of the equivalence scale. This function, while essentially pragmatic, is highly flexible and depends on only two easily-interpreted parameters. Subsection 2.2 compares the use of alternative units of analysis, and reviews the difficulty arising from the fact that for heterogeneous populations the basic equity principle involved in the ‘principle of transfers’ is not consistent with an alternative principle of ‘Pareto indifference’ or anonymity. Subsection 2.3 presents the Atkinson inequality measure and its associated social welfare function, while subsection 2.4 describes the poverty measures used.²

2.1 Adult Equivalence Scales

Let y_i denote the income of the i^{th} household, for $i = 1, \dots, N$. The number of individuals in the household is n_i , while the household’s demographic structure is described by the vector d_i . This vector provides the number of individuals in various demographic groups based on age and gender classifications. Using these definitions, the adult equivalent size of household i may be expressed as:³

$$m_i = m(n_i, d_i) \quad (1)$$

This size is normalised so that $m(1, d = \text{adult}) = 1$. Household income is adjusted to obtain the equivalent income or ‘living standard’, given by:

$$z_i = \frac{y_i}{m_i} \quad (2)$$

A household consisting of one adult with an income of y therefore has the same ‘living standard’ as an n -person household with an income of $y \times m(n, d)$. Further progress requires the form of $m_i = m(n_i, d_i)$ to be specified

If there are $n_{k,i}$ individuals of demographic type $k = 1, \dots, K$ in the i^{th} household, the adult equivalent size may be written as:

$$m_i = \left(\sum_{k=1}^K \theta_k n_{k,i} \right)^\alpha \quad (3)$$

The term α is regarded as a measure of economies of scale within the household. This formulation is an extension of the simple form, n_i^α , used by Buhmann et al (1988) and Coulter et al (1992) and modified by, for example, Cutler and Katz (1992), Banks and

² In fact a wider range of inequality measures were obtained, including generalised Gini measures. The results are not reported here as the behaviour is similar to that of the Atkinson measures.

³ The scales are considered to be independent of prices.

Johnson (1994) and Jenkins and Cowell (1994) who distinguished the number of adults, $n_{a,i}$, and children, $n_{c,i}$, such that:

$$m_i = (n_{a,i} + \theta n_{c,i})^\alpha \quad (4)$$

The parameter, θ , measures the size of children relative to adults, while α again reflects economies of scale in consumption. This two-parameter form is used below.⁴

2.2 Units of Analysis

A number of empirical studies have taken the household itself as the basic unit of analysis, usually with little justification. This approach simply assigns to each household, $i=1, \dots, N$ the equivalent income, z_i , of that household and therefore makes no further allowance for the household's demographic structure. In calculating measures of inequality and poverty, the equivalent incomes of households are all each given the same weight of $1/N$. While this approach appears to have little rationale, it is included here for comparative purposes.

An alternative approach is to use the 'adult equivalent person' as the unit of analysis.⁵ This approach assigns to each of the m_i adult equivalent persons in household i , an equivalent income of z_i/m_i . The income concept and the unit of analysis are treated consistently, ensuring that each individual's contribution to inequality and poverty depends on the demographic structure of the household to which they belong. An adult in a one-person household for example will 'count for one', whereas the same adult in a multi-adult household, will count for 'less than one'.

This approach also satisfies the basic equity principle, associated with the principle of transfers, such that a transfer of income from a poorer to a relatively richer household, which leaves the position of the richer household unchanged, causes inequality to rise. The fulfilment of this principle enables Lorenz and Generalised Lorenz curve analyses to be conducted from the resulting distribution.⁶

A third approach is to treat the individual as the basic unit of analysis.⁷ This approach assigns to each of the n_i individuals in household i an equivalent income of z_i/n_i . Every individual effectively 'counts for one', irrespective of the demographic nature of the household to which they belong. This approach consequently has the property of anonymity, in that inequality and poverty measures remain unchanged when one individual in the population is replaced by another individual who has the same living

⁴ It has been used recently by Trigger (2003) to examine poverty, and by Creedy and Scutella (2003) where the emphasis was on the income unit.

⁵ This approach was proposed by Ebert (1997).

⁶ Despite explicitly not treating individuals as the unit, but instead using adult equivalents, this actually leads to a recommendation for equal standards of living; see Ebert (1997, p.242).

⁷ Jorgenson and Slesnick (1984) and Slesnick (1994) use this method, as does Glewwe (1991), who dismisses the use of adult equivalents in a footnote (1991, p.213). It is also preferred by Shorrocks (1997), Danziger and Taussig (1979) and Ringen (1991).

standard but belongs to a different demographically structured household. This property was called the ‘compensation principle’ by Shorrocks (1997) and the ‘Pareto indifference principle’ by Decoster and Ooghe (2002).

The individual unit of analysis does not in general satisfy the equity principle (of transfers). As shown by Glewwe (1991), a transfer of income from a poor to a relatively richer (and larger) household may actually reduce inequality and raise social welfare.⁸ Despite being based on individuals, the anonymity of the individual income unit can lead to a preference for inequality, with the presence of economies of scale causing a large household to be regarded as being ‘more efficient’ at generating welfare.

An important implication is that in the context of heterogeneous populations, the basic equity principle inherent in the principle of transfers and the concept of Lorenz dominance (whereby one Lorenz curve lies unambiguously closer to the diagonal of equality) are no longer equivalent. This equivalence is a fundamental component of welfare analysis for homogeneous populations. Consequently, the choice between individuals and adult equivalents as the basic unit of analysis involves a choice between two incompatible value judgements. They can in principle lead to opposite conclusions about the effects on inequality of a tax policy change.⁹

2.3 Social Welfare and Inequality

Social welfare is regarded as an additive function over the equivalent income of households, z_i , where $i = 1, \dots, N$. If the household itself is treated as the unit of analysis, where each household is assigned its own equivalent income, social welfare per household is simply:

$$W_I = \frac{1}{N} \sum_{i=1}^N V(z_i) \quad (5)$$

where $V(z)$ is increasing and concave.

If the unit of analysis is the individual, so that the principle of anonymity (referred to alternatively in terms of compensation, or Pareto indifference) applies, social welfare per individual is given by:

$$W_I = \frac{1}{\sum_{i=1}^N n_i} \sum_{i=1}^N n_i V(z_i) \quad (6)$$

Finally, if the unit of analysis is the adult equivalent person, where the principle of transfers is satisfied, social welfare per equivalent adult is:

⁸ Transfers of money do not correspond to transfers of ‘living standard’ units between individuals. Glewwe (1991, p.213) used a numerical example with three households. Decoster and Ooghe (2002, pp.3-4) also construct some illustrative examples using three persons.

⁹ Shorrocks (1997) suggested that if concern is with equity, the use of adult equivalents is recommended, whereas if concern is primarily with social welfare, individuals should be the basic income unit. This places the disinterested economist in the position of being required to report results using both approaches.

$$W_E = \frac{1}{\sum_{i=1}^N m_i} \sum_{i=1}^N m_i V(z_i) \quad (7)$$

Each of the three welfare measures is simply a weighted sum, over all N households, of a function V of the equivalent income of each household, z . The only difference concerns the choice of the weights.¹⁰

The form of the additive welfare function discussed above is known to be consistent with the Atkinson inequality measure, A . The Atkinson measure is defined as the proportional difference between the equally-distributed equivalent income, \tilde{z} , and the arithmetic mean income, \bar{z} . Hence, \tilde{z} is the living standard of a household which, if received by every 'unit of analysis' in the population, produces the same social welfare as the actual distribution, and:¹¹

$$A = 1 - \frac{\tilde{z}}{\bar{z}} \quad (8)$$

Although this may be used with any form of V , the most common form is:

$$V(z) = \frac{z^{1-\varepsilon}}{1-\varepsilon} \quad (9)$$

where $\varepsilon \neq 1$ is the degree of constant relative inequality aversion of a disinterested judge. For $\varepsilon = 1$, the function becomes $V(z) = \log z$. In the case of the individual-based welfare function:

$$\tilde{z} = \frac{1}{\sum_{i=1}^N n_i} \left\{ \sum_{i=1}^N n_i z_i^{1-\varepsilon} \right\}^{1/(1-\varepsilon)} \quad (10)$$

The welfare function can be written in 'abbreviated' form in terms of the arithmetic mean and the measure of inequality, using $W = \bar{z}(1-A)$. This has the convenient interpretation that welfare is equal to average income less a term, $\bar{z}A$, which can be regarded as a measure of the 'cost of inequality'.¹²

¹⁰ In practice, microsimulation models assign a sample weight to each household so that appropriate population values can be obtained. The weights are often those provided by the statistical agency which collects the data, but they may also be modified for specific purposes. See Creedy and Tuckwell (2003) for an example of survey reweighting for microsimulation purposes in New Zealand.

¹¹ There is clearly potential for confusing the uses of the term 'equivalent' here, in terms of equally distributed equivalents and adult equivalent incomes.

¹² In this form W is the equally distributed equivalent income, though strictly abbreviated welfare per 'person' is $\tilde{z}^{1-\varepsilon} / (1-\varepsilon)$. However, the trade-off between equity and mean income is the same in each case. On abbreviated welfare functions, see Lambert (2001).

2.4 Poverty

The poverty measures examined here are based on the class introduced by Foster et al (1984). If the poverty line, below which individuals are judged to be in poverty, is z_p , the poverty measure, P_r , is defined, in terms of individuals, as:

$$P_r = \frac{1}{\sum_{i=1}^n n_i} \left[\sum_{z_i \leq z_p} n_i \left(\frac{z_p - z_i}{z_p} \right)^r \right] \quad (11)$$

It would be possible to define poverty measures in terms of households or equivalent adults, but the present analysis concentrates on individuals. The measure P_0 corresponds to the widely used headcount measure of poverty, while P_1 is equal to P_0 multiplied by $1 - \bar{z}_p / z_p$, where \bar{z}_p is the average of those below the poverty line. Hence P_1 depends on the average depth of poverty as well as the number of individuals below the poverty line. It can be shown that P_2 depends on the coefficient of variation of those in poverty, as well as P_1 and P_0 . In the case of poverty measures, there is no direct link with a social welfare function, as there is with the inequality measures.¹³

¹³ For example, it would be convenient if social welfare could be expressed as average 'income', less the cost of inequality, less the cost of poverty. For further discussion and references, see Creedy (1997).

3 Empirical Analysis for New Zealand

This section analyses the sensitivity of New Zealand's inequality and poverty measures to the parameters of the equivalence scale, $m_i = (n_{a,i} + \theta n_{c,i})^\alpha$ and to the chosen unit of analysis. The analysis is conducted using data on the weekly expenditure of households, as opposed to incomes. The use of expenditure data may be thought to eliminate to some extent the effects of short term variations in income; on the use of expenditure rather than income data, see Blundell and Preston (1994, 1997) and Attanasio and Japelli (1997).¹⁴ The main emphasis of the present paper is to consider the sensitivity of measures to alternative scales and units of analysis, rather than attempting to provide an exhaustive study of inequality and poverty.¹⁵ From the Household Economic Survey, household expenditure data for the years 1995, 1996, 1997, 1998 and 2001, were adjusted to 2001 prices using the consumer price index (CPI).¹⁶ The surveys were then pooled to form one large data base containing the weekly total expenditure of each household along with information about household structure.

3.1 Inequality Measures

The results of the sensitivity analysis for the Atkinson inequality measure are displayed in Figures 1 to 6, which show inequality against α , the economies of scale parameter, for four values of θ , the weight attached to children. Figures 1 to 3 use the individual as the unit of analysis, while Figures 4 to 6 use the equivalent adult. For both types of income unit, three levels of aversion to inequality are considered. Inequality obviously increases as the degree of inequality aversion, ε , is raised.

Coulter et al (1992) found that increasing the value of α has two opposing effects on measures of inequality. The first is the concentration effect whereby α is inversely related to inequality. As the value of α is increased from low values, economies of scale are reduced and as a result equivalent income will fall proportionately more for relatively larger households. It is known that income and total expenditure are positively correlated with household size. This implies that relatively richer households incur proportionately greater falls in equivalent income. The rise in α therefore has an equalising effect.

Over low values of α , Figures 1 to 6 all display the inverse relationship produced by the concentration effect. However, over higher values of α , inequality is seen to rise with α , producing a U-shaped inequality profile. The positive relationship between α and inequality may be attributed to a reranking effect, whereby the rank-order of households (when ranked by equivalent income) changes. The proportionately larger fall in equivalent income for the larger households, as α increases from a relatively high value, eventually leads to the kind of reranking identified by Coulter et al (1992). The figures show that over

¹⁴ Trigger (2003) reviews alternative approaches to the 'income' concept.

¹⁵ For example, in calculating the inequality measures, the unweighted sample observations are used. In a study in which precise levels were the main focus, it may be desirable to use the survey weights; however, the vast majority of these lie within a narrow range; see Creedy and Tuckwell (2003).

¹⁶ Unfortunately no surveys were carried out in 1999 and 2000.

Inequality Sensitivity - Unit of Analysis: Individual

Figure 1 - $\varepsilon = 0.2$

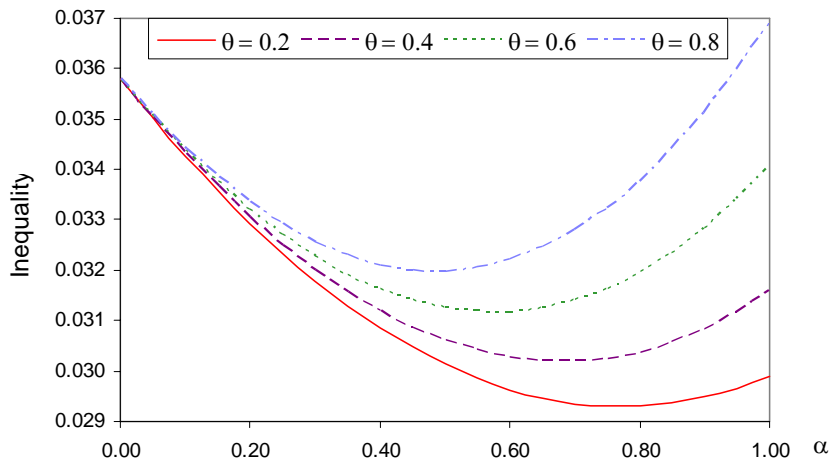


Figure 2 - $\varepsilon = 0.6$

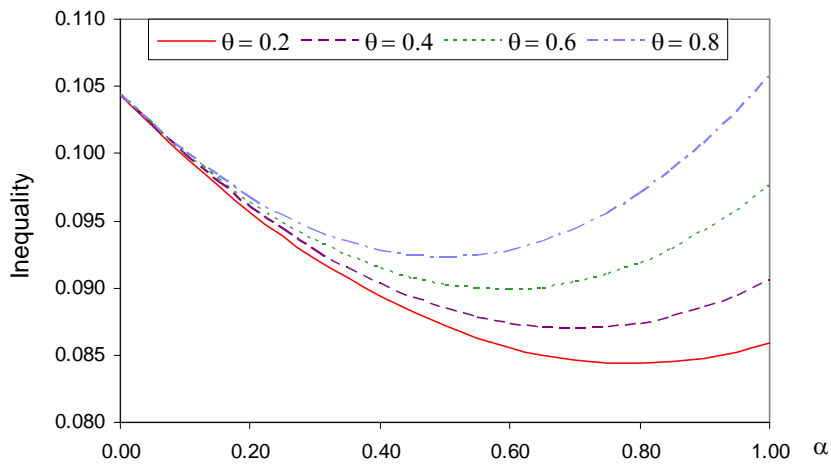
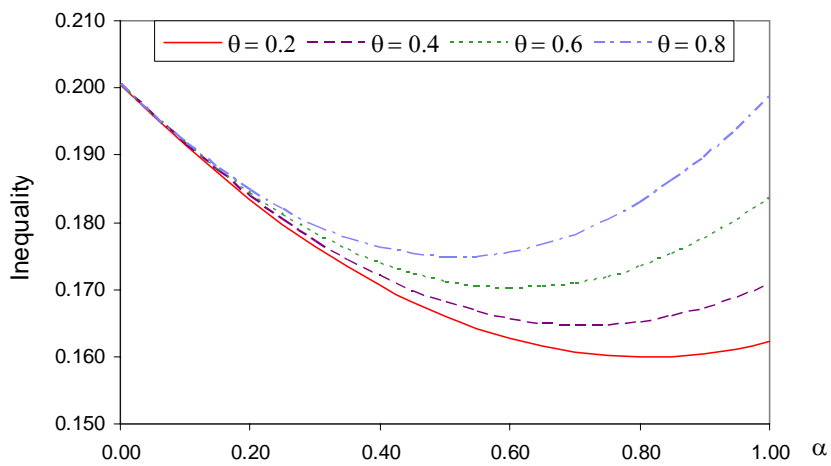


Figure 3 - $\varepsilon = 1.2$



Inequality Sensitivity - Unit of Analysis: Equivalent Adult

Figure 4 - $\varepsilon = 0.2$

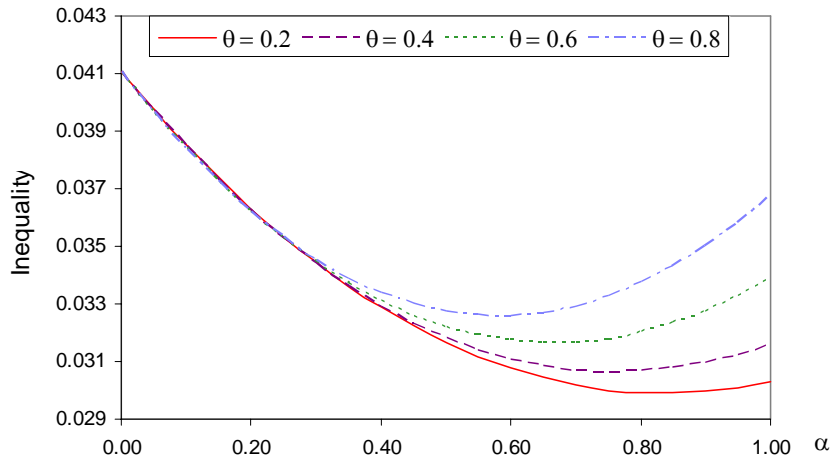


Figure 5 - $\varepsilon = 0.6$

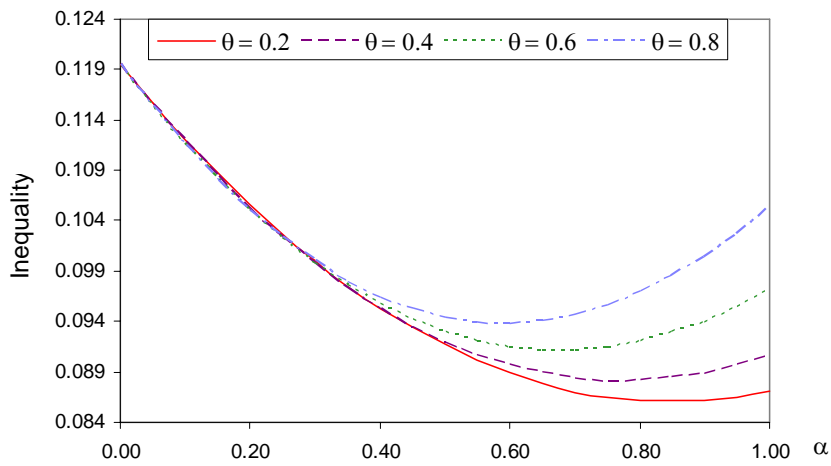
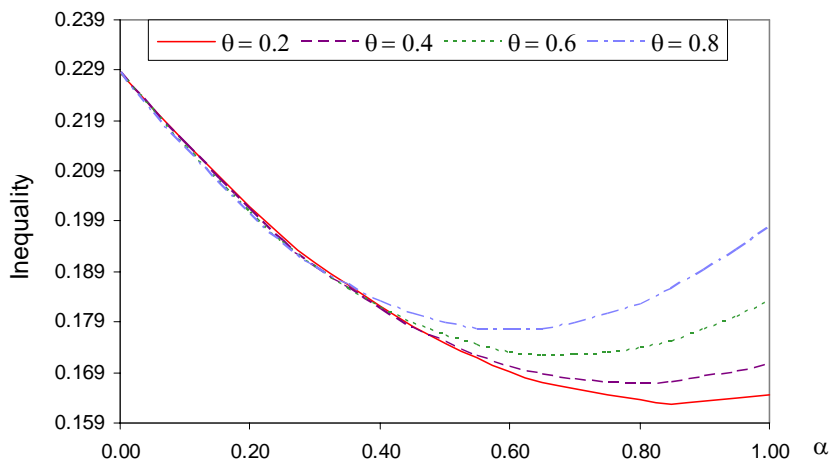


Figure 6 - $\varepsilon = 1.2$

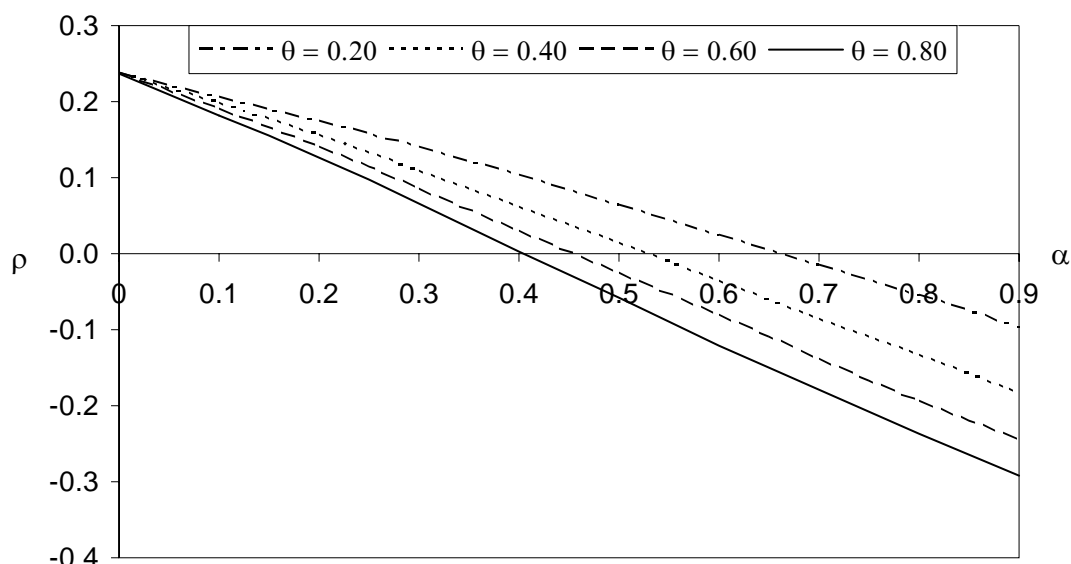


higher values of α , the reranking effect dominates the concentration effect, thereby causing inequality to rise. As observed by Jenkins and Cowell (1994), the reranking effect increases with the parameter θ and as a result, the inequality profiles for higher values of θ show much greater curvature.

The phenomenon of reranking is in fact closely related to the situation in which the correlation between equivalent income and household size becomes negative. Coulter et al (1992, p.1073) state that the reranking occurs, for the case where $\theta=1$, when the scale parameter α exceeds the inverse of the elasticity of household size with respect to income.¹⁷ Although total income, y_i , is known to be positively correlated with household size, n_i , the correlation between equivalent income, z_i , and household size, n_i , is parameter dependent. The appendix derives the condition under which the correlation coefficient, ρ , between z_i and n_i is negative, for the case where income y and household size are jointly lognormally distributed and $\theta=1$. This turns out to depend on the regression coefficient in the log-linear relationship, and so is precisely the same as the ‘elasticity’ condition mentioned above.

However, the present empirical analysis allows also for variations in the weight attached to children. Figure 7 shows the correlation coefficient, ρ , between z_i and n_i as α varies, for four values of θ and for the case where the unit of analysis is the individual. The correlation is initially positive, but clearly falls as economies of scale are reduced, and it eventually becomes negative. Furthermore, the correlation falls faster for higher values of the parameter θ . This in turn causes the correlation to turn negative (introducing reranking) earlier, so that the profile of inequality turns up earlier for higher values of θ . This is clearly reflected in Figures 1 to 6.

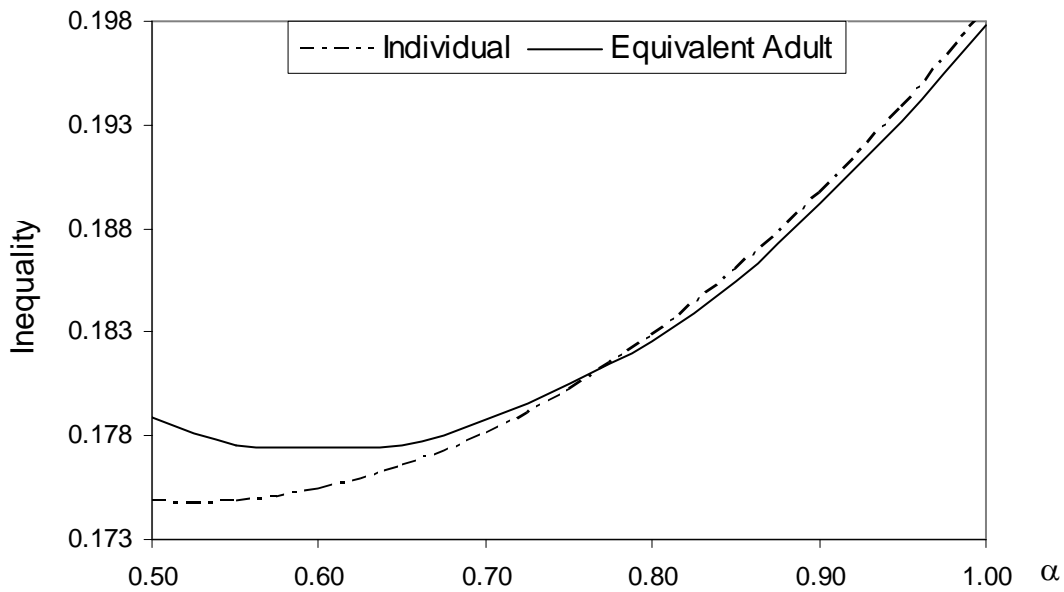
Figure 7 – Correlation Between Equivalent Income, z_i and Household Size, n_i



¹⁷ They suggest that this finding follows from a result established by Kakwani (1980) on the relationship between Lorenz and concentration curves.

The correlation also affects comparisons between the inequality profiles for different income units. For any given household, $m_i \leq n_i$, which leads the equivalent adult unit to give proportionately more weight to smaller households when compared with the individual unit. As α rises and the correlation between z_i and n_i falls, smaller households enjoy increasingly larger equivalent incomes relative to larger households. Consequently, as α rises, inequality measures based on the equivalent adult unit fall relative to those based on the individual unit. This is clearly seen in Figure 8, which shows the inequality measures for the equivalent adult and individual units in the case of an inequality aversion coefficient of 1.2 when the weight attached to children is $\theta = 0.8$.

Figure 8 – Inequality Measures, $\varepsilon = 1.2$ and $\theta = 0.8$



Another finding from the sensitivity analysis is that, for all values of α and for a given unit of analysis, inequality is positively related to the weight attached to children, θ . This result is independent of the inequality aversion coefficient and the chosen unit of analysis. This feature was suggested by Banks and Johnson (1994), but Cowell and Jenkins (1994, pp. 892-893) argued that the relationship need not necessarily be monotonic, although it ‘may be difficult to characterise precisely from theoretical analysis alone’. However, some insight may be obtained as follows. The adult equivalent size of a household may be written, where for convenience subscripts for the household have been omitted, as:

$$m = l^\alpha \tag{12}$$

where:

$$l = n_a + \theta n_c \tag{13}$$

Assuming that the number of adults and the number of children are independent of each other, the variance of l is described by:¹⁸

$$\sigma_l^2 = \sigma_{na}^2 + \theta^2 \sigma_{nc}^2 \tag{14}$$

¹⁸ Allowing for a positive correlation strengthens the effect of θ .

Hence σ^2_l rises with θ .

Taking logs of equation (12) gives $\log m = \alpha \log l$ and the variance of $\log m$ is thus:

$$\sigma^2_{\log m} = \alpha^2 \sigma^2_{\log l} \quad (15)$$

Movements in σ^2_l and $\sigma^2_{\log l}$ are monotonic, so the rise in σ^2_l and hence in $\sigma^2_{\log l}$ as a result of an increase in θ leads $\sigma^2_{\log m}$ to increase. It is then necessary to consider the effect of such an increase on the dispersion of equivalent income, given by $z = \frac{y}{m}$. Taking logs gives $\log z = \log y - \log m$. The variance of logarithms of equivalent income is therefore:

$$\sigma^2_{\log z} = \sigma^2_{\log y} + \sigma^2_{\log m} - 2 \text{cov}_{\log y, \log m} \quad (16)$$

Hence, the dispersion, measured by the variance of logarithms of equivalent income, $\sigma^2_{\log z}$, rises with $\sigma^2_{\log m}$, which has been seen to rise with θ . But the covariance term is also affected positively by θ . Hence, although a positive effect has been found using the present data, it is possible in principle, over some range of parameter values, for the dispersion of equivalent income to fall as the weight attached to children increases.

3.2 Poverty Measures

The variations in P_0 and P_1 are displayed in Figures 9 to 24, which show poverty against α (the economies of scale parameter) for four values of θ (the weight attached to children). Figures 9 to 16 use the individual as the unit of analysis, while Figures 17 to 24 use the child as the unit of analysis, to gauge child poverty. For each poverty measure and unit of analysis, four absolute poverty lines, in terms of equivalent expenditure, are considered, of \$150, \$165, \$180 and \$195 per week. Clearly, poverty rises as the absolute poverty line is increased.¹⁹ As with the measures of inequality, poverty also rises as a greater weight is attached to children.

All figures show that poverty strictly rises with α .²⁰ Jointly considering the behaviour of the inequality and poverty measures, it may appear strange that inequality can fall at the same time as poverty rises, which is observed over low values of α . However the two measures reflect separate effects on the distribution of equivalent income, z , of changes in the scale parameter. Changes in poverty are dominated by shifts in the distribution of equivalent income, while inequality changes are dominated by changes in its dispersion.

¹⁹ The use of absolute poverty lines differs from the variable poverty lines used by Coulter et al, who allow the poverty line to depend on equivalent adult size. Consequently their results are not directly comparable with those shown here.

²⁰ This is a feature of the present data, as the relationship is not necessarily monotonic.

Poverty Sensitivity – Unit of Analysis: Individual, Poverty Measure: P_0

Figure 9 - $z_p = \$150$

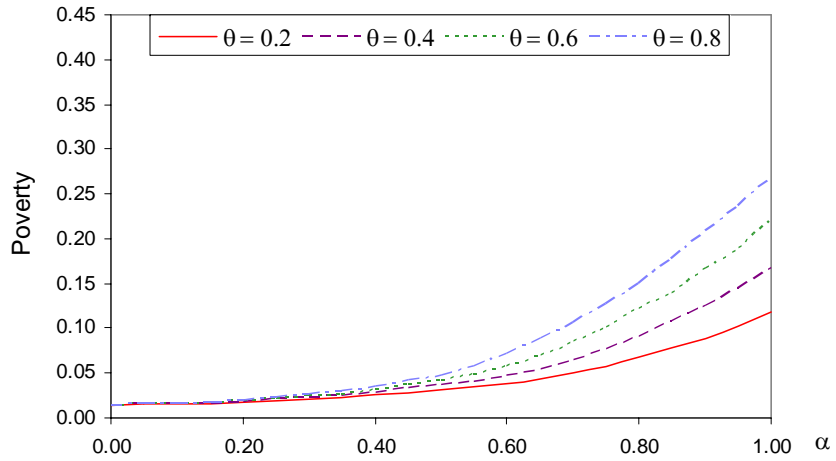


Figure 10 - $z_p = \$165$

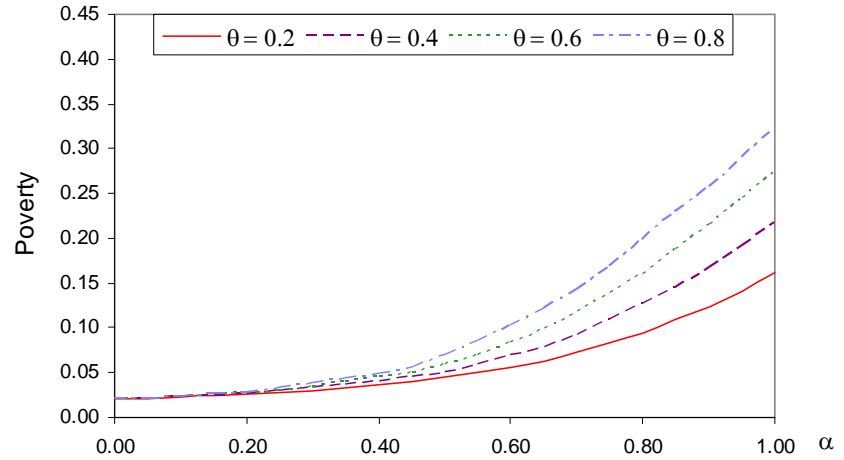


Figure 11 - $z_p = \$180$

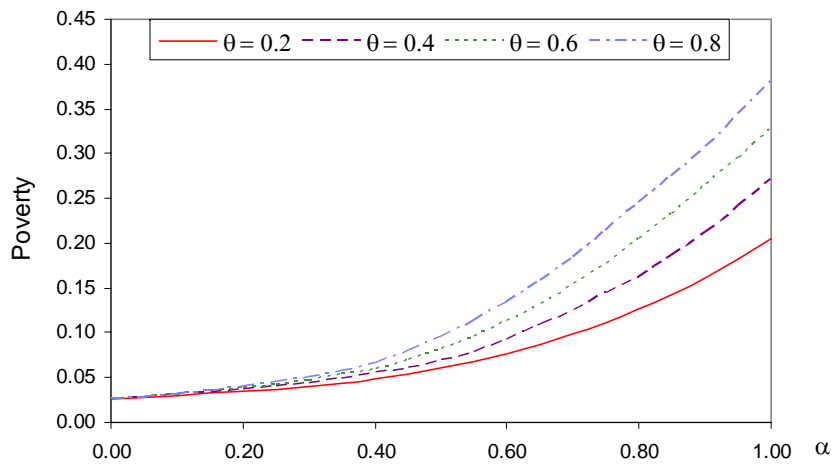
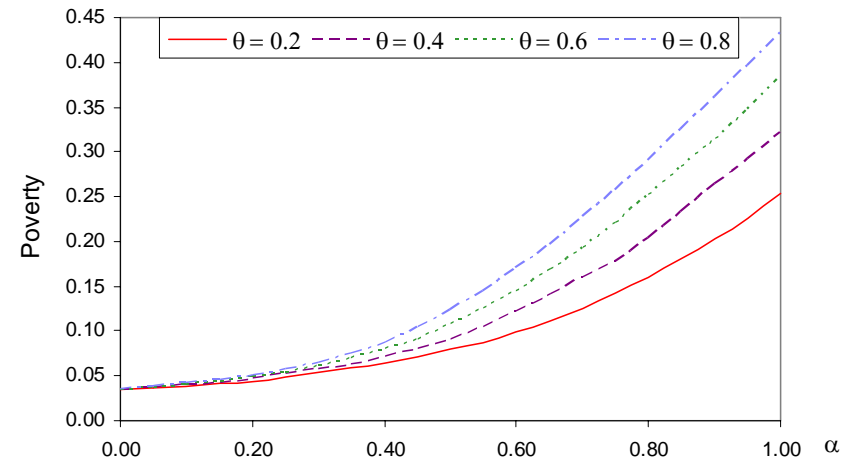


Figure 12 - $z_p = \$195$



Poverty Sensitivity – Unit of Analysis: Individual, Poverty Measure: P_1

Figure 13 - $z_p = \$150$

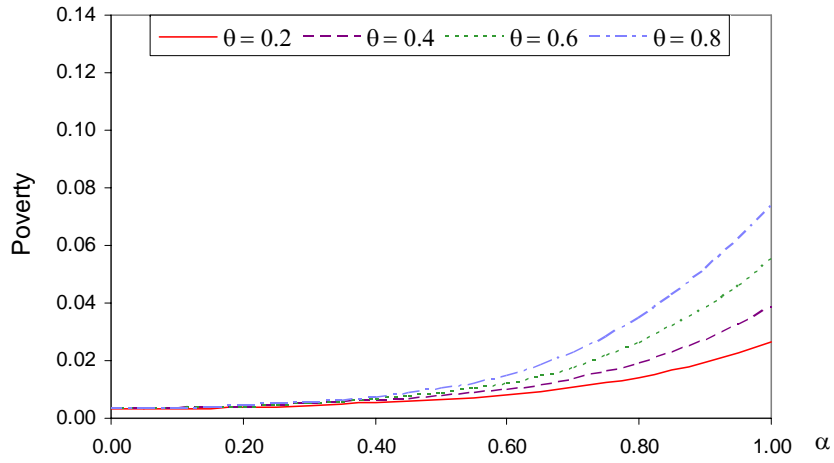


Figure 14 - $z_p = \$165$

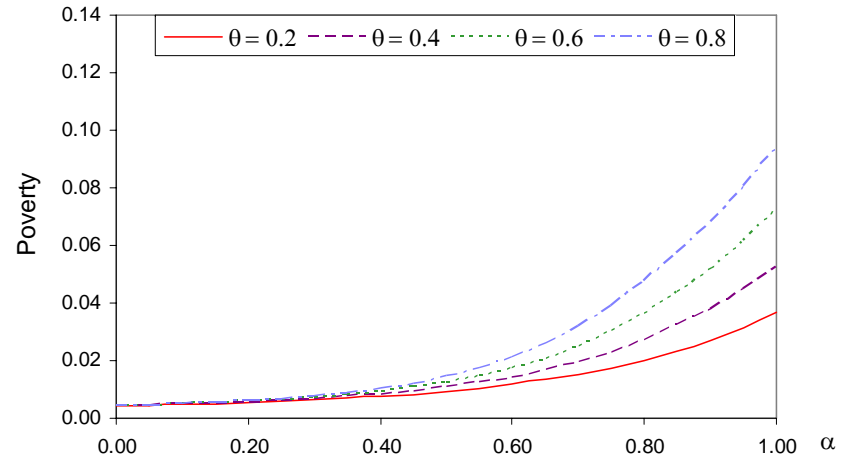


Figure 15 - $z_p = \$180$

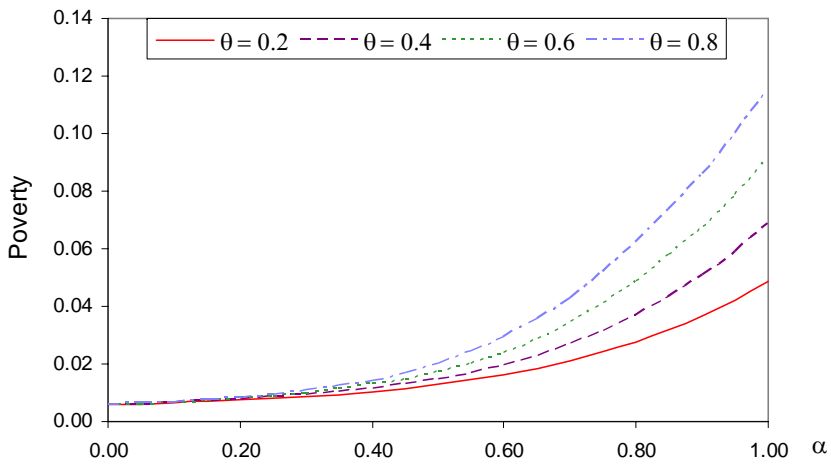
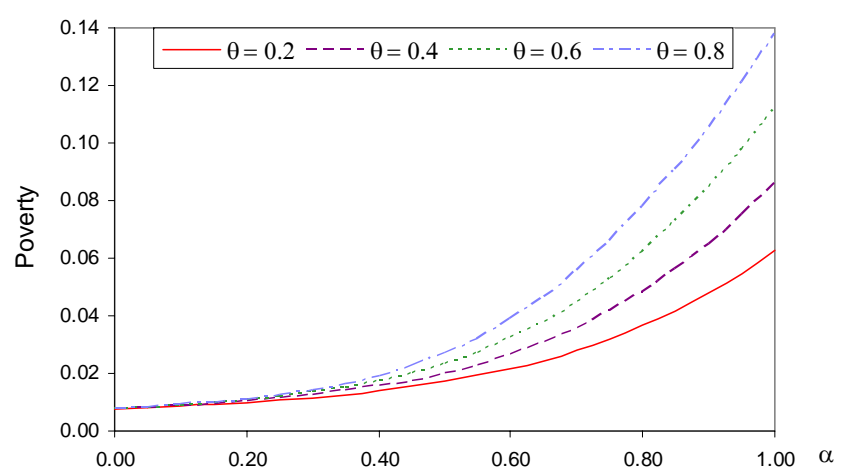


Figure 16 - $z_p = \$195$



Poverty Sensitivity – Unit of Analysis: Child, Poverty Measure: P_0

Figure 17 - $z_p = \$150$

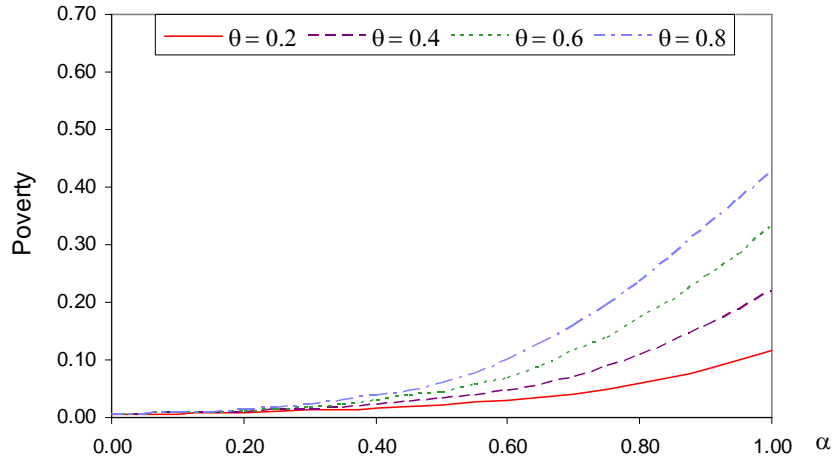


Figure 18 - $z_p = \$165$

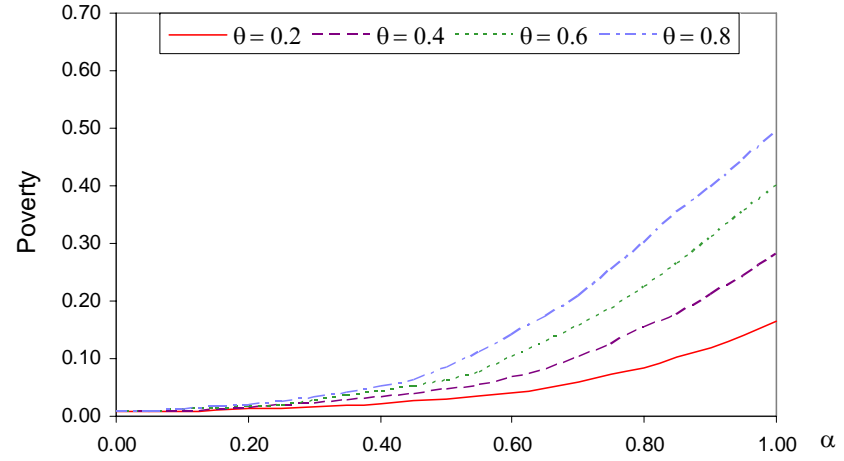


Figure 19 - $z_p = \$180$

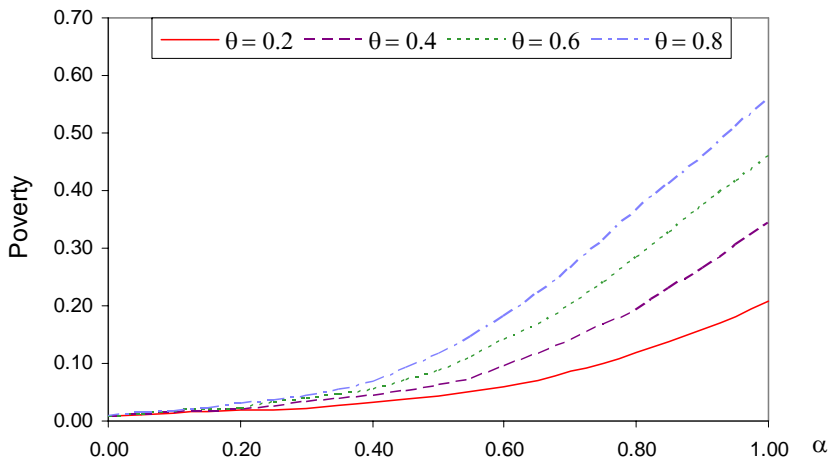
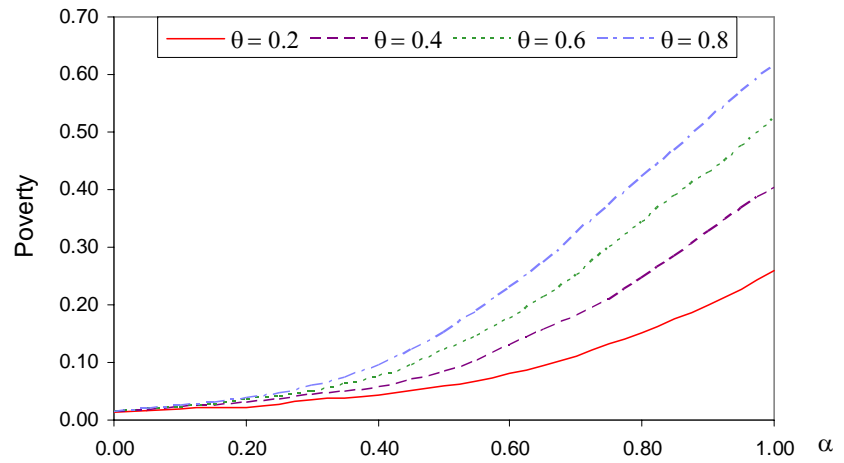


Figure 20 - $z_p = \$195$



Poverty Sensitivity – Unit of Analysis: Child, Poverty Measure: P_1

Figure 21 - $z_p = \$150$

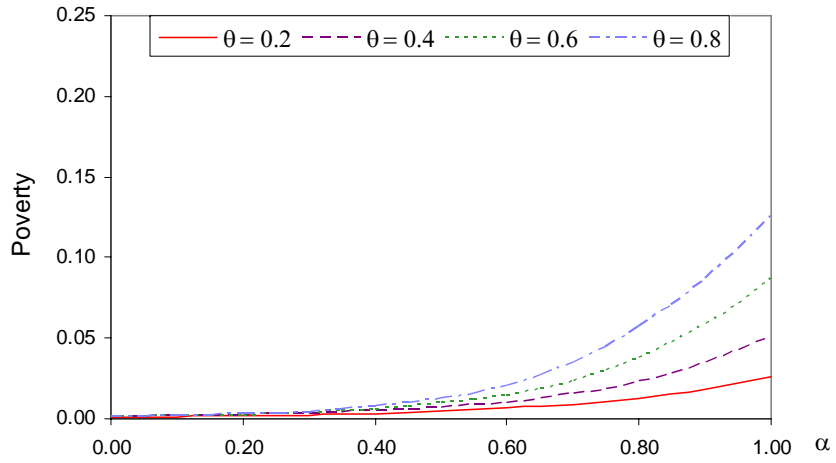


Figure 22 - $z_p = \$165$

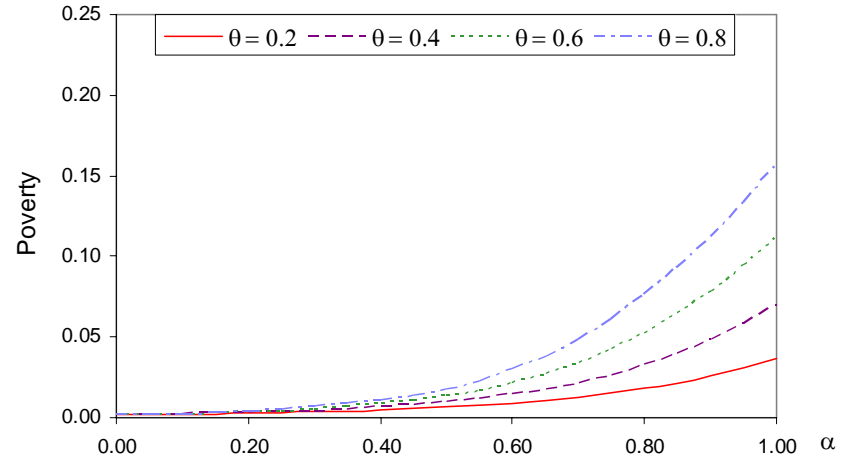


Figure 23 - $z_p = \$180$

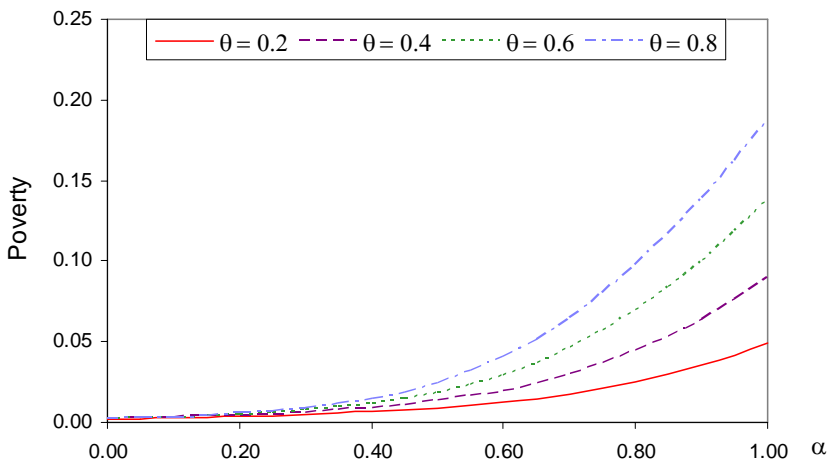
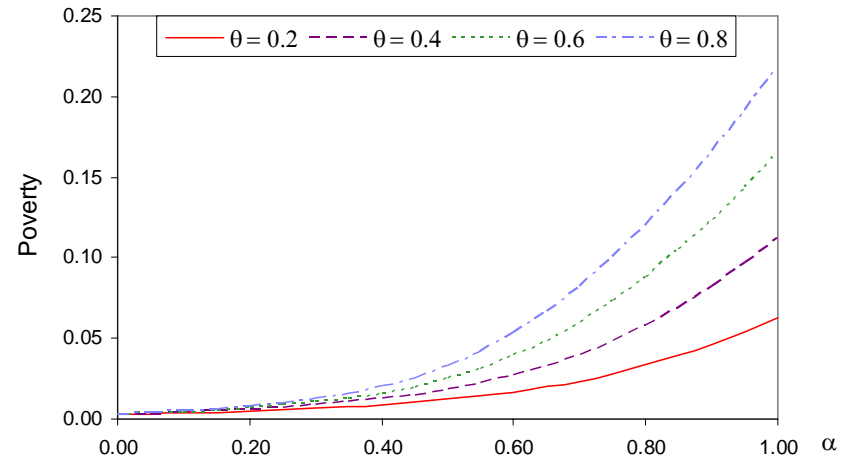
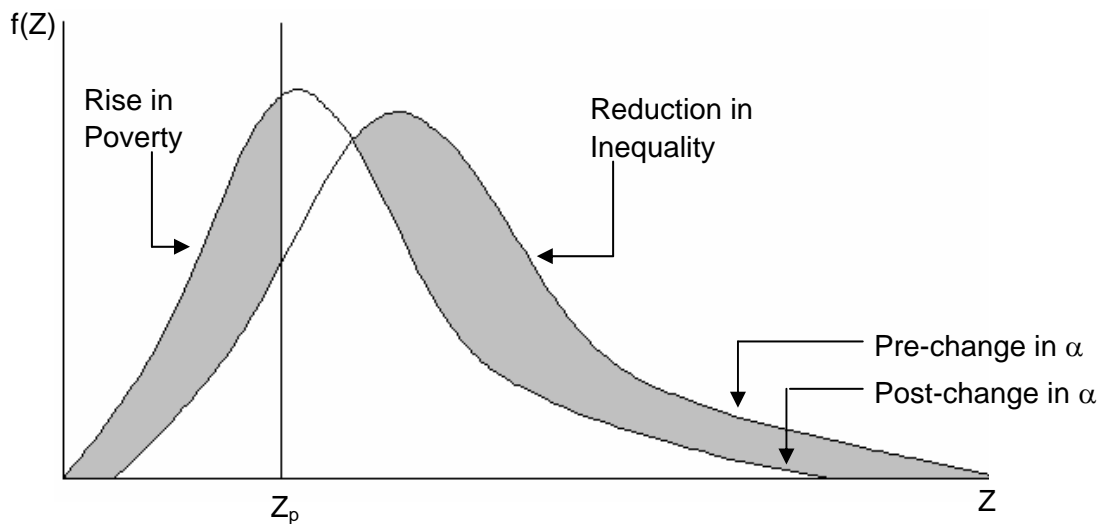


Figure 24 - $z_p = \$195$



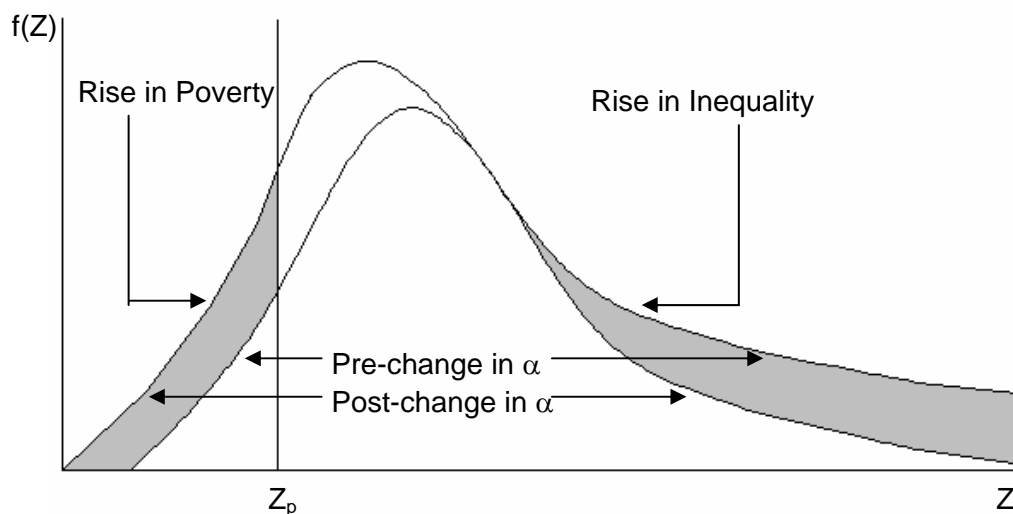
When α is increased over low values, the concentration effect causes the distribution of equivalent income to become less skewed as inequality falls. At the same time poverty rises, as a greater area of the distribution falls below the poverty line. The behaviour of the distribution of equivalent income, z , over low values of α is shown in Figure 25.

Figure 25: Behaviour of the Frequency Distribution of Equivalent Income, z , When α is Increased over Low Values



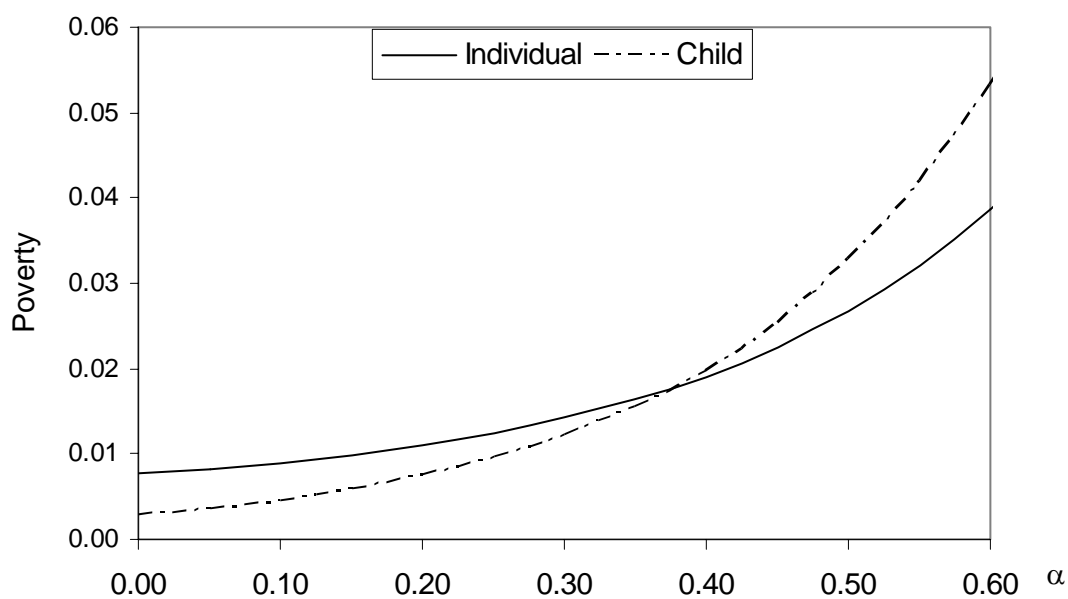
Once reranking (associated with the negative correlation between equivalent income and household size, discussed above) takes effect, increases in α cause inequality to rise, and the distribution of equivalent income becomes increasingly skewed. Again, poverty rises as the distribution continues to shift leftwards. Figure 26 shows how the distribution of equivalent income changes as α is increased over higher values.

Figure 26: Behaviour of the Frequency Distribution of Equivalent Income, z , When α is increased over High Values



It is also of interest to consider whether poverty is higher where the child, as opposed to the individual, is used as the unit of analysis. From the figures, it is clear that poverty is indeed higher at the higher ranges of α . It is known that over the higher ranges there is a negative correlation between the equivalent income and the household size. Therefore larger households, those with more children, are likely to have lower equivalent incomes over the relevant range. A focus on only the children as the population group thus places relatively more units below any given poverty line. However, for lower values of α it has been seen above that there is a positive correlation between equivalent income and household size. Hence it is possible for poverty to be lower when the focus is on children only, compared with the use of all individuals. An example is shown in Figure 27, in which the P_1 poverty measure is provided for both the individual and child units of analysis for a poverty line of \$195 where the weight attached to children is $\theta = 0.8$.

Figure 27 – Poverty Measures, P_1 , $z_p = \$195$ and $\theta = 0.8$



4 Equivalence Scales and Direct Taxation

Previous sections have examined inequality and poverty measures based on the distribution of total household expenditure. The present section considers the role of equivalence scales and the unit of analysis in the context of the redistributive effect of direct taxation.

Aronson and Lambert (1994) decomposed the redistributive effect of taxation, L , in terms of the Gini coefficient, into three components, which they describe as the vertical, horizontal, and reranking effects. The vertical effect measures the progressivity of the effective tax schedule, which incorporates no horizontal or reranking effects and is derived from the actual tax schedule by allocating to each individual the average tax paid by the respective pre-tax equals. The horizontal effect relates to the unequal treatment of equals, and reranking captures the presumably unintended inequitable treatment of unequals by the tax system.²¹ The concept of reranking is therefore quite different from that discussed earlier, which was interpreted in terms of a negative correlation between equivalent income and household size.

Using slightly different notation from above, suppose that the tax and transfer system is such that post-tax expenditure, y , is given by $y = x - T(x)$. Divide the population into N groups. Within each group individuals have similar (or 'near equal') pre-tax values of x , x_k for $k = 1, \dots, K$. Groups are ranked in ascending order. Aronson et al (1994) showed that the reduction in the Gini measure of inequality, L , is given by:

$$L = V - H - R \quad (17)$$

where L , the redistributive effect, is the difference between Gini measures of pre- and post-tax incomes:

$$L = G_x - G_y \quad (18)$$

The vertical redistribution, V , is the difference between the Gini measure of pre-tax income and the between-group Gini measure of post-tax income, obtained when each individual's pre-tax income is adjusted by the average tax paid by their respective pre-tax equals (that is, each individual is given the average post-tax income in the group). Hence:

$$V = G_x - G_{B,y} \quad (19)$$

The horizontal inequity, H , is given by:

$$H = \sum_{k=1}^K a_{k,y} G_{k,y} \quad (20)$$

²¹ The treatment of Aronson and Lambert (1994) was in terms of income taxation, but the method has been applied to indirect taxes by Decoster et al (1997a, b). An application of the approach suggested below to indirect taxes is in Creedy (2002) and Creedy and van de Ven (2001a) which apply the decomposition in a lifetime context.

where $a_k = N_k^2 \mu_k / N^2 \mu$ and is the product of the population and income shares of group k and $G_{k,y}$ is the within-group Gini inequality measure. Finally, R is the measure of reranking, given by:²²

$$R = G_y - C_y \quad (21)$$

where C_y is the concentration measure of post-tax income, obtained as a Gini-type measure, but with post-tax incomes ranked by x .

In practice few exact pre-tax equals are observed in survey data, so the problem arises of selecting an appropriate group class width. This issue was examined by van de Ven et al (2001), who showed that the measured vertical effect initially increases as the class width is increased, and then falls after reaching a maximum. This suggests a strategy whereby the class width used to combine individuals into groups of near-equals is chosen as the value that maximises the estimated vertical effect. The reranking measure, R , can be obtained directly using the ungrouped values and is therefore not affected by the choice of class width. The horizontal effect can then be obtained as a residual using $H = V - R - L$.

In the present context the issue involves the effect of alternative adult equivalence scales on the various components of redistribution. In order to concentrate on direct taxes and transfers, the following results were obtained using information on the pre-tax annual incomes and disposable incomes of households in the 2001 Household Economic Survey.

Figure 28 shows, for four values of θ , the variation in reranking, expressed as a percentage of the redistributive effect, when individuals are regarded as the basic unit of analysis (income per adult equivalent is weighted by the number of individuals in the household). These results show that the degree of reranking is relatively low. The horizontal inequity measures were found to be negligible, being in virtually all cases less than 0.1 percent of redistribution; for this reason they are not reported here. The profiles of reranking with variations in α are U-shaped for the lower value of θ and become J-shaped for higher values. For the range of values displayed here, reranking is lower for lower θ , the difference increasing for the higher α . However, as θ is reduced further, below the lowest profile shown, the degree of reranking begins to increase: hence a reranking minimising set of equivalence scales exists for which $\theta > 0$.

Figure 29 shows a similar pattern of reranking in the case where the number of equivalent adults is used as the weight for each household, that is, the basic income unit is the 'equivalent adult'. While the variations in reranking are similar to those found in Figure 28, the values, as a percentage of redistribution, are systematically slightly lower.

Reranking of unadjusted incomes is of course a deliberate aim of the tax and transfer system. This is precisely because the size and composition of households are considered as relevant non-income characteristics; value judgements about the desirable redistribution arising from taxes and transfers are closely linked with such differences. In considering the tax 'treatment of equals', the equivalence scales determine the meaning attached to 'equality'. It may be suggested that, since reranking works against redistribution, an implicit set of equivalence scales, reflecting the value judgements of

²² See Atkinson (1979) and Plotnick (1981)

policy makers, is found as the set that minimises reranking.²³ Such reranking cannot be expected to be zero, given that some can also arise as the result of government policy that is tangential to equity objectives. For example, unemployment benefits may be designed to encourage labour market participation, or certain types of income may be treated differently on efficiency grounds. It was mentioned above that a reranking-minimising set of scales exists: where individuals are used as weights, this arises for θ at around 0.05 and for α at around 0.45. These values, as shown below, are quite different from those generally used in empirical studies of inequality and poverty.

Figure 28 - Reranking with Individuals as Income unit

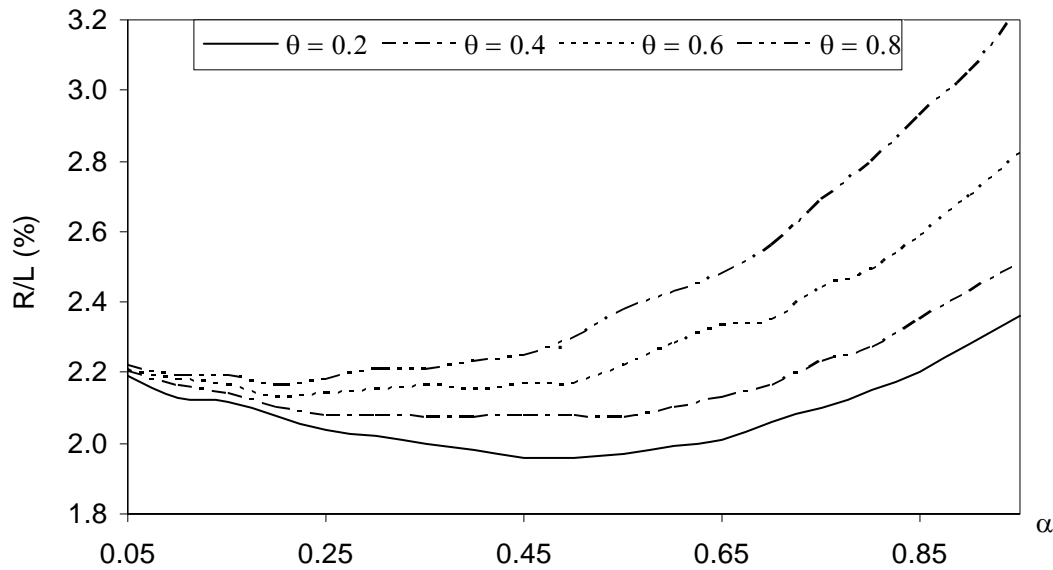
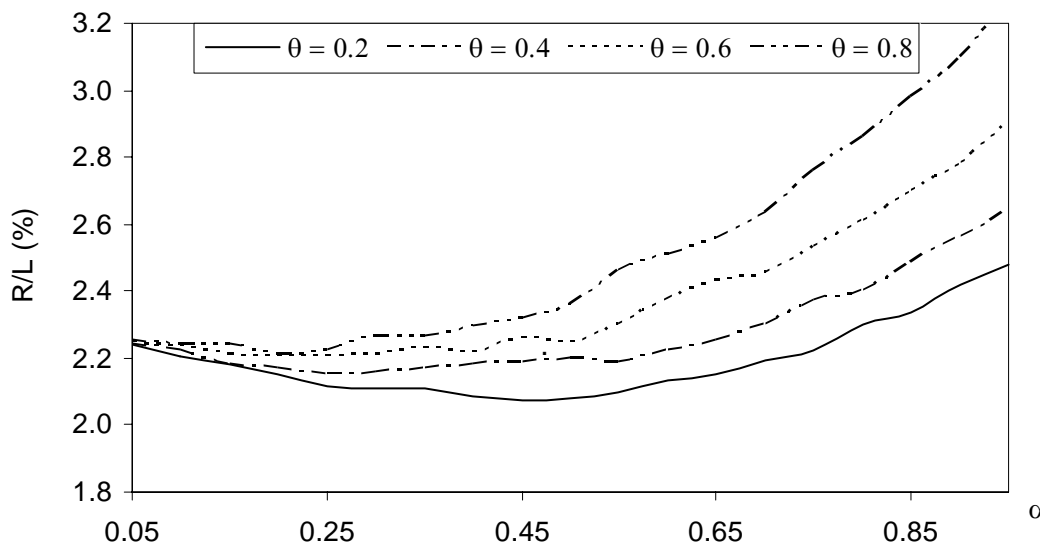


Figure 29 – Reranking with Equivalent Adults as Income unit



²³ van de Ven and Creedy (2003) found that the implicit scales are likely to be in the region of the reranking-minimising scales, though they may not be precisely the same.

5 Alternative Equivalence Scales

Inequality and poverty measures were analysed in section 3 using the equivalence scale:

$$m_i = (n_{a,i} + \theta n_{c,i})^\alpha \quad (22)$$

However, this scale is just one of many which are used in policy evaluation. This section contrasts the inequality and poverty measures which arise from using different equivalence scales. Table 1 describes the 29 equivalence scales analysed; there is no suggestion that this is exhaustive as they consist of an arbitrary selection concentrating largely on New Zealand and Australia. The scales are grouped according to the regions for which they were designed and the sources from which they were obtained.²⁴

As the equivalence scales are based on a variety of different approaches and functional forms, they are not directly comparable. To overcome this, for each scale the equivalent sizes, m_i , were used to fit the two-parameter form. This was achieved by carrying out regressions using:

$$\log m_i = \beta + \alpha \log(n_{a,i} + \theta n_{c,i}) + \psi_i \quad (23)$$

As this is nonlinear in the parameters, regressions were carried out for a range of θ values, and the value producing the highest R^2 was taken as the estimate, with the corresponding α value. Table 1 shows the value of R^2 from these regressions together with the estimates of the parameters, θ and α . Where NA is given in the R^2 column (1, 2, 15 and 16), the coefficients directly apply to the two-parameter functional form, so no estimation was required. As shown by the high values of R^2 , the two-parameter form of equivalence scales provides a very good fit for all scales analysed; the lowest value of R^2 is 0.956.

Table 1 shows substantial variation in the parameter estimates, θ and α . The weight attached to children, θ , ranges from 0.300 (scale no. 25) to 0.916 (scale no. 15), while the parameter reflecting economies of scale, α , ranges from 0.395 (scale no. 14) to 1.014 (scale no. 13). Due to the nonlinear nature of the poverty and inequality profiles, observed variations in α and θ provide little indication as to the variation in inequality and poverty measures that would result from the different equivalence scales. For this reason, each scale was in turn applied to the New Zealand expenditure data, from which inequality and poverty measures were then calculated.

Figure 30 shows the inequality profiles produced using the equivalent adult unit of analysis and an inequality aversion coefficient of 1.2, for four different values of θ . Plotted on this figure are the measures of inequality produced using each of the 29 equivalence scales. There is clearly a considerable range of inequality measures resulting from these scales. The two extremes are provided by scales 25 and 11, where inequality for the latter is 18 percent higher than the former. In terms of inequality comparisons over time and for tax policy microsimulation studies, this order of magnitude is very large. Figure 30 shows that a number of the scales differ mainly in the size of α and lie roughly on the relatively flat section of the inequality profiles, where inequality is not particularly

²⁴ As mentioned above, the scales are price independent, but of course differences in relative prices between countries may be thought to affect their applicability to more than one country.

Table 1 – Equivalence Scales Analysed and Regression Results

Scale No.	Title	R ²	θ	α
New Zealand				
<i>Brashares, Edith and Aynsley, Maryanne (October 1990) Income Adequacy Standards for New Zealand</i>				
1	Jensen's 1978 Equivalence Scale (Annex 5, p.1)	NA	0.781	0.737
2	Jensen's 1988 Equivalence Scale (Annex 5, p. 2)	NA	0.730	0.621
<i>Michelini, Claudio (April 1999) New Zealand Household Consumption Equivalence Scales from Quasi-Unit Record Data</i>				
3	Commodity-specific and household-type equivalence scales: ELES model (p.14, Table 1 - Total Expenditure)	0.999	0.490	0.711
4	Estimates of the household Equivalence Scales for total consumption when the equivalence parameter m_o is commodity-invariant - PS-AID(θ_i) (p.17, Table 2 - PS-AID(θ_i))	0.987	0.620	0.949
5	Estimates of the household Equivalence Scales for total consumption when the equivalence parameter m_o is commodity-invariant - PS-QAID(θ_i) (p.17, Table 2 - PS-QAID(θ_i))	0.991	0.650	0.896
6	Estimates of the household Equivalence Scales for total consumption when the equivalence parameter m_o is commodity-invariant - PS-QAID(θ_i) - H (p.17, Table 2 - PS-QAID(θ_i) - H)	0.956	0.720	0.622
7	Estimates of the household Equivalence Scales for total consumption when the equivalence parameter m_o is commodity-invariant - EPS(α)-QAID(θ_i) (p.17, Table 2 - EPS(α)-QAID(θ_i))	0.999	0.670	0.782
8	Estimates of the household Equivalence Scales for total consumption when the equivalence parameter m_o is commodity-invariant - EPS(α)-QAID(θ_i) ^d (p.17, Table 2 - EPS(α)-QAID(θ_i) ^d)	0.999	0.670	0.775
9	Estimates of the household Equivalence Scales for total consumption when the equivalence parameter m_o is commodity-invariant - EPS(β, λ)-QAID(θ_i) (p.17 Table 2 - EPS(β, λ)-QAID(θ_i))	1.000	0.580	0.799
10	Estimates of the household Equivalence Scales for total consumption when the equivalence parameter m_o is commodity-invariant - EPS(β, λ)-QAID(θ_i) ^d (p.17 Table 2 - EPS(β, λ)-QAID(θ_i) ^d)	0.998	0.630	0.797
11	Commodity-specific and household-type equivalence scales obtained from the PS-AID(θ_{ij}) model (p.18, Table 3 - PS-AID(θ_{ij}), Tot.Expenditure)	0.992	0.890	0.904
12	Commodity-specific and household-type equivalence scales obtained from the PS-QAID(θ_{ij}) model (p.18, Table 3 - PS-QAID(θ_{ij}), Tot.Expenditure)	0.963	0.890	0.825
13	Commodity-specific and household-type equivalence scales obtained from the EPS(α)-QAID(θ_{ij}) model (p.20, Table 4 - EPS(α)-QAID(θ_{ij}), Tot.Expenditure)	0.957	0.670	1.014
14	Commodity-specific and household-type equivalence scales obtained from the EPS(β, λ)-QAID(θ_{ij}) model (p.20, Table 4 - EPS(β, λ)-QAID(θ_{ij}), Tot.Expenditure)	0.986	0.890	0.395

Scale No.	Title	R ²	θ	α
15	Heuristic Household Equivalence Scales - Easton, 1980 (p.21, Table 6 - Easton, 1980)	NA	0.916	0.606
16	Heuristic Household Equivalence Scales - Smith, 1989 (p.21, Table 6 - Smith, 1989)	NA	0.713	0.972
United Kingdom				
<i>Brashares, Edith and Aynsley, Maryanne (October 1990) Income Adequacy Standards for New Zealand</i>				
17	Townsend's Equivalence Scale (Annex 5, p.49, Table 38 - Townsend)	0.995	0.890	0.551
18	The United Kingdom Supplementary Benefit Equivalence Scale 1968/69 (Annex 5, p.49, Table 38 - Supplementary Benefit)	0.997	0.650	0.658
<i>Van de Ven, Justin (November 18, 2003) Demand Based Equivalence Scale Estimates for Australia and the UK</i>				
19	Equivalence Scales by Estimation Method for Engel Estimates for the UK (p.15, Table 3 - UK, Engel)	0.999	0.470	0.928
20	Equivalence Scales by Estimation Method for Rothbarth Estimates for the UK (p.15, Table 3 - UK, Rothbarth)	0.997	0.370	0.876
21	Equivalence Scales by Estimation Method for DS(a) Estimates for the UK (p.15, Table 3 - UK, DS(a))	1.000	0.630	0.576
22	Equivalence Scales by Estimation Method for DS(b) Estimates for the UK (p.15, Table 3 - UK, DS(b))	1.000	0.640	0.501
Australia				
<i>Brashares, Edith and Aynsley, Maryanne (October 1990) Income Adequacy Standards for New Zealand</i>				
23	Henderson Equivalence Scale (Annex 5, p.40, Table 30 - Head Working - All Costs)	0.989	0.810	0.562
<i>Van de Ven, Justin (November 18, 2003) Demand Based Equivalence Scale Estimates for Australia and the UK</i>				
24	Equivalence Scales by Estimation Method for Engel Estimates for Australia (p.15, Table 3 - Australia, Engel)	0.999	0.530	1.013
25	Equivalence Scales by Estimation Method for Rothbarth Estimates for Australia (p.15, Table 3 - Australia, Rothbarth)	0.994	0.300	0.886
26	Equivalence Scales by Estimation Method for DS(a) Estimates for Australia (p.15, Table 3 - Australia, DS(a))	1.000	0.600	0.676
27	Equivalence Scales by Estimation Method for DS(b) Estimates for Australia (p.15, Table 3 - Australia, DS(b))	0.999	0.470	0.639
OECD				
28	The OECD scale (p.172)	0.998	0.700	0.884
29	The Modified OECD scale (p.172)	0.994	0.580	0.763

responsive to changes in α . The use of only those scales would give the misleading impression that the choice of scales is not important.

It is of interest to consider the commodity specific equivalence scales produced by Michelini (1999) for New Zealand: these are scales 11, 12, 13 and 14. They all produce relatively high inequality measures. However, scale 14 is placed on the downward sloping segment of its inequality profile, while the others are on the upward sloping segment of their profiles. A comparison of those scales alone may also give the impression that inequality is not sensitive to a large variation in α , whereas intermediate values would give substantially lower measures of inequality. The scales derived from the Rothbarth estimates for both the United Kingdom and Australia, compiled by van de Ven (scales 20 and 25), give relatively lower measures of inequality.

Figure 30 – Inequality, Unit of Analysis: Equivalent Adult, $\varepsilon = 1.2$

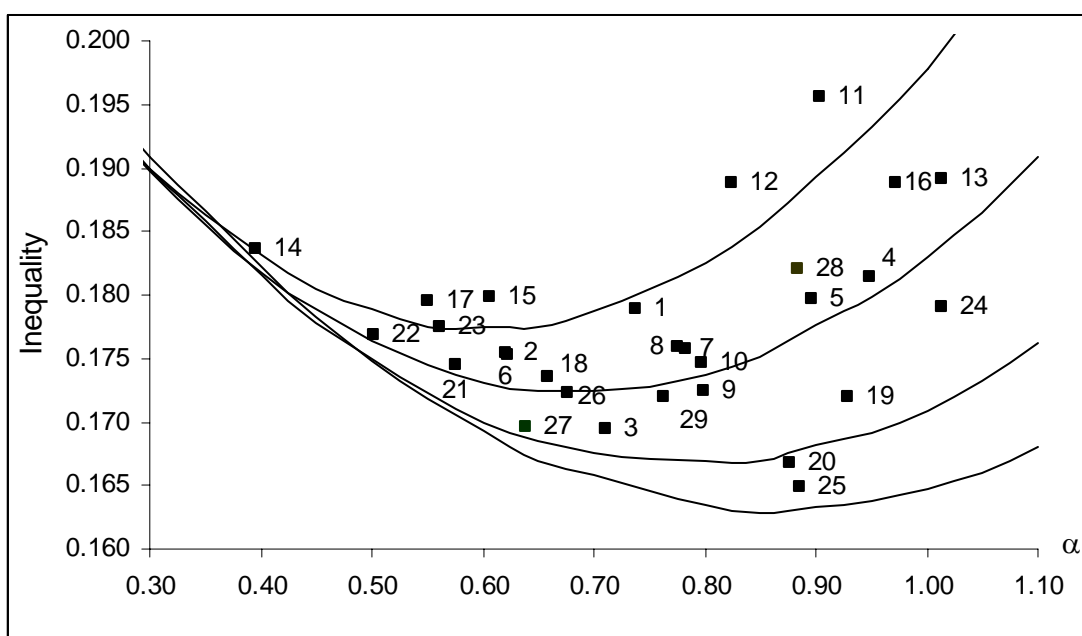
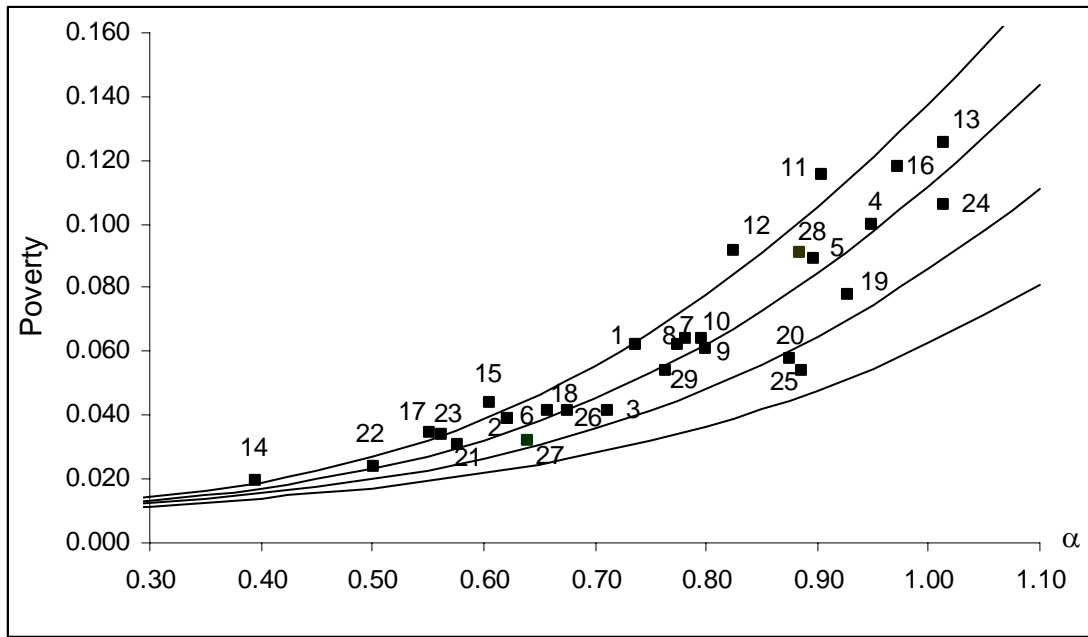


Figure 31 shows the poverty curves of the P_1 poverty measure, using the individual as the unit of analysis and a poverty line of \$195 per week. Plotted on this figure are the measures of poverty produced using each of the 29 equivalence scales. The variation in poverty is substantial. The scales giving the largest difference in inequality do not give the largest poverty differential. This is obtained for scales 14 and 13: poverty increases by over 540 percent when moving from the former to the latter scales. It can also be seen that, whereas the Michelini scales 13 and 14 produce similar inequality measures – being on different sides of the U-shaped profiles – they produce substantially different poverty measures. It is clear that considerable care needs to be taken in comparing results for alternative scales, and in particular much caution is required before declaring that analyses are not affected by the choice of scale.

Figure 31 – Poverty - P_1 , Unit of Analysis: Individual, $z_p = \$195$



6 Conclusions

This paper has examined the sensitivity of several inequality and poverty measures to the choices of the adult equivalence scale, used to adjust total household income, and the unit of analysis, along with the choice of poverty line. In considering alternative scales, extensive use was made of a highly flexible two-parameter functional form of the adult equivalence scale, allowing for economies of scale and a separate weight added to children. Three units of analysis (or weights in computing summary measures), namely the household, the equivalent adult and the individual were examined. The use of individuals was seen to be consistent with an anonymity principle, while the use of the number of equivalent adults is consistent with the principle of transfers.

The main empirical comparisons were for New Zealand, using pooled information about total household expenditure, from several Household Economic Surveys. The results demonstrated the sensitivity of New Zealand's inequality and poverty measures to the parameters of the equivalence scales and, in addition, to the chosen unit of analysis. Profiles of inequality against the economies of scale parameter, for a given weight attached to children, were found to be U-shaped, consistent with other studies. The role of the correlation between the total expenditure per adult equivalent and the size of the household was found to be crucial in generating the U-shape. A negative correlation (despite the positive correlation between total household expenditure and household size) is more likely, the lower is the weight attached to children and the higher is the economies of scale parameter. It was shown that a negative correlation is equivalent to the 'reranking', identified by Coulter et al (1992), that arises as the scale parameter increases. The profiles of inequality and poverty for individuals and equivalent adults as the unit of analysis (or weights) were found to intersect over a range of parameter values. For poverty measures, the profiles of poverty with the economies of scale parameter were upward sloping, with a higher scale parameter increasing poverty over the whole range in all cases.

The effect of alternative equivalence scales and income units on the reranking arising from the direct tax system were also examined. Using a decomposition of the redistributive effect of taxation, horizontal inequity was found to be extremely small in all cases. Profiles of reranking for an increasing scale parameter were found to be J-shaped. A reranking minimising set of parameters were investigated, and were found to be substantially lower than the scales commonly used in New Zealand.

Finally, a wide range of equivalence scales designed for New Zealand, the United Kingdom, Australia and the OECD were examined. The two-parameter form was found to provide a very good fit to these scales. The scales were applied to the New Zealand data and the resulting inequality and poverty measures were contrasted.

The results demonstrate that considerable care needs to be taken in the choice of adult equivalence scales and the income unit. Applied studies of inequality and poverty often use only a single set of scales, claiming that results are not affected. However, given the different patterns of variation in summary measures found, much caution is required before declaring that analyses are not affected by the choice of scale.

Appendix: Covariance Between Adult Equivalent Income and Number of Persons

This appendix examines the covariance between household size (the number of persons, n_i) and equivalent income $z_i = y_i / m_i$, in the simplified case where $m_i = n_i^\alpha$. First, suppose that income and the number of persons in the income unit are jointly log-normally distributed as:

$$\Lambda(y, n | \mu_y, \mu_n, \sigma_y^2, \sigma_n^2, \rho_{yn}) \quad (1)$$

Further results require the following general properties of the lognormal distribution. If x is $\Lambda(x | \mu, \sigma^2)$:

$$E(x) = \exp(\mu + \sigma^2 / 2) \quad (2)$$

The power x^δ is distributed as;

$$\Lambda(\delta\mu, \delta^2\sigma^2) \quad (3)$$

and for two variables jointly distributed as $\Lambda(x, y | \mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho_{xy})$, the ratio x/y is distributed as:

$$\Lambda\left(\frac{x}{y} \middle| \mu_x - \mu_y, \sigma_x^2 + \sigma_y^2 - 2\rho_{xy}\sigma_x\sigma_y\right) \quad (4)$$

The covariance between household size and equivalent income is, by definition:

$$\text{Cov}(z, n) = E\left(\frac{y}{n^{\alpha-1}}\right) - E\left(\frac{y}{n^\alpha}\right)E(n) \quad (5)$$

Using the three properties give above, average adult equivalent income is:

$$E\left(\frac{y}{n^\alpha}\right) = \exp\left\{\mu_y - \alpha\mu_n + \frac{1}{2}(\sigma_y^2 + \sigma_n^2 - 2\alpha\rho_{yn}\sigma_y\sigma_n)\right\} \quad (6)$$

A similar result holds for $E\left(\frac{y}{n^{\alpha-1}}\right)$. It can be shown, after some manipulation, that:

$$\text{Cov}(z, n) = (\exp A)(\exp B - 1) \quad (7)$$

where:

$$A = \mu_y + (\alpha - 1)\mu_n + \frac{1}{2}(\sigma_y^2 + (1 + \alpha^2)\sigma_n^2 - 2\alpha\rho_{yn}\sigma_y\sigma_n) \quad (8)$$

and:

$$B = \rho_{yn} \sigma_y \sigma_n - \alpha \sigma_n^2 \quad (9)$$

Thus the covariance is positive if $\exp B - 1 > 0$, that is if $B > 0$. Hence there is a positive correlation between adult equivalent income and the number of adults if:

$$\rho_{yn} \frac{\sigma_y}{\sigma_n} > \alpha \quad (10)$$

This condition is therefore simply interpreted in terms of the size of the regression coefficient in the linear regression relationship between logarithms of household size and income. It is clearly equivalent to the elasticity condition mentioned by Coulter et al (1992).

References

- Aronson, J.R. and Lambert, P.J. (1994) Decomposing the Gini coefficient to reveal vertical, horizontal and reranking effects of income taxation. *National Tax Journal*, 47, pp. 273-294.
- Aronson, J.R., Johnson, P. and Lambert, P.J. (1994) Redistributive effect and unequal income tax treatment. *The Economic Journal*, 104, pp. 262-270.
- Atkinson, A.B. (1979) Horizontal equity and the distribution of the tax burden. In: Aaron, H.J. and Boskins, M.J. (eds), *The Economics of Taxation*. Washington: The Brookings Institution.
- Attanasio, O. P. and Japelli, T. (1997) The life cycle hypothesis and consumption inequality. Institute For Fiscal Studies Working Paper, W97/17.
- Banks, J. and Johnson, P. (1994) Equivalence scale relativities revisited. *Economic Journal*, 104, pp. 883-890.
- Blundell, R. and Preston, I. (1994) Income or consumption in the measurement of inequality and poverty. Institute For Fiscal Studies Working Paper, W94/12.
- Blundell, R. and Preston, I. (1997) Consumption inequality and economic uncertainty. Institute For Fiscal Studies Working Paper, W97/15.
- Brashares, E. and Aynsley, M. (1990) Income adequacy standards for New Zealand. New Zealand Treasury.
- Buhmann, B., Rainwater, L., Schmaus, G. and Smeeding, T. M. (1988) Equivalence scales, well-being, inequality, and poverty: Sensitivity estimates across ten countries using the Luxembourg income study (LIS) database, *Review of Income and Wealth*, 34, pp. 115-142.
- Coulter, A. E., Cowell, F. A. and Jenkins, S. P. (1992) Equivalence scale relativities and the extent of inequality and poverty. *Economic Journal*, 102, pp. 1067-1082.
- Cowell, F. A. (1984) The structure of American income inequality. *Review of Income and Wealth*, 30, pp. 351-375.
- Creedy, J. (1997) Labour supply and social welfare when utility depends on a threshold consumption level. *Economic Record*, 73, pp. 159-168.
- Creedy, J. (2002) The GST and vertical, horizontal and reranking effects of indirect taxation in Australia. *Australian Economic Review*, 35, pp. 380-390 (2002).
- Creedy, J. and van de Ven, J. (2001) Decomposing redistributive effects of taxes and transfers in Australia: Annual and lifetime measures. *Australian Economic Papers*, 40, pp. 185-198.
- Creedy, J. and Scutella, R. (2003) The role of the unit of analysis in policy reform evaluations. Melbourne Institute of Applied Economic and Social Research.

- Creedy, J. and Tuckwell, I. (2003) Reweighting the New Zealand Household Economic Survey for tax microsimulation modelling. New Zealand Treasury Working Paper.
- Cutler, D. M. and Katz, L. (1992) Rising inequality? Changes in the distribution of income and consumption in the 1980s, *American Economic Review*, 82, pp. 546-551.
- Danziger, S. and Taussig, M. K. (1979) The income unit and the anatomy of income distribution. *Review of Income and Wealth*, 25, pp. 365-375.
- Decoster, A., Schokkaert, E. and Van Camp, G. (1997a) Is redistribution through indirect taxes equitable? *European Economic Review*, 41, pp. 599-608.
- Decoster, A., Schokkaert, E. and Van Camp, G. (1997b) Horizontal neutrality and vertical redistribution with indirect taxes. *Research on Economic Inequality*, 7, pp. 219-239.
- Decoster, A. and Ooghe, E. (2002) Weighting with individuals, equivalent individuals or not weighting at all: does it matter empirically? Catholic University of Leuven Centre for Economic Studies Discussion Paper, DPS 02.15.
- Ebert, U. (1997) Social welfare when needs differ: an axiomatic approach. *Economica*, 64, pp. 233-244.
- Foster, J., Greer, J. and Thorbecke, E. (1984) A class of decomposable poverty measures. *Econometrica*, 52, pp. 761-762.
- Glewwe, P. (1991) Household equivalence scales and the measurement of inequality: transfers from the poor to the rich could decrease inequality. *Journal of Public Economics*, 44, pp. 214-216.
- Jenkins, S. P. and Cowell, F. A. (1994) Parametric equivalence scales and scale relativities, *Economic Journal*, 104, pp. 891-900.
- Jorgenson, D. W. and Slesnick, D. T. (1984) Aggregate consumer behaviour and the measurement of inequality. *Review of Economic Studies*, 51, pp. 369-392.
- Kakwani, N. K. (1980) *Income Inequality and Poverty: Methods of Estimations and Policy Application*. Oxford: Oxford University Press.
- Lambert, P. (2001) *The Distribution and Redistribution of Income: A Mathematical Analysis*. Manchester: Manchester University Press.
- Michellini, C. (1999) New Zealand Household consumption equivalence scales from quasi-unit record data. Massey University Department of Applied and International Economics Discussion Paper, no. 99.02.
- Plotnick, R. (1981) A measure of horizontal inequity. *Review of Economics and Statistics*, 63, pp. 283-288.
- Ringen, S. (1991) Households, standard of living and inequality. *Review of Income and Wealth*, 37, pp. 1-13.
- Shorrocks, A. F. (1997) Inequality and welfare evaluation of heterogeneous income distributions. University of Essex Department of Economics Discussion Paper, no. 447.

Slesnick, D. T. (1994) Consumption, needs and inequality. *International Economic Review*, 35, pp. 677-703.

Trigger, D. (2003) Does the way we measure poverty matter? Natsem Discussion Paper, no. 59.

van de Ven, J. Demand based equivalence scale estimates for Australia and the UK. (unpublished paper)

van de Ven, J. and Creedy, J. (2003) Taxation, Reranking and Equivalence Scales. (unpublished paper)

van de Ven, J., Creedy, J. and Lambert, P. J. (2001). Close equals and calculation of the vertical, horizontal and reranking effects of taxation. *Oxford Bulletin of Economics and Statistics*, 63, pp. 381-394.

Whiteford, P. (1985) A family's needs: equivalence scales, poverty and social security. Department of Social Security Research paper No.27. Canberra: Australia.